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Essays on Private Money Creation and Monetary Policy

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Thesis Summary

This dissertation analyzes monetary policy in various dimensions, namely with regard to central bank collateral frameworks, to the introduction of central bank digital currencies and to the consideration of climate-related aspects.

Chapter 2 and 3 study different characteristics of collateral requirements in central bank lending operations to private banks. First, we provide a theoretical analysis of optimal central bank collateral standards in the presence of subjective expectations on the side of the banks. We show how beliefs among banks can lead to an under- and overvaluation of assets which are pledgable at central bank lending facilities. Specifically, optimism on the side of the banks can lead to excessive lending and bank default, whereas pessimism can trigger deficient lending and productivity losses. The optimal monetary policy, in the form of the haircut on the provided collateral, reacts to such subjective beliefs to provide the optimal level of liquidity to banks. The central bank counteracts growing optimism through stricter collateral requirements and steers against growing pessimism by setting looser standards on the eligible collateral. Under uncertainty about the beliefs, the central bank has, in general, a tendency to loosen its collateral framework in order to avoid an underprovision of liquidity if sufficiently pessimistic beliefs realize. Second, we analyze the impact of different leverage constraints on the banks' incentives for the monitoring of borrowers. In particular, we compare capital constraints following from regulatory, unweighted capital requirements to liquidity constraints following from central bank collateral requirements in reserve lending operations. Our analysis shows that, independent of the monitoring technology considered, liquidity-constrained banks have more incentives to monitor in any situation, compared to capital-constrained banks. For liquidity-constrained banks, monitoring of borrowers does not only improve the expected profits under the same leverage (the "return channel"), but it also allows for a leverage expansion, as the bank assets improve in their collateral value (the "collateral leverage channel"). Furthermore, we illustrate that the superior effect of

central bank collateral requirements on the banks' monitoring incentives is also unique in comparison to contingent (e.g., risk-weighted) capital requirements.

Chapter 4 provides a theoretical analysis of the optimal monetary policy in the presence of a central bank digital currency (CBDC). It studies the short- and long-term implications in the specific context of heightened liquidity risk for banks with an interest-bearing CBDC that is equivalent to bank deposits as medium-of-exchange. We show that, in the short-term, the central bank can make use of the increased liquidity risk stemming from digital bank runs, by setting tight collateral requirements and penalties for illiquidity, to incentivize banks to engage into the monitoring of borrowers. Such a monetary policy is only optimal until liquidity risks become sufficiently prominent, so that tight collateral requirements and penalties for illiquidity would render banking non-viable. In the long run, with a sufficient demand for the CBDC, the optimal monetary policy is thus characterized by loose collateral standards and no illiquidity penalties.

Chapter 5 studies a potential role of monetary policy in the transition to a low-carbon economy. Specifically, it analyzes a new concept of climate risk-adjusted refinancing operations (CAROs) conducted by the central bank. We show that CAROs are a suitable instrument to correct allocations in the financial market and ultimately in the real economy that are distorted, from a central bank perspective. In the considered framework, such distortions arise from private agents having subjective beliefs about climate risk, which enters in the form of new environmental regulations. There exists a nexus between monetary and fiscal policy, as through appropriate subsidies for climate risk mitigation, fiscal policy can reduce the need for CAROs. The optimal design of CAROs is also outlined for environments where the central bank has concerns about financial stability.

Kurzfassung

Diese Dissertation widmet sich verschiedenen Dimensionen der Geldpolitk, nämlich den seitens der Zentralbank angewendeten Besicherungsregeln, der Einführung von digitalem Zentralbankgeld und der Berücksichtigung von klimabezogenen Überlegungen.

Kapitel 2 und 3 studieren verschiedene Aspekte der Besicherungsregelungen, welche von der Zentralbank für ihre Kreditgeschäfte mit Banken festgesetzt werden. Zunächst geben wir eine theoretische Analyse optimaler Besicherungsanforderungen in der Gegenwart von subjektiven Erwartungen auf Seiten der Banken. Wir zeigen, dass die Überzeugungen von Banken zu einer Unter- und Überbewertung von Wertschriften führen können, welche bei der Zentralbank als Besicherung in ihren Kreditfazilitäten akzeptiert werden. Im Speziellen, Optimismus auf Seiten von Banken kann zu exzessiver Kreditvergabe und Bankenausfällen führen, wohingegen Pessimismus restriktive Kreditvergabe and Produktivitätsverluste verursachen kann. Die optimale Geldpolitik, in Form des Bewertungsabschlags auf das gelieferte Pfand, reagiert auf solche subjektiven Überzeugungen, um eine optimale Liquiditätsversorgung der Banken sicherzustellen. Die Zentralbank steuert wachsendem Optimismus durch strengere Besicherungsanforderungen entgegen und begegnet zunehmendem Pessimismus durch eine Lockerung der Besicherungsstandards. Bei Unsicherheit über die Überzeugungen der Banken tendiert die Zentralbank im Allgemeinen zu einer Lockerung der Besicherungsstandards, um einer Liquiditätsunterversorgung im Falle von ausreichend grossem Pessimismus vorzubeugen. Zweitens analysieren wir die Auswirkungen von verschiedenen Leverage-Beschränkungen auf die Anreize von Banken zur Überwachung von Kreditnehmern. Insbesondere werden kapitalwirksame Beschränkungen, initiiert durch regulatorische, nicht-gewichtete Kapitalanforderungen, mit liquiditätswirksamen Beschränkungen, initiiert durch die Besicherungsanforderungen der Zentralbank, verglichen. Die Analyse zeigt, dass, unabhängig von der Überwachungstechnologie, liquiditätswirksame Beschränkungen in jedem Fall zu grösseren Überwachungsanreizen führen

als kapitalwirksame Beschränkungen. Mit liquiditätswirksamen Beschränkungen führt die Überwachung von Kreditnehmern nicht nur zu höheren erwarteten Profiten bei gleichbleibendem Leverage (der "Rendite-Kanal"), sondern sie erlaubt auch eine Ausweitung des Leverage, da die Vermögenswerte der Bank in ihrem Besicherungswert steigen (der "pfandbezogene Leverage-Kanal"). Wir zeigen ausserdem, dass der Effekt von Besicherungsanforderungen der Zentralbank auf die Überwachungsanreize der Banken auch im Vergleich mit bedingten (z.B., risiko-gewichteten) Kapitalanforderungen einzigartig ist.

Kapital 4 liefert eine theoretische Analyse der optimalen Geldpolitik in der Gegenwart digitalen Zentralbankgeldes (DZG). Es werden die kurzfristigen wie auch langfristigen Folgen studiert, wobei stets das erhöhte Liquiditätsrisiko auf Seiten der Banken mit zinsbehaftetem DZG betont wird, welches in seinen Transaktionseigenschaften identisch mit Bankeinlagen ist. Es wird gezeigt, dass die Zentralbank in der kurzen Frist das erhöhte Liquiditätsrisiko, durch strenge Besicherungsstandards und Strafen für Illiquidität, ausnutzen kann, um Banken ausreichend Anreize zur Überwachung ihrer Kreditnehmer zu geben. Solch eine Geldpolitik ist optimal bis das Liquiditätsrisiko so sehr in den Vordergrund rückt, dass strenge Anforderungen für die Besicherung und Illiquiditätsstrafen das Bankgeschäft nicht mehr tragfähig gestalten. In der langen Frist, mit ausreichend grosser Nachfrage nach DZG, ist die optimale Geldpolitik daher durch lockere Besicherungsstandards und Straffreiheit für Illiquidtät gekennzeichnet.

Kapitel 5 studiert eine mögliche Rolle der Geldpolitik im Übergang zu einer weniger kohlenstoffintensiven Ökonomie. Im Speziellen wird ein neues Konzept der für das Klimarisiko angepassten Refinanzierungsoperationen (KAROs) vorgestellt, welche durch die Zentralbank durchgeführt werden. Es wird gezeigt, dass KAROs ein geeignetes Instrument sind, um Allokationen am Finanzmarkt und letztlich in der realen Ökonomie zu korrigieren, welche von der Zentralbank als verzerrt betrachtet werden. In dem betrachteten Model entstehen solche Verzerrungen durch die subjektiven Einschätzungen von privaten Subjekten betreffend dem Klimarisiko, welches durch neue Umweltregulierungen entsteht. Es existiert ein Nexus zwischen der Geldpolitik und der Fiskalpolitik, da letztere durch entsprechende Subventionen für die Minderung von Klimarisiken die Notwendigkeit für KAROs reduzieren kann. Die optimale Ausgestaltung von KAROs wird auch für Situationen beschrieben, in welchen die Zentralbank Bedenken betreffend der Finanzstabilität hat.

Chapter 1

Introduction

During the last two decades, the status quo of monetary policy has been challenged over and over. With the onset of the financial crisis 2007/08, central banks had to deviate from conventional policies in order to stabilize the financial system. As liquidity needs among private banks were largely increasing, central banks reacted by providing broad access to their lending facilities and by easing the standards for liquidity provisions in an unprecedented manner. In particular, the collateral requirements came into focus, as central banks started to grant loans to private banks against assets, which, due to risk concerns, had never been accepted in central bank operations before. Since the financial crisis, central banks have used various other unconventional measures, with the aim of providing sufficient monetary stimulus to pave the way for an economic recovery and for a revival of inflation. Refinancing rates at central banks have reached negative territory, yield curves have been flattened through large-scale asset purchase programs, and currency appreciations have been counteracted by massive interventions at foreign exchange markets. While in some countries unconventional measures could be (partially) removed in the meantime, the outbreak of the Covid-19 crisis has incentivized central banks to return to their expansive monetary policy or to maintain it.

In the past few years, central banks have been confronted with various other challenges questioning the current conduct of monetary policy. Two of them are digitization and climate change.

Technological innovations allow us to design and execute transactions more efficiently, in particular by using more and more digital channels. As societies live in increasingly digitized environments, the call for an electronic form of national currencies that is accessible to the general public has become louder. Moreover, as of today, the public is restricted to physical cash in the form of banknotes and coins, while private banks, for instance, can already hold national currency in an electronic form as deposits at central banks. The introduction of digital public money, often referred to as "central bank digital currency" (CBDC), because it would be issued by the central bank, is therefore supported by at least two arguments. On the one hand, holding money would become more convenient for the general public. On the other hand, libertarian thinking advocates equal opportunities for the general public, which entails the same right to access money as banks, for instance. So far, it is unclear which economic implications a CBDC would have, and what the optimal design of monetary policy would be in an environment with a CBDC.

Current central bank policies are also under pressure, as they often ignore—or even counteract—ongoing efforts to promote the transition to a low-carbon economy. In particular, central banks' collateral framework and central banks' asset purchase programs have been found to not appropriately support less-carbon intensive economic activities up to now. Several proposals have been made how climate-related aspects could be incorporated into different monetary policy instruments. The economic implications of such adjustments in the conduct of monetary policy are, however, not fully understood yet. It remains unclear how such changes affect the central banks' ability to fulfill their mandate, which is generally based on price stability.

In the following, we explain in detail why collateral requirements play an important role for central bank operations today, and how the introduction of central bank digital currencies and the integration of climate considerations may shape the conduct of monetary policy in the future. We also outline the contribution of this thesis.

1.1 Central Bank Collateral Frameworks

Central banks traditionally lend liquidity, in the form of reserves to private banks, on a secured basis. Some central banks, as the Swiss National Bank (SNB), for instance, are restricted by law to secured lending operations (The Federal Assembly of the Swiss Confederation, 2003). Besides the legal reasons for such restrictions, there are various economic incentives for central banks to provide liquidity only against collateral. As outlined by Bindseil et al. (2017), unsecured lending by the central bank would imply more granular risk management procedures, ultimately at the expense of the taxpayers. Terms and conditions would have to take the counterparty risk into account, which would require intensive monitoring of all eligible banks. In addition, as due to the intensive risk management the lending process itself may become more lengthy, unsecured lending may also slow down the implementation of monetary policy, with costs for the financial system and the real economy. Finally, unsecured lending may also endanger the independence of the central bank if the increased risk exposure materializes in the form of losses, which potentially taxpayers would have to compensate.

In general, the collateral standards chosen by central banks are conservative. Central banks only lend against high-quality assets which satisfy certain criteria regarding the creditworthiness of the issuer and the issuer country, for instance.[1] During the financial crisis of 2007/08, certain central banks decided to also accept collateral of lower quality. For example, the European Central Bank (ECB) adjusted its collateral policy on October 15, 2008, by broadening the set of eligible assets, in particular by accepting assets with lower credit ratings.[2] With such an unconventional policy measure central banks could appropriately react to the widespread liquidity needs in the banking sector. Nevertheless, the loosening of collateral standards during the financial crisis has triggered an intensive debate among academics and policy makers about the economic effects of (changes in) central banks' collateral framework. In particular, the increased risk exposure, with all its potential adverse consequences, and the "collateral premium", leading to distortions in the financial market and ultimately the real economy, have been discussed (Nyborg, 2017). The collateral premium represents an increase in the valuation of eligible assets that solely originates from the fact that these assets are pledgable at the central bank. The collateral premium guarantees issuers of pledgable assets (relatively) more favorable financing conditions. Accordingly, through its collateral framework, the central bank may induce a shift in resources to sectors and firms which issue assets that can be used as collateral in central bank operations. Policy makers responded to this critique by emphasizing the fundamental guidelines which central banks follow in the determination of the collateral standards (Bindseil et al., 2017).

We contribute to the debate by examining two other channels through which the central bank collateral framework can affect the allocation in the real economy. One the one

[1] Other criteria include the currency denomination, the liquidity in the market (i.e., the outstanding amount) or the domiciliation of the exchange at which the respective assets can be traded. See, for example, the SNB's collateral framework (Swiss National Bank, 2004).

[2] For example, see https://www.ecb.europa.eu/press/pr/date/2008/html/pr081015.en.html (accessed on April 20, 2021) for the measures taken by the ECB in October 2008 to increase the opportunities for liquidity provisions.

hand, we show how beliefs in the economy can influence the valuation of pledgable assets, ultimately leading to a change in the liquidity provision by the central bank, and in bank lending to the real economy. On the other hand, we show that both capital requirements and central bank collateral requirements impact, through the ensuing leverage constraints, the incentives for banks to monitor borrowers. However, compared to capital regulation, central bank collateral requirements are unique as they lead in any situation to higher monitoring incentives.

1.2 Central Bank Digital Currencies

In the past few years, central banks have become more and more active in researching and developing potential solutions for central bank digital currencies (Boar et al., 2020; Boar and Wehrli, 2021). The rising efforts by central banks came with a change of mind. While in 2017 and 2018, the speeches by official representatives predominantly had a negative stance as to the need for CBDCs, this view changed in 2019 (Auer et al., 2020). As banks and other financial institutions already have access to electronic national currency in the form of deposits at the central bank, so-called "wholesale" CBDC, which are only accessible to banks and other eligible financial institutions, are generally considered to be less controversial than "retail" CBDCs, which are electronic central bank money accessible to the general public. In this dissertation, we focus on the latter type.

With such a retail CBDC[3], the general public does not only enjoy a higher convenience for holding money, but it is also equated with other agents in the economy, such as banks, for instance, who already have access to electronic central bank money today.

Moreover, academics and policy makers are currently investigating the economic implications of such a central bank digital currency. Among the pros, an intensification of bank competition on the deposit market (Andolfatto, 2021; Keister and Sanches, 2019), the mitigation of financial stability concerns (Berentsen and Schär, 2018) and a better conduct of monetary policy (Bordo and Levin, 2017) are the most prominent arguments. First, a well-designed, interest-bearing CBDC can enter into competition with bank deposits and may incentivize banks to pay higher interest or engage into innovation to maintain deposit funding. Second, the risk of digital bank runs—if depositors want to convert bank deposits into CBDC on a large scale—may induce banks to take less risk in order to avoid that

[3]From now on, we will use the term "CBDC" as a shortcut for a retail central bank digital currency.

bank depositors withdraw their funds. Third, with a CBDC, the pass-through of monetary policy may strengthen and the central bank can affect the interest rates in the economy more directly, as the general public holds electronic central bank money.

On the other side, a CBDC may also create risks for the individual bank, the financial system and ultimately for the real economy. It is often argued that a CBDC, acting as a close substitute to bank deposits, can incentivize bank depositors to convert their funds into CBDC, thus raising funding concerns for private banks. If the deposit funding cannot be appropriately replaced, banks may scale down their lending and investment activities, which would lead to a credit crunch in the worst case (Keister and Sanches, 2019). Moreover, if deposit transfers from private banks to the central bank take place on a large scale, for instance in the form of digital bank runs, financial stability may be endangered (Cecchetti and Schoenholtz, 2018). While the central bank can mitigate some effects of large scale transfers into CBDC by acting as lender of last resort for banks, negative implications for the real economy cannot be fully ruled out.

We contribute to this debate by providing a theoretical analysis that sheds light on the short- and long-term implications of a central bank digital currency. Our findings indicate that, in the presence of a CBDC, the central bank can use the increased liquidity risk for banks to incentivize them to monitor their borrowers, by applying tight collateral requirements and illiquidity penalties. Such a monetary policy can only be maintained up to the point where liquidity risks become so prominent that tight collateral requirements would render banking non-viable.

1.3 Climate-related Monetary Policies

In December 2015, the 197 parties to the United National Framework Convention on Climate Change negotiated the "Paris Agreement", which represented a clear signal for limiting the global temperature rise. In order to restrict global temperature increases below 2°C above the pre-industrial level, it is key to make the global economy cleaner, more efficient, and reflective of the social costs of greenhouse gas emissions. Member countries of the UNFCCC have adopted country-specific plans determining climate-related targets and policies, so-called nationally determined contributions. These actions highlight the increasing awareness of governments and societies about the risks posed by climate change and, consequently, of the need for an appropriate response.

Fiscal policies are at the core of the national strategies and are pivotal for managing the transition to a low-carbon economy. Still, the call for an integration of climate considerations into financial supervision and monetary policy has become louder. Financial authorities and central banks are asked to take a more active role in steering finance towards low-carbon activities. Such a requirement is often justified by a policy coherence argument, as financial regulation and monetary policy should not counteract efforts undertaken by governments. Indeed, central bank policies, in the form of asset purchase programs, for instance, do not seem to have adequately supported low-carbon sectors in the past (Matikainen et al., 2017).

Several proposals have been made regarding how central banks could integrate climate considerations into their monetary policy instruments. For example, central banks could use "green collateral frameworks" in their lending operations (Monnin, 2018) or engage into "green asset purchases" (Volz, 2017). In both cases, assets issued by less carbon-intensive or less climate-risk exposed entities should be included to a greater extent than today. In addition, Campiglio (2016) suggests that central banks could apply differentiated reserve requirements that vary with climate-related aspects of the assets held by the individual bank. Due to the expansionary monetary policies applied by central banks after the financial crisis as well as during the Covid-19 crisis, the reserve deposits of private banks have grown tremendously. Banks (are forced to) hold liquidity, in the form of central bank reserves, to an unprecedented degree. Thus, some of the proposals, such as differentiated reserve requirements, for instance, would currently not be effective.

We contribute to the debate about climate-related monetary policy operations by providing a theoretical analysis of a new concept, the climate risk-adjusted refinancing operations, in short CAROs. We show that CAROs are a suitable instrument to correct allocations in the financial markets and the real economy, which are distorted from a central bank perspective. We characterize the optimal design of CAROs, also in the context of financial stability concerns. Finally, we show how climate risk mitigation investments in the economy matter for central bank intervention in the form of CAROs.

1.4 Outline of the Thesis

This thesis relies on a baseline model that features the dual role of banks, providing credit *and* money to the real economy, and this model accounts for central banks' liquidity provisions to banks. It is used in different variations to study several aspects of monetary policy relating to central bank collateral requirements, central bank digital currencies and climate risk considerations. Each chapter responds analytically and partially through numerical simulations to the following questions:

- Chapter 2: How should central bank collateral requirements react to changing beliefs of banks? What is the optimal monetary policy if there is uncertainty about the actual beliefs in the economy?
- Chapter 3: What are the effects of central bank collateral requirements on banks' monitoring incentives? Do banks' monitoring incentives depend on leverage constraints? Is there a difference between capital-driven and liquidity-driven leverage constraints?
- Chapter 4: What are the economic benefits and risks following from the introduction of a central bank digital currency? How does a CBDC affect the conduct of monetary policy in the short- and long-term?
- Chapter 5: How do climate risk-adjusted refinancing operations affect the allocation in the financial market and the real economy? What is the optimal design of CAROs, also when accounting for concerns about financial stability? Which role do climate risk mitigation technologies play?

In chapter 2, we model subjective beliefs of banks about firm productivity and, ultimately, about loan repayment. We study how these beliefs affect the central bank's choice of collateral standards in its lending facilities. Banks are liquidity-constrained, as the access to central bank reserves is subject to collateral requirements. Through the expected value of bank loans as collateral, beliefs influence the initial lending decision of banks. Optimism on the side of banks can lead to excessive lending and bank default, whereas pessimism can trigger deficient lending and productivity losses. With an appropriate haircut on collateral, the central bank can perfectly neutralize such belief distortions and always induce the socially optimal allocation. If banks become more optimistic (pessimistic), the central bank optimally sets a larger (smaller) haircut on bank loans used as collateral for reserve loans. In the presence of uncertainty about beliefs, the central bank's incentives to set looser collateral standards increase, as this avoids deficient bank lending due to sufficiently pessimistic beliefs. Specifically, if uncertainty about beliefs is sufficiently large, the main objective of monetary policy switches from avoiding bank default to mitigating productivity losses.

Chapter 3 uses a simple model that illustrates the different effect of capital constraints and liquidity constraints on bank monitoring. Capital constraints emerge from regulatory (unweighted) capital requirements, while liquidity constraints emerge from the central bank's collateral requirements in its reserve lending facilities. Banks demand liquidity in the form of central bank reserves, as they must settle interbank liabilities arising from deposit transfers among banks. Monitoring incentives are twofold: First, bank monitoring increases the chances for a high loan repayment, which we refer to as the *return channel*. Second, with a higher expected repayment by borrowers, also the collateral value of bank loans increases. In the presence of liquidity constraints, monitoring then improves the banks' access to liquidity at the central bank and, ultimately, allows them to expand loan supply and deposit issuance in the first place. We dub this the *collateral leverage channel*. Based on the collateral leverage channel, liquidity-constrained bankers always have more incentives to monitor than capital-constrained bankers.

Chapter 4 examines how the introduction of an interest-bearing central bank digital currency accessible to the public impacts bank activities and monetary policy. At any time, depositors can switch from bank deposits to CBDC as a safe medium of exchange. As banks might face digital runs, either because depositors have a preference for CBDC or fear bank insolvency, monetary policy can use collateral requirements and penalties for illiquidity to initially increase bankers' monitoring incentives. This leads to higher aggregate productivity. However, the mass of households holding CBDC will increase over time, causing additional liquidity risk for banks. After a certain period, monetary policy with tight collateral requirements generating liquidity risk for banks and exposing bankers to illiquidity penalties would render banking non-viable and prompt the central bank to abandon such policies. In such circumstances, the bankers' monitoring incentives would revert to low levels. Accordingly, a CBDC can yield short-term welfare gains at best.

Finally, in chapter 5, we follow up on the statements of policy makers have argued that markets are not pricing climate risk appropriately yet, which may lead to a misallocation of resources and financial instability. Climate risk-adjusted refinancing operations (CAROs) conducted by the central bank are one possible instrument to address this issue. CAROs are characterized by interest rates on reserve loans, which depend on the climate risk exposure of the assets held by the borrowing bank. If private agents and the central bank have differing beliefs about the likelihood of the transition to a low-carbon economy, the allocation emerging without CAROs is, from the central bank's perspective, suboptimal and may lead to financial instability. We find that an appropriate design of CAROs allows the central bank to influence bank lending in a way that induces the optimal allocation under its beliefs and eliminates financial instability. Moreover, we show that investment into climate risk mitigation reduces the need for central bank intervention, and that CAROs can be used to achieve specific climate-related allocation targets.

In chapters 3, 4 and 5, I contributed to the overall writing and the development of the model.

Chapter 2

Monetary Policy under Subjective Beliefs of Banks: Optimal Central Bank Collateral Requirements*

Abstract

Banks have subjective beliefs about firm productivity and, ultimately, about loan repayment. We study how these beliefs affect the central bank's choice of collateral standards in its lending facilities. Banks are liquidity-constrained, as the access to central bank reserves is subject to collateral requirements. Through the expected value of bank loans as collateral, beliefs influence the initial lending decision of banks. Optimism on the side of banks can lead to excessive lending and bank default, whereas pessimism can trigger deficient lending and productivity losses. With an appropriate haircut on collateral, the central bank can perfectly neutralize such belief distortions and always induce the socially optimal allocation. If banks become more optimistic (pessimistic), the central bank optimally sets a larger (smaller) haircut on bank loans used as collateral for reserve loans. With uncertainty about beliefs, the central bank's incentives to set looser collateral standards increase, as this avoids deficient bank lending in the presence of sufficiently pessimistic beliefs. Specifically, if uncertainty about beliefs is sufficiently large, the main objective of monetary policy switches from avoiding bank default to mitigating productivity losses.

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2.1 Introduction

Central bank interest rates are considered to be the main instruments in the conduct of monetary policy. Central banks set interest rates on loans and deposits of reserves, i.e., on public money that is solely available to banks, to achieve their targets referring to price stability and economic activity. The costs of borrowing reserves at the central bank influences, for instance, the banks' decision about loan financing to the real economy or interest payments on bank deposits (Kakes and Sturm, 2002). Besides interest rates, the pricing of central bank reserves is also influenced by various other factors, which have gained considerable importance at the latest after the financial crisis of 2007/08. For example, the ECB's decision in 2008 to allow banks to pledge assets of lower quality in its lending facilities has initiated a general discussion about central banks' choice of collateral standards (Nyborg, 2017).[1] A central point of this discussion was the so-called "collateral premium", namely an increase in the market value of pledgable assets that solely originates from the fact that these assets can be used as collateral in central bank operations.[2] The collateral premium may lead to improved financing conditions for the issuer and thus influence the allocation of resources in the economy. In our work, we show that beliefs among banks, which lead to an under- or overvaluation of pledgable assets, can also have an influence on the real economic allocation through central banks' collateral framework. Our findings show that central banks may ultimately be incentivized to adjust collateral standards based on the beliefs in the economy.

Our analysis is based on the observation that private agents and the central bank repeatedly disagree about the future path of macroeconomic variables. For instance, as documented by Caballero and Simsek (2020), the beliefs of the market and the Federal Reserve regarding the appropriate future interest rate policy are constantly misaligned. The authors show that such a pattern may emerge if the market acts in an opinionated way, not considering the central bank's information as superior and, as a consequence, not willing to learn from central bank announcements. Instead, agents build their own subjective beliefs about economic fundamentals and update their beliefs solely based on the observed data. In its choice of interest rates, the central bank must thus take the

[1] For example, see https://www.ecb.europa.eu/press/pr/date/2008/html/pr081015.en.html (accessed on January 13, 2021) for the measures taken by the ECB in October 2008 to increase the possibility for liquidity provisions.

[2] See Mésonnier et al. (2017) and Van Bekkum et al. (2018), for instance.

continuous disagreement by private agents into account to ensure that its monetary policy achieves the desired effect. In this paper, we show that beliefs in the economy may also influence the central bank in its choice of other monetary policy instruments, namely the central bank's collateral framework. Similar to Caballero and Simsek (2020), we focus on opinionated banks in our setting, which disregard the central bank's information and stick to their own subjective beliefs about firm productivity and, ultimately, about loan repayment. We show how the central bank collateral requirements depend on the banks' beliefs and which role uncertainty on the side of the central bank about beliefs plays.

Our framework embeds a banking sector and a central bank. Banks grant loans to firms, which are financed through deposit issuance. Banks must settle interbank liabilities at the central bank by using reserves. In our model, interbank liabilities arise from deposit transfers among banks, which occur from transactions on the goods markets. The central bank provides reserves through secured loans, where the pledgable assets are given by the loans that banks provided to firms. Monetary policy comprises the interest rates on reserve loans and reserve deposits, and the haircut on the bank loans used as collateral in central bank lending operations. We model an economy without any price rigidities, so that the neutrality of money applies, i.e., the interest policies applied by the central bank influence prices but not the real allocation. On the contrary, the central bank's collateral framework has a real effect, as it determines banks' access to liquidity in the form of reserves, which is crucial for the banks' decision of issuing deposits and granting loans. Banks are only constrained by liquidity, so that an improved access to liquidity leads to more deposit issuance and loan financing to firms in the first place. In our setting, we distinguish between loan- and bond-financed firms. A higher haircut, leading to a lower provision of central bank reserves and less bank lending, comes at the benefit of bond-financed firms, i.e., all firms in the economy that can access financing at financial markets via bond issuance and do not rely on loan financing from banks. In turn, a smaller haircut on bank loans used as collateral for central bank loans leads to more bank lending and increases the leverage of banks, while reducing the access to finance for bond-financed firms. Banks face limited liability and constraints on equity financing. Further, the loan returns are risky, as the operations of firms are subject to productivity shocks. With a sufficiently large leverage, banks can thus be exposed to a solvency risk. Bank default is costly, as the resolution of defaulting banks requires real resources, which must be provided by the government sector.

In our model, the central bank aims at maximizing welfare, so that, when choosing the haircut on bank loans, it must consider two potential effects: First, any adjustment of the haircut may affect aggregate production, as the share of production in the two sectors changes. If the productivity of loan- and bond-financed firms differs, a change of the haircut translates directly into a change of aggregate production. Second, if the haircut set by the central bank is sufficiently small, banks can leverage enough to be exposed to a solvency risk and default on the liabilities towards depositors whenever the loan returns are low. This, in turn, reduces the production output available for consumption and, ultimately, welfare, as the resolution of bank default requires real resources. Thus, the central bank must, in its choice of collateral standards, trade off productivity losses and default costs, still accounting for the banks' beliefs about firm productivity and, ultimately, about loan repayment. While the central bank has rational beliefs and thus knows the true probability distribution of productivity shocks in the economy, banks have their own subjective beliefs about firm productivity. We allow for optimistic and pessimistic banks, which believe that, compared to the true probability distribution, firms in the loan-financed sector are more and less productive, respectively. Accordingly, there might be situations where, based on their beliefs, banks want to grant more or fewer loans to firms than socially optimal.

The optimal monetary policy in our baseline model, where agents are sufficiently optimistic about production in the loan-financed sector, is simple. Whenever default costs are small enough, the central bank aims at maximizing bank lending and allowing for bank default, which is achieved by setting a sufficiently small haircut in its lending facilities. In turn, if costs associated with bank default are large enough, the central bank aims at restricting bank lending and thereby eliminating bank default, which is achieved by setting a sufficiently large haircut on the collateral provided for reserve loans.

The optimal haircut set by the central bank varies with banks' beliefs about firm productivity. The reason is that banks' beliefs influence the collateral value of bank loans and thus shape their expectation about the access to central bank liquidity. Compared to the true probability distribution, growing optimism on the side of banks leads to an overvaluation of bank loans, causing banks to expect a greater access to liquidity from the central bank and incentivizing them to provide more deposit-financed loans in the first place. To counteract the effects of growing optimism and to restore the optimal level of bank lending, the central bank must tighten collateral requirements by applying a larger haircut. The central bank thereby brings the banks' expectation about the access to reserves back to the original level and eliminates the banks' incentives to grant more loans than before. A similar mechanism applies for growing pessimism among banks. If banks believe that the productivity of loan-financed firms is lower than before, the value of bank loans in the market decreases. Accordingly, the banks' expectation about the access to liquidity lowers, so that they initially issue less deposits and grant less loans. The central bank can steer against the banks' pessimistic beliefs by loosening collateral standards through a smaller haircut. If the central bank reduces the haircut adequately, this restores banks' incentives to grant the socially optimal level of loan financing. Given the mechanism described above, if beliefs are known, the central bank can completely neutralize belief distortions on the side of banks and always induce the socially optimal allocation through its collateral framework.

The central bank's choice of the collateral requirements becomes more challenging if it faces uncertainty about the beliefs in the economy. Without knowing the actual beliefs, the central bank chooses the haircut on bank loans to maximize *expected* welfare in our framework. The central bank faces a trade-off between loose collateral standards, leading to excessive lending and costs due to bank default for more optimistic beliefs, and tight collateral requirements, leading to deficient lending and lower aggregate output for more pessimistic beliefs. We find that compared to any situation where beliefs are known, the central bank becomes less strict in its choice of the collateral framework. Specifically, it prefers bank default—compared to restrictions on bank lending—already for a higher level of default costs. The larger the uncertainty, i.e., the further the most pessimistic and most optimistic types of beliefs are apart, the more the central bank is incentivized to prefer bank default to deficient bank lending. The reason is that the more distinct the possible types of beliefs, the more costly it is for the central bank to avoid bank default for the optimistic beliefs. Instead, the central bank allows for default in the presence of more optimistic banks, while reducing deficient bank lending for more pessimistic beliefs.

The paper is organized as follows: Section 2.2 relates our paper to the literature, section 2.3 introduces our model and outlines the individually optimal behaviour of agents, and section 2.4 provides the equilibrium analysis. Section 2.5 discusses the optimal monetary policy if the central bank knows banks' beliefs perfectly, whereas section 2.6 provides a study of the optimal monetary policy in the presence of belief uncertainty. Section 2.7 discusses the optimal monetary policy in the presence of bank regulation, of information signaling, and of mistakes made by the central bank. Section 2.8 concludes.

2.2 Relation to the Literature

Our paper relates to four strands of the literature, namely the impact of non-rational expectations on macroeconomic policies, optimal monetary policies in the presence of uncertainty, private money creation, and central bank collateral frameworks.

A vast literature discusses how non-rational beliefs among private agents may curtail or amplify the effect of macroeconomic policies. Woodford (2013) reviews various modeling approaches without the hypothesis of rational expectations. A large part of this literature assumes bounded rationality of market participants and aims at finding optimal policies adressing this friction. The bounded rationality of agents has been studied in different ways, for instance, in the form of learning (e.g., Eusepi and Preston (2011)), level k-thinking (e.g., García-Schmidt and Woodford (2019)) or cognitive discounting (e.g., Gabaix (2020)). A closely related literature assumes agents are rational but are not perfectly informed about each other's beliefs, and illustrates how the resulting coordination problems can lead to aggregate behavior that resembles some forms of bounded rationality (Woodford, 2001; Morris and Shin, 2014; Angeletos and Huo, 2021). Our paper is in line with the literature on bounded rationality and information frictions, which generally assumes that the policy maker—in our setting the central bank—is fully rational. In modeling the beliefs in the economy, our work is close to Caballero and Simsek (2020), as banks act in an opinionated way and have their own subjective beliefs, following from the fact that banks interpret data differently than the central bank, for instance. In other words, banks and the central bank agree to disagree. In this regard, our paper is also related to Simsek (2013), which studies the impact of belief disagreements among private traders on collateral constraints for credit financing.

As we study the optimal monetary policy also in settings where the central bank is uncertain about the banks' beliefs, our work is closely connected to the literature that studies robust macroeconomic polices. For example, Woodford (2010) studies the optimally robust monetary policy in the form of interest rates in the presence of so-called near-rational expectations, i.e., the agents' expectations can be generated by the true economic model and are not "too far" away from rational expectations.

Our approach also follows a recent literature that models the dual role of banks—providing loans and creating money in the form of bank deposits (e.g., Faure and Gersbach (2017), Faure and Gersbach (2018) and Benigno and Robatto (2019)). In contrast to the existing literature, we model banks as liquidity-constrained and focus on the real effects of the central bank's collateral framework. We also show how the central bank can use collateral standards in its lending facilities to neutralize belief distortions among banks.

There is a substantial literature on the central bank collateral framework and its possible impact (for an overview see Bindseil (2004), and Bindseil et al. (2017), for instance).

Nyborg (2017) documents the weakening of collateral standards in the ECB's liquidity provisions after the financial crisis 2007/08. The fact that central bank collateral requirements matter for banks' liquidity holdings has been documented by Bindseil et al. (2009), for instance. In their analysis of the ECB's weekly refinancing operations between 2000 and 2001, the authors find evidence that collateral haircuts have been set imperfectly by the ECB, leaving different collateral with different opportunity costs. Fecht et al. (2011) also study the liquidity pricing in the repo transactions with the ECB and find some indication that the collateral available to the individual bank matters for the access to liquidity. Fuhrer et al. (2016) study transactions on the Swiss Franc repo market and find that collateral scarcity matters for a banks' re-use of collateral in the acquisition of liquidity. Our work relates to this literature, as we describe optimal central bank haircut rules in the lending operations to banks and show how they vary along the banks' beliefs. We also describe the optimal haircut rule on collateral if the central bank is uncertain about the banks' beliefs.

2.3 Model

2.3.1 Macroeconomic environment

We focus on a monetary economy, where trades are settled instantaneously by using money in the form of bank deposits. There are five types of agents—firms, households, investors, banks, and a government sector, including the central bank—and two goods—a capital good and a consumption good. Households and investors are endowed with the capital good which they can sell to firms for the production of the consumption good. Firms finance capital good purchases from households and investors either by demanding loans from banks or by issuing bonds at the financial markets. The model features private and public money creation. Private money takes the form of bank deposits which are issued by banks when granting loans to firms. Public money, in turn, is represented by reserves which banks can obtain from the central bank by demanding collateralized reserve loans and which are used by banks to settle interbank liabilities.[3] Liabilities arise when deposits are transferred from one bank to another, which occurs due to trading partners at the good markets holding deposit accounts with different banks. In our model, good markets and asset markets are perfectly competitive.

[3] For simplification we abstract from cash. For environments, where cash is only available through a conversion of bank deposits, this is without loss of generality because holding the alternative form of money, namely bank deposits, yields a positive interest.

Firm productivity, and thus loan repayment, is subject to idiosyncratic shocks. Banks have beliefs about the probability distribution of shocks, which may deviate from the true one. The model thus accounts for optimistic (pessimistic) banks that, compared to the true distribution, overestimate (underestimate) the probability of positive productivity shocks. The beliefs about productivity shocks determine the beliefs about repayment by borrowers and thus the expected value of bank loans. Since bank loans serve as collateral for reserve loans from the central bank, banks' beliefs about the value of bank loans translate into expectations about the access to liquidity at the central bank. The central bank sets the nominal interest rates on reserve loans and reserve deposits, as well as the haircut on bank loans when used as collateral for reserve loans. With the haircut, the central bank can directly affect the banks' access to liquidity.

We model banks as being constrained by liquidity, so that both their beliefs about loan repayment and the haircut set by the central bank matter for the banks' initial decision about loan supply and deposit issuance. Banks operate with limited equity financing and provide loan financing, which is generally risky. Thus, if the leverage becomes sufficiently large in the course of loan financing, banks are exposed to a solvency risk. Deposits issued by banks are insured by the government through guarantees. We impose an implicit deposit insurance, so that in the case of a bank default, depositors are bailed out by the government. As the government covers bank losses in the case of default, the deposits held by households are safe. Bank default, however, has real costs, as the resolution of a defaulting bank requires efforts that must be compensated with resources in the form of the consumption good. Throughout our analysis, we assume that the consolidated budget of the government sector, including the central bank, is balanced.

2.3.2 Summary of events

As we focus on a monetary economy with instantaneous settlement of transactions, the timing of interactions among agents is of great importance for our analysis. Figure 2.1 summarizes the events in our static framework.

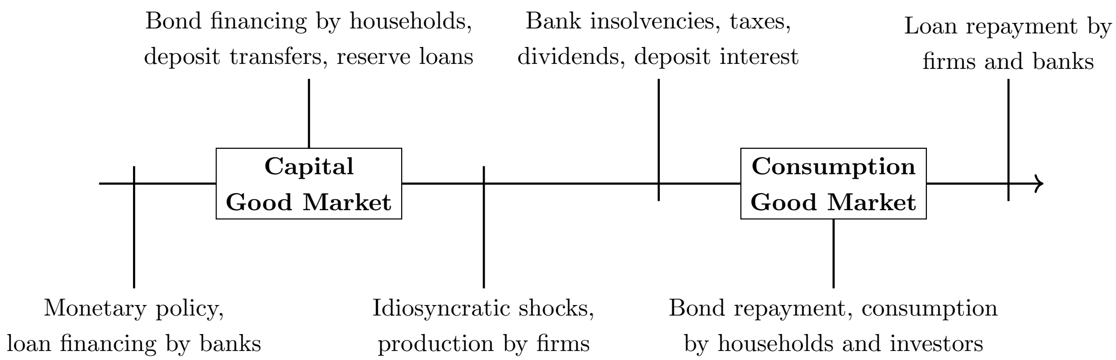

Figure 2.1: Timeline.

Note that all trades are settled by using bank deposits and that prices are in terms of the unit of account of the underlying currency. We use the consumption good as the numeraire of the economy. In the following subsections, we introduce the optimization problems of firms, households, investors, and banks and characterize their optimal choices. The proofs relating to the stated results can be found in appendix 6.1.5.

2.3.3 Firms

Firms are profit-maximizing, protected by limited liability, and penniless. They purchase the capital good from households and investors to produce the consumption good. There are two types of firms, which we index by L and B. Firms of each type are ex-ante identical and exist in a continuum with mass normalized to one, so that we can focus on a representative firm for each type. Firms of type L can only obtain funds (i.e., deposits) through loans by banks, whereas firms of type B can only raise funds in a frictionless bond market.[4]

The firm of type $f \in \{L, B\}$ acquires capital good $K^f \geq 0$ on the markets from households and investors at a nominal price $Q > 0$ and transforms it into consumption good $A_s^f K^f$, where $A_s^f \geq 0$ represents the available linear technology that depends on the idiosyncratic productivity shock s. The latter can be either positive ($s = \overline{s}$) or negative ($s = \underline{s}$), and thus it holds that $A_{\overline{s}}^f \geq A_{\underline{s}}^f$. The idiosyncratic productivity shocks are independent

[4]A possible justification for the assumption that L firms can only obtain loans from banks is that these firms suffer from moral hazard and banks are the only agents that can alleviate moral hazard by monitoring. The restriction that firms of type B can only access funds via the bond market serves the purpose of simplifying the subsequent analysis and can be relaxed.

and identically distributed (i.i.d.) across firms. A positive shock occurs with probability $\eta \in (0,1)$. Private agents—firms, households, investors and banks—have subjective beliefs which potentially deviate from the true ones. Specifically, they believe that the individual firm experiences a positive productivity shock with probability $\eta_m = \eta m$, where $m \in (0, 1/\eta)$ is the distortion factor. If the parameter m is larger (smaller) than one, we call agents and their beliefs optimistic (pessimistic).[5]

The produced consumption good $A_s^f K^f$ is sold to households and investors at a nominal price $P > 0$. The revenues, in the form of bank deposits, are then used to meet the repayment obligation towards external creditors, which, depending on the type of the firm, are banks or bond investors. The repayment is determined by the interest rate $r_s^f > 0$, which typically differs between loans and bonds. Accounting for limited liability, the expected profits of firm f are given in nominal terms by $\mathbb{E}_m[\{PA_s^f - (1 + r_s^f)Q\}^+]K^f$, where we use the notation $\{X\}^+ = \max\{X, 0\}$. Note that, due to the firms' subjective beliefs, the expectation operator is indexed by the distortion factor m. As firms are profit-maximizing, it follows that firm f's optimization problem is given in real terms by

$$\max_{K^f \geq 0} \mathbb{E}_m[\{A_s^f - (1 + r_s^f)q\}^+]K^f, \tag{2.1}$$

where the capital good price, denoted by a lowercase letter, is in terms of the consumption good, i.e., $q := Q/P$.

Due to limited liability, there exists no optimal, finite demand of capital good, if firm f is exposed to excess returns in at least one state. In contrast, without excess returns, firm f will be indifferent between all amounts of capital good put into production. A formal summary is provided in the following lemma.

Lemma 2.3.1 (Optimal Choice of Firms)
The optimal demand of capital good by firm $f \in \{L, B\}$ is characterized by $K^f = +\infty$ if and only if $A_s^f > (1 + r_s^f)q$ for some $s \in \{\underline{s}, \overline{s}\}$, and $K^f \in [0, +\infty)$ otherwise.

Two remarks regarding the relationship of repayment rates and firm productivity are in order. First, in any competitive equilibrium we consider, the capital good market must

[5] The analysis can (under additional assumptions on the trading of bank loans) also be conducted with differing beliefs among agents. To reduce complexity and highlight the relevance of banks' beliefs for the conduct of monetary policy, we focus however on the case where beliefs are shared by firms, households, investors and banks.

clear, which ultimately requires an optimal, finite demand of capital good on the side of firms. From lemma 2.3.1, we know that firms demand a finite amount of capital good if and only if the repayment obligations on external funds weakly exceed the revenues from production, i.e., if $A_s^f \leq (1 + r_s^f)q$ for all f, s. Second, while agents have subjective beliefs about the probability distribution of productivity shocks, we assume that they know the economic model in all other respects perfectly. Accordingly, in equilibrium, agents' behavior cannot be subject to predictable errors. In other words, up to their beliefs, agents are fully rational, which rules out firm default in equilibrium. Formally, this means that it holds $A_s^f \geq (1 + r_s^f)q$ for all f, s. Based on the previous two observations, we can conclude that in any competitive equilibrium it must hold that $A_s^f = (1 + r_s^f)q$ for all f, s and firms make zero profits.

We make specific assumptions on firm productivity. First, for simplicity, we assume that bond-financed firms operate without any risk. Second, we assume that a loan-financed firm is, under the true beliefs, more productive in expectation than a bond-financed firm. This guarantees that loan-financed firms and banks—as their source of financing—are relevant for maximizing aggregate production and, ultimately, welfare. Third, when a loan-financed firm experiences a negative productivity shock, it is less productive than a bond-financed firm. As explained in subsection 2.4.2, the latter assumption guarantees that banks can be exposed to a solvency risk.

Assumption 2.3.1 (Firm Productivities)
$A_{\bar{s}}^B = A_{\underline{s}}^B := A^B$, $\mathbb{E}[A_s^L] \geq A^B$ *and* $A^B > A_{\underline{s}}^L$.

It follows directly from assumption 2.3.1 that a loan-financed firm is more productive than a bond-financed firm if it incurs a positive productivity shock, i.e., $A_{\bar{s}}^L > A^B$.

2.3.4 Households

There is a continuum of identical households with mass normalized to one, so that we can focus on a representative household. The household is endowed with capital good $K > 0$, which can be sold to firms at a nominal price $Q > 0$. The revenues are in the form of deposits and can be invested in bonds, which yield a rate of return $r^B > 0$. Deposits, in turn, are credited with interest according to the rate $r^D > 0$. The share of funds held in the form of deposits is denoted by $\gamma \in [0, 1]$. The household owns firms which distribute any available profits Π as dividends. Taking governmental taxes T^H, which are assumed to be lump-sum, and dividends Π into account, the household uses deposits credited with interest $\gamma(1+r^D)QK$ and the revenues from bond investments $(1-\gamma)(1+r^B)QK$ to purchase an amount C^H of the consumption good from firms at the nominal price $P > 0$. The household maximizes utility, which we assume to be linearly increasing in consumption. Hence, the household's optimization problem is given in real terms by

$$\max_{\gamma\in[0,1]} [\gamma(1+r^D) + (1-\gamma)(1+r^B)]qK + \tau^H + \pi, \tag{2.2}$$

where the taxes and the profits, denoted by lowercase letters, are in terms of the consumption good, i.e., $\tau^H := T^H/P$ and $\pi := \Pi/P$.

Based on the assumption of linear utility, the optimal choice of the household is of knife-edge type. The available funds are invested in the asset which yields the highest return. The following lemma provides the formal details.

Lemma 2.3.2 (Optimal Choice of the Household)
$\gamma = 1$ *(*$\gamma = 0$*) if* $r^D > (<) r^B$ *and* $\gamma \in [0,1]$ *otherwise.*

2.3.5 Investors

Investors are identical and exist in a continuum of unit mass, so that we can focus on a representative investor. The investor is endowed with capital good $E > 0$, which can be sold to firms at a nominal price $Q > 0$. The revenues take the form of deposits and can be invested into equity across all active banks or bonds. The rate of return on equity for a particular bank is given by $r_s^E > 0$, which depends on the idiosyncratic shock s incurred by the respective bank. Bonds, in turn, are subject to a deterministic rate of return $r^B > 0$. The share of funds used for equity financing is denoted by $\zeta \in [0,1]$. Taking governmental taxes T^I, which are assumed to be lump-sum, into account, the investor uses equity returns $\zeta(1+\mathbb{E}_m[r_s^E])QE$ and the revenues from bond investments $(1-\zeta)(1+r^B)QE$ to purchase an amount C_m^I of the consumption good from firms at the nominal price $P > 0$. The investor maximizes the utility, which we assume to be linearly increasing in consumption. Hence, the investor's optimization problem is in real terms given by

$$\max_{\zeta\in[0,1]} [\zeta(1+\mathbb{E}_m[r_s^E]) + (1-\zeta)(1+r^B)]qE + \tau^I, \tag{2.3}$$

where the taxes, denoted by a lowercase letter, are in terms of the consumption good, i.e., $\tau^I := T^I/P$. The expectation about the return on bank equity depends on the subjective beliefs of the investor, which may be deviate from the true one. Accordingly, the expectation operator in (2.3) is indexed by the distortion factor m.

Due to the assumption of linear utility, the investor's optimal choice is of knife-edge type. The available funds are used to invest into the asset which yields the highest expected return. To simplify our analysis, we assume that in the case of indifference ($\mathbb{E}_m[r_s^E] = r^B$), the investor uses all funds to invest into equity ($\zeta = 1$).

Lemma 2.3.3 (Optimal Choice of the Investor)
$\zeta = 1$ ($\zeta = 0$) if and only if $\mathbb{E}_m[r_s^E] \geq (<) r^B$.

2.3.6 Government sector

The government sector consists of the central bank and the government. The central bank provides banks with liquidity in the form of reserves, which banks use to settle interbank liabilities. Reserves can be borrowed from the central bank via collateralized loans. The only pledgable assets available to banks are the loans provided to firms. The value of these bank loans is reduced by a haircut $\psi \in [0, 1]$, which is determined by the central bank. The ensuing borrowing constraint on the side of banks is introduced in subsection 2.3.7. Reserve deposits at the central bank are credited with interest according to the rate $r_{CB}^D > 0$, while reserve loans require a repayment determined by the rate $r_{CB}^L > 0$. For simplicity, we assume that both interest rates are equal.

Assumption 2.3.2 (Reserve Rates)
$r_{CB}^D = r_{CB}^L$.

In our setting, the central bank chooses the interest rate r_{CB}^D and the haircut ψ in order to maximize utilitarian welfare. Details are provided in section 2.5, where we also characterize the optimal monetary policy.

Banks can face a solvency risk if, in the course of loan financing to firms, the leverage becomes sufficiently large; a detailed discussion is provided in subsection 2.3.7. In any equilibrium we consider, default by firms is ruled out (see subsection 2.3.3), so that banks are the only agents in our economy who can default on their liabilities. As the government insures deposits through governmental guarantees, it must balance bank losses, which in the aggregate and in nominal terms are denoted by $\Pi^{b,-}$.[6] The government finances bank losses through lump-sum taxes on households and investors. The resolution of bank default requires efforts which reduce the production output available for consumption by households and investors. Taxes are thus also required to cover the default costs, which in the aggregate and in nominal terms are given by $P\Lambda$ and are further characterized in the equilibrium analysis (see section 2.4). Finally, the government must use taxes to cover losses of the central bank, while it can distribute central bank profits by using transfers. We denote nominal central bank profits/ losses by Π^{CB}. Throughout our analysis, we assume that the consolidated budget of the central bank and the government is balanced, so that governmental lump-sum taxes or transfers are given in nominal terms by $T = \Pi^{b,-} - P\Lambda + \Pi^{CB}$.

2.3.7 Banks

There is a continuum of ex-ante identical banks with mass normalized to one, so that we can focus on a representative bank. Banks are only active if they receive equity financing $E^b > 0$ from investors. Banks and firms are matched one-to-one, which leads to banks holding non-diversified loan portfolios and being fully exposed to the idiosyncratic risk of the financed firm. When a bank is established (i.e., $E^b > 0$), the bank provides loan financing to the matched firm. The decision about loan supply L^b then determines the loans-to-equity ratio $\varphi = L^b/E^b$ and the bank's deposit financing $D^b = L^b - E^b$ once investors used (parts of) their deposits to acquire bank equity.

Transactions on the market for the capital good lead to liabilities between banks when the counterparties to a transaction hold accounts at different banks and, as a consequence, interbank deposit flows occur.[7] We assume that for each bank, a share $\alpha \in (0, 1]$ of deposits is temporarily outflowing as capital good transactions are settled. The bank will become liable for the amount of deposits going to other banks, as it adds liabilities to other banks. We assume that the bank's interbank liabilities are equally distributed across all other banks and are settled in real time, i.e., the central bank applies a gross settlement procedure. Thus, deposit outflows have to be fully settled and cannot be netted later with

[6] In the case of default, the bank only defaults on the deposit funding. The reserve loans can always be repaid, even in the case of default, and thus bank losses only represent the unmet liabilities towards depositors. The fact that reserves can always be repaid rests on the assumption of a representative bank (see subsection 2.3.7).

[7] We abstract from interbank deposit flows that are due to transactions on the market for the consumption good, as this would provide no additional insights, but would complicate our analysis.

claims on other banks from deposit inflows. The bank must therefore borrow an amount $L^{CB} = \alpha(L^b - E^b)$ from the central bank.[8] The latter secures its claim $(1 + r^L_{CB})L^{CB}$ by demanding collateral, which, in our setting, corresponds to the loans that banks provide to firms. The nominal value of loans, as expected by the bank before the realization of the idiosyncratic shocks, is given by $(1 + \mathbb{E}_m[r^L_s])L^b$. Due to the agents' subjective beliefs, the expected value of bank loans may divert from the true one. Specifically, with a distortion factor m larger (smaller) than one, banks are optimistic (pessimistic) and, based on their expectation about repayment by borrowers, they value loans higher (lower) than under the true probability distribution. The central bank applies a haircut $\psi \in [0, 1]$ on the bank loans provided as collateral, so that $(1 - \psi)(1 + \mathbb{E}_m[r^L_s])L^b$ is the overall value of all pledgable assets, also referred to as the "collateral capacity" of the bank.

In the case of illiquidity, the bank defaults and the government seizes all assets, eliminating all potential revenues from banking. Thus, the bank's decision about loan supply and deposit issuance will always comply with the liquidity constraint

$$(1 - \psi)(1 + \mathbb{E}_m[r^L_s])L^b \geq \alpha(1 + r^L_{CB})(L^b - E^b),$$

which, using the loans-to-equity ratio $\varphi = L^b/E^b$, is equivalent to

$$(1 - \psi)(1 + \mathbb{E}_m[r^L_s])\varphi \geq \alpha(1 + r^D_{CB})(\varphi - 1),$$

where we also exploited the equality of interest rates on reserve loans and reserve deposits ($r^L_{CB} = r^D_{CB}$), following from assumption 2.3.2. The bank's beliefs about future loan repayment determine the expectation about the access to liquidity at the central bank and thus the initial decision to grant loans and finance them through deposit issuance. Using the previously outlined liquidity constraint, we can derive the maximum possible loans-to-equity ratio for which the bank is liquid. For a monetary policy (r^D_{CB} and ψ) that satisfies $\alpha(1 + r^D_{CB}) > (1 - \psi)(1 + \mathbb{E}_m[r^L_s])$, this leverage, denoted by $\varphi^L_m(\psi)$, is determined through

[8] We assume that the amount of deposits used by investors to acquire bank equity does not lead to deposit outflows on the market for the capital good. Accordingly, the relevant amount of deposits, of which a share $\alpha \in (0, 1]$ is temporarily flowing out, is given by $D^b = L^b - E^b$. Due to the equality of interest rates on reserve deposits and reserve loans (see assumption 2.3.2), we can then, without loss of generality, assume that the bank does not borrow more reserves than the required amount $\alpha(L^b - E^b)$.

the condition

$$(1-\psi)(1+\mathbb{E}_m[r_s^L])\varphi_m^L(\psi) = \alpha(1+r_{CB}^D)[\varphi_m^L(\psi)-1]$$

and thus leads to the maximum loans-to-equity ratio

$$\varphi_m^L(\psi) = \frac{\alpha(1+r_{CB}^D)}{\alpha(1+r_{CB}^D)-(1-\psi)(1+\mathbb{E}_m[r_s^L])}.$$

Note that the ratio $\varphi_m^L(\psi)$ is indexed by the distortion factor m, as the bank's expectation about liquidity access depends on its beliefs about loan repayment. For a monetary policy satisfying $\alpha(1+r_{CB}^D) \leq (1-\psi)(1+\mathbb{E}_m[r_s^L])$, the bank remains liquid for any loans-to-equity ratio, which we denote by $\varphi_m^L(\psi) = +\infty$.

As the bank can borrow reserves L^{CB} from the central bank and deposit reserves D^{CB} at the central bank, the balance sheet identity $L^b + D^{CB} = L^{CB} + D^b + E^b$ applies. We focus on a representative bank, so that deposit outflows equal deposit inflows. Accordingly, once the capital good transactions have been settled, reserve loans and reserve deposits must match, i.e., formally it holds that $L^{CB} = D^{CB}$. Using $L^{CB} = \alpha(L^b - E^b)$, the bank's assets are given by $L^b + D^{CB} = (1+\alpha)L^b - \alpha E^b$ and the assets-to-equity ratio $\tilde{\varphi} = (L^b + D^{CB})/E^b$ reads $\tilde{\varphi} = (1+\alpha)L^b/E^b - \alpha = (1+\alpha)\varphi - \alpha$. For the subsequent analysis, we will mostly focus on the loans-to-equity ratio φ, as it allows for a more natural representation and analysis of the bank's optimization problem. For convenience, we will then refer to φ as the bank leverage and to $\tilde{\varphi}$ as the integrated bank leverage, which specifically accounts for the reserve holdings of the bank.

The interest rates on reserve deposits and reserve loans equal (see assumption 2.3.2). To derive the rate of return on the bank's equity financing, we can thus focus, without loss of generality, on the balance sheet identity in reduced form that is given by $L^b = D^b + E^b$, ignoring reserve deposits and reserve loans. The loans yield a return that is determined by the rate $r_s^L > 0$, which depends on the idiosyncratic shock $s \in \{\underline{s}, \overline{s}\}$ of the financed firm, whereas deposits are credited with interest according to the deterministic rate $r^D > 0$. Banking operations are protected by limited liability, so that the nominal bank equity returns satisfy $(1+r_s^E)E^b = \{(1+r_s^L)L^b - (1+r^D)D^b\}^+$. These returns depend on the loan rate and therefore on the idiosyncratic shock $s \in \{\underline{s}, \overline{s}\}$ of the financed firm. We made use of the notation $\{X\}^+ = \max\{X, 0\}$ again. The rate of return per unit of bank equity then follows as $r_s^E(\varphi) := \{(r_s^L - r^D)\varphi + 1 + r^D\}^+ - 1$, where we exploited the definition of the bank leverage $\varphi = L^b/E^b$ and the fact that deposits $D^b = L^b - E^b$ are the residual funding source for loans, besides equity.

As loans are risky, the bank is exposed to a solvency risk if the leverage becomes sufficiently large in the course of loan financing to firms. For interest rates satisfying $r^D > r^L_{\underline{s}}$, the maximum leverage φ^S, which guarantees solvency of the bank in all states, is determined by

$$1 + r^E_{\underline{s}}(\varphi^S) = 0 \qquad \Leftrightarrow \qquad (r^L_{\underline{s}} - r^D)\varphi^S + 1 + r^D = 0 \qquad \Leftrightarrow \qquad \varphi^S = \frac{1 + r^D}{r^D - r^L_{\underline{s}}}.$$

For interest rates that satisfy $r^D \leq r^L_{\underline{s}}$, there is no bank leverage which exposes the bank to a solvency risk, as it holds that $1 + r^E_{\underline{s}}(\varphi) \geq 0$ for all $\varphi \geq 1$. We denote this case by $\varphi^S = +\infty$.

The bank maximizes the shareholder value, so that the optimization problem is given by

$$\max_{\varphi \in [1, \varphi^L_m(\psi)]} \mathbb{E}_m[r^E_s(\varphi)]. \tag{2.4}$$

The expectation operator in (2.4) is indexed by the distortion factor m, as the bank's beliefs about the idiosyncratic shock of the financed firm may deviate from the true ones.

In the analysis of the bank's optimal choice of leverage, we have to take into account that the bank is protected by limited liability and may face a solvency risk. First, we focus on the situation where solvency of the bank is always guaranteed, because equity financing or the haircut on bank loans used as collateral for reserve loans is sufficiently large, for instance. Formally, in any such situation, it holds that $\varphi^L_m(\psi) \leq \varphi^S$. Note that the expected rate of return on bank loans is given by $\mathbb{E}_m[r^L_s]$, whereas the interest rate on deposits is given by r^D. Thus, when granting loans funded with deposits yields profits (losses) in expectation, as it holds that $\mathbb{E}_m[r^L_s] > (<) r^D$, the bank chooses the maximum (minimum) possible leverage $\varphi = \varphi^L_m(\psi)$ ($\varphi = 1$). In other words, it supplies the maximum (minimum) possible amount of loans. If it holds that the expected interest rate on loans equals the rate of return on deposits, $\mathbb{E}_m[r^L_s] = r^D$, the bank makes zero profits by granting loans to firms which are financed through deposit issuance, and is thus indifferent between all leverages. To simplify our equilibrium analysis, we assume that in the case of indifference ($\mathbb{E}_m[r^L_s] = r^D$), the bank also chooses the maximum possible bank leverage $\varphi = \varphi^L_m(\psi)$.

Second, we consider the situation where the bank may face a solvency risk if, in the course of loan financing, the leverage grows sufficiently. Formally, such a situation is only possible if it holds that $\varphi_m^L(\psi) > \varphi^S$. The interest rate on deposits is given by r^D, whereas the expected rate of return from loans is *without* a solvency risk ($\varphi \leq \varphi^S$) given by $\mathbb{E}_m[r_s^L]$ and *with* a solvency risk ($\varphi > \varphi^S$) given by $\eta_m r_{\bar{s}}^L$. There are two types of environments in which the bank chooses the maximum possible leverage $\varphi = \varphi_m^L(\psi)$. First, even without the benefits from limited liability, financing loans with deposits is profitable ($\mathbb{E}_m[r_s^L] \geq r^D$), and thus induces the bank to grant as many loans as possible funded with deposits. Second, financing loans with deposits is not profitable without benefiting from limited liability ($\mathbb{E}_m[r_s^L] < r^D$), but the bank can leverage sufficiently, so that, with limited liability, the expected profits under the maximum leverage exceed the ones of financing loans only with equity, i.e., it holds that

$$\eta_m[(r_{\bar{s}}^L - r^D)\varphi_m^L(\psi) + 1 + r^D] > 1 + \mathbb{E}_m[r_s^L] \quad \Leftrightarrow \quad \varphi_m^L(\psi) > \frac{(1 + \mathbb{E}_m[r_s^L])/\eta_m - 1 - r^D}{r_{\bar{s}}^L - r^D}.$$

The latter condition clearly requires returns satisfying $r_{\bar{s}}^L > r^D$, namely that in the presence of a positive productivity shock of the financed firm, the interest rate on loans exceeds the interest rate on deposits. In any other environment with the possibility of a solvency risk ($\varphi_m^L(\psi) > \varphi^S$), the bank chooses to finance loans only with equity ($\varphi = 1$). The following lemma summarizes the above explanations.

Lemma 2.3.4 (Optimal Choice of the Bank)
Without the possibility of a solvency risk, i.e., if $\varphi_m^L(\psi) \leq \varphi^S$, the bank's optimal choice of leverage is given by $\varphi = \varphi_m^L(\psi)$ ($\varphi = 1$) if and only if it holds that $\mathbb{E}_m[r_s^L] \geq (<) r^D$.

With the possibility of a solvency risk, i.e., if $\varphi_m^L(\psi) > \varphi^S$, the bank's optimal choice of leverage is given by $\varphi = \varphi_m^L(\psi)$ if and only if it holds that $\mathbb{E}_m[r_s^L] \geq r^D$, or $r_{\bar{s}}^L > r^D$ and $\varphi_m^L(\psi) > [(1 + \mathbb{E}_m[r_s^L])/\eta_m - 1 - r^D]/(r_{\bar{s}}^L - r^D)$, and $\varphi = 1$ otherwise.

We also account for an interbank market, where banks are matched one-to-one and can borrow from, lend to and deposit with each other. Interbank loans are also collateralized through bank loans granted to firms, where the value of bank loans pledged for interbank loans is reduced by the haircut $\tilde{\psi} \in [0, 1]$. The collateral provided for an interbank loan can be rehypothecated when demanding a reserve loan from the central bank. The interest rates on interbank loans and interbank deposits are equal. Moreover, the bank cannot apply a different pricing on interbank deposits and deposits held by households and firms. Thus, the prevailing interest rate on the interbank market is given by $r^D > 0$.

Note that the bank granting an interbank loan must rehypothecate the pledged assets whenever the borrowing bank transfers interbank deposits to settle liabilities with other banks. The reason is that an interbank loan is completely financed with interbank deposits held by the borrowing bank. Whenever the latter must meets its liabilities with other banks, it can use the interbank deposits and transfer them to the banks it is liable to. This, however, requires the bank granting the interbank loan to hold enough liquidity to meet the liabilities arising from a transfer of interbank deposits.[9] Any bank granting an interbank loan cannot share the liquidity it obtained from the central bank or other banks. It needs this liquidity to settle its own interbank liabilities with other banks. Thus, the only way a bank can provide interbank loans and guarantee that enough liquidity is available to settle the liabilities emerging from the transfer of interbank deposits is that the bank loans pledged by the borrowing bank are completely rehypothicated at the central bank.

As stated in the following lemma, we can then establish a relationship between the terms and conditions for liquidity from the central bank, as captured by r^D_{CB} and ψ, and the standards on liquidity provision through the interbank market, namely r^D and $\tilde{\psi}$.

Lemma 2.3.5 (Interbank Market)
$(1 + r^D)(1 - \psi) = (1 + r^D_{CB})(1 - \tilde{\psi})$.

Given a monetary policy r^D_{CB} and ψ, we can deduce that collateral requirements on the interbank market which are looser than the ones at the central bank ($\tilde{\psi} < \psi$) lead to an interest rate on the interbank market (and ultimately on bank deposits) which is higher than the interest rate on reserves ($r^D > r^D_{CB}$), and vice versa. Moreover, with identical collateral standards at the central bank and on the interbank market, the interest rates on deposits and reserves are equal.

Corollary 2.3.1 (Deposit Rate)
With $\tilde{\psi} = \psi$, it holds that $r^D = r^D_{CB}$.

[9] Note that we model a continuum of banks and assume that interbank liabilities of the individual bank are equally distributed across all other banks and that banks are matched one-to-one on the interbank market. It thus follows that the individual bank granting interbank loans will face a complete outflow of interbank deposits when the borrowing bank uses them to settle its liabilities with other banks.

For tractability of the model, we assume that the haircut on the interbank market is identical to the haircut set by the central bank ($\tilde{\psi} = \psi$). It then follows from corollary 2.3.1 that the interest rates on deposits and reserves match ($r^D = r^D_{CB}$).

Assumption 2.3.3 (Haircuts)
$\tilde{\psi} = \psi$.

2.4 Equilibrium Analysis

2.4.1 Equilibrium definition

In what follows, we focus on competitive equilibria, which are defined hereafter. We use the notation $Y := \mathbb{E}[A^L_s]K^L + A^B K^B$ to represent the aggregate production output. Note that due to the assumption that productivity shocks are i.i.d. across firms, it holds, based on the law of large numbers, that under true beliefs, the expected production of loan-financed firms equals their aggregate production. Following the outline in subsections 2.3.4 and 2.3.5, aggregate consumption by households and investors is, under true beliefs, given by $C^H = [\gamma(1+r^D)+(1-\gamma)(1+r^B)]qK+\tau^H+\pi$ and $C^I = [\zeta(1+\mathbb{E}[r^E_s])+(1-\zeta)(1+r^B)]qE+\tau^I$, respectively.

Definition 2.4.1 (Competitive Equilibrium)
Given a monetary policy $r^D_{CB} > 0$ and $\psi \in [0, 1]$, a competitive equilibrium is a set of prices $P > 0$ and $Q > 0$, interest rates $r^L_s > 0$, $r^B > 0$, $r^D > 0$ and $r^E_s > 0$, with $s \in \{\underline{s}, \overline{s}\}$, and a set of choices K^L, K^B, γ, ζ, and φ, so that

(i) given P, Q, r^L_s, with $s \in \{\underline{s}, \overline{s}\}$, the choice K^L maximizes the expected profits of the loan-financed firm,

(ii) given P, Q, and r^B, the choice K^B maximizes the profits of the bond-financed firm,

(iii) given P, Q, r^D and r^B, the choice γ maximizes the utility of the household,

(iv) given P, Q, r^E_s, with $s \in \{\underline{s}, \overline{s}\}$, and r^B, the choice ζ maximizes the utility of the investor,

(v) *given* r^D_{CB}, ψ, r^L_s, *with* $s \in \{\underline{s}, \overline{s}\}$, *and* r^D, *the choice* φ *maximizes the expected profits of the bank,*

(vi) *the equity, loan, capital good and consumption good markets clear, i.e.,* $E^b = \zeta QE$, $QK^L = \varphi E^b$, $K^L + K^B = K + E$ *and* $C^H + C^I = Y$.

Note that, in the definition of a competitive equilibrium, we did not account for the clearing of the deposit market, as it clears by the construction of the model.

2.4.2 Equilibrium properties

We first highlight some general equilibrium properties, relating to interest rates, prices, bank leverage, default costs and welfare. We then proceed by providing the necessary conditions for the existence of an equilibrium and the bank's exposure to a solvency risk.

Interest rates. In subsection 2.3.3, we outlined that, up to the distribution of productivity shocks, agents know the economic model perfectly. Accordingly, in equilibrium, the interest rates on loans and bonds must be directly linked to firm productivity, so that firm default is ruled out, i.e., it holds that

$$(1 + r^L_s)q = A^L_s, \quad \text{with} \quad s \in \{\underline{s}, \overline{s}\}, \qquad \text{and} \qquad (1 + r^B)q = A^B, \tag{2.5}$$

where $q = Q/P$ represents the price of the capital good in terms of the consumption good. Note that, based on assumption 2.3.1, bond-financed firms do not face productivity shocks, since it holds that $A^B_{\overline{s}} = A^B_{\underline{s}} := A^B$, and thus bonds are subject to a deterministic repayment. Moreover, from corollary 2.3.1 and assumption 2.3.3, we know that the interest rates on deposits and reserves equal ($r^D = r^D_{CB}$). From lemma 2.3.2, it follows that whenever the household invests in deposits and bonds ($0 < \gamma < 1$), the interest rates on deposits and bonds must be equal ($r^D = r^B$). Using the conditions (2.5), it then follows

$$(1 + r^D)q = (1 + r^D_{CB})q = A^B. \tag{2.6}$$

For the polar cases, where either banks issue no deposits or bond-financed firms do not operate, so that households hold no deposits or do not invest into bonds ($\gamma \in \{0, 1\}$), we impose that the interest rates on deposits and bonds are still equal ($r^D = r^B$).[10] From

[10] In the appendix 6.1.1, we outline the flow consistency of our model for the cases where households hold deposits and bonds ($0 < \gamma < 1$).

assumption 2.3.1, we know that it holds $A^L_{\bar{s}} > A^B > A^L_{\underline{s}}$, so that, using the conditions (2.5) and (2.6), we an conclude that the interest rate on loans is higher (lower) than the interest rate on deposits if the financed firm incurs a positive (negative) productivity shock. Formally, it holds that

$$(1 + r^L_{\bar{s}})q = A^L_{\bar{s}} > (1 + r^D)q = (1 + r^D_{CB})q = A^B > (1 + r^L_{\underline{s}})q = A^L_{\underline{s}}. \tag{2.7}$$

Thus, when leveraging sufficiently by financing loans with deposits, banks are exposed to a solvency risk, namely banks default, if loan repayment is low, as the financed firm incurs a negative productivity shock.

Prices. From the previously established deposit pricing condition (2.6), we know that the prices in our economy, namely P and Q, must satisfy

$$\frac{P}{Q} = \frac{1 + r^D_{CB}}{A^B}. \tag{2.8}$$

Given a capital good price Q and firm productivity A^B, the consumption good price P and the interest rate r^D_{CB} on reserves are positively correlated. An increase in the interest rate r^D_{CB} induces an increase in the consumption good price P. Similarly, given a capital good price Q and the interest rate r^D_{CB}, the consumption good price P is negatively correlated with the productivity of bond-financed firms A^B. A productivity decrease induces thus also an increase in the consumption good price.

Bank leverage. With the equilibrium conditions (2.5) on firms' repayment obligations, and the deposit pricing condition (2.6), we can express the leverage ratios φ^S and $\varphi^L_m(\psi)$, both introduced in subsection 2.3.7, using model primitives. Focusing on the leverage φ^S, note that it follows from condition (2.7) that the interest rates on deposits and loans satisfy $r^D > r^L_{\underline{s}}$. Accordingly, we know from the outline in subsection 2.3.7 that φ^S satisfies

$$\varphi^S = \frac{1 + r^D}{r^D - r^L_{\underline{s}}} = \frac{(1 + r^D)q}{(1 + r^D)q - (1 + r^L_{\underline{s}})q} = \frac{A^B}{A^B - A^L_{\underline{s}}}. \tag{2.9}$$

Given the productivity A^B of bond-financed firms, an increase of the productivity $A^L_{\underline{s}}$ of loan-financed firms for a negative productivity shock increases the leverage threshold φ^S. Similarly, given the productivity of loan-financed firms for a negativity productivity shock, an increase of the productivity of bond-financed firms lowers the leverage threshold φ^S.

We can also express the leverage ratio $\varphi^L_m(\psi)$ in terms of economic fundamentals. Note that for a monetary policy (r^D_{CB} and ψ) satisfying $\alpha(1+r^D_{CB}) > (1-\psi)(1+\mathbb{E}_m[r^L_s])$, it follows, using conditions (2.5) and (2.6), that

$$\varphi^L_m(\psi) = \frac{\alpha(1+r^D_{CB})q}{\alpha(1+r^D_{CB})q - (1-\psi)(1+\mathbb{E}_m[r^L_s])q} = \frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_m[A^L_s]}, \tag{2.10}$$

whereas for a monetary policy satisfying $\alpha(1+r^D_{CB}) \leq (1-\psi)(1+\mathbb{E}_m[r^L_s])$, we maintain our previous definition $\varphi^L_m(\psi) = +\infty$.

In equilibrium, the equity market and the loan market clear, so that it must formally hold that $E^b = \zeta QE$ and $QK^L = \varphi E^b$. Both market clearing conditions allow us to relate the bank leverage, the investors' equity financing decision, and real bank lending. The equilibrium leverage is given by $\varphi = K^L/(\zeta E)$ or, equivalently, real bank lending satisfies $K^L = \varphi\zeta E$. With the clearing of the capital good market, $K^L + K^B = K + E$, we know that real bond-financing is the residual of the total capital good endowment in the economy that is not used by loan-financed firms and thus is given by $K^B = K + E - K^L = K + E - \varphi\zeta E$.

Default costs. In our setting, bank default arises if the leverage of the bank is sufficiently large ($\varphi > \varphi^S$), and the financed firm incurs a negative productivity shock ($s = \underline{s}$). In terms of the consumption good, the costs of resolving bank default scale with the real amount of bank assets after repayment and are in the aggregate given by

$$\Lambda = (1-\eta)\lambda(1+r^L_{\underline{s}})qK^L = (1-\eta)\lambda A^L_{\underline{s}} K^L, \tag{2.11}$$

where $\lambda \in (0,1)$ is used for scaling purposes and referred to as default cost parameter.[11] Note that in the presence of solvency risk, the default costs created by a single defaulting bank are given by $\lambda(1+r^L_{\underline{s}})qK^L$, which, using the equilibrium conditions (2.5) on firms' repayment rates, read as $\lambda A^L_{\underline{s}} K^L$. As productivity shocks are i.i.d. across firms, and banks are matched one-to-one with firms, a mass $1-\eta$ of banks defaults in the presence of solvency risk. We accounted for this fact in the specification of the aggregate default costs Λ.

Welfare. Throughout our analysis, we focus on utilitarian welfare. Based on our assumption of linear utility for households and investors, welfare, which is denoted by W, is represented by aggregate consumption, so that it holds that $W = C^H + C^I$. Lemma 2.4.1 provides a characterization of welfare, using primitives of the model.

Lemma 2.4.1 (Welfare)
In equilibrium, welfare is $W = \{\mathbb{E}[A^L_s] - \mathbb{1}\{\varphi > \varphi^S\}(1-\eta)\lambda A^L_{\underline{s}}\}K^L + A^B(K + E - K^L)$.

[11] For an analysis of various other default cost specifications, see Malherbe (2020), for instance.

In our framework, aggregate consumption, and thus utilitarian welfare, comprises aggregate production $\mathbb{E}[A_s^L]K^L + A^B K^B$ and costs due to bank default $(1-\eta)\lambda A_{\underline{s}}^L K^L$. Note that bank default occurs only if banks operate under a sufficiently high leverage ($\varphi > \varphi^S$). Moreover, the amount of capital good used by bond-financed firms equals the total capital good endowment in the economy less the capital good used by loan-financed firms, i.e., $K^B = K + E - K^L$.

Existence and solvency risk. In the following, we provide necessary conditions for the existence of an equilibrium and the bank's exposure to a solvency risk. We restrict private agents to be sufficiently optimistic about the productivity of loan-financed firms. Specifically, we assume that firms, households, investors and banks always believe that a loan-financed firms is at least as productive on average as a bond-financed firm.

Assumption 2.4.1 (Beliefs)
$\mathbb{E}_m[A_s^L] \geq A^B$.

In appendix 6.1.3, we provide the equilibrium analysis and a characterization of the optimal monetary policy in the case of sufficiently pessimistic agents, namely when the distortion factor m satisfies $\mathbb{E}_m[A_s^L] < A^B$.

First, note that under assumption 2.4.1, it follows that the expected loan rate weakly exceeds the deposit rate ($\mathbb{E}_m[r_s^L] \geq r^D$). This follows from the equilibrium link between productivity and the firms' repayment obligations, namely $(1+r_s^L)q = A_s^L$ for all s and $(1+r^D)q = (1+r^B)q = A^B$. From lemma 2.3.4, we then know that the bank always chooses the maximum possible leverage. Moreover, due to the expected loan return exceeding the interest payments on deposits, the expected rate of return on bank equity will also be weakly larger than the interest rate on bonds ($\mathbb{E}_m[r_s^E] \geq r^B$). Using lemma 2.3.3, we can conclude that the investor uses all funds to provide equity financing for the bank.

Lemma 2.4.2 (Bank Leverage and Equity Financing)
It holds that $\varphi = \varphi_m^L(\psi)$ *and* $\zeta = 1$.

In any competitive equilibrium, the equity market must clear, so that the equity financing of banks is given by $E^b = QE$. Moreover, the clearing of the loan market, $QK^L = \varphi E^b$, allows us to express real bank lending as $K^L = \varphi_m^L(\psi)E$. The existence of an equilibrium depends crucially on the clearing of the capital good market. Specifically, we require that the loans granted by banks do not allow firms to acquire more than the entire capital good in the economy. Formally, it must hold that $QK^L = \varphi_m^L(\psi)QE \leq Q(K+E)$, which, using $\varphi^M := 1 + K/E$ to denote the maximum feasible bank leverage, is equivalent to $\varphi_m^L(\psi) \leq \varphi^M$. The latter condition leads us to a smallest feasible haircut ψ_m^M which, if implemented by the central bank, allows loan-financed firms to exactly acquire the entire capital good in the economy. Any haircut larger than ψ_m^M restricts the capital good purchases of the loan-financed sector below the maximum feasible ones and implicitly shifts capital good for production to bond-financed firms. The central bank's choice of the haircut has thus a direct influence on the capital allocation in the economy.

As outlined in subsection 2.3.7, the bank is exposed to a solvency risk whenever the bank's leverage is sufficiently high to surpass the threshold φ^S. Since under assumption 2.4.1, the bank always attains the maximum possible leverage, solvency risk exists whenever it holds that $\varphi_m^L(\psi) > \varphi^S$. Equation (2.9) provides the leverage ratio φ^S expressed using economic fundamentals. The condition $\varphi_m^L(\psi) > \varphi^S$ allows us to derive a critical haircut ψ_m^S, so that for any haircut ψ smaller than ψ_m^S the bank is exposed to solvency risk, as the leverage of the bank exceeds the threshold φ^S. The formal details are provided in the following proposition.

Proposition 2.4.1 (Existence and Solvency Risk)
A competitive equilibrium exists if it holds that

$$\psi \geq \psi_m^M := 1 - \frac{\alpha A^B}{\mathbb{E}_m[A_s^L](1+E/K)},$$

and the bank is exposed to a solvency risk if it holds that

$$\psi < \psi_m^S := 1 - \frac{\alpha A_{\underline{s}}^L}{\mathbb{E}_m[A_s^L]}.$$

Besides the capital allocation in the economy, the choice of the haircut by the central bank also affects the bank's exposure to a solvency risk and thus the occurrence of bank default. These two channels are at the core of the subsequent analysis of the optimal monetary policy.

2.5 Optimal Monetary Policy

In this section, we characterize the optimal monetary policy as represented by the interest rate on reserves r^D_{CB} and the haircut ψ that applies to bank loans pledged as collateral for reserve loans. We also highlight the effect of economic fundamentals and the banks' beliefs about firm productivity and loan repayment, as captured by the distortion factor m, on the optimal monetary policy. In this section, it is assumed that the central bank perfectly knows the beliefs in the economy when deciding about the monetary policy.

While the central bank's choice of the haircut always influences the capital allocation in the economy, the central bank can only trigger or eliminate bank default whenever there is the possibility for a solvency risk. For what follows, we will focus on the case, where, at least under the maximum feasible bank leverage, the bank faces a solvency risk.

Assumption 2.5.1 (Solvency Risk)
$\varphi^M > \varphi^S$ *or , equivalently,* $\psi^M_m < \psi^S_m$.

In our setting, the central bank is maximizing utilitarian welfare. We first observe that the interest rate on reserves r^D_{CB} affects the prices in our economy (see equation (2.8) in subsection 2.4.2). From lemma 2.4.1, in turn, we know that independent of the banks' exposure to a solvency risk, welfare is not influenced by the rate of return on reserves r^D_{CB}. This is a manifestation of the neutrality of money. To the contrary, the haircut ψ set by the central bank generally influences the capital allocation in the economy as well as the banks' exposure to a solvency risk, through its impact on bank lending. With the haircut on bank loans used as collateral, the central bank can regulate the bank's access to liquidity, namely their ability to borrow reserves. As the liquidity constraint, which depends on the haircut ψ, influences the bank's initial decision to grant loans and finance them through deposit issuance, the central bank is able to affect bank lending. Taking the irrelevance of the interest rate r^D_{CB} for the real allocation into account, the optimization problem of the central bank is formally given by

$$\max_{\psi\in[0,1]} W = \max_{\psi\in[0,1]} \{\mathbb{E}[A^L_s] - \mathbb{1}\{\varphi > \varphi^S\}(1-\eta)\lambda A^L_{\underline{s}}\}K^L + A^B(K + E - K^L),$$

where we used lemma 2.4.1 to express welfare W. With the previous results on the existence of an equilibrium and the bank's exposure to a solvency risk (see proposition 2.4.1), we can rewrite the optimization problem of the central bank, as outlined in the following lemma.

Lemma 2.5.1 (The Central Bank's Optimization Problem)
The optimization problem of the central bank is

$$\max_{\psi \in [\psi_m^M, 1]} \{\mathbb{E}[A_s^L] - A^B - \mathbb{1}\{\psi < \psi_m^S\}(1-\eta)\lambda A_{\underline{s}}^L\}\varphi_m^L(\psi).$$

The optimal monetary policy in the form of the haircut ψ depends on the productivity in the two production sectors, A^B and A_s^L, with $s \in \{\underline{s}, \overline{s}\}$, the default costs, as proxied by the parameter λ, and the beliefs, as captured by the distortion factor m.

In its choice of the haircut, the central bank must generally trade off productivity losses and costs due to bank default. From assumption 2.3.1, we know that under the true beliefs, a loan-financed firm is weakly more productive in expectation than a bond-financed firm ($\mathbb{E}[A_s^L] \geq A^B$). Thus, the central bank has no incentive to choose a haircut larger than ψ_m^S. Restricting the bank leverage below φ^S, by setting a haircut higher than ψ_m^S, only reduces bank lending with negative effects for aggregate production and ultimately welfare, but does not yield any benefit.

Furthermore, we know, based on assumption 2.5.1, that bank lending is not maximized under the haircut ψ_m^S, as it holds that $\varphi^M > \varphi^S$ or, equivalently, $\psi_m^M < \psi_m^S$. Any haircut lower than ψ_m^S will lead to costs due to bank default but further extend loan financing by banks. A necessary condition for the optimality of any haircut lower than ψ_m^S is that under true beliefs the expected productivity difference of loan-financed and bond-financed firms is positive, even when taking the costs of bank default into account. Formally, it must hold that

$$\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L > 0 \qquad \Leftrightarrow \qquad \lambda < \lambda^S := \frac{\mathbb{E}[A_s^L] - A^B}{(1-\eta)A_{\underline{s}}^L}.$$

With a default cost parameter satisfying $\lambda < \lambda^S$, welfare is for any situation with bank default maximized for the smallest feasible haircut ψ_m^M. Accordingly, we can conclude that the central bank optimally chooses the haircut ψ_m^S, restricting bank lending and ruling out bank default, instead of the haircut ψ_m^M, maximizing bank lending but allowing for bank default, if it holds that

$$\{\mathbb{E}[A_s^L] - A^B\}\varphi^S \geq \{\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L\}\varphi^M,$$

from which we can derive a condition on the default cost parameter λ, as represented by equation (2.12) in proposition 2.5.1. Note that, based on assumption 2.5.1, it holds that $\lambda^M < \lambda^S$. Accordingly, we can conclude that the central bank chooses the haircut ψ_m^S, restricting bank lending and ruling out bank default, if and only if default costs are sufficiently large, i.e., it holds that $\lambda \geq \lambda^M$. Otherwise, the central bank optimally chooses the smallest feasible haircut ψ_m^M, maximizing bank lending and allowing for bank default. The previous explanations are summarized in the following proposition.

Proposition 2.5.1 (Optimal Monetary Policy)
The central bank optimally restricts liquidity, so that banks are not exposed to a solvency risk, by setting the haircut ψ_m^S, if and only if default costs are sufficiently large, i.e., it holds that

$$\lambda \geq \lambda^M := \left(1 - \varphi^S/\varphi^M\right)\lambda^S. \tag{2.12}$$

Otherwise, the central bank optimally sets the haircut ψ_m^M, maximizing bank lending and allowing for bank default.

The more optimistic agents are about productivity shocks in the loan-financed sector (i.e., m is increasing), the higher the expected value of bank loans and thus the larger the liquidity access expected by banks, which in turn leads to more loan financing in the first place. To restrict bank lending to the optimal level, the central bank must counteract the effect of agents' more optimistic beliefs by implementing tighter collateral standards in the form of a larger haircut on bank loans. Thus, the haircuts ψ_m^S and ψ_m^M, as provided in proposition 2.4.1, both increase with m. Similarly, with growing pessimism (i.e., m is decreasing), the central bank applies looser collateral requirements, adjusting the respective haircut downwards. With perfect information, the central bank can completely eliminate any belief distortions of private agents and induce the desired bank leverage (φ^S or φ^M).

Corollary 2.5.1 (Optimal Monetary Policy and Beliefs)
Suppose the central bank implements the monetary policy according to proposition 2.5.1. Then, the optimal haircut increases (decreases) with more optimistic (pessimistic) beliefs, i.e., it holds that

$$\frac{\partial \psi_m^S}{\partial m} = \frac{\alpha A_{\bar{s}}^L \eta (A_{\bar{s}}^L - A_{\underline{s}}^L)}{(\mathbb{E}_m[A_s^L])^2} > 0 \qquad \textit{and} \qquad \frac{\partial \psi_m^M}{\partial m} = \frac{\alpha A^B \eta (A_{\bar{s}}^L - A_{\underline{s}}^L)}{(\mathbb{E}_m[A_s^L])^2 (1 + E/K)} > 0.$$

Whenever the central bank optimally aims at restricting bank lending and thereby ruling out bank default, it chooses the haircut ψ_m^S which is independent of the productivity in the bond-financed sector (see proposition 2.4.1). The productivity of loan-financed firms, in turn, influences the haircut in two ways: On the one hand, an increase of the productivity for any state s leads to a higher expected value of bank loans, increasing the bank's collateral capacity. On the other hand, an increase of the productivity in the low productivity state ($s = \underline{s}$) leads to a higher leverage ratio $\varphi^S = A^B/(A^B - A_{\underline{s}}^L)$ guaranteeing the solvency of banks. If the productivity in the high productivity state increases, the value of expected bank loans increases but the leverage threshold φ^S is left unchanged. Thus, the central bank must increase the optimal haircut ψ_m^S to counteract the increase in the valuation of bank loans. If the productivity in the low productivity state increases, the value of bank loans and the critical leverage threshold φ^S both increase. The first effect incentivizes the central bank to increase the optimal haircut ψ_m^S, while the second effect incentivizes the central bank to decrease the haircut. It turns out that the second effect dominates the first one and the central bank, ultimately, lowers the optimal haircut ψ_m^S if the productivity in the low productivity state increases (see corollary 2.5.2).

If the central bank aims at implementing maximum bank lending and thereby allowing for bank default, it chooses the haircut ψ_m^M which decreases with the productivity A^B in the bond-financed sector. The haircut ψ_m^M also increases with an improved productivity of loan-financed firms in any state. A higher productivity in the loan-financed sector increases the bank's collateral capacity and allows the bank, ceteris paribus, to borrow more reserves at the central bank and to extend loan supply as well as deposit issuance in the first place. To restrict bank leverage again to the maximum feasible one φ^M, the optimal haircut ψ_m^M must increase if the productivity in the loan-financed sector increases for any state. The details are provided in the following corollary.

Corollary 2.5.2 (Optimal Monetary Policy and Productivity)
Suppose the central bank implements the monetary policy according to proposition 2.5.1. Then, the haircut ψ_m^S does not vary with the productivity of bond-financed firms, but increases (decreases) with the productivity of loan-financed firms in the high (low) productivity state, i.e.,

$$\frac{\partial \psi_m^S}{\partial A^B} = 0, \qquad \frac{\partial \psi_m^S}{\partial A_{\bar{s}}^L} = \frac{\alpha \eta_m A_{\underline{s}}^L}{(\mathbb{E}_m[A_s^L])^2} > 0, \qquad \text{and} \qquad \frac{\partial \psi_m^S}{\partial A_{\underline{s}}^L} = -\frac{\alpha \eta_m A_{\bar{s}}^L}{(\mathbb{E}_m[A_s^L])^2} < 0.$$

In turn, the haircut ψ_m^M declines with a higher productivity of bond-financed firms, i.e.,

$$\frac{\partial \psi_m^M}{\partial A^B} = -\frac{\alpha}{\mathbb{E}_m[A_s^L](1 + E/K)} < 0,$$

and increases with the productivity of loan-financed firms in both states, i.e.,

$$\frac{\partial \psi_m^M}{\partial A_{\bar{s}}^L} = \frac{\alpha A^B \eta_m}{(\mathbb{E}_m[A_s^L])^2(1 + E/K)} > 0 \qquad \text{and} \qquad \frac{\partial \psi_m^M}{\partial A_{\underline{s}}^L} = \frac{\alpha A^B (1 - \eta_m)}{(\mathbb{E}_m[A_s^L])^2(1 + E/K)} > 0.$$

2.6 Optimal Monetary Policy with Uncertainty about Beliefs

In this section, we analyze the optimal monetary policy if the central bank is uncertain about the beliefs in the economy. Formally, the central bank cannot perfectly observe the actual distortion factor $m \in (0, 1/\eta)$. We study the optimal monetary policy in a setting where there are two potential types of beliefs that realize with positive probabilities, not necessarily being uniform. We derive analytical results and study related simulations.

For the subsequent analysis, we make two assumptions on the costs of bank default. First, we assume that default costs are sufficiently large, i.e., $\lambda \geq \lambda^M$, so that, with perfect knowledge about the actual beliefs m in the economy, the optimal monetary policy would restrict bank lending in order to rule out bank default, which is achieved by implementing the haircut ψ_m^S leading to the bank leverage $\varphi_m^L(\psi_m^S) = \varphi^S$. Second, for tractability, we make the assumption that the loan-financed sector is in expectation more productive than the bond-financed sector, even when accounting for costs due to bank default, i.e., it holds that $\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L > 0$ or, equivalently, $\lambda < \lambda^S$.[12]

Assumption 2.6.1 (Default Costs)
$\lambda^S > \lambda \geq \lambda^M$.

The collateral requirements, in the form of the haircut, are set before the actual beliefs in the economy can be observed. In its choice of the haircut, the central bank aims at maximizing expected welfare and thus faces a general trade-off between restricting loan financing for some beliefs and allowing for bank default in the presence of other beliefs. A particular choice of the haircut that maximizes expected welfare will maximize actual welfare only for a specific distortion factor, say $\hat{m}$, as it leads to the bank leverage $\varphi_{\hat{m}}^L(\psi_{\hat{m}}^S) = \varphi^S$ only for beliefs that are described by the distortion factor $m = \hat{m}$. In this particular case, not only *expected* welfare but also *actual* welfare is maximized with the haircut choice $\psi_{\hat{m}}^S$. However, for more pessimistic beliefs, i.e., for the actual distortion factor m satisfying $m < \hat{m}$, the chosen haircut will induce a bank leverage $\varphi_m^L(\psi_{\hat{m}}^S) < \varphi^S$ and thereby cause a decline in aggregate production, compared to any situation where the haircut is chosen without uncertainty about beliefs. Similarly, for more optimistic beliefs, i.e., for the actual distortion factor m satisfying $m > \hat{m}$, banks attain the leverage $\varphi_m^L(\psi_{\hat{m}}^S) > \varphi^S$, so that some banks default, as the financed firm incurs a negative productivity shock. In its choice of monetary policy, the central bank must thus account for the fact that for some beliefs that may realize in the economy the chosen haircut will induce a suboptimal level of bank lending, either leading to deficient bank lending and productivity losses, or to excessive bank lending and bank default.

The beliefs in the economy can only be of two types, namely one of the two distortion factors $\underline{m} \in (0, 1/\eta)$ and $\overline{m} \in (0, 1/\eta)$, satisfying $\underline{m} < \overline{m}$, prevails. From the central bank's perspective, the distortion factors $\underline{m}$ and $\overline{m}$ realize with probability $p \in (0,1)$ and $1-p$, respectively. Accordingly, the expected welfare, which the central bank aims to maximize with its choice of the haircut ψ, is given by

$$\mathbb{E}[W_m(\psi)] = pW_{\underline{m}}(\psi) + (1-p)W_{\overline{m}}(\psi).$$

Using our previous results on equilibrium welfare (see lemma 2.4.1) as well as the conditions for the existence of an equilibrium and the bank's exposure to a solvency risk (see propo-

[12]In appendix 6.1.2, we outline the optimal monetary policy when the latter assumption is not satisfied, so that default costs are sufficiently large, i.e., $\lambda \geq \lambda^S$.

sition 2.4.1), we can derive a reduced form of the central bank's optimization problem, as provided in the following lemma.

Lemma 2.6.1 (The Central Bank's Optimization Problem with Uncertainty) *The optimization problem of the central bank is*

$$\max_{\psi\in[\psi^M_{\overline{m}},1]} (\mathbb{E}[A^L_s] - A^B)\mathbb{E}[\varphi^L_m(\psi)] - (1-\eta)\lambda A^L_{\underline{s}}\mathbb{E}[\mathbb{1}\{\psi < \psi^S_m\}\varphi^L_m(\psi)].$$

First, note that the smallest feasible haircut $\psi^S_{\underline{m}}$, which guarantees the clearing of the capital good market in the presence of the more pessimistic beliefs $\underline{m}$, satisfies $\psi^M_{\underline{m}} < \psi^M_{\overline{m}}$. Without knowing the actual beliefs in the economy, the central bank is unable to set the any haircut smaller than $\psi^M_{\overline{m}}$ (for instance, $\psi^M_{\underline{m}}$), as such a haircut would not allow the clearing of the capital good market if indeed the more optimistic beliefs $\overline{m}$ in the economy prevail. Thus, the smallest possible haircut the central bank can set is given by $\psi^M_{\overline{m}}$, which we already addressed in the formulation of the central bank's optimization problem (see lemma 2.6.1).

Second, note that the haircuts $\psi^S_{\underline{m}}$ and $\psi^S_{\overline{m}}$, which rule out bank default in the presence of the more pessimistic and optimistic beliefs, respectively, satisfy $\psi^S_{\underline{m}} < \psi^S_{\overline{m}}$. Accordingly, choosing the haircut $\psi^S_{\overline{m}}$ guarantees that bank default does not occur, independent of the actual beliefs in the economy. The central bank has no incentive to set any haircut larger than $\psi^S_{\overline{m}}$, since this restricts bank lending but does not yield any benefit, as ruling out bank default, for instance. Thus, the central bank chooses the optimal haircut from the closed set ranging from $\psi^M_{\overline{m}}$ to $\psi^S_{\overline{m}}$.

Third, it turns out that the optimal monetary policy depends on how close or distinct the two types of beliefs are. With sufficiently distinct beliefs $\underline{m}$ and $\overline{m}$, the smallest possible haircut $\psi^M_{\overline{m}}$ does not expose banks to a solvency risk if the more pessimistic beliefs $\underline{m}$ realize. In such a situation, it formally holds that $\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) \leq \varphi^S$. Thus, in the presence of the more pessimistic beliefs $\underline{m}$, banks never experience default, independent of the haircut set by the central bank. Instead, if beliefs $\underline{m}$ and $\overline{m}$ are sufficiently close, so that it holds $\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) > \varphi^S$, banks can be exposed to a solvency risk, at least under the smallest feasible haircut $\psi^M_{\overline{m}}$, also if the more pessimistic beliefs $\underline{m}$ prevail. From the condition $\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) = \varphi^S$, we can derive the belief threshold $\tilde{m}$ satisfying $\tilde{m} < \overline{m}$. We then classify the beliefs $\underline{m}$ and $\overline{m}$ as distinct (close) if it holds $\underline{m} \leq \tilde{m}$ ($\underline{m} > \tilde{m}$). The details are provided in the following lemma.

Lemma 2.6.2 (Belief Differences)
The beliefs $\underline{m}$ and $\overline{m}$ are distinct (close) if it holds that $\underline{m} \leq \tilde{m}$ ($\tilde{m} > \underline{m}$), where

$$\tilde{m} = \delta\overline{m} - \frac{A^L_{\underline{s}}(1-\delta)}{\eta(A^L_{\overline{s}} - A^L_{\underline{s}})}, \qquad \textit{with} \qquad \delta = \frac{(1+E/K)A^L_{\underline{s}}}{A^B}.$$

Since $\delta < 1$, it follows that $\tilde{m} < \overline{m}$.

We now characterize the optimal monetary policy in any situation where the beliefs, as represented by the distortion factors $\underline{m}$ and $\overline{m}$, are distinct (see lemma 2.6.2). With such beliefs, banks do not face a solvency risk in the presence of the more pessimistic beliefs, even if the central bank implements the smallest feasible haircut $\psi^M_{\overline{m}}$. Accordingly, in its choice of the haircut, the central bank must trade off default costs in the presence of the more optimistic beliefs and restricted loan financing in the presence of the more pessimistic beliefs. Due to our linear production technologies, the central bank in effect only chooses between accepting and eliminating bank default in case the more optimistic beliefs realize, by setting the haircuts $\psi^M_{\overline{m}}$ and $\psi^S_{\overline{m}}$, respectively. The former haircut is the smallest feasible haircut for the central bank, as it just guarantees the existence of an equilibrium in the presence of the more optimistic beliefs $\overline{m}$. The haircut $\psi^S_{\overline{m}}$, in turn, eliminates solvency risk in the presence of more optimistic beliefs. The central bank deviates from the objective of ruling out solvency risk, i.e., it implements the haircut $\psi^M_{\overline{m}}$ instead of the haircut $\psi^S_{\overline{m}}$, whenever the default costs are sufficiently low. Formally, when the default cost parameter λ satisfies $\lambda < \lambda^M_{BU}$, with λ^M_{BU} being provided in proposition 2.6.1, the central bank decides to induce the maximum bank lending and accept bank default for the more optimistic beliefs.

Proposition 2.6.1 (Optimal Monetary Policy with Uncertainty - Distinct Beliefs)
Suppose the beliefs $\underline{m}$ and $\overline{m}$ are distinct ($\underline{m} \leq \tilde{m}$). Then, the central bank optimally chooses

(i) the haircut $\psi^M_{\overline{m}}$, accepting bank default for the more optimistic beliefs $\overline{m}$, if and only if it holds that

$$\lambda < \lambda^M_{BU} := \lambda^M + \lambda^S \frac{p}{1-p} \frac{\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})}{\varphi^M},$$

(ii) the haircut $\psi^S_{\overline{m}}$, ruling out bank default for any type of beliefs, otherwise.

We can show that, compared to any situation without uncertainty about beliefs, the central bank accepts default of banks already at higher costs, as measured by the parameter λ. With certainty, in the presence of the more optimistic beliefs $\overline{m}$, the central bank accepts bank default by setting the smallest feasible haircut $\psi^M_{\overline{m}}$, whenever default costs satisfy $\lambda < \lambda^M$ (see proposition 2.5.1). In turn, with uncertainty about beliefs, the central bank must decide about the haircut on bank loans without knowing whether the beliefs in the economy are given by $\underline{m}$ or $\overline{m}$. Proposition 2.6.1 outlines that for sufficiently distinct beliefs, the central bank implements the smallest feasible haircut $\psi^M_{\overline{m}}$ whenever default costs satisfy $\lambda < \lambda^M_{BU}$. As stated in corollary 2.6.1, in the presence of belief uncertainty, the central bank chooses the smallest feasible haircut already at higher default costs, compared to any situation where it knows with certainty that beliefs are represented by the distortion factor $\overline{m}$. Formally, this means that the critical default cost parameters satisfy $\lambda^M_{BU} > \lambda^M$. This result is based on the fact that for a positive likelihood of the more pessimistic beliefs $\underline{m}$, the central bank must not only trade off restricted bank lending and default costs in the presence of the more optimistic beliefs $\overline{m}$, as in the case without uncertainty, but must also account for restricted bank lending in case the more pessimistic beliefs $\underline{m}$ realize. This incentivizes the central bank to prefer bank default over restricted bank lending already for higher default costs, compared to the situation without belief uncertainty.

Corollary 2.6.1 (Optimal Monetary Policy with Uncertainty - Distinct Beliefs) *Suppose the central bank faces uncertainty about beliefs and the beliefs are distinct ($m \leq \tilde{m}$). Then, compared to any situation where the beliefs $\overline{m}$ realize with certainty, the central bank chooses the smallest feasible haircut $\psi^M_{\overline{m}}$ already at higher default costs, i.e., $\lambda^M_{BU} > \lambda^M$.*

Next, we focus on the case where the possible beliefs are sufficiently close, i.e., $\underline{m} > \tilde{m}$, and outline the optimal monetary policy. In any such environment, even in the presence of the more pessimistic beliefs, banks are exposed to a solvency risk if the central bank chooses the smallest feasible haircut $\psi^M_{\overline{m}}$, i.e., it holds that $\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) > \varphi^S$. In its choice of the haircut, the central bank then has to take into account that banks may default independent of the actual beliefs in the economy. Due to our linear production technologies in the economy, we can state that there are three possible haircuts the central bank actually chooses. First, the central bank may allow for bank default independent of the beliefs in the economy by setting the smallest feasible haircut $\psi^M_{\overline{m}}$. Such a monetary policy is always optimal if the default costs are sufficiently small, i.e., $\lambda < \lambda^M_{BU}$, or, in other words, ruling out bank default for any type of beliefs yields a lower welfare, and the alternative of eliminating bank default for the more pessimistic beliefs does not yield a welfare gain either, which, following proposition 2.6.2, is captured by the inequality $\lambda^S_{BU} \leq \lambda^M_{BU}$. Second, the central bank may allow for default of banks in the presence of the more optimistic beliefs, but rule it out for the more pessimistic beliefs, which is achieved by setting the haircut $\psi^S_{\underline{m}}$. Such a monetary policy is optimal if the default costs are sufficiently small, i.e., $\lambda < \lambda^S_{BU}$, or, in other words, if eliminating bank default for any beliefs yields a lower welfare, and if the alternative of accepting bank default independent of beliefs does not yield a welfare gain either, i.e., $\lambda^M_{BU} < \lambda^S_{BU}$. Third, whenever it holds $\max\{\lambda^M_{BU}, \lambda^S_{BU}\} \leq \lambda$, the central bank optimally chooses to rule out bank default independent of the actual beliefs in the economy, which is achieved by setting the haircut $\psi^S_{\overline{m}}$, as it yields a higher welfare compared to the alternatives, where bank default is accepted at least for one particular type of beliefs.

Proposition 2.6.2 (Optimal Monetary Policy with Uncertainty - Close Beliefs) *Suppose the beliefs $\underline{m}$ and $\overline{m}$ are close ($\underline{m} > \tilde{m}$). Then, the central bank optimally chooses*

(i) the haircut $\psi^M_{\overline{m}}$, accepting bank default for any beliefs, if and only if it holds that

$$\lambda < \lambda^M_{BU} := \lambda^M + \lambda^S \left(\frac{\varphi^S}{\varphi^M} - \frac{p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S}{p\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) + (1-p)\varphi^M} \right),$$

and

$$\lambda^M_{BU} \geq \lambda^S_{BU} := \lambda^M + \lambda^S \left(\frac{\varphi^S}{\varphi^M} - \frac{p[\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) - \varphi^S] + (1-p)\varphi^S}{(1-p)\varphi^L_{\overline{m}}(\psi^S_{\underline{m}})} \right),$$

(ii) the haircut $\psi^S_{\underline{m}}$, ruling out bank default for the more pessimistic beliefs $\underline{m}$, if and only if it holds that $\lambda < \lambda^S_{BU}$ and $\lambda^M_{BU} < \lambda^S_{BU}$, and

(iii) the haircut $\psi^S_{\overline{m}}$, ruling out bank default for any beliefs, if and only if it holds that $\max\{\lambda^M_{BU}, \lambda^S_{BU}\} \leq \lambda$.

Next, we provide simulations to illustrate the effect of the beliefs ($\underline{m}$ and $\overline{m}$) and the probability distribution of beliefs (p) on the optimal monetary policy. The parameter specification depicted in table 2.1 represents our baseline, which is also in line with assumption 2.6.1.[13]

Parameter	A^B	$A^L_{\overline{s}}$	$A^L_{\underline{s}}$	η	$\underline{m}$	$\overline{m}$	E/K	α	λ
Value	1	1.6	0.8	0.5	0.5	1.5	0.1	0.5	0.3

Table 2.1: Parameter specification for simulations.

For figures 2.2 to 2.5, the graphs on the left-hand side show the critical default cost parameters λ^M_{BU} (green line) and λ^S_{BU} (red line) in dependence of the considered model parameter, as well as the assumed default cost parameter λ (black line) and the critical default cost parameter λ^M (blue line) in the case without uncertainty about beliefs. The graphs on the right-hand side depict the optimal haircut ψ chosen by the central bank (black dashed line), the smallest feasible haircut $\psi^M_{\overline{m}}$ (orange solid line), the haircut $\psi^S_{\overline{m}}$ (green solid line) guaranteeing solvency of banks for any type of beliefs, and the haircut $\psi^S_{\underline{m}}$ (red solid line) ruling out bank default only for the more pessimistic beliefs $\underline{m}$.

First, we study the effect of increasing uncertainty about the beliefs in the economy, as measured by the spread between the two possible distortion factors. We assume that the distortion factors $\underline{m}$ and $\overline{m}$ are symmetrically centered around one and we only vary the spread $\overline{m} - \underline{m}$. We find that with increasing uncertainty about beliefs (i.e., the spread $\overline{m} - \underline{m}$ is growing from 0 to 0.5), the optimal monetary policy first rules out bank default independent of the actual beliefs in the economy, then allows for bank default in the presence of the more optimistic beliefs and ultimately, with sufficiently large uncertainty, allows for bank default independent of the actual beliefs in the economy. Focusing on figure 2.2, the graph on the left-hand side shows that for a sufficiently small spread $\overline{m} - \underline{m}$, both critical default cost parameters λ^M_{BU} (green line) and λ^S_{BU} (red line) are below the assumed default cost parameter λ, i.e., it holds that $\max\{\lambda^M_{BU}, \lambda^S_{BU}\} \leq \lambda$. From proposition 2.6.2, we know that in such a situation, the optimal monetary policy is represented by the haircut $\psi^S_{\overline{m}}$, eliminating bank default independent of the actual beliefs in the economy. Accordingly, the graph on the right-hand side of figure 2.2 shows that for sufficiently small belief uncertainty, as captured by the spread $\overline{m} - \underline{m}$, the optimal haircut ψ (black dashed

[13] In appendix 6.1.4, we provide computational results for the case where the central bank has a uniform prior over infinitely many different types of beliefs.

line) coincides with the smallest possible haircut $\psi^S_{\overline{m}}$ (green solid line). This policy is optimal until the beliefs are sufficiently different, so that eliminating bank default for both types of beliefs becomes too costly in terms of welfare because in the presence of the more pessimistic beliefs, bank lending would be restricted too much. The central bank thus optimally allows for bank default in the presence of the more optimistic beliefs $\overline{m}$, but still rules it out for the more pessimistic beliefs $\underline{m}$, by setting the haircut $\psi^S_{\underline{m}}$. This policy turns out to be optimal until the two possible types of beliefs are so distinct that setting the haircut $\psi^S_{\overline{m}}$ does not trigger solvency risk for banks in case the more pessimistic beliefs prevail. This threshold in terms of the spread $\overline{m} - \underline{m}$ can be identified, using the graph on the right-hand side of figure 2.2, as follows: If the haircut $\psi^S_{\underline{m}}$ (red line) is below the smallest feasible haircut $\psi^S_{\overline{m}}$ (orange line), it is clear that in the presence of the more pesimistic beliefs, banks cannot face any solvency risk, even if the central bank sets the smallest feasible haircut. From this point on, choosing the haircut $\psi^S_{\underline{m}}$ is not an option for the central bank anymore. Accordingly, focusing on the graph on the left-hand side of figure 2.2, the red line representing the critical default cost parameter λ^S_{BU} ends at the critical spread $\overline{m} - \underline{m}$ where beliefs become sufficiently distinct, i.e., $\underline{m} \leq \tilde{m}$. For any spread $\overline{m} - \underline{m}$ larger than the critical threshold, the parameter λ^S_{BU} is not relevant for the decision of the central bank. The optimal monetary policy is now described by proposition 2.6.1. In its choice of the haircut, the central bank must assess whether maximizing bank lending but accepting bank default (i.e., setting the haircut $\psi^M_{\overline{m}}$) or restricting bank lending and ruling out bank default (i.e., setting the haircut $\psi^S_{\overline{m}}$) maximizes expected welfare.

Specifically, the central bank aims at inducing maximum bank lending by implementing the haircut $\psi^M_{\overline{m}}$ if and only if default costs are sufficiently small ($\lambda < \lambda^M_{BU}$), and aims at ruling out bank default by choosing the haircut $\psi^S_{\overline{m}}$ otherwise. As outlined in corollary 2.6.1, if beliefs are sufficiently distinct ($\underline{m} \leq \tilde{m}$), the central bank chooses, compared to the case without uncertainty and the more optimistic beliefs $\overline{m}$, the smallest feasible haircut $\psi^M_{\overline{m}}$ already at higher default costs. Formally, it holds that $\lambda^M_{BU} > \lambda^M$. Focusing on the graph on the left-hand side of figure 2.2, we see that for moderate spreads $\overline{m} - \underline{m}$, the central bank chooses the smallest feasible haircut $\psi^M_{\overline{m}}$ and prefers eliminating bank default over restricting bank lending. In turn, if the difference in beliefs, as measured by the spread $\overline{m} - \underline{m}$, is large enough, the central bank jumps back to the regime of avoiding bank default but accepting restrictions in bank lending. This is based on the fact that with an increasing spread $\overline{m} - \underline{m}$, λ^M_{BU} converges to λ^M as well as based on the fact that, according to our specification, it holds that $\lambda > \lambda^M$. The latter condition can be observed in the graph on the left-hand side of figure 2.2, as the black line is above the blue one. Thus, while for moderate differences in the beliefs, it holds that $\lambda^M_{BU} > \lambda > \lambda^M$, this turns into $\lambda \geq \lambda^M_{BU} > \lambda^M$ with sufficiently different beliefs. Overall, we can conclude that the restrained behavior of the central bank—not allowing for bank default in any case—disappears with increasing uncertainty about beliefs. Formally, this means that in its choice of the haircut, the central bank shifts from $\psi^S_{\overline{m}}$, eliminating bank default in general, to $\psi^S_{\underline{m}}$, ruling out bank default only in the presence of the more pessimistic beliefs, and ultimately shifts to $\psi^M_{\overline{m}}$, accepting bank default if the more optimistic beliefs in the economy realize. However, it has to be noted that the central bank never accepts default for both types of beliefs. In other words, it only chooses the smallest feasible haircut $\psi^M_{\overline{m}}$ when beliefs are already sufficiently distinct and banks cannot be exposed to a solvency risk in the presence of the more pessimistic beliefs anymore. The objective to avoid bank default only prevails when belief differences become extreme and the choice of haircut policy is close to the one without belief uncertainty and more optimistic beliefs. The reason for the reversal of monetary policy with growing belief differences is the limited ability of the central bank to mitigate the effects of the more pessimistic beliefs, as it cannot set a haircut that is lower than $\psi^M_{\overline{m}}$. At some point, avoiding restrictions in bank lending for the more pessimistic beliefs while allowing for bank default in the presence of the more optimistic beliefs does not yield a welfare gain, compared to the monetary policy of simply avoiding bank default for the more optimistic beliefs and "ignoring" the outcomes with the more pessimistic beliefs. In other words, setting the haircut $\psi^M_{\overline{m}}$ instead of $\psi^S_{\overline{m}}$ has only a small effect on bank lending if the more pessimistic beliefs $\underline{m}$ prevail, but generates large costs due to bank default if the more optimistic beliefs realize. Figures 2.3 and 2.4 provide the simulation results when varying the uncertainty about beliefs by changing only the more optimistic beliefs $\overline{m}$ and pessimistic beliefs $\underline{m}$, respectively. In both cases, we can deduce the same patterns in the optimal monetary policy, as outlined before.

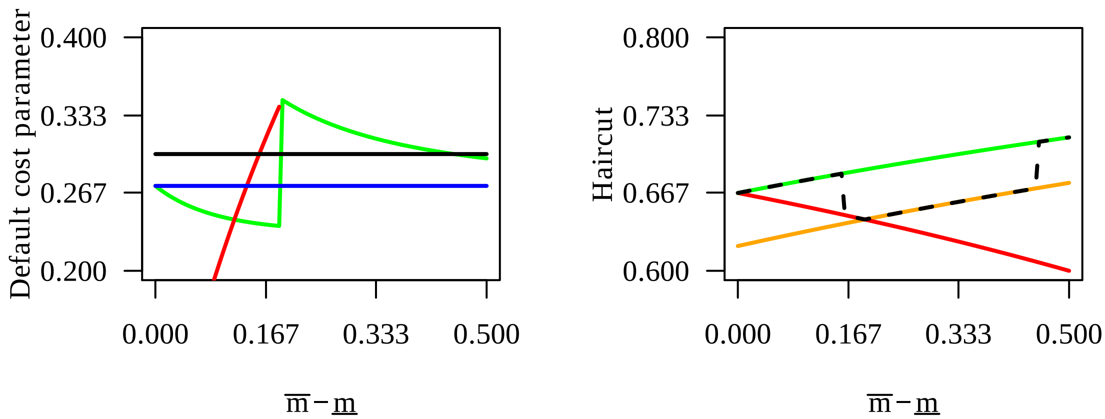

Figure 2.2: Varying uncertainty about beliefs with $\underline{m}$ and $\overline{m}$ symmetrically centered around one.

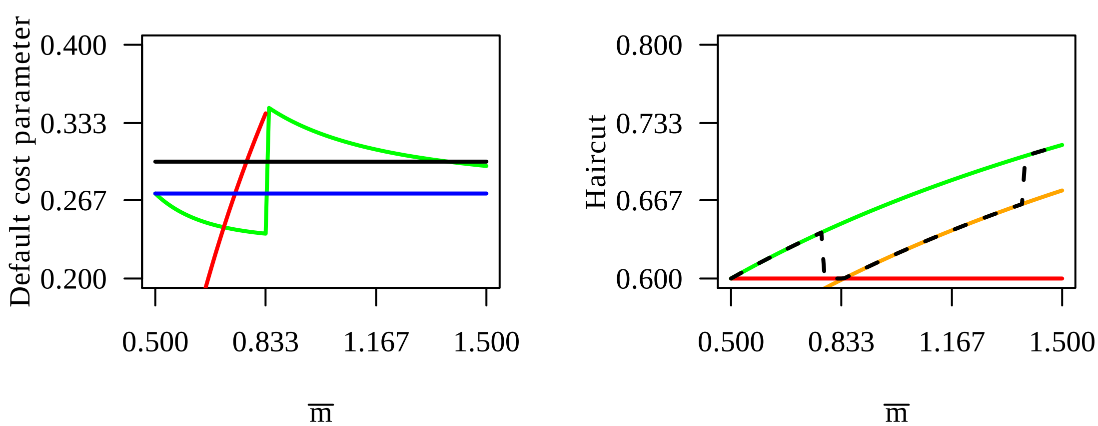

Figure 2.3: Varying uncertainty about beliefs with $\underline{m} = 0.5$ and increasing $\overline{m}$.

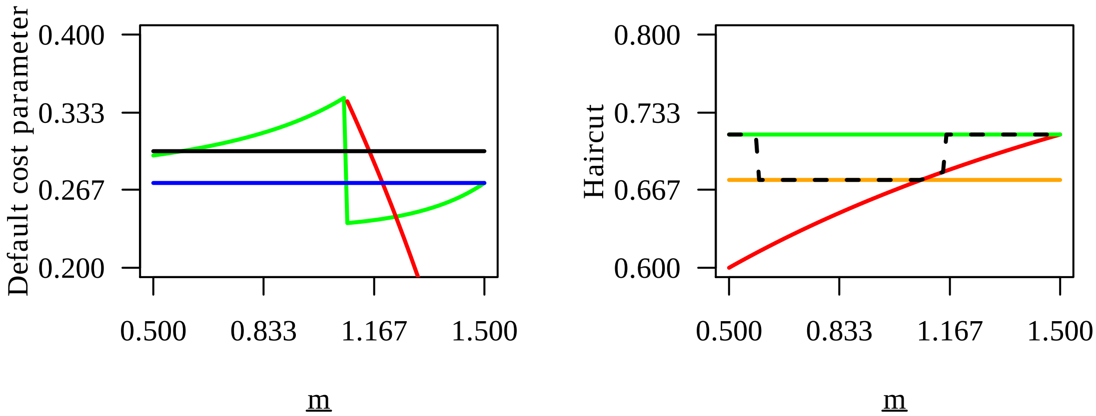

Figure 2.4: Varying uncertainty about beliefs with $\overline{m} = 1.5$ and increasing $\underline{m}$.

Second, we study the effect of the probability distribution of beliefs, as measured by the parameter p, on the optimal monetary policy. In its choice of the haircut, the central bank maximizes expected welfare, where the expected costs of deficient bank lending in the presence of the more pessimistic beliefs $\underline{m}$, are not only influenced by the beliefs themselves, but also by the probability p with which these beliefs emerge. When the probability that the more pessimistic beliefs realize is sufficiently small, the central bank follows the monetary policy which would be optimal without belief uncertainty. Specifically, it sets the haircut $\psi^S_{\overline{m}}$ that rules out bank default if the more optimistic beliefs $\overline{m}$ realize. In the right-hand side graph of figure 2.5, this pattern can be observed, as for a small probability p, the optimal haircut ψ (black dashed line) coincides with the haircut $\psi^S_{\overline{m}}$ (green line). In turn, if the probability for the more pessimistic beliefs is sufficiently large, the central bank optimally chooses to deviate from the avoidance of bank default and focus on the avoidance of bank lending restrictions, which it achieves by setting the smallest feasible haircut $\psi^M_{\overline{m}}$. Note that in this particular example, we rely on our baseline specification, so that the beliefs are sufficiently distinct, i.e., $\underline{m} \leq \tilde{m}$. Accordingly, the central bank cannot choose the haircut $\psi^S_{\underline{m}}$ and banks are not exposed to a solvency risk if the more pessimistic beliefs prevail.

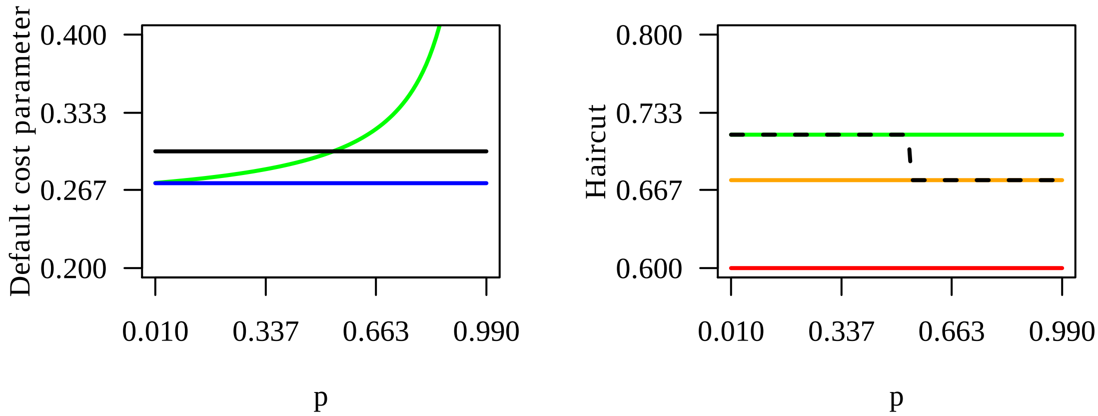

Figure 2.5: Varying probability p for beliefs $\underline{m}$.

2.7 Discussion

2.7.1 Bank regulation

The optimal monetary policy is independent of bank regulation in the form of capital requirements. On that account, note that capital requirements for banks lead to a regulatory leverage constraint. In other words, given the equity financing of banks, bank regulation limits loan financing to a maximum amount. Three possible scenarios may then emerge in our economy. First, capital requirements are sufficiently loose, so that with abundant liquidity, banks can grant loan financing beyond the optimal level. Then, the central bank can induce the optimal allocation by regulating bank lending adequately, which is achieved by restricting the access to liquidity through the appropriate haircut (see proposition 2.5.1). Second, the capital requirements are such that they exactly induce the optimal capital allocation in the economy. Then, the central bank should not further restrict loan financing but allow banks to indeed attain the optimal level of bank lending. This is achieved by setting the optimal haircut (see proposition 2.5.1). Third, capital requirements are sufficiently tight, so that bank lending is restricted below the optimal level. Then, it is optimal for the central bank not to restrict liquidity in a way that reduces bank lending even further. Without loss of generality, the central bank should in this case also implement the monetary policy which is optimal in the absence of bank regulation (see proposition 2.5.1).

2.7.2 Central bank signaling

We ruled out the possibility for the central bank to signal its information to private agents in the economy. However, in our setting, private agents have subjective beliefs and act in an opinionated way, so that even if the central bank signals its information, beliefs will not adjust in the economy. Only if we allow for private agents having distorted beliefs but not acting opinionated, such signaling by the central bank can be effective in eliminating the belief distortions in the economy.

2.7.3 Central bank mistakes

In our analysis, we abstracted from the possibility that the central bank makes mistakes, as we imposed that it knows the true probability distribution of productivity shocks. If the central bank is also subject to errors, monetary policy may induce a level of bank lending that is different from the optimal one. In other words, if the central bank decides about the collateral requirements in the form of the haircut without being perfectly informed about the true probability distribution of productivity shocks, it may erroneously choose a collateral framework that induces a suboptimal level of bank lending, leading to deficient loan financing to firms and productivity losses, or excessive lending and bank default.

2.8 Conclusion

We develop a simple framework that allows to study the optimal collateral framework in central bank lending facilities, while accounting for belief distortions in the economy leading to an over- and undervaluation of the pledged assets. Banks are liquidity-constrained, so that the central bank's choice of the haircut has a direct influence on banks' lending decisions. A larger haircut on bank loans, which can be used as collateral for reserve loans from the central bank, incentivizes banks to grant fewer loans and issue fewer deposits in the first place. Similarly, a lower haircut allows banks to borrow more reserves from the central bank and gives banks incentives to extend loan financing and deposit issuance.

When setting the haircut on bank loans in order to maximize welfare, the central bank generally trades off productivity losses due to deficient bank lending and costs due to bank default following from excessive bank lending. In our baseline model, where agents are sufficiently optimistic about the productivity of the loan-financed sector, the optimal monetary policy maximizes bank lending and allows for bank default if the default costs are sufficiently small. In turn, if the costs associated with bank default are large enough, the central bank aims at restricting bank lending, thereby eliminating a solvency risk for banks. If banks become more optimistic, the expected value of bank loans increases, causing banks to expect, ceteris paribus, an improved access to central bank liquidity. The central bank can counteract the belief changes by setting a larger haircut, thus restraining the access to reserves back to its optimal level. Similarly, growing pessimism in the economy leads to a devaluation of bank loans, ultimately requiring the central bank to loosen collateral standards in order to restore the optimal level of bank lending in the economy.

We also investigated the effect of uncertainty about beliefs on the side of the central bank. In the presence of belief uncertainty, the central bank is choosing the collateral framework in order to maximize the expected welfare in the economy. It faces the same trade-off as under certainty, namely that deficient lending leads to productivity losses, while excessive lending leads to costly bank default. However, under uncertainty about beliefs, the central bank cannot achieve the optimal capital allocation in the economy for any possible beliefs that may realize. Instead, it has to find the right balance of costs due to bank default, emerging when more optimistic beliefs realize, and productivity losses due to a suboptimal level of bank lending, emerging when more pessimistic beliefs realize. We find that with increasing uncertainty about beliefs, the restrained behavior of the central bank to avoid bank default vanishes and avoiding productivity losses due to deficient lending ultimately becomes the central bank's main objective of the central bank. Under uncertainty about beliefs, the central bank tends to become less restrictive in its choice of collateral standards.

Our simple framework allows for numerous extensions to assess the robustness of our findings. A first generalization may be represented by the introduction of strictly concave production technologies for both sectors. Second, the developed framework can be embedded into a dynamic setting, particularly accounting for an updating process of beliefs. Moreover, one may want to study various alternative formulations of default costs. These extensions are left for future research.

Chapter 3

Leverage Constraints and Bank Monitoring: Bank Regulation versus Monetary Policy*

Abstract

We use a simple model that illustrates the different effects of capital constraints and liquidity constraints on bank monitoring. Capital constraints emerge from regulatory (unweighted) capital requirements, while liquidity constraints emerge from the central bank's collateral requirements in its reserve lending facilities. Banks demand liquidity in the form of central bank reserves, as they must settle interbank liabilities arising from deposit transfers among banks. Monitoring incentives are twofold: First, bank monitoring increases the chances for a high loan repayment, to which we refer as the *return channel*. Second, with higher expected repayment by borrowers, also the collateral value of bank loans increases. In the presence of liquidity constraints, monitoring then improves the banks' access to liquidity at the central bank and, ultimately, allows them to expand loan supply and deposit issuance in the first place. We dub this the *collateral leverage channel*. Based on the collateral leverage channel, liquidity-constrained bankers always have more incentives to monitor than capital-constrained bankers.

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3.1 Introduction

A classical foundation for the existence of banks is their unique, or at least superior, ability to monitor borrowers (Diamond, 1984; Holmstrom and Tirole, 1997). Banks' monitoring activities may reduce moral hazard on the side of potential borrowers with projects offering a positive net present value, to the point that loans to them become economically viable. In many countries, banks play a major role in the allocation of investment funds and the investment returns (De Fiore and Uhlig, 2011).[1] Accordingly, their behavior has a strong impact on the level and the fluctuation of economic activities.

It is thus important to ask how well banks pursue their monitoring activities and which factors actually determine banks' monitoring incentives. Monitoring is influenced by bank regulation and monetary policy through the impact on capital and liquidity constraints, ultimately determining the banks' possibilities to leverage. While these two channels are usually examined and discussed in isolation, in this paper we develop a model in which we can analyze the two, compare them and evaluate to which extent they are substitues or play a distinctive and unique role in controlling bank monitoring.

Our main insights are as follows. First, monetary policy via collateral requirements in central bank lending operations is a distinct channel to impact bank monitoring that cannot be replicated by standard capital requirements (unweighted or weighted). Second, this *collateral leverage channel* we identify only operates properly if available central bank reserves are sufficiently scarce. These results may inform two current debates: The design of central bank collateral frameworks and the effectiveness of monetary policy in times when there are large amounts of central bank reserves. This will be detailed in subsection 3.1.1.

We use a simple model that features a perfectly competitive banking sector, a bank regulator and a central bank. Banks fulfill a dual role in our economy, as they provide credit, in the form of loans to firms, and money, in the form of bank deposits. The latter is the predominant form of today's money and constitutes the only medium of exchange in our economy. Banks can monitor firms in order to avoid any opportunistic behavior. We consider two different monitoring technologies. In our baseline model, bank monitoring increases the chances for a high loan repayment. In an extension, we also consider an

[1]When comparing the United States (US) and the Euro Area (EA), De Fiore and Uhlig (2011) find that the importance of banks in providing financing to non-financial corporations in the real economy is particularly prominent in the EA, as the bank-to-bond finance ratio is 5.48, compared to 0.66 in the US.

alternative monitoring technology that increases the loan repayment only in bad states.

The loans granted by banks are financed through deposits and equity. Bank equity financing is limited and the bank regulator imposes (unweighted) capital requirements. Thus, banks may be capital-constrained. Moreover, as loan repayment is risky, highly levered banks face a solvency risk. Solvency risk, in turn, may cause underinvestment by bankers in monitoring, as they do not obtain the entire benefits of monitoring. The underinvestment is most pronounced when monitoring only increases returns in bad states, in which the bank defaults anyway. Hence, with a solvency risk, banks may choose shirking instead of monitoring in order to economize on the monitoring costs.

Bank deposits are used by firms to buy investment goods from households, who, in turn, use the received bank deposits later to buy consumption goods. As a result of these transactions, deposits are transferred among banks, giving rise to interbank liabilities that, following today's institutional arrangement, must be settled at the central bank by using reserves. Banks can obtain liquidity in the form of reserves by borrowing from the central bank. In our setting, monetary policy thus comprises two instruments: interest rates on reserve loans and reserve deposits, as well as the collateral requirements for reserve loans. Banks can pledge their firm loans when borrowing reserves, but these assets are reduced in value through a haircut set by the central bank. With sufficiently tight collateral requirements, namely a haircut on bank loans large enough, banks become liquidity-constrained.

We focus on a classical economy without any price rigidities. Accordingly, any interest rate policy of the central bank is irrelevant for the real allocation, i.e., money is neutral. In contrast, the haircut has a direct impact on banks' ability to borrow reserves and, in the presence of liquidity constraints, influences the banks' incentives to engage into loan financing and deposit issuance. A bank only issues deposits and provides more loans to firms if it is certain that it can borrow the reserves required to settle the interbank liabilities resulting form deposit transfers. Otherwise, it would default prematurely.

We show that in our model, the monitoring incentives for banks are twofold: First, monitoring increases the chances for a high loan repayment (or in the extension, the loan repayment in bad states) and thus increases the expected profits of the bank, which we refer to as the *return channel* of monitoring. Second, the higher expected loan repayment increases the collateral value of bank loans, which, ceteris paribus, allows banks to borrow more reserves at the central bank. For liquidity-constrained banks, this implies that they can extend loan supply and deposit issuance in the first place, leading to a higher bank leverage and higher expected profits for the bank. We refer to this second effect as the *collateral leverage channel* of monitoring.

Due to the collateral leverage channel, liquidity-constrained banks have, under any circumstance, more incentives to monitor than capital-constrained banks. We illustrate that the effect of central bank collateral requirements on the banks' monitoring incentives is also unique in comparison with standard contingent (e.g., risk-weighted) capital requirements. Contingent capital requirements give rise to the *regulatory leverage channel* of monitoring that represents a third channel for the benefits from monitoring. We show that one can construct a particular form of contingent capital requirements that can replicate the monitoring benefits following from the collateral leverage channel. However, this particular form of contingent capital requirements may be difficult to implement by the bank regulator, as it would require instant responses to monitoring activities and may contradict other objectives, such as reducing the risk-taking incentives of banks, for instance.

3.1.1 Contribution to debates

Our analysis may inform two current policy and academic debates. First, with our analysis, we contribute to the current debate about central bank collateral frameworks. With the financial crisis 2007/08, central banks adopted various unconventional measures in order to incentivize banks to maintain the credit supply to the real economy. On the one hand, central banks set short-term interest rates on reserves at unprecedented lows and exercised additional downward pressure on long-term interest rates through large-scale asset purchases, so-called "quantitative easing". On the other hand, central banks lowered the collateral standards in their lending activities to facilitate the banks' access to liquidity. The possible distortions resulting from a deterioration of collateral requirements during the financial crisis have been widely discussed (Nyborg, 2017; Bindseil et al., 2017). To a large extent, the discussion centered around the so-called "collateral premium", i.e., an increase in the valuation of assets, which is solely due to the fact that these assets can be pledged in liquidity operations of the central bank. The collateral premium can ultimately induce distortions in the real allocation, as it benefits the respective issuers.

Despite potential distortions, there are at least three justifications for central bank collateral requirements (see e.g. Bindseil et al. (2017)): With unsecured liquidity provisions, the central bank would face an increased risk of losses, would require more resources to manage its risk exposure, and might reduce the efficiency of monetary policy implementation. First, losses for the central bank are problematic, as they can harm its reputation and even question its independence. Second, a greater use of resources reduces the central bank's profits and thus comes at the taxpayers' expense. Third, diligent lending of public money without collateral requirements requires that the creditworthiness of each counterparty is evaluated, a time-consuming process that may slow down the implementation of central bank policies and ultimately causes real economic losses. We show with our analysis that, besides the previously-mentioned reasons, central bank collateral requirements can also have an important function in maintaining the banks' incentives to monitor.

Second, our work may also inform current debates on potential risks of large central bank balance sheets. The results indicate that through the collateral leverage channel, central banks can affect the banks' monitoring incentives and thus ultimately their monitoring activities. This channel is, however, only active if banks are indeed constrained by liquidity. The unconventional measures adopted by many central banks since the financial crisis 2007/08, such as large-scale asset purchases, for instance, led to the fact that banks currently hold large amounts of reserves, eliminating any liquidity constraints. According to our analysis, with current monetary policies leading to large reserve holdings of banks, a particular effect of central bank collateral frameworks is lost.

As shown in the paper, the collateral leverage channel can be replicated by a particular construction of contingent (e.g., risk-dependent) capital requirements. However, such a replication may be difficult or impossible to implement. On the one hand, risk-dependent capital requirements have the primary purpose to reduce or eliminate excessive risk-taking and typically, the *regulatory* leverage channel for such purposes differs from the *collateral* leverage channel. On the other hand, bank regulation is much more rule-based than monetary policy, leaving bank regulators with less discretion to adjust their regulatory instruments than central bankers when collateral values change, for example.

The remainder of the paper is structured as follows: Section 3.2 relates our work to the existing literature. Section 3.3 introduces the model, discusses the optimal choice of the individual agents and provides the equilibrium analysis. Section 3.4 concludes.

3.2 Relation to the Literature

Our paper relates to three strands of the literature. First, our model features the dual role of banks, providing credit and money to the real economy. We thus rely on the fast-growing literature that emphasizes private money creation by banks, as Faure and Gersbach (2017), Faure and Gersbach (2018) and Benigno and Robatto (2019), for instance. We develop a model to study the differing impact of capital and liquidity constraints, imposed by bank regulation and monetary policy, respectively, on the banks' monitoring incentives.

Second, our baseline model features a monitoring technology in the spirit of Holmstrom and Tirole (1997), as monitoring rules out any opportunistic behavior of the borrower. Monitoring increases the chances for high firm productivity. In an extension to our model (see appendix 6.2.2), we study an alternative monitoring technology that leaves the probability distribution of productivity shocks unchanged but raises firm productivity in low productivity states.

Third, we relate to a large literature that studies the effect of collateral standards, on asset prices (e.g., Brumm et al. (2015)) or on credit constraints and their relevance for business cycles (e.g., Bernanke et al. (1999), Kiyotaki and Moore (1997), Guerrieri and Iacoviello (2017)), for example. We contribute by illustrating how collateral requirements imposed by the central bank can affect banks' monitoring activities and how this impact may differ from the one induced by the bank regulator. Our analysis thus complements the existing literature that analyzed central bank collateral frameworks, mostly from a policy or empirical perspective, see Bindseil (2004), Bindseil et al. (2017), Chailloux et al. (2008) and Nyborg (2017), for instance.

3.3 Model

3.3.1 Macroeconomic environment

Our economy features four types of agents—firms, households, bankers, and a government sector, comprising a bank regulator and a central bank—and two goods—a capital good and a consumption good. Transactions are settled instantaneously by using money in the form of bank deposits. Households and bankers are endowed with the capital good which they can sell to firms for the production of the consumption good. Bankers establish banks by committing to use their proceeds from capital good sales for equity financing. Firms finance capital good purchases from households and bankers either by demanding loan financing from banks or by issuing bonds at financial markets. Based on the type of external financing, we differentiate between loan-financed and bond-financed firms. The model features private and public money creation. Private money takes the form of bank deposits which are issued by banks when granting loans to firms. Public money, in turn, is represented by reserves which banks can obtain from the central bank by demanding collateralized reserve loans and that are used by banks to settle interbank liabilities.[2] The latter arise when, in the course of transactions on the good markets, deposits are transferred from one bank to another. Good markets and asset markets are perfectly competitive.

Firms in the loan-financed sector produce subject to idiosyncratic shocks. Moreover, the expected productivity of loan-financed firms is influenced by bank monitoring. In equilibrium, firm productivity and loan repayment are directly linked, so that bank monitoring matters for loan repayment and affects the expected value of bank loans. These loans serve as collateral for reserve loans from the central bank, leading to the fact that the banks' monitoring decision also affects their access to liquidity. The central bank sets the interest rates on reserve loans and reserve deposits, and the haircut on bank loans when used as collateral for reserve loans.

Banks are either constrained by capital or liquidity, i.e., either the capital requirements imposed by the bank regulator or the haircut set by the central bank matter for banks' initial decision about deposit issuance and loan supply to firms. We impose a one-to-one matching of banks and firms, so that the loan portfolio of the individual bank is fully exposed to the idiosyncratic risk of the financed firm. As banks operate with limited equity financing, they are exposed to a solvency risk whenever, in the course of loan financing, the leverage becomes sufficiently large. Bank deposits are safe as they are insured by the government through guarantees. Throughout our analysis, we assume that the governmental budget is balanced, so that the government distributes central bank profits as transfers and finances central bank losses through taxes.

3.3.2 Timeline

As we focus on a monetary economy with instantaneous settlement of transactions, the timing of interactions among agents matters for the model analysis. Figure 3.1 outlines the events in our static setting.

[2]For simplification, we abstract from cash. For environments, where cash is only available through a conversion of bank deposits, this is without loss of generality because holding the alternative form of money, i.e., bank deposits, yields a positive interest.

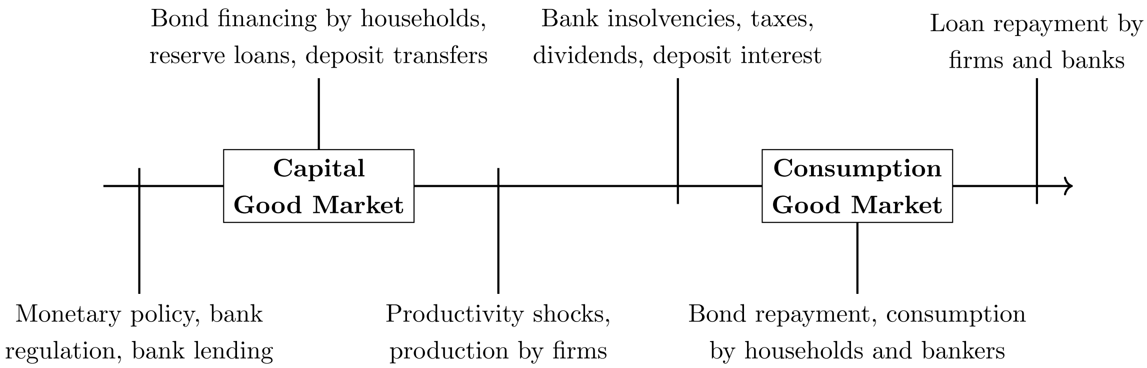

Figure 3.1: Timeline.

We note that all trades are settled by using bank deposits and prices are in terms of the unit of account of the underlying currency. The consumption good acts as the numeraire of the economy. In the following subsections, we outline the agents' optimization problems and characterize the optimal choices. The proofs relating to the stated results can be found in appendix 6.2.3.

3.3.3 Firms

Firms are profit-maximizing, protected by limited liability and penniless. They purchase the capital good from households and bankers to produce the consumption good. There are two types of firms, which we index by L and B. Firms of each type are ex-ante identical and exist in a continuum with mass normalized to one, so that we can focus on a representative firm for each type. Firms of type L are plagued by moral hazard and can only obtain funds through loans by banks. Firms of type B, in turn, can raise funds in a frictionless bond market.[3]

The loan-financed firm purchases the capital good $K^L \geq 0$ from households and bankers at the nominal price $Q > 0$ and uses the capital good to produce the consumption good with the risky technology $A_s^L K^L$, where the marginal productivity $A_s^L \geq 0$ is affected by the idiosyncratic shock s. The productivity can be either low ($s = l$) or high ($s = h$), so that it holds that $A_h^L > A_l^L$. The idiosyncratic productivity shocks are independent and identically distributed (i.i.d.) across firms. A positive shock occurs with probability

[3]The assumption that firms of type B can only raise funds by issuing bonds at financial markets is made for simplification and can be relaxed.

$\eta_m \in (0,1)$, which depends on the monitoring activity m of the matched banker. Bankers can engage into costly monitoring ($m = 1$) or shirking ($m = 0$). Monitoring by the matched banker increases the probability for a positive productivity shock, i.e., $\eta_1 = \eta_0 + \triangle$ with $\triangle \in (0, 1 - \eta_0)$.

The bond-financed firm, in turn, purchases capital good $K^B \geq 0$ from households and bankers at the nominal price $Q > 0$ and uses the capital good to produce consumption good with the riskless technology $A^B K^B$, where the marginal productivity satisfies $A^B > 0$.

Both types of firms sell the produced consumption good to households and bankers at a nominal price $P > 0$. The revenues, in the form of bank deposits, are then used to repay the external funds QK^f, with $f \in \{L, B\}$, where $Q > 0$ denotes the nominal capital good price. Depending on the type of the firm, external financing takes the form of loans or bonds. The repayment of loans is determined by the interest rate $r_s^L > 0$, whereas the repayment of bonds depends on the interest rate $r^B > 0$. Typically, the interest rates on loans and bonds will differ. Accounting for the fact that firms are profit-maximizing and subject to limited liability, it follows that the optimization problems of the loan-financed firm and bond-financed firm are given in real terms by

$$\max_{K^L \geq 0} \mathbb{E}_m[\{A_s^L - (1 + r_s^L)q\}^+]K^L \qquad \text{and} \qquad \max_{K^B \geq 0} \{A^B - (1 + r^B)q\}^+ K^B, \tag{3.1}$$

where we make use of the notation $\{X\}^+ = \max\{X, 0\}$, and apply the notation $q := Q/P$ to represent the capital good price in terms of the consumption good. Note that the expectation operator in (3.1) is indexed by the banker's monitoring activity m, as the latter affects the probability distribution of productivity shocks.

Due to limited liability, there exists no optimal, finite demand for the capital good if the respective firm is exposed to excess returns in at least one state, i.e., for the loan-financed firm, this means $A_s^L > (1+r_s^L)q$ for some s, whereas for the bond-financed firm, this means $A^B > (1 + r^B)q$. We denote this case by $K^L = +\infty$ or $K^B = +\infty$, respectively. In contrast, without excess returns, i.e., for the loan-financed firm, this means $A_s^L \leq (1+r_s^L)q$ for all s, whereas for the bond-financed firm, this means $A^B \leq (1 + r^B)q$, the firms will be indifferent between any amount of capital good put into production, i.e., $K^L \in [0, +\infty)$ and $K^B \in [0, +\infty)$, respectively.

Lemma 3.3.1 (Optimal Choice of Firms)

The loan-financed firm optimally chooses the capital good $K^L = +\infty$ *if and only if* $A_s^L >$

$(1 + r_s^L)q$ for some s, and $K^L \in [0, +\infty)$ otherwise. The bond-financed firm optimally chooses capital good $K^B = +\infty$ if and only if $A^B > (1 + r^B)q$, and $K^B \in [0, +\infty)$ otherwise.

In any competitive equilibrium we study, the capital good market must clear, which ultimately requires an optimal, finite demand of capital good on the side of firms.[4] From lemma 3.3.1, we know that firms demand a finite amount of capital good if and only if the repayment obligations on external funds weakly exceed the revenues from production, i.e., $A_s^L \leq (1 + r_s^L)q$ for all s and $A^B \leq (1 + r^B)q$. We assume that the agents in our model are rational, so that, in equilibrium, their behavior cannot be subject to predictable errors. As a consequence, no competitive equilibrium can feature firm default. Ensuring a finite demand of capital good and ruling out firm default then implies that, in equilibrium, it holds $A_s^L = (1 + r_s^L)q$ for all s and $A^B = (1 + r^B)q$. As a consequence, firms make zero profits in equilibrium.

For our analysis, we make specific assumptions on firm productivity: First, we assume that a loan-financed firm is more productive on average than a bond-financed firm, even if the matched banker does not monitor. This guarantees that the loan-financed sector is relevant in maximizing aggregate production and, ultimately, welfare. Second, when a loan-financed firm experiences a negative shock, it is less productive than a bond-financed firm. The latter assumption allows us to introduce solvency risk on the side of banks, see subsection 3.3.6.

Assumption 3.3.1 (Firm Productivities)
$\mathbb{E}_0[A_s^L] > A^B$ and $A^B > A_l^L$.

It follows directly from assumption 3.3.1 that a loan-financed firm is strictly more productive than a bond-financed firm if it incurs a positive productivity shock, i.e., it holds that $A_h^L > A^B$.

3.3.4 Households

There is a continuum of identical households with mass normalized to one, so that we can focus on a representative household. The household is endowed with capital good $K > 0$, which can be sold to firms at a nominal price $Q > 0$. The revenues are in the form of

[4] In appendix 6.2.1, we provide the definition of a competitive equilibrium in our framework.

deposits and can be invested in bonds, which are subject to a rate of return $r^B > 0$. Deposits, in turn, are credited with interest according to the rate $r^D > 0$. The share of funds held in the form of deposits is denoted by $\gamma \in [0, 1]$. The household owns firms which distribute any available profits Π as dividends. Taking governmental taxes or transfers T, which are assumed to be lump-sum, and dividends Π into account, the household uses deposits credited with interest $\gamma(1 + r^D)QK$ and the revenues from bond investments $(1-\gamma)(1+r^B)QK$ to purchase an amount C^H of the consumption good from firms at the nominal price $P > 0$. The household maximizes utility, which we assume to be linearly increasing in consumption. Hence, the household's optimization problem is given in real terms by

$$\max_{\gamma \in [0,1]} [\gamma(1 + r^D) + (1 - \gamma)(1 + r^B)]qK + \tau + \pi, \tag{3.2}$$

where the taxes and the profits, denoted by lowercase letters, are in terms of the consumption good, i.e., $\tau := T/P$ and $\pi := \Pi/P$.

The optimal choice of the household is of knife-edge type. Whenever the rate of return on deposits exceeds the one on bonds ($r^D > r^B$), the household holds all revenues from capital good sales in the form of deposits ($\gamma = 1$). Similarly, whenever the bond return exceeds the return on deposits ($r^D < r^B$), the household invests all funds into bonds ($\gamma = 0$). And finally, when the interest rates on both assets equal ($r^D = r^B$), the household is indifferent between holding deposits and investing into bonds ($\gamma \in [0, 1]$). The household's optimal choice is summarized in the following lemma.

Lemma 3.3.2 (Optimal Choice of the Household)
$\gamma = 1$ *(*$\gamma = 0$*) if* $r^D > (<) r^B$ *and* $\gamma \in [0, 1]$ *otherwise.*

We focus on environments where the interest rates on deposits and bonds equal, i.e., $r^D = r^B$.[5] As a consequence, the household is always indifferent between holding funds in deposits or bonds.

3.3.5 Government sector

Banks grant loans to firms and ultimately fund them with deposits and equity. Accordingly, banks operate under a certain leverage (i.e., loans-to-equity ratio), which we denote by

[5]In subsection 3.3.8, we further outline under which conditions this identity holds.

φ. The bank regulator imposes a capital requirement for banks, leading to a regulatory leverage constraint $\varphi \leq \varphi^R$, where $\varphi^R \in [1, +\infty)$ represents the regulatory maximum leverage following from the capital requirements.

The central bank provides banks with liquidity in the form of reserves, which banks use to settle interbank liabilities. Reserves can be borrowed from the central bank via collateralized loans. The only pledgeable assets available to banks are the bank loans provided to firms. The value of these bank loans is reduced by a haircut $\psi \in [0, 1]$, which is chosen by the central bank. In subsection 3.3.6, we provide the ensuing borrowing constraint on the side of banks. Reserve deposits at the central bank are credited with interest according to the rate $r^D_{CB} > 0$, while reserve loans require a repayment that follows from the rate $r^L_{CB} > 0$. For simplicity, we assume that the two interest rates equal.

Assumption 3.3.2 (Reserve Rates)
$r^D_{CB} = r^L_{CB}$.

Banks can face a solvency risk if, in the course of loan financing to firms, the leverage becomes sufficiently large; a detailed discussion is provided in subsection 3.3.6. In any equilibrium we consider, default by firms is ruled out (see subsection 3.3.3), so that banks are the only agents in our economy which can default on their liabilities. The government insures deposits through guarantees, so that it must impose taxes on households to balance bank losses, which in the aggregate and in nominal terms, are denoted by $\Pi^{b,-}$.[6] The government also uses taxes to cover losses of the central bank, while it can distribute central bank profits by using transfers. We denote nominal central bank profits or losses by Π^{CB}. As we assume that the governmental budget is balanced, lump-sum taxes or transfers are given in nominal terms by $T = \Pi^{b,-} + \Pi^{CB}$.

In our setting, the government aims at maximizing utilitarian welfare. We introduce the optimization problem and characterize the optimal mix of bank regulation and monetary policy in subsection 3.3.8. We also discuss the optimal bank regulation, taking monetary policy as given, as well as the optimal monetary policy, taking bank regulation as given.

[6] We focus in our analysis on a representative bank, which leads to the fact that under equal reserve rates (see assumption 3.3.2), the repayment obligation for reserve loans always matches the claim for reserve deposits. An insolvent bank thus only defaults on the deposit funding and bank losses only consist of the unmet liabilities towards depositors.

3.3.6 Bankers

There is a continuum of ex-ante identical bankers with mass normalized to one, so that we can focus on a representative banker. Bankers are endowed with capital good $E > 0$, which they can sell to firms at the nominal price $Q > 0$. The banker commits to using the entire proceeds from capital good sales to establish a bank with equity financing $E^b = QE$.[7] Banks are matched one-to-one with firms, so that the individual bank holds a non-diversified loan portfolio and is fully exposed to the idiosyncratic risk of the financed firm. The decision about loan supply L^b to the matched firm pins down the loans-to-equity ratio $\varphi = L^b/E^b$ and the bank's deposit financing $D^b = L^b - E^b$ after capital good transactions have been settled and the banker used the proceeds to acquire equity shares of the owned bank.

The banker can also decide to engage into monitoring of the financed firm, which increases the probability of a positive idiosyncratic shock and thus the chances for a high loan repayment (see subsection 3.3.3). Monitoring is costly, as it causes a non-monetary utility loss κL^b for the banker, which scales with the granted loan amount.[8] The parameter $\kappa > 0$ measures the monitoring efforts per unit of loan financing. The monitoring decision itself is denoted by $m \in \{0, 1\}$, where zero (one) represents shirking (monitoring).

The bank has a demand for liquidity in the form of central bank reserves because transactions on the capital good market lead to interbank deposit flows.[9] Specifically, we assume that a share $\alpha \in (0, 1]$ of deposits $D^b = L^b - E^b$ is temporarily outflowing.[10] The interbank liabilities following from the deposit outflows must be settled without netting the deposit inflows, i.e., the central bank applies a gross settlement procedure. The bank's reserve borrowing and reserve deposits then satisfy $L^{CB} \geq \alpha D^b$ and $D^{CB} \geq \alpha D^b$. As the interest rates on reserve loans and reserve deposits equal (see assumption 3.3.2), borrowing reserves is profit-neutral for the bank and we can, without loss of generality, assume that it holds that $L^{CB} = \alpha D^b$. Moreover, as we focus on a representative bank, deposit outflows

[7] This assumption is without loss of generality, as based on assumption 3.3.1 and the direct link between interest rates and firm productivity in equilibrium, no other asset (i.e., bond or deposit) yields a higher expected return than bank equity. The banker is risk-neutral, so that only the expected return is relevant for the investment decision.

[8] The assumption that monitoring costs scale with the amount of loan financing is of technical nature, as it simplifies the analysis of the banker's optimization problem.

[9] We abstract from deposit flows due to transactions on the consumption good market, since this solely complicates the analysis but does not yield further insights.

[10] We implicitly assume that the deposits of the banker do not cause deposit outflows, but remain at the bank and are used to acquire equity shares directly after the settlement of capital good transactions.

must match deposit inflows, so that, after capital good transactions have been settled, it holds $D^{CB} = L^{CB} = \alpha D^b$ and the balance sheet identity $L^b + D^{CB} = D^b + L^{CB} + E^b$ applies. Using the structure of reserve deposits, the bank's assets satisfy $L^b + D^{CB} = (1+\alpha)L^b - \alpha E^b$ and the assets-to-equity ratio is given by $\tilde{\varphi} = (L^b + D^{CB})/E^b = (1+\alpha)\varphi - \alpha$. For what follows, we will focus on the loans-to-equity ratio φ, as it allows for a more natural representation of the banker's optimization problem. For simplicity, we will in the following refer to φ as the *bank leverage* and to $\tilde{\varphi}$ as the *integrated bank leverage*, accounting specifically for the reserve holdings of the bank. As outlined in subsection 3.3.5, the bank is also subject to a regulatory leverage constraint $\varphi \leq \varphi^R$, with $\varphi^R \in [1, +\infty)$.

The interest rate on loans granted by the bank is given by $r_s^L > 0$, which depends on the idiosyncratic shock s of the financed firm. Deposits are credited with interest according to the rate $r^D > 0$. The nominal equity returns are then given by

$$(1 + r_s^E)E^b = \left\{(1 + r_s^L)L^b + (1 + r_{CB}^D)D^{CB} - (1 + r^D)D^b - (1 + r_{CB}^L)L^{CB}\right\}^+,$$

where we use $\{X\}^+ = \max\{X, 0\}$ to account for the limited liability of the bank. Using the structure of deposit financing, reserve loans and reserve deposits, it follows that the nominal equity returns are given by

$$(1 + r_s^E)E^b = \left\{(1 + r_s^L)L^b + [(1 + r_{CB}^D)\alpha - (1 + r^D) - (1 + r_{CB}^L)\alpha](L^b - E^b)\right\}^+.$$

With assumption 3.3.2, which imposes the equality of interest rates on reserves ($r_{CB}^D = r_{CB}^L$), and the definition of the bank leverage $\varphi = L^b/E^b$, we obtain that the rate of return on bank equity is given by

$$r_s^E(\varphi) := \{(r_s^L - r^D)\varphi + 1 + r^D\}^+ - 1.$$

Based on the explanations in subsection 3.3.3 and 3.3.4, we know that, in equilibrium, the interest rates on loans and deposits satisfy $r_s^L = A_s^L/q - 1$ for all s, and $r^D = r^B = A^B/q - 1$. Accordingly, the equilibrium rate of return on bank equity can be expressed with economic fundamentals, i.e., it holds that

$$r_s^E(\varphi) := \{(A_s^L - A^B)\varphi + A^B\}^+/q - 1. \tag{3.3}$$

It follows with our assumptions on firm productivity (see assumption 3.3.1 in subsection

3.3.3) that only with a low productivity of the financed firm ($s = l$), the bank is making losses on loans that have been funded with deposits. We can derive a maximum leverage, denoted by φ^S, which guarantees solvency of the bank in all states. This leverage is obtained by setting the equity return in the low productivity state to zero, i.e.,

$$1 + r_l^E(\varphi^S) = 0 \quad \Leftrightarrow \quad (A_l^L - A^B)\varphi^S + A^B = 0 \quad \Leftrightarrow \quad \varphi^S := \frac{A^B}{A^B - A_l^L}. \tag{3.4}$$

Based on assumption 3.3.1, we know that this leverage threshold is finite, i.e., it holds that $\varphi^S < +\infty$.

When capital good transactions are settled, the bank requires liquidity in the form of reserves which it can borrow from the central bank by pledging the bank loans granted to the matched firm. At that point in time, productivity shocks have not realized yet, so that the expected value of bank loans is given by $(1 + \mathbb{E}_m[r_s^L])L^b$. The central bank applies a haircut $\psi \in [0, 1]$ on the value of bank loans, so that the overall collateral available to the bank, also referred to as *collateral capacity*, is given by $(1 - \psi)(1 + \mathbb{E}_m[r_s^L])L^b$. Taking the repayment obligation on reserve loans into account, the reserve borrowing L^{CB} of the bank cannot exceed the collateral capacity, which leads to the liquidity constraint

$$(1 - \psi)(1 + \mathbb{E}_m[r_s^L])L^b \geq (1 + r_{CB}^L)L^{CB}.$$

With assumption 3.3.2, which states the equality of interest rates on reserves ($r_{CB}^D = r_{CB}^L$), the structure of reserve loans $L^{CB} = \alpha(L^b - E^b)$, and the definition of the bank leverage $\varphi = L^b/E^b$, we can reformulate the latter inequality as

$$(1 - \psi)(1 + \mathbb{E}_m[r_s^L])\varphi \geq \alpha(1 + r_{CB}^D)(\varphi - 1).$$

We can then define a maximum leverage, up to which liquidity of the bank is guaranteed. This leverage, denoted by $\varphi_m^L(\psi)$, is determined through the binding liquidity constraint, i.e.,

$$(1 - \psi)(1 + \mathbb{E}_m[r_s^L])\varphi_m^L(\psi) = \alpha(1 + r_{CB}^D)[\varphi_m^L(\psi) - 1],$$

so that

$$\varphi_m^L(\psi) = \frac{\alpha(1 + r_{CB}^D)}{\alpha(1 + r_{CB}^D) - (1 - \psi)(1 + \mathbb{E}_m[r_s^L])}. \tag{3.5}$$

The banker's monitoring decision m affects the leverage threshold $\varphi_m^L(\psi)$, as monitoring increases the expected productivity and thus the expected loan repayment of the financed firm. A higher valuation of bank loans increases the collateral capacity of the bank, allowing it to borrow, ceteris paribus, more reserves at the central bank. The improved access to central bank reserves then translates into an expansion of loan supply and deposit issuance in the first place, i.e., the maximum leverage is increasing with bank monitoring ($\varphi_1^L(\psi) > \varphi_0^L(\psi)$). Further, note that the banker never chooses a leverage larger than $\varphi_m^L(\psi)$, as it leads to illiquidity with certainty, in which case the government seizes all bank assets and thus the potential returns on bank equity are eliminated.

We allow for an active interbank market, where the bank can borrow, lend as well as deposit at other banks. The interbank loans are collateralized through bank loans, which are reduced in value by the same haircut $\psi \in [0, 1]$ as applied by the central bank. When paying interest on deposits, banks cannot differentiate between deposits held by other banks and deposits held by households. Accordingly, the deposit rate prevailing on the interbank market is given by $r^D > 0$. It follows that independent of whether the bank is constrained by capital or liquidity, the deposit rate equals the central bank rate, as stated in the following lemma.

Lemma 3.3.3 (Deposit Rate)
$r^D = r_{CB}^D$.

The identical pricing of deposits and reserves has two implications in our economy. First, we can deduce how in equilibrium, the capital good price Q and the consumption good price P form, i.e., based on the equilibrium conditions $r^D = r^B = A^B/q - 1$ and assumption 3.3.3, it holds that

$$r_{CB}^D = A^B/q - 1 \qquad \Leftrightarrow \qquad \frac{P}{Q} = \frac{1 + r_{CB}^D}{A^B}. \tag{3.6}$$

An increase of the interest rate r_{CB}^D on reserves leads to an increase in the consumption good price P or a decrease in the capital good price Q or both. Second, using equation

(3.5) and the equilibrium condition (3.6), we can express the maximum leverage $\varphi_m^L(\psi)$ guaranteeing liquidity of the bank using economic fundamentals, i.e., it holds that

$$\varphi_m^L(\psi) = \frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_m[A_s^L]}. \tag{3.7}$$

The banker uses the returns on bank equity $[1 + r_s^E(\varphi)]E^b$ to purchase consumption good C_s^B at the nominal price $P > 0$. The banker is maximizing the expected utility, which we assume to be linearly increasing in consumption. Accordingly, the optimization problem of the banker is given in real terms by

$$\max_{\substack{\varphi \in [1, \overline{\varphi}_m(\theta)], \\ m \in \{0,1\}}} \{1 + \mathbb{E}_m[r_s^E(\varphi)] - m\kappa\varphi\} qE, \tag{3.8}$$

where we made use of the definitions $E^b = QE$ and $\varphi = L^b/E^b$ to obtain $m\kappa L^b = m\kappa\varphi QE$. We also introduced the notation $\overline{\varphi}_m(\theta) := \min\{\varphi^R, \varphi_m^L(\psi)\}$ to represent the maximum possible bank leverage, where $\theta := (\varphi^R, \psi)$ denotes the policy measures implemented by the bank regulator and the central bank. Note that the expectation operator in (3.8) is indexed by the monitoring activity m, as the monitoring decision affects the probability distribution of productivity shocks for the financed firm and thus the expected equity returns.

We now discuss the optimal choice of the banker, as summarized in the following lemma. First, note that the banker always optimally chooses the maximum leverage, i.e., it holds that $\varphi = \overline{\varphi}_m(\theta)$. The reason is that, in equilibrium, the interest rates on loans and deposits are directly linked to firm productivity, i.e., it holds that $r_s^L = A_s^L/q - 1$ for all s and $r^D = r^B = A^B/q - 1$, and that, based on assumption 3.3.1, a loan-financed firm is, even without monitoring by the banker, more productive in expectation than a bond-financed firm, i.e., it holds that $\mathbb{E}_0[A_s^L] > A^B$.

Second, note that the banker's optimal monitoring decision generally depends on the exposure to solvency risk. Due to limited liability, the banker does not fully internalize the benefits of monitoring if the bank defaults for a negative productivity shock of the financed firm. Moreover, through the central bank collateral requirements, the maximum possible leverage $\overline{\varphi}_m(\theta)$ may vary with the banker's monitoring activity m, while the solvency leverage threshold φ^S is not affected by monitoring. Accordingly, we have to differentiate between three cases: (I) no solvency risk, i.e., the banker is not exposed to a solvency risk, independent of the monitoring decision, (II) "partial" solvency risk, the banker faces solvency risk only with monitoring, and (III) "full" solvency risk, i.e., the banker is always exposed to a solvency risk, independent of the monitoring decision. As the maximum possible leverage $\overline{\varphi}_m(\theta)$ weakly increases with monitoring, the banker can never face a situation where there exists a solvency risk only without monitoring. In the decision about monitoring, the banker must trade off the benefits against the costs in the form of the non-monetary utility loss. The benefits from monitoring are generally twofold: First, monitoring increases the probability for a high productivity of the financed firm and thus a high loan repayment. We refer to this effect as the *return channel* of monitoring. Second, monitoring may allow the bank to leverage more, i.e., expand deposit issuance and loan supply. The reason is that monitoring increases the expected value of bank loans and thus the collateral capacity of the bank, allowing it to borrow more reserves from the central bank. We refer to this effect as the *collateral leverage channel* of monitoring. This channel is only active if the bank is liquidity-constrained, at least without monitoring. In this case, the maximum possible bank leverage varies with monitoring, i.e., it holds that $\overline{\varphi}_0(\theta) < \overline{\varphi}_1(\theta)$. In contrast, if the bank is only constrained by the capital requirements imposed by the bank regulator, independent of the monitoring decision, i.e., $\overline{\varphi}_0(\theta) = \overline{\varphi}_1(\theta) = \varphi^R$, the collateral leverage effect is not at work. In that case, the banker's monitoring decision is only influenced by the benefits following from the return channel and by the monitoring costs.

Lemma 3.3.4 (Optimal Choice of the Banker)
In equilibrium, the banker's optimal choice of leverage is given by $\varphi = \overline{\varphi}_m(\theta)$ and the banker's optimally monitors (i.e., $m = 1$) if and only if

(I) without solvency risk, i.e., $\overline{\varphi}_m(\theta) \leq \varphi^S$ for all m, it holds that $\mathcal{M}_N(\theta) \geq 0$, where

$$\mathcal{M}_N(\theta) := \triangle(A_h^L - A_l^L) + (\mathbb{E}_0[A_s^L] - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right] - \kappa q,$$

(II) with partial solvency risk, i.e., $\overline{\varphi}_1(\theta) > \varphi^S \geq \overline{\varphi}_0(\theta)$*, it holds that* $\mathcal{M}_P(\theta) \geq 0$*, where*

$$\mathcal{M}_P(\theta) := \triangle(A_h^L - A^B) + (1-\eta_0)(A^B - A_l^L) - \frac{(1-\eta_1)A^B}{\overline{\varphi}_1(\theta)} + (\mathbb{E}_0[A_s^L] - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right] - \kappa q,$$

(III) with full solvency risk, i.e., $\overline{\varphi}_m(\theta) > \varphi^S$ *for all* m*, it holds that* $\mathcal{M}_F(\theta) \geq 0$*, where*

$$\mathcal{M}_F(\theta) := \triangle(A_h^L - A^B) + \frac{\triangle A^B}{\overline{\varphi}_1(\theta)} + \eta_0(A_h^L - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right] - \kappa q.$$

In the three different situations that depend on the bank's exposure to solvency risk, the monitoring costs in the banker's decision about monitoring are always given by κq. The benefits stemming from the return channel differ in the three cases: Without solvency risk, the banker internalizes all the expected productivity gains of the financed firm, so that the benefits from the return channel are given by $\triangle(A_h^L - A_l^L)$. With partial solvency risk or even full solvency risk, the banker does not internalize all direct effects of monitoring, as the bank defaults if the financed firm incurs a negative productivity shock. In these two cases, the benefits from the return channel are given by

$$\triangle(A_h^L - A^B) + (1-\eta_0)(A^B - A_l^L) - \frac{(1-\eta_1)A^B}{\overline{\varphi}_1(\theta)} \qquad \text{and} \qquad \triangle(A_h^L - A^B) + \frac{\triangle A^B}{\overline{\varphi}_1(\theta)},$$

respectively. The benefits from monitoring can, however, also emerge from the collateral leverage channel, which in the cases of no or partial and full solvency risk takes the form

$$(\mathbb{E}_0[A_s^L] - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right] \qquad \text{and} \qquad \eta_0(A_h^L - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right],$$

respectively. Note that the collateral leverage channel is not active, i.e., the latter terms vanish in the banker's monitoring decision, if the collateral requirements set by the central bank are sufficiently loose, so that the banker is never constrained by liquidity. In any such case, the maximum possible bank leverage satisfies $\overline{\varphi}_0(\theta) = \overline{\varphi}_1(\theta) = \varphi^R$.

3.3.7 Equilibrium properties

In this subsection, we first provide necessary conditions for the existence of a competitive equilibrium and for the bank's exposure to a solvency risk. Then, we characterize welfare, using economic fundamentals, and further discuss the monitoring decision of the banker.

Existence and solvency risk. The existence of an equilibrium crucially depends on the clearing of the capital good market. Specifically, an equilibrium exists only if banks do not grant more loans than are needed to purchase the entire capital good in the economy, i.e., it holds that $QK^L = L^b = \overline{\varphi}_m(\theta)QE \leq Q(K+E)$ or, with the notation $\varphi^M := 1+K/E$ equivalently, $\overline{\varphi}_m(\theta) \leq \varphi^M$. From the latter inequality, we can derive a condition on the capital requirements or the collateral requirements, depending which are binding for the bank. First, if the bank is constrained by capital, i.e., $\varphi^R \leq \varphi^L_m(\psi)$, it must hold $\varphi^R \leq \varphi^M$. In turn, if the bank is constrained by collateral, i.e., $\varphi^L_m(\psi) < \varphi^R$, the haircut set by the central bank must satisfy $\varphi^L_m(\psi) \leq \varphi^M$. The latter condition can be used to derive the smallest feasible haircut ψ^M_m, which, if implemented by the central bank, allows banks to provide as much loan financing as needed to allow loan-financed firms to acquire the entire capital good in the economy. Any haircut lower than ψ^M_m conflicts with the clearing condition for the capital good market and thus does not permit an equilibrium, whereas any haircut larger than ψ^M_m guarantees the existence of an equilibrium, but restricts the bank leverage below the maximum feasible one, i.e., $\varphi^L_m(\psi) < \varphi^M$.

If an equilibrium exists, i.e., $\varphi^R \leq \varphi^M$ or $\psi \geq \psi^M_m$, the bank is exposed to a solvency risk if the attained leverage is sufficiently large to exceed the leverage threshold guaranteeing solvency, i.e., $\overline{\varphi}_m(\theta) > \varphi^S$. Clearly, this is only possible if the regulatory leverage constraint is sufficiently loose, i.e., $\varphi^R > \varphi^S$, and the haircut set by the central bank is sufficiently small so that $\varphi^L_m(\psi) > \varphi^S$. We can use the condition $\varphi^L_m(\psi) = \varphi^S$ to derive the smallest possible haircut ψ^S_m guaranteeing solvency of the bank in all states. For any haircut ψ lower than ψ^S_m, the bank is exposed to a solvency risk, assuming that capital requirements are also sufficiently loose ($\varphi^R > \varphi^S$). Proposition 3.3.1 summarizes the previous explanations.

Proposition 3.3.1 (Existence and Solvency Risk)
A competitive equilibrium exists only if $\varphi^R \leq \varphi^M$ or $\varphi^L_m(\psi) \leq \varphi^M$, where the latter in-

equality is equivalent to

$$\psi \geq \psi_m^M := 1 - \frac{\alpha A^B}{\mathbb{E}_m[A_s^L](1 + E/K)},$$

and the bank is exposed to a solvency risk only if $\varphi^R > \varphi^S$ *and* $\varphi_m^L(\psi) > \varphi^S$*, where the latter inequality is equivalent to*

$$\psi < \psi_m^S := 1 - \frac{\alpha A_l^L}{\mathbb{E}_m[A_s^L]}.$$

The banker's monitoring decision m *follows from lemma 3.3.4.*

The smallest feasible haircut ψ_m^M and the smallest possible haircut ψ_m^S guaranteeing solvency of the bank both depend on the monitoring activity m. Note that bank monitoring increases the probability for a positive idiosyncratic shock of the respective firm ($\eta_1 = \eta_0 + \triangle$), and thereby also increases the expectation about loan repayment, i.e.,

$$(1 + \mathbb{E}_1[r_s^L])q = \mathbb{E}_1[A_s^L] = \mathbb{E}_0[A_s^L] + \triangle(A_h^L - A_l^L) = (1 + \mathbb{E}_0[r_s^L])q + \triangle(A_h^L - A_l^L).$$

The smallest feasible haircut ψ_m^M and the smallest possible haircut ψ_m^S guaranteeing solvency of banks both increase with monitoring (i.e., $\psi_1^M > \psi_0^M$ and $\psi_1^S > \psi_0^S$), as monitoring increases the collateral value of bank loans, but leaves the maximum feasible bank leverage φ^M as well as the leverage threshold for solvency φ^S unchanged. To keep bank lending at the maximum feasible level or at the level which rules out solvency risk, the central bank must steer against the monitoring-induced, increased collateral value of bank loans by setting stricter collateral requirements in the form of a higher haircut.

Welfare. The following lemma provides a characterization of utilitarian welfare, using economic fundamentals. Due to our assumption of linear utility for households and bankers, utilitarian welfare comprises aggregate consumption as well as bankers' utility losses due to monitoring, i.e., welfare denoted by W satisfies $W = C^H + C_m^B - m\kappa\varphi qE$, where $C_m^B = \mathbb{E}_m[C_s^B] = (1 + \mathbb{E}_m[r_s^E])qE$ represents aggregate consumption by bankers.[11] Welfare is generally affected by three factors: the regulatory maximum leverage φ^R and the haircut ψ, with at least one of them limiting bank leverage and thus determining the

[11] Note that banks and firms are matched one-to-one and the idiosyncratic productivity shocks are i.i.d. across firms. Thus, by the law of large numbers, expected consumption by the banker equals aggregate consumption by bankers.

capital allocation between loan-financed and bond-financed firms, and the monitoring activity of bankers m, influencing the productivity in the loan-financed sector. The banker's monitoring decision may also be shaped by the policy measures in the form of the regulatory maximum leverage φ^R and the haircut ψ (see lemma 3.3.4).

Lemma 3.3.5 (Welfare)
In equilibrium, welfare is given by $W_m(\theta) = (\mathbb{E}_m[A_s^L] - A^B - m\kappa q)\overline{\varphi}_m(\theta)E + A^B(K+E)$.

Monitoring. We now further discuss the banker's monitoring decision, as outlined in lemma 3.3.4, by contrasting two situations: In the first one, the banker is solely constrained by the capital requirements imposed by the bank regulator, as collateral requirements set by the central bank are sufficiently loose, i.e., the maximum possible leverage satisfies $\overline{\varphi}_0(\theta) = \overline{\varphi}_1(\theta) = \varphi^R$. In the second situation, in turn, the banker is constrained by liquidity, at least without monitoring, i.e., it holds that $\overline{\varphi}_0(\theta) < \overline{\varphi}_1(\theta) \leq \varphi^R$. Note that monitoring weakly increases the maximum possible bank leverage ($\overline{\varphi}_1(\theta) \geq \overline{\varphi}_0(\theta)$). Thus, in the first (second) situation, the haircut ψ set on bank loans used as collateral must satisfy $\overline{\varphi}_0(\theta) = \varphi_0^L(\psi) \geq (<)\varphi^R$ or, equivalently,

$$\frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_0[A_s^L]} \geq (<)\varphi^R \qquad \Leftrightarrow \qquad \psi \leq (>)\tilde{\psi}_0(\varphi^R) := 1 - \frac{\alpha A^B}{\mathbb{E}_0[A_s^L]}\frac{\varphi^R - 1}{\varphi^R},$$

where we used equation (3.5) to express the leverage threshold $\varphi_m^L(\psi)$ with model primitives.

Note that the banker is also constrained by liquidity with monitoring of the financed firm, whenever the collateral requirements are sufficiently tight, i.e., it holds that $\overline{\varphi}_1(\theta) = \varphi_1^L(\psi) < \varphi^R$ or, equivalently,

$$\frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_1[A_s^L]} < \varphi^R \qquad \Leftrightarrow \qquad \psi > \tilde{\psi}_1(\varphi^R) := 1 - \frac{\alpha A^B}{\mathbb{E}_1[A_s^L]}\frac{\varphi^R - 1}{\varphi^R},$$

where we again used the representation of $\varphi_m^L(\psi)$ following from equation (3.5). The collateral value of bank loans, and thus the borrowing limit for reserves, increases with monitoring. So, we can conclude that if there are liquidity constraints with monitoring, they will also be present without monitoring, i.e., it holds that $\tilde{\psi}_0^S(\varphi^R) < \tilde{\psi}_1^S(\varphi^R)$.

Next, we further characterize the banker's monitoring decision for any environment where the collateral requirements set by the central bank are sufficiently loose, so that the banker is only constrained by the capital requirements set by the bank regulator. The formal details are provided in the following corollary.

Corollary 3.3.1 (Monitoring Decision without Liquidity Constraints)
If collateral requirements set by the central bank are sufficiently loose, i.e., $\psi \leq \tilde{\psi}_0(\varphi^R)$, so that $\overline{\varphi}_0(\theta) = \overline{\varphi}_1(\theta) = \varphi^R$, the banker optimally monitors (i.e., $m = 1$) if and only if

(I) without solvency risk, i.e., $\varphi^R \leq \varphi^S$, it holds that $\tilde{\mathcal{M}}_N \geq 0$, where

$$\tilde{\mathcal{M}}_N := \triangle(A_h^L - A_l^L) - \kappa q,$$

(II) with full solvency risk, i.e., $\varphi^R > \varphi^S$, it holds that $\tilde{\mathcal{M}}_F(\varphi^R) \geq 0$, where

$$\tilde{\mathcal{M}}_F(\varphi^R) := \triangle(A_h^L - A^B) + \frac{\triangle A^B}{\varphi^R} - \kappa q.$$

Furthermore, it holds that $\lim_{\varphi^R \searrow \varphi^S} \tilde{\mathcal{M}}_F(\varphi^R) = \tilde{\mathcal{M}}_N$. Environments with partial solvency risk (see case (II) in lemma 3.3.4) do not exist with sufficiently loose collateral requirements.

First, note that with sufficiently loose collateral requirements, the collateral leverage channel of monitoring is not active. Thus, the banker's monitoring decision is only shaped through the benefits following from the return channel of monitoring and the costs associated with monitoring.

Second, based on corollary 3.3.1, we know that there is no environment with partial solvency risk and that for a regulatory leverage φ^R approaching the leverage threshold for solvency φ^S, the banker's incentives for monitoring in the presence of solvency risk $\tilde{\mathcal{M}}_F(\varphi^R)$ converge to those without solvency risk, i.e., it holds that $\lim_{\varphi^R \searrow \varphi^S} \tilde{\mathcal{M}}_F(\varphi^R) = \tilde{\mathcal{M}}_N$.

Third, note that in the case without solvency risk, the regulatory maximum leverage φ^R is irrelevant for the banker's monitoring decision. The banker monitors without solvency risk if and only if $\triangle(A_h^L - A_l^L) \geq \kappa q$. In contrast, with solvency risk, the regulatory maximum leverage influences the banker's monitoring decision. Specifically, with increasing leverage, the banker's incentives to monitor decrease, i.e., it holds that

$$\frac{\partial \tilde{\mathcal{M}}_F(\varphi^R)}{\partial \varphi^R} = -\frac{\triangle A^B}{(\varphi^R)^2} < 0.$$

Knowing that the banker's incentives increase with decreasing leverage, we can first conclude that there exists no leverage that induces the banker to monitor if it holds that $\triangle(A_h^L - A_l^L) < \kappa q$. Second, for $\triangle(A_h^L - A^B) \geq \kappa q$, the banker always monitors with solvency risk, independent of the regulatory maximum leverage φ^R. Third and last, if it holds that $\triangle(A_h^L - A_l^L) \geq \kappa q > \triangle(A_h^L - A^B)$, there exists a leverage $\varphi^* > \varphi^S$, with

$$\tilde{\mathcal{M}}_F(\varphi^*) = \triangle(A_h^L - A^B) + \frac{\triangle A^B}{\varphi^*} - \kappa q = 0 \quad \Leftrightarrow \quad \varphi^* = \frac{\triangle A^B}{\kappa q - \triangle(A_h^L - A^B)}, \tag{3.9}$$

so that for any regulatory maximum leverage $\varphi^R \leq \varphi^*$, the banker monitors.

In the subsequent analysis, we focus on situations where monitoring is socially optimal, but the costs associated with monitoring and the exposure to a solvency risk may incentivize the banker to shirk.

Assumption 3.3.3 (Monitoring Costs)
$\triangle(A_h^L - A_l^L) > \kappa q > \triangle(A_h^L - A^B)$.

To simplify the comparison of monitoring incentives of capital-constrained and liquidity-constrained bankers, we also rule out environments with a partial solvency risk if collateral requirements may not be sufficiently loose. We achieve this by assuming that loan-financed firms that experience a negative idiosyncratic shock do not produce any output, i.e., it holds that $A_l^L = 0$. As a result, banks face solvency risk whenever they fund loans with deposits, i.e., whenever it holds that $\varphi = \overline{\varphi}_m(\theta) > \varphi^S = 1$, where the latter equality follows directly from equation (3.4).

Assumption 3.3.4 (Solvency Risk)
$A_l^L = 0$, *so that* $\varphi^S = 1$.

We can show that with and without solvency risk, the collateral leverage channel increases the incentives for the banker to monitor. The collateral leverage channel is, however, only active if the banker is liquidity-constrained, at least without monitoring. In the case without solvency risk, the increased monitoring incentives due to the collateral leverage channel are irrelevant, as based on assumption 3.3.3, the banker always monitors. Hence, proposition 3.3.2 details the increased incentives for monitoring following from the collateral leverage channel only in the case with solvency risk. Based on assumption 3.3.4, we know that banks are always exposed to a solvency risk if they finance loans to firms with deposits (i.e., $\varphi > 1$) to some extent. Note that, in proposition 3.3.2, we only analyze the incentives for monitoring but do not impose that the bank has to attain the same leverage with capital constraints and liquidity constraints, respectively. Both dimensions, the banker's monitoring activity and the bank's leverage, will then be jointly considered in subsection 3.3.8, where we outline the optimal bank regulation and the optimal monetary policy.

Proposition 3.3.2 (Collateral Leverage Channel of Monitoring)
For given capital requirements leading to the regulatory maximum leverage φ^R, tight collateral requirements set by the central bank, i.e., the haircut satisfies $\psi > \tilde{\psi}_0(\varphi^R)$, increase, compared to loose collateral requirements, i.e., the haircut satisfies $\psi \leq \tilde{\psi}_0(\varphi^R)$, in the presence of solvency risk the banker's incentives to monitor, as it holds that

$$\mathcal{M}_F(\theta) - \tilde{\mathcal{M}}_F(\varphi^R) = \eta_0(A_h^L - A^B)\left[1 - \frac{\varphi_0^L(\psi)}{\overline{\varphi}_1(\theta)}\right] + \triangle A^B\left[\frac{1}{\overline{\varphi}_1(\theta)} - \frac{1}{\varphi^R}\right] > 0,$$

where for any haircut $\psi \leq \tilde{\psi}_1^S(\varphi^R)$, it holds that $\overline{\varphi}_1(\theta) = \varphi^R$, and $\overline{\varphi}_1(\theta) = \varphi_1^L(\psi) < \varphi^R$ otherwise. Furthermore, it holds that $\lim_{\psi \searrow \tilde{\psi}_0(\varphi^R)} \mathcal{M}_F(\theta) - \tilde{\mathcal{M}}_F(\varphi^R) = 0$.

It follows directly from proposition 3.3.2 that there exist environments where the banker attains the same leverage with a capital constraint and a liquidity constraint, but the collateral leverage channel is decisive in incentivizing the banker to monitor. Formally, this follows from the fact that for any haircut $\tilde{\psi}_0^S(\varphi^R) < \psi \leq \tilde{\psi}_1^S(\varphi^R)$, the banker attains the regulatory maximum leverage with monitoring ($\overline{\varphi}_1(\theta) = \varphi^R$), and is liquidity-constrained without monitoring ($\overline{\varphi}_0(\theta) = \varphi_0^L(\psi) < \varphi^R$), so that the collateral leverage channel is active, i.e., it holds that

$$\mathcal{M}_F(\theta) - \tilde{\mathcal{M}}_F(\varphi^R) = \eta_0(A_h^L - A^B)\left[1 - \frac{\varphi_0^L(\psi)}{\varphi^R}\right] > 0.$$

Note that the latter expression is maximized for the haircut $\psi = \tilde{\psi}_1^S(\varphi^R)$, which just allows the bank to attain the maximum regulatory leverage φ^R with monitoring.

3.3.8 Optimal bank regulation and optimal monetary policy

In our economy, the government aims at maximizing utilitarian welfare, which can be achieved through an appropriate bank regulation and monetary policy. The bank regulator imposes capital requirements leading to a regulatory maximum leverage φ^R, while the central bank sets the interest rate r^D_{CB} on reserves and the collateral requirements in the form of the haircut ψ on bank loans, determining the banks' access to liquidity.

We start by observing that the interest rate r^D_{CB} on reserves does not affect welfare, as it is irrelevant for the banker's monitoring decision and the capital allocation, see lemma 3.3.4 and lemma 3.3.5. It then follows with the equilibrium condition (3.6) that the interest rate r^D_{CB} only influences prices in our economy. This is a manifestation of the neutrality of money, i.e., the interest rate policy of the central bank has no effect on the real allocation.

Note that the capital and collateral requirements, as captured by the regulatory maximum leverage φ^R and the haircut ψ, both influence bank leverage and thus the allocation of capital among loan-financed and bond-financed firms. In addition, they may influence the monitoring decision m of the banker (see lemma 3.3.4). Formally, the optimization problem of the government is given by

$$\max_{\theta\in\Theta_m} W_m(\theta) = \max_{\theta\in\Theta_m} (\mathbb{E}_m[A^L_s] - A^B - m\kappa q)\overline{\varphi}_m(\theta)E + A^B(K+E),$$

where we used lemma 3.3.5 to express welfare $W_m(\theta)$ and applied the notation $\Theta_m := [1,+\infty) \times [\psi^M_m, 1]$ to represent the set of feasible policy measures, which itself depends on the monitoring decision of the banker. Not only the monitoring activity m is influenced by the haircut ψ set by the central bank and the regulatory maximum leverage φ^R imposed by the bank regulator, also the central bank's set of feasible haircuts $[\psi^M_m, 1]$ is affected by the banker's monitoring activity m. As outlined in subsection 3.3.7, the smallest feasible haircut increases with monitoring, i.e., it holds that $\psi^M_1 > \psi^M_0$. Thus, if the banker monitors (i.e., $m = 1$), the central bank finds itself unable to set any haircut ψ lower than ψ^M_1.

As stated in subsection 3.3.4, we only focus on situations where the interest rates on deposits and bonds equal ($r^D = r^B$). This, however, requires that banks issue deposits and bond-financed firms operate. Accordingly, we need to exclude situations where the bank regulator or the central bank restrict banks to fully fund loans with equity (i.e., $\varphi^R = 1$ or $\psi = 1$), and where the bank regulator and the central bank allow banks to attain the maximum feasible leverage (i.e., $\varphi^R = \varphi^M$ and $\psi = \psi_m^M$), as this would rule out production by bond-financed firms. The conditions $\varphi^R > 1$ and $\psi < 1$ are not restrictive, as based on assumption 3.3.1, a loan-financed firm is more productive in expectation than a bond-financed firm, even without monitoring, and thus the government always prefers to allow as much loan financing as possible. In fact, the optimal policies of the bank regulator and the central bank never include a regulatory maximum leverage $\varphi^R = 1$ or a haircut $\psi = 1$. In contrast, based on assumption 3.3.1, there are situations where the bank regulator or the central bank would prefer to allow banks to attain the maximum feasible bank leverage (i.e., $\varphi^R = \varphi^M$ and $\psi = \psi_m^M$). We rule out such cases, but allow the regulatory maximum leverage and the haircut to be arbitrary close to the polar measures, i.e., in these cases the regulatory maximum leverage is given by $\varphi^R = \varphi^M - \epsilon$ and the haircut is given by $\psi = \psi_m^M + \epsilon$ with $\epsilon \to 0$. The optimal policies discussed in the following are thus only ϵ-optimal in certain situations, namely if the government wants banks to attain the maximum feasible bank leverage. To ease our notation, we will use the limit $\varphi^R = \varphi^M$ and $\psi = \psi_m^M$ to represent the ϵ-optimal policies of the bank regulator and the central bank.

We first study the optimal bank regulation in the presence of sufficiently loose collateral requirements set by the central bank, so that the bank is liquidity-constrained under no circumstances, i.e., we assume the central bank sets a haircut ψ that satisfies $\psi \leq \tilde{\psi}_0(\varphi^R)$. Given this particular monetary policy, we know that welfare is maximized if bank lending is at its maximum and bankers monitor. This reasoning follows from assumption 3.3.1, stating that even without monitoring, a loan-financed firm is more productive than a bond-financed firm, and assumption 3.3.3, which ensures that monitoring is socially optimal, i.e., the productivity gains outweigh the monitoring costs. However, the costs associated with monitoring and the exposure to a solvency risk may lead to shirking of the banker under a sufficiently large leverage. As outlined in the previous subsection, we know that there exists a critical leverage φ^* that satisfies

$$\tilde{\mathcal{M}}_F(\varphi^*) = \triangle(A_h^L - A^B) + \frac{\triangle A^B}{\varphi^*} - \kappa q = 0 \qquad \Leftrightarrow \qquad \varphi^* = \frac{\triangle A^B}{\kappa q - \triangle(A_h^L - A^B)} > \varphi^S = 1.$$

The banker then monitors in the presence of loose collateral requirements, whenever the regulatory maximum leverage satisfies $\varphi^R \leq \varphi^*$. As a result, the bank regulator chooses capital requirements such that banks can attain the maximum feasible leverage, i.e., $\varphi^R = \varphi^M$, whenever $\varphi^M \leq \varphi^*$. This policy maximizes bank lending and induces bankers to monitor. Even if bankers do not monitor under the maximum feasible bank leverage, i.e., $\varphi^M > \varphi^*$, it may be optimal for the bank regulator to implement capital requirements that lead banks to attain the maximum feasible leverage ($\varphi^R = \varphi^M$). This is the case whenever maximum bank lending and no monitoring yields a higher welfare than reducing bank leverage to φ^* and thereby inducing monitoring, which is captured by condition (3.10) in proposition 3.3.3. In all other situations, the bank regulator will optimally choose to implement the regulatory maximum leverage $\varphi^R = \varphi^*$, restricting bank lending but inducing bankers to monitor.

Proposition 3.3.3 (Optimal Bank Regulation without Liquidity Constraints) *Suppose the central bank sets sufficiently loose collateral requirements, so that the bank is never liquidity-constrained, i.e., the haircut satisfies $\psi \leq \tilde{\psi}_0(\varphi^R)$.*

Then, the bank regulator optimally sets $\varphi^R = \varphi^M$ whenever (i) $\varphi^M \leq \varphi^*$, so that bank lending is maximized and the banker monitors, or (ii) $\varphi^M > \varphi^*$, so that bank lending is maximized and the banker does not monitor, but restricting the bank leverage to induce monitoring does not yield a welfare gain, i.e.,

$$\frac{\varphi^M}{\varphi^*} \geq 1 + \frac{\triangle(A_h^L - A_l^L) - \kappa q}{\mathbb{E}_0[A_s^L] - A^B}. \tag{3.10}$$

Otherwise, the bank regulator optimally sets $\varphi^R = \varphi^*$, so that bank lending is not maximized but the banker monitors.

Next, we describe the optimal monetary policy for environments where capital requirements are sufficiently loose, so that the banker is always liquidity-constrained. Specifically, the regulatory maximum leverage following from the capital requirements set by the bank regulator satisfies $\varphi^R \geq \varphi_m^L(\psi)$, where ψ is the haircut chosen by the central bank and m is the banker's monitoring decision under the prevailing collateral requirements. Under these circumstances, the optimal monetary policy, which is formally outlined in the next proposition, follows in its logic the optimal bank regulation in the presence of loose collateral requirements.

Proposition 3.3.4 (Optimal Monetary Policy without Capital Constraints) *Suppose the bank regulator sets sufficiently loose capital requirements, so that the banker is never capital-constrained, i.e., the regulatory maximum leverage satisfies $\varphi^R \geq \varphi_m^L(\psi)$.*

Then, the central bank optimally sets $\psi = \psi_1^M$ whenever $\varphi^M \leq \varphi^{**}$, so that bank lending is maximized and the banker monitors. Moreover, the central bank optimally sets $\psi = \psi_0^M$ whenever $\varphi^M > \varphi^{**}$, so that bank lending is maximized and the banker does not monitor, but reducing the bank leverage to induce monitoring does not yield a welfare gain, i.e.,

$$\frac{\varphi^M}{\varphi^{**}} \geq 1 + \frac{\triangle(A_h^L - A_l^L) - \kappa q}{\mathbb{E}_0[A_s^L] - A^B}, \tag{3.11}$$

where $\varphi^{**} = \varphi_1^L(\psi^{**}) > \varphi^S$, with ψ^{**} satisfying $\mathcal{M}_F(\theta^{**}) = 0$, where $\theta^{**} = (\varphi^R, \psi^{**})$. Otherwise, the central bank optimally sets $\psi = \psi^{**}$, so that bank lending is not maximized but the banker monitors.

Finally, we derive the optimal mix of bank regulation and monetary policy. Due to the collateral leverage channel of monitoring (see proposition 3.3.2), it follows that a liquidity-constrained banker monitors under a larger leverage compared to capital-constrained banker, i.e., it holds that $\varphi^{**} > \varphi^*$. We can thus conclude that it is optimal to restrict bank leverage by imposing collateral requirements instead of capital requirements. The optimal mix of bank regulation and monetary policy is thus represented by the regulatory maximum leverage satisfying $\varphi^R \geq \varphi_m^L(\psi)$ and the haircut ψ following proposition 3.3.4.

Corollary 3.3.2 (Optimal Bank Regulation and Optimal Monetary Policy) *It holds that $\varphi^{**} > \varphi^*$. Accordingly, it is optimal to set sufficiently loose capital requirements, i.e., $\varphi^R \geq \varphi_m^L(\psi)$, and collateral requirements, in the form of the haircut ψ, according to proposition 3.3.4.*

3.3.9 Contingent capital requirements

From our previous analysis, we can deduce that the monitoring incentives of bankers depend on whether they are capital- or liquidity-constrained. In the latter case, bankers have increased incentives to monitor. It thus seems that compared to capital requirements, collateral requirements are special to some extent. This conclusion is certainly true when comparing collateral requirements to unweighted capital requirements.

However, if capital requirements are contingent, such that they ultimately vary with the monitoring activity, they can have a similar (or even the same) effect on the monitoring incentives. We refer to the monitoring benefits induced through contingent capital requirements as the *regulatory leverage channel* of monitoring. Contingent capital requirements already exist in the form of risk-dependent capital requirements implemented by bank regulators, for instance. In the following, we explore how contingent, risk-dependent capital requirements fit into our current framework.

Let σ_m denote the measure of risk which depends on the banker's monitoring activity. A lower parameter σ_m represents a lower risk exposure. The capital requirements set by the bank regulator are then assumed to be contingent on the risk exposure of the individual banker, i.e., the regulatory maximum leverage satisfies $\varphi^R(\sigma_m)$. A risk measure might, for example, be the standard deviation of loan returns; acknowledging that, in practice, risk is often measured differently, using the value-at-risk, for instance. Note that loan returns are, in equilibrium, directly linked to the productivity of firms, i.e., in the loan-financed sector, it holds that $(1 + r_s^L)q = A_s^L$ for all s. Based on assumption 3.3.4, the standard deviation of loan returns is then given by

$$\sigma_m = \sqrt{\eta_m(1-\eta_m)}A_h^L/q.$$

Note that it holds that $\eta_1(1-\eta_1) = \eta_0(1-\eta_0)+\triangle(1-2\eta_0-\triangle)$, so that for $\eta_0 > (\leq)(1-\triangle)/2$, the standard deviation decreases (increases) with bank monitoring, i.e., $\sigma_1 < (\geq)\sigma_0$.

We can always find a schedule for the risk-dependent capital requirements, so that the *regulatory* leverage channel is identical to the *collateral* leverage channel of monitoring, i.e., there exists a $\varphi^R(\cdot)$ such that $\varphi_m^L(\psi) = \varphi^R(\sigma_m)$ for all m. For $\eta_0 > (\leq)(1-\triangle)/2$, mimicking the collateral leverage channel with contingent capital requirements allows implicitly for less (more) risk-taking on the side of banks.

It thus follows that, under certain conditions, our analysis could also be interpreted as a comparison of non-contingent and contingent capital requirements regarding their effect on monitoring incentives. The collateral leverage channel could then be interpreted as the regulatory leverage channel. Using the standard deviation as a risk measure, we could illustrate that contingent capital requirements may indeed be used to replicate the collateral leverage channel of monitoring. However, such a design of contingent capital requirements may not necessarily be in line with their primary objectives. Collateral requirements implemented by the central bank have thus a unique effect on bankers' monitoring incentives.

3.4 Conclusion and Discussion

The unique, or at least superior, ability to monitor is seen as a classical justification for the existence of banks. As banks play a central role in the allocation of funds (and thus resources) in our economy, it is important to understand the fundamental forces shaping banks' monitoring incentives. We develop a simple model that allows to study the monitoring incentives of banks in environments where banks are capital- and/ or liquidity-constrained. In our baseline model, the monitoring technology considered is in the spirit of Holmstrom and Tirole (1997), as it avoids any opportunistic behavior of the bank borrowers, which are firms in our setting.

We show that capital constraints, following from regulatory (unweighted) capital requirements, and liquidity constraints, following from collateral requirements in central bank lending facilities, have different effects on the monitoring incentives of bankers. Specifically, we show that the benefits from monitoring are twofold: First, monitoring leads to greater chances for a high loan repayment and thus, ceteris paribus, it leads to increased expected profits of the bank. We dub this effect the *return channel* of monitoring. Second, as monitoring increases the expected value of bank loans, these loans increase in their collateral value, allowing the respective bank to borrow more reserves. This, in turn, induces any liquidity-constrained bank to grant more loans and issue more deposits in the first place, leading to higher expected profits for the bank as the leverage increases. We refer to this effect as *the collateral leverage channel* of monitoring. This channel, however, is only active if bankers are liquidity-constrained. Accordingly, we find that liquidity-constrained bankers have more incentives to monitor than capital-constrained bankers under any circumstance. We also show that the effect of central bank collateral requirements on bankers' monitoring incentives is also unique in comparison with contingent (e.g., risk-dependent) capital requirements. While such capital requirements lead to a regulatory leverage channel that can replicate the collateral leverage channel, such action may contradict other objectives, as discouraging the banks' risk-taking.

The model we developed has a simple structure and can be extended in various ways: First, the production structure can be modeled in a more general way, assuming strictly concave technologies in at least one of the sectors, for instance. Second, bank default is frictionless in our economy, i.e., there are no further costs arising from banks defaulting on their liabilities. A more realistic version of our model would account for such costs, arising from default resolution, for instance. Third, in our framework, the collateral leverage channel was established through the collateral-enhancing effect of bank monitoring. However, such a leverage channel can also be established in other ways, with monitoring concerning a bank's in-house processes: For example, if banks have access to costly monitoring technologies that reduce their liquidity demand, a similar leverage channel emerges, induced by (binding) central bank collateral requirements.

Chapter 4

Monetary Policy with a Central Bank Digital Currency: The Short and the Long Term*

Abstract

We examine how the introduction of an interest-bearing central bank digital currency accessible to the public impacts bank activities and monetary policy. At any time, depositors can switch from bank deposits to CBDC as a safe medium of exchange. As banks might face digital runs, either because depositors have a preference for CBDC or fear bank insolvency, monetary policy can use collateral requirements and penalties for illiquidity to initially increase bankers' monitoring incentives. This leads to higher aggregate productivity. However, the mass of households holding CBDC will increase over time, causing additional liquidity risk for banks. After a certain period, monetary policy with tight collateral requirements generating liquidity risk for banks and exposing bankers to illiquidity penalties would render banking non-viable and prompt the central bank to abandon such policies. In such circumstances, the bankers' monitoring incentives would revert to low levels. Accordingly, a CBDC can yield short-term welfare gains at best.

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4.1 Introduction

Whether and how governments should introduce a publicly available digital form of their national currency is a widely debated issue in academia and policymaking. Various countries are experiencing a decline in the use of cash and privately issued digital currencies are attracting increasing attention, thus making the issue doubly relevant. Figure 4.1 depicts the share of cash in the narrowest monetary aggregate M1 for a sample of developed countries over the last four decades until the recent outbreak of Covid-19, illustrating the diminishing importance of cash in some countries.[1]

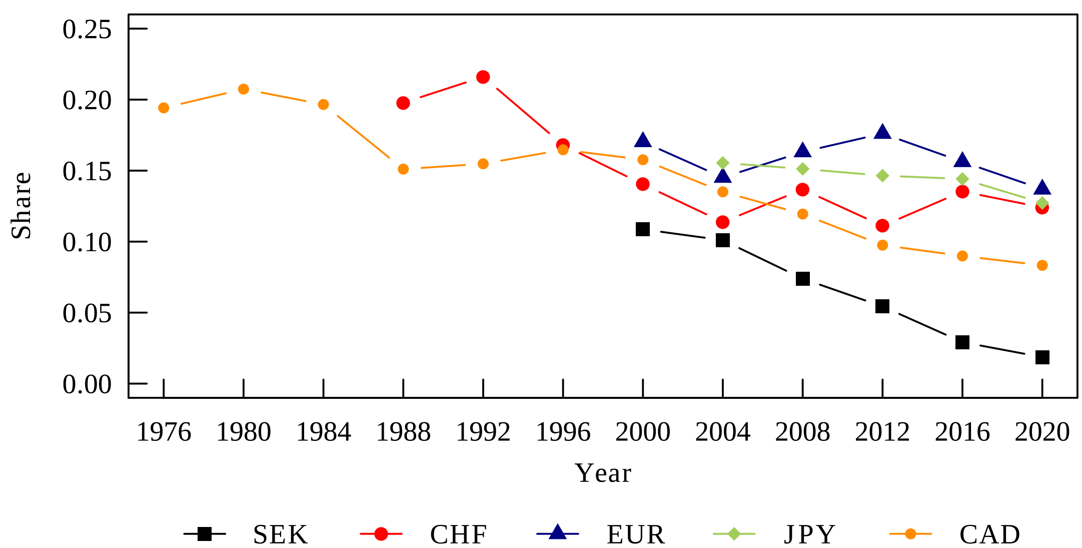

Figure 4.1: Share of cash in the monetary aggregate M1, average of monthly reported figures; *Source:* Bank of Canada, Bank of Japan, European Central Bank, Sveriges Riksbank, and Swiss National Bank; accessed April 7, 2020.

Several central banks have announced plans to assess the implications of this new form of national currency, which, as it would be issued by the respective central bank, is often referred to as "central bank digital currency", or simply "CBDC".[2] Unresolved issues relate not only to the functional and technical design, but also to the economic consequences.

[1] Covid-19 may have a strong impact on the use of cash and accelerate current trends initiated by technological innovation (Brown et al., 2020).

[2] For example, the Sveriges Riksbank (https://www.riksbank.se/en-gb/payments--cash/e-krona/, accessed April 7, 2020) and the Bank of Canada (https://www.bankofcanada.ca/research/digital-currencies-and-fintech/, accessed April 7, 2020).

A central bank digital currency will naturally compete with the predominant types of money currently used by the public, i.e., deposits at private banks and cash. Substitutability will generally depend on how a CBDC compares to the existing monies regarding the three main functions of money: unit of account, medium of exchange, and store of value (Hicks, 1967). Differences are conceivable in the latter two dimensions. The low, and still declining, use of cash indicates that, compared to bank deposits, banknotes and coins are considered as of now to be the inferior medium of exchange. Exceptions are situations in which the infrastructure required for electronic transaction settlement is not available or anonymity is desired. Moreover, outside crisis periods—when bank default is not considered an important issue—the fact that cash does not bear interest also makes it an inferior store of value, in comparison to bank deposits. A central bank digital currency can therefore represent a near substitute for cash—non-interest-bearing but allowing for transactions of any kind, albeit with higher transaction costs than deposits with private banks—or as a near substitute for bank deposits—with interest payments and a similar ability to serve as a medium of exchange. In the latter case, a CBDC may even be considered superior to deposits as a store of value since it does not comprise default risk on the part of the issuer.

Arguably, with a central bank digital currency acting as a new substitute for bank deposits and with no restrictions to the right of converting deposits into CBDC, using a digital infrastructure at negligible cost, a deposit insurance scheme of the kind we have in our current monetary system would be less needed. The aim of this paper is thus to analyze how the introduction of a central bank digital currency would impact bank activities and monetary policy. We analyze the welfare implications of these institutional changes (CBDC and no deposit insurance) and provide a welfare comparison with today's monetary system, where bank deposits, as the principal form of money, are insured through governmental guarantees. We characterize the optimal monetary policy in the presence of a central bank digital currency and discuss its effectiveness in both the short and the long term. We also compare welfare in an economy with central bank digital currency and optimal monetary policy to welfare achieved by a (constrained) social planner.

The model developed in this paper aims to reproduce several stylized features of the current monetary system present in many developed countries. First, private banks supply money by financing loans and other investments with deposits. Accordingly, banks' investment and financing decisions, influenced by bank regulation, strongly affect the supply of money. Second, deposits represent a claim on the legal tender, which at present is cash. Hence, deposit withdrawals are effected by using the legal tender. Interbank liabilities such as those emerging from deposit transfers, are settled by using interest-bearing reserves, a digital form of the national currency only available to banks. Third, the central bank only provides banks with cash and reserves, commonly referred to as liquidity. Hence, monetary policy as the organization and execution of liquidity provisions influences banks' investment and financing decisions and thus private money creation.[3]

In the presence of a deposit insurance scheme or an interest-bearing CBDC, cash is generally an inferior medium of exchange and store of value.[4] Accordingly, it will not concern us any further here. In our model for an alternative monetary system, the CBDC represents the only legal tender. Thus, any deposit withdrawal can be interpreted as a deposit transfer from a private bank to the central bank. Following today's institutional arrangement, deposit transfers are settled with reserves that can be borrowed from, and deposited with, the central bank. Liquidity provisions take the form of collateralized loans, such that monetary policy includes the choice of interest rates for reserves and the CBDC, and also the choice of the collateral framework that determines the eligible assets and their valuation (Bindseil et al., 2017).

Our model accounts for standard banking elements. In particular, bankers act as delegated monitors able to alleviate moral hazard on the side of borrowers. However, bankers are also subject to moral hazard, which leaves them with the alternative between costly monitoring or shirking. Moral hazard arises from the fact that monitoring is costly and, in the presence of a solvency risk, bankers do not fully internalize the benefits from monitoring. Bankers face non-pecuniary penalties, imposed by the central bank, if they become illiquid. Finally, depositors face switching costs when transferring funds between private banks or between a private bank and the central bank. These are motivated by the substantial effort involved in undertaking a transfer. In our model, fiat money in the form of bank deposits has value due to three reasons: First, firms can only acquire investment goods if they obtain loans from banks and thus there are large gains from using money to buy these goods. Second, depositors can avoid default risk at banks by switching to CBCDs.

[3]Liquidity can be provided in different ways; see Bindseil (2004) on how monetary policy has evolved over time.

[4]We rule out situations in which transactions cannot be settled electronically or anonymity is desired. Moreover, in describing today's system we do not account for any legal tender. But in the presence of a deposit insurance scheme, banknotes and coins are not demanded, so we can do this without any loss of generality.

Third, all money is destroyed at the end of the economy.[5] Throughout our analysis, we focus on a perfectly competitive banking sector. This assumption is made on purpose in order to single out the role of the bankers' monitoring incentives and their connection to monetary policy in the presence of a CBDC. The only inefficiency in our model that may emerge, in particular without a proper central bank intervention, is represented by shirking of bankers. Compared to the existing literature, we thus describe an additional effect of a CBDC by showing how and when monetary policy with a CBDC can increase bankers' incentives to monitor and eliminate shirking.

Depositors can switch from bank deposits to CBDC as a safe medium of exchange at any time. We model digital bank runs arising either because depositors have a preference for CBDC or fear bank insolvency. We refer to the former type of bank runs as CBDC-induced, as they arise only because the central bank issues money to the public, which is equivalent to bank deposits as a medium of exchange and store of value. Deposits can be converted into CBDC without the consent of bankers or the central bank. Thus, following a bank run, bankers may face a liability vis-à-vis the central bank that exceeds their collateral capacity as determined by the central bank. In this case, the bank becomes illiquid and defaults, and the respective banker will face a illiquidity penalty scaling with the liability vis-à-vis the central bank that is not covered by the available collateral. While bankers cannot influence the likelihood of CBDC-induced bank runs, they can engage in the costly monitoring of borrowers to alleviate moral hazard, which increases the probability of success for the financed project and ultimately reduces the likelihood of bank insolvency. Monitoring thus not only increases the expected loan repayment, but also reduces the likelihood of a illiquidity penalty. Hence, a monetary policy with tight collateral requirements, generating liquidity risk and exposing bankers to penalties for illiquidity, can increase bankers' monitoring incentives and lead to higher aggregate productivity. In this case, monetary policy mimics the famous dictum of Bagehot (1873), claiming that central banks should provide emergency liquidity assistance to banks only against good collateral and by charging a penalty rate. However, due to recurrent bank insolvencies and positive switching costs, the mass of households holding CBDC will increase over time and cause additional liquidity risk for banks. Thus, as the likelihood of CBDC-induced bank runs increases, the chances of bankers being exposed to illiquidity penalties will also increase,

[5]Imposing a deposit-in-advance constraint is not necessary since deposits are interest-bearing. For an alternative set of assumptions to ensure the value of bank deposits in a finite horizon model see Faure and Gersbach (2017).

while the chances of earning returns from loan financing will decrease. After a certain period, monetary policy with tight collateral requirements would render banking non-viable and prompt the central bank to abandon such policies, so that monitoring incentives will revert to low levels.

We provide necessary conditions for the optimality of tight collateral requirements and characterize the optimal monetary policy explicitly under specific assumptions on firm productivity and switching costs. We find that the illiquidity penalties that suffice to incentivize bankers to monitor, decrease with banks' equity-to-deposit ratio and increase with the probability of success for the financed project without monitoring by the banker. The higher banks' equity financing, the larger the returns from monitoring skimmed by bankers and the lower the illiquidity penalties necessary to incentivize bankers to monitor. Similarly, the higher the probability of success for the financed project without monitoring by bankers, the higher the expected returns without monitoring skimmed by bankers will be, so that there are correspondingly fewer incentives for bankers to monitor. Accordingly, the illiquidity penalty required to incentivize bankers to monitor will increase.

We also compare this alternative monetary system (a CBDC and no deposit insurance) with the current monetary system, where bank deposits are the principal form of money, which is often insured by such things as governmental guarantees. Most notably, if there are no switching costs and monetary policy is optimal, the alternative system will never entail welfare losses compared with today's monetary system. However, through the use of its collateral framework, the central bank can improve bankers' monitoring incentives and ultimately increase welfare. This effect exists at most for a finite period of time as in the presence of solvency risk, tight collateral requirements will at some point render banking non-viable. In this respect, introducing a central bank digital currency involves risks for both the individual bank and for the banking system as a whole. Since banks' liquidity demand is likely to rise with a CBDC, the rules for liquidity provisions by the central bank, including the collateral framework, come to the fore. In modeling the current and alternative monetary system, we abstract from cash and thus implicitly assume that the described effect of monetary policy on the bankers' monitoring incentives is only at work with a CBDC. Two remarks are in order. First, while we abstract from cash for tractability, our results continue to hold in any environment where switching costs for cash are higher than for CBDC and the latter is subject to positive interest payments. Second, we can establish an equivalence result for cash and CBDC if converting bank deposits into any of these two monies leads to the same switching costs and the central bank pays no interest on the CBDC. Monetary policy has then the same effect on the bankers' monitoring incentives in an environment with cash and/ or CBDC. Finally, we also outline potential remedies to avoid (digital) bank runs and discuss their implication for the effectiveness of monetary policy in our framework to incentivize bankers to monitor.

Welfare in a competitive equilibrium with optimal monetary is also compared with welfare achieved by a (constrained) social planner. The unconstrained social planner having complete information about agents' activities can achieve the first-best welfare by reallocating endowments between agents in order to rule out solvency risk for bankers, which guarantees a welfare-maximizing monitoring decision by bankers and avoids switching costs incurred by depositors in the case of bank insolvency. Accordingly, any competitive equilibrium without solvency risk and with loose collateral requirements representing the optimal monetary policy, i.e., no liquidity risk and no illiquidity penalties for bankers, yields the first-best welfare. The constrained social planner having limited information about agents' activities and being restricted to payments contingent on idiosyncratic states can only achieve the second-best welfare: Bankers' monitoring decision can be aligned with the objective of maximizing welfare, but solvency risk for bankers and thus switching costs incurred by depositors in the case of bank insolvency cannot be eliminated. Any competitive equilibrium with solvency risk and tight collateral requirements representing the optimal monetary policy, i.e., liquidity risk and illiquidity penalties for bankers, yields welfare which is generally lower than the second-best welfare, due to penalties for illiquidity imposed on bankers and lost monitoring activities by illiquid bankers.

The paper is structured as follows: Section 4.2 relates our work to the existing literature. Section 4.3 introduces the model and discusses the optimal choices of the individual agents. Section 4.4 provides the equilibrium analysis, while section 4.5 outlines the optimal monetary policy. Subsequently, section 4.6 provides the welfare comparison with today's monetary system, and section 4.7 investigates the dynamic effects of our model. Section 4.8 discusses various model assumptions and outlines potential remedies to avoid digital bank runs, and section 4.9 concludes.

4.2 Relation to the Literature

The introduction of a central bank digital currency is a widely debated issue (Barontini and Holden, 2019; Boar et al., 2020; Boar and Wehrli, 2021). Of all the forms of CBDCs discussed, we focus in this paper on a near substitute for bank deposits with equivalent properties as a medium of exchange.[6] Moreover, similar to reserves held by banks with the central bank, the central bank digital currency held by the public is interest-bearing. Then, the only difference between banks and the public is that banks can borrow national currency from the central bank, while the public cannot.

A series of papers discusses the pros and cons of such central bank digital currencies. The main advantages are considered to be a disciplining effect on commercial banks (Berentsen and Schär, 2018), financial inclusion and an increase of financial stability (Crawford et al., 2021; Berentsen and Schär, 2018), as well as a better conduct of monetary policy (Bordo and Levin, 2017). However, Cecchetti and Schoenholtz (2018) argue that a CBDC may also generate financial instability. Among others, Engert et al. (2017) claim that a central bank digital currency could improve competitiveness in payments. Similarly, Kahn et al. (2018) consider the mitigation of competition problems in the banking sector to be the strongest argument in favor of introducing a CBDC. A comprehensive overview of the potential implications of central banks issuing digital currencies for the public can be found in Pichler et al. (2020).

A few theoretical papers have already assessed possible effects of a central bank digital currency. Andolfatto (2021) shows that in an overlapping generation framework with imperfectly competitive banks, a central bank digital currency not only increases financial inclusion but also raises deposit rates through increased competition. This positive competition effect is also at work in Chiu et al. (2019), who calibrate their model for the US economy and find that, subject to suitable interest rate setting, a central bank digital currency may raise bank lending and output significantly. Barrdear and Kumhof (2021) develop a dynamic stochastic general equilibrium model to show how a central bank digital currency lowers the real policy rate and thereby stimulates the economy. Similarly, Keister and Sanches (2019) find a central bank digital currency to be generally welfare-improving, while noting that there may also be instances in which the funding costs of banks increase, so that lending and ultimately welfare are reduced. The introduction of a CBDC would also pose technical and organizational challenges. The former are addressed by Böhme (2019) and Auer and Böhme (2020), while the latter are discussed by Bindseil (2019). Moreover, Agur et al. (2021) discusses the optimal design of a CBDC, taking preferences for anonymity and security as well as network effects into account.

This paper relates to the growing literature discussing the creation of money by private

[6] An overview of possible types of central bank digital currencies can be found in Bech and Garratt (2017) and Kumhof and Noone (2018).

agents. Models of bank money creation have been developed by Faure and Gersbach (2017) and Benigno and Robatto (2019), which both feature macroeconomic risk. In this model, we focus instead on idiosyncratic risk, while introducing a second form of money, the CBDC, and modeling moral hazard on the part of bankers and bank borrowers.

As we compare monetary systems with and without CBDC, our work is also closely connected to the literature on monetary architectures and the equivalence of monies. Brunnermeier and Niepelt (2019), for example, establish some general conditions for the equivalence of public and private money, without focusing on particular institutional arrangements that rule the process of money creation. Faure and Gersbach (2018), in turn, compare monetary architectures in which money is solely created by the central bank with today's monetary system, which relies particularly on private money creation in the form of deposit issuance by banks.

In our model, bank borrowers, represented by firms, are prone to moral hazard. We introduce a monitoring technology for bankers in the spirit of Holmstrom and Tirole (1997) that prevents any opportunistic behavior by firms. We therefore relate to a large literature that subscribes to this interpretation of monitoring and, specifically, its application to banking (see, for example, Gersbach and Rochet (2017)). Moreover, we introduce moral hazard on the part of bankers, allowing them to engage in costly monitoring or shirking. Note that in our model depositors do not need to monitor bankers as in the classic paper by Calomiris and Kahn (1991), since they can always switch to CBDC and are not impacted by bank defaults.

4.3 Model

4.3.1 Macroeconomic environment

The model features four agents: households, firms, bankers, and a central bank. Households and bankers are endowed with a capital good, which is used by firms to produce a unique consumption good. We consider a monetary economy where transactions are settled instantaneously by using bank deposits or CBDC.[7] The latter only enters the economy when depositors switch from private banks to the central bank. Bankers grant loans, financed with equity and deposits, and can monitor borrowers. Loans are demanded by firms, as

[7]Firms are subject to limited commitment to repayment, which can be overcome by bank loans, securing repayment and allowing firms to acquire capital goods instantaneously using money in the form of bank deposits or CBDC.

they are penniless and need to finance the acquisition of the capital good in the markets instantaneously, i.e., before output is produced and sold. Monitoring of firms increases their expected productivity and ultimately their expected loan repayment, but it also requires costly efforts on the part of bankers. Markets are competitive. By assuming that each banker is matched with one firm and one household, we can account for idiosyncratic risk.

Bankers face runs from households if the latter prefer CBDC to deposits or bankers become insolvent. Households can execute their deposit transfers at any time without the consent of bankers or the central bank. Thus, the demand for reserves by the individual banker may exceed the collateral capacity determined by the central bank. Then, the banker will become illiquid and default. In the case of illiquidity, the central bank seizes all available bank assets and the banker faces a non-pecuniary illiquidity penalty. Firms are exposed to idiosyncratic productivity shocks, so that sufficiently high bank leverage may expose banks to solvency risk, i.e., the revenues from loan financing are insufficient to service the liabilities vis-à-vis depositors and the central bank. Depending on the returns from the assets seized, the central bank generates losses or profits which, as the central bank operates under a balanced budget, are financed through taxes or distributed by using transfers.

4.3.2 Summary of events and notation

We model a monetary economy in which transactions are settled instantaneously so that the timing of events is of great importance for our analysis. Until section 4.7 we consider a static framework with the following three subsequent stages, summarized in figure 4.2.

Stage I. The central bank sets the loan rates for reserves, the deposit rates for reserves and the CBDC, and by setting a haircut determines the valuation of bankers' collateral, i.e., the loans granted by bankers to firms, and the illiquidity penalties for bankers. Each banker is matched with one firm and one household. The banker provides the firm with loan financing, decides on future monitoring activities and demands reserves from the central bank. The firm uses the deposits acquired to purchase capital good on the markets. Bankers use all their deposits for the equity financing of banking operations. As capital good is sold to firms, bankers may experience a CBDC-induced bank run if the matched household prefers CBDC to deposits and thus initially opens an account with the central bank instead of with the matched banker. If the collateral capacity of a banker is insufficient to obtain the reserves required to service the deposit transfer to the central bank, the banker will become illiquid and default. The assets of the respective banker are then seized by the central bank, and the banker will face a illiquidity penalty imposed by the central bank.

Stage II. Any liquid banker executes the monitoring activity previously decided on. The idiosyncratic productivity shocks realize and firms transform the capital good into the consumption good. Bankers can face insolvency, in which case depositors may transfer their funds to the central bank. If the household possessing funds with an insolvent banker chooses to hold CBDC instead of deposits, the respective banker will only default on the central bank. If the banker's collateral capacity is insufficient to cover the liabilities vis-à-vis the central bank, the banker will face a illiquidity penalty imposed by the central bank.

Stage III. Solvent bankers credit deposits with interest and pay out dividends on the equity financing. Central bank losses are financed through taxes, while profits are distributed by using transfers. Households and bankers use their funds to purchase the consumption good (in figure 4.2 abbreviated by using "cons. good") on the markets. Firms use the revenues from sales to meet their repayment obligations on the outstanding loans. Similarly, bankers repay their borrowed reserves to the central bank.

Stage I	**Stage II**	**Stage III**
• Monetary policy decided	• Monitoring by banks	• Interest payments
• Bank loans, reserve loans	• Production by firms	• Taxes and transfers
• Capital good market	• Bank insolvency	• Cons. good market
• CBDC-induced runs	• Insolvency-caused runs	• Loan repayment

Figure 4.2: Summary of events.

From the perspective of the individual bank, a bank run is caused either by a household preferring CBDC to deposits or by bank insolvency, which, in its turn, will result from a negative productivity shock for the financed firm in the presence of sufficiently high bank leverage. Hence, for each triplet (banker, firm and household), the returns on deposits, loans, and equity can at most depend on the type of household, the idiosyncratic productivity shock for the firm, and bank leverage. For the subsequent description of the model it will be useful here to formally introduce the multivariate state

$\mathbf{z} := (\varphi, h, s) \in \mathcal{Z} := [1, +\infty) \times \mathcal{H} \times \mathcal{S}$, with φ denoting the leverage of a representative bank, $h \in \mathcal{H} := \{\underline{h}, \overline{h}\}$ denoting the type of household, where $\underline{h}$ ($\overline{h}$) indexes a household that initially opens an account with the central bank (with the matched banker), and $s \in \mathcal{S} := \{\underline{s}, \overline{s}\}$ denotes the idiosyncratic productivity shock for the matched firm, where $\underline{s}$ ($\overline{s}$) indexes a negative (positive) productivity shock.

In the following subsections, we introduce the optimization problems of the individual agents, outline the optimal choices, and characterize the various equilibria. The proofs relating to the results of the following sections can be found in appendix 6.3.

4.3.3 Households

There is a continuum of households with unit mass. A mass $\mu \in [0, 1]$ of households initially opens an account with the central bank, while the residual mass $1 - \mu$ of households opens an account with private bankers. Each type of household is identical with respect to its behavior on the markets, so that we can focus on a representative household for each type. The household $h \in \mathcal{H}$ maximizes utility, which we assume to be linear and strictly increasing in consumption, and is endowed with capital good $K > 0$, which is sold on the markets to firms at the nominal price $Q > 0$. Depending on where the household initially opens an account—with the central bank or with a private banker—the proceeds from capital good sales, QK, are held as CBDC, denoted by $D^h_{CB} \geq 0$, or as deposits, denoted by $D^h \geq 0$. Based on the previous outline, it holds that the initial allocation across the two monies, CBDC and deposits, satisfies $D^h_{CB} = QK\mathbb{1}\{h = \underline{h}\}$ and $D^h = QK\mathbb{1}\{h = \overline{h}\}$.

Households face a portfolio allocation problem, since deposits with bankers are subject to a potentially stochastic rate of return $r^D_{\mathbf{z}} \geq 0$, while the holdings of CBDC yield a deterministic rate of return $r^D_{CB} > 0$ set by the central bank. Depending on the realized state $\mathbf{z} \in \mathcal{Z}$, each household $h \in \mathcal{H}$ can choose to hold the funds $D^h_{CB} + D^h = QK$ as CBDC, denoted by $D^h_{CB,\mathbf{z}} \geq 0$, or as deposits, denoted by $D^h_{\mathbf{z}} \geq 0$.[8] Households own firms which operate under limited liability and without equity financing, so that households receive firm profits Π^f as dividends.[9] After accounting for taxes and transfers T^h, household $h \in \mathcal{H}$ uses the funds credited with interest $D^h_{CB,\mathbf{z}}(1 + r^D_{CB}) + D^h_{\mathbf{z}}(1 + r^D_{\mathbf{z}})$, and firm profits Π^f,

[8] We assume that a transfer of funds between private bankers is at least as costly as a transfer of funds to the central bank. So, even if we neglect solvency risk, households can never be better off by transferring deposits to another banker rather than to the central bank.

[9] Without macroeconomic risk, the aggregate firm profits do not depend on firm-specific productivity shocks.

to finance the purchase of consumption good $C^h_{\mathbf{z}}$ from firms on the markets at the nominal price $P > 0$.[10] As utility is strictly increasing in consumption, the budget constraint is binding and given by $PC^h_{\mathbf{z}} = D^h_{CB,\mathbf{z}}(1 + r^D_{CB}) + D^h_{\mathbf{z}}(1 + r^D_{\mathbf{z}}) + \Pi^f + T^h$.

Any transfer of funds from a private bank to the central bank, or vice versa, is associated with costs, which, in our model, take the form of a non-monetary utility loss $\nu > 0$.[11] Thus, based on the previous outline, household $h \in \mathcal{H}$ faces for each state $\mathbf{z} \in \mathcal{Z}$ the portfolio allocation problem

$$\max_{D^h_{CB,\mathbf{z}} \geq 0} [D^h_{CB,\mathbf{z}}(1 + r^D_{CB}) + (QK - D^h_{CB,\mathbf{z}})(1 + r^D_{\mathbf{z}})]/P - \nu \mathbb{1}\{D^h_{CB,\mathbf{z}} \neq D^h_{CB}\},$$

A household will only shift funds between a private banker and the central bank if the alternative money yields excess returns leading to a utility gain sufficient to offset the utility loss resulting from the transfer of funds. Thus, the optimal choice of households between deposits and CBDC is of a knife-edge type. If a household is indifferent between deposits and CBDC, we assume that it will stay with its initial choice. The following lemma summarizes the optimal choice for both types of household. In what follows, we use the notation $\tilde{\nu} := \nu/(qK)$, with $q := Q/P$ denoting the capital good price in terms of the consumption good.

Lemma 4.3.1 (Optimal Choice of Households)
$D^{\overline{h}}_{CB,\mathbf{z}} = QK$ *iff* $r^D_{\mathbf{z}} < r^D_{CB} - \tilde{\nu}$, $D^{\underline{h}}_{CB,\mathbf{z}} = QK$ *iff* $r^D_{\mathbf{z}} \leq r^D_{CB} + \tilde{\nu}$, *and* $D^h_{CB,\mathbf{z}} = 0$ *otherwise.*

4.3.4 Firms

Firms operate with identical production functions and exist in a continuum with unit mass, so that we can focus on a representative firm. To produce the consumption good, the firm purchases capital good $K^f \geq 0$ on the markets from households and bankers at the nominal price $Q > 0$. The firm operates without equity financing, relying on external financing in the form of bank loans $L^f = QK^f$ to finance the acquisition of capital good. The loans are subject to repayment determined by the potentially stochastic interest rate $r^L_s > 0$.[12] For

[10]Since there is no aggregate risk, the price of the consumption good will be deterministic.

[11]Such losses can be justified with the effort involved in engineering a transfer, e.g., account opening and closing.

[12]We can assume, without loss of generality, that the interest rate on loans varies at most with the firm's idiosyncratic productivity shock $s \in \mathcal{S}$, i.e., $r^L_{\mathbf{z}} = r^L_s$ for all $\mathbf{z} \in \mathcal{Z}$, as this represents an equilibrium outcome.

convenience, we will occasionally use the notation $R_s^L := 1 + r_s^L$ to represent the interest factor on loans.

We assume that the firm operates with limited liability, so that in the case of default, the matched banker can never seize more than the available production output $Y_s^f = A_s K^f$, where A_s represents the idiosyncratic productivity of the firm. Since $\underline{s}$ ($\overline{s}$) represents a negative (positive) productivity shock, it holds that $A_{\overline{s}} > A_{\underline{s}} \geq 0$. The expected productivity of the firm depends on the monitoring activities $m(h)$ of the matched banker, which may vary with the type of the matched household. We assume that the idiosyncratic productivity shock is distributed with probabilities $\eta_{s|m(h)} \in (0,1)$, where $m(h) \in \{0,1\}$ denotes the monitoring decision of the matched banker, with value zero (one) representing shirking (monitoring). In our model, monitoring increases the likelihood of a positive productivity shock, i.e., it holds that $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0} > 0$. In what follows, we denote the monitoring activities by $\underline{m} := m(\underline{h})$ and $\overline{m} := m(\overline{h})$. To indicate that the firm's expectation depends on the banker's monitoring activities, we index the expectation operator $\mathbb{E}[\cdot]$ by the monitoring decisions $\mathbf{m} := (\underline{m}, \overline{m})$. The production output is sold on the markets to households and bankers at the nominal price $P > 0$. The firm maximizes profits, so that its optimization problem is in real terms given by

$$\max_{K^f \geq 0} \mathbb{E}_{\mathbf{m}}[\max\{A_s - (1 + r_s^L)q, 0\}]K^f.$$

Whenever the firm faces excess returns from production in one of the states, there exists no optimal finite demand for capital good, as the firm's profits will grow with the amount of capital good. The optimal choice of the firm is summarized in the following lemma.

Lemma 4.3.2 (Optimal Choice of the Firm)
$K^f = +\infty$ iff $A_s > (1 + r_s^L)q$ for some $s \in \mathcal{S}$, and $K^f \in [0, +\infty)$ otherwise.

4.3.5 Central bank

The central bank has three instruments for conducting monetary policy: interest rates on reserves and the CBDC, collateral requirements, and penalties for illiquidity on bankers. The details are as follows: The central bank lends reserves used by bankers to hold reserve deposits or to service deposit transfers. Bankers' reserve holdings and the public's CBDC holdings are both credited with the same (deterministic) interest rate $r^D_{CB} > 0$[13], while reserve loans lead to a repayment obligation on the part of bankers that is determined by the (deterministic) interest rate $r^L_{CB} > 0$. In what follows, we will also make use of the notation $R^D_{CB} := 1 + r^D_{CB}$ and $R^L_{CB} := 1 + r^L_{CB}$ to denote the interest factors on reserve deposits/ CBDC and reserve loans, respectively. For simplicity, we assume that the interest rates on reserve deposits, CBDC, and reserve loans equal.[14]

Assumption 4.3.1 (Central Bank Rates)
$r^D_{CB} = r^L_{CB}$.

Deposit transfers are settled using reserves that the individual banker can borrow from the central bank while depositing assets as collateral. The collateral capacity of the individual banker is given by $(1 + \psi)L^b$, where L^b denotes the loans provided by the banker to the matched firm and $\Psi := 1 + \psi \geq 0$ represents the valuation of these loans by the central bank, following from the central bank's choice of the haircut $\psi \geq -1$. For simplicity, we will also refer to Ψ as the haircut on the provided collateral.

Every household can transfer funds from a banker to the central bank, i.e., convert deposits into CBDC at any time, without the consent of the respective banker or the central bank. Thus, the deposit transfers may lead to a banker having liabilities vis-à-vis the central bank that exceed the banker's collateral capacity. If the collateral capacity is insufficient to cover the repayment obligation on the reserve loan $L^b_{CB,\mathbf{z}}$ required to settle deposit transfers, the respective banker will become illiquid and default, in which case the central bank seizes all available assets and imposes a illiquidity penalty on the banker.[15] The latter scales with any outstanding claim of the central bank in excess of the banker's collateral capacity. Specifically, the banker experiences a utility loss in the form of $\phi \max\{(1+r^D_{CB})L^b_{CB,\mathbf{z}} - (1+\psi)L^b, 0\} = \phi \max\{R^D_{CB}L^b_{CB,\mathbf{z}} - \Psi L^b, 0\}$, where $\phi > 0$ represents a scaling parameter also chosen by the central bank.

[13]Introducing two different deposit rates for reserves and the CBDC, while preserving the unrestricted right of converting deposits leads to arbitrage opportunities for bankers.

[14]A spread between central bank rates can be accommodated in our framework and does not alter our results qualitatively, as it generates central bank profits which, assuming a balanced budget, are distributed using transfers to households and bankers before the purchase of the consumption good.

[15]Note that we abstract here from the possibility of interbank borrowing, which constitutes for the individual banker an alternative way of obtaining liquidity. However, in the case of bank insolvency, interbank borrowing is not effective in reducing the respective banker's liabilities vis-à-vis the central bank and the resulting penalties for illiquidity. Thus, integrating an interbank market into our framework does not impair the subsequently illustrated effect of monetary policy on the bankers' monitoring incentives.

As the central bank operates under a balanced budget, its losses are financed through taxes while its profits are distributed by using transfers. In the following, we denote aggregate taxes and transfers in nominal terms by T and nominal central bank profits and losses by Π^{CB}. The assumption of a balanced budget then implies $T = \Pi^{CB}$.

4.3.6 Bankers

There is a continuum of identical bankers with unit mass, so that we can focus on a representative banker. Each banker maximizes utility, which is linear and strictly increasing in consumption, and is endowed with capital good $E > 0$, which is sold on the markets to firms at the nominal price $Q > 0$. The banker can decide whether to open an account with the central bank and hold the proceeds from capital good sales as CBDC or to conduct banking operations with limited liability and financed with bank equity $E^b = QE$. If indifferent, the banker is assumed to engage in banking operations. In this case, the banker supplies loan financing $L^b \geq E^b$ to the matched firm, where loans are, at the outset, completely funded with deposits. As soon as the banker has sold the endowment of capital good and received deposits in return, all funds are used to provide equity financing for the banking operations. Since the amount of equity financing is fixed, the loan supply implicitly determines the leverage $\varphi := L^b/E^b$, which must comply with a regulatory leverage constraint, i.e., $\varphi \leq \varphi^r$, where $\varphi^r \in [1, +\infty)$ represents the regulatory maximum leverage.

The banker can demand reserves $L^b_{CB,\mathbf{z}} \geq 0$ from the central bank, either to hold reserves $D^b_{CB,\mathbf{z}} \geq 0$ with the central bank or to service deposit transfers. Thus, for any state $\mathbf{z} \in \mathcal{Z}$, the balance sheet identity $L^b + D^b_{CB,\mathbf{z}} = D^b_{\mathbf{z}} + L^b_{CB,\mathbf{z}} + E^b$ applies, where $D^b_{\mathbf{z}}$ denotes the total supply of deposits to the matched household. For state $\mathbf{z} \in \mathcal{Z}$, the nominal returns on equity are then given by $R^{E,+}_{\mathbf{z}} E^b$, where we define $R^{E,+}_{\mathbf{z}} := \max\{R^L_s L^b + R^D_{CB} D^b_{CB,\mathbf{z}} - R^D_{\mathbf{z}} D^b_{\mathbf{z}} - R^D_{CB} L^b_{CB,\mathbf{z}}, 0\}/E^b$. If the repayment obligations on reserve loans, $R^D_{CB} L^b_{CB,\mathbf{z}} = (1 + r^D_{CB}) L^b_{CB,\mathbf{z}}$, exceed the collateral capacity, ψL^b, the banker will become illiquid and default, in which case the central bank seizes all available assets and imposes the penalty $R^{E,-}_{\mathbf{z}} E^b$ on the banker, where $R^{E,-}_{\mathbf{z}} := \phi \max\{R^D_{CB} L^b_{CB,\mathbf{z}} - \Psi L^b, 0\}/E^b$. The assumption of competitive markets implies that interest rates must form in such a way that the banker generates no returns in excess of the outside option, i.e., holding CBDC at the central bank, which yields the return $R^D_{CB} E^b$. Arbitrage opportunities on the deposit market, and hence excess returns for the banker, are only ruled out if in each state without default the deposit rate equals the central bank rate, as expressed in the following lemma. Otherwise, the price-taking behavior imposed on the banker is not incentive-compatible.

Lemma 4.3.3 (Deposit Interest Rate)
For any state $\mathbf{z} \in \mathcal{Z}$*, where the banker does not default, it holds that* $r^D_{\mathbf{z}} = r^D_{CB}$.

From lemma 4.3.3, the positive switching costs ($\nu > 0$), and the fact that the deposit rate falls short of the central bank rate if the banker defaults, we know that the household that initially opens an account with the central bank will never transfer funds to a banker. Thus, the banker does not experience any deposit inflows. Using assumption 4.3.1, which states the equality of central bank interest factors, we can then, without loss of generality, assume that the banker does not hold any reserve deposits, i.e., it holds that $D^b_{CB,\mathbf{z}} = 0$.

Furthermore, the matching of one banker and one household enables us to express the demand for reserve loans as $L^b_{CB,\mathbf{z}} = \xi_{\mathbf{z}}(L^b - E^b)$, where $\xi_{\mathbf{z}} = \max\{\xi_h, \xi_{\varphi,s}\}$ represents the bank run indicator, with $\xi_h = \mathbb{1}\{h = \underline{h}\}$ indicating a CBDC-induced bank run and $\xi_{\varphi,s} \in \{0, 1\}$ indicating a run following bank insolvency. From the balance sheet identity, we can then infer that the supply of deposits is given by the residual, i.e., $D^b_{\mathbf{z}} = (1-\xi_{\mathbf{z}})(L^b-E^b)$. The banker faces solvency risk if and only if, in the presence of a negative productivity shock for the financed firm ($s = \underline{s}$), the returns from loan financing, $R^L_{\underline{s}} L^b = (1+r^L_{\underline{s}})L^b$, are not sufficient to cover the liabilities vis-à-vis the matched household as the only potential depositor and the central bank, $R^D_{CB}(L^b - E^b) = (1 + r^D_{CB})(L^b - E^b)$, i.e., for interest rates satisfying $r^D_{CB} > r^L_{\underline{s}}$ the banker is exposed to a solvency risk if and only if it holds that

$$R^L_{\underline{s}}\varphi < R^D_{CB}(\varphi - 1) \qquad \Leftrightarrow \qquad \varphi > \varphi^S := \frac{R^D_{CB}}{R^D_{CB} - R^L_{\underline{s}}} = \frac{1 + r^D_{CB}}{r^D_{CB} - r^L_{\underline{s}}} > 0,$$

where we have used the definition of the leverage $\varphi = L^b/E^b$. For interest rates satisfying $r^D_{CB} \leq r^L_{\underline{s}}$, the banker faces never a solvency risk, so that we define $\varphi^S := +\infty$ if $r^D_{CB} \leq r^L_{\underline{s}}$. Depositors incur switching costs, so bank insolvency does not necessarily trigger a bank run. Instead, the matched household may prefer a bail-in, i.e., stay with the banker and accept a deposit rate that is lower than the central bank rate if the deposit rate is still sufficiently high for the switching costs associated with a transfer of funds to the central bank to lead to a higher utility loss for the household. Specifically, for interest rates satisfying $r^D_{CB} - \tilde{\nu} > r^L_{\underline{s}}$, depositors will shift their funds to the central bank if and only if

$$R^L_{\underline{s}}\varphi < (R^D_{CB} - \tilde{\nu})(\varphi - 1) \qquad \Leftrightarrow \qquad \varphi > \varphi^R := \frac{R^D_{CB} - \tilde{\nu}}{R^D_{CB} - \tilde{\nu} - R^L_{\underline{s}}} = \frac{1 + r^D_{CB} - \tilde{\nu}}{r^D_{CB} - \tilde{\nu} - r^L_{\underline{s}}} > 0.$$

For interest rates satisfying $r^D_{CB} - \tilde{\nu} \leq r^L_{\underline{s}}$, depositors will always accept a bail-in in the case of bank insolvency, so that we define $\varphi^R := +\infty$ if $r^D_{CB} - \tilde{\nu} \leq r^L_{\underline{s}}$. Thus, bank runs due to insolvency occur if and only if the financed firm incurs a negative productivity shock and bank leverage is sufficiently high to incentivize depositors to shift their funds to the central bank, i.e., it holds that $\xi_{\varphi,s} = \mathbb{1}\{s = \underline{s} \wedge \varphi > \varphi^R\}$. Accordingly, we can characterize the bank run indicator as $\xi_{\mathbf{z}} = \mathbb{1}\{h = \underline{h} \vee (s = \underline{s} \wedge \varphi > \varphi^R)\}$. The banker faces liquidity risk if and only if the repayment obligation on central bank loans, $R^D_{CB}L^b_{CB,\mathbf{z}}$, exceeds the collateral capacity, ΨL^b, determined by the central bank, i.e., if and only if it holds that

$$\Psi\varphi < R^D_{CB}(\varphi - 1) \quad \Leftrightarrow \quad \varphi > \varphi^L := \frac{R^D_{CB}}{R^D_{CB} - \Psi} = \frac{1 + r^D_{CB}}{r^D_{CB} - \psi} > 0,$$

where we have used the definition of the leverage $\varphi = L^b/E^b$ and the fact that $L^b_{CB,\mathbf{z}} = \xi_{\mathbf{z}}(L^b - E^b)$, with $\xi_{\mathbf{z}} \in \{0, 1\}$. For monetary policy instruments satisfying $R^D_{CB} \leq \Psi$ or, equivalently, $r^D_{CB} \leq \psi$, the banker can never become illiquid and thus we define $\varphi^L := +\infty$ if $r^D_{CB} \leq \psi$.

Using the previous results, we can further characterize equity returns and illiquidity penalties as $R^{E,+}_{\mathbf{z}} = \max\{(R^L_s - R^D_{CB})\varphi + R^D_{CB}, 0\}$ and $R^{E,-}_{\mathbf{z}} = \phi\max\{(\xi_{\mathbf{z}}R^D_{CB} - \Psi)\varphi + \xi_{\mathbf{z}}R^D_{CB}\}$, respectively. Note that the banker is only exposed to equity returns if the banker is not facing illiquidity, i.e., if there is no CBDC-induced bank run ($h = \overline{h}$) or there is a CBDC-induced bank run but the leverage is not sufficiently high to cause liquidity risk ($h = \underline{h}$ and $\varphi \leq \varphi^L$). The banker also decides on the monitoring activities $m(h) \in \{0, 1\}$, which may depend on the occurrence of a CBDC-induced bank run or, equivalently, may vary with the type of matched household $h \in \mathcal{H}$. To indicate that the banker's expectation depends on the monitoring activities, we index the expectation operator $\mathbb{E}[\cdot]$ by the monitoring decisions $\mathbf{m} := (\underline{m}, \overline{m})$, where we again use $\underline{m} := m(\underline{h})$ and $\overline{m} := m(\overline{h})$ as a shortcut. Monitoring requires effort on the part of the banker, which, in our model, takes the form of a non-monetary utility loss $\kappa > 0$ that scales with the amount of loan financing L^b.[16]

[16] The assumption that monitoring efforts scale with the amount of loan financing is technical in nature, as it simplifies the analysis of the banker's optimization problem.

The banker uses the returns on equity, $\zeta_{\mathbf{z}} R_{\mathbf{z}}^{E,+} E^b$, with $\zeta_{\mathbf{z}} = 1 - \mathbb{1}\{h = \underline{h} \wedge \varphi > \varphi^L\}$ being the liquidity indicator, to finance the purchase of consumption good $C_{\mathbf{z}}^b$ on the markets from firms at the nominal price $P > 0$. As utility is strictly increasing in consumption, the budget constraint is binding and given by $PC_{\mathbf{z}}^b = \zeta_{\mathbf{z}} R_{\mathbf{z}}^{E,+} E^b$. The optimization problem of the banker in real terms is then given by

$$\max_{\substack{\varphi \in [1,\varphi^r], \\ m(h) \in \{0,1\}}} \mathbb{E}_{\mathbf{m}}[\zeta_{\mathbf{z}} R_{\mathbf{z}}^{E,+} - R_{\mathbf{z}}^{E,-} - m(h)\kappa\varphi]qE,$$

where we used $E^b = QE$ and applied again the notation $q = Q/P$ to represent the capital good price in terms of the consumption good. The following lemma summarizes the banker's optimal choice in equilibrium, where bankers make in expectation zero excess returns from conducting banking operations, compared to their outside option of simply holding CBDC.

Lemma 4.3.4 (Optimal Choice of the Banker)
The banker chooses leverage φ and monitoring activities $m(h)$, with $h \in \mathcal{H}$, such that

(i) $1 \leq \varphi \leq \min\{\varphi^L, \varphi^S, \varphi^r\}$ *iff*

$$\mathbb{E}_{\mathbf{m}}[R_s^L] = R_{CB}^D + \overline{m}\kappa,$$

where $\underline{m} = \overline{m} = \mathbb{1}\{\triangle(R_{\overline{s}}^L - R_{\underline{s}}^L) \geq \kappa\}$,

(ii) $\varphi^L < \varphi = \varphi^r \leq \varphi^S$ *iff* $\phi \in (0,1]$ *and*

$$\mathbb{E}_{\overline{m}}[R_s^L | h = \overline{h}] = R_{CB}^D \left(1 + \frac{\mu}{1-\mu}\frac{1}{\varphi^r}\right) + \frac{\mu\phi}{1-\mu}\left(R_{CB}^D \frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \overline{m}\kappa,$$

where $\underline{m} = 0$ *and* $\overline{m} = \mathbb{1}\{\triangle(R_{\overline{s}}^L - R_{\underline{s}}^L) \geq \kappa\}$,

(iii) $\varphi^S < \varphi = \varphi^r \leq \varphi^L$ *iff*

$$R_{\overline{s}}^L = R_{CB}^D \left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}},$$

where $\underline{m} = \overline{m} = \mathbb{1}\{\triangle[R_{\overline{s}}^L - R_{CB}^D(\varphi^r - 1)/\varphi^r] \geq \kappa\}$, *(iv)* $\max\{\varphi^S, \varphi^L\} < \varphi = \varphi^r \leq \varphi^R$ *iff* $\mu\phi \leq \mu + (1-\mu)\eta_{\underline{s}|\overline{m}}$ *and*

$$R^L_{\overline{s}} = R^D_{CB}\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{\mu\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(R^D_{CB}\frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}},$$

with $\underline{m} = 0$ *and* $\overline{m} = \mathbb{1}\{\triangle[R^L_{\overline{s}} - R^D_{CB}(\varphi^r - 1)/\varphi^r] \geq \kappa\}$,

(v) $\max\{\varphi^S, \varphi^L, \varphi^R\} < \varphi = \varphi^r$ *iff* $\phi \in (0, 1]$ *and*

$$R^L_{\overline{s}} = R^D_{CB}\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(R^D_{CB}\frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}},$$

where $\underline{m} = 0$ *and* $\overline{m} = \mathbb{1}\{\triangle[R^L_{\overline{s}} - \phi\Psi - R^D_{CB}(1-\phi)(\varphi^r - 1)/\varphi^r] \geq \kappa\}$.

Note that the loan rates required to incentivize the banker to shoulder on liquidity risk are increasing in the mass of households initially opening an account with the central bank, denoted by μ, and the illiquidity penalty per unit of supplied loan financing, denoted by $\phi\epsilon/\varphi$, where $\epsilon := R^D_{CB}(\varphi - 1) - \Psi\varphi$. Thus, the higher the risk of illiquidity and the higher the utility loss due to penalties in the case of illiquidity, the higher the returns required from loan financing. Similarly, in the presence of solvency risk, the loan rates decrease with the probability of a positive productivity shock, denoted by $\eta_{\overline{s}|\overline{m}}$. Thus, the lower the probability of bank insolvency, the lower the returns from loan financing required to incentivize the banker to shoulder on solvency risk.

4.4 Equilibrium Analysis

4.4.1 Equilibrium definition

In our subsequent analysis we focus on competitive equilibria, which are introduced hereafter. In what follows, we denote the expected consumption of the banker and the household using $C^b := \mathbb{E}_{\mathbf{m}}[C^b_{\mathbf{z}}]$ and $C^h := \mathbb{E}_{\mathbf{m}}[C^h_{\mathbf{z}}]$, with $h \in \mathcal{H}$, respectively. Note that expectations are taken at the first stage, when monetary policy has been decided on and all interest rates are known. Due to the law of large numbers, the aggregate consumption of bankers and households is then given by C^b and $(1-\mu)C^{\overline{h}} + \mu C^{\underline{h}}$, respectively. Also due to the law of large numbers, aggregate production equals expected production, denoted by $Y^f := \mathbb{E}_{\mathbf{m}}[A_s]K^f$, and aggregate firm profits equal expected firm profits, denoted in real terms by $\pi^f := \mathbb{E}_{\mathbf{m}}[\max\{A_s - (1 + r^L_s)q, 0\}]K^f$.

Definition 4.4.1 (Competitive Equilibrium)
Given a monetary policy r^D_{CB}, ψ and ϕ, a competitive equilibrium is a set of interest factors $\{r^D_{\mathbf{z}}, r^L_s\}_{\mathbf{z}\in\mathcal{Z}}$, prices $\{Q, P\}$, and choices $\{D^h_{CB,\mathbf{z}}\}_{\mathbf{z}\in\mathcal{Z}}$, K^f, φ and $m(h)$, with $h \in \mathcal{H}$, such that

(i) given $\{r^D_{CB}, r^D_{\mathbf{z}}, Q, P\}_{\mathbf{z}\in\mathcal{Z}}$, choices $\{D^h_{CB,\mathbf{z}}\}_{\mathbf{z}\in\mathcal{Z}}$ maximize the utility of the household $h \in \mathcal{H}$,

(ii) given $\{r^L_s, Q, P\}_{s\in\mathcal{S}}$, choice K^f maximizes the profits of the firm,

(iii) given $\{r^D_{CB}, r^D_{\mathbf{z}}, r^L_s, \psi, \phi, Q, P\}_{\mathbf{z}\in\mathcal{Z}}$, choices φ and $m(h)$, with $h \in \mathcal{H}$, maximize the utility of the banker,

(iv) the good markets clear, i.e., $K^f = K + E$ and $Y^f = C^b + (1-\mu)C^{\overline{h}} + \mu C^{\underline{h}}$, and

(v) the asset markets clear, i.e., $L^b = L^f$.

Note that the asset markets for deposits, CBDC, equity, and reserves clear by construction of the model. Thus, when analyzing competitive equilibria, we only have to take the clearing of the markets for loans, capital good, and consumption good into account.

4.4.2 Equilibrium properties

First we highlight some general properties of all competitive equilibria in our framework and then proceed to a characterization of the various possible equilibria that differ in terms of bankers' risk exposure. Since in equilibrium the market for capital good clears, i.e., $K^f = K + E$, loan demand is determined and given by $L^f = Q(K + E)$. Loan supply then follows from the clearing of the loan market, i.e., $L^b = L^f$. As the bankers' equity financing is fixed, $E^b = QE$, the equilibrium leverage is given by $\varphi = (K + E)/E$. The regulatory leverage constraint must thus satisfy $\varphi^r \geq (K + E)/E$. From lemma 4.3.4 we know that in any environment where bankers are exposed to risk, bankers will choose $\varphi = \varphi^r$, so that the latter inequality must be binding in equilibrium, i.e., $\varphi^r = (K+E)/E$.

In equilibrium, deposit rates never exceed the central bank rate. Specifically, as stated in lemma 4.3.3, the deposit rate paid by the banker without default equals the central bank rate and will only fall short of the central bank rate if the banker defaults due to illiquidity or insolvency. Thus, due to positive switching costs, households that have initially opened an account with the central bank will never transfer their funds to a private banker. Households that have initially opened an account with a banker will transfer their funds to the central bank if and only if the respective banker defaults due to insolvency. However, one remark is in order: If an insolvent banker can pay a deposit rate that is sufficiently high for the switching costs related to a deposit transfer to lead to a higher utility loss for the matched household, depositors will accept a bail-in in the case of bank insolvency. This specific case arises if bank leverage is sufficiently low, i.e., $\varphi \leq \varphi^R$. CBDC holdings thus satisfy $D^h_{CB,\mathbf{z}} = \xi_{\mathbf{z}} QK$, where $\xi_{\mathbf{z}} = \mathbb{1}\{h = \underline{h} \vee (s = \underline{s} \wedge \varphi > \varphi^R)\}$ is the measure of households. Deposit holdings then represent the residual, i.e., $D^h_{\mathbf{z}} = (1 - \xi_{\mathbf{z}})QK$.

We have specified all equilibrium choices except the banker's monitoring decision, which is described when we characterize the various possible equilibria in our framework. For each equilibrium we provide existence conditions and utilitarian welfare, which, based on our assumption of linear utility, comprises aggregate consumption, utility losses due to monitoring, illiquidity penalties, and switching costs emerging from deposit transfers. The prices and interest rates prevailing in each equilibrium can be found in the respective proof (see appendix 6.3).

Note that, in the presence of liquidity risk, the banker's optimality condition relates the central bank interest rate r^D_{CB} to the real haircut ψ. We can interpret monetary policy then as setting the central bank interest rate or, alternatively, as setting the haircut. In the following, we adopt the latter view, so that various equilibrium conditions include the haircut, which is at the discretion of the central bank.

First consider the situation where bankers face neither a liquidity risk nor a solvency risk. Banking operations, and hence deposits, are safe, so there are no illiquidity penalties for bankers and no deposit transfers. Then welfare simply comprises aggregate consumption and potential utility losses due to monitoring. The banker will monitor if and only if the expected productivity gain exceeds the utility loss due to monitoring. Given the financing of banking operations via external funds, such an equilibrium without risk exists if and only if the collateral requirements of the central bank are sufficiently loose, thus not exposing bankers to liqudity risk, and the productivity losses induced by a negative productivity shock are sufficiently small, thus ruling out solvency risk.

Proposition 4.4.1 (Equilibrium without Risk)
There exists a unique equilibrium without risk iff

$$\frac{\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa q}{1 + E/K} \leq \min\{\Psi q, A_{\underline{s}}\},$$

and it yields welfare $W = (\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa q)(K + E)$, *where the monitoring decision is given by* $\underline{m} = \overline{m} = \mathbb{1}\{\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa q\}$.

Note that, in general, prices follow from the banker's optimality condition, provided in lemma 4.3.4. For example, if bankers face neither a liquidity risk nor a solvency risk, i.e., $1 \leq \varphi \leq \min\{\varphi^L, \varphi^S, \varphi^r\}$, it holds that $\mathbb{E}_{\mathbf{m}}[R^L_s] = R^D_{CB} + \overline{m}\kappa$, where $\underline{m} = \overline{m} = \mathbb{1}\{\triangle(R^L_{\overline{s}} - R^L_{\underline{s}}) \geq \kappa\}$. As shown in the proof of proposition 4.4.1 (see appendix 6.3), in equilibrium loan rates are linked to firm productivity, i.e., $A_s = (1 + r^L_s)q = R^L_s q$, with $s \in \mathcal{S}$. Hence, the banker's optimality condition in nominal terms reads as

$$\mathbb{E}_{\mathbf{m}}[A_s] = (R^D_{CB} + \overline{m}\kappa)q \qquad \Leftrightarrow \qquad \frac{P}{Q} = \frac{R^D_{CB} + \overline{m}\kappa}{\mathbb{E}_{\mathbf{m}}[A_s]} = \frac{1 + r^D_{CB} + \overline{m}\kappa}{\mathbb{E}_{\mathbf{m}}[A_s]},$$

which fully characterizes the prices in our economy. Thus, given a capital good price Q, the consumption good price P is increasing in the central bank interest rate r^D_{CB}, the monitoring efforts κ (if bankers monitor, i.e., if $\overline{m} = 1$), and decreasing in aggregate productivity $\mathbb{E}_{\mathbf{m}}[A_s]$. For the following cases, in which bankers face risk, the price relationships can be derived by the same procedure. In any situation where bankers face liquidity risk, illiquidity penalties and hence the haircut ψ chosen by the central bank will influence the prices in the economy, too.

Second, consider the situation where bankers face a liquidity risk but no solvency risk. Thus, the central bank will adopt tight collateral requirements, so that, in the case of a CBDC-induced bank run, bankers will face a repayment obligation towards the central bank that exceeds their collateral capacity. However, productivity shocks are moderate in this situation, so that bankers not facing a CBDC-induced bank run will remain solvent, even if productivity is low. As in any equilibrium without risk, liquid bankers will monitor if and only if the expected productivity gain exceeds the utility loss due to monitoring. The assets of bankers who become illiquid and default are seized by the central bank, so the respective bankers have no incentive to monitor, ultimately lowering aggregate production output over and against the equilibrium without risk. As liquid banks face no solvency risk, depositors have no incentive to transfer funds to the central bank, so there are no switching costs. Welfare thus comprises aggregate consumption, utility losses due to monitoring, and illiquidity penalties.

Proposition 4.4.2 (Equilibrium with Liquidity Risk)
There exists a unique equilibrium with liquidity risk iff $\varphi^r = (K+E)/E$, $\phi \in (0,1)$ *and*

$$\Psi q < \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa q)}{1-\mu+E/K} \leq A_{\underline{s}},$$

and it yields welfare $W^L = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa q - \mu\phi\epsilon\}(K+E)$, *where the monitoring decision is given by* $\underline{m} = 0$ *and* $\overline{m} = \mathbb{1}\{\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa q\}$ *and it holds that*

$$\epsilon = \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa q) - \Psi q(1-\mu+E/K)}{1-\mu+\mu\phi+E/K}.$$

Third, consider the situation where bankers face a solvency risk but no liquidity risk. No bank run due to either a household preferring CBDC to deposits or bank insolvency will lead to a illiquidity penalty for the banker, because the collateral capacity determined by the central bank suffices to cover any liability towards the central bank emerging from deposit transfers. Solvency risk arises when the productivity losses due to a negative productivity shock are sufficiently large for the revenues from loan financing to be insufficient to meet the liabilities towards the matched household or the central bank. A CBDC-induced bank run does not alter the size of bank liabilities, as the deposit rate and the central bank rate equal without bank default. Hence, the banker's monitoring decision is independent of the type of matched household, or equivalently, the occurrence of a CBDC-induced bank run. Households possessing deposits with insolvent bankers will only transfer their funds to the central bank if a bail-in leads to a higher utility loss than transferring their funds to the central bank and incurring switching costs. Hence, whenever the switching costs ν are lower than a critical level ν^*, households possessing deposits with insolvent bankers will transfer their funds to the central bank. Since bankers face no illiquidity, utilitarian welfare comprises aggregate consumption, utility losses on the part of bankers due to monitoring, and switching costs on the part of depositors.

Proposition 4.4.3 (Equilibrium with Solvency Risk)
There exists a unique equilibrium with solvency risk iff $\varphi^r = (K + E)/E$ *and*

$$A_{\underline{s}} < \frac{\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q}{\eta_{\overline{s}|\overline{m}} + E/K} \leq \Psi q$$

and it yields welfare $W^S = (\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa q)(K + E) - (1 - \mu)\eta_{\underline{s}|\overline{m}}\nu\mathbb{1}\{\nu < \nu^*\}$, *where the monitoring decision is given by* $\underline{m} = \overline{m} = \mathbb{1}\{\triangle A_{\overline{s}} \geq \kappa q(1 + \eta_{\overline{s}|0}K/E)\}$ *and the critical switching cost level satisfies*

$$\nu^* = \left(\frac{\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q}{\eta_{\overline{s}|\overline{m}} + E/K} - A_{\underline{s}}\right)(K + E).$$

Finally, we consider the situation where bankers face both liquidity risk and solvency risk. Negative productivity shocks lead to low loan repayments, which are insufficient for the banker to meet the obligations vis-à-vis the matched household or the central bank. In addition, the central bank imposes tight collateral requirements, such that, if the banker is exposed to a bank run, the resulting liability towards the central bank will exceed collateral capacity. Bankers then default due either to illiquidity or insolvency. Due to switching costs on the part of depositors, bank insolvency does not necessarily trigger a bank run. Only if the switching costs are sufficiently low will households possessing accounts with insolvent bankers shift their funds to the central bank. Thus if switching costs are sufficiently low the banker will incur the same illiquidity penalty in the case of insolvency as in the case of illiquidity.

Compared to the situation where bankers face only a solvency risk, the mass of defaulting bankers will increase due to illiquidity after a CBDC-induced bank run. As illiquid bankers do not monitor, the mass of bankers potentially monitoring will decrease over and against the situation where bankers only face a solvency risk. With tight collateral requirements, bankers will face not only a liquidity risk but also illiquidity penalties that bankers incur in the case of illiquidity or insolvency. While bankers cannot influence the likelihood of a CBDC-induced bank run, they can monitor borrowers in order to increase the likelihood of a positive productivity shock and ultimately decrease the likelihood of bank insolvency. If depositors switch to the central bank in the case of bank insolvency, monitoring will decrease the expected illiquidity penalties. Thus, tight collateral requirements can incentivize bankers to start monitoring.

The following proposition characterizes the equilibrium with both liquidity risk and solvency risk, where depositors accept a bail-in if the respective banker becomes insolvent. Thus, tight collateral requirements do not lead to illiquidity penalties in the case of bank insolvency and therefore only indirectly affect the monitoring incentives, as penalties for illiquidity also influence prices in the economy. Utilitarian welfare comprises aggregate consumption, potential utility losses due to monitoring, and illiquidity penalties, but not switching costs.

Proposition 4.4.4 (Equilibrium with Liquidity and Solvency Risk, and Bail-in) *There exists a unique equilibrium with liquidity and solvency risk and with bail-in iff* $\varphi^r = (K+E)/E$, $\mu\phi < \mu + (1-\mu)\eta_{\underline{s}|\overline{m}}$, *and*

$$\max\left\{A_{\underline{s}} + \frac{\mu\phi(A_{\underline{s}} - \Psi q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}, \Psi q\right\} < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}$$

$$\leq A_{\underline{s}} + \frac{\nu}{K+E} + \frac{\mu\phi(A_{\underline{s}} + \nu/(K+E) - \Psi q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}$$

and it yields welfare $W_B^{LS} = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa q - \mu\phi\epsilon\}(K+E)$, *where the monitoring decision is given by* $\underline{m} = 0$ *and* $\overline{m} = 1$ *iff*

$$\triangle A_{\overline{s}} - \frac{\triangle\mu\phi\Psi q}{\mu\phi + E/K} \geq \kappa q\left[1 + \frac{(1-\mu)\eta_{\overline{s}|0}}{\mu\phi + E/K}\right],$$

and it holds that

$$\epsilon = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q) - \Psi q[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi + E/K}.$$

The following proposition describes the equilibrium with liquidity risk and solvency risk and with deposit transfers of households in the case of bank insolvency. Thus, switching costs are sufficiently low for depositors to prefer switching to the central bank rather than keeping deposits with an insolvent banker and accepting a bail-in. As a consequence, tight collateral requirements lead to illiquidity penalties in the case of bank insolvency and directly affect the monitoring incentives of bankers. The monitoring decision depends, as before, on the expected productivity gain and the utility losses due to monitoring, but now also include the expected reduction of illiquidity penalties, as monitoring reduces the likelihood of bank insolvency. Utilitarian welfare comprises aggregate consumption, potential utility losses on the part of bankers due to monitoring and penalties for illiquidity, and switching costs on the part of depositors.

Proposition 4.4.5 (Equilibrium with Liquidity and Solvency Risk and no Bail-in) *There exists a unique equilibrium with liquidity and solvency risk and without bail-in iff* $\varphi^r = (K+E)/E$, $\phi \in (0,1)$ *and*

$$\max\left\{A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi(A_{\underline{s}} + \frac{\nu}{K+E} - \Psi q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}, \Psi q\right\} < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}$$

and it yields welfare $W_{NB}^{LS} = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa q - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\epsilon\}(K+E) - (1-\mu)\eta_{\underline{s}|\overline{m}}\nu$*, where the monitoring decision is given by* $\underline{m} = 0$ *and* $\overline{m} = 1$ *iff*

$$\triangle A_{\overline{s}} - \triangle\phi\Psi q\frac{1+E/K}{\phi+E/K} \geq \kappa q\left[1 + \frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi+E/K}\right],$$

and it holds that

$$\epsilon = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q) - \Psi q[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + E/K}.$$

4.5 Optimal Monetary Policy

The central bank aims at maximizing utilitarian welfare in the economy by setting the central bank rate $r_{CB}^D > 0$, the haircut $\psi \geq 0$, and the illiquidity parameter $\phi > 0$. The central bank determines the collateral capacity of bankers and thus decides on bankers' exposure to liquidity risk and illiquidity penalties. As the latter can influence bankers' monitoring decisions, the central bank can use its collateral framework to improve monitoring activities in the economy.

4.5.1 Necessary conditions for tight collateral requirements

In our model, bankers are exposed to two types of risk. Bankers may experience (a) a CBDC-induced bank run leading to illiquidity if the central bank sets tight collateral requirements, and (b) low loan repayment as a consequence of a negative productivity shock for the financed firm leading to insolvency if leverage is sufficiently high. The monitoring of bankers can only influence the likelihood of a positive productivity shock for the financed firm, but not the likelihood of a CBDC-induced bank run. Thus, penalties for illiquidity can only influence bankers' monitoring decisions if there is a solvency risk. If, independently of the productivity shock for the financed firm, a banker is able to service the liabilities towards the matched household or the central bank it will never be optimal to apply tight collateral requirements. In such a case, tight collateral requirements would expose bankers to illiquidity penalties and potentially even reduce monitoring activities if some banks become illiquid, but they would have no positive effects. Hence, without solvency risk, tight collateral requirements lead to welfare loss due to illiquidity penalties for bankers and, if liquid bankers monitor, due to lower aggregate production.

But with solvency risk and sufficiently low switching costs, tight collateral requirements can improve bankers' monitoring activities, as bank insolvency triggers a bank run and ultimately exposes the respective banker to a illiquidity penalty. The likelihood of insolvency induced by a negative productivity shock for the financed firm can be reduced through monitoring. Thus, tight collateral requirements prevent bankers from shirking. However, since tight collateral requirements lead to illiquidity following a CBDC-induced bank run, there are also negative consequences, particularly utility losses on the part of bankers due to illiquidity penalties. On that account, tight collateral requirements are only optimal if the aggregate productivity gains resulting from the improved monitoring activities of liquid bankers are sufficient to offset the monitoring efforts and the penalties for illiquid bankers, as stated in the following proposition.

Proposition 4.5.1 (Optimal Monetary Policy)
Tight collateral requirements, i.e., $(1 + r^D_{CB})K > (1 + \psi)(K + E) \geq 0$ *and* $\phi > 0$*, are optimal, if bankers shirk with loose collateral requirements, i.e.,* $\triangle A_{\overline{s}} < \kappa q(1 + \eta_{\overline{s}|0} K/E)$*, if tight collateral requirements incentivize bankers to monitor, i.e., there exists* $r^D_{CB} > 0$*,* $\psi \geq 0$ *and* $\phi \in (0, 1)$*, such that*

$$\triangle A_{\overline{s}} - \triangle \phi (1 + \psi) q \frac{1 + E/K}{\phi + E/K} \geq \kappa q \left[1 + \frac{(1 - \phi)(1 - \mu)\eta_{\overline{s}|0}}{\phi + E/K} \right],$$

if banking with liquidity risk and solvency risk is viable (left hand side of inequality) and if implementing tight collateral requirements is welfare improving (right hand side of inequal-

ity), i.e.,

$$\underline{\chi}(\phi,\psi) < \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} < \overline{\chi}(\phi,\psi),$$

where

$$\underline{\chi}(\phi,\psi) := \max\left\{A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\underline{s}|1}+\mu]\phi[A_{\underline{s}} + \nu/(K+E) - (1+\psi)q]}{(1-\mu)\eta_{\overline{s}|1} + E/K}, (1+\psi)q\right\}$$

and

$$\overline{\chi}(\phi,\psi) := \Psi q + \{(1-\mu)[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa q - (\eta_{\underline{s}|1} - \eta_{\underline{s}|0}\mathbb{1}\{\nu < \nu^*\})\nu/(K+E)]\}$$
$$\times \{[(1-\mu)\eta_{\overline{s}|1} + E/K]^{-1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]^{-1}\phi^{-1}\},$$

with the critical switching cost level $\nu^* = [\eta_{\overline{s}|0}A_{\overline{s}}/(\eta_{\overline{s}|0} + E/K) - A_{\underline{s}}](K+E)$*. Otherwise, loose collateral requirements are optimal, i.e.,* $(1+\psi)(K+E) \geq (1+r^D_{CB})K > 0$ *and* $\phi > 0$*.*

4.5.2 The central bank's optimization problem

In our model, the central bank has to choose between loose and tight collateral requirements. As shown in proposition 4.5.1, the central bank will only choose tight collateral requirements exposing bankers to liquidity risk and illiquidity penalties if bankers' monitoring incentives can be improved, banking with liquidity risk and solvency risk is viable, and tight collateral requirements are welfare-improving, i.e., productivity gains following from bankers' improved monitoring incentives offset monitoring effort and illiquidity penalties. We can show that when tight collateral requirements are optimal, choosing optimal monetary policy essentially boils down to choosing the penalty parameter ϕ. The nominal central bank rate R^D_{CB} and the nominal haircut Ψ, both influencing the prices in the economy, are then chosen to ensure that the real haircut ψ satisfies a pre-specified condition, which itself varies with the illiquidity penalty parameter ϕ. The details are summarized in the following lemma, which follows directly from proposition 4.5.1.

Lemma 4.5.1 (Optimal Monetary Policy)
If tight collateral requirements are optimal (see proposition 4.5.1), the optimal penalty parameter $\hat{\phi}$ *satisfies*

$$\hat{\phi} \in \arg\min_{\phi \in (0,1)} \phi \tilde{\epsilon}(\phi) \quad \textit{subject to} \quad \max\{\underline{\gamma}_1(\phi), \underline{\gamma}_2\} \leq \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\},$$

where

$$\tilde{\epsilon}(\phi) = \frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa q) - \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\}[(1-\mu)\eta_{\overline{s}|1} + E/K]}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K},$$

$$\underline{\gamma}_1(\phi) = A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\overline{s}|1} + E/K][A_{\underline{s}} + \nu/(K+E)] - (1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa q)}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi}, \underline{\gamma}_2 = 0,$$

$$\overline{\gamma}_1 = \frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} \quad \textit{and} \quad \overline{\gamma}_2(\phi) = \frac{\phi + E/K}{1 + E/K}\left[\frac{A_{\overline{s}}}{\phi} - \frac{\kappa q}{\triangle \phi}\left(1 + \frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi + E/K}\right)\right].$$

The optimal central bank rate $\hat{r}^D_{CB} > 0$ *and the optimal haircut* $\hat{\psi} \geq 0$ *satisfy*

$$(1 + \hat{r}^D_{CB})K > (1 + \hat{\psi})(K + E) \geq 0 \quad \textit{and} \quad (1 + \hat{\psi})q = \min\{\overline{\gamma}_1, \overline{\gamma}_2(\hat{\phi})\}.$$

4.5.3 An explicit solution

In the following, we provide sufficient conditions for tight collateral requirements and characterize optimal monetary policy explicitly in the case where negative productivity shocks are extreme and there are no switching costs. Most notably, in such an environment the central bank will choose optimally any positive nominal central bank rate while setting the nominal haircut to zero and using the illiquidity penalty parameter to incentivize bankers to monitor. The following corollary provides the details.

Corollary 4.5.1 (Optimal Monetary Policy)
Suppose $\kappa q < \triangle A_{\overline{s}} < \kappa q(1 + \eta_{\overline{s}|0} K/E)$. *Then, if negative productivity shocks are extreme, i.e.,* $A_{\underline{s}} = 0$, *if there are no switching costs, i.e.,* $\nu = 0$, *and if the risk exposure of bankers*

is low, i.e., $\mu > 0$ *and* $\eta_{\underline{s}|1} > 0$ *are sufficiently small, such that* $\mu\eta_{\overline{s}|1} < \eta_{\overline{s}|0}$ *and*

$$\frac{(1-\mu)\eta_{\overline{s}|0}A_{\overline{s}}}{(1-\mu)\eta_{\overline{s}|1} + E/K} < \frac{(1-\mu)(\triangle A_{\overline{s}} - \kappa q)}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\hat{\phi}},$$

then tight collateral requirements are optimal, and the optimal monetary policy satisfies

$$\hat{r}^D_{CB} > 0, \quad \hat{\psi} = 0, \quad \text{and} \quad \hat{\phi} = \frac{\kappa q(1-\mu)\eta_{\overline{s}|0} - (\triangle A_{\overline{s}} - \kappa q)E/K}{\triangle A_{\overline{s}} - \kappa q[(1-\mu)\eta_{\underline{s}|0} + \mu]}.$$

Note that, with the imposed condition $\triangle A_{\overline{s}} > \kappa q$, the optimal illiquidity penalty parameter decreases with the equity-to-deposits ratio E/K. The higher the equity financing of banks, the more returns from monitoring can be skimmed by bankers and, thus, the higher are the incentives of bankers to monitor. Hence, bankers are bound to be less incentivized through the use of penalties for illiquidity. The optimal penalty parameter $\hat{\phi}$ increases with the probability of a positive productivity shock without monitoring, denoted by $\eta_{\overline{s}|0}$. Clearly, the higher the probability of a positive productivity shock without monitoring, the lower the returns from monitoring and, hence, the lower the incentives for bankers to engage in costly monitoring. As a consequence, illiquidity penalties required to incentivize bankers to monitor must increase.

4.5.4 Social planner solutions

As outlined before, through the use of the collateral framework, the central bank can under certain conditions incentivize bankers to monitor and thereby induce a welfare gain. However, it needs to be clarified how utilitarian welfare in a competitive equilibrium with an optimal monetary policy compares to the first-best (second-best) utilitarian welfare achieved by a unconstrained (constrained) social planner. The unconstrained social planner has complete information about agents' activities, and can reallocate the endowments of capital good among households and bankers as well as impose (distribute) taxes (transfers) contingent on macroeconomic and idiosyncratic states. The constrained social planner, in turn, has incomplete information about agents' activities and cannot observe bankers' monitoring activities. The constrained social planner can only impose (distribute) taxes (transfers) contingent on macroeconomic and idiosyncratic states but not reallocate the endowments of the capital good. The first-best and second-best welfare are analyzed in the presence of loose collateral requirements, i.e., bankers face no liquidity risk and illiquidity penalties. Note that welfare in such an environment is maximized if households do not incur switching costs and the welfare gain due to the productivity increase induced by monitoring offsets bankers' utility losses due to monitoring.

When bankers face no risk, households do not incur switching costs and bankers' monitoring decision maximizes welfare, i.e., bankers monitor if and only if the welfare gain due to the productivity increase induced by monitoring $\triangle(A_{\overline{s}} - A_{\underline{s}})(K+E)$ offsets the utility losses due to monitoring $\kappa q(K+E)$ (see proposition 4.4.1). Thus any competitive equilibrium without a risk for bankers yields the first-best welfare. From proposition 4.4.3 it follows that also any competitive equilibrium with a solvency risk but no liquidity risk for bankers yields the first-best welfare if households accept a bail-in the case of bank insolvency, i.e., switching costs are sufficiently high so that $\nu \geq \nu^*$, with ν^* provided in proposition 4.4.3, and bankers' monitoring decision is welfare-maximizing, i.e., $\triangle A_{\overline{s}} \geq \kappa q(1 + \eta_{\underline{s}|0} K/E)$ if and only if $\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa q$.

Proposition 4.5.2 (Competitive Equilibrium without Liquidity Risk)
When bankers face no risk, the competitive equilibrium yields the first-best welfare. Households do not incur switching costs and bankers' monitoring decision maximizes welfare, i.e., bankers monitor if the welfare gain due to the productivity increase induced by monitoring offsets bankers' utility losses due to monitoring, i.e., $\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa q$.

When bankers face a solvency risk, the competitive equilibrium yields the first-best welfare if (a) depositors accept a bail-in in the case of bank insolvency, i.e., switching costs are sufficiently high so that $\nu \geq \nu^*$*, with* ν^* *provided in proposition 4.4.3, and (b) monitoring by bankers maximizes welfare, i.e.,* $\triangle A_{\overline{s}} \geq \kappa q(1+\eta_{\underline{s}|0} K/E)$ *if and only if* $\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa q$.

Note that when bankers face no risk, the competitive equilibrium yields the first-best welfare as there are no switching costs incurred by households and bankers' monitoring decision maximizes welfare. Thus the unconstrained social planner can always achieve the first-best welfare by reallocating households' and bankers' endowments of the capital good, so that bankers face no solvency risk.

Proposition 4.5.3 (Unconstrained Social Planner Solution)
The social planner can always achieve the first-best welfare by reallocating the capital good between households and bankers, so that bankers are not exposed to a solvency risk.

We now turn to the equilibrium implemented by a constrained social planner which has perfect but incomplete information about agents' activities and in particular cannot observe bankers' monitoring activities. In contrast to the unconstrained social planner, the constrained social planner can only impose (distribute) taxes (transfers) contingent on macroeconomic and idiosyncratic states. The constrained social planner can therefore not eliminate any solvency risk faced by bankers, and the potential switching costs in the case of bank insolvency on the part of households, but use contingent taxes and transfers to ensure that bankers' monitoring decision is welfare-maximizing. On that account, we assume that the constrained social planner imposes (distributes) taxes (transfers) depending on a bank's observed loan returns or, equivalently, the idiosyncratic productivity shock for the financed firm. Thus we denote these taxes (transfers) in real terms by $\tau_s := T_s/P$.

Based on the previous remarks, the constrained social planner does not need to apply any taxes or transfers if bankers do not face a risk, since the competitive equilibrium yields the first-best welfare. Similarly, the constrained social planner does not need to become active if bankers face a solvency risk, but households accept a bail-in in the case of bank insolvency and bankers' monitoring decision is welfare-maximizing as stated in proposition 4.5.2. Note that in any environment with a solvency risk for bankers, inefficiencies compared to the first-best welfare can arise for two reasons: Either because households' switching costs are sufficiently low so that they convert deposits in the case of bank insolvency and thus incur utility losses or because bankers' monitoring decision is not welfare-maximizing. The constrained social planner can, in contrast to the unconstrained social planner, not eliminate solvency risk for bankers and thus not avoid households incurring switching costs. However, the constrained social planner can use the contingent taxes and transfers to align bankers' monitoring incentives with the objective of maximizing utilitarian welfare. With contingent taxes and transfers, bankers' optimization problem in real terms is given by

$$\max_{\substack{\varphi\in[1,\varphi^r],\\ m(h)\in\{0,1\}}} \mathbb{E}_{\mathbf{m}}[\zeta_{\mathbf{z}} R^{E,+}_{\mathbf{z}} - R^{E,-}_{\mathbf{z}} - m(h)\kappa\varphi + \tau_s\varphi]qE.$$

Following the proof of lemma 4.3.4, in the presence of solvency risk bankers' monitoring decision reads as $\underline{m} = \overline{m} = \mathbb{1}\{\triangle[R^L_{\overline{s}} - R^D_{CB}(\varphi^r - 1)/\varphi^r] \geq \kappa - \triangle\tau_{\overline{s}}\}$, where we have assumed, without loss of generality, $\tau_{\underline{s}} = 0$. It is irrelevant whether the constrained social planner distributes transfers to bankers that monitor, imposes taxes on bankers that do not monitor or both. Following the proof of proposition 4.4.3, we can state that in equilibrium bankers' monitoring decision is given by

$$\underline{m} = \overline{m} = \mathbb{1}\{\triangle A_{\overline{s}} \geq \kappa q(1 + \eta_{\underline{s}|0} K/E) - \triangle \tau_{\overline{s}} q\}.$$

The aim of the constrained social planner is then to choose $\tau_{\overline{s}}$, so that bankers' monitoring activity equals the welfare-maximizing monitoring activity $\underline{m} = \overline{m} = \mathbb{1}\{\triangle(A_{\overline{s}} - A_{\overline{s}}) \geq \kappa q\}$. The details are stated in proposition 4.5.4. If switching costs are small so that $\nu < \nu^*$, with ν^* provided in proposition 4.4.3, the constrained social planner can only implement the second-best welfare, as households convert deposits into CBDC in the case of bank insolvency and thus incur switching costs.

Proposition 4.5.4 (Constrained Social Planner Solution)
When bankers face a solvency risk, the constrained social planner maximizes welfare by applying the contingent taxes and transfers of the form $\tau_{\underline{s}} = 0$ and

$$\tau_{\overline{s}} = \max\{\kappa(1 + \eta_{\overline{s}|0} K/E)/\triangle - A_{\overline{s}}/q, 0\}.$$

If switching costs are sufficiently high, so that households accept a bail-in in the case of bank insolvency, i.e., $\nu \geq \nu^*$, with ν^* provided in proposition 4.4.3, the constrained social planner can achieve the first-best welfare. Otherwise, the constrained social planner can only achieve the second-best welfare, as solvency risk for bankers and the resulting switching costs for households cannot be eliminated.

The question which remains to be answered is how utilitarian welfare in a competitive equilibrium with a central bank that aims at maximizing welfare, through the use of its collateral framework, compares to the first-best and second-best welfare. First, note that the central bank can, under certain circumstances, incentivize bankers to monitor, when exposing them to liquidity risk and illiquidity penalties. While monitoring leads to a welfare gain through the induced productivity increase, the imposed penalties for illiquidity and the lost monitoring activities by illiquid bankers yield a welfare loss. On that account, the central bank can in general only implement a third-best welfare, as stated in the following proposition.

Proposition 4.5.5 (Competitive Equilibrium with Liquidity Risk)
Suppose bankers face a solvency risk and switching costs are sufficiently low, so that households convert deposits into CBDC in the case of bank insolvency, i.e., $\nu < \nu^*$ *with* ν^* *provided in proposition 4.4.3. If it is optimal for the central bank to apply tight collateral requirements (for the necessary conditions see proposition 4.5.1), the resulting welfare is in general only third-best and the welfare loss compared to the second-best welfare is given by*

$$-\mu[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa q](K + E) - [(1 - \mu)\eta_{\underline{s}|1} + \mu]\hat{\phi}\epsilon(\hat{\phi})(K + E),$$

where $\hat{\phi}$ *follows from lemma 4.5.1. If* $\mu \to 0$ *and* $\eta_{\underline{s},1} \to 0$*, utilitarian welfare in a competitive equilibrium with tight collateral requirements as optimal monetary policy approaches the second-best welfare and, with negligible switching costs, i.e.,* $\nu \to 0$*, the first-best welfare.*

4.6 Comparison with Today's Monetary System

In today's monetary system, bank deposits are the predominant form of money. They are often insured, for instance by governmental guarantees. Thus, in the case of bank insolvency, depositors generally do not have to convert deposits into cash or into any other safe asset. Nor do bankers face penalties in the case of their bank defaulting and claims being made on the deposit insurance. A monetary system with CBDC as the only legal tender and no deposit insurance scheme is equivalent to today's monetary system in terms of the real allocation in the economy, if there are no switching costs associated with converting deposits into CBDC and bankers do not face illiquidity penalties. Hence, within our framework we can replicate the real allocation emerging in today's monetary system by setting switching costs to zero, i.e., $\nu = 0$, and by focusing on loose collateral requirements, i.e., $(1 + \psi)(K + E) \geq (1 + r^D_{CB})K$.

As outlined in section 4.5, introducing a central bank digital currency and abolishing deposit insurances while establishing the unrestricted right of converting deposits into CBDC may enable the central bank, through the use of its collateral framework, to improve bankers' monitoring incentives. However, this effect of monetary policy only exists in the presence of solvency risk. Without solvency risk, households holding deposits will never shift their funds to the central bank, so there are no switching costs, and the alternative system yields the same welfare as today's monetary system. The same result applies if bankers face a solvency risk but households face sufficiently high switching costs to ensure that, in the case of bank insolvency, they will accept a bail-in and not transfer funds to the central bank.

Finally, consider the situation where bankers face a solvency risk and switching costs are sufficiently low for households holding deposits with insolvent bankers not to accept a bail-in and thus to shift their funds to the central bank. If loose collateral requirements ruling out liquidity risk and illiquidity penalties for bankers, are optimal, the alternative monetary system will yield a welfare loss compared to today's monetary system due to positive switching costs on the part of depositors. In the extreme case where there are no switching costs, the alternative system with loose collateral requirements will yield the same welfare as today's monetary system. When tight collateral requirements are optimal, i.e., when bankers' monitoring activities can be improved through illiquidity penalties and the resulting productivity gains offset utility losses due to monitoring efforts and penalties for illiquidity, the institutional changes will lead, with sufficiently low switching costs, to a welfare gain over and against today's monetary system. Hence, introducing an interest-bearing central bank digital currency, as a medium of exchange equivalent to bank deposits, abolishing deposit insurances and establishing the unrestricted right of converting deposits into CBDC will only entail welfare losses if bankers face a solvency risk and bankers' monitoring incentives cannot be improved through tight collateral requirements. The previous observations are summarized in the following proposition.

Proposition 4.6.1 (Comparison with Today's Monetary System)
Without solvency risk or with solvency risk and bail-ins, a CBDC will never entail welfare losses compared with today's monetary system. With solvency risk and no bail-ins, a CBDC will lead to a welfare gain compared with today's monetary system if tight collateral requirements are optimal and switching costs are sufficiently low; otherwise, a CBDC will entail a welfare loss due to positive switching costs on the part of depositors.

4.7 A Dynamic Perspective

We now consider a dynamic version of our model with discrete time, denoted by $t \in \mathbb{N}_0$. In particular, we focus on an endowment economy where households and bankers do not save and receive the same endowment $K > 0$ and $E > 0$, respectively, at the beginning of each period. Each period can be separated into the three stages of our static framework. Moreover, we focus on the particular case of sufficiently small switching costs ν, where bank insolvency will trigger a bank run. We use this simple setup to illustrate the fundamental forces at work.

First note that, as stated in the following proposition, the mass of households possessing accounts with the central bank only changes over time if bankers face a solvency risk. In the case of insolvency, a household possessing deposits with the respective banker will transfer the funds to the central bank. Due to positive switching costs and the fact that deposit rates never exceed the central bank rate, households, once they have opened an account with the central bank, continue to hold CBDC. Thus, with solvency risk, the mass of households holding CBDC will increase over time. Without solvency risk, the mass of households possessing an account with the central bank will remain constant over time.

Proposition 4.7.1 (Households with Central Bank Accounts)
The mass of households possessing an account with the central bank evolves according to $\mu_{t+1} = \mu_0$ *without solvency risk, and according to* $\mu_{t+1} = (1-\mu_t)\eta_{\underline{s}|\overline{m}} + \mu_t$ *with solvency risk.*

In our model, the mass of households possessing an account with the central bank is closely connected to the mass of defaulting bankers, as outlined in the following proposition.

Proposition 4.7.2 (Bank Default)
The mass of defaulting bankers is given by $\sigma_t = \mu_0$ *if only liquidity risk is present,* $\sigma_t = \eta_{\underline{s}|\overline{m}}$ *if only solvency risk is present, and* $\sigma_t = (1-\mu_t)\eta_{\underline{s}|\overline{m}} + \mu_t$ *if both liquidity risk and solvency risk are present.*

From proposition 4.4.5 we can infer that an equilibrium with liquidity risk, following from tight collateral requirements, solvency risk, and no bail-ins, can at most exist for a finite period of time. Specifically, note that there exists no sequence $\{\psi_t\}_{t\in\mathbb{N}_0}$ such that for all $t \in \mathbb{N}_0$

$$\max\left\{A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu_t)\eta_{\underline{s}|\overline{m}} + \mu_t]\phi[A_{\underline{s}} + \frac{\nu}{K+E} - (1+\psi_t)q]}{(1-\mu_t)\eta_{\overline{s}|\overline{m}} + E/K}, (1+\psi_t)q\right\}$$
$$< \frac{(1-\mu_t)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q)}{(1-\mu_t)\eta_{\overline{s}|\overline{m}} + E/K},$$

where $\mu_{t+1} = (1 - \mu_t)\eta_{\underline{s}|\overline{m}} + \mu_t$. With solvency risk, the mass of households possessing accounts with the central bank converges to one, i.e., $\lim_{t\to\infty} \mu_t = 1$, such that the right-hand side approaches zero while the left-hand side remains positive for any $\psi_t \geq -1$. Hence, with constant endowments of households and bankers, tight collateral requirements can only be maintained for a finite period of time. After this period, tight collateral requirements would render banking non-viable in our economy. As a consequence, the central bank can only use its collateral framework to improve monitoring activities by bankers for a finite period of time without rendering banking non-viable. We summarize this observation in the following proposition.

Proposition 4.7.3 (Viability of Banking)
Suppose bankers face both solvency risk and liquidity risk, i.e., the central bank applies tight collateral requirements. Then there exists a period $\tilde{t} \in \mathbb{N}_0$ subsequent to which banking will be non-viable.

The corollary implies that the central bank faces a dilemma over time when it introduces a central bank digital currency. To induce monitoring by bankers, tight collateral requirements would be needed, but at some point this renders banking non-viable since bankers face a growing liquidity risk that reduces their chances of earning sufficient returns on their endowments in the good state and of offsetting utility losses when they default. As a consequence, the central bank will optimally choose loose collateral requirements and stop punishing default by banks, so that monitoring ceases.

4.8 Discussion

4.8.1 Model assumptions

In modeling the current and alternative monetary system we made some simplifying assumption. First, we abstracted from cash which, as of today, still represents the second most important form of money, after deposits at private banks. Second, when investigating the dynamics of our model, we were relying on constant capital good endowments across agents, and thus on a time-invariant equity financing of banks. In what follows, we discuss the relevance of these two assumptions in detail.

Cash. By abstracting from cash, it implicitly followed that the described effect of monetary policy on bankers' monitoring incentives only exists in the alternative monetary system featuring a CBDC. However, with cash, monetary policy can also affect bankers' monitoring in the current monetary system. In fact, the optimal monetary policy in the current system would be similar to the one in our alternative economy with only a CBDC. As cash is not interest-bearing and potentially associated with higher switching costs than a CBDC, the switching behavior of depositors will, however, generally change with the introduction of a CBDC. If the CBDC is, compared to cash, associated with less switching costs or positive interest payments, depositors will convert their funds at private banks into CBDC already for lower bank losses, i.e.,, they want to avoid a bail-in already for lower losses than in any environment where they only have access to cash. With a CBDC, bank runs then occur faster. Accordingly, there exist situations where depositors only withdraw their funds at private banks with a CBDC but not with cash. In such situations, cash would never be demanded and the central bank is only through the introduction of a CBDC able to affect bankers' monitoring incentives. Such situations are thus captured by our framework.

We can establish an equivalence result whenever introducing a CBDC does not affect depositors' switching behavior. In particular, this includes the special case where cash and CBDC are associated with the same switching costs and the CBDC does not exhibit interest payments.[17] As introducing a CBDC does not affect depositors' switching behavior, it is also irrelevant for the central bank's ability to increase bankers' monitoring incentives through tight collateral requirements and illiquidity penalties. Monetary policy works the same way in the current as well as the alternative monetary system.

Only in the case, where a CBDC should be associated, compared to cash, with higher switching costs, or be exposed to negative interest rate payments, accounting for cash would severely limit the validity of our conclusions. In any such situation, the CBDC would never be demanded. Nevertheless, the central bank can exploit the depositors' conversion of funds at private banks into cash, by using tight collateral requirements and illiquidity penalties, to increase bankers' monitoring incentives.

Capital accumulation. In section 4.7, we investigated temporal effects within our framework, while restricting the capital good endowments of households and bankers to be constant. Accordingly, this dynamic perspective featured a time-invariant equity financing

[17]Note that in our setting, central bank reserves and CBDC have been subject to the same interest rate. Thus, when accounting for a CBDC, which is not interest-bearing, we either have to assume that reserves do also not feature interest payments or reserves are interest-bearing but there exist no possibilities for agents to exploit the interest rate differential between reserves and the CBDC.

of banks. We concluded that, due to over time increasing liquidity risk, tight collateral requirements and illiquidity penalties imposed by the central bank lead in the long run to the non-viability of the banking sector. This conclusion rests on the condition that banks are also in the long-term exposed to a solvency risk. Our conclusion that banking becomes, in the presence of tight collateral requirements and illiquidity penalties, over time non-viable is only misleading if the banks would at some point in time manage, through sufficiently large equity financing, be able to avoid solvency risk. The literature provides several reasons why we can consider the elimination of solvency risk in the long run as rather unlikely and our conclusions provided in section 4.7 continue to hold. On the one hand, banks' possibilities to increase their equity financing are limited, for instance due to the avoidance of share dilution (Goetz et al., 2021) or due to dividend payouts (Gambacorta et al., 2020; Fama and French, 2002). On the other hand, there is evidence that banks have limited incentives to diversify and thus reduce risk (Acharya et al., 2006).

4.8.2 Potential remedies

Throughout our analysis, we aimed at showing how and when the central bank is able to exploit digital bank runs in the presence of a CBDC, by imposing tight collateral requirements and illiquidity penalties, in order to increase bankers' monitoring incentives. In the following, we want to discuss potential measures that eliminate or at least reduce the risk of such bank runs, and we outline how such measures affect the effectiveness of the previously outlined monetary policy.

Holding limits. One such measure that could eliminate the risk of digital bank runs are limits on the amounts that can be held in the form of CBDC by an individual agent. The overall amount of deposits that can be converted into CBDC would then be limited. The central bank could, by imposing sufficiently tight collateral requirements, still expose banks to the same liquidity risk as without transfer limits.

Interest payments. The incentives for depositors to convert funds into CBDC can also be weakened by reducing the interest rate on the CBDC. If the latter is sufficiently low (potentially even negative), depositors will in our framework prefer a bail-in at private banks to a bank run. Without runs, however, banks face no liquidity risk, so that the central bank will be unable to affect bankers' monitoring decision. Bindseil (2019), for example, proposed a more subtle remuneration scheme for a CBDC. The envisioned CBDC is subject to a two-tier interest rate system, so that any amount above a certain threshold is subject to a reduced interest rate. This clearly reduces the incentives for depositors to convert large funds held at private banks into CBDC. In such a two-tier remuneration system, the our postulated effect of monetary policy on bankers' monitoring incentives can still prevail, if depositors decide in the case of default to convert at least some of their funds at private banks into CBDC. However, as in the case of holding limits, the collateral requirements set by the central bank must be sufficiently tight, to expose banks, also in the presence of lower transfers, to a liquidity risk.

4.9 Conclusion

While a CBDC may entail various benefits for society, such as financial inclusion or higher deposit rates resulting from increased competition among banks, they also entail risks for the banking system, potentially impairing the viability of banking or causing financial instability. Thus, the integration of a CBDC into our current monetary system poses several challenges to policymakers, and the economic consequences of such a new form of national currency are still unclear.

We examine how the introduction of an interest-bearing central bank digital currency (CBDC) impacts bank activities and monetary policy. As depositors can switch from bank deposits to CBDC as a safe medium of exchange at any time, banks face digital runs, either because depositors have a preference for CBDC or because they fear bank insolvency. By setting appropriate collateral requirements (and illiquidity penalties) optimal monetary policy can initially increase monitoring incentives for bankers, which leads to higher aggregate productivity. We provide necessary conditions for the optimality of tight collateral requirements and characterize the optimal monetary policy explicitly under specific assumptions on firm productivity and switching costs.

As the mass of households holding CBDC increases, monetary policy with tight collateral requirements generating liquidity risk for banks and exposing bankers to illiquidity penalties would after some time render banking non-viable, thus prompting the central bank to deviate from these policies. Under these circumstances, monitoring incentives will revert to low levels. Hence, the central bank faces a dilemma when introducing a central bank digital currency. While in the short term tight collateral requirements can be used to incentivize bankers to monitor, in the long term they will endanger the viability of banking. Introducing a central bank digital currency therefore involves risks for the entire banking system. Since banks' liquidity demand is likely to rise with a CBDC, the rules for liquidity provisions by the central bank, including the collateral framework, come to the fore.

We also compare this alternative monetary system (CBDC and no deposit insurance) with the current monetary system, where bank deposits are the principal form of money, often insured by such things as governmental guarantees. Most notably, without switching costs and with an optimal monetary policy, a CBDC will never entail welfare losses over and against today's monetary system. However, it may enable the central bank, through the use of its collateral framework, to improve the monitoring incentives for bankers and ultimately to increase welfare.

We compare welfare in a competitive equilibrium with welfare achieved by an unconstrained and constrained social planner. The unconstrained social planner has complete information about agents' activities. Any competitive equilibrium without solvency risk and with loose collateral requirements representing the optimal monetary policy yields the first-best welfare. By reallocating endowments between agents the unconstrained social planner can achieve the first-best welfare as solvency risk for bankers is ruled out, which guarantees a welfare-maximizing monitoring decision by bankers and avoids switching costs incurred by depositors in the case of bank insolvency. The constrained social planner has limited information about agents' activities and is restricted to taxes and transfers contingent on idiosyncratic states. In contrast to the unconstrained social planner, the constrained social planner can only achieve the second-best welfare: Bankers' monitoring decision can be aligned with the objective of maximizing welfare but solvency risk for bankers and thus switching costs incurred by depositors in the case of bank insolvency cannot be eliminated. Any competitive equilibrium with solvency risk and tight collateral requirements representing the optimal monetary policy, i.e., liquidity risk and illiquidity penalties for bankers, yields welfare which is generally lower than the second-best welfare due to penalties for illiquidity imposed on bankers and lost monitoring activities by illiquid bankers.

Several features of our model can be studied in greater detail and are of particular interest when further analyzing the economic consequences of central bank digital currencies. First, we focused on a particular institutional rule that enables agents to convert bank deposits into CBDC any time, specifically without the consent of the respective banker or the central bank. While this institutional setup enables the central bank to expose bankers to illiquidity penalties and ultimately to improve bankers' monitoring incentives, a comparison with other institutional setups has yet to be made. On this account, an in-depth study of various institutional rules accompanying the introduction of a central bank digital currency is urgently required. Second, our framework abstracts from the interbank market, which may however allow individual banks, whose solvency is not questioned, facing CBDC-induced bank runs to avoid illiquidity by borrowing from other banks. Whereas in the present paper we only provide an intuition of the impact of the interbank market, a more analytical analysis may be valuable.

Chapter 5

CAROs: Climate Risk-Adjusted Refinancing Operations*

Abstract

Policy makers have argued that markets are not pricing climate risk appropriately yet, which may lead to a misallocation of resources and financial instability. Climate risk-adjusted refinancing operations (CAROs) conducted by the central bank are one possible instrument to address this issue. CAROs are characterized by interest rates on reserve loans, which depend on the climate risk exposure of the assets held by the borrowing bank. If private agents and the central bank have differing beliefs about the likelihood of the transition to a low-carbon economy, the allocation emerging without CAROs is, from the central bank's perspective, suboptimal and may lead to financial instability. We find that an appropriate design of CAROs allows the central bank to influence bank lending in a way that induces the optimal allocation under its beliefs and eliminates financial instability. Moreover, we show that investment into climate risk mitigation reduces the need for central bank intervention, and that CAROs can be used to achieve specific climate-related allocation targets.

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5.1 Introduction

Climate risk is now widely recognized as a source of financial risk among academics, financial authorities, and financial market participants[1]. However, financial markets do not seem to fully integrate this fact yet. Central bankers claim that there is evidence financial markets are not pricing climate risks adequately, and often publicly highlight this market failure.[2] Such an inaccurate pricing of assets leads to distorted investment decisions as well as a potential build-up of financial risks that can even endanger financial stability, with adverse consequences for the real economy. For example, banks may suffer unexpected losses due to stranded assets and, as a consequence, may be fettered in their role as financial intermediaries.

In this context, a fiscal measure such as a carbon tax would be an effective instrument not only to internalize the climate damage associated with economic activities, but also to reduce the mispricing of assets and the potential risk of financial instability.[3] While the debate on policy measures promoting the transition to a low-carbon economy has largely focused on the fiscal dimension, the call for action by financial and monetary authorities has become stronger. Financial supervisors and central banks are both urged to adopt measures that include climate-related aspects, such as the exposure to climate risk. Regardless of the introduction of fiscal measures, mitigating the mispricing of climate risks lies within the mandate of financial supervisors and central banks to guarantee the stability of the financial system (NGFS, 2018).

Climate-related aspects can enter both financial supervision and monetary policy. Today, certain financial market participants, such as private banks, already face regulation, in the form of risk-weighted capital requirements, for instance. Accounting for climate risk in the currently used risk assessment procedures is thus a straightforward way to integrate climate considerations into a regulatory framework.[4] To the extent that climate risks en-

[1]See Battiston et al. (2017), NGFS (2019), Lagarde (2020) and Fink (2020), for instance.

[2]See Rudebusch et al. (2019) and Schnabel (2020), for instance.

[3]Potential fiscal measures include, among others, carbon taxes (Nordhaus, 2013; Weitzman, 2014; Borissov et al., 2019), cap-and-trade systems for emission certificates (Gersbach and Winkler, 2011; Goulder and Schein, 2013; Greaker and Hagem, 2014), subsidies for clean investments (Acemoglu et al., 2012, 2016; Gerlagh et al., 2018; Greaker et al., 2018; Ramstein et al., 2019) and feed-in tariffs (Proença and Aubyn, 2013).

[4]Volz (2017) proposes a climate-oriented bank regulation in the form of differentiated capital requirements depending on the type of lending conducted by the individual bank. Such an approach would, for example, foresee higher risk weights and thus capital requirements for loans to emission-intensive and carbon-dependent sectors.

danger the financial stability and thus the effectiveness of monetary policy, central banks should also implement appropriate measures. We contribute to this discussion by outlining a potential way for central banks to account for climate-related aspects, such as climate risk, in their refinancing operations.

We study a climate-oriented monetary policy where the central bank uses differentiated interest rates in its refinancing operations, which depend on the climate risk exposure of individual bank's assets. We analyze this type of monetary policy operations in an environment characterized by private and public agents having differing beliefs about climate risk. Our analysis aims at answering the following questions: What are the implications of belief differences between private agents and the government for the real economy? From a central bank perspective, what is the optimal monetary policy in the presence of such differences? How is the optimal monetary policy affected by climate risk mitigation, concerns about financial instability and climate-related targets?

Various other forms of climate-oriented monetary policies have been suggested (NGFS, 2020). Campiglio (2016) discusses differentiated reserve requirements, which take the carbon footprint of the asset portfolio held by the individual financial institution into account. Such differentiated reserve requirements based on the composition of a bank's asset holdings are also discussed by Volz (2017) and Fender et al. (2019). Monnin (2018), in turn, calls for an integration of climate risk into the collateral framework used in central bank refinancing operations. Green quantitative easing, namely asset purchases by central banks that are directed towards low-carbon financial assets is another possibility (Volz, 2017). The monetary policy we consider, namely bank-specific interest rates in central bank refinancing operations, uses climate risk exposure as the conditional factor, but can be also applied more broadly: Other climate-related measures of financial assets, such as a taxonomy, could be considered as a conditional factor. The central bank policy we discuss is thus closely connected to recent proposals of green targeted long-term refinancing operations (TLTROs), see van't Klooster and van Tilburg (2020) or Batsaikhan and Jourdan (2021). Green TLTROs allow central banks to provide liquidity on a long-term basis, while inducing banks to apply more favorable financing conditions for green activities.

Our analysis is based on a static general equilibrium framework that embeds a banking sector, a government sector, comprising a central bank, and two types of loan-financed production sectors that differ in their exposure to climate risk, i.e., a riskless and a risky sector. Banks grant loans to firms which they finance through equity and deposit issuance (i.e., money creation). Moreover, banks need liquidity in the form of central bank reserves to settle interbank liabilities arising from deposit transfers among banks. The liquidity borrowed from the central bank is priced according to the individual bank's exposure to climate risk, which ultimately depends on the composition of its loan portfolio. We refer to such liquidity provisions by the central bank as "climate risk-adjusted refinancing operations", in short CAROs. Our economy either remains in business as usual or shifts to low-carbon activities, as more stringent environmental regulations are put in place. Private agents have subjective beliefs about climate risk, which lead them to attach a likelihood to the transition that may be different from the government's. We extend our baseline model by introducing investment into climate risk mitigation by firms and accounting for costly bank recapitalization, which may be necessary if banks incur sufficiently high losses in the transition. In our framework, bank recapitalization represents a proxy for financial stability.

The belief differences between private agents and the government lead to the fact that, in equilibrium, the allocation of loans is distorted from a governmental perspective. Specifically, if private agents attach a lower probability to the transition than the government, bank lending to the more climate risk-exposed production sector is excessive. As the government aims at maximizing expected welfare, taxing (subsidizing) loans to the sector which benefits (looses) from the distorted beliefs of agents, is optimal. We show that such a tax/subsidy can be implemented through CAROs conducted by the central bank. A differentiated interest rate policy on reserves allows the central bank to influence the allocation of loans in the economy, through the liquidity costs for banks. For example, if the government finds more likely that the transition occurs, compared to private agents, the central bank can counteract the belief-driven effect on the allocation of loans by setting higher marginal liquidity costs for loans allocated to the more climate risk-exposed sector. The marginal liquidity cost factors associated with loans to the two production sectors are thus at the core of the considered climate-oriented monetary policy in our setting.

As mentioned above, the central bank chooses its monetary policy to maximize the, from its point of view, expected welfare, which in our baseline model depends only on the allocation of loans (or equivalently, of capital) across production sectors. We find that the central bank can fully eliminate the belief-driven distortion of the loan allocation and induce the allocation which would emerge if private agents shared the government's beliefs and the central bank does not intervene. If agents attach a lower (higher) probability to the transition than the government, the optimal marginal liquidity cost factors set by the central bank are higher (lower) for the risky sector than for the riskless sector. We can show that the intensity of central bank intervention, as measured by the absolute difference of the marginal cost factors, increases with the belief differences between private agents and the government.

We consider several extensions to our baseline model. First, we introduce the possibility for firms to invest into climate risk mitigation technologies (CRMT). Within this setting, we can show that a higher CRMT investment reduces the intensity of the optimal central bank intervention, for any possible belief of private agents and of the government. Thus, fiscal policies in the form of a subsidy for CRMT investment can help to reduce the need for monetary policy to correct the assessment of climate risk by private agents, which is erroneous from a government perspective.

Second, we account for concerns about financial stability by modeling bank recapitalization, which is required if bank losses in the transition scenario are sufficiently large, such that the initial equity financing of banks is wiped out and shareholders must inject new equity. With costs of bank recapitalization, the central bank faces a trade-off between ruling out financial instability and correcting the belief-driven distortion of the loan allocation. This trade-off emerges from the fact that eliminating financial instability requires a shift of capital to the riskless sector that is larger than the one induced by correcting belief distortions and maximizing expected output in the economy. Accordingly, two monetary policy regimes can be identified. In the first regime, the central bank resolves concerns about financial stability by ruling out bank recapitalization. Specifically, it sets the marginal liquidity cost factor for loans to the more climate risk-exposed sector high enough to induce a sufficient shift of loans towards the less climate risk-exposed sector. In the second regime, the central bank accepts bank recapitalization in the transition but corrects the capital allocation. The choice between the two regimes is driven by a welfare comparison. We also show that if the central bank is equipped with an additional tool, in the form of quantity restrictions on reserve loans, the optimal monetary policy can at the same time rule out concerns about financial stability and correct the belief-driven distortion of the loan allocation. It turns out that, under the optimal monetary policy, the central bank may allow banks to make positive profits through the borrowing of reserves, i.e., the interest rate on reserve loans is lower than the interest rate on reserve deposits. This is the case whenever, with costly reserve borrowing at the central bank, banks would make losses that are high enough to require a recapitalization in the transition. It is then optimal for the central bank to provide an implicit subsidy to banks, by allowing them to generate profits through the borrowing of reserves, in order to prevent costly injections of new equity by shareholders. Whenever borrowing reserves is profitable, the central bank must implement quantity restrictions on reserve loans, as otherwise banks would demand an infinite amount.

Third and last, abstracting from the welfare-maximizing objective of the central bank, we also characterize the monetary policy that is needed to achieve a pre-specified target in the form of loan allocation in the economy. The less loans should be allocated to a particular sector, the higher the respective liquidity cost factor must be. Such a pre-specified target may not only be derived from climate risk considerations, but also from other sustainability objectives. For instance, the central bank may want to ensure coherence with fiscal policies and contribute to the transition to a low-carbon economy, providing support to close the green investment gap. In this particular case, the pre-specified target may represent the share of loans that banks should grant to green projects.

As a final remark, our model assumes that the loan rate on reserves varies with the climate risk exposure of the borrowing bank's asset holdings, while the deposit rate on reserves is uniform for all banks. This approach is equivalent to allowing the deposit rate on reserves to depend on the borrowing bank's climate risk exposure, while keeping the loan rate on reserves constant. The latter specification may be particularly relevant in situations where banks hold large amounts of reserves that are not matched by reserve loans from the central bank, e.g, due to large scale asset purchases by central banks (so-called "quantitative easing").

The paper is organized as follows: Section 5.2 relates our paper to the existing literature. Section 5.3 introduces the model and discusses the optimal choices of the individual agents. Section 5.4 studies the competitive equilibrium in our baseline model. Section 5.5 discusses the impact of CRMT investment by firms, while section 5.6 addresses concerns about financial stability. Monetary policies achieving climate-related targets are chracterized in section 5.7. Section 5.8 outlines an alternative formulation of the considered central bank policy, and discusses the application of CAROs in situations where banks hold large amount of reserves that do not originate from reserve borrowing at the central bank. Section 5.9 concludes.

5.2 Relation to Literature

Our paper relates to four strands of the literature. First, it contributes to the growing number of proposals for a green monetary policy, of which many have already been discussed in section 5.1. Importantly, our paper can also be seen as a formal analysis to understand the functioning of green TLTROs, as currently proposed by van‘t Klooster and van Tilburg (2020) and Batsaikhan and Jourdan (2021).

Second, our paper is also related to the literature on the impact of targeted long-term refinancing operations and their ability to shift resources to the desired sectors. For instance, the ECB TLTROs applied in the aftermath of the financial crisis are deemed to have significantly reduced the funding costs of banks, ultimately at the benefit of the real economy. Evidence is, for instance, provided by Andreeva and García-Posada (2021) who show that credit standards ease and loan margins narrow with a bank’s uptake of TLTROs. In addition, Benetton and Fantino (2018) find that banks which used TLTROs facilities decreased their lending rates, compared to non-participating banks. They also show that market concentration and counterparty characteristics (small versus large firms, for instance) play an important role for the effect of TLTROs on the real economy. Further, as shown by Afonso and Sousa-Leite (2020), country characteristics, such as a more or less vulnerable economy, affect the pass-through of targeted long-term refinancing operations.

Third, we rely on the literature investigating the impact of climate risk on financial stability, which also plays a key role in our analysis of the optimal design of CAROs. Battiston et al. (2017), for instance, evaluate the impact of climate policies favoring (discouraging) green (brown) economic activities on the valuation of financial assets. Climate policy-induced shocks to the financial system and the pass-through to the real economy, with a specific focus on the amplification mechanisms, are also studied by Stolbova et al. (2018).

Fourth, our paper is connected to the literature on private money creation, as it accounts for the dual role of banks, providing both credit and money, in the form of bank deposits, to the real economy. Recent contributions are Faure and Gersbach (2021) and Benigno and Robatto (2019), for instance. Our monetary architecture is particularly close to the one described in Faure and Gersbach (2017) who emphasize the hierarchical structure of many modern monetary systems and analyze various stylized elements: First, the money stock available to the public mainly takes the form of deposits and is only to a minor extent in the form of cash. Second, deposits are created by commercial banks when granting loans or purchasing assets. Third, the central bank issues reserves to commercial banks that use them to settle claims between each other, which can, for example, arise from interbank deposits flows.

5.3 Model

5.3.1 Macroeconomic environment

We develop a static general equilibrium model featuring firms, households, banks and a government sector, including a central bank, as well as two goods—a capital good and a consumption good. Households are endowed with the capital good, which they sell to firms for production of the consumption good. Production of some firms is exposed to climate risk and accordingly we distinguish between riskless and risky firms. Climate risk enters our model through a positive probability of the transition to a low-carbon economy induced, for instance, by more stringent environmental regulations. The decision about the introduction of such regulations is external to our model. The economy features two macroeconomic states: The business as usual scenario without further regulations, and the transition scenario. Throughout our analysis, we allow for differences in beliefs of private agents—firms, households and banks—and beliefs of the government about the likelihood of each scenario.

We focus on a monetary economy where trades are settled instantaneously by using private money in the form of bank deposits.[5] Firms are penniless and must acquire from external creditors the funds (i.e., deposits) needed to finance the capital good purchases from households. Due to moral hazard, repayment of firms can only be enforced by banks, so that production is fully financed with bank loans. When granting loans, banks issue deposits, which are, after the capital good sales have been settled, held by households. Parts of these deposits are used for investments into bank equity. Banks operate under unlimited liability and may experience losses, as loan repayment is risky. If bank losses are sufficiently large, banks must be recapitalized, i.e., households, as the only shareholders, must inject new equity. In our baseline model, bank recapitalization is frictionless. We also provide an extension where new equity injections lead to additional costs, which are not internalized by bank managers and shareholders in the initial equity financing decision. We use this setup to study the effect of financial stability concerns on monetary policy.

In our setup, banks must settle interbank liabilities at the central bank by using reserves. Liabilities between banks arise from interbank deposit flows following from transactions on the good markets. The needed liquidity, in the form of reserves, can be borrowed from the

[5] We abstract from cash. In environments, where cash is only available through a conversion of bank deposits, this is without loss of generality, as the alternative money (i.e., bank deposits) is interest-bearing.

central bank. The interest rate on reserve loans, as set by the central bank, depends on the climate risk exposure of the loan portfolio held by the borrowing bank. By applying different liquidity cost factors on loans to riskless and risky firms, the central bank can influence the loan allocation to firms in the economy. Monetary policy is chosen by the central bank to maximize expected welfare, while the governmental budget is balanced throughout our analysis.

5.3.2 Timeline

As we focus on a monetary economy where trades are settled instantaneously, the timing of interactions among agents is important for our analysis. Figure 5.1 summarizes the events in our static framework.

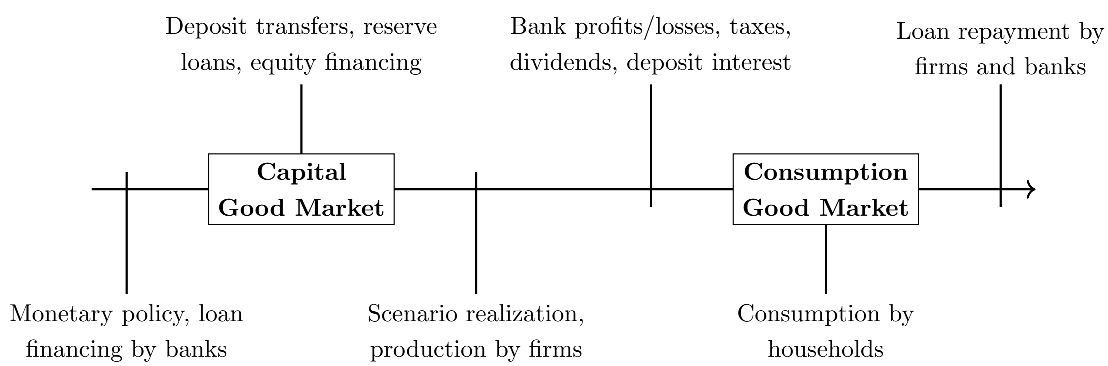

Figure 5.1: Timeline.

5.3.3 Firms

There exist two types of firms, which differ in their exposure to climate risk: Firms are either riskless (indexed by l) or risky (indexed by h). Each type of firm exists in a continuum with mass normalized to one, so that we can focus on a representative firm for each type. Firms are penniless and thus must acquire external funds in the form of deposits to finance the capital good purchases before production starts. Firms are prone to moral hazard and can only raise funds through loans from banks, as banks are the only agents in the economy that can eliminate moral hazard by monitoring. For the subsequent analysis, we assume that bank monitoring is costless and fully eliminates moral hazard.

The riskless firm purchases capital good $K_l \geq 0$ from households at a nominal price $Q > 0$.[6] It produces the consumption good with the strictly concave and deterministic technology $A_l K_l^\alpha$, where $A_l > 0$ denotes the total factor productivity and $\alpha \in (0,1)$ represents the capital intensity. The produced consumption good is then sold to households at a nominal price $P > 0$. The revenues, in the form of deposits, are used to repay bank loans QK_l, which are subject to the interest rate $r_l^L > 0$. The firm operates with unlimited liability and maximizes profits, so that the optimization problem is in real terms given by

$$\max_{K_l \geq 0} \; A_l K_l^\alpha - (1 + r_l^L) q K_l, \tag{5.1}$$

where the capital good price is in terms of the consumption good, i.e., $q := Q/P$. The riskless firm demands an optimal amount K_l of the capital good if and only if the marginal return from production equals the repayment obligation per unit of the capital good, i.e., $\alpha A_l K_l^{\alpha-1} = (1 + r_l^L)q$. The following lemma outlines the resulting optimal demand of the capital good by the riskless firm.

Lemma 5.3.1 (Optimal Choice of the Riskless Firm)
The optimal demand of capital good by the riskless firm is given by

$$K_l = \left[\frac{\alpha A_l}{(1 + r_l^L) q} \right]^{\frac{1}{1-\alpha}}. \tag{5.2}$$

The risky firm purchases capital good $K_h \geq 0$ from households at a nominal price $Q > 0$. It produces the consumption good according to $A_{h,s} K_h^\alpha$, where $A_{h,s} > 0$ represents the stochastic total factor productivity, which depends on the scenario s, and $\alpha \in (0,1)$ denotes the capital intensity. The scenario is given either by business as usual ($s = b$) or by the transition to a low-carbon economy ($s = t$). Private agents—firms, household and banks—believe that the transition occurs with probability $\eta_p \in (0,1)$. In the transition scenario, more stringent environmental regulations are introduced by an official authority, whose

[6] Integrating fixed labor as second production input is straightforward, but does not yield additional insights. If labor is assumed to be mobile, further assumptions must be made to maintain the relevance of production by riskless *and* risky firms, such as heterogeneous consumption goods and aggregation, for instance.

decision making is external to our model.[7] The risky firm sells the produced consumption good $A_{h,s}K_h^\alpha$ to households at the nominal price $P > 0$. The revenues, in the form of deposits, are used to repay bank loans QK_h, which are subject to the interest rate $r_{h,s}^L > 0$ that depends on the scenario s. The risky firm operates with unlimited liability and maximizes expected profits, so that the optimization problem is in real terms given by

$$\max_{K_h \geq 0} \mathbb{E}_p[A_{h,s}K_h^\alpha - (1 + r_{h,s}^L)qK_h]. \tag{5.3}$$

The firm demands an optimal amount K_h of the capital good if and only if the expected marginal return from production equals the expected repayment obligation per unit of the capital good, i.e., $\alpha\mathbb{E}_p[A_{h,s}]K_h^{\alpha-1} = (1 + \mathbb{E}_p[r_{h,s}^L])q$. The following lemma outlines the resulting optimal demand of the capital good by the risky firm.

Lemma 5.3.2 (Optimal Choice of the Risky Firm)
The optimal demand of capital good by the risky firm is given by

$$K_h = \left[\frac{\alpha\mathbb{E}_p[A_{h,s}]}{(1 + \mathbb{E}_p[r_{h,s}^L])q}\right]^{\frac{1}{1-\alpha}}. \tag{5.4}$$

We impose a specific structure of loan rates, which ensures that, in each scenario, the marginal return of production equals the repayment obligation per unit of the capital good. This assumption simplifies the introduction of bank recapitalization, as outlined in section 5.6.

Assumption 5.3.1 (Repayment of the Risky Firm)
$(1 + r_{h,s}^L)q = \alpha A_{h,s}K_h^{\alpha-1}$ *for all* s.

With assumption 5.3.1, the aggregate firm profits in scenario s are under optimal choices of riskless and risky firms given, in real terms, by

$$\begin{aligned} \pi_s &= A_l K_l^\alpha - (1 + r_l^L)qK_l + A_{h,s}K_h^\alpha - (1 + r_{h,s}^L)qK_h \\ &= (1 - \alpha)[A_l K_l^\alpha + A_{h,s}K_h^\alpha] \geq 0. \end{aligned} \tag{5.5}$$

[7] Even when integrating the decision about the introduction of additional regulations, the probabilistic structure for the realization of the transition can be maintained. For instance, the decision maker may not perfectly observe the support for such regulations.

5.3.4 Households

Households are identical and exist in a continuum with mass normalized to one, so that we can focus on a representative household. The household is endowed with capital good $K > 0$, which can be sold to firms at the nominal price $Q > 0$. The revenues from capital good sales take the form of deposits, which are credited with interest according to the rate $r^D > 0$. Deposits can be used to invest into bank equity, which yields the rate of return r_s^E in scenario s. The share of funds invested into bank equity is denoted by $\gamma \in [0, 1]$. Households own firms and thus receive profits Π_s as dividends. After accounting for governmental taxes or transfers T_s, the household uses the equity returns $\gamma(1+r_s^E)QK$, the deposits credited with interest $(1-\gamma)(1+r^D)QK$, and the firm profits Π_s received as dividends to purchase an amount C_s of the consumption good at the nominal price $P > 0$ from firms. The household is maximizing the expected utility, which we assume to be linear and strictly increasing in consumption. Thus, the optimization problem of the household is given, in real terms, by

$$\max_{\gamma \in [0,1]} \mathbb{E}_p[\{\gamma(1+r_s^E) + (1-\gamma)(1+r^D)\}qK + \tau_s + \pi_s], \tag{5.6}$$

where taxes and profits are in terms of the consumption good, i.e., $\tau_s := T_s/P$ and $\pi_s := \Pi_s/P$. The expectation operator in (5.6) is indexed by "p", as like all other private agents, the household has subjective beliefs about the transition, which are captured by the probability η_p. Due to the assumption of linear utility, the household invests the funds in the asset which yields the highest expected rate of return. The following lemma outlines the household's optimal choice.

Lemma 5.3.3 (Optimal Choice of the Household)
$\gamma = 1$ *(*$\gamma = 0$*) if* $\mathbb{E}_p[r_s^E] > (<) r^D$*, and* $\gamma \in [0,1]$ *otherwise.*

5.3.5 Government sector

The government sector comprises the central bank and the government. Via uncollateralized loans, the central bank provides liquidity to banks in the form of reserves, which the banks use to settle interbank liabilities.[8] Reserves can be deposited at the central bank

[8] To isolate the effects of differentiated interest rates in central bank lending facilities, we abstract from potential collateral requirements. However, our analysis is also applicable to any environment where the central bank engages only into secured lending, but banks are not constrained by collateral.

and are credited with interest according to the rate $r^D_{CB} > 0$. The repayment of reserve loans, in turn, is determined by the interest rate $r^L_{CB}(\zeta) > 0$, which depends on the share $\zeta \in [0, 1]$ of loans granted to riskless firms by the borrowing bank. Specifically, we assume that the interest rates on reserves satisfy

$$1 + r^L_{CB}(\zeta) = (1 + r^D_{CB})[1 + \zeta\kappa_l + (1 - \zeta)\kappa_h] \quad \text{subject to} \quad \zeta\kappa_l + (1 - \zeta)\kappa_h \geq 0, \tag{5.7}$$

with $\kappa_l \in \mathbb{R}$ and $\kappa_h \in \mathbb{R}$ representing the liquidity cost factors on bank loans granted to riskless firms and risky firms, respectively. Due to the constraint $\zeta\kappa_l + (1 - \zeta)\kappa_h \geq 0$, the loan rate on reserves always weakly exceeds the deposit rate on reserves, i.e., $r^L_{CB}(\zeta) \geq r^D_{CB}$, so that liquidity is costly for banks. To simplify the subsequent analysis, we reformulate equation (5.7) to

$$r^L_{CB}(\zeta) = r^D_{CB}[1 + \zeta\tilde{\kappa}_l + (1 - \zeta)\tilde{\kappa}_h] \quad \text{with} \quad \tilde{\kappa}_l = \frac{\kappa_l(1 + r^D_{CB})}{r^D_{CB}} \quad \text{and} \quad \tilde{\kappa}_h = \frac{\kappa_h(1 + r^D_{CB})}{r^D_{CB}}.$$

Given that $\tilde{\kappa}_l$ ($\tilde{\kappa}_h$) is a rescaling of κ_l (κ_h), we will also refer to $\tilde{\kappa}_l$ ($\tilde{\kappa}_h$) as the liquidity cost factor on riskless (risky) loans.

In our setting, the central bank aims at maximizing the expected welfare, not knowing which scenario realizes, by choosing the interest rate r^D_{CB} and the cost factors κ_l and κ_h. The belief of the government sector, including the central bank's, about the likelihood of the transition is given by the probability $\eta_g \in (0, 1)$. Thus, the optimization problem of the central bank is given by

$$\max_{\substack{r^D_{CB} > 0 \\ \kappa_l, \kappa_h \in \mathbb{R}}} \mathbb{E}_g[W_s] \qquad \text{subject to} \qquad \zeta\kappa_l + (1 - \zeta)\kappa_h \geq 0, \tag{5.8}$$

where W_s denotes welfare in scenario s. The government has a passive role as it only distributes (finances) central bank profits (losses) Π^{CB}_s by using governmental transfers (taxes) T_s. Throughout our analysis, we impose that the consolidated budget of the government sector is balanced, so that taxes and transfers are given by $T_s = \Pi^{CB}_s$.

Two remarks regarding the potential spread on central bank interest rates (i.e., $r^L_{CB} > r^D_{CB}$) are in order. First, we can always find an optimal monetary policy that rules out a spread on central bank rates (i.e., $\zeta\kappa_l + (1 - \zeta)\kappa_h = 0$) and thus implies zero liquidity costs for banks. In fact, in the presence of financial stability concerns, any optimal monetary policy implies zero liquidity costs for banks (see section 5.6). Second, even if monetary policy induces a spread on central bank interest rates, this does not affect the real allocation and, importantly, not the ability of banks to repay their reserve loans to the central bank. The reason is that central bank profits, emerging from the spread on central bank rates, are distributed to households through transfers. As we abstract from cash, these transfers represent for households an increase on their deposit accounts and for banks an inflow of deposits. Deposit flows are matched by reserve flows (for a detailed description, see subsection 5.3.6), so that the distribution of transfers also increases the reserve holdings of banks. The latter exactly matches the missing amount of reserves needed to cover the repayment of reserve loans.

5.3.6 Banks

Banks are identical and exist in a continuum with mass normalized to one, so that we can focus on a representative bank. Banks are only active if they receive a positive amount of equity financing $E > 0$ from households. The bank grants loans to riskless and risky firms, which are denoted by $L_l \geq 0$ and $L_h \geq 0$, respectively. The total loan volume is then given by $L = L_l + L_h$ and the share of loans granted to riskless firms satisfies $\zeta = L_l/L$. The supply of loans and the equity financing determine the amount of deposit financing $D = L - E$, once the capital good sales have been settled and households used (parts of) their deposits to invest into bank equity.

Deposits are credited with interest according to the rate $r^D > 0$, whereas loans yield a return determined by the interest rates $r_l^L > 0$ and $r_{h,s}^L > 0$, respectively. The repayment by risky firms is uncertain, as it depends on the scenario realized, business as usual versus transition. The bank can borrow reserves L^{CB} from the central bank, which requires a repayment determined by the interest rate $r_{CB}^L(\zeta) > 0$, which depends on the portfolio allocation, as measured by the share ζ of loans granted to riskless firms. The bank can deposit reserves D^{CB} at the central bank, which yield a rate of return $r_{CB}^D > 0$. Therefore, the balance sheet identity $L + D^{CB} = D + L^{CB} + E$ applies and, taking the returns of the various assets and liabilities into account, the nominal equity returns in scenario s are given by

$$(1 + r_s^E)E = (1 + r_l^L)L_l + (1 + r_{h,s}^L)L_h + (1 + r_{CB}^D)D^{CB} - (1 + r^D)D - (1 + r_{CB}^L(\zeta))L^{CB}. \tag{5.9}$$

The bank demands liquidity in the form of reserves, as transactions on the good markets lead to deposit flows among banks, which entail interbank liabilities. The latter must be settled at the central bank by using reserves, where settlement occurs on a gross basis, i.e., the liabilities from deposit outflows cannot be netted with the claims from deposit inflows. We assume that in the course of transactions on the capital good market, a share $\psi \in (0, 1]$ of deposits is temporarily outflowing.[9] Note that when the capital good market is active, deposits equal loans, and households acquire bank equity only after all capital good transactions have been settled. Accordingly, the reserve loans demanded by the bank must satisfy $L^{CB} \geq \psi L$. The pricing of reserves chosen by the central bank is such that the loan rate is weakly exceeding the deposit rate (see equation (5.7) in subsection 5.3.5), i.e., $r_{CB}^L(\zeta) \geq r_{CB}^D$ for all $\zeta \in [0, 1]$. Thus, we can assume, without loss of generality, that the liquidity demand on the side of the bank is given by $L^{CB} = \psi L$. Since we focus on a representative bank, deposit outflows always match deposit inflows, such that after all capital good transactions have been settled, reserve loans must equal reserve deposits, i.e., $L^{CB} = D^{CB}$. Using the definition of the deposit financing after capital good transactions have been settled, $D = L - E$, and the definition of the share of riskless loans in the bank's loan portfolio, $\zeta = L_l/L$, the nominal equity returns (see equation (5.9)) can be rewritten as

$$(1 + r_s^E)E = [(1 + r_l^L)\zeta + (1 + r_{h,s}^L)(1 - \zeta)]L - (1 + r^D)(L - E) - [r_{CB}^L(\zeta) - r_{CB}^D]\psi L. \tag{5.10}$$

As reserve deposits and reserve loans satisfy $D^{CB} = L^{CB} = \psi L$, the bank's assets are given by $L + D^{CB} = (1 + \psi)L$, so that the bank leverage reads $\varphi = (L + D^{CB})/E = (1 + \psi)L/E$. After capital good transactions have been settled, deposit financing is given by $D = L - E$, so that the bank leverage can also be written as $\varphi = (1 + \psi)(1 + D/E)$. Banking operations

[9] We abstract from deposit flows due to transactions on the consumption good market, as including them does not yield further insights, but complicates the analysis.

are subject to capital requirements leading to a regulatory leverage constraint. The bank's decision about loan supply, leading to the leverage φ, must satisfy the constraint $\varphi \leq \varphi^R$, where $\varphi^R \in [1, +\infty)$ is the regulatory maximum leverage.

Using the definition of the bank leverage $\varphi = (1+\psi)L/E$, we can derive the rate of return on equity as a function of the bank leverage φ and the portfolio allocation share ζ, i.e., from equation (5.10), it follows that

$$r_s^E(\varphi,\zeta) := (1+\psi)^{-1}\left\{[(1+r_l^L)\zeta + (1+r_{h,s}^L)(1-\zeta)]\varphi - (1+r^D)[\varphi-(1+\psi)] - \psi[r_{CB}^L(\zeta) - r_{CB}^D]\varphi\right\} - 1,$$

which can be rewritten as

$$r_s^E(\varphi,\zeta) = (1+\psi)^{-1}[r_l^L\zeta + r_{h,s}^L(1-\zeta) - r^D - \psi(r_{CB}^L(\zeta) - r_{CB}^D)]\varphi + r^D. \tag{5.11}$$

We also allow for an active interbank market, where the bank can borrow from, lend to and deposit with other banks. We assume that the bank cannot differentiate between deposit holdings of other banks and deposit holdings of households and firms. Thus, the interest rate on interbank deposits is given by r^D. An active interbank market, which rules out arbitrage opportunities for banks, exists if and only if the interest rate on the interbank deposits equals the interest rate on reserve deposits at the central bank.

Lemma 5.3.4 (Interbank Market)
$r^D = r_{CB}^D$.

Then, using lemma 5.3.4 and the functional form of the interest rate on reserve loans, namely $r_{CB}^L(\zeta) = r_{CB}^D[1 + \zeta\tilde{\kappa}_l + (1-\zeta)\tilde{\kappa}_h]$, the rate of return on bank equity, stated in equation (5.11), translates into

$$r_s^E(\varphi,\zeta) = (1+\psi)^{-1}[r_l^L\zeta + r_{h,s}^L(1-\zeta) - r_{CB}^D\Psi(\zeta)]\varphi + r_{CB}^D, \tag{5.12}$$

where we used the notation $\Psi(\zeta) := 1 + \psi[\zeta\tilde{\kappa}_l + (1-\zeta)\tilde{\kappa}_h]$. The bank operates with unlimited liability and maximizes the shareholder value by choosing the leverage and the loan portfolio allocation. Its optimization problem is thus given by

$$\max_{\substack{\varphi \in [1, \varphi^R], \\ \zeta \in [0,1]}} \mathbb{E}_p[r_s^E(\varphi, \zeta)]. \tag{5.13}$$

The expectation operator in (5.13) is indexed by "p", as banks share the same subjective beliefs as all other private agents about the likelihood of the transition, which is captured by the probability η_p.

We now discuss the optimal choice of the bank, focusing first on the optimal leverage. As the leverage is given by $\varphi = (1 + \psi)(1 + D/E)$, we know that any leverage greater than $1 + \psi$ implies that the bank is partly financing loans with deposits. For its decision to finance loans with deposits, and thus its decision about the leverage, the bank must evaluate the expected repayment of loans, the interest payment on deposits and the liquidity costs arising from reserve borrowing. From equation (5.12), which describes the rate of return on bank equity, we know that the expected rate of return from granting loans financed with deposits is given by

$$r_l^L \zeta + \mathbb{E}_p[r_{h,s}^L](1 - \zeta) - r_{CB}^D \Psi(\zeta)$$
$$= [r_l^L - r_{CB}^D(1 + \psi\tilde{\kappa}_l)]\zeta + [\mathbb{E}_p[r_{h,s}^L] - r_{CB}^D(1 + \psi\tilde{\kappa}_h)](1 - \zeta),$$

where we used the definition $\Psi(\zeta) := 1 + \psi[\zeta\tilde{\kappa}_l + (1 - \zeta)\tilde{\kappa}_h]$. Financing loans with deposits generates costs for the bank, due to interest payments on deposits and costly reserve borrowings. Reserves are needed, as deposits are transferred between banks in the course of transactions on the capital good market. The costs of financing one unit of loans to riskless and risky firms with deposits are therefore given by the deposit rate r_{CB}^D and the marginal liquidity costs $r_{CB}^D\psi\tilde{\kappa}_l$ and $r_{CB}^D\psi\tilde{\kappa}_h$, respectively. If the expected loan rate in one of the sectors, r_l^L and $\mathbb{E}_p[r_{h,s}^L]$ respectively, exceeds the deposit rate and the marginal costs of reserve borrowing, i.e., if it holds that

$$r_l^L > r_{CB}^D(1 + \psi\tilde{\kappa}_l) \qquad \text{or} \qquad \mathbb{E}_p[r_{h,s}^L] > r_{CB}^D(1 + \psi\tilde{\kappa}_h),$$

the bank can increase the expected rate of return on bank equity by extending loan financing and deposit issuance, leading to a higher leverage. Similarly, the bank makes losses by financing loans with deposits if the expected loan rates, r_l^L and $\mathbb{E}_p[r_{h,s}^L]$, are insufficient to cover the financing costs $r^D_{CB}(1+\psi\tilde{\kappa}_l)$ and $r^D_{CB}(1+\psi\tilde{\kappa}_h)$, respectively. In this case, the bank increases the expected rate of return on bank equity by reducing the loan supply and deposit issuance, leading to a lower leverage. From the previous observations, we can conclude that the bank chooses the maximum (minimum) leverage $\varphi = \varphi^R$ ($\varphi = 1$) if it holds that

$$\max\{r^L_l - r^D_{CB}\psi\tilde{\kappa}_l, \mathbb{E}_p[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h\} > (<) r^D_{CB}.$$

In all other situations, the bank makes zero profit by granting loans financed with deposits and thus is indifferent between all leverages, i.e., $\varphi \in [1, \varphi^R]$.

Next, we discuss the optimal portfolio allocation of the bank, as captured by the share ζ of loans granted to riskless firms. The portfolio allocation of the bank depends on the expected rate of return from loans to riskless and risky firms, and the associated marginal liquidity costs. Specifically, if, after accounting for the costs of deposit financing and costly reserve borrowing, the rate of return on loan financing to riskless firms is higher (lower) than the expected rate of return on loan financing to risky firms, i.e., if it holds that

$$\begin{aligned} & r^L_l - r^D_{CB}(1+\psi\tilde{\kappa}_l) > (<) \mathbb{E}_p[r^L_{h,s}] - r^D_{CB}(1+\psi\tilde{\kappa}_h) \\ \Leftrightarrow \quad & r^L_l - r^D_{CB}\psi\tilde{\kappa}_l > (<) \mathbb{E}[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h, \end{aligned}$$

the bank chooses to provide only loan financing to riskless (risky) firms, i.e., $\zeta = 1$ ($\zeta = 0$). In all other situations, the bank is indifferent between loan financing to riskless and to risky firms, i.e., $\zeta \in [0, 1]$. The optimal choice of the bank is summarized in the following lemma.

Lemma 5.3.5 (Optimal Choice of the Bank)
The bank's optimal choice of the leverage is given by $\varphi = \varphi^R$ ($\varphi = 1$) if it holds that

$$\max\{r^L_l - r^D_{CB}\psi\tilde{\kappa}_l, \mathbb{E}_p[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h\} > (<) r^D_{CB},$$

and $\varphi \in [1, \varphi^R]$ otherwise. The bank's optimal choice of the portfolio allocation is given by $\zeta = 1$ ($\zeta = 0$) if it holds that

$$r^L_l - r^D_{CB}\psi\tilde{\kappa}_l > (<) \mathbb{E}_p[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h,$$

and $\zeta \in [0, 1]$ otherwise.

5.4 Equilibrium Analysis

5.4.1 Equilibrium definition

In the subsequent analysis, we focus on competitive equilibria. For what follows, we use the notation $Y_s := A_l K_l^\alpha + A_{h,s} K_h^\alpha$ to represent the aggregate production output in scenario s.

Definition 5.4.1 (Competitive Equilibrium)
Given a monetary policy $r_{CB}^D > 0$, $\kappa_l \in \mathbb{R}$ and $\kappa_h \in \mathbb{R}$, a competitive equilibrium is a set of prices $P > 0$ and $Q > 0$, interest rates $r^D > 0$, $r_l^L > 0$, $r_{h,s}^L > 0$ and $r_s^E > 0$, with $s \in \{b, t\}$, and choices K_l, K_h, γ, φ and ζ, so that

(i) given P, Q and r_l^L, the choice K_l maximizes the profits of the riskless firm,

(ii) given P, Q, $r_{h,s}^L$, with $s \in \{b, t\}$, the choice K_h maximizes the expected profits of the risky firm,

(iii) given P, Q, r^D and r_s^E, with $s \in \{b, t\}$, the choice γ maximizes the utility of the household,

(iv) given r_{CB}^D, κ_l, κ_h, r^D, r_l^L, $r_{h,s}^L$, with $s \in \{b, t\}$, the choices φ and ζ maximize the shareholder value of the bank,

(v) the equity, loan, capital good and consumption good markets clear, i.e., $E = \gamma Q K$, $QK_l = L_l$, $QK_h = L_h$, $K_l + K_h = K$ and $C_s = Y_s$.

Note that in the definition of a competitive equilibrium, we do not account for the deposit market, as it clears by construction of the model.

5.4.2 Equilibrium properties

We first show that, in equilibrium, riskless and risky firms both obtain loans, and we describe the prevailing interest rates and prices. We then provide properties relating to bank leverage and welfare, and finally outline the capital allocation in the decentralized equilibrium.

Loan demand. In equilibrium, both sectors obtain a positive amount of loan financing and produce. This is due to the fact that riskless and risky firms operate with technologies that satisfy the Inada conditions, i.e., the marginal return from production is strictly increasing with lower input of capital good. As marginal productivities are directly linked to loan rates (see subsection 5.3.3), we can deduce that for any possible interest rates on loans, both types of firms obtain loan financing. A higher loan rate in one sector simply leads to less demand for bank loans by this respective sector, but will remain positive in any case.

Lemma 5.4.1 (Loan Demand)
In equilibrium, riskless and risky firms obtain loans, i.e., it holds that $\zeta \in (0,1)$.

Interest rates. Using the fact that in equilibrium riskless and risky firms both demand loan financing, and by the assumption that in equilibrium, perfect competition leads to banks making zero expected profits by financing loans with deposits, we can further characterize the interest rates in our economy. Specifically, we can relate the loan rates in the two sectors to each other, and the loan rates to the interest rate on reserve deposits. First, given that in equilibrium, both types of firms demand loan financing, as shown in lemma 5.4.1, the bank must be indifferent between granting loans to riskless and to risky firms, which, using lemma 5.3.5, implies

$$r_l^L - r_{CB}^D \psi \tilde{\kappa}_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D \psi \tilde{\kappa}_h. \tag{5.14}$$

The expected loan returns adjusted for the marginal liquidity costs—hereinafter referred to as *adjusted loan rates*—must be identical across sectors. Otherwise, the bank would have no incentive to grant loans to the two types of firms. If the liquidity cost factors κ_l and κ_h equal, so that loans to both sectors are subject to the same marginal liquidity costs, the expected loan rates in both sectors equal too, i.e., it holds that $r_l^L = \mathbb{E}_p[r_{h,s}^L]$.[10] In turn, if riskless and risky loans have a differing impact on the liquidity costs, i.e., $\kappa_l \neq \kappa_h$, the expected loan rates from the two sectors will not be identical. The sector for which a lower liquidity cost factor applies will benefit from relatively better loan financing conditions, in terms of a lower interest rate on loans. For example, note that with cost factors satisfying

[10] Recall that $\tilde{\kappa}_l = \kappa_l(1 + r_{CB}^D)/r_{CB}^D$ and $\tilde{\kappa}_h = \kappa_h(1 + r_{CB}^D)/r_{CB}^D$. Thus, $\kappa_l = \kappa_h$ implies $\tilde{\kappa}_l = \tilde{\kappa}_h$.

$\kappa_l < \kappa_h$, it follows from equation (5.14) that loan rates in both sectors satisfy

$$r_l^L = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D \psi(\tilde{\kappa}_h - \tilde{\kappa}_l) < \mathbb{E}_p[r_{h,s}^L],$$

leaving riskless firms with better terms for bank loans than risky firms. Second, we assume perfect competition among banks, leading to zero expected profits from financing loans with deposits in equilibrium. In other words, the bank must be indifferent in equilibrium between all possible leverages, i.e., $\varphi \in [1, \varphi^R]$. Using lemma 5.3.5, this translates into the condition

$$\max\{r_l^L - r_{CB}^D \psi \tilde{\kappa}_l, \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D \psi \tilde{\kappa}_h\} = r_{CB}^D,$$

which, using the equality of adjusted loan rates (see equation (5.14)), leads to

$$r_l^L - r_{CB}^D \psi \tilde{\kappa}_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D \psi \tilde{\kappa}_h = r_{CB}^D.$$

The latter two conditions relate the adjusted loan rates in the two sectors to the interest rate on reserve deposits.

Corollary 5.4.1 (Loan Rates)
In equilibrium, the loan rates satisfy $r_l^L = r_{CB}^D(1 + \psi \tilde{\kappa}_l)$ *and* $\mathbb{E}_p[r_{h,s}^L] = r_{CB}^D(1 + \psi \tilde{\kappa}_h)$.

Note that interest rates on loans are linked to firm productivity, see subsection 5.3.3. For loans to riskless firms, we know from the first-order condition that it holds that $(1 + r_l^L)q = \alpha A_l K_l^{\alpha - 1}$. From assumption 5.3.1, we know that for loans to risky firms, it holds that $(1 + r_{h,s}^L)q = \alpha A_{h,s} K_h^{\alpha-1}$.

Prices. From corollary 5.4.1, we can deduce the formation of prices P and Q in our economy, see corollary 5.4.2. Note that the price ratio P/Q is positively correlated with the interest rate r_{CB}^D on reserve deposits. Thus, an increase of r_{CB}^D leads to an increase of the consumption good price P or a decrease of the capital good price Q or both.

Corollary 5.4.2 (Prices)
In equilibrium, the prices P and Q satisfy

$$\frac{P}{Q} = \frac{(1 + r_{CB}^D)(1 + \psi \kappa_h)}{\alpha \mathbb{E}_p[A_{h,s}] K_h^{\alpha - 1}}.$$

Bank leverage. Using the definition of bank leverage, $\varphi = (1+\psi)L/E$, and the definition of the share of loans allocated to the riskless sector, $\zeta = L_l/L$, we can express the amount of loan financing granted to riskless firms as $L_l = \zeta(1+\psi)^{-1}\varphi E$. Similarly, the loan supply to the risky firm is given by $L_h = (1-\zeta)(1+\psi)^{-1}\varphi E$. Due to the clearing of the equity market, i.e., $E = \gamma QK$, and the loan market, i.e., $QK_l = L_l$ and $QK_h = L_h$, we know that the amount of capital good used in production by riskless and risky firms is given by $K_l = \zeta(1+\psi)^{-1}\varphi\gamma K$ and $K_h = (1-\zeta)(1+\psi)^{-1}\varphi\gamma K$, respectively. With the clearing of the capital good market, i.e., $K_l + K_h = K$, we then obtain that the equilibrium leverage is given by $\varphi = (1+\psi)/\gamma$ and the capital good used by firms in the riskless and risky sector satisfies $K_l = \zeta K$ and $K_h = (1-\zeta)K$, respectively. As the bank is facing the regulatory leverage constraint $\varphi \leq \varphi^R$, the existence of an equilibrium is only guaranteed if $\varphi^R \geq (1+\psi)/\gamma$.

Welfare. Throughout our analysis, we focus on utilitarian welfare. Due to our assumption of linear utility for the household, welfare comprises aggregate consumption. As the scenario, business as usual versus transition, affects the productivity in the risky sector, welfare generally depends on the state s and is given by $W_s = C_s$. The following lemma provides a characterization of welfare in terms of economic fundamentals.

Lemma 5.4.2 (Welfare)
In equilibrium, welfare is given by $W_s = [A_l\zeta^\alpha + A_{h,s}(1-\zeta)^\alpha]K^\alpha$.

Capital allocation. The demand for capital good and thus the demand for loan financing in each of the sectors depends on the respective repayment obligation as determined by the loan rate (see lemma 5.3.1 and lemma 5.3.2). For both types of firms, it holds that a higher interest rate on loans reduces the demand for loan financing and, ultimately, the amount of the capital good used in production. Equation (5.14) relates the equilibrium loan rates in the two sectors. Specifically, the adjusted loan rates must be equal, i.e.,

$$r_l^L - r_{CB}^D\psi\tilde{\kappa}_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D\psi\tilde{\kappa}_h.$$

The sector for which a lower liquidity cost factor applies benefits from relatively better terms on bank loans in the form of a lower loan rate. With identical liquidity cost factors, i.e., if $\kappa_l = \kappa_h$, both sectors face identical conditions for loan financing, i.e., $r_l^L = \mathbb{E}_p[r_{h,s}^L]$, and the allocation of capital among the sectors is only driven by the relative expected productivity of riskless and risky firms, i.e., $\mathbb{E}_p[A_{h,s}]/A_l$. In turn, if, for instance, loans to risky firms are subject to higher marginal liquidity costs than loans to riskless firms, i.e., $\kappa_l < \kappa_h$, the riskless sector is facing more favorable conditions for loan financing compared to the risky sector. Compared to the case of equal liquidity cost factors, riskless firms will demand more loan financing in equilibrium and thus receive a larger share of the capital good available in the economy. The equilibrium share ζ of capital good allocated to the riskless sector, as stated in the following proposition, captures the previously described forces driving the capital allocation, namely the relative expected productivity and the impact of marginal liquidity costs on loan financing conditions.

Proposition 5.4.1 (Capital Allocation)
In equilibrium, the share of capital good allocated to the riskless sector is given by

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l} \frac{1 + \psi\kappa_l}{1 + \psi\kappa_h} \right)^{\frac{1}{1-\alpha}} \right]^{-1} . \tag{5.15}$$

Note that the expected productivity of risky firms, and thus the relative expected productivity of the two sectors, is affected by the beliefs of private agents about the likelihood of the transition. Specifically, the higher the probability η_p that agents attach to the transition, the lower the expected productivity of risky firms and the higher the share ζ of capital good allocated to riskless firms.

5.4.3 Optimal monetary policy

We now study the optimal monetary policy that maximizes expected welfare. Without knowing the scenario realization, the central bank chooses the interest rate $r_{CB}^D > 0$ on reserve deposits, and the costs factors $\kappa_l \in \mathbb{R}$ and $\kappa_h \in \mathbb{R}$, which ultimately determine the interest rate $r_{CB}^L(\zeta)$ on reserve loans. Note that the interest rate r_{CB}^D does not affect welfare (see lemma 5.4.2), and only influences the prices in our economy (see corollary 5.4.2). Thus, the neutrality of money applies in our model and any positive interest rate $r_{CB}^D > 0$ represents an optimal choice for the central bank. The government sector, including the central bank, has its own beliefs about the introduction of more stringent environmental regulations and thus the occurrence of the transition. These beliefs translate into the probability η_g that the government associates with the transition, which may differ from the probability η_p that private agents have. Formally, the optimization problem of the central bank is given by

$$\max_{\kappa_l,\kappa_h\in\mathbb{R}} \{A_l\zeta^\alpha + \mathbb{E}_g[A_{h,s}](1-\zeta)^\alpha\}K^\alpha \qquad \text{subject to} \qquad \zeta\kappa_l + (1-\zeta)\kappa_h \geq 0,$$

where we made use of lemma 5.4.2 to represent welfare W_s.

The cost factors κ_l and κ_h implemented by the central bank influence the capital allocation ζ in the economy, as shown in proposition 5.4.1. The capital allocation is also influenced by the beliefs of private agents. For example, the less private agents believe that the transition realizes (i.e., the lower η_p), the more capital good is allocated to the risky sector (i.e., the lower ζ). The central bank uses its interest policy on reserve loans, determined by the cost factors κ_l and κ_h, to induce the capital allocation that would emerge without central bank intervention if private agents shared the beliefs of the government sector. In other words, the central bank corrects the capital allocation for the belief differences between private agents and the government sector. Note that the central bank is restricted in its choice of the cost factors κ_l and κ_h, as liquidity must be costly for banks in order to avoid arbitrage opportunities, i.e., it holds that $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h \geq 0$, where ζ_g represents the optimal capital allocation. The allocation ζ_g is indexed by "g", as it crucially depends on the beliefs in the government sector (see Proposition 5.4.2).

Proposition 5.4.2 (Optimal Monetary Policy)
The central bank optimally chooses cost factors κ_l and κ_h, so that

$$\kappa_h = a\kappa_l + \frac{a-1}{\psi} \qquad \textit{and} \qquad \zeta_g\kappa_l + (1-\zeta_g)\kappa_h \geq 0,$$

where

$$a := \frac{\mathbb{E}_p[A_{h,s}]}{\mathbb{E}_g[A_{h,s}]} \qquad \textit{and} \qquad \zeta_g := \left[1 + \left(\frac{\mathbb{E}_g[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

If $\eta_g > (<)\eta_p$, it follows that $a > (<)1$ and therefore $\kappa_h > (<)\kappa_l$.

If compared to the government sector, private agents underestimate the likelihood of the transition, i.e., $\eta_g > \eta_p$, the central bank implements cost factors that satisfy $\kappa_h > \kappa_l$. Thus compared to riskless firms, risky firms face worse conditions for loan financing, as loans to risky firms increase relatively more the liquidity costs of the bank. Similarly, the central bank discourages loan financing to riskless firms by setting cost factors that satisfy $\kappa_l > \kappa_h$, whenever compared to the government sector, private agents overestimate the likelihood of the transition, $\eta_p > \eta_g$. If the beliefs of private agents match the ones of the government sector, the central bank does not have to intervene, so that it optimally sets identical cost factors, $\kappa_l = \kappa_h$, resulting in the capital allocation

$$\zeta_g = \zeta_p := \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

We now focus on the intensity of central bank intervention as measured by the difference between the cost factors, $|\kappa_h - \kappa_l|$.

We can show that difference of cost factors $\kappa_h - \kappa_l$ increases (decreases) with η_g (η_p), the probability associated by the government sector (private agents) to the transition. This implies that whenever beliefs satisfy $\eta_g > \eta_p$, so that $\kappa_h - \kappa_l > 0$, the intensity of central bank intervention, as measured by the absolute difference of cost factors $|\kappa_h - \kappa_l|$, increases with η_g and decreases with η_p. In turn, if beliefs satisfy $\eta_g < \eta_p$, cost factors are such that $\kappa_h - \kappa_l < 0$, and the intensity of central bank intervention decreases with η_g and increases with η_p.

Corollary 5.4.3 (Optimal Monetary Policy and Beliefs)
If the central bank chooses the monetary policy according to proposition 5.4.2, the difference between the optimal cost factors, $\kappa_h - \kappa_l$, increases with the beliefs η_g of the government sector and decreases with the beliefs η_p of private agents.

5.5 Climate Risk Mitigation

In this section, we extend our baseline model by accounting for the adoption of a climate risk mitigation technology (CRMT) by risky firms. Specifically, firms in the risky sector can invest parts of the acquired capital good to reduce their exposure to risk, which ultimately increases their total factor productivity in the transition scenario. For what follows, we use $i \in [0, 1]$ to denote the share of capital good used for CRMT investment, so that the amount of capital good used for production is given by $(1 - i)K_h$. The optimization problem of the risky firm is in real terms then given by

$$\max_{\substack{K_h \geq 0, \\ i \in [0,1]}} \mathbb{E}_p[A_{h,s}(i)((1-i)K_h)^\alpha - (1+r^L_{h,s})qK_h], \tag{5.16}$$

where $A_{h,s}(i)$ represents the total factor productivity in scenario s that now depends on the CRMT investment. The risky firm demands an optimal amount of capital good if the marginal productivity equals the repayment obligation per unit of capital good, i.e., if it holds that $\alpha\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha K_h^{\alpha-1} = (1+\mathbb{E}_p[r^L_{h,s}])q$. The firm chooses the share of capital good invested into CRMT optimally if the expected return from investment is maximized, i.e., if it holds that $\partial(\mathbb{E}[A_{h,s}(i)](1-i)^\alpha)/\partial i = 0$.

For the subsequent analysis, we make specific assumptions on the CRMT investment. First, CRMT investment does not affect the productivity in the business as usual scenario. Second, the marginal effect of CRMT investment on productivity in the transition scenario scales with the expected productivity.

Assumption 5.5.1 (CRMT)
$\partial A_{h,b}(i)/\partial i = 0$ *and* $\partial A_{h,t}(i)/\partial i = \mathbb{E}_p[A_{h,s}(i)]\beta(1-i)^{\beta-1}$, *where* $\beta > 0$.

The following lemma outlines the optimal choice of the risky firm, namely the demand of capital good K_h and the share i of capital good devoted to CRMT investment.

Lemma 5.5.1 (Optimal Choice of the Risky Firm with CRMT Investment)
The optimal demand of capital good and the optimal CRMT investment by the risky firm are given by

$$K_h = \left[\frac{\alpha\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{(1+\mathbb{E}_p[r^L_{h,s}])q}\right]^{\frac{1}{1-\alpha}} \qquad \textit{and} \qquad i = \max\left\{1 - \left(\frac{\alpha}{\eta_p\beta}\right)^{\frac{1}{\beta}}, 0\right\}.$$

Note that the share i increases with the probability η_p associated by private agents to the transition and the CRMT parameter β, whereas it decreases with the capital intensity α. CRMT investment only affects the productivity in the transition scenario (see assumption 5.5.1). Thus, a higher likelihood for the transition, as given by the probability η_p, incentivizes firms to increase CRMT investment. A higher β increases the marginal return from CRMT investment, so that firms are incentivized to devote more resources to it in terms of capital good (i.e., i is increasing). In turn, a higher capital intensity α increases the marginal return from production, so that firms optimally invest less into CRMT and produce more.

We now outline the capital allocation in the decentralized economy with CRMT investment by risky firms. The share of capital good allocated to riskless firms is similar to the one in our baseline model, as it depends on the relative expected productivity in both sectors and the cost factors applied by the central bank. The only difference is represented by the impact of CRMT investment on the total factor productivity in the risky sector. Formally, the expected productivity of risky firms $\mathbb{E}_p[A_{h,s}]$ is now replaced by the term $\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha$. While CRMT investment increases the expected total factor productivity by reducing the exposure to risk, it reduces the amount of capital good available for production to $(1-i)K_h$.

Proposition 5.5.1 (Capital Allocation with CRMT Investment)
The share of capital good allocated to riskless firms is given by

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)\right]^{-1}.$$

The optimal monetary policy chosen by the central bank is similar to the one outlined in proposition 5.4.2. In fact, the optimal cost factors and the resulting capital allocation have the same structure as before. However, the capital allocation, which from the goverment's perspective is optimal, and the central bank intervention now depend also on the CRMT investment by risky firms.

Proposition 5.5.2 (Optimal Monetary Policy with CRMT Investment)
The central bank optimally chooses cost factors κ_l and κ_h such that

$$\kappa_h = a(i)\kappa_l + \frac{a(i)-1}{\psi} \qquad \text{and} \qquad \zeta_g\kappa_l + (1-\zeta_g)\kappa_h \geq 0,$$

where

$$a(i) = \frac{\mathbb{E}_p[A_{h,s}(i)]}{\mathbb{E}_g[A_{h,s}(i)]} \qquad \text{and} \qquad \zeta_g = \left[1 + \left(\frac{\mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha}{A_l}\right)\right]^{-1}.$$

If $\eta_g > (<)\eta_p$, it follows $a(i) > (<)1$ and therefore $\kappa_h > (<)\kappa_l$.

Finally, we are interested in the effect of CRMT investment on the intensity of central bank intervention, as measured by the absolute difference between the cost factors $|\kappa_h - \kappa_l|$. If risky firms devote a larger share of capital good to CRMT, their expected total factor productivity increases, i.e., $\mathbb{E}_p[A_{h,s}(i)]$ increases with i. We can then deduce that if the government assigns a higher (lower) probability to the transition than private agents, i.e., if $\eta_g > (<)\eta_p$, the policy parameter $a(i)$ decreases (increases) with the share i of capital good devoted to CRMT investment.

From proposition 5.5.2, we know that the policy parameter $a(i)$ is larger (smaller) than one if beliefs satisfy $\eta_g > (<)\eta_p$. We can conclude that, independent of the beliefs of private agents and the government, the policy parameter $a(i)$ is moving closer to one with increasing CRMT investment (i.e., risky firms choose a larger i). Accordingly, the intensity of central bank intervention, as measured by $|\kappa_h - \kappa_l|$, is always decreasing with CRMT investment.

Corollary 5.5.1 (Optimal Monetary Policy and CRMT Investment)
For beliefs satisfying $\eta_g > (<)\eta_p$, it holds that $\partial a(i)/\partial i < (>)0$. If the central bank chooses the monetary policy according to proposition 5.5.2, CRMT investment always reduces the intensity of central bank intervention, as measured by the absolute difference between cost factors $|\kappa_h - \kappa_l|$.

If the government assigns a higher (lower) probability to the transition than private agents, i.e., if beliefs satisfy $\eta_g > (<)\eta_p$, the share of capital good devoted to CRMT investment in the decentralized equilibrium is lower (higher) than the government believes to be optimal, i.e.,

$$i_g = 1 - \left(\frac{\alpha}{\eta_g \beta}\right)^{\frac{1}{\beta}} > (<) i_p = 1 - \left(\frac{\alpha}{\eta_p \beta}\right)^{\frac{1}{\beta}}.$$

An appropriate subsidy (tax) on CRMT investment can incentivize risky firms to use a share i_g of capital good for CRMT investment and, ultimately, reduce the need for the central bank to intervene.

5.6 Bank Recapitalization

In this section, we extend our baseline model by allowing for costs arising from bank recapitalization. The latter represents a proxy for financial instability in our framework.

Banking operations are generally risky as loan repayment is uncertain but the costs arising from interest payments on deposits and reserve borrowing at the central bank are deterministic. Specifically, deposit contracts cannot be conditioned on the prevailing scenario, i.e., whether the economy remains in the business as usual or shifts to low-carbon activities. As banks operate with unlimited liability, the households, which are the only shareholders of banks in our model, may be required to inject new equity whenever the initial equity financing has been wiped out. We refer to this process as "bank recapitalization".

Maximum leverage without bank recapitalization. Formally, the bank experiences losses if the leverage φ is sufficiently large and loan repayment of risky firms in the transition scenario, $s = t$, is not sufficient for the bank to meet the promises towards depositors and the central bank. Due to perfect competition, the bank is, in equilibrium, making zero expected profits from granting loans to firms funded with deposits. Accordingly, the equity return in the business as usual scenario can never be negative, i.e., in the business as usual scenario, bank recapitalization cannot occur. For interest rates satisfying $r_l^L \zeta + r_{h,t}^L(1-\zeta) < r_{CB}^D \Psi(\zeta)$ in the transition scenario, the maximum leverage $\varphi^S(\zeta)$ without bank recapitalization, is determined by setting the equity return to zero, i.e., $\varphi^S(\zeta)$ satisfies $1 + r_t^E(\varphi^S(\zeta), \zeta) = 0$. Using equation (5.12) to express the equity rate of return, the latter condition reads as

$$(1+\psi)^{-1}[r_l^L \zeta + r_{h,t}^L(1-\zeta) - r_{CB}^D \Psi(\zeta)]\varphi^S(\zeta) + 1 + r_{CB}^D = 0,$$

so that

$$\varphi^S(\zeta) = \frac{(1+r_{CB}^D)(1+\psi)}{r_{CB}^D \Psi(\zeta) - r_l^L \zeta - r_{h,t}^L(1-\zeta)}. \tag{5.17}$$

Using the previous results on the equilibrium loan rates (see corollary 5.4.1), and the link between loan returns and firm productivity (see subsection 5.3.1), we can express the leverage ratio $\varphi^S(\zeta)$ using economic fundamentals, as provided in the following lemma.

Lemma 5.6.1 (Maximum Leverage without Bank Recapitalization)
The maximum leverage, ruling out bank recapitalization, is given by

$$\varphi^S(\zeta) = \frac{\mathbb{E}_p[A_{h,s}](1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)}. \tag{5.18}$$

Costs of bank recapitalization. New equity injections have real costs, as they require negotiation and organization with shareholders. These costs are not internalized by shareholders, which are households in our model, and by banks. The costs of recapitalization scale with the amount of loans granted to the risky sector, as these ultimately cause the costly bank recapitalization. The aggregate costs in terms of the consumption good are given by

$$\lambda(1+r^L_{h,t})qK_h = \lambda\alpha A_{h,t}K_h^\alpha = \lambda\alpha A_{h,t}(1-\zeta)^\alpha K^\alpha =: \Lambda(\zeta), \tag{5.19}$$

where we used assumption 5.3.1, stating $(1+r^L_{h,s})q = \alpha A_{h,s}K_h^{\alpha-1}$ for all s, and the equilibrium allocation of capital as derived in subsection 5.4.2, leading to $K_h = (1-\zeta)K$. The parameter $\lambda \in (0,\overline{\lambda})$ is solely used for scaling purposes. We assume that the costs of bank recapitalization cannot exceed the output of the risky sector, as expected under government beliefs, i.e., $\Lambda(\zeta) < \mathbb{E}_g[A_{h,s}](1-\zeta)^\alpha K^\alpha$. Otherwise, the central bank would find it never optimal to allow for production by the risky sector in the presence of bank recapitalization. Thus, there also exists an upper bound for the parameter λ that is determined by

$$\overline{\lambda}\alpha A_{h,t}(1-\zeta)^\alpha K^\alpha = \mathbb{E}_g[A_{h,s}](1-\zeta)^\alpha K^\alpha \qquad \Leftrightarrow \qquad \overline{\lambda} = \frac{\mathbb{E}_g[A_{h,s}]}{\alpha A_{h,t}}.$$

Welfare. Due to our assumption of linear utility for the household, welfare comprises aggregate consumption and, in the case of bank recapitalization, also the costs associated with new equity injections. As the scenario business as usual versus transition affects the productivity in the risky sector and potentially leads to bank recapitalization, welfare is given by

$$W_s^\lambda = C_s - \Lambda(\zeta)\mathbb{1}\{\varphi > \varphi^S(\zeta) \wedge s = t\}.$$

The costs $\Lambda(\zeta)$ due to new equity injections arise only if the bank chooses a leverage that exposes it to a solvency risk, i.e., $\varphi > \varphi^S(\zeta)$, and if indeed more stringent environmental regulations are put in place, i.e., $s = t$. The following lemma provides a characterization of welfare in terms of model primitives.

Lemma 5.6.2 (Welfare with Bank Recapitalization)
Equilibrium welfare is $W_s^\lambda = \{A_l\zeta^\alpha + A_{h,s}(1-\zeta)^\alpha[1 - \lambda\alpha\mathbb{1}\{\varphi > \varphi^S(\zeta) \wedge s = t\}]\}K^\alpha$.

We now discuss the impact of monetary policy and beliefs on bank recapitalization. First, note that the equilibrium capital allocation, as captured by the share ζ of capital good allocated to riskless firms and outlined in proposition 5.4.1, depends on the cost factors κ_l and κ_h, which are chosen by the central bank. It then follows from lemma 5.6.1 that the maximum leverage which rules out bank recapitalization also depends on these cost factors, both directly and indirectly, via the capital allocation ζ. Moreover, the costs of bank recapitalization $\Lambda(\zeta)$ also depend, through the capital allocation, on the costs factors chosen by the central bank. Accordingly, in its choice of the cost factors, the central bank must account for the effect of its policy on the capital allocation as well as on the occurrence and the associated costs of bank recapitalization.

In proposition 5.6.1, we provide the comparative statics on the maximum leverage ruling out bank recapitalization with respect to the monetary policy. We find that the leverage threshold $\varphi^S(\zeta)$ always decreases with an increasing cost factor κ_l on loans to the riskless sector. With an increasing κ_l and a fixed κ_h, the loan financing conditions for riskless firms worsen compared to the one for risky firms, leading to a larger share of loans to the risky sector within banks' portfolio in equilibrium. Banks are therefore exposed to more risk, so that the critical leverage threshold $\varphi^S(\zeta)$ ruling out bank recapitalization decreases. In addition, we find that the same leverage threshold increases with the cost factor κ_h on loans to the risky sector only if a sufficiently large share of capital is already allocated to the riskless sector, i.e., $\zeta \geq 1 - \alpha$. An increasing κ_h and a fixed κ_l, lead to a worsening of loan financing conditions for risky firms, compared to riskless firms, so that in equilibrium, the latter receive even more funds from banks. This, in turn, reduces the risk exposure of banks, resulting in a higher maximum leverage $\varphi^S(\zeta)$ that rules out bank recapitalization. Finally, we show that for cost factors satisfying $\kappa_l \to -1/\psi$ or $\kappa_h \to +\infty$, banks are not facing recapitalization, i.e., the maximum leverage $\varphi^S(\zeta)$ is approaching infinity.

We also provide comparative statics on the costs of bank recapitalization with respect to the monetary policy in the form of the costs factors κ_l and κ_h. An increase in κ_l (κ_h) leads to a higher (lower) share of capital allocated to risky firms and thus to higher (lower) bank recapitalization costs $\Lambda(\zeta)$. For the extreme case, where $\kappa_l \to -1/\psi$ or $\kappa_h \to +\infty$, only riskless firms produce (i.e., $\zeta \to 1$), so that there are no costs of bank recapitalization.

Proposition 5.6.1 (Monetary Policy and Recapitalization)
The maximum leverage ruling out bank recapitalization varies with the monetary policy in the form of the cost factors κ_l and κ_h according to

$$\frac{\partial \varphi^S(\zeta)}{\partial \kappa_l} < 0, \qquad \text{and} \qquad \frac{\partial \varphi^S(\zeta)}{\partial \kappa_h} < (\geq) 0 \qquad \text{if and only if} \qquad \zeta < (\geq) 1 - \alpha.$$

Moreover, it holds that $\lim_{\kappa_l \to -1/\psi} \varphi^S(\zeta) = \lim_{\kappa_h \to +\infty} \varphi^S(\zeta) = +\infty$.

The costs of bank recapitalization vary with the monetary policy in the form of the cost factors κ_l and κ_h according to

$$\frac{\partial \Lambda(\zeta)}{\partial \kappa_l} > 0 \qquad \text{and} \qquad \frac{\partial \Lambda(\zeta)}{\partial \kappa_h} < 0.$$

Moreover, it holds that $\lim_{\kappa_l \to -1/\psi} \Lambda(\zeta) = \lim_{\kappa_h \to +\infty} \Lambda(\zeta) = 0$.

The occurrence of bank recapitalization and the associated costs also depend on the beliefs of private agents. Specifically, the higher the probability η_p that agents attach to the transition, the higher the maximum leverage $\varphi^S(\zeta)$ ruling out bank recapitalization and the lower the costs $\Lambda(\zeta)$ in the case of bank recapitalization. The intuition behind this result is that the more agents believe that the transition will occur, the lower the expected productivity of risky firms. In equilibrium, this leads to more production by riskless firms and thus to more loan financing to the riskless sector. Banks therefore become safer, so that bank recapitalization occurs only at a higher leverage, i.e., $\varphi^S(\zeta)$ is increasing with η_p. Since the costs of bank recapitalization $\Lambda(\zeta)$ scale with the amount of loans granted to the risky sector, the belief-driven increase in production of riskless firms also decreases bank recapitalization costs.

Proposition 5.6.2 (Beliefs and Recapitalization)
The maximum leverage $\varphi^S(\zeta)$ ruling out bank recapitalization increases with the beliefs η_p of private agents, whereas the bank recapitalization costs $\Lambda(\zeta)$ decrease with the beliefs η_p

of private agents, i.e.,

$$\frac{\partial \varphi^S(\zeta)}{\partial \eta_p} > 0 \qquad and \qquad \frac{\partial \Lambda(\zeta)}{\partial \eta_p} < 0.$$

Optimal Monetary Policy. As in section 5.3, the central bank aims at maximizing expected welfare by choosing the cost factors κ_l and κ_h. The neutrality of money with regard to the interest rate policy of the central bank still applies. Specifically, the interest rate r^D_{CB} on reserve deposits does not affect the real allocation and thus welfare, but only prices (see corollary 5.4.2 and lemma 5.6.2). We showed that the beliefs of private agents affect the capital allocation as well as the occurrence and costs of bank recapitalization. In its choice of the monetary policy, the central bank thus generally faces two externalities following from private agents' beliefs. First, from a central bank perspective, beliefs of private agents lead to a distortion of the capital allocation, such that one of the sectors receives, without central bank intervention, more capital good for production than it would receive under the government's beliefs. Second, private agents' beliefs can trigger bank recapitalization and reduce welfare by inducing costly equity injections, or, if bank recapitalization also exist under the government's beliefs, private agents' distorted beliefs can lead to an increase of such costs. The central bank can use the cost factors κ_l and κ_h to steer the capital allocation in the economy and thereby aim at eliminating the previously mentioned two externalities arising from agents assessing the likelihood of the transition differently from the government. However, due to the constraint on the cost factors, namely that liquidity must remain costly for banks, the monetary policy may not always be able to eliminate both externalities. In fact, the central bank faces generally a trade-off between reducing capital distortions and ruling out bank recapitalization. In subsection 5.6.1, we show that once the restriction on the cost factors is relaxed and once an additional central bank tool in the form of quantity restrictions for reserve loans is introduced, the monetary policy can always eliminate both externalities following from the belief difference between private agents and the government.

We can distinguish three regimes for the optimal monetary policy. In the first regime, bank recapitalization does not occur under the capital allocation induced by government beliefs and no central bank intervention, as captured by the share ζ_g. Then, the optimal monetary policy only corrects for the impact of private agents' beliefs on the capital allocation, so that the optimal central bank policy is characterized by proposition 5.4.2. However, in an economy where bank recapitalization is costly, the central bank always has incentives to choose cost factors κ_l and κ_h, not only to induce the capital allocation ζ_g, but also to maximize the leverage threshold $\varphi^S(\zeta_g)$ ruling out bank recapitalization. The highest leverage threshold $\varphi^S(\zeta_g)$ is obtained by minimizing liquidity costs for bank, as the lower the costs for borrowing reserves at the central bank, the lower the financing costs per unit of loans funded with deposits, as measured by $r^D_{CB}\Psi(\zeta_g) = r^D_{CB}[1+\zeta_g\kappa_l + (1-\zeta_g)\kappa_h]$. Accordingly, the leverage threshold $\varphi^S(\zeta_g)$ is maximized for costs factors satisfying $\zeta_g\kappa_l+(1-\zeta_g)\kappa_h = 0$.[11]

Proposition 5.6.3 (Optimal Monetary Policy without Recapitalization)
The optimal monetary policy follows proposition 5.4.2 with $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h = 0$, if there is no bank recapitalization under the allocation ζ_g, i.e., $\varphi = (1+\psi)/\gamma \leq \varphi^S(\zeta_g)$.

Now suppose that under a monetary policy foreseeing cost factors κ_l and κ_h, which induce the capital allocation ζ_g and minimize liquidity costs as $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h = 0$, banks are in the transition exposed to recapitalization, i.e., $\varphi = (1+\psi)/\gamma > \varphi^S(\zeta_g)$. Then, the central bank must decide between the second and third monetary policy regime. In the second regime, the central bank implements cost factors $\hat{\kappa}_l$ and $\hat{\kappa}_h$, that lead to a capital allocation $\hat{\zeta}$, which rules out recapitalization of banks, i.e., $\varphi = (1+\psi)/\gamma = \varphi^S(\hat{\zeta})$. From proposition 5.6.1, we know that there always exists such cost factors that sufficiently discourage loan financing to risky firms, compared to loan financing to riskless firms, in order to make banks safer and rule out recapitalization in the transition. Specifically, the required allocation $\hat{\zeta}$ to rule out bank recapitalization satisfies $\hat{\zeta} > \zeta_g$. Moreover, we can show that it is optimal for the central bank to also minimize liquidity costs, i.e., $\hat{\zeta}\hat{\kappa}_l + (1-\hat{\zeta})\hat{\kappa}_h = 0$, as this leads to the smallest possible distortion in the capital allocation. In other words, allowing for positive liquidity costs would require the central bank to induce, through the choice of the cost factors, a larger shift of capital towards riskless firms. Since without bank recapitalization, welfare is maximized for the capital allocation ζ_g, a greater distortion away from ζ_g cannot be optimal.

Lemma 5.6.3 (Monetary Policy Ruling Out Bank Recapitalization)
Suppose that for cost factors κ_l and κ_h inducing ζ_g (see proposition 5.4.2) and satisfying $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h = 0$, it holds that $\varphi = (1+\psi)/\gamma > \varphi^S(\zeta_g)$. Then, there exist costs factors

[11] In section 5.3, bank recapitalization was frictionless, so that the maximum leverage $\varphi^S(\zeta)$ and the effect of liquidity costs on $\varphi^S(\zeta)$ were irrelevant. Thus, the optimal monetary policy, outlined in proposition 5.4.2, allowed for any positive spread between deposit rates, i.e., $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h \geq 0$.

$\hat{\kappa}_l$ and $\hat{\kappa}_h$, with $\hat{\zeta}\hat{\kappa}_l + (1-\hat{\zeta})\hat{\kappa}_h = 0$, that implement the capital allocation $\hat{\zeta}$ satisfying $\varphi = (1+\psi)/\gamma = \varphi^S(\hat{\zeta})$.

In the third monetary policy regime, the central bank chooses to accept bank recapitalization in the transition scenario but corrects for the belief-driven distortion of the capital allocation. The rule for the optimal cost factors is similar to the one in proposition 5.4.2. However, the central bank must now account for the costly bank recapitalization, which only arises due to loan financing to the risky sector. From the central bank's perspective, the expected productivity of the risky sector must be adjusted for the costs associated with new equity injections in the transition. It is therefore lower than without bank recapitalization. We use the notation $\mathbb{E}_g^\lambda[A_{h,s}] := \mathbb{E}_g[A_{h,s}] - \eta_g \lambda \alpha A_{h,t}$ to represent the productivity in the risky sector, as expected under government beliefs and taking the costs of recapitalization of banks into account. As a result, the policy parameter $a_\lambda = \mathbb{E}_p[A_{h,s}]/\mathbb{E}_g^\lambda[A_{h,s}]$ depends on the recapitalization costs and is thus indexed by λ.

The central bank decides between the second and third regime, depending on which one yields the highest expected welfare. Formally, the central bank then prefers the second regime, ruling out bank recapitalization, over the third regime, accepting bank recapitalization and correcting the capital allocation, if it holds that

$$\mathbb{E}_g[W_s^\lambda(\hat{\zeta})] = \mathbb{E}_g[W_s(\hat{\zeta})] \geq \mathbb{E}_g[W_s^\lambda(\zeta_g^\lambda)]$$

$$\Leftrightarrow \qquad A_l[(\hat{\zeta})^\alpha - (\zeta_g^\lambda)^\alpha] \geq \mathbb{E}_g^\lambda[A_{h,s}](1-\zeta_g^\lambda)^\alpha - \mathbb{E}_g[A_{h,s}](1-\hat{\zeta})^\alpha.$$

Of course, if it holds that $\zeta_g^\lambda > \hat{\zeta}$, expected welfare under the third regime—accepting bank recapitalization and correcting the belief-driven capital distortion—can never be higher than expected welfare under the second regime—ruling out bank recapitalization, i.e., if it holds that $\mathbb{E}_g[W_s^\lambda(\zeta_g^\lambda)] < \mathbb{E}_g[W_s(\hat{\zeta})]$. The details of the third monetary policy regime are provided in the following proposition.

Proposition 5.6.4 (Optimal Monetary Policy with Bank Recapitalization) *Suppose that for cost factors κ_l and κ_h inducing ζ_g (see proposition 5.4.2) and satisfying $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h = 0$, it holds that $\varphi = (1+\psi)/\gamma > \varphi^S(\zeta_g)$. Then, with $\mathbb{E}_g[W_s^\lambda(\zeta_g^\lambda)] >$*

$\mathbb{E}_g[W_s(\hat{\zeta})]$, *the central bank optimally chooses cost factors* κ_l *and* κ_h *such that*

$$\kappa_h = a_\lambda \kappa_l + \frac{a_\lambda - 1}{\psi} \qquad \text{and} \qquad \zeta_g^\lambda \kappa_l + (1 - \zeta_g^\lambda)\kappa_h \geq 0,$$

where

$$a_\lambda = \frac{\mathbb{E}_p[A_{h,s}]}{\mathbb{E}_g^\lambda[A_{h,s}]} \qquad \text{and} \qquad \zeta_g^\lambda = \left[1 + \left(\frac{\mathbb{E}_g^\lambda[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

Otherwise, i.e., $\mathbb{E}_g[W_s(\hat{\zeta})] \geq \mathbb{E}_g[W_s^\lambda(\zeta_g^\lambda)]$, *the central bank implements cost factors* $\hat{\kappa}_l$ *and* $\hat{\kappa}_h$ *that satisfy* $\hat{\zeta}\hat{\kappa}_l + (1-\hat{\zeta})\hat{\kappa}_h = 0$ *and* $\varphi = (1+\psi)/\gamma = \varphi^S(\hat{\zeta})$.

5.6.1 Quantity restrictions on reserve loans

In our previous analysis of the optimal monetary policy, we imposed that liquidity must always remain costly for banks, i.e., $\zeta\kappa_l + (1-\zeta)\kappa_h \geq 0$, in order to avoid arbitrage opportunities. For the optimal monetary policy, the latter constraint is binding, so that liquidity costs for banks are minimized. This, in turn, reduces the monetary policy instruments to essentially one cost factor, either κ_l or κ_h, as they are co-linear due to the binding constraint on liquidity costs. As a consequence, the central bank may not be able to fully eliminate both externalities following from the beliefs of private agents. If we remove the constraint on liquidity costs, the central bank has two independent instruments, which allow it to always correct for belief-driven capital distortions and avoid bank recapitalization. However, in some situations, reserve borrowing may become profitable for banks, as the optimal cost factors satisfy $\kappa_l < 0$ and $\kappa_h < 0$. To prevent arbitrage opportunities for banks, the central bank must limit the amount of reserves that the individual bank can borrow. In what follows, we denote the maximum amount of reserve loans by $\overline{L}^{CB}$. The central bank can then always fully eliminate both externalities, namely the capital distortion and the occurrence of bank recapitalization. The central bank optimally chooses cost factors, which on the one hand implement the capital allocation ζ_g—which from a central bank perspective is the optimal allocation—and, on the other hand, rules out bank recapitalization, i.e., cost factors are chosen such that $\varphi = (1+\psi)/\gamma = \varphi^S(\zeta_g)$. The following proposition outlines the optimal monetary policy without the constraint of costly liquidity and with quantity restrictions on reserve loans. It also provides the necessary and sufficient conditions under which the quantity restriction on reserve loans is indeed effective, as captured by inequality (5.20) in proposition 5.6.5.

Proposition 5.6.5 (Optimal Monetary Policy with Restrictions on Reserves) *The central bank optimally chooses cost factors κ_l and κ_h such that*

$$\kappa_l = \frac{\mathbb{E}_g[A_{h,s}]\gamma}{\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1}{\psi}$$

and

$$\kappa_h = \frac{\mathbb{E}_p[A_{h,s}]\gamma}{\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1}{\psi}.$$

The amount of reserve loans must be restricted to the maximum $\overline{L}^{CB} = \psi QK$ if and only if liquidity is not costly, i.e., $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h < 0$ or, equivalently,

$$\frac{\zeta_g}{1-\zeta_g} < \frac{(1-\eta_p)(A_{h,b}-A_{h,t})(1-\gamma) - A_{h,t}\gamma}{\mathbb{E}_g[A_{h,s}]\gamma}. \tag{5.20}$$

The central bank must implement quantity restrictions on reserve loans if liquidity is not priced in a way that it is costly for banks. From inequality (5.20) in proposition 5.6.5, it follows that this is the case if, for instance, the share ζ_g of capital good received by riskless firms or the share γ of funds used by investors for equity financing are sufficiently small. In both cases, banks are highly risky and incur large losses in the transition, requiring the central bank to provide a subsidy to banks by allowing them to generate profits through reserve borrowing, i.e., $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h < 0$. If the transition realizes, these profits are sufficient for banks to compensate the losses originating from loan financing to the risky sector, in a way that bank recapitalization is ruled out. Distributing these implicit subsidies is welfare improving, as it avoids new equity injections, whose costs are not internalized by households and banks.

Note that, in the presence of quantity restriction on reserve loans, the optimal cost factors κ_l and κ_h, as chosen by the central bank, increase if agents' beliefs about the transition, captured by the probability η_p, are growing. Formally, it holds that

$$\frac{\partial\kappa_l}{\partial\eta_p} = \frac{\mathbb{E}_g[A_{h,s}]\gamma}{\psi(1-\eta_p)^2(A_{h,b}-A_{h,t})(1-\zeta_g)} > 0$$

and

$$\frac{\partial \kappa_h}{\partial \eta_p} = \frac{A_{h,t}\gamma}{\psi(1-\eta_p)^2(A_{h,b}-A_{h,t})(1-\zeta_g)} > 0.$$

However, note that, for an increasing probability η_p, the cost factor κ_l for loans to riskless firms increases more than the cost factor κ_h on loans to risky firms, i.e., it holds that

$$\frac{\partial \kappa_l}{\partial \eta_p} > \frac{\partial \kappa_h}{\partial \eta_p}.$$

Thus, if agents' beliefs about transitioning to a low-carbon economy grow, loan financing to the risky sector is discouraged less than before, under the optimal monetary policy, relative to loan financing to the riskless sector.

5.7 Targets

In this section, we look at the possibility for the central bank to implement a target allocation of loans and, ultimately, of production input in the form of the capital good in the economy. We denote this target allocation by the share $\zeta_t \in (0,1)$. Such a target can, for example, be derived from a policy coherence argument according to which the central bank aims at contributing to the transition to a low-carbon economy. An alternative interpretation is that the central bank aims at mitigating climate risk and the desired level of climate risk is achieved through the target allocation ζ_t.

For the subsequent analysis, we assume that the central bank deviates from its welfare-maximizing objective and solely cares about implementing the target allocation. The central bank achieves this goal by choosing the appropriate cost factors κ_l and κ_h, while keeping liquidity generally costly for banks, i.e. $\zeta_t\kappa_l + (1-\zeta_t)\kappa_h \geq 0$. Proposition 5.7.1 outlines the optimal monetary policy in the form of the cost factors κ_l and κ_h that lead to the target allocation ζ_t.

Proposition 5.7.1 (Optimal Monetary Policy with a Target)
The central bank optimally chooses the cost factors κ_l and κ_h such that

$$\kappa_h = a_t\kappa_l + \frac{a_t-1}{\psi} \qquad \text{and} \qquad \zeta_t\kappa_l + (1-\zeta_t)\kappa_h \geq 0,$$

where it holds that

$$a_t = \frac{\mathbb{E}_p[A_{h,s}]}{A_l}\left(\frac{\zeta_t}{1-\zeta_t}\right)^{1-\alpha}.$$

If $\zeta_t > (<)\zeta_p$*, it follows that* $a_t > (<)1$ *and therefore that* $\kappa_h > (<)\kappa_l$*.*

First, we focus on the case where under the target allocation, more capital is shifted to riskless firms than under the beliefs of private agents and without central bank intervention ($\kappa_l = \kappa_h$), so that it holds that $\zeta_t > \zeta_p$. Then, the central bank must implement cost factors that satisfy $\kappa_h > \kappa_l$, so that risky firms face relatively worse loan financing conditions, compared to riskless firms. Second, if the central bank aims at a target allocation that foresees less capital for riskless firms than in the decentralized equilibrium without central bank intervention, i.e., $\zeta_t < \zeta_p$, the optimal cost factors satisfy $\kappa_l > \kappa_h$. Such a monetary policy penalizes riskless firms, relative to risky firms, when demanding loans from banks.

The intensity of central bank intervention, as measured by the difference of costs factors $\kappa_h - \kappa_l$, is now influenced by the target allocation ζ_t and the beliefs η_p of private agents. In contrast to section 5.3, the beliefs η_g of the government sector do not play a role anymore. Whenever the central bank sets a target $\zeta_t > \zeta_p$, the cost factors satisfy $\kappa_h - \kappa_l > 0$, and the intensity of central bank intervention, as measured by the absolute difference between the cost factors $|\kappa_h - \kappa_l|$, increases with ζ_t and decreases with η_p. In turn, if the target allocation satisfies $\zeta_t < \zeta_p$, the cost factors satisfy $\kappa_h - \kappa_l < 0$, and the intensity of central bank intervention decreases with ζ_t and increases with η_p.

Corollary 5.7.1 (Optimal Monetary Policy, Beliefs and Targets)
If the central bank chooses the monetary policy according to proposition 5.7.1, the difference between the optimal cost factors $\kappa_h - \kappa_l$ *increases with the target* ζ_t *of the central bank and decreases with the belief* η_p *of private agents.*

5.8 Discussion

As an alternative to the loan rate on reserves varying with the climate risk exposure of banks' asset holdings, we could also allow for a deposit rate on reserves that varies with banks' asset allocation. Both approaches yield the same result in our model. Formally, setting a constant interest rate r_{CB}^D on reserve deposits and choosing the cost factors κ_l

and κ_h, such that the loan rate on reserves satisfies $r_{CB}^L(\zeta) = r_{CB}^D[1 + \zeta\tilde{\kappa}_l + (1-\zeta)\tilde{\kappa}_h]$, is equivalent to setting a constant interest rate r_{CB}^L on reserve loans and choosing the cost factors κ_l and κ_h such that the deposit rate on reserves is given by $r_{CB}^D(\zeta) = r_{CB}^L[1 - \zeta\tilde{\kappa}_l - (1-\zeta)\tilde{\kappa}_h]$.

The latter approach may be particularly relevent when banks holds large amounts of reserves without borrowing from the central bank. In such situations, banks may face no or only a small demand for reserve loans from the central bank, as liquidity in the form of central bank reserves is relatively abundant. Then, only the deposit rate on reserves, but not the loan rate on reserves, is the relevant policy instrument. In many countries banks currently hold large amounts of reserves, that are not matched with loans from the central bank. This situation is a consequence of the expansionary monetary policy central banks adopted in the aftermath of the financial crisis in 2007/08 and in the current Covid-19 crisis. Due to large scale asset purchases by central banks, so-called "quantitative easing", banks acquired tremendous amounts of reserves. Liquidity seems to be abundant and there is no or only little need to approach central banks for reserve borrowing. Accordingly, the loan rate on reserves is of minor relevance and the deposit rate on reserves emerged as key interest rate.

5.9 Conclusion

It has been argued that financial market participants fail to adequately account for climate risk and thereby contribute to a mispricing of assets, which leads to a misallocation of resources, a build-up of financial risks and potentially even to financial instability. There is an ongoing debate on to which extent central banks can and should intervene by adopting a climate-oriented monetary policy to correct a potential market failure. Several monetary policy instruments taking climate considerations into account have been proposed. In this paper, we study the effect of a new concept, the climate risk-adjusted refinancing operations, in short CAROs, on resource allocation and financial stability.

We developed a static general equilibrium framework that allows us to study CAROs in environments with different beliefs between private agents and the government about the likelihood of the transition. From a central bank's perspective, without intervention, the different beliefs of private agents lead to a resource allocation in the decentralized equilibrium that is suboptimal. In our baseline model, we show that by using appropriate liquidity cost factors on loans to riskless and risky firms, the central bank can induce the allocation which is optimal under its beliefs.

We extend our baseline model by introducing climate risk mitigation technologies (CRMT), by accounting for financial stability concerns and by featuring climate-related allocation targets, following, for instance, from a policy coherence argument regarding fiscal policies. We find that CRMT investment decreases the need for the central bank to intervene, no matter the beliefs of private agents and the government. Accounting for financial stability concerns, beliefs of private agents lead to a second externality next to the distorted capital allocation from the central bank's perspective, as they trigger bank recapitalization or increase its costs. This generally leads to a trade-off for the central bank between correcting the capital allocation and eliminating bank recapitalization. However, we also show that if the central bank is equipped with an additional monetary policy instrument in the form of quantity restrictions on reserve loans, it can always resolve both belief-driven externalities. Finally, we show that CAROs can be used to achieve any target allocation in the economy, which might follow from a coherence argument with fiscal policies.

Our analysis is a first attempt to formally analyze central bank refinancing operations taking climate risk into account. Similar to CAROs, the pricing of central bank reserves can be conditioned on other characteristics of bank assets. In particular, if central bank operations should take climate considerations into account, other criteria may be used, such as emission intensity or a taxonomy. Our framework can also be extended along other dimensions. First, we did not account for capital accumulation—and potentially for other dynamics—as we focused on a static environment. Second, we used a classical setup without any price rigidities and thus cannot study how CAROs and the resulting economic effects are linked to inflation. Third, we restricted firms to relying on loans from banks and did not account for other sources of financing, such as from the financial markets. The investigation of these aspects is left to future research.

Chapter 6

Appendices

6.1 Appendix for Chapter 2

6.1.1 Flow consistency

In the following, we detail the flow consistency of our model for the case where banks issue deposits and bond-financed firms operate ($0 < \gamma < 1$). Note that bond investments are, as any other transaction in our economy, settled by using deposits. Thus, firms issuing bonds receive deposits from households and investors, which they can use to purchase capital good. However, deposits enter the economy only through loan financing by banks. Thus, before bonds can be purchased, loans must have been granted to firms and the respective firms must have used (some of) these deposits to purchase capital good from households and investors.

On this account, transactions on the good markets and bond financing proceed as follows. Loan-financed firms purchase capital good K^L, so that households and investors receive deposits in the amount QK^L. If households and investors decide to invest into bonds, (some of) these deposits are used to purchase bonds, so that bond-financed firms can acquire capital good. With the purchase of capital good by bond-financed firms, deposits flow back to households and investors. Clearly, if the deposits available to purchase bonds are less than the overall amount of required bond financing, QK^B, bond issuance and capital good purchase by bond-financed firms must be organized in several rounds. Assuming that firms, households and investors always use all their deposits at hand to settle transactions, the minimum number of rounds is given by

$$\sigma_1 = \left\lceil \frac{QK^B}{\xi QK^L} \right\rceil,$$

where $\lceil x \rceil$ denotes the least integer greater than or equal to x, and ξ is the share of deposits in the economy available for bond financing. If households and investors both invest into bonds, ξ takes the value of one, as all deposits in the economy can be used to purchase bonds issued by firms. If either households or banks are willing to invest into bonds and both sold already capital good to loan-financed firms, ξ takes a value less than one and essentially depends on the amount of deposits available to the respective bond investor, which, in turn, depends on how much capital good has been sold in the first place to loan-financed firms.

A similar process of transaction settlement must take place on the market for consumption good. Households and investors must use the available deposits to purchase consumption good from bond-financed firms, which then use the proceeds to meet the repayment obligations on bonds. The total amount of deposits in the economy, credited with interest, is given by $(1 + r^D)QK^L$, so that the purchase of consumption good and bond repayment must be organized in at least

$$\sigma_2 = \left\lceil \frac{PA^B K^B}{(1 + r^D)QK^L} \right\rceil,$$

rounds. The parameters σ_1 and σ_2 are irrelevant for our model analysis, but have been derived to illustrate that our model is flow consistent, particularly when taking the assumption that any kind of transaction is settled instantaneously by using bank deposits into account.

6.1.2 Optimal monetary policy with uncertainty about beliefs and large costs of bank default

In this section, we provide additional results on the optimal monetary policy in the case where the central bank is uncertain about the beliefs in the economy. As in section 2.6, the beliefs can be of two types, as captured by the distortion factors $\underline{m} \in (0, 1/\eta)$ and $\overline{m} \in (0, 1/\eta)$, satisfying $\underline{m} < \overline{m}$. In section 2.6, we only provided the analytical results for the case where the expected productivity of loans-financed firms is, even after accounting for costs due to bank default, higher than the productivity of bond-financed firms, i.e., $\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L > 0$, or, equivalently, $\lambda < \lambda^S$ (see assumption 2.6.1). The analytical results derived under this assumption are provided in proposition 2.6.1, corollary 2.6.1 and proposition 2.6.2. In the following, we provide the results on the optimal monetary if the previous assumption does not hold and default costs are sufficiently large. Thus, when taking default costs into account, loan-financed firms are in expectation weakly less productive than bond-financed firms.

Assumption 6.1.1 (Default Costs)
$\mathbb{E}_[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L \leq 0$ *or, equivalently,* $\lambda \geq \lambda^S$.

From section 2.4, we know that the critical default cost parameters satisfy $\lambda^S > \lambda^M = (1-\varphi^S/\varphi^M)\lambda^S$. Using proposition 2.5.1 and assumption 6.1.1, it follows that, with perfect knowledge about the beliefs m, the central bank would optimally set the haircut ψ_m^S, restricting bank leverage below the maximum feasible level and eliminating bank default. Under assumption 6.1.1, the central bank's optimization problem is still described by lemma 2.6.1.

We make similar observations as in section 2.6. First, the smallest feasible haircuts in the case of the more pessimistic and the more optimistic beliefs, respectively, satisfy $\psi_{\underline{m}}^M < \psi_{\overline{m}}^M$. Ex-ante, before the actual beliefs in the economy are revealed, the central bank cannot choose any haircut that is smaller than $\psi_{\overline{m}}^M$, as such a haircut would rule out the existence of an equilibrium if indeed, the more optimistic beliefs $\overline{m}$ realize. Accordingly, the smallest possible haircut the central bank can choose is $\psi_{\overline{m}}^M$.

Second, note that the haircuts ruling out solvency risk for the banks in the presence of the more optimistic and more pessimistic beliefs, respectively, satisfy $\psi_{\underline{m}}^S < \psi_{\overline{m}}^S$. With the haircut $\psi_{\overline{m}}^S$, bank default is eliminated independent of the beliefs. Based on assumption 2.3.1, a loan-financed firm is, under true beliefs, weakly more productive in expectation than a bond-financed firm. Accordingly, the central bank has no incentive to set a haircut that is larger than $\psi_{\overline{m}}^S$, as it only restricts bank lending but yields no benefit, such as eliminating solvency risk, for instance.

Third, the central bank will never choose a haircut that triggers bank default for both types of beliefs, as, based on our assumption on default costs (see assumption 6.1.1), such a monetary policy is clearly welfare-reducing compared to any monetary policy that simply eliminates bank default for both types of beliefs. Note that bank default occurs independent of beliefs if the chosen haircut ψ satisfies $\psi < \psi^S_{\underline{m}}$. Such a haircut choice is only feasible if it holds that $\psi^S_{\underline{m}} > \psi^M_{\overline{m}}$, where, based on our previous explanation, $\psi^M_{\overline{m}}$ is the smallest possible haircut the central bank can choose. For the analysis of the central bank's optimal haircut choice, as outlined in the following proposition, we can thus focus on the closed set $\Psi := [\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}, \psi^S_{\overline{m}}]$.

Proposition 6.1.1 (Optimal Monetary Policy with Uncertainty - Large Costs)
If it holds that $p\varphi^L_{\underline{m}}(\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}) > p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S$*, there exists a*

$$\hat{\psi} \in \arg\max_{\psi \in \Psi} \lambda^M_{BU}(\psi) := \lambda^S \left\{ 1 - \frac{\varphi^S}{\varphi^L_{\overline{m}}(\psi)} + \frac{p}{1-p} \frac{\varphi^L_{\underline{m}}(\psi) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})}{\varphi^L_{\overline{m}}(\psi)} \right\},$$

with $\lambda^M_{BU}(\hat{\psi}) > \lambda^S$*, so that the central bank optimally chooses* $\hat{\psi}$ *whenever* $\lambda < \lambda^M_{BU}(\hat{\psi})$*, accepting bank default for the more optimistic beliefs* $\overline{m}$*. Otherwise, the central bank optimally chooses the haircut* $\psi^S_{\overline{m}}$*, eliminating bank default for all possible beliefs.*

In the case where $\lambda = \lambda^S$*, the central bank optimally chooses the haircut* $\psi = \max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}$ *if and only if* $p\varphi^L_{\underline{m}}(\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}) > p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S$*, accepting bank default for the more optimistic beliefs* $\overline{m}$*. Otherwise, the central bank optimally chooses the haircut* $\psi^S_{\overline{m}}$*, eliminating bank default for all possible beliefs.*

6.1.3 Pessimism

In this section, we provide the model analysis in the presence of sufficiently pessimistic beliefs. Specifically, private agents—firms, households, investors, and banks—believe that a loan-financed firm is on average weakly less productive than a bond-financed firm.

Assumption 6.1.2 (Beliefs)
$\mathbb{E}_m[A^L_s] < A^B$.

Two fundamental questions are whether banks are willing to finance loans with deposits and whether investors are willing to provide equity financing. First, note that due to the equilibrium conditions on the firms' repayment obligations—see conditions (2.5) in subsection 2.4.2 and the equality of deposit and bond rate—assumption 2.4.1 implies that the expected loan rate is lower than the deposit rate, i.e., it holds that $\mathbb{E}_m[r^L_s] < r^D$.

From lemma 2.3.4, we know that in any such situation, the bank is only willing to grant loans and finance them with deposits if it makes profits if the financed firm incurs a positive productivity shock, i.e., $r^L_{\overline{s}} > r^D$, and it can leverage sufficiently, i.e., $\varphi^L_m(\psi) > [(1+\mathbb{E}_m[r^L_s])/\eta_m - 1 - r^D]/(r^L_{\overline{s}} - r^D)$. The first condition $r^L_{\overline{s}} > r^D$ translates with the equilibrium conditions $(1+r^L_s)q = A^L_s$, with $s \in \{\underline{s}, \overline{s}\}$, and $(1+r^B)q = (1+r^D)q = A^B$ (see conditions (2.5) and (2.6) in subsection 2.4.2) into $A^L_{\overline{s}} > A^B$, which is always satisfied, based on our assumptions on firm productivity (see assumption 2.3.1 in subsection 2.3.3). The second condition translates with the previous equilibrium conditions into

$$\varphi^L_m(\psi) > \frac{\mathbb{E}_m[A^L_s] - \eta_m A^B]}{\eta_m(A^L_{\overline{s}} - A^B)},$$

which, after some rearranging, yields the condition $\psi < \hat{\psi}_m$, with $\hat{\psi}_m$ being provided in lemma 6.1.1.

Second, the investor is providing equity financing for the bank ($\zeta = 1$) if and only if the expected rate of return on bank equity weakly exceeds the interest rate on bonds, i.e., $\mathbb{E}_m[r^E_s] \geq r^B$. Using the equilibrium conditions $(1+r^L_s)q = A^L_s$, with $s \in \{\underline{s}, \overline{s}\}$, and $(1+r^B)q = (1+r^D)q = A^B$, the latter inequality translates into $\psi < \tilde{\psi}_m$, with $\tilde{\psi}_m$ being provided in lemma 6.1.1.

Lemma 6.1.1 (Bank Leverage and Equity Financing)
The bank chooses the maximum (minimum) leverage $\varphi = \varphi^L_m(\psi)$ ($\varphi = 1$) if and only if it holds that

$$\psi < (\geq)\hat{\psi}_m := 1 - \frac{\alpha(A^L_{\underline{s}} - \eta_m A^L_{\overline{s}})}{\mathbb{E}_m[A^L_s](1-\eta_m)},$$

and the investor provides (no) equity financing $\zeta = 1$ ($\zeta = 0$) if and only if it holds that

$$\psi \leq (>)\tilde{\psi}_m := 1 - \frac{\alpha(A^B - \eta_m A^L_{\overline{s}})}{\mathbb{E}_m[A^L_s](1-\eta_m)}. \tag{6.1}$$

Furthermore, it holds that $\tilde{\psi}_m < \hat{\psi}_m$.

In section 2.4, we outlined that the clearing of the equity market and the capital good market leads us to real bank lending $K^L = \varphi\zeta E$. Using lemma 6.1.1, we can then deduce that real bank lending is given by $K^L = \mathbb{1}\{\psi \leq \tilde{\psi}_m\}\varphi^L_m(\psi)E$.

Optimal monetary policy. We now characterize the optimal monetary policy as represented by the interest rate on reserves r_{CB}^D and the haircut ψ that applies to bank loans pledged as collateral for reserve loans. As in section 2.3, the central bank perfectly knows the beliefs in the economy when deciding about the monetary policy and chooses its instruments in order to maximize utilitarian welfare. Again, the interest rate on reserves r_{CB}^D affects, in conjunction with firm productivity A^B in the bond-financed sector, the prices in our economy (see equation (2.8) in subsection 2.4.2) but not the real allocation. With the haircut on bank loans used as collateral, the central bank can regulate the banks' access to liquidity, i.e., their ability to borrow reserves. As the liquidity constraint, which depends on the haircut ψ, influences the banks' initial decision to grant loans financed through deposit issuance, the central bank is able to affect bank lending and the allocation of capital good in the economy. Taking the irrelevance of the interest rate r_{CB}^D for the real allocation into account, the optimization problem of the central bank is formally given by

$$\max_{\psi\in[0,1]} W = \max_{\psi\in[0,1]} \{\mathbb{E}[A_s^L] - \mathbb{1}\{\varphi > \varphi^S\}(1-\eta)\lambda A_{\underline{s}}^L\}K^L + A^B(K + E - K^L),$$

where real bank lending is given by $K^L = \mathbb{1}\{\psi \leq \tilde{\psi}_m\}\varphi_m^L(\psi)E$.

We can rewrite the optimization problem of the central bank, as outlined in the following lemma. First, we exploit that the fact that bank lending only occurs if the haircut satisfies $\psi \leq \tilde{\psi}_m$ (see lemma 6.1.1). Specifically, note that the condition for equity financing by the investor ($\psi \leq \tilde{\psi}_m$) is stricter than the condition for banks granting loans funded with deposits ($\psi < \hat{\psi}_m$). Second, we use a result from our analysis in section 2.3, stating that the bank is exposed to a solvency risk if and only if $\psi < \psi_m^S$ (see proposition 2.4.1). Third, we can show that, in the presence of sufficiently pessimistic beliefs (see assumption 6.1.2), whenever the bank is issuing deposits and the investor is providing equity financing, the bank is exposed to a solvency risk. Formally, the critical haircuts satisfy $\tilde{\psi}_m < \psi_m^S$. These three observations allow us to provide an alternative characterization of the central bank's optimization problem, as stated in the following lemma.

Lemma 6.1.2 (The Central Bank's Optimization Problem - Pessimism)
The central bank's optimization problem is

$$\max_{\psi\in[\psi_m^M,1]} \{\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L\}\mathbb{1}\{\psi \leq \tilde{\psi}_m\}\varphi_m^L(\psi).$$

With sufficiently pessimistic beliefs in the economy, the central bank faces generally two options for the implementation of its monetary policy. First, it can set loose collateral requirements in the form of a small haircut, allowing banks to leverage sufficiently and ultimately incentivizing them to grant loans funded with deposits. Second, it can set strict collateral requirements in the form of a large haircut, ruling out equity financing by investors and ultimately bank lending. The first option is preferred over the second one whenever the costs associated with bank default are sufficiently small. Formally, it must hold that $\lambda < \lambda^S$. The first option, however, is only possible if the central bank can indeed set sufficiently loose collateral requirements, while ensuring the existence of an equilibrium. Formally, the first option is feasible whenever the smallest feasible haircut satisfies $\psi_m^M < \tilde{\psi}_m$.

Proposition 6.1.2 (Optimal Monetary Policy — Pessimism)
The central bank optimally sets the haircut ψ_m^M if and only if banks can leverage enough, so that they have incentives to finance loans with deposits and investors are willing to provide equity financing, i.e., it holds that $\psi_m^M < \tilde{\psi}_m$, and default costs are sufficiently small, i.e., it holds that $\lambda < \lambda^S$. Otherwise, the central bank optimally sets the haircut $\psi = 1$, thus eliminating bank default.

Whenever the central bank optimally aims at restricting bank lending to its minimum, so that the optimal haircut is given by $\psi = 1$, monetary policy is independent of the beliefs or economic fundamentals. Instead, if the central bank aims at maixmizing bank lending and thereby accepts bank default, the optimal haircut ψ_m^M varies with beliefs and economic fundamentals, as firm productivity, for instance. For more details, see corollaries 2.5.1 and 2.5.2 in section 2.5.

6.1.4 Simulations for a continuous belief set

In this subsection, we assume that the central bank faces uncertainty of beliefs, where the set of potential distortion factors is continuous and given by $\mathcal{M} := [\underline{m}, \overline{m}]$, with $\underline{m} > 0$ and $\overline{m} < 1/\eta$. The most pessimistic beliefs $\underline{m}$ are, however, such that they still comply with assumption 2.4.1 in section 2.4. Specifically, private agents must still believe that the loan-financed sector is weakly more productive than the bond-financed one under the most pessimistic beliefs $\underline{m}$.

The default costs are sufficiently large, so that with knowledge of the actual beliefs in the economy, the central bank would optimally eliminate bank default. In other words, assumption 2.6.1 in section 2.6 also applies for the subsequent analysis. The central bank has a uniform prior about the possible distortion factors in the set $\mathcal{M}$ and chooses the haircut ψ to maximize expected welfare

$$\int_{\underline{m}}^{\overline{m}} \frac{W_m(\psi)}{\overline{m} - \underline{m}} dm,$$

where, based on lemma 2.6.1, for a specific haircut ψ set by the central bank and a distortion factor m, welfare $W_m(\psi)$ is given by

$$W_m(\psi) = \{\mathbb{E}[A_s^L] - \mathbb{1}\{\psi < \psi_m^S\}(1-\eta)\lambda A_{\underline{s}}^L\}\varphi_m^L(\psi)E + A^B(K + E - \varphi_m^L(\psi)E).$$

We can further simplify the central bank's optimization problem by focusing only on those terms which depend on the haircut ψ, so that the optimization problem is ultimately given by

$$\max_{\psi \in [\psi_{\overline{m}}^M, 1]} \int_{\underline{m}}^{\overline{m}} \left\{\mathbb{E}[A_s^L] - A^B - \mathbb{1}\{\psi < \psi_m^S\}\lambda(1-\eta)A_{\underline{s}}^L\right\} \frac{\varphi_m^L(\psi)}{\overline{m} - \underline{m}} dm.$$

In the following, we provide simulations that illustrate the dependence of the optimal haircut ψ on the default costs (λ), the set of possible beliefs ($\underline{m}$, $\overline{m}$), and the productivity in the bond-financed sector (A^B). We provide results for small and large default costs. Specifically, we assume a default cost parameter $\lambda = 0.3$ and $\lambda = 0.5$, respectively. If not stated otherwise, the baseline for the parameter specification is the one provided in table 2.1 in section 2.6.

In the following graphs, the orange solid line illustrates the smallest feasible haircut $\psi_{\overline{m}}^M$, the dashed black line represents the optimal haircut ψ and the dotted green line depicts the smallest possible haircut $\psi_{\overline{m}}^S$ guaranteeing solvency of banks in all states if the most optimistic beliefs realize, as captured by the distortion factor $\overline{m}$. The graphs on the left hand side follow from simulations with low default costs (i.e., $\lambda = 0.3$), whereas the graphs on the right hand side follow from simulations with high default costs (i.e., $\lambda = 0.5$).

First, we study the effect of belief uncertainty on the optimal monetary policy in terms of the haircut ψ on bank loans. Figure 6.1 illustrates the effect of increasing uncertainty about beliefs, as represented by the spread between distortion factors $\overline{m} - \underline{m}$, with the lower bound $\underline{m}$ and the upper bound $\overline{m}$ being symmetrically centered around one. It can be observed that the central bank switches, with beliefs becoming sufficiently different, i.e., with the spread $\overline{m} - \underline{m}$ being sufficiently large, from the avoidance of bank default (achieved by setting the haircut $\psi^S_{\overline{m}}$) to the avoidance of deficient bank lending (achieved by setting the haircut $\psi^M_{\overline{m}}$). This, however, only holds if default costs are sufficiently low (i.e., $\lambda = 0.3$). With high default costs, which in our setting are represented by the default cost parameter $\lambda = 0.5$, the central bank does not deviate from its objective of ruling out bank default for any considered spread of distortion factors. Similar effects exist if the varying uncertainty about beliefs only stems from a different upper or lower bound on beliefs, see figure 6.2 and figure 6.3, respectively.

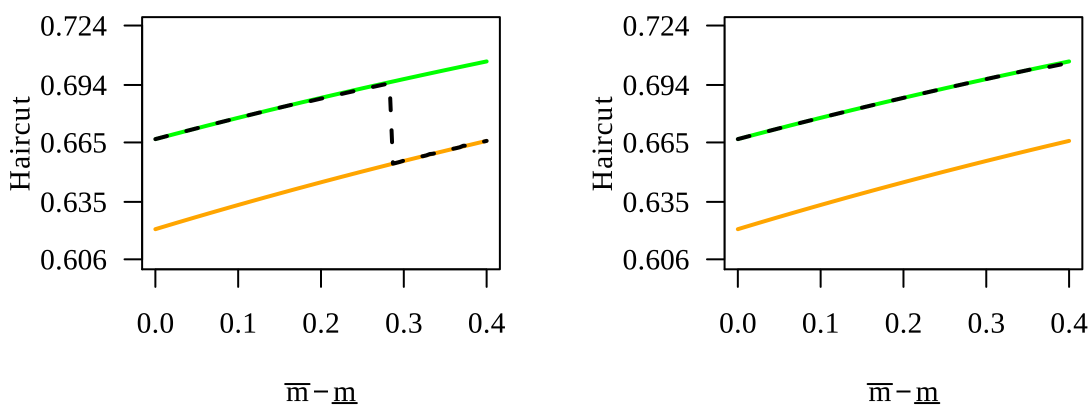

Figure 6.1: Varying uncertainty about a continuous belief set ranging from $\underline{m}$ to $\overline{m}$ symmetrically centered around one.

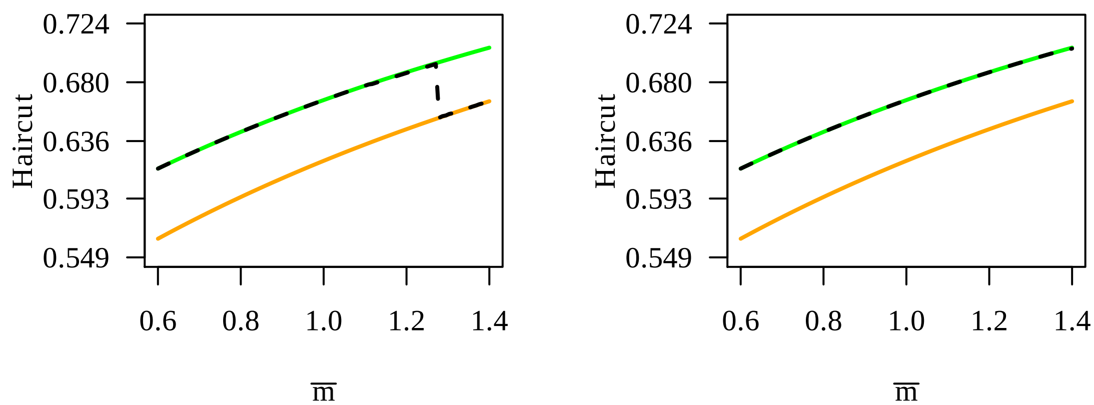

Figure 6.2: Varying uncertainty about a continuous belief set ranging from $\underline{m} = 0.5$ to $\overline{m}$.

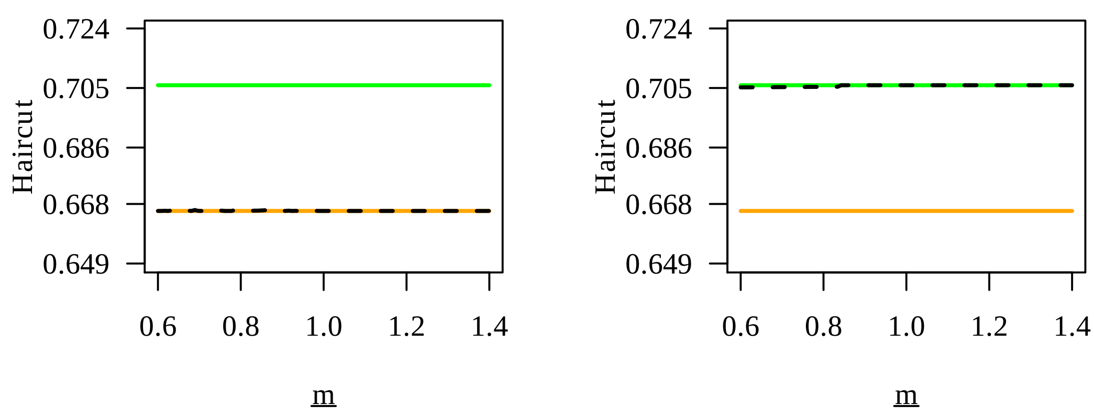

Figure 6.3: Varying uncertainty about a continuous belief set ranging from $\underline{m}$ to $\overline{m} = 1.5$.

Last, we study how the costs of deficient lending, as measured by the productivity difference $\mathbb{E}[A_s^L] - A^B$, influence the optimal monetary policy. Specifically, we analyze the effect of the productivity of bond-financed firms, denoted by A^B, on the central bank's choice of the haircut. It follows that with a relatively large productivity difference, i.e., if deficient lending is relatively costly compared to bank default, the central bank wants to avoid restrictions on bank lending and sets the smallest feasible haircut $\psi_{\overline{m}}^M$. In turn, if the productivity difference $\mathbb{E}[A_s^L] - A^B$ is sufficiently small, the central bank switches its objective to the avoidance of bank default and accordingly sets a haircut $\psi_{\overline{m}}^S$. This effect, however, only exists if default costs are sufficiently small (graph on the left hand side). If default costs are large, i.e., $\lambda = 0.5$, it follows that the central bank always wants to eliminate bank default for all potential distortion factors and thus sets the haircut $\psi_{\overline{m}}^S$ (see graph on the right hand side).

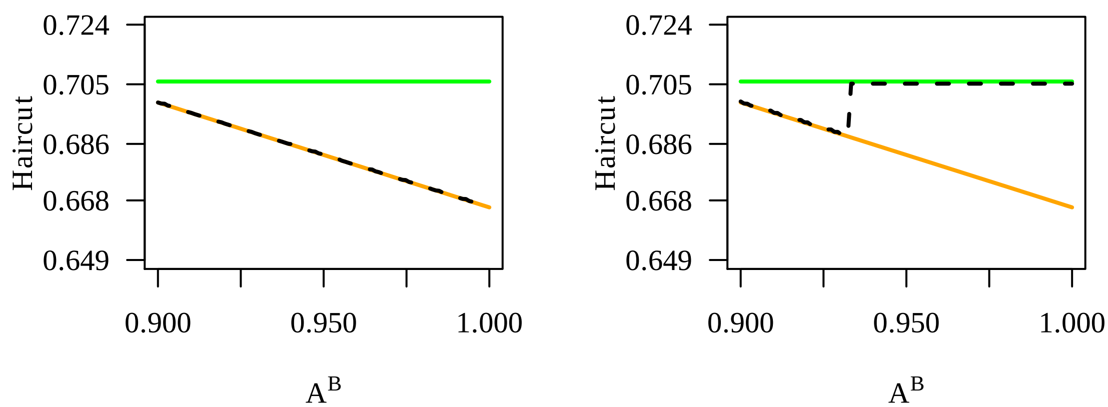

Figure 6.4: Varying productivity A^B of bond-financed firms.

6.1.5 Proofs

Proof of Lemma 2.3.1. Firms are penniless and operate under limited liability, so that they are fully protected from losses. Accordingly, if the firm $f \in \{L, B\}$ is facing excess returns in one of the states, i.e., $A_s^f > (1+r_s^f)q$ for some $s \in \{\underline{s}, \overline{s}\}$, the expected profits are increasing with the input K^f of capital good to production. Thus, there exists no optimal, finite demand for capital good by firm f, which we denote by $K^f = +\infty$. In contrast, without excess returns, i.e., $A_s^f \leq (1 + r_s^f)q$ for all $s \in \{\underline{s}, \overline{s}\}$, the firm f is making zero profits for any production input due to limited liability. Accordingly, the firm is indifferent between any amount of capital good put into production and the optimal demand is given by $K^f \in [0, \infty)$. ■

Proof of Lemma 2.3.2. Due to our assumption of linear utility, the household ultimately aims at maximizing consumption $C^H = [\gamma(1 + r^D) + (1 - \gamma)(1 + r^B)]qK + \tau^H + \pi$. The optimal choice of the household is thus of knife-edge type, namely the household invests the revenues from capital good sales in the asset which yields the highest return. If the deposit rate exceeds the bond rate ($r^D > r^B$), the household only holds deposits and if the bond rate exceeds the deposit rate ($r^D < r^B$), the household only invests into bonds. Otherwise ($r^D = r^B$), the household is indifferent between holding deposits and investing into bonds ($\gamma \in [0, 1]$). ■

Proof of Lemma 2.3.3. Due to our assumption of linear utility, the investor ultimately aims at maximizing consumption $C_m^I = [\zeta(1 + \mathbb{E}_m[r_s^E]) + (1 - \zeta)(1 + r^B)]qE + \tau^I$. The optimal choice of the investor is thus of knife-edge type, namely the investor uses the revenues from capital good sales to invest into the asset which yields the highest expected return. The investor's optimal choice satisfies $\zeta = 1$ if the expected rate of return on equity exceeds the one on bonds ($\mathbb{E}_m[r_s^E] > r^B$), and $\zeta = 0$, if the bond rate exceeds the expected rate of return on equity ($\mathbb{E}_m[r_s^E] < r^B$). Otherwise, the investor is indifferent between investing into bank equity and investing into bonds. In this particular case ($\mathbb{E}_m[r_s^E] = r^B$), we assume for simplicity that the investor uses all funds for investment into bank equity ($\zeta = 1$). ■

Proof of Lemma 2.3.4. First, we focus on the situation where the bank cannot face a solvency risk, as it holds that $\varphi_m^L(\psi) \leq \varphi^S$. In this case, the protection from losses through limited liability is not relevant, so that the expected rate of return on bank equity is given by

$$\mathbb{E}_m[r_s^E(\varphi)] = \mathbb{E}_m[(r_s^L - r^D)\varphi + 1 + r^D] - 1 = (\mathbb{E}_m[r_s^L] - r^D)\varphi + r^D.$$

The expected rate of return on bank equity is maximized for the leverage $\varphi = \varphi_m^L(\psi)$ if the loan and deposit rates satisfy $\mathbb{E}_m[r_s^L] > r^D$, and $\varphi = 1$ if it holds that $\mathbb{E}_m[r_s^L] < r^D$. If the expected interest rate on loans equals the interest rate on deposits ($\mathbb{E}_m[r_s^L] = r^D$), the bank is indifferent between all leverages and the optimal choice is given by $\varphi \in [1, \varphi_m^L(\psi)]$. For simplicity, we assume that in any situation where the bank is indifferent, it chooses the maximum leverage, so that it holds that $\varphi = \varphi_m^L(\psi)$. Accordingly, we can state that, without the possibility of solvency risk, the bank chooses $\varphi = \varphi_m^L(\psi)$ ($\varphi = 1$) if and only if it holds that $\mathbb{E}_m[r_s^L] \geq (<) r^D$.

Second, we focus on the situation where the bank can face a solvency risk, as it holds that $\varphi_m^L(\psi) > \varphi^S$. A necessary condition for solvency risk is that the bank is making losses for a negative productivity shock of the financed firm. Thus, the interest rates on deposits and loans must satisfy $r^D > r_{\underline{s}}^L$. Taking the limited liability into account, with the possibility of a solvency risk, the expected rate of return on bank equity satisfies

$$\begin{aligned}\mathbb{E}_m[r_s^E(\varphi)] &= \mathbb{E}_m[\max\{(r_s^L - r^D)\varphi + 1 + r^D, 0\}] - 1 \\ &= \eta_m[(r_{\bar{s}}^L - r^D)\varphi + 1 + r^D] + \mathbb{1}\{\varphi \leq \varphi^S\}(1-\eta_m)[(r_{\underline{s}}^L - r^D)\varphi + 1 + r^D] - 1.\end{aligned}$$

If financing loans with deposits is profitable, even without benefiting from limited liability, i.e., $\mathbb{E}_m[r_s^L] \geq r^D$, the expected rate of return on bank equity is maximized for the largest possible leverage which guarantees liquidity, i.e., $\varphi = \varphi_m^L(\psi)$. This is due to the fact that if the financed firm incurs a negative productivity shock ($s = \underline{s}$), the bank makes losses (as $r^D > r_{\underline{s}}^L$) until the leverage is sufficiently high, so that the bank defaults and is protected from additional losses due to limited liability, while the bank makes always profits if the financed firm incurs a positive productivity shock, as it holds that $r_{\bar{s}}^L > r^D$.

Similarly, the expected rate of return on bank equity is maximized for $\varphi = \varphi_m^L(\psi)$ if without the benefits from limited liability financing loans with deposits is not profitable ($\mathbb{E}_m[r_s^L] < r^D$), but there are excess returns from loan financing if the financed firm incurs a positive productivity shock ($r_{\bar{s}}^L > r^D$), and the bank can leverage sufficiently, so that the expected equity return under the maximum leverage and default in the case where the financed firm incurs a negative productivity shock ($s = \underline{s}$) outweighs the expected equity return when financing loans solely with equity ($\varphi = 1$), i.e., it holds that

$$\eta_m[(r_{\bar{s}}^L - r^D)\varphi_m^L(\psi) + 1 + r^D] - 1 > \mathbb{E}_m[r_s^L] \quad \Leftrightarrow \quad \varphi_m^L(\psi) > \frac{(1 + \mathbb{E}_m[r_s^L])/\eta_m - (1 + r^D)}{r_{\bar{s}}^L - r^D}.$$

In all other cases with the possibility of solvency risk ($\varphi^L_m(\psi) > \varphi^S$), the expected rate of return on bank equity is maximized for the smallest possible leverage $\varphi = 1$. ■

Proof of Lemma 2.3.5. From lemma 2.3.4, we know that the bank is either financing loans with deposits and is liquidity-constrained as it chooses the maximum possible leverage $\varphi = \varphi^L_m(\psi)$ or finances loans solely with equity ($\varphi = 1$) and does not require any liquidity. We first focus on the situation where banks issue deposits and leverage as much as possible without risking liquidity. Note that the liquidity demand of the bank is given by $L^{CB} = \alpha(L^b - E^b)$. Thus, when borrowing liquidity from the central bank, the bank faces the liquidity constraint

$$(1-\psi)(1+\mathbb{E}_m[r^L_s])L^b \geq (1+r^D_{CB})L^{CB}.$$

The repayment of the borrowed liquidity is determined by the interest rate on reserves r^D_{CB}. In turn, when borrowing liquidity on the interbank market, the bank faces the liquidity constraint

$$(1-\tilde{\psi})(1+\mathbb{E}_m[r^L_s])L^b \geq (1+r^D)L^{CB},$$

where the repayment of interbank loans is determined by the interest rate r^D. As the bank is liquidity-constrained, the interbank market can only be active if the liquidity supply from other banks is weakly exceeding the liquidity supply from the central bank, i.e.,

$$\frac{(1-\psi)(1+\mathbb{E}_m[r^L_s])L^b}{1+r^D_{CB}} \leq \frac{(1-\tilde{\psi})(1+\mathbb{E}_m[r^L_s])L^b}{1+r^D}$$

$$\Leftrightarrow \quad (1+r^D)(1-\psi) \leq (1+r^D_{CB})(1-\tilde{\psi}). \tag{6.2}$$

Like reserves, interbank deposits are used to settle interbank liabilities. The bank which granted an interbank loan must therefore ensure that if the interbank deposits, which have been created when the interbank loan was granted, are transferred to other banks, the liquidity (in the form of reserves) to settle the resulting interbank liability is available. If interbank deposits are transferred, the bank can use the pledged bank loans and rehypothecate them, namely use it as collateral at the central bank to borrow reserves. The maximum amount of liquidity that can be obtained by the bank, using the collateral

$(1+\mathbb{E}_m[r_s^L])L^b$ associated with interbank loans, is given by $(1-\psi)(1+\mathbb{E}_m[r_s^L])L^b/(1+r_{CB}^D)$. Hence, when interbank loans are granted, it must hold that

$$\frac{(1-\tilde{\psi})(1+\mathbb{E}_m[r_s^L])L^b}{1+r^D} \geq \frac{(1-\psi)(1+\mathbb{E}_m[r_s^L])L^b}{1+r_{CB}^D}$$

$$\Leftrightarrow \qquad (1+r^D)(1-\psi) \geq (1+r_{CB}^D)(1-\tilde{\psi}). \tag{6.3}$$

From equations (6.2) and (6.3), it follows that $(1+r^D)(1-\psi) = (1+r_{CB}^D)(1-\tilde{\psi})$. For any situation where the bank is financing loans only with equity, it issues no deposits, so that liquidity in the form of central bank reserves is irrelevant. We thus assume that when the bank chooses the smallest possible leverage $\varphi = 1$, it also holds $(1+r^D)(1-\psi) = (1+r_{CB}^D)(1-\tilde{\psi})$. ■

Proof of Corollary 2.3.1. Lemma 2.3.5 states $(1+r^D)(1-\psi) = (1+r_{CB}^D)(1-\tilde{\psi})$. For any $\psi \in [0,1)$, imposing $\tilde{\psi} = \psi$ then yields that the interest rates on deposits and reserves are equal ($r^D = r_{CB}^D$). For $\psi = 1$, the central bank does not provide any liquidity, so that the bank will finance loans only with equity and without deposits. Accordingly, the interest rate on deposits does not play any role for the real allocation in the economy. In the case $\psi = 1$, we thus assume that it also holds that $r^D = r_{CB}^D$. ■

Proof of Lemma 2.4.1. Due to our assumption of linear utility, utilitarian welfare represents aggregate consumption. Welfare $W = C^H + C^I$ can then be rewritten as

$$W = [\gamma(1+r^D) + (1-\gamma)(1+r^B)]qK + \tau^H + \pi$$
$$+ [\zeta(1+\mathbb{E}[r_s^E(\varphi)]) + (1-\zeta)(1+r^B)]qE + \tau^I,$$

where we used the expression for aggregate consumption of households and investors (see subsection 2.4.2).

First, note that, in our model, it holds that the interest rates on bonds, deposits and reserves are equal, i.e., $r^B = r^D = r_{CB}^D$. Moreover, using the conditions on the firms' repayment obligations, i.e., $(1+r_s^L)q = A_s^L$, with $s \in \{\underline{s}, \overline{s}\}$, and $(1+r^B)q = A^B$ (see conditions (2.5) in subsection 2.4.2), firms make zero profits ($\pi = 0$). Welfare thus reads as

$$W = (1 + r^D_{CB})qK + \tau^H + [\zeta(1 + \mathbb{E}[r^E_s(\varphi)]) + (1 - \zeta)(1 + r^D_{CB})]qE + \tau^I.$$

Second, we focus on the governmental taxes. Note that there is a representative bank, so that after the transactions on the capital good market have been settled, reserve deposits and reserve loans are equal. Moreover, based on assumption 2.3.2, the interest rates on reserve deposits and reserve loans equal. Thus, reserve loans do not bear any risk, since the bank's balance of reserve deposits always matches the repayment obligation on reserve loans, independent of the idiosyncratic productivity shock incurred by the financed firm. The central bank thus makes neither profits nor losses ($\Pi^{CB} = 0$), and the taxes imposed by the government must cover only liabilities arising from the deposit insurance and the costs due to the resolution of bank default. Specifically, the governmental taxes T in nominal terms are given by

$$T = \Pi^{b-} - P\Lambda = \{(1 - \eta)[(r^L_{\underline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}]E^b - P(1 - \eta)\lambda A^L_{\underline{s}} K^L\}\mathbb{1}\{\varphi > \varphi^S\},$$

where we used

$$\Pi^{b-} = (1 - \eta)[(r^L_{\underline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}]E^b\mathbb{1}\{\varphi > \varphi^S\}$$

to represent aggregate nominal bank losses in the case of default. If banks are exposed to a solvency risk ($\varphi > \varphi^S$), a mass $1 - \eta$ of banks is defaulting, as the financed firms incur a negative productivity shock ($s = \underline{s}$). The expression for the resolution costs of bank default $P\Lambda$ follows from (2.11) in subsection 2.4.2. Using $T = T^H + T^I$, welfare is then given by

$$\begin{aligned} W = {} & (1 + r^D_{CB})qK + [\zeta(1 + \mathbb{E}[r^E_s(\varphi)]) + (1 - \zeta)(1 + r^D_{CB})]qE \\ & + \{(1 - \eta)[(r^L_{\underline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}]E^b/P - (1 - \eta)\lambda A^L_{\underline{s}} K^L\}\mathbb{1}\{\varphi > \varphi^S\}. \end{aligned}$$

Third, the expected rate of return on bank equity is given by

$$\begin{aligned} \mathbb{E}[r^E_s(\varphi)] &= \mathbb{E}[\max\{(r^L_s - r^D_{CB})\varphi + 1 + r^D_{CB}, 0\}] - 1 \\ &= \eta[(r^L_{\bar{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}] + (1 - \eta)[(r^L_{\underline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}]\mathbb{1}\{\varphi \leq \varphi^S\} - 1, \end{aligned}$$

where we used that, based on corollary 2.3.1 and assumption 2.3.3, the interest rates on deposits and reserves are equal ($r^D = r^D_{CB}$). With the clearing condition for the equity market ($E^b = \zeta QE$), utilitarian welfare reads as

$$\begin{aligned} W &= (1 + r^D_{CB})qK + \{\eta[(r^L_{\overline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}] \\ &\quad + (1 - \eta)[(r^L_{\underline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}]\}\zeta\mathbb{1}\{\varphi \leq \varphi^S\}qE + (1 - \zeta)(1 + r^D_{CB})qE \\ &\quad + \{(1 - \eta)[(r^L_{\underline{s}} - r^D_{CB})\varphi + 1 + r^D_{CB}]\zeta qE - (1 - \eta)\lambda A^L_{\underline{s}} K^L\}\mathbb{1}\{\varphi > \varphi^S\} \\ &= (1 + r^D_{CB})qK + (\mathbb{E}[r^L_s] - r^D_{CB})\varphi\zeta qE + (1 + r^D_{CB})qE - (1 - \eta)\lambda A^L_{\underline{s}} K^L \mathbb{1}\{\varphi > \varphi^S\}. \end{aligned}$$

Using the equilibrium leverage $\varphi = K^L/(\zeta E)$, the conditions $(1+r^B)q = A^B$ and $(1+r^L_s)q = A^L_s$, with $s \in \{\underline{s}, \overline{s}\}$, and $(1 + r^D_{CB})q = A^B$, as stated in subsection 2.4.2, welfare translates into

$$\begin{aligned} W &= A^B K + (\mathbb{E}[A^L_s] - A^B)K^L + A^B E - (1 - \eta)\lambda A^L_{\underline{s}} K^L \mathbb{1}\{\varphi > \varphi^S\} \\ &= \{\mathbb{E}[A^L_s] - \mathbb{1}\{\varphi > \varphi^S\}(1 - \eta)\lambda A^L_{\underline{s}}\}K^L + A^B(K + E - K^L). \end{aligned}$$

■

Proof of Lemma 2.4.2. In equilibrium, firm productivity and firms' repayment rates are linked. Specifically, from conditions (2.5) in subsection 2.4.2, we know that it holds $(1 + r^L_s)q = A^L_s$, with $s \in \{\underline{s}, \overline{s}\}$, and $(1 + r^B)q = A^B$. Moreover, deposit rate and bond rate equal (see condition (2.6) in subsection 2.4.2), so that it holds $(1 + r^D)q = A^B$. With assumption 2.4.1, stating $\mathbb{E}_m[A^L_s] \geq A^B$, we can then conclude that it holds $\mathbb{E}_m[r^L_s] \geq r^D$. Using lemma 2.3.4, it then follows that the bank always chooses the maximum possible leverage $\varphi = \varphi^L_m(\psi)$.

The expected rate of return on bank equity is then given by

$$\begin{aligned} \mathbb{E}_m[r^E_s] &= \mathbb{E}_m[\{(r^L_s - r^D)\varphi^L_m(\psi) + 1 + r^D\}^+] - 1 \\ &= \mathbb{E}_m[\{(A^L_s - A^B)\varphi^L_m(\psi) + A^B\}^+]/q - 1, \end{aligned}$$

where we used $(1 + r^L_s)q = A^L_s$ and $(1 + r^D)q = A^B$. With $(1 + r^B)q = A^B$ and the fact that, based on assumption 2.3.1, it holds $\mathbb{E}_m[A_s^L] \geq A^B$, we know that the rate of return on bank equity as expected by the investor weakly exceeds the rate of return on bonds, i.e., it holds that $\mathbb{E}_m[r_s^E] \geq r^B$. Using lemma 2.3.3, it then follows that the investor uses all available funds to invest into bank equity ($\zeta = 1$). ■

Proof of Proposition 2.4.1. Bank lending must comply with the clearing of the capital good market, so that it holds that $L^b = QK^L = \varphi_m^L(\psi)QE \leq Q(K+E)$ or, equivalently, $\varphi_m^L(\psi) \leq 1 + K/E$. Using (2.10) to express $\varphi_m^L(\psi)$ in terms of the economic fundamentals, we obtain that the inequality $\varphi_m^L(\psi) \leq 1 + K/E$ reads as

$$\frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_m[A_s^L]} \leq 1 + K/E \quad \Leftrightarrow \quad \alpha A^B \leq (1+K/E)\{\alpha A^B - (1-\psi)\mathbb{E}_m[A_s^L]\}.$$

Rearranging yields $(1-\psi)\mathbb{E}_m[A_s^L](1+K/E) \leq \alpha A^B K/E$, which finally leads to another lower bound on the haircut that is given by

$$\psi \geq \frac{\mathbb{E}_m[A_s^L](1+E/K) - \alpha A^B}{\mathbb{E}_m[A_s^L](1+E/K)} \qquad \Leftrightarrow \qquad \psi \geq \psi_m^M := 1 - \frac{\alpha A^B}{\mathbb{E}_m[A_s^L](1+E/K)}.$$

Using equations (2.9) and (2.10), which are provided in subsection 2.4.2, the inequality $\varphi_m^L(\psi) > \varphi^S$ translates into

$$\frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_m[A_s^L]} > \frac{A^B}{A^B - A_{\underline{s}}^L} \qquad \Leftrightarrow \qquad \alpha(A^B - A_{\underline{s}}^L) > \alpha A^B - (1-\psi)\mathbb{E}_m[A_s^L].$$

Rearranging yields $(1-\psi)\mathbb{E}_m[A_s^L] < \alpha A_{\underline{s}}^L$, which finally provides us with a lower bound on the haircut that is given by

$$\psi < \frac{\mathbb{E}_m[A_s^L] - \alpha A_{\underline{s}}^L}{\mathbb{E}_m[A_s^L]} \qquad \Leftrightarrow \qquad \psi < \psi_m^S := 1 - \frac{\alpha A_{\underline{s}}^L}{\mathbb{E}_m[A_s^L]}.$$

■

Proof of Lemma 2.5.1. As the central bank aims at maximizing utilitarian welfare, its optimization problem is generally given by

$$\max_{\psi \in [0,1]} \{\mathbb{E}[A_s^L] - \mathbb{1}\{\varphi > \varphi^S\}(1-\eta)(1-\lambda)A_{\underline{s}}^L\}K^L + A^B(K+E-K^L),$$

where we used lemma 2.4.1 to express welfare.

First, from the outline in subsection 2.4.2 and lemma 2.4.2, we know that it holds that $K^L = \varphi_m^L(\psi)E$.

Second, based on proposition 2.4.1, it holds that $\varphi = \varphi_m^L(\psi) > \varphi^S$ if and only if $\psi < \psi_m^S$.

Third, we know from proposition 2.4.1 that $\varphi = \varphi_m^L(\psi) \leq \varphi^M := 1 + K/E$ if and only if $\psi \geq \psi_m^M$. Accordingly, the central bank is unable to choose any haircut smaller than ψ_m^M.

Omitting all terms that do not depend on the haircut ψ, we can then conclude that the optimization problem of the central bank is

$$\max_{\psi \in [\psi_m^M, 1]} \{\mathbb{E}[A_s^L] - A^B - \mathbb{1}\{\psi < \psi_m^S\}(1-\eta)\lambda A_{\underline{s}}^L\}\varphi_m^L(\psi).$$

■

Proof of Proposition 2.5.1. First, note that without a solvency risk, welfare is maximized for the haircut ψ_m^S, as, based on assumption 2.3.1, a loan-financed firm is weakly more productive on average than a bond-financed firm ($\mathbb{E}_m[A_s^L] \geq A^B$).

Second, from lemma 2.5.1, we know that the central bank then chooses any haircut ψ lower than ψ_m^S if it holds that

$$\{\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L\}\varphi_m^L(\psi) > \{\mathbb{E}[A_s^L] - A^B\}\varphi^S,$$

where we used $\varphi^S = \varphi_m^L(\psi^S)$. Due to the linear structure of the production technologies and the default costs, we can deduce that with a solvency risk, welfare is maximized for the smallest feasible haircut ψ_m^M. Using the notation, we can thus conclude that the central bank only chooses the haircut ψ_m^M, instead of the haircut ψ_m^S, if it holds that

$$\begin{aligned} & \{\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L\}\varphi^M > \{\mathbb{E}[A_s^L] - A^B\}\varphi^S \\ \Leftrightarrow \quad & \{\mathbb{E}[A_s^L] - A^B\}(\varphi^M - \varphi^S) > (1-\eta)\lambda A_{\underline{s}}^L\varphi^M \\ \Leftrightarrow \quad & \frac{\mathbb{E}[A_s^L] - A^B}{(1-\eta)A_{\underline{s}}^L}\left(1 - \varphi^S/\varphi^M\right) =: \lambda^M > \lambda. \end{aligned}$$

A necessary condition for the central bank to optimally choose the haircut ψ_m^M is that the expected productivity difference between the loan-financed sector and the bond-financed sector is positive, even when accounting for the costs originating from bank default, i.e.,

$$\mathbb{E}[A_s^L] - A^B - (1-\eta)\lambda A_{\underline{s}}^L > 0 \qquad \Leftrightarrow \qquad \frac{\mathbb{E}[A_s^L] - A^B}{(1-\eta)A_{\underline{s}}^L} =: \lambda^S > \lambda.$$

We can state $\lambda^M = (1 - \varphi^S/\varphi^M)\lambda^S$. Based on assumption 2.5.1, we know that $\varphi^M > \varphi^S$ and therefore $\lambda^M < \lambda^S$. The condition $\lambda < \lambda^S$ is thus no further restriction for the central bank's choice of the haircut ψ_m^M. Hence, we know that the central bank chooses the haircut ψ_m^S if and only if $\lambda \geq \lambda^M$, and the haircut ψ_m^M otherwise. ■

Proof of Corollary 2.5.1. The haircut ψ_m^S, restricting bank lending and eliminating bank default, satisfies

$$\psi_m^S = 1 - \frac{\alpha A_{\underline{s}}^L}{\mathbb{E}_m[A_s^L]} = 1 - \frac{\alpha A_{\underline{s}}^L}{\eta_m(A_{\overline{s}}^L - A_{\underline{s}}^L) + A_{\underline{s}}^L} = 1 - \frac{\alpha A_{\underline{s}}^L}{\eta m(A_{\overline{s}}^L - A_{\underline{s}}^L) + A_{\underline{s}}^L},$$

where we used the definition $\eta_m = \eta m$, with $m \in (0, 1/\eta)$. The haircut depends on the productivity $\mathbb{E}_m[A_s^L]$ of loan-financed firms, as expected by the bank, and hence depends on the beliefs, as represented by the distortion factor m. Specifically,

$$\frac{\partial \psi_m^S}{\partial m} = -\frac{-\alpha A_{\underline{s}}^L \eta(A_{\overline{s}}^L - A_{\underline{s}}^L)}{[\eta m(A_{\overline{s}}^L - A_{\underline{s}}^L) + A_{\underline{s}}^L]^2} = \frac{\alpha A_{\underline{s}}^L \eta(A_{\overline{s}}^L - A_{\underline{s}}^L)}{[\eta m(A_{\overline{s}}^L - A_{\underline{s}}^L) + A_{\underline{s}}^L]^2} = \frac{\alpha A_{\underline{s}}^L \eta(A_{\overline{s}}^L - A_{\underline{s}}^L)}{(\mathbb{E}_m[A_s^L])^2} > 0.$$

The haircut ψ_m^M, maximizing bank lending and allowing for bank default, satisfies

$$\psi_m^M = 1 - \frac{\alpha A^B}{\mathbb{E}_m[A_s^L](1 + E/K)} = 1 - \frac{\alpha A^B}{[\eta m(A_{\overline{s}}^L - A_{\underline{s}}^L) + A_{\underline{s}}^L](1 + E/K)},$$

where we again used the definition $\eta_m = \eta m$, with $m \in (0, 1/\eta)$. This haircut also depends on the productivity $\mathbb{E}_m[A_s^L]$ of loan-financed firms, as expected by the bank, and hence depends on the beliefs, as represented by the distortion factor m. Specifically,

$$\frac{\partial \psi_m^M}{\partial m} = -\frac{-\alpha A^B \eta(A_{\overline{s}}^L - A_{\underline{s}}^L)(1 + E/K)}{[\eta m(A_{\overline{s}}^L - A_{\underline{s}}^L) + A_{\underline{s}}^L]^2(1 + E/K)^2} = \frac{\alpha A^B \eta(A_{\overline{s}}^L - A_{\underline{s}}^L)}{(\mathbb{E}_m[A_s^L])^2(1 + E/K)} > 0.$$

We can thus conclude that the optimal haircut set by the central bank, either ψ_m^S or ψ_m^M, increases if agents become more optimistic (i.e., m is increasing) and decreases if agents become more pessimistic (i.e., m is decreasing). ■

Proof of Corollary 2.5.2. The haircut ψ_m^S, restricting bank lending and eliminating bank default, is independent of the productivity of bond-financed firms, i.e.,

$$\frac{\partial \psi_m^S}{\partial A^B} = 0.$$

However, it varies with the productivity of loan-financed firms in both states. On the one hand, the derivative of the haircut with respect to $A_{\bar{s}}^L$, the productivity of loan-financed firms in the high productivity state, is given by

$$\frac{\partial \psi_m^S}{\partial A_{\bar{s}}^L} = -\frac{(-\alpha)\eta_m A_{\underline{s}}^L}{(\mathbb{E}_m[A_s^L])^2} = \frac{\alpha \eta_m A_{\underline{s}}^L}{(\mathbb{E}_m[A_s^L])^2} > 0.$$

On the other hand, the derivative with respect to $A_{\underline{s}}^L$, the productivity of loan-financed firms in the low productivity state, is given by

$$\frac{\partial \psi_m^S}{\partial A_{\underline{s}}^L} = -\frac{\mathbb{E}_m[A_s^L]\alpha - \alpha A_{\underline{s}}^L(1-\eta_m)}{(\mathbb{E}_m[A_s^L])^2} = -\frac{\alpha \eta_m A_{\bar{s}}^L}{(\mathbb{E}_m[A_s^L])^2} < 0.$$

The haircut ψ_m^M, maximizing bank lending and allowing for bank default, depends on the productivity of bond-financed firms. The derivative with respect to A^B is given by

$$\frac{\partial \psi_m^M}{\partial A^B} = -\frac{\alpha}{\mathbb{E}_m[A_s^L](1+E/K)} < 0.$$

The haircut also depends on the productivity of loan-financed firms in both states. On the one hand, the derivative with respect to $A_{\bar{s}}^L$ is given by

$$\frac{\partial \psi_m^M}{\partial A_{\bar{s}}^L} = -\frac{(-\alpha)A^B\eta_m(1+E/K)}{[\mathbb{E}_m[A_s^L](1+E/K)]^2} = \frac{\alpha A^B \eta_m}{(\mathbb{E}_m[A_s^L])^2(1+E/K)} > 0.$$

On the other hand, the derivative with respect to $A_{\underline{s}}^L$ is given by

$$\frac{\partial \psi_m^M}{\partial A_{\underline{s}}^L} = -\frac{(-\alpha)A^B(1-\eta_m)(1+E/K)}{[\mathbb{E}_m[A_s^L](1+E/K)]^2} = \frac{\alpha A^B(1-\eta_m)}{(\mathbb{E}_m[A_s^L])^2(1+E/K)} > 0.$$

■

Proof of Lemma 2.6.1. First, from the outline in subsection 2.4.2 and lemma 2.4.2, we know that for any type of beliefs $m \in \{\underline{m}, \overline{m}\}$, it holds that $K^L = \varphi^L_m(\psi)E$.

Second, based on proposition 2.4.1, it holds that $\varphi = \varphi^L_m(\psi) > \varphi^S$ if and only if $\psi < \psi^S_m$.

Third, we know from proposition 2.4.1 that $\varphi = \varphi^L_m(\psi) \leq \varphi^M := 1 + K/E$ if and only if $\psi \geq \psi^M_m$. As it holds that $\psi^M_{\underline{m}} < \psi^M_{\overline{m}}$, the central bank can, under uncertainty about beliefs, not set any haircut smaller than $\psi^M_{\overline{m}}$.

As the central bank aims at maximizing expected utilitarian welfare, its optimization problem is generally given by

$$\max_{\psi \in [\psi^M_{\overline{m}}, 1]} p\{\mathbb{E}[A^L_s] - \mathbb{1}\{\psi > \psi^S_{\overline{m}}\}(1-\eta)(1-\lambda)A^L_{\underline{s}}\}\varphi^L_{\overline{m}}(\psi)E + pA^B(K + E - \varphi^L_{\overline{m}}(\psi)E)$$

$$+ (1-p)\{\mathbb{E}[A^L_s] - \mathbb{1}\{\psi > \psi^S_{\underline{m}}\}(1-\eta)(1-\lambda)A^L_{\underline{s}}\}\varphi^L_{\underline{m}}(\psi)E$$

$$+ (1-p)A^B(K + E - \varphi^L_{\underline{m}}(\psi)E),$$

where we used lemma 2.4.1 to express welfare.

Omitting all terms that do not depend on the haircut ψ, we can then conclude that the optimization problem of the central bank is given by

$$\max_{\psi \in [\psi^M_{\overline{m}}, 1]} (\mathbb{E}[A^L_s] - A^B)\mathbb{E}[\varphi^L_m(\psi)] - (1-\eta)\lambda A^L_{\underline{s}}\mathbb{E}[\mathbb{1}\{\psi < \psi^S_m\}\varphi^L_m(\psi)].$$

■

Proof of Lemma 2.6.2. We say beliefs are distinct if under the smallest feasible haircut $\psi^M_{\overline{m}}$ the bank is not exposed to a solvency risk in the presence of the more pessimistic beliefs $\underline{m}$, as it holds that $\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) \leq \varphi^S$. By using the equations (2.9) and (2.10) in subsection 2.4.2, the latter inequality can be rewritten as

$$\frac{\alpha A^B}{\alpha A^B - (1-\psi^M_{\overline{m}})\mathbb{E}_{\underline{m}}[A^L_s]} \leq \frac{A^B}{A^B - A^L_{\underline{s}}} \quad \Leftrightarrow \quad \alpha(A^B - A^L_{\underline{s}}) \leq \alpha A^B - (1-\psi^M_{\overline{m}})\mathbb{E}_{\underline{m}}[A^L_s],$$

which is equivalent to

$$(1-\psi^M_{\overline{m}})\mathbb{E}_{\underline{m}}[A^L_s] \leq \alpha A^L_{\underline{s}} \qquad \Leftrightarrow \qquad \frac{\alpha A^B \mathbb{E}_{\underline{m}}[A^L_s]}{\mathbb{E}_{\overline{m}}[A^L_s](1+E/K)} \leq \alpha A^L_{\underline{s}},$$

where we, based on proposition 2.4.1, used

$$\psi^M_{\overline{m}} = 1 - \frac{\alpha A^B}{\mathbb{E}_{\overline{m}}[A^L_s](1+E/K)}.$$

Using the definition $\eta_m = \eta m$, further rearranging of the latter inequality yields

$$\begin{aligned} &\mathbb{E}_{\underline{m}}[A^L_s] \leq \mathbb{E}_{\overline{m}}[A^L_s](1+E/K)A^L_{\underline{s}}/A^B \\ \Leftrightarrow \quad & A^L_{\underline{s}} + \eta\underline{m}(A^L_{\overline{s}} - A^L_{\underline{s}}) \leq [A^L_{\underline{s}} + \eta\overline{m}(A^L_{\overline{s}} - A^L_{\underline{s}})](1+E/K)A^L_{\underline{s}}/A^B. \end{aligned}$$

Using the notation $\delta = (1+E/K)A^L_{\underline{s}}/A^B$, we obtain

$$A^L_{\underline{s}}(1-\delta) \leq \eta(A^L_{\overline{s}} - A^L_{\underline{s}})(\delta\overline{m} - \underline{m}) \qquad \Leftrightarrow \qquad \frac{A^L_{\underline{s}}(1-\delta)}{\eta(A^L_{\overline{s}} - A^L_{\underline{s}})} \leq \delta\overline{m} - \underline{m},$$

or, equivalently,

$$\underline{m} \leq \tilde{m} := \delta\overline{m} - \frac{A^L_{\underline{s}}(1-\delta)}{\eta(A^L_{\overline{s}} - A^L_{\underline{s}})}.$$

Note that $\delta < 1$, as

$$(1+E/K)\frac{A^L_{\underline{s}}}{A^B} < 1 \qquad \Leftrightarrow \qquad (1+E/K)A^L_{\underline{s}} < A^B \qquad \Leftrightarrow \qquad A^L_{\underline{s}}E/K < A^B - A^L_{\underline{s}},$$

which then translates into

$$\frac{A^L_{\underline{s}}}{A^B - A^L_{\underline{s}}} < \frac{K}{E} \qquad \Leftrightarrow \qquad 1 + \frac{A^L_{\underline{s}}}{A^B - A^L_{\underline{s}}} < 1 + \frac{K}{E} \qquad \Leftrightarrow \qquad \frac{A^B}{A^B - A^L_{\underline{s}}} < 1 + \frac{K}{E},$$

and finally reads as $\varphi^S < \varphi^M$, which, based on assumption 2.5.1, is always satisfied. ■

Proof of Proposition 2.6.1. By assumption the beliefs $\underline{m}$ and $\overline{m}$ are distinct, i.e., it holds that $\underline{m} \leq \tilde{m}$.

First, note that there is no reason for the central bank to set a haircut larger than $\psi^S_{\overline{m}}$ which eliminates solvency risk for the more optimistic beliefs $\overline{m}$, as it only induces more restrictions on loan financing without any additional benefits, such as eliminating bank default, for instance. Using the fact that even under the smallest feasible haircut $\psi^M_{\overline{m}}$, the bank is not exposed to a solvency risk in case the more pessimistic beliefs $\underline{m}$ realize, we know that the central bank chooses a haircut $\psi \in [\psi^M_{\overline{m}}, \psi^S_{\overline{m}})$ if and only if

$$(\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi) + (1-p)\varphi^L_{\overline{m}}(\psi)] - (1-p)(1-\eta)\lambda A^L_{\underline{s}}\varphi^L_{\overline{m}}(\psi)$$
$$> (\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^L_{\overline{m}}(\psi^S_{\overline{m}})].$$

With assumption 2.6.1, we know that it holds that $\mathbb{E}[A^L_s] - A^B - (1-\eta)\lambda A^L_{\underline{s}} > 0$ and thus welfare with a solvency risk for banks (i.e., the left-hand side of the latter inequality) is maximized for $\psi = \psi^M_{\overline{m}}$. Accordingly, we can state that the central bank chooses $\psi = \psi^M_{\overline{m}}$ if and only if it holds that

$$(\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) + (1-p)\varphi^M] - (1-p)\lambda(1-\eta)A^L_{\underline{s}}\varphi^M$$
$$> (\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S],$$

where we used $\varphi^M = \varphi^L_{\overline{m}}(\psi^M_{\overline{m}})$ and $\varphi^S = \varphi^L_{\overline{m}}(\psi^S_{\overline{m}})$. Rearranging of the latter inequality yields

$$(1-p)(1-\eta)\lambda A^L_{\underline{s}}\varphi^M < (\mathbb{E}[A^L_s] - A^B)\{p[\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})] + (1-p)[\varphi^M - \varphi^S]\}$$

and further simplifies to

$$\lambda < \frac{\mathbb{E}[A^L_s] - A^B}{(1-\eta)A^L_{\underline{s}}} \frac{p[\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})] + (1-p)(\varphi^M - \varphi^S)}{(1-p)\varphi^M}.$$

Using the definitions $\lambda^S = (\mathbb{E}[A^L_s] - A^B)/[(1-\eta)A^L_{\underline{s}}]$ and $\lambda^M = \left(1 - \varphi^S/\varphi^M\right)\lambda^S$, the latter inequality reads

$$\lambda < \lambda^S\left(1 - \frac{\varphi^S}{\varphi^M} + \frac{p}{1-p}\frac{\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})}{\varphi^M}\right) = \lambda^M + \lambda^S\frac{p}{1-p}\frac{\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})}{\varphi^M}.$$

■

Proof of Corollary 2.6.1. Suppose the types of possible beliefs are distinct ($\underline{m} \leq \tilde{m}$). Then, it follows from proposition 2.6.1 that the central bank chooses the smallest feasible haircut $\psi^M_{\overline{m}}$ if and only if

$$\lambda < \lambda^M_{BU} = \lambda^M + \lambda^S \frac{p}{1-p} \frac{\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})}{\varphi^M}.$$

From proposition 2.5.1, we know that under perfect information and in the presence of the more optimistic beliefs $\overline{m}$, the central bank chooses the smallest feasible haircut $\psi^M_{\overline{m}}$ if and only if $\lambda < \lambda^M$. Note that it holds that

$$\frac{p}{1-p} \frac{\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})}{\varphi^M} > 0 \qquad \Leftrightarrow \qquad \varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) > \varphi^L_{\underline{m}}(\psi^S_{\overline{m}}),$$

which is satisfied as, based on assumption 2.5.1, it holds that $\psi^S_{\overline{m}} > \psi^M_{\overline{m}}$. Accordingly, we can conclude that it holds that $\lambda^M_{BU} > \lambda^M$ and under belief uncertainty, the central bank chooses the smallest feasible haircut $\psi^M_{\overline{m}}$ already at a higher default cost parameter, compared to the case without uncertainty. ■

Proof of Proposition 2.6.2. By assumption the beliefs $\underline{m}$ and $\overline{m}$ are close, i.e., it holds that $\underline{m} > \tilde{m}$.

Then, adopting a haircut $\psi \in [\psi^M_{\overline{m}}, \psi^S_{\underline{m}})$, i.e., accepting bank default for any possible type of beliefs in the economy, is welfare-improving compared to the situation without any solvency risk if and only if

$$\{\mathbb{E}[A^L_s] - A^B - (1-\eta)\lambda A^L_{\underline{s}}\}[p\varphi^L_{\underline{m}}(\psi) + (1-p)\varphi^L_{\overline{m}}(\psi)]$$
$$> (\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^L_{\overline{m}}(\psi^S_{\overline{m}})].$$

With assumption 2.6.1, we know that it holds that $\mathbb{E}[A^L_s] - A^B - (1-\eta)\lambda A^L_{\underline{s}} > 0$, so that welfare with a solvency risk for banks (i.e., the left-hand side of the latter inequality) is maximized for the smallest feasible haircut $\psi = \psi^M_{\overline{m}}$. Then, the central bank chooses the haircut $\psi = \psi^M_{\overline{m}}$ if and only if

$$\lambda < \frac{\mathbb{E}[A^L_s] - A^B}{(1-\eta)A^L_{\underline{s}}} \frac{\underline{\pi}[\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})] + (1-\underline{\pi})[\varphi^L_{\overline{m}}(\psi^M_{\overline{m}}) - \varphi^L_{\overline{m}}(\psi^S_{\overline{m}})]}{\pi\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) + (1-\pi)\varphi^L_{\overline{m}}(\psi^M_{\overline{m}})}.$$

Using $\varphi^L_{\overline{m}}(\psi^S_{\overline{m}}) = \varphi^S$ and $\varphi^L_{\overline{m}}(\psi^M_{\overline{m}}) = \varphi^M$, the latter inequality reads as

$$\lambda < \frac{\mathbb{E}[A^L_s] - A^B}{(1-\eta)A^L_{\underline{s}}} \frac{p[\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})] + (1-p)(\varphi^M - \varphi^S)}{p\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) + (1-p)\varphi^M}$$

and further simplifies to

$$\lambda < \frac{\mathbb{E}[A^L_s] - A^B}{(1-\eta)A^L_{\underline{s}}} \left(1 - \frac{p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S}{p\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) + (1-p)\varphi^M}\right).$$

We can then state that the central bank prefers the smallest feasible haircut $\psi^M_{\overline{m}}$ over the smallest possible haircut $\psi^S_{\overline{m}}$, eliminating bank default for any beliefs, if and only if

$$\begin{aligned}
& \lambda < \frac{\mathbb{E}[A^L_s] - A^B}{(1-\eta)A^L_{\underline{s}}} \left(1 - \frac{p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S}{p\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) + (1-p)\varphi^M}\right) \\
\Leftrightarrow \quad & \lambda < \lambda^S \left(1 - \frac{\varphi^S}{\varphi^M} + \frac{\varphi^S}{\varphi^M} - \frac{p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S}{p\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) + (1-p)\varphi^M}\right) \\
\Leftrightarrow \quad & \lambda < \lambda^M + \lambda^S \left(\frac{\varphi^S}{\varphi^M} - \frac{p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S}{p\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) + (1-p)\varphi^M}\right),
\end{aligned}$$

where we used the definitions $\lambda^S = (\mathbb{E}[A^L_s] - A^B)/[(1-\eta)A^L_{\underline{s}}]$ and $\lambda^M = \lambda^S\left(1 - \varphi^S/\varphi^M\right)$. The central bank may also choose a haircut $\psi \in [\psi^S_{\underline{m}}, \psi^S_{\overline{m}})$ to only accept bank default in the presence of the more optimistic beliefs $\overline{m}$, but not in the case of the more pessimistic beliefs $\underline{m}$. Note that beliefs satisfy $\underline{m} > \tilde{m}$ or, equivalently, $\varphi^L_{\underline{m}}(\psi^M_{\overline{m}}) > \varphi^S$, so that it holds that $\psi^S_{\underline{m}} > \psi^M_{\overline{m}}$. The central bank prefers a haircut $\psi \in [\psi^S_{\overline{m}}, \psi^S_{\underline{m}})$ over the haircut $\psi^S_{\overline{m}}$, eliminating solvency risk for any type of beliefs, if and only if

$$\{\mathbb{E}[A_s^L]-A^B\}p\varphi_{\underline{m}}^L(\psi)+\{\mathbb{E}[A_s^L]-A^B-(1-\eta)\lambda A_{\underline{s}}^L\}(1-p)\varphi_{\overline{m}}^L(\psi)$$
$$>(\mathbb{E}[A_s^L]-A^B)[p\varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)+(1-p)\varphi_{\overline{m}}^L(\psi_{\overline{m}}^S)].$$

From assumption 2.6.1, we know that it holds that $\mathbb{E}[A_s^L]-A^B-(1-\eta)\lambda A_{\underline{s}}^L>0$, so that the left-hand side of the latter inequality is maximized for the haircut $\psi=\psi_{\underline{m}}^S$. Using $\psi=\psi_{\underline{m}}^S$, the latter inequality yields

$$\lambda<\frac{\mathbb{E}[A_s^L]-A^B}{(1-\eta)A_{\underline{s}}^L}\frac{\underline{\pi}[\varphi_{\underline{m}}^L(\psi_{\underline{m}}^S)-\varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)]+(1-\underline{\pi})[\varphi_{\overline{m}}^L(\psi_{\underline{m}}^S)-\varphi_{\overline{m}}^L(\psi_{\overline{m}}^S)]}{(1-\pi)\varphi_{\overline{m}}^L(\psi_{\underline{m}}^S)}.$$

With $\varphi_{\overline{m}}^L(\psi_{\overline{m}}^S)=\varphi^S$ and $\varphi_{\underline{m}}^L(\psi_{\underline{m}}^S)=\varphi^S$, the latter inequality reads as

$$\lambda<\frac{\mathbb{E}[A_s^L]-A^B}{(1-\eta)A_{\underline{s}}^L}\frac{p[\varphi^S-\varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)]+(1-p)(\varphi_{\overline{m}}^L(\psi_{\underline{m}}^S)-\varphi^S)}{(1-p)\varphi_{\overline{m}}^L(\psi_{\underline{m}}^S)}$$

and further simplifies to

$$\lambda<\frac{\mathbb{E}[A_s^L]-A^B}{(1-\eta)A_{\underline{s}}^L}\left(1-\frac{p[\varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)-\varphi^S]+(1-p)\varphi^S}{(1-p)\varphi_{\overline{m}}^L(\psi_{\underline{m}}^S)}\right).$$

We can then state that the central bank prefers the haircut $\psi_{\underline{m}}^S$ to the smallest possible haircut $\psi_{\overline{m}}^S$, eliminating bank default for any type of beliefs, if and only if

$$\lambda<\frac{\mathbb{E}[A_s^L]-A^B}{(1-\eta)A_{\underline{s}}^L}\left(1-\frac{p[\varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)-\varphi^S]+(1-p)\varphi^S}{(1-p)\varphi_{\overline{m}}^L(\psi_{\underline{m}}^S)}\right)$$

$$\Leftrightarrow\qquad\lambda<\lambda^S\left(1-\frac{\varphi^S}{\varphi^M}+\frac{\varphi^S}{\varphi^M}-\frac{p[\varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)-\varphi^S]+(1-p)\varphi^S}{(1-p)\varphi_{\overline{m}}^L(\psi_{\underline{m}}^S)}\right)$$

$$\Leftrightarrow\qquad\lambda<\lambda^M+\lambda^S\left(\frac{\varphi^S}{\varphi^M}-\frac{p[\varphi_{\underline{m}}^L(\psi_{\overline{m}}^S)-\varphi^S]+(1-p)\varphi^S}{(1-p)\varphi_{\overline{m}}^L(\psi_{\underline{m}}^S)}\right),$$

where we used the definitions $\lambda^S=(\mathbb{E}[A_s^L]-A^B)/[(1-\eta)A_{\underline{s}}^L]$ and $\lambda^M=\lambda^S\left(1-\varphi^S/\varphi^M\right)$ again.

We can conclude that the central bank optimally chooses the smallest feasible haircut $\psi^M_{\overline{m}}$ if and only if $\lambda < \lambda^M_{BU}$ and $\lambda^S_{BU} \leq \lambda^M_{BU}$, where

$$\lambda^S_{BU} := \lambda^M + \lambda^S \left(\frac{\varphi^S}{\varphi^M} - \frac{p[\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) - \varphi^S] + (1-p)\varphi^S}{(1-p)\varphi^L_{\overline{m}}(\psi^S_{\underline{m}})} \right).$$

Instead, if $\lambda < \lambda^S_{BU}$ and $\lambda^M_{BU} < \lambda^S_{BU}$, the central bank chooses the smallest possible haircut, eliminating bank default only in the case of the more pessimistic beliefs $\underline{m}$. Otherwise, i.e., $\lambda^M_{BU} \leq \lambda$ and $\lambda^S_{BU} \leq \lambda$, the central bank chooses the smallest possible haircut $\psi^S_{\underline{m}}$, ruling out bank default for any beliefs in the economy. ■

Proof of Proposition 6.1.1. First, note that independent of the actual beliefs in the economy, bank default is eliminated if the central bank sets a haircut $\psi \in [\psi^S_{\overline{m}}, 1]$. The central bank never chooses a haircut larger than $\psi^S_{\overline{m}}$, as this would simply restrict bank lending further but not yield any additional benefits, as eliminating bank default, for instance. Moreover, the central bank will never choose a haircut that triggers bank default for both types of beliefs, as, based on our assumption on default costs (see assumption 6.1.1), such a monetary policy is welfare-reducing compared to any monetary policy that simply eliminates bank default for both types of beliefs. Note that bank default occurs independent of the actual beliefs in the economy if the haircut ψ chosen by the central bank satisfies $\psi < \psi^S_{\underline{m}}$. Such a monetary policy is only feasible if it holds that $\psi^S_{\underline{m}} > \psi^M_{\overline{m}}$, where $\psi^M_{\overline{m}}$ is the smallest feasible haircut. With assumption 6.1.1, we can thus focus for the analysis of the central bank's haircut choice on the set $\Psi := [\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}, \psi^S_{\overline{m}}]$. The central bank chooses a haircut $\psi \in [\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}, \psi^S_{\overline{m}})$, and thereby accepts default of banks in the presence of the more optimistic beliefs, if and only if

$$(\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi) + (1-p)\varphi^L_{\overline{m}}(\psi)] - (1-p)(1-\eta)\lambda A^L_{\underline{s}}\varphi^L_{\overline{m}}(\psi)$$
$$> (\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^L_{\overline{m}}(\psi^S_{\overline{m}})],$$

which simplifies to

$$\frac{(\mathbb{E}[A^L_s] - A^B)}{(1-\eta)A^L_{\underline{s}}}\{p[\varphi^L_{\underline{m}}(\psi) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})] + (1-p)[\varphi^L_{\overline{m}}(\psi) - \varphi^L_{\overline{m}}(\psi^S_{\overline{m}})]\} > \lambda(1-p)\varphi^L_{\overline{m}}(\psi).$$

Using $\varphi^L_{\overline{m}}(\psi^S_{\overline{m}}) = \varphi^S$ and $\lambda^S = (\mathbb{E}[A^L_s] - A^B)/[(1-\eta)\lambda A^L_{\underline{s}}]$, the latter inequality reads

$$\lambda < \lambda^M_{BU}(\psi) := \lambda^S \left\{ 1 - \frac{\varphi^S}{\varphi^L_{\overline{m}}(\psi)} + \frac{p}{1-p} \frac{\varphi^L_{\underline{m}}(\psi) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})}{\varphi^L_{\overline{m}}(\psi)} \right\}.$$

Note that, based on assumption 6.1.1, it holds that $\lambda \geq \lambda^S$. A haircut $\psi \in [\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}, \psi^S_{\overline{m}})$ satisfies $\lambda^S < \lambda^M_{BU}(\psi)$ if and only if

$$\frac{\varphi^S}{\varphi^L_{\overline{m}}(\psi)} < \frac{p}{1-p} \frac{\varphi^L_{\underline{m}}(\psi) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})}{\varphi^L_{\overline{m}}(\psi)} \qquad \Leftrightarrow \qquad (1-p)\varphi^S + p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) < p\varphi^L_{\underline{m}}(\psi).$$

The right-hand side of the latter inequality is maximized for $\psi = \max\{\varphi^M_{\overline{m}}, \varphi^S_{\underline{m}}\}$. Thus, we can state that, if $p\varphi^L_{\underline{m}}(\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}) > p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S$, there exists a

$$\hat{\psi} \in \arg\max_{\psi \in \Psi} \lambda^M_{BU}(\psi), \qquad \text{with} \qquad \lambda^M_{BU}(\hat{\psi}) > \lambda^S,$$

so that the central bank chooses $\hat{\psi}$ whenever $\lambda < \lambda^M_{BU}(\hat{\psi})$, and $\psi^S_{\overline{m}}$ otherwise. In the special case where $\lambda = \lambda^S$, we can derive a more simple monetary policy rule. The central bank chooses a haircut $\psi \in [\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}, \psi^S_{\overline{m}})$, and thereby accepts default of banks in the presence of the more optimistic beliefs $\overline{m}$, if and only if

$$(\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi) + (1-p)\varphi^L_{\overline{m}}(\psi)]E - (1-p)(1-\eta)\lambda A^L_{\underline{s}}\varphi^L_{\overline{m}}(\psi)E$$
$$> (\mathbb{E}[A^L_s] - A^B)[p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^L_{\overline{m}}(\psi^S_{\overline{m}})]E,$$

which simplifies to

$$\frac{(\mathbb{E}[A^L_s] - A^B)}{(1-\eta)A^L_{\underline{s}}}\{p[\varphi^L_{\underline{m}}(\psi) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})] + (1-p)[\varphi^L_{\overline{m}}(\psi) - \varphi^L_{\overline{m}}(\psi^S_{\overline{m}})]\} > \lambda(1-p)\varphi^L_{\overline{m}}(\psi).$$

Using $\lambda = \lambda^S = (\mathbb{E}[A^L_s] - A^B)/[(1-\eta)A^L_{\underline{s}}]$, we find that the central bank chooses a haircut $\psi \in [\max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}, \psi^S_{\overline{m}})$ if and only if

$$p[\varphi^L_{\underline{m}}(\psi) - \varphi^L_{\underline{m}}(\psi^S_{\overline{m}})] > (1-p)\varphi^L_{\overline{m}}(\psi^S_{\overline{m}}) \quad \Leftrightarrow \quad p\varphi^L_{\underline{m}}(\psi) > p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^L_{\overline{m}}(\psi^S_{\overline{m}}).$$

The left-hand side of the latter inequality is maximized for $\psi = \max\{\psi^M_{\overline{m}}, \psi^S_{\underline{m}}\}$. Thus, we can state that the central bank chooses the haircut $\psi = \max\{\varphi^M_{\overline{m}}, \varphi^S_{\underline{m}}\}$ if $p\varphi^L_{\underline{m}}(\max\{\varphi^M_{\overline{m}}, \varphi^S_{\underline{m}}\}) >$

$p\varphi^L_{\underline{m}}(\psi^S_{\overline{m}}) + (1-p)\varphi^S$, and the haircut $\psi^S_{\overline{m}}$ otherwise. ■

Proof of Lemma 6.1.1. In equilibrium, firm productivity is linked to the interest rates on loans, bonds and deposits, i.e., it holds that $(1+r^L_s)q = A^L_s$ for all $s \in \{\underline{s}, \overline{s}\}$ and $(1+r^D)q = (1+r^B)q = A^B$, see conditions (2.5) and (2.6) in subsection 2.4.2. Accordingly, assumption 6.1.2, stating $\mathbb{E}_m[A^L_s] < A^B$, implies that loan rates and the deposit rate satisfy $\mathbb{E}_m[r^L_s] < r^D$. From lemma 2.3.4, we can then deduce that the bank chooses the maximum leverage if and only if $r^L_{\overline{s}} > r^D$ and $\varphi^L_m(\psi) > [(1+\mathbb{E}_m[r^L_s])/\eta_m - 1 - r^D]/(r^L_{\overline{s}} - r^D)$. The first condition $r^L_{\overline{s}} > r^D$ is in equilibrium equivalent to $A^L_{\overline{s}} > A^B$, which is always satisfied, based on assumption 2.3.1. The second condition translates with the equilibrium conditions $(1+r^L_s)q = A^L_s$ for all $s \in \{\underline{s}, \overline{s}\}$ and $(1+r^D)q = (1+r^B)q = A^B$ into

$$\varphi^L_m(\psi) > \frac{\mathbb{E}_m[A^L_s] - \eta_m A^B}{\eta_m(A^L_{\overline{s}} - A^B)}.$$

Using the equation (2.10) in subsection 2.4.2, which expresses the leverage ratio $\varphi^L_m(\psi)$ using economic fundamentals, the latter inequality translates into

$$\frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_m[A^L_s]} > \frac{\mathbb{E}_m[A^L_s] - \eta_m A^B}{\eta_m(A^L_{\overline{s}} - A^B)}$$

$$\Leftrightarrow \quad \alpha A^B \eta_m(A^L_{\overline{s}} - A^B) > (\mathbb{E}_m[A^L_s] - \eta_m A^B)\{\alpha A^B - (1-\psi)\mathbb{E}_m[A^L_s]\}$$

$$\Leftrightarrow \quad (1-\psi)(\mathbb{E}_m[A^L_s] - \eta_m A^B)\mathbb{E}_m[A^L_s] > \alpha A^B(\mathbb{E}_m[A^L_s] - \eta_m A^L_{\overline{s}})$$

$$\Leftrightarrow \quad \psi < \hat{\psi}_m := 1 - \frac{\alpha A^B(\mathbb{E}_m[A^L_s] - \eta_m A^L_{\overline{s}})}{(\mathbb{E}_m[A^L_s] - \eta_m A^B)\mathbb{E}_m[A^L_s]}.$$

The investor provides equity financing ($\zeta = 1$) if and only if the expected rate of return on bank equity exceeds the interest rate on bonds, i.e., $\mathbb{E}_m[r^E_s] \geq r^B$, translates into

$$\eta_m[(r^L_{\overline{s}} - r^D)\varphi^L_m(\psi) + 1 + r^D] - 1 \geq r^B \qquad \Leftrightarrow \qquad \varphi^L_m(\psi) \geq \frac{(1+r^B)/\eta_m - (1+r^D)}{r^L_{\overline{s}} - r^D}.$$

Using the equilibrium conditions $(1+r^L_s)q = A^L_s$, with $s \in \{\underline{s}, \overline{s}\}$, and $(1+r^D)q =$

$(1 + r^B)q = A^B$, the latter inequality reads

$$\frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_m[A_s^L]} \geq \frac{(1-\eta_m)A^B}{\eta_m(A_{\bar{s}}^L - A^B)}.$$

Further rearranging yields

$$\begin{aligned} & \alpha\eta_m(A_{\bar{s}}^L - A^B) \geq (1-\eta_m)\{\alpha A^B - (1-\psi)\mathbb{E}_m[A_s^L]\} \\ \Leftrightarrow \quad & (1-\psi)(1-\eta_m)\mathbb{E}_m[A_s^L] \geq \alpha(A^B - \eta_m A_{\bar{s}}^L) \\ \Leftrightarrow \quad & (1-\eta_m)\mathbb{E}_m[A_s^L] - \alpha(A^B - \eta_m A_{\bar{s}}^L) \geq \psi(1-\eta_m)\mathbb{E}_m[A_s^L] \end{aligned}$$

and finally leads to another upper bound on the haircut that is given by

$$\psi \leq \frac{(1-\eta_m)\mathbb{E}_m[A_s^L] - \alpha(A^B - \eta_m A_{\bar{s}}^L)}{(1-\eta_m)\mathbb{E}_m[A_s^L]} \qquad \Leftrightarrow \qquad \psi \leq \tilde{\psi}_m := 1 - \frac{\alpha(A^B - \eta_m A_{\bar{s}}^L)}{(1-\eta_m)\mathbb{E}_m[A_s^L]}.$$

Note that $\tilde{\psi}_m < \hat{\psi}_m$ is equivalent to

$$\begin{aligned} & \frac{\alpha A^B(\mathbb{E}_m[A_s^L] - \eta_m A_{\bar{s}}^L)}{(\mathbb{E}_m[A_s^L] - \eta_m A^B)\mathbb{E}_m[A_s^L]} < \frac{\alpha(A^B - \eta_m A_{\bar{s}}^L)}{(1-\eta_m)\mathbb{E}_m[A_s^L]} \\ \Leftrightarrow \quad & (1-\eta_m)A^B(\mathbb{E}_m[A_s^L] - \eta_m A_{\bar{s}}^L) < (A^B - \eta_m A_{\bar{s}}^L)(\mathbb{E}_m[A_s^L] - \eta_m A^B) \\ \Leftrightarrow \quad & \eta_m A^B A_{\bar{s}}^L < \eta_m A^B \mathbb{E}_m[A_s^L] - \eta_m (A^B)^2 \\ \Leftrightarrow \quad & \eta_m \mathbb{E}_m[A_s^L](A_{\bar{s}}^L - A^B) < \eta_m A^B (A_{\bar{s}}^L - A^B). \end{aligned}$$

The latter inequality translates into $\mathbb{E}_m[A_s^L] < A^B$, which, based on assumption 6.1.2, is always satisfied. ■

Proof of Lemma 6.1.2. First, note that, based on the outline in subsection 2.4.2 and lemma 6.1.1, we can state that real bank lending is given by $K^L = \varphi\zeta E = \mathbb{1}\{\psi \leq \tilde{\psi}_m\}\varphi_m^L(\psi)E$.

Second, note that $\tilde{\psi}_m \leq \psi_m^S$ is equivalent to

$$\frac{\alpha A^L_{\underline{s}}}{\mathbb{E}_m[A^L_s]} \leq \frac{\alpha(A^B - \eta_m A^L_{\bar{s}})}{\mathbb{E}_m[A^L_s](1-\eta_m)} \quad \Leftrightarrow \quad (1-\eta_m)A^L_{\bar{s}} \leq A^B - \eta_m A^L_{\bar{s}} \quad \Leftrightarrow \quad \mathbb{E}_m[A^L_s] \leq A^B.$$

The latter is always satisfied, based on assumption 6.1.2. Using lemma 2.4.1, we can conclude that for a specific haircut, welfare is given by

$$W_m(\psi) = \{\mathbb{E}[A^L_s] - (1-\eta)\lambda A^L_{\underline{s}}\}\mathbb{1}\{\psi \leq \tilde{\psi}\}\varphi^L_m(\psi)E + A^B(K + E - \mathbb{1}\{\psi \leq \tilde{\psi}_m\}\varphi^L_m(\psi)E).$$

Third, from proposition 2.4.1, we know that the central bank can only set haircuts weakly higher than ψ_m^M, as otherwise the capital good market does not clear.

Omitting all terms which do not depend on the haircut ψ chosen by the central bank, we can then conclude that the central bank's optimization problem is given by

$$\max_{\psi \in [\psi_m^M, 1]} \{\mathbb{E}_m[A^L_s] - A^B - (1-\eta)\lambda A^L_{\underline{s}}\}\mathbb{1}\{\psi \leq \tilde{\psi}_m\}\varphi^L_m(\psi).$$

■

Proof of Proposition 6.1.2. Note that the central bank can only incentivize banks to grant loans funded with deposits and incentivize investors to provide equity financing for banks if the smallest feasible haircut ψ_m^M satisfies $\psi_m^M \leq \tilde{\psi}_m$. Suppose the latter condition holds. Then, it is only optimal for the central bank to provide the incentives for bank lending, i.e., setting a haircut ψ lower than $\tilde{\psi}_m$, if it holds that $\mathbb{E}[A^L_s] - A^B - (1-\eta)\lambda A^L_{\underline{s}} > 0$ or, equivalently, $\lambda < \lambda^S = (\mathbb{E}[A^L_s] - A^B)/[(1-\eta)A^L_{\underline{s}}]$. Instead, if it holds that $\psi_m^M > \tilde{\psi}_m$ or $\lambda \geq \lambda^S$, the central bank chooses to restrict bank lending and rule out bank default, by setting the haircut $\psi = 1$. Any other haircut ψ satisfying $\psi > \tilde{\psi}_m$ would also be a feasible policy for the central bank to restrict bank lending and rule out bank default. ■

6.2 Appendix for Chapter 3

6.2.1 Equilibrium definition

Throughout our analysis, we focus on competitive equilibria as defined hereafter. We use the notation $C_m^B = \mathbb{E}_m[C_s^B] = (1 + \mathbb{E}_m[r_s^E])qE$ to represent aggregate consumption by bankers and $Y_m = \mathbb{E}_m[A_s^L]K^L + A^B K^B$ to represent aggregate production. As idiosyncratic productivity shocks are i.i.d. across firms and we assume a continuum of firms, we obtain by the law of large numbers that aggregate production by the loan-financed sector equals expected production of the loan-financed firm. Moreover, banks and firms are matched one-to-one, so that the expected consumption by the banker equals aggregate consumption of bankers.

Definition 6.2.1 (Competitive Equilibrium)
Given a monetary policy $r_{CB}^D > 0$ and $\psi \in [0, 1]$, a competitive equilibrium is a set of prices $P > 0$ and $Q > 0$, interest rates $r^D > 0$, $r_s^L > 0$, with $s \in \{\underline{s}, \overline{s}\}$, and $r^B > 0$, and choices K^L, K^B, γ, φ and m, such that

- *given P, Q and r_s^L, with $s \in \{\underline{s}, \overline{s}\}$, the choice K^L maximizes the expected profits of the loan-financed firm,*

- *given P, Q and r^B, the choice K^B maximizes the expected profits of the bond-financed firm,*

- *given P, Q, r^D and r^B, the choice γ maximizes the utility of the household,*

- *given P, Q, r_{CB}^D, ψ, r_s^L, with $s \in \{\underline{s}, \overline{s}\}$, and r^D, the choices φ and m maximize the expected utility of the banker,*

- *the loan, bond, capital good and consumption good markets clear, i.e., $QK^L = \varphi QE$, $QK^B = \gamma QK$, $K^L + K^B = K + E$ and $C^H + C_m^B = Y_m$.*

6.2.2 Alternative monitoring technology

In this section, we study the collateral leverage channel, assuming a different monitoring technology. Specifically, monitoring does not increase the probability for a positive idiosyncratic shock but directly affects the productivity in the case where a negative productivity shock realizes. In what follows, we outline the changes in the setup for loan-financed firms and bankers, and then discuss the resulting equilibrium properties. The alternative monitoring technology does not lead to changes for bond-financed firms, households and the government sector, including the bank regulator and the central bank.

Loan-financed firms

The loan-financed firm uses the capital good $K^L \geq 0$ to produce consumption good with the risky technology $A^L_{s,m}K^L$, where the marginal productivity $A^L_{s,m} \geq 0$ is not only affected by an idiosyncratic shock s, but also by the monitoring activity m of the matched banker. The productivity can be either low ($s = l$) or high ($s = h$), so that it holds that $A^L_{h,m} > A^L_{l,m}$. The idiosyncratic productivity shocks are i.i.d. across firms, where a positive idiosyncratic shock occurs with probability $\eta \in (0, 1)$. Bankers can engage into costly monitoring ($m = 1$) or shirking ($m = 0$). Monitoring by the matched banker limits the impact of a negative idiosyncratic productivity shock. Formally, monitoring has the following effect on the productivity of the loan-financed firm: $A^L_{h,1} = A^L_{h,0}$ and $A^L_{l,1} = A^L_{l,0} + \triangle$, where $\triangle > 0$.

The external funds QK^L borrowed by the firm from the matched bank requires a repayment that is determined by the interest rate $r^L_{s,m} > 0$, which depends on the idiosyncratic shock s of the firm and the monitoring activity m of the matched banker. Accounting for the fact that firms are profit-maximizing and subject to limited liability, it follows that the optimization problem of the loan-financed firm is given in real terms by

$$\max_{K^L \geq 0} \mathbb{E}[\{A^L_{s,m} - (1 + r^L_{s,m})q\}^+]K^L, \tag{6.4}$$

where we use the notation $q := Q/P$ to denote the capital good price in terms of the consumption good.

Due to limited liability, there exists no optimal, finite demand of capital good if the firm is exposed to excess returns in at least one state. In contrast, without excess returns, the firm will be indifferent between any amount of capital good put into production. The previous explanations are formally summarized in the following lemma.

Lemma 6.2.1 (Optimal Choice of the Loan-Financed Firm)
The loan-financed firm optimally chooses capital good $K^L = +\infty$ *if and only if* $A^L_{s,m} > (1 + r^L_{s,m})q$ *for some* s*, and* $K^L \in [0, +\infty)$ *otherwise.*

In equilibrium, the optimal demand for capital good must be finite and firm default cannot arise due to the rationality of all agents in the economy. Accordingly, it must hold that in equilibrium, $(1 + r^L_{s,m})q = A^L_{s,m}$ for all s, m.

We make specific assumptions on firm productivity: First, we assume that a loan-financed firm is more productive on average than a bond-financed firm, even if the matched banker does not monitor. This assumption guarantees that the loan-financed firms—and thus banks—are needed to maximize aggregate production and ultimately welfare. Second, only when a loan-financed firm experiences a negative idiosyncratic shock, it is less productive than a bond-financed firm, even if the matched banker monitors the loan-financed firm. The latter assumption allows us to introduce solvency risk on the side of banks, as outlined in subsection 6.2.2.

Assumption 6.2.1 (Firm Productivities)
$\mathbb{E}[A^L_{s,0}] > A^B$*, and* $A^B > A^L_{l,1}$.

Note that, based on assumption 6.2.1, we implicitly imposed an upper bound on the effect of monitoring, as the condition $A^B > A^L_{l,1}$ translates into $\triangle < A^B - A^L_{l,0}$. Moreover, it follows from assumption 6.2.1 that independent of the monitoring activity by the matched banker, a loan-financed firm is strictly more productive than a bond-financed firm if it incurs a positive productivity shock, i.e., it holds that $A^L_{h,m} > A^B$ for all m.

Bankers

The interest rate on loans granted by the bank is given by $r^L_{s,m} > 0$, which depends on the idiosyncratic shock s as well as on the banker's monitoring decision m. Deposits are credited with interest according to the rate $r^D > 0$. The nominal equity returns are then given by

$$(1 + r^E_{s,m})E^b = \left\{(1 + r^L_{s,m})L^b + (1 + r^D_{CB})D^{CB} - (1 + r^D)D^b - (1 + r^L_{CB})L^{CB}\right\}^+,$$

where we use $\{X\}^+ = \max\{X, 0\}$ to account for the limited liability of the bank. Using the structure of deposit financing, $D^b = L^b - E^b$, reserve loans and reserve deposits, $L^{CB} = D^{CB} = \alpha(L^b - E^b)$ (for a derivation, see subsection 3.3.6), it follows that the nominal equity returns are given by

$$(1 + r^E_{s,m})E^b = \left\{(1 + r^L_{s,m})L^b + [(1 + r^D_{CB})\alpha - (1 + r^D) - (1 + r^L_{CB})\alpha](L^b - E^b)\right\}^+.$$

Using assumption 3.3.2, which imposes the equality of interest rates on reserves ($r^D_{CB} = r^L_{CB}$), and using the definition of bank leverage $\varphi = L^b/E^b$, we obtain the rate of return on bank equity

$$r^E_{s,m}(\varphi) := \{(r^L_{s,m} - r^D)\varphi + 1 + r^D\}^+ - 1.$$

Based on the explanations in subsection 3.3.3 and 3.3.4, we know that, in equilibrium, the interest rates on loans and deposits satisfy $r^L_{s,m} = A^L_{s,m}/q - 1$ for all s and $r^D = r^B = A^B/q - 1$. Accordingly, the equilibrium rate of return on bank equity can be expressed using economic fundamentals, i.e., it holds that

$$r^E_{s,m}(\varphi) := \{(A^L_{s,m} - A^B)\varphi + A^B\}^+/q - 1. \tag{6.5}$$

It follows with our assumptions on firm productivity (see assumption 6.2.1) that only in the presence of a low productivity ($s = l$), the bank is making losses on loans funded with deposits. We can derive a maximum leverage, denoted by φ^S_m, which guarantees solvency of the bank in all states. This leverage is obtained by setting the equity return in the low productivity state to zero, i.e.,

$$1 + r^E_{l,m}(\varphi^S_m) = 0 \quad \Leftrightarrow \quad (A^L_{l,m} - A^B)\varphi^S_m + A^B = 0 \quad \Leftrightarrow \quad \varphi^S_m := \frac{A^B}{A^B - A^L_{l,m}}. \tag{6.6}$$

Note that the leverage threshold φ^S_m depends on the banker's monitoring activity m, as the latter increases the productivity of the financed firm whenever it incurs a negative shock, i.e., it holds that $A^L_{l,1} = A^L_{l,0} + \triangle$ with $\triangle > 0$. Thus, with monitoring, the bank can leverage more, i.e., issue more deposits and provide more loan financing, until it is exposed to a solvency risk ($\varphi^S_1 > \varphi^S_0$).

When capital good transactions are settled, the bank requires liquidity in the form of reserves which it can borrow from the central bank by pledging the bank loans granted to the matched firm. At that point in time, productivity shocks have not realized yet, so that the expected value of bank loans is given by $(1+\mathbb{E}[r^L_{s,m}])L^b$. The central bank applies a haircut $\psi \in [0,1]$ on the value of bank loans, so that the overall collateral available to the bank, also referred to as the "collateral capacity", is given by $(1-\psi)(1+\mathbb{E}[r^L_{s,m}])L^b$. Taking the repayment obligation on reserve loans into account, the reserve borrowing L^{CB} of the bank cannot exceed the bank's collateral capacity, which leads to the liquidity constraint

$$(1-\psi)(1+\mathbb{E}[r^L_{s,m}])L^b \geq (1+r^L_{CB})L^{CB}.$$

Using the structure of reserve loans, $L^{CB} = \alpha(L^b - E^b)$, and the definition of the bank leverage, $\varphi = L^b/E^b$, we can reformulate the latter inequality as

$$(1-\psi)(1+\mathbb{E}[r^L_{s,m}])\varphi \geq \alpha(1+r^D_{CB})(\varphi-1),$$

where we also made use of assumption 3.3.2, stating the equality of interest rates on reserves deposits and reserve loans ($r^D_{CB} = r^L_{CB}$). We can then define a maximum leverage, up to which liquidity of the bank is guaranteed. This leverage, denoted by $\varphi^L_m(\psi)$, is determined by the binding liquidity constraint, i.e.,

$$(1-\psi)(1+\mathbb{E}[r^L_{s,m}])\varphi^L_m(\psi) = \alpha(1+r^D_{CB})[\varphi^L_m(\psi)-1],$$

so that

$$\varphi^L_m(\psi) = \frac{\alpha(1+r^D_{CB})}{\alpha(1+r^D_{CB}) - (1-\psi)(1+\mathbb{E}[r^L_{s,m}])}. \tag{6.7}$$

The banker's monitoring decision m affects the leverage threshold $\varphi^L_m(\psi)$, as monitoring increases the productivity and ultimately the loan repayment of the financed firm in the presence of a negative idiosyncratic shock. Higher loan repayment in one state increases the valuation of bank loans and finally the collateral capacity of the bank, allowing it to borrow more reserves at the central bank. Thus, the bank grants more loans, funded with deposits, in the first place, i.e., the maximum leverage is increasing with bank monitoring ($\varphi^L_1(\psi) > \varphi^L_0(\psi)$). The bank never chooses a leverage larger than $\varphi^L_m(\psi)$, as it would lead to illiquidity with certainty, in which case the government would seize all bank assets and thus eliminate the potential returns on bank equity. The bank is also subject to a regulatory leverage $\varphi \leq \varphi^R$, where $\varphi^R \in [1, +\infty)$ denotes the regulatory maximum leverage.

Using equation (6.7) and the equilibrium condition (3.6) in subsection 3.3.6, we can express the maximum leverage $\varphi_m^L(\psi)$ guaranteeing liquidity of the banker, using model primitives, i.e., it holds that

$$\varphi_m^L(\psi) = \frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}[A_{s,m}^L]}. \tag{6.8}$$

The banker uses the returns on bank equity $[1 + r_{s,m}^E(\varphi)]E^b$ to purchase consumption good C_s^B at the nominal price $P > 0$. The banker is maximizing the expected utility, which we assume to be linearly increasing in consumption. Accordingly, the optimization problem of the banker is given in real terms by

$$\max_{\substack{\varphi \in [1, \overline{\varphi}_m(\theta)],\\ m \in \{0,1\}}} \{1 + \mathbb{E}[r_{s,m}^E(\varphi)] - m\kappa\varphi\} qE,$$

where we made use of the definitions $E^b = QE$ and $\varphi = L^b/E^b$ to obtain $m\kappa L^b = m\kappa\varphi QE$. As in subsection 3.3.6, we apply the notation $\overline{\varphi}_m(\theta) = \min\{\varphi^R, \varphi_m^L(\psi)\}$, where $\theta = (\varphi^R, \psi)$ represents the policy measures imposed by the bank regulator and the central bank.

We now outline the banker's optimal choice in equilibrium. First, we focus on the banker's optimal choice of the leverage or, in other words, the optimal loan supply and deposit issuance. Based on assumption 6.2.1, we know that the expected productivity of a loan-financed firm is higher than the productivity of a bond-financed firm, even without monitoring by the matched banker ($\mathbb{E}[A_{s,0}^L] > A^B$). Interest rates on loans and deposits are directly linked to firm productivity in equilibrium, namely, it holds that $r_{s,m}^L = A_{s,m}^L/q - 1$ for all s and $r^D = r^B = A^B/q - 1$. Accordingly, the expected loan repayment is larger than the interest payment on deposits, incentivizing the banker to attain the maximum leverage, i.e., $\varphi = \overline{\varphi}_m(\theta)$.

Next, we turn to the banker's monitoring decision, which generally depends on three factors: (i) the monitoring-induced increase of loan repayment for a negative idiosyncratic productivity shock of the financed firm, to which we refer to as the *return channel* of monitoring, (ii) the monitoring-induced increase of collateral capacity, allowing any liquidity-constrained bank to expand deposit issuance and loan supply, to which we refer to as the *collateral leverage channel* of monitoring, and (iii) the monitoring costs. If, independent of the monitoring decision, the banker is not exposed to a solvency risk (case (I) in lemma

6.2.2), the banker internalizes all the expected benefits $(1-\eta)\triangle$ from higher loan repayment due to monitoring in the presence of a negative idiosyncratic shock. In turn, if the banker is exposed to a solvency risk (cases (II) and (III) in lemma 6.2.2), the banker defaults for a low productivity of the financed firm and thus expects no benefits from higher loan repayment due to monitoring. In other words, the return channel is not active. Solvency risk thus reduces the banker's incentives to monitor and ultimately may even induce the banker to shirk. However, if the bank is liquidity-constrained, monitoring also increases the valuation of bank loans and thereby the collateral capacity, allowing the bank to expand deposit issuance and loan supply, which increases the expected profits of the bank. This collateral leverage channel is only active if the banker is liquidity-constrained at least without monitoring, i.e., $\overline{\varphi}_0(\theta) = \varphi_0^L(\psi) < \overline{\varphi}_1(\theta) \leq \varphi^R$. In contrast, if, independent of the monitoring decision, the banker is only constrained by capital, i.e., $\overline{\varphi}_0(\theta) = \overline{\varphi}_1(\theta) = \varphi^R$, the banker's decision about monitoring only involves the benefits following from the return channel and the monitoring costs. The following lemma summarizes the previous explanations on the banker's optimal choice.

Lemma 6.2.2 (Optimal Choice of the Banker)
In equilibrium, the banker's optimal choice of leverage is given by $\varphi = \overline{\varphi}_m(\theta)$ and the banker's optimal monitoring decision is given by $m = 1$ if and only if

(I) without solvency risk, i.e., $\overline{\varphi}_m(\theta) \leq \varphi_m^S$ for all m, it holds that $\mathcal{M}_N(\theta) \geq 0$, where

$$\mathcal{M}_N(\theta) := (1-\eta)\triangle + (\mathbb{E}[A_{s,0}^L] - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right] - \kappa q,$$

(II) with partial solvency risk, i.e., $\overline{\varphi}_0(\theta) > \varphi_0^S$ and $\overline{\varphi}_1(\theta) \leq \varphi_1^S$, it holds that $\mathcal{M}_P(\theta) \geq 0$, where

$$\mathcal{M}_P(\theta) := -(1-\eta)\left[A^B - A_{l,1}^L - \frac{A^B}{\overline{\varphi}_1(\theta)}\right] + \eta(A_{h,0}^L - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right] - \kappa q,$$

(III) with full solvency risk, i.e., $\overline{\varphi}_m(\theta) > \varphi_m^S$ for all m, it holds that $\mathcal{M}_F(\theta) \geq 0$, where

$$\mathcal{M}_F(\theta) := \eta(A_{h,0}^L - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right] - \kappa q.$$

Equilibrium properties

We first provide necessary conditions for the existence of a competitive equilibrium and the bank's exposure to a solvency risk. Then, we characterize welfare, using economic fundamentals, and provide further details on the monitoring decision of the banker.

Existence and solvency risk. The existence of an equilibrium crucially depends on the clearing of the capital good market. Specifically, an equilibrium only exists if loan-financed firms do not receive more funds from banks than needed to purchase the entire capital good in the economy, i.e. $L^b = QK^L = \overline{\varphi}_m(\theta)E \leq K + E$ or, with the notation $\varphi^M := 1 + K/E$, equivalently, $\overline{\varphi}_m(\theta) \leq \varphi^M$. From the latter inequality, we can derive a condition on the capital requirements or the collateral requirements, depending which ones are binding. First, if the banker is constrained by capital, i.e., $\varphi^R \leq \varphi^L_m(\psi)$, it must hold that $\varphi^R \leq \varphi^M$. In turn, if the banker is constrained by liquidity, i.e., $\varphi^L_m(\psi) \leq \varphi^R$, the collateral requirements in the form of the haircut must be such that $\varphi^L_m(\psi) \leq \varphi^M$. From the latter condition, we can derive a smallest feasible haircut ψ^M_m, which, if implemented, allows bankers to provide as much loan financing as needed to allow loan-financed firms to acquire the entire capital good in the economy. Any haircut lower than ψ^M_m conflicts with the clearing condition for the capital good market and thus does not permit an equilibrium, whereas any haircut larger than ψ^M_m restricts the bank leverage below the maximum feasible, i.e. $\varphi^L_m(\psi) < \varphi^M$, but guarantees the existence of an equilibrium.

If an equilibrium exists, i.e., $\varphi^R \leq \varphi^M$ or $\psi \geq \psi^M_m$, the banker is exposed to a solvency risk if the attained leverage is sufficiently large to exceed the leverage guaranteeing solvency in all states, i.e., $\overline{\varphi}_m(\theta) > \varphi^S_m$. Clearly, this is only possible if the capital requirements, leading to the regulatory maximum leverage, are sufficiently loose, i.e., $\varphi^R > \varphi^S_m$, and the haircut set by the central bank is sufficiently small to achieve $\varphi^L_m(\psi) > \varphi^S_m$. We can use the condition $\varphi^L_m(\psi) > \varphi^S$ to derive the smallest possible haircut ψ^S_m guaranteeing solvency of the bank in all states: For any haircut ψ satisfying $\psi < \psi^S_m$, the banker is exposed to a solvency risk, assuming that capital requirements are sufficiently loose and it holds $\varphi^R > \varphi^S_m$. Proposition 6.2.1 provides the details.

Proposition 6.2.1 (Existence and Solvency Risk)

A competitive equilibrium exists only if $\varphi^R \leq \varphi^M$ or $\varphi^L_m(\psi) \leq \varphi^M$, where the latter in-

equality is equivalent to

$$\psi \geq \psi_m^M := 1 - \frac{\alpha A^B}{\mathbb{E}[A_{s,m}^L](1 + E/K)},$$

where the banker is exposed to a solvency risk only if $\varphi^R > \varphi_m^S$ *and* $\varphi_m^L(\psi) > \varphi_m^S$*, where the latter inequality is equivalent to*

$$\psi < \psi_m^S := 1 - \frac{\alpha A_{l,m}^L}{\mathbb{E}[A_{s,m}^L]}.$$

The banker's monitoring decision m *follows from lemma 6.2.2.*

The smallest feasible haircut ψ_m^M and the smallest possible haircut ψ_m^S guaranteeing solvency of banks both depend on the monitoring activity m. Note that bank monitoring increases productivity in the presence of negative idiosyncratic shock, i.e., $A_{h,1}^L = A_{h,0}^L$ and $A_{l,1}^L = A_{l,0}^L + \triangle$, and thereby also increases the expected loan repayment, i.e., $\mathbb{E}[A_{s,1}^L] = \mathbb{E}[A_{s,0}^L] + (1 - \eta)\triangle$. The smallest feasible haircut ψ_m^M increases with monitoring, i.e., $\psi_1^M > \psi_0^M$, as monitoring increases the collateral value of bank loans, but leaves the maximum feasible bank leverage φ^M unchanged. In contrast, the smallest possible haircut ψ_m^S guaranteeing solvency of banks decreases with monitoring, i.e.,

$$\psi_0^S = 1 - \frac{\alpha A_{l,0}^L}{\mathbb{E}[A_{s,0}^L]} > \psi_1^S = 1 - \frac{\alpha A_{l,1}^L}{\mathbb{E}[A_{s,1}^L]} = 1 - \frac{\alpha(A_{l,0}^L + \triangle)}{\mathbb{E}[A_{s,0}^L] + (1 - \eta)\triangle}.$$

Monitoring increases the expected value of bank loans and would lead on its own to a higher critical haircut. However, bank monitoring also increases the necessary leverage for which the bank defaults, i.e., $\varphi_1^S > \varphi_0^S$, which by itself would lead to a lower critical haircut. It turns out that the second effect of monitoring dominates the first and the smallest possible haircut guaranteeing solvency of banks is actually decreasing with bank monitoring, i.e., it holds that $\psi_0^S > \psi_1^S$. This result contrasts the one obtained with the monitoring technology used in section 3.3.

Welfare. The following lemma provides a characterization of utilitarian welfare using economic fundamentals. Due to our assumption of linear utility, utilitarian welfare comprises aggregate consumption as well as utility losses due to monitoring by bankers, i.e., welfare, denoted by W, satisfies $W = C^H + C^B - m\kappa q\varphi E$, where $C^B = \mathbb{E}_m[C_s^B] =$

$(1 + \mathbb{E}[r^E_{s,m}(\varphi)])qE$ denotes aggregate consumption by bankers.[1] Welfare is generally affected by three factors: the regulatory maximum leverage φ^R and the haircut ψ, both limiting bank leverage and thus the capital allocation between loan-financed and bond-financed firms, as well as the monitoring activity of bankers m, influencing the productivity of loan-financed firms. Note that the monitoring decision of the banker may also be influenced by the policy measures θ, the regulatory maximum leverage φ^R and the haircut ψ (see lemma 6.2.2).

Lemma 6.2.3 (Welfare)
In equilibrium, welfare is given by $W_m(\theta) = (\mathbb{E}[A^L_{s,m}] - A^B - m\kappa q)\overline{\varphi}_m(\theta)E + A^B(K + E)$.

Monitoring. We proceed as in section 3.3 by contrasting two situations: In the first, the banker is solely constrained by capital, as collateral requirements set by the central bank are sufficiently loose, i.e., it holds that $\overline{\varphi}_0(\theta) = \overline{\varphi}_1(\theta) = \varphi^R$. In the second situation, the banker is constrained by liquidity at least without monitoring, i.e., it holds that $\overline{\varphi}_0(\theta) < \overline{\varphi}_1(\theta) \leq \varphi^R$. In the first (second) situation, the haircut ψ set on bank loans used as collateral must be sufficiently small (large), so that it holds that $\overline{\varphi}_0(\theta) = \varphi^L_0(\psi) \geq (<)\varphi^R$ or, equivalently,

$$\frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}[A^L_{s,0}]} \geq (<)\varphi^R \qquad \Leftrightarrow \qquad \psi \leq (>)\tilde{\psi}_0(\varphi^R) := 1 - \frac{\alpha A^B}{\mathbb{E}[A^L_{s,0}]}\frac{\varphi^R - 1}{\varphi^R},$$

where we exploited equation (6.8) to represent $\varphi^L_m(\psi)$ using model primitives. Note that the banker is constrained by liquidity with monitoring whenever the collateral requirements are sufficiently tight, i.e., it holds that $\overline{\varphi}_1(\theta) = \varphi^L_1(\psi) < \varphi^R$ or, equivalently,

$$\frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}[A^L_{s,1}]} < \varphi^R \qquad \Leftrightarrow \qquad \psi > \tilde{\psi}_1(\varphi^R) := 1 - \frac{\alpha A^B}{\mathbb{E}[A^L_{s,1}]}\frac{\varphi^R - 1}{\varphi^R},$$

where we again used the representation of the leverage $\varphi^L_m(\psi)$, following from equation (6.8). Bank monitoring increases the collateral value of bank loans and allows the bank to borrow more reserves from the central bank. Thus, when the bank is liquidity-constrained with monitoring, it is also liquidity-constrained without monitoring. Formally, it holds

[1] Note that the idiosyncratic productivity shocks are i.i.d. across firms, and banks and firms exist each in a continuum, and as they are matched one-to-one. Thus, by the law of large numbers, the expected consumption by the banker equals the aggregate consumption of bankers.

that $\tilde{\psi}_1^S(\varphi^R) > \tilde{\psi}_0^S(\varphi^R)$.

Next, we describe the banker's monitoring decision in the presence of sufficiently loose collateral requirements set by the central bank, so that the banker is never constrained by liquidity but only by capital. In other words, the haircut set by the central bank is sufficiently small, so that it satisfies $\psi \leq \tilde{\psi}_0(\varphi^R)$.

Corollary 6.2.1 (Monitoring Decision without Liquidity Constraints) *Suppose the collateral requirements set by the central bank are sufficiently loose, so that the bank is never liquidity-constrained, i.e., $\psi \leq \tilde{\psi}_0(\varphi^R)$. Then, it holds that $\overline{\varphi}_0(\theta) = \overline{\varphi}_1(\theta) = \varphi^R$ and the banker optimally monitors (i.e., $m = 1$), if and only if*

(I) without solvency risk, i.e., $\varphi^R \leq \varphi_0^S$, it holds that $\tilde{\mathcal{M}}_N \geq 0$ where

$$\tilde{\mathcal{M}}_N := (1-\eta)\triangle - \kappa q,$$

(II) with partial solvency risk, i.e., $\varphi_1^S \geq \varphi^R > \varphi_0^S$, it holds that $\tilde{\mathcal{M}}_P(\varphi^R) \geq 0$, where

$$\tilde{\mathcal{M}}_P(\varphi^R) := -(1-\eta)(A^B - A_{l,1}^L) + \frac{(1-\eta)A^B}{\varphi^R} - \kappa q,$$

(III) with full solvency risk, i.e., $\varphi^R > \varphi_1^S$, it holds that $\tilde{\mathcal{M}}_F \geq 0$, where

$$\tilde{\mathcal{M}}_F := -\kappa q.$$

Furthermore, it holds that $\lim_{\varphi^R \searrow \varphi_0^S} \tilde{\mathcal{M}}_P(\varphi^R) = \tilde{\mathcal{M}}_N$ and $\lim_{\varphi^R \nearrow \varphi_1^S} \tilde{\mathcal{M}}_P(\varphi^R) = \tilde{\mathcal{M}}_F$.

Note that without a solvency risk or with full exposure to a solvency risk, the banker's monitoring decision is not affected by the capital requirements in the presence of loose collateral requirements. Without solvency risk, the banker monitors whenever the benefits following from the return channel are sufficient to cover the monitoring costs, i.e., whenever it holds that $(1-\eta)\triangle \geq \kappa q$. With a full exposure to solvency risk, in turn, the banker does not enjoy any benefits from the increased productivity of the financed firm, but only incurs costs when monitoring. Accordingly, the banker monitors in this case only if there are no monitoring costs, i.e., whenever it holds that $\kappa = 0$. Finally, with partial exposure to a solvency risk, the banker's incentives depend on the regulatory maximum leverage following from the capital requirements. Specifically, a loosening of capital requirements decreases the banker's incentives to monitor, i.e., it holds that

$$\frac{\partial \mathcal{M}_P(\theta)}{\partial \varphi^R} = -\frac{(1-\eta)A^B}{(\varphi^R)^2} < 0.$$

From the latter result and the fact that $\lim_{\varphi^R \searrow \varphi_0^S} \tilde{\mathcal{M}}_P(\varphi^R) = \tilde{\mathcal{M}}_N$, we know that in the presence of partial solvency risk, the banker is never monitoring if $(1-\eta)\triangle \leq \kappa q$, and is always monitoring if $(1-\eta)\triangle > \kappa q$ and $\varphi^R \leq \varphi^*$, where

$$\tilde{\mathcal{M}}_P(\varphi^*) = -(1-\eta)(A^B - A^L_{l,1}) + \frac{(1-\eta)A^B}{\varphi^*} - \kappa q = 0 \quad \Leftrightarrow \quad \varphi^* = \frac{(1-\eta)A^B}{\kappa q + (1-\eta)(A^B - A^L_{l,1})}.$$

We are particularly interested in situations where monitoring is socially optimal but the costs associated with monitoring and the exposure to a solvency risk induce the banker to shirk in the absence of the collateral leverage channel. From lemma 3.3.5, we know that the condition $(1-\eta)\triangle \geq \kappa q$ guarantees that monitoring is socially optimal.

Assumption 6.2.2 (Monitoring Costs)
$(1-\eta)\triangle \geq \kappa q$.

We now want to analyze the banker's monitoring decision in the presence of sufficiently loose capital requirements, such that the banker is never constrained by capital but only by liquidity, i.e. it holds that $\varphi^R \geq \varphi^L_m(\psi)$ for all m. We thereby again focus on the three situations, differing in the banker's exposure to a solvency risk; see cases (I)-(III) in lemma 6.2.2. First, we can show that under assumption 6.2.2, the banker always monitors without solvency risk, even without taking the benefits following from the collateral leverage channel into account. The reason is that the expected benefits of a higher loan repayment are sufficient to exceed the monitoring costs ($(1-\eta)\triangle \geq \kappa q$). Second, these direct effects of monitoring are not internalized by the banker if there is a solvency risk. In the cases with partial and full exposure to solvency risk, lemma 6.2.4 thus provides the conditions on the haircut ψ, so that the monitoring benefits following from the collateral leverage channel are sufficient to incentivize the banker to monitor.

Lemma 6.2.4 (Monitoring Decision without Capital Constraints)
Suppose that the capital requirements set by the bank regulator are sufficiently loose, so

that the banker is never constrained by capital, i.e., $\varphi^R \geq \varphi^L_m(\psi)$*. Then, it holds that* $\overline{\varphi}_m(\theta) = \varphi^L_m(\psi)$ *for all* m *and*

(I) with no solvency risk, i.e., $\psi \geq \psi^S_0$*, the banker always monitors,*

(II) with partial solvency risk, i.e., $\psi^S_0 > \psi \geq \psi^S_1$*, there exists a critical haircut* $\psi^{**} = \min\left\{\psi^S_0 \geq \psi \geq \psi^S_1 : \mathcal{M}_P(\theta) \geq 0\right\}$*, so that the banker monitors if and only if* $\psi \geq \psi^{**}$*,*

(III) with full solvency risk, i.e., $\psi^S_1 > \psi$*, the banker monitors if and only if*

$$\psi \leq \hat{\psi} := 1 - \frac{\chi\alpha A^B}{\chi\mathbb{E}[A^L_{s,0}] + (1-\eta)\triangle}, \qquad \textit{where} \qquad \chi := \frac{\kappa q}{\eta(A^L_{h,0} - A^B)}.$$

We are particularly interested in case (III) of lemma 6.2.2 and lemma 6.2.4, where independent of the monitoring decision, the banker is exposed to a solvency risk, i.e., it holds that $\psi^S_1 > \psi$. Note that, in the presence of monitoring, the banker can only be exposed to a solvency risk if it holds that

$$\psi^M_1 < \psi^S_1 \qquad \Leftrightarrow \qquad \frac{E}{K} < \frac{A^B - A^L_{l,1}}{A^L_{l,1}}.$$

We now further detail when the critical haircut $\hat{\psi}$, which in the presence of a full exposure to a solvency risk induces the banker to monitor, can be achieved indeed, as it weakly exceeds the smallest feasible haircut ψ^M_1, and when the condition $\psi \leq \hat{\psi}$ does not constitute an additional condition, as it holds that $\hat{\psi} \geq \psi^S_1$.

Lemma 6.2.5 (Collateral Leverage Channel of Monitoring)
It holds that $\psi^M_1 \leq \hat{\psi}$ *if and only if*

$$\frac{E}{K} \leq \frac{1-\chi}{\chi}\frac{(1-\eta)\triangle}{\mathbb{E}[A^L_{s,1}]},$$

and $\hat{\psi} \geq \psi_1^S$ *if and only if*

$$\chi \leq \frac{(1-\eta)\triangle A_{l,1}^L}{A^B \mathbb{E}[A_{s,1}^L] - A_{l,1}^L \mathbb{E}[A_{s,0}^L]}.$$

The parameter χ *follows from lemma 6.2.4.*

Optimal bank regulation and optimal monetary policy

As in section 3.3, the government aims at maximizing welfare by setting the appropriate bank regulation and monetary policy. Also with the alternative technology, the neutrality of money applies, so that the optimization problem of the government is formally given by

$$\max_{\theta \in \Theta_m} W_m(\theta) = \max_{\theta \in \Theta_m} (\mathbb{E}[A_{s,m}^L] - A^B - m\kappa q)\overline{\varphi}_m(\theta)E + A^B(K+E),$$

where we used lemma 6.2.3 to express welfare $W_m(\theta)$ and again applied the notation $\Theta_m := [1, +\infty) \times [\psi_m^M, 1]$ to represent the set of feasible policy measures, which itself depends on the monitoring activity m of the banker. In particular, not only is the monitoring activity m influenced by the policy measures, but also the central bank's set of feasible haircuts $[\psi_m^M, 1]$ is affected by the monitoring activity m. As outlined before, the smallest feasible haircut increases with monitoring ($\psi_1^M > \psi_0^M$). Thus, if bankers monitor ($m = 1$), the central bank finds itself unable to set any haircut ψ lower than ψ_1^M.

We first discuss the optimal bank regulation in the presence of sufficiently loose collateral requirements set by the central bank, i.e., the haircut satisfies $\psi \leq \tilde{\psi}_0(\varphi^R)$, so that the banker is only constrained by capital. The logic of the optimal bank regulation exactly follows the one in section 3.3.

Proposition 6.2.2 (Optimal Bank Regulation without Liquidity Constraints) *Suppose the central bank sets sufficiently loose collateral requirements, so that the banker is never constrained by liquidity, i.e., the haircut satisfies* $\psi \leq \tilde{\psi}_0(\varphi^R)$.

Then, the bank regulator optimally sets capital requirements leading to the regulatory maximum leverage $\varphi^R = \varphi^M$ *whenever (i)* $\varphi^M \leq \varphi^*$*, such that bank lending is maximized and the banker monitors, or (ii)* $\varphi^M > \varphi^*$*, such that bank lending is maximized and the banker does not monitor, but reducing bank leverage to induce monitoring does not yield a welfare* *gain, i.e.,*

$$\frac{\varphi^M}{\varphi^*} \geq 1 + \frac{(1-\eta)\triangle}{\mathbb{E}[A^L_{s,0}] - A^B}, \tag{6.9}$$

Otherwise, the bank regulator optimally implements capital requirements leading to the regulatory maximum leverage $\varphi^R = \varphi^*$*, restricting bank leverage below the maximum feasible one and thereby inducing monitoring.*

We now discuss the optimal monetary policy, assuming that the banker is only constrained by liquidity. In other words, capital requirements set by the bank regulator are sufficiently loose, i.e., the regulatory maximum leverage satisfies $\varphi^R \geq \varphi^L_m(\psi)$. For what follows, we use the notation $\hat{\varphi} = \varphi^L_0(\hat{\psi})$.

Proposition 6.2.3 (Optimal Monetary Policy without Capital Constraints) *Suppose that the bank regulator sets sufficiently loose capital requirements, so that the banker is never constrained by capital, i.e.,* $\varphi^R \geq \varphi^L_m(\psi)$*.*

Then, the central bank optimally chooses the smallest feasible haircut $\psi = \psi^M_1$ *whenever (i)* $\psi^M_1 \geq \psi^{**}$*, or (ii)* $\psi^S_1 > \psi^M_1$ *and* $\hat{\psi} \geq \psi^M_1$*, such that bank lending is maximized and the banker monitors.*

The central bank optimally chooses the haircut $\psi = \psi^M_0$ *whenever (i)* $\psi^{**} > \psi^M_1$ *and* $\psi^M_0 > \hat{\psi}$*, such that the banker does not monitor, but reducing the bank leverage to induce monitoring does not yield a welfare gain, i.e.,*

$$\frac{\varphi^M}{\varphi^{**}} \geq \ 1 + \frac{(1-\eta)\triangle - \kappa q}{\mathbb{E}[A^L_{s,0}] - A^B}.$$

The central bank optimally chooses the haircut $\psi = \hat{\psi}$ *whenever* $\psi^{**} > \psi^M_1$*,* $\psi^S_1 > \psi^M_0$ *and* $\hat{\psi} \geq \psi^M_0$*, such that the banker does not monitor, but reducing the bank leverage to induce monitoring does not yield a welfare gain, i.e.,*

$$\frac{\hat{\varphi}}{\varphi^{**}} \geq \ 1 + \frac{(1-\eta)\triangle - \kappa q}{\mathbb{E}[A^L_{s,0}] - A^B}.$$

Otherwise, the central bank optimally chooses the haircut $\psi = \psi^{**}$ *to limit the bank leverage* *below the maximum feasible and thereby inducing monitoring.*

We now outline the optimal mix of bank regulation and monetary policy.

Corollary 6.2.2 (Optimal Bank Regulation and Optimal Monetary Policy) *It is optimal to set capital requirements and collateral requirements such that*

(i) $\varphi^R \geq \varphi_1^L(\psi)$ *and* $\psi = \psi_1^M$ *whenever* $\psi_1^M \geq \psi^{**}$, *or* $\psi_1^S > \psi_1^M$ *and* $\hat{\psi} \geq \psi_1^M$,

(ii) $\varphi^R \geq \varphi_0^L(\psi)$ *and* $\psi = \psi_0^M$ *whenever* $\psi^{**} > \psi_1^M$, $\psi_1^S > \psi_0^M > \hat{\psi}$, *and*

$$\frac{\varphi^M}{\varphi^{**}} \geq \; 1 + \frac{(1-\eta)\triangle - \kappa q}{\mathbb{E}[A_{s,0}^L] - A^B},$$

(iii) $\varphi^R = \varphi^M$ *and* $\psi \leq \tilde{\psi}_0(\varphi^R)$ *whenever* $\psi^{**} > \psi_1^M$, $\psi_1^S > \psi_0^M$, $\hat{\psi} \geq \psi_0^M$, *and*

$$\frac{\varphi^M}{\varphi^{**}} \geq \; 1 + \frac{(1-\eta)\triangle - \kappa q}{\mathbb{E}[A_{s,0}^L] - A^B},$$

(iv) $\varphi^R \geq \varphi_1^L(\psi)$ *and* $\psi = \psi^{**}$ *otherwise.*

6.2.3 Proofs

Proof of Lemma 3.3.1. Firms are penniless and operate under limited liability, so that they are fully protected from losses. Accordingly, if the loan-financed firm is facing excess returns in one of the states, i.e., $A_s^L > (1 + r_s^L)q$ for some s, the expected profits are increasing with the input K^L of capital good to production. Thus, there exists no optimal, finite demand for the capital good by the loan-financed firm, which we denote by $K^L = +\infty$. In contrast, without excess returns, i.e., $A_s^L \leq (1 + r_s^L)q$ for all s, the loan-financed firm is making zero profits for any production input due to limited liability. Accordingly, the firm is indifferent between any amount of capital good put into production, and the optimal demand is given by $K^L \in [0, +\infty)$.

Similarly, there exists no optimal, finite demand of capital good by the bond-financed firm if it holds that $A^B > (1 + r^B)q$, which we denote by $K^B = +\infty$. In turn, if it holds that $A^B \leq (1 + r^B)q$, the bond-financed firm is indifferent between any input of capital good to production, i.e., $K^B \in [0, +\infty)$. ■

Proof of Lemma 3.3.2. Due to our assumption of linear utility, the household maximizes consumption $C^H = [\gamma(1+r^D) + (1-\gamma)(1+r^B)]qK + \tau + \pi$. The optimal choice of the household is thus of knife-edge type, i.e., the household invests the revenues from capital good sales in the asset which yields the highest return. In other words, the household maximizes utility by only holding deposits ($\gamma = 1$) if the deposit rate exceeds the bond rate ($r^D > r^B$), and by only investing into bonds ($\gamma = 0$) if the bond rate exceeds the deposit rate ($r^D < r^B$). Otherwise ($r^D = r^B$), the household is indifferent between deposits and bonds ($\gamma \in [0, 1]$). ■

Proof of Lemma 3.3.3. Note that reserves can be borrowed from the central bank at an interest rate r^L_{CB} and can be deposited at the central bank at an interest rate r^D_{CB}. The interest rate for interbank loans is given by $r^L_{IB} > 0$, whereas the interest rate on interbank deposits is given by r^D_{IB}. We assume that the bank cannot differentiate between deposits held by other banks and deposits from households and firms, so that it holds that $r^D_{IB} = r^D$. Interbank loans are only demanded if $r^L_{IB} \leq r^L_{CB}$, whereas interbank deposits are only attractive to the bank if $r^D \geq r^D_{CB}$. Otherwise, the bank would only deposit at the central bank. The liquidity provided on the interbank market through loans L^{IB} to other banks is matched by interbank deposits D^{IB} held by the borrowing banks. Thus, it holds that $L^{IB} = D^{IB}$. Interbank deposits are fully withdrawn by the borrowing banks if these banks must settle deposit outflows due to transactions on the capital good market. The lending bank must settle the outflow of interbank deposits by using reserves in the amount $D^{CB} = D^{IB}$, which this bank must borrow from the central bank by demanding loans L^{CB}. The revenues from interbank lending are given by $r^L_{IB} L^{IB}$, whereas the costs of interbank lending are given by $r^D D^{IB} + L^{CB} - r^D_{CB} D^{CB}$. Using $L^{IB} = D^{IB}$ and $L^{CB} = D^{CB} = D^{IB}$, the bank only offers interbank loans and deposits if

$$r^L_{IB} \geq r^D + r^L_{CB} - r^D_{CB} \qquad \Leftrightarrow \qquad r^L_{IB} \geq r^D,$$

where we used the equality of central bank rates ($r^L_{CB} = r^D_{CB}$), following from assumption 3.3.2. Since the interbank market is active only if $r^D \geq r^D_{CB}$ and $r^L_{IB} \leq r^L_{CB}$, we can conclude that the interest rates satisfy $r^L_{IB} = r^D = r^D_{CB}$. ■

Proof of Lemma 3.3.4. First, we focus on the banker's optimal choice of the leverage. The banker's expected utility is given by

$$\begin{aligned}\{1+\mathbb{E}_m[r_s^E(\varphi)]-m\kappa\varphi\}qE &= \{\mathbb{E}_m[\{(A_s^L-A^B)\varphi+A^B\}^+]-m\kappa q\varphi\}E \\ &= \eta_m[(A_h^L-A^B)\varphi+A^B]E \\ &\quad + \mathbb{1}\{\varphi\le\varphi^S\}(1-\eta_m)[(A_l^L-A^B)\varphi+A^B]E - m\kappa q\varphi E.\end{aligned}$$

Based on assumption 3.3.1, even without monitoring, the expected productivity of a loan-financed firm exceeds the productivity of a bond-financed firm, i.e., it holds that $\mathbb{E}_0[A_s^L] > A^B$. Accordingly, for any monitoring decision m, the banker maximizes the expected return from banking operations by choosing the maximum possible leverage, i.e., $\varphi = \overline{\varphi}_m(\theta)$.

Second, we focus on the banker's optimal monitoring decision. This monitoring decision crucially depends on whether there is solvency risk or not. First, let us focus on the case where, independent of the monitoring decision, the banker is not exposed to a solvency risk, i.e., it holds that $\overline{\varphi}_m(\theta) \le \varphi^S$ for all m. Then, the banker monitors (i.e., $m = 1$) if and only if

$$\begin{aligned} &\{\mathbb{E}_1[(A_s^L-A^B)\overline{\varphi}_1(\theta)+A^B]-\kappa q\overline{\varphi}_1(\theta)\}E \ge \{\mathbb{E}_0[(A_s^L-A^B)\overline{\varphi}_0(\theta)+A^B]\}E \\ \Leftrightarrow\quad &(\mathbb{E}_1[A_s^L]-A^B)\overline{\varphi}_1(\theta)-(\mathbb{E}_0[A_s^L]-A^B)\overline{\varphi}_0(\theta)\ge\kappa q\overline{\varphi}_1(\theta),\end{aligned}$$

which can be further rearranged to

$$\begin{aligned} &(\mathbb{E}_1[A_s^L]-\mathbb{E}_0[A_s^L])\overline{\varphi}_1(\theta)+(\mathbb{E}_0[A_s^L]-A^B)[\overline{\varphi}_1(\theta)-\overline{\varphi}_0(\theta)]\ge\kappa q\overline{\varphi}_1(\theta) \\ \Leftrightarrow\quad &\mathcal{M}_N(\theta) := \triangle(A_h^L-A_l^L)+(\mathbb{E}_0[A_s^L]-A^B)\left[1-\frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right]-\kappa q\ge 0.\end{aligned}$$

Second, we focus on the case where the banker is exposed to a solvency risk only with monitoring, i.e., it holds that $\overline{\varphi}_1(\theta) > \varphi^S \ge \overline{\varphi}_0(\theta)$. Then, the banker monitors (i.e., $m = 1$)

if and only if

$$\{\eta_1[(A_h^L - A^B)\overline{\varphi}_1(\theta) + A^B] - \kappa q\overline{\varphi}_1(\theta)\}E \geq \{(\mathbb{E}_0[A_s^L] - A^B)\overline{\varphi}_0(\theta) + A^B\}E$$

$$\Leftrightarrow \quad \eta_1[(A_h^L - A^B)\overline{\varphi}_1(\theta) + A^B] - (\mathbb{E}_0[A_s^L] - A^B)\overline{\varphi}_0(\theta) - A^B \geq \kappa q\overline{\varphi}_1(\theta)$$

$$\Leftrightarrow \quad (\eta_1 - \eta_0)(A_h^L - A^B)\overline{\varphi}_1(\theta) + (\mathbb{E}_0[A_s^L] - A^B)[\overline{\varphi}_1(\theta) - \overline{\varphi}_0(\theta)]$$

$$- (1 - \eta_0)(A_l^L - A^B)\overline{\varphi}_1(\theta) - (1 - \eta_1)A^B \geq \kappa q\overline{\varphi}_1(\theta)$$

$$\Leftrightarrow \quad \mathcal{M}_P(\theta) := \triangle(A_h^L - A^B) + (\mathbb{E}_0[A_s^L] - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right]$$

$$+ (1 - \eta_0)(A^B - A_l^L) - \frac{(1 - \eta_1)A^B}{\overline{\varphi}_1(\theta)} - \kappa q \geq 0.$$

Third, we focus on the case where, independent of the monitoring decision, the banker is exposed to a solvency risk, i.e., it holds that $\overline{\varphi}_m(\theta) > \varphi^S$ for all m. Then, the banker monitors (i.e., $m = 1$) if and only if

$$\{\eta_1[(A_h^L - A^B)\overline{\varphi}_1(\theta) + A^B] - \kappa q\overline{\varphi}_1(\theta)\}E \geq \eta_0[(A_h^L - A^B)\overline{\varphi}_0(\theta) + A^B]E$$

$$\Leftrightarrow \quad \eta_1[(A_h^L - A^B)\overline{\varphi}_1(\theta) + A^B] - \eta_0[(A_h^L - A^B)\overline{\varphi}_0(\theta) + A^B] \geq \kappa q\overline{\varphi}_1(\theta)$$

$$\Leftrightarrow \quad \eta_1(A_h^L - A^B)\overline{\varphi}_1(\theta) - \eta_0(A_h^L - A^B)\overline{\varphi}_0(\theta) + \triangle A^B \geq \kappa q\overline{\varphi}_1(\theta)$$

$$\Leftrightarrow \quad (\eta_1 - \eta_0)(A_h^L - A^B)\overline{\varphi}_1(\theta) + \eta_0(A_h^L - A^B)[\overline{\varphi}_1(\theta) - \overline{\varphi}_0(\theta)] + \triangle A^B \geq \kappa q\overline{\varphi}_1(\theta)$$

$$\Leftrightarrow \quad \mathcal{M}_F(\theta) := \triangle(A_h^L - A^B) + \eta_0(A_h^L - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right] + \frac{\triangle A^B}{\overline{\varphi}_1(\theta)} - \kappa q \geq 0.$$

Note that the banker cannot face situations where there is solvency risk only without monitoring, i.e., where it holds that $\overline{\varphi}_0(\theta) > \varphi^S \geq \overline{\varphi}_1(\theta)$. The reason is that the maximum possible leverage $\varphi_m^L(\psi)$ increases with monitoring (i.e., $\overline{\varphi}_0(\theta) \leq \overline{\varphi}_1(\theta)$), while the leverage threshold for solvency φ^S is unaffected by monitoring. ■

Proof of Proposition 3.3.1. First, note that in any competitive equilibrium, the capital good market must clear. Accordingly, bank lending cannot exceed the funds needed to purchase the entire endowment in the economy, i.e., it must holds that $QK^L = L^b = \overline{\varphi}_m(\theta)QE \leq Q(K+E)$ or, equivalently, $\overline{\varphi}_m(\theta) \leq 1 + K/E := \varphi^M$. By definition, $\overline{\varphi}_m(\theta) = \min\{\varphi^R, \varphi^L_m(\psi)\}$, so that the latter inequality implies $\varphi^R \leq \varphi^M$ or $\varphi^L_m(\psi) \leq \varphi^M$. Using the structure of $\varphi^L_m(\psi)$, as provided in equation (3.7), the latter inequality can be rewritten as

$$\begin{aligned}
& \frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_m[A^L_s]} \leq 1 + K/E \\
\Leftrightarrow \qquad & \alpha A^B \leq \{\alpha A^B - (1-\psi)\mathbb{E}_m[A^L_s]\}(1 + K/E) \\
\Leftrightarrow \qquad & (1-\psi)\mathbb{E}_m[A^L_s](1 + K/E) \leq \alpha A^B K/E,
\end{aligned}$$

which further simplifies to

$$(1-\psi) \leq \frac{\alpha A^B}{\mathbb{E}_m[A^L_s](1 + E/K)} \qquad \Leftrightarrow \qquad \psi \geq \psi^M_m := 1 - \frac{\alpha A^B}{\mathbb{E}_m[A^L_s](1 + E/K)}.$$

Thus, ψ^M_m represents the smallest feasible haircut the central bank can choose.

Again using $\overline{\varphi}_m(\theta) = \min\{\varphi^R, \varphi^L_m(\psi)\}$, the banker can only be exposed to a solvency risk if $\varphi^R > \varphi^S$ and $\varphi^L_m(\psi) > \varphi^S$. Using the structure of φ^S and $\varphi^L_m(\psi)$, as provided in equation (3.4) and equation (3.7), respectively, the latter inequality can be rewritten as

$$\begin{aligned}
& \frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}_m[A^L_s]} > \frac{A^B}{A^B - A^L_l} \\
\Leftrightarrow \qquad & \alpha(A^B - A^L_l) > \alpha A^B - (1-\psi)\mathbb{E}_m[A^L_s] \\
\Leftrightarrow \qquad & (1-\psi)\mathbb{E}_m[A^L_s] > \alpha A^L_l \\
\Leftrightarrow \qquad & \psi < \psi^S_m := 1 - \frac{\alpha A^L_l}{\mathbb{E}_m[A^L_s]}.
\end{aligned}$$

■

Proof of Lemma 3.3.5. Due to our assumption of linear utility for households and bankers, utilitarian welfare comprises aggregate consumption and utility losses due to monitoring, i.e.,

$$W = [\gamma(1 + r^D) + (1 - \gamma)(1 + r^B)]qK + \tau + \pi + (1 + \mathbb{E}_m[r_s^E(\varphi)] - m\kappa\varphi)qE.$$

In equilibrium, the interest rates on bonds and deposits satisfy $r^D = r^B = A^B/q - 1$ (for a derivation see subsections 3.3.4 and 3.3.3), so that firms make zero profits, i.e., $\pi = 0$, and welfare translates into

$$W = A^B K + \tau + \mathbb{E}_m[\{(A_s^L - A^B)\varphi + A^B\}^+]E - m\kappa\varphi qE,$$

where we used equation (3.3) in subsection 3.3.6, stating that the rate of return on bank equity is given by $r_s^E(\varphi) = \{(A_s^L - A^B)\varphi + A^B\}^+/q - 1$. The government uses taxes to cover central bank losses and bank losses in the case of default, while it distributes central bank profits through transfers, i.e., it holds that $T = \Pi^{b,-} + \Pi^{CB}$. Note that as we focus on a representative bank, deposit outflows match deposit inflows. Together with the equal interest rates on reserves deposits and reserve loans (see assumption 3.3.2), we can then conclude that the central bank makes neither profits nor losses, i.e., $\Pi^{CB} = 0$. Then, taxes must only cover bank losses in the case of default, so that government taxes satisfy in real terms

$$\begin{aligned}\tau = \pi^{b,-} &= \mathbb{1}\{\varphi > \varphi^S\}(1 - \eta_m)[(A_l^L - A^B)\varphi + A^B]E \\ &= \mathbb{E}_m[\{(A_s^L - A^B)\varphi + A^B\}^-]E,\end{aligned}$$

where we make use of the notation $\{X\}^- = \min\{X, 0\}$. Welfare then simplifies to

$$W = A^B K + \mathbb{E}_m[(A_s^L - A^B)\varphi + A^B]E - m\kappa\varphi qE,$$

which, using the bank's optimal leverage choice $\varphi = \overline{\varphi}_m(\theta)$ (see lemma 3.3.4), finally reads as

$$W_m(\theta) = (\mathbb{E}_m[A_s^L] - A^B - m\kappa q)\overline{\varphi}_m(\theta)E + A^B(K + E).$$

■

Proof of Corollary 3.3.1. The results follow directly from lemma 3.3.4 by using $\overline{\varphi}_0(\theta) =$

$\overline{\varphi}_1(\theta) = \varphi^R$, which follows from the assumption that the central bank implements sufficiently loose collateral requirements, i.e., $\psi \leq \tilde{\psi}_0(\varphi^R)$. Note that in any such situation, case (II) in lemma 3.3.4 cannot arise, where there is partial solvency risk, namely where banker is exposed to a solvency risk only with monitoring. Either the banker faces a solvency risk or not, so that we are left with the cases (I) and (III) of lemma 3.3.4.

We can then conclude that, if it holds $\psi \leq \tilde{\psi}_0(\varphi^R)$, so that $\overline{\varphi}_0(\theta) = \overline{\varphi}_1(\theta) = \varphi^R$, the banker optimally monitors (i.e., $m = 1$) if and only if

(I) without solvency risk, i.e., $\varphi^R \leq \varphi^S$, it holds $\tilde{\mathcal{M}}_N \geq 0$, where

$$\tilde{\mathcal{M}}_N := \triangle(A_h^L - A_l^L) - \kappa q,$$

(II) with full solvency risk, i.e., $\varphi^R > \varphi^S$, it holds $\tilde{\mathcal{M}}_F(\varphi^R) \geq 0$, where

$$\tilde{\mathcal{M}}_F(\varphi^R) := \triangle(A_h^L - A^B) + \frac{\triangle A^B}{\varphi^R} - \kappa q.$$

Furthermore, it holds that $\lim_{\varphi^R \searrow \varphi^S} \tilde{\mathcal{M}}_F(\varphi^R) = \tilde{\mathcal{M}}_N$, as

$$\begin{aligned}
\lim_{\varphi^R \searrow \varphi^S} \tilde{\mathcal{M}}_F(\varphi^R) &= \lim_{\varphi^R \searrow \varphi^S} \triangle(A_h^L - A^B) + \frac{\triangle A^B}{\varphi^R} - \kappa q \\
&= \triangle(A_h^L - A^B) + \frac{\triangle A^B}{\varphi^S} - \kappa q \\
&= \triangle(A_h^L - A^B) + \triangle(A^B - A_l^L) - \kappa q \\
&= \triangle(A_h^L - A_l^L) - \kappa q,
\end{aligned}$$

where we made use of $\varphi^S = A^B/(A^B - A_l^L)$ which is provided by equation (3.4) in subsection 3.3.6. ■

Proof of Proposition 3.3.2. Based on assumption 3.3.3, the banker monitors in any case if there is no solvency risk, in particular no matter whether the leverage constraint stems from capital requirements or collateral requirements. Formally, this means that $\mathcal{M}_N(\theta) \geq 0$ for all $\theta = (\varphi^R, \psi)$ and $\tilde{\mathcal{M}}_N \geq 0$. However, whenever the banker is exposed to a solvency risk, it matters for the monitoring incentives if the banker is constrained by capital or liquidity, i.e.,

$$\mathcal{M}_F(\theta) - \tilde{\mathcal{M}}_F(\varphi^R) = \eta_0(A_h^L - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right] + \triangle A^B \left[\frac{1}{\overline{\varphi}_1(\theta)} - \frac{1}{\varphi^R}\right].$$

Note that we assume $\tilde{\psi}_0(\varphi^R) < \psi$, so that $\overline{\varphi}_0(\theta) = \varphi_0^L(\psi) < \varphi^R$ and thus

$$\mathcal{M}_F(\theta) - \tilde{\mathcal{M}}_F(\varphi^R) = \eta_0(A_h^L - A^B)\left[1 - \frac{\varphi_0^L(\psi)}{\overline{\varphi}_1(\theta)}\right] + \triangle A^B \left[\frac{1}{\overline{\varphi}_1(\theta)} - \frac{1}{\varphi^R}\right].$$

Moreover, note that for $\psi \leq \tilde{\psi}_1(\varphi^R)$ it holds that $\overline{\varphi}_1(\theta) = \varphi^R$ and otherwise $\overline{\varphi}_1(\theta) = \varphi_1^L(\psi) < \varphi^R$. Furthermore, note that

$$\lim_{\psi \searrow \tilde{\psi}_0(\varphi^R)} \mathcal{M}_F(\theta) - \tilde{\mathcal{M}}_F(\varphi^R) = \eta_0(A_h^L - A^B)\left[1 - \frac{\varphi^R}{\varphi^R}\right] + \triangle A^B \left[\frac{1}{\varphi^R} - \frac{1}{\varphi^R}\right] = 0,$$

as it holds that $\lim_{\psi \searrow \tilde{\psi}_0(\varphi^R)} \varphi_0^L(\psi) = \varphi^R$. ■

Proof of Proposition 3.3.3. As the central bank implements sufficiently loose collateral requirements, i.e., the haircut satisfies $\psi \leq \tilde{\psi}_0(\varphi^R)$, so that $\overline{\varphi}_0(\theta) = \overline{\varphi}_1(\theta) = \varphi^R$, we know, using lemma 3.3.5, that welfare is given by

$$W_m(\varphi^R) = (\mathbb{E}_m[A_s^L] - A^B - m\kappa q)\varphi^R E + A^B(K + E),$$

where the banker's monitoring decision is described by corollary 3.3.1. From corollary 3.3.1 and assumption 6.2.2, we know that the banker monitors whenever it holds that $\varphi^R \leq \varphi^*$, where φ^* is described by equation (3.9). Then, the bank regulator maximizes welfare by implementing the regulatory maximum leverage $\varphi^R = \varphi^M$ if it holds (i) $\varphi^M \leq \varphi^*$, so that bank lending is maximized and the banker monitors, or (ii) $\varphi^M > \varphi^*$, so that bank lending is maximized and the banker does not monitor, but reducing the bank leverage to induce monitoring does not yield a welfare gain, i.e., $W_0(\varphi^M) \geq W_1(\varphi^*)$ or, equivalently,

$$
\begin{aligned}
& (\mathbb{E}_0[A_s^L] - A^B)\varphi^M E + A^B(K+E) \geq (\mathbb{E}_1[A_s^L] - A^B - \kappa q)\varphi^* E + A^B(K+E) \\
\Leftrightarrow \quad & (\mathbb{E}_0[A_s^L] - A^B)\varphi^M \geq (\mathbb{E}_1[A_s^L] - A^B - \kappa q)\varphi^* \\
\Leftrightarrow \quad & \frac{\varphi^M}{\varphi^*} \geq \frac{\mathbb{E}_1[A_s^L] - A^B - \kappa q}{\mathbb{E}_0[A_s^L] - A^B}.
\end{aligned}
$$

Using $\mathbb{E}_1[A_s^L] = \mathbb{E}_0[A_s^L] + \triangle(A_h^L - A_l^L)$, the latter inequality further simplifies to

$$\frac{\varphi^M}{\varphi^*} \geq 1 + \frac{\triangle(A_h^L - A_h^L) - \kappa q}{\mathbb{E}_0[A_s^L] - A^B}.$$

Note that, based on assumption 3.3.1, even without monitoring by the matched banker, a loan-financed firm is more productive than a bond-financed firm, i.e., it holds that $\mathbb{E}_0[A_s^L] > A^B$. Accordingly, under the assumption that $\varphi^M > \varphi^*$, the bank regulator maximizes welfare without monitoring by setting the capital requirements such that $\varphi^R = \varphi^M$. Similarly, welfare with monitoring is maximized by setting the capital requirements such that $\varphi^R = \varphi^*$. Hence, we only need to compare welfare $W_0(\varphi^M)$ and $W_1(\varphi^*)$.

In all other situations, the bank regulator optimally sets capital requirements such that the regulatory maximum leverage is given by $\varphi^R = \varphi^*$, restricting bank leverage below the maximum feasible and thereby inducing the banker to monitor. ■

Proof of Proposition 3.3.4. As the bank regulator implements sufficiently loose capital requirements, i.e., $\varphi^R \geq \varphi_m^L(\psi)$, so that $\overline{\varphi}_m(\theta) = \varphi_m^L(\psi)$ for all m, we know, using lemma 3.3.5, that welfare is given by

$$W_m(\psi) = (\mathbb{E}_m[A_s^L] - A^B - m\kappa q)\varphi_m^L(\psi)E + A^B(K+E),$$

where the banker's monitoring decision is described by lemma 3.3.4. First, note that based on assumption 3.3.3, there exists a critical haircut ψ^{**} such that for $\theta^{**} = (\varphi^R, \psi^{**})$ it holds that

$$\mathcal{M}_F(\theta^{**}) = \triangle(A_h^L - A^B) + \eta_0(A_h^L - A^B)\left[1 - \frac{\varphi_0^L(\psi^{**})}{\varphi_1^L(\psi^{**})}\right] + \frac{\triangle A^B}{\varphi_1^L(\psi^{**})} - \kappa q = 0,$$

where we used $\overline{\varphi}_m(\theta) = \varphi_m^L(\psi)$ for all m, as it holds that $\varphi^R \geq \varphi_m^L(\psi)$. For what follows, we will make use of the notation $\varphi^{**} = \varphi_1^L(\psi^{**})$.

Furthermore, note that it holds that $\lim_{\psi \nearrow \psi_1^S(\varphi^R)} \mathcal{M}_F(\theta) = \mathcal{M}_N(\theta_1^S) > 0$, with $\theta_1 = (\varphi^R, \psi_1^S)$, where, based on lemma 3.3.4, for sufficiently loose collateral requirements implying $\varphi^R \geq \varphi_m^L(\psi)$ for all m, it holds that

$$\mathcal{M}_N(\theta_1^S) = \triangle(A_h^L - A_l^L) + (\mathbb{E}_0[A_s^L] - A^B)\left[1 - \frac{\varphi_0^L(\psi_1^S)}{\varphi^S}\right] - \kappa q,$$

where we used the fact that $\varphi_1^L(\psi_1^S) = \varphi^S$. Now observe that it holds that

$$\begin{aligned}
\lim_{\psi \nearrow \psi_1^S(\varphi^R)} \mathcal{M}_F(\theta) &= \triangle(A_h^L - A^B) + \eta_0(A_h^L - A^B)\left[1 - \frac{\varphi_0^L(\psi_1^S)}{\varphi_1^L(\psi_1^S)}\right] + \frac{\triangle A^B}{\varphi_1^L(\psi_1^S)} - \kappa q \\
&= \triangle(A_h^L - A^B) + \eta_0(A_h^L - A^B)\left[1 - \frac{\varphi_0^L(\psi_1^S)}{\varphi^S}\right] + \frac{\triangle A^B}{\varphi^S} - \kappa q \\
&= \triangle(A_h^L - A^B) + \eta_0(A_h^L - A^B)\left[1 - \frac{\varphi_0^L(\psi_1^S)}{\varphi^S}\right] + \triangle(A^B - A_l^L) - \kappa q \\
&= \triangle(A_h^L - A_l^L) + \eta_0(A_h^L - A^B)\left[1 - \frac{\varphi_0^L(\psi_1^S)}{\varphi^S}\right] - \kappa q \\
&= \mathcal{M}_N(\theta_1^S),
\end{aligned}$$

where we made use of $\varphi_1^L(\psi_1^S) = \varphi^S$ and $\varphi^S = A^B/(A^B - A_l^L)$, the latter following from equation (3.4) in subsection 3.3.6.

We can then conclude that the banker always monitors if it holds that $\varphi < \varphi^{**}$ and it is optimal for the central bank to set $\psi = \psi_1^M$ whenever $\varphi^M \leq \varphi^{**}$, so that bank lending is maximized and the banker monitors. Moreover, it is optimal for the central bank to set $\psi = \psi_0^M$ whenever $\varphi^M > \varphi^{**}$, so that bank lending is maximized and the banker does not monitor, and reducing the bank leverage to induce monitoring does not yield a welfare gain, i.e., it holds that $W_0(\psi_0^M) \geq W_1(\psi^{**})$ or, equivalently,

$$
\begin{aligned}
& (\mathbb{E}_0[A_s^L] - A^B)\varphi^M E + A^B(K+E) \geq (\mathbb{E}_1[A_s^L] - A^B - \kappa q)\varphi^{**} E + A^B(K+E) \\
\Leftrightarrow \quad & (\mathbb{E}_0[A_s^L] - A^B)\varphi^M \geq (\mathbb{E}_1[A_s^L] - A^B - \kappa q)\varphi^{**} \\
\Leftrightarrow \quad & \frac{\varphi^M}{\varphi^{**}} \geq \frac{\mathbb{E}_1[A_s^L] - A^B - \kappa q}{\mathbb{E}_0[A_s^L] - A^B} \\
\Leftrightarrow \quad & \frac{\varphi^M}{\varphi^{**}} \geq 1 + \frac{\triangle(A_h^L - A_l^L) - \kappa q}{\mathbb{E}_0[A_s^L] - A^B},
\end{aligned}
$$

where we made use of $\mathbb{E}_1[A_s^L] = \mathbb{E}_0[A_s^L] + \triangle(A_h^L - A_l^L)$. Note that, based on assumption 3.3.1, even without monitoring by the matched banker, a loan-financed firm is more productive than a bond-financed firm, i.e., it holds that $\mathbb{E}_0[A_s^L] > A^B$. Accordingly, under the assumption that $\varphi^M > \varphi^{**}$, the central bank maximizes welfare without monitoring by setting the haircut $\psi = \psi_0^M$. Similarly, welfare with monitoring is maximized by setting the haircut such that $\psi = \psi^{**}$. Hence, we only need to compare welfare $W_0(\psi_0^M)$ and $W_1(\psi^{**})$.

In all other situations, the central bank optimally sets the haircut $\psi = \psi^{**}$ to reduce bank leverage below the maximum feasible and thereby inducing the banker to monitor. ∎

Proof of Corollary 3.3.2. We start by showing that it holds that $\varphi^{**} = \varphi_1^L(\psi^{**}) > \varphi^*$. Note that by the definition of φ^{**} and φ^*, we obtain

$$\mathcal{M}_F(\theta^{**}) = 0 = \tilde{\mathcal{M}}_F(\varphi^*),$$

where $\theta^{**} = (\varphi^R, \psi^{**})$. The latter equation reads as

$$\triangle(A_h^L - A^B) + \eta_0(A_h^L - A^B)\left[1 - \frac{\varphi_0^L(\psi^{**})}{\varphi_1^L(\psi^{**})}\right] + \frac{\triangle A^B}{\varphi_1^L(\psi^{**})} - \kappa q = \triangle(A_h^L - A^B) + \frac{\triangle A^B}{\varphi^*} - \kappa q$$

and can be further simplified to

$$\eta_0(A_h^L - A^B)\left[1 - \frac{\varphi_0^L(\psi^{**})}{\varphi_1^L(\psi^{**})}\right] = \triangle A^B \left[\frac{1}{\varphi^*} - \frac{1}{\varphi_1^L(\psi^{**})}\right].$$

The left-hand side of the latter condition is strictly positive, so that we can conclude that it holds that $\varphi^* < \varphi_1^L(\psi^{**}) = \varphi^{**}$. Note that the difference between the two critical leverage ratios φ^* and φ^{**} originates from the collateral leverage channel of monitoring.

A liquidity-constrained banker monitors under higher leverage ratios than a capital-constrained banker. Based on assumption 3.3.1 and assumption 3.3.3, we know that more loan financing and monitoring by the banker both increase welfare. Accordingly, it is optimal to only constrain the bank by liquidity, through the implementation of sufficiently tight collateral requirements, while capital requirements set by the bank regulator should be sufficiently loose not to constrain the banker. Specifically, the capital requirements should lead to a regulatory maximum leverage $\varphi^R \geq \varphi_m^L(\psi)$ (e.g., $\varphi^R = \varphi^M$), where the haircut ψ should be set according to proposition 3.3.4. ■

Proof of Lemma 6.2.1. Firms are penniless and operate under limited liability, so that they are fully protected from losses. Accordingly, if the loan-financed firm is facing excess returns in one of the states, i.e., $A_{s,m}^L > (1 + r_{s,m}^L)q$ for some s, the expected profits are increasing with the input K^L of capital good to production. Thus, there exists no optimal, finite demand for capital good by the loan-financed firm, which we denote by $K^L = +\infty$. In contrast, without excess returns, i.e., $A_{s,m}^L \leq (1+r_{s,m}^L)q$ for all s, the loan-financed firm is making zero profits for any production input due to limited liability. Accordingly, the firm is indifferent between any amount of capital good put into production and the optimal demand is given by $K^L \in [0, +\infty)$. ■

Proof of Lemma 6.2.2. The expected utility of the banker is given by

$$\begin{aligned}\{1 + \mathbb{E}[r_{s,m}^E(\varphi)] - m\kappa\varphi\}qE &= \mathbb{E}[\{(A_{s,m}^L - A^B)\varphi + A^B\}^+]E - m\kappa\varphi qE \\ &= \eta[(A_{h,m}^L - A^B)\varphi + A^B]E \\ &\quad + \mathbb{1}\{\varphi \leq \varphi_m^S\}(1-\eta)[(A_{l,m}^L - A^B)\varphi + A^B]E - m\kappa q\varphi E.\end{aligned}$$

First, we focus on the banker's choice of leverage φ or, in other words, the decision about deposit issuance and loan supply. From assumption 6.2.1, we know that, even without monitoring by the banker, the loan-financed firm is more productive on average than the bond-financed firm, i.e., it holds that $\mathbb{E}[A_{s,0}^L] > A^B$. Thus, the banker optimally always leverages as much as possible, i.e., $\varphi = \overline{\varphi}_m(\theta)$.

Next, we focus on the banker's monitoring decision. The banker's incentives crucially depend on the exposure to a solvency risk, so that we must differentiate three situations. First, in any situation where, independent of the monitoring decision, the banker is not exposed to a solvency risk, i.e., it holds for all m that $\overline{\varphi}_m(\theta) \leq \varphi_m^S$, the banker decides to monitor (i.e., $m = 1$) if and only if

$$\begin{aligned}
& \{(\mathbb{E}[A_{s,1}^L] - A^B)\overline{\varphi}_1(\theta) + A^B - \kappa q \overline{\varphi}_1(\theta)\}E \geq \{(\mathbb{E}[A_{s,0}^L] - A^B)\overline{\varphi}_0(\theta) + A^B\}E \\
\Leftrightarrow \quad & (\mathbb{E}[A_{s,1}^L] - A^B)\overline{\varphi}_1(\theta) - (\mathbb{E}[A_{s,0}^L] - A^B)\overline{\varphi}_0(\theta) \geq \kappa q \overline{\varphi}_1(\theta) \\
\Leftrightarrow \quad & \mathbb{E}[A_{s,1}^L] - A^B - (\mathbb{E}[A_{s,0}^L] - A^B)\frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)} \geq \kappa q.
\end{aligned}$$

The latter inequality can be further rearranged to

$$\begin{aligned}
& \mathbb{E}[A_{s,1}^L] - A^B - (\mathbb{E}[A_{s,0}^L] - A^B) - (\mathbb{E}[A_{s,0}^L] - A^B)\left[\frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)} - 1\right] \geq \kappa q \\
\Leftrightarrow \quad & \mathbb{E}[A_{s,1}^L] - \mathbb{E}[A_{s,0}^L] + (\mathbb{E}[A_{s,0}^L] - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right] \geq \kappa q \\
\Leftrightarrow \quad & \mathcal{M}_N(\theta) := (1-\eta)\triangle + (\mathbb{E}[A_{s,0}^L] - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right] - \kappa q \geq 0,
\end{aligned}$$

where we used $A_{h,1}^L = A_{h,0}^L$ and $A_{l,1}^L = A_{l,0}^L + \triangle$.

Second, in any situation where the banker is exposed to a solvency risk only without monitoring, i.e., it holds that $\overline{\varphi}_0(\theta) > \varphi_0^S$ and $\overline{\varphi}_1(\theta) \leq \varphi_1^S$, the banker decides to monitor (i.e., $m = 1$) if and only if

$$\begin{aligned}
& \{(\mathbb{E}[A_{s,1}^L] - A^B)\overline{\varphi}_1(\theta) + A^B - \kappa q \overline{\varphi}_1(\theta)\}E \geq \eta[(A_{h,0}^L - A^B)\overline{\varphi}_0(\theta) + A^B]E \\
\Leftrightarrow \quad & (\mathbb{E}[A_{s,1}^L] - A^B)\overline{\varphi}_1(\theta) - \eta(A_{h,0}^L - A^B)\overline{\varphi}_0(\theta) + (1-\eta)A^B \geq \kappa q \overline{\varphi}_1(\theta) \\
\Leftrightarrow \quad & (\mathbb{E}[A_{s,1}^L] - A^B) - \eta(A_{h,0}^L - A^B)\frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)} + \frac{(1-\eta)A^B}{\overline{\varphi}_1(\theta)} \geq \kappa q.
\end{aligned}$$

The latter inequality can be rewritten to

$$(\mathbb{E}[A^L_{s,1}] - A^B) + (1-\eta)(A^B - A^L_{l,0}) - \eta(A^L_{h,0} - A^B)\frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)} + \frac{(1-\eta)A^B}{\overline{\varphi}_1(\theta)} \geq \kappa q + (1-\eta)(A^B - A^L_{l,0})$$

which, using $(1-\eta)(A^B - A^L_{l,0}) = \eta(A^L_{h,0} - A^B) - (\mathbb{E}[A^L_{s,0}] - A^B)$, translates into

$$(\mathbb{E}[A^L_{s,1}] - A^B) - (\mathbb{E}[A^L_{s,0}] - A^B) + \eta(A^L_{h,0} - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right] + \frac{(1-\eta)A^B}{\overline{\varphi}_1(\theta)} \geq \kappa q + (1-\eta)(A^B - A^L_{l,0}).$$

With $\mathbb{E}[A^L_{s,1}] = \mathbb{E}[A^L_{s,0}] + (1-\eta)\triangle$, and $A^L_{l,1} = A^L_{l,0} + \triangle$, the latter inequality simplifies to

$$\mathcal{M}_P(\theta) := \eta(A^L_{h,0} - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right] - (1-\eta)\left[A^B - A^L_{l,1} - \frac{A^B}{\overline{\varphi}_1(\theta)}\right] - \kappa q \geq 0.$$

Third, in any situation where, independent of the monitoring decision, the banker is exposed to a solvency risk, i.e., $\overline{\varphi}_m(\theta) > \varphi^S_m$ for all m, the banker monitors, i.e., $m = 1$, if and only if

$$\{\eta[(A^L_{h,1} - A^B)\overline{\varphi}_1(\theta) + A^B] - \kappa q\overline{\varphi}_1(\theta)\}E \geq \eta[(A^L_{h,0} - A^B)\overline{\varphi}_0(\theta) + A^B]E$$

$$\Leftrightarrow \quad \eta(A^L_{h,1} - A^B)\overline{\varphi}_1(\theta) - \eta(A^L_{h,0} - A^B)\overline{\varphi}_0(\theta) \geq \kappa q\overline{\varphi}_1(\theta)$$

$$\Leftrightarrow \quad \eta(A^L_{h,1} - A^B) - \eta(A^L_{h,0} - A^B)\frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)} \geq \kappa q.$$

Using $A^L_{h,1} = A^L_{h,0}$, the latter inequality further simplifies to

$$\mathcal{M}_F(\theta) := \eta(A^L_{h,0} - A^B)\left[1 - \frac{\overline{\varphi}_0(\theta)}{\overline{\varphi}_1(\theta)}\right] - \kappa q \geq 0.$$

Note that the banker can never face a situation where solvency risk only exists with monitoring, i.e., where it holds that $\overline{\varphi}_0(\theta) \leq \varphi^S_0$ and $\overline{\varphi}_1(\theta) > \varphi^S_1$. This is straightforward if, independent of the monitoring decision, the banker is always constrained by capital, i.e., $\overline{\varphi}_0(\theta) = \overline{\varphi}_1(\theta) = \varphi^R$, as it can never hold that $\varphi_0^S \geq \varphi^R > \varphi_1^S$ with $\varphi_1^S > \varphi_0^S$. Next, we show that such a situation cannot arise either if the banker is constrained by liquidity only, i.e., when it holds that $\overline{\varphi}_m(\psi) = \varphi_m^L(\psi)$ for all m. Specifically, we show that it cannot hold that $\varphi_0^L(\psi) \leq \varphi_0^S$ and $\varphi_1^L(\psi) > \varphi_1^S$. On that account, note that

$$\begin{aligned}
& \varphi_m^L(\psi) \leq \varphi_m^S \\
\Leftrightarrow \quad & \frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}[A_{s,m}^L]} \leq \frac{A^B}{A^B - A_{l,m}^L} \\
\Leftrightarrow \quad & \alpha(A^B - A_{l,m}^L) \leq \alpha A^B - (1-\psi)\mathbb{E}[A_{s,m}^L] \\
\Leftrightarrow \quad & (1-\psi)\mathbb{E}[A_{s,m}^L] \leq \alpha A_{l,m}^L \\
\Leftrightarrow \quad & \psi \geq \psi_m^S := 1 - \frac{\alpha A_{l,m}^L}{\mathbb{E}[A_{s,m}^L]},
\end{aligned}$$

where we made use of equations (6.6) and (6.8) to express the leverage ratios φ_m^S and $\varphi_m^L(\psi)$ in terms of the economic fundamentals. It thus holds that $\varphi_m^L(\psi) > \varphi_m^S$ if and only if $\psi < \psi_m^S$. Note further that

$$\psi_0^S = 1 - \frac{\alpha A_{l,0}^L}{\mathbb{E}[A_{s,0}^L]} > \psi_1^S = 1 - \frac{\alpha A_{l,1}^L}{\mathbb{E}[A_{s,1}^L]} = 1 - \frac{\alpha(A_{l,0}^L + \triangle)}{\mathbb{E}[A_{s,0}^L] + (1-\eta)\triangle},$$

where we used $A_{h,1}^L = A_{h,0}^L$ and $A_{l,1}^L = A_{l,0}^L + \triangle$. It thus follows that $\psi < \psi_1^S$ only if $\psi < \psi_0^S$, leading us to the conclusion that the banker can never face a situation where there is only solvency risk with monitoring. ■

Proof of Proposition 6.2.1. First, note that in any competitive equilibrium, the capital good market must clear. Accordingly, bank lending cannot exceed the funds needed to purchase the entire endowment in the economy, i.e., $QK^L = L^b = \overline{\varphi}_m(\theta)QE \leq Q(K+E)$ or, equivalently, $\overline{\varphi}_m(\theta) \leq 1 + K/E := \varphi^M$. As $\overline{\varphi}_m(\theta) = \min\{\varphi^R, \varphi_m^L(\psi)\}$, the latter inequality requires that $\varphi^R \leq \varphi^M$ or $\varphi_m^L(\psi) \leq \varphi^M$. Using the structure of $\varphi_m^L(\psi)$, as provided in (6.8), the latter inequality can be rewritten as

$$\begin{aligned} & \frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}[A^L_{s,m}]} \leq 1 + K/E \\ \Leftrightarrow \qquad & \alpha A^B \leq \{\alpha A^B - (1-\psi)\mathbb{E}[A^L_{s,m}]\}(1+K/E) \\ \Leftrightarrow \qquad & (1-\psi)\mathbb{E}[A^L_{s,m}](1+K/E) \leq \alpha A^B K/E, \end{aligned}$$

which further simplifies to

$$(1-\psi) \leq \frac{\alpha A^B}{\mathbb{E}[A^L_{s,m}](1+E/K)} \qquad \Leftrightarrow \qquad \psi \geq \psi^M_m := 1 - \frac{\alpha A^B}{\mathbb{E}[A^L_{s,m}](1+E/K)}.$$

Thus, ψ^M_m represents the smallest feasible haircut the central bank can choose.

Again using $\overline{\varphi}_m(\theta) = \min\{\varphi^R, \varphi^L_m(\psi)\}$, the banker can only be exposed to a solvency risk if $\varphi^R > \varphi^S_m$ and $\varphi^L_m(\psi) > \varphi^S_m$. Using the structure of φ^S_m and $\varphi^L_m(\psi)$, as provided by equations (6.6) and (6.8), the latter inequality can be rewritten as

$$\begin{aligned} & \frac{\alpha A^B}{\alpha A^B - (1-\psi)\mathbb{E}[A^L_{s,m}]} > \frac{A^B}{A^B - A^L_{l,m}} \\ \Leftrightarrow \qquad & \alpha(A^B - A^L_{l,m}) > \alpha A^B - (1-\psi)\mathbb{E}[A^L_{s,m}] \\ \Leftrightarrow \qquad & (1-\psi)\mathbb{E}[A^L_{s,m}] > \alpha A^L_{l,m} \\ \Leftrightarrow \qquad & \psi < \psi^S_m := 1 - \frac{\alpha A^L_{l,m}}{\mathbb{E}[A^L_{s,m}]}. \end{aligned}$$

■

Proof of Lemma 6.2.3. Due to our assumption of linear utility for households and bankers, utilitarian welfare comprises aggregate consumption and utility losses due to monitoring, i.e.,

$$W = [\gamma(1+r^D) + (1-\gamma)(1+r^B)]qK + \tau + \pi + (1 + \mathbb{E}[r^E_{s,m}(\varphi)] - m\kappa\varphi)qE.$$

In equilibrium, the interest rates on bonds and deposits satisfy $r^D = r^B = A^B/q - 1$, so that firms make zero profits, i.e., $\pi = 0$, and welfare translates into

$$W = A^B K + \tau + \mathbb{E}[\{(A^L_{s,m} - A^B)\varphi + A^B\}^+]E - m\kappa\varphi q E,$$

where we used equation (6.5) in subsection 3.3.6, stating that the rate of return on bank equity is given by $r^E_{s,m}(\varphi) = \{(A^L_{s,m} - A^B)\varphi + A^B\}^+/q - 1$. The government uses taxes to cover central bank losses and bank losses in the case of default, while it distributes central bank profits through transfers, i.e., $T = \Pi^{b,-} + \Pi^{CB}$. Note that as we focus on a representative bank, deposit outflows match deposit inflows. Moreover, the interest rates on reserves deposits and reserve loans equal (see assumption 3.3.2). Thus, the central bank makes neither profits nor losses, i.e., $\Pi^{CB} = 0$, and taxes must only cover bank losses in the case of default, so that in real terms, governmental taxes satisfy

$$\tau = \pi^{b,-} = \mathbb{1}\{\varphi > \varphi^S\}(1-\eta)[(A^L_{l,m} - A^B)\varphi + A^B]E$$
$$= \mathbb{E}[\{(A^L_{s,m} - A^B)\varphi + A^B\}^-]E,$$

where we make use of the notation $\{X\}^- = \min\{X, 0\}$. Welfare then simplifies to

$$W = A^B K + \mathbb{E}[(A^L_{s,m} - A^B)\varphi + A^B]E - m\kappa\varphi q E,$$

which, using $\varphi = \overline{\varphi}_m(\theta)$ (see lemma 6.2.2), finally reads as

$$W_m(\theta) = (\mathbb{E}[A^L_{s,m}] - A^B - m\kappa q)\overline{\varphi}_m(\theta)E + A^B(K + E).$$

■

Proof of Corollary 6.2.1. The results follow directly from lemma 6.2.2 by using $\overline{\varphi}_0(\theta) = \overline{\varphi}_1(\theta) = \varphi^R$, which follows from the assumption that the central bank implements sufficiently loose collateral requirements, i.e., the haircut set by the central bank satisfies $\psi \leq \tilde{\psi}_0(\varphi^R)$, so that $\overline{\varphi}_0(\theta) = \overline{\varphi}_1(\theta) = \varphi^R$. Using lemma 6.2.2, the banker then optimally monitors (i.e., $m = 1$) if and only if

(I) without solvency risk, i.e., $\varphi^R \leq \varphi_0^S$, it holds that $\tilde{\mathcal{M}}_N \geq 0$, where

$$\tilde{\mathcal{M}}_N := (1-\eta)\triangle - \kappa q,$$

(II) with partial solvency risk, i.e., $\varphi_1^S \geq \varphi^R > \varphi_0^S$, it holds that $\tilde{\mathcal{M}}_P(\varphi^R) \geq 0$, where

$$\tilde{\mathcal{M}}_P(\varphi^R) := -(1-\eta)(A^B - A_{l,1}^L) + \frac{(1-\eta)A^B}{\varphi^R} - \kappa q,$$

(III) with full solvency risk, i.e., $\varphi^R > \varphi_1^S$, it holds that $\tilde{\mathcal{M}}_F \geq 0$, where

$$\tilde{\mathcal{M}}_F := -\kappa q.$$

Furthermore, it holds that $\lim_{\varphi^R \searrow \varphi_0^S} \tilde{\mathcal{M}}_P(\varphi^R) = \tilde{\mathcal{M}}_N$ and $\lim_{\varphi^R \nearrow \varphi_1^S} \tilde{\mathcal{M}}_P(\varphi^R) = \tilde{\mathcal{M}}_F$, as

$$\begin{aligned}
\lim_{\varphi^R \searrow \varphi_0^S} \tilde{\mathcal{M}}_P(\varphi^R) &= -(1-\eta)(A^B - A_{l,1}^L) + \frac{(1-\eta)A^B}{\varphi_0^S} - \kappa q \\
&= -(1-\eta)(A^B - A_{l,1}^L) + (1-\eta)(A^B - A_{l,0}^L) - \kappa q \\
&= (1-\eta)(A_{l,1}^L - A_{l,0}^L) - \kappa q \\
&= (1-\eta)\triangle - \kappa q \\
&= \tilde{\mathcal{M}}_N,
\end{aligned}$$

where we used $\varphi_0^S = A^B/(A^B - A_{l,0}^L)$, following from equation (6.6). ■

Proof of Lemma 6.2.4. Based on the assumption that the bank regulator sets sufficiently loose capital requirements, i.e. $\varphi^R \geq \varphi_m^L(\psi)$, we know that the maximum possible leverage satisfies $\overline{\varphi}_m(\theta) = \varphi_m^L(\psi)$ for all m. First, we focus on the case where no matter the monitoring decision, the banker is not exposed to a solvency risk, i.e., $\varphi_m^S \geq \varphi_m^L(\psi)$ for all m, or, equivalently, $\psi \geq \psi_m^S$ for all m. As we know that it holds that $\psi_0^S > \psi_1^S$, the inequality $\psi \geq \psi_m^S$ is satisfied for all m whenever $\psi \geq \psi_0^S$. We know from lemma 6.2.2 that in any such situation, the banker monitors (i.e., $m = 1$) if and only if $\mathcal{M}_N(\theta) \geq 0$, where

$$\mathcal{M}_N(\theta) = (1-\eta)\triangle + (\mathbb{E}[A^L_{s,0}] - A^B)\left[1 - \frac{\varphi^L_0(\psi)}{\varphi^L_1(\psi)}\right] - \kappa q.$$

Based on assumption 6.2.1, stating $\mathbb{E}[A^L_{s,0}] > A^B$, and assumption 6.2.2, stating that $(1-\eta)\triangle \geq \kappa q$, it follows that $\mathcal{M}_N(\theta) \geq 0$ for any ψ, so that without solvency risk, the banker always monitors.

Second, we focus on the situation where the banker is exposed to a solvency risk only without monitoring, i.e., $\varphi^L_0(\psi) > \varphi^S_0$ and $\varphi^L_1(\psi) \leq \varphi^S_1$, or, equivalently, $\psi^S_0 > \psi \geq \psi^S_1$. We know from lemma 6.2.2 that in any such situation, the banker monitors (i.e., $m = 1$) if and only if $\mathcal{M}_P(\theta) \geq 0$, where

$$\mathcal{M}_P(\theta) = \eta(A^L_{h,0} - A^B)\left[1 - \frac{\varphi^L_0(\psi)}{\varphi^L_1(\psi)}\right] - (1-\eta)\left[A^B - A^L_{l,1} - \frac{A^B}{\varphi^L_1(\psi)}\right] - \kappa q.$$

Note that, using $A^L_{l,1} = A^L_{l,0} + \triangle$, we can rearrange the inequality $\mathcal{M}_P(\theta) \geq 0$ to

$$(1-\eta)\triangle + \eta(A^L_{h,0} - A^B)\left[1 - \frac{\varphi^L_0(\psi)}{\varphi^L_1(\psi)}\right] - (1-\eta)\left[A^B - A^L_{l,0} - \frac{A^B}{\varphi^L_1(\psi)}\right] \geq \kappa q.$$

With $\mathbb{E}[A^L_{s,0}] - A^B = \eta(A^L_{h,0} - A^B) + (1-\eta)(A^L_{l,0} - A^B)$, the latter inequality further simplifies to

$$(1-\eta)\triangle + (\mathbb{E}[A^L_{s,0}] - A^B)\left[1 - \frac{\varphi^L_0(\psi)}{\varphi^L_1(\psi)}\right] + (1-\eta)\frac{(A^L_{l,0} - A^B)\varphi^L_0(\psi) + A^B}{\varphi^L_1(\psi)} \geq \kappa q.$$

Note that

$$(\mathbb{E}[A^L_{s,0}] - A^B)\left[1 - \frac{\varphi^L_0(\psi)}{\varphi^L_1(\psi)}\right] + (1-\eta)\frac{(A^L_{l,0} - A^B)\varphi^L_0(\psi) + A^B}{\varphi^L_1(\psi)} \geq 0$$

$$\Leftrightarrow \quad (\mathbb{E}[A^L_{s,0}] - A^B)[\varphi^L_1(\psi) - \varphi^L_0(\psi)] + (1-\eta)[(A^L_{l,0} - A^B)\varphi^L_0(\psi) + A^B] \geq 0,$$

where the latter inequality is satisfied for any haircut ψ sufficiently close to ψ_0^S, i.e.,

$$\lim_{\psi \nearrow \psi_0^S} (\mathbb{E}[A_{s,0}^L] - A^B)[\varphi_1^L(\psi) - \varphi_0^L(\psi)] + (1-\eta)[(A_{l,0}^L - A^B)\varphi_0^L(\psi) + A^B]$$

$$= (\mathbb{E}[A_{s,0}^L] - A^B)[\varphi_1^L(\psi_0^S) - \varphi_0^L(\psi_0^S)] + (1-\eta)[(A_{l,0}^L - A^B)\varphi_0^L(\psi_0^S) + A^B]$$

$$= (\mathbb{E}[A_{s,0}^L] - A^B)[\varphi_1^L(\psi_0^S) - \varphi_0^S] + (1-\eta)[(A_{l,0}^L - A^B)\varphi_0^S + A^B]$$

$$= (\mathbb{E}[A_{s,0}^L] - A^B)[\varphi_1^L(\psi_0^S) - \varphi_0^S] + (1-\eta)[(A_{l,0}^L - A^B)\varphi_0^S + A^B]$$

$$= (\mathbb{E}[A_{s,0}^L] - A^B)[\varphi_1^L(\psi_0^S) - \varphi_0^S] + (1-\eta)[-A^B + A^B]$$

$$= (\mathbb{E}[A_{s,0}^L] - A^B)[\varphi_1^L(\psi_0^S) - \varphi_0^S] > 0,$$

where we used $\varphi_0^L(\psi_0^S) = \varphi_0^S$ and $\varphi_0^S = A^B/(A^B - A_{l,0}^L)$. Using assumption 6.2.2, stating $(1-\eta)\triangle \geq \kappa q$, we can then conclude that there exists a set of haircuts in the interval $(\psi_0^S, \psi_1^S]$ which induces the banker to monitor. Specifically, the banker monitors for any haircut $\psi \geq \psi^{**}$, where $\psi^{**} = \min\{\psi_0^S \geq \psi \geq \psi_1^S : \mathcal{M}_P(\theta) \geq 0\}$.

Third and last, we focus on the situation, where independent of the monitoring decision, the banker is exposed to a solvency risk, i.e., $\varphi_m^L(\psi) > \varphi_m^S$ for all m, or, equivalently, $\psi_m^S > \psi$ for all m. Since we know that it holds that $\psi_0^S > \psi_1^S$, the inequality $\psi_m^S > \psi$ is satisfied for all m whenever $\psi_1^S > \psi$. We then know from lemma 6.2.2 that the banker monitors (i.e., $m = 1$) if and only if $\mathcal{M}_F(\theta) \geq 0$, where

$$\mathcal{M}_F(\theta) = \eta(A_{h,0}^L - A^B)\left[1 - \frac{\varphi_0^L(\psi)}{\varphi_1^L(\psi)}\right] - \kappa q.$$

We know that for any haircut ψ sufficiently close to one, the banker does not monitor, as it holds that $\mathcal{M}_F(\theta) < 0$. However, if the haircut ψ is sufficiently small, the banker may decide to monitor, i.e., formally, it must hold that

$$\mathcal{M}_F(\theta) \geq 0 \qquad \Leftrightarrow \qquad 1 - \frac{\varphi_0^L(\psi)}{\varphi_1^L(\psi)} \geq \frac{\kappa q}{\eta(A_{h,0} - A^B)} := \chi \qquad \Leftrightarrow \qquad 1 - \chi \geq \frac{\varphi_0^L(\psi)}{\varphi_1^L(\psi)}.$$

Based on assumption 6.2.1, we know that $\chi > 0$. Using the structure of $\varphi_m^L(\psi)$, as outlined in equation (6.8), the latter inequality reads as

$$1-\chi \geq \frac{\dfrac{\alpha A^B}{\alpha A^B-(1-\psi)\mathbb{E}[A^L_{s,0}]}}{\dfrac{\alpha A^B}{\alpha A^B-(1-\psi)\mathbb{E}[A^L_{s,1}]}} \qquad \Leftrightarrow \qquad 1-\chi \geq \frac{\alpha A^B-(1-\psi)\mathbb{E}[A^L_{s,1}]}{\alpha A^B-(1-\psi)\mathbb{E}[A^L_{s,0}]}.$$

The latter inequality further simplifies to

$$\Leftrightarrow \qquad (1-\chi)\{\alpha A^B-(1-\psi)\mathbb{E}[A^L_{s,0}]\} \geq \alpha A^B-(1-\psi)\mathbb{E}[A^L_{s,1}]$$

$$\Leftrightarrow \qquad (1-\psi)\{\mathbb{E}[A^L_{s,1}]-(1-\chi)\mathbb{E}[A^L_{s,0}]\} \geq \chi\alpha A^B$$

$$\Leftrightarrow \qquad (1-\psi)\{\chi\mathbb{E}[A^L_{s,0}]+(1-\eta)\triangle\} \geq \chi\alpha A^B$$

$$\Leftrightarrow \qquad 1-\frac{\chi\alpha A^B}{\chi\mathbb{E}[A^L_{s,0}]+(1-\eta)\triangle} := \hat{\psi} \geq \psi,$$

where we used $A^L_{h,1}=A^L_{h,0}$ and $A^L_{l,1}=A^L_{l,0}+\triangle$. ■

Proof of Lemma 6.2.5. From Lemma 6.2.4, we know that in the situation where the banker is fully exposed to a solvency risk, i.e., $\psi^S_1>\psi$, the banker monitors if and only if

$$\psi \leq \hat{\psi} := 1-\frac{\chi\alpha A^B}{\chi\mathbb{E}[A^L_{s,0}]+(1-\eta)\triangle}, \qquad \text{with} \qquad \chi := \frac{\kappa q}{\eta(A^L_{h,0}-A^B)}.$$

First, we want to know under which conditions the smallest feasible haircut with monitoring by the banker ψ^M_1 is indeed smaller than the critical haircut $\hat{\psi}$. On that account, note that it holds that

$$\psi^M_1 \leq \hat{\psi}$$

$$\Leftrightarrow \qquad 1-\frac{\alpha A^B}{\mathbb{E}[A^L_{s,1}](1+E/K)} \leq 1-\frac{\chi\alpha A^B}{\chi\mathbb{E}[A^L_{s,0}]+(1-\eta)\triangle}$$

$$\Leftrightarrow \qquad \frac{\chi\alpha A^B}{\chi\mathbb{E}[A^L_{s,0}]+(1-\eta)\triangle} \leq \frac{\alpha A^B}{\mathbb{E}[A^L_{s,1}](1+E/K)},$$

which further simplifies to

$$\begin{aligned}
& \chi\mathbb{E}[A^L_{s,1}](1+E/K) \leq \chi\mathbb{E}[A^L_{s,0}] + (1-\eta)\triangle \\
\Leftrightarrow \quad & \chi\{\mathbb{E}[A^L_{s,0}] + (1-\eta)\triangle\} + \chi\mathbb{E}[A^L_{s,1}]E/K \leq \chi\mathbb{E}[A^L_{s,0}] + (1-\eta)\triangle \\
\Leftrightarrow \quad & \chi\mathbb{E}[A^L_{s,1}]E/K \leq (1-\chi)(1-\eta)\triangle \\
\Leftrightarrow \quad & E/K \leq \frac{(1-\chi)(1-\eta)\triangle}{\chi\mathbb{E}[A^L_{s,1}]}.
\end{aligned}$$

Second, we assess when the condition $\psi \leq \hat{\psi}$ is less restrictive than the condition $\psi^S_1 > \psi$. On that account, note that it holds that

$$\begin{aligned}
& \hat{\psi} \geq \psi^S_1 \\
\Leftrightarrow \quad & 1 - \frac{\chi\alpha A^B}{\chi\mathbb{E}[A^L_{s,0}] + (1-\eta)\triangle} \geq 1 - \frac{\alpha A^L_{l,1}}{\mathbb{E}[A^L_{s,1}]} \\
\Leftrightarrow \quad & \frac{\alpha A^L_{l,1}}{\mathbb{E}[A^L_{s,1}]} \geq \frac{\chi\alpha A^B}{\chi\mathbb{E}[A^L_{s,0}] + (1-\eta)\triangle},
\end{aligned}$$

which further simplifies to

$$\begin{aligned}
& A^L_{l,1}\{\chi\mathbb{E}[A^L_{s,0}] + (1-\eta)\triangle\} \geq \chi A^B\mathbb{E}[A^L_{s,1}] \\
\Leftrightarrow \quad & (1-\eta)\triangle A^L_{l,1} \geq \chi(A^B\mathbb{E}[A^L_{s,1}] - A^L_{l,1}\mathbb{E}[A^L_{s,0}]) \\
\Leftrightarrow \quad & \frac{(1-\eta)\triangle A^L_{l,1}}{A^B\mathbb{E}[A^L_{s,1}] - A^L_{l,1}\mathbb{E}[A^L_{s,0}]} \geq \chi.
\end{aligned}$$

■

Proof of Proposition 6.2.2. Based on assumption 6.2.1, stating that even without monitoring, a loan-financed firm is in expectation more productive than a bond-financed firm, and based on assumption 6.2.2, ensuring that monitoring is socially optimal, welfare increases with loan financing and monitoring by the banker. The banker always monitors if

$\varphi^R \leq \varphi^*$. Accordingly, it is optimal for the bank regulator to implement capital requirements, such that $\varphi^R = \varphi^M$ whenever (i) $\varphi^M \leq \varphi^*$, such that bank lending is maximized and the banker monitors, or (ii) $\varphi^M > \varphi^*$, so that bank lending is maximized and the banker does not monitor, but reducing bank leverage to induce monitoring does not yield a welfare gain, i.e., it holds that $W_0(\varphi^M) \geq W_1(\varphi^*)$ or, equivalently,

$$(\mathbb{E}[A^L_{s,0}] - A^B)\varphi^M E + A^B(K+E) \geq (\mathbb{E}[A^L_{s,1}] - A^B - \kappa q)\varphi^* E + A^B(K+E)$$

$$\Leftrightarrow \quad (\mathbb{E}[A^L_{s,0}] - A^B)\varphi^M \geq (\mathbb{E}[A^L_{s,1}] - A^B - \kappa q)\varphi^*$$

$$\frac{\varphi^M}{\varphi^*} \geq \frac{\mathbb{E}[A^L_{s,1}] - A^B - \kappa q}{\mathbb{E}[A^L_{s,0}] - A^B}$$

$$\frac{\varphi^M}{\varphi^*} \geq 1 + \frac{(1-\eta)\triangle - \kappa q}{\mathbb{E}[A^L_{s,0}] - A^B},$$

where we used $A_{l,1} = A_{l,0} + \triangle$, implying $\mathbb{E}[A^L_{s,1}] = \mathbb{E}[A^L_{s,0}] + (1-\eta)\triangle$. Based on asssumption 3.3.1, a loan-financed firm is in expectation, independent of the banker's monitoring decision, more productive than a bond-financed firm. Accordingly, under the condition $\varphi^M > \varphi^*$, the welfare without monitoring by the banker is maximized for $\varphi^R = \varphi^M$, whereas welfare with monitoring is maximized for $\varphi^R = \varphi^*$. We therefore only need to compare welfare $W_0(\varphi^M)$ and $W_1(\varphi^*)$.

In all other situations, it is optimal for the bank regulator to implement capital requirements leading to the regulatory maximum leverage $\varphi^R = \varphi^*$, restricting bank leverage below the maximum feasible and thereby inducing the banker to monitor. ■

Proof of Proposition 6.2.3. Based on assumption 6.2.1, stating that even without monitoring, a loan-financed firm is in expectation more productive than a bond-financed firm, and based on assumption 6.2.2, ensuring that monitoring is socially optimal, we know that welfare increases with loan financing and monitoring by bankers. From lemma 6.2.4, it follows that the banker monitors whenever (i) $\psi \geq \psi^S_0$, (ii) $\psi^S_1 > \psi \geq \psi^S_1$ and $\psi \geq \psi^{**}$, where $\psi^{**} = \min\{\psi^S_0 > \psi \geq \psi^S_1 : \mathcal{M}_P(\theta) \geq 0\}$, and (iii) $\psi^S_1 > \psi$ and $\hat{\psi} \geq \psi$, where

$$\hat{\psi} = 1 - \frac{\chi\alpha A^B}{\chi\mathbb{E}[A^L_{s,0}] + (1-\eta)\triangle}, \qquad \text{with} \qquad \chi := \frac{\kappa q}{\eta(A^L_{h,0} - A^B)}.$$

Note that it holds that $\psi_0^S > \psi^{**} \geq \psi_1^S$. Accordingly, we can state that the central bank optimally sets the haircut $\psi = \psi_1^M$ whenever (i) $\psi_1^M \geq \psi^{**}$ or (ii) $\psi_1^S > \psi_1^M$ and $\hat{\psi} \geq \psi_1^M$. Next, we study the alternative cases. First, focus on the situation where $\psi^{**} > \psi_1^M$. Then three situations can arise: Either it holds (iii) $\psi_0^M \geq \psi_1^S$, or (iv) $\psi_1^S > \psi_0^M$ and $\psi_0^M > \hat{\psi}$, or (v) $\psi_0^S > \psi_0^M$ and $\hat{\psi} \geq \psi_0^M$. In the cases (iii) and (iv), the banker does not monitor for any feasible haircut lower than ψ^{**}. Thus, the central bank has to decide between maximizing bank lending by setting the haircut $\psi = \psi_0^M$ but having bankers not monitoring, or reducing bank leverage below the maximum feasible by setting the haircut $\psi = \psi^{**}$ but having bankers monitoring. In the cases (iii) and (iv), the central bank implements the haircut $\psi = \psi_0^M$ whenever it holds that $W_0(\psi_0^M) \geq W_1(\psi^{**})$ or, equivalently,

$$
\begin{aligned}
& (\mathbb{E}[A_{s,0}^L] - A^B)\varphi^M E + A^B(K+E) \geq (\mathbb{E}[A_{s,1}^L] - A^B - \kappa q)\varphi^{**} E + A^B(K+E) \\
\Leftrightarrow \quad & (\mathbb{E}[A_{s,0}^L] - A^B)\varphi^M \geq (\mathbb{E}[A_{s,1}^L] - A^B - \kappa q)\varphi^{**} \\
\Leftrightarrow \quad & \frac{\varphi^M}{\varphi^{**}} \geq \frac{\mathbb{E}[A_{s,1}^L] - A^B - \kappa q}{\mathbb{E}[A_{s,0}^L] - A^B} \\
\Leftrightarrow \quad & \frac{\varphi^M}{\varphi^{**}} \geq 1 + \frac{(1-\eta)\triangle - \kappa q}{\mathbb{E}[A_{s,0}^L] - A^B},
\end{aligned}
$$

where we used $\mathbb{E}[A_{s,1}^L] = \mathbb{E}[A_{s,0}^L] + (1-\eta)\triangle$ and applied the notation $\varphi^{**} = \varphi_1^L(\psi^{**})$. In case (v), the central bank finds itself unable to set the smallest feasible haircut $\psi = \psi_0^M$, as it would actually induce monitoring, but with monitoring, the haircut $\psi = \psi_0^M$ would not permit clearing of the capital good market. Thus, the central bank can only set a haircut sufficiently close to, but above $\hat{\psi}$ in order to maximize bank lending and without inducing monitoring. In case (v), the central bank then decides to set the haircut $\psi = \hat{\psi} - \epsilon$ with

$\epsilon \to 0$ whenever it holds that

$$
\begin{aligned}
& (\mathbb{E}[A^L_{s,0}] - A^B)\varphi^L_0(\hat{\psi})E + A^B(K+E) \geq (\mathbb{E}[A^L_{s,1}] - A^B - \kappa q)\varphi^{**}E + A^B(K+E) \\
\Leftrightarrow \quad & (\mathbb{E}[A^L_{s,0}] - A^B)\varphi^L_0(\hat{\psi}) \geq (\mathbb{E}[A^L_{s,1}] - A^B - \kappa q)\varphi^{**} \\
\Leftrightarrow \quad & \frac{\hat{\varphi}}{\varphi^{**}} \geq \frac{\mathbb{E}[A^L_{s,1}] - A^B - \kappa q}{\mathbb{E}[A^L_{s,0}] - A^B} \\
\Leftrightarrow \quad & \frac{\hat{\varphi}}{\varphi^{**}} \geq 1 + \frac{(1-\eta)\triangle - \kappa q}{\mathbb{E}[A^L_{s,0}] - A^B},
\end{aligned}
$$

where we used $\mathbb{E}[A^L_{s,1}] = \mathbb{E}[A^L_{s,0}] + (1-\eta)\triangle$ and applied the notation $\hat{\varphi} = \varphi^L_0(\hat{\psi})$.

In all other cases, the central bank optimally sets the haircut $\psi = \psi^{**}$ to restrict bank leverage below the maximum feasible and thereby inducing the banker to monitor. ∎

Proof of Corollary 6.2.2. We first show that a liquidity-constrained banker monitors under higher leverage ratios than a capital-constrained banker. Note that it holds that

$$\mathcal{M}_P(\theta^{**}) = 0 = \tilde{\mathcal{M}}(\varphi^*),$$

where $\theta^{**} = (\varphi^R, \psi^{**})$. The latter equation is equivalent to

$$
\begin{aligned}
\eta(A^L_{h,0} - A^B)\left[1 - \frac{\varphi^L_0(\psi^{**})}{\varphi^L_1(\psi^{**})}\right] - (1-\eta)\left[A^B - A^L_{l,1} - \frac{A^B}{\varphi^L_1(\psi^{**})}\right] - \kappa q \\
= -(1-\eta)(A^B - A^L_{l,1}) + \frac{(1-\eta)A^B}{\varphi^*} - \kappa q.
\end{aligned}
$$

Rearranging the latter equation yields

$$\eta(A^L_{h,0} - A^B)\left[1 - \frac{\varphi^L_0(\psi^{**})}{\varphi^L_1(\psi^{**})}\right] = (1-\eta)A^B\left[\frac{1}{\varphi^*} - \frac{1}{\varphi^L_1(\psi^{**})}\right].$$

Since the left-hand side of the this equality is always positive for $\psi \in [0,1)$, we can conclude that $\varphi^{**} = \varphi^L_1(\psi^{**}) > \varphi^*$. It thus follows that it is generally optimal to constrain the banker by liquidity rather than by capital. Accordingly, in most situations, the optimal bank regulation is characterized by sufficiently loose capital requirements, i.e., $\varphi^R \geq \varphi^L_m(\psi)$

(e.g., $\varphi = \varphi^M$), and collateral requirements in the form of the haircut that follow the monetary policy described in proposition 6.2.3. There is only one exception: If it holds that $\psi^{**} > \psi_1^M > \hat{\psi}$ and the central bank cannot implement the smallest feasible haircut $\psi = \psi_0^M$ with inducing monitoring, i.e., $\psi_1^S > \psi_0^M$ and $\hat{\psi} \geq \psi_0^M$, it follows that it is better to constrain the banker by capital rather than liquidity, as in the former case, bank lending can be maximized. In this particular case, the collateral requirements set by the central bank should satisfy $\psi \leq \tilde{\psi}_0(\varphi^R)$, while the capital requirements satisfy $\varphi^R = \varphi^M$. ■

6.3 Appendix for Chapter 4

Proof of Lemma 4.3.1. Note that the initial CBDC holdings satisfy $D^h_{CB} = QK\mathbb{1}\{h = \underline{h}\}$. Thus, the household that initially opens an account with a banker ($h = \overline{h}$) transfers deposits to the central bank iff the excess returns on CBDC suffice to offset the switching costs, that is $\nu < QK(r^D_{CB} - r^D_{\mathbf{z}})$. Using the notation $\tilde{\nu} := \nu/(QK)$, the latter condition translates into $r^D_{\mathbf{z}} < r^D_{CB} - \tilde{\nu}$. Similarly, the household that initially opens an account with the central bank ($h = \underline{h}$) will keep the funds at the central bank iff $QK(r^D_{\mathbf{z}} - r^D_{CB}) \leq \nu$ or, equivalently, $r^D_{\mathbf{z}} \leq r^D_{CB} + \tilde{\nu}$. ■

Proof of Lemma 4.3.2. Suppose $A_s > (1 + r^L_s)q$ for some state $s \in \mathcal{S}$. As the firm operates with limited liability, its expected profits grow with the amount of capital good K^f. Thus, there exists no optimal finite demand for capital good. We denote this case by $K^f = +\infty$. Instead, if $A_s \leq (1+r^L_s)q$ for all states $s \in \mathcal{S}$, the firm will generate zero profits for any production input in each state and is thus indifferent in its demand for capital good $K^f \in [0, +\infty)$. ■

Proof of Lemma 4.3.3. Note that deposit flows are matched by reserve flows and reserves are credited with the real interest rate r^D_{CB}. To rule out arbitrage opportunities and thus excess returns for the banker, the deposit rate must satisfy $r^D_{\mathbf{z}} = r^D_{CB}$ for any state $\mathbf{z} \in \mathcal{Z}$ where the banker does not default. Suppose $r^D_{\mathbf{z}} > r^D_{CB}$. Then the banker benefits by setting a deposit rate $\tilde{r}^D_{\mathbf{z}} < r^D_{\mathbf{z}}$, as it would not be matched with any household, but finances loans with equity and central bank loans, where the latter are subject to the repayment rate $r^D_{CB} < r^D_{\mathbf{z}}$.

Similarly, suppose $r^D_{CB} > r^D_{\mathbf{z}}$. Then the banker profits from setting a deposit rate $\tilde{r}^D_{\mathbf{z}} < r^D_{CB}$ and $\tilde{r}^D_{\mathbf{z}} > r^D_{\mathbf{z}}$, as all households of type $h = \overline{h}$ would initially open an account with this banker. The latter then generates riskless profits due to the interest rate spread $r^D_{CB} - \tilde{r}^D_{\mathbf{z}} > 0$. ■

Proof Lemma 4.3.4. We address each case separately. (i) Consider the situation where the banker faces neither liquidity risk nor solvency risk, i.e., $1 \leq \varphi \leq \min\{\varphi^L, \varphi^S, \varphi^r\}$. Then the banker will monitor, given the type of matched household $h \in \mathcal{H}$, iff

$$\mathbb{E}_1[(R^L_s - R^D_{CB})\varphi + R^D_{CB}|h]qE \geq \mathbb{E}_0[(R^L_s - R^D_{CB})\varphi + R^D_{CB}|h]qE + \kappa\varphi qE$$

or, equivalently, $\mathbb{E}_1[R_s^L|h] - \mathbb{E}_0[R_s^L|h] \geq \kappa$. Using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, we can state that the banker will monitor independently of the type of household, i.e., $\underline{m} = \overline{m} = 1$, iff $\triangle(R_{\overline{s}}^L - R_{\underline{s}}^L) \geq \kappa$. The banker's expected utility from conducting banking operations with a leverage $1 \leq \varphi \leq \min\{\varphi^L, \varphi^S, \varphi^r\}$ is given by $\mathbb{E}_{\mathbf{m}}[(R_s^L - R_{CB}^D)\varphi + R_{CB}^D - m(h)\kappa\varphi]qE$. Due to competitive markets, the utility expected from banking must equal the utility from holding CBDC, i.e., $R_{CB}^D QE$. Using the fact that $\underline{m} = \overline{m}$, the banker will thus only choose a leverage ratio $1 \leq \varphi \leq \min\{\varphi^L, \varphi^S, \varphi^r\}$ if $\mathbb{E}_{\mathbf{m}}[R_s^L] = R_{CB}^D + \overline{m}\kappa$.

(ii) Consider the situation where the banker faces a liquidity risk but no solvency risk, i.e., $\varphi^L < \varphi \leq \varphi^S$. The banker will monitor iff the matched household opens an account with the banker, as otherwise the banker will become illiquid and defaults. Thus it holds that $\underline{m} = 0$. For $h = \overline{h}$, the banker will monitor, i.e., $\overline{m} = 1$, iff

$$\mathbb{E}_1[(R_s^L - r_{CB}^D)\varphi + R_{CB}^D|h = \overline{h}]qE \geq \mathbb{E}_0[(R_s^L - R_{CB}^D)\varphi + R_{CB}^D|h = \overline{h}]qE + \kappa\varphi qE$$

or, equivalently, $\mathbb{E}_1[R_s^L|h = \overline{h}] - \mathbb{E}_0[R_s^L|h = \overline{h}] \geq \kappa$. Using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, we can state that the banker will monitor iff $h = \overline{h}$ and $\triangle(R_{\overline{s}}^L - R_{\underline{s}}^L) \geq \kappa$. Using the fact that $\underline{m} = 0$, we know that the utility expected from conducting banking operations is given by

$$\{(1-\mu)\mathbb{E}_{\overline{m}}[(R_s^L - R_{CB}^D)\varphi + R_{CB}^D|h = \overline{h}] - (1-\mu)\overline{m}\kappa\varphi - \mu\phi[(R_{CB}^D - \Psi)\varphi - R_{CB}^D]\}qE.$$

Due to competitive markets, the latter must equal the utility from holding CBDC, i.e $R_{CB}^D QE$. Thus, the banker will only choose $\varphi^L < \varphi \leq \varphi^S$ with $\varphi = \varphi^r$ if

$$\mathbb{E}_{\overline{m}}[R_s^L|h = \overline{h}] = R_{CB}^D\left(1 + \frac{\mu}{1-\mu}\frac{1}{\varphi}\right) + \frac{\mu\phi}{1-\mu}\left(R_{CB}^D\frac{\varphi-1}{\varphi} - \Psi\right) + \overline{m}\kappa,$$

and there is no incentive for the banker to reduce the supply of loans, i.e.,

$$(1-\mu)\mathbb{E}_{\overline{m}}[R_s^L - R_{CB}^D - \overline{m}\kappa|h = \overline{h}] - \mu\phi(R_{CB}^D - \Psi) \geq 0.$$

Both conditions are only satisfied for $\phi \leq 1$.

(iii) Consider the situation where the banker faces a solvency risk but no liquidity risk, i.e., $\varphi^S < \varphi \leq \varphi^L$. The banker will monitor, given the type of matched household $h \in \mathcal{H}$, iff

$$\eta_{\overline{s}|1}[(R_{\overline{s}}^L - R_{CB}^D)\varphi + R_{CB}^D]qE \geq \eta_{\overline{s}|0}[(R_{\overline{s}}^L - R_{CB}^D)\varphi + R_{CB}^D]QE + \kappa\varphi qE,$$

which, using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, can be rewritten as $\triangle[R^L_{\overline{s}} - R^D_{CB}(\varphi-1)/\varphi] \geq \kappa$. Using the fact that the banker's monitoring decision does not depend on the type of matched household, i.e., $\underline{m} = \overline{m}$, we know that the banker's expected utility from conducting banking operations is given by $\{\eta_{\overline{s}|\overline{m}}[(R^L_{\overline{s}} - R^D_{CB})\varphi + R^D_{CB}] - \overline{m}\kappa\varphi\}qE$. Due to competitive markets, the utility expected from conducting banking operations must equal the utility from holding CBDC, i.e., $R^D_{CB}QE$. Thus, with $\underline{m} = \overline{m}$ we can deduce that the banker will choose $\varphi^S < \varphi \leq \varphi^L$ with $\varphi < \varphi^r$ if

$$R^L_{\overline{s}} = R^D_{CB}\left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi}\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}$$

and there is no incentive to adjust the supply of loans, i.e., $\eta_{\overline{s}|\overline{m}}(R^L_{\overline{s}} - R^D_{CB}) - \overline{m}\kappa = 0$, which, however, contradicts the former equation. Hence the banker will only choose a leverage $\varphi^S < \varphi \leq \varphi^L$ if $\varphi = \varphi^r$,

$$R^L_{\overline{s}} = R^D_{CB}\left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}$$

and $\eta_{\overline{s}|\overline{m}}(R^L_{\overline{s}} - R^D_{CB}) - \overline{m}\kappa > 0$, which follows directly from the previous equation.

(iv) Consider the situation where the banker faces both a liquidity risk and a solvency risk, i.e., $\varphi > \max\{\varphi^L, \varphi^S\}$. In the case of bank insolvency, depositors will prefer a bail-in over a transfer of funds to the central bank, i.e., $\varphi \leq \varphi^R$. The banker will monitor iff matched with a household that opens an account with the banker ($h = \overline{h}$), as otherwise the banker will become illiquid and defaults. Thus it holds that $\underline{m} = 0$. In addition, we can state $\overline{m} = 1$ iff $\eta_{\overline{s}|1}[(R^L_{\overline{s}} - R^D_{CB})\varphi + R^D_{CB}]qE \geq \eta_{\overline{s}|0}[(R^L_{\overline{s}} - R^D_{CB})\varphi + R^D_{CB}]qE + \kappa\varphi qE$. Using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, the latter inequality can be rewritten as $\triangle[(R^L_{\overline{s}} - R^D_{CB})\varphi + R^D_{CB}]qE \geq \kappa\varphi qE$ or, equivalently, $\triangle[R^L_{\overline{s}} - R^D_{CB}(\varphi-1)/\varphi] \geq \kappa$. Using the fact that $\underline{m} = 0$ and the fact that illiquidity penalties will only arise if the banker is matched with a household that opens an account with the central bank ($h = \underline{h}$) but not in the case of bank insolvency as the matched household prefers a bail-in to a transfer of funds, the banker's expected utility from conducting banking operations is given by

$$\{(1-\mu)\eta_{\overline{s}|\overline{m}}[(R^L_{\overline{s}} - R^D_{CB})\varphi + R^D_{CB}] - (1-\mu)\overline{m}\kappa\varphi - \mu\phi[R^D_{CB}(\varphi-1) - \psi\varphi]\}qE.$$

Due to competitive markets, the expected utility from conducting banking operations must equal the utility from holding CBDC, i.e., $R^D_{CB}QE$. Thus the banker will only choose

$\max\{\varphi^L, \varphi^S\} < \varphi$ with $\varphi = \varphi^r$ if

$$R^L_{\overline{s}} = R^D_{CB}\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi}\right) + \frac{\mu\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(R^D_{CB}\frac{\varphi-1}{\varphi} - \psi\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}$$

and there is no incentive to reduce the supply of loans, i.e.,

$$(1-\mu)(\eta_{\overline{s}|\overline{m}}R^L_{\overline{s}} - \eta_{\overline{s}|\overline{m}}R^D_{CB} - \overline{m}\kappa) - \mu\phi(R^D_{CB} - \Psi) \geq 0,$$

Both conditions are only satisfied for $\mu\phi \leq \mu + (1-\mu)\eta_{\underline{s}|\overline{m}}$.

(v) Consider the situation where the banker faces both a liquidity risk and a solvency risk, i.e., $\varphi > \max\{\varphi^L, \varphi^S\}$, and in the case of bank insolvency depositors transfer their funds to the central bank, i.e., $\varphi > \varphi^R$. The banker will monitor iff matched with a household that opens an account with the banker ($h = \overline{h}$) as otherwise the banker will become illiquid and default. Thus it holds that $\underline{m} = 0$. In addition, we can state that $\overline{m} = 1$ iff

$$\eta_{\overline{s}|1}[(R^L_{\overline{s}} - R^D_{CB})\varphi + R^D_{CB}]qE - \eta_{\underline{s}|1}\phi[(R^D_{CB} - \Psi)\varphi - R^D_{CB}]qE$$
$$\geq \eta_{\overline{s}|0}[(R^L_{\overline{s}} - R^D_{CB})(\varphi-1) + R^L_{\overline{s}}]qE - \eta_{\underline{s}|0}\phi[(R^D_{CB} - \Psi)\varphi - R^D_{CB}]qE + \kappa\varphi qE.$$

Using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, the latter inequality can be rewritten as

$$\triangle[(R^L_{\overline{s}} - R^D_{CB})\varphi + R^D_{CB}]qE - \triangle\phi[(\Psi - R^D_{CB})\varphi + R^D_{CB}]QE \geq \kappa\varphi qE,$$

or equivalently, $\triangle[R^L_{\overline{s}} - \phi\Psi - R^D_{CB}(1-\phi)(\varphi-1)/\varphi] \geq \kappa$. The banker's expected utility from conducting banking operations is given by

$$\{(1-\mu)\eta_{\overline{s}|\overline{m}}[(R^L_{\overline{s}} - R^D_{CB})\varphi + R^D_{CB}] - (1-\mu)\overline{m}\kappa\varphi$$
$$- \phi[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu][R^D_{CB}(\varphi-1) - \Psi\varphi]\}qE.$$

Due to competitive markets, the utility expected from conducting banking operations must equal the utility from holding CBDC, i.e., $R^D_{CB}QE$. Thus the banker will only choose

$\max\{\varphi^L, \varphi^S, \varphi^R\} < \varphi$ with $\varphi < \varphi^r$ if

$$R^L_{\overline{s}} = R^D_{CB}\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi}\right) + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(R^D_{CB}\frac{\varphi - 1}{\varphi} - \Psi\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}},$$

and there is no incentive to reduce the supply of loans, i.e.,

$$(1-\mu)(\eta_{\overline{s}|\overline{m}} R^L_{\overline{s}} - \eta_{\overline{s}|\overline{m}} R^D_{CB} - \overline{m}\kappa) - \phi[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](R^D_{CB} - \Psi) \geq 0.$$

Both conditions are satisfied only for $\phi \leq 1$. So far, we have established the conditions for the banker's choice of leverage and monitoring. Since the previous conditions are mutually exclusive, these conditions are necessary and sufficient. ■

Proof of Proposition 4.4.1. Consider the situation where bankers face neither a liquidity risk nor a solvency risk, i.e., $\varphi \leq \min\{\varphi^L, \varphi^S\}$. From lemma 4.3.4 we know that the banker will choose leverage $1 \leq \varphi \leq \min\{\varphi^L, \varphi^S, \varphi^r\}$ iff

$$\mathbb{E}_{\mathbf{m}}[R^L_s] = R^D_{CB} + \overline{m}\kappa. \tag{6.10}$$

Furthermore, the banker's monitoring decision is independent of the type of household and given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle(R^L_{\overline{s}} - R^L_{\underline{s}}) \geq \kappa\}$. As banks are not defaulting, the central bank makes zero profits, i.e., $\pi^{CB} = 0$, where $\pi^{CB} := \Pi^{CB}/P$ denotes central bank profits in terms of the consumption good. Moreover, in equilibrium, the demand for capital good is finite, such that, with lemma 4.3.2, we can deduce $A_s \leq (1 + r^L_s)q$, with $s \in \mathcal{S}$. In addition, due to rational expectations of firms and bankers, it must hold that $A_s = (1 + r^L_s)q = R^L_s q$ for all $s \in \mathcal{S}$. Thus firms make zero profits, i.e., $\pi^f = 0$. Since the central bank and firms make zero profits, there are no taxes and transfers, i.e., $\tau = 0$, where $\tau := T/P$ denotes taxes and transfers in terms of the consumption good.

Without taxes and transfers and zero firm profits, the expected consumption of the banker and the household is given by $C^b = \mathbb{E}_{\mathbf{m}}[(R^L_s - R^D_{CB})\varphi + R^D_{CB}]qE$ and $C^h = R^D_{CB}qK$, with $h \in \mathcal{H}$, respectively. The banker's monitoring decision is given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa q\}$.

Due to our assumption of linear utility, utilitarian welfare comprises aggregate consumption and potential utility losses on the part of bankers due to monitoring. Note that, if the banker faces no risk, there will be no illiquidity penalties for bankers and no switching costs on the part of depositors. Using the fact that $\underline{m} = \overline{m}$, utilitarian welfare is given by

$$W = \mathbb{E}_{\mathbf{m}}[(R_s^L - R_{CB}^D)\varphi + R_{CB}^D]qE - \overline{m}\kappa\varphi qE + R_{CB}^D qK.$$

Making use of equilibrium conditions $\varphi = (K + E)/E$ and $A_s = R_s^L Q$, with $s \in \mathcal{S}$, utilitarian welfare further simplifies to $W = (\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa q)(K + E)$.

Liquidity risk and solvency risk are ruled out iff $\varphi \leq \min\{\varphi^L, \varphi^S\}$. Using the definition of φ^L and φ^S, we can state that the banker does not face any risk iff

$$\varphi \leq \min\left\{\frac{r_{CB}^D}{r_{CB}^D - \psi}, \frac{r_{CB}^D}{r_{CB}^D - r_{\underline{s}}^L}\right\}.$$

Using equilibrium leverage $\varphi = (K + E)/E$, we know that liquidity risk and solvency risk are ruled out iff

$$\frac{K + E}{E} \leq \min\left\{\frac{R_{CB}^D}{R_{CB}^D - \Psi}, \frac{R_{CB}^D}{R_{CB}^D - R_{\underline{s}}^L}\right\} \quad \Leftrightarrow \quad \frac{K}{K + E} \leq \min\left\{\frac{\Psi}{R_{CB}^D}, \frac{R_{\underline{s}}^L}{R_{CB}^D}\right\}.$$

Using equilibrium condition $A_s = R_s^L Q$, with $s \in \mathcal{S}$, and the fact that based on equation (6.10), in equilibrium, the real central bank rate satisfies $R_{CB}^D = \mathbb{E}_{\mathbf{m}}[R_s^L] - \overline{m}\kappa$, we know that the banker will face neither a liquidity risk nor a solvency risk iff

$$\frac{K}{K + E} \leq \frac{\min\{\Psi q, A_{\underline{s}}\}}{\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa q} \quad \Leftrightarrow \quad \frac{\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa q}{1 + E/K} \leq \min\{\Psi q, A_{\underline{s}}\}.$$

■

Proof of Proposition 4.4.2. Consider the situation where bankers face a liquidity risk but no solvency risk, i.e., $\varphi^L < \varphi \leq \varphi^S$. From lemma 4.3.4 we know that the banker will choose leverage $\varphi^L < \varphi \leq \varphi^S$ iff $\varphi = \varphi^r$, $\phi < 1$ and

$$\mathbb{E}_{\overline{m}}[R_s^L | h = \overline{h}] = R_{CB}^D \left(1 + \frac{\mu}{1 - \mu}\frac{1}{\varphi^r}\right) + \frac{\mu\phi}{1 - \mu}\left(R_{CB}^D \frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \overline{m}\kappa. \tag{6.11}$$

Using equilibrium leverage $\varphi = (K+E)/E$, we know that such an equilibrium with liquidity risk but without solvency risk exists only if $\varphi^r = (K + E)/E$. Furthermore, the banker will only monitor if the matched household opens an account with the banker ($h = \overline{h}$), as otherwise the banker will become illiquid and default, i.e., $\underline{m} = 0$, and $\overline{m} = \mathbb{1}\{\triangle(R_{\underline{s}}^L -$

$R^L_{\underline{s}}) \geq \kappa\}$. As banks are defaulting due to illiquidity when they are matched with a household that opens an account with the central bank ($h = \underline{h}$), the central bank's profits or losses in terms of the consumption are given by

$$\pi^{CB} = \mu[\mathbb{E}_0[R^L_s|h = \underline{h}]L^b - R^D_{CB}(L^b - E^b)]/P = \mu[\mathbb{E}_0[R^L_s|h = \underline{h}]q(K + E) - R^D_{CB}qK],$$

where we have used the banker's equity financing $E^b = QE$, the equilibrium loan supply $L^b = Q(K+E)$ and the fact that bankers who become illiquid and default do not monitor, i.e., $\underline{m} = 0$. Moreover, in equilibrium, the demand for capital good is finite, such that, with lemma 4.3.2, we can deduce $A_s \leq (1+r^L_s)q = R^L_s q$, with $s \in \mathcal{S}$. In addition, due to rational expectations of firms and bankers, it must hold that $A_s = R^L_s Q$ for all $s \in \mathcal{S}$. Thus firms make zero profits, i.e., $\pi^f = 0$. Since the central bank operates under a balanced budget, it holds that $\pi^{CB} = \tau$.

With zero firm profits, the expected consumption of the banker and the household is given by

$$C^b = (1-\mu)\mathbb{E}_{\overline{m}}[(R^L_s - R^D_{CB})\varphi + R^D_{CB}|h = \overline{h}]qE \quad \text{and} \quad C^h = R^D_{CB}qK + \tau^h,$$

with $h \in \mathcal{H}$, respectively. The banker's monitoring decision, if matched with a household that opens an account with the banker ($h = \overline{h}$), is given by $\overline{m} = \mathbb{1}\{\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa q\}$.

Due to our assumption of linear utility, utilitarian welfare comprises aggregate consumption, potential utility losses on the part of bankers due to monitoring, and bankers' illiquidity penalties. Note that, if the banker faces liquidity risk but no solvency risk, there are no switching costs on the part of depositors. Using the fact that $\underline{m} = 0$, the welfare is given by

$$\begin{aligned} W^L = {} & (1-\mu)\mathbb{E}_{\overline{m}}[(R^L_s - R^D_{CB})\varphi + R^D_{CB}|h = \overline{h}]qE - (1-\mu)\overline{m}\kappa\varphi qE \\ & - \mu\phi[R^D_{CB}(\varphi - 1) - \Psi\varphi]qE + R^D_{CB}qK + (1-\mu)\tau^{\overline{h}} + \mu\tau^{\underline{h}}, \end{aligned}$$

which, using equilibrium leverage $\varphi = (K+E)/E$, further simplifies to

$$\begin{aligned} W^L = {} & (1-\mu)\mathbb{E}_{\overline{m}}[R^L_s|h = \overline{h}]q(K+E) - (1-\mu)\overline{m}\kappa\varphi qE \\ & - \mu\phi[R^D_{CB}qK - \Psi q(K+E)] + (1-\mu)\tau^{\overline{h}} + \mu(R^D_{CB}qK + \tau^{\underline{h}}). \end{aligned}$$

With $\tau = (1-\mu)\tau^{\overline{h}} + \mu\tau^{\underline{h}}$ and $\tau = \pi^{CB}$, utilitarian welfare is given by

$$W^L = (1-\mu)\mathbb{E}_{\overline{m}}[R_s^L|h=\overline{h}]q(K+E) - (1-\mu)\overline{m}\kappa\varphi qE - \mu\phi[R_{CB}^D qK - \Psi q(K+E)]$$
$$+ \mu R_{CB}^D qK + \mu[\mathbb{E}_0[R_s^L|h=\underline{h}]q(K+E) - R_{CB}^D qK],$$

which, using $A_s = R_s^L q$, with $s \in \mathcal{S}$, then reads as

$$W^L = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa q\}(K+E) - \mu\phi[R_{CB}^D qK - \Psi q(K+E)].$$

To fully characterize utilitarian welfare, we derive in the following the real central bank rate r_{CB}^D prevailing in equilibrium. First note that, using the equilibrium condition $A_s = R_s^L q$, with $s \in \mathcal{S}$, equation (6.11) can be rewritten as

$$\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] = R_{CB}^D Q\left(1 + \frac{\mu}{1-\mu}\frac{1}{\varphi^r}\right) + \frac{\mu\phi}{1-\mu}\left(R_{CB}^D Q\frac{\varphi^r - 1}{\varphi^r} - \Psi q\right) + \overline{m}\kappa q.$$

Rearranging yields

$$(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa q) + \mu\phi\Psi q = R_{CB}^D q(1-\mu+\mu\phi+\mu(1-\phi)/\varphi^r),$$

such that we can deduce that, in equilibrium, the real central bank rate satisfies

$$R_{CB}^D q = \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa q) + \mu\phi\Psi q}{(1-\mu) + \mu\phi + \mu(1-\phi)/\varphi^r}. \tag{6.12}$$

Thus, it holds that

$$R_{CB}^D qK - \Psi q(K+E) = \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa q)K + \mu\phi\psi qK}{(1-\mu) + \mu\phi + \mu(1-\phi)/\varphi^r} - \psi q(K+E)$$
$$= \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa q)K - \psi q[(1-\mu)K + E]}{(1-\mu) + \mu\phi + \mu(1-\phi)E/(K+E)}$$
$$= \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa q) - \psi q(1-\mu+E/K)}{1-\mu+\mu\phi+E/K}(K+E).$$

Hence welfare is given by $W^L = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa q - \mu\phi\epsilon\}(K+E)$, where

$$\epsilon := \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa q) - \psi q(1-\mu+E/K)}{1-\mu+\mu\phi+E/K}.$$

Liquidity risk exists iff $\varphi > \varphi^L$, while solvency risk is ruled out iff $\varphi \leq \varphi^S$. Using the definition of φ^L and φ^S and equilibrium leverage $\varphi = (K+E)/E$, we can state that the banker will face a liquidity risk but no solvency risk iff

$$\frac{R^D_{CB}}{R^D_{CB} - \psi} < \varphi \leq \frac{R^D_{CB}}{R^D_{CB} - R^L_{\underline{s}}} \quad \Leftrightarrow \quad \frac{\Psi}{R^D_{CB}} < \frac{K}{K+E} \leq \frac{R^L_{\underline{s}}}{R^D_{CB}}.$$

Using the equilibrium condition (6.12), we know that

$$\frac{\Psi}{R^D_{CB}} < \frac{K}{K+E} \qquad \Leftrightarrow \qquad \frac{\Psi q[(1-\mu)+\mu\phi+\mu(1-\phi)/\varphi^r]}{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa q) + \mu\phi\Psi q} < \frac{K}{K+E},$$

where the latter can be further rearranged to give

$$\Psi q(K+E)[(1-\mu)+\mu\phi+\mu(1-\phi)/\varphi^r] < [(1-\mu)(\mathbb{E}_{\mathbf{m}}[A_s|h=\overline{h}] - \overline{m}\kappa q) + \mu\phi\Psi q]K.$$

Note that, using $\varphi^r = (K+E)/E$,

$$\begin{aligned}(K+E)[(1-\mu)+\mu\phi+\mu(1-\phi)/\varphi^r] &= [(1-\mu)+\mu\phi](K+E) + \mu(1-\phi)E \\ &= (1-\mu+\mu\phi)K + E.\end{aligned}$$

Hence, the previous inequality translates into

$$\begin{aligned} & \Psi q[(1-\mu+\mu\phi)K+E] < [(1-\mu)(\mathbb{E}_{\mathbf{m}}[A_s|h=\overline{h}] - \overline{m}\kappa q) + \mu\phi\Psi q]K \\ \Leftrightarrow \quad & \Psi q[(1-\mu)K+E] < (1-\mu)(\mathbb{E}_{\mathbf{m}}[A_s|h=\overline{h}] - \overline{m}\kappa q)K \\ \Leftrightarrow \quad & \Psi q(1-\mu+E/K) < (1-\mu)(\mathbb{E}_{\mathbf{m}}[A_s|h=\overline{h}] - \overline{m}\kappa q) \\ \Leftrightarrow \quad & \Psi q < \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}] - \overline{m}\kappa q)}{1-\mu+E/K}.\end{aligned}$$

Similarly, using the equilibrium conditions $A_s = R^L_s Q$, with $s \in \mathcal{S}$, and (6.12), we know that the banker will face no solvency risk iff

$$\frac{K}{K+E} \leq \frac{R^L_{\underline{s}}}{R^D_{CB}} \quad \Leftrightarrow \quad \frac{K}{K+E} \leq \frac{A_{\underline{s}}[(1-\mu)+\mu\phi+\mu(1-\phi)/\varphi^r]}{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa q)+\mu\phi\Psi q},$$

which can be rearranged to

$$[(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa q)+\mu\phi\Psi q]K \leq A_{\underline{s}}(K+E)[(1-\mu)+\mu\phi+\mu(1-\phi)/\varphi^r]$$

and using $(K+E)[(1-\mu)+\mu\phi+\mu(1-\phi)/\varphi^r] = (1-\mu+\mu\phi)K+E$, as previously derived, further simplifies to

$$\mu\phi\Psi qK \leq A_{\underline{s}}[(1-\mu+\mu\phi)K+E]-(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa q)K$$

or, equivalently,

$$\Psi q \leq \frac{A_{\underline{s}}}{\mu\phi}(1-\mu+\mu\phi+E/K)-\frac{1-\mu}{\mu\phi}(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa q).$$

As shown before, the liquidity risk and solvency risk condition both constrain the real haircut. Thus we can verify when the liquidity risk condition will be stricter than the solvency risk condition, i.e.,

$$\frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa q)}{1-\mu+E/K} \leq \frac{A_{\underline{s}}}{\mu\phi}(1-\mu+\mu\phi+E/K)-\frac{1-\mu}{\mu\phi}(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa q),$$

which can be rearranged to

$$\mu\phi\frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa q)}{1-\mu+E/K} \leq A_{\underline{s}}(1-\mu+\mu\phi+E/K)-(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa q)$$

and

$$\mu\phi\frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa q)}{1-\mu+E/K}+(1-\mu+E/K)\frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa q)}{1-\mu+E/K}$$
$$\leq A_{\underline{s}}(1-\mu+\mu\phi+E/K).$$

Further rearranging yields

$$(1-\mu+\mu\phi+E/K)\frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa q)}{1-\mu+E/K} \leq A_{\underline{s}}(1-\mu+\mu\phi+E/K)$$

$$\Leftrightarrow \quad \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa q)}{1-\mu+E/K} \leq A_{\underline{s}}.$$

Note that with the liquidity risk condition we can deduce $\Psi q < A_{\underline{s}}$. For $\Psi q \geq A_{\underline{s}}$, we can show that the solvency risk condition contradicts the liquidity risk condition. Thus the banker will face a liquidity risk but no solvency risk iff

$$\Psi q < \frac{(1-\mu)(\mathbb{E}_{\overline{m}}[A_s|h=\overline{h}]-\overline{m}\kappa q)}{1-\mu+E/K} \leq A_{\underline{s}}.$$

■

Proof of Proposition 4.4.3. Consider the situation where bankers face a solvency risk but no liquidity risk, i.e., $\varphi^S < \varphi \leq \varphi^L$. From lemma 4.3.4 we know that the banker will choose a leverage $\varphi^S < \varphi \leq \varphi^L$ iff $\varphi = \varphi^r$ and

$$R^L_{\overline{s}} = R^D_{CB}\left(1+\frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right)+\frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}. \tag{6.13}$$

Using equilibrium leverage $\varphi = (K+E)/E$, we know that such an equilibrium with solvency risk but without liquidity risk only exists if $\varphi^r = (K+E)/E$. Furthermore, the banker's monitoring decision is given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle[R^L_{\overline{s}} - R^D_{CB}(\varphi^r-1)/\varphi^r] \geq \kappa\}$. As banks are only defaulting due to insolvency, i.e., when the financed firm incurs a negative productivity shock, the central bank's losses in real terms are given by

$$\pi^{CB} = \eta_{\underline{s}|\overline{m}}[R^L_{\underline{s}}L^b - R^D_{CB}(L^b-E^b)]/P = \eta_{\underline{s}|\overline{m}}[R^L_{\underline{s}}q(K+E) - R^D_{CB}qK],$$

where we have used the banker's equity financing $E^b = QE$, the equilibrium loan supply $L^b = Q(K+E)$, and the fact that the banker's monitoring decision is independent of the type of household, i.e., $\underline{m} = \overline{m}$. In equilibrium, the demand for capital good is finite, such that, with lemma 4.3.2, we can deduce $A_s \leq (1+r^L_s)q = R^L_s q$, with $s \in \mathcal{S}$. In addition, due to rational expectations of firms and bankers, it must hold that $A_s = R^L_s q$ for all $s \in \mathcal{S}$. Thus firms make zero profits, i.e., $\pi^f = 0$. Since the central bank operates under a balanced budget, it holds that $\pi^{CB} = \tau$.

With zero firm profits, the expected consumption by the banker and the household is given by

$$C^b = \eta_{\overline{s}|\overline{m}}[(R^L_{\overline{s}} - R^D_{CB})\varphi + R^D_{CB}]qE \quad \text{and} \quad C^h = R^D_{CB}qK + \tau^h,$$

with $h \in \mathcal{H}$, respectively. The banker's monitoring decision is given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle[A_{\overline{s}} - R^D_{CB}qK/(K+E)] \geq \kappa q\}$. To fully characterize the banker's monitoring decision, we derive in the following the real central bank interest factor R^D_{CB} prevailing in equilibrium. First note that, using equilibrium condition $A_s = R^L_s Q$, with $s \in \mathcal{S}$, (6.13) can be rewritten as

$$A_{\overline{s}} = R^D_{CB}q\left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{\overline{m}\kappa q}{\eta_{\overline{s}|\overline{m}}}.$$

Rearranging yields

$$\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q = R^D_{CB}q(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r),$$

such that we can finally deduce that, in equilibrium, the real central bank rate satisfies

$$R^D_{CB}q = \frac{\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q}{\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r}. \tag{6.14}$$

We can then state that the banker will monitor, independently of the type of matched household iff

$$\triangle\left[A_{\overline{s}} - \frac{\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q}{\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r}\frac{K}{K+E}\right] \geq \kappa q.$$

Rearranging yields

$$\triangle\left[A_{\overline{s}}(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E) - (\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q)K\right] \geq \kappa q(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E).$$

Note that, using $\varphi^r = (K+E)/K$,

$$(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E) = \eta_{\overline{s}|\overline{m}}(K+E) + \eta_{\underline{s}|\overline{m}}E = E + \eta_{\overline{s}|\overline{m}}K.$$

Thus, the latter inequality reads as

$$\triangle\left[A_{\overline{s}}(E+\eta_{\overline{s}|\overline{m}}K)-(\eta_{\overline{s}|\overline{m}}A_{\overline{s}}-\overline{m}\kappa q)K\right]\geq\kappa Q(E+\eta_{\overline{s}|\overline{m}}K)$$

$$\Leftrightarrow\quad\triangle A_{\overline{s}}E\geq\kappa q(E+\eta_{\overline{s}|\overline{m}}K-\triangle\overline{m}K).$$

Exploiting $\triangle := \eta_{\overline{s}|1}-\eta_{\overline{s}|0}$ and setting $\overline{m}=1$, as the condition, if satisfied, implies monitoring, we know that the banker will monitor iff

$$\triangle A_{\overline{s}}E\geq\kappa q[E+\eta_{\overline{s}|1}K-(\eta_{\overline{s}|1}-\eta_{\overline{s}|0})K]\qquad\Leftrightarrow\qquad\triangle A_{\overline{s}}\geq\kappa q(1+\eta_{\overline{s}|0}K/E).$$

Due to our assumption of linear utility, utilitarian welfare comprises aggregate consumption, potential utility losses on the part of bankers due to monitoring, and potential switching costs on the part of depositors. Note that, if the banker faces no liquidity risk, there are no illiquidity penalties for bankers. Whether depositors transfer funds to the central bank if the respective banker becomes insolvent depends on the leverage. Specifically, depositors will switch in the case of insolvency iff $\varphi^R<\varphi$. Utilitarian welfare is then given by

$$W^S=\eta_{\overline{s}|\overline{m}}[(R^L_{\overline{s}}-R^D_{CB})\varphi+R^D_{CB}]qE-\overline{m}\kappa\varphi qE$$
$$+(1-\mu)(R^D_{CB}qK+\tau^{\overline{h}})+\mu(R^D_{CB}qK+\tau^{\underline{h}})-(1-\mu)\eta_{\underline{s}|\overline{m}}\nu\mathbb{1}\{\varphi^R<\varphi\},$$

which, using equilibrium leverage $\varphi=(K+E)/E$, further simplifies to

$$W^S=\eta_{\overline{s}|\overline{m}}(r^L_{\overline{s}}-\overline{m}\kappa)q(K+E)+(1-\mu)(\eta_{\underline{s}|\overline{m}}R^D_{CB}qK+\tau^{\overline{h}})$$
$$+\mu(\eta_{\underline{s}|\overline{m}}R^D_{CB}qK+\tau^{\underline{h}})-(1-\mu)\eta_{\underline{s}|\overline{m}}\nu\mathbb{1}\{\varphi^R<\varphi\}.$$

Note that only a mass $\eta_{\underline{s}|\overline{m}}$ of households that initially open an account with a banker (mass $1-\mu$) will potentially transfer funds and thus incur switching costs. With $\tau=(1-\mu)\tau^{\overline{h}}+\mu\tau^{\underline{h}}$ and $\tau=\pi^{CB}$, welfare is given by

$$W^S=\eta_{\overline{s}|\overline{m}}(R^L_{\overline{s}}-\overline{m}\kappa)q(K+E)+\eta_{\underline{s}|\overline{m}}R^D_{CB}qK$$
$$+\eta_{\underline{s}|\overline{m}}[R^L_{\underline{s}}q(K+E)-R^D_{CB}QK]-(1-\mu)\eta_{\underline{s}|\overline{m}}\nu\mathbb{1}\{\varphi^R<\varphi\},$$

which, using $A_s = R^L_s Q$, with $s \in \mathcal{S}$, then reads as

$$W^S = (\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa q)(K+E) - (1-\mu)\eta_{\underline{s}|\overline{m}}\nu\mathbb{1}\{\varphi^R < \varphi\}.$$

We will specify the switching condition $\varphi^R < \varphi$ at a later stage. First note that liquidity risk is ruled out iff $\varphi \leq \varphi^L$, while solvency risk exists iff $\varphi > \varphi^S$. Using the definition of φ^L and φ^S, we can state that the banker will face a solvency risk but no liquidity risk iff

$$\frac{R^D_{CB}}{R^D_{CB} - R^L_{\underline{s}}} < \varphi \leq \frac{R^D_{CB}}{R^D_{CB} - \Psi}.$$

Using equilibrium leverage $\varphi = (K+E)/E$, the latter inequalities translate into

$$\frac{R^D_{CB}}{R^D_{CB} - R^L_{\underline{s}}} < \frac{K+E}{E} \leq \frac{R^D_{CB}}{R^D_{CB} - \Psi} \qquad \Leftrightarrow \qquad \frac{R^L_{\underline{s}}}{R^D_{CB}} < \frac{K}{K+E} \leq \frac{\Psi}{R^D_{CB}}.$$

Using equilibrium conditions $A_s = R^L_s Q$, with $s \in \mathcal{S}$, and (6.14), we know that the banker will face a solvency risk but not liquidity risk iff

$$\frac{A_{\underline{s}}(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)}{\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q} < \frac{K}{K+E} \leq \frac{\Psi q(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)}{\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q},$$

which can be rewritten as

$$A_{\underline{s}}(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E) < (\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q)K \leq \Psi q(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E).$$

Note that, using $\varphi^r = (K+E)/E$, it holds that $(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E) = \eta_{\overline{s}|\overline{m}}K + E$. Then the latter inequalities read as

$$A_{\underline{s}}(\eta_{\overline{s}|\overline{m}}K + E) < (\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q)K \leq \Psi q(\eta_{\overline{s}|\overline{m}}K + E)$$

or, equivalently,

$$A_{\underline{s}} < \frac{\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q}{\eta_{\overline{s}|\overline{m}} + E/K} \leq \Psi q.$$

Finally, we need to specify when bank insolvency will trigger a bank run, i.e., when $\varphi^R < \varphi$. Using equilibrium leverage $\varphi = (K+E)/E$ and the definition of φ^R, we know that $\varphi^R < \varphi$

iff

$$\frac{R^D_{CB} - \tilde{\nu}}{R^D_{CB} - \tilde{\nu} - R^L_{\underline{s}}} < \frac{K+E}{E} \qquad \Leftrightarrow \qquad \frac{R^L_{\underline{s}}}{R^D_{CB} - \tilde{\nu}} < \frac{K}{K+E},$$

where $\tilde{\nu} := \nu/(QK)$. Using the equilibrium conditions $A_s = R^L_s Q$, with $s \in \mathcal{S}$, and (6.14), the latter inequality translates into

$$\frac{A_{\underline{s}}}{R^D_{CB} q - \nu/K} < \frac{K}{K+E} \quad \Leftrightarrow \quad \frac{A_{\underline{s}}(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)}{(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q) - \nu(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)/K} < \frac{K}{K+E}.$$

Rearranging yields

$$\begin{aligned}
& A_{\underline{s}}(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E) < [(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q) - \nu(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)/K]K \\
\Leftrightarrow \quad & A_{\underline{s}}(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E) < (\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q)K - \nu(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r) \\
\Leftrightarrow \quad & \nu < \frac{(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q)K}{\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r} - A_{\underline{s}}(K+E) \\
\Leftrightarrow \quad & \nu < \left(\frac{(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q)K}{(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E)} - A_{\underline{s}} \right)(K+E),
\end{aligned}$$

which, using $(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E) = \eta_{\overline{s}|\overline{m}} K + E$, leads to

$$\nu < \nu^* := \left(\frac{\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q}{\eta_{\overline{s}|\overline{m}} + E/K} - A_{\underline{s}} \right)(K+E).$$

Thus utilitarian welfare is given by $W^S = (\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa q)(K+E) - (1-\mu)\eta_{\underline{s}|\overline{m}}\nu\mathbb{1}\{\nu < \nu^*\}$.
■

Proof of Proposition 4.4.4. Consider the situation where bankers face both a solvency and a liquidity risk, i.e $\max\{\varphi^L, \varphi^S\} < \varphi$. However, bank insolvency will not trigger a bank run, i.e., $\varphi \leq \varphi^R$. Then we know from lemma 4.3.4 that the banker will choose $\max\{\varphi^L, \varphi^S\} < \varphi \leq \varphi^R$ iff $\varphi = \varphi^r$, $\mu\phi < \mu + (1-\mu)\eta_{\underline{s}|\overline{m}}$ and

$$R^L_{\overline{s}} = R^D_{CB}\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{\mu\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(R^D_{CB}\frac{\varphi^r - 1}{\varphi^r} - \psi\right) + \frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}. \quad (6.15)$$

Using equilibrium leverage $\varphi = (K+E)/E$, we know that such an equilibrium with liquidity risk and solvency risk only exists if $\varphi^r = (K+E)/E$. Furthermore, the banker's monitoring decision is given by $\underline{m} = 0$ and $\overline{m} = \mathbb{1}\{\triangle[R^L_{\overline{s}} - R^D_{CB}(\varphi^r - 1)/\varphi^r] \geq \kappa\}$. Banks are defaulting due to illiquidity and insolvency, such that the central bank's profits and losses in terms of the consumption good are given by

$$\begin{aligned}\pi^{CB} &= [\mu\mathbb{E}_0[r^L_s|h=\underline{h}] + (1-\mu)\eta_{\underline{s}|\overline{m}}R^L_{\underline{s}}]L^b/P - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]R^D_{CB}(L^b - E^b)/P \\ &= [\mu\mathbb{E}_0[r^L_s|h=\underline{h}] + (1-\mu)\eta_{\underline{s}|\overline{m}}r^L_{\underline{s}}]q(K+E) - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]R^D_{CB}qK,\end{aligned}$$

where we have used the banker's equity financing $E^b = QE$, the equilibrium loan supply $L^b = Q(K+E)$ and the fact that the banker will not monitor if the matched household initially opens an account with the central bank ($h = \underline{h}$), i.e., $\underline{m} = 0$. Moreover, in equilibrium, the demand for capital good is finite, such that, with lemma 4.3.2, we can deduce $A_s \leq (1 + r^L_s)q = R^L_s q$, with $s \in \mathcal{S}$. In addition, due to rational expectations of firms and bankers, it must hold that $A_s = R^L_s q$ for all $s \in \mathcal{S}$. Thus firms make zero profits, i.e., $\pi^f = 0$. Since the central bank operates under a balanced budget, the taxes and transfers in real terms are given by $\tau = \pi^{CB}$.

With zero firm profits, expected consumption by the banker and the household is given by

$$C^b = (1-\mu)\eta_{\overline{s}|\overline{m}}[(R^L_{\overline{s}} - R^D_{CB})\varphi + R^D_{CB}]qE \quad \text{and} \quad C^h = R^D_{CB}qK + \tau^h,$$

with $h \in \mathcal{H}$, respectively. The banker's monitoring decision is given by $\underline{m} = 0$ and $\overline{m} = \mathbb{1}\{\triangle[A_{\overline{s}} - R^D_{CB}qK/(K+E)] \geq \kappa q\}$. To fully characterize the banker's monitoring decision, we derive in the following the real central bank interest factor R^D_{CB} prevailing in equilibrium. First note that, using $A_s = R^L_s q$, with $s \in \mathcal{S}$, (6.15) can be rewritten as

$$A_{\overline{s}} = R^D_{CB}q\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{\mu\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(R^D_{CB}q\frac{\varphi^r - 1}{\varphi^r} - \Psi q\right) + \frac{\overline{m}\kappa q}{\eta_{\overline{s}|\overline{m}}}.$$

Rearranging the latter equation yields

$$(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + \mu\phi\Psi q = R^D_{CB}q\{(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu(1-\phi)]/\varphi^r\},$$

such that we can deduce that, in equilibrium, the real central bank rate satisfies

$$R^D_{CB}q = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + \mu\phi\psi q}{(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r}. \tag{6.16}$$

With (6.16), we can state that the banker will monitor iff $h = \overline{h}$ and

$$\triangle\left[A_{\overline{s}} - \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + \mu\phi\psi q}{(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r}\frac{K}{K+E}\right] \geq \kappa q.$$

Note that, using $\varphi^r = (K+E)/E$,

$$\begin{aligned}\{(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r\}(K+E)&\\ = [(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi](K+E) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu(1-\phi)]E&\\ = [(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi]K + E.&\end{aligned}$$

Then the latter inequality translates into

$$\triangle A_{\overline{s}} - \triangle\frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + \mu\phi\psi Q}{(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi + E/K} \geq \kappa q.$$

Setting $\overline{m} = 1$, as the condition, if satisfied, implies monitoring, and further rearranging yields

$$\begin{aligned}&\triangle A_{\overline{s}}[(1-\mu)\eta_{\overline{s}|1} + \mu\phi + E/K] - \triangle[(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q) + \mu\phi\Psi q]\\ &\qquad\geq \kappa q[(1-\mu)\eta_{\overline{s}|1} + \mu\phi + E/K]\\ \Leftrightarrow\quad &\triangle A_{\overline{s}}(\mu\phi + E/K) - \triangle\mu\phi\Psi q \geq \kappa q[(1-\mu)\eta_{\overline{s}|1} + \mu\phi + E/K - (1-\mu)\triangle],\end{aligned}$$

which, using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, finally reads as

$$\triangle A_{\overline{s}} - \frac{\triangle\mu\phi\Psi q}{\mu\phi + E/K} \geq \kappa q\left[1 + \frac{(1-\mu)\eta_{\overline{s}|0}}{\mu\phi + E/K}\right].$$

Due to our assumption of linear utility, utilitarian welfare comprises aggregate consumption, potential utility losses on the part of bankers due to monitoring, and illiquidity penalties. Note that, as bank insolvency does not trigger a bank run, there are no switching costs for depositors. Of course, the latter requires $\varphi \leq \varphi^R$, which we will further specify at a later stage. Utilitarian welfare with liquidity and solvency risk and with bail-in in the case of bank insolvency, is then given by

$$\begin{aligned} W_B^{LS} =& (1-\mu)\{\eta_{\overline{s}|\overline{m}}[(R_{\overline{s}}^L - R_{CB}^D)\varphi + R_{CB}^D] - \overline{m}\kappa\varphi - \mu\phi[R_{CB}^D(\varphi - 1) - \Psi\varphi]\}qE \\ & + (1-\mu)(R_{CB}^D qK + \tau^{\overline{h}}) + \mu(R_{CB}^D qK + \tau^{\underline{h}}), \end{aligned}$$

which, using equilibrium leverage $\varphi = (K+E)/E$, translates into

$$\begin{aligned} W_B^{LS} =& (1-\mu)(\eta_{\overline{s}|\overline{m}} R_{\overline{s}}^L - \overline{m}\kappa)q(K+E) - \mu\phi[R_{CB}^D qK - \Psi q(K+E)] \\ & + (1-\mu)(\eta_{\underline{s}|\overline{m}} R_{CB}^D qK + \tau^{\overline{h}}) + \mu(R_{CB}^D qK + \tau^{\underline{h}}), \end{aligned}$$

With $\tau = (1-\mu)\tau^{\overline{h}} + \mu\tau^{\underline{h}}$ and $\tau = \pi^{cb}$, welfare is given by

$$\begin{aligned} W_B^{LS} =& (1-\mu)(\eta_{\overline{s}|\overline{m}} R_{\overline{s}}^L - \overline{m}\kappa)q(K+E) - \mu\phi[R_{CB}^D qK - \Psi q(K+E)] \\ & + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]R_{CB}^D qK \\ & + [\mu\mathbb{E}_0[R_s^L | h = \underline{h}] + (1-\mu)\eta_{\underline{s}|\overline{m}} r_{\underline{s}}^L]q(K+E) \\ & - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]R_{CB}^D qK, \end{aligned}$$

which, using $A_s = R_s^L q$, with $s \in \mathcal{S}$, reads as

$$W_B^{LS} = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa q\}(K+E) - \mu\phi[R_{CB}^D qK - \Psi q(K+E)],$$

where, with equilibrium condition (6.16),

$$\begin{aligned} R^D_{CB}qK - \Psi q(K+E) &= \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + \mu\phi\Psi q}{(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r}K - \Psi q(K+E) \\ &= \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q)K - \Psi q[E + (1-\mu)\eta_{\overline{s}|\overline{m}}K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}E/(K+E) + \mu\phi + \mu(1-\phi)E/(K+E)} \\ &= \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) - \Psi q[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi + E/K}(K+E). \end{aligned}$$

Hence, welfare is given by $W^{LS}_B = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa q - \mu\phi\epsilon\}(K+E)$, where

$$\epsilon = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) - \Psi q[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi + E/K}.$$

Liquidity risk exists iff $\varphi^L < \varphi$, while solvency risk exists iff $\varphi^S < \varphi$. Using the definition of φ^L and φ^S and the equilibrium leverage $\varphi = (K+E)/E$, we can state that the banker will face liquidity risk and solvency risk iff

$$\max\left\{\frac{R^D_{CB}}{R^D_{CB} - \Psi}, \frac{R^D_{CB}}{R^D_{CB} - R^L_{\underline{s}}}\right\} < \frac{K+E}{E} \qquad \Leftrightarrow \qquad \max\left\{\frac{\Psi}{R^D_{CB}}, \frac{R^L_{\underline{s}}}{R^D_{CB}}\right\} < \frac{K}{K+E}.$$

Using equilibrium conditions $A_s = R^L_s q$, with $s \in \mathcal{S}$, and (6.16), we know that the banker will face liquidity risk and solvency risk iff

$$\frac{\max\{\Psi q, A_{\underline{s}}\}[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r]}{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + \mu\phi\Psi q} < \frac{K}{K+E}.$$

The liquidity risk condition

$$\Psi q\frac{[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r]}{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + \mu\phi\Psi q} < \frac{K}{K+E}.$$

can be further rearranged as

$$\begin{aligned} &\Psi q(K+E)[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r] \\ &\qquad < K[(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + \mu\phi\Psi q]. \end{aligned}$$

Note that, using $\varphi^r = (K+E)/E$,

$$
\begin{aligned}
(K+E)&[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r] \\
&= (1-\mu)\eta_{\overline{s}|\overline{m}}(K+E) + (1-\mu)\eta_{\underline{s}|\overline{m}}E + \mu\phi(K+E) + \mu(1-\phi)E \\
&= [(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi]K + E.
\end{aligned}
$$

Then the latter inequality translates into

$$
\begin{aligned}
& \Psi q[(1-\mu)\eta_{\overline{s}|\overline{m}}K + \mu\phi K + E] < K[(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + \mu\phi\Psi q] \\
\Leftrightarrow\quad & \Psi q[(1-\mu)\eta_{\overline{s}|\overline{m}}K + E] < K(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) \\
\Leftrightarrow\quad & \Psi q[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K] < (1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) \\
\Leftrightarrow\quad & \Psi q < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.
\end{aligned}
$$

The solvency risk condition is given by

$$
A_{\underline{s}}\frac{[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r]}{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + \mu\phi\Psi q} < \frac{K}{K+E}.
$$

and can be rearranged to

$$
\begin{aligned}
A_{\underline{s}}(K+E)&[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r] \\
&< K[(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + \mu\phi\Psi q].
\end{aligned}
$$

Using, as before,

$$
(K+E)[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r] = [(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi]K + E,
$$

we obtain

$$
\begin{aligned}
& A_{\underline{s}}[(1-\mu)\eta_{\overline{s}|\overline{m}}K + \mu\phi K + E] < K[(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + \mu\phi\Psi q] \\
\Leftrightarrow \quad & A_{\underline{s}}[(1-\mu)\eta_{\overline{s}|\overline{m}}K + E] + \mu\phi(A_{\underline{s}} - \Psi q)K < K[(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q)] \\
\Leftrightarrow \quad & A_{\underline{s}}[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K] + \mu\phi(A_{\underline{s}} - \Psi q) < [(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q)] \\
\Leftrightarrow \quad & A_{\underline{s}} + \frac{\mu\phi(A_{\underline{s}} - \Psi q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K} < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.
\end{aligned}
$$

Thus the banker will face liquidity risk and solvency risk iff

$$
\max\left\{A_{\underline{s}} + \frac{\mu\phi(A_{\underline{s}} - \Psi q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}, \Psi q\right\} < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.
$$

Bank insolvency does not trigger a bank run iff $\varphi \leq \varphi^R$, which, using equilibrium leverage $\varphi = (K+E)/E$ and the definition of φ^R, translates into

$$
\frac{K+E}{E} \leq \frac{R^D_{CB} - \tilde{\nu}}{R^D_{CB} - \tilde{\nu} - R^L_{\underline{s}}} \qquad \Leftrightarrow \qquad \frac{K}{K+E} \leq \frac{R^L_{\underline{s}}}{R^D_{CB} - \tilde{\nu}},
$$

where $\tilde{\nu} := \nu/(QK)$. Using equilibrium conditions $A_s = R^L_s q$, with $s \in \mathcal{S}$, and (6.16), we can state $\varphi \leq \varphi^R$ iff

$$
\frac{K}{K+E} \leq \frac{A_{\underline{s}}\alpha}{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + \mu\phi\Psi q - \nu\alpha/K},
$$

where $\alpha := (1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}/\varphi^r + \mu\phi + \mu(1-\phi)/\varphi^r$. Rearranging yields

$$
(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q)K \leq A_{\underline{s}}\alpha(K+E) + \nu\alpha - \mu\phi\Psi qK.
$$

Using $\alpha(K+E) = [(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi]K + E$ as before, the latter inequality translates into

$$
(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q)K \leq \left(A_{\underline{s}} + \frac{\nu}{K+E}\right)\{[(1-\mu)\eta_{\overline{s}|\overline{m}} + \mu\phi]K + E\} - \mu\phi\Psi qK,
$$

which can be rewritten as

$$\frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}}-\overline{m}\kappa q)}{(1-\mu)\eta_{\overline{s}|\overline{m}}+E/K}\leq A_{\underline{s}}+\frac{\nu}{K+E}+\frac{\mu\phi(A_{\underline{s}}+\nu/(K+E)-\Psi q)}{(1-\mu)\eta_{\overline{s}|\overline{m}}+E/K}.$$

■

Proof of Proposition 4.4.5. Consider the situation where bankers face both a liquidity and a solvency risk and bank insolvency triggers a bank run, i.e., $\min\{\varphi^L,\varphi^S,\varphi^R\}<\varphi$. Then we know from Lemma 4.3.4 that the banker will choose $\max\{\varphi^L,\varphi^S,\varphi^R\}<\varphi$ iff $\varphi=\varphi^r$, $\phi<1$ and

$$R^L_{\overline{s}}=R^D_{CB}\left(1+\frac{(1-\mu)\eta_{\underline{s}|\overline{m}}+\mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right)+\frac{[(1-\mu)\eta_{\underline{s}|\overline{m}}+\mu]\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(R^D_{CB}\frac{\varphi^r-1}{\varphi^r}-\Psi\right)+\frac{\overline{m}\kappa}{\eta_{\overline{s}|\overline{m}}}, \tag{6.17}$$

Using equilibrium leverage, we know that such an equilibrium with liquidity risk and solvency risk only exists if $\varphi^r=(K+E)/E$. Furthermore, the banker will monitor iff matched with a household that initially holds deposits ($h=\overline{h}$) and if $\triangle[R^L_{\overline{s}}-\phi\Psi-R^D_{CB}(1-\phi)(\varphi^r-1)/\varphi^r]\geq\kappa$. Banks are defaulting due to illiquidity and insolvency, such that the central bank's profits and losses in terms of the consumption good are given by

$$\begin{aligned}\pi^{CB}&=[\mu\mathbb{E}_0[R^L_s|h=\underline{h}]+(1-\mu)\eta_{\underline{s}|\overline{m}}R^L_{\underline{s}}]L^b/P-[(1-\mu)\eta_{\underline{s}|\overline{m}}+\mu]R^D_{CB}(L^b-E^b)/P\\&=[\mu\mathbb{E}_0[R^L_s|h=\underline{h}]+(1-\mu)\eta_{\underline{s}|\overline{m}}R^L_{\underline{s}}]q(K+E)-[(1-\mu)\eta_{\underline{s}|\overline{m}}+\mu]R^D_{CB}qK,\end{aligned}$$

where we have used the banker's equity financing $E^b=QE$, the equilibrium loan supply $L^b=Q(K+E)$, and the fact that the banker will not monitor if the matched household initially opens an account with the central bank ($h=\underline{h}$), i.e., $\underline{m}=0$. Moreover, in equilibrium, the demand for capital good is finite, such that, with lemma 4.3.2, we can deduce $A_s\leq R^L_sQ$, with $s\in\mathcal{S}$. In addition, due to rational expectations of firms and bankers it must hold that $A_s=R^L_sQ$ for all $s\in\mathcal{S}$. Hence, firms make zero profits, i.e., $\pi^f=0$. Since the central bank operates under a balanced budget, taxes and transfers in real terms are given by $\tau=\pi^{CB}$.

With zero firm profits, expected consumption by the banker and the household is given by

$$C^b = (1-\mu)\eta_{\overline{s}|\overline{m}}[(R^L_{\overline{s}} - R^D_{CB})\varphi + R^D_{CB}]qE \quad \text{and} \quad C^h = R^D_{CB}qK + \tau^h,$$

with $h \in \mathcal{H}$, respectively. The banker will monitor iff $h = \overline{h}$ and $\triangle[A_{\overline{s}} - \phi\Psi q - R^D_{CB}q(1-\phi)K/(K+E)] \geq \kappa q$. To fully characterize the banker's monitoring decision, we derive in the following the real central bank rate prevailing in equilibrium. First note that, using equilibrium condition $A_s = R^L_s Q$, with $s \in \mathcal{S}$, (6.17) can be rewritten as

$$A_{\overline{s}} = R^D_{CB}q\left(1 + \frac{(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi}{(1-\mu)\eta_{\overline{s}|\overline{m}}}\left(R^D_{CB}q\frac{\varphi^r - 1}{\varphi^r} - \Psi q\right) + \frac{\overline{m}\kappa q}{\eta_{\overline{s}|\overline{m}}}.$$

Rearranging yields

$$\begin{aligned}(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\Psi q \\ = R^D_{CB}q\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r\}\end{aligned}$$

such that we can deduce that, in equilibrium, the real central bank rate satisfies

$$R^D_{CB}q = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\Psi q}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r}. \tag{6.18}$$

Using (6.18), we can state that the banker will monitor iff $h = \overline{h}$ and

$$\triangle\left[A_{\overline{s}} - \phi\Psi q - \frac{\{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\Psi q\}(1-\phi)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r}\frac{K}{K+E}\right] \geq \kappa q.$$

Note that, using $\varphi^r = (K+E)/E$,

$$\begin{aligned}&\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r\}(K+E) \\ &= \{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}(K+E) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)E \\ &= \{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}K + E.\end{aligned}$$

Then the latter inequality translates into

$$\triangle \left[A_{\overline{s}} - \phi\Psi q - \frac{\{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\Psi q\}(1-\phi)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + E/K} \right] \geq \kappa q.$$

Rearranging yields

$$\begin{aligned} &\triangle(A_{\overline{s}} - \phi\Psi q)\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + E/K\} \\ &\quad - \triangle(1-\phi)\{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\Psi q\} \\ &\qquad \geq \kappa q\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + E/K\}, \end{aligned}$$

which, in turn, simplifies to

$$\begin{aligned} \triangle A_{\overline{s}}(\phi + E/K) - \triangle\Psi q(1 + E/K) &\geq \kappa q\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + E/K\} \\ &\quad - \kappa q(1-\phi)(1-\mu)\triangle\overline{m}. \end{aligned}$$

Setting $\overline{m} = 1$, as the condition, if satisfied, implies monitoring, and using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$ yields

$$\triangle A_{\overline{s}}(\phi + E/K) - \triangle\phi\Psi q(1 + E/K) \geq \kappa q[\phi + E/K + (1-\phi)(1-\mu)\eta_{\overline{s}|0}],$$

which then translates into

$$\triangle A_{\overline{s}} - \triangle\phi\Psi q \frac{1 + E/K}{\phi + E/K} \geq \kappa q \left[1 + \frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi + E/K} \right].$$

Utilitarian welfare comprises aggregate consumption, potential utility losses on the part of bankers due to monitoring and illiquidity penalties, and switching costs on the part of depositors. Utilitarian welfare with liquidity and solvency risk and without bail-ins in case of bank insolvency is then given by

$$W_{NB}^{LS} = (1-\mu)\{\eta_{\overline{s}|\overline{m}}[(R_{\overline{s}}^{L} - R_{CB}^{D})\varphi + R_{CB}^{D}] - \overline{m}\kappa\varphi$$

$$- [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi[R_{CB}^{D}(\varphi - 1) - \psi\varphi]\}qE$$

$$+ (1-\mu)(R_{CB}^{D}qK + \tau^{\overline{h}}) + \mu(R_{CB}^{D}qK + \tau^{\underline{h}}) - (1-\mu)\eta_{\underline{s}|\overline{m}}\nu,$$

which, using equilibrium leverage $\varphi = (K+E)/E$, reads as

$$W_{NB}^{LS} = (1-\mu)\eta_{\overline{s}|\overline{m}}R_{\overline{s}}^{L}q(K+E) - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi[R_{CB}^{D}qK - \Psi q(K+E)]$$

$$+ (1-\mu)(\eta_{\underline{s}|\overline{m}}R_{CB}^{D}qK + \tau^{\overline{h}}) + \mu(R_{CB}^{D}qK + \tau^{\underline{h}}) - (1-\mu)\eta_{\underline{s}|\overline{m}}\nu,$$

With $\tau = (1-\mu)\tau^{\overline{h}} + \mu\tau^{\underline{h}}$ and $\tau = \pi^{cb}$, the welfare is given by

$$W_{B}^{LS} = (1-\mu)\eta_{\overline{s}|\overline{m}}R_{\overline{s}}^{L}q(K+E) - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\{\phi[R_{CB}^{D}qK - \psi q(K+E)] + R_{CB}^{D}qK\}$$

$$+ [\mu\mathbb{E}_0[r_s^L|h = \underline{h}] + (1-\mu)\eta_{\underline{s}|\overline{m}}r_{\underline{s}}^{L}]Q(K+E)$$

$$- [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]r_{CB}^{D}QK,$$

which, using $A_s = R_s^L q$, with $s \in \mathcal{S}$, reads as

$$W_{B}^{LS} = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa q\}(K+E) - \mu\phi[R_{CB}^{D}qK - \Psi q(K+E)],$$

where, with the equilibrium condition (6.18),

$$R^D_{CB}qK - \Psi q(K+E)$$

$$= \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\Psi q}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r}K - \Psi q(K+E)$$

$$= \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q)K - \Psi q[E + (1-\mu)\eta_{\overline{s}|\overline{m}}K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)E/(K+E)}$$

$$= \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) - \Psi q[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + E/K}(K+E).$$

The welfare is then given by

$$W^{LS}_{NB} = \{\mathbb{E}_{\mathbf{m}}[A_s] - (1-\mu)\overline{m}\kappa q - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\epsilon\}(K+E) - (1-\mu)\eta_{\underline{s}|\overline{m}}\nu,$$

where

$$\epsilon = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) - \Psi q[(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K]}{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + E/K}.$$

In the case of bank insolvency, depositors will shift their funds to the central bank iff $\varphi^R < \varphi$, which we further specify at a later stage.

Liquidity risk exists iff $\varphi^L < \varphi$, while solvency risk exists iff $\varphi^S < \varphi$. Using the definition of φ^L and φ^S and equilibrium leverage $\varphi = (K+E)/E$, we can state that bankers face liquidity risk and solvency risk iff

$$\max\left\{\frac{R^D_{CB}}{R^D_{CB} - \Psi}, \frac{R^D_{CB}}{R^D_{CB} - R^L_{\underline{s}}}\right\} < \frac{K+E}{E} \quad \Leftrightarrow \quad \max\left\{\frac{\Psi}{R^D_{CB}}, \frac{R^L_{\underline{s}}}{R^D_{CB}}\right\} < \frac{K}{K+E}.$$

Using the equilibrium conditions $A_s = R^L_s q$, with $s \in \mathcal{S}$, and (6.18), we know that the banker will face liquidity risk and solvency risk iff

$$\frac{\max\{\Psi q, A_{\underline{s}}\}\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}}{(1-\mu)(\eta_{\overline{s}|\overline{m}}A_{\overline{s}} - \overline{m}\kappa q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\Psi q} < \frac{K}{K+E}.$$

The liquidity risk condition is given by

$$\Psi q \frac{\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}}{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\Psi q} < \frac{K}{K+E},$$

which can be rearranged to

$$\Psi q(K+E)\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}$$

$$< K[(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\Psi q].$$

Note that, using $\varphi^r = (K + E/K)$,

$$(K+E)\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}$$

$$= (1-\mu)\eta_{\overline{s}|\overline{m}}(K+E) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)E + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi(K+E)$$

$$= \{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}K + E.$$

Then the latter inequality translates into

$$\Psi q\{(1-\mu)\eta_{\overline{s}|\overline{m}} K + E\} < (1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q)K \quad \Leftrightarrow \quad \Psi q < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.$$

Following the same procedure, the solvency risk condition

$$A_{\underline{s}} \frac{\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}}{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\Psi q} < \frac{K}{K+E},$$

can be rearranged such that

$$A_{\underline{s}}(K+E)\{(1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\}$$

$$< K[(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\Psi q],$$

and finally reads as

$$A_{\underline{s}} + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi(A_{\underline{s}} - \Psi q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K} < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.$$

Thus the banker will face both liquidity risk and solvency risk iff

$$\max\left\{A_{\underline{s}} + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi(A_{\underline{s}} - \Psi q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}, \Psi q\right\} < \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.$$

Finally, bank insolvency will trigger a bank run iff $\varphi^R < \varphi$, which, using equilibrium leverage $\varphi = (K+E)/E$ and the definition of φ^R, translates into

$$\frac{R^D_{CB} - \tilde{\nu}}{R^D_{CB} - \tilde{\nu} - R^L_{\underline{s}}} < \frac{K+E}{E} \quad \Leftrightarrow \quad \frac{R^L_{\underline{s}}}{R^D_{CB} - \tilde{\nu}} < \frac{K}{K+E}.$$

Using equilibrium conditions $A_s = R^L_s q$, with $s \in \mathcal{S}$, and (6.18), we can state $\varphi^R < \varphi$ iff

$$\frac{A_{\underline{s}}\alpha}{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q) + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\Psi q - \nu\alpha/K} < \frac{K}{K+E},$$

where $\alpha := (1-\mu)\eta_{\overline{s}|\overline{m}} + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi + [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu](1-\phi)/\varphi^r$. Rearranging yields

$$(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q)K \leq A_{\underline{s}}\alpha(K+E) + \nu\alpha - [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\Psi q K.$$

Using $\alpha(K+E) = [(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}\phi + \mu\phi]K + E$ as before, the latter inequality translates into

$$(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q)K \leq \left(A_{\underline{s}} + \frac{\nu}{K+E}\right)\{[(1-\mu)\eta_{\overline{s}|\overline{m}} + (1-\mu)\eta_{\underline{s}|\overline{m}}\phi + \mu\phi]K + E\}$$
$$- [(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi\Psi q K.$$

which can be rewritten as

$$\frac{(1-\mu)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K} \leq A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\underline{s}|\overline{m}} + \mu]\phi(A_{\underline{s}} + \nu/(K+E) - \Psi q)}{(1-\mu)\eta_{\overline{s}|\overline{m}} + E/K}.$$

■

Proof of Proposition 4.5.1. First note that if the banker does not face a solvency risk, it is never optimal for the central bank to trigger liquidity risk by setting tight collateral requirements. The latter will only lead to illiquidity following a CBDC-induced bank run, while it does not have any positive effect. In particular, monitoring incentives for liquid bankers remain identical to the situation with loose collateral requirements. We denote the monitoring decision of the banker without solvency risk and without liquidity risk by $\mathbf{m} = (\underline{m}, \overline{m})$. From proposition 4.4.1 we know that $\underline{m} = \overline{m}$. Similarly, the monitoring decision without solvency risk, but with liquidity risk due to tight collateral requirements, is denoted by $\mathbf{m}_L = (\underline{m}_L, \overline{m}_L)$. Illiquid bankers will not monitor, such that $\underline{m}_L = 0$. From proposition 4.4.2 we know that liquid bankers have the same monitoring incentives as in the situation with loose collateral requirements, such that $\overline{m}_L = \overline{m}$. Without solvency risk, the change in welfare induced by tight collateral requirements is given by

$$W - W^L = (\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa q)(K + E) - [\mathbb{E}_{\mathbf{m}_L}[A_s] - (1 - \mu)\overline{m}_L \kappa q - \mu\phi\epsilon](K + E),$$

where according to proposition 4.4.2

$$\epsilon = \frac{(1 - \mu)(\mathbb{E}_{\overline{m}_L}[A_s|h = \overline{h}] - \overline{m}_L \kappa q) - \Psi q(1 - \mu + E/K)}{1 - \mu + \mu\phi + E/K}.$$

Using $\overline{m} = \overline{m}_L$, it follows that

$$W - W^L = \mu(\mathbb{E}_{\overline{m}}[A_s|h = \underline{h}] - \mathbb{E}_0[A_s|h = \underline{h}] - \overline{m}\kappa q + \phi\epsilon)(K + E) \geq 0.$$

Accordingly, tight collateral requirements, i.e., $\phi > 0$ and $(1 + r^D_{CB})K = R^D_{CB}K > \Psi(K + E) = (1 + \psi)(K + E)$, exposing the banker to liquidity risk and illiquidity penalties, are never optimal if there is no solvency risk.

In what follows we focus on the situation where the banker faces a solvency risk. Using the existence conditions provided in proposition 4.4.3, bankers will face a solvency risk iff

$$A_{\underline{s}} < \frac{\eta_{\overline{s}|\overline{m}_S} A_{\overline{s}} - \overline{m}_S \kappa q}{\eta_{\overline{s}|\overline{m}_S} + E/K},$$

where the banker's monitoring decision in the presence of solvency risk and in the absence of liquidity risk is denoted by $\mathbf{m}_S = (\underline{m}_S, \overline{m}_S)$. From proposition 4.4.3 we know that $\underline{m}_S = \overline{m}_S$. Clearly, if bankers monitor without being exposed to liquidity risk and the ensuing illiquidity penalties, i.e., $\overline{m}_S = 1$, tight collateral requirements are never optimal, i.e., they induce a welfare loss as bankers face penalties for illiquidity and bankers that become illiquid do not monitor. Thus, tight collateral requirements can only induce a welfare gain if bankers shirk without liquidity risk. According to proposition 4.4.3, this translates into the condition $\triangle A_{\overline{s}} < \kappa q(1+\eta_{\overline{s}|0}K/E)$. We denote the monitoring decision of the banker in the presence of solvency risk and liquidity risk by $\mathbf{m}_{LS} = (\underline{m}_{LS}, \overline{m}_{LS})$. Illiquid bankers do not monitor, so $\underline{m}_{LS} = 0$. According to proposition 4.4.5, in the presence of both solvency risk and liquidity risk liquid bankers will only monitor, i.e., $\overline{m}_{LS} = 1$, if there exists $\phi \in (0,1)$ and $\psi \geq 0$ such that

$$\triangle A_{\overline{s}} - \triangle\phi\Psi q\frac{1+E/K}{\phi+E/K} \geq \kappa q\left[1+\frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi+E/K}\right].$$

Note that, based on the existence conditions provided in proposition 4.4.5, the illiquidity penalty parameter ϕ must be smaller than one. In what follows, we assume that the latter inequality holds, i.e., $\overline{m}_{LS} = 1$. Based on proposition 4.4.5, an equilibrium with solvency risk and liquidity risk and without bail-ins in the case of bank insolvency will then exist iff

$$\underline{\chi}(\phi,\psi) < \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}}-\kappa q)}{(1-\mu)\eta_{\overline{s}|1}+E/K}$$

where

$$\underline{\chi}(\phi,\psi) := \max\left\{A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\underline{s}|1}+\mu]\phi(A_{\underline{s}}+\nu/(K+E)-\Psi q)}{(1-\mu)\eta_{\overline{s}|1}+E/K}, \Psi q\right\}.$$

Note that the latter inequality is sufficient for the previously introduced existence condition of an equilibrium with solvency risk only where bankers do not monitor, i.e., $\overline{m}_S = 0$. Using $\overline{m}_{LS} = 1$ and $\overline{m}_S = 0$, in the presence of a solvency risk the welfare change induced by tight collateral requirements is given by

$$\begin{aligned} W^{LS}_{NB} - W^S = {} & \{\mathbb{E}_{\mathbf{m}_{LS}}[A_s] - (1-\mu)\overline{m}_{LS}\kappa q - [(1-\mu)\eta_{\underline{s}|\overline{m}_{LS}}+\mu]\phi\}(K+E) \\ & - (1-\mu)\eta_{\underline{s}|\overline{m}_{LS}}\nu - (\mathbb{E}_{\mathbf{m}_S}[A_s] - \overline{m}_S\kappa q)(K+E) + (1-\mu)\eta_{\underline{s}|\overline{m}_S}\nu\mathbb{1}\{\nu<\nu^*\}, \end{aligned}$$

where, based on proposition 4.4.5,

$$\epsilon = \frac{(1-\mu)(\eta_{\overline{s}|\overline{m}_{LS}}A_{\overline{s}}-\kappa q) - \Psi q[(1-\mu)\eta_{\overline{s}|\overline{m}_{LS}}+E/K]}{(1-\mu)\eta_{\overline{s}|\overline{m}_{LS}} + [(1-\mu)\eta_{\underline{s}|\overline{m}_{LS}}+\mu]\phi + E/K}$$

and, following proposition 4.4.3,

$$\nu^* := \left(\frac{\eta_{\overline{s}|\overline{m}_S} A_{\overline{s}}}{\eta_{\overline{s}|\overline{m}_S} + E/K} - A_{\underline{s}} \right) (K + E).$$

Setting $\mathbf{m}_S = (0,0)$ and $\mathbf{m}_{LS} = (0,1)$, it follows that

$$W_{NB}^{LS} - W^S = \{\mathbb{E}_{(0,1)}[A_s] - (1-\mu)\kappa q - [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi\}(K+E)$$
$$- (1-\mu)\eta_{\underline{s}|1}\nu - \mathbb{E}_{(0,0)}[A_s](K+E) + (1-\mu)\eta_{\underline{s}|0}\nu \mathbb{1}\{\nu < \nu^*\},$$

where

$$\epsilon = \frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa q) - \Psi q[(1-\mu)\eta_{\overline{s}|1} + E/K]}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K}$$

and

$$\nu^* := \left(\frac{\eta_{\overline{s}|0} A_{\overline{s}}}{\eta_{\overline{s}|0} + E/K} - A_{\underline{s}} \right) (K + E).$$

Note that

$$\mathbb{E}_{(0,1)}[A_s] = \mu[\eta_{\underline{s}|0} A_{\underline{s}} + \eta_{\overline{s}|0} A_{\overline{s}}] + (1-\mu)[\eta_{\underline{s}|1} A_{\underline{s}} + \eta_{\overline{s}|1} A_{\overline{s}}]$$

and

$$\mathbb{E}_{(0,0)}[A_s] = \eta_{\underline{s}|0} A_{\underline{s}} + \eta_{\overline{s}|0} A_{\overline{s}}.$$

Then the welfare change, induced by tight collateral requirements, is given by

$$W_{NB}^{LS} - W^S = (1-\mu)[(\eta_{\overline{s}|1} - \eta_{\overline{s}|0}) A_{\overline{s}} + (\eta_{\underline{s}|1} - \eta_{\underline{s}|0}) A_{\underline{s}} - \kappa q](K+E)$$
$$- [(1-\mu) + \mu]\phi\epsilon(K+E) - (1-\mu)(\eta_{\underline{s}|1} - \eta_{\underline{s}|0} \mathbb{1}\{\nu < \nu^*\})\nu,$$

which, using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, further simplifies to

$$W_{NB}^{LS} - W^S = \{(1-\mu)[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa q] - [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi\epsilon\}(K+E) \\ - (1-\mu)(\eta_{\underline{s}|1} - \eta_{\underline{s}|0}\mathbb{1}\{\nu < \nu^*\})\nu.$$

Tight collateral requirements are welfare-improving if $W_{NB}^{LS} - W^S > 0$ or equivalently

$$(1-\mu)[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa q - (\eta_{\underline{s}|1} - \eta_{\underline{s}|0}\mathbb{1}\{\nu < \nu^*\})\nu/(K+E)] > [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi\epsilon.$$

Using the structure of ϵ, the latter inequality translates into

$$\{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K\} \\ \times \{(1-\mu)[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa q - (\eta_{\underline{s}|1} - \eta_{\underline{s}|0}\mathbb{1}\{\nu < \nu^*\})\nu/(K+E)]\} \\ > [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi\{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q) - \Psi q[(1-\mu)\eta_{\overline{s}|1} + E/K]\},$$

which can be further rearranged as

$$\left\{1 + \frac{(1-\mu)\eta_{\overline{s}|1} + E/K}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi}\right\}\{(1-\mu)[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa q - (\eta_{\underline{s}|1} - \eta_{\underline{s}|0}\mathbb{1}\{\nu < \nu^*\})\nu/(K+E)]\} \\ > (1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q) - \Psi q[(1-\mu)\eta_{\overline{s}|1} + E/K].$$

and finally reads as

$$\{[(1-\mu)\eta_{\overline{s}|1} + E/K]^{-1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]^{-1}\phi^{-1}\} \\ \times \{(1-\mu)[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa q - (\eta_{\underline{s}|1} - \eta_{\underline{s}|0}\mathbb{1}\{\nu < \nu^*\})\nu/(K+E)]\} \\ > \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} - \Psi q.$$

Using the definition

$$\overline{\chi}(\phi,\psi) := \Psi q + \{(1-\mu)[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa q - (\eta_{\underline{s}|1} - \eta_{\underline{s}|0}\mathbb{1}\{\nu < \nu^*\})\nu/(K+E)]\}$$
$$\times \{[(1-\mu)\eta_{\overline{s}|1} + E/K]^{-1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]^{-1}\phi^{-1}\},$$

we can state that tight collateral requirements will induce a welfare gain if

$$\frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} < \overline{\chi}(\phi,\psi).$$

■

Proof of Lemma 4.5.1. If tight collateral requirements are optimal, i.e., the conditions stated in proposition 4.5.1 apply, the optimization problem of the central bank is given by

$$\max_{r^D_{CB}>0,\Psi\geq 0,\phi>0} W^{LS}_{NB} - W^S$$

$$\text{subject to} \qquad \underline{\chi}(\phi,\psi) < \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} \tag{6.19}$$

$$\kappa q\left[1 + \frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi + E/K}\right] \leq \triangle A_{\overline{s}} - \triangle\phi\Psi q\frac{1+E/K}{\phi+E/K}. \tag{6.20}$$

Note that

$$W^{LS}_{NB} - W^S = \{(1-\mu)[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa q] - [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi\epsilon(\phi,\psi)\}(K+E),$$

where

$$\epsilon(\phi,\psi) = \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q) - \Psi q[(1-\mu)\eta_{\overline{s}|1} + E/K]}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K}.$$

Hence the optimization problem of the central bank can be rewritten as

$$\min_{r^D_{CB}>0,\Psi\geq 0,\phi>0} \phi\epsilon(\phi,\psi) \quad \text{subject to} \quad (6.19) \quad \text{and} \quad (6.20).$$

Next we analyze the constraint (6.19), which reads as

$$\max\left\{A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi(A_{\underline{s}} + \nu/(K+E) - \Psi q)}{(1-\mu)\eta_{\overline{s}|1} + E/K}, \Psi q\right\}$$
$$< \frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa q)}{(1-\mu)\eta_{\overline{s}|1} + E/K}.$$

Clearly, if $A_{\underline{s}} + \nu/(K+E) \leq \Psi q$, the condition translates into

$$\Psi q < \frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa q)}{(1-\mu)\eta_{\overline{s}|1} + E/K}.$$

For $\Psi q < A_{\underline{s}} + \nu/(K+E)$, the condition reads as

$$A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi(A_{\underline{s}} + \nu/(K+E) - \Psi q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} < \frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa q)}{(1-\mu)\eta_{\overline{s}|1} + E/K},$$

which can be rearranged to

$$[(1-\mu)\eta_{\overline{s}|1} + E/K][A_{\underline{s}} + \nu/(K+E)] + \{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi(A_{\underline{s}} + \nu/(K+E) - \Psi q)\}$$
$$< (1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa q).$$

Further rearranging yields

$$A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\overline{s}|1} + E/K][A_{\underline{s}} + \nu/(K+E)] - (1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa q)}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi} < \Psi q.$$

Thus the real haircut must satisfy

$$A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\overline{s}|1} + E/K][A_{\underline{s}} + \nu/(K+E)] - (1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa q)}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi}$$
$$< \Psi Q < \frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa q)}{(1-\mu)\eta_{\overline{s}|1} + E/K}. \tag{6.21}$$

Now we focus on constraint (6.20), which can be rewritten as

$$\Psi q \leq \frac{\phi + E/K}{1 + E/K}\left[A_{\overline{s}} - \frac{\kappa q}{\triangle}\left(1 + \frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi + E/K}\right)\right]. \tag{6.22}$$

Note that it holds that $\Psi q \geq 0$, as the central bank chooses a haircut $\psi \geq -1$. From proposition 4.5.1 we know that, if tight collateral requirements are optimal, there exists a $\tilde{\phi} \in (0,1)$ such that the right-hand side of the latter inequality is zero. Moreover, the right-hand side of the latter inequality is increasing with ϕ. Furthermore, note that $\phi\epsilon(\phi,\psi)$ is decreasing with Ψq. Hence the central bank will choose the highest possible real haircut satisfying (6.21) and (6.22). Thus we define the lower bounds

$$\underline{\gamma}_1(\phi) := A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu)\eta_{\overline{s}|1} + E/K][A_{\underline{s}} + \nu/(K+E)] - (1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q)}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi},$$

and $\underline{\gamma}_2 := 0$, and the upper bounds

$$\overline{\gamma}_1 := \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} \text{ and } \overline{\gamma}_2(\phi) := \frac{\phi + E/K}{1 + E/K}\left[\frac{A_{\overline{s}}}{\phi} - \frac{\kappa q}{\triangle\phi}\left(1 + \frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi + E/K}\right)\right].$$

Then we can rewrite the central bank's optimization problem as

$$\min_{\phi\in(0,1)} \phi\tilde{\epsilon}(\phi) \quad \text{subject to} \quad \max\{\underline{\gamma}_1(\phi), \underline{\gamma}_2\} \leq \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\},$$

where

$$\tilde{\epsilon}(\phi) = \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q) - \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\}[(1-\mu)\eta_{\overline{s}|1} + E/K]}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K}.$$

The central bank rate $r^D_{CB} > 0$ and the haircut $\psi \geq 0$ must then be chosen such that $(1 + r^D_{CB})K = R^D_{CB}K > \Psi(K+E) = (1+\psi)(K+E) \geq 0$ and $\Psi q = \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\}$. ■

Proof of Corollary 4.5.1. First note that with a high bank leverage, i.e., $E/K \to 0$, we know from proposition 4.4.3 that without tight collateral requirements, and thus without liquidity risk and illiquidity penalties, bankers will shirk as $\triangle A_{\overline{s}} < \lim_{E/K\to 0} \kappa q(1 + \eta_{\overline{s}|0}K/E) = +\infty$. Second, note that if $\triangle A_{\overline{s}} > \kappa q$, we know there exists a $\tilde{\phi} \in (0,1)$ and

$\tilde{\Psi} = 0$, such that

$$\triangle A_{\overline{s}} = \kappa q \left[1 + \frac{(1-\tilde{\phi})(1-\mu)\eta_{\overline{s}|0}}{\tilde{\phi} + E/K} \right].$$

From proposition 4.5.1, we know that, with $A_{\underline{s}} = \nu = 0$, it follows $\underline{\chi}(\phi, \psi) = \Psi q$ and

$$\overline{\chi}(\phi, \psi) = \Psi q + (1-\mu)[\triangle A_{\overline{s}} - \kappa q] \left\{ \frac{1}{(1-\mu)\eta_{\overline{s}|1} + E/K} + \frac{1}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi} \right\}.$$

Note that for $\mu, \eta_{\underline{s}|1} \to 0$, the monetary policy $\tilde{\phi} \in (0,1)$ and $\tilde{\Psi} = 0$, which, as shown before, incentivizes bankers to monitor, yields $\underline{\chi}(\tilde{\phi}, \tilde{\psi}) = 0$ and $\overline{\chi}(\tilde{\phi}, \tilde{\psi}) = +\infty$. Thus, banking with liquidity risk and solvency risk is viable, and tight collateral requirements are welfare-improving. Hence, for a sufficiently small risk exposure of bankers, i.e., small μ and $\eta_{\underline{s}|1}$, it is optimal for the central bank to apply tight collateral requirements exposing bankers to liquidity risk and illiquidity penalties. Specifically, we require that

$$\frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} < \Psi q + \frac{(1-\mu)(\triangle A_{\overline{s}} - \kappa q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} + \frac{(1-\mu)(\triangle A_{\overline{s}} - \kappa q)}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\hat{\phi}} \tag{6.23}$$

where $\hat{\phi}$ represents the optimal illiquidity penalty parameter.

With lemma 4.5.1 we now deduce that the optimal monetary policy $\hat{r}^D_{CB} > 0$, $\hat{\psi} \geq 0$ and $\hat{\phi} > 0$ satisfies

$$\hat{\phi} \in \arg \min_{\phi \in [\tilde{\phi}, 1)} \phi \tilde{\epsilon}(\phi) \quad \text{subject to} \quad \max\{\underline{\gamma}_1, \underline{\gamma}_2(\phi)\} \leq \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\},$$

where, using $A_{\underline{s}} = \nu = 0$, it follows that

$$\tilde{\epsilon}(\phi) = \frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa q) - \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\}[(1-\mu)\eta_{\overline{s}|1} + E/K]}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K},$$

$$\underline{\gamma}_1(\phi) = -\frac{(1-\mu)(\eta_{\overline{s}|1} A_{\overline{s}} - \kappa Q)}{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi} < 0, \quad \underline{\gamma}_2 = 0,$$

$$\overline{\gamma}_1 = \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q)}{(1-\mu)\eta_{\overline{s}|1} + E/K} \text{ and } \overline{\gamma}_2(\phi) = \frac{\phi + E/K}{1 + E/K}\left[A_{\overline{s}} - \frac{\kappa q}{\triangle}\left(1 + \frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi}\right)\right].$$

Note that in the limit the constraint $\max\{\underline{\gamma}_1, \underline{\gamma}_2(\phi)\} \leq \min\{\overline{\gamma}_1, \overline{\gamma}_2(\phi)\}$ is always satisfied, so the central bank faces an unconstrained optimization problem. If μ is sufficiently small, such that $\mu\eta_{\overline{s}|1} < \eta_{\overline{s}|0}$, we can deduce that $\overline{\gamma}_1 > \overline{\gamma}_2(\phi)$ for all $\phi \in (0,1)$. Hence the optimization problem reads as

$$\hat{\phi} \in \arg\min_{\phi\in[\tilde{\phi},1)} \phi\tilde{\epsilon}(\phi),$$

where, using $\overline{\gamma}_1 > \overline{\gamma}_2(\phi)$, it follows that

$$\tilde{\epsilon}(\phi) = \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q) - \overline{\gamma}_2(\phi)[(1-\mu)\eta_{\overline{s}|1} + E/K]}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K}.$$

Taking the derivative of $\phi\tilde{\epsilon}(\phi)$ with respect to ϕ yields

$$\begin{aligned}
\frac{\partial\phi\tilde{\epsilon}(\phi)}{\partial\phi} &= \tilde{\epsilon}(\phi) + \frac{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q) - \left[\overline{\gamma}_2(\phi) + \phi\frac{\partial\overline{\gamma}_2(\phi)}{\partial\phi}\right][(1-\mu)\eta_{\overline{s}|1} + E/K]}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K} \\
&\quad - \frac{\{(1-\mu)(\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q) - \overline{\gamma}_2(\phi)[(1-\mu)\eta_{\overline{s}|1} + E/K]\}[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi}{\{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K\}^2} \\
&= \tilde{\epsilon}(\phi)\left(2 - \frac{[(1-\mu)\eta_{\underline{s}|1} + \mu]\phi}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K}\right) \\
&\quad - \frac{(1-\mu)\eta_{\overline{s}|1}\phi\frac{\partial\overline{\gamma}_2(\phi)}{\partial\phi}}{(1-\mu)\eta_{\overline{s}|1} + [(1-\mu)\eta_{\underline{s}|1} + \mu]\phi + E/K}.
\end{aligned}$$

Note that the derivative of $\overline{\gamma}_2(\phi)$ with respect to ϕ is given by

$$\frac{\partial\overline{\gamma}_2(\phi)}{\partial\phi} = \frac{\phi(1+E/K)-(\phi+E/K)(1+E/K)}{\phi^2(1+E/K)^2}\left[A_{\overline{s}} - \frac{\kappa q}{\triangle}\left(1+\frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi}\right)\right]$$

$$+\frac{\phi+E/K}{1+E/K}\left[\frac{\kappa q}{\triangle}\frac{(-\phi)(1-\mu)\eta_{\overline{s}|0}-(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi^2}\right]$$

$$= -\frac{E/K}{\phi^2(1+E/K)}\left[A_{\overline{s}} - \frac{\kappa Q}{\triangle}\left(1+\frac{(1-\phi)(1-\mu)\eta_{\overline{s}|0}}{\phi}\right)\right]$$

$$-\frac{\phi+E/K}{1+E/K}\left[\frac{\kappa q}{\triangle}\frac{(1-\mu)\eta_{\overline{s}|0}}{\phi^2}\right].$$

Note that $\overline{\gamma}_2(\phi) \geq 0$ for all $\phi \geq \tilde{\phi}$. Thus for all $\phi \geq \tilde{\phi}$ we know that the first derivative of $\overline{\gamma}_2(\phi)$ with respect to ϕ is negative and $\tilde{\epsilon}(\phi) > 0$ for all $\phi \in (0,1)$, so we can conclude that the derivative of $\phi\tilde{\epsilon}(\phi)$ is positive and hence the optimal monetary policy is characterized by $\hat{\phi} = \tilde{\phi}$, $\hat{\Psi} = \tilde{\Psi} = 0$ and $\hat{r}^D_{CB} > 0$. Note that $\tilde{\phi}$ is determined by the equation

$$\triangle A_{\overline{s}} = \kappa q\left[1+\frac{(1-\tilde{\phi})(1-\mu)\eta_{\overline{s}|0}}{\tilde{\phi}+E/K}\right],$$

which can be rearranged to

$$\tilde{\phi}[\triangle A_{\overline{s}} - \kappa q + \kappa q(1-\mu)\eta_{\overline{s}|0}] = \kappa q(1-\mu)\eta_{\overline{s}|0} - (\triangle A_{\overline{s}} - \kappa q)E/K,$$

which finally reads as

$$\tilde{\phi} = \frac{\kappa q(1-\mu)\eta_{\overline{s}|0} - (\triangle A_{\overline{s}} - \kappa q)E/K}{\triangle A_{\overline{s}} - \kappa q[(1-\mu)\eta_{\underline{s}|0}+\mu]}.$$

Given optimal monetary policy, constraint (6.23) translates into

$$\frac{(1-\mu)\eta_{\overline{s}|0}A_{\overline{s}}}{(1-\mu)\eta_{\overline{s}|1}+E/K} < \frac{(1-\mu)(\triangle A_{\overline{s}} - \kappa q)}{[(1-\mu)\eta_{\underline{s}|1}+\mu]\hat{\phi}}.$$

■

Proof of Proposition 4.5.2. Focusing on competitive equilibria without a liquidity risk for bankers, note that the first-best utilitarian welfare is achieved if households do not incur switching costs and bankers monitor if the welfare gain through the induced productivity increase offsets the bankers' utility losses due to monitoring. In any competitive equilibrium without a solvency risk for bankers, households do not incur switching costs as they are not exposed to bank insolvency. In addition, bankers' monitoring decision is given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa q\}$ (see propositon 4.4.1) and thus is welfare-maximizing because bankers monitor if the welfare gain due to the productivity increase induced by monitoring $\triangle(A_{\overline{s}} - A_{\underline{s}})(K + E)$ offsets the utility losses for bankers due to monitoring $\kappa q(K + E)$. Thus any competitive equilibrium without a risk for bankers yields the first-best welfare. From proposition 4.4.3 we know that the first-best welfare is also achieved in any competitive equilibrium with a solvency risk for bankers if the following two conditions are met: First, households do not convert their deposits into CBDC in the case of a bank insolvency and thus do not incur switching costs. In other words, households accept a bail-in, which occurs if switching costs are sufficiently high, i.e., $\nu \geq \nu^*$, with ν^* provided in proposition 4.4.3. Second, bankers' monitoring decision must be welfare maximizing, i.e., bankers should monitor only if the welfare gain due to the productivity increase, $\triangle(A_{\overline{s}} - A_{\underline{s}})(K + E)$, induced by monitoring offsets bankers' utility losses due to monitoring, $\kappa q(K+E)$. With proposition 4.4.3 we know that it must hold $\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa q$ if and only if $\triangle A_{\overline{s}} \geq \kappa q(1 + \eta_{\overline{s}|0} K/E)$. ■

Proof of Proposition 4.5.3. As the social planner can reallocate the capital good among households and bankers, proposition 4.5.3 follows directly from proposition 4.5.2 stating that any competitive equilibrium without solvency risk yields the first-best welfare, as households do not incur switching costs and bankers' monitoring decision is welfare-maximizing. ■

Proof of Proposition 4.5.4. Note that with a solvency risk faced by bankers, the constrained social planner can only align the bankers' monitoring incentives with the objective of maximizing utilitarian welfare but not eliminate solvency risk for bankers and thus not the switching costs faced by households in the case of bank insolvency. It is then the aim of the constrained social planner through the use of taxes and transfers depending on the idiosyncratic productivity shock for the financed firm to align bankers' monitoring decision with the objective of maximizing welfare. With contingent taxes and transfers τ_s, the banker's optimization problem in real terms is given by

$$\max_{\substack{\varphi\in[1,\varphi^r],\\ m(h)\in\{0,1\}}} \mathbb{E}_{\mathbf{m}}[\zeta_{\mathbf{z}} R_{\mathbf{z}}^{E,+} - R_{\mathbf{z}}^{E,-} - m(h)\kappa\varphi + \tau_s\varphi]qE.$$

Consider the situation where the banker faces a solvency risk but no liquidity risk, i.e., $\varphi^S < \varphi \leq \varphi^L$. The banker will monitor, given the type of matched household $h \in \mathcal{H}$, iff

$$\eta_{\overline{s}|1}[(R_{\overline{s}}^L - R_{CB}^D)\varphi + R_{CB}^D]qE + \mathbb{E}_1[\tau_s|h]\varphi qE$$
$$\geq \eta_{\overline{s}|0}[(R_{\overline{s}}^L - R_{CB}^D)\varphi + R_{CB}^D]qE + \mathbb{E}_0[\tau_s|h]\varphi qE + \kappa\varphi qE,$$

which, using $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, can be rewritten as $\triangle[R_{\overline{s}}^L - R_{CB}^D(\varphi-1)/\varphi] \geq \kappa - \triangle\tau_{\overline{s}} + \triangle\tau_{\underline{s}}$. Using the fact that the banker's monitoring decision does not depend on the type of matched household, i.e., $\underline{m} = \overline{m}$, we know that the banker's expected utility from conducting banking operations is given by $\{\eta_{\overline{s}|\overline{m}}[(R_{\overline{s}}^L - R_{CB}^D)\varphi + R_{CB}^D] - \overline{m}\kappa\varphi + \mathbb{E}_{\mathbf{m}}[\tau_s]\varphi\}qE$. Due to competitive markets, the utility expected from conducting banking operations must equal the utility from holding CBDC, i.e., $R_{CB}^D QE$. Thus, with $\underline{m} = \overline{m}$ we can deduce that the banker will choose $\varphi^S < \varphi \leq \varphi^L$ with $\varphi < \varphi^r$ if

$$R_{\overline{s}}^L = R_{CB}^D\left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi}\right) + \frac{\overline{m}\kappa - \mathbb{E}_{\mathbf{m}}[\tau_s]}{\eta_{\overline{s}|\overline{m}}}$$

and there is no incentive to adjust the supply of loans, i.e., $\eta_{\overline{s}|\overline{m}}(R_{\overline{s}}^L - R_{CB}^D) - \overline{m}\kappa + \mathbb{E}_{\mathbf{m}}[\tau_s] = 0$, which, however, contradicts the former equation. Hence the banker will only choose a leverage $\varphi^S < \varphi \leq \varphi^L$ if $\varphi = \varphi^r$,

$$R_{\overline{s}}^L = R_{CB}^D\left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}}\frac{1}{\varphi^r}\right) + \frac{\overline{m}\kappa - \mathbb{E}_{\mathbf{m}}[\tau_s]}{\eta_{\overline{s}|\overline{m}}}$$

and $\eta_{\overline{s}|\overline{m}}(R_{\overline{s}}^L - R_{CB}^D) - \overline{m}\kappa + \mathbb{E}_{\mathbf{m}}[\tau_s] > 0$, which follows directly from the previous equation.

Using equilibrium leverage $\varphi = (K+E)/E$, we know that such an equilibrium with solvency risk but without liquidity risk only exists if $\varphi^r = (K+E)/E$. Furthermore, the banker's monitoring decision is given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle[R_{\overline{s}}^L - R_{CB}^D(\varphi^r - 1)/\varphi^r] \geq \kappa - \triangle\tau_{\overline{s}} + \triangle\tau_{\underline{s}}\}$. As banks are only defaulting due to insolvency, i.e., when the financed firm incurs a negative productivity shock, the central bank's losses in real terms are given by

$$\pi^{CB} = \eta_{\underline{s}|\overline{m}}[R_{\underline{s}}^L L^b - R_{CB}^D(L^b - E^b)]/P = \eta_{\underline{s}|\overline{m}}[R_{\underline{s}}^L q(K+E) - R_{CB}^D qK],$$

where we have used the banker's equity financing $E^b = QE$, the equilibrium loan supply $L^b = Q(K+E)$, and the fact that the banker's monitoring decision is independent of the type of household, i.e., $\underline{m} = \overline{m}$. Moreover, in equilibrium, the demand for capital good is finite, such that, with lemma 4.3.2, we can deduce $A_s \leq (1 + r_s^L)q = R_s^L q$, with $s \in \mathcal{S}$. In addition, due to rational expectations of firms and bankers, it must hold $A_s = R^L sq$ for all $s \in \mathcal{S}$. Hence, firms make zero profits, i.e., $\pi^f = 0$. With contingent taxes and transfers implemented by the social planner and a balanced budget for the central bank, it holds that $\pi^{CB} - \mathbb{E}_{\mathbf{m}}[\tau_s]\varphi qE = \tau$.

With zero firm profits, the expected consumption by the banker and the household is given by

$$C^b = \eta_{\overline{s}|\overline{m}}[(R_{\overline{s}}^L - R_{CB}^D)\varphi + R_{CB}^D]qE \quad \text{and} \quad C^h = R_{CB}^D qK + \tau^h,$$

with $h \in \mathcal{H}$, respectively. The banker's monitoring decision is given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle[A_{\overline{s}} - R_{CB}^D qK/(K+E)] \geq (\kappa - \triangle\tau_{\overline{s}} + \triangle\tau_{\underline{s}})q\}$. To fully characterize the banker's monitoring decision, we derive in the following the real central bank interest factor R_{CB}^D prevailing in equilibrium. First note that, using equilibrium condition $A_s = R_s^L q$, with $s \in \mathcal{S}$, (6.13) can be rewritten as

$$A_{\overline{s}} = R_{CB}^D q \left(1 + \frac{\eta_{\underline{s}|\overline{m}}}{\eta_{\overline{s}|\overline{m}}} \frac{1}{\varphi^r}\right) + \frac{\overline{m}\kappa q - \mathbb{E}_{\mathbf{m}}[\tau_s]q}{\eta_{\overline{s}|\overline{m}}}.$$

Rearranging yields

$$\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q + \mathbb{E}_{\mathbf{m}}[\tau_s]q = R_{CB}^D Q(\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r),$$

such that we can finally deduce that, in equilibrium, the real central bank rate satisfies

$$R_{CB}^D q = \frac{\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q + \mathbb{E}_{\mathbf{m}}[\tau_s]q}{\eta_{\overline{s}|\overline{m}} + \eta_{\underline{s}|\overline{m}}/\varphi^r}.$$

We can then state that the banker will monitor, independently of the type of matched household iff

$$\triangle\left[A_{\overline{s}}-\frac{\eta_{\overline{s}|\overline{m}}A_{\overline{s}}-\overline{m}\kappa q+\mathbb{E}_{\mathbf{m}}[\tau_s]q}{\eta_{\overline{s}|\overline{m}}+\eta_{\underline{s}|\overline{m}}/\varphi^r}\frac{K}{K+E}\right]\geq(\kappa-\triangle\tau_{\overline{s}}+\triangle\tau_{\underline{s}})q.$$

Rearranging yields

$$\triangle\left[A_{\overline{s}}(\eta_{\overline{s}|\overline{m}}+\eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E)-(\eta_{\overline{s}|\overline{m}}A_{\overline{s}}-\overline{m}\kappa q+\mathbb{E}_{\mathbf{m}}[\tau_s]q)K\right]$$
$$\geq(\kappa-\triangle\tau_{\overline{s}}+\triangle\tau_{\underline{s}})Q(\eta_{\overline{s}|\overline{m}}+\eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E).$$

Note that, using $\varphi^r=(K+E)/K$,

$$(\eta_{\overline{s}|\overline{m}}+\eta_{\underline{s}|\overline{m}}/\varphi^r)(K+E)=\eta_{\overline{s}|\overline{m}}(K+E)+\eta_{\underline{s}|\overline{m}}E=E+\eta_{\overline{s}|\overline{m}}K.$$

Thus, the latter inequality reads as

$$\triangle\left[A_{\overline{s}}(E+\eta_{\overline{s}|\overline{m}}K)-(\eta_{\overline{s}|\overline{m}}A_{\overline{s}}-\overline{m}\kappa q+\mathbb{E}_{\mathbf{m}}[\tau_s]q)K\right]\geq(\kappa-\triangle\tau_{\overline{s}}+\triangle\tau_{\underline{s}})q(E+\eta_{\overline{s}|\overline{m}}K)$$

$$\Leftrightarrow\quad\triangle A_{\overline{s}}E\geq\kappa q(E+\eta_{\overline{s}|\overline{m}}K-\triangle\overline{m}K)+\triangle\mathbb{E}_{\mathbf{m}}[\tau_s]qK-\triangle(\tau_{\overline{s}}-\tau_{\underline{s}})q(E+\eta_{\overline{s}|\overline{m}}K).$$

Exploiting $\triangle:=\eta_{\overline{s}|1}-\eta_{\overline{s}|0}$ and setting $\overline{m}=1$, as the condition, if satisfied, implies monitoring, we know that the banker will monitor iff

$$\triangle A_{\overline{s}}E\geq\kappa q[E+\eta_{\overline{s}|1}K-(\eta_{\overline{s}|1}-\eta_{\overline{s}|0})K]+\triangle(\eta_{\overline{s}|1}\tau_{\overline{s}}+\eta_{\underline{s}|1}\tau_{\underline{s}})qK$$
$$-\triangle(\tau_{\overline{s}}-\tau_{\underline{s}})q(E+\eta_{\overline{s}|1}K)$$

$$\Leftrightarrow\quad\triangle A_{\overline{s}}\geq\kappa q(1+\eta_{\overline{s}|0}K/E)-\triangle\tau_{\overline{s}}q+\triangle\tau_{\underline{s}}q(1+K/E).$$

Without loss of generality, we can set $\tau_{\underline{s}}=0$ as it is irrelevant whether the constrained social planner imposes taxes on the non-monitoring bankers, distributes transfer to the monitoring bankers or both. Thus the constrained social planner must choose $\tau_{\overline{s}}$ such that $\triangle A_{\overline{s}}\geq\kappa q(1+\eta_{\overline{s}|0}K/E)-\triangle\tau_{\overline{s}}q$ if and only if $\triangle(A_{\overline{s}}-A_{\underline{s}})\geq\kappa q$. In what follows, we show that it always holds $\triangle(A_{\underline{s}}-A_{\underline{s}})\geq\kappa q$ if $\triangle A_{\overline{s}}\geq\kappa q(1+\eta_{\overline{s}|0}K/E)$. Thus, whenever the banker monitors in the presence of solvency risk, monitoring is also welfare-maximizing. In other words, the constrained social planner must never apply taxes to prevent the banker from monitoring because it would be not welfare-maximizing. Suppose the banker faces solvency risk and is monitoring without any taxes or transfers applied by the constrained social planner. Then we know from proposition 4.4.3 that it holds

$$\triangle A_{\overline{s}} \geq \kappa q(1+\eta_{\overline{s}|0}K/E) \quad \text{and} \quad A_{\underline{s}} < \frac{\eta_{\overline{s}|1}A_{\overline{s}} - \kappa q}{\eta_{\overline{s}|1} + E/K}.$$

The latter inequality can be rearranged to

$$\begin{aligned} & \kappa q < \eta_{\overline{s}|1}A_{\overline{s}} - (\eta_{\overline{s}|1} + E/K)A_{\overline{s}} \\ \Leftrightarrow \quad & \kappa q < \eta_{\overline{s}|1}(A_{\overline{s}} - A_{\underline{s}}) - A_{\underline{s}}E/K \\ \Leftrightarrow \quad & \kappa q < (\eta_{\overline{s}|1} - \eta_{\overline{s}|0})(A_{\overline{s}} - A_{\underline{s}}) - A_{\underline{s}}E/K + \eta_{\overline{s}|0}(A_{\overline{s}} - A_{\underline{s}}) \end{aligned}$$

and finally, with the notation $\triangle := \eta_{\overline{s}|1} - \eta_{\overline{s}|0}$, reads as

$$\kappa q < \triangle(A_{\overline{s}} - A_{\underline{s}}) - A_{\underline{s}}E/K + \eta_{\overline{s}|0}(A_{\overline{s}} - A_{\underline{s}}). \tag{6.24}$$

Suppose now that monitoring by bankers is not welfare-maximizing, i.e., $\triangle(A_{\overline{s}} - A_{\underline{s}}) < \kappa q$. Then we know with (6.24) that it must hold

$$-A_{\overline{s}}E/K + \eta_{\overline{s}|0}(A_{\overline{s}} - A_{\underline{s}}) > 0 \quad \Leftrightarrow \quad K/E > \frac{A_{\underline{s}}}{\eta_{\overline{s}|0}(A_{\overline{s}} - A_{\underline{s}})}. \tag{6.25}$$

Since the banker is monitoring, we know it holds that

$$\triangle A_{\overline{s}} \geq \kappa q(1+\eta_{\overline{s}|0}K/E),$$

which with the inequality (6.25) implies

$$\triangle A_{\overline{s}} > \kappa q\left(1 + \eta_{\overline{s}|0}\frac{A_{\underline{s}}}{\eta_{\overline{s}|0}(A_{\overline{s}} - A_{\underline{s}})}\right) \qquad \Leftrightarrow \qquad \triangle A_{\overline{s}} > \kappa q\left(1 + \frac{A_{\underline{s}}}{(A_{\overline{s}} - A_{\underline{s}})}\right),$$

which further simplifies to

$$\triangle A_{\overline{s}}(A_{\overline{s}} - A_{\underline{s}}) > \kappa q\left(A_{\overline{s}} - A_{\underline{s}} + A_{\underline{s}}\right) \qquad \Leftrightarrow \qquad \triangle(A_{\overline{s}} - A_{\underline{s}}) > \kappa q,$$

where the latter represents a contradiction to the previous assumption of monitoring not being welfare-maximizing. Accordingly, whenever the banker monitors in the presence of solvency risk, monitoring is also welfare-maximizing. As a consequence, we know that the constrained social planner must only apply contingent taxes and transfers if the banker faces solvency risk and does not monitor, i.e., $\triangle A_{\overline{s}} < \kappa q(1+\eta_{\overline{s}|0}K/E)$, although monitoring would be welfare-maximizing, i.e., $\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa q$. If we assume that the banker chooses in the case of indifference the welfare-maximizing monitoring activity, the optimal transfer for monitoring bankers set by the constrained social planner satisfies

$$\tau_{\overline{s}} = \max\{\kappa(1 + \eta_{\overline{s}|0}K/E)/\triangle - A_{\overline{s}}/q, 0\}.$$

We still need to check whether the transfer applied by the constrained social planner is feasible. As in our model only households are taxed, the following constraint applies:

$$\begin{aligned}
& R^D_{CB}qK + \tau \geq 0 \\
\Leftrightarrow \quad & R^D_{CB}qK + \pi^{CB} - \mathbb{E}_{\mathbf{m}}[\tau_s]\varphi qE \geq 0 \\
\Leftrightarrow \quad & R^D_{CB}qK + \eta_{\underline{s}|\overline{m}}[R^L_{\underline{s}}q(K+E) - R^D_{CB}qK] - \eta_{\overline{s}|\overline{m}}\tau_{\overline{s}}q(K+E) \geq 0 \\
\Leftrightarrow \quad & R^D_{CB}(\varphi-1)/\varphi + \eta_{\underline{s}|\overline{m}}[R^L_{\underline{s}} - R^D_{CB}(\varphi-1)/\varphi] \geq \eta_{\overline{s}|\overline{m}}\tau_{\overline{s}} \\
\Leftrightarrow \quad & \eta_{\underline{s}|\overline{m}}R^L_{\underline{s}} + \eta_{\overline{s}|\overline{m}}R^D_{CB}(\varphi-1)/\varphi \geq \eta_{\overline{s}|\overline{m}}\tau_{\overline{s}} \\
\Leftrightarrow \quad & \eta_{\underline{s}|\overline{m}}R^L_{\underline{s}} + \eta_{\overline{s}|\overline{m}}R^L_{\overline{s}} - \eta_{\overline{s}|\overline{m}}[R^L_{\overline{s}} - R^D_{CB}(\varphi-1)/\varphi] \geq \eta_{\overline{s}|\overline{m}}\tau_{\overline{s}}.
\end{aligned}$$

As the banker's monitoring decision is given by $\underline{m} = \overline{m} = \mathbb{1}\{\triangle[R^L_{\overline{s}} - R^D_{CB}(\varphi^r - 1)/\varphi^r] \geq \kappa - \triangle\tau_{\overline{s}}\}$, we can also express the optimal tax applied by the constrained social planner as

$$\tau_{\overline{s}} = \kappa/\triangle - R^L_{\overline{s}} + R^D_{CB}(\varphi-1)/\varphi,$$

which then can be rewritten as $R^L_{\overline{s}} - R^D_{CB}(\varphi-1)/\varphi = \kappa/\triangle - \tau_{\overline{s}}$. Then the latter inequality translates into

$$\mathbb{E}_{\mathbf{m}}[R^L_s] - \eta_{\overline{s}|\overline{m}}[\kappa/\triangle - \tau_{\overline{s}}] \geq \eta_{\overline{s}|\overline{m}}\tau_{\overline{s}} \qquad \Leftrightarrow \qquad \triangle\mathbb{E}_{\mathbf{m}}[R^L_s] \geq \eta_{\overline{s}|\overline{m}}\kappa.$$

Setting $\underline{m} = \overline{m} = 1$ and using $A_s = R_s^L q$, with $s \in \mathcal{S}$, the latter condition reads as

$$\begin{aligned} & \triangle(\eta_{\overline{s}|1} A_{\overline{s}} + \eta_{\underline{s}|1} A_{\underline{s}}) \geq \eta_{\overline{s}|1} \kappa q \\ \Leftrightarrow \quad & \triangle[\eta_{\overline{s}|1}(A_{\overline{s}} - A_{\overline{s}}) + A_{\underline{s}}] \geq \eta_{\overline{s}|1} \kappa q \\ \Leftrightarrow \quad & \eta_{\overline{s}|1}[\triangle(A_{\overline{s}} - A_{\overline{s}}) - \kappa Q] \geq -\triangle A_{\underline{s}}, \end{aligned}$$

which holds true as monitoring by bankers was assumed to be welfare-maximizing, i.e., $\triangle(A_{\overline{s}} - A_{\underline{s}}) \geq \kappa q$. ■

Proof of Proposition 4.5.5. Suppose bankers face solvency risk. Then we know from proposition 4.5.4 and 4.4.3 that utilitarian welfare achieved by the constrained social planner is given by

$$W^{CS} = (\mathbb{E}_{\mathbf{m}}[A_s] - \overline{m}\kappa q)(K + E) - (1 - \mu)\eta_{\underline{s}|\overline{m}} \nu \mathbb{1}\{\nu < \nu^*\},$$

where $\underline{m} = \overline{m} = \mathbb{1}\{A_{\overline{s}} \geq \kappa q(1 + \eta_{\overline{s}|0} K/E) - \triangle \tau_{\overline{s}} q\}$ with $\tau_{\overline{s}}$ satisfying

$$\tau_{\overline{s}} = \max\{\kappa(1 + \eta_{\overline{s}|0} K/E)/\triangle - A_{\overline{s}}/q, 0\}.$$

Now suppose tight collateral requirements are optimal (see proposition 4.5.1), where the optimal illiquidity penalty parameter $\hat{\phi}$ follows from lemma 4.5.1. With proposition 4.5.1 we can immediately deduce that this implies monitoring is welfare-maximizing, i.e., $\triangle(A_{\overline{s}} - A_{\underline{s}})$. The constrained social planner will therefore implement contingent transfers $\tau_{\overline{s}}$, so that $\underline{m} = \overline{m} = 1$ and welfare is given by

$$W^{CS} = (\mathbb{E}_1[A_s] - \kappa q)(K + E) - (1 - \mu)\eta_{\underline{s}|1} \nu \mathbb{1}\{\nu < \nu^*\}.$$

Then we know from proposition 4.4.5 that the welfare in the competitive equilibrium with optimal monetary policy is given by

$$W_{NB}^{LS} = \{\mu \mathbb{E}_0[A_s] + (1 - \mu)\mathbb{E}_1[A_s] - (1 - \mu)\kappa q - [(1 - \mu)\eta_{\underline{s}|1} + \mu]\hat{\phi}\epsilon(\hat{\phi})\}(K + E) - (1 - \mu)\eta_{\underline{s}|1}\nu.$$

Suppose $\nu < \nu^*$, with ν^* provided in proposition 4.4.3, so that households convert deposits in the case of bank insolvency. Then, the difference between utilitarian welfare in the competitive equilibrium with optimal monetary policy and second-best welfare is given by

$$\mu(\mathbb{E}_1[A_s] - \mathbb{E}_0[A_s] - \kappa q)(K + E) - [(1 - \mu)\eta_{\underline{s}|1} + \mu]\hat{\phi}\epsilon(\hat{\phi})\}(K + E),$$

which, using $\mathbb{E}_1[A_s] - \mathbb{E}_0[A_s] = \triangle(A_{\overline{s}} - A_{\underline{s}})$, translates into

$$\mu[\triangle(A_{\overline{s}} - A_{\underline{s}}) - \kappa q](K + E) - [(1 - \mu)\eta_{\underline{s}|1} + \mu]\hat{\phi}\epsilon(\hat{\phi})\}(K + E).$$

Clearly, for any $\mu > 0$ there is a welfare loss in the competitive equilibrium with optimal monetary policy compared to the constrained social planner solution due to lost monitoring activities by illiquid bankers. In addition, there is a welfare loss due to the imposed illiquidity penalties. For $\mu \to 0$ and $\eta_{\underline{s}|1} \to 0$ utilitarian welfare in the competitive equilibrium with optimal monetary policy approaches utilitarian welfare achieved by the constrained social planner. If in addition switching costs are negligible, i.e., $\nu \to 0$, welfare in the competitive equilibrium approaches the first-best welfare. ■

Proof of Proposition 4.6.1. Note that using our framework, which features a CBDC and no deposit insurance scheme, we can replicate the real allocation emerging in today's monetary system with a deposit insurance scheme. Today's monetary system can be captured in our model through an environment in which there are no costs for converting deposits into CBDC and there are no penalties for bankers if they default on liabilities towards the central bank. Thus using our framework, we can replicate the real allocation emerging in today's monetary system by setting switching costs to zero, i.e., $\nu = 0$, and focusing on loose collateral requirements, i.e., $\Psi(K + E) \geq R^D_{CB}K$, which rules out illiquidity penalties for bankers.

Without solvency risk, households will never transfer funds from private bankers to the central bank. Thus, even if there are costs for converting deposits into CBDC, the alternative system with a CBDC and no deposit insurance scheme yields the same welfare as today's monetary system. We obtain the same result if bankers face a solvency risk but switching costs are sufficiently high, such that households holding deposits with an insolvent banker accept a bail-in and do not transfer funds to the central bank.

Last, consider the situation where bankers face a solvency risk and switching costs are sufficiently low, so that households holding deposits with insolvent bankers will not accept a bail-in and shift their funds to the central bank. Then, if loose collateral requirements are optimal, the alternative monetary system yields a welfare loss compared to today's monetary system, due to switching costs on the part of depositors. If tight collateral requirements are optimal, the alternative system may yield a welfare gain compared to today's monetary system if the switching costs are sufficiently low. ■

Proof of Proposition 4.7.1. Note that we assume sufficiently small switching costs ν, such that bank insolvency will trigger a bank run. Households will then only transfer their funds from a banker to the central bank if the respective banker defaults due to insolvency. Otherwise, the mass of households holding accounts with the central bank will stay constant over time, i.e., $\mu_{t+1} = \mu_0$. In turn, if bankers face a solvency risk, i.e., they will default if the financed firm experiences a negative productivity shock ($s = \underline{s}$), a household which has a deposit with a banker will shift the funds to the central bank and, due to positive switching costs, stays with the central bank in the following periods. Thus, with solvency risk the mass of households holding accounts with the central bank will evolve in accordance with $\mu_{t+1} = (1 - \mu_t)\eta_{\overline{s}|\overline{m}} + \mu_t$. ■

Proof of Proposition 4.7.2. We denote the mass of defaulting bankers in period $t \in \mathbb{N}_0$ by σ_t. Without liquidity and solvency risk, no banker will default, i.e., $\sigma_t = 0$. With liquidity risk only, bankers experiencing a CBDC-induced bank run or, equivalently, matched with a household that holds an account with the central bank, will default. The mass of such households in the economy is given by $\mu_t = \mu_0$ and stays constant over time as there is no solvency risk (see proposition 4.7.1). Thus, with liquidity risk only, the mass of defaulting bankers is constant and is given by $\sigma_t = \mu_0$. With solvency risk only, bankers will default if the financed firm incurs a negative productivity shock ($s = \underline{s}$), which occurs with probability $\eta_{\underline{s}|\overline{m}}$. Note that with solvency risk only, the monitoring decision is independent of the type of matched household, as stated in proposition 4.4.3. Thus, with solvency risk only, the mass of defaulting bankers is constant and is given by $\sigma_t = \eta_{\underline{s}|\overline{m}}$. With liquidity risk *and* solvency risk, the mass of defaulting bankers is given by $\sigma_t = \mu_t + (1 - \mu_t)\eta_{\underline{s}|\overline{m}}$, for the reasons stated above. From proposition 4.7.1, we know that with solvency risk, the mass of households possessing accounts with the central bank is converging to one, i.e., $\lim_{t\to\infty} \mu_t = 1$, such that in the presence of both liquidity and solvency risk the mass of defaulting bankers also approaches one, i.e., $\lim_{t\to\infty} \sigma_t = 1$. ■

Proof of Proposition 4.7.3. From proposition 4.4.5 we know that an equilibrium with liquidity risk following from tight collateral requirements, solvency risk, and no bail-ins, will exist iff

$$\max\left\{A_{\underline{s}} + \frac{\nu}{K+E} + \frac{[(1-\mu_t)\eta_{\underline{s}|\overline{m}} + \mu_t]\phi(A_{\underline{s}} - \Psi_t q) + \frac{\nu}{K+E}}{(1-\mu_t)\eta_{\overline{s}|\overline{m}} + E/K}, \Psi_t q\right\}$$

$$< \frac{(1-\mu_t)(\eta_{\overline{s}|\overline{m}} A_{\overline{s}} - \overline{m}\kappa q)}{(1-\mu_t)\eta_{\overline{s}|\overline{m}} + E/K},$$

where $\mu_{t+1} = (1-\mu_t)\eta_{\underline{s}|\overline{m}} + \mu_t$. Specifically, note that there exists no sequence $\{\Psi_t\}_{t\in\mathbb{N}_0}$ such that for all $t \in \mathbb{N}_0$ the above inequality is satisfied: With solvency risk, the mass of households holding accounts with the central bank converges to one, i.e., $\lim_{t\to\infty}\mu_t = 1$, such that the right-hand side approaches zero while the left-hand side remains positive for any $\Psi_t \geq 0$. Hence, with constant endowments of households and bankers, tight collateral requirements can only be maintained for a finite period of time without rendering banking non-viable, i.e., there exists a period $\tilde{t} \in \mathbb{N}_0$ subsequent to which tight collateral requirements will lead to non-viability of banking. ∎

6.4 Appendix for Chapter 5

Proof of Lemma 5.3.1. For the optimization problem of the riskless firm, which is given by (5.1), the first-order condition with respect to K_l is given by

$$\alpha A_l K_l^{\alpha-1} = (1 + r_l^L)q.$$

Rearranging then yields the optimal demand of the capital good by the riskless firm

$$K_l = \left[\frac{\alpha A_l}{(1 + r_l^L)q}\right]^{\frac{1}{1-\alpha}}.$$

■

Proof of Lemma 5.3.2. For the optimization problem of the risky firm, which is given by (5.3), the first-order condition with respect to K_h is given by

$$\alpha \mathbb{E}_p[A_{h,s}] K_h^{\alpha-1} = (1 + \mathbb{E}_p[r_{h,s}^L])q.$$

Rearranging then yields the optimal demand of the capital good by the risky firm

$$K_h = \left[\frac{\alpha \mathbb{E}_p[A_{h,s}]}{(1 + \mathbb{E}_p[r_{h,s}^L])q}\right]^{\frac{1}{1-\alpha}}.$$

■

Proof of Lemma 5.3.3. Due to the assumption of linear utility, the household maximizes expected consumption $\mathbb{E}_p[\{\gamma(1+r_s^E)+(1-\gamma)(1+r^D)\}qK+\tau_s+\pi_s]$. Thus, its optimal choice is of knife-edge type, as the household holds the asset which yields the highest expected return. Specifically, the household invests only into bank equity, i.e., $\gamma = 1$, if the expected rate of return on equity strictly exceeds the interest rate on deposits, i.e., $\mathbb{E}_p[r_s^E] > r^D$, and only holds deposits, i.e., $\gamma = 0$, if the interest rate on deposits exceeds the expected equity rate of return, i.e., $\mathbb{E}_p[r_s^E] < r^D$. If the returns on bank equity and deposits equal, i.e., $\mathbb{E}_p[r_s^E] = r^D$, the household is indifferent, i.e., $\gamma \in [0, 1]$. ■

Proof of Lemma 5.3.4. Note that reserves can be borrowed from the central bank at an interest rate $r^L_{CB}(\zeta)$ and can be deposited at the central bank at an interest rate r^D_{CB}. The interest rate for interbank loans is given by $r^L_{IB} > 0$, whereas the interest rate on interbank deposits is given by r^D_{IB}. We assume that the bank cannot differentiate between deposits held by other banks and deposit from households and firms, so that it holds $r^D_{IB} = r^D$. Interbank loans are only demanded if $r^L_{IB} \leq r^L_{CB}(\zeta)$, whereas interbank deposits are only attractive to the bank if $r^D \geq r^D_{CB}$. Otherwise, the bank would only deposit at the central bank. The liquidity provided on the interbank market through loans L^{IB} to other banks are matched by interbank deposits D^{IB} held by the borrowing banks. Thus, it holds $L^{IB} = D^{IB}$. Interbank deposits are fully withdrawn by the borrowing banks if the latter must settle deposit outflows due to transactions on the capital good market. The lending bank must settle the outflow of interbank deposits by using reserves in the amount $D^{CB} = D^{IB}$, which itself must borrow from the central bank by demanding loans L^{CB}. The revenues from interbank lending are given by $r^L_{IB}L^{IB}$, whereas the costs of interbank lending are given by $r^D D^{IB} + r^L_{CB}(\zeta)L^{CB} - r^D_{CB}D^{CB}$. Using $L^{IB} = D^{IB}$ and $L^{CB} = D^{CB} = D^{IB}$, the bank only offers interbank loans and deposits if

$$r^L_{IB} \geq r^D + r^L_{CB}(\zeta) - r^D_{CB} \qquad \Leftrightarrow \qquad r^D_{CB} - r^D \geq r^L_{CB}(\zeta) - r^L_{CB}.$$

Since the interbank market is active only if $r^D \geq r^D_{CB}$ and $r^L_{IB} \leq r^L_{CB}(\zeta)$, we can conclude that the interest rates satisfy $r^L_{IB} = r^L_{CB}(\zeta)$ and $r^D = r^D_{CB}$. ■

Proof of Lemma 5.3.5. Note that the expected rate of return on bank equity is given by

$$\begin{aligned}\mathbb{E}_p[r^E_s(\varphi,\zeta)] &= \mathbb{E}_p[(1+\psi)^{-1}\{r^L_l\zeta + r^L_{h,s}(1-\zeta) - r^D_{CB}\Psi(\zeta)\}\varphi + r^D_{CB}] \\ &= (1+\psi)^{-1}\{(r^L_l - r^D_{CB}\psi\tilde{\kappa}_l)\zeta + (\mathbb{E}_p[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h)(1-\zeta) - r^D_{CB}\}\varphi + r^D_{CB},\end{aligned}$$

where we used the definition $\Psi(\zeta) = 1 + \psi[\zeta\tilde{\kappa}_l + (1-\zeta)\tilde{\kappa}_h]$. The equity rate of return is maximized for the maximum (minimum) possible leverage, i.e., $\varphi = \varphi^R$ ($\varphi = 1$), if the expected return per unit of loan financing, funded with deposits, is positive (negative), i.e., for some $\zeta \in [0,1]$

$$(r^L_l - r^D_{CB}\psi\tilde{\kappa}_l)\zeta + (\mathbb{E}_p[r^L_{h,s}] - r^D_{CB}\psi\tilde{\kappa}_h)(1-\zeta) - r^D_{CB} > (<)0$$

or, equivalently,

$$\max\{r_l^L - r_{CB}^D\psi\tilde{\kappa}_l, \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D\psi\tilde{\kappa}_h\} > (<) r_{CB}^D.$$

Otherwise, i.e., if $\max\{r_l^L - r_{CB}^D\psi\tilde{\kappa}_l, \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D\psi\tilde{\kappa}_h\} = r_{CB}^D$, the bank is indifferent between any leverage, i.e., $\varphi \in [1, \varphi^R]$.

The bank optimally grants loan financing to the sector, which yields the highest expected return, taking the revenues from loan repayment and the costs from interest payments on deposits as well as from the borrowing of reserves at the central bank into account. That is, the bank optimally chooses to grant loans only to the riskless (risky) firms, i.e., $\zeta = 1$ ($\zeta = 0$), if

$$r_l^L - r_{CB}^D(1+\psi\tilde{\kappa}_l) > (<) \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D(1+\psi\tilde{\kappa}_h)$$

$$\Leftrightarrow \quad r_l^L - r_{CB}^D\psi\tilde{\kappa}_l > (<) \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D\psi\tilde{\kappa}_h.$$

In all other cases, i.e., $r_l^L - r_{CB}^D\psi\tilde{\kappa}_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D\psi\tilde{\kappa}_h$, the bank is indifferent between loan financing to riskless and risky firms, i.e., $\zeta \in [0,1]$. ■

Proof of Lemma 5.4.1. According to lemma 5.3.5, it is optimal for the bank to grant loan financing to both sectors if it holds $r_l^L - r_{CB}^D(1+\psi\tilde{\kappa}_l) = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D(1+\psi\tilde{\kappa}_h)$. Using the definition of $\tilde{\kappa}_l$ and $\tilde{\kappa}_h$, the latter inequality translates into

$$r_l^L - r_{CB}^D + (1+r_{CB}^D)\psi\kappa_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D + (1+r_{CB}^D)\psi\kappa_h$$

$$\Leftrightarrow \qquad 1 + r_l^L - (1+r_{CB}^D)(1+\psi\kappa_l) = 1 + \mathbb{E}_p[r_{h,s}^L] - (1+r_{CB}^D)(1+\psi\kappa_h).$$

Multiplying both sides of the inequality with the real capital good price $q = Q/P$, using the first-order condition of the riskless firm, i.e., $(1+r_l^L)q = \alpha A_l K_l^{\alpha-1}$, and using assumption 5.3.1, i.e., $(1+r_{h,s}^L)q = \alpha A_{h,s} K_h^{\alpha-1}$ for all s, it follows

$$\alpha A_l K_l^{\alpha-1} - (1+r_{CB}^D)(1+\psi\kappa_l) = \alpha A_{h,s} K_h^{\alpha-1} - (1+r_{CB}^D)(1+\psi\kappa_h).$$

Suppose the bank only grants loans $L_l = \epsilon$ to the riskless firm, so that due to the clearing of the capital good market, i.e., $K_h = K - \epsilon$, it holds that loan financing to risky firms is given by $L_h = QK_h = Q(K-\epsilon)$. As the capital allocation satisfies $K_l = \epsilon$ and $K_h = K - \epsilon$, we know that the left-hand side of the latter equation tends to infinity for ϵ approaching zero, while the right-hand side is finite. Granting only loans to the risky sector is not optimal for the bank, as, according to lemma 5.3.5, it should in such a situation only grant loans to the riskless sector. Similarly, the right-hand side converges to infinity for ϵ approaching K, while the left-hand side is finite. Granting only loans to the riskless sector is not optimal for the bank, as, according to lemma 5.3.5, it should in such a situation only grant loans to the risky sector. We can therefore conclude that in equilibrium it is never optimal for the bank to grant loan financing to only one sector. ■

Proof of Corollary 5.4.1. First, from lemma 5.4.1 we know that, in equilibrium, both riskless and risky firms demand loans, i.e., $\zeta \in (0,1)$. Second, from lemma 5.3.5, we know that the bank is willing to grant loans to both types of firms if and only if the adjusted loan rates equal, i.e., $r_l^L - r_{CB}^D \psi \tilde{\kappa}_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D \psi \tilde{\kappa}_h$. Third, due to perfect competition among banks, financing loans with deposits must in equilibrium yield zero expected profits, i.e.,

$$r_l^L \zeta + r_{h,s}^L (1-\zeta) = r_{CB}^D \Psi(\zeta)$$

$$\Leftrightarrow \qquad (r_l^L - r_{CB}^D \psi \tilde{\kappa}_l)\zeta + (\mathbb{E}_p[r_{h,s}^L] - r_{CB}^D \psi \tilde{\kappa}_h)(1-\zeta) = r_{CB}^D,$$

where we used the definition $\Psi(\zeta) = 1 + \psi[\zeta \tilde{\kappa}_l + (1-\zeta)\tilde{\kappa}_h]$. With equal adjusted loan rates, $r_l^L - r_{CB}^D \psi \tilde{\kappa}_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D \psi \tilde{\kappa}_h$, it follows that loan rates satisfy

$$r_l^L - r_{CB}^D \psi \tilde{\kappa}_l = \mathbb{E}_p[r_{h,s}^L] - r_{CB}^D \psi \tilde{\kappa}_h = r_{CB}^D,$$

ultimately leading to $r_l^L = r_{CB}^D(1 + \psi \tilde{\kappa}_l)$ and $\mathbb{E}_p[r_{h,s}^L] = r_{CB}^D(1 + \psi \tilde{\kappa}_h)$. ■

Proof of Corollary 5.4.2. Note that, from corollary 5.4.1, it follows that $\mathbb{E}_p[r_{h,s}^L] = r_{CB}^D(1 + \psi \tilde{\kappa}_h)$. Using the definition of $\tilde{\kappa}_h$, the latter condition translates into

$$\mathbb{E}_p[r_{h,s}^L] = r_{CB}^D + (1 + r_{CB}^D)\psi \kappa_h \qquad \Leftrightarrow \qquad 1 + \mathbb{E}_p[r_{h,s}^L] = (1 + r_{CB}^D)(1 + \psi \kappa_h).$$

Multiplying both sides of the equation with the real capital good price q and using $(1 +$

$r^L_{h,s})q = \alpha A_{h,s} K_h^{\alpha-1}$ for all s, the latter condition reads as

$$\alpha \mathbb{E}_p[A_{h,s}] K_h^{\alpha-1} = (1 + r^D_{CB})(1 + \psi\kappa_h)q.$$

Using $q = Q/P$ it follows that prices P and Q must satisfy

$$\alpha \mathbb{E}_p[A_{h,s}] K_h^{\alpha-1} = \frac{Q}{P}(1 + r^D_{CB})(1 + \psi\kappa_h) \qquad \Leftrightarrow \qquad \frac{P}{Q} = \frac{(1 + r^D_{CB})(1 + \psi\kappa_h)}{\alpha \mathbb{E}_p[A_{h,s}] K_h^{\alpha-1}}.$$

■

Proof of Lemma 5.4.2. From subsection 5.4.2, we know that welfare is in scenario $s \in \{b, t\}$ generally given by $W_s = C_s$. Using the structure of the household's consumption, welfare reads

$$W_s = [\gamma(1 + r^E_s) + (1 - \gamma)(1 + r^D)]qK + \tau_s + \pi_s.$$

First, note that the rate of return on bank equity is, based on equation (5.12), given by

$$r^E_s(\varphi, \zeta) = (1 + \psi)^{-1}[r^L_l \zeta + r^L_{h,s}(1 - \zeta) - r^D_{CB}\Psi(\zeta)]\varphi + r^D_{CB}.$$

Using the equilibrium leverage $\varphi = (1+\psi)/\gamma$ and the definition $\Psi(\zeta) = 1+\psi[\zeta\tilde{\kappa}_l+(1-\zeta)\tilde{\kappa}_h]$, the equity rate of return reads as

$$r^E_s(\varphi, \zeta) = [(r^L_l - r^D_{CB}\psi\tilde{\kappa}_l)\zeta + (r^L_{h,s} - r^D_{CB}\psi\tilde{\kappa}_h)(1 - \zeta) - r^D_{CB}]/\gamma + r^D_{CB}.$$

Second, based on lemma 5.3.4, the interest rates on deposits and reserves equal, i.e., $r^D = r^D_{CB}$.

Third, due to the fact that reserve loans are costly, the central bank generates profits, which in nominal terms are given by

$$\Pi^{CB}_s = r^L_{CB}(\zeta)L^{CB} - r^D_{CB}D^{CB} = [r^L_{CB}(\zeta) - r^D_{CB}]L^{CB} = [\zeta\tilde{\kappa}_l + (1 - \zeta)\tilde{\kappa}_h]\psi r^D_{CB}QK,$$

where we used the equality of reserve loans and reserve deposits, i.e., $L^{CB} = D^{CB}$, the structure of reserve loans, i.e., $L^{CB} = \psi L$, and the fact that, in equilibrium, bank loans are given by $L = QK$. Because we impose a balanced budget for the government and the central bank, central bank profits are distributed by the government through transfers, i.e.,

$$\tau_s = \pi_s^{CB} = [\zeta\tilde{\kappa}_l + (1-\zeta)\tilde{\kappa}_h]\psi r_{CB}^D qK.$$

Fourth, from the outline in subsection 5.3.3, we know that the aggregate firm profits are characterized through equation (5.5), i.e., $\pi_s = (1-\alpha)(A_l K_l^\alpha + A_{h,s} K_h^\alpha)$.

Thus, welfare in scenario $s \in \{b, t\}$ reads as

$$\begin{aligned} W_s = \gamma\{1 + [(r_l^L - r_{CB}^D\psi\tilde{\kappa}_l)\zeta + (r_{h,s}^L - r_{CB}^D\psi\tilde{\kappa}_h)(1-\zeta) - r_{CB}^D]/\gamma + r_{CB}^D\}qK \\ + (1-\gamma)(1 + r_{CB}^D)qK + [\zeta\tilde{\kappa}_l + (1-\zeta)\tilde{\kappa}_h]\psi r_{CB}^D qK \\ + (1-\alpha)(A_l K_l^\alpha + A_{h,s} K_h^\alpha) \end{aligned}$$

and simplifies to

$$W_s = [(1 + r_l^L)\zeta + (1 + r_{h,s}^L)(1-\zeta)]qK + (1-\alpha)(A_l K_l^\alpha + A_{h,s} K_h^\alpha).$$

Using the first-order condition for the optimization problem of the riskless firm, i.e., $(1 + r_l^L)q = \alpha A_l K_l^{\alpha-1}$, assumption 5.3.1, which states that loan rates for risk firms satisfy $(1 + r_{h,s}^L)q = \alpha A_{h,s} K_h^{\alpha-1}$ for all s, and the capital allocation across riskless and risky firms, i.e., $K_l = \zeta K$ and $K_h = (1-\zeta)K$, welfare is finally given by

$$W_s = A_l K_l^\alpha + A_{h,s} K_h^\alpha = [A_l\zeta^\alpha + A_{h,s}(1-\zeta)^\alpha]K^\alpha.$$

■

Proof of Proposition 5.4.1. From corollary 5.4.1, we know that the interest rates on loans satisfy

$$r_l^L = r_{CB}^D(1 + \psi\tilde{\kappa}_l) \qquad \text{and} \qquad \mathbb{E}_p[r_{h,s}^L] = r_{CB}^D(1 + \psi\tilde{\kappa}_h),$$

so that, using the definition of $\tilde{\kappa}_l$ and $\tilde{\kappa}_h$, it follows

$$1 + r_l^L = (1 + r_{CB}^D)(1 + \psi\kappa_l) \qquad \text{and} \qquad 1 + \mathbb{E}_p[r_{h,s}^L] = (1 + r_{CB}^D)(1 + \psi\kappa_h).$$

From the latter two equations, we then obtain

$$\frac{1+r_l^L}{1+\psi\kappa_l}=\frac{1+\mathbb{E}_p[r_{h,s}^L]}{1+\psi\kappa_h}\qquad\Leftrightarrow\qquad\frac{1+r_l^L}{1+\mathbb{E}_p[r_{h,s}^L]}=\frac{1+\psi\kappa_l}{1+\psi\kappa_h}.$$

Using the first-order condition $(1+r_l^L)q=\alpha A_l K_l^{\alpha-1}$, and assumption 5.3.1, stating that $(1+r_{h,s}^L)q=\alpha A_{h,s}K_h^{\alpha-1}$ for all s, it follows

$$\frac{A_l K_l^{\alpha-1}}{\mathbb{E}_p[A_{h,s}]K_h^{\alpha-1}}=\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\qquad\Leftrightarrow\qquad\left(\frac{K_h}{K_l}\right)^{1-\alpha}=\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}.$$

Using $K_l=\zeta K$ and $K_h=(1-\zeta)K$, as derived in subsection 5.4.2, we obtain

$$\frac{1-\zeta}{\zeta}=\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\qquad\Leftrightarrow\qquad\zeta=\left[1+\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

■

Proof of Proposition 5.4.2. Note that the central bank faces the optimization problem

$$\max_{\kappa_l,\kappa_h\in\mathbb{R}}\{A_l\zeta^\alpha+\mathbb{E}_g[A_{h,s}](1-\zeta)^\alpha\}K^\alpha\qquad\text{subject to}\qquad\zeta\kappa_l+(1-\zeta)\kappa_h\geq 0,$$

where the share ζ of capital good allocated to riskless firms satisfies

$$\zeta=\left[1+\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

The optimal allocation follows by taking the derivatives of expected welfare with respect to κ_l and κ_h, and setting them to zero. With $\mu\geq 0$ denoting the Lagrange multiplier on the constraint, the optimality conditions are given by the two first-order conditions

$$\alpha A_l\zeta^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_l}-\alpha\mathbb{E}_g[A_{h,s}](1-\zeta)^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_l}-\mu\zeta-\mu(\kappa_l-\kappa_h)\frac{\partial\zeta}{\partial\kappa_l}=0,$$

$$\alpha A_l\zeta^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_h}-\alpha\mathbb{E}_g[A_{h,s}](1-\zeta)^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_h}-\mu(1-\zeta)-\mu(\kappa_l-\kappa_h)\frac{\partial\zeta}{\partial\kappa_h}=0,$$

and the complementary slackness condition $\mu[\zeta\kappa_l+(1-\zeta)\kappa_h]=0$. Note that the two first-order conditions can be rewritten as

$$\alpha A_l \zeta^{\alpha-1} - \alpha \mathbb{E}_g[A_{h,s}](1-\zeta)^{\alpha-1} = \mu(\kappa_l - \kappa_h) + \mu\zeta \left(\frac{\partial \zeta}{\partial \kappa_l}\right)^{-1},$$

$$\alpha A_l \zeta^{\alpha-1} - \alpha \mathbb{E}_g[A_{h,s}](1-\zeta)^{\alpha-1} = \mu(\kappa_l - \kappa_h) + \mu(1-\zeta) \left(\frac{\partial \zeta}{\partial \kappa_h}\right)^{-1}.$$

First, we show that the Lagrange multiplier μ equals always zero. Suppose to the contrary that $\mu > 0$. Then, equating the two first-order conditions yields

$$\zeta \frac{\partial \zeta}{\partial \kappa_h} = (1-\zeta)\frac{\partial \zeta}{\partial \kappa_l}. \tag{6.26}$$

The derivatives of the capital allocation share ζ with respect to κ_l and κ_h are given by

$$\begin{aligned}
\frac{\partial \zeta}{\partial \kappa_l} = &-\frac{1}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}} \frac{\psi \mathbb{E}_p[A_{h,s}]}{A_l(1+\psi\kappa_h)} \\
&\times \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-2} \\
= &\frac{-\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \frac{\psi}{1+\psi\kappa_l} \\
= &\frac{-\zeta(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_l}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \zeta}{\partial \kappa_h} = &\frac{1}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}} \frac{\psi A_l \mathbb{E}_p[A_{h,s}](1+\psi\kappa_l)}{A_l^2(1+\psi\kappa_h)^2} \\
&\times \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-2} \\
= &\frac{\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \frac{\psi}{1+\psi\kappa_h} = \frac{\zeta(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_h}.
\end{aligned}$$

Using the latter two results, the condition (6.26) translates into

$$\frac{\zeta^2(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_h} = \frac{-\zeta(1-\zeta)^2}{1-\alpha}\frac{\psi}{1+\psi\kappa_l} \qquad \Leftrightarrow \qquad \frac{\zeta}{1+\psi\kappa_h} = \frac{\zeta-1}{1+\psi\kappa_l}$$

and further simplifies to

$$\zeta(1+\psi\kappa_l) = (\zeta-1)(1+\psi\kappa_h) \quad \Leftrightarrow \quad \psi\{\zeta\kappa_l + (1-\zeta)\kappa_h\} = -1.$$

The latter equation contradicts the complementary slackness condition, which implies for any positive Lagrange multiplier (i.e., $\mu > 0$), $\zeta\kappa_l + (1-\zeta)\kappa_h = 0$. Thus, we can conclude that the Lagrange multiplier is always zero, i.e., $\mu = 0$. The two first-order conditions are therefore identical and given by

$$\alpha A_l \zeta^{\alpha-1} - \alpha \mathbb{E}_g[A_{h,s}](1-\zeta)^{\alpha-1} = 0.$$

This optimality condition translates into

$$A_l \zeta^{\alpha-1} = \mathbb{E}_g[A_{h,s}](1-\zeta)^{\alpha-1} \qquad \Leftrightarrow \qquad \frac{1-\zeta}{\zeta} = \left(\frac{\mathbb{E}_g[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}.$$

Further rearranging yields that the optimal capital allocation satisfies

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_g[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right]^{-1} =: \zeta_g.$$

Using the capital allocation in the decentralized equilibrium (see proposition 5.4.1), which is given by

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1},$$

we can deduce that the optimal monetary policy must satisfy

$$\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h} = \frac{\mathbb{E}_g[A_{h,s}]}{A_l} \qquad \Leftrightarrow \qquad \frac{1+\psi\kappa_h}{1+\psi\kappa_l} = \frac{\mathbb{E}_p[A_{h,s}]}{\mathbb{E}_g[A_{h,s}]} =: a.$$

Rearranging then yields

$$1 + \psi\kappa_h = (1 + \psi\kappa_l)a \quad \Leftrightarrow \quad \psi\kappa_h = \psi a\kappa_l + a - 1 \quad \Leftrightarrow \quad \kappa_h = a\kappa_l + \frac{a-1}{\psi}.$$

In addition, the cost factors must satisfy the constraint $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h \geq 0$. Note that whenever $\eta_g > (<)\eta_p$ it follows $a > (<)1$ and therefore $\kappa_h > (<)\kappa_l$. ■

Proof of Corollary 5.4.3. Based on proposition 5.4.2, we know that the optimal cost factors κ_l and κ_h satisfy

$$\kappa_h = a\kappa_l + \frac{a-1}{\psi} \qquad \Leftrightarrow \qquad \frac{1+\psi\kappa_h}{1+\psi\kappa_l} = a, \qquad \text{with} \qquad a = \frac{\mathbb{E}_p[A_{h,s}]}{\mathbb{E}_g[A_{h,s}]}.$$

Note that a increases with η_g and decreases with η_p, so that we can conclude that the difference between the optimal cost factors $\kappa_h - \kappa_l$ increases with η_g and decreases with η_p. ■

Proof of Lemma 5.5.1. The first-order condition of the optimization problem of the risky firm (see equation (5.16)) with respect to capital good K_h is given by $\alpha\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha K_h^{\alpha-1} = (1+\mathbb{E}_p[r^L_{h,s}])q$. The latter condition can be rearranged to

$$K_h^{1-\alpha} = \frac{\alpha\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{(1+\mathbb{E}_p[r^L_{h,s}])q} \qquad \Leftrightarrow \qquad K_h = \left[\frac{\alpha\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{(1+\mathbb{E}_p[r^L_{h,s}])q}\right]^{\frac{1}{1-\alpha}},$$

which gives the optimal demand of capital good by the risky firm. The first-order condition with respect to the share i of capital good devoted to CRMT investment is for an interior solution given by

$$\mathbb{E}_p\left[\frac{\partial A_{h,s}(i)}{\partial i}\right](1-i)^\alpha = \alpha\mathbb{E}_p[A_{h,s}(i)](1-i)^{\alpha-1} \quad \Leftrightarrow \quad i = 1 - \frac{\alpha\mathbb{E}_p[A_{h,s}(i)]}{\mathbb{E}_p[\partial A_{h,s}(i)/\partial i]}.$$

Using assumption 5.5.1, we get that the share i of capital good devoted to CRMT investment simplifies to

$$i = 1 - \frac{\alpha\mathbb{E}_p[A_{h,s}(i)]}{\eta_p\mathbb{E}_p[A_{h,s}(i)]\beta(1-i)^{\beta-1}} \ \Leftrightarrow \ 1-i = \frac{\alpha}{\eta_p\beta(1-i)^{\beta-1}} \ \Leftrightarrow \ i = 1 - \left(\frac{\alpha}{\eta_p\beta}\right)^{\frac{1}{\beta}}.$$

We can conclude that $i < 1$, but we have to account for the fact that risky firms may not devote any capital good to CRMT investment if $\alpha > \eta_p \beta$. Thus, the optimal share i of capital good devoted to CRMT investment is generally given by

$$i = \max\left\{1 - \left(\frac{\alpha}{\eta_p \beta}\right)^{\frac{1}{\beta}}, 0\right\}.$$

■

Proof of Proposition 5.5.1. From corollary 5.4.1, we know that the interest rates on loans satisfy

$$r_l^L = r_{CB}^D(1 + \psi\tilde{\kappa}_l) \qquad \text{and} \qquad \mathbb{E}_p[r_{h,s}^L] = r_{CB}^D(1 + \psi\tilde{\kappa}_h),$$

so that, using the definition of $\tilde{\kappa}_l$ and $\tilde{\kappa}_h$, it follows

$$1 + r_l^L = (1 + r_{CB}^D)(1 + \psi\kappa_l) \qquad \text{and} \qquad 1 + \mathbb{E}_p[r_{h,s}^L] = (1 + r_{CB}^D)(1 + \psi\kappa_h).$$

From the latter two equations, we then obtain

$$\frac{1 + r_l^L}{1 + \psi\kappa_l} = \frac{1 + \mathbb{E}_p[r_{h,s}^L]}{1 + \psi\kappa_h} \qquad \Leftrightarrow \qquad \frac{1 + r_l^L}{1 + \mathbb{E}_p[r_{h,s}^L]} = \frac{1 + \psi\kappa_l}{1 + \psi\kappa_h}.$$

Using the first-order condition $(1 + r_l^L)q = \alpha A_l K_l^{\alpha-1}$, and assumption 5.3.1 together with the fact that risky firms can invest into CRMT, both leading to $(1 + r_{h,s}^L)q = \alpha A_{h,s}(i)(1 - i)^\alpha K_h^{\alpha-1}$ for all s, it follows

$$\frac{A_l K_l^{\alpha-1}}{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha K_h^{\alpha-1}} = \frac{1 + \psi\kappa_l}{1 + \psi\kappa_h} \quad \Leftrightarrow \quad \left(\frac{K_h}{K_l}\right)^{1-\alpha} = \frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1 + \psi\kappa_l}{1 + \psi\kappa_h}.$$

Using $K_l = \zeta K$ and $K_h = (1 - \zeta)K$, as derived in subsection 5.4.2, we obtain

$$\frac{1-\zeta}{\zeta} = \left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1 + \psi\kappa_l}{1 + \psi\kappa_h}\right)^{\frac{1}{1-\alpha}}$$

$$\Leftrightarrow \qquad \zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1 + \psi\kappa_l}{1 + \psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

■

Proof of Proposition 5.5.2. Note that the central bank faces the optimization problem

$$\max_{\kappa_l,\kappa_h\in\mathbb{R}} \{A_l\zeta^\alpha + \mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha(1-\zeta)^\alpha\}K^\alpha,$$

subject to $\zeta\kappa_l + (1-\zeta)\kappa_h \geq 0$, where the share ζ of capital good allocated to the riskless firm satisfies

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

The optimal allocation follows by taking the derivatives of expected welfare with respect to κ_l and κ_h, and setting them to zero. With $\mu \geq 0$ denoting the Lagrange multiplier on the constraint, the optimality conditions are given by the two first-order conditions

$$\alpha A_l\zeta^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_l} - \alpha\mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha(1-\zeta)^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_l} - \mu\zeta - \mu(\kappa_l-\kappa_h)\frac{\partial\zeta}{\partial\kappa_l} = 0,$$

$$\alpha A_l\zeta^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_h} - \alpha\mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha(1-\zeta)^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_h} - \mu(1-\zeta) - \mu(\kappa_l-\kappa_h)\frac{\partial\zeta}{\partial\kappa_h} = 0,$$

and the complementary slackness condition $\mu[\zeta\kappa_l + (1-\zeta)\kappa_h] = 0$. Note that the two first-order conditions can be rewritten as

$$\alpha A_l\zeta^{\alpha-1} - \alpha\mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha(1-\zeta)^{\alpha-1} = \mu(\kappa_l-\kappa_h) + \mu\zeta\left(\frac{\partial\zeta}{\partial\kappa_l}\right)^{-1},$$

$$\alpha A_l\zeta^{\alpha-1} - \alpha\mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha(1-\zeta)^{\alpha-1} = \mu(\kappa_l-\kappa_h) + \mu(1-\zeta)\left(\frac{\partial\zeta}{\partial\kappa_h}\right)^{-1}.$$

First, we show that the Lagrange multiplier μ equals always zero. Suppose to the contrary that $\mu > 0$. Then, equating the two first-order conditions yields

$$\zeta\frac{\partial\zeta}{\partial\kappa_h} = (1-\zeta)\frac{\partial\zeta}{\partial\kappa_l}. \tag{6.27}$$

The derivatives of the capital allocation share ζ with respect to κ_l and κ_h are given by

$$\frac{\partial \zeta}{\partial \kappa_l} = -\frac{1}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}}\frac{\psi\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l(1+\psi\kappa_h)}$$
$$\times\left[1+\left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-2}$$
$$= \frac{-\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\frac{\psi}{1+\psi\kappa_l}$$
$$= \frac{-\zeta(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_l}$$

and

$$\frac{\partial \zeta}{\partial \kappa_h} = \frac{1}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}}\frac{\psi A_l\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha(1+\psi\kappa_l)}{A_l^2(1+\psi\kappa_h)^2}$$
$$\times\left[1+\left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-2}$$
$$= \frac{\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\frac{\psi}{1+\psi\kappa_h}$$
$$= \frac{\zeta(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_h}.$$

Using the latter two results, the condition (6.27) translates into

$$\frac{\zeta^2(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_h} = \frac{-\zeta(1-\zeta)^2}{1-\alpha}\frac{\psi}{1+\psi\kappa_l} \qquad \Leftrightarrow \qquad \frac{\zeta}{1+\psi\kappa_h} = \frac{\zeta-1}{1+\psi\kappa_l}$$

and further simplifies to

$$\zeta(1+\psi\kappa_l) = (\zeta-1)(1+\psi\kappa_h) \quad \Leftrightarrow \qquad \psi\{\zeta\kappa_l + (1-\zeta)\kappa_h\} = -1.$$

The latter equation contradicts the complementary slackness condition, which implies for any positive Lagrange multiplier (i.e., $\mu > 0$), $\zeta\kappa_l + (1-\zeta)\kappa_h = 0$. Thus, we can conclude that the Lagrange multiplier is always zero, i.e., $\mu = 0$. The two first-order conditions are therefore identical and given by

$$\alpha A_l \zeta^{\alpha-1} - \alpha \mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha (1-\zeta)^{\alpha-1} = 0.$$

This optimality condition can be rearranged to

$$A_l \zeta^{\alpha-1} = \mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha (1-\zeta)^{\alpha-1} \quad \Leftrightarrow \quad \frac{1-\zeta}{\zeta} = \left(\frac{\mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha}{A_l} \right)^{\frac{1}{1-\alpha}}.$$

Further rearranging yields that the optimal capital allocation is given by the share

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_g[A_{h,s}]}{A_l} \right)^{\frac{1}{1-\alpha}} \right]^{-1} =: \zeta_g.$$

Using the capital allocation in the decentralized equilibrium (see proposition 5.4.1), which is given by

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l} \frac{1+\psi\kappa_l}{1+\psi\kappa_h} \right)^{\frac{1}{1-\alpha}} \right]^{-1},$$

we can deduce that the optimal monetary policy must satisfy

$$\frac{\mathbb{E}_p[A_{h,s}(i)](1-i)^\alpha}{A_l} \frac{1+\psi\kappa_l}{1+\psi\kappa_h} = \frac{\mathbb{E}_g[A_{h,s}(i)](1-i)^\alpha}{A_l}$$

$$\Leftrightarrow \qquad \frac{1+\psi\kappa_h}{1+\psi\kappa_l} = \frac{\mathbb{E}_p[A_{h,s}(i)]}{\mathbb{E}_g[A_{h,s}(i)]} =: a(i).$$

Rearranging then yields

$$1 + \psi\kappa_h = (1+\psi\kappa_l)a(i) \quad \Leftrightarrow \quad \psi\kappa_h = \psi a(i)\kappa_l + a(i) - 1 \quad \Leftrightarrow \quad \kappa_h = a(i)\kappa_l + \frac{a(i)-1}{\psi}.$$

In addition, the cost factors must satisfy the constraint $\zeta_g \kappa_l + (1-\zeta_g)\kappa_h \geq 0$. Note that whenever beliefs satisfy $\eta_g > (<) \eta_p$ it follows $a(i) > (<) 1$ and therefore $\kappa_h > (<) \kappa_l$. ■

Proof of Corollary 5.5.1. Note that it holds

$$a(i) = \frac{(1-\eta_p)A_{h,b} + \eta_p A_{h,t}(i)}{(1-\eta_g)A_{h,b} + \eta_g A_{h,t}(i)},$$

where we used $A_{h,b} := A_{h,b}(i)$ for all i, following from assumption 5.5.1, which states $\partial A_{h,b}(i)/\partial i = 0$. The parameter $a(i)$ varies with CRMT investment according to

$$\begin{aligned}
\frac{\partial a(i)}{\partial i} &= \frac{\mathbb{E}_g[A_{h,s}(i)]\eta_p \frac{\partial A_{h,t}(i)}{\partial i} - \mathbb{E}_p[A_{h,s}(i)]\eta_g \frac{\partial A_{h,t}(i)}{\partial i}}{(\mathbb{E}_g[A_{h,s}(i)])^2} \\
&= \frac{\partial A_{h,t}(i)}{\partial i} \frac{\eta_p[(1-\eta_g)A_{h,b} + \eta_g A_{h,t}(i)] - \eta_g[(1-\eta_p)A_{h,b} + \eta_p A_{h,t}(i)]}{(\mathbb{E}_g[A_{h,s}(i)])^2} \\
&= \frac{\partial A_{h,t}(i)}{\partial i} \frac{(\eta_p - \eta_g)A_{h,b}}{(\mathbb{E}_g[A_{h,s}(i)])^2}.
\end{aligned}$$

Based on assumption 5.5.1, we know that $\partial A_{h,t}(i)/\partial i > 0$, so that we can conclude $\partial a(i)/\partial i < (>)0$ for $\eta_g > (<)\eta_p$.

From proposition 5.5.2, we know that if beliefs satisfy $\eta_g > (<)\eta_p$, it holds $a(i) > (<)1$ and therefore $\kappa_h > (<)\kappa_l$. Accordingly, we can deduce that the difference of cost factors $\kappa_h - \kappa_l$ is positive (negative) for beliefs satisfying $\eta_g > (<)\eta_p$ and decreases (increases) with higher CRMT investment, i.e., for a larger share i. Thus, we can conclude that CRMT investment reduces, independent of the beliefs, the intensity of central bank intervention, as measured by the absolute difference between cost factors $|\kappa_h - \kappa_l|$. ■

Proof of Lemma 5.6.1. From equation (5.17) in subsection 5.3.6, we know that the maximum leverage, which rules out bank recapitalization in the transition scenario, is given by

$$\varphi^S(\zeta) = \frac{(1 + r^D_{CB})(1+\psi)}{r^D_{CB}\Psi(\zeta) - r^L_l \zeta - r^L_{h,t}(1-\zeta)}.$$

To express this leverage using economic fundamentals, we first use the fact that, in equilibrium, banks must, due to perfect competition, make in expectation zero profits from granting loans funded with deposits (see subsection 5.4.2), i.e., $r^L_l \zeta + \mathbb{E}_p[r^L_{h,s}](1-\zeta) = r^D_{CB}\Psi(\zeta)$.

Then, using $\mathbb{E}_p[r^L_{h,s}] = r^L_{h,t} + (1-\eta_p)(r^L_{h,b} - r^L_{h,t})$, we get

$$\varphi^S(\zeta) = \frac{(1 + r^D_{CB})(1+\psi)}{(1-\eta_p)(r^L_{h,b} - r^L_{h,t})(1-\zeta)}.$$

Moreover, from corollary 5.4.1, we know that the interest rate on loans to the risky sector satisfies

$$\mathbb{E}_p[r^L_{h,s}] = r^D_{CB}(1 + \psi\tilde{\kappa}_h) \qquad \Leftrightarrow \qquad 1 + \mathbb{E}_p[r^L_{h,s}] = (1 + r^D_{CB})(1 + \psi\kappa_h),$$

where we used the definition $\tilde{\kappa}_h = \kappa_h(1 + r^D_{CB})/r^D_{CB}$. Accordingly, we obtain

$$\varphi^S(\zeta) = \frac{(1 + \mathbb{E}_p[r^L_{h,s}])(1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(r^L_{h,b} - r^L_{h,t})(1-\zeta)}.$$

Using assumption 5.3.1, which states $(1+r^L_{h,s})q = \alpha A_{h,s} K_h^{\alpha-1}$ for all s, the latter expression translates into

$$\varphi^S(\zeta) = \frac{\mathbb{E}_p[A_{h,s}](1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b} - A_{h,t})(1-\zeta)},$$

where we used

$$\frac{1 + \mathbb{E}_p[r^L_{h,s}]}{r^L_{h,b} - r^L_{h,t}} = \frac{1 + \mathbb{E}_p[r^L_{h,s}]}{(1 + r^L_{h,b}) - (1 + r^L_{h,t})} = \frac{(1 + \mathbb{E}_p[r^L_{h,s}])q}{(1 + r^L_{h,b})q - (1 + r^L_{h,t})q}$$

that further simplifies to

$$\frac{\alpha\mathbb{E}_p[A_{h,s}]K_h^{\alpha-1}}{\alpha A_{h,b}K_h^{\alpha-1} - \alpha A_{h,t}K_h^{\alpha-1}} = \frac{\mathbb{E}_p[A_{h,s}]}{A_{h,b} - A_{h,t}}.$$

■

Proof of Lemma 5.6.2. From subsection 5.4.2 we know that welfare in scenario $s \in \{b, t\}$ is generally given by

$$W^\lambda_s = C_s - \Lambda(\zeta)\mathbb{1}\{\varphi > \varphi^S(\zeta) \wedge s = t\} = W_s - \Lambda\mathbb{1}\{\varphi > \varphi^S(\zeta) \wedge s = t\}.$$

Using lemma 5.4.2, which provides W_s in terms of economic fundamentals, and the costs of bank recapitalization, i.e., $\Lambda(\zeta) = \lambda\alpha A_{h,t}(1-\zeta)^\alpha K^\alpha$, we know that welfare is given by

$$W_s^\lambda = \{A_l\zeta^\alpha + A_{h,s}(1-\zeta)^\alpha[1 - \lambda\alpha\mathbb{1}\{\varphi > \varphi^S(\zeta) \wedge s = t\}]\}K^\alpha.$$

■

Proof of Proposition 5.6.1. From lemma 5.6.1, we know that the maximum leverage ruling out bank recapitalization is given by

$$\varphi^S(\zeta) = \frac{\mathbb{E}_p[A_{h,s}](1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)}.$$

First, taking the derivative of $\varphi^S(\zeta)$ with respect to κ_l yields

$$\begin{aligned}\frac{\partial\varphi^S(\zeta)}{\partial\kappa_l} &= \frac{\mathbb{E}_p[A_{h,s}](1+\psi)(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})}{[(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)]^2}\frac{\partial\zeta}{\partial\kappa_l} \\ &= \frac{\mathbb{E}_p[A_{h,s}](1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)^2}\frac{\partial\zeta}{\partial\kappa_l} \\ &= \frac{\varphi^S(\zeta)}{1-\zeta}\frac{\partial\zeta}{\partial\kappa_l}.\end{aligned}$$

Note that it holds

$$\begin{aligned}\frac{\partial\zeta}{\partial\kappa_l} &= \frac{-\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}}\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{\psi}{1+\psi\kappa_h} \\ &= \frac{-\psi\zeta^2}{(1-\alpha)(1+\psi\kappa_l)}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \\ &= \frac{-\psi\zeta^2}{(1-\alpha)(1+\psi\kappa_l)}\frac{1-\zeta}{\zeta} \\ &= \frac{-\psi\zeta(1-\zeta)}{(1-\alpha)(1+\psi\kappa_l)} < 0.\end{aligned}$$

Accordingly, we obtain $\partial\varphi^S(\zeta)/\partial\kappa_l < 0$. Second, taking the derivative of $\varphi^S(\zeta)$ with respect to κ_h yields

$$\frac{\partial \varphi^S(\zeta)}{\partial \kappa_h} = \frac{-\mathbb{E}_p[A_{h,s}](1+\psi)(1-\eta_p)(A_{h,b}-A_{h,t})\left[\psi(1-\zeta)-(1+\psi\kappa_h)\frac{\partial \zeta}{\partial \kappa_h}\right]}{[(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)]^2},$$

which further simplifies to

$$\begin{aligned}
\frac{\partial \varphi^S(\zeta)}{\partial \kappa_h} &= \frac{\mathbb{E}_p[A_{h,s}](1+\psi)\left[(1+\psi\kappa_h)\frac{\partial \zeta}{\partial \kappa_h}-\psi(1-\zeta)\right]}{(1+\psi\kappa_h)^2(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)^2} \\
&= \frac{\varphi^S(\zeta)}{(1+\psi\kappa_h)(1-\zeta)}\left[(1+\psi\kappa_h)\frac{\partial \zeta}{\partial \kappa_h}-\psi(1-\zeta)\right] \\
&= \varphi^S(\zeta)\left[\frac{1}{1-\zeta}\frac{\partial \zeta}{\partial \kappa_h}-\frac{\psi}{1+\psi\kappa_h}\right].
\end{aligned}$$

Note that it holds

$$\begin{aligned}
\frac{\partial \zeta}{\partial \kappa_h} &= \frac{\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}}\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{(1+\psi\kappa_l)\psi}{(1+\psi\kappa_h)^2} \\
&= \frac{\psi\zeta^2}{(1-\alpha)(1+\psi\kappa_h)}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \\
&= \frac{\psi\zeta^2}{(1-\alpha)(1+\psi\kappa_h)}\frac{1-\zeta}{\zeta} \\
&= \frac{\psi\zeta(1-\zeta)}{(1-\alpha)(1+\psi\kappa_h)} > 0.
\end{aligned}$$

Thus, we obtain

$$\frac{1}{1-\zeta}\frac{\partial \zeta}{\partial \kappa_h} \geq \frac{\psi}{1+\psi\kappa_h} \quad \Leftrightarrow \quad \frac{\zeta}{1-\alpha}\frac{\psi}{1+\psi\kappa_h} \geq \frac{\psi}{1+\psi\kappa_h} \quad \Leftrightarrow \quad \zeta \geq 1-\alpha.$$

Then, it follows

$$\frac{\partial \varphi^S(\zeta)}{\partial \kappa_h} < (\geq) 0 \qquad \text{if and only if} \qquad \zeta < (\geq) 1-\alpha.$$

Third, note that for $\kappa_l \to -1/\psi$ the share ζ of capital good allocated to riskless firms is approaching one. From the structure of $\varphi^S(\zeta)$, we can conclude that $\lim_{\kappa_l \to -1/\psi} \varphi^S(\zeta) = +\infty$.

Fourth, we consider the case where the cost factor κ_h approaches infinity. Note that it follows from the structure of $\varphi^S(\zeta)$ that we only need to evaluate the limit of $(1+\psi\kappa_h)(1-\zeta)$ to obtain the limit of $\varphi^S(\zeta)$. Moreover, it holds that

$$\lim_{\kappa_h \to +\infty}(1+\psi\kappa_h)(1-\zeta) = \lim_{\kappa_h \to +\infty}(1+\psi\kappa_h)\zeta\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}},$$

since

$$\zeta = \left[1+\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1} \quad \Leftrightarrow \quad 1-\zeta = \zeta\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}.$$

Further rearranging yields

$$\begin{aligned}
&\lim_{\kappa_h \to +\infty}(1+\psi\kappa_h)\zeta\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \\
&\qquad = \lim_{\kappa_h \to +\infty}\zeta\left[\frac{\mathbb{E}_p[A_{h,s}](1+\psi\kappa_l)}{A_l}\right]^{\frac{1}{1-\alpha}}(1+\psi\kappa_h)^{-\frac{\alpha}{1-\alpha}} \\
&\qquad = \left[\frac{\mathbb{E}_p[A_{h,s}](1+\psi\kappa_l)}{A_l}\right]^{\frac{1}{1-\alpha}}\lim_{\kappa_h \to +\infty}\frac{\zeta}{(1+\psi\kappa_h)^{\frac{\alpha}{1-\alpha}}}.
\end{aligned}$$

It follows from the structure of the equilibrium share ζ of capital good allocated to the riskless sector that in the limit all capital good is used for production by riskless firms, $\lim_{\kappa_h \to +\infty}\zeta = 1$. We therefore obtain that $\lim_{\kappa_h \to +\infty}(1+\psi\kappa)(1-\zeta) = 0$ and furthermore

$$\lim_{\kappa_h \to +\infty}\varphi^S(\zeta) = \lim_{\kappa_h \to +\infty}\frac{\mathbb{E}_p[A_{h,s}](1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)} = +\infty.$$

Finally, let us focus on the costs of bank recapitalization $\Lambda(\zeta) = \lambda\alpha A_{h,t}(1-\zeta)^\alpha K^\alpha$. As we showed before, it holds $\partial\zeta/\partial\kappa_l < 0$ and $\partial\zeta/\partial\kappa_h > 0$. Thus, we can conclude

$$\frac{\partial\Lambda(\zeta)}{\partial\kappa_l} > 0 \qquad \text{and} \qquad \frac{\partial\Lambda(\zeta)}{\partial\kappa_h} < 0.$$

Moreover, based on $\lim_{\kappa_l \to -1/\psi} \zeta = \lim_{\kappa_h \to +\infty} \zeta = 1$, we further know that it holds $\lim_{\kappa_l \to -1/\psi} \Lambda(\zeta) = \lim_{\kappa_h \to +\infty} \Lambda(\zeta) = 0$. ■

Proof of Proposition 5.6.2. The maximum leverage $\varphi^S(\zeta)$ ruling out bank recapitalization varies with the beliefs η_p of private agents according to

$$\frac{\partial \varphi^S(\zeta)}{\partial \eta_p} = \frac{-(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})^2(1-\zeta)(1+\psi)}{\{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)\}^2}$$

$$- \frac{\mathbb{E}_p[A_{h,s}](1+\psi)(1+\psi\kappa_h)(A_{h,b}-A_{h,t})\left[-(1-\zeta)-(1-\eta_p)\frac{\partial \zeta}{\partial \eta_p}\right]}{\{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)\}^2}$$

$$= (1+\psi)\frac{-(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta)+\mathbb{E}_p[A_{h,s}]\left[1-\zeta+(1-\eta_p)\frac{\partial \zeta}{\partial \eta_p}\right]}{(1+\psi\kappa_h)(1-\eta_p)^2(A_{h,b}-A_{h,t})(1-\zeta)^2}$$

$$= (1+\psi)\frac{A_{h,t}(1-\zeta)+(1-\eta_p)\mathbb{E}_p[A_{h,s}]\dfrac{\partial \zeta}{\partial \eta_p}}{(1+\psi\kappa_h)(1-\eta_p)^2(A_{h,b}-A_{h,t})(1-\zeta)^2}.$$

Since it holds

$$\frac{\partial \zeta}{\partial \eta_p} = -\left[1+\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-2}\frac{1}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\times \frac{A_{h,t}-A_{h,b}}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}$$

$$= \frac{(A_{h,b}-A_{h,t})\zeta^2}{(1-\alpha)\mathbb{E}_p[A_{h,s}]}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}$$

$$= \frac{(A_{h,b}-A_{h,t})\zeta(1-\zeta)}{(1-\alpha)\mathbb{E}_p[A_{h,s}]} > 0,$$

we know that $\partial \varphi^S(\zeta)/\partial \eta_p > 0$. As the bank recapitalization costs are given by $\Lambda(\zeta) = \lambda\alpha A_{h,t}(1-\zeta)^\alpha K^\alpha$, we can conclude with $\partial \zeta/\partial \eta_p > 0$ that it holds $\partial \Lambda(\zeta)/\partial \eta_p < 0$. ■

Proof of Proposition 5.6.3. From lemma 5.6.2, we know that whenever the leverage satisfies $\varphi \leq \varphi^S(\zeta)$, no bank recapitalization oocurs and therefore it holds $W_s^\lambda = W_s$. We know that the optimal monetary policy maximizing the expected utilitarian welfare $\mathbb{E}_g[W_s]$ is characterized by proposition 5.4.2. This optimal monetary policy induces the capital allocation ζ_g by implementing cost factors that satisfy

$$\kappa_h = a\kappa_l + \frac{a-1}{\psi} \qquad \text{and} \qquad \zeta_g\kappa_l + (1-\zeta_g)\kappa_h \geq 0, \qquad \text{with} \qquad a = \frac{\mathbb{E}_p[A_{h,s}]}{\mathbb{E}_g[A_{h,s}]}.$$

From the outline in subsection 5.4.2, we know that the equilibrium leverage is given by $\varphi = (1+\psi)/\gamma$, so that under the optimal monetary policy bank recapitalization is only ruled out if

$$\varphi = (1+\psi)/\gamma \leq \varphi^S(\zeta_g) = \frac{\mathbb{E}_p[A_{h,s}](1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)}.$$

The maximum leverage $\varphi^S(\zeta)$ is highest for the lowest possible cost factor κ_h on risky loans, which is obtained by imposing $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h = 0$. ■

Proof of Lemma 5.6.3. By assumption, we know that with cost factors inducing the capital allocation ζ_g (see proposition 5.4.2) and satisfying $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h = 0$, bank recapitalization occurs in the transition, i.e., $\varphi = (1+\psi)/\gamma > \varphi^S(\zeta_g)$. From lemma 5.6.1, we know that for the cost factors $\kappa_l \to -1/\psi$ or $\kappa_h \to \infty$, bank recapitalization does not occur, i.e., $\lim_{\kappa_l\to-1/\psi}\varphi^S(\zeta) = \lim_{\kappa_h\to+\infty}\varphi^S(\zeta) = +\infty$. As $\varphi^S(\zeta)$ is a continuous function in κ_l and κ_h, we can conclude that there exist cost factors $\hat{\kappa}_l$ and $\hat{\kappa}_h$ inducing the capital allocation $\hat{\zeta}$ with $\varphi = (1+\psi)/\gamma = \varphi^S(\hat{\zeta})$ and satisfying $\hat{\zeta}\hat{\kappa}_l + (1-\hat{\zeta})\hat{\kappa}_h = 0$. ■

Proof of Proposition 5.6.4. Note that the central bank faces the optimization problem

$$\max_{\kappa_l,\kappa_h\in\mathbb{R}} [A_l\zeta^\alpha + (\mathbb{E}_g[A_{h,s}] - \eta_g\lambda\alpha A_{h,t}\mathbb{1}\{\varphi > \varphi^S(\zeta)\})(1-\zeta)^\alpha]K^\alpha,$$

$$\text{subject to} \quad \zeta\kappa_l + (1-\zeta)\kappa_h \geq 0,$$

where the share ζ of capital allocated to riskless firms satisfies

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

By assumption, we know that with cost factors inducing the capital allocation ζ_g (see proposition 5.4.2) and satisfying $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h = 0$, bank recapitalization occurs in the transition, i.e., $\varphi = (1+\psi)/\gamma > \varphi^S(\zeta_g)$. Thus, the central bank needs to decide whether it wants to rule out bank recapitalization or accept bank recapitalization but correct the capital allocation. In the first regime, the central bank sets the cost factors $\hat{\kappa}_l$ and $\hat{\kappa}_h$ inducing the capital allocation $\hat{\zeta} > \zeta_g$ with $\varphi = (1+\psi)/\gamma = \varphi^S(\hat{\zeta})$ and satisfying $\hat{\zeta}\hat{\kappa}_l + (1-\hat{\zeta})\hat{\kappa}_h = 0$. Note that it is optimal for the central bank to minimize liquidity costs, as otherwise it would have to set costs factors inducing a capital allocation $\hat{\hat{\zeta}} > \hat{\zeta} > \zeta_g$. However, without bank recapitalization welfare only depends on the allocation of capital allocation, so that the capital allocation $\hat{\hat{\zeta}}$ yields a lower welfare then the capital allocation $\hat{\zeta}$. In the second regime, the central bank accepts bank recapitalization in the transition and corrects the belief-driven capital distortion, while accounting for the costs arising from equity injections by shareholders. Formally, the central bank then faces within this regime the optimization problem

$$\max_{\kappa_l,\kappa_h\in\mathbb{R}} [A_l\zeta^\alpha + \mathbb{E}_g^\lambda[A_{h,s}](1-\zeta)^\alpha]K^\alpha \qquad \text{subject to} \qquad \zeta\kappa_l + (1-\zeta)\kappa_h \geq 0,$$

The optimal allocation follows by taking the derivatives of welfare with respect to κ_l and κ_h, and setting them to zero. With $\mu \geq 0$ denoting the Lagrange multiplier on the constraint, the optimality conditions are given by the two first-order conditions

$$\alpha A_l\zeta^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_l} - \alpha\mathbb{E}_g^\lambda[A_{h,s}](1-\zeta)^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_l} - \mu\zeta - \mu(\kappa_l-\kappa_h)\frac{\partial\zeta}{\partial\kappa_l} = 0,$$

$$\alpha A_l\zeta^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_h} - \alpha\mathbb{E}_g^\lambda[A_{h,s}](1-\zeta)^{\alpha-1}\frac{\partial\zeta}{\partial\kappa_h} - \mu(1-\zeta) - \mu(\kappa_l-\kappa_h)\frac{\partial\zeta}{\partial\kappa_h} = 0,$$

and the complementary slackness condition $\mu[\zeta\kappa_l + (1-\zeta)\kappa_h] = 0$. Note that the two first-order conditions can be rewritten as

$$\alpha A_l \zeta^{\alpha-1} - \alpha \mathbb{E}_g^{\lambda}[A_{h,s}](1-\zeta)^{\alpha-1} = \mu(\kappa_l - \kappa_h) + \mu\zeta \left(\frac{\partial \zeta}{\partial \kappa_l}\right)^{-1},$$

$$\alpha A_l \zeta^{\alpha-1} - \alpha \mathbb{E}_g^{\lambda}[A_{h,s}](1-\zeta)^{\alpha-1} = \mu(\kappa_l - \kappa_h) + \mu(1-\zeta) \left(\frac{\partial \zeta}{\partial \kappa_h}\right)^{-1}.$$

First, we show that the Lagrange multiplier μ equals always zero. Suppose to the contrary that $\mu > 0$. Then, equating the two first-order conditions yields

$$\zeta \frac{\partial \zeta}{\partial \kappa_h} = (1-\zeta)\frac{\partial \zeta}{\partial \kappa_l}. \tag{6.28}$$

The derivatives of the capital allocation share ζ with respect to κ_l and κ_h are given by

$$\begin{aligned}
\frac{\partial \zeta}{\partial \kappa_l} &= -\frac{1}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}} \frac{\psi \mathbb{E}_p[A_{h,s}]}{A_l(1+\psi\kappa_h)} \\
&\qquad \times \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-2} \\
&= \frac{-\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \frac{\psi}{1+\psi\kappa_l} \\
&= \frac{-\zeta(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_l}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \zeta}{\partial \kappa_h} &= \frac{1}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{\alpha}{1-\alpha}} \frac{\psi A_l \mathbb{E}_p[A_{h,s}](1+\psi\kappa_l)}{A_l^2(1+\psi\kappa_h)^2} \\
&\qquad \times \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-2} \\
&= \frac{\zeta^2}{1-\alpha}\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}} \frac{\psi}{1+\psi\kappa_h} = \frac{\zeta(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_h}.
\end{aligned}$$

Using the latter two results, the condition (6.28) translates into

$$\frac{\zeta^2(1-\zeta)}{1-\alpha}\frac{\psi}{1+\psi\kappa_h} = \frac{-\zeta(1-\zeta)^2}{1-\alpha}\frac{\psi}{1+\psi\kappa_l} \qquad \Leftrightarrow \qquad \frac{\zeta}{1+\psi\kappa_h} = \frac{\zeta-1}{1+\psi\kappa_l}$$

and further simplifies to

$$\zeta(1+\psi\kappa_l) = (\zeta-1)(1+\psi\kappa_h) \quad \Leftrightarrow \quad \psi\{\zeta\kappa_l + (1-\zeta)\kappa_h\} = -1.$$

The latter equation contradicts the complementary slackness condition, which implies for any positive Lagrange multiplier (i.e., $\mu > 0$), $\zeta\kappa_l + (1-\zeta)\kappa_h = 0$. Thus, we can conclude that the Lagrange multiplier is always zero, i.e., $\mu = 0$. The two first-order conditions are therefore identical and given by

$$\alpha A_l \zeta^{\alpha-1} - \alpha \mathbb{E}_g^\lambda[A_{h,s}](1-\zeta)^{\alpha-1} = 0.$$

This optimality condition can be rearranged to

$$A_l\zeta^{\alpha-1} = \mathbb{E}_g^\lambda[A_{h,s}](1-\zeta)^{\alpha-1} \qquad \Leftrightarrow \qquad \frac{1-\zeta}{\zeta} = \left(\frac{\mathbb{E}_g^\lambda[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}.$$

Further rearranging yields that the optimal capital allocation is given by the share

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_g^\lambda[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right]^{-1} =: \zeta_g^\lambda.$$

Using the capital allocation in the decentralized equilibrium (see proposition 5.4.1), which is given by

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1},$$

we can deduce that the optimal monetary policy must satisfy

$$\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h} = \frac{\mathbb{E}_g^\lambda[A_{h,s}]}{A_l} \qquad \Leftrightarrow \qquad \frac{1+\psi\kappa_h}{1+\psi\kappa_l} = \frac{\mathbb{E}_p[A_{h,s}]}{\mathbb{E}_g^\lambda[A_{h,s}]} =: a_\lambda.$$

Rearranging then yields

$$1+\psi\kappa_h=(1+\psi\kappa_l)a_\lambda \quad\Leftrightarrow\quad \psi\kappa_h=\psi a_\lambda\kappa_l+a_\lambda-1 \quad\Leftrightarrow\quad \kappa_h=a_\lambda\kappa_l+\frac{a_\lambda-1}{\psi}.$$

The central bank decides between the first and the second regime based on a welfare comparison. It implements the monetary policy inducing $\hat{\zeta}$ (ζ_g^λ) if and only if $\mathbb{E}_g[W_s(\hat{\zeta})] \geq (<)\mathbb{E}_g[W_s^\lambda(\zeta_g^\lambda)]$. ∎

Proof of Proposition 5.6.5. Note that the capital allocation in the decentralized equilibrium is provided proposition 5.4.1 and given by

$$\zeta=\left[1+\left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

One the hand, the central bank aims at inducing the capital allocation ζ_g, which it finds given its belief and without bank recapitalization to be optimal one, where

$$\zeta_g=\left[1+\left(\frac{\mathbb{E}_g[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right].$$

Equating ζ and ζ_g yields

$$\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}=\frac{\mathbb{E}_g[A_{h,s}]}{A_l} \qquad\Leftrightarrow\qquad \frac{1+\psi\kappa_h}{1+\psi\kappa_l}=\frac{\mathbb{E}_p[A_{h,s}]}{\mathbb{E}_g[A_{h,s}]}=:a.$$

Rearranging then yields

$$1+\psi\kappa_h=(1+\psi\kappa_l)a \quad\Leftrightarrow\quad \psi\kappa_h=\psi a\kappa_l+a-1 \quad\Leftrightarrow\quad \kappa_h=a\kappa_l+\frac{a-1}{\psi}.$$

On the other hand, the central bank aims at eliminating bank recapitalization. When implementing ζ_g, bank recapitalization is ruled out whenever it holds $\varphi=(1+\psi)/\gamma=$

$\varphi^S(\zeta_g)$ or, equivalently,

$$(1+\psi)/\gamma = \frac{\mathbb{E}_p[A_{h,s}](1+\psi)}{(1+\psi\kappa_h)(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)}$$

$$\Leftrightarrow \qquad 1+\psi\kappa_h = \frac{\mathbb{E}_p[A_{h,s}]\gamma}{(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)}$$

$$\Leftrightarrow \qquad \kappa_h = \frac{\mathbb{E}_p[A_{h,s}]\gamma}{\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1}{\psi}.$$

Combining the two previous conditions on κ_h, we obtain

$$a\kappa_l + \frac{a-1}{\psi} = \frac{\mathbb{E}_p[A_{h,s}]\gamma}{\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1}{\psi}$$

$$\Leftrightarrow \qquad \kappa_l = \frac{\mathbb{E}_p[A_{h,s}]\gamma}{a\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1}{\psi}$$

$$\Leftrightarrow \qquad \kappa_l = \frac{\mathbb{E}_p[A_{h,s}]\gamma}{a\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1}{\psi}$$

$$\Leftrightarrow \qquad \kappa_l = \frac{\mathbb{E}_g[A_{h,s}]\gamma}{\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1}{\psi},$$

where we used $a = \mathbb{E}_p[A_{h,s}]/\mathbb{E}_g[A_{h,s}]$. Note that banks receive an implicit subsidy by borrowing reserves, so that the central bank must implement quantity restrictions on reserve loans, if $r^L_{CB}(\zeta_g) < r^D_{CB}$ or, equivalently, $\zeta_g\kappa_l + (1-\zeta_g)\kappa_h < 0$. The latter inequality reads as

$$\frac{\zeta_g\mathbb{E}_p[A_{h,s}]\gamma}{a\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{\zeta_g}{\psi}$$
$$+ \frac{(1-\zeta_g)\mathbb{E}_p[A_{h,s}]\gamma}{\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} - \frac{1-\zeta_g}{\psi} < 0,$$

which further simplifies to

$$\begin{aligned}
& \frac{\mathbb{E}_p[A_{h,s}]\gamma[\zeta_g + a(1-\zeta_g)]}{a\psi(1-\eta_p)(A_{h,b}-A_{h,t})(1-\zeta_g)} < \frac{1}{\psi} \\
\Leftrightarrow \quad & \frac{\zeta_g + a(1-\zeta_g)}{1-\zeta_g} < \frac{a(1-\eta_p)(A_{h,b}-A_{h,t})}{\mathbb{E}_p[A_{h,s}]\gamma} \\
\Leftrightarrow \quad & \frac{\zeta_g}{1-\zeta_g} < a\left[\frac{(1-\eta_p)(A_{h,b}-A_{h,t})}{\mathbb{E}_p[A_{h,s}]\gamma} - 1\right]
\end{aligned}$$

and finally reads as

$$\begin{aligned}
& \frac{\zeta_g}{1-\zeta_g} < a\left[\frac{(1-\eta_p)(A_{h,b}-A_{h,t})}{\mathbb{E}_p[A_{h,s}]\gamma} - \frac{\mathbb{E}_p[A_{h,s}]\gamma}{\mathbb{E}_p[A_{h,s}]\gamma}\right] \\
\Leftrightarrow \quad & \frac{\zeta_g}{1-\zeta_g} < \frac{(1-\eta_p)(A_{h,b}-A_{h,t})(1-\gamma) - A_{h,t}\gamma}{\mathbb{E}_g[A_{h,s}]\gamma}.
\end{aligned}$$

■

Proof of Proposition 5.7.1. Suppose the central bank aims at setting cost factors such that it induces the capital allocation ζ_t. From proposition 5.4.1, we know that the capital allocation in the decentralized equilibrium is given by

$$\zeta = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1}.$$

Equating ζ_t and ζ yields that the cost factors κ_l and κ_h must satisfy

$$\begin{aligned}
& \zeta_t = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\frac{1+\psi\kappa_l}{1+\psi\kappa_h}\right)^{\frac{1}{1-\alpha}}\right]^{-1} \\
\Leftrightarrow \quad & \frac{1+\psi\kappa_h}{1+\psi\kappa_l} = \frac{\mathbb{E}_p[A_{h,s}]}{A_l}\left(\frac{\zeta_t}{1-\zeta_t}\right)^{1-\alpha} =: a_t.
\end{aligned}$$

Rearranging then leads to

$$1 + \psi\kappa_h = (1+\psi\kappa_l)a_t \quad \Leftrightarrow \quad \psi\kappa_h = \psi a_t\kappa_l + a_t - 1$$

and finally

$$\kappa_h = a_t \kappa_l + \frac{a_t - 1}{\psi} \qquad \Leftrightarrow \qquad \kappa_h = a_t \kappa_l + \frac{a_t - 1}{\psi}.$$

Note that the cost factors must also satisfy the constraint $\zeta_t \kappa_l + (1 - \zeta_t)\kappa_h \geq 0$. Whenever it holds $\zeta_t = \zeta_p$, we know that

$$\zeta_t = \left[1 + \left(\frac{\mathbb{E}_p[A_{h,s}]}{A_l}\right)^{\frac{1}{1-\alpha}}\right]^{-1} \qquad \Leftrightarrow \qquad \left(\frac{\zeta_t}{1 - \zeta_t}\right)^{1-\alpha} = \frac{\mathbb{E}_p[A_{h,s}]}{A_l},$$

and thus $a_t = 1$. Accordingly, whenever $\zeta_t = \zeta_p$, there is no central bank intervention and cost factors equal, i.e., $\kappa_l = \kappa_h$. We can also conclude that for any $\zeta_t > (<)\zeta_p$, it holds $a_t > (<)1$ and therefore $\kappa_h > (<)\kappa_l$. ■

Proof of Corollary 5.7.1. From proposition 5.7.1, we know that the optimal cost factors satisfy

$$\kappa_h = a_t \kappa_l + \frac{a_t - 1}{\psi} \quad \Leftrightarrow \quad \frac{1 + \psi \kappa_h}{1 + \psi \kappa_l} = a_t, \quad \text{with} \quad a_t = \frac{\mathbb{E}_p[A_{h,s}]}{A_l}\left(\frac{\zeta_t}{1 - \zeta_t}\right)^{1-\alpha}.$$

Note that a_t increases with ζ_t and decreases with η_p, so that we can conclude that the difference between the optimal cost factors $\kappa_h - \kappa_l$ increases with ζ_t and decreases with η_p. ■

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