The role of risk measures in making seismic upgrading decisions

Journal Article

Author(s):
Bodenmann, Lukas; Galanis, Panagiotis; Broccardo, Marco; Stojadinovic, Bozidar

Publication date:
2020-11-01

Permanent link:
https://doi.org/10.3929/ethz-b-000453596

Originally published in:
Earthquake Spectra 36(4), https://doi.org/10.1177/8755293020919423

The Role of Risk Measures in Making Seismic Upgrading Decisions

Lukas Bodenmann,[a] **Panagiotis Galanis,**[b] **M.EERI, Marco Broccardo,**[c,e] **M.EERI, and Bozidar Stojadinovic**[d]**, M.EERI**

Risk measures are tools that enable consistent measurement of financial risk and quantify the risk exposure to an associated hazard. In finance there is a broad spectrum of risk measures, which reflect different asset performance goals and the risk appetite of the decision-maker. In this study, the authors leverage advancements in financial risk management to examine the role of risk measures to quantify the seismically induced financial risk, measure the benefit of seismic upgrading, and relate the benefit of seismic risk reduction to a degree of the implemented seismic upgrade. The findings demonstrate that the relation between the financial benefits of a seismic upgrade, quantified using risk measures that consider the full range of earthquake events, and the degree of the seismic upgrade are concave, i.e. the incremental financial benefit reduces gradually with increasing degree of seismic upgrading. The opposite holds if the risk measures consider only the high-severity low-likelihood events. Therefore, the study shows that the selection of the risk measure plays a crucial role in determining the target degree of seismic upgrading. Equivalently, quantifying the financial benefits of seismic risk mitigation using different risk measures might lead to different seismic upgrading decisions for the same structure.

INTRODUCTION

The majority of the typical European and US building stock has been constructed prior to the existence of modern seismic code provisions (Comerio and Anagnos, 2012; Wenk, 2008).

[a] Doctoral Student, Chair of Structural Dynamics and Earthquake Engineering, ETH Zurich, Switzerland,

[b] Senior Research Associate, ETH Risk Center, Zurich, Switzerland

[c] Senior Research Associate, Chair of Structural Dynamics and Earthquake Engineering, ETH Zurich, Switzerland,

[d] Professor, Chair of Structural Dynamics and Earthquake Engineering, ETH Zurich, Switzerland,

[e] Senior Research Associate, Swiss Seismological Service, ETH Zurich, Switzerland,

Moreover, in many regions, the market penetration rate of seismic insurance is low (Bevere and Grollimund, 2012). As a consequence, in case of a high intensity seismic event, homeowners are exposed to high risk while lacking any form of financial protection. This high seismic risk can be significantly reduced by designing a new building according to modern design codes or by upgrading the structural characteristics of an existing building to achieve the desired seismic performance. However, communicating the benefits of seismic upgrading, or modern seismic design, to the different stakeholders (e.g. homeowners, insurance companies and government authorities) is difficult because a clear systematic framework to classify seismic performance as acceptable or not in financial terms[1] has as-yet not been established.

A prerequisite to an economically efficient seismic risk mitigation is a coherent and well-defined measurement of the underlying risk exposure and a systematic approach in defining the notion of "satisfactory seismic performance." Risk measurement aims, in general terms, to quantify the risk associated with a single position[2] or a portfolio[3] and is crucial in defining the limits of "acceptable" risk exposure. An "unacceptable" risk exposure could be mitigated in a variety of ways including risk avoidance, risk transfer, and/or risk reduction. A broad set of financial risk measures is used in financial risk management to quantify the financial risk exposure in probabilistic terms. These measures provide a systematic approach to compare the risk exposure of individual building properties (or different building designs/configurations) or of building portfolios (e.g. a building portfolio owned by a real estate holding company).

A risk neutral decision maker exposed to uncertainty would select the risk mitigation strategy that maximizes the expected net gains computed by subtracting the risk mitigation costs from the expected received gains corresponding to the earthquake losses avoided through

---

[1] Note that the PEER-PBEE framework (Cornell and Krawinkler, 2000; Moehle and Deierlein, 2004), while systematic, has focused on classifying seismic performance as satisfactory or not primarily in engineering terms, and secondarily in financial terms. Common implementations use engineering seismic performance measures and thresholds (e.g. inter-story drift ratios associated with discrete building performance states) for a set of discrete earthquake intensity levels associated with estimates of exceedance probabilities (e.g. 10% or 2% in a 50-year-long time interval). Although extensions of this engineering framework to financial quantification of losses, as proposed in (Porter et al., 2001; Yang et al., 2009) and implemented in the FEMA P-58 project (Applied Technology Council (ATC), 2012), exist, there is no clear consensus in the earthquake design community on defining the measures that would indicate inappropriate seismic performance in financial terms in a way similarly to financial risk assessment in other industries such as banking (Basel Committee on Banking Supervision (BCBS), 2019) and insurance (Swiss Financial Market Supervisory Authority (FINMA), 2017).

[2] In finance, a position corresponds to the amount of a financial instrument held/owned by a physical person or legal entity. In this study, a single position is an individual building property (which corresponds to a tangible asset for the owner) associated with a defined value that could be impaired due to seismically induced damage.

[3] A portfolio corresponds to a collection of single positions i.e. in this study a collection of individual building properties.

risk mitigation and rental income, as done in a conventional cost-benefit analysis (VSP Associates, 1992). The relevant risk measure for a risk neutral decision maker is the expected value. However, decision makers concerned with seismic risk mitigation must consider rare but potentially catastrophic earthquakes. These decision makers have, in most cases, a risk averse attitude, corresponding to a behavioral phenomenon in which the decision maker's perception and judgment of risk is systematically distorted[4], resulting in decisions that might be viewed as excessively conservative when compared to those obtained from conventional cost-benefit analyses that maximize the expected net gains. Risk measures that focus on the events in the tails of loss probability distributions are more relevant for decision makers concerned with mitigation of rare catastrophic risk[5].

Historically, structural design codes have been developed with a primary focus on the design of new buildings. Guidelines for seismic evaluation and retrofit of existing buildings have been developed only in the recent decades, addressing the principal earthquake engineering aspects of the problem. However, the fundamental question of efficient resource allocation for optimal seismic risk mitigation, more specifically, to what extent a non-code conforming building should be retrofitted such that the undertaking is financially acceptable, remains open and can potentially only be answered in a broad exchange between professionals and the public (Porter, 2016). This study aims to provide further insight for such discussions by showing the relation between the financial benefits of a seismic upgrade of an individual existing structure (obtained by avoiding earthquake losses due to seismic upgrading) and the degree of seismic upgrade, quantified using risk measures. The study demonstrates the crucial role risk measures play in quantifying the seismic risk exposure of an existing building and in measuring the seismic risk reduction achieved by seismic upgrading.

REVIEW OF RISK MEASURES

Risk measures are a common tool employed in quantitative risk modeling, as they facilitate communication of risk exposure to stakeholders and, thus, are useful in decision-making and/or

---

[4] An in-depth discussion and more exact definition of risk aversion in the context of uncertain earthquake events is out of scope of the present study. For a more thorough discussion on this the reader is referred to (Cha and Ellingwood, 2013a, 2013b; Kunreuther and Kleffner, 1992).

[5] The authors note that risk aversion is not commonly associated with risk measures as the latter do not incorporate any utility function associated to uncertain outcomes. However, the authors consider that risk measures focusing only on tail events are more relevant for decision makers that demonstrate a non-risk neutral behavior. A discussion can be found in (Acerbi, 2002).

defining the risk appetite for individuals and corporations (Alibrandi and Mosalam, 2017; Mosalam et al., 2018; Rockafaller and Royset, 2012). For that reason, regulatory frameworks either implicitly (ATC, 2009) or explicitly (FINMA, 2017) define acceptable limits in risk exposure using risk measures, quantify the required safety buffers against low likelihood high severity events (McNeil et al., 2005), and define objective functions for optimization problems related to decision-making. Risk measures have also been employed to communicate the safety standards and acceptable risk levels for anthropogenic activities (Broccardo et al., 2017; Jonkman et al., 2003; Vrijling et al., 1995). In the context of earthquake engineering, modern seismic design provisions are based mainly on risk measures that define acceptable levels of the probability of collapse of an individual building structure (ATC, 2009; Galanis and Moehle, 2015).

A risk measure, $\rho(\cdot)$, is a mapping assigning a real number to one or a set of random variables. In this study, the random variable is the seismically induced financial loss (referred to as loss in this study), generically denoted as L. Specifically, L is defined here as a non-negative continuous random variable, with cumulative distribution function (CDF) $F(l)$, and complementary cumulative distribution function (CCDF) $G(l) = 1 - F(l)$ (the so-called loss exceedance probability curve or loss curve). Specifically, in this study, $\rho(L)$ maps the random loss associated with an individual building structure exposed to earthquake hazard to a real number, which is used for decision making. Given the random variable L (with $l \in \mathbb{R}^+$), popular financial risk measures used in finance, real-estate and insurance industry as well as in earthquake engineering are presented below.

EXPECTED LOSS RISK MEASURE

Expected Loss (EL) The expected loss, EL, is defined as:

$$\rho(L) \equiv \mathrm{EL}(L) = \int_{\mathbb{R}^+} G(l)dl\,. \tag{1}$$

Expected loss is one of the most popular measures to quantify risk. By its definition, it is a measure of the expected outcomes. This is somewhat of a contradiction, since the main objective of risk measurement is to deal with the “unexpected” events. Nevertheless, EL provides valuable information about risk exposure and is widely applied in cost-benefit analysis (VSP Associates, 1992), in risk-neutral decision making, as well as in insurance

(Wuthrich, 2017). Figure 1a provides a graphical illustration of the expected loss risk measure based on a generic loss exceedance curve $G(l)$.

TAIL RISK MEASURES

Value-at-Risk (VaR_α). Given a confidence level $\alpha \in [0,1]$, the Value-at-Risk of the random variable L is defined as the smallest number l such that the probability that $L > l$ is no larger than $(1-\alpha)$, i.e. the loss exceedance probability is $(1-\alpha)$:

$$\rho(L) \equiv \mathrm{VaR}_\alpha(L) = \inf\{l \in \mathbb{R}^+ : G(l) \leq 1-\alpha\}. \tag{2}$$

VaR_α is the most widely used risk measure in finance and insurance (BCBS, 2019; FINMA, 2017). Simply put, VaR_α is the α-quantile of $F(l)$. As discussed in (McNeil et al., 2005), one of the main criticisms regarding VaR_α is that it is not a "what-if" risk measure, but that it rather provides the information about the severity of losses that occur with probability smaller or equal than $(1-\alpha)$.

Expected Shortfall (ES_α). Given a confidence level $\alpha \in [0,1]$, the Expected Shortfall of the random variable L is defined as:

$$\rho(L) \equiv \mathrm{ES}_\alpha(L) = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_u\,(L)du\,. \tag{3}$$

Since L is a continuous random variable, Equation 3 can be alternatively expressed as:

$$\mathrm{ES}_\alpha(L) = \mathrm{VaR}_\alpha(L) + \frac{1}{1-\alpha}\int_{\mathrm{VaR}_\alpha(L)}^{\infty} G(l)dl\,. \tag{4}$$

If L is continuous, $\mathrm{ES}_\alpha(L)$ is an average of $\mathrm{VaR}_u(L)$ for all $u \geq \alpha$. ES_α looks further into the tail of $F(l)$ and is a "what-if" risk measure. In other words, it expresses the expected value of the seismic loss given that the loss incurred is greater or equal to the threshold $\mathrm{VaR}_\alpha(L)$. The aforementioned risk measures are illustrated in Figure 1b, to provide a visual interpretation of the mathematical definitions stated above.

Figure 1. Schematic illustration of three risk measures based on a generic loss exceedance probability curve $G(l)$: (a) Expected Loss EL; and (b) Value-at-Risk (VaR_α) and Expected Shortfall (ES_α) at confidence level $\alpha = 0.98$.

Loss-at-Frequency (LaF_α). The LaF_α is closely related to $\mathrm{VaR}_\alpha(L)$. Given a confidence level $\alpha \in [0,1]$, the Loss-at-Frequency is defined as:

$$\rho(L) \equiv \mathrm{LaF}_{\alpha}(L) = \inf\{l \in \mathbb{R}^{+}: \lambda(l) \leq 1-\alpha\}, \tag{5}$$

where $\lambda(l)$ is the rate of exceedance of the loss level l. This measure is not formally defined in the financial risk quantification field: instead, it is defined here to encapsulate the earthquake engineering (and the insurance industry) practice of communicating the risk exposure of a structure in terms of the loss corresponding to a certain return period. Strictly speaking, $\mathrm{LaF}_{\alpha}(L)$ is not consistent with the formal definition of a risk measure because the mapping is with respect to the rate of exceedance and not with respect to the loss exceedance probability $G(l)$. However, in the context of low probability Poissonian events (typically used in earthquake engineering to model a sequence of earthquake events (Der Kiureghian, 2005)) $\lambda(l) \cong G(l)$, which implies $\mathrm{LaF}_{\alpha}(L) \cong \mathrm{VaR}_{\alpha}(L)$. Given the close relation with $\mathrm{VaR}_{\alpha}(L)$, $\mathrm{LaF}_{\alpha}(L)$ shares the same criticisms as VaR_{α}. In particular, for a selected confidence level α, $\mathrm{LaF}_{\alpha}(L)$ gives merely the minimum loss threshold, l=$\mathrm{LaF}_{\alpha}(L)$, for the events with exceedance rate $\lambda(l) = \alpha$, but not the expected loss $L \geq l$=$\mathrm{LaF}_{\alpha}(L)$ given the same events.

Another risk measure used extensively in seismic due-diligence for building property transactions and in mortgage issuance is called Probable Maximum Loss (PML). Historically, PML has had numerous different definitions, leading to confusion among stakeholders and difficulties in its application. For this reason, the American Society of Testing Materials (ASTM) developed standards for seismic risk assessment (ASTM, 2016a, 2016b), where the terms Probable Loss and Scenario Loss are defined. Probable Loss refers to the loss associated with a certain rate of exceedance (or a return period) and is identical to $\mathrm{LaF}_{\alpha}(L)$ defined above. For Scenario Loss, the ASTM standards (ASTM, 2016a, 2016b) recommend using the Scenario Expected Loss and Scenario Upper Loss, defined below for completeness (Thiel et al., 2012).

SCENARIO BASED RISK MEASURES

Scenario Expected Loss (SEL_{τ}): This measure corresponds to the expected loss due to specific seismic scenario event associated with a certain return period τ, often chosen to be consistent with the Design Basis Earthquake (DBE) return period of 475 years, corresponding to a confidence level $\alpha = 99.79\%$, or an exceedance level $(1-\alpha) = 0.21\%$. A comprehensive scenario-based loss assessment builds up on hazard disaggregation (Bazzurro and Cornell, 1999) to define a set of earthquake events with corresponding magnitude and source to site distance. Within this specific framework, a scenario is defined by selecting a seismic intensity measure, im_{τ}, (e.g., peak ground acceleration, spectral acceleration at a certain vibration period) associated to a given rate of exceedance $\lambda(im_\tau) = 1/\tau$. Given im_τ, SEL_τ is defined as:

$$\rho(L) \equiv \mathrm{SEL}_\tau(L|im_\tau) = \int_0^\infty G(l|im_\tau)\mathrm{d}l \ , \tag{6}$$

where $G(l|im_\tau)$ refers to the CCDF of loss random variable L conditioned on an intensity measure $IM = im_\tau$ with return period τ.

Scenario Upper Loss (SUL_τ): Similar to SEL_τ, this is a scenario-based measure which refers to the 90% quantile (ASTM, 2016a) of the seismic loss distribution conditioned on an event with a certain return period τ (i.e. $= im_\tau$), defined as:

$$\rho(L) \equiv \mathrm{SUL}_\tau(L|im_\tau) = \inf\{(l \in \mathbb{R}^+ : G(l \mid im_\tau) \leq 1 - 0.9\} \ . \tag{7}$$

QUANTIFICATION OF SEISMICALLY INDUCED LOSSES

A framework to compute the seismically induced financial loss exceedance probability curve $G(l)$, and the rate of exceedance of seismically induced financial losses $\lambda(l)$ used in this study is based on the Pacific Earthquake Engineering Research Center (PEER) probabilistic performance-based earthquake engineering (PBEE) framework (Cornell and Krawinkler, 2000; Moehle and Deierlein, 2004). Thus, the total probability theorem is used to calculate the mean annual rate of exceedance of seismically induced financial losses as:

$$\lambda(l) = \sum_{dm} \int_{im} G(l|dm)|dG(dm|im)||d\lambda(im)| \, , \tag{8}$$

where $L = l$ is the seismically induced financial loss[6], $DM = dm$ is a damage measure (e.g. damage states ranging from minor damage to complete damage), $IM = im$ is an earthquake intensity measure (e.g. spectral acceleration at the fundamental building vibration period), $|dG(dm|im)| = P(DM = dm|im)$ is the so-called vulnerability function, $G(l|dm)$ is the cost function, a conditional CCDF expressing the probability that the financial loss is greater that l conditioned on a specific damage state, and $\lambda(im)$ is the annual rate of exceedance of the earthquake intensity measure im that quantifies the earthquake hazard. The intensity measure employed in the present study is the elastic, 5% damped, spectral acceleration S_{ae}(T) at the

[6] The PEER PBEE framework (Cornell and Krawinkler, 2000; Moehle and Deierlein, 2004) defines a generic decision variable *DV*, that is specialized in this study to be the seismically induced financial loss *L*.

elastic first-mode vibration period T of the examined structure. The seismic hazard data to determine $\lambda(im)$ are obtained from open-source databases provided by the (EFEHR, 2014) or the (USGS, 2014).

The employed methodology builds on and extends the work described in (Galanis et al., 2018). To make the present study in large parts self-contained, to highlight extensions and to provide further insight, the main parts are described below. For additional details the interested reader is referred to the stated reference. The methodology quantifies seismically induced financial losses for a generic building structure, whose seismic response is represented using an equivalent single-degree-of-freedom (SDoF) model derived using the properties of the first elastic vibration mode of the structure. The response is characterized by an elastic-perfectly-plastic response envelope. The yield point of this static pushover response envelope is defined by the elastic stiffness k_y, derived from the elastic first-mode vibration period of the structure T, and the yielding base shear strength F_y, expressed as yielding spectral pseudo-acceleration $a_y=F_y/M_s$ where M_s is the seismic mass of the structure. The resulting yielding displacement $d_y=F_y/k_y$ is used to normalize the maximum inelastic displacement of the SDoF model d_u and compute the displacement ductility capacity $\mu_c=d_u/d_y$. Seismic vulnerability of this generic building structure is defined using damage grades DG_k with thresholds $\mu_{lim,k}$ specified in terms of structural displacement ductility capacity μ_C following (Lagomarsino and Giovinazzi, 2006).

Assuming a lognormal distribution function for the vulnerability $|dG(dm|im)|$, the probability of exceeding a certain damage grade takes the mathematical form of:

$$P\big(DG_k\big|S_{ae}(\mathrm{T})\big) = \Phi\left[\frac{\ln\big(S_{ae}(\mathrm{T})\big) - \ln\big(\hat{s}_{lim,k}\big)}{\sqrt{\beta_{Dk}^2 + \beta_{Mk}^2}}\right], \qquad (9)$$

where Φ is the standard normal CDF, $\hat{s}_{lim,k}$ represents the median capacity of the structure corresponding to damage grade DG_k in units of the considered im, and factors β_{Dk}, the dispersion due to ground motion record-to-record variability, and β_{Mk}, the dispersion due to model variability, quantify the associated aleatory and epistemic uncertainties. The values of these three parameters are derived from the relation between the lateral strength reduction factor R and the structural displacement ductility demand μ for an inelastic SDoF system with a certain elastic period of vibration T, the so-called $R-\mu-T$ relation. The tool SPO2IDA by (Vamvatsikos and Cornell, 2006) allows for a rapid estimation of the 16%, 50% and 84%-

fractiles of the strength reduction factor R as a function of the displacement ductility μ, as illustrated in Figure 2 for a generic building structure with μ_c = 3 and T = 0.6s. Estimation of not only a central tendency but also the fractiles of the strength reduction factor extend the original framework presented in (Galanis et al., 2018).

Figure 2. For a generic building structure with μ_c = 3 and T = 0.6, the 16%, 50% and 84%-fractiles of lateral strength reduction factor R given ductility μ are shown. The blue dotted lines and points indicate the 50% fractile of R given the damage state thresholds $\mu_{lim,k}$.

The median capacity of the structure corresponding to a certain damage grade DG_k is:

$$\hat{s}_{lim,k} = R_{50\%}\left(\mathrm{T}, \mu_{lim,k}\right) \cdot a_y \;, \tag{10}$$

where a_y refers to the yielding spectral pseudo-acceleration and $R_{50\%}\left(\mathrm{T}, \mu_{lim,k}\right)$ is the 50%-fractile of R given $\mu_{lim,k}$ for a certain T, as illustrated using the dotted lines in Figure 2. The ground motion record-to-record variability is estimated as:

$$\beta_{Dk} = \frac{1}{2} \ln \left[\frac{R_{84\%}\left(\mathrm{T}, \mu_{lim,k}\right)}{R_{16\%}\left(\mathrm{T}, \mu_{lim,k}\right)} \right] \;. \tag{11}$$

The dispersion due to modelling uncertainties is taken as a discrete function of the strength reduction factors (ATC 2012):

$$\beta_{M,k} = \begin{cases} 0.25 \text{ if } R_{50\%}\left(\mathrm{T}, \mu_{lim,k}\right) \leq 2 \\ 0.35 \text{ if } R_{50\%}\left(\mathrm{T}, \mu_{lim,k}\right) = 4 \\ 0.50 \text{ if } R_{50\%}\left(\mathrm{T}, \mu_{lim,k}\right) \geq 6 \end{cases} . \tag{12}$$

The dispersion values for other strength reduction factor values can be determined using linear interpolation.

The seismically induced financial losses caused by the damage of a certain grade that is incurred by the building in a given earthquake event are expressed as a portion of the Present Building Property Value (PBPV), which is assumed to remain constant during the considered time horizon. This approach, used in (Galanis et al., 2018), makes it possible to normalize and compare the losses in different seismic hazard environments that also entail different construction and repair costs. Finally, the cost function $G(l|dm)$ is assumed to follow a beta distribution, $\mathrm{Beta}(L|DG_k; \alpha, \beta)$, where random variable L has finite support [0,1] and parameters α and β based on mean damage ratio and coefficient of variation as defined in (Dolce et al., 2006).

In combination with the earthquake hazard (i.e. the earthquake occurrence rate $\lambda(im)$), the integration of vulnerability and cost function results in the mean annual loss exceedance rate $\lambda(l)$ (Eq. 8). The financial loss induced by a randomly selected earthquake event is defined as the random variable S, called the severity. According to (Der Kiureghian, 2005; Galanis et al., 2018) the CCDF of S can be estimated as:

$$P(S > l) = \lambda(l)/\lambda(0)\,, \tag{13}$$

where $\lambda(0)$ is the mean annual rate of exceedance of losses greater than zero.

Two types of loss exceedance probability curves are used in this study to represent the seismically induced financial losses in terms of, first, a single loss occurrence in the considered time horizon, and, second, as losses aggregated over that time horizon. The Occurrence Exceedance Probability (OEP) (Budinger, 2013) loss curve is the CCDF of the largest loss $L_t = \max_{0 \leq t' < t} L(t')$ that could be triggered by a single earthquake event occurring in a certain time horizon t. The thinning property of the Poisson process is used to calculate OEP. Defining T_l as the time to the first excursion of loss level l, the OEP for a certain time horizon t is evaluated as:

$$\mathrm{OEP}(l|t) = P(L_t > l|t) = 1 - P(L_t \leq l|t) = P(T_l \leq t) = 1 - e^{-\lambda(l)t}\,, \tag{14}$$

with the mean annual loss exceedance rate $\lambda(l)$ resulting from Eq. 8. It follows that $\mathrm{OEP}(l|t)$ coincides with the seismically induced financial loss curves commonly used in earthquake engineering and defined as $P(L > l \text{ in } t \text{ years}) = 1 - P(\text{no } L > l \text{ events in } t \text{ years}) = 1 - e^{-\lambda(l)t}$.

The second type of loss exceedance probability curve used in this study is the Aggregate Exceedance Probability curve (AEP) (Budinger, 2013). It refers to the CCDF of Aggregated Seismic Losses, ASL defined as the sum of losses for a given time horizon t:

$$ASL|t = \sum_{n=1}^{N(t)} S_n\,, \tag{15}$$

where S_n is the severity of an earthquake of intensity $IM > im$, and $N(t)$ is the number of earthquakes of intensity $IM > im$, taken in this study as a uniform Poisson process with a cumulative rate $\Lambda(t) = \lambda(im)t$. Under this assumption, $ASL|t$ has a compound Poisson distribution, $CompPoi(\Lambda(t), S)$. Then the AEP loss curve is defined as:

$$\mathrm{AEP}(asl|t) = P(ASL > asl\,|t) = \sum_{n=1}^{\infty} [S^{n*}|N(t) = n]\, P\big(N(t) = n|\Lambda(t)\big), \qquad (16)$$

where $P(n|\Lambda(t))$ corresponds to the probability of n occurrences of earthquakes that cause a loss within the time period t and S^{n*} is the severity distribution S (Eq. 13) convolved n times with itself. Note that this procedure implicitly assumes that every time a seismically induced financial loss is incurred, the structure is instantaneously restored to its original state without any deterioration of its performance. It is further assumed that the uncertainties are "renewed" after each earthquake event (Der Kiureghian, 2005).

MEASURING SEISMIC RISK IN DIFFERENT HAZARD ENVIRONMENTS

The framework described above was implemented to compute the seismically induced financial losses for individual existing building structures located in different hazard environments. The same existing building structure is assumed to be located at the geographic coordinates listed in Table 1. The 5% damped elastic pseudo-acceleration uniform hazard spectra $\mathrm{S_{ae}}(\mathrm{T})$ for these locations are provided in open-access databases (EFEHR, 2014; USGS, 2014). The seismic hazard at the chosen locations ranges from very low (Zurich) to high (Los Angeles), as is evident from the uniform hazard spectra and the seismic hazard curves shown in Figure 3.

Table 1. Geographical coordinates of the building locations considered in this study.

	Zurich	Lisbon	Seattle	Reggio. C.	Los Angeles
Longitude	8.58	-9.14	-122.35	15.68	-118.25
Latitude	47.40	38.72	47.60	38.10	34.00

An existing building structure investigated in this study is vulnerable because it lacks the strength and the detailing compared to new structures complying with modern seismic design provisions. Specifically, the existing building considered has a fundamental elastic vibration period $\mathrm{T} = 0.6s$, a small base shear strength specified by a yielding pseudo-acceleration $a_y = 0.05\mathrm{g}$, and a small inelastic deformation capacity specified by a displacement ductility capacity $\mu_C = 2$ that defines the damage grade thresholds $\mu_{lim,k}$.

Figure 3. For the considered building locations stated in Table 1: (a) Uniform hazard elastic pseudo-acceleration spectra (UHS) for probability of exceedance of 10% in a 50-year long time horizon; (b)

Seismic hazard curves illustrating the probability of exceeding a certain $S_{ae}(T)$ in a 50-year long time period at a vibration period of 0.6s.

Two graphs in Figure 4 illustrate the OEP and AEP seismic loss curves (as defined above) for an existing building located in the five different hazard environments considered in this study for a 50-year time horizon. Note that the AEP loss curve accounts for possible multiple earthquake events occurring in the considered time horizon: thus, the losses could exceed the PBPV, i.e. the normalized loss value could exceed the value of 1, necessitating the $[0, \infty)$ support for the AEP loss curve. Note that the building is assumed to be instantaneously repaired to its original condition and PBPV after the occurrence of each earthquake event. On the other hand, the OEP loss curve is associated with the one single largest event that can occur during the considered time horizon. Therefore, the support of the OEP loss curve is [0,1]. Note also that the AEP and OEP loss curves develop two inflection points with similar horizontal coordinates for all hazard environments, one at loss values of about 0.1PBPV and the other at loss values between 0.60 and 0.80 of PBPV. This similarity stems from considering the same normalized support [0-1], and an identical building with identical vulnerability (e.g., the loss given a specific damage state and intensity measure is the same for all hazard environments, whereas the vertical coordinate, namely the probability of exceedance in 50 years, reflects the difference in seismic hazard of the site locations).

Figure 4. Seismic loss curves in terms of (a) Occurrence Exceedance Probability (OEP) and (b) Aggregated Exceedance Probability (AEP) for an existing building for a 50-year time horizon. The seismic losses (on the horizontal axis) are expressed as a portion of the PBPV.

Table 2 reports the values of nine different risk measures expressed as a percentage of PBPV. Expected Loss is evaluated on an annual basis, whereas the other measures are evaluated for an exceedance level of 10% in 50 years. It is important to note that Table 2 compares risk exposure both in terms of the seismic hazard environment (Zurich vs. Los Angeles) and in terms of the type of risk measure employed (e.g. EL vs. $ES_{\alpha=90\%}$). Ordering of different locations in terms of the seismic risk exposure differs according to the risk measure considered. Namely, based on the data in Table 2, an existing building in Seattle has a distinctly lower seismic risk exposure than the same building in Reggio Calabria according to risk measures Nr.1, 2, 4 and 6, but using risk measures Nr. 3, 5, and 7, the existing building in Seattle and Reggio Calabria appears to have approximately the same seismic risk exposure. This is because the risk measures Nr. 3, 5, and 7 are occurrence-based and saturate near a complete loss of 100% of PBPV.

Table 2. Risk measures evaluated for the same existing building situated in the five different hazard environments, expressed in terms of percentages of PBPV.

Nr.	Risk Measure	Zurich	Lisbon	Seattle	Reggio	Los Angeles
1	EL ($L_{t\,=\,1yr}$)	0.21%	0.97%	1.99%	2.37%	3.58%
2	EL ($ASL_{t\,=\,1yr}$)	0.21%	0.97%	2.01%	2.41%	3.66%
3	$\mathrm{VaR}_{\alpha=0.9}$ ($L_{t\,=\,50yr}$)	35.1%	96.4%	99.2%	99.3%	99.7%
4	$\mathrm{VaR}_{\alpha=0.9}$ ($ASL_{t\,=\,50yr}$)	37.3%	124.6%	218.3%	253.5%	346.1%
5	$\mathrm{ES}_{\alpha=0.9}$ ($L_{t\,=\,50yr}$)	77.8%	98.7%	99.7%	99.7%	99.9%
6	$\mathrm{ES}_{\alpha=0.9}$ ($ASL_{t\,=\,50yr}$)	84.2%	181.8%	286.1%	316.9%	422.1%
7	$\mathrm{LaF}_{\alpha=0.998}$ ($L_{t\,=\,1yr}$)	35.2%	96.4%	99.2%	99.3%	99.7%
8	SEL_{τ} ($L \mid im_{\tau\,=\,475yr}$)	34.0%	73.1%	87.4%	85.7%	87.4%
9	SUL_{τ} ($L \mid im_{\tau\,=\,475yr}$)	90.4%	99.1%	99.4%	99.4%	99.4%

Note that considering only the losses incurred in events characterized by a specific earthquake return period, for example 475 years (or exceedance probability of 10% in 50 years), assuming an underlying Poisson distribution of earthquake occurrence, may provide misleading information about the seismic risk exposure because this approach focuses only on seismic events of a certain occurrence frequency and neglects the remainder of the seismic hazard exposure. For example, using $\mathrm{SEL}_{\tau}(L)$ (risk measures Nr. 8 in Table 2), the seismic risk exposure of a building in Seattle is slightly higher than that in Reggio Calabria, contrary to a common trend. This is due to the shape of the hazard curves illustrated in the Figure 3b, where the considered intensity measure at this specific exceedance probability level is higher for Seattle than Reggio Calabria for the Design Basis Event, even though the situation is reversed for the Frequent and the Maximum Considered events (i.e. focusing on risk measures 1 and 2).

EVALUATING THE BENEFIT OF SEISMIC UPGRADE USING DIFFERENT RISK MEASURES

The aforementioned observations underline the need to examine the financial benefit of seismic upgrading not only by comparing different seismic hazard environments but also considering that seismic upgrade benefits could differ significantly if they are quantified using different risk measures. The relationship between the avoided earthquake losses and the degree of seismic upgrade dsu[7] is explored to illustrate this point, following the approach used in (Galanis et al., 2018). The degree of seismic upgrade dsu ranges between 0 and 1, i.e., 0% and

[7] Lower case font is used to highlight that the degree of seismic upgrade is a deterministic variable.

100% of the full seismic upgrade. In particular, it is assumed that the existing building without a seismic upgrade has $dsu = 0$, while a modern code-compliant building has $dsu = 1$. Stemming from the assumption that the existing and the upgraded buildings have the same geometry and yielding material characteristics, the yielding displacement is assumed to be the same for all buildings with different dsu. The yielding acceleration capacity of the modern code-compliant building is evaluated based on the $S_{ae}(T)$ with a probability of exceedance of 10% in 50 years and applying a behavior factor of $q = 2$ to approximate for some inelastic response of the structure:

$$a_y(dsu = 1) = S_{ae,10\%\ 50\ Years}(T)/q\ . \quad (17)$$

The structural displacement ductility capacity of modern code-compliant building is taken as $\mu_c = 6$, while that of the existing building is $\mu_c = 2$. The parameters for the existing and modern code-compliant buildings in different seismic hazard environments are stated in Table 3. The static pushover response curves for buildings with intermediate dsu values are derived assuming a proportional increase of the displacement ductility capacity μ_c and the yielding acceleration capacity a_y, leading to the following definition of $a_y(dsu)$:

$$a_y(dsu) = a_y(0) + dsu \cdot \left(a_y(dsu) - a_y(0)\right), \quad (18)$$

$$\mu_c(dsu) = \mu_c(0) + dsu \cdot \left(\mu_c(dsu) - \mu_c(0)\right), \quad (19)$$

The fundamental vibration period of these buildings is evaluated assuming constant yield displacement leading to the following relationship:

$$T(dsu) = T(0) \cdot \sqrt{a_y(0)/a_y(dsu)}\ . \quad (20)$$

Table 3. Structural characteristics of the existing (dsu=0) and the modern code-compliant (dsu=1) buildings for the different hazard environments considered in this study.

Site location	Existing building (dsu = 0)			Code-compliant building (dsu = 1)		
	a_y [g]	μ_C	T [sec]	a_y [g]	μ_C	T [sec]
Zurich	0.05	2	0.60	0.05	6	0.60
Lisbon	0.05	2	0.60	0.17	6	0.33
Seattle	0.05	2	0.60	0.32	6	0.24
Reggio C.	0.05	2	0.60	0.36	6	0.22
Los Angeles	0.05	2	0.60	0.48	6	0.19

Risk measures $\rho(L|dsu)$ are calculated for the buildings specified in Table 3 as functions of the dsu value using the introduced framework to quantify seismic losses and the hazard data used in the previous section. Given a risk measure ρ, the Upgrading Benefit for a given *dsu* is:

$$\mathrm{UB}_{\rho}(dsu) = \rho(L|dsu = 0) - \rho(L|dsu)\,, \tag{21}$$

the difference between the risk measure values for the existing and the upgraded building. The Normalized Upgrading Benefit for a given *dsu* is:

$$\mathrm{NUB}_{\rho}(dsu) = \frac{\mathrm{UB}_{\rho}(dsu)}{\mathrm{UB}_{\rho}(dsu = 1)}\,, \tag{22}$$

the Upgrading Benefit normalized by the Upgrading Benefit of a full upgrade. This quantity indicates to what extent are earthquake losses avoided by upgrading the existing building to a certain dsu compared to the earthquake loss avoided by upgrading the structure such that it fully complies with modern seismic design codes, i.e. has the same earthquake risk exposure as a new structure[8].

EXPECTED LOSS RISK MEASURE

The expected value risk measure for a loss cumulative probability distribution assigns weights proportional to the frequency of events. Thus, more frequent events could have a significant contribution to the losses. Therefore, the expected value of the ASL (defined in Equation 15 and shown in Figure 4b due to the full range of earthquake events (in terms of occurrence frequency) is of interest in the considered time horizon, i.e. $\rho(L|dsu) \equiv \mathrm{EL}(ASL|dsu)$ is used to account for multiple small losses due multiple frequent events. Note that the mean expected cumulative loss measure, $\mathrm{NUB}_{\mathrm{EL}}(dsu)$, is independent of the time horizon considered due to the normalization in Equation 22.

Figure 5 illustrates that, for all hazard environments considered, the $\mathrm{NUB}_{\mathrm{EL}}(dsu)$ curves are concave. This finding corroborates the results presented in (Galanis et al., 2018) for a building archetype located in Zurich and L'Aquila. The marginal benefits of seismic upgrading correspond to the slope of the $\mathrm{NUB}_{\mathrm{EL}}(dsu)$ curves. Thus, a concave curve indicates a large marginal $\mathrm{NUB}(dsu)$ for small dsu values and a gradually decreasing marginal $\mathrm{NUB}(dsu)$ as

[8] Note that modern code-compliant structures have a finite but acceptably small earthquake risk exposure defined explicitly or implicitly in modern seismic design provisions. Setting this acceptable risk exposure for a single building structure is important, but a discussion of what this value is and how this value should be set it is out of the scope of this study.

the dsu values increase. Conversely, a convex $\mathrm{NUB}(dsu)$ curve shape corresponds to a small marginal $\mathrm{NUB}(dsu)$ for small dsu values and increasing marginal $\mathrm{NUB}(dsu)$ for larger dsu values, meaning that the highest marginal $\mathrm{NUB}(dsu)$ is obtained for $dsu = 1$, a full retrofit. It is notable that even though the seismic hazard exposure in the considered seismic hazard environments is very different, the $\mathrm{NUB}_{\mathrm{EL}}(dsu)$ curves are fairly similar. Specifically, a seismic upgrade with a degree of only 30% makes it possible to reduce 65% (Zurich) to 75% (Los Angeles) of the expected losses. Normalizing with respect to the structural characteristics (Eq. 18-20) and the seismic performance (Eq. 22) of a modern code-compliant structure, together with the expected value of a random variable as the risk metric, leads to similar concave curves.

Figure 5. Normalized Upgrading Benefit in terms of the Expected Loss risk measure based on ASL for the hazard environments considered in this study.

TAIL RISK MEASURES

In this subsection, the financial benefits of seismic upgrading are evaluated using the tail risk measures $\mathrm{VaR}_{\alpha}(L)$ and $\mathrm{ES}_{\alpha}(L)$. Contrary to the Expected Loss risk measure, the tail risk measures are calculated based on high-severity events with occurrence likelihood below a certain exceedance threshold. Therefore, $\mathrm{VaR}_{\alpha}(L)$ and $\mathrm{ES}_{\alpha}(L)$ are evaluated using the loss occurrence exceedance probability (OEP) curve, as shown in Figure 4a.

Value-at-Risk evaluated using an annual loss occurrence exceedance probability curve is illustrated in Figure 6 for the seismic hazard environments of Lisbon and Los Angeles versus a range of exceedance levels $(1 - \alpha)$ (where α is the confidence level). The vertical axis represents the Value-at-Risk as a proportion of PBPV, such that a value of 0 means there are no losses and a value of 1 means that the losses equal to the value of the building (i.e. it is extensively damaged or collapsed). Each curve in Figure 6 corresponds to a different *dsu*.

Figure 6. $\mathrm{VaR}_{\alpha}(L|dsu)$ plotted as a function of exceedance level $(1 - \alpha)$ for five different *dsu* levels and the seismic hazard environments of: (a) Lisbon and (b) Los Angeles.

Figure 7 illustrates the NUB quantified using $\mathrm{VaR}_{\alpha}(L|dsu)$, denoted as $\mathrm{NUB}_{\mathrm{VaR}}(dsu|\alpha)$, for different levels of dsu and for confidence levels α equal to 98%, 99% and 99.9%. All the $\mathrm{NUB}(dsu)$ curves for the seismic hazard environments considered are concave for the 98% and 99% confidence levels, showing that the seismic upgrading marginal benefit is larger for small dsu levels. Data in Figure 7a indicates that an upgrade of 30% reduces $\mathrm{VaR}_{\alpha=98\%}$ between 85% and 95% for Seattle, Reggio Calabria and Los Angeles seismic hazard environments, which is significantly higher than the $\mathrm{NUB}_{\mathrm{EL}}$ indicated in Figure 5 for the same dsu level. Note that the $\mathrm{NUB}_{\mathrm{VaR}}(dsu|\alpha = 99\%)$ curve shapes for Seattle, Reggio Calabria and Los Angeles are no longer strictly concave in Figure 7b, specifically for a dsu of 10% the reduction in $\mathrm{VaR}_{\alpha=99\%}$ is only 10% in the case of Los Angeles (compared to a 40% reduction in EL indicated in Figure 5). For the high confidence level of 99.9% shown in Figure 7c, the $\mathrm{NUB}(dsu)$ curve shape is convex for all hazard environments except for Zurich, indicating that seismic upgrading marginal benefit is larger for larger dsu levels, and is the largest for a full upgrade. For the low hazard environment of Zurich, the curve shape remains concave for dsu levels above 15% in Figure 7c. Only if the confidence level is increased further (e.g. to 99.99%) the corresponding $\mathrm{NUB}_{\mathrm{VaR}}(dsu|\alpha)$ curve shape becomes convex. The relation between confidence level, hazard environment and $\mathrm{NUB}(dsu)$ curve shape is explained in more detail below for the Expected Shortfall risk measure.

Figure 7 Normalized Upgrading Benefit in terms of the Value-at-Risk risk measure evaluated using an annual loss occurrence exceedance probability curve for all hazard environments considered at a confidence level α of: (a) 98%; (b) 99% and (c) 99.9%.

The relationship between NUB quantified using the Expected Shortfall risk measure and dsu is illustrated in Figure 8 for confidence levels α equal to 98%, 99% and 99.9%. As shown in Figure 1, ES_{α} are the expected earthquake-induced financial losses given that these have exceeded the VaR_{α} at the same confidence level α (annual exceedance level (1-α)), for which the results are shown in Figure 7.

Figure 8 Normalized Upgrading Benefit in terms of the Expected Shortfall risk measure evaluated using an annual loss occurrence exceedance probability curve for all hazard environments considered, at a confidence level α of: (a) 98%; (b) 99% and (c) 99.9%.

The $\mathrm{NUB}_{\mathrm{ES}}(dsu|\alpha)$ curves shown in Figure 8 lead to several observations. First, the shape of the $\mathrm{NUB}(dsu)$ curves at a confidence level of $\alpha = 98\%$ is strictly concave for all hazard environments except for very small dsu in Los Angeles. Second, the reduction in $\mathrm{NUB}_{\mathrm{ES}}(dsu|\alpha = 98\%)$ for a dsu of 30% is smaller than that of $\mathrm{NUB}_{\mathrm{VaR}}(dsu|\alpha = 98\%)$ indicated in Figure 7a. Finally, increasing the confidence level beyond 99% (i.e. extending the observation beyond the 100-year long time horizon and focusing only on the events far in the tail of the loss exceedance probability distribution) $\mathrm{NUB}(dsu)$ curves start to change their shape from concave to convex for most of the considered hazard environments, starting from the low dsu levels.

As indicated in Figure 7 for VaR_{α} and in Figure 8 for ES_{α}, there seems to exist a confidence level α above which the corresponding $\mathrm{NUB}(dsu)$ curve shape changes from concave to convex. This statement is examined thoroughly on the example of ES_{α} , for which $\mathrm{NUB}_{\mathrm{ES}}(dsu|\alpha)$ is disaggregated into its underlying parts, namely the Upgrading Benefit $\mathrm{UB}_{\mathrm{ES}}(dsu|\alpha)$ and the risk measure itself $\mathrm{ES}_{\alpha}(L|dsu)$. Figure 9 illustrates these three quantities with respect to the exceedance level $(1-\alpha)$ for the hazard environments of Lisbon (Figure 9a, c and e) and Los Angeles (Figure 9b, d and f). Similar results are obtained for other hazard environments.

Function $\mathrm{UB}_{\mathrm{ES}}(dsu|\alpha)$ attains the maximum for the confidence level $\alpha_{\max}$ (Figure 9c and d), which corresponds to the inflection point of the $\mathrm{NUB}_{\mathrm{ES}}(dsu|\alpha)$ curve (Figure 9e and f). Two important observations can be drawn from these plots. First, confidence level $\alpha_{\max}$ is almost invariant with respect to the value of dsu for a given hazard environment. Second, confidence level $\alpha_{\max}$ depends on the hazard environment, i.e. lower values of $\alpha_{\max}$ are related to more hazardous environments (e.g. for Lisbon $\alpha_{\max} \approx 99\%$ and for Los Angeles $\alpha_{\max} \approx 96.5\%$). These two observations have an important consequence: for a given hazard environment there exists an $(1-\alpha_{\max})$ exceedance level for which the $\mathrm{UB}_{\mathrm{ES}}(dsu|\alpha)$ is maximized regardless of the dsu chosen. The reason for the peak in $\mathrm{UB}_{\mathrm{ES}}(dsu|\alpha)$ at some α_{max} lies in the bounded nature of the underlying loss curve $\mathrm{OEP}(l|t)$, i.e. the loss due to a single event cannot exceed PBPV. This implies an inflection point in the $\mathrm{ES}_{\alpha}(L|dsu)$ curves (Figure 9a and b) that occurs at approximately the same value of loss, 85% of PBPV, regardless of the value of dsu. Finally, as shown in Figure 9e and 9f the $\mathrm{NUB}_{\mathrm{ES}}(dsu|\alpha)$ is almost constant for exceedance levels higher than $(1-\alpha_{\max})$, whereas for lower exceedance levels there is a steep drop in the normalized benefit of partial upgrades. This relates directly to the results shown in Figure 8, where the $\mathrm{NUB}_{\mathrm{ES}}(dsu|\alpha)$ curve shape for Lisbon at confidence level 99% is still strictly concave, whereas the curve shape at the same confidence level for Los Angeles has already a convex shape over a substantial range of dsu. Thus, for exceedance levels higher than $(1-\alpha_{\max})$ the $\mathrm{NUB}_{\mathrm{ES}}(dsu|\alpha)$ curve shape is strictly concave and the marginal benefit is large for small dsu values and gradually decreases as the dsu values increase. For the hazard environment of Lisbon any partial upgrading is effective in reducing the average losses given that the losses exceeded the 100-year loss (or $\mathrm{VaR}_{\alpha=99\%}$). Conversely, partial upgrading becomes rapidly less effective in reducing the average losses given that the losses exceeded the 200-year loss (or $\mathrm{VaR}_{\alpha=99.5\%}$) and finally, by only looking at average losses given the exceedance of the 1000-year loss (or $\mathrm{VaR}_{\alpha=99.9\%}$) the $\mathrm{NUB}_{\mathrm{ES}}(dsu|\alpha)$ curve shape is strictly convex, indicating that any partial seismic upgrade becomes less effective than the full upgrade.

Figure 9. Risk measure Expected Shortfall $\mathrm{ES}_{\alpha}(L|dsu)$, Upgrading Benefit $\mathrm{UB}_{\mathrm{ES}}(dsu|\alpha)$ and Normalized Upgrading Benefit $\mathrm{NUB}_{\mathrm{ES}}(dsu|\alpha)$ plotted as a function of exceedance level $(1-\alpha)$ for the seismic hazard environments of Lisbon and Los Angeles and five different *dsu* levels. The dashed black lines refer to the exceedance level $1-\alpha_{\max}$ when $\mathrm{UB}_{\mathrm{ES}}(dsu|\alpha)$ attains its maximum.

In online Appendix A the benefit of seismic upgrading is further evaluated for the remaining tail risk measure defined above $\mathrm{LaF}_{\alpha}(L)$, as well as for the scenario-based risk measures $\mathrm{SEL}_{\tau}(L)$, $\mathrm{SUL}_{\tau}(L)$.

The methodology used in the present study to estimate seismically induced financial losses and to derive the results discussed above leans on several assumptions, which are explained in detail in (Galanis et al., 2018). Discrete damage grade thresholds are identified directly on the SDoF model elasto-perfectly-plastic force-displacement response and, thus, depend on the definition of SDoF yielding displacement and displacement ductility that is consistent with the damage in the actual building structure. The SPO2IDA tool used to assess vulnerability is not fully consistent with the damage and cost functions because the datasets from recent earthquakes are not complete or have not been consistently processed. Thus, several analyses have been performed to explore the sensitivity of the $\mathrm{NUB}(dsu)$ curve for the Expected Loss and Expected Shortfall risk measures. Using different $R-\mu-T$ relations to compute the median building capacity corresponding to a certain damage grade (Eq. 10), the definition of the discrete damage state thresholds and related cost functions, as well as modified values of structural displacement ductility μ_C of the existing building ($dsu=0$) are shown to have little effect on the principal observations presented above. Specifically, the concave nature of the $\mathrm{NUB}(dsu)$ curves for the Expected Loss and Expected Shortfall (below a certain confidence level) risk measures is confirmed. The reader is referred to online Appendix B for a more detailed presentation of these results.

CONCLUSIONS

Seismic upgrade is an up-front investment without an immediate commensurate gain of the present building property value. Thus, communicating the benefits of seismic upgrading to different stakeholders is difficult without a framework to express acceptable seismic performance of a structure in financial terms.

The Normalized Upgrade Benefit $\mathrm{NUB}(dsu)$ of a seismic upgrade with a degree dsu (Eq. 22) provided insight into how earthquake losses can be reduced using seismic upgrading. A concave $\mathrm{NUB}(dsu)$ curve makes it possible to justify partial seismic upgrades by arguing that the incremental benefits of increasing the degree of seismic upgrade are diminishingly smaller. Conversely, a convex $\mathrm{NUB}(dsu)$ curve justifies a full seismic upgrade, the one that maximizes the marginal $\mathrm{NUB}(dsu)$ with respect to dsu.

The convexity or concavity of the $\mathrm{NUB}(dsu)$ curves reveals how the choice of a particular risk measure and confidence level affects the selection of an optimal degree of seismic upgrade. Namely, $\mathrm{NUB}(dsu)$ curves become convex for high confidence levels for all tail risk measures, justifying full seismic upgrades. The opposite is true for lower confidence levels, when partial seismic upgrades can be justified based on marginally larger avoided losses for comparatively smaller retrofit costs. It is particularly important to note that the $\mathrm{NUB}(dsu)$ curves for the Expected Loss risk measures are concave for all hazard environments (with almost the same shape). A sensitivity study revealed that these curves are not sensitive to several important assumptions made in the present study. Thus, $\mathrm{NUB}_{\mathrm{EL}}(dsu)$ curves uniformly indicate that partial seismic upgrades should be financially quite effective, providing marginally more loss avoidance at lower, i.e. less expensive, upgrade levels.

Considering that the Expected Loss risk measure is one of the most commonly used risk measures to support risk-neutral risk mitigation decisions (e.g. in cost-benefit analyses), it is not surprising that conclusions from such studies often contradict full seismic upgrading decisions based on structural engineering code provisions. These provisions are based on tail risk measures and long return periods, potentially resulting in convex $\mathrm{NUB}(dsu)$ curves. This would push the seismic upgrading decision towards the most conservative full seismic upgrade[8], implicitly introducing an attitude that resembles risk aversion[5]. Therefore, the selection of the risk measure (and the associated confidence level) to quantify the earthquake losses avoided by seismic upgrading of an existing building is crucial in formulating systematic seismic risk mitigation policies. Simultaneously, this opens an opportunity to base decisions on seismic upgrading of individual existing structures on shorter return periods commensurate with the expected remaining service life time of the structure, making it easier to financially justify partial seismic upgrades. This strategy could be particularly interesting for regions with low seismic hazard exposure, where the NUB(dsu) curves remain concave even at high confidence levels. An example framework employing the expected loss NUB curves and the time horizon as inputs that affects decision-making of different seismic upgrading can be found in Galanis et al. (2018).

Discussion on the role of risk measures in making seismic upgrading decision is currently largely missing in the literature (with only a few notable exceptions (Cha and Ellingwood, 2013a; Goda and Hong, 2008; Rockafaller and Royset, 2012)). Given the high importance of seismic upgrading in the overall community seismic risk mitigation process, and the need to clearly explain the financial benefits seismic risk mitigation to the public[9], the authors hope that the findings of the present study could serve as a basis for such a discussion.

ACKNOWLEDGEMENTS

The authors thank the reviewers for constructive feedback that improved this paper. The second author, was supported by the RiskLab Switzerland and the Chair of Structural Dynamics and Earthquake Engineering at ETH Zurich. The third author was supported by the Swiss Seismological Service.

REFERENCES

Acerbi C (2002) Spectral measures of risk: A coherent representation of subjective risk aversion. *Journal of Banking and Finance* 26(7): 1505–1518. DOI: 10.1016/S0378-4266(02)00281-9.

Alibrandi U and Mosalam KM (2017) A Decision Support Tool for Sustainable and Resilient Building Design. *Risk and Reliability Analysis: Theory and Applications* -: 509–536.

American Society for Testing and Materials (ASTM) (2016a) *Standard Guide for Seismic Risk Assessment of Buildings. ASTM E2026 - 16a*. West Conshohocken, PA.

American Society for Testing and Materials (ASTM) (2016b) *Standard Practice for Probable Maximum Loss (PML) Evaluations for Earthquake Due-Diligence Assessments, ASTM E2557 - 16a*. West Conshohocken, PA.

Applied Technology Council (ATC) (2009) *Quantification of Building Seismic Performance Factors. FEMA P-695. Federal Emergency Management Agency*. Washington D.C.

Applied Technology Council (ATC) (2012) *Seismic Performance Assessment of Buildings Volume 1 –*

[9] An example of such a discussion lead to a policy adopted by the New Zealand government after the 2011 Christchurch earthquake (Tailrisk Economics, 2014).

Methodology. *FEMA P-58-1*. *Federal Emergency Management Agency*. Washington D.C.

Basel Committee on Banking Supervision (BCBS) (2019) *The Basel Framework*. *Bank for International Settlements*. Basel, Switzerland.

Bazzurro P and Cornell CA (1999) Disaggregation of seismic hazard. *Bulletin of the Seismological Society of America* 89(2): 501–520. DOI: 10.1785/0120060093.

Bevere L and Grollimund B (2012) *Lessons from recent major earthquakes*. *Swiss Reinsurance Company*. Zurich, Switzerland.

Broccardo M, Danciu L, Stojadinovic B, et al. (2017) Individual and societal risk metrics as parts of a risk governance framework for induced seismicity. In: *16th World Conference on Earthquake Engineering*, Santiago, Chile, 2017.

Budinger G (2013) *Catastrophe Modelling: Guidance for Non-Catastrophe Modellers*. *Lloyd's Market Association*. London.

Cha EJ and Ellingwood BR (2013a) Seismic risk mitigation of building structures: The role of risk aversion. *Structural Safety* 40: 11–19. DOI: 10.1016/j.strusafe.2012.06.004.

Cha EJ and Ellingwood BR (2013b) The role of risk aversion in nuclear plant safety decisions. *Structural Safety* 44: 28–36. DOI: 10.1016/j.strusafe.2013.05.002.

Comerio MC and Anagnos T (2012) Los Angeles Inventory: Implications for Retrofit Policies for Nonductile Concrete Buildings. In: *15th World Conference on Earthquake Engineering*, Lisbon, Portugal, 2012.

Cornell CA and Krawinkler H (2000) *Progress and Challenges in Seismic Performance Assessment*. *PEER Center News*.

Der Kiureghian A (2005) Non-ergodicity and PEER's framework formula. *Earthquake Engineering and Structural Dynamics* 34(13): 1643–1652. DOI: 10.1002/eqe.504.

Dolce M, Kappos A, Masi A, et al. (2006) Vulnerability assessment and earthquake damage scenarios of the building stock of Potenza (Southern Italy) using Italian and Greek methodologies. *Engineering Structures* 28(3): 357–371. DOI: 10.1016/j.engstruct.2005.08.009.

European Facilities for Earthquake Hazard and Risk (EFEHR) (2014) Hazard Data Access. Available at: http://www.efehr.org/en/hazard-data-access/hazard-curves/ (accessed 22 June 2017).

Galanis P, Sycheva A, Mimra W, et al. (2018) A framework to evaluate the benefit of seismic upgrading. *Earthquake Spectra* 34(2). Earthquake Engineering Research Institute: 527–548. DOI: 10.1193/120316EQS221M.

Galanis PH and Moehle JP (2015) Development of collapse indicators for risk assessment of older-type reinforced concrete buildings. *Earthquake Spectra* 31(4): 1991–2006. DOI: 10.1193/080613EQS225M.

Goda K and Hong HP (2008) Application of cumulative prospect theory: Implied seismic design preference. *Structural Safety* 30(6): 506–516. DOI: 10.1016/j.strusafe.2007.09.007.

Jonkman SN, Van Gelder PHAJM and Vrijling JK (2003) An overview of quantitative risk measures for loss of life and economic damage. *Journal of Hazardous Materials*. Elsevier. DOI: 10.1016/S0304-3894(02)00283-2.

Kunreuther H and Kleffner AE (1992) Should earthquake mitigation measures be voluntary or required? *Journal of Regulatory Economics* 4(4): 321–333. DOI: 10.1007/BF00134925.

Lagomarsino S and Giovinazzi S (2006) Macroseismic and mechanical models for the vulnerability and damage assessment of current buildings. *Bulletin of Earthquake Engineering* 4(4): 415–443. DOI: 10.1007/s10518-006-9024-z.

McNeil AJ, Frey R and Embrechts P (2005) *Quantitative Risk Management: Concepts, Techniques, and Tools*. *Princeton Series in Finance*. Princeton University Press. DOI: 10.1198/jasa.2006.s156.

Moehle J and Deierlein GG (2004) A framework methodology for performance-based earthquake engineering. In: *13th World Conference on Earthquake Engineering*, 2004, pp. 3812–3814. DOI: 10.1061/9780784412121.173.

Mosalam KM, Alibrandi U, Lee H, et al. (2018) Performance-based engineering and multi-criteria decision analysis for sustainable and resilient building design. *Structural Safety* 74: 1–13. DOI: 10.1016/j.strusafe.2018.03.005.

Porter KA (2016) Safe enough? A building code to protect our cities and our lives. *Earthquake Spectra* 32(2): 677–695. DOI: 10.1193/112213EQS286M.

Porter KA, Kiremidjian AS and LeGrue JS (2001) Assembly-based vulnerability of buildings and its use in performance evaluation. *Earthquake Spectra* 17(2): 291–312. DOI: 10.1193/1.1586176.

Rockafaller RT and Royset JO (2012) Risk Measures in Engineering Design under Uncertainty. In: *12th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP12*, Vancouver, Canada, 2012.

Swiss Financial Market Supervisory Authority (FINMA) (2017) *Swiss Solvency Test (SST)*. Bern, Switzerland.

Tailrisk Economics (2014) *Error Prone Bureaucracy. Earthquake strenghtening policy formulation in New Zealand 2003-13: A Study in Failure*. Wellington.

Thiel CC, Kosonen TE and Stivers DA (2012) SEL versus SUL: Managing seismic risk in commercial real estate investments. *Structural Design of Tall and Special Buildings* 21(6): 389–404. DOI: 10.1002/tal.601.

United States Geological Survey (USGS) (2014) Unified Hazard Tool. Available at: https://earthquake.usgs.gov/hazards/interactive/ (accessed 22 June 2017).

Vamvatsikos D and Cornell CA (2006) Direct estimation of the seismic demand and capacity of oscillators with multi-linear static pushovers through IDA. *Earthquake Engineering and Structural Dynamics* 35(9): 1097–1117. DOI: 10.1002/eqe.573.

Vrijling JK, van Hengel W and Houben RJ (1995) A framework for risk evaluation. *Journal of Hazardous Materials* 43(3): 245–261. DOI: 10.1016/0304-3894(95)91197-V.

VSP Associates (1992) *A Benefit-Cost Model for Seismic Rehabilitation of Buildings. FEMA 227*. *Federal Emergency Management Agency*. Washington D.C.

Wenk T (2008) *Seismic retrofitting of structures. Strategies and collection of examples in Switzerland. Environmental studies No. 0832. Federal Office for the Environment*. Bern, Switzerland.

Wuthrich M V. (2017) Non-Life Insurance: Mathematics and Statistics. *SSRN Electronic Journal*. DOI: 10.2139/ssrn.2319328.

Yang TY, Moehle J, Stojadinovic B, et al. (2009) Seismic performance evaluation of facilities: Methodology and implementation. *Journal of Structural Engineering* 135(10): 1146–1154. DOI: 10.1061/(ASCE)0733-9445(2009)135:10(1146).