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Piping Criteria for Hydraulically Stable Anisotropic Slopes

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Abstract: Piping has been a documented cause of collapse of multiple tailings dams, hydraulic structures and natural slopes. Important limitation of the existing piping criteria for a sloped ground is that they are treating soils as hydraulically isotropic, which is rarely the case in real life problems. Another obstacle for wider application of these criteria in engineering practice is that they have not been translated into an adequate definition of the safety factor against piping. This paper provides rigorous yet simple piping criteria and safety factors for slopes built of hydraulically stable anisotropic materials, as well as the safety factor against instability of an infinite anisotropic slope with a slope-parallel flow. It has been demonstrated why it is important to account for anisotropy, and how the proposed analytical expressions can be applied to practical problems with calculated flow nets and piezometric field measurements. **DOI: 10.1061/(ASCE)GT.1943-5606.0002629.**

Author keywords: Piping; Anisotropy; Slopes; Tailings dams; Earth dams; Safety.

Introduction

Piping in excavations, hydraulic, and coastal structures has been a source of significant damage, concern for designers, and a subject of numerous research efforts. Discussing piping in dams, Terzaghi and Peck (1948) stated, "[Piping] may be due to scour or subsurface erosion that starts at springs near the downstream toe and proceeds upstream along the base of the structure or some bedding plane. Failure occurs as soon as the upstream or intake end of the eroded hole approaches the bottom of the reservoir. The mechanics of this type of piping defy theoretical approach." The latter statement is rather unusual for Terzaghi, who managed to apply theoretical approaches to practically all significant problems in soil mechanics (Terzaghi 1943). It reflects the complexity of the problem, which more than 70 years later is still a subject of significant research efforts and lacks rigorous theoretical solutions applicable to general soil conditions encountered in practice. The author came across this gap during his investigation of a piping-triggered mudslide in Switzerland, which served as a motivation for this research.

A comprehensive review of the literature devoted to piping in natural slopes is given by Harrison (2014), who noted that, in spite of the numerous case studies (e.g., Williams 1966; Eisbacher and Clague 1981; Hungr and Smith 1985; Hagerty 1991; Evans and Savigny 1994; Crosta and di Prisco 1999; Cavers 2003; Fox et al. 2007), many of them lack a quantitative description of the underlying mechanisms. Piping has been a documented cause of collapse of 17 tailings dams (8% of all failures) in 1915–2016 (ICOLD 2001; Bowker and Chambers 2015), which is significant, especially considering that tailings dams are usually designed to prevent water from reaching the downstream surface of the wall (Klohn 1979). Furthermore, in additional 30 dams collapse was explained by a slope failure due to seepage, where phenomena like internal erosion could also play a role. In hydraulic structures, piping and erosion represent even a higher risk: Richards and Reddy (2007) compiled more than 250 cases of piping (or related) failures, which is about one-third of all dam failures.

Development of a simple theoretical framework for piping criteria has been complicated by the fact that unfavorable grain size distribution can lead to internal hydraulic instability of soil, resulting in lower critical hydraulic gradients (e.g., Kenney and Lau 1985; Skempton and Brogan 1994; Tomlinson and Vaid 2000; Wan and Fell 2004; Richards and Reddy 2010; Moffat and Fannin 2011; Li and Fannin 2012; Chang and Zhang 2013). As a consequence, numerical studies of piping in dams and natural slopes are carried out using high-level finite elements methods (e.g., Cividini and Gioda 2004; Lei et al 2017), discrete elements methods (e.g., Tao and Tao 2017), or computational fluid/solid dynamics modeling (e.g., Alcérreca-Huerta and Oumeraci 2018), and are not yet directly suitable for practical applications.

For slopes built of hydraulically stable materials, piping analysis is more straightforward, and a number of theoretical piping criteria (with different levels of rigor) have been proposed in the literature (e.g., Iverson and Major 1986; Ghiassian and Ghareh 2008; Van Beek et al 2014; Tao and Tao 2017; Kirca and Kilci 2018). There are, however, two important limitations that complicate application of the existing criteria to practical problems.

The first limitation is that the above criteria are dealing with hydraulically isotropic slopes, which is hardly ever the case in real life problems. Hicock and Armstrong (1985) demonstrated that for modeling seepage-induced instability of natural slopes, assumption of isotropic soil is not valid. Natural slopes are built of layers, which may have different inclination either due to tectonic uplift/subduction [Fig. 1(a)] or due to their alluvial/colluvial/glacial deposition [Fig. 1(b)]. While in the former case the strata can have a wide range of inclinations, in the latter one the layers are likely to be parallel to the underlying rock surface (Harrison 2014). Terzaghi (1943) demonstrated that in case of layers with different permeability coefficients, permeability in the direction parallel to the layers can be orders of magnitude higher than the one perpendicular to the layers. Also for manmade structures, like compacted embankments and tailings dams [Fig. 1(c)], the horizontal permeability appears to be 4 to 10 times larger than the vertical one, even when great care has been taken to minimize horizontal stratification (Klohn 1979). As will be demonstrated in this paper, this magnitude of hydraulic

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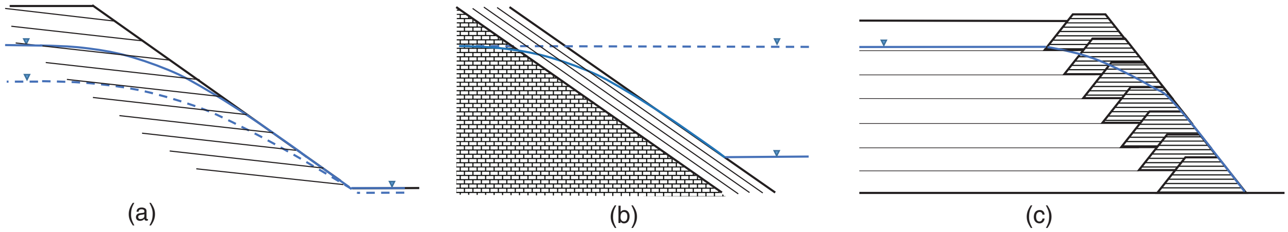

Fig. 1. (Color) Examples of a flow in hydraulically anisotropic slopes: (a) rise of water in inclined strata; (b) rapid drawdown in alluvial sediments; and (c) insufficient drainage in a tailings dam.

anisotropy can have a dramatic effect on slope susceptibility to piping.

The second limitation of the existing piping criteria is that they have not been translated into an adequate definition of the safety factors. Terzaghi (1943) defined a load-based safety factor against piping (heave) at the bottom of the cofferdam $F_p = W'/U_e$ as a ratio between the effective weight W' of the critical sand prism and the excess water pressure U_e at the bottom of the prism. For a uniform vertical flow, this safety factor reduces to the local, load-based safety factor against erosion $F_e = i_T/i$, where i is the local hydraulic gradient; $i_T = \gamma'/\gamma_w$ is the Terzaghi critical hydraulic gradient; γ' is the effective unit weight of soil; and γ_w is the unit weight of water (Terzaghi 1922). The existing theoretical studies of piping in isotropic slopes either do not define safety factors at all, or define them as classical load-based safety factors, which are known to have certain limitations even for isotropic materials. For example, for certain flow path inclinations, the effective weight can contribute to driving forces rather than to resisting ones. For anisotropic materials, the situation is further complicated by the fact that the driving seepage force is not parallel to the flow line. In this paper we'll provide classic load-based safety factors for a general anisotropic case, but also explore an alternative, strength-based safety factor definition, which has a more straightforward mechanical interpretation and is independent of the total versus effective equilibrium considerations. For a slope-parallel flow, this strength-based definition will also provide a safety factor against instability of an infinite anisotropic slope, which can be useful for design of tailings dams (Jantzer and Knutsson 2010).

The goal of this paper is to propose simple piping criteria and safety factors for slopes built of hydraulically stable anisotropic materials, and to demonstrate (1) why it is important to account for anisotropy and (2) how the proposed analytical expressions can be applied to practical problems with field piezometric measurements or the calculated flow net.

Because the general term "piping" covers a number of possible mechanisms (Richards and Reddy 2007), it is important to clarify upfront, what kind of piping is being considered here, and what are the corresponding assumptions and limitations. We investigate what Richards and Reddy (2007) call "the classic backwards-erosion style of piping, where a roof of competent soil or some other structure allows the formation of a bridged opening." Formation of an "erosion pipe" (Crosta and di Prisco 1999) starts close to the slope surface, where the critical flow path exits with a sufficiently high hydraulic gradient. Erosion then progresses backward into the slope, provided the seepage force is sufficiently high to overcome gravity and friction at the upstream end of the pipe (with earth pressures reduced by soil arching). Accordingly, the proposed local piping criterion (analogous to Terzaghi's erosion condition) will define piping initiation/progression at the location where the flow path exits the slope surface or enters the erosion channel. We shall also formulate an integrated criterion (analogous to Terzaghi's heave condition), and demonstrate its limitations and advantages. A particular case of a uniform flow in an infinite homogeneous slope will be investigated in more detail to demonstrate importance of anisotropy and to compare between different safety factor definitions.

Slope stability is not a part of this study and the piping is assumed to take place in a stable slope. This assumption can be justified by the fact that stability analysis is a standard procedure in design of dams and analysis of natural slopes, and is normally performed before investigating their susceptibility to piping. Furthermore, global stability of the slope can be higher than the local stability of a soil element along the critical flow path. This can happen, for example, due to a concentrated non-uniform flow, or due to a locally weaker cementation between the grains.

With the main focus on anisotropy and safety, the presented framework is currently limited to piping in materials, which (1) are cohesionless and (2) when subjected to a vertical flow, satisfy Terzaghi's condition $i_{cr} = i_T = \gamma'/\gamma_w$. While for hydraulically stable soils these two assumptions are conservative (e.g., Fleshman and Rice 2014), for unstable soils the latter one will result in an overestimation of the safety factors. Extending the proposed approach to internally unstable soils is, therefore, an important next step for enabling its wider application in practice.

Seepage in Hydraulically Anisotropic Slopes

The flow in a hydraulically anisotropic media is described by the total head h and the flux vector q [Fig. 2(a)]:

$$h = z + \frac{u}{\gamma_w}; \qquad q_{x'} = -k_{x'}\frac{\partial h}{\partial x'}; \qquad q_{z'} = -k_{z'}\frac{\partial h}{\partial z'} \tag{1}$$

where $k_{x'}$ and $k_{z'}$ are permeability coefficients in the principal hydraulic directions x' and z', inclined by angle β to the x and z axes, respectively

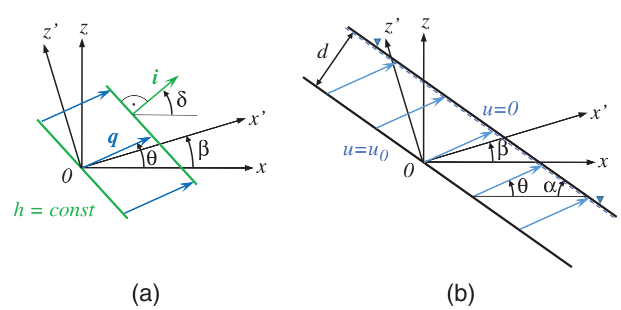

Fig. 2. (Color) Flow in a hydraulically anisotropic material: (a) local flow net; and (b) pore pressures and flow paths in an infinite slope.

$$x' = x\cos\beta + z\sin\beta; \qquad z' = -x\sin\beta + z\cos\beta$$
$$x = x'\cos\beta - z'\sin\beta; \qquad z = x'\sin\beta + z'\cos\beta \tag{2}$$

The continuity of flow results in the following differential equation

$$k_{x'}\frac{\partial^2 h}{\partial x'^2} + k_{z'}\frac{\partial^2 h}{\partial z'^2} = 0 \tag{3}$$

Directions in coordinates xz of the hydraulic gradient and the flow line are given by

$$\delta = \operatorname{atan}\frac{\partial h}{\partial z'}\bigg/\frac{\partial h}{\partial x'} + \beta \tag{4}$$

and

$$\theta = \operatorname{atan}\frac{q_{z'}}{q_{x'}} + \beta = \operatorname{atan}\frac{1}{r}\frac{\partial h}{\partial z'}\bigg/\frac{\partial h}{\partial x'} + \beta \tag{5}$$

respectively, where

$$r = \frac{k_{x'}}{k_{z'}} \tag{6}$$

is the anisotropy ratio. As expected for a flow in anisotropic materials (with $r \neq 1$), direction of the gradient δ (and thus of the seepage force) is related to, but does not coincide with the direction of the flow line θ

$$\delta = \operatorname{atan}(r\tan(\theta - \beta)) + \beta \tag{7}$$

All the aforementioned expressions are local and valid for any general 2D seepage boundary value problem. Solution of such a problem for a particular case of an infinite anisotropic slope is derived in the next section, to be later used in the safety factor parametric studies.

Uniform Flow in an Anisotropic Infinite Slope

Consider an infinite layer of saturated hydraulically anisotropic soil of constant thickness d, inclined by angle α to horizontal [Fig. 2(b)]. Consistent with the infinite slope assumption, pore water pressures on the internal boundary of the layer is $u = u_0$; on the external one, $u = 0$.

Solution of the differential Eq. (3) with boundary conditions

$$h = z + \frac{u_0}{\gamma_w} = z'\cos\beta + x'\sin\beta + \frac{u_0}{\gamma_w}, \quad \text{at} \quad z' = -x'\tan(\alpha+\beta)$$
$$h = z = z'\cos\beta + x'\sin\beta, \quad \text{at} \quad z' = \frac{d}{\cos(\alpha+\beta)} - x'\tan(\alpha+\beta) \tag{8}$$

for the internal and external boundaries, is given by

$$h = \frac{u_0}{\gamma_w d} + x'\sin\beta + z'\cos\beta - \frac{u_0}{\gamma_w d}(x'\sin(\alpha+\beta) + z'\cos(\alpha+\beta))$$
$$h = \frac{u_0}{\gamma_w d} + z - \frac{u_0}{\gamma_w d}(x\sin\alpha + z\cos\alpha) \tag{9}$$

Components of the hydraulic gradient are

$$i_{x'} = \frac{\partial h}{\partial x'} = \sin\beta - \bar{u}\frac{\sin(\alpha+\beta)}{\cos\alpha};$$
$$i_{z'} = \frac{\partial h}{\partial z'} = \cos\beta - \bar{u}\frac{\cos(\alpha+\beta)}{\cos\alpha}$$
$$i_x = \frac{\partial h}{\partial x} = -\bar{u}\tan\alpha; \qquad i_z = \frac{\partial h}{\partial z} = 1 - \bar{u} \tag{10}$$

where

$$\bar{u} = \frac{u_0\cos\alpha}{\gamma_w d} \tag{11}$$

is the normalized pore water pressure on the internal boundary of the slope.

Directions of the hydraulic gradient and of the flow (in coordinates xz) are given by

$$\delta = \operatorname{atan}\frac{\bar{u} - 1}{\bar{u}\tan\alpha} \tag{12}$$

and

$$\theta = \operatorname{atan}\frac{\bar{u}(1 - \tan\alpha\tan\beta) - 1}{r\bar{u}(\tan\alpha + \tan\beta) - r\tan\beta} + \beta \tag{13}$$

respectively. Of particular interest are the two special cases: $\beta = -\alpha$ (slope-parallel strata)

$$\theta = \operatorname{atan}\frac{\bar{u}(1+\tan^2\alpha) - 1}{r\tan\alpha} - \alpha = \operatorname{atan}\frac{\bar{u} - r + (r-1)\cos^2\alpha}{\bar{u}\tan\alpha + (r-1)\sin\alpha\cos\alpha} \tag{14}$$

and $\beta = 0$ (horizontal strata)

$$\theta = \operatorname{atan}\frac{\bar{u} - 1}{\bar{u}r\tan\alpha} \tag{15}$$

As expected, for isotropic materials ($r = 1$), Eqs. (14) and (15) become identical.

For arbitrary anisotropy orientation β, Eq. (13) can be resolved with respect to $\bar{u}$ as a function of the direction of flow θ:

$$\bar{u} = \frac{1}{1 - \tan\alpha\tan\beta}\frac{1 - r\tan\beta\tan(\theta-\beta)}{1 - r\tan(\alpha+\beta)\tan(\theta-\beta)} \tag{16}$$

Expressions for normalized pore water pressures $\bar{u}$ required to achieve horizontal $\theta = 0$ and slope-parallel $\theta = -\alpha$ flow for each of the two special cases of anisotropy are presented in Table 1. The slope-parallel ($\theta = -\alpha$) flow condition is given by

$$\bar{u}_\alpha = \frac{1}{1 - \tan\alpha\tan\beta}\frac{1 + r\tan\beta\tan(\alpha+\beta)}{1 + r\tan^2(\alpha+\beta)} \tag{17}$$

It follows that a flow with a nonzero component normal to the slope (a prerequisite for piping) can only take place when the normalized pore pressure exceeds $\bar{u}_\alpha$.

Table 1. Normalized pore water pressure required to achieve horizontal and slope-parallel flow

	Anisotropy direction	
Flow direction	$\beta = 0$	$\beta = -\alpha$
$\theta = 0$	$\bar{u} = 1$	$\bar{u} = r - (r-1)\cos^2\alpha$
$\theta = -\alpha$	$\bar{u}_\alpha = (1 + r\tan^2\alpha)^{-1}$	$\bar{u}_\alpha = \cos^2\alpha$

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Piping Criteria

Terzaghi (1922) identified two types of piping for vertical flow in horizontal terrain: backward erosion and heave. Backward erosion initiates when the vertical hydraulic gradient at the soil surface exceeds the critical value $i > i_T = \gamma'/\gamma_w$, while heave occurs when this local condition is not necessarily satisfied at the soil surface, but the excess water pressure U_e at the bottom of the critical soil prism exceeds its effective weight W' (Terzaghi 1943). For a uniform vertical flow in a homogeneous soil these two conditions become identical. In this section we apply similar concepts to define an initiation and a heave-type criteria for inclined flow in a sloped anisotropic ground, and to demonstrate their equivalence for a uniform flow.

Local Initiation/Progression Criterion

Initiation of piping as a backward erosion takes place locally at the slope surface, where the critical flow path exits the slope. To derive Terzaghi's local criterion $i_z \geq i_T = \gamma'/\gamma_w$, the vertical critical seepage force dS_{cr} (acting on a soil element of the volume dV at the ground surface) is found from the limiting equilibrium with the effective weight $dW' = \gamma' dV$. Piping initiation will take place if the existing seepage force $dS = i_z \gamma_w dV$ becomes larger than dS_{cr}. In this 1D setup, the friction on the vertical boundaries of the soil element is neglected. For a nonvertical flow, however, the friction along the flow path is proportional to the effective weight dW' and cannot be neglected. Therefore, in the limiting equilibrium of the soil element at the slope surface [Fig. 3(a)], the third force has to be considered in addition to the effective weight dW' and the critical seepage force dS_{cr}: the effective soil reaction dR'. Because in the limiting equilibrium the friction is fully mobilized, reaction dR' is inclined by angle φ' (effective angle of internal friction) to the normal to the flow line θ. Note, that in the case of anisotropic flow, the seepage force dS is not parallel to the flow line.

Limiting equilibrium equations can be derived from the polygon of effective forces in Fig. 3(b)

$$dR' \sin(\varphi' + \theta) = dS_{cr} \cos\delta; \quad dW' = dS_{cr} \sin\delta + dR' \cos(\varphi' + \theta) \tag{18}$$

where the inclination of the seepage force δ to horizontal is given by Eq. (7).

Substituting $dW' = \gamma' dV$ and eliminating dR' provides the local criterion for initiation of the backward erosion in an anisotropic slope, at the point where the critical path exits at the slope surface

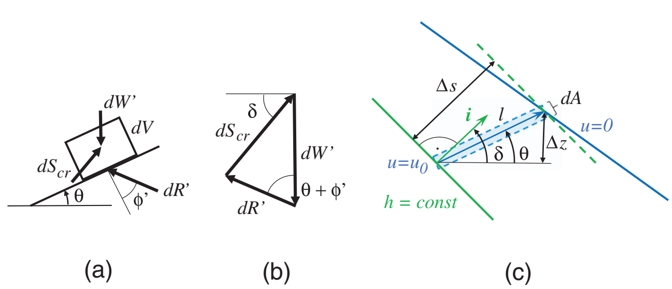

Fig. 3. (Color) Local erosion and piping in a hydraulically anisotropic slope: (a) a soil element of volume dV subjected to local erosion; (b) the polygon of forces; and (c) calculation of an average hydraulic gradient along a straight flow line.

$$dS \geq dS_{cr} = \gamma' dV \frac{\sin(\varphi' + \theta)}{\cos(\phi' + \theta - \delta)} \tag{19}$$

which after substitution of $dS = |i|\gamma_w dV$ can be formulated in terms of hydraulic gradients

$$|i| \geq i_{cr} = i_T \frac{\sin(\varphi' + \theta)}{\cos(\varphi' + \theta - \delta)} \tag{20}$$

where θ = local inclination of the flow line; $|i| = \sqrt{i_x^2 + i_z^2}$ is the absolute value of the exit hydraulic gradient; $\delta = \mathrm{atan}(r \tan(\theta - \beta)) + \beta$ is the inclination of the local gradient vector; and $i_T = \gamma'/\gamma_w$ is the Terzaghi critical hydraulic gradient. For retrogression of the backward erosion into the slope, the same local condition (20) should always be satisfied at the upstream end of the erosion channel supported by soil arching.

Integrated Heave-Type Criterion

Terzaghi's heave condition for a vertical flow $U_e > W'$ can be derived by integrating the critical ($dS_{cr} = dW' = \gamma' dV$) and acting ($dS = i_z \gamma_w dV$) seepage forces over the depth of a soil column, with uplift becoming possible when the effective stress at the bottom of the column $\sigma_z' = \int (dS_{cr} - dS)$ becomes negative. Neglecting friction at the sides of the column is normally justified either by soil liquefaction or by the large area of the column. Adapting this heave condition for a case of an inclined flow in a slope requires that the resultant of the driving forces along a portion of the flow line exiting the slope becomes larger than the resultant of the resisting forces $S \geq S_{cr}$

$$S = \gamma_w \Delta A \sqrt{\left(\int_0^l i_x dl\right)^2 + \left(\int_0^l i_z dl\right)^2}$$
$$S_{cr} = \gamma_w \Delta A \sqrt{\left(\int_0^l i_{cr} \cos\delta dl\right)^2 + \left(\int_0^l i_{cr} \sin\delta dl\right)^2} \tag{21}$$

where l = length of a flow path portion; and ΔA = area of the elementary pipe cross-section.

For curved flow paths the above integration has to be carried out numerically. For a straight flow path, inclinations θ and δ are constant along the path; therefore, i_{cr} calculated from Eq. (20) is also constant, and the resultant resisting force in the second Eq. (21) becomes

$$S_{cr} = i_{cr} \gamma_w l \Delta A = i_T \frac{\sin(\varphi' + \theta)}{\cos(\varphi' + \theta - \delta)} \gamma_w l \Delta A \tag{22}$$

where l = length of the straight flow path [Fig. 3(c)]. The resultant driving force S can be found using $i_z/i_x = \tan\delta$ and the relationship for the full differential of the total head

$$dh = i_x dx + i_z dz = i_x dl \cos\theta + i_z dl \sin\theta$$
$$= i_x dl(\cos\theta + \tan\delta \sin\theta) \tag{23}$$

leading to

$$i_x = \frac{dh}{dl} \frac{\cos\delta}{\cos(\delta - \theta)}; \qquad i_z = \frac{dh}{dl} \frac{\sin\delta}{\cos(\delta - \theta)} \tag{24}$$

which after substitution into in the first Eq. (21), where inclinations θ and δ are constant along the entire flow path give

$$S = \frac{\Delta h}{\cos(\delta - \theta)} \gamma_w \Delta A = \frac{u_0/\gamma_w - l \sin\theta}{\cos(\delta - \theta)} \gamma_w \Delta A \tag{25}$$

where Δh = total head drop between the beginning of the straight portion of the flow path, with the pore water pressure $u = u_0$ and its exit at the surface of the slope, where $u = 0$.

The integrated heave type criterion for a straight flow path is obtained by substituting S_{cr} from Eq. (22) and S from Eq. (25) into inequality $S \geq S_{cr}$

$$\bar{u}_z \geq \frac{1}{i_T} + \frac{\tan\delta + \cot\theta}{\tan\delta + \cot(\varphi' + \theta)} \tag{26}$$

where

$$\bar{u}_z = \frac{u_0}{\gamma' l \sin\theta}; \qquad \delta = \mathrm{atan}(r\tan(\theta - \beta)) + \beta \tag{27}$$

In contrast to the classical Terzaghi's heave criterion for horizontal ground, practical significance of the integrated criterion Eq. (26) is rather limited. With exception to rather low slope inclinations, sapping in natural slopes (Hagerty 1991), and poorly compacted layers in tailings dams (Klohn 1979), the affected portion of the slope is likely to become unstable before this condition is satisfied. The main practical value of this criterion is that for homogeneous soils it can replace Eq. (20) as a convenient approximate local initiation condition. Indeed, it does not require calculation of local hydraulic gradients at the slope surface and can be easily applied to anisotropic flow nets, where the exit portion of a flow path can be approximated by a straight line. Measuring the length l and inclination θ of this straight portion, and the pore water pressure u_0 at its upstream end, is sufficient for checking if the piping can originate at the exit.

Finally, for a uniform flow in an anisotropic infinite slope shown in Fig. 2, horizontal and vertical components of the hydraulic gradient are defined in Eq. (10) for the entire length $l = d/\sin(\alpha + \theta)$ of the straight flow lines, whose inclination θ is defined in Eq. (13). In this case, the hydraulic gradient distribution is uniform and the local initiation criterion [Eq. (20)] can be formulated in terms of the normalized pore water pressure $\bar{u}$

$$|i| = \sqrt{(1-\bar{u})^2 + \bar{u}^2\tan^2\alpha} \geq i_{cr} = i_T \frac{\sin(\varphi' + \theta)}{\cos(\varphi' + \theta - \delta)} \tag{28}$$

where δ and θ are given by Eqs. (12) and (13)

$$\delta = \mathrm{atan}\frac{\bar{u} - 1}{\bar{u}\tan\alpha}; \qquad \theta = \mathrm{atan}\frac{\bar{u}(1 - \tan\alpha\tan\beta) - 1}{r\bar{u}(\tan\alpha + \tan\beta) - r\tan\beta} + \beta;$$
$$i_T = \frac{\gamma'}{\gamma_w}; \qquad \bar{u} = \frac{u_0\cos\alpha}{\gamma_w d}; \qquad r = \frac{k_{x'}}{k_{z'}} \tag{29}$$

It can be shown that because in an infinite slope $l = d/\sin(\alpha + \theta)$ and $\tan\delta = (\bar{u} - 1)/(\bar{u}\tan\alpha)$, the heave criterion Eq. (26) also reduces to [Eq. (28)].

Validation of the Piping Criteria

In this section we consider three particular cases which validate the piping criterion [Eq. (28)] for uniform flow against some theoretical and experimental results for isotropic materials ($r = 1$).

Horizontal Layer ($\alpha = 0°$)

Substitution of $\alpha = 0°$ and $r = 1$ into Eq. (29) gives $\delta = \theta = 90°$, i.e., the upward vertical flow, for which the piping criterion [Eq. (28)] degenerates to

$$i = i_v \geq i_{v,cr} = i_T \tag{30}$$

which is identical to Terzaghi's criterion for the upward piping in hydraulically stable soils (as expected, independent of the friction angle φ').

Horizontal Flow ($\theta = 0°$)

Substitution of $\theta = 0°$ and $r = 1$ into Eq. (29) gives $\bar{u} = 1$ and $\delta = 0°$ for any $\alpha > 0°$ and the piping criterion [Eq. (28)] becomes

$$i = i_h \geq i_{h,cr} = i_T \tan\varphi' \tag{31}$$

den Adel et al., (1988) and Skempton and Brogan (1994) reported the ratio of $i_{h,cr}/i_{v,cr} = 0.7$ for hydraulically stable sandy gravels, which according to Eqs. (30) and (31) corresponds to $\varphi' = 35°$. Ahlinhan and Achmus (2010) measured the ratios of $i_{h,cr}/i_{v,cr} = 0.61$ for hydraulically stable fine sand at the relative density $I_D = 0.56$, and $i_{h,cr}/i_{v,cr} = 0.75$ for $I_D = 0.79$. The corresponding angles of internal friction are calculated using Eqs. (30) and (31) as $\varphi' = 31.4°$ and $\varphi' = 37.0°$, respectively. According to Bolton (1986), this is a reasonable range of the friction angles for the corresponding range of relative densities, considering rather small confining pressures in the tests.

Slope-Parallel Flow ($\theta = -\alpha$)

Substitution of $\theta = -\alpha$ and $r = 1$ into Eq. (29) gives

$$\bar{u} = \cos^2\alpha; \qquad \delta = -\alpha; \qquad i = \sin\alpha$$

and the piping criterion in Eq. (28) becomes

$$i_{-\alpha} = \sin\alpha \geq i_{-\alpha,cr} = i_T \frac{\sin(\varphi' - \alpha)}{\cos\varphi'} \tag{32}$$

This condition can be rewritten as

$$F_i = \frac{i_T}{1 + i_T}\frac{\tan\varphi'}{\tan\alpha} = \frac{\gamma'\tan\varphi'}{\gamma\tan\alpha} \geq 1 \tag{33}$$

where F_i can be recognized as the conventional safety factor for an isotropic infinite slope. It follows that for a slope-parallel flow the proposed piping condition is identical to the instability criterion for an infinite slope. Therefore, as expected for hydraulically stable materials, if the slope is stable, no piping can take place parallel to this slope. The equivalence of the piping and slope stability criteria for the slope-parallel flow is not coincidental. Later in the paper it will be demonstrated that it also holds for anisotropic slopes.

Load-Based Safety Factors against Piping

Definitions

Conventional load-based definition of the local safety factor against initiation of the piping at the slope surface is the ratio between the critical and the exit hydraulic gradients, and for the proposed condition in Eq. (19) we obtain

$$F_{p,l} = \frac{dS_{cr}}{dS} = \frac{i_{cr}}{|i|} = \frac{i_T}{|i|}\frac{\sin(\varphi' + \theta)}{\cos(\varphi' + \theta - \delta)}$$
$$\delta = \mathrm{atan}(r\tan(\theta - \beta)) + \beta \tag{34}$$

where θ = local inclination of the flow line; i = local hydraulic gradient; δ = inclination of the local gradient vector; and $i_T = \gamma'/\gamma_w$ is the Terzaghi critical hydraulic gradient.

The safety factor against heave for curved flow paths of length l is defined using Eq. (21)

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$$F_{p,l} = \frac{S_{cr}}{S} = \sqrt{\frac{(\int_0^l i_{cr}\cos\delta dl)^2 + (\int_0^l i_{cr}\sin\delta dl)^2}{(\int_0^l i_x dl)^2 + (\int_0^l i_z dl)^2}} \quad (35)$$

where i_{cr} is defined in condition (20). The safety factor against heave for straight flow paths of length l and inclination θ [Fig. 3(c)] is obtained using Eqs. (22) and (25)

$$F_{p,l} = \frac{S_{cr}}{S} = \frac{1}{\bar{u}_z - 1/i_T}\frac{\tan\delta + \cot\theta}{\tan\delta + \cot(\varphi' + \theta)}$$

$$\bar{u}_z = \frac{u_0}{\gamma' l \sin\theta}; \qquad \delta = \mathrm{atan}(r\tan(\theta - \beta)) + \beta \quad (36)$$

As aforementioned, for practical applications in homogeneous soils where the flow net is available, this definition can be used to replace the local safety factor [Eq. (34)], with the length l, inclination θ, and the pore water pressure u_0 obtained from the flow net. Eq. (36) is only valid for $\theta \neq 0$; for horizontal flow $\theta = 0$ it degenerates to

$$F_{p,l} = \frac{\gamma' l}{u_0(\tan\delta + \cot\varphi')}; \qquad \tan\delta = \frac{(1-r)\tan\beta}{1 + r\tan^2\beta} \quad (37)$$

Finally, for a uniform flow in an anisotropic infinite slope the safety factor against piping/heave is obtained from criterion in Eq. (28)

$$F_{p,l} = \frac{i_{cr}}{|i|} = \frac{i_T}{\bar{u} - 1 + \bar{u}\tan\alpha\cot(\varphi' + \theta)} \quad (38)$$

with parameters defined in Eq. (29). Because in an infinite slope $l = d/\sin(\alpha + \theta)$ and $\tan\delta = (\bar{u} - 1)/\bar{u}\tan\alpha$, all safety factor definitions in Eqs. (34), (36), and (38) become identical.

Parametric Study

The purpose of this parametric study is to demonstrate effects of soil anisotropy on the safety factor against piping. Because all the aforementioned safety factors depend both on the flow direction θ and either on the hydraulic gradient or the pore water pressure, the parametric study can become cumbersome and difficult to visualize. To avoid this, we are going to use the load-based safety factor $F_{p,l}$ for an infinite slope because it can be expressed as a function of the flow direction θ only, by substituting $\bar{u}$ from Eq. (16) into Eq. (38). Dependency of the safety factor on the anisotropy orientation β is shown in Fig. 4, where $i_T = 1$; $\alpha = 15°, 30°$; $\theta = -\alpha, 0$; and $r = 1, 3, 5, 10$. Friction angle is chosen as $\tan\varphi' = 2\tan\alpha$, which satisfies $F_i = 1.0$ in Eq. (33), i.e., the corresponding isotropic slope would be at the limiting equilibrium. This means that an anisotropic slope with the same parameters is stable (as demonstrated later in the paper). From Fig. 4 it follows that for a horizontal flow $\theta = 0$, the load-based safety factor against piping can become lower than unity for lower values of angle β, and larger values of slope inclination α and anisotropy ratio r.

Dependency of the safety factor on the flow direction θ is shown in Fig. 5 for $i_T = 1$, $\tan\varphi' = 2\tan\alpha$; two slope inclinations $\alpha = 15°, 30°$; two anisotropy orientations $\beta = -\alpha, 0$; and five anisotropy ratios $r = 1, 2, 3, 5, 10$. It can be seen that the slope-parallel anisotropy orientation is more unfavorable in terms of piping than the horizontal anisotropy orientation. It can also be observed that for larger slope inclinations α and anisotropy ratios r, piping $(F_{p,l} < 1)$ can occur at lower angles θ of the flow line inclinations.

Strength-Based Safety Factor against Piping

A general limitation of the load-based safety factors in piping problems is that although the same piping criteria can be derived using

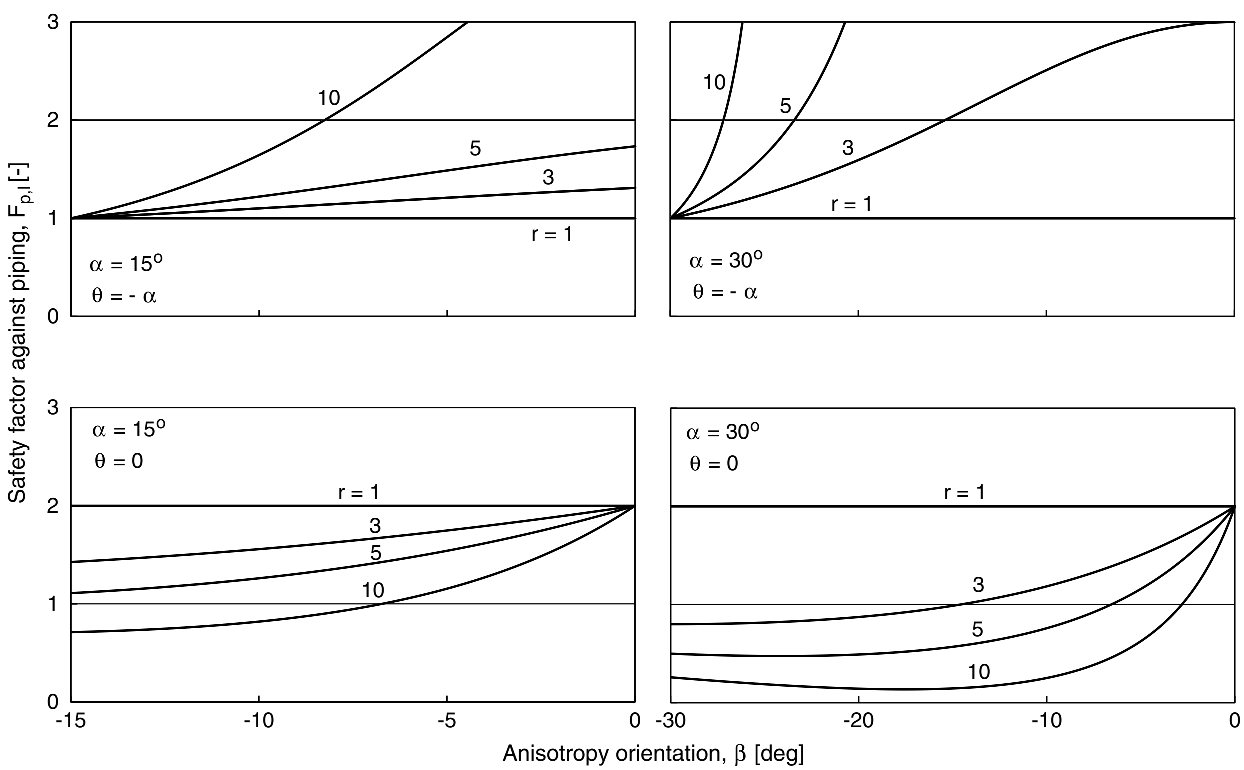

Fig. 4. Load-based safety factor against piping as a function of anisotropy orientation (slope parallel flow in the top plots and horizontal flow in the bottom plots).

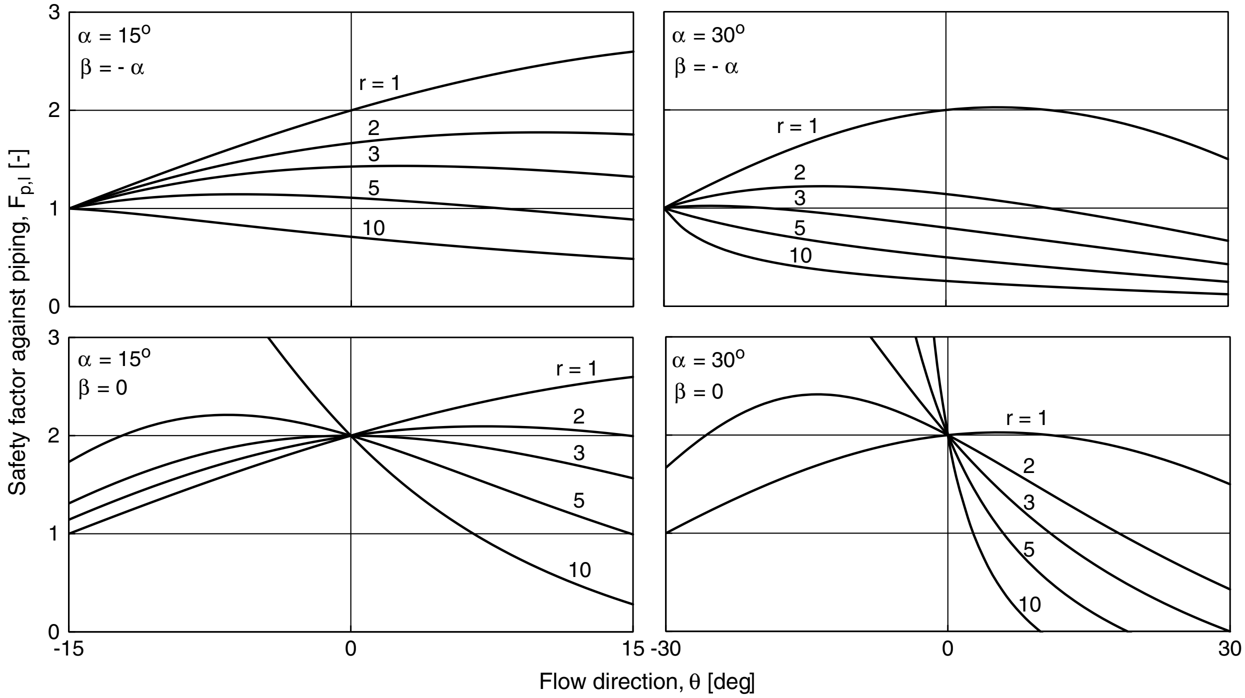

Fig. 5. Load-based safety factor against piping as a function of flow direction.

either total or effective forces, the corresponding safety factors are not identical. Also, in contrast to the classical case of a vertical flow in a horizontal layer, where the load-based safety factor against piping can be interpreted as the ratio between the resisting effective weight and the driving seepage force, interpretation in the case of an inclined flow in a sloped ground is less straightforward. Firstly, depending on the flow path inclination, the effective weight can contribute to driving force rather than to resisting one. Secondly, in anisotropic soils the driving seepage force is not parallel to the flow line.

On the other hand, the general case also presents an opportunity to resolve the above issues, because in contrast to the classical one, resistance to piping does not rely here purely on gravity, but also on friction. This allows for introducing an alternative, strength-based safety factor, which is independent of driving versus resisting and total versus effective forces considerations.

Strength-Based Piping Criteria

The local piping initiation/progression condition [Eq. (20)] was formulated as a load-based criterion, using existing and critical hydraulic gradients, the latter derived from the effective limiting equilibrium Eq. (18) of a soil element in Fig. 3(a). In order to formulate the strength-based local criterion, it is still convenient to use the equilibrium of effective forces, but instead of looking for the critical seepage force dS_{cr} as a function of the existing friction ϕ', we shall determine the mobilized angle of internal friction φ'_m at which the existing seepage force $dS = i\gamma_w dV$ can bring the soil element to failure [Fig. 6(a)]. In this formulation, the piping will take place when the mobilized friction φ'_m exceeds the existing friction φ'. From the force polygon in Fig. 6(a)

$$dR' \sin(\varphi'_m + \theta) = dS \cos\delta; \quad dW' = dS \sin\delta + dR' \cos(\varphi'_m + \theta) \tag{39}$$

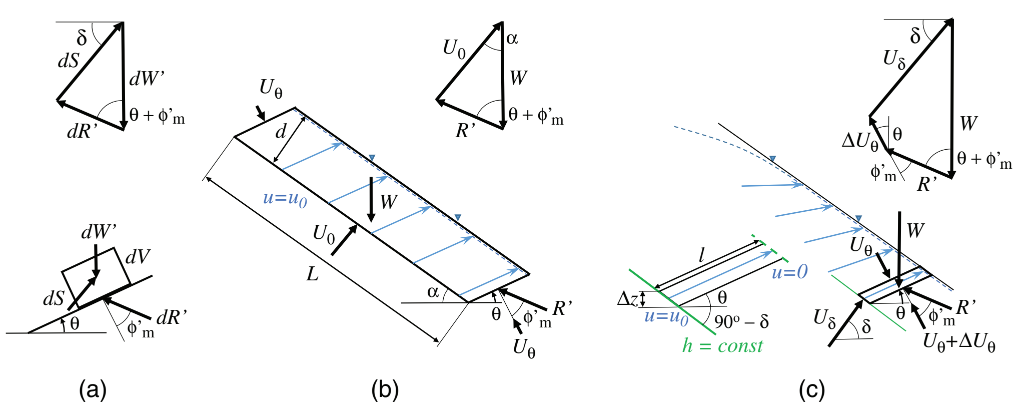

Fig. 6. (Color) Stability analysis: (a) a soil element; (b) a semiinfinite slope with a uniform flow; and (c) a straight portion of a daylighting flow path within a nonuniform flow.

where the inclination of the seepage force δ to horizontal is given by Eq. (7). Eliminating dR' and using $dW' = \gamma' dV$, $dS = i\gamma_w dV$ and $i_T = \gamma'/\gamma_w$ gives the strength-based piping initiation condition for a soil element subjected to a seepage force in an anisotropic slope

$$\varphi'_m = \delta_s - \theta \geq \varphi'; \qquad \delta_s = \text{atan}\frac{-i_x}{i_T + i_z} = \text{atan}\frac{-|i|\cos\delta}{i_T + |i|\sin\delta} \tag{40}$$

which is identical to the load-based initiation criterion in Eq. (20). For the uniform anisotropic flow in an infinite slope shown in Fig. 2, horizontal and vertical components of the hydraulic gradient are defined in Eq. (10), so that the strength-based initiation criterion becomes

$$\varphi'_m = \text{atan}\frac{\bar{u}\tan\alpha}{i_T + 1 - \bar{u}} - \theta \geq \varphi' \tag{41}$$

with θ and $\bar{u}$ defined by Eq. (29).

Next, we formulate the strength-based heave criterion for a uniform flow in an infinite slope. Here, instead of using equilibrium of effective forces, it is more convenient to consider equilibrium of total forces and pore water pressures. Consider a soil layer of the width d and length $L \gg d$, inclined by angle α and subjected to the uniform pore water pressure $u = u_0 \geq d\gamma\cos\alpha$ on its internal boundary [Fig. 6(b)].

Direction of the flow in the layer θ can be determined from Eq. (13). Equilibrium of the layer against sliding along the flow line can be presented graphically as a polygon of total forces [Fig. 6(b)], where $W = \gamma L d$ is the total weight of the layer; $U_0 = u_0 L$ is the force applied by the water pressure on the internal boundary (inclined by angle α); R' is the effective reaction force on the sliding surface (inclined by angle θ). Note that at the limiting equilibrium, there is no soil reaction on the internal boundary of the slope. The force U_θ, applied by the water pressure on the sliding surface is proportional to d and, therefore, can be neglected compared to other forces, which are all proportional to $L > d$. It follows that the inclination of the effective reaction force R' to the normal to the sliding surface is equal to the mobilized angle of internal friction φ'_m, and from the force polygon in Fig. 6(b)

$$\frac{U_0}{\sin(\varphi'_m + \theta)} = \frac{W}{\sin(\varphi'_m + \theta + \alpha)} \tag{42}$$

which can be rewritten as

$$\frac{u_0\cos\alpha}{\gamma d} = \frac{\tan(\varphi'_m + \theta)}{\tan(\varphi'_m + \theta) + \tan\alpha} \tag{43}$$

and resolved with respect to the mobilized friction

$$\tan\varphi'_m = \frac{\bar{u}\tan\alpha - (i_T + 1 - \bar{u})\tan\theta}{(i_T + 1 - \bar{u}) + \bar{u}\tan\alpha\tan\theta}; \qquad \bar{u} = \frac{u_0\cos\alpha}{\gamma_w d} \tag{44}$$

Eq. (44) can be simplified as

$$\varphi'_m = \text{atan}\frac{\bar{u}\tan\alpha}{i_T + 1 - \bar{u}} - \theta \geq \varphi' \tag{45}$$

providing the strength-based heave condition for a semiinfinite slope, which, as expected, is identical to the strength-based piping initiation criterion [Eq. (41)].

Finally, we formulate the strength-based heave condition for a more general case, of a nonuniform flow in a slope, where a flow path exiting the slope surface can be approximated by a straight line, with length l, inclination θ, and the pore water pressure u_0 at its upstream end obtained from the flow net [Fig. 6(c)]. Considering the equilibrium of total forces acting on a soil pipe of the height Δz around the flow path, from the force polygon in Fig. 6(c) we can write the following equilibrium equations

$$R'\sin(\varphi'_m + \theta) = U_\delta\cos\delta - \Delta U_\theta\sin\theta$$
$$W = R'\cos(\varphi'_m + \theta) + U_\delta\sin\delta + \Delta U_\theta\cos\theta \tag{46}$$

where W = total weight of the soil pipe; U_δ = pore pressure acting at the pipe entrance normal to the equipotential line; and ΔU_θ = net pore pressure acting normal to the flow path

$$W = \gamma l\Delta z\frac{\cos(\delta - \theta)}{\cos\delta}; \qquad U_\delta = u_0\Delta z\frac{1}{\cos\delta}; \qquad \Delta U_\theta = \gamma_w l\Delta z \tag{47}$$

and R' = effective reaction force, which can be eliminated from Eq. (46)

$$W = U_\delta\frac{\cos(\varphi'_m + \theta - \delta)}{\sin(\varphi'_m + \theta)} + \Delta U_\theta\frac{\sin\varphi'_m}{\sin(\varphi'_m + \theta)} \tag{48}$$

and after substitution of Eq. (47) and certain trigonometric simplifications gives

$$\bar{u}_z = \frac{1}{i_T} + \frac{\tan\delta + \cot\theta}{\tan\delta + \cot(\varphi'_m + \theta)} \tag{49}$$

where

$$\bar{u}_z = \frac{u_0}{\gamma' l\sin\theta}; \qquad \delta = \text{atan}(r\tan(\theta - \beta)) + \beta \tag{50}$$

Eq. (49) is equivalent to the load-based heave condition [Eq. (26)] and can be converted into the strength-based heave criterion:

$$\varphi'_m = \delta_s - \theta \geq \varphi'$$
$$\delta_s = \arctan\left(\frac{\bar{u}_z - i_T^{-1}}{\cot\theta + (1 + i_T^{-1} - \bar{u}_z)\tan\delta}\right) \tag{51}$$

For a uniform flow in an infinite slope, with $l = d/\sin(\alpha + \theta)$ and $\tan\delta = (\bar{u} - 1)/(\bar{u}\tan\alpha)$, the strength-based piping initiation and heave criteria [Eqs. (41), (45), and (51)] are identical to each other and to the corresponding load-based criteria.

Strength-Based Definition of the Safety Factor against Piping

While the load and strength-based piping initiation/progression and heave criteria are identical, the corresponding safety factors are not. In contrast to the load-based approach, where the safety factor is defined as the ratio between the resisting and driving forces, the strength-based approach uses the ratio between the existing and mobilized strength. For the piping initiation/progression condition [Eq. (40)], this results in

$$F_{p,s} = \frac{\tan\varphi'}{\tan\varphi'_m} = \frac{\tan\varphi'}{\tan(\delta_s - \theta)} \tag{52}$$

where

$$\delta_s = \text{atan}\frac{-|i|\cos\delta}{i_T + |i|\sin\delta}; \qquad \delta = \text{atan}(r\tan(\theta-\beta)) + \beta;$$
$$i_T = \frac{\gamma'}{\gamma_w}; \qquad r = \frac{k_{x'}}{k_{z'}} \tag{53}$$

For the heave condition in Eq. (51), the strength-based safety factor is defined as

$$F_{p,s} = \frac{\tan\varphi'}{\tan\varphi'_m} = \frac{\tan\varphi'}{\tan(\delta_s - \theta)} \tag{54}$$

where

$$\delta_s = \arctan\left(\frac{\bar{u}_z - i_T^{-1}}{\cot\theta + (1 + i_T^{-1} - \bar{u}_z)\tan\delta}\right); \qquad \bar{u}_z = \frac{u_0}{\gamma' l \sin\theta}$$
$$\delta = \text{atan}(r\tan(\theta-\beta)) + \beta; \qquad i_T = \frac{\gamma'}{\gamma_w}; \qquad r = \frac{k_{x'}}{k_{z'}} \tag{55}$$

which for a particular case of $\theta = 0$ degenerates to

$$F_{p,s} = \frac{\gamma' l - u_0\tan\delta}{u_0\cot\varphi'}; \qquad \tan\delta = \frac{(1-r)\tan\beta}{1 + r\tan^2\beta} \tag{56}$$

Finally, the strength-based safety factor against piping/heave in an infinite slope with a uniform flow is obtained using the corresponding condition [Eq. (45)]

$$F_{p,s} = \frac{\tan\varphi'}{\tan\varphi'_m} = \frac{\tan\varphi'}{\tan(\delta_s - \theta)} = F_i\frac{i_T + 1}{i_T}\frac{\tan\alpha}{\tan(\delta_s - \theta)} \tag{57}$$

where

$$\delta_s = \text{atan}\frac{\bar{u}\tan\alpha}{i_T + 1 - \bar{u}}; \qquad \theta = \text{atan}\frac{\bar{u}(1 - \tan\alpha\tan\beta) - 1}{r\bar{u}(\tan\alpha + \tan\beta) - r\tan\beta} + \beta$$
$$F_i = \frac{i_T}{1 + i_T}\frac{\tan\varphi'}{\tan\alpha}; \qquad i_T = \frac{\gamma'}{\gamma_w}; \qquad \bar{u} = \frac{u_0\cos\alpha}{\gamma_w d}; \qquad r = \frac{k_{x'}}{k_{z'}} \tag{58}$$

with F_i being the conventional safety factor for an isotropic infinite slope.

Advantage of the strength-based definition [Eq. (57)] is that it is not just limited to piping but, as shown in the next section, can be also used to describe another type of instability.

Safety Factor against Instability of an Anisotropic Infinite Slope

For a hydraulically anisotropic infinite slope, the strength-based piping criterion in Eqs. (42)–(45) is derived using the limiting equilibrium considerations that are identical to those for a classical stability analysis of a block with a sliding surface inclined by angle θ. Therefore, for a slope-parallel flow, substituting $\theta = -\alpha$ and $\bar{u} = \bar{u}_\alpha$ from Eq. (17) into Eq. (45) will provide the instability criterion for an anisotropic infinite slope. The corresponding strength-based safety factor is derived by substituting $\theta = -\alpha$ and $\bar{u} = \bar{u}_\alpha$ into Eq. (57)

$$F_a = \frac{\tan\varphi'}{\tan\varphi'_m} = \frac{F_i}{i_T}\left(1 + i_T - \frac{1 + \tan^2\alpha}{1 - \tan\alpha\tan\beta}\frac{1 + r\tan\beta\tan(\alpha+\beta)}{1 + r\tan^2(\alpha+\beta)}\right) \tag{59}$$

where F_i = conventional safety factor for an isotropic infinite slope defined in Eq. (58).

As expected, for two particular cases: (1) $r = 1$ and arbitrary β and (2) a slope-parallel anisotropy $\beta = -\alpha$ and arbitrary r; Eq. (59) degenerates into $F_a = F_i$. For a horizontally oriented anisotropy $\beta = 0$ (typical for tailings dams) and arbitrary r

$$F_a = \left[1 + \frac{1}{i_T}\frac{(r-1)\tan^2\alpha}{1 + r\tan^2\alpha}\right]F_i \tag{60}$$

According to Jantzer and Knutsson (2010), the rule of thumb for design of tailings dams in Sweden requires that for a case of the slope-parallel flow, the safety factor for the slope stability should be at least 1.5. Because designers conventionally assume that the dam wall is isotropic, this requirement implies that $F_i = 1.5$. From Eq. (60) it follows, however, that for a typical range of $r = 4 - 10$ (Klohn 1979), the dam will have 20% to 50% of additional hidden safety.

General dependency of the normalized safety factor F_a/F_i on the anisotropy orientation β is identical to that shown in the top two plots of Fig. 4 for $i_T = 1$, two slope inclinations $\alpha = 15°, 30°$ and four anisotropy ratios $r = 1, 3, 5, 10$. It follows that for $r > 1$, the anisotropic slope is always more stable than the isotropic one ($F_a > F_i$). It can be also shown that for $r < 1$, the opposite is true (i.e., $F_a < F_i$).

Parametric Study

The strength-based piping/heave safety factor $F_{p,s}$ for an infinite slope can be expressed as a function of the flow direction θ by substituting $\bar{u}$ from Eq. (16) into the expression in Eq. (58) for δ_s. Dependency of the safety factor on the anisotropy orientation β is shown in Fig. 7 for $i_T = 1$; two slope inclinations $\alpha = 15°, 30°$; two flow directions $\theta = -\alpha, 0$; and four anisotropy ratios $r = 1, 3, 5, 10$. Note that Eqs. (57) and (58) are also valid for $r < 1$, but the case of $r > 1$ is of a higher practical significance. Friction angle is again chosen to satisfy $F_i = 1.0$, i.e., as aforementioned, the corresponding anisotropic infinite slope with slope-parallel flow is stable.

Comparing Fig. 7 with Fig. 4, it can be concluded that for a slope-parallel flow $\theta = -\alpha$ and $r > 1$, the load-based safety factors for piping are higher than the strength-based ones. For a horizontal flow, the load and strength-based safety factors for piping are not dramatically different, and it is not possible to claim that one is consistently more conservative than another.

In both cases, an increase in the slope inclination α and anisotropy ratio r leads to a decrease in the safety factor, which can become lower than unity, in particular for lower values of β. The only difference is that unlike the load-based safety factor, the strength-based one can become negative for larger values of the anisotropy ratio $(r = 5, 10)$.

Dependency of the safety factor on the flow direction θ is shown in Fig. 8, where $i_T = 1$, $F_i = 1.0$; $\alpha = 15°, 30°$; $\beta = -\alpha, 0$; and $r = 1, 2, 3, 5, 10$. Similar to the load-based safety factors in Fig. 5, in can be seen that the slope-parallel anisotropy orientation is more susceptible to piping than the horizontal anisotropy orientation, and piping $(F_{p,s} < 1)$ is achieved at smaller inclinations θ of the flow lines for larger slope inclinations α and anisotropy ratios r. However, in spite of these qualitative similarities, the values of the load-based safety factor in Fig. 5 and strength-based safety factor in Fig. 8 can differ significantly.

Discussion: Which Safety Factor Definition to Use?

The observed strong dependency of safety factors in Figs. 4, 5, 7, and 8 on anisotropic parameters β and r highlights importance of

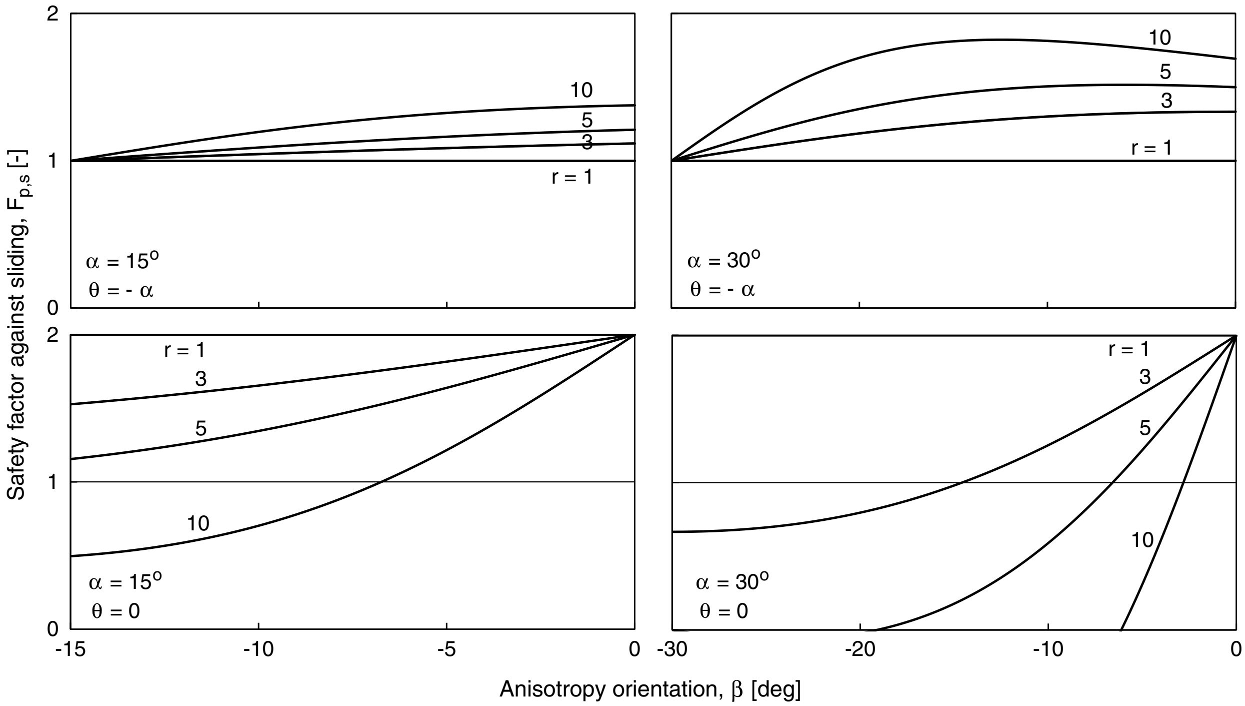

Fig. 7. Strength-based safety factor against piping as a function of anisotropy orientation.

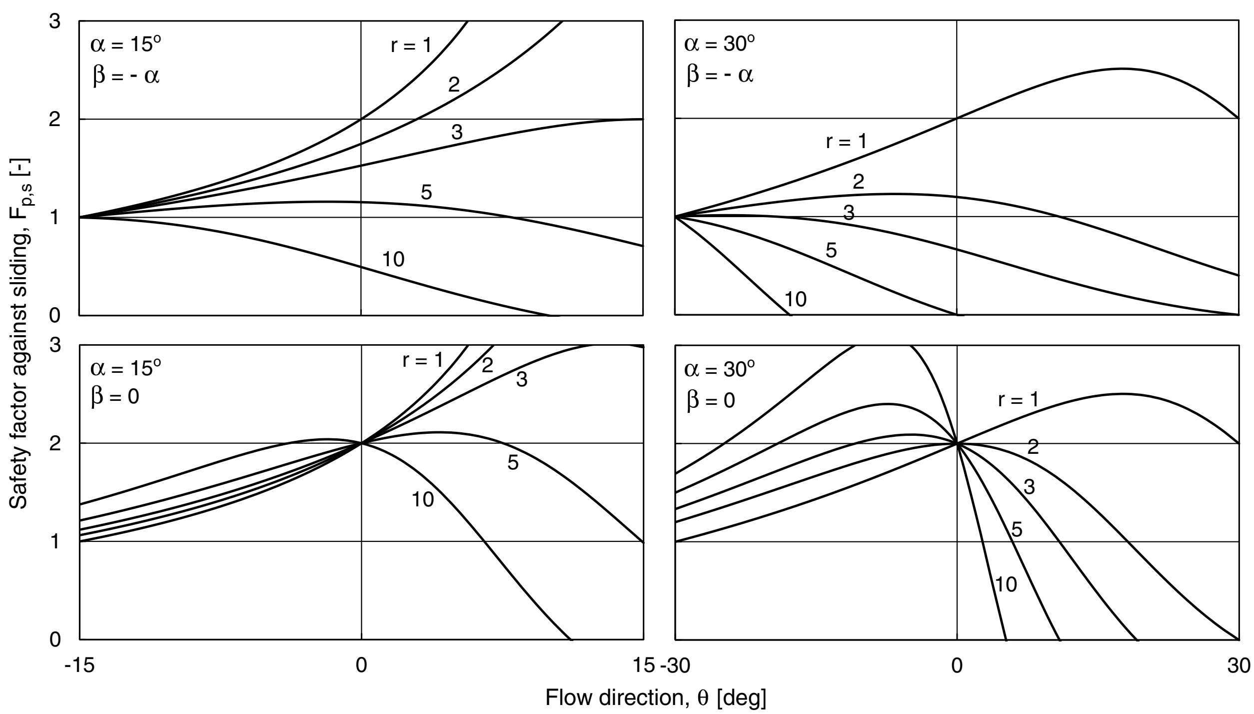

Fig. 8. Strength-based safety factor against piping as a function of flow direction.

accounting for soil anisotropy in piping analysis. At the same time, the observed quantitative difference between the two definitions of the safety factor against piping requires a decision to be made, which safety factor definition is more appropriate for design and analysis.

The strength-based definition of the safety factor against piping has been demonstrated to have certain advantages over the load-based definition

- it is independent of how different forces are attributed to driving versus resisting ones;

Table 2. Load and strength-based safety factors against piping for four different flow directions in an isotropic slope ($r = 1$)

Safety factor	Flow direction			
	$\theta = \pi/2$; $\alpha = 0$	$\theta = \pi/2 - \alpha$	$\theta = 0$	$\theta = -\alpha$
$\boldsymbol{F_{p,l}}$	i_T/i_z	0	$(i_T + 1)F_i$	$(i_T + 1)F_i - i_T$
$\boldsymbol{F_{p,s}}$	0	0	$(i_T + 1)F_i$	F_i

- it is independent of total versus effective forces being considered in the formulation; and
- it can be used not only in the piping but sometimes also in stability analysis.

Nevertheless, the strength-based definition cannot be categorically recommended for use in the piping analysis, for two related reasons. The first one is that existing minimum safety factor requirements are historically based on a vast experience gained from applying the Terzaghi safety factor definition, which uses the ratio between effective forces acting on the soil element.

In this context, the strength-based definition could still be used if it could be demonstrated to be consistently more conservative than the load-based one, which is not the case (see Figs. 5 and 8). This becomes even more obvious in direct comparison between the two definitions for a particular case of an infinite isotropic ($r = 1$) slope with uniform flow, where load and strength-based safety factors [Eqs. (38) and (58)] are given, respectively, by

$$F_{p,l} = \left(i_T + 1 + \frac{i_T}{F_i}\frac{\tan\theta}{\tan\alpha}\right)\frac{1 - \tan\alpha\tan\theta}{1 + \tan^2\theta}F_i$$
$$F_{p,s} = \frac{(i_T + 1)\tan\alpha}{\tan(\alpha + \theta) - (i_T + 1)\tan\theta}F_i \qquad (61)$$

Comparison between these two safety factor definitions for four particular cases of the flow direction is shown in Table 2, and for arbitrary flow directions, with $i_T = 1$, $F_i = 1.0$ and $\alpha = 15°$, $20°$, $30°$, in Fig. 9. It follows that apart from some special cases, the two definitions produce very different values. For $\theta < \alpha$ the strength-based definition $F_{p,s}$ seems to be more conservative, but for most of the $\theta > \alpha$ range it appears to be less conservative to a very significant extent. Furthermore, the strength-based safety factor against piping has a limitation of not converging to Terzaghi's safety factor for the upward flow in a horizontal layer.

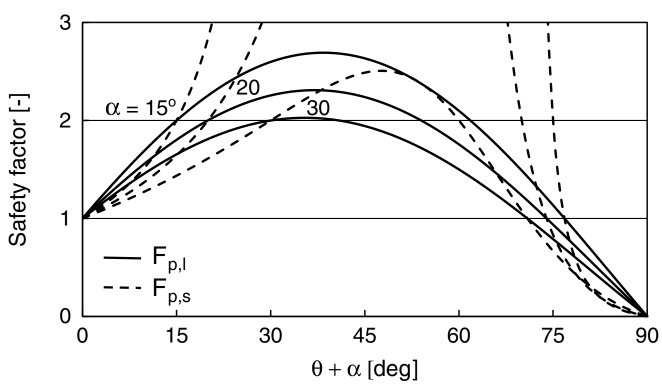

Fig. 9. Load-based (solid lines) and strength-based (dashed lines) safety factors against piping in isotropic slopes as a function of the flow direction.

It can be suggested, that the best strategy would be to perform piping analysis using both safety factor definitions and to require that both of them satisfy the requirements. As demonstrated in the following section, the additional effort is minimal, since both definitions require the same geometric and soil parameters. If exit hydraulic gradients are available, local safety factors should be used, otherwise they can be approximated by the integrated ones.

Applications

To demonstrate application of the proposed criteria to practical cases, two conceptual examples are presented below: (1) a rapid drawdown in a natural slope [Fig. 1(b)], and (2) piping in a tailings dam [Fig. 1(c)]. To enable a comparison, the idealized geometry of both examples is similar. Pore water pressures and flow paths for the two examples are shown in Figs. 10(a and b), respectively, with the safety factors summarized in Table 3.

Rapid Drawdown in a Natural Slope

Consider an artificial reservoir with a natural 1V:2H slope [$\alpha = 26.57°$, Fig. 10(a)], built of a weathered rock covered by the layer of slope debris of thickness $d = 8.95$ m. The weathered rock is isotropic, with permeability coefficient $k = 10^{-6}$ m/s; permeability of the slope debris is anisotropic, oriented parallel to the slope ($\beta = -\alpha$), with the slope-parallel permeability coefficient $k_{x'} = 10^{-6}$ m/s, and the anisotropy ratio $r = 3$. The horizontal base is assumed to have permeability $k \ll 10^{-6}$ m/s. Angle of internal friction of the slope debris $\varphi' = 45°$; its submerged unit weight $\gamma' = \gamma_w = 9.81$ kN/m^3, so that $i_T = 1$; $F_i = 1.0$. Initially, the water level in the reservoir and in the slope stands at the elevation 50 m. After rapid drawdown the water level in the reservoir drops to

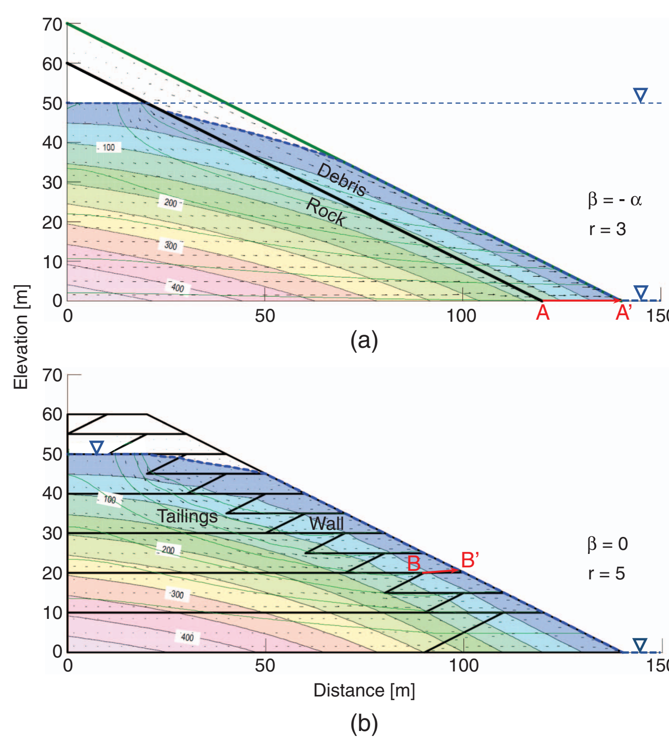

Fig. 10. (Color) Application examples (pore pressure distributions from a SEEP/W analysis): (a) rapid drawdown in a reservoir with a slope-parallel sediment layer; and (b) tailings dam with horizontally compacted layers on an impermeable base.

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Table 3. Flow parameters and safety factors for application examples in Fig. 10

	Anisotropy direction and ratio	
	$\beta = -\alpha$	$\beta = 0$
Parameters	$r = 3$	$r = 5$
$\theta_{measured}$	0°	3.10°
$u_{0,measured}$	125 kPa	115 kPa
F_{pl}	0.999	1.46
F_{ps}	0.998	1.61
$u_{0,uniform}$	137 kPa	113.5 kPa
$F_{pl,uni}$	—	1.48
$F_{ps,uni}$	—	1.64

zero, causing flow in the slope, with contours of the water pressures and flow paths obtained using software SEEP/W (GEO-SLOPE 2020) and shown in Fig. 10(a).

The contours of the pore water pressure are not parallel to the slope so that the infinite slope conditions are not satisfied and the corresponding safety factors against piping cannot be used. Local safety factors against piping initiation are difficult to use, because exit hydraulic gradients on the slope surface are not available. Instead, because the flow paths are straight and the soil is homogeneous, we can use integrated heave safety factors [Eqs. (36), (54), and (55)] as a practical tool to analyze piping initiation. The lowest safety factors are likely to be achieved along the flow path AA′, at the boundary with the base material in Fig. 10(a). Because inclination of this flow path is $\theta = 0$, the safety factors are calculated using Eqs. (37) and (56), which are special cases of Eqs. (36) and (54), respectively. The length of the flow path AA′ is $l = 20$ m and the pore water pressure at the point A is $u_0 = 125$ kPa [Fig. 10(a)], which after substitution into Eqs. (37) and (56) give

$$\tan\delta = \frac{(1-r)\tan\beta}{1+r\tan^2\beta} = \frac{4}{7}$$

$$F_{p,l} = \frac{\gamma' l}{u_0(\tan\delta + \cot\varphi')} = 0.999;$$

$$F_{p,s} = \frac{\gamma' l - u_0\tan\delta}{u_0\cot\varphi'} = 0.998$$

It follows that piping can initiate at the point A′. Applying Eqs. (36), (54), and (55) to other flow paths (with corresponding values of u_0 and θ) confirms that the flow path AA′ is critical for piping. Clearly, this analysis only makes sense if the slope remains stable, which can be due to cohesion in the debris material, neglected at the boundary with the base.

Tailings Dam Analysis

A tailings dam is built on an impermeable base [Fig. 10(b)]. Inclination of the tailings dam wall is 1V:2H ($\alpha = 26.57°$); the average horizontal width of the wall is 20 m; and in the narrowest place it is 10 m ($d = 4.47$ m). Tailings are hydraulically isotropic, with permeability coefficient linearly decreasing with depth due to self-compaction: $k = (1 + z/10)\cdot 10^{-6}$ m/s, where z is the elevation (in meters) above the base. Permeability of the compacted wall is anisotropic, oriented horizontally ($\beta = 0$), with the horizontal permeability coefficient $k_{x'} = 10^{-6}$ m/s and the anisotropy ratio $r = 5$. Angle of internal friction of the wall material is $\varphi' = 45°$; its submerged unit weight $\gamma' = \gamma_w = 9.81$ kN/m^3, so that $i_T = 1$; $F_{si} = 1.0$. The water level in the dam has reached the elevation of 50 m, with contours of the water pressure and flow paths obtained using SEEP/W (GEO-SLOPE 2020) and shown in Fig. 10(b).

Similar to the previous example, due to the lack of data about exit hydraulic gradients on the wall surface, we are going to use integrated heave safety factors [Eqs. (36), (54), and (55)] as a convenient approximation of the corresponding local safety factors against piping initiation. In the flow net in Fig. 10(b), all the flow paths in the bottom half of the wall look similar, and for a typical flow path BB′ at the narrow section in the midheight, we obtain its length ($l = 9$ m), inclination ($\theta = 3.8°$) and the pore water pressure at its upstream end B ($u_0 = 58$ kPa), which after substitution into Eqs. (37) and (56) together with $\beta = 0$ give

$$\bar{u}_z = \frac{u_0}{\gamma' l \sin\theta} = 10.1; \qquad \tan\delta = r\tan\theta = 0.33;$$

$$\delta_s = \arctan\left(\frac{\bar{u}_z - i_T^{-1}}{\cot\theta + (1 + i_T^{-1} - \bar{u}_z)\tan\delta}\right) = 0.633;$$

$$F_{p,l} = \frac{1}{\bar{u}_z - 1/i_T}\frac{\tan\delta + \cot\theta}{\tan\delta + \cot(\varphi' + \theta)} = 1.40;$$

$$F_{p,s} = \frac{\tan\varphi'}{\tan(\delta_s - \theta)} = 1.57$$

It follows that both safety factors against piping along the flow path BB′ are larger than unity, but the load-based one is smaller than the minimum safety factor of 1.5 normally required for such structures. It can be concluded that this structure cannot be considered safe, and that, for this particular example, using the strength-based safety factor would be misleading. Again, this analysis only makes sense if the dam remains stable.

It can be also noticed, that in contrast to the previous example in Fig. 10(a), in Fig. 10(b) the contours of the pore water pressure are almost parallel to the slope; therefore, the infinite slope conditions are likely to be satisfied. This can be validated by substituting the "measured" pore water pressure $u_0 = 58$ kPa into Eq. (11), to obtain the normalized pore water pressure $\bar{u} = 1.18$, which after substitution unto Eq. (13) gives the estimate of the flow path inclination for an infinite slope $\theta = 3.53°$. This inclination is indeed very close to $\theta = 3.8°$ "measured" along the path BB' in Fig. 10(b). Substituting $\theta = 3.53°$ and $\bar{u} = 1.18$ into Eqs. (38), (57), and (58), we obtain the load and strength-based safety factors against piping in an infinite slope with uniform flow

$$F_{p,l} = \frac{i_T}{\bar{u} - 1 + \bar{u}\tan\alpha\cot(\varphi' + \theta)} = 1.42$$

$$\delta_s = \text{atan}\frac{\bar{u}\tan\alpha}{i_T + 1 - \bar{u}} = 0.642; \qquad F_{p,s} = \frac{\tan\varphi'}{\tan(\delta_s - \theta)} = 1.58$$

which are slightly higher than those derived above using the heave safety factors for the flow path BB′, which actually looks slightly more susceptible to piping in Fig. 10(b).

Importantly, the latter calculations demonstrate that when the infinite slope conditions are likely to be met, it is not necessary to derive a flow net for the safety factor calculations. In fact, inserting a piezometer and measuring the pore water pressure u_0 at the certain distance d from the slope surface allows for calculating from Eq. (58) both the normalized pressure $\bar{u}$ and the inclination of the flow path θ. Substituting these parameters into Eqs. (38) and (57) will produce the load and strength-based safety factors against piping. To validate the infinite slope assumption, several piezometric measurements should be performed along the slope (always at the same distance d from the surface).

Summary and Conclusions

This paper is an attempt to adapt the classical approach to piping initiation and heave analysis (Terzaghi 1922, 1943) to hydraulically anisotropic slopes. This endeavor encountered some challenges, but also presented interesting opportunities. The main challenge arises from the fact that in anisotropic materials, hydraulic gradient vector is not parallel to the flow path, while the critical gradient is affected by the soil friction, anisotropy orientation and the ratio between the principal permeability coefficients. This complicates not only the formulation of the local piping initiation criterion, but also its practical applicability, because anisotropic exit gradients are not easily obtained from a flow net. Another challenge is that application of the classical heave condition to sloped ground appears to be mostly limited to slopes with strength inhomogeneity, because the homogeneous ones are likely to become unstable before this condition is satisfied.

The main opportunity, presented by adaptation of Terzaghi's approach to an inclined flow, is that it allowed formulating not only traditional load-based safety factors against piping and heave, but also alternative strength-based safety factors, independent of driving versus resisting, and total versus effective forces considerations. This is facilitated by the fact that, in contrast to the classical case of the vertical flow, here resistance to piping initiation does not rely only on gravity, but also on soil friction. In turn, the proposed strength-based formulation provided an opportunity to derive a safety factor against instability of an infinite anisotropic slope with a slope-parallel flow, which gives some insights into design of tailings dams. Finally, although the classical heave criterion did not appear to be particularly useful for homogeneous slopes, it provided in such slopes an opportunity to overcome the challenge of finding exit hydraulic gradients at the slope surface. It has been shown, that when applied to a straight portion of a daylighting flow path, the heave criterion can serve as a reasonable approximation of the local piping initiation condition, which appeared useful in practical applications.

Two application examples, of a natural slope and a tailings dam, have shown that because the safety factors are defined in a closed form, their application is rather straightforward, provided a corresponding flow net is available. In case when the infinite slope conditions are likely to be satisfied, it is not necessary to derive the flow net for the safety calculations: piezometer measurements of the pore water pressure u_0 at the certain distance d from the slope surface are sufficient. In order to validate the infinite slope assumption, at least two measurements should be performed along the slope (always at the same distance d from the surface).

The main conclusion, derived from parametric studies, is that not accounting for soil anisotropy can lead to a significant overestimation of safety factors against piping, in particular for higher anisotropy ratios and inclinations of anisotropic strata. Another important conclusion is that in spite of many advantages of the strength-based safety factor definition, it cannot be recommended as a single safety factor against piping for design and analysis, because the existing minimum safety factor requirements are based on experience gained from applying load-based safety factors. The recommended strategy would be to perform piping analysis using both safety factor definitions and to require that both of them have sufficiently large values. The additional effort is likely to be minimal, since both definitions result in relatively simple closed form expressions and require the same geometric and soil parameters.

The main limitation of the study is that it only considers cohesionless stable materials satisfying Terzaghi's condition $i_{cr} = i_T = \gamma'/\gamma_w$ for a vertical flow in horizontal strata. While for hydraulically stable soils these two assumptions are conservative, for internally unstable soils the latter one will result in an overestimation of the safety factors. Therefore, extending the proposed approach to internally unstable soils is important for enabling its wider application in practice. Because all the safety factor expressions proposed in this paper are functions of i_T, it seems that as a first attempt, this parameter could be simply replaced by the true value of i_{cr} obtained from lab tests with upward flow. This may, however, be an oversimplification of a very complex problem, and in any case would require experimental and numerical validation.

Data Availability Statement

All data, models, and code generated or used during the study appear in the published article.

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