Two-degree-of-freedom surrogate model for performance-based seismic design

TWO-DEGREE-OF-FREEDOM SURROGATE MODEL FOR PERFORMANCE-BASED SEISMIC DESIGN

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Abstract: *Structures designed using modern performance-based seismic design procedures are expected to present different structural behaviors at different earthquake intensity levels, i.e. seismic hazard levels. The structural behaviors can be classified into flexure, shear, sliding, and rocking categories. Controversially, most current design methods are rooted in the single-degree-of-freedom surrogate model that only addresses one structural behavior: flexure. The surrogate is central in the design method because the statistics of its response to multiple ground motions are used as a proxy of the expected structural response (e.g., through design equations, R-μ-T relations, or the seismic design spectra). This paper proposes a new seismic design method that enables simultaneous consideration of multiple structural behaviors (flexure, shear, sliding, and rocking) at different seismic hazard levels. It is based on a surrogate model with two degrees of freedom so as to represent different structural behaviors and their interaction. The objective is to bring the conventional single-degree-of-freedom and the new two-degree-of-freedom surrogates into a unified risk-based framework for performance-based seismic design. We present the core of this unified framework herein, which consists of the equations that define each two-degree-of-freedom surrogate model and the transition between the behavior modes supported by the model. Furthermore, we present an application example of the proposed method for a seismic design with multiple performance objectives which are attained by the structure developing different behavior modes, e.g., flexure then sliding.*

1. Introduction

The seismic design framework aims to guide structural engineers in building structures that are safe against earthquakes. Structural safety relates to risk metrics such as the probability of a collapse in the lifetime of a structure. The current seismic design frameworks try to address a large range of structures in a rather simple way. Herein, we highlight two aspects simplified in the current seismic design frameworks: 1) structures often present complex behavior, with two or more behaviors occurring simultaneously or consecutively; and 2) the need to cover the entire risk spectrum, currently assessed at one discrete earthquake return period. Both simplifications have been the focus of recent research

Statistical relationships between capacity and demand of single-degree-of-freedom (SDOF) oscillators are available to design a structure that has a predominant behavior, namely: flexure (and yielding), sliding, or rocking. For example, Ruiz-Garcia & Miranda (2007) describe the response of simple elastic-perfect-plastic oscillators to different ground motions. Meanwhile, the SPO2IDA tool (Vamvatsikos & Cornell 2005) covers different yielding backbone curves. For rocking blocks, Reggiani Manzo & Vassiliou (2021) developed closed-form expressions for overturning displacement capacity and Kazantzi, Lachanas & Vamvatsikos (2021) developed equations for the statistics of the rocking angle given peak ground velocity (PGV) intensity. For sliding, O'Reilly et al. (2022) developed design equations and a risk-based design framework for single friction pendulum bearings. Furthermore, studies have shown that we can do full-range risk assessment in preliminary design. For example, the Yield Frequency Spectra (Vamvatsikos & Aschheim 2016), the uniform risk spectra for rocking oscillators (Reggiani Manzo et al. 2022), and the base-isolation design example given by O'Reilly et al. (2022).

Structures that present two behaviors simultaneously or consecutively were also studied in depth. The work of Naem & Kelly (1999) covers the multiple-degrees-of-freedom (MDOF) structures on top of sliding and shear bearings. Yim & Chopra (1985) and Psycharis (1981) studied the behavior of flexible MDOF structures on top of a rocking base. We can cite many other and more recent studies that cover multiple-behavior structures (e.g., Acikgoz & DeJong 2016; Vassiliou Truniger & Stojadinović 2015; Vassiliou, Tsiavos & Stojadinović 2013; Kikuchi, Black & Aiken 2008).

Nevertheless, until today, no single design framework covers the full-range risk-based design approach and structures presenting multiple behaviors. Our work aims to bring those concepts together into a unified risk- and performance-based seismic design framework.

This paper presents the theoretical basis of this new design framework, which are the equations of motion of the structure under design, and the conditions to initiate different behaviors. Those equations are the basis to form surrogate models, both single and two-degrees-of-freedom (2DOF) systems that represent the dynamics of the designed structure. We end by presenting a design example in which we use the risk-based approach and the proposed 2DOF surrogate models.

2. Methodology

We propose a set of surrogate models and transition equations to perform seismic design accounting for multiple behaviors. The structure under design is a multiple-degrees-of-freedom system (MDOF) with *N* lumped masses m_i, assembled into a diagonal mass matrix $\boldsymbol{m_s}$. The structure is fixed to a rigid base plate of mass m_0 and width *2B*. Each structural mass is associated with a horizontal degree of freedom u_i, that is parallel to the base plate and relative to its vertical centerline. The vector of displacements u_i is named $\boldsymbol{u_s}$. The height of each structure's mass relative to the base is described by the vector of heights $\boldsymbol{\iota_h}$. The structure stiffness is described by the elastic stiffness matrix $\boldsymbol{k_s}$. The structure's damping matrix $\boldsymbol{c_s}$ is assembled by superposition of modal damping (Chopra 2017, p. 445) with a fixed damping ratio for all modes (ζ). The system moves in its own plane and is excited by a horizontal ground motion ($\ddot{u}_g$).

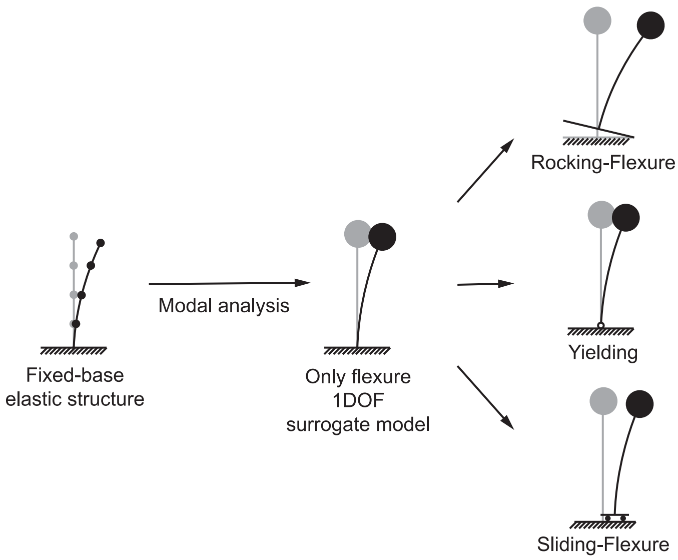

Figure 1. Multiple-behavior surrogates for seismic design and transitions among them.

Initially, the response of this MDOF structure to the horizontal excitation is elastic bending. We use the response of a single degree of freedom (SDOF) surrogate to estimate the response of the MDOF structure. But, as soon as a trigger condition for non-linear behavior is met, one of three different additional responses can be expected: rocking, yielding, or sliding (Figure 1). In each case, a two-behavior surrogate is needed to estimate the response of the MDOF structure. In the following, we present the equations of motion for such 2DOF surrogate models.

2.1 Equations of motion of the fixed-base structure and its SDOF surrogate

The fixed-base MDOF structure described previously has the equation of motion (Chopra 2017):

$$\boldsymbol{m_s}\ddot{\boldsymbol{u}}_{\boldsymbol{s}} + \boldsymbol{c_s}\dot{\boldsymbol{u}}_{\boldsymbol{s}} + \boldsymbol{k_s}\boldsymbol{u_s} = -\boldsymbol{m_s}\boldsymbol{\iota}\ddot{u}_g \tag{1}$$

where $\boldsymbol{\iota}$ is a column vector of ones. We apply modal analysis ($\boldsymbol{k_s\Phi} = \boldsymbol{m_s\Phi\Omega^2}$) to obtain the matrix of eigen vectors $\boldsymbol{\Phi} = [\boldsymbol{\phi_1\phi_2} \dots \boldsymbol{\phi_n}]$ and the matrix of natural frequencies squared $\boldsymbol{\Omega^2} = diag(\omega_1^2, \omega_2^2, \dots, \omega_n^2)$. We pre-multiply Equation 1 by $(\boldsymbol{\Phi^T m_s \Phi})^{-1}\boldsymbol{\Phi^T}$ and apply the modal coordinate change $\boldsymbol{u_s} = \boldsymbol{\Phi q}$. The result is:

$$\ddot{\boldsymbol{q}}_{\boldsymbol{s}} + 2\zeta\boldsymbol{\Omega}\dot{\boldsymbol{q}}_{\boldsymbol{s}} + \boldsymbol{\Omega^2 q_s} = -\boldsymbol{\Gamma}\ddot{u}_g \tag{2}$$

where $\boldsymbol{\Gamma} = (\boldsymbol{\Phi}^T\boldsymbol{m_s\Phi})^{-1}(\boldsymbol{\Phi}^T\boldsymbol{m_s\iota})$. As $\boldsymbol{\Omega}$ is a diagonal matrix, the set of *N* equations contained in Equation 2 are independent. To build a surrogate model, we consider only the first mode coordinate and apply a coordinate scaling $\mathrm{q}_1 = \Gamma_1 \mathrm{D}_1$, resulting in:

$$\ddot{D}_1 + 2\zeta\omega_1\dot{D}_1 + \omega_1^2 D_1 = -\ddot{u}_g \tag{3}$$

2.2 Transition to multiple behavior

For the fixed-base elastic MDOF structure, the condition to start rocking is:

$$\mp\boldsymbol{\iota}_{\boldsymbol{h}}^{\boldsymbol{T}}\boldsymbol{m_s}(\ddot{\boldsymbol{u}}_{\boldsymbol{s}} + \boldsymbol{\iota}\ddot{u}_g) > \boldsymbol{\iota^T m_s}(\boldsymbol{\iota}B \mp \boldsymbol{u_s})g + m_0 gB \tag{4}$$

where g is the gravity, and the upper and lower signs indicate rocking around right and left pivot respectively. We account for only the first mode degree of freedom to obtain the following condition for the surrogate to start rocking:

$$\mp(m_1^* h_1^* \ddot{D}_1 + L_0^r \ddot{u}_g) > m_{tot} gB \mp m_1^* g D_1 \tag{5}$$

where $L_0^r = \sum_{i=0}^N m_i h_i$, $m_{tot} = \sum_{i=0}^N m_i$, $m_1^* = \left(\sum_{i=1}^N m_i \phi_{i,1}\right)^2 / \sum_{i=1}^N m_i \phi_{i,1}^2$, and $h_1^* = \sum_{i=1}^N m_i h_i \phi_{i,1} / \sum_{i=1}^N m_i \phi_{i,1}$.

In parallel, the simplified condition for the fixed-base MDOF structure to start yielding is:

$$\boldsymbol{\iota^T k_s u_s} > V_{b,y} \tag{6}$$

where $V_{b,y}$ is the base shear at yield, which can be determined by a modal pushover analysis (Chopra & Goel 2002). The condition for the surrogate model to start yielding is:

$$\omega_1^2 D_1 > V_{b,y} / m_1^* \tag{7}$$

The translation of the pushover curve from MDOF structure's parameters to SDOF system's parameters is explained in detail in the work of Chopra & Goel (2002).

Lastly, also in parallel, the condition for the MDOF structure to start sliding is:

$$\boldsymbol{\iota^T m_s}(\ddot{\boldsymbol{u}}_{\boldsymbol{s}} + \boldsymbol{\iota}\ddot{u}_g) + m_0\ddot{u}_g > m_{tot}\mu_f g \tag{8}$$

where μ_f is the base friction coefficient. We account only the first mode to obtain the condition for the surrogate model to start sliding:

$$m_1^*\ddot{D}_1 + m_{tot}\ddot{u}_g > m_{tot}\mu_f g \tag{9}$$

2.3 2DOF rocking-flexure surrogate equations of motion

The equations of motion of a MDOF structure that rocks on its base are presented by Acikgoz & DeJong (2016). The derivation of those equations and the conversion to a 2DOF surrogate model are presented in Silva & Stojadinović (2023) in the same notation as herein. Also, Silva & Stojadinović (2023) analyze different assumptions for the impact model and possible simplifications to the surrogate model. For completeness and brevity, we re-write the simplified equations of motion of the 2DOF surrogate model of a structure that rocks on its base:

$$I_\theta \ddot{\theta} + m_1^* h_1^* \ddot{D}_1 - L_0^r g\theta \pm M_r = -L_0^r \ddot{u}_g \tag{10a}$$

$$h_1^* \ddot{\theta} + \ddot{D}_1 + 2\zeta\omega_1 \dot{D}_1 + \omega_1^2 D_1 = -\ddot{u}_g \tag{10b}$$

where $I_\theta = \sum_{i=0}^{N} m_i (h_i^2 + B^2)$ and $M_r = m_{tot} g B$. In Equation 10, it is assumed small angles and small displacements, therefore $sin\,\theta \approx \theta$, $cos\,\theta \approx 1$, $B - D_1 \approx B$ and the Coriolis forces are neglected.

2.4 Yielding SDOF surrogate equation of motion

As soon as the condition of Equation 6 is met, the structure yields and its equation of motion is non-linear. The elastic modal shapes cease to be orthogonal to the now non-linear stiffness matrix $\boldsymbol{k_s}$ which creates modal coupling. Nevertheless, such modal coupling is weak for regular fixed-base buildings behaving inelastically (Chopra 2017; Chopra & Goel 2002) and the modal pushover procedure can be applied to define a first-mode inelastic SDOF surrogate model. The equation of motion of this model is:

$$\ddot{D}_1 + 2\zeta\omega_1 \dot{D}_1 + f(D_1, D_y) = -\ddot{u}_g \tag{11}$$

where $f(D_1, D_y)$ is the inelastic force-displacement function (Chopra 2017, p. 822).

2.5 2DOF sliding-flexure surrogate equations of motion

The equation of motion of the 2DOF surrogate model that slides on its base is (Naem & Kelly 1999, p. 31-34):

$$(m_1^*/m_{tot})\ddot{D}_1 + \ddot{u}_0 + f_b(u_0, \dot{u}_0, \mu_f, u_{y,b}, T_b, \zeta_b) = -\ddot{u}_g \tag{12a}$$

$$\ddot{u}_0 + \ddot{D}_1 + 2\zeta\omega_1 \dot{D}_1 + \omega_1^2 D_1 = -\ddot{u}_g \tag{12b}$$

where $f_b(u_0, \dot{u}_0, \mu_f, u_{y,b}, T_b, \zeta_b)$ is the force in the isolation system, that can be described by a yield function (e.g., Sayani & Ryan 2009) or by a Bouc-Wen model (e.g., Vassiliou, Tsiavos & Stojadinović 2013).

3. Preliminary seismic design example

We want to design a 3-story hospital building in San Jose (California) at a site type D (hazard curves are shown in Figure 2). The building structural system is a reinforced concrete (RC) frame, either as fixed-base or seismically isolated using single friction pendulum sliding bearings. The main performance objective is: the mean annual frequency (MAF) of the roof exceeding the yield displacement is 0.002 (1/500). If the building is base-isolated, an additional performance objective is: the MAF of the base isolation reaching its maximum displacement is 0.0004 (1/2500).

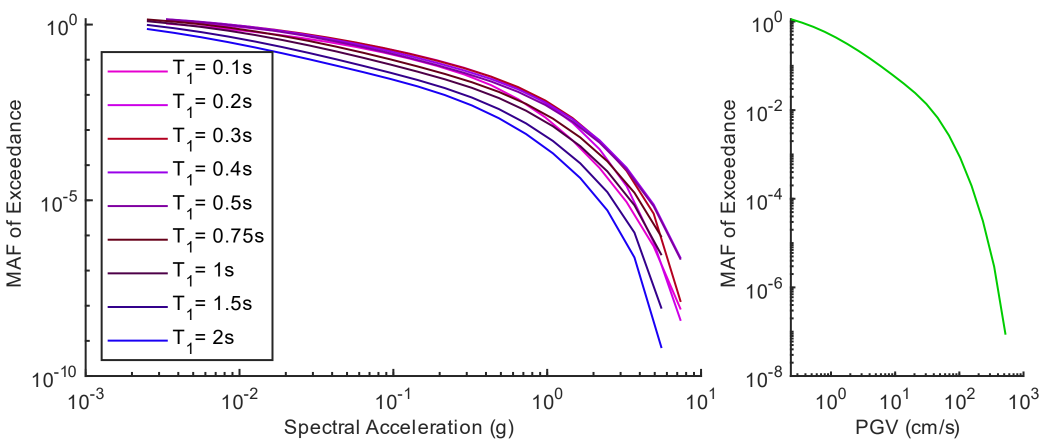

Figure 2. Hazard Curves for San Jose, California at a site type D. Data from the 2018 US National Seismic Hazard Model (Petersen et al. 2020).

3.1 Seismic design tools

The seismic design is displacement based and it follows the premise that the yield displacement is a stable parameter during the design iterations (Priestley 2000; Aschheim 2002; Aschheim, Hernandez-Montes & Vamvatsikos 2019; Silva, Tsiavos & Stojadinović 2023). A good initial guess for the yield displacement of an

RC frame is 0.55% of its height (Hernández-Montes & Aschheim 2019). Therefore, the yield displacement of the surrogate is: $D_y^* = N_{story} \cdot h_{story} \cdot 0.0055/\Gamma_1 = 3 \cdot 3.2 \cdot 0.0055/1.24 = 0.0426m$.

The design of the fixed-base inelastic structure is performed with the Yield Frequency Spectra (Vamvatsikos & Aschheim 2016). We compute the YFS with the hazard curves for San Jose (Figure 2) and the probabilistic estimation of inelastic displacement ratios from Ruiz-Garcia and Miranda (2007). The resultant YFS are in Figure 3 with the performance objective marked with a red X.

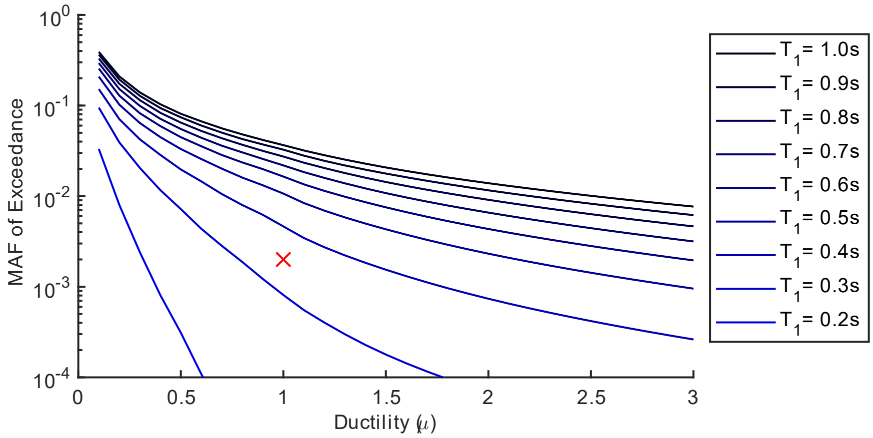

Figure 3. Yield Frequency Spectra for San Jose (site type D), California, for a SDOF yield displacement of 0.0426m.

To design the base isolated building, we use the 2DOF surrogate model described in Section 2.5 with a Bouc-Wen spring for the base isolation force-displacement function (f_b). Equation 12 becomes:

$$(m_1^*/m_{tot})\ddot{D}_1 + \ddot{u}_0 + 2\omega_b\zeta_b\dot{u}_0 + \omega_b^2 u_0 + \left((\omega_b^2/\alpha_{BW}) - \omega_b^2\right)u_{y,b}z_0 = -\ddot{u}_g \quad (13a)$$

$$\ddot{u}_0 + \ddot{D}_1 + 2\zeta\omega_1\dot{D}_1 + \omega_1^2 D_1 = -\ddot{u}_g \quad (13b)$$

$$\dot{z}_0 u_{y,b} = \dot{u}_0 - \gamma_{BW}\,|\dot{u}_0|\,z_0\,|z_0|^{n_{BW}-1} - \beta_{BW}\dot{u}_0\,|z_0|^{n_{BW}} \quad (13c)$$

where $\omega_b = \sqrt{\alpha_{BW}k_i/m_{tot}}$ is the base nominal frequency, $k_i = m_{tot}g\mu_f/u_{y,b}$ is the initial stiffness, α_{BW} is the post-yield stiffness ratio, $u_{y,b}$ is the base's yield displacement, ζ_b is the base's damping ratio and n_{BW}, γ_{BW}, and β_{BW} are the Bouc-Wen model's parameters. We choose a single friction pendulum (SFP) isolation system, therefore we adopt: $u_{y,b} = 0.0005m$, $\zeta_b = 0$, $\gamma_{BW} = 0.5$, $\beta_{BW} = 0.5$, $n_{BW} = 8$, $\alpha_{BW} = m_{tot}\,\omega_b^2 / k_i$. For the 3-story frame building, we estimate $m_1^*/m_{tot} = 0.6$, and we fix the structure's damping ratio $\zeta = 0.02$. We run the non-linear dynamic analysis (solution of Equation 13) for multiple options of structural period ($T_1 = [0.1\ 0.2\ 0.3\ 0.4\ 0.5\ 0.6\ 0.7\ 0.8\ 0.9\ 1.0]s$), base period ($T_b = 2\pi/\omega_b = [3\ 3.5\ 4\ 4.5\ 5\ 5.5\ 6.0]s$), friction coefficient ($\mu_f = [0.03\ 0.04\ 0.05\ 0.06\ 0.07\ 0.08\ 0.09\ 0.1\ 0.14\ 0.18]$), and for a ground motion set of 105 earthquake records scaled by 0.5, 1.0, 2.0, 3.0, 4.0. Those records are classified as ordinary, with no-pulse neither long-duration characteristics (Kazantzi, Lachanas & Vamvatsikos 2021; Reggiani Manzo et al. 2022). For each 2DOF system we perform two cloud analysis: one for $max(|D_1|)$ and $S_d(T_1)$ (spectral displacement) and other for $max(|u_0|)$ and PGV (peak ground velocity). Based on the cloud analysis, we obtain the parameters c_i and β of the following equations:

$$ln(max(|D_1|)) = c_1 + c_2\,ln\left(S_d(T_1)\right) + \varepsilon\beta_{ln(D1)} \quad (14a)$$

$$ln(max(|u_0|)) = c_3 + c_4\,ln(PGV) + \varepsilon\beta_{ln(u0)} \quad (14b)$$

where β is the dispersion and $\varepsilon \sim N(0,1)$. With the cloud linear models, we build fragility functions (Shome et al. 1998; Jalayer 2003). In the following step, the MAF of exceeding different performance parameters is computed by integrating the product of the respective fragility function with the derivative of the hazard curve. The whole procedure is illustrated in Figure 4.

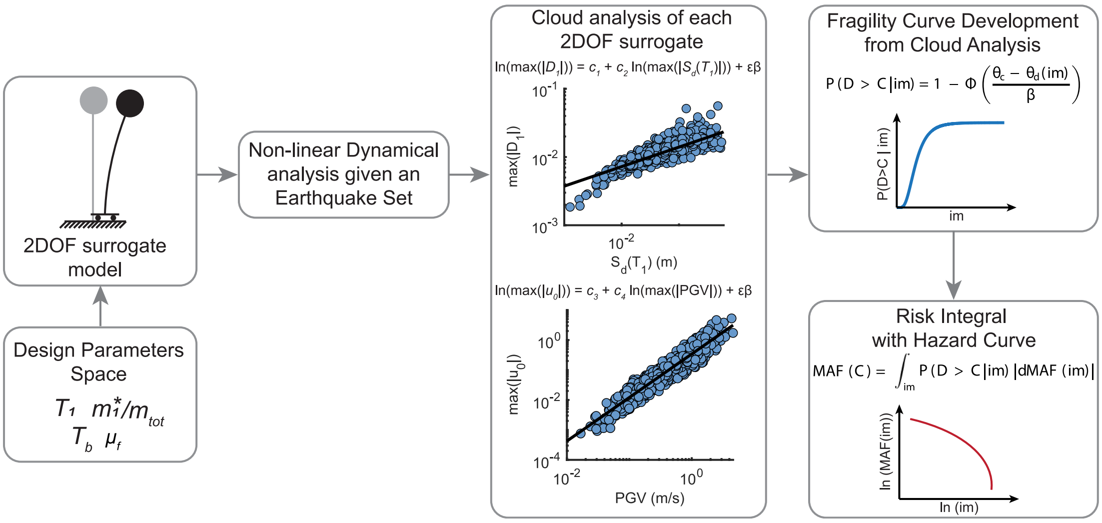

Figure 4. 2DOF surrogate model use for risk-based seismic design.

3.2 Preliminary design decision

The possible design solutions for a fixed-base building, according to Figure 3, require T_1 ~ 0.3s. Conversely, the minimum spectral acceleration at yield is $C_y^* = u_y^* (2\pi/T_1)^2/g = 1.9g$. Therefore, the building lateral load resisting system should be able to carry about 2 times its total weight.

For the base-isolated structure there are many possible design solutions as there are many parameters under design. The first step is to choose the base isolation size. We choose the SFP size of 1m, therefore the base isolation performance objective is MAF(u_0>0.5m) = 1/2500. From the 700 combinations of design parameters (T_1, T_b, μ_f), only 290 satisfy the aforementioned performance objective. For those, we plot the MAF(D_y^*) in Figure 4.

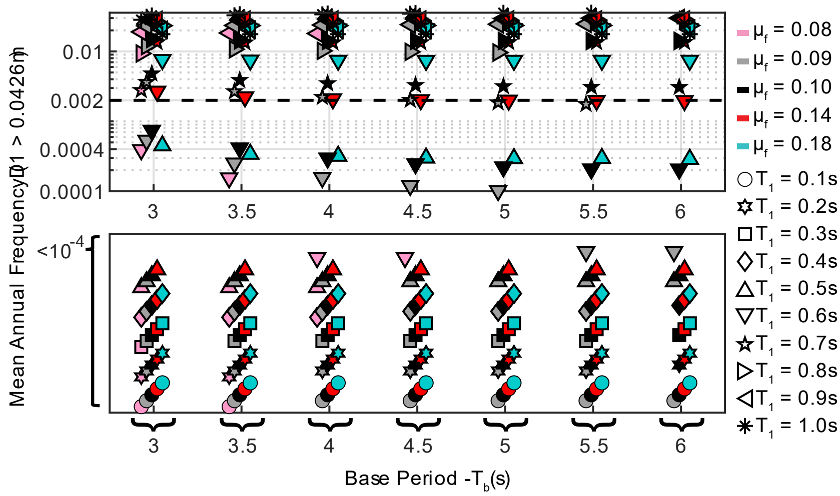

Figure 4. Mean Annual Frequency of D_1>0.0426m for different structural periods (T_1), base friction coefficient (μ_f) and base period (T_b). Given by the 2DOF surrogate model and a site type D in San Jose (California).

In Figure 4, the friction coefficient must be high to accommodate the base displacement restriction (i.e., $\mu_f > 0.08$). Moreover, the highest period that satisfies $MAF(D_y^*) < 0.002$ is T_1 = 0.7s. This is already an improvement since $C_y^* = 0.35g$ in this case (5.7 times lower than the fixed-base building). A good design candidate is: T_1 = 0.6s, μ_f = 0.08, and T_b = [3.0 – 4.5]s. In Figure 4, this design option is represented by a pink downward-pointing triangle. This solution is very attractive given the $MAF(D_y^*) < 0.0004$, i.e., the superstructure remains elastic for a 2500-year return period event, which is ideal for a hospital building.

3.3. Discussion

It is still possible to optimize the preliminary design options with the tools we used in this design example. For the fixed-base building, one can find which T_1 between 0.3s and 0.4s satisfies the performance objective. For the base-isolated building, one can increase the size of the isolator to possibly reduce the friction coefficient, which would help to reduce the demand on the superstructure and lead to a more economical design of the superstructure.

In addition, there are further applications of the proposed tools. With the use of surrogate models for different behavior structures, it is possible to compare, in the preliminary design phase, the seismic risk and performance of two different structural systems (e.g., Sayani & Ryan 2009). With some further steps, it is possible to create a simplified life cycle assessment of the design options (Terzic, Merrfield & Mahin 2012) to convince shareholders to adopt a long-term less risky structure. That is made possible using multi-behavior surrogates in only a few steps, but bearing in mind the trade-off between accuracy and simplicity.

Lastly, we can develop probabilistic seismic design equations with the 2DOF surrogate models to improve the current design practice (e.g., for SDOF, Ruiz-Garcia & Miranda 2007; Vamvatsikos & Cornell 2005, O'Reilly et al. 2022; Kazantzi, Lachanas & Vamvatsikos 2021). Those equations are part of the toolset necessary to move toward fully risk- and performance-based seismic design.

4. Conclusion

This paper introduces a new seismic design method for structures presenting multiple behaviors based on two-degrees-of-freedom (2DOF) surrogate models. We show how a multiple-degrees-of-freedom (MDOF) structure that may present different simultaneous or consecutive behavior modes during its seismic response can be represented by different surrogate models and we introduce the conditions for transition between different behaviors and models. The surrogates are based on modal analysis and simplifications of the equations of motion. We use the proposed surrogates in a seismic design example to compare two different design solutions: a fixed-base yielding structure and a sliding base-isolated structure. The surrogate simplicity allows us to estimate the seismic performance of both structures (i.e., the mean annual frequency of exceeding capacity parameters) in the preliminary design phase. Therefore, the proposed design method is risk-based. The main contribution of this work is to bring together different design methods and models to a unified framework that comprises many possible behaviors and novel risk-based preliminary seismic design techniques.

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