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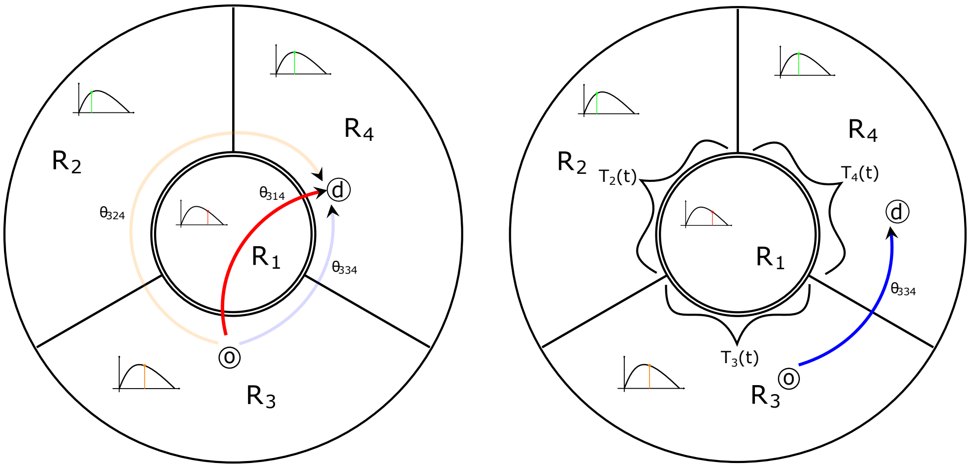

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Abstract

Traffic management with congestion pricing is a measure for mitigating traffic congestion in a protected corridor. Hence, the level of service in a city center (i.e., the protected corridor) increases and leads to a reduction in travel times and travel delays. So far, implementations in the field are restricted to static pricing, i.e., the price is fixed and not responsive to the prevailing regional traffic conditions. Dynamic pricing overcomes these limitations but also influences in real time the user's route choices. Consequently, this measure can be utilized to aim for optimal traffic distribution in the network. The proposed framework models a large-scale network where every region is considered as homogeneous, allowing for application of Macroscopic Fundamental Diagram (MFD). We compute Dynamic System Optimum (DSO) and Dynamic User Equilibrium (DUE) of the macroscopic model by formulating a linear optimization problem and utilizing the Dijkstra algorithm and a multinomial Logit model, respectively. Finally, we derive the optimal pricing functions for an exogenous demand scenario by applying the concept of elasticities. We test our framework to a case study in Zurich, Switzerland, and showcase the optimal price functions for the multi-region model.

Keywords

Optimal congestion pricing; Multi-region urban modeling; Macroscopic Fundamental Diagram (MFD); Optimal route-choice

1 Introduction and problem statement

In recent decades traffic congestion has emerged, especially in urban transportation networks. Hence, this is leading to negative impacts on the performance level of such networks, the environment, and also on social aspects. To tackle this problem, a promising approach is congestion pricing (often also called road pricing). Besides systems that are focusing on the pricing of one lane (Hot-Occupancy-Lanes), tolls at the border of a protected region have been shown as beneficial. Such fixed corridor (or cordon) pricing systems are already applied in reality (e.g., London, Singapore, or Stockholm) and show a decrease in the Vehicle Kilometers Traveled (VKT). Nevertheless, most of the pricing schemes are fixed over time and do not adjust to current traffic states (i.e., the current vehicle accumulations) in a system. The most advanced cordon pricing system so far is implemented in Singapore, where tolls are revised four times a year by evaluating the deviation of speed measurements (Eliasson, 2017).

Hence, the question arises if dynamic congestion pricing can improve the performance of a transportation network. This research topic has gained rising attention in the last years. The optimal toll problem has been tackled for microscopic and macroscopic traffic models. On the microscopic level, the pricing of all network links (first-best pricing problem) has been claimed as impractical due to high operational costs and low acceptance rate of the users. Also, the theoretical modeling of large networks has to tackle high computational complexity. Therefore, recent research focuses on the second-best pricing problem, where only a subset of the links is utilized for pricing. Considering such a small toy network, represented as a graph, Meng *et al.* (2012) and Chung *et al.* (2012) are formulating optimization problems to derive optimal tolls with a bi-level cellular particle swarm optimization and (also considering demand uncertainties) a mixed-integer problem, respectively. Both works focus on the determination of prices by utilizing the total distance traveled on priced arcs in a network. An optimal speed-based pricing design using the average travel speed has been proposed by Liu *et al.* (2013). Finally, a joint model incorporating the travel distance and time (Joint distance and time toll (JDTT)) was introduced by Liu *et al.* (2014). Methodologies of the aforementioned papers are operating at the link level, which remains challenging when one considers a city center corridor with a high number of links (holds for the first and second-best toll problem). Besides, sophisticated modeling incorporates a dynamic traffic assignment, which makes the computation of dynamic traffic equilibria (i.e., Dynamic User Equilibrium (DUE) and Dynamic System Optimum (DSO)) relatively expensive. Nevertheless, this is an essential procedure to evaluate the applied dynamic pricing scheme.

To account for given drawbacks of microscopic modeling of congestion pricing, other works have focused on a macroscopic approach with the utilization of multi-region models and Macroscopic Fundamental Diagram (MFD). To the authors' best knowledge, one of the first works considering MFD to obtain optimal pricing was published by Simoni *et al.* (2015). As no feedback-control strategy was applied, Gu *et al.* (2018) investigate several pricing methodologies with a simulation-based optimization and feedback control. The work combines a microscopic simulator with a Proportional-Integral (PI) control utilizing the MFD. This approach allows maintaining a protected region at the critical vehicle accumulation (corresponding to the maximum vehicle flow) and calculating prices based on the link-based distance and time travelled. Nevertheless, the iterative approach introduces a heavy dependency on a simulator to derive the MFD and the link-based prices; moreover, no comparison to traffic equilibria is performed.

Although optimal pricing solutions could be applied in the field, the users elasticities to a specific toll is not considered. The concept of elasticity gives the relation between the percentage of change in the demand and change in price. Hence, the quantities give insights to the reaction of users to a specific toll price. Olszewski and Xie (2005) derived the elasticity of different pricing systems (e.g., specific bridge sections in New York, motorways in Spain, cordon pricing in Singapore). The results indicate that elasticity is dependent on the type of pricing system (road section, specific infrastructure type, cordon), time of the day, and also if all or specific vehicles are priced. Sarlas *et al.* (2013) extended the findings with an analysis in Athens (Greece). To this end, if the elasticities of users in the system are available, the concept can be utilized to determine optimal toll prices, if the following information is present: the DSO and DUE of the transportation network for a known exogenous demand are known and hence allows the calculation of the system-, and user-optimal transfer flows.

In the present work, we focus on a multi-region-network based on Sirmatel and Geroliminis (2018) and Genser and Kouveals (2019) to find the optimal macroscopic pricing scheme with given elasticities. The defined urban regions are considered as homogeneous with different characteristics (i.e., size, capacity, average trip length) in the heterogeneous traffic network. A well-defined MFD characterizes every region with a recently proposed method by Ambühl *et al.* (2018). The determination of DSO is solved by reformulating the nonlinear model into a linear program by applying several approximations based on the work by Genser and Kouvelas (2020) with a Linear Rolling Horizon Optimization (LRHO); i.e., the optimal splitting rates are determined. Implementation of the DUE is based on the utilization of Djikstra algorithm to find the shortest paths and a multinomial Logit (MLN) model to determine the user's route choices. To determine the optimal time-varying tolls, the concept of elasticity is deployed by utilizing the transfer flows of DUE and DSO scenarios to model the change in demand and users costs.

The remainder of this paper is organized as follows: Section 2 introduces the macroscopic modeling based on the so-called P/L model. Section 5 elaborates on the derivation of DSO with a linear approach. Furthermore, DUE with Djikstra algorithm and MLN are introduced. The optimal toll derivation is presented in Section 4 and elaborates on the offline computation with known elasticities. The methodology is applied to a case study in Zurich, Switzerland, with results for DSO, DUE as well as the optimal tolls. The paper closes with a conclusion and future work in Section 6.

2 Macroscopic multi-region modeling

A multi-region-network partitioned into homogeneous regions is introduced, defined by $\mathcal{R} = \{1, 2, \ldots, K\}$, where K is the number of regions. Every region from $\mathcal{R}$ is modeled with a well-defined MFD, represented by the function $G(N_I(t))$. $N_I(t)$ denotes the accumulation of a region I at time t. Consequently, the dynamic equations can be defined in continuous time as follows:

$$\frac{dN_{II}(t)}{dt} = Q_{II}(t) - M_{II}(t) + \sum_{H \in \mathcal{N}_I} M_{HII}(t), \tag{1}$$

$$\frac{dN_{IJ}(t)}{dt} = Q_{IJ}(t) - \sum_{H \in \mathcal{N}_I} M_{IHJ}(t) + \sum_{H \in \mathcal{N}_I; H \neq J} M_{HIJ}(t), \tag{2}$$

where indices $I \in \mathcal{R}$, $H \in \mathcal{N}_I$ and $J \in \mathcal{R}$ represent the origin, stop-over, and destination region, respectively. Variables $N_{II}(t)$ and $N_{IJ}(t)$ denote accumulations of region I that have final destination region I and J, respectively. $\mathcal{N}_I$ is a set that contains all neighboring regions of I. Internal demand within one region is defined by $Q_{II}(t)$; moreover, demands with origin I and destination J are denoted by $Q_{IJ}(t)$. Note that $Q_{II}(t)$ and $Q_{IJ}(t)$ are exogenous signals. Intra- and inter-regional flows are computed by functions $M_{II}(t)$ and $M_{IHJ}(t)$ representing internal flows in a region and transfer flows from region I to H (with final destination J), respectively, defined as follows:

$$M_{II}(t) = \frac{N_{II}(t)}{N_I(t)} G(N_I(t)), \tag{3}$$

$$M_{IHJ}(t) = \theta_{IHJ}(t)\frac{N_{IJ}(t)}{N_I(t)}G(N_I(t)). \tag{4}$$

Variable $\theta_{IHJ}(t)$ represents the route choices at time t; for its computation, an implementation of Dijkstra shortest path algorithm in combination with a MNL model is utilized for the computation of DUE. To find the optimal route guidance (i.e., DSO splitting rates), a linear optimization problem is solved (see Section 3 for the derivation).

The sequence of regions a user can traverse in the proposed model is not arbitrary. If the indices IHJ are parametrized with $I = J$, paths are restricted (e.g. $IHJ = 131$). This assumption does not allow for unrealistic path choices and improves the quality of the model. Note that the transfer flows need to be restricted by (5). The minimum among incoming transfer flow or maximum region capacity is considered, preventing a region from accepting incoming flows that exceed capacity (overflow). The latter is modeled with function $C_{IHJ}(N_H(t))$ (the reader is referred to Sirmatel and Geroliminis (2018) for the modeling of function $C(\cdot)$).

$$\tilde{M}_{IHJ}(t) = \min\big(C_{IHJ}(N_H(t)), \theta_{IHJ}(t)\frac{N_{IJ}(t)}{N_I(t)}G(N_I(t))\big). \tag{5}$$

Nevertheless, the constraint is omitted throughout this work.

Elements of set $\mathcal{R}$ are considered as homogeneous and can, therefore, be characterized by a well-defined MFD. Previous works are using mathematical relationships for modeling an MFD that is represented as an polynomial of degree n (e.g. in Geroliminis and Daganzo (2008) the approximation takes the form of $G(N_I(t)) = (aN_I^3(t) + bN_I^2(t) + cN_I(t))/\bar{L}$, where coefficients a, b, c are derived from measurement data and $\bar{L}$ denotes the average trip length). Furthermore, other approximations, such as an exponential function are used. However, the function parameters lack physical meaning and might introduce problems with the application of optimization procedures. Instead of assuming a functional relationship, another approach is to estimate the MFD from measurement data. Nevertheless, the quality of data or difficulties in data acquisition might lead to unreasonable approximations Ambühl *et al.* (2018). In the current work, for modeling function $G(\cdot)$ the novel procedure developed by Ambühl *et al.* (2018) is utilized; represented by an approximation of a trapezoidal diagram with the properties of smoothness, concavity, and continuity, defined by:

$$G(N_I) = -\lambda \ln \left(\exp\left(-\frac{a\frac{N_I}{L_{I,n}}}{\lambda}\right) + \exp\left(-\frac{q_{out}}{\lambda}\right) + \exp\left(-\frac{(\frac{N_{I,\text{jam}}}{L_{I,n}} - \frac{N_I}{L_{I,n}})b}{\lambda}\right)\right). \quad (6)$$

Function $G(\cdot)$ is the estimated outflow [veh/s] with respect to N_I; q_{out} denotes the maximum outflow (capacity) in [veh/s]; parameter $L_{I,n}$ denotes the network length of a region in $\mathcal{R}$. The utilized approach for deriving the MFDs proposes a function that is dependent on the density k. Consequently, our approach needs to convert the input N_I and jam accumulation $N_{I,jam}$, by applying $N_I/L_{I,n}$ for density k [veh/m] and $N_{I,\text{jam}}/L_{I,n}$ for jam density κ [veh/m]. a and b define the slopes of free-flow speed and congestion propagation, respectively; finally, λ serves as smoothing parameter. Note that variable names for free flow speed a and congestion propagation b are different than the work in Ambühl *et al.* (2018), because the parameters are utilized for an entire urban region and not a single intersection.

To model a realistic demand-supply system, the simulation plant receives consistent demand patterns as trapezoids. A trapezoid is defined as a symmetric shape by specifying the rising time t_r [s], falling time t_f [s] (where $t_r = t_f$), time that the demand remains constant t_c [s], and demand magnitude Q_t in [veh/sec]. Often these parameters are found by generating random numbers that satisfy the given application requirements. In current work, an optimization procedure from Kosmatopoulos and Kouvelas (2009) is utilized to find appropriate parameters t_r, t_f, t_c, and Q_t, producing a desired simulation scenario (e.g. two congested and two uncongested regions). By setting a target accumulation per region on the MFD curves, different scenarios for testing the optimal route guidance determination can be generated efficiently (Genser and Kouveals, 2019).

3 Equilibra derivation

At first, the derivation of DSO is introduced. The multi-region model is formulated with several nonlinearities (e.g. formulation of MFD function $G(\cdot)$, fraction of accumulations $N_{IJ}(t)/N_I(t)$, etc.). Hence, an NMPC is applied in several other studies focusing on optimal control (Sirmatel and Geroliminis, 2018; Tajalli and Hajbabaie, 2018; Hajiahmadi *et al.*, 2013). This work formulates the problem as a linear model to allow the application of an LRHO. The application of an LRHO implies the utilization of a linear model. Therefore, the nonlinearities are removed by applying several approximations based on Genser and

Kouvelas (2020).

First, the model parameters $\alpha_{II}(k)$ and $\alpha_{IJ}(k)$ are introduced, which are updated every time a predicted solution is applied to simulation plant; i.e., the parameters remain constant over the prediction horizon and are updated when rolling the prediction horizon. $\alpha_{II}(k)$ and $\alpha_{IJ}(k)$ are defined as follows:

$$\alpha_{II}(k) = \frac{N_{II}(k)}{N_I(k)}, \quad \alpha_{IJ}(k) = \frac{N_{IJ}(k)}{N_I(k)}. \tag{7}$$

Secondly, MFD functions $G_I(\cdot)$ are approximated with a number of piece-wise affine (PWA) functions; $l = \{1, 2, ..., L\}$ denotes the index of PWA function and L the total number of functions, chosen for an accurate approximation. In the following, each piece-wise linear MFD function is indicated by $G_I^l(\cdot)$.

Thirdly, Kouvelas *et al.* (2017) introduces new decision variables

$$f_{II}(k) = \theta_{III}(k) G_I^l(N_I(k)) \alpha_{II}(k), \tag{8}$$

and

$$f_{IH}(k) = G_I^l(N_I(k)) \sum_{J \in \mathcal{R}} \theta_{IHJ}(k) \alpha_{IJ}(k), \tag{9}$$

where $f_{II}(k)$ and $f_{IH}(k)$ define decision variables for internal and transfer flows, respectively. The right sides of equations (8) and (9) show the remaining nonlinearities by the product of decision variables $\theta_{III}(k)$ and $\theta_{IHJ}(k)$, respectively. The introduction of $f_{II}(k)$ and $f_{IH}(k)$ allow to complete the linearization of the problem. As in Kouvelas *et al.* (2017) after the methodology was applied to find the optimal perimeter control, a transformation from $f_{II}(k)$ and $f_{IH}(k)$ to the original control variables is used.

Nevertheless, variables $f_{II}(k)$ and $f_{IH}(k)$ only consider internal flows, and transfer flows to a neighboring region H; i.e., the information about final destination J is not available. In our approach to determine the optimal splitting rates $\theta_{III}(k)$ and $\theta_{IHJ}(k)$ this information is necessary to ensure that the summation of flow proportions on every possible path from I to J is correct, as well as for the transformation to the original decision variables. Therefore, we introduce one additional decision variable $f_{IHJ}(k)$ that is constrained by

$$\sum_{J \in \mathcal{R}} f_{IHJ}(k) = f_{IH}(k), \ \forall I, H \in \mathcal{R} \tag{10}$$

to ensure that splitting rates can be constrained correctly and calculation of the original decision signals $\theta_{III}(k)$ and $\theta_{IHJ}(k)$ can be obtained. Note that for $\theta_{III}(k)$ the result does not influence optimal route choices, as the splitting rate corresponds to users traveling from origin I, over I, to a final destination I. Hence, the splitting rate must be $\theta_{III}(k) = 1$. Nevertheless, the decision signals are included in the algorithm and the results need to be validated.

An LRHO procedure is introduced and utilized to solve the optimal route guidance problem:

$$\max_{N_I(k), f_{II}(k), f_{IH}(k)} T_c \cdot \sum_{k=k_p}^{k_p+N_p-1} \sum_{I \in \mathcal{R}} \Big[f_{II}(k) + f_{IH}(k) \Big] \tag{11}$$

$$\text{s.t. } N_I(k+1) = N_I(k) + T_c \Big(Q_I(k) - f_{II}(k) - \tag{12}$$

$$\sum_{H \in \mathcal{N}_I} f_{IH}(k) + \sum_{H \in \mathcal{N}_I} f_{HI}(k) \Big) \tag{13}$$

$$0 \leq f_{II}(k) \leq \alpha_{II} G_I^l(N_I(k)) \tag{14}$$

$$0 \leq f_{IHJ}(k) \tag{15}$$

$$\sum_{H \in \mathcal{N}_I} f_{IHJ}(k) \leq \alpha_{IJ}(k) G_I^l(N_I(k)) \tag{16}$$

$$0 \leq N_I(k) \leq N_{I,\text{jam}} \tag{17}$$

$$k = k_p, k_p + 1, ..., k_p + N_p - 1 \tag{18}$$

$$\forall I, J \in \mathcal{R}, H \in \mathcal{N}_I \tag{19}$$

Note that all constraints in equations (11)–(19) are linear, and consequently, the problem can be solved with low computational power as a linear program.

The derivation of DUE is based on finding the shortest paths with Djikstra algorithm and a MLN model. To model the inputs for these algorithms, the costs of a trip in the network have to be available. Therefore, we first calculate the travel time of a trip from a region I to a neighbor H by utilizing the signals from the macroscopic model. The travel time $\tau_{IH}(k)$ can be defined as follows:

$$\tau_{IH}(k) = \tau_I(k) + \tau_J(k) = \frac{\bar{L}_I \cdot N_I(k)}{G_I(N_I(k) \cdot \bar{L}_I} + \frac{\bar{L}_H \cdot N_H(k)}{G_H(N_H(k) \cdot \bar{L}_H}, \tag{20}$$

where $\tau_I(k)$ and $\tau_H(k)$ are approximated by the fraction of average trip lengths $\bar{L}_I$, $\bar{L}_h$ and the corresponding estimated speeds (by utilizing the outflow $G_I(N_I(k))$, average trip length $\bar{L}_I$, and vehicle accumulation of a region N_I). Note that all elements for $\tau_{IH}(k)$ of a given network with arbitrary topology can be compiled in a travel time matrix $\mathrm{T}(k)$.

To transform the elements of $\mathrm{T}(k)$ into generalized costs that users experience when traveling through the network, we utilize the Value of Time (VOT). Hence, the generalized cost matrix $C(k)$ can be defined by simply multiplying

$$C(k) = \mathrm{T}(k)V(k), \tag{21}$$

where $V(k)$ is a matrix of VOTs for all trips from I to H in the network. Generalized costs $C(k)$ are then utilized to calculate the shortest paths with Djikstra algorithm (represent minimum users costs) and derive the route choices with the MLN, which is defined as follows:

$$\theta_{IHJ}(k) = \frac{\exp(\mu U_{H,IJ})}{\sum_{H \in \mathcal{N}} \exp(\mu U_{H,IJ})}, \tag{22}$$

where $U_{H,IJ}$ defines the utility function for individuals going from I to J to an alternative, here a neighbor region H. Essentially, $U_{H,IJ}$ is modeled with a deterministic term, which is the corresponding element for a pair (I, H) from the generalized cost matrix $C(k)$, and an error term that is omitted in this work; finally, μ denotes a scaling parameter. This definition of the MNL is motivated by Ben-Akiva and Bierlaire (1999). The derived quantities of DUE and DSO are utilized for the determination of toll prices in Section 4.

4 Price determination with the concept of elasticities

This work utilizes the macroscopic multi-region model to derive optimal tolls for every region boundary in the network. Recent works that have been published, e.g., Gu *et al.* (2018), focus on a Proportional-Integral (PI) scheme utilizing MFD to allow for maintaining a protected region at the critical vehicle accumulation (corresponding to maximum vehicular flow) and calculating prices based on the link-based distance and time travelled. Here, on the contrary, we utilize the concept of elasticity to derive an optimal price matrix. If one assumes the elasticities and cost matrix of DSO as known, at every time step, the concept allows for the calculation of additional costs that a user should experience to lead the network to the optimal state.

First, we introduce the concept of elasticity E which can be defined by the fraction of partial derivative of demand Q and price P (at any point) and the product of price-demand ratio:

$$E = \frac{\partial Q}{\partial P}\frac{P}{Q}. \tag{23}$$

Olszewski and Xie (2005) have further introduced a simplified calculation of elasticities by utilizing the arithmetic average of demand and price quantities before and after a price change. Hence, the elasticity can be derived by

$$E = \frac{(Q_2 - Q_1)(P_2 + P_1)}{(P_2 - P_1)(Q_2 + Q_1)}, \tag{24}$$

where variables with index 1 and 2 represent demands and prices before and after the price change, respectively. We modify this concept for our problem by modeling the demand with transfer flows $M_{IH}(k)$, i.e., the flows that want to leave from region I towards their destination J and have to decide on which region to traverse based on the current costs and travel times. Hence, all transfer flows that leave a region I and traverse via region H are defined as:

$$M_{IH}(k) = \sum_{J\in\mathcal{R}} M_{IHJ}(k). \tag{25}$$

The price that a user has to pay for the travel is represented by the generalized cost matrices $C_I H(k)$. Therefore we utilize (24) and modify it to:

$$E_{IH}(k) = \frac{(M_{IH,\mathrm{SO}}(k) - M_{IH,\mathrm{UE}}(k))(C_{IH,\mathrm{SO}} + C_{IH,\mathrm{UE}})}{(C_{IH,\mathrm{SO}} - C_{IH,\mathrm{UE}})(M_{IH,\mathrm{SO}}(k) - M_{IH,\mathrm{UE}}(k))}. \tag{26}$$

Note that the indices SO and UE represent the corresponding quantities computed by DSO and DUE, respectively. If elasticity $E_{IH}(k)$ is assumed as known and DSO has already been computed by the proposed methodology from Section 5, we can rearrange (26) to derive the new cost matrix $C_{IH,\mathrm{P}}(k)$, which denotes the generalized costs that users need to experience so that system optimum is achieved.

$$C_{IH,\mathrm{P}}(k) = \frac{C_{IH,\mathrm{SO}}(k)\Big(M_{IH,\mathrm{UE}}(k)\Big(E_{IH}(k)+1\Big) + M_{IH,\mathrm{SO}}(k)\Big(E_{IH}(k)-1\Big)\Big)}{M_{IH,\mathrm{UE}}(k)\Big(E_{IH}(k)-1\Big) + M_{IH,\mathrm{SO}}(k)\Big(E_{IH}(k)+1\Big)}. \tag{27}$$

Finally, we can derive the tolls matrix $P(k)$ which contains all toll prices $p_{IH}(k)$ by subtracting the generalized cost matrix $C_{IH,\mathrm{P}}(k)$ from $C_{IH,\mathrm{UE}}(k)$

$$P(k) = C_{IH,\mathrm{P}}(k) - C_{IH,\mathrm{UE}}(k). \tag{28}$$

Note that $P(k)$ is not bounded by any means, and in that sense can give not only extremely high but also negative prices, which means that there should be an economic incentive for people to travel through this toll (so that the systems operates in DSO). Nevertheless, in practice, the derived prices are bounded by operational constraints, e,g, by following condition:

$$p_{\min} \leq p_{IH}(k) \leq p_{\max}, \; \forall p_{IH}(k) \in P(k), \tag{29}$$

where $p_{\min}$ denotes the minimum permissible price, which should be set to 0 for practical reasons and $p_{\max}$ denotes the maximum permissible price, dependent on the, e.g., each country's currency or gross domestic product. Note that the smaller the space for all elements $p_{IH}(k)$ is chosen, the bigger the deviation from the DSO will be.

5 Case study

Content of the result section is not shown here for journal publication reasons.

6 Conclusion

The paper presents the derivation of optimal price functions for a multi-region network with homogeneous regions, characterized by well-defined MFD functions. First, the optimal routing information (splitting rates) is derived with an LRHO optimization problem, providing network system optimum, which can be utilized as ideal target for the determination of dynamic pricing functions. To relax the nonlinear optimization problem, a recent linearization methodology was implemented that allows the application of LRHO. The proposed method from the literature was extended and utilized for obtaining optimal splitting rates in the multi-region-network. Accumulation trajectories are utilized to show the system improvement of the methodology with TTS as a performance indicator. The results are compared to DUE scenario, which is derived by utilization of Dijkstra route choice algorithm and a MLN. The proposed linear program reduces TTS significantly and guarantees an optimal and fast solution as opposed to nonlinear formulations.

To determine the optimal pricing functions, the concept of elasticities is applied. With utilization of the generalized cost matrices of DUE and DSO (derived with the VOT), a price matrix is computed offline that shows the dynamic pricing functions for the simulation scenario. Furthermore, the average prices and activation times of all tolls are calculated to give an impression on which tolls are from great importance to reach the system optimum for a given demand scenario.

Future research should focus on a sensitivity analysis of the proposed traffic management settings. The performance evaluation can be further extended by comparing the TTS-improvement with a nonlinear system and also how sensitive the controller is to parameters such as the control time step and prediction horizon. Based on recent research, the simulation plant should be extended with a trip length model (for now only average trip lengths are considered) that allows extensive analysis of users' travel times in the system. Furthermore, a weighting of the different regions can be applied in the optimization procedure to account for different region parameters (i.e., size, storage capacity, etc.). This improves the quality of the modeling further and also contributes to a more detailed evaluation of the proposed methodology. Besides, the online computation of the pricing methodology by formulating an optimization problem and incorporating the elasticity concept into the simulation plant is considered.

7 References

Ambühl, L., A. Loder, M. C. Bliemer, M. Menendezc and K. W. Axhausen (2018) A functional form with a physical meaning for the macroscopic fundamental diagram, *Transportation Research Part B: Methodological.*

Ben-Akiva, M. and M. Bierlaire (1999) *Discrete Choice Methods and their Applications to Short Term Travel Decisions*, 5–33, Springer US, Boston, MA.

Chung, B. D., T. Yao, T. L. Friesz and H. Liu (2012) Dynamic congestion pricing with demand uncertainty: A robust optimization approach, *Transportation Research Part B: Methodological*, **46** (10) 1504 – 1518, ISSN 0191-2615.

Eliasson, J. (2017) Determination and evaluation of traffic congestion costs, *Handbook of Transport Economics, forthcoming, Routledge.*

Genser, A. and A. Kouveals (2019) Dynamic congestion pricing for multi-region net-works:

A traffic equilibria approach, *19th Swiss Transport Research Conference (STRC 2019)*.

Genser, A. and A. Kouvelas (2020) Optimum route guidance in multi-region networks. a linear approach, *99th Annual Meeting of the Transportation Research Board, Washington, D.C., USA, January 12-16*.

Geroliminis, N. and C. Daganzo (2008) Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings, *Transportation Research Part B: Methodological*, **42** (9) 759–770.

Gu, Z., S. Shafiei, Z. Liu and M. Saberi (2018) Optimal distance- and time-dependent area-based pricing with the network fundamental diagram, *Transportation Research Part C: Emerging Technologies*, **95**, 1 – 28, ISSN 0968-090X.

Hajiahmadi, M., V. L. Knoop, B. De Schutter and H. Hellendoorn (2013) Optimal dynamic route guidance: A model predictive approach using the macroscopic fundamental diagram, *16th International IEEE Conference on Intelligent Transportation Systems (ITSC 2013)*, 1022–1028.

Kosmatopoulos, E. B. and A. Kouvelas (2009) Large scale nonlinear control system fine-tuning through learning, *IEEE Transactions on Neural Networks*, **20** (6) 1009–1023, June 2009, ISSN 1045-9227.

Kouvelas, A., M. Saeedmanesh and N. Geroliminis (2017) A linear formulation for model predictive perimeter traffic control in cities, *IFAC PapersOnLine*, **50-1**, 8543–8548.

Liu, Z., Q. Meng and S. Wang (2013) Speed-based toll design for cordon-based congestion pricing scheme, *Transportation Research Part C: Emerging Technologies*, **31**, 83 – 98, ISSN 0968-090X.

Liu, Z., S. Wang and Q. Meng (2014) Optimal joint distance and time toll for cordon-based congestion pricing, *Transportation Research Part B: Methodological*, **69**, 81 – 97, ISSN 0191-2615.

Meng, Q., Z. Liu and S. Wang (2012) Optimal distance tolls under congestion pricing and continuously distributed value of time, *Transportation Research Part E: Logistics and Transportation Review*, **48** (5) 937 – 957, ISSN 1366-5545. Selected papers from the 14th ATRS and the 12th WCTR Conferences, 2010.

Olszewski, P. and L. Xie (2005) Modelling the effects of road pricing on traffic in Singapore, *Transportation Research Part A: Policy and Practice*, **39** (7-9) 755–772.

Sarlas, G., V. Papathanasopoulou and C. Antoniou (2013) Simulation-based analysis of road-pricing prospects for athens, greece, *Journal of Urban Planning and Development*, **139** (3) 206–215.

Simoni, M., A. Pel, R. Waraich and S. Hoogendoorn (2015) Marginal cost congestion pricing based on the network fundamental diagram, *Transportation Research Part C: Emerging Technologies*, **56**, 221 – 238, ISSN 0968-090X.

Sirmatel, I. I. and N. Geroliminis (2018) Economic model predictive control of large-scale urban road networks via perimeter control and regional route guidance, *IEEE Transactions on Intelligent Transportation Systems*, **19** (4) 1112–11210.

Tajalli, M. and A. Hajbabaie (2018) Dynamic speed harmonization in connected urban street networks, *Computer-Aided Civil and Infrastructure Engineering*, **33**, 510–523.