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Conference Paper

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Publication date:
2022

Permanent link:
https://doi.org/10.3929/ethz-b-000557257

3rd International Conference on Natural Hazards & Infrastructure
5-7 July 2022, Athens, Greece

Finite element analysis of the dynamic response of a free-standing cylindrical column: A statistical approach

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ABSTRACT

This paper presents a modeling approach for predicting the shake-table response of a rocking and sliding column. A three-dimensional finite element (FE) model was developed and validated statistically against experimental results, which involved testing the column under a set of 115 bidirectional ground motions. The free-standing body had a circular cross-section and was allowed to slide and rock in all directions. Both the rocking body and the shake table were modeled using elastic elements. The contact surface was simulated using Coulomb friction for the tangential behavior and stiff contact for the normal direction. Friction was the main energy dissipation mechanism. Rocking is characterized as a chaotic and unpredictable problem, with experiments being non-repeatable. Therefore, this study employs a statistical approach to validate the numerical results. This is achieved by using the cumulative distribution function (CDF) for the main response quantity (i.e., maximum displacement at the top of the column) instead of comparing the numerical and experimental results one by one for each test (deterministic comparison). It was proved that the model performs poorly in the deterministic validation but demonstrates satisfying agreement with the experimental results when validated statistically. Important modeling parameters, such as the solution time step and the value of the integrated numerical damping, were elucidated through an extensive sensitivity analysis, employing non-linear time-history analyses. It was shown that a small change of the value of these parameters leads to a different individual rocking oscillation but only marginally influences the statistical response.

Keywords: rocking columns, three-dimensional finite element modeling, statistical validation, sensitivity analysis

INTRODUCTION

Rocking structures are the ones that uplift when they are subjected to dynamic excitation. Uplifting occurs when the ratio of the acceleration of the excitation ($\ddot{u}_g$) divided by the gravity acceleration (g) is larger than the slenderness of the block ($\tan\alpha$), provided that the friction coefficient at the base is high enough to prevent sliding (Fig. 1). This uplifting effect acts as a fuse, limiting the inertial forces transmitted to the superstructure. After uplift, a rocking oscillator demonstrates negative stiffness, making the description of such systems significantly different from the well-known elastic single-degree-of-freedom systems.

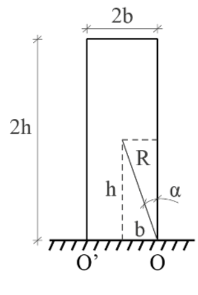

Figure 1. *Geometric properties of a rocking block.*

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The first analytical study of this phenomenon is dated back to 1885 (Makris, 2014). However, contemporary research of rocking structures started after 1963, when Housner published his seminal paper entitled “The behavior of inverted pendulum structures during earthquakes” where two main properties of the rocking structures were elucidated: i) out of two geometrically similar rocking blocks (same α) the larger one (larger R) can survive an excitation which will topple the smaller one; ii) ground motions containing pulses with longer period have a higher overturning potential (Housner, 1963). The rocking oscillator has been used to describe the dynamic behavior of free-standing equipment (Konstantinidis and Makris, 2009, 2010; Di Sarno et al., 2019; D’Angela et al., 2021; Linde et al., 2020), masonry structures (Stefanou et al., 2011; DeJong, 2012; Tondelli et al., 2016; Casapulla et al., 2017; Kalliontzis & Schultz, 2017; Kalliontzis et al., 2020a, 2020b; Mehrotra & DeJong, 2018; Mouzakis et al., 2002) and monumental structures (Funari et al., 2020; Papantonopoulos et al., 2002; Vassiliou and Makris, 2012, Drosos and Anastasopoulos, 2014; Konstantinidis and Makris, 2005). Rocking oscillators could be used as kinematic isolation bearings in bridges or buildings. Such kinematic bearings were implemented in the former USSR and New Zealand. In buildings, to limit the inertial forces transmitted to the superstructure, either a soft-rocking-story (Bantilas et al., 2020; Bachmann et al., 2017, 2019; Dar et al., 2018) or an uplifting wall could be used (Makris and Aghagholizadeh, 2017; Di Egidio et al., 2020). In bridges, kinematic bearings could be used as rocking piers (Makris and Vassiliou, 2013, 2014; Dimitrakopoulos and Giouvanidis, 2015; Giouvanidis and Dimitrakopoulos, 2017b; Thomaidis et al., 2020, 2022; Giouvanidis and Dong, 2020; Kashani et al., 2018; Thiers-Moggia and Málaga-Chuquitaype, 2020; Xie et al., 2019; Zhang et al. 2019). The combination of rocking structures with external dampers, restraining tendons or inerters was suggested in (Makris and Vassiliou, 2015; Vassiliou and Makris, 2015; Makris and Aghagholizadeh, 2019; Pan and Málaga-Chuquitaype, 2020; Thiers-Moggia and Málaga-Chuquitaype 2019, 2020) . The influence of the flexibility of the rocking body was also studied, both analytically and experimentally (Acikgoz and DeJong, 2012; Vassiliou et al., 2014, 2015; Truniger et al. 2015).

The negative stiffness of the rocking oscillator makes it significantly different from the conventional single-degree-of-freedom (SDOF) oscillator, hence, many assumptions commonly used in the seismic design and analysis of SDOF oscillators cannot be applicable to rocking oscillators (DeJong and Dimitrakopoulos, 2014; Dimitrakopoulos and Paraskeva, 2015; Dimitrakopoulos and DeJong, 2012; Dimitrakopoulos and Giouvanidis, 2015; Giouvanidis and Dimitrakopoulos, 2017a, 2018; Kazantzi et al., 2021; Lachanas and Vamvatsikos, 2021; Makris and Konstantinidis, 2003; Makris and Zhang, 2001; Zhang and Makis, 2001; Reggiani Manzo and Vassiliou, 2020, 2021).

The analytical model proposed by Housner describes the planar rocking response of a rigid body when subjected to one-directional excitation. However, under realistic conditions, rocking structures are subjected to bidirectional excitation (or three-directional when the vertical acceleration is considered) (Chatzis & Smith, 2012a, 2012b; Mathey et al., 2016; Vassiliou et al., 2017; Vassiliou, 2018, Konstantinidis and Makris, 2007; Zulli et al., 2012, Bachmann et al., 2019). Under these conditions, an unanchored body may rock, uplift, translate with the ground, and/or wobble. When it is not restrained, it may also slide out of its initial position (Bao & Konstantinidis, 2020).

This study aims at developing a practical three-dimensional finite element model for predicting the response of free-standing cylindrical rocking columns. The accuracy of the proposed model is assessed by statistically comparing numerical and experimental results. The experimental results comprise 115 shake table tests, using a slender steel column with circular cross-section. The large number of shake-table tests allows for such a statistical validation procedure. The specimen was subjected to two-dimensional excitation and was free to slide, rock and wobble in all directions. As the column is free to slide and wobble out of its initial position, its dynamic response is qualitatively similar to free-standing internal building components.

THE STATISTICAL VALIDATION APPROACH

Rocking is often characterized as “chaotic”, in the sense that the experimental response of a rocking oscillator to a single ground motion is oftentimes non-predictable and non-repeatable. Therefore, validating numerical models in a deterministic way is impossible, since no experimental test can be used as a benchmark for the comparison with the numerical model.

Bachmann et al. (2017) and Del Guidice et al. (2020, 2021a, 2021b) claimed that validating a numerical model using a single ground motion is a sufficient but not a necessary validation procedure. The seismic response is inherently stochastic since the excitation is stochastic. Therefore, a statistical (and not a deterministic, one-by-one) validation of the numerical model is proposed, which compares the statistical distributions of the main response quantities of the model and the experiments. This procedure requires an experimental benchmark dataset, where the same (or identical) specimens are excited by an ensemble of spectrally-compatible ground motions. Subsequently, a numerical model is used to create the corresponding numerical dataset, using the same ensemble of excitations. The validity of the numerical model is assessed by comparing the Cumulative Distribution Function (CDF) of these two datasets for the same response quantity (i.e., maximum displacement at the top). This validation test is weaker (and easier to pass), yet adequate for earthquake engineering applications. It is worth mentioning that Yim, Chopra and Penzien where the first ones to state that rocking should be studied in the statistical sense (Yim et al., 1980).

Both the Finite Element (FEM) and the Discrete Element (DEM) method was used in the past for the description of the rocking problem (Agalianos et al., 2017; Thomaidis et al., 2018; Pappas et al., 2017; Sieber et al., 2020). A recent blind prediction contest organized by ETH Zurich, the University of Bristol and the Pacific Earthquake Engineering Research (PEER) Center, shed light on the efficiency of numerical models and the adopted numerical parameters used for modeling a rocking podium structure (Vassiliou et al., 2020). Unlike the tests discussed in this paper, the tests of the blind prediction contest concerned a rocking podium structure that was restrained not to slide or wobble out of its original position. Thirteen contestants participated, using FEM, DEM, and analytical rigid-body models (Zhong & Christopoulos, 2020; Malomo et al., 2020). The main outcomes of the blind prediction were that: i) There is no basis for recommending FEM or DEM to model the response of wobbling structures since the accuracy of these models depends on the calibration of the model and the corresponding modeling assumptions; ii) Using Rayleigh damping to model energy dissipation in rocking structures is both inaccurate and inconsistent to the physical problem; iii) The winning participants accurately captured the Cumulative Distribution Function of the maxima of the responses to each set of excitations, but their accuracy in predicting the response to each individual ground motion was significantly lower.

EXPERIMENTAL BENCHMARK DATASET

The numerical model proposed in the present study was validated using an experimental benchmark dataset, which is described in detail in (Vassiliou et al., 2021), and also briefly presented in this section for reasons of completeness. The experimental campaign was designed at ETH Zurich and performed at EQUALS Lab, University of Bristol (Fig. 2). A total of 115 shake table tests of cylindrical and rectangular free-standing rocking bodies was conducted, with the specimens being free to slide and rock in all directions. The results of these experiments were selected as a benchmark database to assess the efficiency of the numerical model since they include a large number of spectrally-compatible ground motions applied on the same rocking specimens. The rocking specimens were not chosen to represent specific free-standing rocking equipment but a class of free-standing rocking bodies. They were designed to remain elastic after each test, so they could be excited with a large number of earthquake excitations to create a database suitable for statistical validation. The specimens were made of round, hollow steel pipes with different dimensions and slenderness (Fig. 2). The rocking response was induced by a bi-directional dynamic excitation using a shake table. The applied ground motions were synthesized using a spectral version of the Rezaeian and Der Kiureghian stochastic ground motion model (Rezaeian & Der Kiureghian, 2008; Broccardo & Dabaghi, 2017). The 1989 Loma Prieta UCSC Lick Observatory ground motion record was used as a seed ground motion to generate an ensemble of 115 ground motions. The ground motions were time-scaled (i.e., the frequency of ground motions increased by 2) without changing the amplitude. Therefore, in the prototype scale, the columns are 4 times larger. Out of the 3 cylindrical specimens tested in (Vassiliou et al. 2020), the most slender was selected to be modeled in the present numerical study.

NUMERICAL METHODOLOGY

The general-purpose finite element software ABAQUS (Abaqus, 2019) was utilized. The model comprised the cylindrical rocking body, the flat moving base (which simulates the shake table), interface elements between the two aforementioned parts and a rectangular cap plate on top of the column. The interface elements were stiff in the normal direction, allowing no penetration between the rocking column and the base. The lateral response of the interface elements was governed by Coulomb friction, with a friction coefficient of μ = 0.3 (Katsamakas and Vassiliou, 2021, 2022). The translational motion along the z axis and all rotational motions of the base were fixed. The base moved parallel to x and y axis, applying the ground motion (Fig. 3).

The rocking body was separated into two parts with different mesh sizes (Fig. 3). The lower part of the rocking column had an inform mesh of 5 mm, equal to the thickness of the wall of the hollow section. The elements of the mesh of the upper part of the rocking body had dimensions of 5x15x15 mm. The lower and the upper part were connected with a tie constraint, meaning that they move and rotate as one body. The mesh of the base was compatible with the one of the columns close to the contact area and became coarser away from it. This mesh configuration is considered adequate to avoid mesh-related errors and was also proved as time-efficient. 8-node (brick) finite elements were used both for the rocking column and for the flat base. Both the column and the base were modeled with an elastic material since the stresses developed during testing were significantly lower than the yielding point. The modulus of elasticity was set to E = 200 GPa and the Poisson's ratio to v = 0.3. The displacement of the specimen was monitored with a reference point at the top of the column. An implicit integration scheme was utilized. The proposed numerical model considers one main damping mechanism; friction, which is considered through the friction coefficient. Rayleigh damping is set to zero since this energy dissipation mechanism is inconsistent with the physical problem and could make the model inaccurate (Vassiliou et al., 2020). The two critical numerical parameters that affect the accuracy of the model were the solution time step (dt) and the numerical energy dissipation included in the solution algorithm (α_{HHT}) (Hilber et al., 1977). To shed light into the influence of these values on the proposed numerical model, an extensive sensitivity analysis was performed employing non-linear time-history analyses.

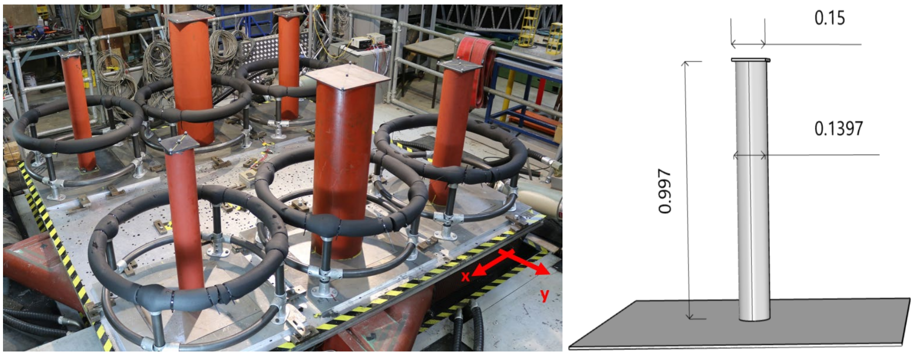

Figure 2. *Left; Free-standing rocking column specimens on the shake table at EQUALS lab, University of Bristol, Right; Schematic representation of the rocking column (dimensions are in meters).*

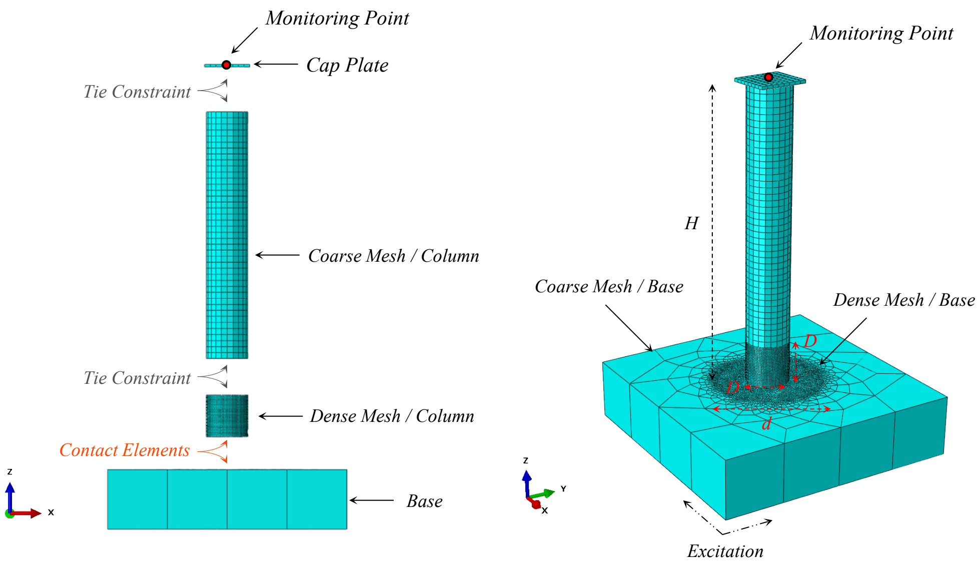

Figure 3. *Description of the developed numerical model.*

Regarding the solution time step (*dt*), the following values were considered in the sensitivity analysis; 0.02, 10^{-3}, 10^{-4} seconds, following the conclusions of (Vassiliou et al. 2017). For the numerical energy dissipation included in the solution algorithm (α_{HHT}) the following values were considered; 0, -0.05, -0.1, -0.2, -0.333. The α_{HHT} parameter controls damping of the solution algorithm by damping-out the high-frequency components of the computed response without markedly affecting the low-frequency response components. Therefore, the α_{HHT} parameter only affects the numerical response in higher vibration modes. The α_{HHT} parameter follows the Hilber-Hughes-Taylor numerical dissipation for time integration algorithms (Hilber et al., 1977). According to the documentation of the software, $\alpha_{HHT} = 0$ gives zero dissipation, whereas $\alpha_{HHT} = -0.333$ gives maximum dissipation. The other values of α_{HHT} correspond to in-between cases of moderate energy dissipation.

RESULTS

Deterministic comparison

When the numerical results are compared to each other one by one, the influence of the time step (*dt*) and the energy dissipation included in the solution algorithm (α_{HHT}) is significant (Fig. 4). Even though correlation between the results of the various models can be observed, no clear trend emerges. A modification of *dt* or α_{HHT} leads to a different maximum displacement (u_{max}), and, oftentimes, to a prediction of overturn, that is not predicted by the other group of analyses. Decreasing the time step (*dt*) leads to a significant increase in the solution time. Increasing α_{HHT} (i.e. less numerical damping) leads to a small increase in the solution time. Based on the above, the numerical model with $dt = 10^{-3}\ s$ and $\alpha_{HHT} = -0.2$ was selected and compared with the experimental results. A relatively loose correlation is observed, with the corresponding correlation coefficient being equal to $R=0.49$ (assuming that overturn is set to a displacement of 250 mm).

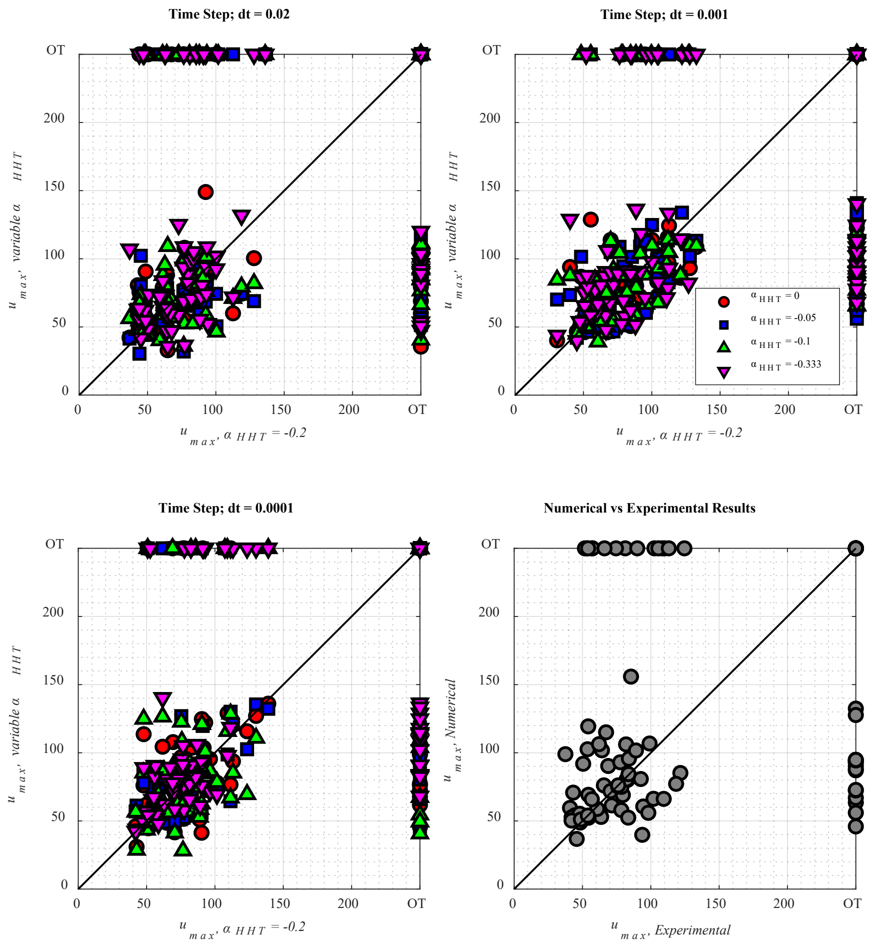

Figure 4. *Deterministic comparison of the numerical results. Top-left, Top-right, Bottom-left; Comparison of the numerical results for different dt and* α_{HHT} *values; Bottom-right; Comparison between experimental and numerical results for* $dt = 10^{-3}\ s$ *and* $\alpha_{HHT} = -0.2$.

Most importantly, the numerical model is oftentimes unable to predict overturning. It is noted that, based on this one-by-one comparison of the experimental and the numerical results, the numerical model seems inaccurate but is not biased since it does not systematically under- or over-predict the response.

Statistical comparison

When the numerical results are statistically assessed using the cumulative distribution function (CDF), clear trends emerge (Fig. 5). This is in agreement to the conclusions of Yim, Chopra and Penzien, who observed similar trends for the planar rocking response (Yim et al., 1980). The influence of the time step *(dt)* is moderate and becomes insignificant when it is smaller than 10^{-3} seconds. Therefore, the use of a time step equal to, or smaller than $dt = 10^{-3}$ seconds is suggested for this specific system. With the time step being small enough, the influence of the selected value of α_{HHT} becomes marginal and practically insignificant. Similarly to the deterministic comparison, the numerical model with $dt = 10^{-3}\ s$ and $\alpha_{HHT} = -0.2$ was selected and compared with the experimental results. The CDF of the numerical model is closely correlated to the experimental one, with the numerical always being inside the 95% CI of the experimental. The probability of overturning of the rocking body is equal to 1 minus the value of the last point of the CDF. For the specific rocking body under the considered group of ground motions, the numerical model gives a probability of overturning equal to 43.5%, whereas the experimental results give 44.4%. Hence, the numerical model accurately predicts the statistical response of the cylindrical column.

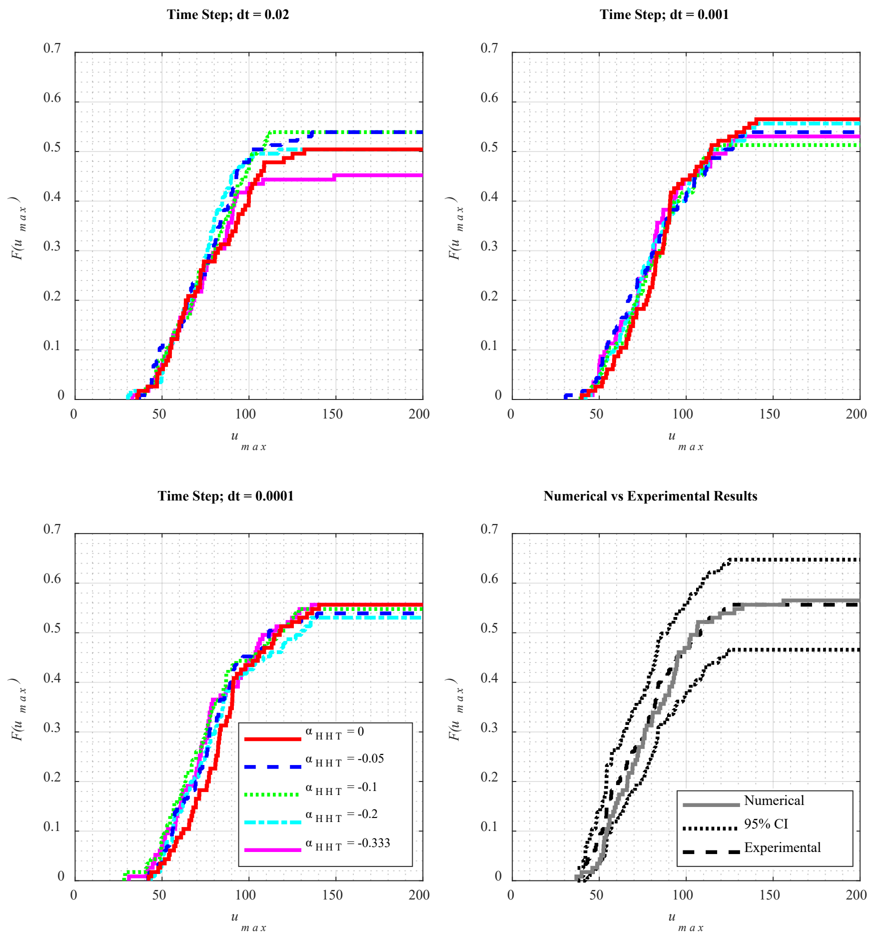

Figure 5. *Statistical comparison of the numerical results. Top-left, Top-right, Bottom-left; Comparison of the numerical results for different dt and* α_{HHT} *values; Bottom-right; Comparison between experimental and numerical results for* $dt = 10^{-3}\ s$ *and* $\alpha_{HHT} = -0.2$.

CONCLUSIONS

The presented numerical model simulates the response of a slender, free-standing cylindrical column. The column was free to slide and rock in all directions, whereas the cylindrical cross-section led to significant wobbling. A three-dimensional numerical model was developed, aiming at predicting the experimentally observed response, in terms of maximum displacement at the top of the column (u_{max}).
The main energy dissipation mechanism of the numerical model was interfacial friction (between the rocking body and the base), which was explicitly considered in the numerical model. The value of the solution time step (*dt*) and of the numerical damping of the solution algorithm (α_{HHT}) were varied numerically and their influence was assessed with a large number of non-linear time-history analyses. These parameters significantly influence the deterministic response of the model; however, their influence on the statistical response is marginal and statistically insignificant. It was shown that the solution time step should be smaller than (or equal to) $dt = 10^{-3}$. If so, the value of α_{HHT} does not affect significantly the statistical response. Based on the above, the following set of parameters was selected; $dt = 10^{-3}$ and $\alpha_{HHT} = 0.2$. When compared to the experimental results, the model performs poorly based on its ability to predict the maximum to an *individual* ground motion. However, it performs well when it is evaluated based on its ability to predict the *CDF* of the maxima of the responses to a set of ground motions. The experimental and the numerical CDF curves were closely correlated, with the numerical always being inside the 95% CI of the experimental.

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