mNARX - A novel surrogate model for the uncertainty quantification of dynamical systems

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Risk, Safety & Uncertainty Quantification

mNARX - A novel surrogate model for the uncertainty quantification of dynamical systems

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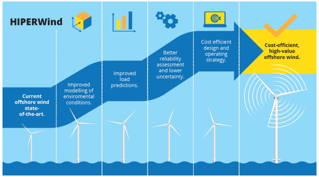
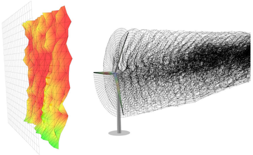
Motivation

Uncertainty quantification of (offshore) wind turbines

- Design for ultimate and fatigue limit state
- Subject to wind loads with high aleatory uncertainty
- Responses are time series



https://www.hiperwind.eu



Modified from Perez-Becker at al. (2021). Energies 14(3):783.

Many runs of computationally expensive simulators required.
Need a fast surrogate!

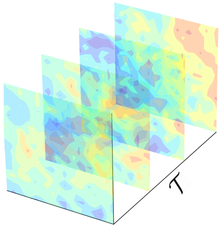
Surrogate modelling for dynamical systems

Setup

- Computational model $\mathcal{M}$ with time-dependent exogenous input $\boldsymbol{x}$ and output $\boldsymbol{y}$:

$$\boldsymbol{x} : \mathcal{T} \to \mathbb{R}^M, \boldsymbol{y} : \mathcal{T} \to \mathbb{R}$$

- Discrete time axis $\mathcal{T} = \{0, \delta t, 2\delta t, \ldots, N\delta t\}$

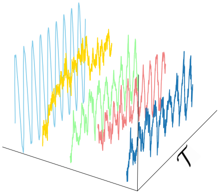


Objective

- Replace computational model with a fast-to-evaluate surrogate $\hat{\mathcal{M}}$

$$y(t) = \mathcal{M}(\boldsymbol{x}(\mathcal{T} \leq t)) \approx \tilde{\mathcal{M}}(\boldsymbol{x}(\mathcal{T} \leq t))$$

- Surrogate is built on a limited number of model runs ($\approx \mathcal{O}(10^2)$)



Challenge

- High-dimensional input
- Highly nonlinear and non-smooth response

Multistep surrogate modelling

Rationale

- Using the original input can result in a complex nonlinear problem
- Constructing the surrogate on a more informative manifold $\zeta \in \mathbb{R}^{N \times M_\zeta}$ can simplify the problem:

$$\tilde{\mathcal{M}} : \zeta(\mathcal{T} \leq t) \rightarrow y(t) \text{ where } \zeta = \mathcal{F}(\boldsymbol{x})$$

We propose

Manifold Nonlinear AutoRegressive with eXogenous input (mNARX) modelling - A multistep surrogate modelling approach

1) Input preprocessing	2) Manifold construction	3) Surrogate training
▶ Dealing with high dimensionality in $\boldsymbol{x}$ ▶ Upsampling, scaling, etc.	▶ Incremental process ▶ Incorporate prior knowledge of the system	▶ Built on the manifold ▶ Use of autoregressive surrogate

S. Schär et al. (2023). Emulating the dynamics of complex systems using autoregressive models on manifolds (mNARX)

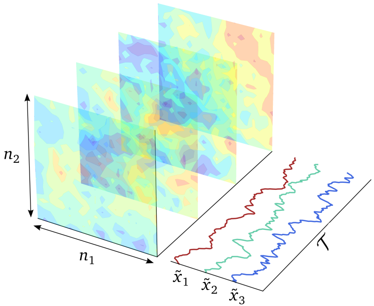
Input preprocessing

Reduce dimensionality of the system excitation $\boldsymbol{x}$ along non-temporal coordinates:

$$\tilde{\boldsymbol{x}} = \mathcal{G}(\boldsymbol{x})$$

where $\boldsymbol{x} \in \mathbb{R}^{N \times M}$ and $\tilde{\boldsymbol{x}} \in \mathbb{R}^{N \times m}$ such that $m \ll M$

- Original time scale $\mathcal{T}$ is preserved
- E.g. N-dimensional discrete cosine transform (DCT)
- Many more methods available

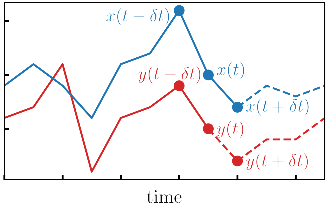


ARX modelling

AutoRegressive with eXogenous inputs (ARX) models predict new values of a time series based on

- Past values of the same series
- Current and past values of exogenous time series

$$\hat{y}(t+\delta t) = \tilde{\mathcal{M}}(\hat{y}(t), \hat{y}(t-\delta t), \dots), \boldsymbol{x}(t+\delta t), \boldsymbol{x}(t), \boldsymbol{x}(t-\delta t), \dots)$$



Polynomial nonlinear ARX models are well-established

S. A. Billings (2013). Nonlinear system identification

- Simple parametrization
- Training takes just a few seconds
- Very fast to evaluate

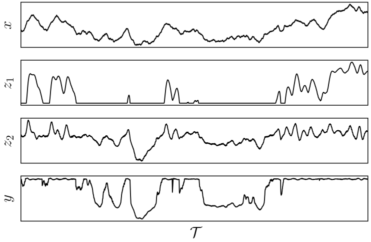
Manifold construction

- Manifold ζ includes of features z_i called auxiliary quantities:

$$\zeta = \{\boldsymbol{x}, \boldsymbol{z}_1, \ldots, \boldsymbol{z}_n\}$$

- Auxiliary quantities are constructed incrementally

$$\boldsymbol{z}_1(t) = \mathcal{F}_1(\boldsymbol{x}(\mathcal{T} \le t), \boldsymbol{z}_1(\mathcal{T} < t))$$
$$\boldsymbol{z}_2(t) = \mathcal{F}_2(\boldsymbol{z}_1(\mathcal{T} \le t), \boldsymbol{x}(\mathcal{T} \le t), \boldsymbol{z}_2(\mathcal{T} < t))$$
$$\vdots$$
$$\boldsymbol{z}_n(t) = \mathcal{F}_n(\boldsymbol{z}_1(\mathcal{T} \le t), \ldots, \boldsymbol{z}_{n-1}(\mathcal{T} \le t), \boldsymbol{x}(\mathcal{T} \le t), \boldsymbol{z}_n(\mathcal{T} < t))$$



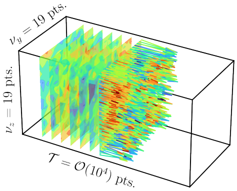
- Transform $\mathcal{F}$ can be an ARX model
- Auxiliary quantities can depend on each other
- E.g. control system outputs, moving averages or integrals/derivatives

Case study

Input turbulence box

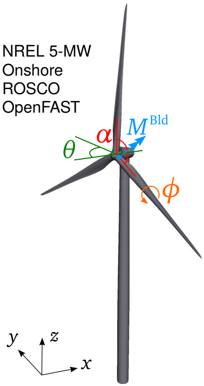
$$\boldsymbol{v} : \mathcal{T} \rightarrow \mathbb{R}^{\nu_w \times \nu_y \times \nu_z}$$

- *Movie* of wind speeds
- Wind speed components ν_w
- Discrete spatial grid $\nu_y \times \nu_z$



Computational model

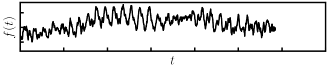
Turbine	NREL 5-MW
Type	Onshore
Controller	ROSCO
Simulator	OpenFAST



Quantity of interest

$$M^{\text{Bld}} : \mathcal{T} \rightarrow \mathbb{R}$$

- Flapwise blade root bending moment M^{Bld}
- Sensitive to blade pitch ϕ and azimuth α



mNARX for wind turbine simulations

1. Reduce turbulence box to low frequency spatial coefficients:

$$\boldsymbol{\xi}(t) = \text{DCT}(\boldsymbol{v}_x(t))$$

2. Build surrogate for blade pitch:

$$\hat{\phi}(t) = \tilde{\mathcal{M}}(\boldsymbol{\xi}(\mathcal{T} \leq t), \hat{\phi}(\mathcal{T} < t))$$

3. Build surrogate for rotor speed:

$$\hat{\omega}(t) = \tilde{\mathcal{M}}(\boldsymbol{\xi}(\mathcal{T} \leq t), \hat{\phi}(\mathcal{T} \leq t), \hat{\omega}(\mathcal{T} < t))$$

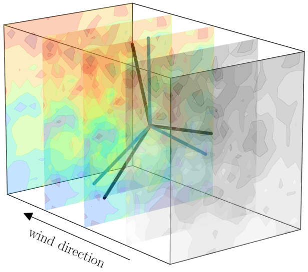
4. Reconstruct rotor azimuth α and its harmonics h:

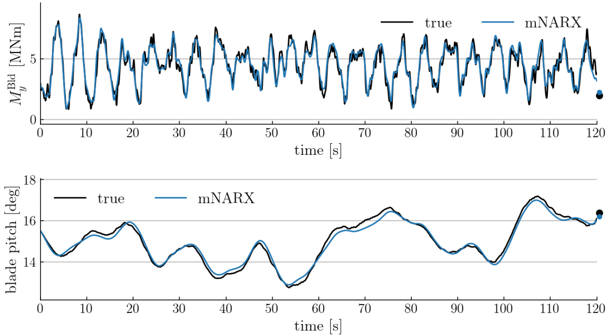
$$\hat{\alpha}(t) = \int_0^t \hat{\omega}(\tau)\mathrm{d}\tau \text{ and } \hat{\boldsymbol{h}} = \{\cos(\hat{\boldsymbol{\alpha}}), \ldots, \cos(4\hat{\boldsymbol{\alpha}}), \ldots, \sin(\hat{\boldsymbol{\alpha}}), \ldots, \sin(4\hat{\boldsymbol{\alpha}})\}$$

5. Build final surrogate:

$$\hat{M}^{\text{Bld}}(t) = \tilde{\mathcal{M}}(\boldsymbol{\xi}(\mathcal{T} \leq t), \hat{\phi}(\mathcal{T} \leq t), \hat{\boldsymbol{h}}(\mathcal{T} \leq t), \hat{M}^{\text{Bld}}(\mathcal{T} < t))$$

Final surrogate performance





Summary & Conclusion

- Surrogating complex dynamical systems following a multistep approach can be beneficial since
  - even simple model structures can yield accurate results
  - it allows to incorporate prior knowledge
  - it requires relatively few data ($\mathcal{O}(10^2)$ 10-min simulations used)
- The surrogate provides stable predictions in time (no drift) over a wide range of operating conditions and long time horizons
- Huge speedup: $\mathcal{O}(10^4)$ faster than the simulator
- mNARX is a universal algorithm that is not restricted to wind turbine simulations
- Automating construction of mNARX surrogate is work in progress

Chair of Risk, Safety & Uncertainty Quantification

www.rsuq.ethz.ch

Read the preprint

Read the EU report

The Uncertainty Quantification Software

www.uqlab.com

www.uqpylab.uq-cloud.io

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