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Arora, Rohit; Sowlati, Taraneh; Mortyn, Joel; Roeser, Dominik; Griess, Verena 

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By Rohit Arora^a, Taraneh Sowlati^a, Joel Mortyn^b, Dominik Roeser^c and Verena C. Griess^d

^aDepartment of Wood Science, the University of British Columbia, Vancouver, BC, Canada; Email: rohita23@mail.ubc.ca and taraneh.sowlati@ubc.ca

^bInventory and Analysis, Western Forest Products Inc, Vancouver, BC, Canada; Email: jmortyn@westernforest.com

^cDepartment of Forest Resources Management, University of British Columbia, Vancouver, BC, Canada; Email: dominik.roeser@ubc.ca

^dDepartment of Environmental System Science, ETH Zürich, Zurich, Switzerland. Email: verena.griess@usys.ethz.ch

Abstract

The competitiveness of forest companies is strongly affected by the costs associated with getting the raw material to the mills. As harvesting costs contribute significantly to this cost, mathematical programming models were developed to optimize the scheduling of harvest activities within and between cut blocks to reduce the overall cost. However, the precedence relationship between harvesting activities occurring concurrently across multiple cut blocks has not been considered in the existing literature. In this paper, a mixed-integer linear programming model is developed to optimize the scheduling of harvesting activities, considering the precedence relationship among harvesting activities. The objective of the model is to minimize the total costs. The model determines the start time and end time of each harvesting activity at each cut block, considering the movement time of machines between cut blocks. The model is applied to the case of a large forest company in British Columbia, Canada. The model's harvesting cost is only 1.37% higher than the lowest possible harvesting cost, and only 3 assigned machines have an idle time. The detailed harvesting schedule is generated based on the start time, the end time, and the operating time for each activity at each cut block.

Introduction

The cost associated with providing sawmills with logs plays a significant role in the competitiveness of forest companies in the global market (Working Forest Staff 2020). The so called “delivered cost of logs” include stumpage cost, harvesting cost, and transportation cost. The Global Timber and Sawmill Cost Benchmarking report released by Forest Economic Advisors (FEA) in 2019 highlighted the fact that regions with a low delivered log costs had a competitive advantage over regions with higher delivered costs of logs (Working Forest Staff 2020). For instance, a reduction in the cost of delivered logs in Germany due to spruce bark beetle increased the exports of lumber by 10% between 2018 and 2019 (Wood Resources International LLC 2019). At the global level, the delivered cost of logs has

increased by 7% from 2016 to 2019. Meanwhile, for some countries such as Lithuania, Latvia, and some regions of Canada and Russia, the value of increase has been more than 20%. For instance, this cost has increased by almost 70% in the coastal part of British Columbia (BC), Canada, during the same period (Girvan and Taylor 2020). Therefore, there is pressure on forest companies in these regions to reduce the delivered cost of logs to remain competitive in the global market (Working Forest Staff 2020).

The harvesting cost accounts for almost 35–50% of the delivered log cost (Visser 2010; Marques et al. 2014; Girvan and Taylor 2020). Forest harvesting includes all the required operations to convert a stand of trees into merchantable logs as per industrial requirements. The first step is to cut the trees, which is known as felling. This operation can be performed manually using a chainsaw, or by use of machines such as a feller-buncher and harvester (Schiess and Krogstad 2004). There are three harvesting systems to handle the felled trees: cut to length, tree length, and whole tree systems. In the cut to length system, the processing of felled trees, i.e., delimbing and bucking, occurs at the felling site, and logs are transported to a landing zone or roadside. In the tree length system, the trees are only topped and delimbed at the felling site, and the rest of the processing takes place at the roadside or landing zone. In the whole tree system, no processing takes place at the felling site. In all systems, logs are usually loaded into trucks using loaders at the landing zone or roadside (MacDonald 1999; Castro et al. 2016).

Machine cost is the major component of the forest harvesting costs, and its contribution has increased up to 70% due to mechanization of harvesting operations in the past few decades (Visser 2010). According to 80% of the harvesting contractors in Interior BC, the cost of harvesting machinery is the biggest challenge (Church 2018). The major portion of the machine cost is the fixed capital cost of the machine (Zhang et al. 2016). To offset this fixed capital cost, it is necessary to have high efficiency and utilization of the machine. Also, most forest companies have a limited number of machines that need to be moved from one harvesting block to another during harvesting. This movement cost accounts for 6–10% of harvesting costs depending on the size of harvest areas and the distance between them (Väätäinen et al. 2006; Conrad 2014; Santos et al. 2019). To improve the utilization of machines while considering the movement of equipment between cut blocks and assignment of harvesting crews, operational level plans are modelled and optimized in the literature (Karlsson et al. 2003; Bredström et al. 2010; Frisk et al. 2016; Victor and Cancela 2018; Santos et al. 2019).

Operational level forest harvest planning determines how harvesting activities are carried out at each cut block and how resources should be allocated to achieve the goals of any higher-level plans such as the amount of harvesting to be done in the next five years. Mathematical models have been developed to tackle this problem, which aim to minimize the harvesting cost (Epstein et al. 2006; Santos et al. 2019), maximize the profit (Vera et al. 2003) or minimize the movement cost of machines (Victor and Cancela 2018).

One group of previous studies considered only one harvesting activity (i.e. yarding) (Vera et al. 2003; Epstein et al. 2006; Legües et al. 2007). These studies assumed that there were sufficient yarding machines, so that no machine had to be moved between cut blocks. Therefore, machine movement was not considered in these papers. The goal was to determine the optimum location for the placement of yarding machines, and the roads to be constructed for the flow of timber from the forest to the final destination in order to minimize the total costs (Epstein et al. 2006) or maximize the net profit (Vera et al. 2003; Legües et al. 2007). Vera et al. (2003) used Lagrangian relaxation to solve this problem, while Epstein et al. (2006) and Legües et al. (2007) used heuristics and metaheuristics methods, respectively, for solving the optimization models.

In another group of studies, machine movement was considered, but harvesting was taken into account in a more general way (Karlsson et al. 2003; Bredström et al. 2010; Frisk et al. 2016; Victor and Cancela 2018; Santos et al. 2019). In all these studies, a limited number of machines was assumed to be available for harvesting, and machines had to move from one cut block to another. Therefore, the cost of machine movement from one cut block to another was considered. Victor and Cancela (2018) addressed the problem of determining the sequence of cut blocks for harvesting to reduce the machine movement costs. The authors defined the objective function as the minimization of total machine movement costs. They developed a mathematical programming model based on the Multi Depot Multiple Traveling Salesman Problem. The model was applied to a case study in Uruguay and was solved using the CPLEX solver (CPLEX 2021). Karlsson et al. (2003) and Bredström et al. (2010) combined crew assignment decisions and machine movement decisions in their work. The primary decisions were to determine which crew should be assigned to each cut block and where machines should move after completing their operation in one cut block. Karlsson et al. (2003) developed a mathematical model for a planning horizon of 6 weeks with weekly time steps. The authors defined harvesting schedules for each crew, which determined the cut blocks to be harvested and the sequence in which the crew had to move from one cut block to another. The model prescribed the optimum schedule for each harvesting crew to minimize the total cost. The authors applied their model to a case study in Sweden. Bredström et al. (2010) tackled the same problem in a slightly different way. They used a 2-phase approach to solve their model. In the first phase, the model assigned the cut blocks to machines, and in the second phase, a route for each machine was generated, similar to the Traveling Salesman Problem. The planning horizon of this model was one year with seasonal time steps. The model was applied to a Swedish forest company to minimize the total costs of harvesting. Frisk et al. (2016) and Santos et al. (2019) integrated the machine movement decisions with detailed harvest scheduling decisions in their work. The primary decisions were to determine the starting and ending time of harvesting at each cut block and where machines should move after completing their operation in one cut block. The objective function of both studies was to minimize the total costs. The planning horizon of detailed scheduling was less than 6

weeks with daily time steps. Frisk et al. (2016) solved the full model using a solution approach based on the decomposition and aggregation technique and applied their model to the case of Swedish companies. Santos et al. (2019) constructed 13 problem instances with increasing levels of complexity to test their model. They solved this model using commercial software tools. In all studies that involved machine movement, it was assumed that machines of all harvesting activities move together as a group from one cut block to another.

A similar harvest scheduling problem is tackled in the agricultural literature, where contractors move their machines from one farm to another farm to perform agricultural harvesting activities. The most common problem encountered by these contractors is the scheduling of machines for executing multiple activities at multiple farms with a precedence relationship between activities. Mathematical programming models were developed in previous studies to schedule agricultural activities that contractors carried out at various farms (Basnet et al. 2006; Orfanou et al. 2013; Edwards et al. 2015; Guan et al. 2018). The objective function of all these studies was to minimize the duration of harvesting at all agricultural farms and the decisions regarding the detailed scheduling and machine movement were included in the models. The models were either applied to a problem generated by authors (Basnet et al. 2006), or a real case study (Orfanou et al. 2013; Edwards et al. 2015; Guan et al. 2018). In all these studies, the authors did not consider any planning horizon and assumed that all agricultural farms had to be harvested completely to make sure that farms were ready for the next agriculture cycle (Guan et al. 2018). However, in forest harvesting, it is not possible to harvest all cut blocks within a short period of time as the duration of forest harvesting is much longer than agricultural harvesting (Basnet et al. 2006).

Although many previous studies focused on operational level planning, some aspects of forest harvesting problems have not been addressed in the literature. Harvesting at the operational level includes many activities, such as felling and yarding. There is a precedence relationship between these activities; for instance, yarding cannot start before felling. Hence, it is important to schedule each harvesting activity at each cut block, considering these precedence relationships in a way that after completing a harvesting activity at a cut block, machines move to the next cut block, while the next activity starts in the current block. These relationships were considered by Corner and Foulds (2005). However, they considered only a single cut block. The objective function of their model was to minimize the time to complete all harvesting activities in the given cut block. Therefore, they did not include any movement of machines in their work.

To the best of our knowledge, no study has addressed the problem of scheduling the forest harvesting activities for multiple cut blocks considering a precedence relationship in the harvesting activities and machine movement for a planning horizon of several months. To overcome this gap in the literature, in this study, we develop a mathematical model to optimize the harvest scheduling at the operational level

for a real case study considering the precedence relationship between harvesting activities and the movement of individual machines of each harvesting activity. The developed model prescribes the starting and ending time for each harvesting activity at each cut block and the movement of machines between the cut blocks. The aim is to minimize the total cost of harvesting, including machine movement cost, and machine idle time cost.

Materials and methods

Case study

The harvesting operations of a forest company in coastal BC is considered as the case study for this research. The forest company owns six sawmills and has a production capacity of more than 2.6 million m³. It has an annual allowable cut (AAC1) of 6 million m³ and performs harvesting on 500 to 700 cut blocks every year. The forest company uses the clear cutting method for harvesting. Harvesting of about 50% of the cut blocks is completely contracted out, with contractors being responsible for all harvesting activities. For the rest of the cut blocks, the forest company either performs the harvesting activities using its own machines or contracts out some of the harvesting activities to small contractors. The case study for this study comprises just over 100 cut blocks, which are connected by a road network. The distance between cut blocks is calculated using the GIS files. The size of cut blocks varies from 0.9 ha to 54.5 ha. The availability of cut blocks for harvesting depends on the time of year. For instance, higher elevation cut blocks are available only during the summer season.

The key activities of forest harvesting include felling, yarding, processing and loading. Each activity is described in detail below:

- Felling: Stands are felled either manually or mechanically. Around 80% of the felling in the case study area is done manually due to steep terrain and the large size of trees. Mechanical felling is commonly used on gentle slopes (0% to 35%) in the second-growth forests.
- Yarding: After felling, the trees are transported to the landing zone. This is achieved using three systems: ground-based, cable-based, and aerial-based yarding systems. The ground-based system is mostly used on gentle slopes (0% –<35%). Around 30–40% of yarding in the case study area is performed by the ground-based system.
- The cable-based yarding system is used on moderate (>35%–60%) and steeper (greater than 60%) slopes. Around 50–60% of yarding in the case study area is done by a cable-based system. The aerial-based yarding is used in areas with no road access. Aerial-based systems are used for 5–10% of yarding in the case study area. The yarding productivity is affected by tree size and yarding distance.

- **Processing:** Felled trees are processed at landing sites or at stump sites. Processing includes two steps: delimbing, which includes removing the top and branches from felled trees, and bucking, which includes cutting the trees into logs of pre-determined lengths. The processing of felled trees is performed manually or using machines. Manual processing is performed at the stump site before yarding, whereas mechanical processing is performed at the landing zone after yarding.
- **Loading:** The last activity is loading the logs onto trucks. It is performed by swing loaders. The productivity of the loading machines is affected by the space at the landing site and the availability of trucks.

There are precedence relationships among harvesting activities; for instance, yarding cannot start before felling. The forest company has a limited number of machines for harvesting, and machines of each harvesting activity move from one cut block to another after the activity is done. An activity has to be completely finished in any cut block before the machine can move to the next cut block. Ground-based machines move on their own between cut blocks for a distance of less than one km, but for other machines, a low-bed trailer is used for movement. All cut blocks are clearcut harvested. The number of machines available for each harvest activity is shown in Table 1.

The forest company intends to optimize the scheduling of harvesting activities to determine each harvesting activity's start and end time at each cut block. This work aims to address this problem by developing a mathematical model for scheduling harvesting activities. The time horizon of this model is three months with weekly time steps.

Mathematical problem formulation

The problem is formulated as a mixed-integer linear programming model as it involves both continuous and integer decision variables. The following sections explain the different components of the mathematical programming model. The sets, parameters and decision variables in the mathematical programming model are shown in Table 2.

Table 1: machines available for harvesting activities (source: oral communication J. Mortyn)

Activity number	Harvesting activity	Number of machines
1	Manual felling and processing	28
2	Mechanical felling	1
3	Ground yarding	7
4	Cable yarding	22
5	Aerial yarding	2
6	Mechanical processing	6
7	Loading	37

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Table 2: Model sets, parameters, and decision variables

Sets	
$i \in I$	Set of cut blocks (0, 1, ..., I)
$k \in K$	Set of harvesting activities (1, 2, ..., K)
$j \in J$	Set of machines (0, 1, ..., J)
$I_k \in I$	Set of cut blocks for activity k
$I_{kp} \in I$	Set of cut blocks where activity k was started in the previous planning horizon
$I_{kn} \in I$	Set of cut blocks where activity k will start in this planning horizon
J_k	Set of machines for activity k
Parameters	
α_{ijk}	Time needed by machine j to perform activity k at cut block i (in weeks)
β_{ijk}	Operating cost for machine j to perform activity k at cut block i (\$/week)
$\gamma_{i'i}$	Distance between cut block i' and i (in km)
δ_k	Speed of movement of machines of activity k (km/week)
λ	Movement cost (\$/km)
τ_{ik}	Penalty cost if activity k has not started during the planning horizon in cut block i
θ_{ijk}	Penalty cost for the extra time required after the planning horizon by machine j to perform activity k at cut block i (\$/week)
η_j	Idle time cost for machine j (\$/week)
π_j	Fixed cost if machine j is moves from the depot (\$)
σ	Planning horizon (in weeks)
μ_{ik}	Minimum time lag required between activity k and the activity proceeding activity k for cut block i (in weeks)
ϕ_{ijk}	Binary parameter. Equals 1 if machine j required extra time after the previous planning horizon to complete phase k at cut block i
ω_{ijk}	Extra time required by machine j after the previous planning horizon to complete phase k at cut block i
M	A large number used in constraints involving binary decision variables
Decision Variables	
X_{ijk}	Start time of machine j to perform activity k at cut block i
Y_{ijk}	End time of machine j to perform activity k at cut block i

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$Z_{i'jk}$	Binary variable will take value 1 if machine j moves from cut block i' to i to perform harvesting activity k , otherwise it will be 0
U_{ik}	Binary variable will take value 1 if activity k has not started at cut block i during the planning horizon, otherwise it will be 0
V_{ijk}	Extra time required after the planning horizon by machine j to perform activity k at cut block i
P_{ijk}	Binary variable will take value 1 if machine j requires extra time after the planning horizon to perform activity k at cut block i
S_j	Start time of operation for machine j
E_j	End time of operation for machine j
I_j	Idle time for machine j

Assumptions

Assumptions for the model development are as follows:

- There are seven harvesting activities
- Each activity uses exclusive machines.
- Cut block 0 is the virtual depot from which machines can move to any cut block.
- The start time for machines performing harvesting activities 1 and 2 is zero because the machines move from the depot, and the travel time between the depot and cut blocks is zero.
- One machine can be assigned to each activity at each cut block.
- For each activity, a subset of cut blocks is defined in which that activity is performed.
- Manual felling and manual processing are conducted by the same person with the same chainsaw.
- For cable yarding, a machine refers to a team of a cable yarder and a loader for stacking.
- For ground yarding, stacking is performed by the hoe chucker itself.
- For aerial yarding, both yarding and loading take place simultaneously, so no separate loader is required for stacking.
- An activity can only begin after all activities that precede it at that cut block have been completed or have started. For instance, loading is preceded by ground yarding, aerial yarding, and cable yarding in case of manual felling. Thus, if in the same cut block, ground yarding and cable yarding take place, then the loading can only start after both types of yarding are completed in that cut block.

Objective function

The objective function of the model is to minimize the total cost and is represented by equation (1). The components of the total cost are represented by equations (2) to (7).

$$\text{Minimize } Z = OC + MC + PC + OTC + IC + FC \quad (1)$$

or

Minimize (Totalcosts) = Minimize (Operating cost of machines + Movement cost of machines + Penalty cost for not performing an activity + Cost of operating after planning horizon + Cost of idle time of machine + Fixed cost of using machines)

$$\text{Operating costs of machines} = \sum_{i \in I} \sum_{k \in K} \sum_{j \in J_k} (Y_{ijk} - X_{ijk}) * \beta_{ijk} \quad (2)$$

$$\text{Movement cost of machines} = \sum_{i, i' \in I, i' \neq i} \sum_{k \in K} \sum_{j \in J_k} Z_{i'ijk} * \gamma_{i'i} * \lambda \quad (3)$$

$$\text{Penalty cost for not performing an activity} = \sum_{i \in I} \sum_{k \in K} U_{ik} * \tau_{ik} \quad (4)$$

$$\text{Cost of operating after planning horizon} = \sum_{i \in I} \sum_{k \in K} \sum_{j \in J_k} (V_{ijk} + P_{ijk}) * \theta_{ijk} \quad (5)$$

$$\text{Costs of idle time of machines} = \sum_{j \in J} I_j * \eta_j \quad (6)$$

$$\text{Fixed cost of using machines} = \sum_{k \in K} \sum_{j \in J_k} \sum_{i \in I - \{0\}} Z_{j0ik} * \pi_j \quad (7)$$

Constraints

In this section, constraints of the model are explained. They are represented by equations (8)-(27).

Constraint set (8) ensures that machine j can move from the depot to only one cut block to perform harvest activity k . In case machine j is not moved to any cut block during the entire planning horizon, then $\sum_{i \in I_k - 0} Z_{0ijk}$ will remain zero.

$$\sum_{i \in I_k - 0} Z_{0ijk} \leq 1, \forall j \in J_k, k \in K \quad (8)$$

Constraint set (9) is added to ensure that at cut block i , only one machine can arrive to perform activity k . In case activity k is not performed at cut block i during the entire planning horizon, then $\sum_{i' \in I_k, i' \neq i} \sum_{j \in J_k} Z_{i'ijk}$ will remain zero.

$$\sum_{i' \in I_k, i' \neq i} \sum_{j \in J_k} Z_{i'ijk} \leq 1 \quad \forall i \in I_k - 0, k \in K \quad (9)$$

Constraint set (10) ensures that machine j can only move from cut block i to cut block h to perform activity k , if it had already moved from cut block i' to cut block i before.

$$\sum_{i' \in I_k - 0, i' \neq i} Z_{i'ijk} \geq \sum_{h \in I_k - 0, h \neq i} Z_{ihjk}, \forall j \in J_k, i \in I_k, k \in K \quad (10)$$

Constraint set (11) guarantees that machine j performing phase k at cut block i at the end of the previous planning horizon will move from depot to the same cut block at the start of the current planning horizon. In case machine j is working at a cut block at the end of the previous planning horizon, then ϕ_{ijk} is 1 and will force Z_{0ijk} to be 1 and X_{jik} to be zero.

$$X_{jik} \leq -M * (\phi_{ijk} - Z_{0ijk}) \quad \forall j \in J_k, i \in I_{kp}, k \in K \quad (11)$$

Constraint set (12) indicates that the start time for machine i to perform activity 1 or activity 2 on cut block j is zero. When machine j moves from the depot to cut block i to perform harvest activity 1 or activity 2, the value of Z_{0ijk} will be 1. Therefore, the start time of machine j will be forced to be zero.

$$X_{jik} \leq M * (1 - Z_{0ijk}) \quad \forall j \in J_k, i \in I_k, k \in \{1, 2\} \quad (12)$$

Constraint set (13) specifies that the start time of machine j for harvest activity k at cut block i is zero, if machine j has not moved to cut block i . When machine j has not moved to cut block i to perform activity k , then $\sum_{i' \in I_k - 0, i' \neq i} Z_{i'ijk}$ will be zero. Therefore, the start time of machine j will be forced to be zero.

$$X_{ijk} \leq M * \sum_{i' \in I_k - 0, i' \neq i} Z_{i'ijk} \quad , \quad \forall j \in J_k, i \in I_k, k \in K \quad (13)$$

Constraint set (14) states that the start time of machine j at cut block i for activity k is greater than the end time of machine j for activity k at cut block i' plus the machine movement time between cut blocks i and i' . In case machine j does not move to cut block i to perform activity k , then the right-hand side will take a large negative value, and this constraint will remain true because in that case, X_{ijk} will take value zero as per equation 13.

$$X_{ijk} \geq (Y_{i'jk} + (\gamma_{i'i} / \delta_k)) - M(1 - Z_{i'ijk}), \quad \forall j \in J_k, i \in I_{kn}, i' \in I_k, i' \neq i, k \in K \quad (14)$$

Constraint set (15) makes sure that if machine j requires extra time after the planning horizon to complete activity k at cut block I , then the end time for it is equal to the extra time required by machine j after the previous planning horizon to complete phase k at cut block i .

$$Y_{ijk} = \omega_{ijk} \quad \forall j \in J_k \quad k \in K, i \in I_{kp} \quad (15)$$

Constraint set (16) ensures that in case machine j moves to cut block i to perform activity k , then the end time of machine j to perform activity k is equal to the start time of machine j at cut block i plus the time needed by machine j to perform activity k at cut block i . On the other hand, in case machine j does not move to cut block i , then $\sum_{i' \in I, i' \neq i} Z_{i'ijk} \alpha_{ijk}$ will become zero, therefore, the end time of machine j is forced to be equal to the start time of machine j .

$$Y_{ijk} = X_{ijk} + \sum_{i' \in I, i' \neq i} Z_{i'ijk} \alpha_{ijk}, \quad \forall j \in J_k, k \in K, i \in I_{kn} \quad (16)$$

Constraint set (17) indicates that the starting time of machine j for activity k at cut block i is greater than the ending time of activity k' which precedes activity k at cut block i plus the minimum time required between activity k and k' . In case machine j does not move to cut block i to perform activity k , then the right-hand side of the constraint will take a large negative value, and this constraint will remain true because in that case, X_{ijk} will take a value of zero as per equation 12

$$X_{ijk} \geq \left(\sum_{j' \in J_k} Y_{ij'k'} + \mu_{ik} \right) - M * \left(1 - \sum_{i' \in I_k, i' \neq i} Z_{i'ijk} \right) \quad \forall i \in I_{kn}, k \in (3, 4, \dots, 8), j \in J_{k+1}, k' \in (K \text{ preceding } k) \quad (17)$$

Constraint set (18) guarantees that machine j can move to cut block i to perform activity k only if machine j' moved to cut block i to perform activity k' in the past. In case no machine j' has moved to cut block i to perform activity k , then $\sum_{i' \in I_k, i' \neq i} \sum_{j' \in J_k} Z_{i'ijk}$ will become zero and it will force $\sum_{i' \in I_k, i' \neq i} Z_{i'ijk+1}$ to remain zero.

$$\sum_{i' \in I_k, i' \neq i} Z_{i'ijk+1} \leq \sum_{i' \in I_k, i' \neq i} \sum_{j' \in J_k} Z_{i'ijk} \quad \forall j \in J_{k+1}, i \in I_{kn}, k \in (3, 4, \dots, 8), k' \in (K \text{ preceding } k) \quad (18)$$

However, this constraint may not work properly if there is more than one preceding activity at cut block i . For cut blocks that have m preceding activities for activity k the updated constraint is shown in equation (19).

$$m * \sum_{i' \in I_k, i' \neq i} Z_{i'ijk+1} \leq \sum_{k' \in (K \text{ preceding } k)} \sum_{i' \in I_k, i' \neq i} \sum_{j' \in J_k} Z_{i'ijk} \quad \forall j \in J_{k+1}, i \in I_{kn}, k \in (3, 4, \dots, 8) \quad (19)$$

Constraint set (20) states that the start time of machine j at cut block i to perform activity k is less than or equal to the planning horizon of the problem.

$$X_{ijk} \leq \sigma \quad \forall j \in J_k, i \in I_k, k \in K \quad (20)$$

Constraint set (21) ensures that if machine j has not moved to cut block i to perform activity k during the planning horizon, then the binary variable associated with the penalty of not starting activity k at cut block i will become 1. When no machine is moved to a cut block, then $\sum_{i' \in I_k, i' \neq i} \sum_{j' \in J_k} Z_{i'ijk}$ will become zero, and therefore it will force U_{ijk} to take the value of 1.

$$U_{ik} = 1 - \sum_{i' \in I_k, i' \neq i} \sum_{j' \in J_k} Z_{i'ijk}, \quad \forall j \in J, k \in K \quad (21)$$

Constraint set (22) calculates the extra time required by machine j to perform activity k at cut block i after the planning horizon σ . In case the ending time of machine j to perform activity k at cut block i is less than the planning horizon, then the $Y_{ijk} - \sigma$ will become negative, and V_{ijk} will take the value of zero.

$$V_{ijk} \geq Y_{ijk} - \sigma \quad \forall j \in J_k, i \in I_k, k \in K \quad (22)$$

Constraint set (23) calculates the value of P_{ijk} which will take the value of 1, if machine j requires extra time after the planning horizon to perform activity k at cut block i . In case the value of V_{ijk} is zero, then equation (23) will force P_{ijk} to be zero.

$$V_{ijk} \leq M \cdot P_{ijk} \quad \forall j \in J_k, i \in I_k, k \in K \quad (23)$$

Start time of operations of a machine: Constraint set (24) determines the start time of operations of machine j . In case machine j moves from the depot to cut block i to perform activity k , then the value of S_j will be less than or equal to the start time of machine j to perform activity k at cut block i . On the other hand, in case machine j does not move from depot to cut block i to perform activity k , then S_j will be forced to take the value of zero as per equation (26).

$$S_j \leq X_{ijk} + M \cdot (1 - Z_{0ijk}) \quad \forall i \in I_k, k \in K, j \in J_k \quad (24)$$

Constraint set (25) determines the end time of operations of machine j . The end time of operations of machine j will be greater than or equal to the ending time of machine j to perform activity k at the last cut block it performed its operation. In case the machine has not moved to any cut block, then the right-hand side will become zero, and E_j can take any value, but the objective function of minimizing cost will force it to be zero.

$$E_j \geq Y_{ijk} \quad \forall i \in I_k, k \in K, j \in J_k \quad (25)$$

Constraint set (26) will ensure the start time of machine j remains zero if it has not moved from the depot.

$$S_j \leq M \cdot Z_{0ijk} \quad \forall i \in I_k, k \in K, j \in J_k \quad (26)$$

Constraint set (27) calculates the idle time of machine j . For machine j , idle time I_j will be equal to the difference between the total time when the machine was in operation given by $(E_j - S_j)$ and the actual operating time of the machine given by $\sum_{i \in I_k} \sum_{k \in K} (Y_{ijk} - X_{ijk})$ plus the total equipment movement time given by $\sum_{i, i' \in I_k, i' \neq i} \sum_{k \in K} \sum_{j \in J_k} Z_{i'ijk} \cdot (\gamma_{i'i} / \delta)$

$$I_j = (E_j - S_j) - \sum_{i \in I_k} \sum_{k \in K} (Y_{ijk} - X_{ijk}) - \sum_{i, i' \in I_k, i' \neq i} \sum_{k \in K} \sum_{j \in J_k} Z_{i'ijk} \cdot (\gamma_{i'i} / \delta) \quad \forall i \in I_k, k \in K, j \in J_k \quad (27)$$

Equations (28) and (29) show the range of decision variables

$$X_{ijk}, Y_{ijk}, V_{ijk}, S_j, E_i \in \mathbb{R}^+ \quad (28)$$

$$U_{jk}, Z_{i'ijk} \in \{0,1\} \quad (29)$$

Data and input parameters

The data on machine cost, productivity, distances between cut blocks and cut block sizes were provided by the forest company. The volume of standing timber in cut blocks varies between 500 m³ to 29,500

m3. The weekly productivity (m3/ week) and the cost of machines (\$/week) used for different harvesting activities were given to us as confidential information. The productivity of each machine varies based on the type of cut block and productive hours of machines as different machines have different utilization rates. The cost of machines includes labor cost, fuel cost and maintenance cost. The capital cost of machines is not included in the operating cost as this is a short-term planning model. The capital cost is considered in strategic and tactical level planning tools used by the forest company for harvest scheduling. The operating time for each machine in a cut block is calculated based on the cut block's volume and the machine's productivity. The penalty cost of not performing an activity at a cut block is assumed to be higher than the highest operating cost to ensure that the model opts for harvesting whenever there is a choice. The cost of completing a harvesting activity after the planning horizon is assumed to be equal to the operating cost of the machine plus 10 \$CAD/ week. In this way, the priority will be given to cut blocks that can be finished within the planning horizon. The fixed cost of machines is a one-time cost that occur when a machine leaves the depot. The value of this cost depends on the harvesting activity that the machine performs. The nominal fixed cost of machines that were applied for each harvesting activity is shown in Table 3. This cost is added so that a minimum number of machines will be used for harvesting which was an important factor for the forest company. The other parameters of the model are shown in Table 4.

Table 3: Fixed costs of using machines for each harvesting activity.

Harvesting activity	Fixed cost (CAD\$/machine)
Manual felling and processing	10
Mechanical felling	5,000
Ground yarding	5,000
Cable yarding	5,000
Aerial yarding	10,000
Mechanical processing	5,000
Loading	5,000

Table 4: Other model parameters

Parameter	Value
Planning horizon (weeks)	12
Idle time cost (CAD\$/week)	50% of operating cost
Movement cost (CAD\$/km)	20

Execution

The developed mixed-integer linear programming (MILP) model was built and executed using the AIMMS 4.77 software (AIMMS 2022). The model was executed on a desktop computer with an Intel® core™ i7-6700 CPU @ 3.41 GHz processor and 16.0 GB RAM. The model was solved using the CPLEX solver (CPLEX 2021). CPLEX uses a branch and bound algorithm to solve the MILP model. The model run involved 30 cut blocks and 103 machines, which included 28 manual fellers. The model included 8,423 non-negative continuous decision variables, 76,538 integer decision variables, and 99,581 constraints.

Three different cases are considered here to assess the results of the model in terms of operating cost and penalty cost as there is no information regarding those costs or any similar study in the past to compare the results. The three cases are explained below:

The ideal operating cost case: The forest company has a number of machines for each harvesting activity, as shown in Table 1. However, the machines are not identical and have different operating costs (\$CAD/m³). In the ideal case, it is assumed that the operating cost (CAD\$/m³) of all the machines for each harvesting activity is equal to the lowest operating cost (CAD\$/m³) of the machines for that harvesting activity. This case provides the lowest possible operating cost of harvesting performed during the planning horizon, if all the machines had the lowest cost. It can be a good benchmark for comparing the operating cost results.

The average operating cost case: In this case, it is assumed that operating cost (CAD\$/m³) of all the machines for each harvesting activity is equal to the average operating cost (CAD\$/m³) of machines that are available for that harvesting activity. This case provides the average operating cost of harvesting performed during the planning horizon, if all the machines had the average cost.

The ideal penalty cost case: In this case, it is assumed that each activity can start at the earliest time at each cut block. In other words, machines of the highest productivity are always available at the cut blocks, and there is no delay in harvesting due to unavailability of machines. This case provides the

maximum number of harvesting activities that can start in the planning horizon or the lowest value of penalty cost.

Results

The AIMMS software found the integer solution after about 4 million iterations in 3.5 hours with an optimality gap of 2.97%. In the best integer solution, only 47 out of 103 machines are used for harvesting. Only 3 out of 47 assigned machines have an idle time. The total cost for the best integer solution calculated by AIMMS is CAD\$ 9,024,876 (Table 5).

Table 6 shows the total operating cost, volume, and the operating cost per volume for each harvesting activity. The manual felling and processing have the highest operating cost because they have started in all cut blocks within the planning horizon. The volume of trees that are felled and processed manually in the planning horizon is 78,923 m³. The total operating cost of aerial yarding is the second highest even though its volume is lower than other harvesting activities such as ground yarding and loading. This high cost is due to the operating cost per m³ (71.54 CAD\$) of aerial yarding being significantly higher than that of other harvesting activities. The total operating cost for other harvesting activities is significantly lower than that for the manual felling and aerial yarding, because in many cut blocks these activities have not started within the planning horizon. The mechanical processing has the lowest total cost because the volume of trees that are mechanically processed in the planning horizon is only 4,104 m³.

Table 5: Components of total costs for a 12-week planning horizon

Cost component	Value (CAD\$)	Percentage of total (%)
Operating cost of machines	2,292,247	25.40
Movement cost of machines	26,642	0.30
Penalty cost for not performing an activity	5,772,478	63.96
Cost of operating after the planning horizon	832,018	9.22
Cost of idle time of machines	1,211	0.01
Fixed cost of using machines	100,280	1.11
Total cost	9,024,876	100

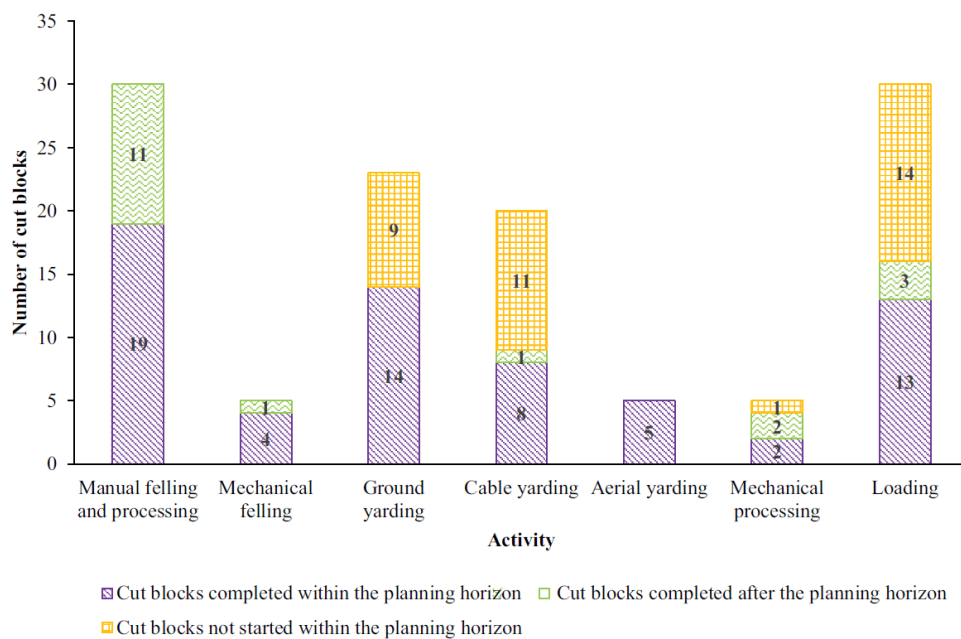
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Table 6: Operating costs for each harvesting activity for a 12 week planning horizon.

Activity	Operating cost (CAD\$)	Volume (m3)	Operating cost per volume (CAD\$/m3)
Manual felling and processing	966,318	78,923	12.24
Mechanical felling	108,533	20,594	5.27
Ground yarding	118,635	23,240	5.10
Cable yarding	295,212	12,228	24.17
Aerial yarding	715,385	10,000	71.54
Mechanical processing	21,630	4,104	5.27
Loading	66,537	23,466	2.84

All assigned harvesting activities are completed within the planning horizon for 13 cut blocks (Figure 1). Activities that have started but not completed during the planning horizon include: manual felling and processing in 11 cut blocks, mechanical felling in 1 cut block, cable yarding in 1 cut block, mechanical processing in 2 cut blocks, and loading in 3 cut blocks.

Figure 1: Harvesting details in 12-week planning horizon



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The operating cost of machines from the model's output is compared with the ideal operating cost case and the average operating cost case for the same amount of harvesting (Figure 2). The cost based on the model is 1.37% (31,596 CAD \$) higher than that of the ideal operating cost case because machines with the lowest operating cost are limited, and it is not possible to achieve the ideal operating cost. For instance, for ground yarding, only one out of seven machines have the lowest operating cost, and for cable yarding, only two out of 22 machines have the lowest operating cost. Therefore, in order to complete these activities at the assigned cut blocks within the planning horizon, machines with higher operating costs must be used. In comparison with the average operating cost case, the model's cost is 10.24% lower.

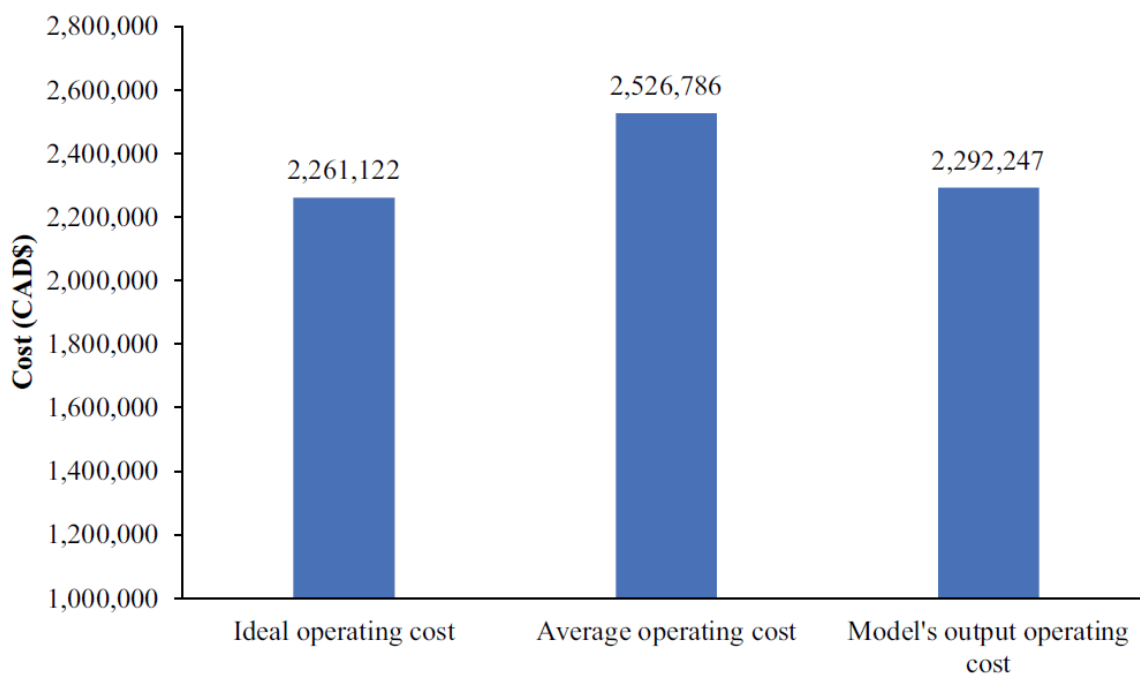


Figure 2: Comparison of operating costs for harvesting performed during the planning horizon (CAD\$).

The harvesting activities that started during the planning horizon as per the best integer solution are compared with the harvesting activities started in the ideal penalty cost case (Figure 3). The harvesting activities started in the model is equal to the harvesting activities started in the ideal penalty cost case for all harvesting activities except the ground yarding. The number of cut blocks in which ground yarding started as per model output is lower than the ideal penalty cost case because in one of the cut blocks, i.e., C14, the preceding activity for ground yarding, i.e., mechanical felling, is completed within the planning horizon in the ideal penalty cost case. In contrast, according to the model's output, extra time is required after the planning horizon to complete the mechanical felling in cut block C14. This disparity occurs because in the ideal penalty cost case, we assume that each activity can start at the

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earliest time, i.e., zero in the case of mechanical felling. As a result, mechanical felling is finished in all cut blocks within the planning horizon in the ideal penalty cost case. However, it is practically not possible to mechanical fell all cut blocks, i.e., 27,935 m³, within the planning horizon with only one feller-buncher. Therefore, it can be concluded that the penalty cost (PC) calculated by the model is optimum and cannot be decreased any further with available machines.

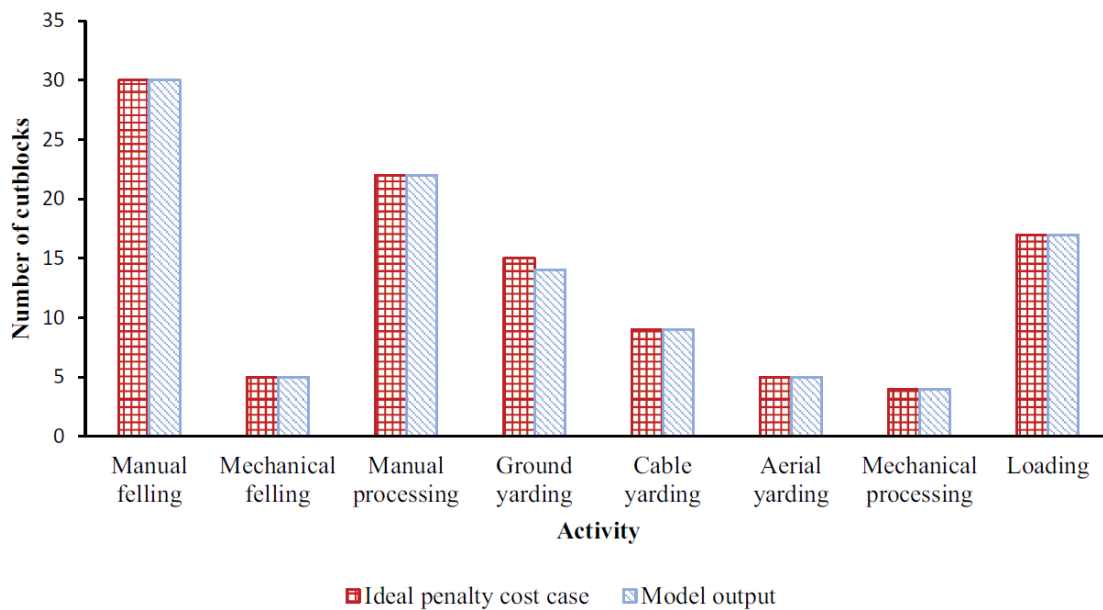


Figure 3: Comparison of the amount of harvesting started during the planning horizon.

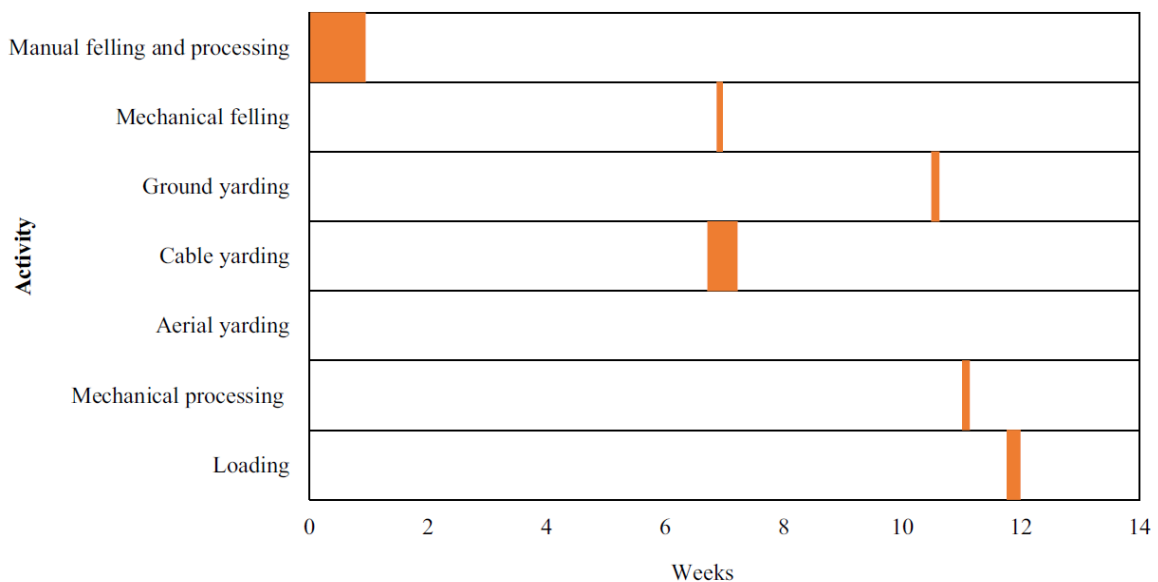


Figure 4: Detailed schedule of cut block C12.

The detailed schedule for each cut block is generated based on the output of the optimization model. The values of start time, the end time, and the operating time for each activity at each cut block are used to generate a detailed schedule for each cut block. The following paragraphs show the detailed schedules derived for two cut blocks: cut block C12 and cut block C24. Similar schedules are generated for all cut blocks but are not shown here for the sake of brevity.

The detailed schedule of cut block C12, in which both manual and mechanical felling takes place, is shown in Figure 4. The trees that are felled manually are processed at the stump site and then they are moved to the landing zone using cable yarding, whereas the trees that are felled mechanically are first moved to the landing zone using ground yarding machines and then they are mechanically processed at the landing zone. The scheduling result for this cut block is in accordance with these precedence relationships. In this cut block, loading is preceded by mechanical processing and cable yarding. As a result, loading does not start until both preceding activities are completed in the cut block.

In cut block C24, all harvesting activities cannot be completed within one planning horizon (Figure 5). It requires an additional 1.15 weeks after the planning horizon 1 to complete the loading activity. In the second planning horizon, loading starts from time 0 in the cut block and finishes in 1.15 weeks. The detailed schedule for each machine is generated based on the output of the optimization model. The values of start time, the end time, the operating time at each cut block, and the movement time between cut blocks are used to generate the detailed schedule for each machine. For instance, the detailed schedule of machine M29 is depicted in Figure 6. This machine finishes its operation in the assigned five cut blocks within the planning horizon with zero idle time. Similar detailed schedules are generated for all machines but are not shown here for the sake of brevity.

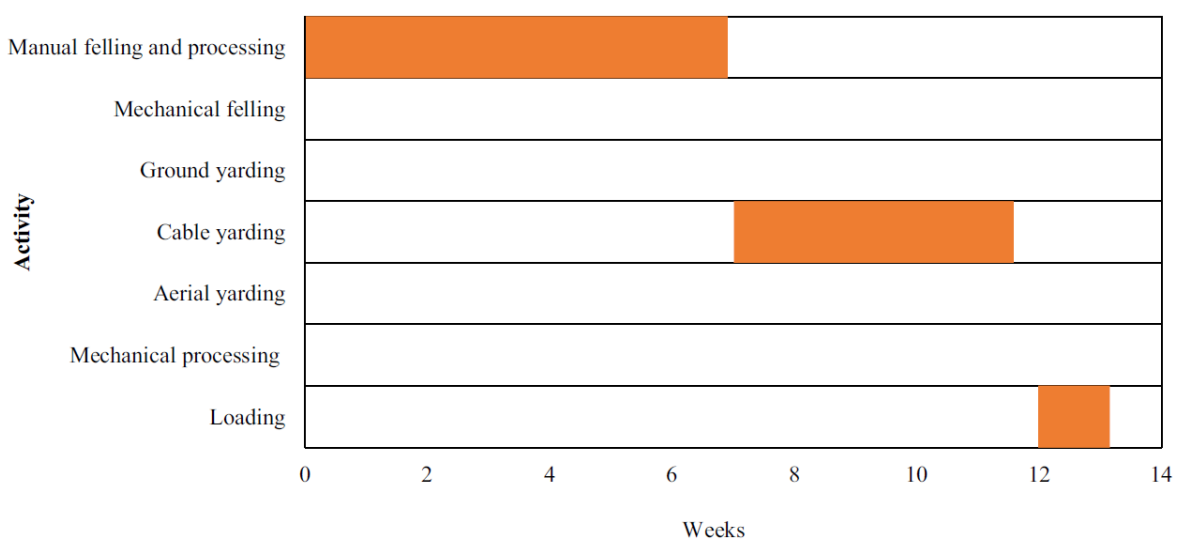


Figure 5: Detailed schedule of cut block C24 in planning horizon 1.

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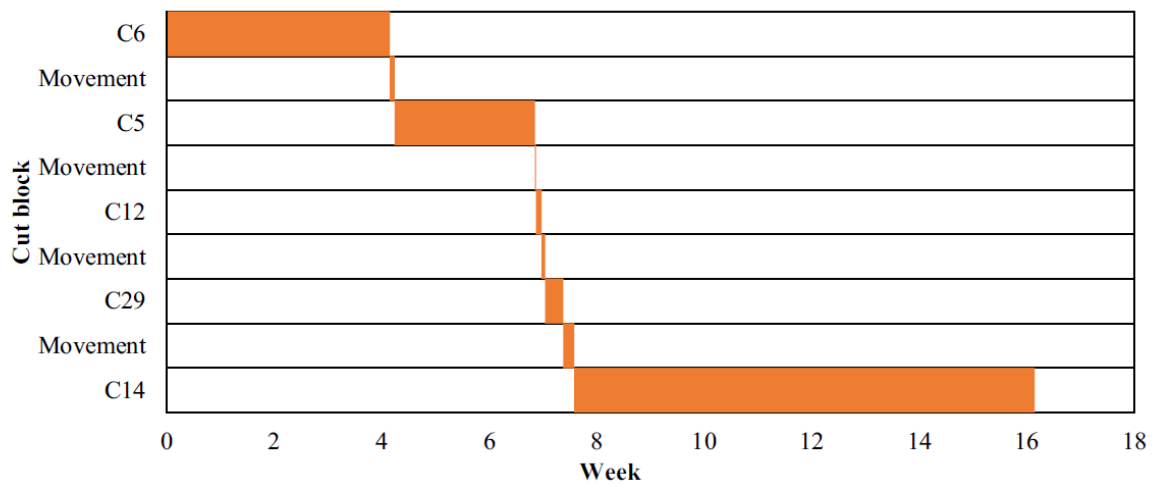


Figure 6: Detailed schedule of machine M29 (mechanical feller).

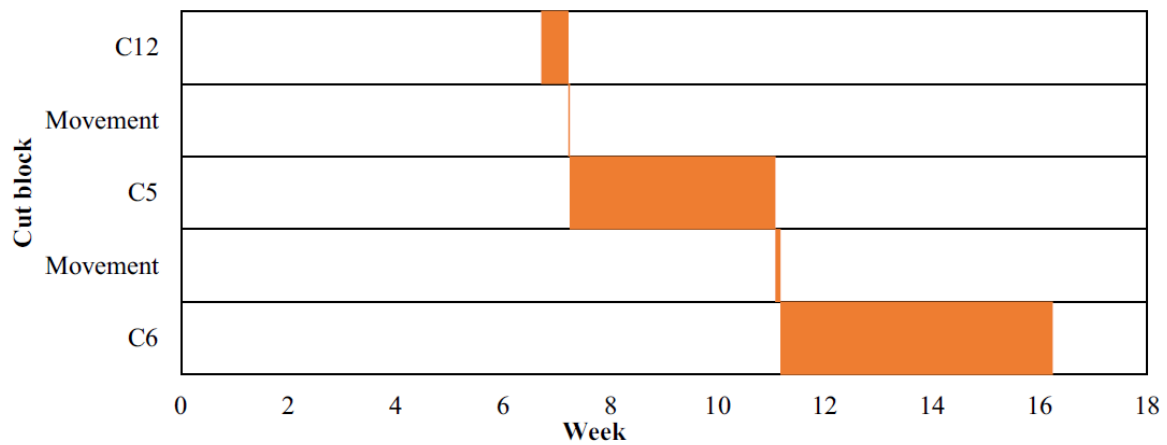


Figure 7: Detailed schedule of machine M31 (cable yarder + loader) in planning horizon 1.

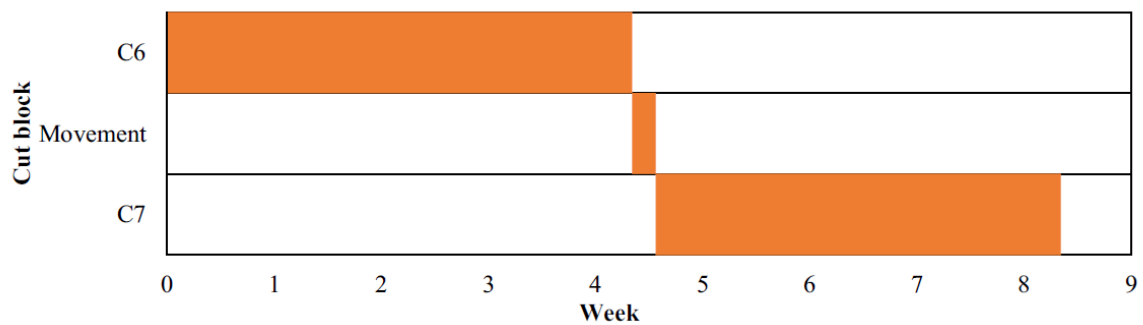


Figure 8: Detailed schedule of machine M31 (cable yarder + loader) in planning horizon 2.

Discussion

In this study, a mixed-integer linear programming model is developed for the detailed scheduling of harvesting activities at the operational level. Unlike previous studies in forest harvest scheduling, the precedence relationship between harvesting activities for multiple cut blocks and the movement of individual machines between cut blocks are considered in this model. Similar to the models for agriculture harvest scheduling (e.g., Basnet et al. 2006), all the precedence relationships between different harvesting activities are met in our study as shown in the detailed scheduling of cut blocks. However, in the agriculture harvest scheduling studies, no planning horizon was considered, and the model was run only once for detailed scheduling to minimize the duration of harvesting. In contrast, the planning horizon is considered in the model developed in this study, and constraints were added to the model to make sure that the outputs of the model implemented for a planning horizon can be used as the inputs for the next planning horizon, which is an important aspect of the planning and one of the contributions of the proposed model. This aspect of the model ensures the continuity of operations of the machines. Therefore, if a machine requires some extra time after the planning horizon to complete its operation in a cut block, then the same machine will be assigned to the same cut block in the next planning horizon, and it will start its operation from time zero in the next planning horizon. For instance, machine M31 (cable yarder + loader) requires 4.34 weeks after the planning horizon 1 to complete its operation in cut block C6 (Figure 7), and it starts its operation in cut block C6 from time 0 in planning horizon 2 (Figure 8).

The developed model can be used for better utilization of machines for forest companies in the harvesting of cut blocks, as it can prescribe delaying the start of some of the harvesting activities to minimize the idle time of machines. For instance, Table 7 compares the schedule of Machine M31 (cable yarder + loader) based on the earliest start time and the model's output to perform the loading at assigned cut blocks. Machine M31 could start its operation at the earliest start time in all the assigned cut blocks; however, it would result in a total idle time of 5.66 weeks for its operation (Table 7).

Table 7: Comparison of schedule of machine M31 (cable yarder + loader) based on the earliest start time and the model's output.

Cut block	Start time (in weeks)		End time (in weeks)		Idle time (in weeks)		Movement time (in weeks)
	Earliest start time	Model output	Earliest start time	Model output	Earliest start time	Model output	
C12	1.05	6.71	1.56	7.22	0	0	0
C5	7.24	7.24	11.07	11.07	5.66	0	0.02
C6	11.16	11.16	16.24	16.24	0	0	0.09

Therefore, by delaying the start time of cut block C12 from week 1.05 to week 6.71 the idle time of machine M31 is reduced from 5.66 weeks to zero. As a result of this approach, only three assigned machines have idle time. Additionally, the model ensures that this delay does not result in any extra penalty cost.

In this paper, we considered different harvesting systems for different types of felling activities. The trees are felled either manually using a chainsaw or mechanically using a feller-buncher. After the manual felling, trees are processed at the stump site or in woods, then they are yarded to the landing zone, where loading takes place. So, the precedence relationship for the manual felling is: 1) manual felling and processing, 2) yarding, and 3) loading. However, after the mechanical felling, the trees are yarded to the landing zone, where mechanical processing takes place, then they are loaded to the trucks. Therefore, the precedence relationship for the mechanical felling is: 1) mechanical felling, 2) yarding, 3) mechanical processing, and 4) loading. In case there is any other different harvesting system, then the precedence relationship constraints (constraint set 17) and machine movement constraints (constraint sets 18 and 19) can easily be modified to accommodate this change, and the preceding activities must be updated only in the input data. Furthermore, the developed model can be modified to accommodate other precedence relationships, such as a harvesting activity starting a few weeks after the start of the previous harvesting activity, or two harvesting activities starting simultaneously in the cut block.

The other approach for solving this problem could be to use a heuristic or metaheuristics method. However, the operating cost obtained by the model was very close to the lowest possible operating cost, and the penalty cost calculated could not be improved any further, as discussed earlier. Also, the solver found the best integer solution with an optimality gap of 2.97% in 3.5 hours which is acceptable for a model with a planning horizon of three months, as the model has to be run once per quarter. Therefore, heuristics were not used for solving the problem.

Conclusions

In this paper, an optimization model was developed to minimize the total costs of harvesting considering precedence relationships among harvesting activities. The model determined the start and the end times for each harvesting activity at each cut block. The model was applied to a real case study of a large forest company in Coastal British Columbia for a three-month planning horizon. The case study included 30 cut blocks and 103 machines. The solver found the integer solution after 3.5 hours with an optimality gap of 2.97%. The results indicated that all the assigned harvesting activities were completed in 13 cut blocks within the planning horizon, while 17 cut blocks required extra time after the planning horizon. The operating cost from the model's output was 1.37% higher than the ideal operating cost case. The penalty cost from the model was optimal, and it could not be improved any further with avail-

able machines. Out of 103 machines, 47 machines were assigned for harvesting and only three machines had an idle time.

In this model, it was assumed that only one machine can be assigned for each harvesting activity at each cut block, and each harvesting activity had an exclusive set of machines. However, in some cases, multiple machines can be assigned for the same harvesting activity at a cut block, and some machines can perform more than one harvesting activity. Therefore, one of the plausible future avenues for this study is to develop a model to determine the number of machines to be assigned for performing each harvesting activity at each cut block considering the interchangeability in the use of machines.

Note

AAC is the maximum average level of timber harvest permitted for forest management areas, it represents a harvest level that balances environmental, economic and social considerations (Government of BC 2021)

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