CubeSat orbit determination using on-board GNSS velocity solutions

Institute of Geodesy and Photogrammetry

CubeSat orbit determination using on-board GNSS velocity solutions

Lukas Müller, Kangkang Chen, Markus Rothacher

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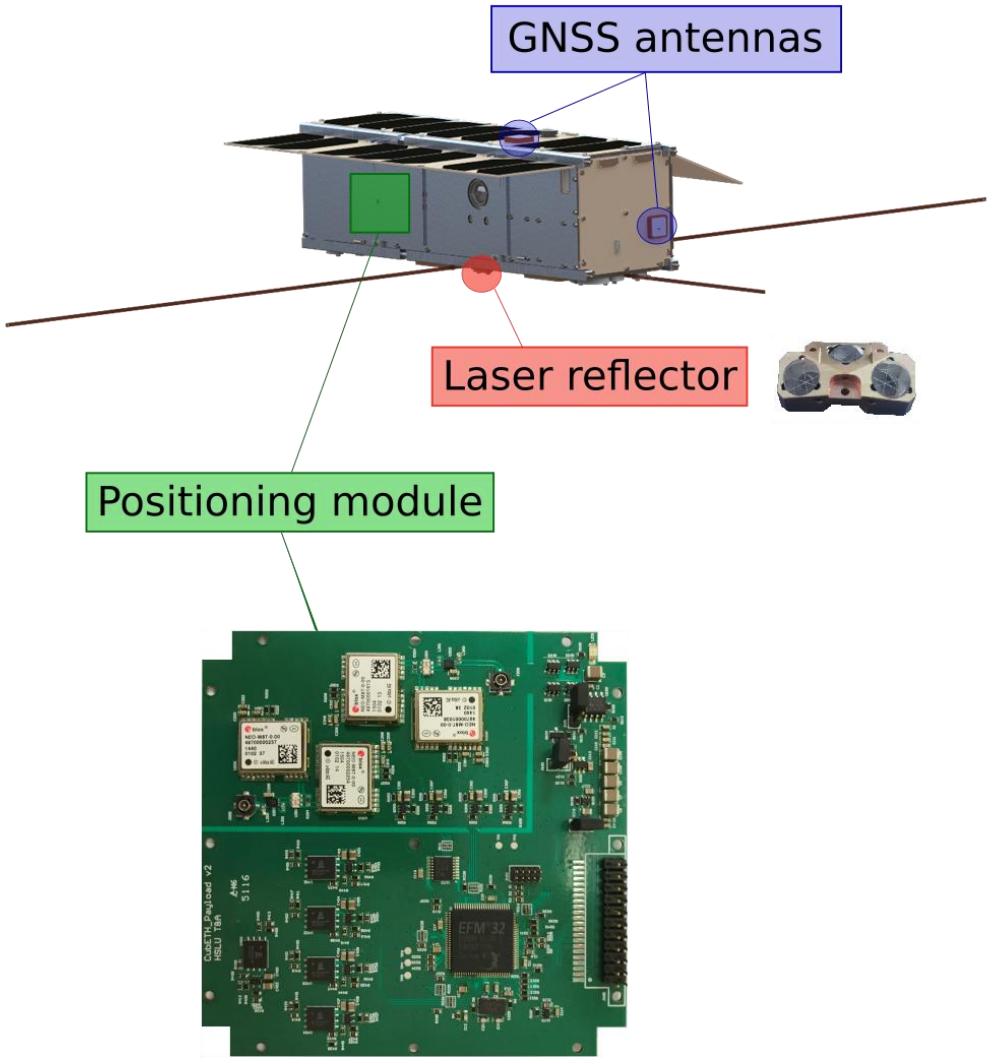
Introduction

- Two 3-unit cube satellites launched in December 2018 and April 2019
- Sun-synchronous orbit with inclination of 97°
- GNSS payload board providing continuous on-board receiver solutions
  - Positions from GNSS code observations
  - Independent velocity solutions
- Analysis of the quality of the on-board solutions by fitting a dynamic orbit model
- Standard approach based on positions only
- Idea: Additional use of velocity information for orbit estimation



Dynamic orbit fitting

- Equation of motion:

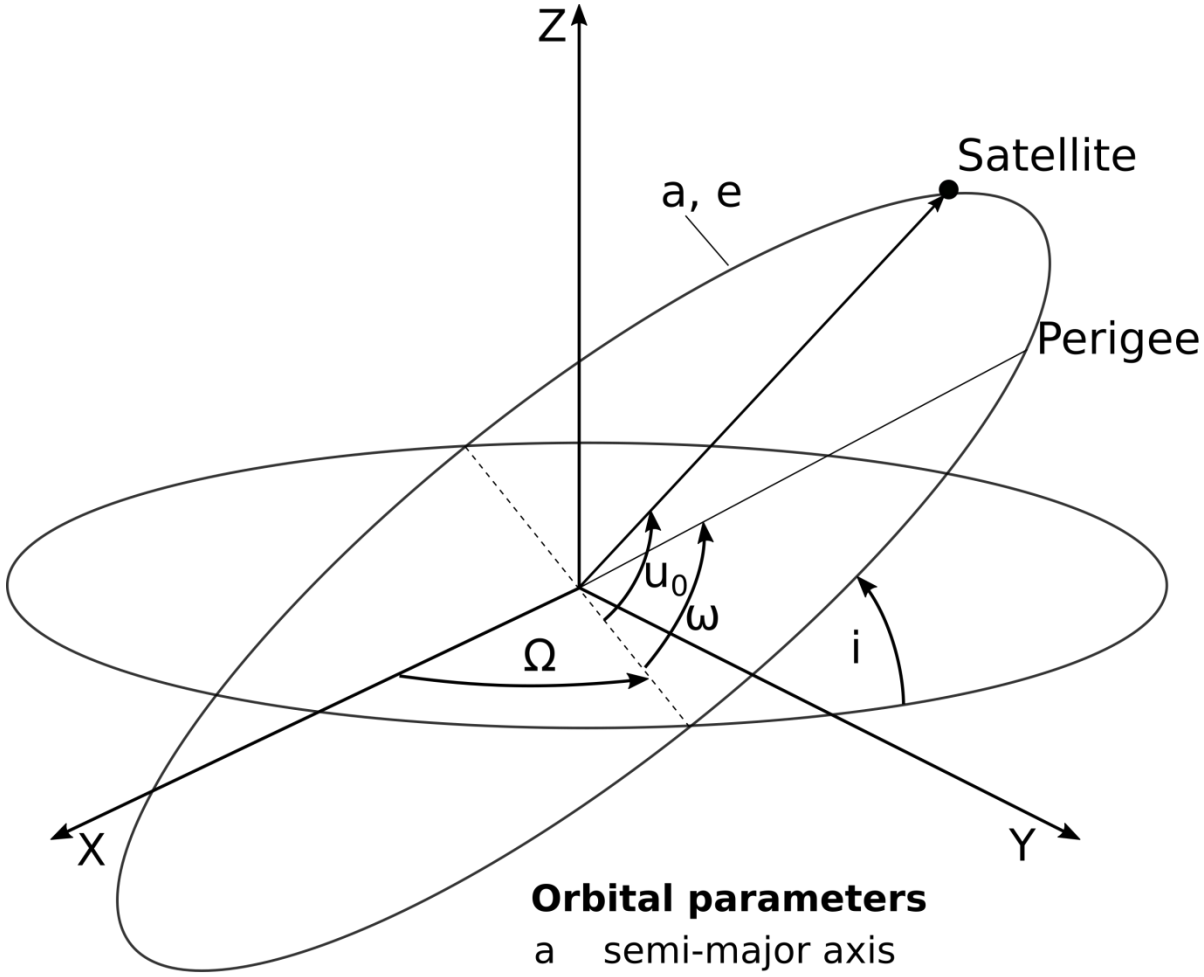
$$\ddot{\boldsymbol{r}} = -GM \cdot \frac{\boldsymbol{r}}{r^3} + k$$

G … gravitational constant
M … mass of the Earth
k … perturbing accelerations

- Orbit determination by numerical integration with initial conditions:

$$\boldsymbol{r}_0 = \boldsymbol{r}(t_0; a, e, i, \Omega, \omega, u_0)$$
$$\boldsymbol{v}_0 = \boldsymbol{v}(t_0; a, e, i, \Omega, \omega, u_0)$$

- Orbital elements determined by least squares adjustment
- Observations $\mathbf{r}_t$ and $\mathbf{v}_t$



Orbital parameters

a	semi-major axis
e	eccentricity
i	inclination
Ω	right ascension of ascending node
ω	argument of perigee
u_0	argument of latitude

Dynamic orbit fitting

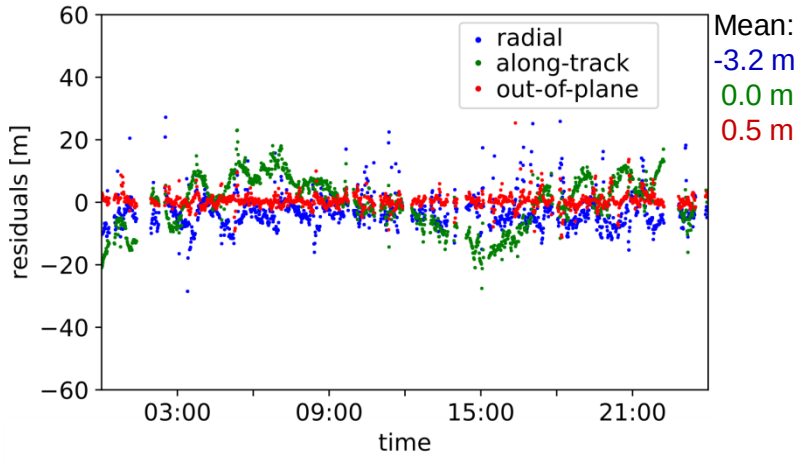
Three different approaches depending on the observations used:

1. Positions-only
2. Velocities-only
3. Positions and velocities

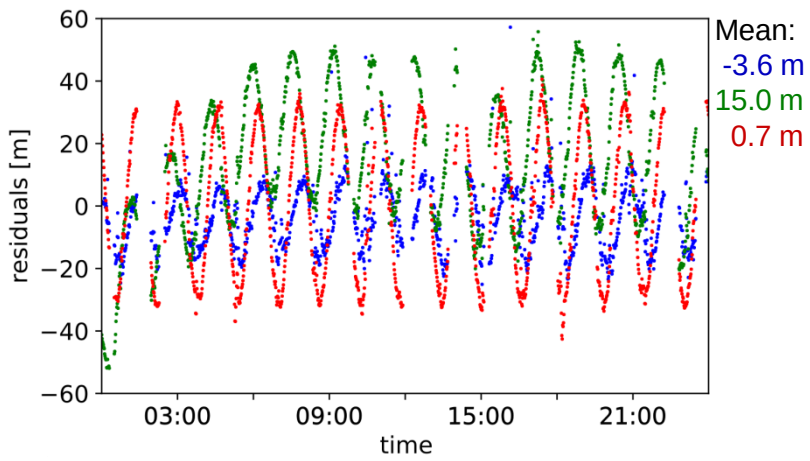
Estimation of 6 Keplerian elements + constant along-track acceleration

Residuals of one-day orbital arc (18/10/2019)

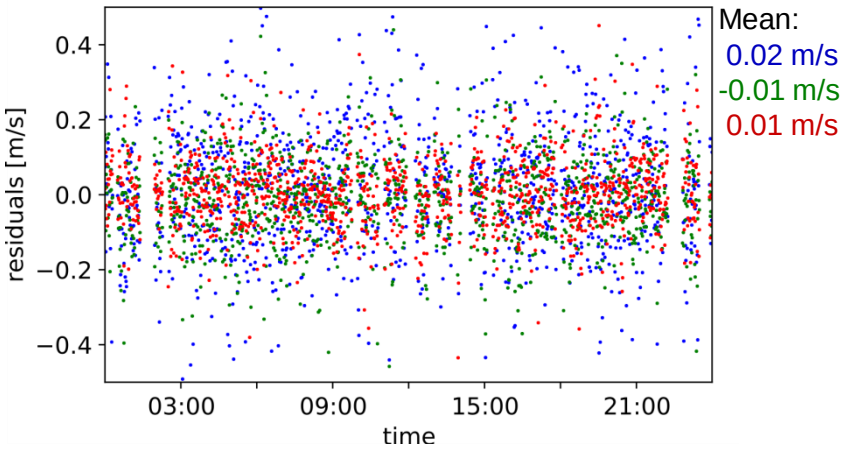
Positions-only approach, position residuals



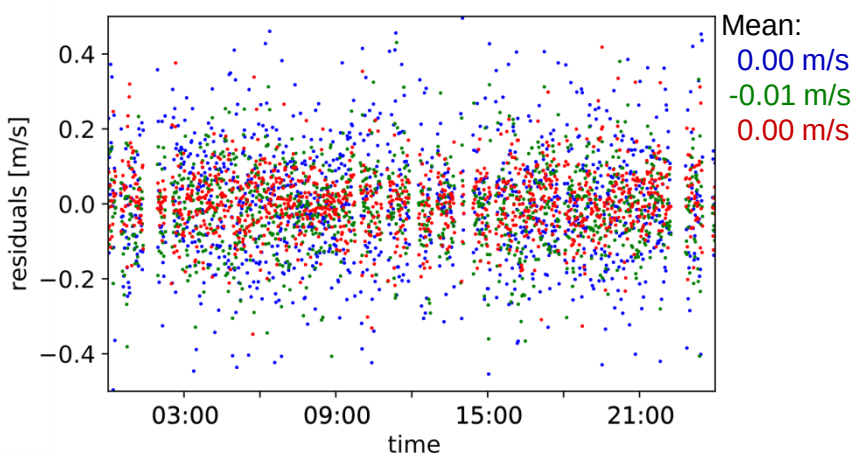
Velocities-only approach, position residuals



Positions-only app., velocity residuals



Velocities-only app., velocity residuals

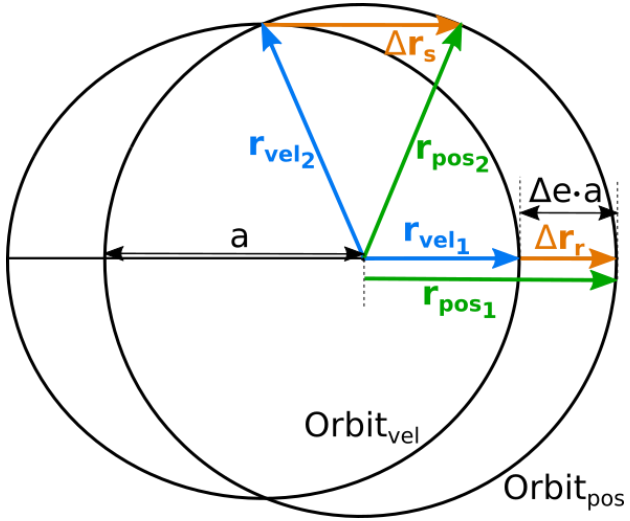


- Once-per-revolution periodicity (96 min) in position residuals
  - Amplitude of about 30 m in along-track and out-of-plane
  - Amplitude of about 15 m in radial direction
- Offsets:
  - Negative radial offset of about 3 m
  - Positive along-track offset of 15 m
  - Positive radial offset of about 0.02 m/s in velocities
  - Negative along-track offset of -0.01 m/s in velocities

Periodicity – Effect of parameter differences

Parameter differences between velocities-only and positions-only approach

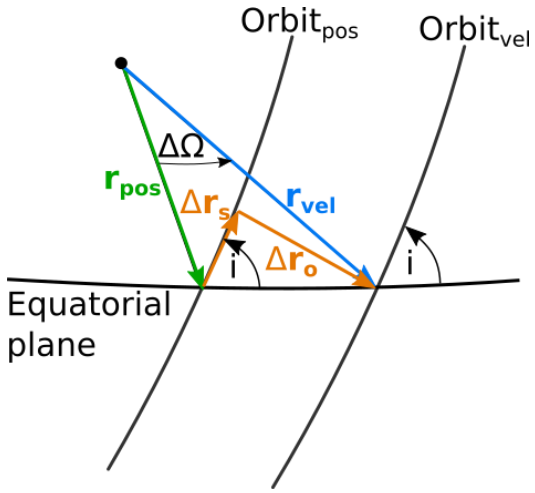
Δa [m]	Δe [10^{-6}]	Δi [10^{-6} rad]	$\Delta\Omega$ [10^{-6} rad]	Δu_0 [10^{-6} rad]	$\Delta\omega$ [10^{-6} rad]
1.0	-1.6	0.2	4.5	3.5	-685.0



Maximum differences in radial direction:

$$\Delta \boldsymbol{r}_r = \Delta e \cdot a = 11.4\, m$$

Amplitude in along-track about twice as large as in radial: $\Delta \boldsymbol{r}_s \approx 23\, m$



Out-of-plane difference:

$$\Delta \boldsymbol{r}_o = \Delta\Omega \cdot \|\boldsymbol{r}\| \cdot \sin(i) = 30.8\, m$$

Along-track difference:

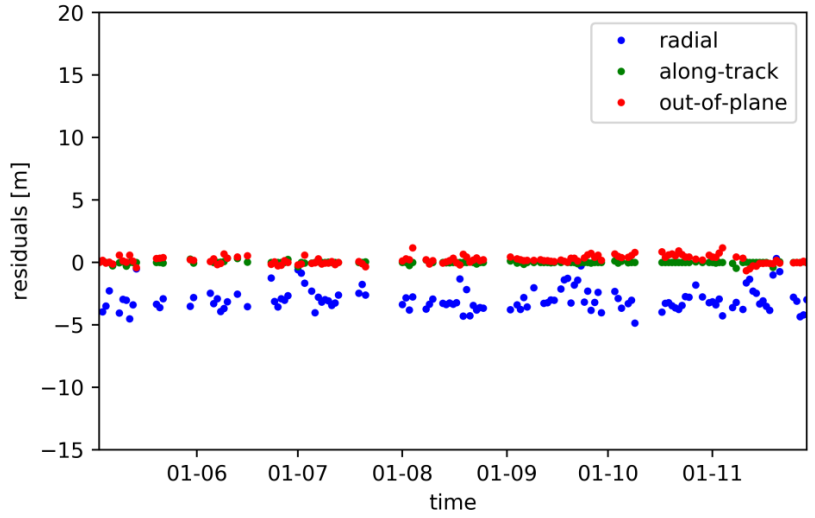
$$\Delta \boldsymbol{r}_s = \Delta\Omega \cdot \|\boldsymbol{r}\| \cdot \cos(i) = -3.8\, m$$

Argument of perigee poorly defined:

- Due to nearly circular orbit
- Only small effect on residuals (few meters)

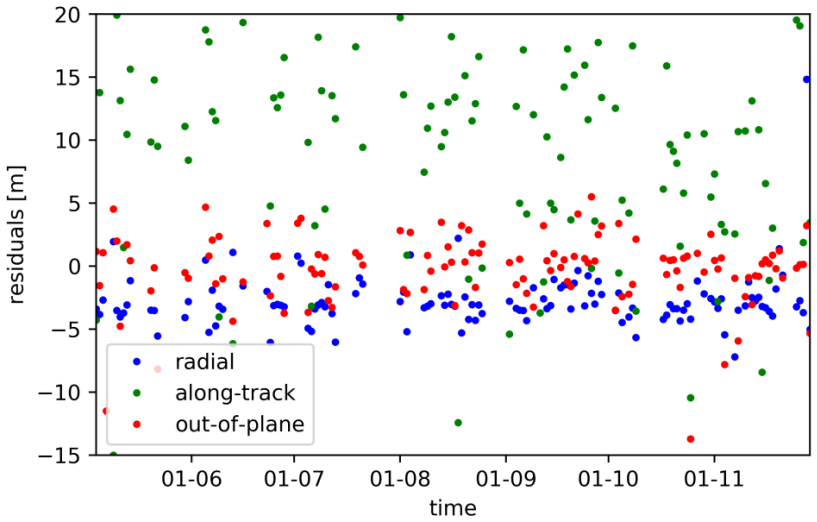
Offsets - Daily mean residuals from May to November 2019

Positions-only app., position residuals



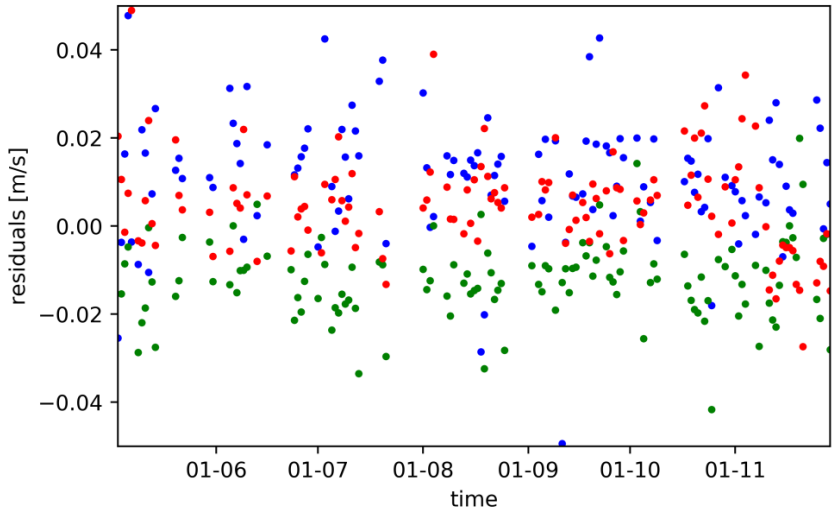
Mean:
-2.9 m
0.0 m
0.2 m

Velocities-only app., position residuals



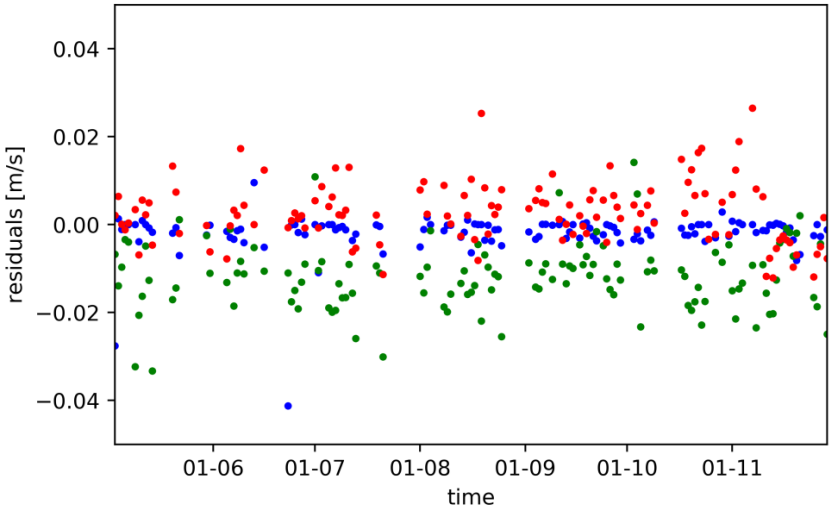
Mean:
-2.9 m
13.3 m
-0.3 m

Positions-only app., velocity residuals



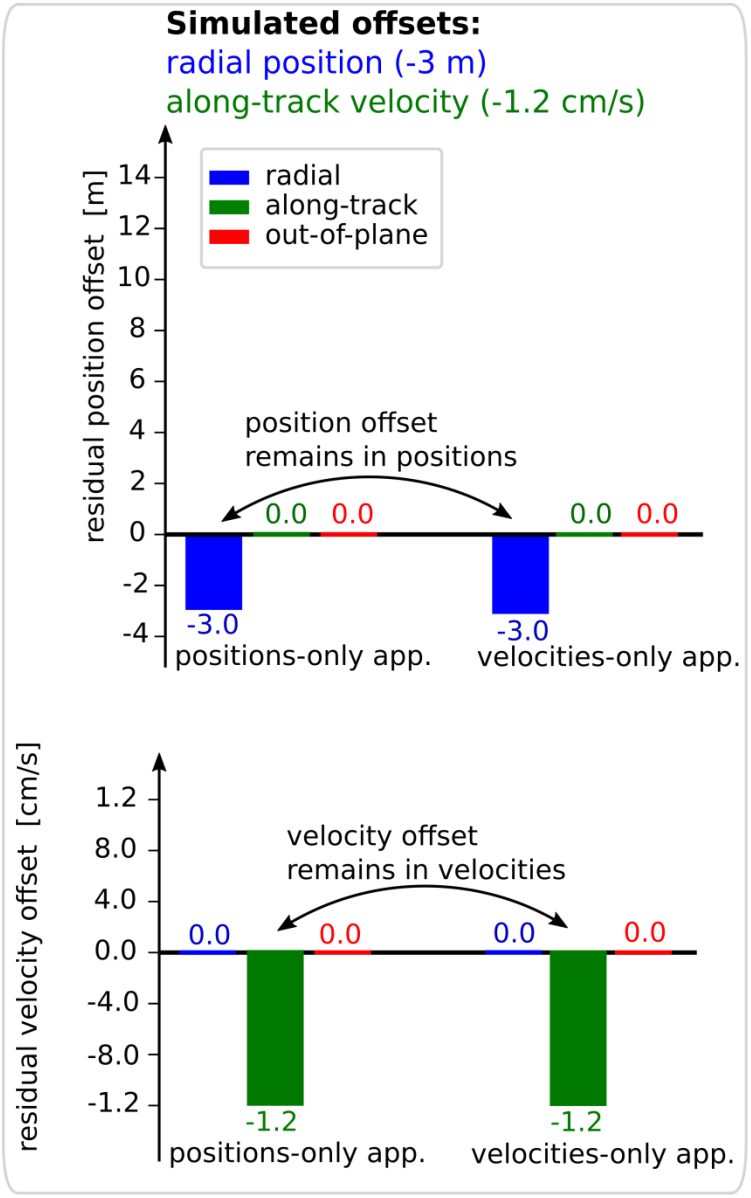
Mean:
0.013 m/s
-0.012 m/s
0.006 m/s

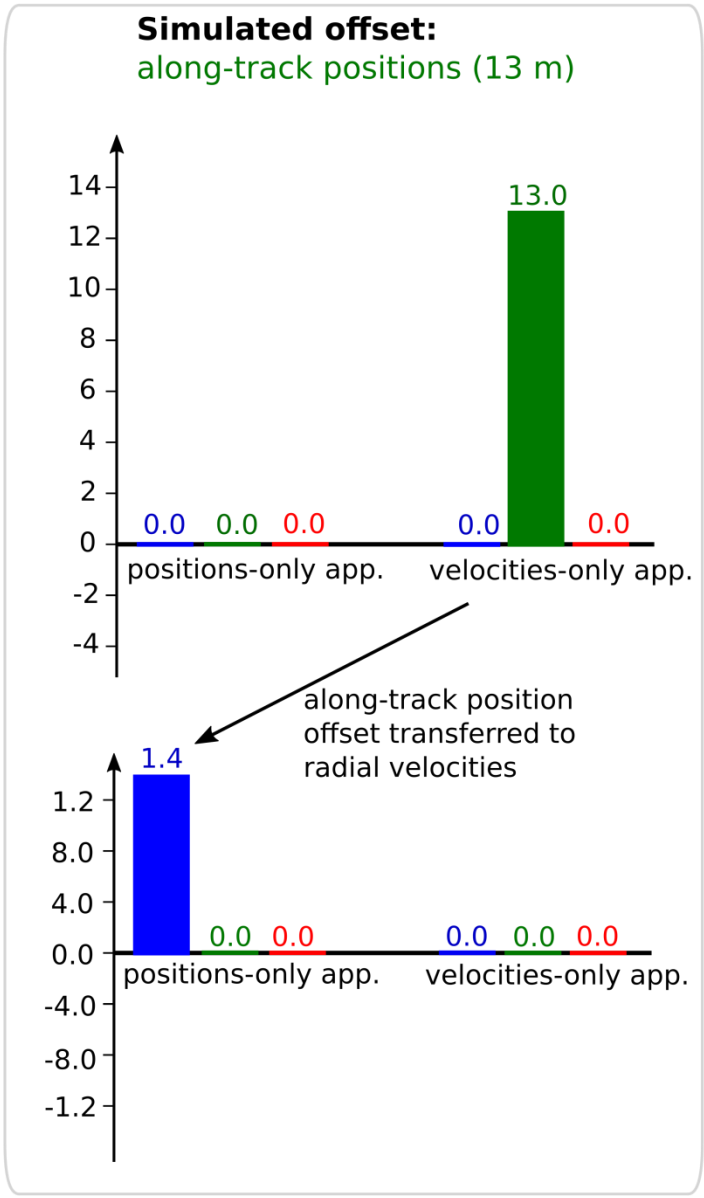
Velocities-only app., velocity residuals

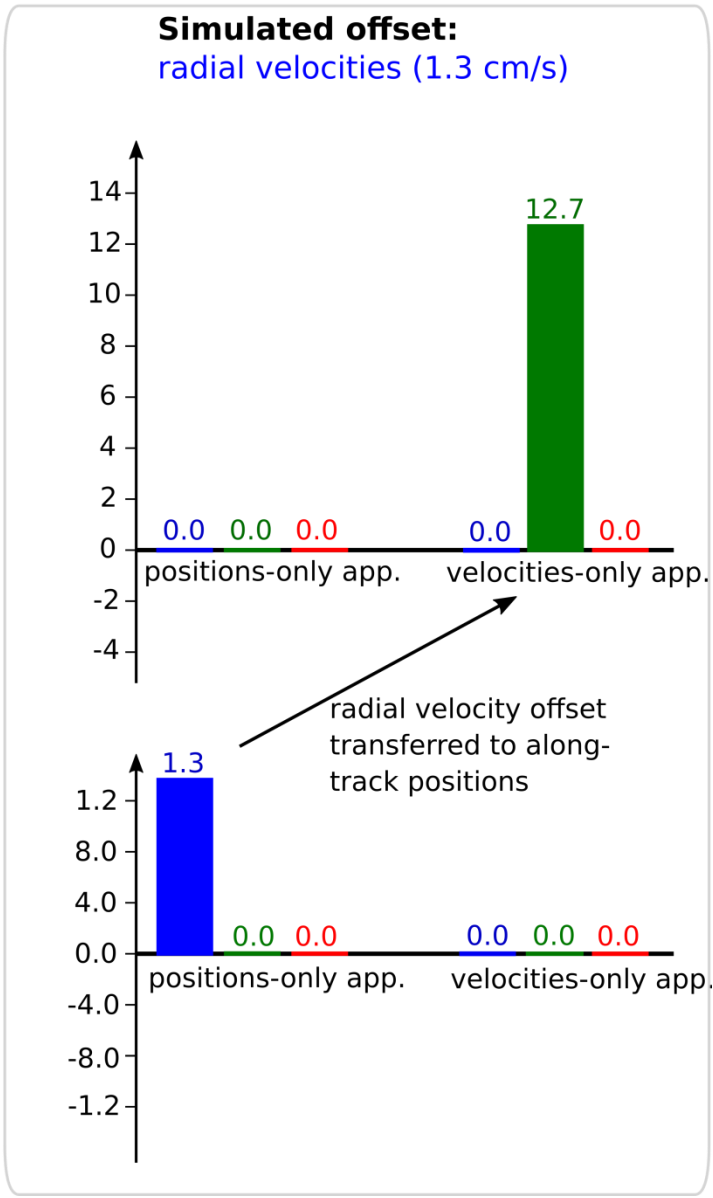


Mean:
-0.004 m/s
-0.012 m/s
0.003 m/s

Offsets - Simulation of measurement biases







Offsets - Relation between along-track offset and radial velocity

Simple approximation in a circular orbit

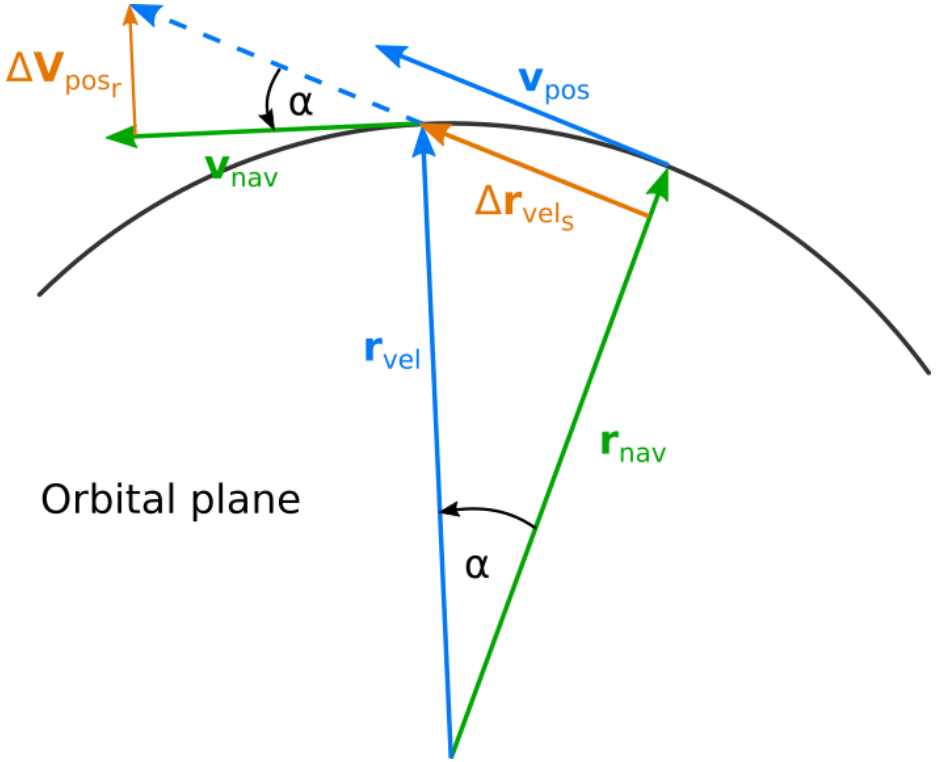
$$\frac{\Delta v_{pos_r}}{\|\boldsymbol{v}\|} = \frac{\Delta r_{vel_s}}{\|\boldsymbol{r}\|}$$

$$\Delta r_{vel_s} \approx 13.3\ m$$

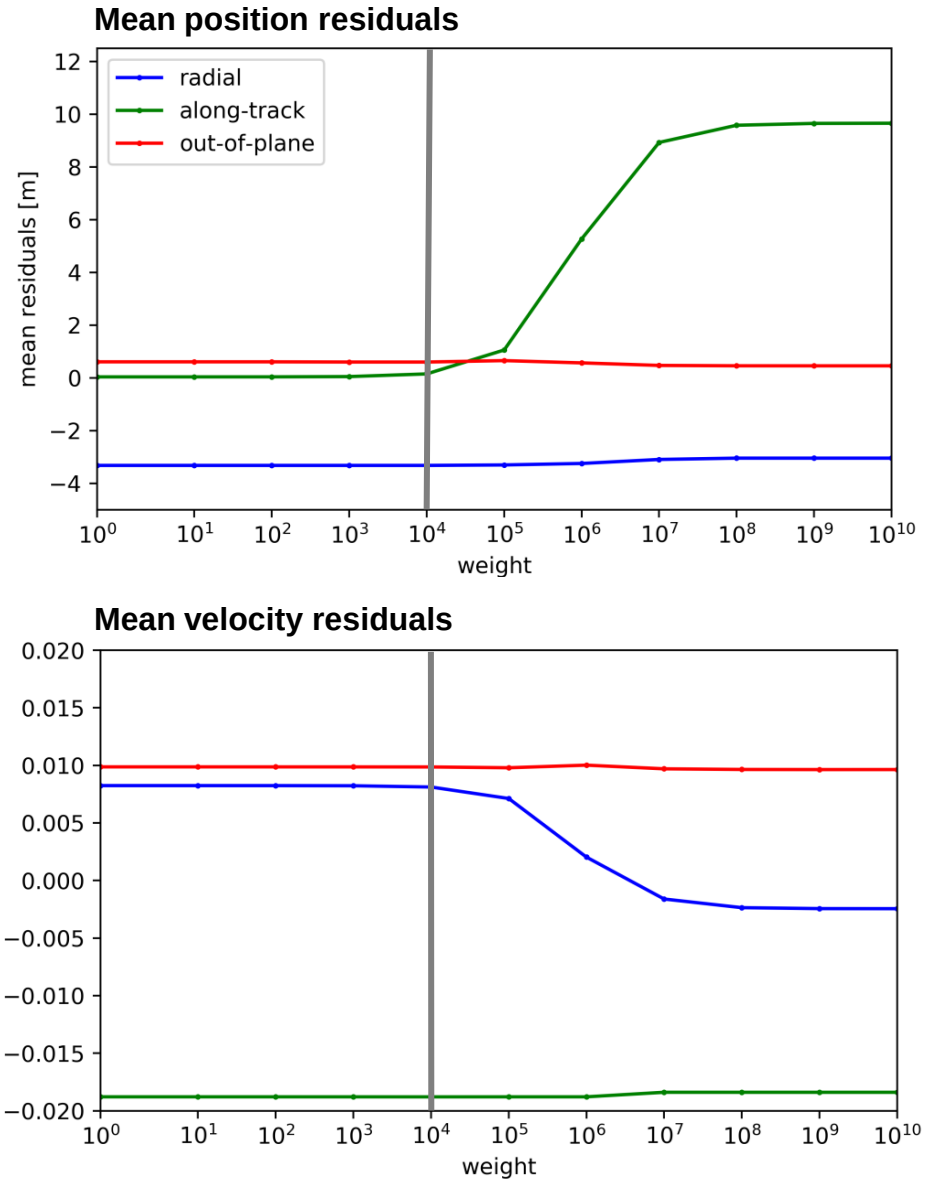
$$\Delta v_{pos_r} = 13.3\ m \cdot \frac{7.8 \cdot 10^3 \frac{m}{s}}{6.9 \cdot 10^6\ m}$$

$$= 0.015\ \mathrm{m/s}$$

⇒Inconsistency between position and velocity vector causes shift in along-track



Relative weighting of velocities in combined approach



- Need for an appropriate relative weighting factor
- Standard deviation of positions about 2-3 m
- Standard deviation of velocities about 0.02-0.03 m/s

⇒Expected relative weight of 10^4

⇒Measurement bias in velocities rather than in positions

Conclusions

- Benefits from the additional use of on-board velocities for orbit determination
  - Independent observations allow to identify and explain systematic biases
  - Better understanding the orbit dynamics
- Residual periodicity
  - Amplitude of 27 m in out-of-plane residuals mainly related to ascending node
  - Amplitude of about 14 m in radial and 28 m in along-track direction related to eccentricity
- Residual offsets
  - Radial offset of -3 m in positions due to measurement bias in positions
  - Along-track velocity offset of -0.012 m/s due to bias in the observed velocities
  - Measurement bias in observed radial velocities of 0.013 m/s more likely than along-track bias of 13 m in positions

Thanks for your attention!