Modeling solid-solid contact in a fully Eulerian phase-field framework

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Modeling solid-solid contact in a fully Eulerian phase-field framework

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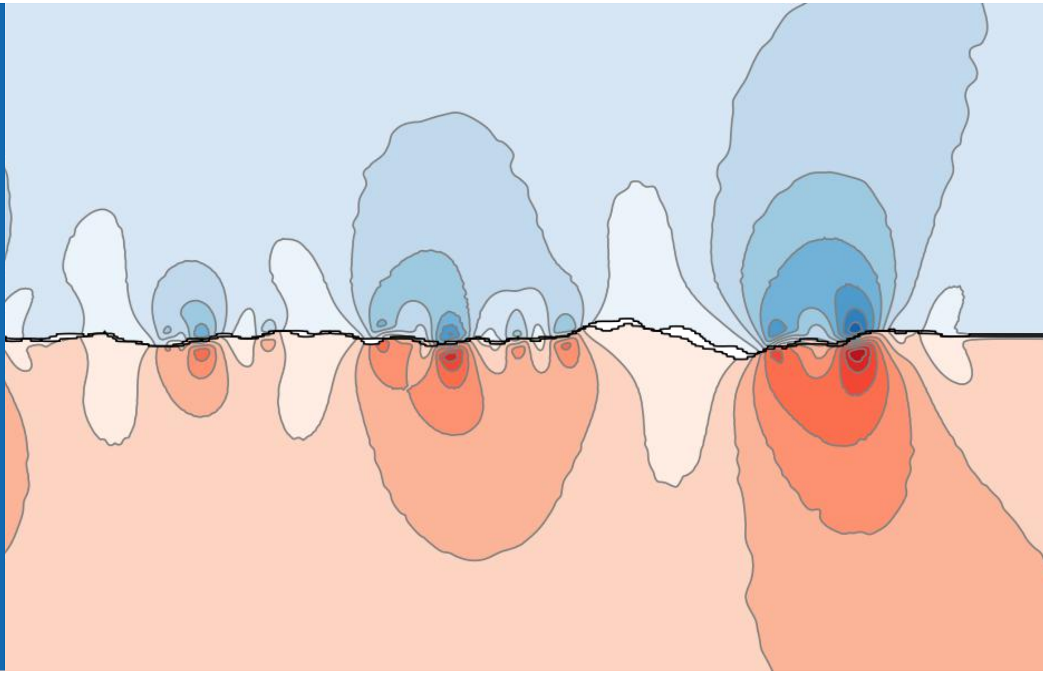
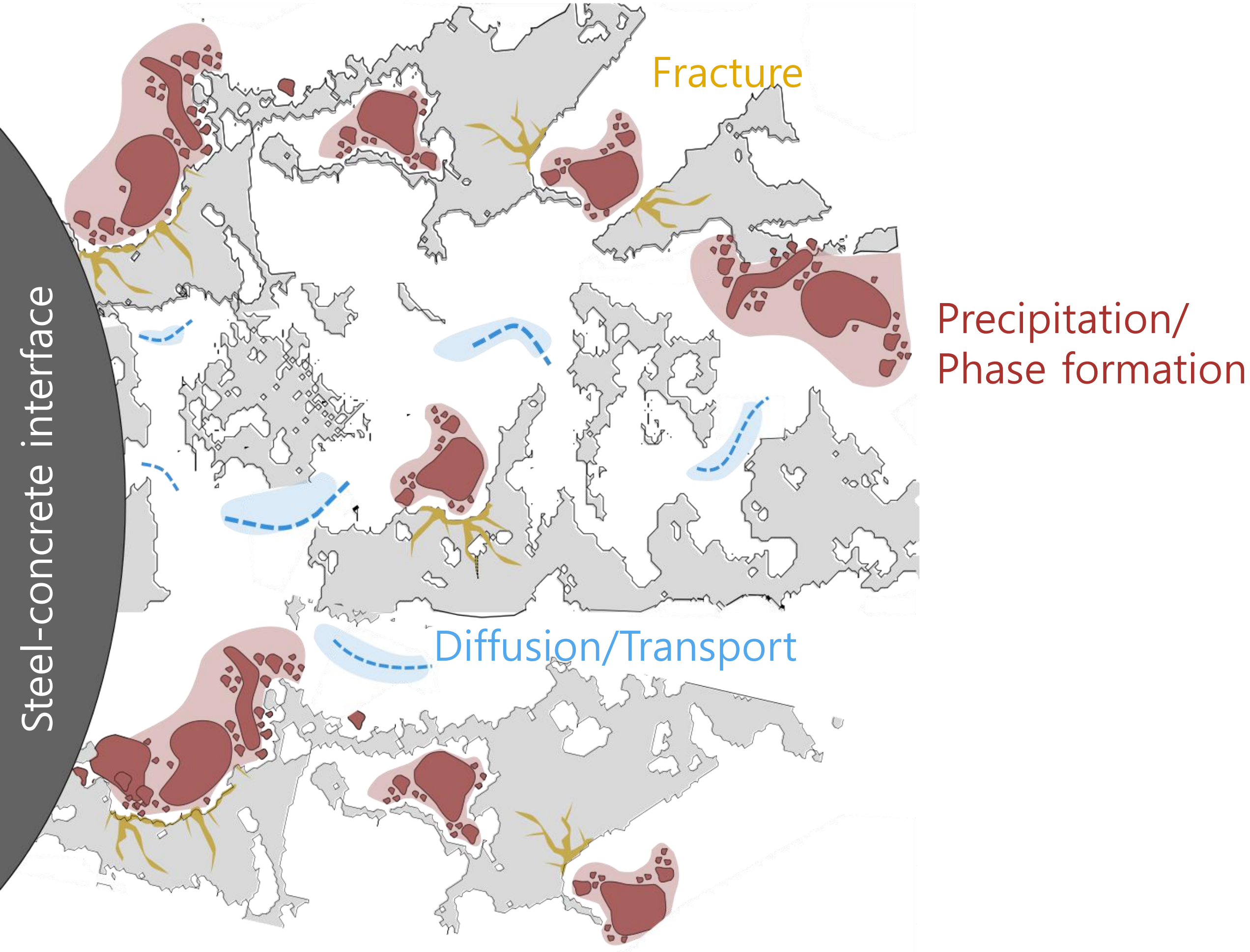
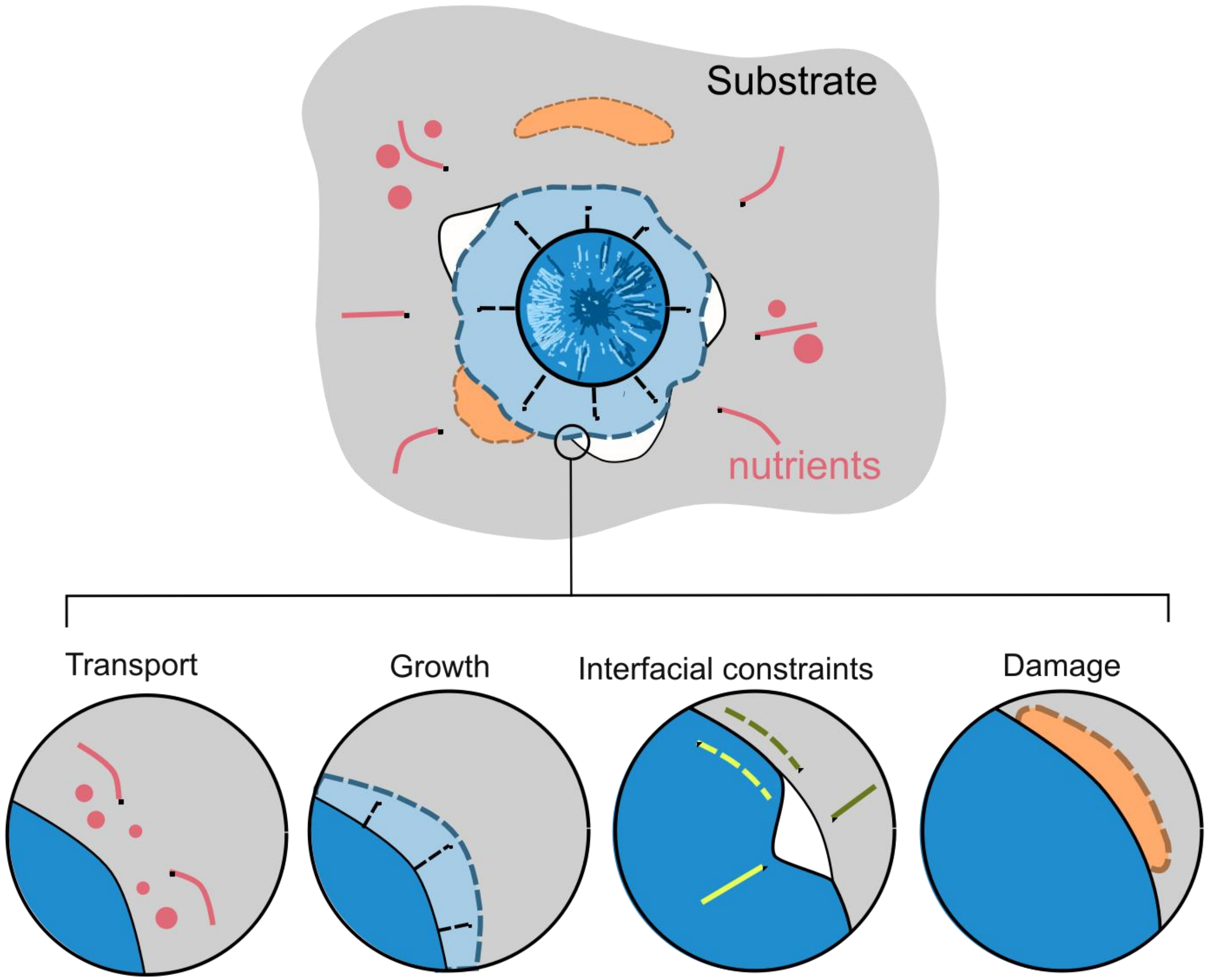


WCCM-PANACM 2024
Contact and interface mechanics: Modeling and computation, July 21-26 2024, Vancouver

Motivation - solid *growth/evolution* in constrained space



Corrosion-driven failure[1]

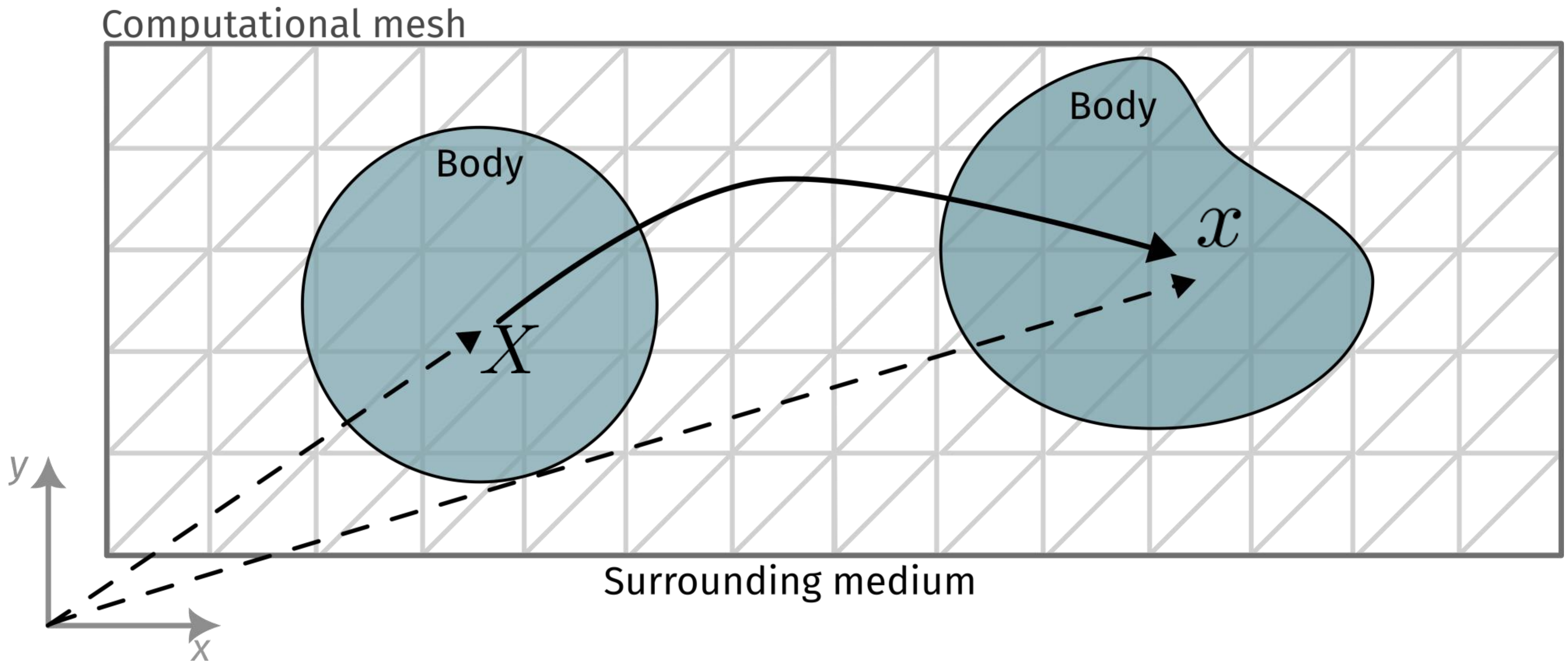


Biological growth/morphogensis[2,3]

- Multiphysical processes lead to interfacial interactions

1. *Pundir et al.*, **JMPS**, 2023,
2. *Amar et al.*, **EPL**, 2014
3. *Zhang et al.*, **PNAS**, 2021

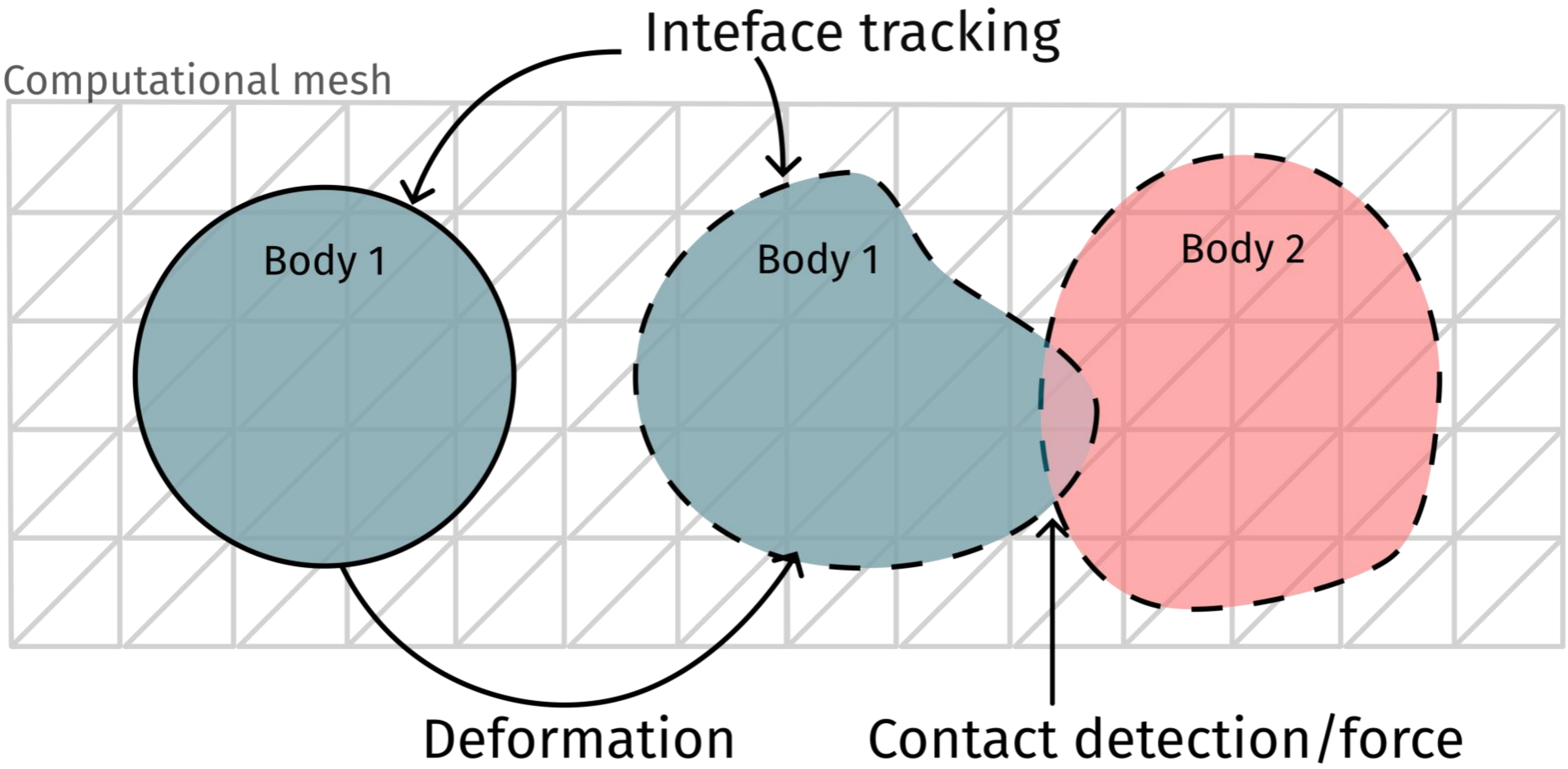
Why *Eulerian* framework?



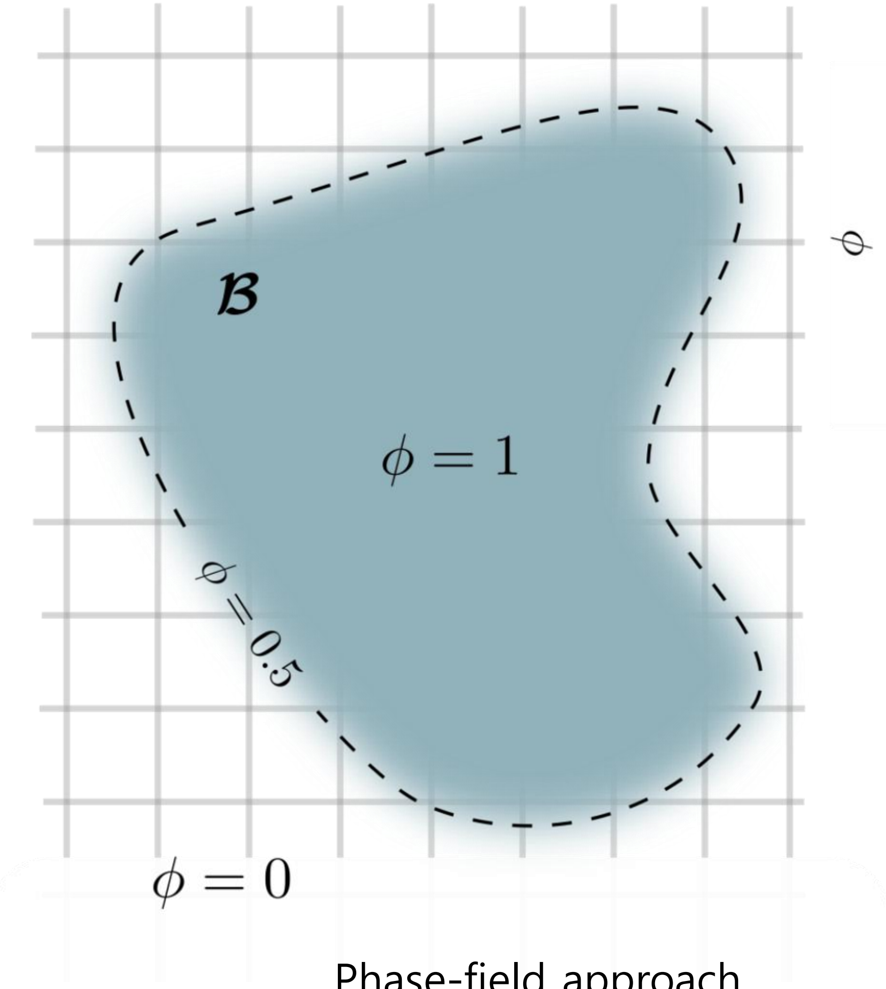
- Multiphysical processes are better described in Eulerian framework

How to model *contact* in Eulerian framework?

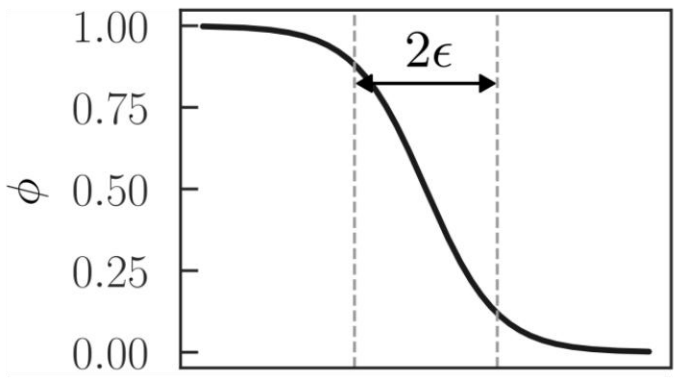
Key ingredients to contact in Eulerian framework



How to *track solids* in Eulerian framework?



Phase-field approach



Interface region

Free energy functional

$$\mathcal{E}_{\mathrm{pf}} = \int_{\Omega} \left(\frac{\epsilon^2}{2} (\nabla \phi)^2 + g(\phi) \right) \, \mathrm{d}\Omega$$

$$\frac{\delta \mathcal{E}_{\mathrm{pf}}}{\delta \phi} = 0$$ → equilibrium interface profile

$$\phi(x) = \frac{1}{2} \left[1 - \tanh\left(\frac{x}{\sqrt{2}\epsilon} \right) \right]$$

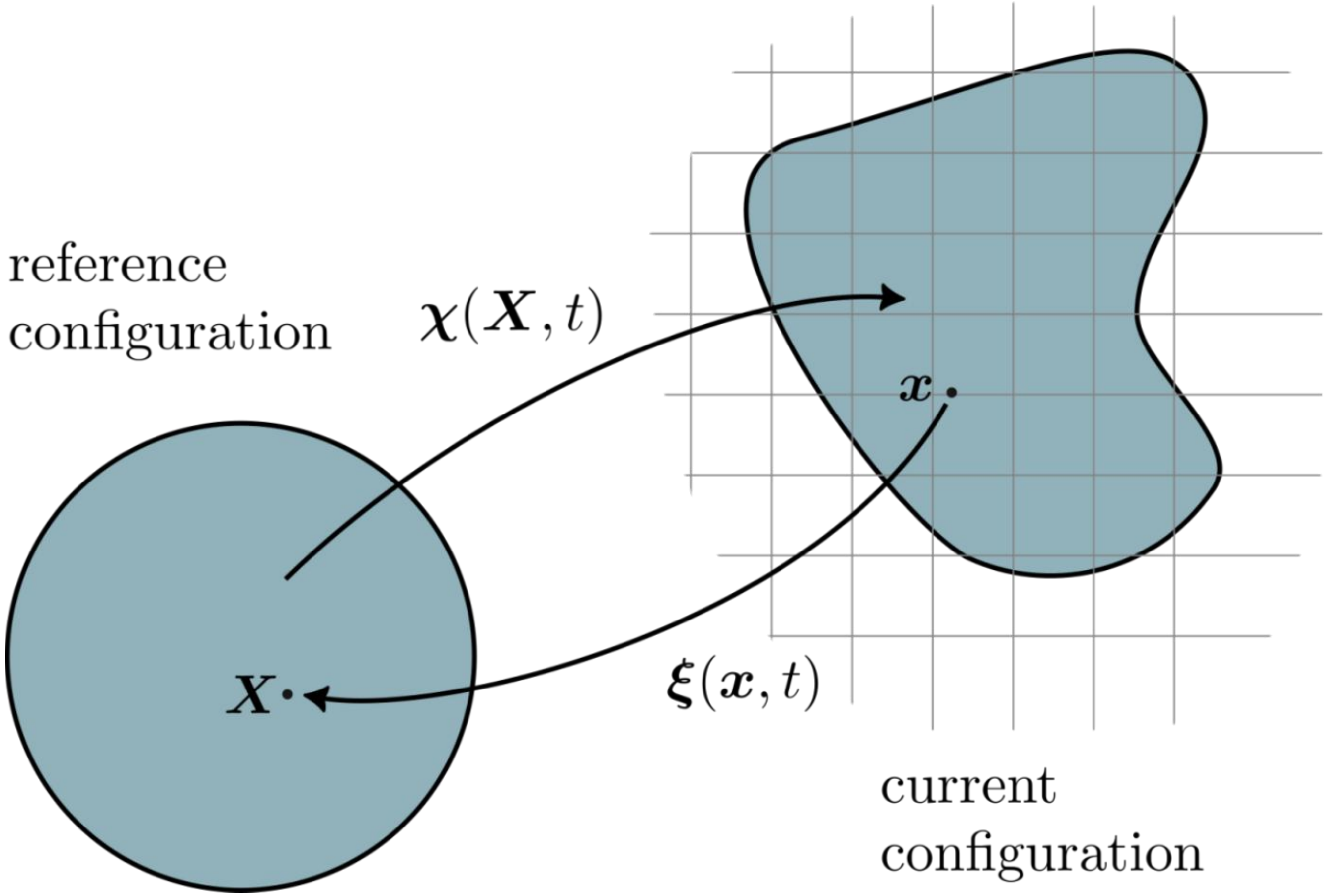
Evolution of phasefield variable

$$\frac{\partial \phi}{\partial t} + \frac{\partial \boldsymbol{u}}{\partial t} . \nabla \phi = \nabla . \left(-\mathcal{M} \nabla \left(\frac{\delta \mathcal{E}_{\mathrm{pf}}}{\delta \phi} \right) \right)$$

$\boldsymbol{u}$ is the displacement field

- Phase-field approach along with Cahn-Hilliard advective evolution

How to describe *elasticity* in Eulerian framework?



Reference map technique[1,2]

Mapping current to reference configuration

$$\xi(\boldsymbol{x}, t) \rightarrow \boldsymbol{X}$$

$$\frac{\partial \xi}{\partial t} + \frac{\partial \boldsymbol{u}}{\partial t} . \nabla \xi = \boldsymbol{0}$$

Constructing deformation gradient

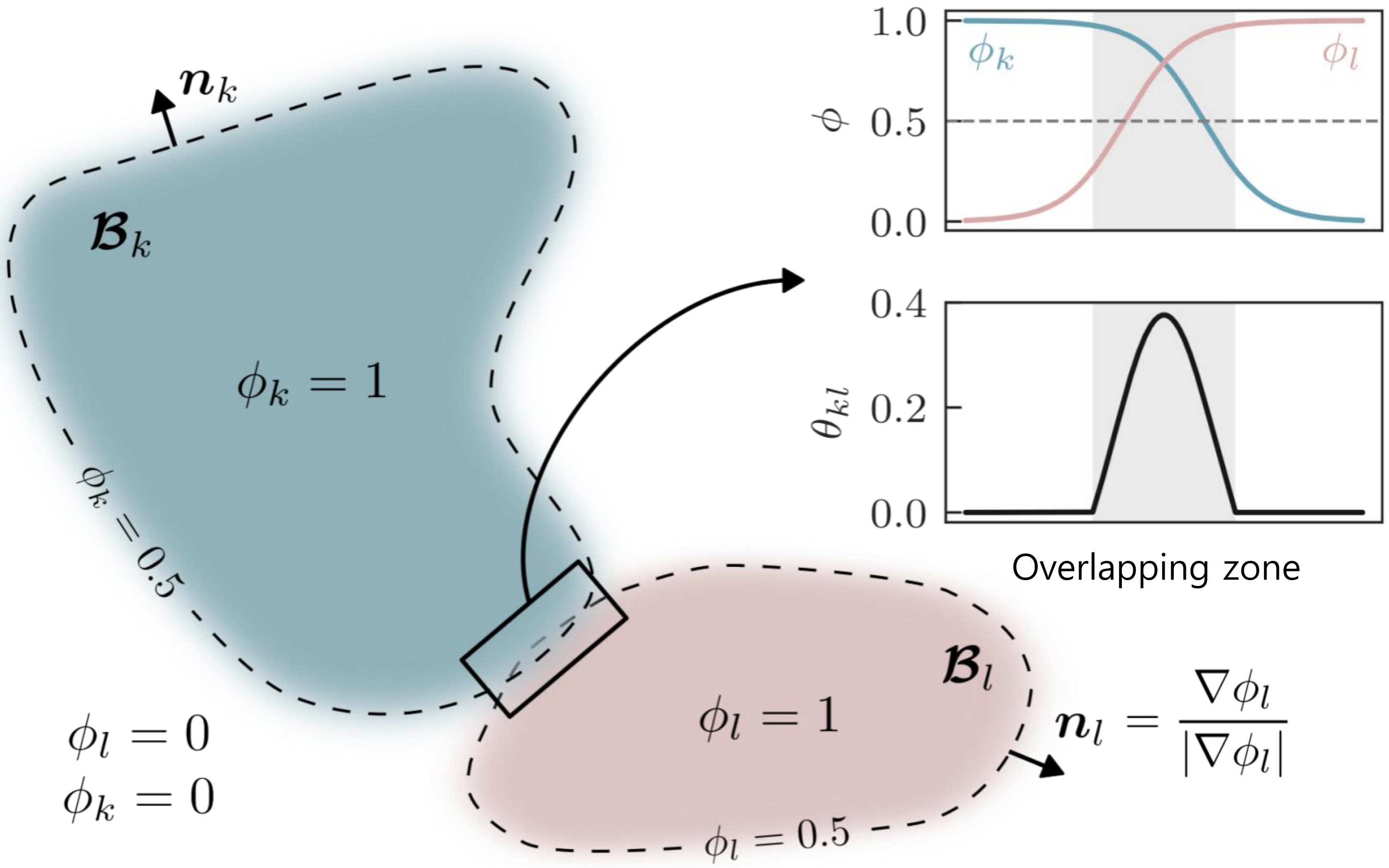
$$\boldsymbol{F}(\boldsymbol{x}, t) = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}} = \left(\frac{\partial \boldsymbol{X}}{\partial \boldsymbol{x}}\right)^{-1} = (\nabla \boldsymbol{\xi}(\boldsymbol{x}, t))^{-1}$$

Using constitutive laws

$$\boldsymbol{\sigma} = f(\boldsymbol{F})$$

1. *Kamrin et al.*, **JMPS**, 2012
2. *Rycroft et al.*, **Journal of Fluid Mechanics**, 2020

How to *detect contact* in Eulerian framework?

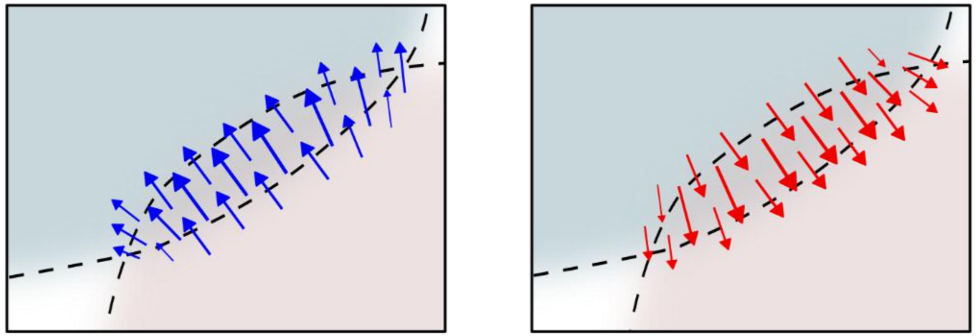


Scalar intersection metric

$$\theta_{kl} = \left\langle \phi_k . \phi_l - \frac{1}{4} \right\rangle^+$$

Contact force field

$\boldsymbol{b}_k = \kappa \cdot \theta_{kl} \cdot \boldsymbol{n}_l$ $\quad$ $\boldsymbol{b}_l = \kappa \cdot \theta_{kl} \cdot \boldsymbol{n}_k$



- Contact detection becomes trivial and contact force as volumetric forces

1. *Lorez et al.*, Eulerian framework for contact between solids represented as phase fields, **CMAME**, 2024

Coupling all the ingredients

Solving for the elasticity

$$\sum_{i=1}^{2} \nabla.(\phi_i \boldsymbol{\sigma}(\boldsymbol{\xi_i})) + \boldsymbol{b}_i = 0$$

Advect phasefields according to displacements

$$\frac{\partial \phi}{\partial t} + \frac{\partial \boldsymbol{u}}{\partial t}.\nabla \phi = -\nabla.\mathcal{M}\nabla \mu$$

$$\mu = \frac{\delta \mathcal{E}_{\text{pf}}}{\delta \phi}$$

Updating the reference map

$$\frac{\partial \xi}{\partial t} + \frac{\partial \boldsymbol{u}}{\partial t}.\nabla \xi = \mathbf{0}$$

System of equations for a single body

$$\underbrace{\begin{bmatrix} \mathbf{K}_{\boldsymbol{u}} & \mathbf{K}_{\boldsymbol{u\xi}} & \mathbf{K}_{\boldsymbol{u\phi}} & 0 \\ & \mathbf{K}_{\boldsymbol{\xi}} & 0 & 0 \\ & & \mathrm{K}_{\phi} & \mathrm{K}_{\phi\mu} \\ & & & \mathrm{K}_{\mu} \end{bmatrix}}_{\mathbf{K}^k} \cdot \underbrace{\begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{\xi} \\ \phi \\ \mu \end{bmatrix}}_{\mathbf{u}^k} - \underbrace{\begin{bmatrix} \boldsymbol{b}(\phi) \\ \mathbf{0} \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{f}^k} = \mathbf{0}$$

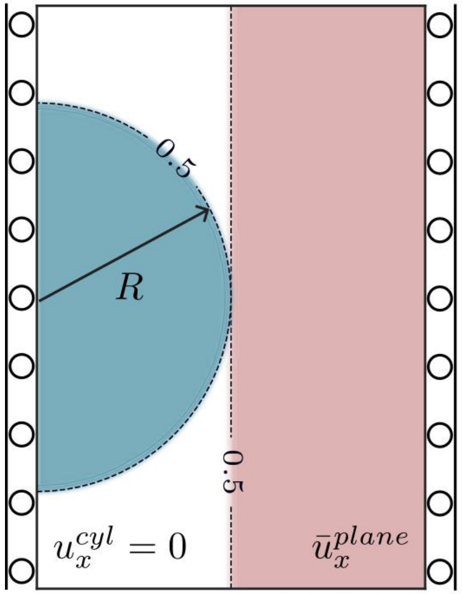
Coupled system for two bodies

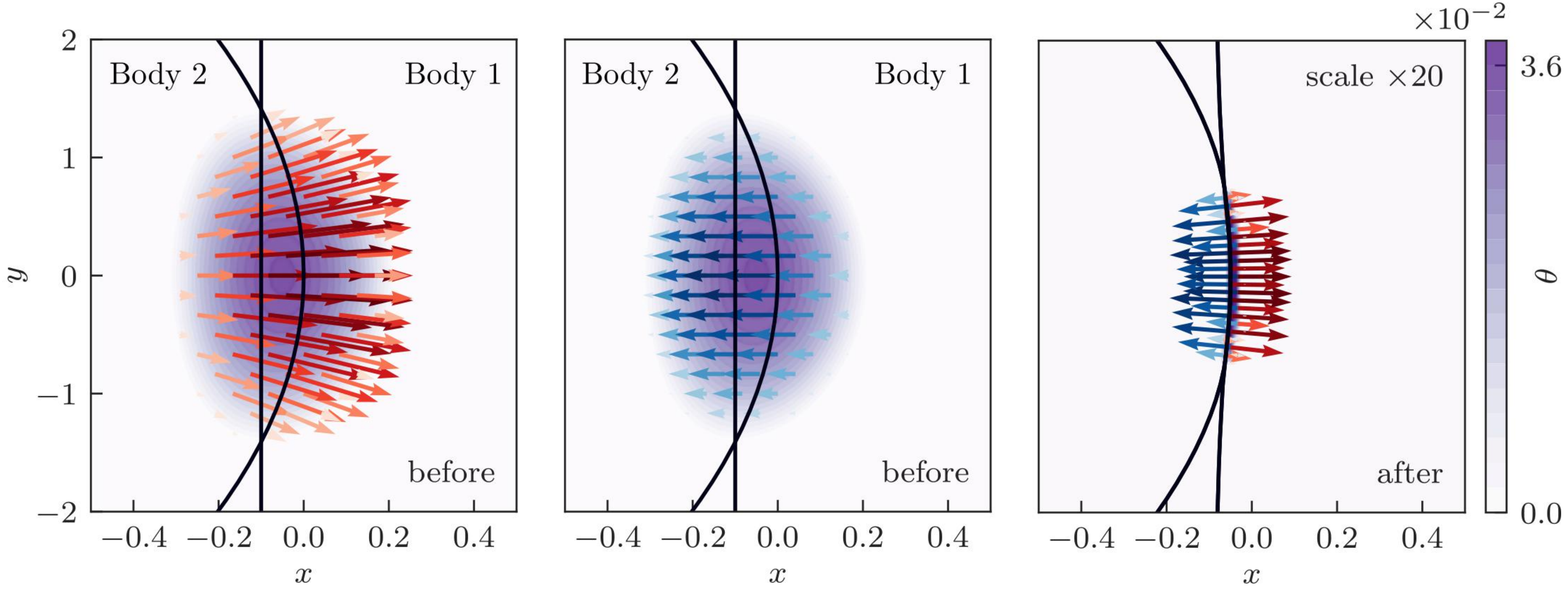
$$\begin{bmatrix} [\mathbf{K}^k] & \mathbf{0} \\ & [\mathbf{K}^l] \end{bmatrix} \cdot \begin{bmatrix} \mathbf{u}^k \\ \mathbf{u}^l \end{bmatrix} - \begin{bmatrix} \mathbf{f}^k \\ \mathbf{f}^l \end{bmatrix} = \mathbf{0}$$

1. *Lorez et al.*, Eulerian framework for contact between solids represented as phase fields, **CMAME**, 2024

Does the method *work*?

Hertz contact

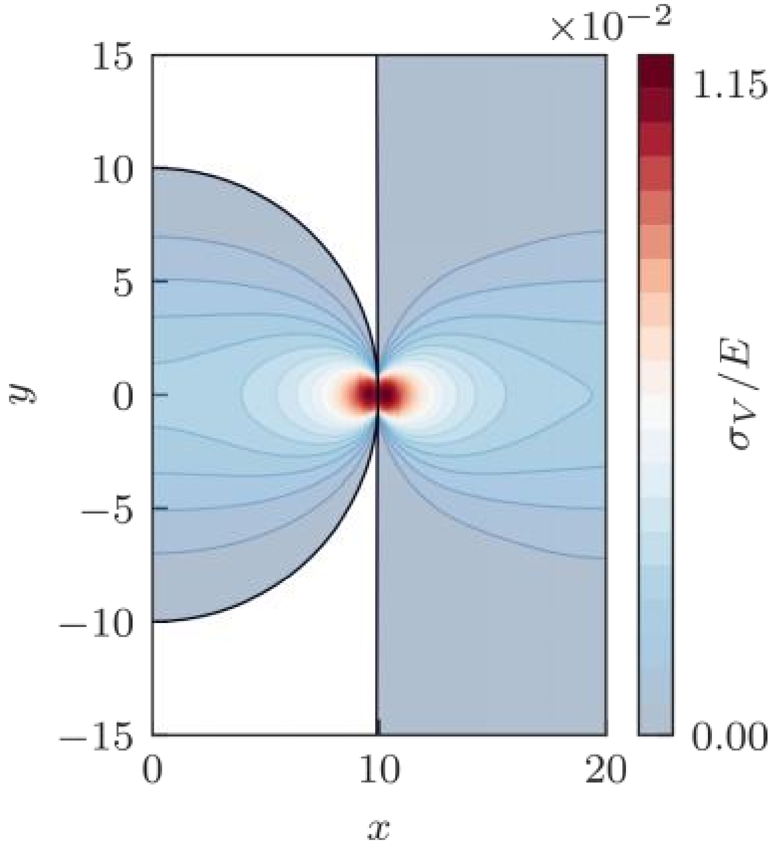




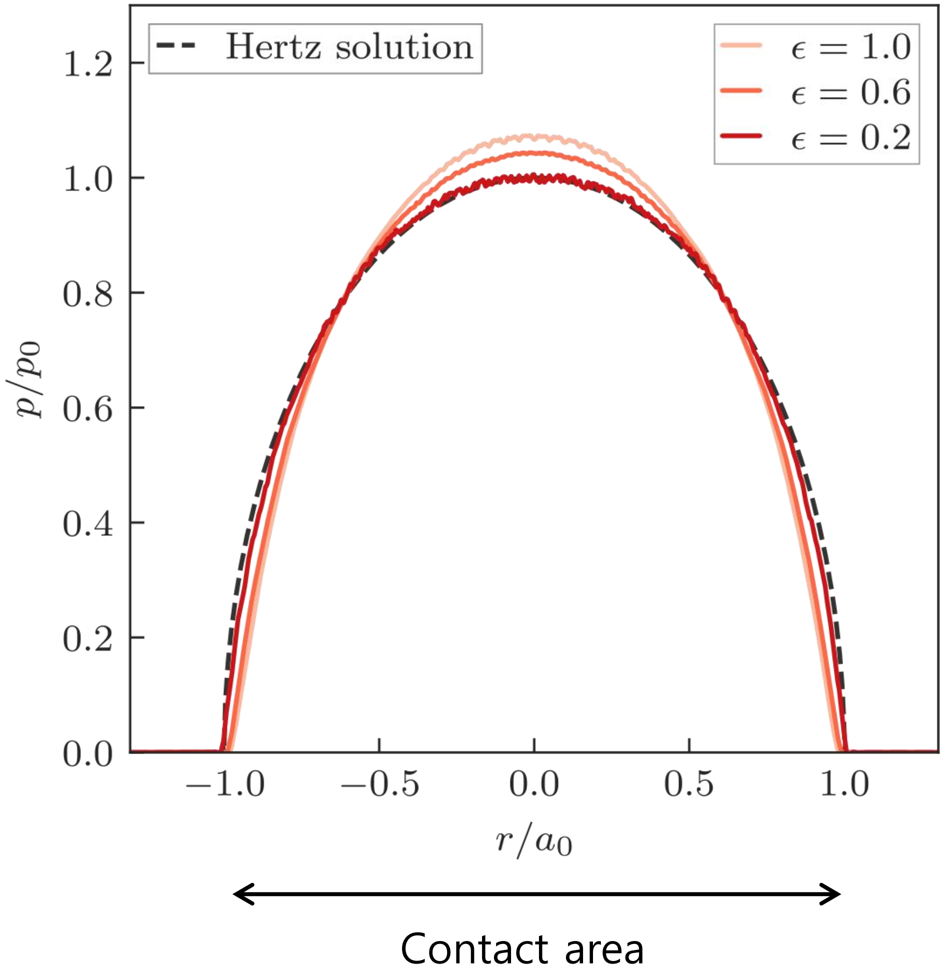
Contact force field (before)

Contact force field (after)

Hertz contact

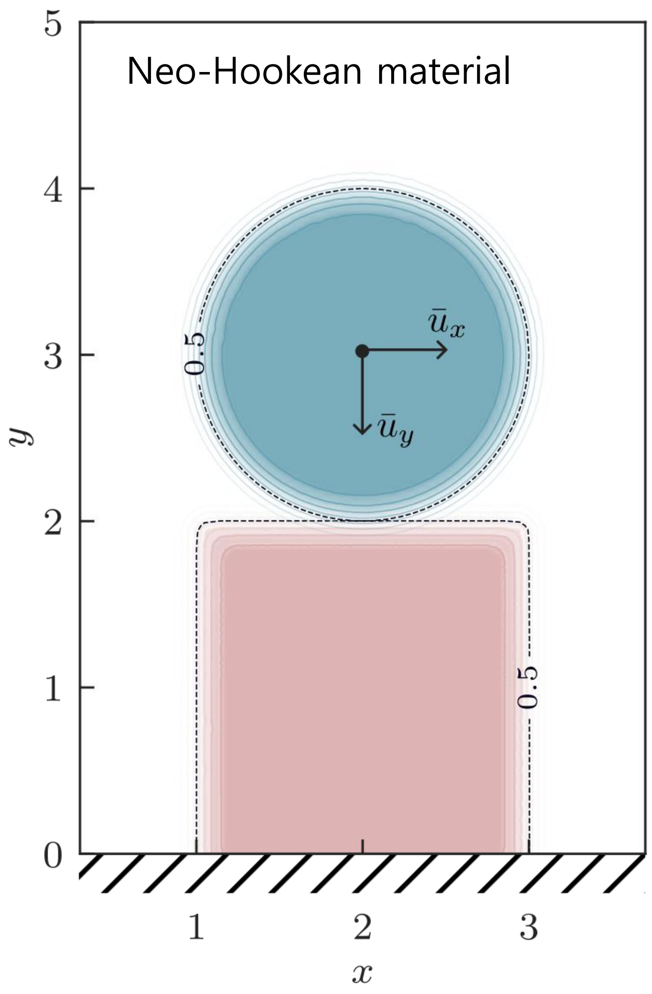


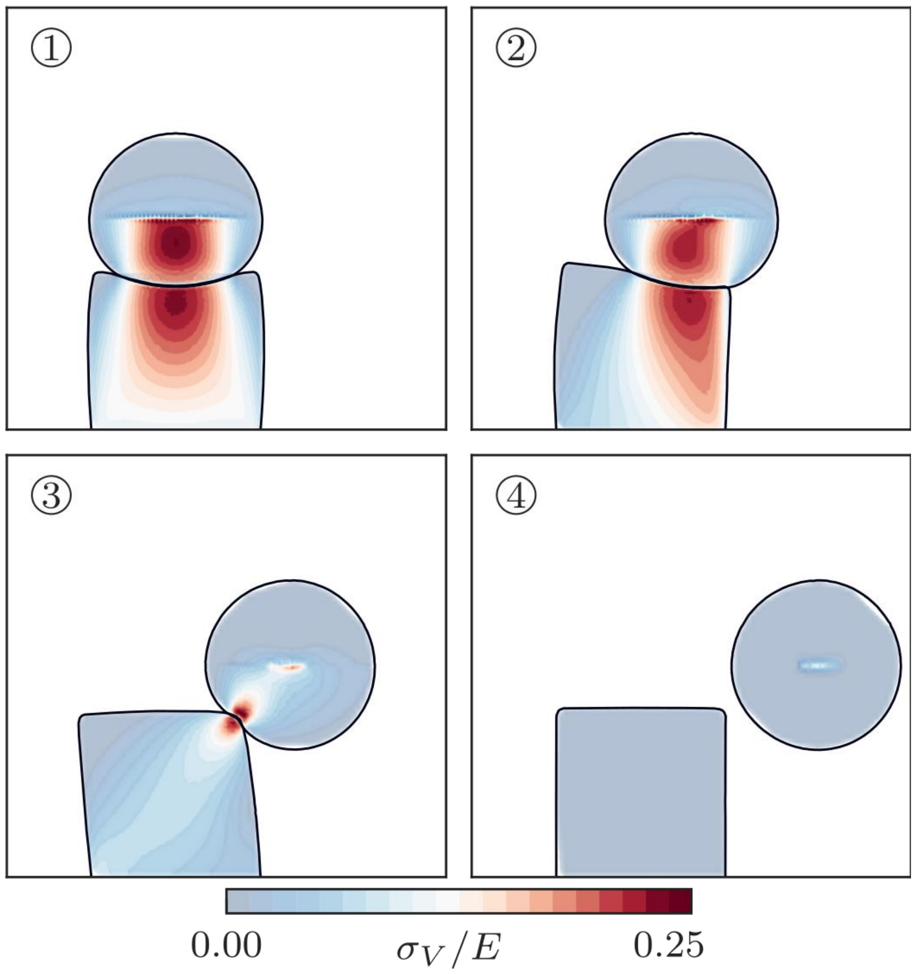
Effect of interface region



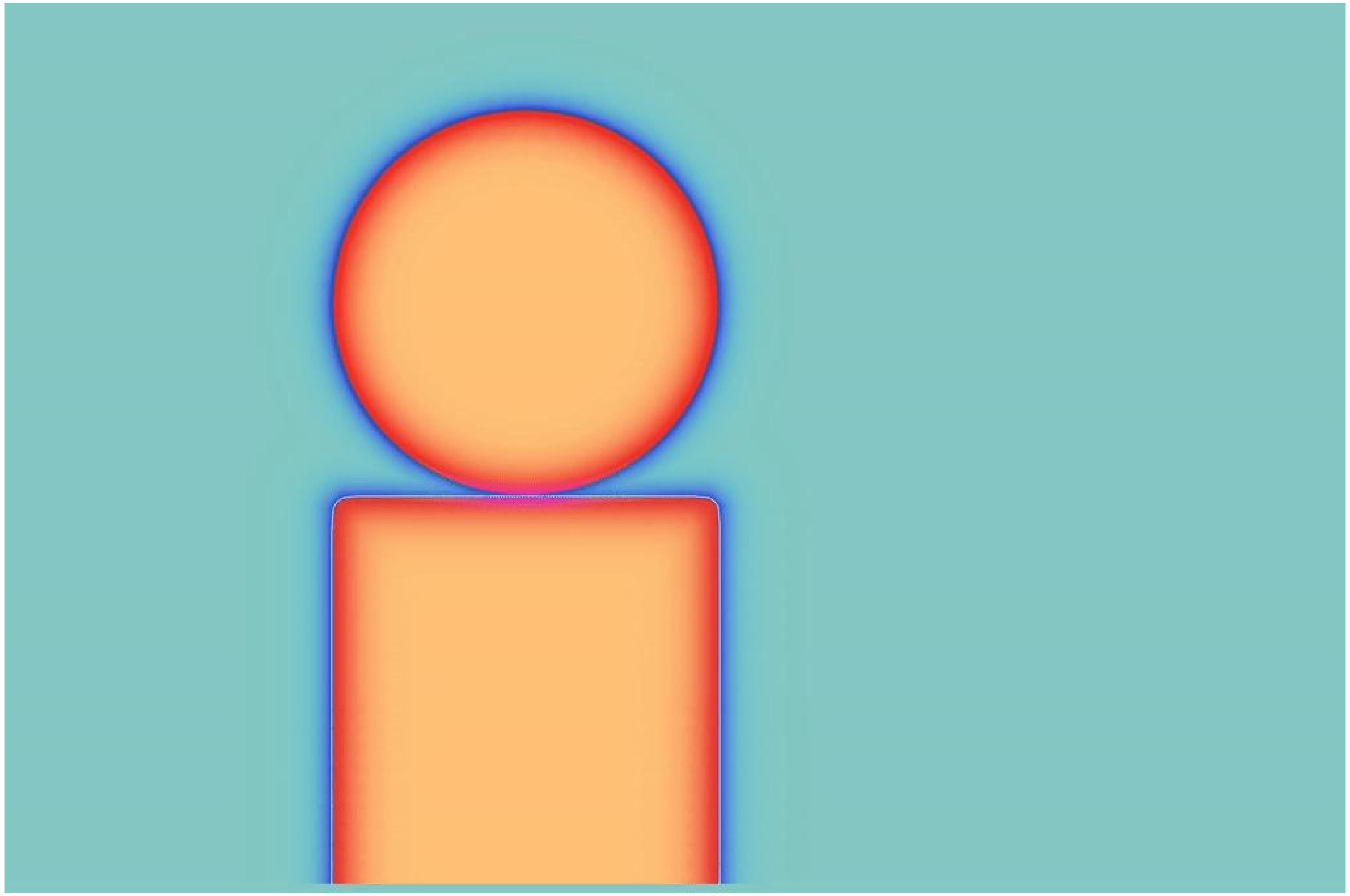
How *robust* is the method?

Can handle *large deformations*

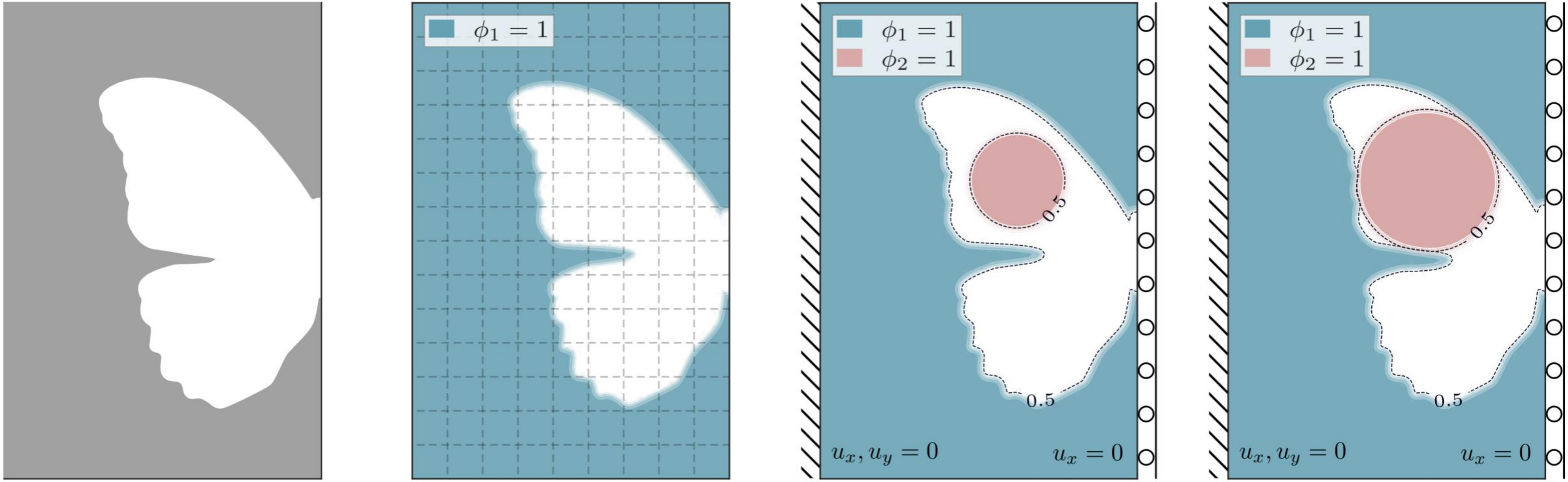




Can handle *large deformations*



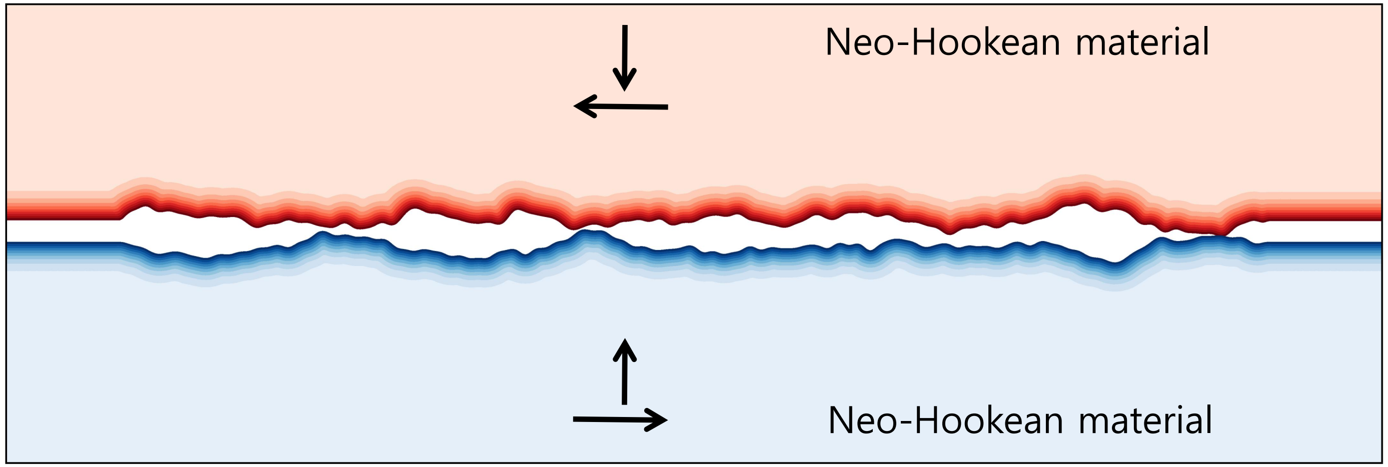
Can handle *evolving surfaces*

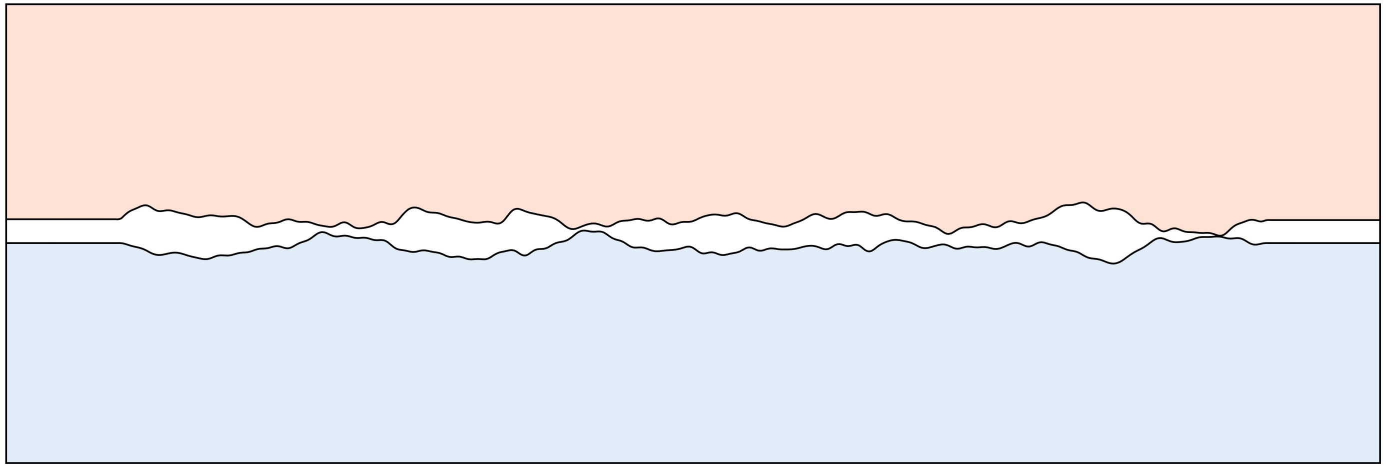


Can handle *evolving surfaces*

Evolution of Von-Mises stress

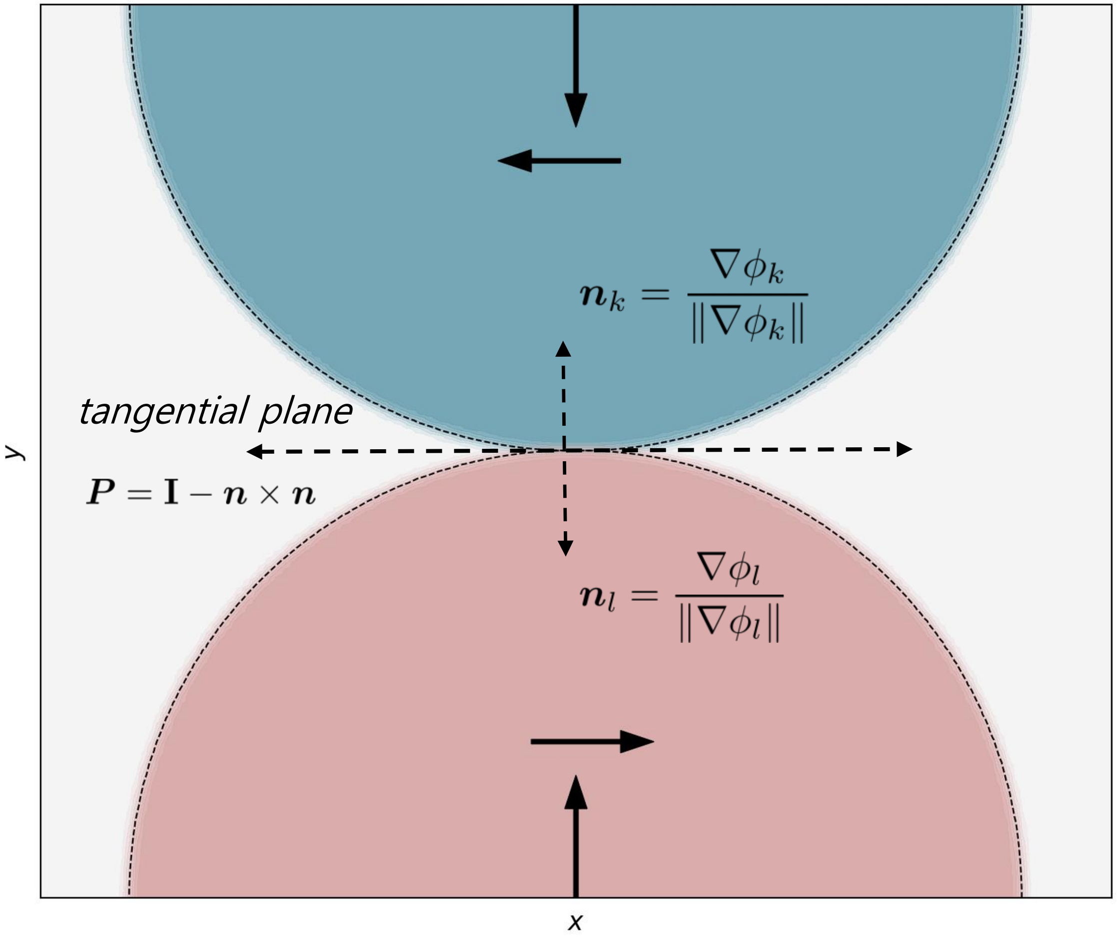
Can handle *complex domains*





Rough-on-Rough contact

Can handle *friction*



Tangential slip

$$\boldsymbol{u}_{\boldsymbol{t}}^{\boldsymbol{ij}} = \mathbf{P} \cdot (\Delta \boldsymbol{u}_{\boldsymbol{i}} - \Delta \boldsymbol{u}_{\boldsymbol{j}})$$

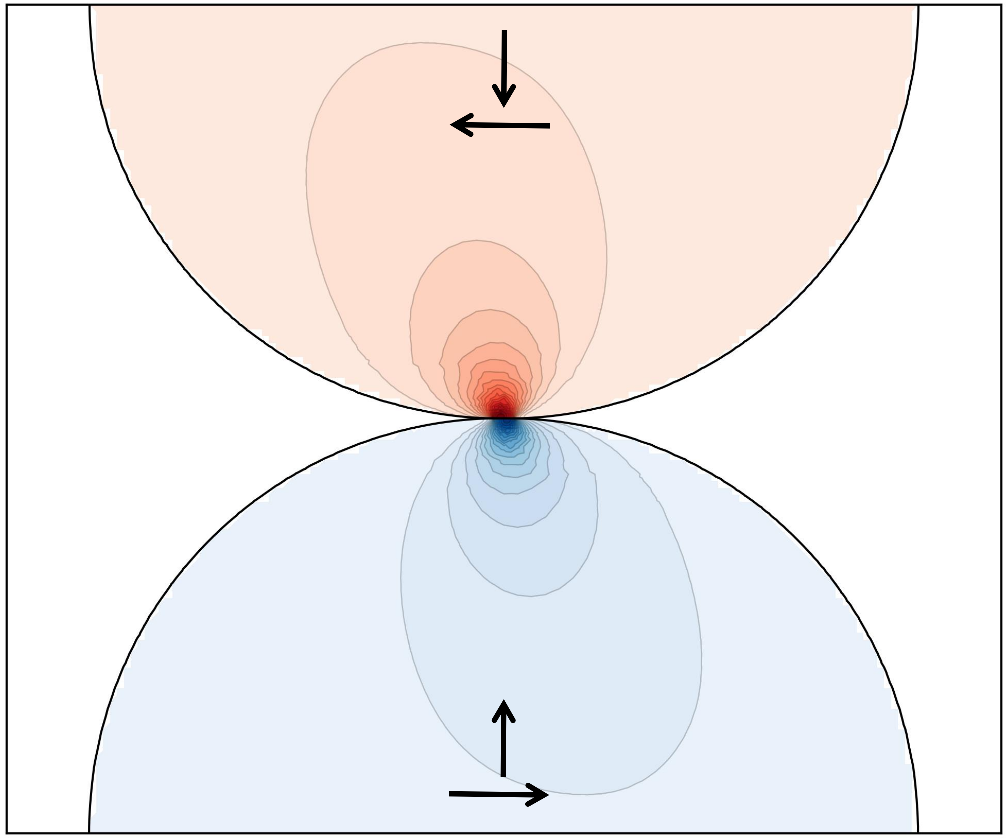
Tangential force

$$\boldsymbol{b}_{\boldsymbol{t}} = \lambda \cdot \mu \boldsymbol{b}_n \cdot \frac{\boldsymbol{u}_{\boldsymbol{t}}}{\|\boldsymbol{u}_{\boldsymbol{t}}\|}, \quad \lambda \in [0, 1]$$

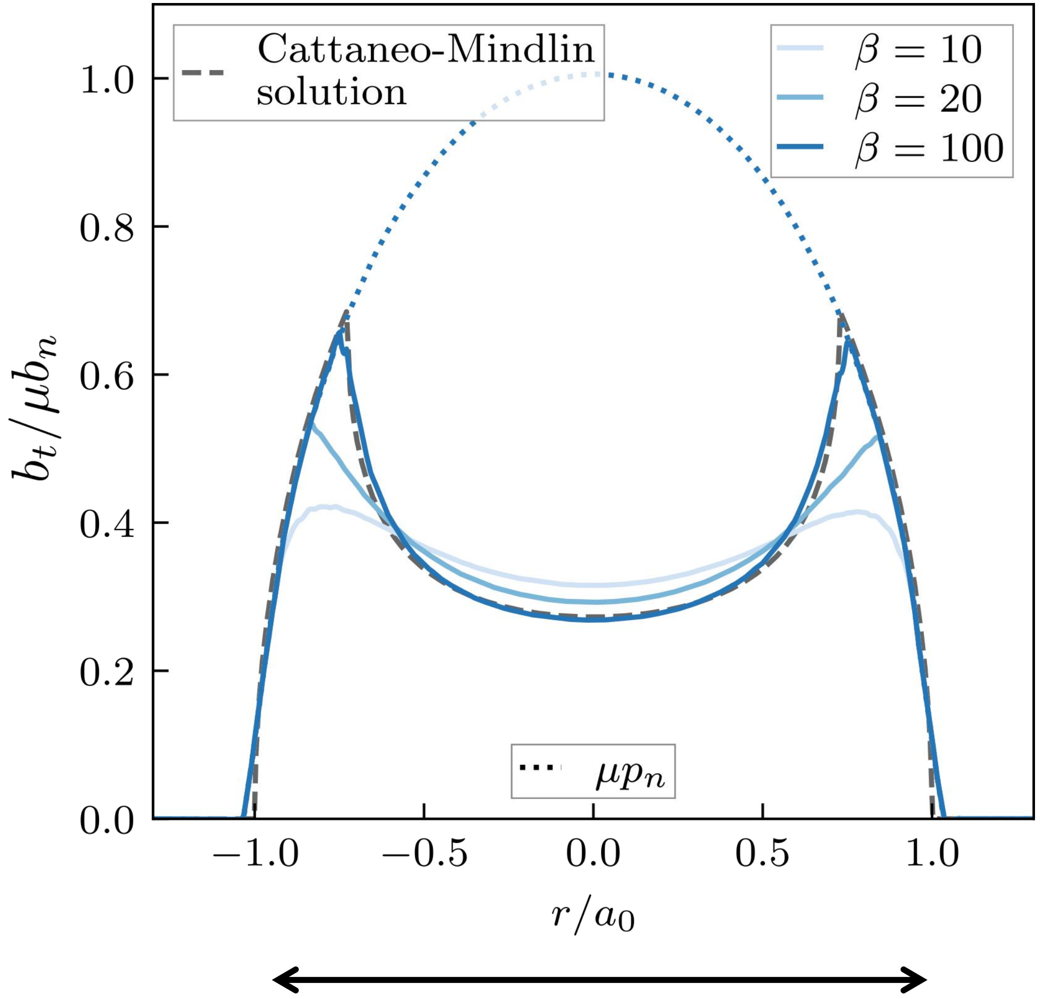
Balance of linear momentum

$$\nabla \cdot (\phi_i \boldsymbol{\sigma}(\boldsymbol{\xi}_{\boldsymbol{i}})) - \boldsymbol{b}_n^i - \boldsymbol{b}_t^i = \mathbf{0}$$

Can handle *friction*



Cattaneo-Mindlin Traction Profile



Conclusion

- A *robust methodology* to handle *interfacial interactions* within *Eulerian* framework.
- Allows *strong coupling* of multiphysical process with contact.
- *Contact detection* phase becomes *trivial*.
- Can handle *large deformation*, *complex surfaces* and *friction.*

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Thank you for your attention.

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Lorez et al., Eulerian framework for contact between solids represented as phase fields, **CMAME**, 2024