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Global and Local Components of Output Gaps

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Abstract

This paper proposes a multi-level dynamic factor model to identify common components in output gap estimates. We pool multiple output gap estimates for 157 countries and decompose them into one global, eight regional, and 157 country-specific cycles. Our approach easily deals with mixed frequencies, ragged edges, and discontinuities in the underlying output gap estimates. To restrict the parameter space in the Bayesian state space model, we apply a stochastic search variable selection approach and base the prior inclusion probabilities on spatial information. Our results suggest that the global and the regional cycles explain a substantial proportion of the output gaps. On average, 18% of a country's output gap is attributable to the global cycle, 24% to the regional cycle, and 58% to the local cycle.

JEL Classification: C11, C32, C52, F44, R11
Keywords: Multi-Level DFM, Bayesian State Space Model, Output Gap Decomposition, Model Combination, Business Cycles, Variable Selection, Spatial Prior.

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1 Introduction

Fiscal and monetary policy authorities use the output gap to determine the cyclical position of the economy, detect structural imbalances, and predict inflationary pressure (see, for instance, Gerlach and Smets, 1999; Sturm and de Haan, 2011; Coibion and Gorodnichenko, 2015). As a result, output gaps are usually estimated at a level which corresponds to the economic aggregate that is affected by policy decisions. Typically, this corresponds to the national level. However, it is increasingly common that counter-cyclical policies are implemented at transnational levels. Countries might coordinate policy actions during crisis periods, or delegate them to a centralized institution if they are members of a monetary or customs union. In these instances, it is important whether cyclical imbalances are unique to one country, shared with nearby countries, or even common to all countries. This information allows policy actions to be taken at the appropriate level, thereby avoiding the issue of pro-cyclical outcomes of policy measures. For instance, in the case of a negative country-specific shock that leads to a deflationary output gap in only one particular country, it would not be efficient for a cross-country institution (for example, the central bank in a currency area) to act. By doing so, it would only contribute to imbalances in countries that are not affected by this shock. Instead, only the government of the affected country should intervene with counter-cyclical policies. If, on the other hand, a shock affects all countries in a region, cross-country measures taken by a centralized institution or coordinated interventions by several countries are probably more efficient in limiting the economic impact. Standard methods for estimating the output gap do not allow to directly identify a possible transnational overlap of business cycles and, therefore, do not provide guidance on the level of government at which policymakers should intervene with measures to stabilize the cycle.

To deal with the problem of allocating responsibility for policy measures, we propose a multi-level dynamic factor model (DFM). We pool a large collection of output gap estimates on the country level and extract the global, regional, and local components of these output gaps. We build upon the multi-level factor model proposed by Kose et al. (2003). Their model extracts common factors from macroeconomic data at different hierarchical levels. It identifies global, regional, and country-specific factors in output, consumption, and investment. Similar decomposition exercises and methodological refinements can be found in Del Negro and Otrok (2008) for GDP growth rates, Moench et al. (2013) for US economic variables, and Bai and Wang (2015) for international bond yields. We extend the literature on multi-level dynamic factor models by focusing on output gaps, thereby providing several contributions to the literature on output gap estimation and the use of spatial information in model selection.

First, we extend the literature on the estimation of multi-level factor models to identify global business cycles. In contrast to the aforementioned literature, we use several output gap estimates as 'data' and identify the factors using linear constraints on the factor loadings. Since the underlying time series share the same unit, which is the deviation of

GDP from potential output in percent, all cyclical components have a clear interpretation as the contribution of a hierarchical level to a country's output gap in percentage points. Furthermore, our model allows for mixed-frequencies, ragged edges, and missing observations in the data such that we are able to include a large set of countries in our analysis. The model can easily be adapted to assess if any aggregate of countries shares a common cycle by changing the hierarchical structure. For instance, by forming country aggregates based on currency unions, trade agreements, language, or income level.

Second, we provide an efficient Bayesian sampling algorithm that introduces sparsity into the large-dimensional vector autoregressive (VAR) coefficient matrices by means of stochastic search variable selection. Since VARs involve a large number of coefficients, it is necessary to use shrinkage or selection algorithms to reduce the dimensionality of the model. For cross-sectional spatial data, LeSage and Krivelyova (1999) and LeSage and Cashell (2015) propose to shrink coefficients by incorporating spatial information in the prior specification. Their priors are based on the belief that neighboring spatial entities exert a stronger influence than non-adjacent entities. We extend their approach in several ways. We employ stochastic search variable selection to determine which coefficients are likely to be different from zero and should therefore be included in the regression. This approach is far more parsimonious as it only requires prior inclusion probabilities instead of normal priors. The inclusion probabilities also allow us to use spatial information in a generic way that requires less tuning. Furthermore, we propose a finer measurement for geographic proximity, namely, a continuous setup based on distances between countries instead of a binary setup based on contiguities.

Third, we provide a comprehensive empirical study on global and local components of output gaps. In a recent paper, Kose et al. (2020) explain that identifying 'global recessions' is compounded by the fact that the definition for national recessions, namely a contraction in output for at least two consecutive quarters, cannot be applied at the global level because a contraction in total global output is extremely rare. We complement this literature with a model that quantifies global, regional, and national recessions. Our data includes five different output gap estimation methods in yearly and quarterly frequency for 157 countries grouped into eight regions and covering the years 1990 to 2020. Our results suggest a strong global cycle because a substantial part of the output gap is shared with all countries. In addition, most regions exhibit a strong common business cycle. The strongest common movement exhibit the countries in North America, Europe, and East and Southeast Asia. In contrast, the regions Sub-Saharan Africa and North Africa and West Asia display almost no significant common fluctuations. On average, 18% of a country's output gap is attributable to the global cycle, 24% to the regional cycle, and 58% to the local cycle. Furthermore, our model provides output gap estimates that are reconciled across multiple output gap estimation methods, frequencies, and countries. The reconciliation across countries is based on the assumption of co-moving output gaps across all countries. The model gives a structure in the cross-country output gaps such that the estimations borrow strength from the estimated output gaps of other countries.

The remainder of the paper is structured as follows. Section 2 presents the multi-level dynamic factor model and discusses the assumptions necessary to identify the parameters. It shows the use of stochastic search variable selection in estimating the VAR coefficients and highlights the benefits of spatial information to determine prior inclusion probabilities. Section 3 presents the data used in the empirical application and discusses the decomposition of output gaps into global, regional, and local cycles. Section 4 concludes.

2 A Large Multi-Level Dynamic Factor Model

2.1 Measurement

We estimate a multi-level dynamic factor model to extract common components from output gaps for various countries. It is assumed that the output gaps are linear functions of a hierarchical structure. Each output gap estimate consists of three factors: a global factor that is common to all countries, a regional factor that is common only to countries in a specific region, and a country-specific factor. Since we attempt to measure the output gap using several models, an additional error term captures the country- and model-specific error. In the absence of intertemporal constraints, the measurement equation is given by

$$\mathbf{y}_t = \mathbf{\Lambda} \mathbf{f}_t + \mathbf{e}_t, \qquad \mathbf{e}_t \sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Sigma}\right), \tag{1}$$

where $\mathbf{y}_t$ is an n-dimensional vector containing output gap estimates that are subject to measurement errors. The factor loadings matrix $\mathbf{\Lambda}$ is of order $n \times q$ and encodes the linear hierarchical constraints. The factors are stacked into a q-dimensional vector $\mathbf{f}_t$ that adheres to the linear constraints implied by the loadings matrix. The errors are distributed normally with mean zero and diagonal covariance matrix $\mathbf{\Sigma}$. The assumption of uncorrelated errors can be made since we assume that correlated innovations are likely to be captured by a common factor at a higher level, due to the hierarchical structure of the factors.

Such a multi-level factor model was proposed by Kose et al. (2003). In contrast to their approach, we restrict the loadings matrix $\mathbf{\Lambda}$ to be binary. Therefore, the factors contribute either fully to a particular output gap estimate or not at all. This has the advantage that the factors have a direct interpretation as additive contributions to a country's output gap in percentage points. From a Bayesian perspective, this specification can be interpreted as assigning zero prior probability to outcomes that do not adhere to the linear constraints. Moreover, this restriction ensures that the world factor influences the output gap of all countries equally, and the regional factors influence the output gap of each country within a region equally. Hence, the factors are identified straightforward as additive components of output gaps.

As a basic example, we take a collection of four countries A, B, C, and D. The countries A and B form one region, C and D form a second region. Assuming that the output gaps for each country are estimated using two methods, the variables involved in equation (1) are given by

$$\underset{(n\times 1)}{\mathbf{y}_t} = \begin{bmatrix} y_{A1,t} \\ y_{A2,t} \\ y_{B1,t} \\ y_{B2,t} \\ y_{C1,t} \\ y_{C2,t} \\ y_{D1,t} \\ y_{D2,t} \end{bmatrix} \quad \underset{(n\times q)}{\mathbf{\Lambda}} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \underset{(q\times 1)}{\mathbf{f}_t} = \begin{bmatrix} f_{ABCD,t} \\ f_{AB,t} \\ f_{CD,t} \\ f_{A,t} \\ f_{B,t} \\ f_{C,t} \\ f_{D,t} \end{bmatrix}.$$

The output gap for country A, estimated with method 1, is then given by $y_{A1,t} = f_{ABCD,t} + f_{AB,t} + f_{A,t} + e_{A1,t}$.

The hierarchical affiliation of each country to global and local cycles is straightforward. Regional cycles, however, can be a matter of discretion and their boundaries should be determined carefully. The inclusion of regional aggregates serves two main purposes. First, they eliminate or reduce cross-correlation in the measurement errors. Second, they allow for the examination of common cyclical components in known aggregates, such as currency or customs unions. It is, therefore, important to determine regional boundaries such that they include homogeneous countries according to economic, political, or geographical criteria.[1]

In addition, we may wish to impose temporal linear constraints on the factors. These are usually of interest if the data is sampled at varying frequencies. Output gap estimates, for instance, are usually sampled at quarterly or annual frequency. Mixed frequencies in the data can be implemented easily by specifying the factors at the highest frequency and letting low-frequency variables also load on lagged factors. In a first step, the mixed-frequency data is coerced to a matrix. All time series are converted to the highest frequency and extended to match the time of the first and last observation across the entire collection. A time series is assigned a value of zero in a specific period if there are missing observations due to publication delays, a gap in the data, or a limited history. Time series with lower frequencies are converted to the highest frequency by registering each observation in the last high-frequency entry of the corresponding low-frequency period. All other entries are filled with zeros. When mixing annual and quarterly data, for instance, the annual observation is registered in the last quarter of the corresponding year and the entries of the first three quarters are set to zero. This results in a matrix of order $T \times n$ where $t = 1, \ldots, T$ is the high-frequency time index and n the number of variables. To impose temporal linear constraints, equation (1) is extended using selection matrices that allow

[1] Francis et al. (2017) measure comovement in business cycles with a factor model and determine the grouping of the countries endogenously. Their results suggest that countries with similar legal institutions and linguistic diversity share a stronger common cycle than physically close countries. However, we focus in this study only on the spatial and regional interdependencies to examine the existence of regional business cycles.

for missing observations and distributed lag matrices that enforce the linear constraints. The extended measurement equation is given by

$$\mathbf{y}_t = \mathbf{S}_t \left(\mathbf{L}_0 \mathbf{\Lambda} \mathbf{f}_t + \mathbf{L}_1 \mathbf{\Lambda} \mathbf{f}_{t-1} + \ldots + \mathbf{L}_s \mathbf{\Lambda} \mathbf{f}_{t-s} + \mathbf{e}_t\right), \qquad \mathbf{e}_t \sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Sigma}\right), \tag{2}$$

where $\mathbf{S}_t$ is a diagonal selection matrix of order $n \times n$, featuring ones on the diagonal if the corresponding value in $\mathbf{y}_t$ is observed and zeros otherwise. Such a selection matrix has been used for instance in Banbura and Modugno (2014) and ensures that the factors are only updated if the data is indeed observed. An annual output gap, for instance, is only registered in the last quarter of a year and set to zero else. The corresponding entry in $\mathbf{S}_t$ is set to one in the last quarter and zero else, ensuring that both sides of the equation equal zero if the annual output gap is not observed. The distributed lag matrices $\mathbf{L}_0, \ldots, \mathbf{L}_s$ are also diagonal matrices that impose linear temporal constraints on the factors (Bai and Wang, 2015; Stock and Watson, 2016). Since annual output gaps are simply an average of the quarterly output gaps, the entries in the distributed lag matrices are either 1, 0, or 0.25. An annual output gap, registered only in the fourth quarter, is given by the average of the factors in the current quarter and the three preceding quarters. A quarterly output gap only loads on the factors in the corresponding quarter.

2.2 State Transition

The state equation determines how the hierarchical components interact. It may be, for instance, that small, globalized economies react stronger to changes in the global cycle than large, self-sustaining countries. The global, regional, and local components at time t are stacked in a vector $\mathbf{f}_t$. To allow for comovements between and within all levels, it is assumed that $\mathbf{f}_t$ follows a vector autoregressive process according to the state equation

$$\mathbf{f}_t = \mathbf{\Phi}_1 \mathbf{f}_{t-1} + \ldots + \mathbf{\Phi}_p \mathbf{f}_{t-p} + \mathbf{v}_t, \qquad \mathbf{v}_t \sim \mathcal{N}\left(\mathbf{0}, \mathbf{\Omega}\right), \tag{3}$$

where $\mathbf{\Phi}_1, \ldots, \mathbf{\Phi}_p$ are $q \times q$ matrices of autoregressive coefficients at lags 1 to p. The autoregressive coefficients are restricted to non-negative values. This is a plausible assumption given that a negative synchronization of business cycles is unlikely. The error term $\mathbf{v}_t$ is normally distributed with mean zero and covariance matrix $\mathbf{\Omega}$.

The estimation and interpretation of $\mathbf{\Phi}_1, \ldots, \mathbf{\Phi}_p$ is demanding due to the large number of autoregressive coefficients involved. To deal with the issue of over-parameterization, we employ a stochastic search variable selection approach that reduces dimensionality by selecting only a few relevant variables (George and McCulloch, 1993). There are several advantages to selecting a subset of variables as opposed to using shrinkage priors. Kuo and Mallick (1998) state that subset selection enables a sparse parameterization, increases the precision of statistical estimates, and allows differentiating between important and negligible predictors. We follow Kuo and Mallick (1998) in embedding indicator variables in the regression equation and rewrite the vector autoregression as in Chan (2019). Defining

$vec([\mathbf{\Phi}_1, \ldots, \mathbf{\Phi}_p]') = \mathbf{\Gamma}\boldsymbol{\theta}$, we can express equation (3) as

$$\mathbf{f}_t = \mathbf{X}_t \mathbf{\Gamma} \boldsymbol{\theta} + \mathbf{v}_t, \tag{4}$$

where $\mathbf{X}_t = \mathbf{I}_q \otimes [\mathbf{f}'_{t-1}, \ldots, \mathbf{f}'_{t-p}]$. $\boldsymbol{\theta}$ contains the correspondingly stacked vector autoregressive coefficients. The diagonal matrix $\mathbf{\Gamma}$ contains pq^2 binary indicator variables γ_{ijr} that follow a Bernoulli distribution with probabilities $\pi_{1,ijr}$. Each entry in $\mathbf{f}_t$ can be expressed equivalent to equation (4) as

$$f_{i,t} = \sum_{j=1}^{q} \sum_{r=1}^{p} \gamma_{ijr} \theta_{ijr} f_{j,t-r} + v_{i,t} \tag{5}$$

by indexing the dependent variable with i, the independent variable with j, and the lag with r. The indicator variable γ_{ijr} determines if the predictor is included ($\gamma_{ijr} = 1$) or excluded ($\gamma_{ijr} = 0$) from the reduced regression model. If $\gamma_{ijr} = 0$, the factor of country i is not influenced by the factor of country j at lag r and the model does not estimate θ_{ijr}. If $\gamma_{ijr} = 1$, $f_{j,t-r}$ influences $f_{i,t}$ and the model estimates the corresponding coefficient θ_{ijr}.

2.3 Spatial Prior Information

In the absence of prior information on how important a predictor is, a prior inclusion probability of 0.5 is typically chosen for all indicators. This reflects a prior belief that all variables are equally likely to be included. However, as O'Hara and Sillanpää (2009) point out, a typical problem with stochastic search variable selection is poor mixing during the sampling process. This issue can be tackled by using informative priors for the probability of a variable being included in the model. As suggested by George and McCulloch (1993), priors can also be used to influence the number of variables included in the model. For output gaps, it is reasonable to form priors based on the assumptions that output gaps are fairly persistent over time and that comovements are usually stronger for geographically close countries and regions. To impose time dependence, we follow the widely used Minnesota prior and assume that own lags as well as more recent lags are more important (Doan et al., 1984; Koop and Korobilis, 2009). Regarding the spatial dependence, we follow LeSage and Krivelyova (1999) and LeSage and Cashell (2015) in assuming that neighboring spatial entities exert a stronger influence than non-adjacent entities.[2] They set the prior mean for adjacent countries to one and for non-adjacent countries to zero. Whereas LeSage and Krivelyova (1999) assume only spatial dependence, LeSage and Cashell (2015) implement spatial as well as time dependence. We extend their procedure by using a stochastic search variable selection approach. This approach allows including the spatial information in a straightforward fashion. Instead of a normal prior that requires care-

[2]For simplicity, we only account for the time and spatial dependence of output gaps. However, other country characteristics as its economic size or a bilateral measure as the trade volume between two countries certainly also determine if two countries exhibit a common cyclical movement and would be an alternative way to define the priors.

ful tuning, we only need to specify the prior probability that a coefficient is included in the reduced model. Furthermore, instead of a binary setup based on contiguities, we use a continuous measure based on geographic distances. This gives us a finer mapping of geographical proximity to inclusion probability.

We determine a prior inclusion probability $\pi_{0,ijr}$ for each autoregressive coefficient. Since we have a total of q global, regional, and local factors that follow a VAR process of order p, there are pq^2 binary indicators. We use the following equation to calculate $\pi_{0,ijr}$ for the dependent variable i and independent variable j

$$\pi_{0,ijr} = \left(\frac{\exp(-\rho d_{ij})}{\sum_j \exp(-\rho d_{ij})} \right)^{\alpha} \frac{1}{r^{\beta}}, \qquad \text{for } j \neq i \text{ and } j, i \neq world \tag{6}$$

where d_{ij} is the shortest distance between any point in the polygons of the spatial entities i and j.[3] The first fraction in the equation, $\frac{\exp(-\rho d_{ij})}{\sum_j \exp(-\rho d_{ij})}$, transforms the distances d_{ij} into probabilities using the normalized exponential function, also referred to as the softmax function. It accounts for the assumption that the dependence between two geographic entities decreases exponentially with their distance. The normalization ensures that the vector of prior probabilities of an entity i sums to one. Note that this vector does not contain the values for the own lag ($j = i$) and the lag of the world factor ($j = world$). Put in other words, the denominator sums over all j except for the own and the world factor. We explain in the next two paragraphs how we calculate the prior inclusion probabilities for the own lags and the world factor. The hyperparameter $\rho > 0$ regulates the degree of shrinkage, depending on the distance: The higher ρ, the stronger the decay with distance. The minus before ρ ensures that the entities with the shortest geographic distance have a higher prior inclusion probability. The hyperparameter $\alpha > 0$ scales the prior inclusion probabilities, which allows a researcher to influence the number of included variables and, hence, the model size. The second fraction in equation (6), $\frac{1}{r^{\beta}}$, introduces a decay over the lag length r. The hyperparameter $\beta > 0$ leads to an exponential decrease of the prior probability with increasing lag length.

The prior inclusion probabilities for the own lags ($j = i$), the world factor as the predictor ($j = world$), and all predictors in the equation with the world factor as the dependent variable ($i = world$) require special treatment because we cannot build upon distances. For the prior inclusion probabilities of the own lags ($j = i$), we impose the belief that output gaps are relatively persistent over time. We use the following equation

$$\pi_{0,iir} = \frac{1}{r^{\beta}} \tag{7}$$

such that the probability of the variable being included is one for the first lag and decays exponentially with increasing lag length. Similarly, we set the prior probability for the world factor as the predictor variable j to $\frac{1}{r^{\beta}}$. We thereby assume that the lagged world

[3] We transform the coordinates of the polygons with the Plate Carree projection, an equidistant cylindrical method.

factor is an important predictor for all factors. In the equation with the world factor as the dependent variable i and $r = 1$, we set the prior for each region j to one divided by the number of regions and the prior for each country j to one divided by the number of countries. For $r > 1$, we divide the values of $r = 1$ by r^β such that a lag decay is introduced. Consequently, we assume that the temporal dependence decreases with the lag length and that the world factor depends more on the lagged regional factors compared to the lagged country-specific factors.

The pq^2-dimensional vector with all prior inclusion probabilities is defined as $\boldsymbol{\pi}_0 = vec([\boldsymbol{\pi}_{0,r=1}, \dots, \boldsymbol{\pi}_{0,r=p}]')$ where

$$\boldsymbol{\pi}_{0,r} = \begin{bmatrix} \pi_{0,i=1,j=1,r} & \pi_{0,i=1,j=2,r} & \cdots & \pi_{0,i=1,j=q,r} \\ \pi_{0,i=2,j=1,r} & \pi_{0,i=2,j=2,r} & \cdots & \pi_{0,i=2,j=q,r} \\ \vdots & \vdots & & \vdots \\ \pi_{0,i=q,j=1,r} & \pi_{0,i=q,j=2,r} & \cdots & \pi_{0,i=q,j=q,r} \end{bmatrix} \quad \text{for } r \in \{1, ..., p\}.$$

With $\alpha = 1$ our specification implies that the sum of each row in $\boldsymbol{\pi}_{0,r=1}$ is three. Consequently, if $p = 1$, our prior belief is that in each autoregressive equation three out of the q factors have a nonzero regression coefficient. For the regions and countries this sum of three consists of a probability of one for the lag of the world factor, one for the own lag, and one in total for the remaining countries and regions where the probability depends on the distances. The sum of each row in $\boldsymbol{\pi}_{0,r}$ decreases with increasing r as we expect that fewer regression coefficients are different from zero. With $\beta = 02$, the sum of each row in $\boldsymbol{\pi}_{0,r=2}$ is equal to 0.75, in $\pi_{0,r=3}$ equal to 0.33, in $\pi_{0,r=4}$ equal to 0.19, and so forth. In appendix A.2 we provide two examples of the probabilities.

2.4 Estimation

Since $\mathbf{S}_t$ and $\mathbf{L}_0, \dots, \mathbf{L}_s$ are known ex ante, the computational task reduces to estimating the dynamic factors $\mathbf{f}_t$ and the parameter $\boldsymbol{\Phi}_1, \dots, \boldsymbol{\Phi}_p$, $\boldsymbol{\Sigma}$, and $\boldsymbol{\Omega}$. We sample the latent states jointly using the efficient and sparse algorithm by Chan and Jeliazkov (2009). To group the parameters in appropriate blocks, we stack the measurement equation (2) over the T time periods.

$$\mathbf{y} = \mathbf{G}\mathbf{f} + \mathbf{e}, \qquad \mathbf{e} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_T \otimes \boldsymbol{\Sigma}), \tag{8}$$

where

$$\underset{nT\times 1}{\mathbf{y}} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_T \end{bmatrix}, \qquad \underset{nT\times q(T+s)}{\mathbf{G}} = \begin{bmatrix} \mathbf{S}_1\mathbf{L}_s\boldsymbol{\Lambda} & \dots & \mathbf{S}_1\mathbf{L}_0\boldsymbol{\Lambda} & & & \\ & \mathbf{S}_2\mathbf{L}_s\boldsymbol{\Lambda} & \dots & \mathbf{S}_2\mathbf{L}_0\boldsymbol{\Lambda} & & \\ & & \ddots & & \ddots & \\ & & & \mathbf{S}_T\mathbf{L}_s\boldsymbol{\Lambda} & \dots & \mathbf{S}_T\mathbf{L}_0\boldsymbol{\Lambda} \end{bmatrix}.$$

Correspondingly, the state equation (3) is stacked according to

$$\mathbf{H}\mathbf{f} = \mathbf{v}, \qquad \mathbf{v} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{T+s} \otimes \mathbf{\Omega}), \tag{9}$$

where

$$\underset{q(T+s)\times q(T+s)}{\mathbf{H}} = \begin{bmatrix} \mathbf{I}_q & & & & & \\ -\mathbf{\Phi}_1 & \mathbf{I}_q & & & & \\ \vdots & \ddots & \mathbf{I}_q & & & \\ -\mathbf{\Phi}_p & \dots & -\mathbf{\Phi}_1 & \mathbf{I}_q & & \\ & \ddots & \ddots & \ddots & \ddots & \\ & & -\mathbf{\Phi}_p & \dots & -\mathbf{\Phi}_1 & \mathbf{I}_q \end{bmatrix}, \qquad \underset{q(T+s)\times 1}{\mathbf{f}} = \begin{bmatrix} \mathbf{f}_{1-s} \\ \vdots \\ \mathbf{f}_0 \\ \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_T \end{bmatrix}.$$

Following Chan and Jeliazkov (2009), the precision matrix $\mathbf{K}$ is given by $\mathbf{H}'(\mathbf{I}_{T+s} \otimes \mathbf{\Omega})^{-1}\mathbf{H}$ and the conditional posterior of the factors is normally distributed with

$$\begin{aligned} \mathbf{f} \sim \mathcal{N}(\mathbf{p}_1, \mathbf{P}_1^{-1}) \quad \text{where} \quad \mathbf{P}_1 &= \mathbf{K} + \mathbf{G}'(\mathbf{I}_T \otimes \mathbf{\Sigma}^{-1})\mathbf{G} \\ \mathbf{p}_1 &= \mathbf{P}^{-1}(\mathbf{G}'(\mathbf{I}_T \otimes \mathbf{\Sigma}^{-1})\mathbf{y}). \end{aligned}$$

This algorithm is computationally very efficient if block-banded and sparse matrix algorithms are used.[4] For further details on the estimation algorithm and the conditional distributions of the remaining parameters see Appendix A.1.

3 Global and Local Cycles

3.1 Data and Output Gaps

We extract the common factors from a large collection of quarterly and annual output gap estimates for various countries. By combining several models for each country, we account for the fact that no true estimates of the unobservable output gaps exist. The HP-filter, for instance, has been criticized by Nelson and Plosser (1982), Cogley and Nason (1995), and more recently by Hamilton (2018) for introducing spurious cycles and depending highly on the smoothing parameters. Pooling several estimates is likely to lead to more robust results, in particular for less developed countries with a more volatile business cycle. We focus on five well-established methods that rely only on real gross domestic product, are applicable to quarterly as well as annual data, and do not require excessive computational effort. Most of these approaches have also been used by Garratt et al. (2014) in their ensemble nowcasts of the output gap.

First, we apply the well-known filter proposed by Hodrick and Prescott (1997), using the parametrization suggested by Ravn and Uhlig (2002). Second, we apply a bandpass

[4]It is faster to compute the banded Cholesky factor of $\mathbf{P}_1$ and solve for $\mathbf{p}_1$ by forward- and backward substitution.

filter proposed by Baxter and King (1999). Third, we use the filter proposed by Hamilton (2018), which takes the two-year forecast error of a projection based on an autoregressive model as the cyclical component of output. Fourth, we apply an unobserved components model following Watson (1986), which performs a decomposition of GDP into a trend with stochastic drift and a cycle that follows a stationary autoregressive process. Lastly, we fit a simple cubic spline to logarithmized GDP. It should be noted that this selection does not necessarily represent the most suitable methods. In particular, the generic parameter assumptions may not be appropriate for every country. To make the results comparable across countries, we choose to omit relational methods, where the output gap is modeled as a function of well observable market outcomes, such as inflation or unemployment (Kuttner, 1994; Gerlach and Smets, 1999; Graff and Sturm, 2012). Due to data limitations, we also do not use output gaps that originate from production function approaches. Table 1 provides an overview of the data by giving the standard deviations of the output gaps by region and estimation method.

Table 1. Ex-Post Variation of Output Gap Estimates

	#	Hodrick-Prescott Filter		Baxter-King Filter		Hamilton Filter		Unobs. Compo-nents		Cubic Spline	
		a	q	a	q	a	q	a	q	a	q
Total	157	3.2	2.3	3.1	2.0	8.4	4.3	9.3	3.5	6.2	3.3
Australia & Oceania	3	1.1	1.4	1.0	1.0	2.4	2.3	3.0	2.2	1.7	1.9
North America	2	1.0	1.5	1.0	1.1	2.4	2.5	3.3	2.6	1.8	2.1
Europe	39	2.9	2.5	2.8	2.1	8.2	4.7	9.7	3.5	6.3	3.7
East & Southeast Asia	16	2.2	1.5	2.3	1.4	5.8	2.6	6.0	1.8	4.2	2.1
North Africa & W. Asia	27	4.9	2.7	3.9	2.4	12.0	5.1	15.1	3.9	9.6	3.5
Sub-Saharan Africa	39	3.3	2.7	3.5	1.5	9.1	3.0	8.0	7.9	5.8	2.4
Latin America & Carib.	24	2.0	2.4	2.5	1.9	5.7	3.9	4.8	4.0	4.2	2.8
South Asia	7	2.1	3.2	2.3	2.1	4.4	4.5	3.8	4.7	2.8	3.3

Notes: Table exhibits standard deviations for annual and quarterly output gap estimates across geographical regions. # indicates the number of countries at each level. Not all countries have quarterly data.

We apply the five methods for all available annual and quarterly time series on real gross domestic product. The annual data originates from the Penn World Table (version 10.0) and is available for 154 countries, the quarterly data originates from the Quarterly National Accounts of the OECD and is available for 47 countries. In total, 157 countries are included. The data starts in the first quarter of 1990 and ends in the fourth quarter of 2020. As outlined in Section 2.1, our approach can handle a data matrix with missing observations.

We classify all countries into 21 geographic subregions according to the United Nations geoscheme, which is based on the M49 coding classification. We then aggregate these subregions further into eight economic regions. Figure 1 shows a map of the regions and appendix A.3 lists the assignment of the countries to the different regions. These regions are based on regional aggregation schemes used by large international institutions and reflect our belief that they share a common cyclical component.

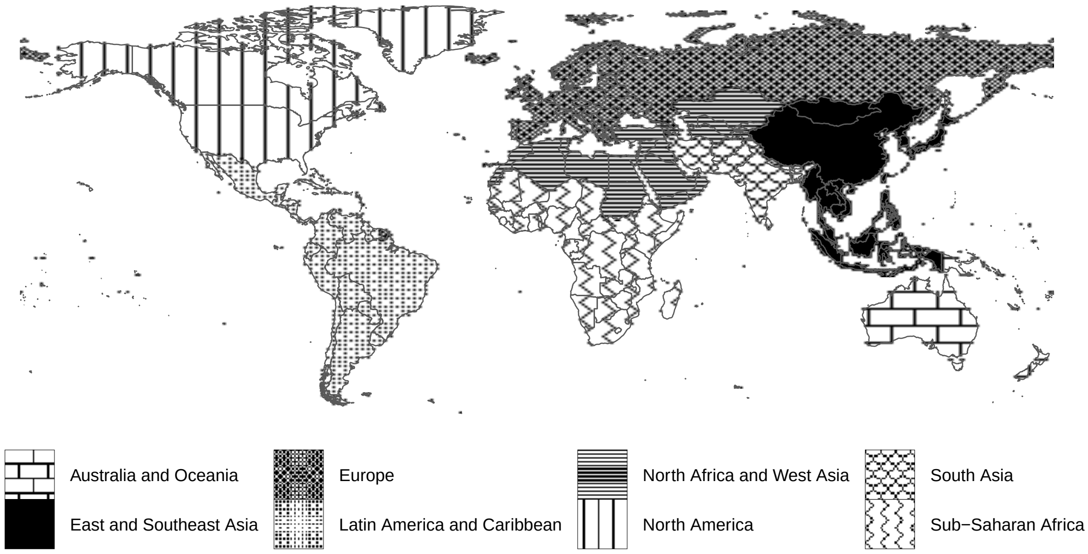

Figure 1: World map showing the division into the eight regions.

3.2 Decomposition of Output Gaps

The multi-level DFM allows us to decompose the (reconciled) output gap of each country into a global, a regional, and a local component. The global factor captures the variation in the output gaps that is common to all countries. The regional factor represents the comovements that are unique to all countries within a specific region. Finally, the local factor captures the country-specific part of the output gap that is not shared with the region and the world as a whole. Since we have restricted the factor loadings to unity, the estimated factors can be interpreted directly as the contribution of the corresponding hierarchical level to a country's output gap in percentage points.

Figure 2 shows the estimated global factor from 1990 to 2020. The factor is significantly different from zero in several years, indicating the existence of a global component in capacity utilization across countries. Furthermore, the global cyclical component is fairly persistent with an autoregressive coefficient of the first lag of 0.63. The Asian financial crisis in 1997 and 1998 and the bursting of the tech bubble in 2000 had only minor impacts on the global component. The global financial crisis, on the other hand, marked a sharp common decline in capacity utilization in each country. By far the strongest common deviation from potential output was caused by the COVID-19 recession, amounting to a value of more than -7 percentage points in the second quarter of 2020.

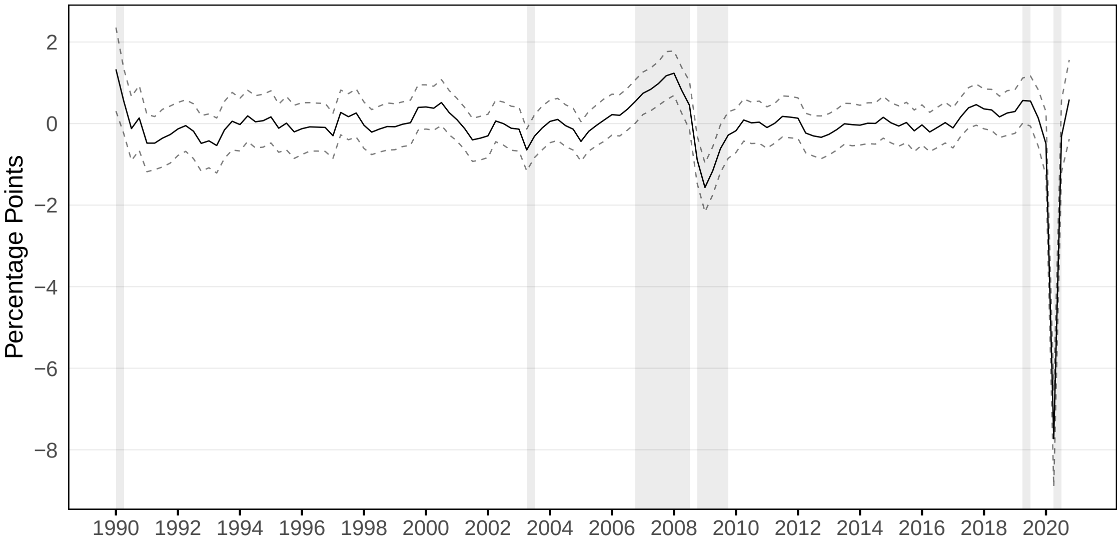

Figure 2: Global Component of Output Gaps. The solid line represents the estimated global factor that measures in percentage points the component of the output gaps that is common to all countries. The dashed lines mark the 95%-confidence interval of the factor. The shaded areas highlight the periods where the factor is significantly different from zero.

Regional components explain the variations in output gaps that are common only to the countries in a specific region. Figure 3 shows the eight regional factors where periods of a significant factor are highlighted in grey. In Figure 4, the regional components are shown together with the global component. While there appears to be a significant common component in some regions, others show no sign of a shared cycle. The strongest regional cycles can be found in North America, Europe, and East and Southeast Asia. The cycles in Latin America and Australia are somewhat smaller but still significantly different from zero in many quarters. The factor for Sub-Saharan Africa is highly volatile and has large standard errors such that no significant common component results in almost all quarters. The regional factors for South Asia and North Africa and West Asia are virtually non-existent.

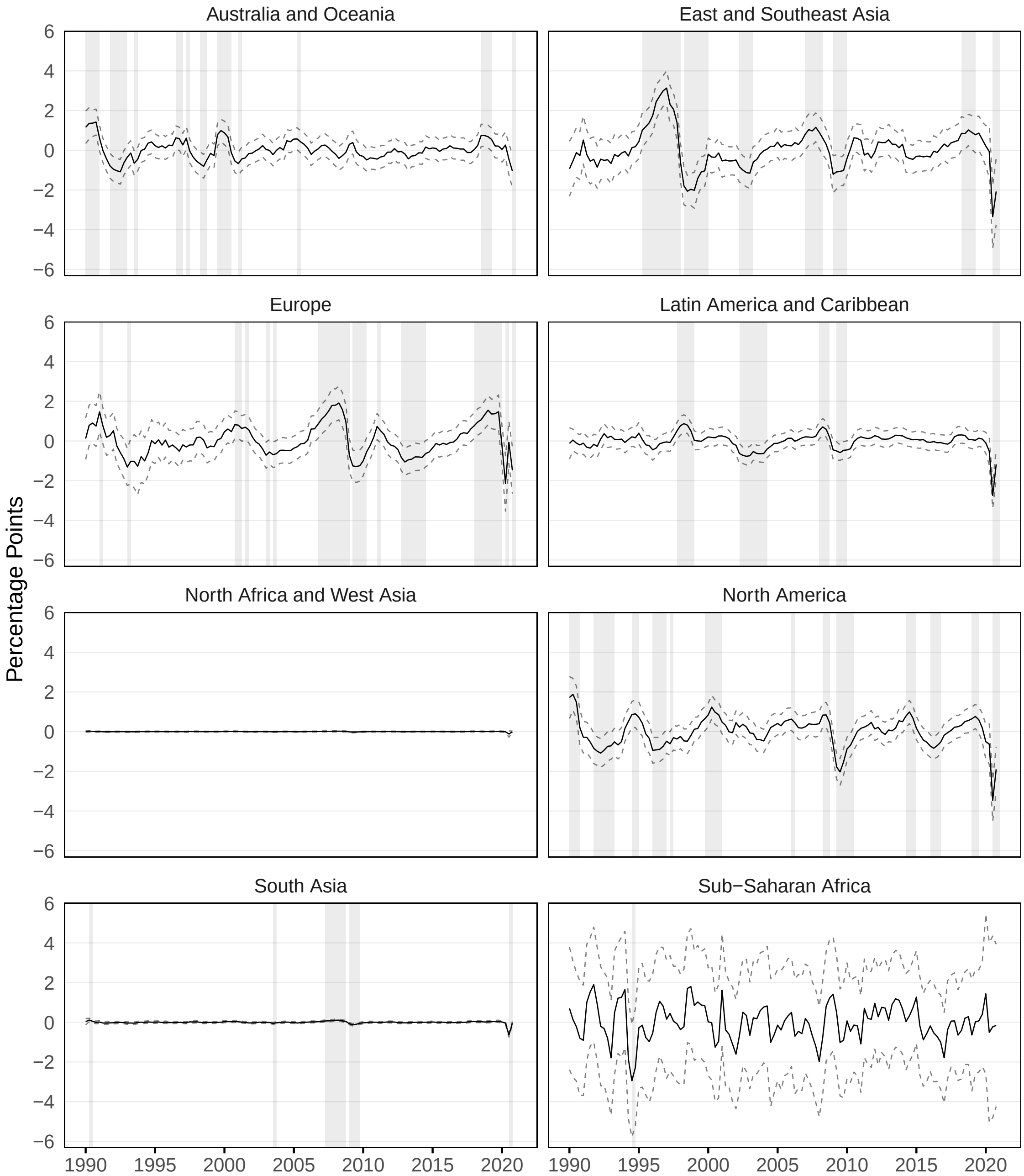

Figure 3: Regional Components of Output Gaps. The solid line represents the estimated regional factors that measures in percentage points the component of the output gaps that are common to all countries within a region. The dashed lines mark the 95%-confidence interval of the factors. The shaded areas highlight the periods where the factor is significantly different from zero.

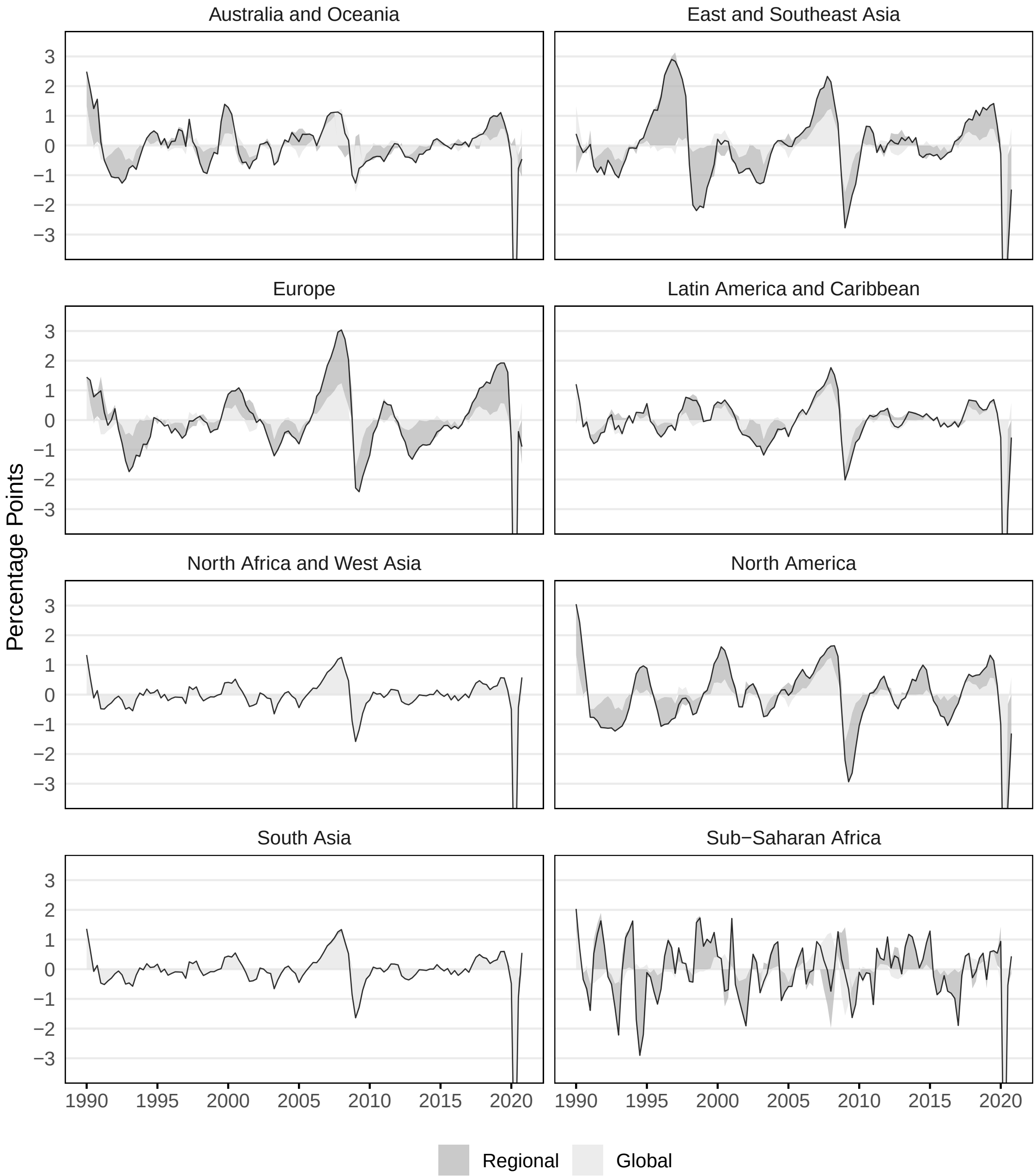

Figure 4: Global and Regional Components of Output Gaps. The lighter area represents the global output gap component that is common to all countries. The darker area represents the region-specific component that measure the part of the output gaps that is common to all countries within a region. The black line shows the sum of the two components. Figures are truncated at -3.5 percentage points to improve readability.

It may well be that some declines in output happen simultaneously in multiple regional components. The global financial crisis, for instance, caused very negative output gaps in 2009 in Europe, North America, and East and Southeast Asia, whereas the decline in Sub-Saharan Africa and Australia was only driven by the global factor. The model also identifies several region-specific shocks. For instance, the Asian financial crisis affected in 1998 in particular the output gaps of countries in East and Southeast Asia, the crisis of the European exchange rate mechanism in 1992 and 1993 is mainly reflected in the European cycle, or the South American economic crisis in 2002 and 2003 is visible in the Latin American factor only. Also the COVID-19 recession led to regional differences, although the global component accounts for a large fraction of the declines in capacity utilization. The regional components of Europe, North America, East and Southeast Asia, and Latin America are more negative in 2020 than the other regional components. The components of Sub-Saharan Africa and North Africa and West Asia, on the other hand, are not significantly different from zero in 2020.

Figure 5 adds the local cycle to show the contributions of each hierarchical level. We limit our exposition here to a selection of eight countries that exhibit interesting decompositions. The shaded areas mark the parts of the output gap attributable to the country-specific, regional, and global cycles. The black line represents the sum of the three factors and thereby the reconciled output gap of the country. Figure 5 visualizes that the global factor usually constitutes a substantial portion of the output gap. On average, the global component accounts for 18% of the output gap, the regional factor for 24%, and the country-specific factor for 58% of the absolute sum of the three components. However, these shares vary substantially between the countries.

The estimated components allow us to study the integration of local cycles in the regional and global business cycles. Of particular interest is the development of countries within the same region. In Europe, for instance, Germany recovers relatively quickly following the global financial crisis and is barely affected by the subsequent sovereign debt crisis. The local cycle even contributes to an inflationary output gap in a multiple quarters while the regional cycle for Europe indicates undercapacities. Greece, on the other hand, recovers very slowly and the local component remains negative for three years. For the United Kingdom, a negative factor results in the year 2019, reflecting the economic uncertainties related to the Brexit negotiations. A volatile and strong local cycle can be found for Brazil. 64% of the absolute sum of the three components is attributable to the country-specific factor. Brazil experienced a severe economic crisis in 2014 that was caused by deteriorating commodity prices and, more importantly, a series of unfortunate macroeconomic policies. The persistently negative output gap was, therefore, limited to the Brazilian economy and is not visible in the regional cycle for Latin America. The local cycle for the United States and Canada are relatively small because the regional cycle captures a large part of the cyclical variation. This points towards a strong cyclical integration of the countries included in the corresponding aggregate. Of the three components, the local cycle accounts for only 37% for the United States and for 23% for Canada. The figure for Japan reveals that the Japanese economy was also hit relatively hard by the global financial crisis. Lastly, China was only marginally affected by the global financial crisis but drifted into a persistent undercapacity in recent years. The Chinese factor is negatively correlated with the sum of the two components of the corresponding higher hierarchical levels – the East and Southeast Asian and the global component. Put in other words, the cyclical phases of China do not coincide at times with the phases of the remaining East and Southeast Asian countries. Given China's large domestic market and the political heterogeneity of the East and Southeast Asia region, this low degree of synchronization is expected.

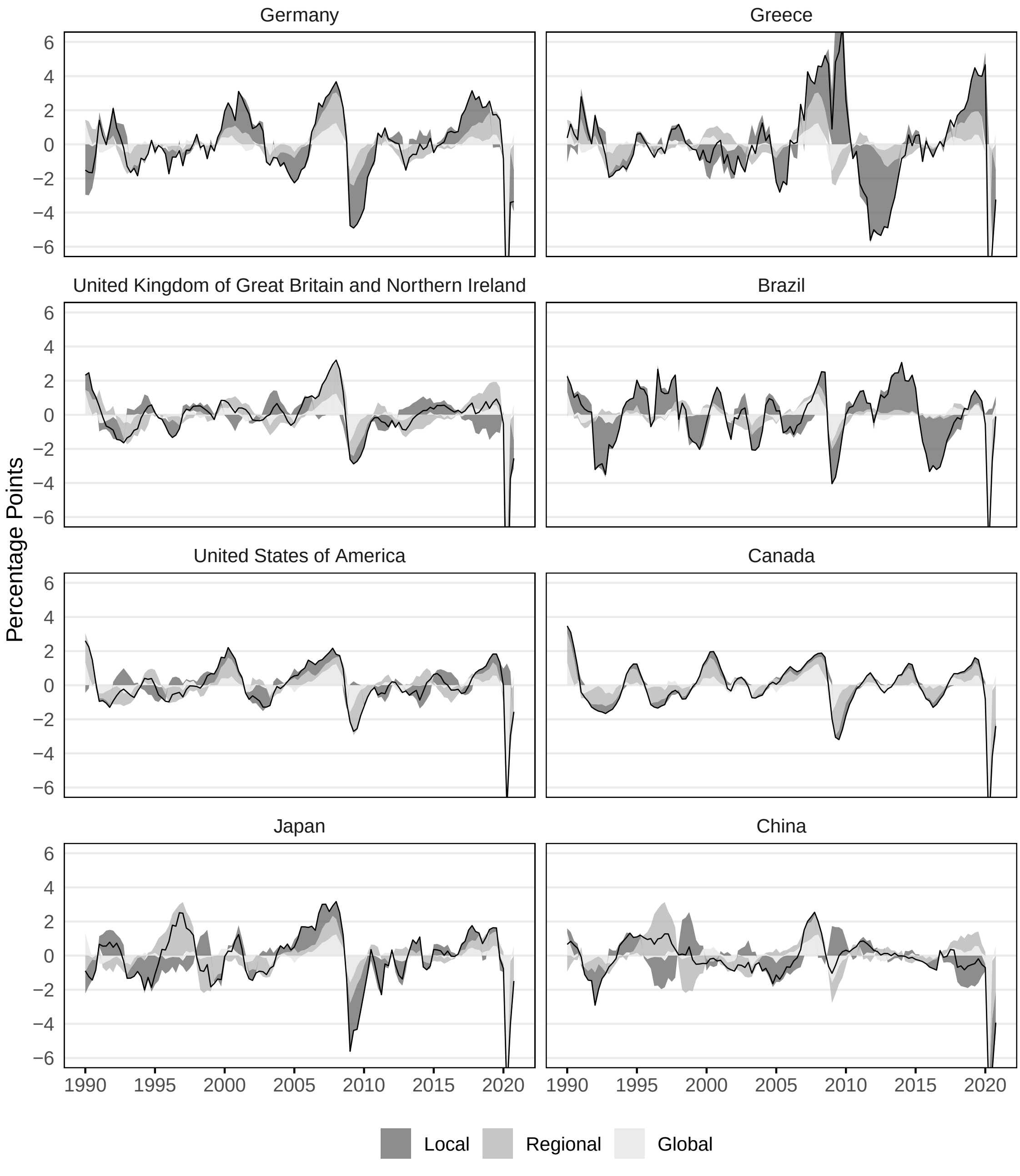

Figure 5: Global, Regional, and Country-Specific Components of Output Gaps. The output gaps of the countries decomposed into three parts: A global component (lightest area), a region-specific component (medium dark area), and a country-specific component (darkest area), all in percentage points. The black line shows the estimated output gap in percent given by the sum of the three components. Figures are truncated at -6 and +6 percentage points to improve readability.

In addition, it is instructive to look at the correlations between global, regional, and local cycles. Table 2 gives the correlations between the global and regional factors. We find, for instance, a relatively high correlation between the Latin American and the North American factor, suggesting a comovement of their business cycles. The European cycle exhibits the highest correlation with the global cycle. This indicates that these countries are highly integrated in the global economy.

Table 2. Correlation Matrix Global and Regional Factors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1) World	1.00								
(2) Australia & Oceania	0.04	1.00							
(3) East & Southeast Asia	0.13	0.30	1.00						
(4) Europe	0.47	0.20	0.24	1.00					
(5) Latin America & Carib.	0.24	-0.07	0.29	0.33	1.00				
(6) North Africa & W. Asia	0.26	0.19	0.41	0.40	0.73	1.00			
(7) North America	0.30	0.44	0.17	0.45	0.54	0.68	1.00		
(8) South Asia	0.26	0.18	0.41	0.38	0.72	1.00	0.67	1.00	
(9) Sub-Saharan Africa	-0.05	-0.10	-0.12	-0.15	0.01	-0.06	-0.06	-0.06	1.00

Notes: Table shows correlations between global and regional output gap components.

The estimated autoregressive coefficients of the factors are informative as well since they explain which cycles explain or predict other cycles. Our results suggest, for instance, that the South Korean cycle lags the Japanese cycle, the Uruguayan lags the Argentinian cycle, and the Taiwanese cycle lags the Chinese cycle.

The coefficients for the world factor as the explanatory variable differ highly between the countries and regions. For instance, the factors of Germany, China, and North America depend highly on the first lag of the world factor, where the value of the coefficients is between 0.3 and 0.6. Other cycles, such as those of the region Australia and Oceania do not depend on the lagged world factor. The own lag is in most equations the most important determinant. On average, the coefficient for the own lag is 0.3 which indicates some persistence of the cycles. We obtain, amongst others, a high own lag coefficient for Germany (0.8) and the United States (0.5).

4 Conclusion

We have proposed a multi-level dynamic factor model to identify common and country-specific components in output gap estimates. Our model combination framework pools information from several output gap estimation methods, which may be subject to mixed frequencies, random patterns of missing observations, and ragged edges. Appropriate restrictions on the factor loadings impose a hierarchical multi-level factor structure such that each output gap estimate consists of a global, a regional, and a local component, as well as an idiosyncratic component that is specific to each model and country.

We show that spatial information can be used to reduce model complexity not only using shrinkage priors, in line with LeSage and Krivelyova (1999), but also using stochastic search variable selection. The prior inclusion probabilities are estimated based on the geographic distances between spatial entities. We provide evidence on suitable tuning parameters and show how to determine important predictors in large vector autoregressions.

Furthermore, we contribute to the literature on the identification of global and regional business cycles by providing a decomposition of a country's output gap into common hierarchical components. This information is useful to fiscal and monetary policy makers that implement counter-cyclical policies at different levels of government. The results provide significant evidence for the existence of a common global cycle and various regional cycles, in particular in North America, Europe, and East and Southeast Asia. The results suggest that, on average, 18% of the output gap is attributable to the global cycle, 24% to the regional cycle, and 58% to the local cycle.

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A Appendix

A.1 Estimation Algorithm

We estimate the model parameters $\mathbf{\Gamma}$ and $\boldsymbol{\theta}$ using the Metropolis-Hastings algorithm and the model parameters $\mathbf{\Sigma}$ and $\mathbf{\Omega}$ as well as the factors $\mathbf{f}_t$ using Gibbs sampling. We discard the first 10,000 draws as burn-in and then save every fifth draw until we have a sample of 5,000 draws from the joint posterior distribution. The factor loadings $\mathbf{\Lambda}$ are assumed to be fixed.

To estimate the vector autoregressive coefficients $\mathbf{\Phi}_1, \dots, \mathbf{\Phi}_p$, it is useful to induce sparsity into the regression. We use indicator variable selection by defining $\mathbf{\Gamma}\boldsymbol{\theta} = vec([\mathbf{\Phi}_1, \dots, \mathbf{\Phi}_p]')$, where the pq^2-dimensional vector $\boldsymbol{\theta}$ holds the unknown regression coefficients. The diagonal matrix $\mathbf{\Gamma}$ contains pq^2 binary indicators γ_{ijr} that determine whether the coefficients are included in the model. Recall the state equation 4 from Section 3.

$$\mathbf{f}_t = \mathbf{X}_t\mathbf{\Gamma}\boldsymbol{\theta} + \mathbf{v}_t, \qquad \mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{\Omega}) \tag{10}$$

with $\mathbf{X}_t = \mathbf{I}_q \otimes [\mathbf{f}'_{t-1}, \dots, \mathbf{f}'_{t-p}]$. If an indicator is equal to zero, the corresponding column in $\mathbf{X}\mathbf{\Gamma}$ is set to zero (see, for instance, Kuo and Mallick, 1998; Korobilis, 2013, for similar approaches). Following Chan (2019), we stack equation (10) over each time period into $\mathbf{x} = [\mathbf{f}'_{1+p-s}, \dots, \mathbf{f}'_T]'$ and $\mathbf{X} = [\mathbf{X}'_{1+p-s}, \dots, \mathbf{X}'_T]'$.

$$\mathbf{x} = \mathbf{X}\mathbf{\Gamma}\boldsymbol{\theta} + \mathbf{v}, \quad \mathbf{v} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{T-p+s} \otimes \mathbf{\Omega}). \tag{11}$$

While it is possible to sample directly from the conditional posterior distributions of $\mathbf{\Gamma}$ and $\boldsymbol{\theta}$, it is computationally more efficient to use a Metropolis algorithm. We simulate candidate values for $\mathbf{\Gamma}$ and $\boldsymbol{\theta}$, using appropriate proposal distributions, and evaluate them for each dependent variable in the VAR. The acceptance probability after burn-in is typically around 30%.

The indicator variables $\mathbf{\Gamma}$ are sampled using an independence chain Metropolis-Hastings algorithm. We use the prior inclusion probabilities, given by the pq^2-dimensional vector $\boldsymbol{\pi}_0$, as proposal distribution. We sample each proposed indicator from a Bernoulli distribution according to

$$\gamma_{ijr,new} \sim \mathcal{B}(\pi_{0,ijr})$$

and construct a proposal indicator matrix $\mathbf{\Gamma}_{new}$. Using the indicator matrix $\mathbf{\Gamma}_{old}$ from the previous iteration and the proposal indicator matrix $\mathbf{\Gamma}_{new}$, we then evaluate the likelihood ratio for each variable $i = 1, \dots, q$. This is feasible since we assume the errors in the state equation to be uncorrelated. For each $i = 1, \dots, q$, we determine the acceptance ratio

$$R_i = \frac{\exp(-\frac{1}{2\omega_i^2}(\mathbf{x}_i - \mathbf{X}_i\mathbf{\Gamma}_{i,new}\boldsymbol{\theta}_i)'(\mathbf{x}_i - \mathbf{X}_i\mathbf{\Gamma}_{i,new}\boldsymbol{\theta}_i))}{\exp(-\frac{1}{2\omega_i^2}(\mathbf{x}_i - \mathbf{X}_i\mathbf{\Gamma}_{i,old}\boldsymbol{\theta}_i)'(\mathbf{x}_i - \mathbf{X}_i\mathbf{\Gamma}_{i,old}\boldsymbol{\theta}_i))},$$

where $\mathbf{x}_i$, and $\mathbf{X}_i$ are the rows corresponding to variable i. We then accept the indicators for variable i with probability $\min(1, R_i)$ and construct the indicator matrix $\mathbf{\Gamma}$.

For the vector autoregressive coefficients $\boldsymbol{\theta}$, we use a random-walk chain Metropolis-Hastings algorithm. Each element in $\boldsymbol{\theta}$ is proposed using

$$\theta_{ijr,new} = \theta_{ijr,old} + \xi,$$

where $\xi \sim \mathcal{N}(0, 0.05)$. Similar to the indicator variables, we evaluate the acceptance ratio for each dependent variable. It might be useful in some cases to enforce stationarity or positivity of the coefficients. In these cases, one simply discards proposed values that do not comply with these criteria during sampling. Furthermore, it increases the numerical stability of the sampler to include a Minnesota-type prior distribution to shrink coefficients at distant lags towards zero.

We assume all non-zero elements in the factor loadings matrix $\mathbf{\Lambda}$ to be equal to one. It greatly speeds up computation to preallocate a sparse matrix and not update it during the sampling procedure. Fixing $\mathbf{\Lambda}$ also solves the rotational problem associated with dynamic factor models. In order to identify the dynamic factors, it is necessary to impose restrictions on the factor loadings. It is feasible to put informative priors on the non-zero elements in $\mathbf{\Lambda}$ in order to solve the scale, sign, and rotational indeterminacies (see, for instance, Drèze and Richard, 1974).

Both $\mathbf{\Omega}$ and $\mathbf{\Sigma}$ are restricted to be diagonal matrices. It is, therefore, assumed that the dynamic factors account for the cross-correlation in the data and the measurement errors across variables and estimation methods are independent.

$$\mathbf{\Omega} = \begin{bmatrix} \omega_1^2 & & & & \\ & \ddots & & & \\ & & \omega_i^2 & & \\ & & & \ddots & \\ & & & & \omega_q^2 \end{bmatrix}, \quad \mathbf{\Sigma} = \begin{bmatrix} \sigma_1^2 & & & & \\ & \ddots & & & \\ & & \sigma_k^2 & & \\ & & & \ddots & \\ & & & & \sigma_n^2 \end{bmatrix}.$$

We draw the diagonal elements σ_k^2 equation-by-equation from an inverse Gamma distribution. The measurement errors $\mathbf{e}_k$ for the kth variable can be retrieved from equation 2 using $\mathbf{y} - \mathbf{Gf}$. Since missing observations and low-frequency variables lead to zeros in the error term, the shape parameter of the inverse gamma distribution is determined by the number of non-zero elements in $\mathbf{e}_k$, given by τ. The conditional posterior distribution is then given by

$$\sigma_k^2 \sim \mathcal{IG}\left(c_{1,k}/2, d_{1,k}/2\right) \quad \text{where} \quad \begin{aligned} c_{1,k} &= c_{0,k} + \tau \\ d_{1,k} &= d_{0,k} + \mathbf{e}_k\mathbf{e}_k', \end{aligned}$$

where τ is the number of non-zero elements in the corresponding time series. The priors are chosen to be uninformative by setting $c_{0,k} = 3$ and $d_{0,k} = 1$.

The covariance matrix of the errors in the state equation, given by $\mathbf{\Omega}$, is assumed to be diagonal because the innovations to each factor are independent and comovements are accounted for by the common factors. As a result, we draw the diagonal elements ω_i^2 also equation-by-equation from an inverse Gamma distribution. The measurement errors $\mathbf{v}_i$ for the ith factor can be retrieved from equation (11) using $\mathbf{x} - \mathbf{X\Gamma}\boldsymbol{\theta}$. The conditional posterior distribution is then given by

$$\omega_i \sim \mathcal{IG}\left(l_{1,i}/2, m_{1,i}/2\right) \quad \text{where} \quad \begin{aligned} l_{1,i} &= l_{0,i} + T + s - p \\ m_{1,i} &= m_{0,i} + \mathbf{v}_i\mathbf{v}_i'. \end{aligned}$$

The priors are chosen to be uninformative by setting $l_{0,i} = 3$ and $m_{0,i} = 1$, leaving the smoothness of the common components up to the underlying data.

A.2 Specification Spatial Prior Information

We use the following parameters to calculate the spatial priors in equation 6: $\rho = 0.01, \beta = 2$, and $\alpha = 1$. To check for the robustness of the results, we evaluated the model with different specifications of these parameters. The changes to the results are qualitatively negligible.

To illustrate the prior inclusion probabilities we provide two examples. For the German factor as the dependent variable, the prior inclusion probabilities $\pi_{0,i=DE,j,r=1}$ for the first lagged factors are the following: Germany and global cycle (each 1). European regional cycle, France, Luxembourg, Denmark, Switzerland, Poland, Netherlands, Austria, Belgium, Czech Republic (each about 0.09), Italy (0.05), Sweden (0.04), Slovenia (0.02), and Croatia (0.01). These probabilities sum to three. The prior inclusion probabilities for the remaining factors are close to zero. For the United States as the dependent variable, the non-zero prior inclusion probabilities $\pi_{0,i=US,j,r=1}$ for the first lagged factors are the following: United States and global cycle (each 1). North American and Latin American regional cycles, Mexico, Canada (each 0.2), Russia and European regional cycle (each 0.07), and Bahamas (0.06). To obtain the probabilities for $r > 1$, the probabilities of $r = 1$ are divided by r^β. As a consequence, the prior inclusion probabilities for the second lag of the world factor as well as for the second own lag are 0.25 ($1/r^\beta$). The probabilities of the remaining second lags sum up to 0.25. Therefore, with $p = 2$ the prior specification implies that we expect in each autoregressive equation 3.75 factors with a nonzero regression coefficient. With $p = 3$ the corresponding sum is 4.08 and with $p = 4$ 4.27.

For the world factor as the dependent variable, the prior inclusion probabilities $\pi_{0,i=WD,j,r=1}$ for the first lagged factors are one for the own lag, one in sum for all regions, and one in sum for all countries. With eight regions and 157 countries, the (rounded) prior inclusion probabilities are 0.13 for each regional cycles and 0.01 for each country. The probabilities are shrunk towards zero with increasing lag length.

A.3 List of Countries

Australia and Oceania (3)
Australia, Fiji, New Zealand

East and Southeast Asia (16)
Brunei Darussalam, Cambodia, China, China, Hong Kong Special Administrative Region, China, Macao Special Administrative Region, Indonesia, Japan, Malaysia, Mongolia, Myanmar, Philippines, Republic of Korea, Singapore, Taiwan (Province of China), Thailand, Viet Nam

Europe (39)
Albania, Austria, Belarus, Belgium, Bosnia and Herzegovina, Bulgaria, Croatia, Czechia, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Moldova (Republic of), Montenegro, Netherlands, North Macedonia, Norway, Poland, Portugal, Romania, Russian Federation, Serbia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Ukraine, United Kingdom of Great Britain and Northern Ireland

Latin America and Caribbean (24)
Argentina, Bahamas, Bolivia (Plurinational State of), Brazil, Cayman Islands, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, El Salvador, Guatemala, Haiti, Honduras, Jamaica, Mexico, Nicaragua, Panama, Paraguay, Peru, Suriname, Trinidad and Tobago, Uruguay, Venezuela (Bolivarian Republic of)

North Africa and West Asia (27)
Algeria, Armenia, Azerbaijan, Bahrain, Cyprus, Egypt, Georgia, Iraq, Israel, Jordan, Kazakhstan, Kuwait, Kyrgyzstan, Morocco, Oman, Palestine, State of, Qatar, Saudi Arabia, Sudan, Syrian Arab Republic, Tajikistan, Tunisia, Turkey, Turkmenistan, United Arab Emirates, Uzbekistan, Yemen

North America (2)
Canada, United States of America

South Asia (7)
Bangladesh, Bhutan, India, Iran (Islamic Republic of), Maldives, Nepal, Pakistan

Sub-Saharan Africa (39)
Angola, Benin, Botswana, Burkina Faso, Burundi, Cabo Verde, Cameroon, Central African Republic, Chad, Congo, Congo (Democratic Republic of the), Côte d'Ivoire, Djibouti, Equatorial Guinea, Eswatini, Ethiopia, Gabon, Gambia, Ghana, Guinea, Kenya, Madagas-

car, Malawi, Mali, Mauritania, Mauritius, Mozambique, Namibia, Niger, Nigeria, Rwanda, Senegal, Sierra Leone, South Africa, Tanzania (United Republic of), Togo, Uganda, Zambia, Zimbabwe