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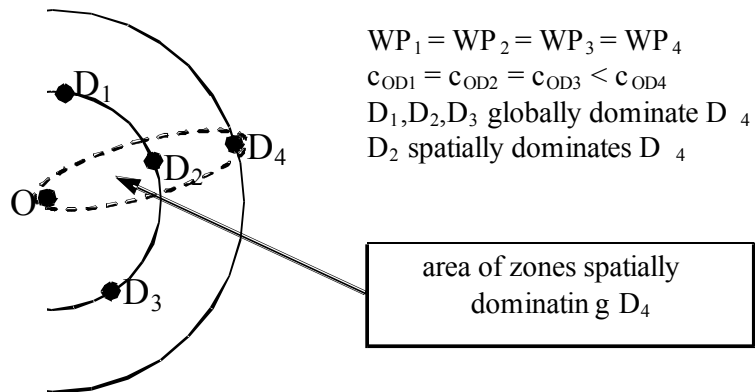
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O = origin
D = destinations
WP = workplaces
 c_{OD} = OD distance



An application of the Constrained Multinomial Logit (CMNL) for modelling dominated choice alternatives

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Abstract

Random utility models are widely used to analyze choice behaviour and predict choices among discrete alternatives in a given set. These models are based on the assumption that an individual's preference for the available alternatives can be described with a utility function and that the individual selects the alternative with the highest utility. The traditional formulation of logit models applied to transport demand assumes compensatory (indirect) utilities based on the trade-off between attributes.

A different strategy has been proposed, which makes those choice alternatives out of the feasible domain, available but undesirable. This approach has the advantage that the model is applied to the entire set of choices, thus gaining on efficiency by avoiding the explicit identification of choice sets for every individual, and secondly, obtaining a model with better properties for the calculation of equilibrium or optimum conditions. Based on this approach, the Constrained Multinomial Logit (CMNL) model was specified, which combines the multinomial logit model with a binomial logit factor that represents soft cut-offs.

Keywords

Random Utility Theory – Choice set – Dominance variables – Constrained Multinomial Logit Model

1. Introduction

Random utility models are widely used to analyze choice behaviour and predict choices among discrete sets of alternatives. These models are based on the assumption that an individual's preference among the available alternatives can be described with a utility function and that the individual selects the alternative with the highest utility (Ben-Akiva and Lerman, 1985; Cascetta, 2001).

The traditional formulation of Logit models applied to transport demand assumes a trade-off between attributes. Some authors have criticised this approach because it fails to recognize attribute thresholds in consumers' behaviour, or in fact the existence of a more generic domain where such compensatory strategy is contained. For example, under the principle of rationality, some alternatives may not be considered because, for example, they violate the income constraint, or because they are dominated by another alternative. The widely used random utility theory and their most used model the Multinomial Logit Model (MNL) can only theoretically deal with such constraint by formulating a complex non linear utility function, but in practice these functions are difficult to identify and data is rarely available to estimate the required parameters. Alternatively, Manski (1977) and several authors have developed MNL models where the set of alternatives is defined for each individual by choice set model, which eliminates all alternatives that not comply with a set of constraints. While this approach is feasible and theoretically well founded, it is hardly applicable in large scale choice problems such as those where the choice domain is the physical space (e.g. trip destination or residential location choice problems). Another option is to build utility functions with implicit perception of the availability of the choice alternatives, which is implemented by introducing a penalty factor in the utility function.

An application of the implicit method was made by Cascetta and Papola (2005), whom studied the trip distribution model assuming that a subset of destination choices is irrelevant for the choice maker because they are completely or highly dominated by other alternative(s). The authors model the dominance effects by introducing new variables in the utility function, which are built from a combination of rules, which generate dominance values that will be assigned to each alternative.

In this paper, we report on the current status of an ongoing project, aiming at applying a different method to model the dominance effect, which uses cut-off factors, instead of assigning dominance values to alternatives. These cut-off factors will represent the probability of an alternative for being dominated by other alternatives. The method, called the Constrained Multinomial Logit, combines the Multinomial Logit model with a binomial Logit factor that represents soft cut-offs (Martinez *et al.*, 2005).

One thing that has come out from an application to the residential location choice context is that (i) the CMNL cut-off may end up being equivalent to the linear model in the case of the dominance variables and (ii) that a linear specification is rejected by a Box-Cox test. More investigation is clearly needed with this data to identify the appropriate specification.

This paper is organised in 5 sections. In section 2 the Constrained Multinomial Logit Model (CMLM) is summarized. Section 3 describes the dominance approach, while in section 4 the mixed strategy is proposed and preliminary estimation results are reported. Finally section 5 reports conclusions and further perspectives.

2. The Constrained Multinomial Logit Model

The constrained version of the Multinomial Logit model, denoted as CMNL, was proposed by Martinez *et al.* (2005) in order to complement the Multinomial Logit model. In this enhanced model, the consumer's choice set is explicitly model for each consumer. Thus, CMNL model maximizes the Gumbel distributed utility of the consumer within a domain that contains only the set of available options for the consumer. Instead of defining explicitly the consumer's choice set, the CMNL model defines a probability for every option d in the complete set of alternatives C , to belong to the n -th consumer's choice set C_n , denoted by $p(d \in C_n, C_n \subset C)$.

The CMNL model is derived upon assuming that the utility function of the n -th consumer is:

$$V_n(Z_d) = V_n^c(Z_d) + \frac{1}{\mu} \ln \phi_n(Z_d) + \varepsilon_{nd} \quad (1)$$

where ε is assumed Gumbel distributed $(0, \mu)$ and the attributes vector Z_d describes the characteristics of alternative d . This utility has two components: the compensatory utility function, V^c , which defines the consumer's trade off between the attributes and theoretically represents the indirect utility function; this is the utility assumed in the classical MNL model. The second term is called the cut-off factor and is defined by $p_n(d \in C_n) = \phi_n(Z_d)$, such that the choice set probability is defined as a function of the attributes of the alternative. Hereafter, we denote $\phi_n(d \in C_n) = \phi_{nd}$ and the consumer's feasible domain D_n .

The assumptions made lead us to the following modified Multinomial Logit probability:

$$P_{nd} = \frac{\phi_{nd} \cdot \exp(\mu V_{nd}^c)}{\sum_{j \in C} \phi_{nj} \cdot \exp(\mu V_{nj}^c)} \quad (2)$$

which defines the CMNL model. Notice that the CMNL probability is defined on the complete set of alternatives C , while the consumer's choice set C_n is implicitly defined by the non-compensatory utility function V_n in (1). Notice also that this function preserves the closed form of the MNL model.

The cut-off factor ϕ_{nd} has the effect that the consumer's utility V_n tends to minus infinite as the choice set probability approaches zero and becomes innocuous as that probability approaches one. This implies that those alternatives having one or more attributes lying out of the consumer's feasible domain have a very low cut-off factor and a very negative utility; hence, although they are chosen this happens with very low probability.

Additionally, the consumer's domain D_n may be defined by a set of criteria, where each criterion is defined by an upper or lower limit for the alternative's attributes. To incorporate this in the model a composite cut-off factor is defined as: $\phi_{nd} = \prod_{k=1}^K \phi_{nkd}^L \cdot \phi_{nkd}^U$, which is composed by a set of K lower and upper elemental cut-offs. Therefore, whichever criterion that makes alternative d in the choice set unfeasible for the consumer, then the probability in (2) tends to zero.

The cutoff is assumed a binomial logit function because it preserves the closed form of the logit probability (2). Then, consider the criteria that alternative d is feasible if $a_{nk} \leq Z_{kd} \leq b_{nk}$, with Z_{kd} any attribute of alternative d . Then, we define:

$$\phi_{nkd}^L = \frac{1}{1 + \left(\frac{1-\eta_k}{\eta_k}\right) \exp(\omega_k (a_{nk} - Z_{kd}))} = \begin{cases} 1 & \text{if } (a_{nk} - Z_{kd}) \rightarrow -\infty \\ \eta_k & \text{if } a_{nk} = Z_{kd} \end{cases} \quad (3)$$

$$\phi_{nkd}^U = \frac{1}{1 + \left(\frac{1-\eta_k}{\eta_k}\right) \exp(\omega_k (Z_{kd} - b_{nk}))} = \begin{cases} 1 & \text{if } (b_{nk} - Z_{kd}) \rightarrow \infty \\ \eta_k & \text{if } b_{nk} = Z_{kd} \end{cases} \quad (4)$$

which are the lower and upper cut-offs respectively. This binomial function has the following parameters: ω defines the rate of change of the utility as the constrained variable approaches the boundary; a and b are the boundaries of the k -th attribute and η is a tolerance parameter. This last parameter takes into account that cut-off is strictly positive ($\phi > 0$), meaning that in fact all alternatives are feasible in this model. Therefore, instead of eliminating alternatives this parameter η limits the maximum probability of choosing any alternative d that lies out of the domain by defining $\eta_{nk} = \phi_{nkd}(Z_{dk} = a_{nk})$ or $\eta_{nk} = \phi_{nkd}(Z_{dk} = b_{nk})$; $\forall d \in C$. We conclude then that the binomial cut-off method imposes soft constraints, which can be justified by considering that consumers perceive constraints and they behave much in the same way as they react to the utility yield by alternatives.

The paper by Martinez *et al.* (2005) mentions a variety of constraints that can be represented by cut-offs and that each cut-off can be used to estimate the economic impact of the corresponding constrain. In this paper, we apply the CMNL model in the residential location choice context, where the feasible domain is defined by the set of non dominated alternatives.

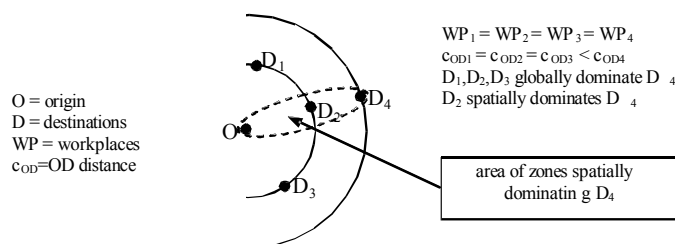
3. The dominance approach

In many choice contexts, it may happen that some alternatives are not taken into account by the decision maker since they are dominated by other alternatives. In general, an alternative d is dominated by another alternative d^* if d is “worse” than d^* , with respect to one or more attributes, without being better with respect to any attribute. The concept of dominance among alternatives is widely recognized in project evaluation, e.g. through Multi-Criteria Decision-Making (MCDM) (Haines and Chankong, 1985) where dominated projects are excluded from the choice set on the basis of the principle of rationality (transferability of preferences). Instead, it has rarely been used explicitly within random utility (RU) theory.

A general approach to extend and apply the concept of dominance among alternatives to RU theory has been proposed (Cascetta et al., 2007), defining (a) when an alternative d is dominated by another alternative d^* ; (b) possible ways of exploiting the dominance information about pairs of alternatives in the choice set generation process and (c) the use of dominance criteria as weights for the sampling probabilities.

Concerning (a) some dominance rules have been proposed. Specifically, in destination choice and residential location choice contexts, it is assumed that an alternative d dominates an alternative d^* (for a decision-maker moving from origin zone o) if the attractiveness of d is greater than that of d^* and at the same time the generalised costs c_{od} are smaller than c_{od^*} (global dominance rule). Moreover, a spatial domination can be constructed based on the concept of intervening opportunities (Stouffer, 1960). It is assumed that d spatially dominates d^* if it dominates d^* in relation to the above conditions and d is along the path to reach d^* from the individual origin o (i.e. if the length of the shortest path odd^* is close to that of the shortest path od^*) (spatial dominance rule). In this case, d represents an intervening opportunity along the path, or bundle of paths, towards d^* . In Fig. 1 an example of spatial domination is reported.

Figure 1 Example of spatial domination



Dominance variables are obtained through combinations of the previous rules. Therefore, a dominance value will be assigned to each alternative. The new variable can be defined in several ways: it can be a Boolean variable, it can be a variable taking values between 0 and 1; it can be the number of times an alternative is dominated by the others as it will be represented in this paper.

Dominance variables can affect the choice set formation process (Cascetta and Papola, 2005). These attributes, which may obviously be introduced whatever the choice set simulation model, have been tested on the IAP (Implicit Availability-Perception) RU model (Cascetta and Papola, 2001). Indeed, in this model, choice set enumeration is avoided by simulating the probability of an alternative d belonging to the choice set, $p(d \in C)$, and by introducing the logarithm of $p(d \in C)$ in the utility of that alternative:

$$U_d^* = U_d + \ln p(d \in C) = \beta_n \hat{a}_n X_{dn} + E[\ln p(d \in C) | Y_d] + \hat{a}_d \quad (5)$$

The rationale of this model is that a lower probability of an alternative d belonging to the choice set reduces the U_d^* and hence the $p(d)$.

The proposed perception attributes have been also introduced directly in the alternatives' utilities of an MNL model in order to test the model predictive ability; it follows that $p(d \in C) | Y_d$ is expressed as:

$$p(d \in C) | Y_d = \frac{\exp(\tilde{a} Y_d)}{\sum_{d' \in C} \exp(\tilde{a} Y_{d'})} \quad \text{then it results that } \ln p(d \in C) | Y_d = \tilde{a} Y_d + \text{cost}, \quad \text{with cost being a}$$

constant, and then the utility becomes:

$$U_d^* = \beta_n \hat{a}_n X_{dn} + \beta_k \tilde{a}_k Y_{dk} + \hat{a}_d \quad (6)$$

where β_n and β_k are coefficients of utility and availability/perception attributes respectively.

4. The proposed methodology

In Cascetta *et al.* (2007) the residential location model was specified as a Multinomial Logit (MNL) model using the following linear utility function:

$$U_{d \in I}^* = U_d + \varepsilon_d = \sum_{n \in N} \alpha_n X_{dn} + \sum_{k \in K} \beta_k Y_{dk} + \varepsilon_d \quad (7)$$

where X_{dn} are the values of the compensatory utility variables in zone d and Y_{dk} are the dominance variables on zone d ; α_n and β_k are the respective parameters; I , N and K are, respectively, the sets of zones (residential location options), utility variables and dominance variables. The last one is the random term assumed distributed identically and independently Gumbel. Then the location choice multinomial logit model is:

$$p(d) = \frac{\exp(U_d)}{\sum_{j \in I} \exp(U_j)} \quad (8)$$

Applying the cut-off method proposed by Martinez *et al.* (2005) and the dominance variables proposed by Cascetta *et al.* (2007), the CMNL model is specified.

The following cut-off factor is defined:

$$\phi(Y_{dk}) = \frac{1}{1 + \exp\{\omega(Y_{dk} - Y_k + \rho)\}} \quad (9)$$

where Y_k is the cut-off level, or the level of dominance above which location choices become irrelevant so they are detracted or ignored from the choice set, or equivalently, their individuals utility is so low or negative that these locations are not considered by the individual. The cut-off function (9) can be interpreted as a binomial logit model where the alternatives options define if those alternatives that violate the maximum dominance level are included or not in the choice set.

Then the utility function is defined as:

$$U_d = V_d + \Phi_d = \sum_{l \in L} \alpha_l X_{dl} + \frac{1}{\mu} \sum_{k \in K} \ln \phi(Y_{dk}) \quad (10)$$

If any of the K criterion for dominance takes a value $Y_{dk} \geq Y_k$, then the cut-off tends to zero and the utility falls to minus infinity, thus making the location option irrelevant although still feasible.

The CMNL model has been calibrated using BIOGEME (Bierlaire, 2007) by estimating the set of parameters that best fits the available sample of the observed residential choices. The calibration procedure starts defining the level of disaggregation of the model and parameter estimates. The most disaggregate level considers the estimation of the following parameter's vector $(\alpha_{nl}, \omega_{nk}, \rho_{nk})_{n \in N, l \in L, k \in K}$, which includes specific parameters for each individuals socioeconomic category n , compensating utility variables l , and for each variable cut-off k . This definition is highly dependent on the available data.

4.1 An application

In 2005 an RP survey was conducted in the canton of Zurich in Switzerland covering the mobility and moving biography of the respondents. A sample of 1100 residents was obtained. Among them 658 respondents were considered useful for our purpose on the basis of those living and working within the canton of Zurich. For each resident included it is known the respondent's residential place and workplace, the age, income, number of household members. Residents considered are both those living in a zone and working in another and those living and working in the same zone of the canton. The sample included also residents working outside the canton of Zurich. The study area has been divided in 182 traffic zones (of which 12 make up the municipality of Zurich) that represent the universal choice set of the model.

The residential location model specified is a Multinomial Logit model and the variables considered are:

$Price_d$	is the average land price of zone d ;
$nStock_d$	is the natural logarithm of the housing stock in zone d ;
$Logsum_{od}^{LM}$	is the logsum of the mode choice model for work purpose for low-medium income residents; (attributes are of the mode choice and reference to these models);
$Logsum_{od}^H$	is the logsum of the mode choice model for work purpose for high income residents;
$lnWorkplaces_serv_d$	is the natural logarithm of the workplaces in services (summation of retail, leisure and services to the households such as education, health) in zone d and it represents a measure of the quality of services to households in the zone itself.

The availability/perception variables have been obtained through a combination of the rules defined in section 3 and they are:

$Dom1_d$	is the number of zones d^* strongly dominating zone d , i.e: <ul style="list-style-type: none"> (a) d^* has average land price lower than d; (b) the distance from the optional respondent's workplace zone d^* ($dist_{od^*}$) is shorter than that to zone d ($dist_{od}$); (c) d^* is along the path to reach the optional respondent's workplace zone d from o: $dist_{od^*} + dist_{d^*d} < 1.2 \cdot dist_{od}$
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STRONG GLOBAL DOMINANCE RULE

$Dom2_d$	is the number of zones d^* for which conditions (a) and (b) simultaneously occurs.
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WEAK GLOBAL DOMINANCE RULE

$Dom3_d$	is the number of zones d^* strongly dominating zone d , i.e. satisfying conditions (b) and (c) simultaneously.
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STRONG SPATIAL DOMINANCE RULE

$Dom4_d$	is the number of zones d^* weakly dominating zone d , i.e. satisfying only condition (b).
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WEAK SPATIAL DOMINANCE RULE

The model estimation has been of increasing complexity.

The first specification is a simple MNL model containing only the compensatory part of the utility function. In Table 1 calibration results are reported, while in Table 2 we report the statistics of the attributes' parameter estimates.

Table 1 First model (compensatory part): Estimation results

Number of estimated parameters	5
Number of observations	657
Null log-likelihood	-3419.032
Cte log-likelihood	-2061.58
Final log-likelihood	-53.971
Rho-square	0.984
Adjusted rho-square	0.983

Table 2 Statistics of the attributes

Name	Value	Robust Std err	Robust t-test	p-value
$\beta_Logsum_{od}^H$	15.3	2.85	5.36	0
$\beta_Logsum_{od}^{LM}$	16.6	2.97	5.58	0
β_Price_d	-0.0016	0.000221	-7.24	0
β_Stock_d	1.12	0.102	10.93	0
$\beta_Workplaces_serv_d$	0.188	0.18	1.04	0.3
$\beta_Logsum_{od}^H$	15.3	2.85	5.36	0

In the second model, the weak spatial dominance indicator has been introduced in a linear way and the estimation results are reported in Table 3. In Table 4 the statistics of the variables are reported.

Table 3 Second model (Linear specification): Estimation results

Number of estimated parameters	6
Number of observations	657
Null log-likelihood	-3419.032
Cte log-likelihood	-2061.58
Final log-likelihood	-47.055
Rho-square	0.986

Adjusted rho-square	0.984
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Table 4 Statistics of the attributes

Name	Value	Robust Std err	Robust t-test	p-value
$\beta_Logsum_{od}^H$	-0.0859	0.012	-7.17	0
$\beta_Logsum_{od}^{LM}$	16.1	2.62	6.16	0
β_Price_d	17.1	2.76	6.2	0
β_Stock_d	-0.00245	0.000313	-7.82	0
$\beta_Workplaces_serv_d$	1.2	0.133	9.01	0
$\beta_Logsum_{od}^H$	-0.172	0.198	-0.87	0.39

A significant improvement from the MNL model is observed. The likelihood ratio test is 13.832, and the 95% threshold for 1 degree of freedom is 3.84.

The third model replaces the linear specification of the dominance term by the sigmoidal form, that is:

$$V_d - \ln(1 + 1000 \exp(\omega_4 Y_{dk}))$$

which is a simplified version of the form (10), obtained after testing various specifications. It appears that the cut-off model is particularly difficult to estimate on this data (see Tables 5 and 6).

Table 5 Third model: Estimation results

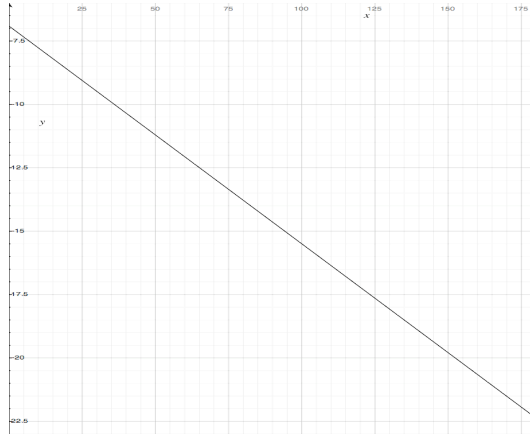
Number of estimated parameters	6
Number of observations	657
Null log-likelihood	-3419.032
Cte log-likelihood	-2061.58
Final log-likelihood	-47.057
Rho-square	0.986
Adjusted rho-square	0.984

Table 6 Statistics of the attributes

Name	Value	Robust Std err	Robust-t-test	p-value
$\beta_Logsum_{od}^H$	16.1	2.62	6.16	0
$\beta_Logsum_{od}^{LM}$	17.1	2.76	6.2	0
β_Price_d	-0.00245	0.000313	-7.82	0
β_Stock_d	1.2	0.133	9.01	0
$\beta_Workplaces_serv_d$	-0.172	0.198	-0.87	0.39
$\beta_Logsum_{od}^H$	0.0859	0.012	7.17	0
ω	0.0859	0.012	7.17	0

Compared to the linear specification of the dominance, no fit improvement is observed. Actually, when the sigmoidal correction is plotted (Fig. 2), it appears to be, almost linear:

Figure 2 Plot of the corrections



In order to test if the dominance indeed affects the utility in a linear way, a Box-Cox specification is estimated:

$$\beta_{\text{dom4}} \frac{Y_{dk}^{\lambda} - 1}{\lambda}$$

In Tables 7 and 8 the estimation results and the statistics of the variables are reported.

Table 7 Fourth model: estimation results

Number of estimated parameters	7
Number of observations	657
Null log-likelihood	-3419.032
Cte log-likelihood	-43.12
Final log-likelihood	6751.826
Rho-square	0.987
Adjusted rho-square	0.985

Table 8 Statistics of the attributes

Name	Value	Robust Std err	Robust-t-test	p-value
$\beta_{\text{Logsum}_{od}^H}$	-0.579	0.0539	-10.74	0
$\beta_{\text{Logsum}_{od}^{LM}}$	16.9	2.66	6.36	0
β_{Price_d}	18	2.68	6.72	0
β_{Stock_d}	-0.00292	0.000324	-9	0
$\beta_{\text{Workplaces}_{serv}_d}$	1.42	0.175	8.1	0
$\beta_{\text{Logsum}_{od}^H}$	-0.328	0.257	-1.28	0.2
λ	0.434	0.0388	11.19	0

In terms of fit, this is clearly the best model. The likelihood ratio test compared to the linear specification is 7.87, beyond the 3.84 threshold. Consequently, the hypothesis that the dominance affects the utility in a linear way can be rejected.

5. Conclusions and further research

In this paper a different method is introduced to model the dominance effect, which uses cut-off factors, instead of assigning dominance values to alternatives. These cut-off factors represent the probability of an alternative for being dominated by other alternatives. The method, called the Constrained Multinomial Logit (CMNL), combines the Multinomial Logit model with a binomial logit factor that represents soft cut-offs.

One thing that has come out from an application to the residential location choice context is that the CMNL cutoff is equivalent to the linear model in the case of the dominance variables.

This means:

- That a simple method to calibrate the parameters is by its equivalent, the linear model. This may be of help in cases where there are several cutoffs combined. The method should consider the fact that these models are equivalent in the vicinity of the constraint, because in the interior of the choice domain they are significantly different. Nevertheless, it is precisely in the region of the vicinity of the domain where these parameters are relevant.
- Some variables, like the dominance we have used in our case, are only defined in the vicinity of the constraint; they play no role in the interior of the choice domain. In such particular cases, the cutoff makes no significant difference with the linear model.

Additionally, the box-cox utility model shows that the dominance affects the utility in a non-linear way.

New dominance variables will be tested with a different data set and they will be defined in the following way:

- maximum threshold: for a trip to have a destination zone d it must not be dominated by another one above some threshold.

-minimum threshold: below which any destination d is not really considered.

These new test will be performed in the context of the destination choice for grocery shopping. The first analyses of all those trips reported in the Mikrozensus 2005, the Swiss National travel survey, for the Canton Zürich showed, that the distance to the stores has the expected effects (sign and size) for the car-based shoppers, but not for those walking to the stores. They also showed that the shoppers prefer the closest stores, which indicates a ranking rather than a trade-off rule, especially for walking. The choice set was constructed using a

time-space prism approach from among the totality of all grocery stores available in the canton.

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