# An automated surrogate modelling approach for the uncertainty quantification of dynamical systems

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**Author(s):**
Schär, Styfen; Marelli, Stefano; Sudret, Bruno

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# An automated surrogate modelling approach for the uncertainty quantification of dynamical systems

S. Schär, S. Marelli and B. Sudret
ETH Zürich, Chair of Risk, Safety and Uncertainty Quantification, styfen.schaer@ibk.baug.ethz.ch

## 1 Problem Statement

**The challenge**: Build a surrogate $\tilde{\mathcal{M}}$ that emulates the response of a complex time-dependent system $\mathcal{M}$ over extended time periods:

$$y(t) = \mathcal{M}(\boldsymbol{x}(\mathcal{T} \leq t)) \approx \tilde{\mathcal{M}}(\boldsymbol{x}(\mathcal{T} \leq t))$$

- Discretized time axis $\mathcal{T} = \{0, \delta t, 2\delta t, \ldots, (N-1)\delta t\}$
- System response $y : \mathcal{T} \to \mathbb{R}$
- Possibly high-dimensional exogenous excitation $\boldsymbol{x} : \mathcal{T} \to \mathbb{R}^M$

**Our approach**: Automated incremental construction of an exogenous input manifold suitable for the construction of autoregressive surrogates

## 2 Autoregressive Modelling

Nonlinear AutoRegressive with eXogenous input (NARX) models take advantage of the temporal coherence of the output and exogenous inputs:

$$y(t) = \tilde{\mathcal{M}}(\boldsymbol{\varphi}(t), \mathbf{c})$$

$$\begin{aligned}\boldsymbol{\varphi}(t) = \{&y(t-\ell_1^y), \ldots, y(t-\ell_{n_y}^y),\\ &x_1(t-\ell_1^{x_1}), \ldots, x_1(t-\ell_{n_{x_1}}^{x_1}), \ldots,\\ &x_{M_x}(t-\ell_1^{x_{M_x}}), \ldots, x_{M_x}(t-\ell_{n_{x_{M_x}}}^{x_{M_x}})\}\end{aligned}$$

- Model parameters $\boldsymbol{c}$
- $\ell_i^y \in \{\delta t, \ldots, (N-1)\delta t\}$
- $\ell_i^{x_j} \in \{0, \delta t, \ldots, (N-1)\delta t\}$

## 3 Manifold NARX

**Exogenous input manifold**

$$\tilde{\mathcal{M}} : \boldsymbol{\zeta}(\mathcal{T} \leq t) \to y(t)$$

- Manifold $\boldsymbol{\zeta}$ enables simpler mapping to $\boldsymbol{y}$
- Incorporates prior knowledge about the system $\mathcal{M}$
- Consists of auxiliary quantities $\boldsymbol{z}_i$

**Auxiliary quantities**

$$\begin{aligned}\boldsymbol{z}_1(t) &= \mathcal{F}_1(\boldsymbol{x}(\mathcal{T} \leq t), \boldsymbol{z}_1(\mathcal{T} < t))\\ \boldsymbol{z}_2(t) &= \mathcal{F}_2(\boldsymbol{x}(\mathcal{T} \leq t), \boldsymbol{z}_1(\mathcal{T} \leq t), \boldsymbol{z}_2(\mathcal{T} < t))\\ &\vdots\\ \boldsymbol{z}_i(t) &= \mathcal{F}_i(\boldsymbol{x}(\mathcal{T} \leq t), \boldsymbol{z}_1(\mathcal{T} \leq t), \ldots, \boldsymbol{z}_{i-1}(\mathcal{T} \leq t), \boldsymbol{z}_i(\mathcal{T} < t))\end{aligned}$$

- Transform $\mathcal{F}_i$ can be an ARX model
- Auxiliary quantities can depend on each other
- Examples are control system outputs, moving averages, integrals or derivates

**Automatic recursive selection of auxiliary quantities**

**Function** SelectFeatures$(\tilde{\boldsymbol{x}}, \boldsymbol{z}, \boldsymbol{y})$
  $\boldsymbol{\zeta} \leftarrow \{\}, \tilde{\boldsymbol{y}} \leftarrow \boldsymbol{y}$
  **while** $\tilde{\boldsymbol{x}} \neq \{\}$ *or* $\boldsymbol{z} \neq \{\}$ **do**
    $\rho \leftarrow$ Correlate $(\{\tilde{\boldsymbol{x}}, \boldsymbol{z}\}, \tilde{\boldsymbol{y}})$
    **if** $\max(|\rho|) < \theta$ **then**
      break
    $\zeta_i \leftarrow \arg\max(|\rho|)$
    **if** $\zeta_i \in \tilde{\boldsymbol{x}}$ **then**
      $\boldsymbol{\zeta} \leftarrow \{\boldsymbol{\zeta}, \zeta_i\}$
      $\tilde{\boldsymbol{x}} \leftarrow \tilde{\boldsymbol{x}} \backslash \zeta_i$
    **else if** $\zeta_i \in \boldsymbol{z}$ **then**
      $\boldsymbol{\zeta}' \leftarrow$ SelectFeatures $(\{\tilde{\boldsymbol{x}}, \boldsymbol{\zeta}\}, \boldsymbol{z} \backslash \zeta_i, \zeta_i)$
      $\tilde{\boldsymbol{x}} \leftarrow \{\tilde{\boldsymbol{x}}, \boldsymbol{z} \cap \boldsymbol{\zeta}'\}$
      $\boldsymbol{z} \leftarrow \boldsymbol{z} \backslash \boldsymbol{z} \cap \boldsymbol{\zeta}'$
      $\boldsymbol{\zeta} \leftarrow \{\boldsymbol{\zeta}, \zeta_i\}$
      $\boldsymbol{z} \leftarrow \boldsymbol{z} \backslash \zeta_i$
    $\tilde{\boldsymbol{y}} \leftarrow \boldsymbol{y} - \tilde{\mathcal{M}}(\boldsymbol{\zeta})$
  **return** $\boldsymbol{\zeta}$

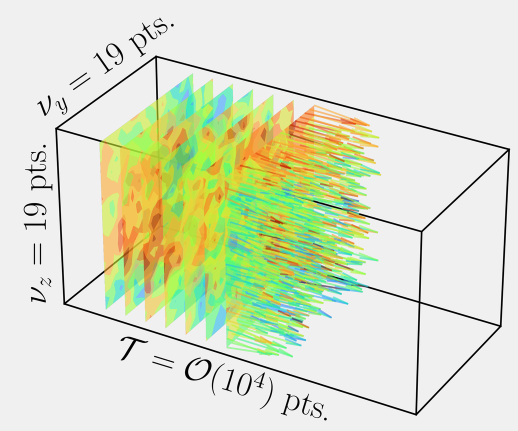
**Function** Correlate$(\boldsymbol{x}, \boldsymbol{y})$
  $\rho \leftarrow \{\}$
  **for** $\boldsymbol{x}_i \leftarrow \boldsymbol{x}_1, \ldots, \boldsymbol{x}_M$ **do**
    $\rho \leftarrow \{\rho, \text{KendallTau}(\boldsymbol{x}_i, \boldsymbol{y})\}$
  **return** $\rho$

- $\tilde{\boldsymbol{x}}$: available features
- $\boldsymbol{z}$: yet unavailable features
- $\tilde{\boldsymbol{y}}$: residuals
- $\rho$: measure of association
- $\theta$: critical correlation value
- $\tilde{\mathcal{M}}$: proxy surrogate

## 4 Case study

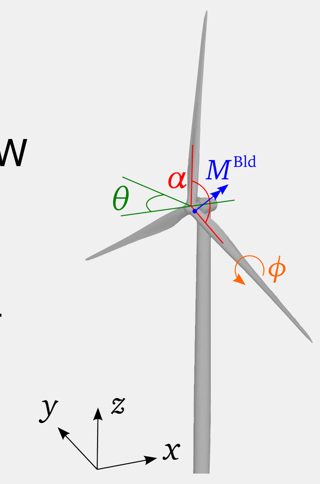
Complex **onshore wind turbine simulator** with control systems

- High-dimensional turbulent wind input: $\boldsymbol{v} : \mathcal{T} \to \mathbb{R}^{3 \times \nu_y \times \nu_z}$
- Quantity of interest: Flapwise blade root bending moment $M^{\text{Bld}} : \mathcal{T} \to \mathbb{R}$



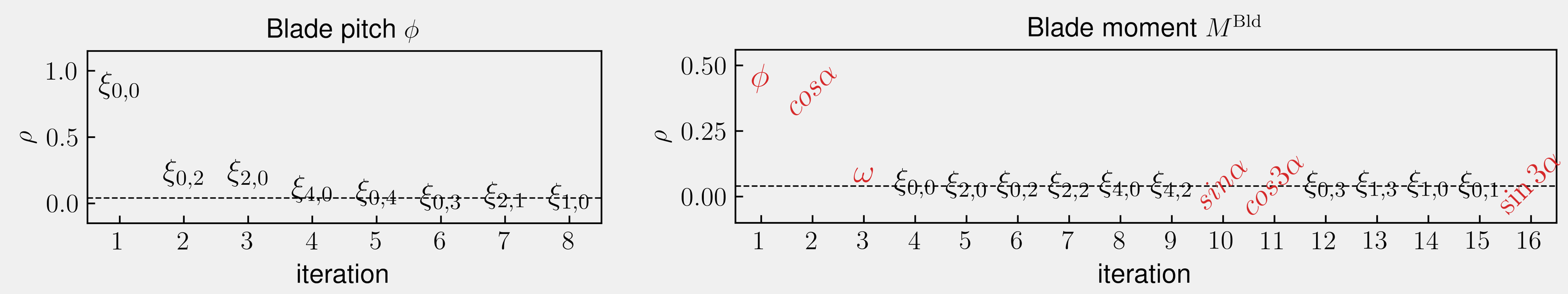

**Computational model**

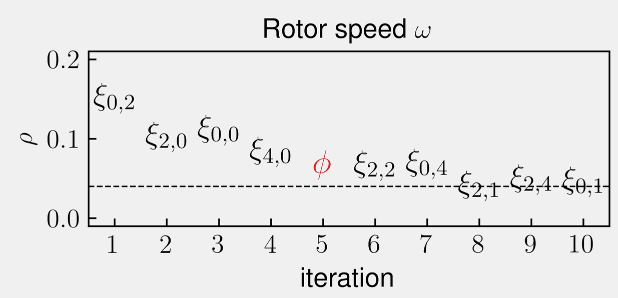
| | |
|---|---|
| Turbine | NREL 5-MW |
| Type | Onshore |
| Controller | ROSCO |
| Simulator | OpenFAST |



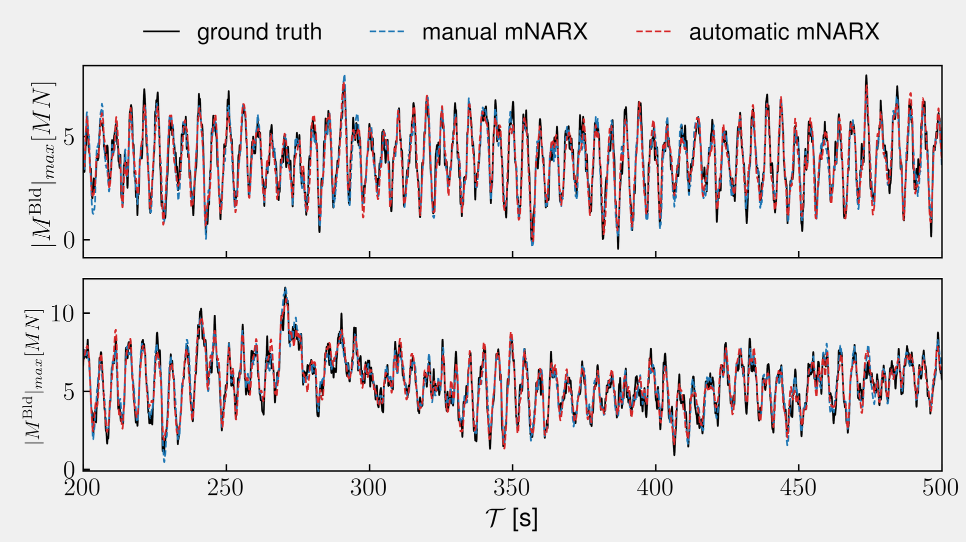

- Spatial compression of $\boldsymbol{v}$ into spectral coefficients $\boldsymbol{\xi}$

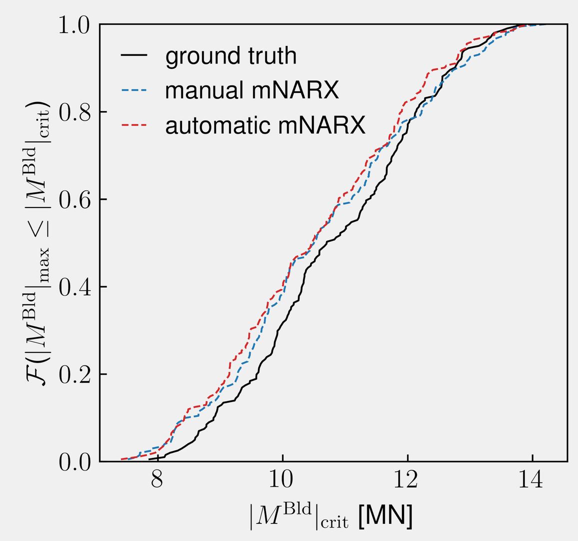
## 5 Results




- Sequence of selected features
- Black: Depend only on exogenous input
- Red: Constructed using ARX model




- (Left) Example traces: Simulated blade moment and the emulated ones using the manual and automatic mNARX surrogates
- (Right) CDF from 300 10-minute simulations and the corresponding emulated data using the manual and automatic mNARX surrogates

## 6 Discussion and Outlook

**Discussion**

- Multistep approach allows accurate emulation of complex dynamical systems
- Relevant features can be automatically selected using a measure of association
- Automatic selection reduces manual effort to construct an mNARX surrogate

**Outlook**

- Investigate the effect of the stopping criteria and measures of association
- Investigate the importance of the proxy surrogate to the final solution

### References

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