ETH zürich

ETH Library

Fighting Free with Free: Freemium vs. Piracy

Working Paper

Author(s):
Dubus, Antoine; Halmenschlager, Christine; Waelbroeck, Patrick

Publication date:
2023-03-02

Permanent link:
https://doi.org/10.3929/ethz-b-000601226

Rights / license:
In Copyright - Non-Commercial Use Permitted

This page was generated automatically upon download from the ETH Zurich Research Collection.
For more information, please consult the Terms of use.

Fighting Free with Free: Freemium vs. Piracy*

Antoine Dubus[†] Christine Halmenschlager[‡] and Patrick Waelbroeck[§]

March 2, 2023

Abstract

In this article, we show how freemium business models can deter piracy. We analyze a simple freemium model in which a firm offers both a free version and a premium version. The firm can restrict the use of the free version. Consumers can choose between the free and the premium version, but can also get an illegal digital copy. More restrictions can increase the number of premium users but divert other users to piracy. On the contrary, fewer restrictions deter online piracy. We show that with a low level of piracy, the firm sets a high level of restrictions on the free version, which makes the traditional premium business model more profitable than the freemium model. We therefore challenge the idea that strong copyright laws are necessary to protect digital markets. We argue that there are market solutions to fight free with free that better segment consumer audiences according to their willingness to pay for digital music.

Keywords: Online piracy, versioning, freemium, streaming, copyright, music.

JEL Classification: L12, L82, L86, O34.

---

*We thank Paul Belleflamme, Joan Calzada, Vincenzo Denicolò, and Yassine Lefouili for their useful comments. We also thank seminar participants at Université Catholique de Louvain, and participants at the EARIE and EPIP conferences.

[†]Department of Management, Technology and Economics, ETH Zürich Leonhardstrasse 21 Switzerland – 8092 Zürich, Switzerland; antoine1dubus@gmail.com.

[‡]CRED, Université Paris-Panthéon-Assas, 12 place du Panthéon, 75230 Paris Cedex 05, France; Christine.Halmenschlager@u-paris2.fr.

[§]i3, Telecom Paris, Institut Polytechnique de Paris, 19 Place Marguerite Perey, 91120 Palaiseau, France; patrick.waelbroeck@telecom-paris.fr.

1 Introduction

Digitization has dramatically decreased the production and distribution costs of physical and digital products (Shapiro et al., 1999; Rifkin, 2014). These developments have created both threats and opportunities for all industries facing digitization, and especially for cultural industries selling information goods including music, books, movies and video games. On the one hand, new business models have been introduced to better monetize different segments of the demand. With freemium services, developed for instance by Spotify and Deezer, consumers can either use a free version financed by ads, or pay a subscription fee for the premium service. [1] On the other hand, cheap reproduction costs have also made it easier for users of new technologies to access pirated digital content without authorization. Online piracy constitutes a significant threat to providers of digital content: pirated TV shows and films represent more than 230 billion views a year, inducing an expected loss of 11.58 billion U.S.D. for the TV and movie industries for the year 2022 in the U.S. only.[3] Online content providers are therefore under a huge pressure to design the best business models to deter online piracy.

In this article, we analyze the interplay between freemium models and digital piracy and we argue that freemium models have created opportunities to monetize consumers with low willingness to pay, thereby reducing digital piracy by fighting free with free.

We consider a firm selling a premium version of a good to consumers. The firm can also decide to offer a free version generating advertising revenues. The firm faces a competitive pressure from pirated versions of the content, for instance from Peer-to-Peer networks, from illegal online servers (such as the defunct Mega Upload website), or from illegal websites that offer free direct streaming services in the case of films and videos. Consumers face two costs when they acquire digital

[1] After becoming the dominant business model in the pre-recorded music industry, freemium business models are being developed in many other industries. In the videogame industry, freemium services and free-to-play (F2P) games[2] allow game developers to attract a large number of users who can test the game for free. Cloud services also propose a free version with limited functionalities: limited online storage, limited number of collaborators who can work on a file (Dropbox for instance) or limited session length (Zoom for instance).

[3] 18 Piracy Statistics To Keep You Away From Trouble in 2022, Web Tribunal, last accessed December 12, 2022.

copies. On the one hand, the copy might be of lower quality than the premium version, and this degradation cost increases with the taste for quality of particular a user. On the other hand, copying induces a fixed reproduction cost, represented for instance by monetary costs (e.g. to have access to a digital locker, or private network), or by ethical and technological costs that people face when they get digital content from illegal sources. Bae and Choi (2006) have highlighted the importance of these two dimensions of piracy, and our framework encompasses as special cases models in which there is only one of these two dimensions (Novos and Waldman, 1984; Belleflamme and Picard, 2007).

We focus on the design of the free version offered by the firm, and analyze how the free and the premium versions of the good can deter piracy. The firm can lower the quality of the free version, for instance by restricting access to some of the content. The strategy of the firm is to choose the level of restrictions of the free version, as well as to set the price of the premium version. We will show that restrictions on the free version constitute a central element of the design of the freemium business model, and that they have an important impact on the profitability of the firm. Restrictions on the free version include among other things the number of hours per month available for users of the free version, the size of the catalog available to free users, or the quality of the good in the case of videos and music.

We characterize the optimal business model of the firm as a response to the pressure exerted by the pirated version of the product. Introducing the free version of the good has two opposite effects on the profits of the firm. On the one hand, some of the users of the premium version are better off consuming the free version and this creates a cannibalization effect that reduces the profits of the firm. On the other hand, some consumers will choose the free version instead of the pirated version and this generates a market-expansion effect resulting from additional advertising revenues. By determining the level of restrictions of the free version and the price of the premium version, the firm can maximize the market-expansion effect while minimizing the cannibalization effect. The optimal price and level of restriction constitute the business model of the firm, which crucially depends on the properties of the pirated good.

We show that when the quality of the pirated good is high, the firm must offer a high quality for the free good to deter piracy. The resulting cannibalization effect on the premium good is strong, and the firm is better off offering only the premium version with a high price: the firm extracts sufficient surplus from consumers with the highest willingness to pay for the product. On the contrary, when the quality of the pirated good is low, the free version represents a credible alternative to the pirated versions for some users. The quality of the free version can be set relatively low and few of the premium users migrate to the free version. In this second business model, the firm benefits from advertising revenues generated from a large share of consumers who do not use the pirated version.

These results have important managerial implications. In the streaming industry, Youtube was initially providing only free services until the introduction of its premium services in 2014, becoming later Youtube Premium. On the contrary, Netflix has focused on premium services, but is now considering to introduce a free version to generate advertising revenues. Part of the difficulties faced by Netflix result from the pressure exerted by online free streaming pirate websites, and our model explains the reluctance of Netflix to offer this freemium model. On the one hand, a low level of restrictions on the free version could turn the tide against digital pirates.[4] On the other hand, offering a free product with a high level of restrictions would improve the conversion of free users to premium users but also diverts some consumers to digital piracy. Our contribution in this article is to show how firms can design a freemium business model to deter digital piracy by fighting free with free. We characterize the optimal business model of the firm by providing the equilibrium properties of the premium and free versions as a best response to the properties of the pirated version.

Secondly, the regulatory landscape is also a crucial factor determining the optimal business model of the firm. The legal risk associated with downloading a movie or an album – that our model captures through the reproduction cost – has an important impact on the willingness of consumers to use the free and the premium versions of the good. Policymakers can directly impact this cost by

[4] This effect has already been argued by observers of the music industry Streaming services turn the tide against digital pirates, Financial Times, September 13, 2013.

setting regulations against pirate websites, and our results show that institutional responses such as 3-strike laws can make freemium business models less relevant by encouraging firms to only offer the premium version, which can harm consumers and even the profit of the firm.

The article is organized as follows. In section 2 we review the literature. We present the setup in section 3 and show how the firm adapts its business model to deter piracy in Section 4. We discuss the robustness of our results to different assumptions on the primitives of the model in Section 5. Section 6 concludes the article.

2 Literature

Digital piracy. Our article contributes to the empirical literature on digital piracy by analyzing the development of freemium services as a response to piracy and to the implementation of stronger copyright enforcement. Most studies consider the substitution between pre-recorded music and digital piracy (Aguiar and Martens, 2016) and between box office sales and digital piracy (Peukert et al., 2017). Few articles study the impact of Subscription VOD (such as Netflix) on digital piracy. Lu et al. (2021) analyze the deployment of Netflix in South Asia.[5] They find that the unavailability of Netflix in Indonesia led to more searches for pirated content, but the effect is relatively lower for original Netflix content.

Godinho de Matos et al. (2018) find a strong substitution between SVOD services such as Netflix and digital piracy. The level of substitution can be modulated by the size of the catalog and the number of restrictions on the premium version. Overall these studies do not consider consumers who can access digital content for free from legal streaming services financed by ads and how this changes their trade-off between the premium version and digital piracy. We argue in this article that research linking sales of legal products or services and digital piracy should account for the availability of freemium services to consumers.

Freemium. The literature on freemium mainly addresses the design of the freemium business model: the number of different versions; the number and the type of re-

[5]Netflix was available at the beginning of 2016 in 40 countries, but not in Indonesia.

strictions on the free version (difficulty or usage) (Aral and Dhillon, 2021; Gu et al., 2018); how to attract new customers (Belo and Li, 2022) or maximize the social engagement of premium users (Bapna et al., 2016). Few articles analyze the effect of ads on the trade-off between the free and the premium version (Huang et al., 2018; Lee et al., 2021) and on the optimality of freemium among other business models (Sato, 2019).

Other articles study the impact of network effects on freemium design (Shi et al., 2019) and on competitors (Boudreau et al., 2021). Finally, Rietveld (2018) shows that in some cases, free versions of video games are less used and bring lower revenues than just selling a single premium version.

Finding the right product mix is a challenging task. In the beginning, Spotify offered 3 versions: a free version financed by ads, a desktop version at USD 4.99 and a premium version with mobile synchronization at USD 9.99. In 2010, the number of restrictions on the free version changed: a song could only be played 5 times and a 10 hours limit restriction was imposed on the free version. Then Spotify respectively lifted these restrictions in 2013 and 2014. In 2014, Spotify revised its versioning strategy by offering only 2 versions: a free version and the mobile subscription at USD 9.99. While the question of choosing the right premium offer has been addressed in the literature (Gu et al., 2018), the literature on the freemium business ignores the effect of piracy on the choice of consumers and on the optimal product mix of the firm and we contribute to the literature on this point.

Versioning. Given that information goods can be easily copied and modified at low marginal costs, it is often optimal to create several versions of a digital product (Shapiro et al., 1999). Freemium models can be seen as an extreme case of versioning in which the low-quality version is free and is therefore not monetized through a direct payment but indirectly through ad revenues. Freemium models encompass purely ad-based business models (Youtube) and premium-only subscription services (Netflix).

Offering a free version can be seen as a strategy similar to introducing damaged goods studied by Deneckere and Preston McAfee (1996). They show that it is profitable to introduce a damaged version of an existing (full-featured) product when the ratio of the valuation of the damaged good to the valuation of the full-featured good is decreasing in the valuation for the full-featured product. Bhargava and Choudhary (2008) extend this result by considering different variable costs and provide conditions on the market shares of the high-quality product compared with those of lower-quality products for versioning to be profitable.

Recently, Chellappa and Mehra (2018) study the choice of the qualities of different versions of a product sold by a monopolist when consumers have a usage cost, i.e. when a higher quality is associated with a disutility due to the complexity of the product or unnecessary features such as bloatware that reduce the user experience. Johnson and Myatt (2018) study a duopoly with asymmetric costs. Each firm can propose several qualities in their product lines and the authors determine under which conditions firms restrict their product lines or not.

Jiménez-Martínez (2019) analyzes a random network with an externality, which increases the value of the premium version. Ads increase the heterogeneity of the valuations of the product by consumers. They find that if the price of the product is low, the firm prefers to only sell the premium version. When the price level is intermediate, the freemium model becomes profitable.

Finally, Belo and Li (2022) study the incentives of a two-sided platform to launch several versions. They argue that since versioning is costly and that there is uncertainty about the extent of indirect network externalities, startups should wait before introducing a new version, while established firms have a clear decision to make between versioning or not, depending on the costs of versioning.

We contribute to the literature on versioning by showing that freemium is optimal in the presence of a competitive fringe, regardless of the assumption on the distribution of the taste parameter or on the costs of producing the different versions. Note that the competitive fringe could be anything that competes with the product line sold by the firm and is not limited to digital piracy: open source software, free video games, or even the existence of a second-hand market or parallel imports also represent a competitive fringe.

3 Description of the Model

Consumers can choose between three options to consume a good: use the free legal version financed by ads, purchase the premium version or get a digital copy. The first two options are legal and generate income for the monopoly, while the latter is illegal.

Qualities of the versions. All three versions differ in quality. The difference of quality between the free and the premium versions is endogenous and is determined by the level of restrictions associated with the free version. More restrictions increase the number of premium users but divert other users to piracy. On the contrary, fewer restrictions deter online piracy. The quality of digital copies is lower than the quality of the premium version and is modeled by an exogenous parameter α. This cost can be related to the sound quality (320 Kbps or higher for the premium service vs. 128 Kbps for many pirated copies) for music files. For software, it is related to fewer features, while for video games, this cost translates the fact that connecting the legal services brings additional value such as connecting to a community.

Piracy cost. We introduce a cost of copying $c \geq 0$. This fixed cost can simply be the monthly fee that a user pays for direct download sites. For instance, users of Megaupload were paying a fee in US dollars per month ($260 for a lifetime) to download files. It is also related to the ethical and technological costs to acquire illegal digital copies on the Internet. It takes time and knowledge to download files from specialized websites and P2P networks, especially in countries where there is close monitoring of file-sharing networks such as France or the US.

More generally, these reproduction costs associated with piracy can be interpreted as monetary, legal, ethical and technological costs that people face when they get digital content from illegal sources.

From a methodological point of view, it is important to introduce a fixed (reproduction) cost in our model. Indeed, without a fixed piracy cost, the freemium and pirated version would compete head to head: among those who do not purchase the premium version, either all consumers use the free version, or they use the pirated version. When introducing the fixed cost to piracy, we will show that consumers are segmented in the following order: consumers using the free version, consumers copying illegal files, and consumers purchasing the premium version.

Consumer utility. Let $p > 0$ denote the price of the premium version, R the level of restrictions, θ the willingness to pay for the good, α the reduced value of the digital copy (with $\alpha < 1$ and $1 - \alpha$ the degradation cost) and U_x the utility of consumers when they choose the option x. Consumers have the three following options:

1. Purchase the premium subscription at price p: $U_p = \theta - p$.

2. Use the free version with restrictions $0 \leq R \leq 1$: $U_f = (1 - R)\,\theta$.

3. Use a digital copy of lower quality $0 < \alpha < 1$: $U_c = \alpha\theta - c$.

4. Not consuming the good: $U_\varnothing = 0$.

A user of the free version generates an exogenous benefit $a > 0$ for the firm, which can be interpreted as an ARPU (average revenue per user) resulting from revenues from advertisements. For music and video streaming services, this ARPU is very small and is hardly negotiable with advertising companies, and can be considered exogenous. This assumption is supported by the literature on online advertising and in particular by Gentzkow (2014) who analyzes the value of targeted advertising by newspapers, and who argues that the price charged by online publishers is pinned-down by exogenous factors determining the willingness to pay of advertisers.[6]

We use the following tie-sorting condition: a consumer prefers the outside option to the free version with the maximal number of restrictions $R = 1$. These outside options include among others: listening to the radio, using an open-source software, reading a book from the public domain, or downloading pirated content from P2P servers such as The Pirate Bay.[7]

[6] Crampes et al. (2009) make a similar assumption, by considering digital companies making revenues from an exogenous return on the advertising displayed to consumers.

[7] This assumption guarantees that the firm does not make money from the segment of con-

Assumptions. We make the two following assumptions

Assumption A1. $a < \underline{a}$

Assumption A2. $c < \alpha$.

Assumption $A1$ requires that a is smaller than a threshold value $\underline{a}$, which allows to ensure a non-negative demand for the premium version. In particular, Assumption $A1$ implies that $a < \min\{\frac{1}{4}, \frac{(4-\alpha)(1-\alpha)\alpha}{\alpha^2+8(1-\alpha)}, 1-\alpha\}$. $a < \frac{1}{4}$ guarantees that a firm covering the unit segment with the free version and generating in this way a profit of a does not make more than the profits of $\frac{1}{4}$ of a monopoly without copies.

Moreover, this specification simplifies the analysis without loss of generality, and corresponds to the current state of online revenues generated by targeted advertising. As we have already noted before, ARPUs are in general very small: for Spotify, the quarterly ARPU is about 1 USD.[8] This value is especially small compared with the prices charged to Spotify premium service (29.97 USD/quarter).

Assumption $A2$ states that at least one consumer derives a strictly positive utility from the copy.[9] We also need to have $c < 1 - \alpha$ to ensure that the demand for the premium version is strictly positive and combining this inequality with Assumption $A1$ implies that $c < \frac{1}{2}$, i.e. the cost of copying cannot be greater than the price of a monopoly facing the outside option.

4 Equilibrium

For the clarity of the exposition we first assume that the consumer taste parameter θ is uniformly distributed on the segment $[0, 1]$. Then, we show that the main results hold with a general distribution function in Section 5.

sumers with the lowest willingness to pay for quality (such that $0 < \theta < \frac{c}{\alpha}$) who prefer the outside option to the free version with $R = 1$. Otherwise, the firm would generate a constant income of ac/α, which does not affect the optimum price and restrictions but changes the comparison between profits in the various cases that we analyze.

[8] Spotify Technology S.A. Announces Financial Results for First Quarter 2022, last accessed January 4, 2023.

[9] When $c > \alpha$, consumers do not copy and the firm chooses the unconstrained freemium model since this form of versioning yields more profit than simply offering the premium version.

4.1 Demand and Profit

The firm maximizes profits by choosing a couple (p, R). Depending on its choices, some options are not available to consumers and others are dominated.

The consumers indifferent between the free and the premium version, between the copy and the free version, and between the copy and the premium version are respectively:

$$\theta_{fp} = \frac{p}{R}, \quad \theta_{cf} = \frac{c}{R+\alpha-1}, \quad \theta_{cp} = \frac{p-c}{1-\alpha}.$$

When the number of restrictions is maximal, that is when $R = 1$, the indifferent consumer between the outside option and the copy version and between the outside option and the premium version are respectively:

$$\theta_{c\varnothing} = \frac{c}{\alpha}, \, \theta_{\varnothing p} = p.$$

We need to analyze four cases that depend on the value of p and R. They are defined in the following table:

	$p \leq c$	$p > c$
$R \leq 1-\alpha$	Case 2	
$R > 1-\alpha$	Case 3	Case 1A & 1B

However, there are only two market configurations to consider.[10] First, when the number of restrictions on the free version is higher than a threshold, there exists a set of parameter values for which some consumers prefer the digital copy to the free version (Case 1A). In all the other cases (cases 1B, 2 and 3), consumers choose between the free and the premium versions and do not use the pirated version. In this second configuration, the free version allows the firm to fully deter piracy.

These two configurations are represented in Figure 1. In Case 1A, there are potential copies, while in cases 1B, 2 and 3, the parameters are such that copies are dominated by the free or the premium versions. However, in all cases, the

[10]See Appendix A.1 for a detailed characterization of the cases.

availability of digital copies exerts a constraint on the range of prices and restrictions that the firm can choose. As we move from Cases 1A and 1B to Cases 2 and 3, the intensity of the competition from free online copies decreases, and therefore the strategy that the firm can use to deter or accommodate piracy will also change. Case 1A brings a new element compared with classical models of end-user piracy since the demand for the copy lies between the two legal versions.

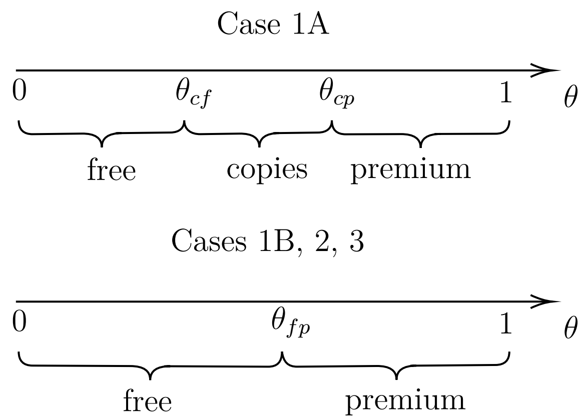

Figure 1: Demands for copies, free, and premium versions.

First configuration (Case 1A). The demand for the premium and the free services are respectively:

$$\begin{aligned} D_{premium} &= 1-\theta_{cp} = 1-\frac{p-c}{1-\alpha} \\ D_{free} &= \theta_{cf} = \frac{c}{R+\alpha-1} \end{aligned}$$

Given that $\frac{p}{p-c} > 1$, it is straightforward to see that condition $R \geq \frac{p(1-\alpha)}{p-c}$ is more restrictive than $R > 1-\alpha$, the limit case when the cost to copy is equal to zero. The firm maximizes profits

$$\Pi = a\frac{c}{R+\alpha-1} + p\left[1-\frac{p-c}{1-\alpha}\right] \text{subject to } R \geq \frac{p\,(1-\alpha)}{p-c}. \tag{1}$$

In this case, the pricing of the premium version does not depend on the introduction of the free version, as the firm is only constrained by the pirate version. Introducing the free version can be considered as a source of extra profits and will always be profitable for the firm.

Second configuration (cases 1B, 2 and 3). When consumers do not use the pirated version, the demand for the premium and the free services are respectively:

$$\begin{aligned} D_{premium} &= 1 - \theta_{fp} = 1 - \frac{p}{R} \\ D_{free} &= \theta_{fp} = \frac{p}{R} \end{aligned}$$

The firm maximizes

$$\Pi = a\frac{p}{R} + p\left[1 - \frac{p}{R}\right].$$

Subject to:[11]

- $1 - \alpha < R < \min\{1, \frac{p(1-\alpha)}{p-c}\}$ (Case 1B),
- $R \geq p$ and $R \leq 1 - \alpha$ (Case 2),
- $R \geq p$, $p \leq c$ and $R \geq 1 - \alpha$ (Case 3).

4.2 Optimal Business Models

There exist four business models that can be optimal for the firm, depending on the primitives of the model a, c, α: the freemium, freemium+, limit-pricing, and monopoly business models. In the freemium model, the firm offers a lower level of restrictions and a lower price than in the freemium+ model. In the two other business models, the firm can deter the copy without versioning strategies and offers only a premium version of its product. In the monopoly business model, the firm can charge its monopoly price whereas in what we call the limit-pricing business model, the firm deters the copy with a limit-pricing strategy. Figure 2 represents these business models.[12]

[11] The conditions characterizing the different cases are provided in Appendix A.2.
[12] The detailed analysis of the graph is available in Appendix A.2.1.

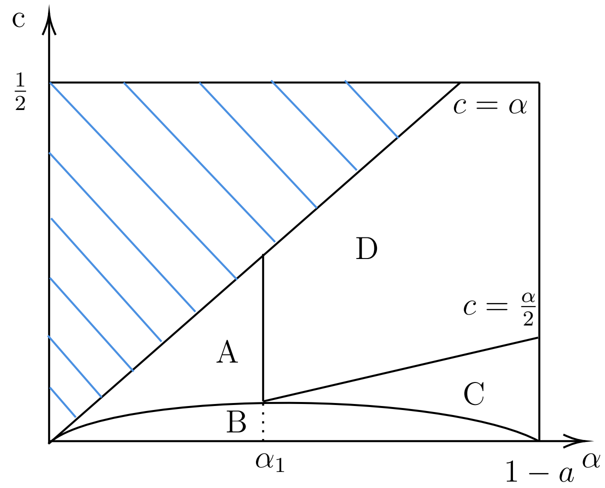

Figure 2: Demands for copies, free, and premium versions.

Proposition 1.

The optimal strategies lead to the following business models:

(*A*) *In area A the freemium model is optimal with*

$$R^* = 1 - \alpha, \; p^* = \frac{a + 1 - \alpha}{2}.$$

(*B*) *In area B, a freemium+ model is optimal, with*

$$R^* = \frac{(a + c + 1 - \alpha)(1 - \alpha)}{a - c + 1 - \alpha}, \; p^* = \frac{a + c + 1 - \alpha}{2}.$$

(*C*) *In area C, the limit-pricing model is optimal, with*

$$R^* = 1, \; p^* = \frac{c}{\alpha}.$$

(*D*) *In area D, the monopoly model is optimal with*

$$R^* = 1, \; p^* = \frac{1}{2}.$$

Proof: see Appendix A.3.

The proof proceeds in two steps: first, we determine optimal profits for cases 1A, 1B, 2 and 3; and secondly, we compare profits in each area.

Interpretation: Market Expansion and Cannibalization Effects We can compare the demand and profit of the optimal freemium model with copies to the demand and profit of a non-discriminatory monopoly facing copies to determine which effect dominates. To understand the trade-off between the different business models in Figure 2, it is useful to analyze the impact of introducing a free version on the profits of a firm. Introducing the free version has two main opposite effects:[13]

1. A market-expansion effect: consumers who were previously copying now use the free version; the firm increases the lower part of the demand function that was previously lost to copies and fights free with free. Note that if the copy is not a threat, a freemium business model will not be optimal.

2. A cannibalization effect: the firm loses demand for its premium version to the free version with restrictions. Consumers who were previously purchasing the premium version now only generate a per consumer.[14]

When the market-expansion effect dominates the cannibalization effect, introducing the free version is optimal, and the firm chooses the level of restrictions to maximize its profits. When the cannibalization effect is stronger than the market-expansion effect, the firm does not introduce the free version and prefers to deter piracy by using limit pricing.

Using these insights, the interpretation of Figure 2 is now straightforward. When the copy exerts a strong competitive pressure (low values of c and high values of α), the firm fights free with free by introducing the free version. In area B, some consumers copy in equilibrium and face the reproduction cost c, therefore the firm can increase its profits by increasing the level of restrictions to $R^* > 1 - \alpha$ (the level of restrictions of the freemium model in area A). The freemium+ generates the highest profits across all business models for all values of α. It is easy to show that the equilibrium demand for the premium version in area B increases when c increases: the firm increases the number of restrictions on the free version and more consumers prefer the premium version. The higher the reproduction cost, the higher the level of restrictions, but only up to a certain

[13] Introducing a free version will also change the equilibrium price of the premium version.

[14] See Belleflamme (2005) and Siebert (2015) for a review.

threshold, since the level of restrictions needs to be smaller than 1. When the copy is not a threat (high values of c or low values of α), the firm is either unconstrained by the copy (area D), or deters piracy with a limit pricing strategy (C).

We can interpret our results in terms of an increase in copyright protection, measured in our model by parameters c and α. Increasing c or decreasing α increases the level of restrictions and reduces the role of the free version to fight the copy. If copyright enforcement is strong enough, the firm does not need to deter the copy with the free version, and the firm does not use a freemium business model. Therefore institutional responses such as 3-strike laws can make freemium business models less relevant.

5 Discussion

We now discuss how our results change with a general distribution of the taste parameter, and when a increases. Consider first a general distribution of the taste parameter θ. It is clear that the firm will always weakly prefer the freemium model to a monopoly model without discrimination when it is feasible to do so (areas A and B). We establish this result in Proposition 2. In areas C and D, the firm is either unconstrained or uses limit pricing without introducing the free version, and the optimal business model will not change in this area with a general distribution function. Therefore, the qualitative features of Figure 2 will not change with a general distribution of the taste parameter.

We now show that the freemium model dominates the monopoly business model, and since the freemium+ model always yields a higher profit than the freemium model, it will dominate both business models when it is achievable. Let $F(\theta)$ be the distribution of the taste parameter with support $[\theta_0, \theta_1]$ and density $f(\theta)$.

The main trade-off identified in the previous section (market expansion vs cannibalization) is still valid with a general distribution of the taste parameter, and Proposition 2 establishes that the market-expansion effect when there is no cost of versioning always dominates the cannibalization effect. Consumers only use the pirated version in Case 1A that we consider in the remaining of this section.

Moreover, in the remaining of the section, we assume that some consumers will use the free version in equilibrium, namely that $1-\alpha < R < \min\{1, \frac{p(1-\alpha)}{p-c}\}$, or $R \geq p$ and $R \leq 1-\alpha$, or also $R \geq p$, $p \leq c$ and $R \geq 1-\alpha$. The consumer indifferent between the free version and the premium version is defined by:

$$\theta_{fp} = \frac{p}{R}$$

This gives a demand for the premium version equal to $1 - F\left(\frac{p}{R}\right)$ and a demand for the free version of $F\left(\frac{p}{R}\right)$. The profit is

$$\Pi_f = aF\left(\frac{p}{R}\right) + \left(1 - F\left(\frac{p}{R}\right)\right)p \tag{2}$$

Consider now the profits of the firm when proposing only the premium version. The consumer who is indifferent between the copy and the premium version is defined by:

$$\theta_{cp} = \frac{p-c}{1-\alpha}$$

The demand for the premium service is $1 - F\left(\frac{p-c}{1-\alpha}\right)$, and the profit is

$$\Pi_p = \left(1 - F\left(\frac{p-c}{1-\alpha}\right)\right)p. \tag{3}$$

Let $p^* = \arg\max_{p>0}\{\Pi_f(p)\}$ and $p^{**} = \arg\max_{p>0}\{\Pi_p(p)\}$. The freemium and freemium+ models generate more profits than offering a single product.

Proposition 2.

Offering a free version is strictly optimal: $\Pi_f(p^*) > \Pi_p(p^{**})$.

Proof. By Lemma 2, $R^* = \frac{p(1-\alpha)}{p-c}$ in the freemium setup. Thus, we have:

$$\begin{aligned}
\Pi_f(p^*) - \Pi_p(p^{**}) &= aF\left(\frac{p^*-c}{1-\alpha}\right) + \left(1 - F\left(\frac{p^*-c}{1-\alpha}\right)\right)p^* \\
&\quad - \left(\left(1 - F\left(\frac{p^{**}-c}{1-\alpha}\right)\right)p^{**} + aF\left(\frac{p^{**}-c}{1-\alpha}\right) - aF\left(\frac{p^{**}-c}{1-\alpha}\right)\right) \\
&= \Pi_f(p^*) - \Pi_f(p^{**}) + aF\left(\frac{p^{**}-c}{1-\alpha}\right) > 0.
\end{aligned}$$

The last inequality follows a revealed preference argument as p^* is optimal for Π_f in (2). □

The term $aF\left(\frac{p^{**}-c}{1-\alpha}\right)$ is the positive market-expansion effect on profits. Thus the difference $\Pi_f(p^*) - \Pi_f(p^{**})$ is the sum of the cannibalization and price effects on profits. The proof of Proposition 2 states that this sum is always positive.

The following Proposition 3 states that introducing the free version always increases the price of the premium service.

Proposition 3.

The optimal price of the premium service is higher with the free version $p^* \geq p^{**}$.

Proof. Proof by contradiction. Suppose that $p^* < p^{**}$. Then

$$\begin{aligned}\Pi_f(p^*) &< aF\left(\frac{p^{**}-c}{1-\alpha}\right) + \left(1 - F\left(\frac{p^*-c}{1-\alpha}\right)\right)p^* \\ &< aF\left(\frac{p^{**}-c}{1-\alpha}\right) + \left(1 - F\left(\frac{p^{**}-c}{1-\alpha}\right)\right)p^{**} = \Pi_f(p^{**})\end{aligned}$$

where the first inequality comes from the fact that $F(.)$ is increasing and the second inequality comes from the optimality of p^{**} for (3). Therefore p^* was not optimal for Π_f. This is a contradiction. □

Finally, we can analyze the impact of an increase in the ARPU a on the business models of the firm. A higher value of a increases the revenues per consumer for the free product, which enhances the benefits from reaching a higher demand with this version of the good. Hence, an increase in a decreases the optimal level of restrictions R in the freemium and freemium+ business models. The cannibalization effect is fiercer as the free version serves a larger demand, and the optimal price for the premium good increases with a, as only consumers with the highest valuation will consume this version. The overall impact on profit is positive.

6 Conclusion

We have analyzed a model of online piracy where a firm uses a special form of versioning that includes two versions of its product: a free version financed by ads where the firm uses restrictions as a strategic variable to reach a specific group of consumers, and a premium version with the highest quality. We have shown that even when free digital content is available online, the firm can completely deter online piracy by offering a free version with a low level of restriction along with the premium version.

Our results have important policy implications. The recent European successes of Spotify and Deezer suggest that freemium models are excellent market-based alternatives to fight online piracy. In the digital video industry, consumers with a low willingness to pay can stream online content for free and pay a monthly subscription fee for premium services such as Video on Demand. Similarly, in the video games industry, free-to-play models offer new ways to players to try out new games for free, and they only purchase premium services and additional content. There are thus market solutions based on freemium models that can fight free with free by better segmenting consumers and audiences according to their willingness to pay for digital content. These results therefore challenge the view that strong copyright laws are necessary to fight online piracy.

References

Aguiar, L. and Martens, B. (2016), 'Digital music consumption on the internet: Evidence from clickstream data', *Information Economics and Policy* **34**, 27–43.

Aral, S. and Dhillon, P. S. (2021), 'Digital paywall design: Implications for content demand and subscriptions', *Management Science* **67**(4), 2381–2402.

Bae, S. H. and Choi, J. P. (2006), 'A model of piracy', *Information Economics and Policy* **18**(3), 303–320.

Bapna, R., Ramaprasad, J. and Umyarov, A. (2016), 'Monetizing freemium communities: Does paying for premium increase social engagement?', *Available at SSRN 2885681* .

Belleflamme, P. (2005), 'Versioning in the information economy: theory and applications', *CESifo Economic Studies* **51**(2-3), 329–358.

Belleflamme, P. and Picard, P. M. (2007), 'Piracy and competition', *Journal of Economics & Management Strategy* **16**(2), 351–383.

Belo, R. and Li, T. (2022), 'Social referral programs for freemium platforms', *Management Science* .

Bhargava, H. K. and Choudhary, V. (2008), 'Research note—when is versioning optimal for information goods?', *Management Science* **54**(5), 1029–1035.

Boudreau, K. J., Jeppesen, L. B. and Miric, M. (2021), 'Competing on freemium: Digital competition with network effects', *Strategic Management Journal* .

Chellappa, R. K. and Mehra, A. (2018), 'Cost drivers of versioning: Pricing and product line strategies for information goods', *Management Science* **64**(5), 2164–2180.

Crampes, C., Haritchabalet, C. and Jullien, B. (2009), 'Advertising, competition and entry in media industries', *The Journal of Industrial Economics* **57**(1), 7–31.

Deneckere, R. J. and Preston McAfee, R. (1996), 'Damaged goods', *Journal of Economics & Management Strategy* **5**(2), 149–174.

Gentzkow, M. (2014), 'Trading dollars for dollars: The price of attention online and offline', *American Economic Review* **104**(5), 481–88.

Godinho de Matos, M., Ferreira, P. and Smith, M. D. (2018), 'The effect of subscription video-on-demand on piracy: Evidence from a household-level randomized experiment', *Management Science* **64**(12), 5610–5630.

Gu, X., Kannan, P. and Ma, L. (2018), 'Selling the premium in freemium', *Journal of Marketing* **82**(6), 10–27.

Huang, J., Reiley, D. and Riabov, N. (2018), 'Measuring consumer sensitivity to audio advertising: A field experiment on pandora internet radio', *Available at SSRN 3166676* .

Jiménez-Martínez, A. (2019), 'Discrimination through versioning with advertising in social networks', *Economic Theory* **67**(3), 525–564.

Johnson, J. P. and Myatt, D. P. (2018), 'The determinants of product lines', *The RAND Journal of Economics* **49**(3), 541–573.

Lee, S., Zhang, J. and Wedel, M. (2021), 'Managing the versioning decision over an app's lifetime', *Journal of Marketing* **85**(6), 44–62.

Lu, S., Rajavi, K. and Dinner, I. (2021), 'The effect of over-the-top media services on piracy search: evidence from a natural experiment', *Marketing Science* **40**(3), 548–568.

Novos, I. E. and Waldman, M. (1984), 'The effects of increased copyright protection: An analytic approach', *Journal of political economy* **92**(2), 236–246.

Peukert, C., Claussen, J. and Kretschmer, T. (2017), 'Piracy and box office movie revenues: Evidence from megaupload', *International Journal of Industrial Organization* **52**, 188–215.

Rietveld, J. (2018), 'Creating and capturing value from freemium business models: A demand-side perspective', *Strategic Entrepreneurship Journal* **12**(2), 171–193.

Rifkin, J. (2014), *The zero marginal cost society: The internet of things, the collaborative commons, and the eclipse of capitalism*, St. Martin's Press.

Sato, S. (2019), 'Freemium as optimal menu pricing', *International Journal of Industrial Organization* **63**, 480–510.

Shapiro, C., Varian, H. R., Carl, S. et al. (1999), *Information rules: A strategic guide to the network economy*, Harvard Business Press.

Shi, Z., Zhang, K. and Srinivasan, K. (2019), 'Freemium as an optimal strategy for market dominant firms', *Marketing Science* **38**(1), 150–169.

Siebert, R. (2015), 'Entering new markets in the presence of competition: Price discrimination versus cannibalization', *Journal of Economics & Management Strategy* **24**(2), 369–389.

A Appendix

A.1 Characterization of Cases $1A$, $1B$, 2 and 3

We need to analyze three cases according to whether the digital copy represents a credible threat to the firm.

A.1.1 Case 1: $p > c$ and $R > 1 - \alpha$

The price of the premium version is higher than the fixed cost of the copy and the quality of the copy is higher than the "quality" of the free version. We need to distinguish 2 sub-cases.

A.1.1.1 Case 1A: $\theta_{cf} \leq \theta_{fp}$. This condition can be written as $R \geq \frac{p(1-\alpha)}{p-c}$. The number of restrictions on the free version is higher than a threshold. There exists a set of parameter values for which some consumers prefer the digital copy. This situation is depicted in Figure 3.

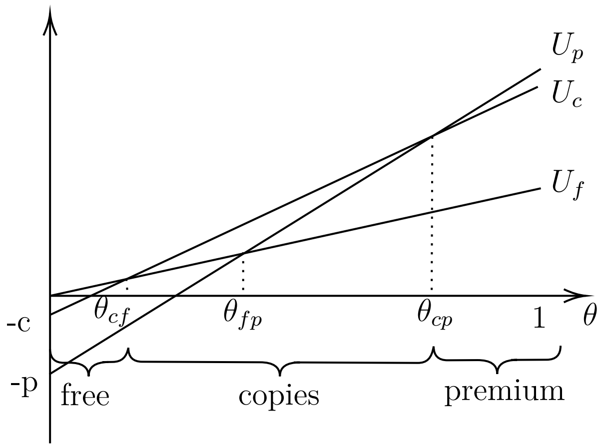

Figure 3: Case 1A

It is easy to show that $\theta_{cf} \leq \theta_{fp}$ implies $\theta_{fp} \leq \theta_{cp}$.

A.1.1.2 Case 1B: $\theta_{cf} > \theta_{fp}$. The number of restrictions is below the threshold defined in Case 1A. Consumers choose between the free and the premium version. It is straightforward to show that $\theta_{cf} > \theta_{fp}$ implies $\theta_{fp} > \theta_{cp}$. This configuration of the parameters is represented in Figure 4.

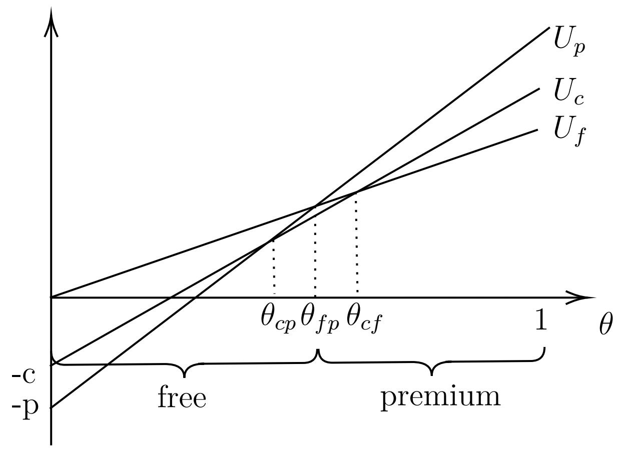

Figure 4: Case 1B

A.1.2 Case 2: $R \leq 1 - \alpha$.

The quality of the copy is always lower than the "quality" of the free version. This case is depicted in Figure 5.

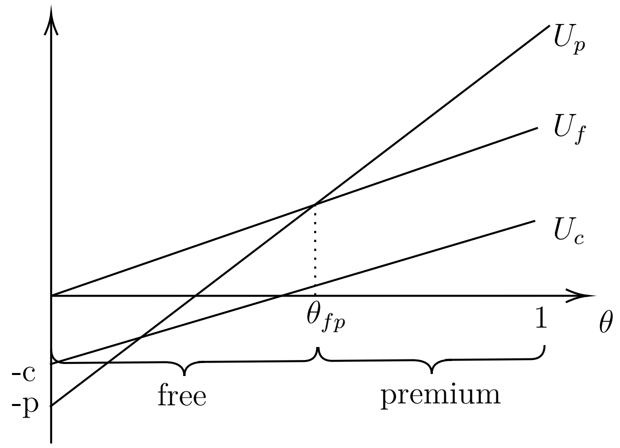

Figure 5: Case 2

The copy is always dominated by the free version because the slope of U_f is greater than the slope of U_c, and U_f intersects the y-axis above the intersection of U_c and the y-axis.

A.1.3 Case 3: $R > 1 - \alpha$ and $p \leq c$.

The quality of the copy is higher than the "quality" of the free version, but the fixed cost of the copy is higher than the price of the premium version. This configuration of the parameters is depicted in Figure 6.

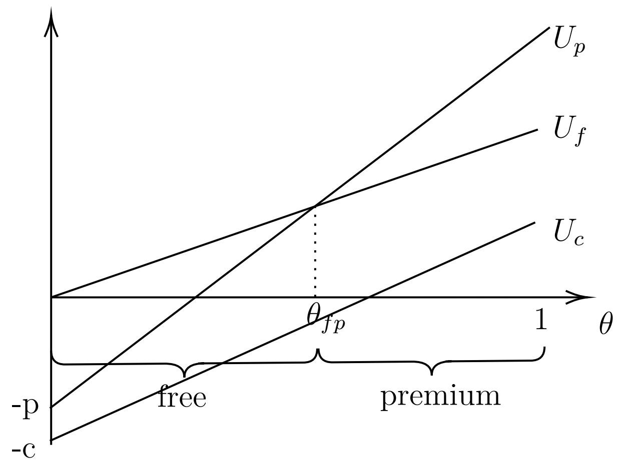

Figure 6: Case 3

The consumer prefers the free version to the copy up to the intersection of the lines U_f and U_c, which is always to the right of the point where the consumer is indifferent between the free version and the premium version. Therefore, the copy is always dominated by the free version.

A.2 Profits in Cases 1B, 2 and 3.

The three sub-cases differ with respect to their respective conditions on R and p. In Case 1B, the constraint $\theta_{cf} > \theta_{fp}$ can be written as:

$$R < \frac{p(1-\alpha)}{p-c}$$

Thus the number of restrictions is such that

$$1-\alpha < R < \min\{1, \frac{p(1-\alpha)}{p-c}\}.$$

The conditions characterizing Case 2 are:

$$R \geq p \text{ and } R \leq 1-\alpha.$$

And finally, in Case 3, the firm sets a high level of restrictions on the free subscription service but at the same time sets a relatively low price for the premium version because of the high cost to copy and the conditions are:

$$R \geq p,\ p \leq c \text{ and } R \geq 1-\alpha.$$

A.2.1 Description of Figure 2

We now describe Figure 2. The shaded area above the diagonal line $c = \alpha$ corresponds to a situation in which $c > \alpha$. In this area, consumers can no longer get a strictly positive utility from the copy. This shaded area is ruled out of our analysis by Assumption $A2$. Below the diagonal $c = \alpha$, we have assigned letters to four areas. In areas A and B, the firm deters the copy by offering a free version of its product. Area A represents the configuration of parameters where the freemium business model is optimal. In area B, the freemium+ business model is optimal. In area D, the firm charges its monopoly price whereas in area C, the firm deters the copy with a limit-pricing strategy.

The frontier between area A and areas C and D is given by the vertical line with abscissa $\alpha_1 = a + \frac{1}{2} - \frac{1}{2}\sqrt{(1-4a)}$. It is easy to check that the line defined by $c = \alpha$ is always above the curve $\frac{\alpha(a+1-\alpha)}{2-\alpha}$. This line $c = \alpha$ intersects the vertical line $\alpha = \alpha_1$ at $c = a + \frac{1}{2} - \frac{1}{2}\sqrt{(1-4a)}$ on the y-axis. This intersection is below $c = \frac{1}{2}$ if $a < \frac{\sqrt{2}-1}{2}$ and above $c = \frac{1}{2}$ otherwise.

A.3 Proof of proposition 1

The optimal strategies lead to the following business models:

(A) $\frac{\alpha(a+1-\alpha)}{2-\alpha} \leq c < \frac{1}{2}$ and $0 < \alpha < a + \frac{1}{2} - \frac{1}{2}\sqrt{(1-4a)}$: the freemium model is optimal with

$$R^* = 1 - \alpha,\ p^* = \frac{a+1-\alpha}{2},\ \textit{and}\ \Pi^* = \frac{(a+1-\alpha)^2}{4(1-\alpha)}$$

(B) $0 < c < \frac{\alpha(a+1-\alpha)}{2-\alpha}$: the freemium+ model is optimal, with

$R^* = \frac{(a+c+1-\alpha)(1-\alpha)}{a-c+1-\alpha}$, $p^* = \frac{a+c+1-\alpha}{2}$, *and* $\Pi^* = \frac{(1-\alpha)^2+(a-c)^2+2(1-\alpha)(a+c)}{4(1-\alpha)}$

(C) $\frac{\alpha(a+1-\alpha)}{2-\alpha} \leq c < \frac{\alpha}{2}$ and $a + \frac{1}{2} - \frac{1}{2}\sqrt{(1-4a)} < \alpha < 1 - a$: the limit-pricing model is optimal, with

$$R^* = 1,\ p^* = \frac{c}{\alpha},\ \textit{and}\ \Pi^* = \frac{c}{\alpha}\left(1 - \frac{c}{\alpha}\right)$$

(D) $\frac{\alpha}{2} \leq c < \frac{1}{2}$ and $a + \frac{1}{2} - \frac{1}{2}\sqrt{(1-4a)} < \alpha < 1 - a$: the monopoly model is optimal with

$$R^* = 1,\ p^* = \frac{1}{2},\ and\ \Pi^* = \frac{1}{4}$$

We proceed in two steps. First, we determine optimal profits for cases $1A$, $1B$, 2 and 3. Secondly, we compare profits in each area.

A.3.1 Step1. Optimal profits

A.3.1.1 Case 1: $p > c$, $R > 1 - \alpha$

Analysis of Case 1A: $1 \geq R \geq \frac{p(1-\alpha)}{p-c}$ In this case, the firm maximizes profits

$$\Pi = a\frac{c}{R+\alpha-1} + p\left[1 - \frac{p-c}{1-\alpha}\right] \text{ subject to } 1 \geq R \geq \frac{p\,(1-\alpha)}{p-c}.$$

The first order condition with respect to R is $\frac{\partial \Pi}{\partial R} = -\frac{ac}{(R+\alpha-1)^2} < 0$. Therefore, the firm sets R at its minimum level $R^* = \frac{p(1-\alpha)}{p-c}$.

Result 1. The optimal profits are:

(i) If $0 < c < \frac{\alpha(a+1-\alpha)}{2-\alpha}$, $\Pi^* = \frac{(1-\alpha)^2 + (a-c)^2 + 2(1-\alpha)(a+c)}{4(1-\alpha)}$;

(ii) If $\frac{\alpha(a+1-\alpha)}{2-\alpha} \leq c < 1/2$, $\Pi^* = \frac{c}{\alpha}\left(1 - \frac{c}{\alpha}\right)$.

Proof. Using this level of restriction R^*, the firm maximizes profits

$$\Pi = a\frac{p-c}{1-\alpha} + p\left[1 - \frac{p-c}{1-\alpha}\right] \tag{4}$$

The unconstrained optimum is $p^* = \frac{a+c+1-\alpha}{2}$.[15] The associated unconstrained optimal number of restrictions is

$$R^* = \frac{(a+c+1-\alpha)\,(1-\alpha)}{a-c+1-\alpha}$$

Note that R^* is increasing with c: when the copy becomes more costly, the free version becomes more attractive and the firm can increase the number of restrictions on the free version. There are two cases to analyze: when $R^* < 1$ (the utility

[15] The level of restrictions is such that the firm faces two segments of the total demand that are separated by $\frac{p}{R}$, instead of the three initial segments (free/copy/premium).

of the free version is positive) and when $R^* = 1$ (the utility of the free version is 0).

First, R^* is strictly smaller than 1 if

$$c < \frac{\alpha\left(a+1-\alpha\right)}{2-\alpha} \tag{5}$$

When this condition (5) is satisfied, we also have $p^* > c$ (since $R^* < 1$ is equivalent to $p^* > \frac{c}{\alpha}$ and that $\alpha < 1$) so that the condition on price of Case 1A is satisfied. The optimal profit is

$$\Pi^* = \frac{\left(1-\alpha\right)^2 + \left(a-c\right)^2 + 2\left(1-\alpha\right)\left(a+c\right)}{4\left(1-\alpha\right)} \tag{6}$$

This proves (i).

Secondly, when condition (5) is not satisfied, the firm sets $R^* = 1$. Given our tie-sorting assumption, the free version is dominated by the outside option and the consumer chooses between the premium version and a copy. In Case 1A, condition $R \geq \frac{p(1-\alpha)}{p-c}$ becomes $\frac{p-c}{1-\alpha} \geq p$ or $p \geq \frac{c}{\alpha}$. Recall that $\frac{c}{\alpha} = \theta_{c\varnothing}$ and $p = \theta_{\varnothing p}$. Then, the indifferent consumer between the copy and the outside option has to be located to the left of the indifferent consumer between the outside option and the premium version. The latter has to be located to the left of the indifferent consumer between the copy and the premium version. The profit of the firm is then: $\Pi = p\left[1 - \frac{p-c}{1-\alpha}\right]$. The optimal unconstrained price, $p^* = \frac{c+1-\alpha}{2}$, is greater than $\frac{c}{\alpha}$ if $c < \frac{\alpha(1-\alpha)}{2-\alpha}$. However, when (5) is not satisfied, we must have $c \geq \frac{\alpha(a+1-\alpha)}{2-\alpha}$. Since $a > 0$, these two inequalities are mutually incompatible, and therefore the firm sets a constrained price $p^* = \frac{c}{\alpha}$ (indeed, $\frac{\partial \Pi}{\partial p}|_{p=\frac{c}{\alpha}} < 0$ for $c > \frac{\alpha(a+1-\alpha)}{2-\alpha}$ and $\frac{c}{\alpha}$ $> c$) and there are no copies in equilibrium. This proves (ii). □

Analysis of Case 1B : $R < \min\{1, \frac{p(1-\alpha)}{p-c}\}$

In this case, recall that the profit function is given by:

$$\Pi = a\frac{p}{R} + p\left[1 - \frac{p}{R}\right] \text{ subject to } 1-\alpha < R \leq \min\{1, \frac{p\left(1-\alpha\right)}{p-c}\}$$

The first order condition with respect to R yields

$$\frac{\partial \Pi}{\partial R} = \frac{p\left(p-a\right)}{R^2} \tag{7}$$

There are two cases to distinguish according to whether (A) $a \leq c$ or (B) $a > c$.

(A) When $a \leq c$, condition $p > c$ defining Case 1 implies $p > a$. Thus (7) is positive. The firm sets R to its maximal value, i.e $R^* = \min\{1, \frac{p(1-\alpha)}{p-c}\}$.

Result 2. The optimal profits are:

(*i*) If $a \leq c < \frac{\alpha(a+1-\alpha)}{2-\alpha}$ and $\alpha \geq 2a$, $\Pi^* = \frac{(1-\alpha)^2+(a-c)^2+2(1-\alpha)(a+c)}{4(1-\alpha)}$;

(*ii*) If $\frac{\alpha(a+1-\alpha)}{2-\alpha} \leq c \leq \frac{\alpha}{2}$ and $\alpha \geq 2a$, $\Pi^* = \frac{c}{\alpha}\left(1 - \frac{c}{\alpha}\right)$;

(*iii*) If ($\frac{\alpha}{2} \leq c < \frac{1}{2}$ and $\alpha \geq 2a$) or ($a \leq c < \frac{1}{2}$ and $\alpha \leq 2a$) , $\Pi^* = \frac{1}{4}$.

Proof. There are two cases to analyse according to whether $R^* = \min\{1, \frac{p(1-\alpha)}{p-c}\} = \frac{p(1-\alpha)}{p-c}$ or $R^* = \min\{1, \frac{p(1-\alpha)}{p-c}\} = 1$.

In the first situation, the firm chooses an optimal price p^* such that $\frac{p^*(1-\alpha)}{p^*-c} \leq 1$, that is $p^* \geq \frac{c}{\alpha}$. The unconstrained optimum price is $p^* = \frac{a+c+1-\alpha}{2}$, that is greater than $\frac{c}{\alpha}$ if $c \leq \frac{\alpha(a+1-\alpha)}{2-\alpha}$. When this condition is satisfied, the associated unconstrained optimal number of restrictions is $R^* = \frac{(a+c+1-\alpha)(1-\alpha)}{a-c+1-\alpha} \leq 1$, and the optimal profit is either $\Pi^* = \frac{(1-\alpha)^2+(a-c)^2+2(1-\alpha)(a+c)}{4(1-\alpha)}$ when $R^* < 1$ or $\Pi^* = \frac{c}{\alpha}\left(1 - \frac{c}{\alpha}\right)$ when $R^* = 1$. On the contrary, when $c > \frac{\alpha(a+1-\alpha)}{2-\alpha}$, the optimal price is constrained to $p^* = \frac{c}{\alpha}$ with $R^* = 1$ and the optimal profit is $\Pi^* = \frac{c}{\alpha}\left(1 - \frac{c}{\alpha}\right)$.

In the second situation, the firm chooses an optimal price p^* such as $\frac{p^*(1-\alpha)}{p^*-c} \geq 1$, that is $p^* \leq \frac{c}{\alpha}$. The firm sets $R^* = 1$, and the free version is dominated by the outside option. Furthermore, by $p^* \leq \frac{c}{\alpha}$, the copy is dominated by the premium. The consumer chooses between the premium and the outside option. The profit becomes $p(1-p)$. The unconstrained optimal price is $p^* = \frac{1}{2}$, which is less than $\frac{c}{\alpha}$ if $c \geq \frac{\alpha}{2}$. When this condition is satisfied, the optimal profit is $\Pi^* = \frac{1}{4}$. When it is not satisfied, that is when $c < \frac{\alpha}{2}$, $p^* = \frac{c}{\alpha}$ and the optimal profit is $\Pi^* = \frac{c}{\alpha}\left(1 - \frac{c}{\alpha}\right)$.

We determine the choice of the firm by comparing the different optimal profits. We have first to notice that the thresholds on c are such that $\frac{\alpha(a+1-\alpha)}{2-\alpha} \geq (\leq) \frac{\alpha}{2}$ iff $\alpha \leq (\geq) 2a$ and that $\frac{\alpha(a+1-\alpha)}{2-\alpha} \geq (\leq) a$ iff $\alpha \geq (\leq) 2a$. It is straightforward to show that for $\alpha \geq 2a$, when $c < \frac{\alpha(a+1-\alpha)}{2-\alpha}$, the optimal profit $\Pi^* = \frac{(1-\alpha)^2+(a-c)^2+2(1-\alpha)(a+c)}{4(1-\alpha)} > \frac{c}{\alpha}\left(1 - \frac{c}{\alpha}\right)$ and when $c \geq \frac{\alpha}{2}$, the optimal profit $\Pi^* = \frac{1}{4} > \frac{c}{\alpha}\left(1 - \frac{c}{\alpha}\right)$. Finally, when $\frac{\alpha(a+1-\alpha)}{2-\alpha} \leq c \leq \frac{\alpha}{2}$, the optimal profit is $\Pi^* = \frac{c}{\alpha}\left(1 - \frac{c}{\alpha}\right)$. And, for $\alpha \leq 2a$, we have $\frac{\alpha}{2} \leq \frac{\alpha(a+1-\alpha)}{2-\alpha} \leq c \leq a$. Thus, the only optimal profit is $\Pi^* = \frac{1}{4}$. This proves (i), (ii), (iii). □

(B) When $a > c$, there are 3 sub-cases to analyze: (B1) $p > a > c$, (B2) $p = a$, (B3) $c < p < a$.

We first show that Case (B3) is impossible. Suppose on the contrary that $c < p < a$. Expression (7) is negative. The firm chooses the lowest possible value for R.[16] We have to make sure that the demand for the premium version is positive, that is $p \leq R$. As R increases, R converges to $\max\{p, 1-\alpha\}$ given that $R > 1-\alpha$ by the definition of Case 1. By Assumption $A1$ ($a < 1-\alpha$), we know that $p < a < 1-\alpha$, so the firm sets $R^* \to 1-\alpha$ and a price $p^* \to \frac{a+1-\alpha}{2}$. However, this price is strictly greater than a, by Assumption $A1$. This is a contradiction.

In subcase (B1), expression (7) is positive. The firm sets $R^* = \min\{1, \frac{p(1-\alpha)}{p-c}\}$. This case can be analyzed in a similar way as Case 1B (A), but with $a > c$. We have three possible prices according to Assumption A1:

- $p^* = \frac{a+c+1-\alpha}{2} > a$,
- $p^* = \frac{1}{2} > a$,
- $p^* = \frac{c}{\alpha}$, this price is possible only for $\frac{c}{\alpha} > a$, if not, the optimal price is constrained $p^* = a$, and the analysis is similar to Subcase (B2).

In Subcase (B2), where $p = a$, the firm can choose any value of R^* such that $R^* \in \left(1-\alpha, \min\left(1, \frac{a(1-\alpha)}{a-c}\right)\right)$. The optimal profit is $\Pi^* = a$.

Results of Case 1B (B) are summarized in Result 3.

Result 3. The optimal profits are:

(i′) If $(0 < c < a$ and $\alpha \geq 2a)$ or $(0 < c < \frac{\alpha(a+1-\alpha)}{2-\alpha}$ and $\alpha \leq 2a)$, $\Pi^* = \frac{(1-\alpha)^2+(a-c)^2+2(1-\alpha)(a+c)}{4(1-\alpha)}$;

(ii′) If $\frac{\alpha(a+1-\alpha)}{2-\alpha} \leq c < a$ and $\alpha \leq 2a$, $\Pi^* = \frac{1}{4}$.

Proof. When $\alpha \geq 2a$, we have $c < a < \frac{\alpha(a+1-\alpha)}{2-\alpha} < \frac{\alpha}{2} < \frac{1}{2}$. Thus, in subcase (B1), the optimal profit is $\Pi^* = \frac{(1-\alpha)^2+(a-c)^2+2(1-\alpha)(a+c)}{4(1-\alpha)}$. This profit is greater than a, the optimal profit in subcase (B2). When $\alpha \leq 2a$, the different thresholds are such that $\frac{\alpha}{2} < \frac{\alpha(a+1-\alpha)}{2-\alpha} < a < \frac{1}{2}$. In subcase (B1), when $0 < c < \frac{\alpha}{2}$, the optimal

[16] If $p = a$, we have already shown that the profit evaluated at this price is $\Pi^* = a$.

profit $\Pi^* = \frac{(1-\alpha)^2+(a-c)^2+2(1-\alpha)(a+c)}{4(1-\alpha)} > \frac{c}{\alpha}\left(1-\frac{c}{\alpha}\right)$, and when $\frac{\alpha}{2} \le c < \frac{\alpha(a+1-\alpha)}{2-\alpha}$, the optimal profit is still $\Pi^* = \frac{(1-\alpha)^2+(a-c)^2+2(1-\alpha)(a+c)}{4(1-\alpha)} > \frac{1}{4}$. Finally, when $\frac{\alpha(a+1-\alpha)}{2-\alpha} \le c < a$, the optimal profit is $\Pi^* = \frac{1}{4} > \frac{c}{\alpha}\left(1-\frac{c}{\alpha}\right)$. Both optimal profits of subcase (B1) are greater than a, the optimal profit in subcase (B2). This proves (i'), (ii''). □

Bringing together Result 2 and Result 3 gives us the following conclusion for Case 1B.

Result 4. The optimal profits of Case 1B are:

(i) If $0 < c < \frac{\alpha(a+1-\alpha)}{2-\alpha}$, $\Pi^* = \frac{(1-\alpha)^2+(a-c)^2+2(1-\alpha)(a+c)}{4(1-\alpha)}$;

(ii) If $\frac{\alpha(a+1-\alpha)}{2-\alpha} \le c \le \frac{\alpha}{2}$ and $\alpha \ge 2a$, $\Pi^* = \frac{c}{\alpha}\left(1-\frac{c}{\alpha}\right)$;

(iii) If $\left(\frac{\alpha(a+1-\alpha)}{2-\alpha} \le c < \frac{1}{2} \text{ and } \alpha \le 2a\right)$ or $(\frac{\alpha}{2} \le c < \frac{1}{2}$ and $\alpha \ge 2a)$, $\Pi^* = \frac{1}{4}$.

We can easily bring together Result 1 of Case 1A and Result 4 of Case 1B to summarize Case 1 (remember that $\frac{c}{\alpha}\left(1-\frac{c}{\alpha}\right) < \frac{1}{4}$):

Result 5. Suppose $p > c$, $R > 1-\alpha$.

$(R5.i)$ If $0 < c < \frac{\alpha(a+1-\alpha)}{2-\alpha}$, the optimum is $p^* = \frac{a+c+1-\alpha}{2}$, $R^* = \frac{(a+c+1-\alpha)(1-\alpha)}{a-c+1-\alpha}$, $\Pi^* = \frac{(1-\alpha)^2+(a-c)^2+2(1-\alpha)(a+c)}{4(1-\alpha)}$;

$(R5.ii)$ If $\frac{\alpha(a+1-\alpha)}{2-\alpha} \le c \le \frac{\alpha}{2}$ and $\alpha \ge 2a$, the optimum is $p^* = \frac{c}{\alpha}$, $R^* = 1$, $\Pi^* = \frac{c}{\alpha}\left(1-\frac{c}{\alpha}\right)$;

$(R5.iii)$ If $\left(\frac{\alpha(a+1-\alpha)}{2-\alpha} \le c < \frac{1}{2} \text{ and } \alpha \le 2a\right)$ or $(\frac{\alpha}{2} \le c < \frac{1}{2}$ and $\alpha \ge 2a)$, the optimum is $p^* = \frac{1}{2}$, $R^* = 1$, $\Pi^* = \frac{1}{4}$.

Result 5 is straightforward and shows that the profits and strategies of Case 1 are the same as those of Case 1B.

A.3.1.2 Case 2: $R \le 1-\alpha$ The profit is given by:

$$\Pi = a\frac{p}{R} + p\left(1-\frac{p}{R}\right) \text{ subject to } R \ge p \text{ and } R \le 1-\alpha$$

The first order condition with respect to R gives $\frac{\partial \Pi}{\partial R} = \frac{p(p-a)}{R^2}$. Therefore if the firm chooses a price $p < a$, the firm can set $R^* = p$ and there is no demand for the premium version. If the firm sets $p = a$ then $R^* \in (a, 1-\alpha)$. In both cases, the profit is $\Pi^* = a$. If the firm chooses a price $p > a$, the firm sets $R^* = 1-\alpha$, and this case is identical to the situation analyzed in Section 2 when $R \leq 1-\alpha$ without the cost of copying: $p^* = \frac{a+1-\alpha}{2}$, $\Pi^* = \frac{(a+1-\alpha)^2}{4(1-\alpha)}$. It is easy to show that $\frac{(a+1-\alpha)^2}{4(1-\alpha)} > a$. Result 6 summarizes Case 2.

Result 6. ($R6$) Suppose $R \leq 1-\alpha$. $\forall c, a, \alpha$, the optimum is $p^* = \frac{a+1-\alpha}{2}$, $R^* = 1-\alpha$, $\Pi^* = \frac{(a+1-\alpha)^2}{4(1-\alpha)}$.

A.3.1.3 Case 3: $p \leq c$, $R \geq 1-\alpha$

The firm chooses p and R in order to maximize:

$$\Pi = a\frac{p}{R} + p(1-\frac{p}{R}) \text{ subject to } R \geq p,\ p \leq c \text{ and } R \geq 1-\alpha.$$

The first derivative of Π with respect to R is $\frac{\partial \Pi}{\partial R} = \frac{p(p-a)}{R^2}$. We break down the analysis of the sign of this derivative according to whether $a \leq c$ or $a > c$.

First, when $a > c$, $p \leq c$ implies $p < a$ and $\frac{\partial \Pi}{\partial R} < 0$ so that $R^* = \max\{1-\alpha, p\} = 1-\alpha$ using Assumption $A1$ with $p < a < 1-\alpha$. The unconstrained optimal price is $\frac{a+1-\alpha}{2} > c$. The set-up of Case 3 constrains the price to $p^* = c$ and the profit is then $\Pi^* = \frac{c(a+1-\alpha-c)}{1-\alpha}$.

Secondly, when $a \leq c$, there are 3 cases to analyze: (A) $p < a$, (B) $p = a$, (C) $p > a$.

Case (A): $p < a$. We use the same argument as above. The price is now constrained by a so that $p^* = a$, with profit $\Pi^* = a$.

Case (B): $p = a$. We have already shown that $\Pi^* = a$ with that price.

Case (C): $p > a$; $\frac{\partial \Pi}{\partial R} > 0$. The firm sets $R^* = 1$. In this situation, the consumer chooses either the outside option or the premium version, the profit is then equal to $p\,(1-p)$. The unconstrained optimal price is $\frac{1}{2}$. However, given that $c < \frac{1}{2}$ implied by Assumption $A3$, the price is constrained to $p^* = c$ and the profit is given by $\Pi^* = (1-c)\,c$.

We now compare $\Pi^* = a$ to $\Pi^* = (1-c)\,c$. The difference between $(1-c)\,c$ and a can be written as the polynomial in c, $-c^2 + c - a$. The two roots are $c_1 = \frac{\left(1-\sqrt{1-4a}\right)}{2}$ and $c_2 = \frac{\left(1+\sqrt{1-4a}\right)}{2}$. It is easy to show that these roots are such that $a < c_1 < \frac{1}{2} < c_2$. Thus for c such that $a \leq c < \frac{1}{2}$, if $c < c_1$, $a > c(1-c)$ and if $c > c_1$, $a < c(1-c)$.

Result 7 summarizes Case 3.

Result 7. Suppose $p \leq c$, $R \geq 1-\alpha$.

($R7.i$) If $0 < c < a$, the optimum is $p^* = c$, $R^* = 1-\alpha$, $\Pi^* = \frac{c(a+1-\alpha-c)}{1-\alpha}$;

($R7.ii$) If $a \leq c \leq \frac{\left(1-\sqrt{1-4a}\right)}{2}$, the optimum is $p^* = a$, $R^* \in (1-\alpha, 1)$, $\Pi^* = a$;

($R7.iii$) If $\frac{\left(1-\sqrt{1-4a}\right)}{2} < c < \frac{1}{2}$, the optimum is $p^* = c$, $R^* = 1$, $\Pi^* = c(1-c)$.

A.3.2 Step2. Comparison between profits

In the second step, we compare all profits in order to determine the equilibrium behavior of the firm.

Result 8. The profit from Case 2 ($R6$) is greater than the profits from Case 3 ($R7$) for all the values of the model.

Proof. ($R6$) vs ($R7.i$): $\frac{(a+1-\alpha)^2}{4(1-\alpha)} - \frac{c(a+1-\alpha-c)}{1-\alpha} = \frac{(2c-a+\alpha-1)^2}{4(1-\alpha)} > 0$.

($R6$) vs ($R7.ii$): $\frac{(a+1-\alpha)^2}{4(1-\alpha)} - a = \frac{(a+\alpha-1)^2}{4(1-\alpha)} > 0$.

($R6$) vs ($R7.iii$): $\frac{(a+1-\alpha)^2}{4(1-\alpha)} - c(1-c)$ has the same sign as the following polynomial in c : $P_1(c) = 4(1-\alpha)(1-c)c - (\alpha - a - 1)^2$ which is negative for $c = 0$. The roots c_1 and c_2 are such that: $0 < c_1 < \frac{\left(1-\sqrt{1-4a}\right)}{2} < c < \frac{1}{2} < c_2$. Thus, $\frac{(a+1-\alpha)^2}{4(1-\alpha)} > c(1-c)$ when $\frac{\left(1-\sqrt{1-4a}\right)}{2} < c < \frac{1}{2}$. □

It remains to compare profits from Case 1 with the profit from Case 2.

($R5.i$) vs ($R6$): $\frac{(1-\alpha)^2+(a-c)^2+2(1-\alpha)(a+c)}{4(1-\alpha)} - \frac{(a+1-\alpha)^2}{4(1-\alpha)} = \frac{(c+2(1-a-\alpha))c}{4(1-\alpha)} > 0$ by Assumption $A1$ ($a < 1-\alpha$). This proves Proposition 1 (B).

($R5.iii$) vs ($R6$): $\frac{(a+1-\alpha)^2}{4(1-\alpha)} - \frac{1}{4}$ has the same sign as the following polynomial in α: $P_2(\alpha) = \alpha^2 - \alpha\,(2a+1) + a(a+2)$, which is positive for $\alpha = 0$. The roots α_1 and α_2 are such that: $0 < 2a < \alpha_1 < 1-a < \alpha_2 < 1$, where $\alpha_1 = a + \frac{1}{2} - \frac{1}{2}\sqrt{1-4a}$. Thus, when $0 < \alpha < \alpha_1$ then $\frac{(a+1-\alpha)^2}{4(1-\alpha)} > \frac{1}{4}$ and when $\alpha_1 < \alpha < 1-a$ then $\frac{(a+1-\alpha)^2}{4(1-\alpha)} < \frac{1}{4}$. Finally, the optimal profit is $\frac{1}{4}$ when ($\alpha > \alpha_1$) and (($\frac{\alpha(a+1-\alpha)}{2-\alpha} \leq c \leq \frac{1}{2}$ and $\alpha \leq 2a$) or ($\frac{\alpha}{2} \leq c \leq \frac{1}{2}$ and $\alpha \geq 2a$)). This proves Proposition 1 (D).

The comparison ($R5.iii$) vs ($R6$) gives us also the following intermediate result: if ($\frac{\alpha(a+1-\alpha)}{2-\alpha} \leq c < \frac{1}{2}$ and $\alpha \leq 2a$) or ($\frac{\alpha}{2} \leq c < \frac{1}{2}$ and $2a < \alpha < a + \frac{1}{2} - \frac{1}{2}\sqrt{1-4a}$) then the optimal profit is $\frac{(a+1-\alpha)^2}{4(1-\alpha)}$.

We have to compare now $(R5.ii)$ vs $(R6)$: $\frac{(a+1-\alpha)^2}{4(1-\alpha)} - \frac{c}{\alpha}\left(1-\frac{c}{\alpha}\right)$ has the same sign as the following polynomial in c: $P_3(c) = \alpha^2(\alpha - a - 1)^2 - 4c(1-\alpha)(\alpha - c)$, which is positive for $c = 0$. When $2a < \alpha < a + \frac{1}{2} - \frac{1}{2}\sqrt{1-4a}$, $P_3(c)$ has no roots and is always positive. In this case $\frac{(a+1-\alpha)^2}{4(1-\alpha)} > \frac{c}{\alpha}\left(1-\frac{c}{\alpha}\right)$. Combining this with the intermediate result above proves Proposition 1 (A).

When $a + \frac{1}{2} - \frac{1}{2}\sqrt{1-4a} < \alpha < 1 - a$, $P_3(c)$ has two roots c_1 and c_2 such that $0 < c_1 < \frac{\alpha}{2} < c_2$. The difficult point here is that the root c_1 may be lower or greater than $\frac{\alpha(a+1-\alpha)}{2-\alpha}$. More precisely, $0 < c_1 < \frac{\alpha(a+1-\alpha)}{2-\alpha}$ when $a < \underline{a}$ where $\underline{a} = \frac{(4-\alpha)(1-\alpha)\alpha}{\alpha^2+8(1-\alpha)}$. Under Assumption 1 the ARPU a is relatively small, that is $a < \underline{a}$. Thus, $\max\{\frac{c}{\alpha}\left(1-\frac{c}{\alpha}\right), \frac{(a+1-\alpha)^2}{4(1-\alpha)}\} = \frac{c}{\alpha}\left(1-\frac{c}{\alpha}\right)$. This concludes the proof of Proposition 1.