Connectivity in the presence of Shadowing in 802.11 Ad Hoc Networks

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Abstract

Connectivity is an important property for QoS Support in Mobile Ad Hoc Networks (MANETs). Recently, there has been a big effort in exploring the critical transmission range (CTR) analytically, based on different network models. While most of these studies rely on a geometric model and come up with asymptotic bounds, their significance regarding finite 802.11 *based MANETs is questionable. In this paper, we investigate connectivity in MANETs from a layered perspective. We first point out how the transmission range affects the end-to-end connection probability in a log-normal shadowing model and compare the results to theoretical bounds and measurements in the path loss model. We then show how connectivity issues behave in* 802.11 *and IP based networks if the fading effect increases. The paper concludes with a analytical model for the link probability in log-normal shadowing environments as a function of the number of nodes, network area, transmission range, path loss exponent and shadowing deviation.*

1 Introduction

Connectivity is typically studied by choosing an appropriate network model that allows for analytical treatment. Most of the work assumes a network to have n nodes distributed according to a certain probability function within an area of $[0, l]^2$. Furthermore a transmission power p_t is assigned to each node. A node v is then said to be directly connected to another node u if the received signal power p_r^v does not drop below a certain threshold β, for a given attenuation function $L(\cdot)$, therefore if $p_r^v = p_t^u \cdot L(\|u-v\|) \geq \beta$. The attenuation function usually takes the form $1/d^\rho$, where ρ is denotes the path loss exponent. This model is referred to as *path loss* model and predicts the received power as a deterministic function of distance, therefore representing the communication range as an ideal circle. In reality, the received power at a certain distance is a random variable due to fading effects. This behavior is reflected by the *shadowing* model:

$$\left[\frac{p_r(d)}{p_t}\right]_{dB} = -10\rho \log_{10} \frac{d}{d_0} + X \qquad (1)$$

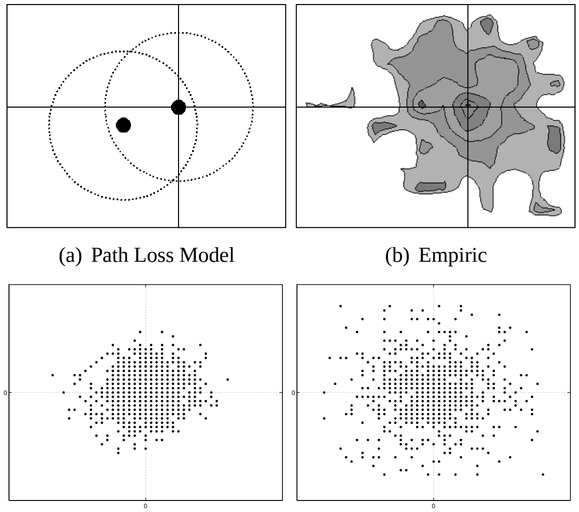
where X is a Gaussian random variable with zero mean and standard deviation σ and ρ is the aforementioned path loss exponent. Figure 1 shows how the transmission area may look in reality and in different models. Table 1 shows some typical values of ρ and σ.

Work on connectivity in ad hoc networks mainly has focused on finding the critical transmission range to assure the network stays connected if the node density grows infinite.

For the path loss model, Gupta and Kumar show in [7] that if the radio transmission range of n nodes uniformly distributed in a disc of unit area is set to $r_c = \sqrt{(ln(n) + c(n))/(\pi n)}$, then the resulting wireless multi-hop network is asymptotically connected with probability one if and only if $c(n) \to +\infty$. In [13] the authors give a non-tight bound for sparse network, by taking the network size into account. Bounds on the connection probability and critical transmission range for a finite ad hoc network were given by [4][11][10] and recently by [14]. While all this work is based on a geometric model, it was shown in [17] that a more accurate modeling of the physical layer is important. The applicability of these results to finite 802.11 based networks is questionable due to the following facts:

- Radio propagation is far from isotropic
- 802.11 asks for symmetric links (ACK-based protocol)
- Packet reception does not scale linearly with the signal strength
- Border effects cannot be neglected

Nevertheless, only a few results considering connectivity under more realistic environments are available. Recently, connection probability has been analyzed in a shadow fading model [2] but without considering the asymmetric link problem. In [5] the authors investigate the relation of connectivity and capacity saying that if the attenuation function does not have a singularity at the origin, then connectivity



(a) Path Loss Model (b) Empiric

(c) Shadowing with σ=2 (d) Shadowing with σ=4

Figure 1. Transmission area

Environment		ρ	σ
Outdoor	Free space	2	4 to 12
	Shadowed/Urban	2.7 to 5	
Indoor	Line-of-sight	1.6 to 1.8	3 to 6
	Obstructed	4 to 6	6.8

Table 1. Some typical values of path loss (ρ) and shadowing deviation (β)

does not scale. Indeed connectivity and capacity are opposite properties. While connectivity demands high node density, per node capacity was shown to decrease as $1/\sqrt{n}$ [6].

In this paper, we investigate connectivity in MANETs from a layered perspective. We first show how the transmission range affects the end-to-end connection probability in a log-normal shadowing model and compare the results to theoretical bounds and measurements in the path loss model. We then show how connectivity issues behave in 802.11 and IP based networks if the fading effect increases. The paper concludes with an analytical model for the link probability in log-normal shadowing environments as a function of the number of nodes, network area, transmission range, path loss exponent and shadowing deviation.

The rest of the paper is organized as follows. The next section shortly describes the network model and the different perspectives. In sections 3 and 4 we analyze end-to-end connection probabilities and critical transmission range with respect to the perspectives just defined. Section 5 compares the results to certain analytical studies given in literature. In section 6 we give a analytical model of the link probability and section 7 concludes the paper.

	Path	Connection	k
GEO	floyd/warshall	geometric	10'000
MAC	floyd/warshall	packet based	5000
IP	AODV	packet based	100

Table 2. Properties of different perspectives

2 Network Model and Connectivity Metric

We consider n nodes to be uniformly distributed in a network of size $[0,l]^2$. Each node has a transmission power p_t assigned. We adjust the receiving threshold to produce an average transmission range of length r [1]. Further parameters are path loss exponent (ρ) and shadowing deviation (σ). We use monte carlo simulation to explore a given range of the multi-dimensional parameter space. All simulations have been done using ns-2 [15]. Due to the excessive amount of simulations needed to get stable probability values we computed them on a linux cluster of about 64 machines using JOpera [16] as a grid engine. For a random source-destination pair we ran k simulations to compute to connection probability as follows:

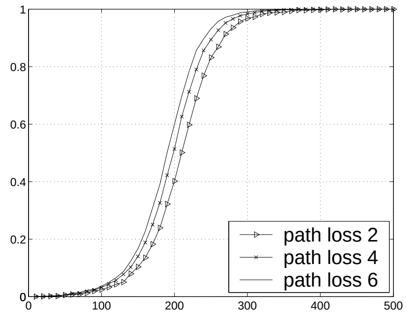
$$\frac{1}{k}\sum_{i=0}^{k} t_i^c/t_i^s$$

where t^c is the total time two nodes have connection during one session and t^s is the simulation time. As mentioned earlier, we highlight connectivity from different perspectives: GEO[2], MAC and IP. On the lowest layer (GEO), connection time is defined in a purely geometric manner. The signal strength p_r perceived at a node v due to radio propagation of node u is computed according to equation (1). Node v is considered a neighbor of u if p_r does not drop below a certain threshold β. We say there is a link between two nodes if both nodes see themselves as neighbors. Therefore, neighbors can be asymmetric whereas links cannot. We define a connection between two random nodes if a path can be found so that every two consecutive nodes have a link in between. This definition of a connection accounts for the fact that most media access protocols are ACK-based.

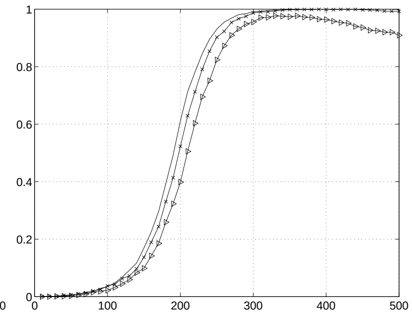
In contrast to the definition of connectivity in the GEO perspective, we use a packet based approach for the MAC and the IP perspective. In the MAC perspective, packets are forwarded along a path, pre-computed by the *Floyd-Warshall*[3] Algorithm. The algorithm to compute the path is based on the topology used in the GEO perspective (therefore it guarantees the signal strengths between any two nodes on the path to be greater or equal the receiving threshold). For a given packet transmission frequency f, each correctly received packet contributes $1/f$ time units to the to-

[1] The expected signal strength received at distance r can be computed as follows: $p_r = p_t\left(\frac{r}{d_0}\right)^{-\rho}\exp\left(\frac{\log(10)^2}{200}\sigma^2\right)$

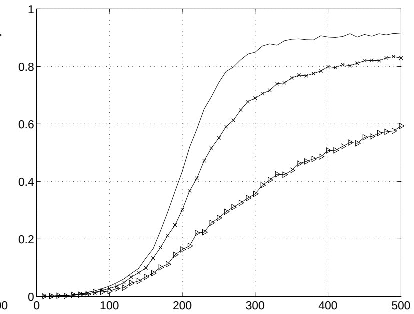
[2] For the GEO perspective we have extended the setdest topology generator to use the shadowing propagation model



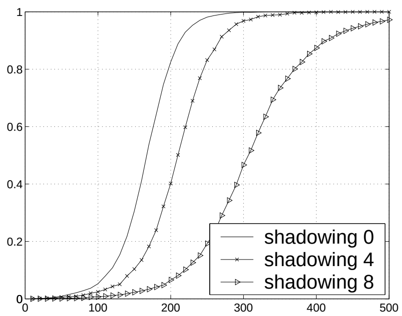
(a) GEO, Impact of ρ, σ=4



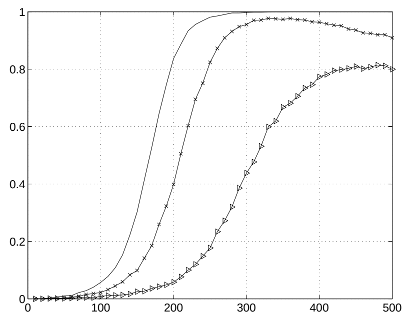
(b) MAC, Impact of ρ, σ=4



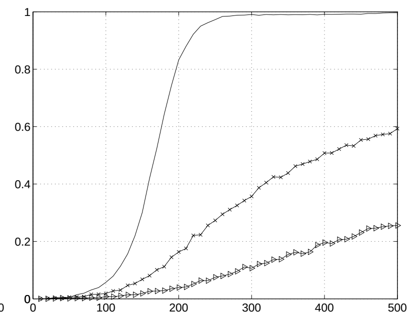
(c) IP, Impact of ρ, σ=4



(d) GEO, Impact of σ, ρ=2



(e) MAC, Impact of σ, ρ=2



(f) IP, Impact of σ, ρ=2

Figure 2. Connection Probability

tal connection time. This approximately defines two nodes u and v to be connected if node v is able to receive a consecutive bit stream sent by node u, potentially transmitted over multiple hops. One issue is regarding the packet transmission frequency f. On the one hand, since connectivity should not be confused with capacity, the bit stream must not exceed the available bandwidth. On the other hand it is known from [8] that for any given signal, the sampling frequency has to be at least twice the signal frequency. In this paper, we decided to use a rather low transmission frequency for probing packets so that they do not conflict with capacity issues. This is reasonable as we only consider static ad hoc networks. Hence, the difference in radio propagation is a purely spatial phenomenon [12] and does not change over time. Furthermore, we *synthetically* manipulate the size of the probing packets to be almost zero and eliminate any additional traffic like ARP or Routing.

The IP perspective differs from the MAC perspective only in a sense that AODV [9] is used to compute the packet forwarding path. Obviously, the IP perspective includes ARP. See table 2 for an overview on three of the perspectives just described.

3 Connectivity

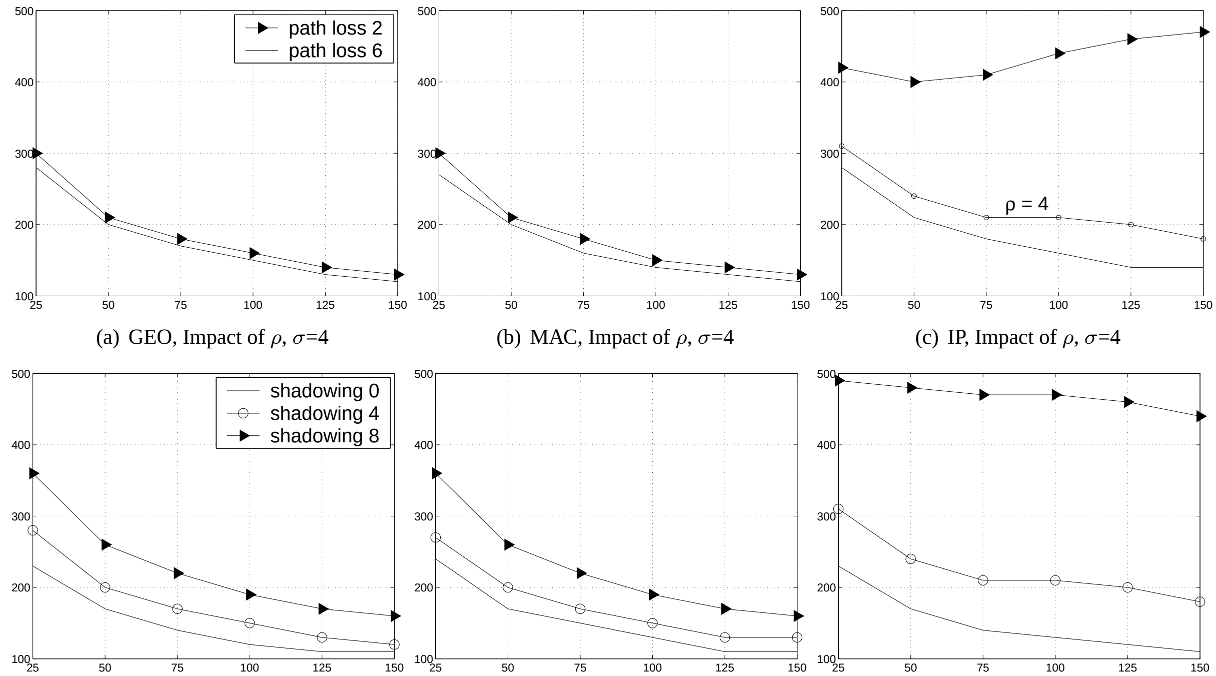
We first study connectivity in one specific network configuration: a network comprising 50 nodes within an area of the size of 1000×1000 meters. Figure 2 shows connectivity as a function of the average transmission range for different perspectives and different propagation parameters (path loss exponent ρ and shadowing deviation σ).

For the GEO perspective, changes in path loss (ρ) do not have an impact on connectivity for σ equal to zero (these results are not shown here), because the radio propagation refers to the ideal circle with radius r (remember that we adjust the receiving threshold β according to $avg\_transmission\_range(p_t, \beta) = r$). But there is some noticeable degradation for a decreasing path loss when σ is set to values greater than zero, see Figure 2a. This behavior can be explained by looking at the shadowing propagation model. According to equation (1), the received signal power can be seen as of the following form:

$$p_r \sim p_t \ \frac{1}{10^{\rho \log_{10}(d) - X/10}} \tag{2}$$

Therefore, small values of ρ can lead to a much more irregular radio propagation (if σ is big enough) than big values. Regarding the impact of shadowing, Figure 2d clearly shows a diminishing connectivity as σ increases. This result stands in contrast to [2], where the author shows for unidirectional links that an increase in σ also increases connectivity. So while irregular radio propagation helps to improve connectivity in networks with unidirectional links, it reduces connectivity if links are supposed to be bi-directional.

Similar to the GEO perspective, connectivity in the MAC perspective degrades for small values of ρ (if $\sigma > 0$) and large values of σ. But unlike in GEO, the probability never reaches 1 if σ is big enough or/and ρ is small, no matter how big the average transmission range is (Figure 2b). This can be explained by the nature of 802.11. Packets in 802.11 can



(a) GEO, Impact of ρ, σ=4 (b) MAC, Impact of ρ, σ=4 (c) IP, Impact of ρ, σ=4

(d) GEO, Impact of σ, ρ=4 (e) MAC, Impact of σ, ρ=4 (f) IP, Impact of σ, ρ=4

Figure 3. Critical Transmission Range for $p_{conn} = 0.5$, (nodes vs. transmission range)

interfere beyond the actual transmission range; typically the so called interference distance is assumed to be twice the transmission range. Since a large shadowing deviation σ not only leads to an irregular transmission range but also to an interference area that is far from a circle, RTS/CTS packets collide more often, especially when σ as well as the average transmission range are big. This finally results in a reduction of connectivity.

These problems are even amplified in the IP perspective (Figure 2c and 2f). If RTS/CTS packets collide, then it might happen that temporarily a route cannot be established. On the other hand, if the shadowing deviation is zero, the curve almost looks like the one in the GEO perspective. Thus, in general AODV is able to find the route.

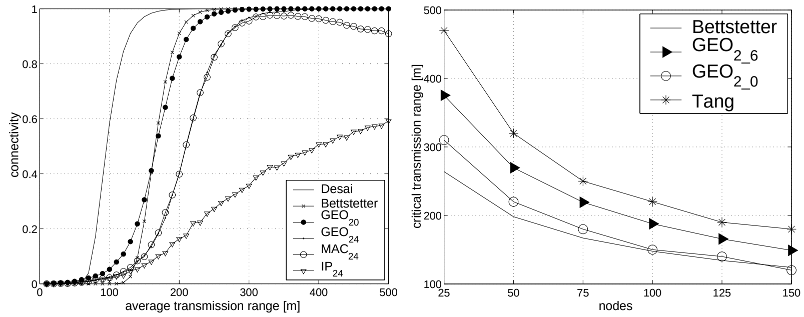
Overall we can say, the higher in the network stack and the more out of shape the transmission range (compared with a perfect circle), the worse connectivity gets.

4 Critical Transmission Range

So far we only considered one fixed network configuration. In order to see how connectivity evolves with a growing number of nodes we have computed the critical transmission range (CTR) for a connection probability of at least 0.5. As mentioned in section 1, CTR have been studied based on the the path loss model for years. Its value is known to decrease as $\sqrt{A \log(n)/n}$ for an increasing n. Figure 3 shows the CTR for different perspectives and propagation parameters in scenarios where the network area is kept of constant size (hence the density is increasing). In general, a similar behavior can be observed as previously in the fixed network configuration (variation in shadowing deviation σ has more impact on the shape of the curve than variation in pathloss exponent). However, it seems that at least for the GEO and the MAC perspective an increasing node density reduces the degradation provoked by irregular radio propagation. This is reasonable as a bigger coverage factor (nodes per transmission range) enhances the link probability. Furthermore, we observe that in contrast to the fixed network configuration where we noticed a slight difference between the GEO and the MAC perspective, these two perspectives almost scale equally for an increasing node density. This is different for the IP perspective. Here we find, that for a σ big enough and/or a small ρ, the transmission range does not decrease further with increasing node density, proving how badly RTS/CTS and routing are affected by irregular radio propagation.

5 Comparison with analytical studies

As mentioned, most analytical studies are not directly applicable to finite ACK-based ad hoc networks. Either their work allows for an asymptotic statement only or they do not cope with the unpredictability of radio signal propagation. Figure 4a compares connection probabilities of several analytical studies with simulation results in a log-normal shadowing environment, viewed under different perspectives. Again, the network is assumed to have 50 nodes within an area of 1000×1000 meters. As we can see, there is already quite a big gap between the two analytical studies.



(a) Connectivity

(b) CTR for $p_{conn} = 0.9$

Figure 4. Analytical studies vs. simulation results

Both, Desai[4] and Bettstetter[1] aim at computing connectivity for the path loss model. However, the simulation results measured under the GEO perspective using a σ equal to 0 (note that this refers to an ideal circle) match quite well to the closed form proposed by Bettstetter. But if the shadowing deviation increases and even routing is used, network connectivity is far from the theoretical bound.

For the critical threshold we observe a similar behavior (Figure 4b). Again we compared the simulation results to the study of the Bettstetter as it matches best. Additionally, we also refer the results of a recent study done by Tang[14], which is based on simulation and curve fitting (for the path loss model as well). From Figure 4b, we again observe that irregularity in radio propagation results in the curve diverging from theoretical bounds. However, the bigger the node density the smaller the gap between the results gets. One reason for this could be, that most analytical studies are based on statistical assumptions, i.e. the distribution of average node distance. These assumptions loose significance if the number of nodes is small. Another reason is that border effects start to have an increasing influence if the node density becomes low.

6 Link Probability

After we have shown simulation results for connectivity and critical transmission range we want now to analytically determine the link probability for the GEO perspective as a function of nodes in the network, network size, average transmission range, path loss exponent and shadowing deviation. Remember that we define a link between two nodes if and only if both participants receive a signal stronger than β. We start the derivation of our model by considering two points u_i and u_j whose coordinates are assumed to be uniformly distributed random variables over $[0, l]^2$, each. The Euclidean distance

$$d(u_i, u_j) = \|u_i - u_j\|_2$$

is again a random variable, whose *probability density function* (pdf) can be approximated by

$$f_d(x) = \frac{2x^3 - 8l\,x^2 + 2\pi l^2 x}{l^4}$$

over the interval $[0, l]$. Note, that we deliberately disregard the distribution's tail since the additional expressions would be too complicated and the error introduced is negligible. Following equation (1) we then introduce a new random variable

$$\delta(u_i, u_j) = -10\rho\big(\log_{10} d(u_i, u_j) - \log_{10} d_0\big)$$

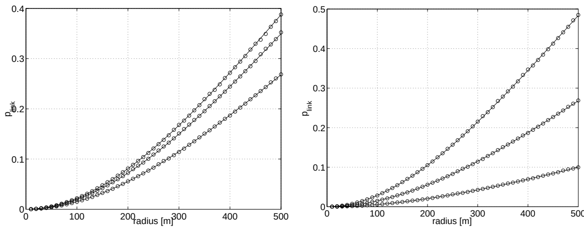
whose distribution can be expressed by the pdf

$$f_\delta(x) = d_0 \frac{\log(10)}{10\rho} 10^{-x/(10\rho)} f_d(d_0 10^{-x/(10\rho)})$$

defined over the interval $[-10\rho \log \frac{l}{d_0}, \infty]$, which is again a good approximation. By continuing according to equation (1), we can finally express the probability of a bidirectional link between any two points u_i and u_j as

$$\begin{aligned}
p_{lnk} &= P(p_r^{u_i} \geq \beta)\, P(p_r^{u_j} \geq \beta) \\
&= \int_0^{\sqrt{2}l} P\big(\big[\frac{p_r^{u_i}(d_{i,j})}{p_t}\big]_{dB} \geq \beta_{dB} \,|\, d_{i,j}\big) \\
&\quad P\big(\big[\frac{p_r^{u_j}(d_{i,j})}{p_t}\big]_{dB} \geq \beta_{dB} \,|\, d_{i,j}\big) P(d(u_i, u_j) = \lambda)\, d\lambda \\
&= \int_0^{\sqrt{2}l} P(X_{i,j} \geq \beta_{dB} + 10\rho \log_{10} \frac{d_{i,j}}{d_0} \,|\, d_{i,j}) \\
&\quad P(X_{j,i} \geq \beta_{dB} + 10\rho \log_{10} \frac{d_{i,j}}{d_0} \,|\, d_{i,j}) P(d = \lambda)\, d\lambda \\
&= \int_{-\infty}^{10\rho \log_{10} \frac{\sqrt{2}l}{d_0}} \bar{\phi}(\beta_{dB} + \lambda_{dB}, \sigma)^2 f_\delta(\lambda_{dB})\, d\lambda_{dB}
\end{aligned}$$

where $\bar{\phi}(x, \sigma) = 1 - (\sqrt{2\pi}\sigma)^{-1} \int_{-\infty}^{x} \exp(-\frac{1}{2}\xi^2/\sigma^2)\, d\xi$ is the complementary cumulative distribution function of a normally distributed random variable with mean zero and



(a) Impact of ρ (b) Impact of σ

Figure 5. Link probability analytical (line) vs. measured (bullets)

variance σ^2. Notice, that the threshold β introduced in section 3, if transformed into the dB-domain, has the form

$$\beta_{dB} = -10\rho \log_{10} \frac{r}{d_0} + \frac{\log(10)}{20}\sigma^2.$$

Figure 5 compares p_{lnk} to results gained through simulation for different values of ρ and σ. Again the network chosen consists of 50 nodes placed within a square of 1000$\times$1000 meters, the sample size (k) is 500. As it can be seen, the analytical results from the model just described tally closely with the results of the simulations.

7 Conclusion

In this paper we have investigated connectivity in ad hoc networks under log-normal shadowing radio propagation. Our study targets networks with ACK-based media access which require symmetric links. Results in this paper are gained through excessive simulation and are given for different network layer perspectives. Already if computed purely geometrically, it was shown that connectivity is significantly reduced as soon as the shadowing deviation becomes bigger than zero and/or the path loss exponent is small. For the MAC and the IP perspective the results get even worse due to packet collision during the RTS/CTS handshake of 802.11. We have also demonstrated how connectivity scales with increasing node density if fading effects increase and we compared the results to analytical studies in the path loss model. In addition to the simulation results we derived the link probability analytically as a function of nodes in the network, network size, average transmission radius, path loss exponent and shadowing deviation. The formula was shown to match quite well with the simulation measurements for different values of shadowing deviation and path loss.

For future work we are planning also to investigate the connectivity/capacity trade-off under irregular radio propagation and the effects of mobility. Additionally, jitter is an important issue to take care of, especially in scenarios where mobility introduces change in signal propagation over time. In general, we think that a more realistic modeling of physical layers is important in network level research.

Acknowledgment

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