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Basic Research, Complexity, and Wage Inequality

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Summary

This thesis consists of two parts: Part I presents an analysis of basic research in a multi-country, multi-industry general equilibrium framework. Part II consists in a formalization of a new task-based framework, which we will call the "task-complexity model", in which we study wage inequality. The two parts can be read independently.

Part I After the introductory chapter, we present in Chapter 2 a multi-country, multi-industry model that formalizes key characteristics of basic research and the innovation process. Governments invest in basic research and target their investments to industries. A government's decision depends on domestic production, which is determined by the general equilibrium forces of trade.

We find (i) that governments' targeting of their basic research investments can be inefficiently concentrated on low-complexity industries, (ii) that basic research investments are inefficiently allocated across countries, and (iii) that there is typically too little aggregate basic research investment. In Chapter 3, we simplify the model and provide three extensions. We conclude in Chapter 4.

Part II After the introductory chapter, we present the task-complexity model in Chapter 6. We propose a new skill-task-assignment and analyze its implications. In Chapter 7, we study uneven technological change in the task-complexity model. We introduce an additional industry—manufacturing—that produces capital. Depending on the production mode, this capital can either be produced by low-skilled labor, in which case we call capital "machines", or only by high-skilled labor, in which case we call capital "robots". Machines and robots both can be used in production as a substitute for low-skilled labor. The two production modes reflect a key difference between the industrial economy of the beginning of the 20^{st} century and today's economy.

Given the mechanism of the model, increased efficiency of the production of *robots* leads to a strong upward pressure on the wage premium earned by the high-skilled and, thus, to rising wage inequality, whereas an efficiency increase in the production of *machines* can lead to both upward or downward pressure on the wage premium and to raising or falling wage inequality.

In Chapter 8, we present two generalizations of the models of the previous chapters. First, we present the task-complexity model with an additional task. Second, we assume that households exhibit non-homothetic preferences. We find that this assumption accentuates the trends towards more wage inequality shown in Chapter 7. We conclude in Chapter 9.

Zusammenfassung

Diese Dissertation besteht aus zwei Teilen: Teil I präsentiert eine Analyse von Grundlagenforschung in einem Gleichgewichtsmodel mit mehreren Ländern und Industrien. Teil II entwickelt ein Gleichgewichtsmodel anhand dessen Lohnungleichheit untersucht wird. Die zwei Teile sind unabhänging voneinander.

Teil I Nach einer Einführung präsentieren wir in Kapitel 2 das Gleichgewichtsmodel, welches die wichtigsten Eigenschaften von Grundlagenforschung formalisiert. Regierungen investieren in die Grundlagenforschung und entscheiden, in welche Industrien sie im Speziellen investieren wollen. Dabei berücksichtigen sie die jeweilige inländische Produktion, welche durch den globalen Handel determiniert wird. Wir erhalten folgende Resultate: (i) Die Investitionen in die Grundlagenforschung der Regierungen sind tendenziell ineffizient auf Industrien mit einem tiefen Komplexitätsgrad in der Produktion konzentriert. (ii) Die Investitionen sind ineffizient über die Länder verteilt. (iii) In der Summe sind die Investitionen zu tief, wenn realistische Annahmen bezüglich der Parameter getroffen werden. In Kapitel 3 präsentieren wir ein vereinfachtes Model, anhand dessen wir weitere Fragestellungen untersuchen. Wir konkludieren in Kapitel 4.

Teil II Wiederum bildet eine Einführung den Beginn dieses zweiten Teils. Danach präsentieren wir das Model in Kapitel 6, dessen Grundannahme eine neuartige Allokation der Fähigkeiten einer Arbeitskraft zu Aufgaben im Produktionsprozess bildet. Die Implikationen dieser Grundannahme werden im Folgenden untersucht. In Kapitel 7 erweitern wir das Model um eine zusätzliche Industrie, welche für die Produktion von Kapital zuständig ist. Wir analysieren diesbezüglich zwei Produktionsmodi. Der eine Produktionsmodus sieht vor, dass für die Produktion von Kapital niedrig-qualifizierte Arbeitskräfte eingesetzt werden können. Das dabei produzierte Kapital nennen wir "Maschinen". Im anderen Produktionsmodus können ausschließlich hoch-qualifizierte Arbeitskräfte für die Produktion von Kapital eingesetzt werden. Dieses Kapital nennen wir "Roboter". Maschinen wie auch Roboter können als Substitut für niedrig-qualifizierte Arbeitskräfte im Produktionsprozess der restlichen Industrien verwendet werden. Die beiden Produktionsmodi von Kapital stehen dabei jeweils repräsentativ für eine industrielle Marktwirtschaft zu Beginn

des 20. Jahrhunderts und für eine heutige Marktwirtschaft. Anhand des Models untersuchen wir die Implikationen von Effizienzgewinne durch technologischen Fortschritt in der Produktion von Kapital. Eine effizientere Herstellung von *Maschinen* kann die Lohnungleichheit zwischen niedrig- und hoch-qualifizierten Arbeitskräfte schwächen oder verstärken. Die Richtung dieses Effekts hängt von der Parameterwahl ab. Wenn die Herstellung von *Roboter* effizienter wird, führt dies eindeutig zu höherer Lohnungleichheit zwischen den unterschiedlich qualifizierten Arbeitskräften.

In Kapitel 8 präsentieren wir zwei Generalisierungen des Models. Erstens zeigen wir eine Version mit zusätzlichen Produktionsprozessen. Zweitens nehmen wir an, dass die Präferenzen der Haushalte Nichthomothetizität aufweisen. Die Annahme verstärkt die Effekte in Richtung Lohnungleichheit, welche in Kapitel 7 gezeigt werden, eindeutig. Wir konkludieren in Kapitel 9.

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Part I.

Basic Research

1. Introduction

> *"During the past 500 years modern science has achieved wonders thanks largely to the willingness of governments, businesses, foundations and private donors to channel billions of dollars into scientific research."*
> (Harari, 2014, p. 301)

Basic research fosters the innovation capability of countries and is considered to be an important source of technological progress (Salter and Martin, 2001). Basic research is hierarchically related to applied research, where it acts as an upstream supplier of so-called "ideas" (Gersbach et al., 2010). Ideas are unfinished concepts, new materials, methods, or discoveries that must be converted into products that can be sold. Commercialization is often the result of applied research performed by private agents.[1] This observation led economists to model applied research as the driver of growth in endogenous growth models. In these models, basic research is often omitted. Considering the importance of basic research for the innovation process and that basic research investment is a policy tool that is at the center of budget debates in industrialized countries, it is surprising how little we know about the mechanisms at work with regard to basic research (Salter and Martin, 2001; David and Metcalfe, 2007).

The OECD defines basic research as *"experimental or theoretical work undertaken primarily to acquire new knowledge of the underlying foundations of phenomena and observable facts, without any particular application or use in view"* (OECD, 2002, p. 30). Gersbach et al. (2015) list six attributes of basic research. It is:

(i) *Embryonic:* Basic research outcomes are early findings without any particular application or use is in view, and without immediate commercial use;

(ii) *Cumulative:* Research builds on previous findings, so that every researcher is "standing on the shoulder of giants";

(iii) *Delayed:* From the first funding until a finding occurs—and until ultimately a commercial product that builds on this finding is available—may involve great delays;

(iv) *Uncertain:* Basic research investments are connected to high uncertainty;

[1] The U.S. Bayh-Dole Act from 1980 allowed universities to acquire patent rights over innovations from federally funded research. We will analyze this special case in Section 2.8.1.

(v) *Hierarchical:* As noted above, basic research is the upstream supplier of ideas for applied research;

(vi) *Comprising two-way spillover:* Although the relation between basic research and applied research is hierarchical, applied research feeds back into basic research, i.e. applied research outcomes may also stimulate new findings in basic research.

We will describe the characteristics of basic research in more detail in Chapter 2. The multi-dimensionality of basic research makes it difficult to understand and to measure. Salter and Martin (2001) state

> *"Currently, we do not have the robust and reliable methodological tools needed to state with any certainty what the benefits of additional public support for science might be, other than suggesting that some support is necessary to ensure that there is a 'critical mass' of research activities"* (p. 529).

This shows that an empirical analysis of basic research entails conceptual problems and measurement difficulties. The economic effects of basic research investments are difficult to measure, as basic research affects the economy in various direct and indirect ways, in the short-term and in the long-term (Salter and Martin, 2001; Gersbach et al., 2015). Basic research creates new knowledge, new instrumentation and methodologies, and ultimately, ideas for new products. But it also trains researchers and students, it generates valuable networks in the global economy, and contributes to introducing new issues to society.[2] The characteristics of basic research make it hard to analyze. This might be why it is seldom addressed by economists although they acknowledge its role in technological advancement and in promoting growth. Hence, we believe that a theoretical approach is needed which takes into account key features of basic research. We will thus develop a framework to study basic research in a global economy that could serve as a basis for structural empirical tests. Most of the models on basic research are built from a closed economy perspective.[3] Thus, it will be useful to take a global perspective to analyze basic research in a general equilibrium trade model.

After our introduction of the key research issues in Chapter 1, we present our general equilibrium approach, which integrates basic research investments of governments into a multi-country, multi-industry, trade setting in Chapter 2. The model allows us to analyze

[2] Salter and Martin (2001) and Gersbach et al. (2015) give an overview of the benefits of basic research.

[3] Exceptions are Park (1998), Gersbach et al. (2014), Gersbach et al. (2013). Park (1998) studies basic research investments and their spillovers in a two-country model. In a small open economy Gersbach et al. (2014) study the interplay between governmental decisions of basic research investment and private sector's decisions of applied research investment. And Gersbach et al. (2013) study a small open economy where the leading technology is either imported by foreign firms or generated by own basic research investments. Depending on the costs and benefits of the two channels, the government decides how much to invest in basic research.

potential inefficiencies of basic research investments of countries in relative and absolute terms. We draw policy implications through social welfare analyses and comparative statics. In Chapter 3, we simplify the model of Chapter 2 for the use in additional research questions. Chapter 4 concludes.

2. A Unified Theory of Public Basic Research*

"It's pretty safe to say, in fact, that hardware, software, networks, and robots would not exist in anything like the volume, variety, and forms we know today without sustained government funding."
(Brynjolfsson and McAfee, 2014, p. 218)

2.1. Introduction

Public investment in basic research is an important policy instrument of governments to foster innovation and economic growth. While the literature analyzing basic research policies mostly assumes a closed economy perspective these investments cannot be fully understood in isolation, as it will be detailed below. In this chapter, we seek to fill the gap by jointly analyzing basic research investments of many countries that engage in international trade.

Basic research provides knowledge, networks, and understanding needed for innovation. It has, however, little commercial value in itself. The main motive for national investments in basic research is thus to support applied research and development in the domestic economy. The costs and benefits associated with these investments critically depend on a country's integration in the world economy. For instance, innovative domestic firms may benefit from supplying their products to the world market. Moreover, innovation requires industry-specific know-how that is built up via production. A country's current specialization in international trade will thus feed back into its potential to innovate in those industries with domestic production. The benefits from basic research investments depend on how much of the knowledge created a national government can expect to be used in domestic production. These effects are of first order importance for basic research policies. We are the first to analyze them in a multi-country general equilibrium setting. This provides a coherent picture of public investment in basic research in a global economy. It also allows to analyze important policy questions that could not be

* This chapter is based on joint research with Hans Gersbach and Ulrich Schetter.

addressed without such a comprehensive framework. In particular, we show that from a global perspective decentralized investments in basic research are inefficient along three dimensions: (i) Basic research is potentially not sufficiently directed to support innovation in complex high-tech industries. (ii) Basic research is too heavily concentrated in industrialized countries. (iii) And there is typically too little global investment in basic research. Our framework also provides a new, global perspective on the Bayh Dole Act: We show that while such policies are never welfare optimal, they may mitigate global underinvestments in basic research.

Model and Key Results

Our model embeds a two-stage innovation process with public basic research and private applied research in a variant of the multi-country, multi-industry general equilibrium model of international trade developed in Schetter (2014, 2018). Basic research generates ideas that are industry-specific and can be taken up by the private sector to develop new varieties of the products in an industry. There is a global patent for each variety, and the owner of that patent is free to choose his location or locations for production, and supply the world market from there. Transportation costs are zero, as are tariffs on imports or exports. Countries differ in their productive knowledge while industries differ in their complexity. Firms can freely choose the quality of their variety which—for a given complexity level of their industry—gives them endogenous control over the production requirements. Quality differentiation is, however, subject to functional minimum requirements that are the more demanding to be satisfied the more complex a product.

To analyze the model we proceed in two steps. We first study the equilibrium for a given set of varieties in each industry. In the second step, we examine basic research investments and applied research by the private sector which develops the varieties. In the ensuing equilibrium in the first step, countries with higher productive knowledge are more diversified. In particular, in countries with the highest productive knowledge, production takes place over the whole range of industries, from complex to simple ones, while countries with low productive knowledge are unable to attract firms in particularly complex industries such as aircraft or high-tech engineering or pharmaceutical, in line with what we observe from the data (Hausmann and Hidalgo, 2011; Bustos et al., 2012; Schetter, 2018). We show in the second step that this pattern of international specialization has profound consequences for countries' investments in basic research.

To study these investments, we start from key characteristics of basic research and stylized facts as documented in Section 2.2. We assume that public basic research impacts the economy indirectly via the generation of ideas that private firms can take up in applied research to develop new varieties in an industry. Ideas diffuse locally and then globally, reflecting the importance of both local effects of basic research and global spillovers through the dissemination of ideas.[1] To commercialize an idea, industry-specific, tacit know-how is needed. It is acquired via domestic production, i.e. a country's manufacturing base (broadly defined) is a pivotal element of its innovation system (Nelson, 1959; Arrow, 1962; Pisano and Shih, 2012; McKinsey Global Institute, 2012; Akcigit et al., 2016). We document stylized facts to corroborate this conjecture in Section 2.2. We allow governments to target their basic research investments to support innovation in specific industries, e.g. by prioritizing certain scientific fields. Such targeting, however, is necessarily imperfect.

National governments decide how many scientists they need to employ in basic research to maximize the well-being of their citizens, which boils down to weighing the costs associated with these investments against the domestic social value of patents for new varieties. We establish a decentralized equilibrium involving government decisions in basic research, applied research activities yielding a distribution of developed varieties across countries, and production patterns and wage patterns across countries.

In equilibrium governments of countries with high productive knowledge face both higher costs and benefits: Scientists earn higher wages in these countries, as they are more productive if they were employed as production workers. In addition the domestic economy is more diversified which allows to commercialize ideas in a large set of industries which gives an improved targeting potential for basic research investments. We show that the latter dominates when basic research is at least as skill intense as production, implying that countries with higher productive knowledge will employ more scientists in basic research.[2] In addition, thanks to their broad manufacturing base, these countries benefit more from knowledge spillovers from the rest of the world, and thus are highly innovative. Their high level of innovation allows these countries to capture a disproportionate share of global profits. These equilibrium results are consistent with salient features in the data. To the best of our knowledge, we are the first to rationalize these basic observations in general equilibrium.

We then compare investments by national governments to the optimal solution of a global social planner, to find that coordinated basic research policies would yield welfare improvements along three dimensions. First, we document that the social planner would distribute investments in basic research more equally across countries. The basic intuition is that developing countries invest little in basic research because their domestic econ-

[1] A large literature documents various forms of international knowledge spillovers and spatial dependence in the diffusion of knowledge (Jaffe et al., 1993; Keller, 2002, 2004; Keller and Yeaple, 2013; Bahar et al., 2014).

[2] This assumption is in line with the cumulative nature of basic research (Scotchmer, 1991, 2004; Nelson, 2004). More generally, it is a weak version of the idea that a stock of knowledge and technical expertise is needed to be able to effectively perform basic research.

omy is not effective in science-driven innovation, implying that they suffer more from knowledge spillovers to the rest of the world compared to industrialized countries. Such spillovers, however, do not matter from a global perspective. As a consequence, a global social planner will be able to stimulate more innovation with the level of aggregate basic research investments achieved in the decentralized solution by correcting the allocation of these investments across countries.

Second, we show that in spite of the inefficiently high concentration of basic research investments in countries with a high productive knowledge, global investments may not be targeted sufficiently towards high-tech industries. This counterintuitive result is rooted in the importance of tacit know-how for innovation and the "nestedness" of countries' exports, i.e. in the fact that countries with a high productive knowledge successfully export varieties in both simple and complex industries while developing countries specialize in the simpler ones.

Third, we show that aggregate investments are typically too low in equilibrium. Hence, the decentralized solution in which each country decides on basic research investments produces too little knowledge for the world. As a consequence, a social planner will increase aggregate investments in basic research and—as a consequence of the first and second inefficiencies—correct the distribution of basic research investments across countries and correct the distribution across industries towards more complex ones. The latter two corrections imply that the social planner is able to stimulate more innovations than in the decentralized equilibrium with a given amount of basic research investments.

Our set-up also has interesting implications for the Bayh Dole Act[3] that incentivizes university researchers to get more engaged in the commercialization of their work. Such incentives arguably come at the cost of lowering their productivity in terms of pure science. Yet, they may be welfare-improving as they allow countries to capture a larger share of the gains from their own basic research. In turn, this induces countries to invest more in basic research and thereby contributes to closing the gap to globally efficient levels of investment in basic research.

Relation to the Literature

Our model is related to a large literature that provides a thorough understanding of basic research and its effects on the overall economy. Let us briefly summarize this literature to show how it guides our modeling choices for the innovation process in the next section, before explaining our contribution to the literature.

Our model can be seen as an extension of an expanding variety model following Romer (1987, 1990) to a multi-country, multi-industry setting with basic and applied research,

[3] See https://www.energy.gov/gc/bayh-dole-act-usc retrieved on the 12^{th} of March, 2018.

international trade, and knowledge diffusion. Accordingly, our work is related to the following strands of literature.

It is closest related to the literature that analyzes basic research investments with theoretical models. This literature mostly considers closed economies (Mansfield, 1995; Aghion and Howitt, 1996; Morales, 2004; Cozzi and Galli, 2009, 2014; Akcigit et al., 2013; Gersbach et al., 2018). Notable exceptions are Gersbach et al. (2013) who consider basic research investments of a small open economy, and Gersbach and Schneider (2015) who consider strategic basic research investments in a two-country model with access to foreign markets. Gersbach and Schneider (2015) also compare equilibrium investments to those of a social planner. Our set-up is very different and substantially richer insofar that we consider the general equilibrium with trade among many countries and an endogenous choice of location for production by private firms. Moreover, we allow that ideas produced by basic research efforts in one country diffuse locally and then globally and thus can be taken up by applied researchers in other countries which is absent in Gersbach and Schneider (2015). Hence, our model provides a framework for a comprehensive account of the effects of basic research investments by countries. The multi-country set-up produces predictions about the distribution of basic research investments across countries and they can be related to cross-country data on these investments. With this connection, our work complements recent papers that assesses various innovation policies in closed-economy models using micro data (Akcigit et al., 2013; Garicano et al., 2016; Atkeson and Burstein, 2018).[4]

We also contribute to the literature analyzing innovation in the global economy that goes back at least to Grossman and Helpman (1991).[5] Recent contributions involve Atkeson and Burstein (2010), who consider a two-country Melitz-type model with product and process innovation. They find approximately the same effects of a change in trade costs on aggregate productivity as in models with product innovation. Arkolakis et al. (2018) develop a variant of a Melitz model where firms can disentangle the location of market entry (innovation) from the location(s) of production. They use a calibrated version of their model to study the implications of a decline in the cost of multinational production. Our model shares the feature that highly innovative countries benefit from extracting a disproportionate share of global profits. In our model, however, this potential depends on governments' investments in basic research, which is the main focus of our work.

The diffusion of ideas from basic research and the ability to commercialize these ideas

[4] We analyze efficient levels of basic research. In that sense, our work is also related to a somewhat older empirical literature that measures the gains from (public) basic research (Mansfield, 1980; Griliches, 1986; Toole, 2012). Hall et al. (2010) provide a survey of the literature on measuring the returns to R&D in general.

[5] At a more general level, our work relates to the literature analyzing the growth effects of international trade, e.g. Acemoglu (2003), Galor and Mountford (2008), and Nunn and Trefler (2010).

domestically are at the heart of the underlying government decision in our model. Our work is thus also, but less closely, related to recent work on the diffusion of ideas (Lucas and Moll, 2014; Buera and Oberfield, 2016). Compared to these papers, we use a considerably simpler idea diffusion model but we focus on the distribution of national basic research policies that generate the ideas.

Organisation of the Chapter

The remainder of this chapter is organized as follows. In Section 2.2 we summarize key characteristics of basic research and present stylized facts that will guide our modeling choices. In Section 2.3, we introduce our model, first the macroeconomic environment and then the innovation process. Sections 2.4 and 2.5 present the equilibria for exogenously given and for decentralized decisions on investments in basic research, respectively. Section 2.6 analyzes the social planner's optimum. Section 2.7 compares the competitive equilibrium and the social planner's solution. Section 2.8 provides extensions and further discussions on complementary policy tools. Section 2.9 concludes.

2.2. Motivating Facts on Basic Research

In this section, we will summarize key characteristics of basic research and present stylized facts that will guide our modeling choices in the next section.
The definition of basic research by the OECD shown in Chapter 1 immediately points to important characteristics of basic research. First, the new knowledge that is the key outcome of basic research resembles a global public good. This observation and the associated lack of appropriability of the gains from basic research by private firms were at the center of the early literature identifying a need for public funding of basic research (Nelson, 1959; Arrow, 1962). Indeed, the major part of basic research is publicly funded and provided (Akcigit et al., 2013; Gersbach et al., 2015). While there are some joint efforts, e.g. at the EU level, the vast majority of basic research funding is provided by national (or even subnational) governments.[6] This may seem surprising given that new knowledge from basic research features key characteristics of a *global* public good. However, a series of influential papers (Jaffe et al., 1993; Anselin et al., 1997; Audretsch and Lehmann, 2004) documents that basic research also has significant local effects on innovation. In particular, basic research provides domestic firms with problem solvers, trained scientists, access to scientific networks and, in general, better access to new knowledge. This fosters

[6] The Horizon 2020 program, for example, the largest EU funding program for research and innovation so far, amounts to EUR 77bn over the period 2014-2020. This compares to total EU-28 expenditures for R&D in the government and higher education sectors of over EUR 100bn in 2015 alone.

the innovativeness and growth of local firms and their competitiveness on the world market.[7] Indeed, Figure 2.1 shows that on balance countries that had a high basic research intensity in the past patent more, and they earn a disproportionate share of global profits as measured by the ratio $\frac{GNI-GDP}{GDP}$.[8] These local effects are a key motive for national governments to invest in basic research.

Second, the definition of basic research also implies that basic research is embryonic in the sense that it has little or no commercial value in itself. New knowledge and ideas from basic research need to be commercialized through private applied research, which, in turn, results in new or improved products or production processes.[9] The use of ideas from basic research, however, requires industry-specific tacit know-how (Nelson, 1959; Arrow, 1962; Akcigit et al., 2013, 2016). Such know-how is mostly acquired through production and there is a rationale for a close proximity of innovation and production activities (Pisano and Shih, 2012; McKinsey Global Institute, 2012). A country's current specialization in production will therefore be an important determinant of the domestic economy's capability to make use of ideas from basic research. This is also reflected in the countries' patenting: As Figure 2.2(a) and (b) show, countries have a higher propensity to actively patent in industries with domestic production. In addition, on balance, countries tend to patent more in industries where they export more. This is true both when considering log exports and patents, normalized by countries' population and industries' size, (Figure 2.2(c)) and when considering log RCA in exporting and patenting (Figure 2.2(d)).

Third, with this relationship between domestic production and innovation in mind, governments may seek to target their basic research investments in order to best support innovation in the domestic economy. Indeed, the idea to optimally target basic research investments to industries or fields of science features prominently in policy debates (European Commission, 2012; Research Prioritisation Project Steering Group, Ireland, 2012). While the generation of new knowledge is highly uncertain by definition, there is some room for prioritizing basic research investments (Cohen et al., 2002). In our theoretical set-up, we will allow governments to target ideas from basic research to certain industries, but this targeting will be imperfect.

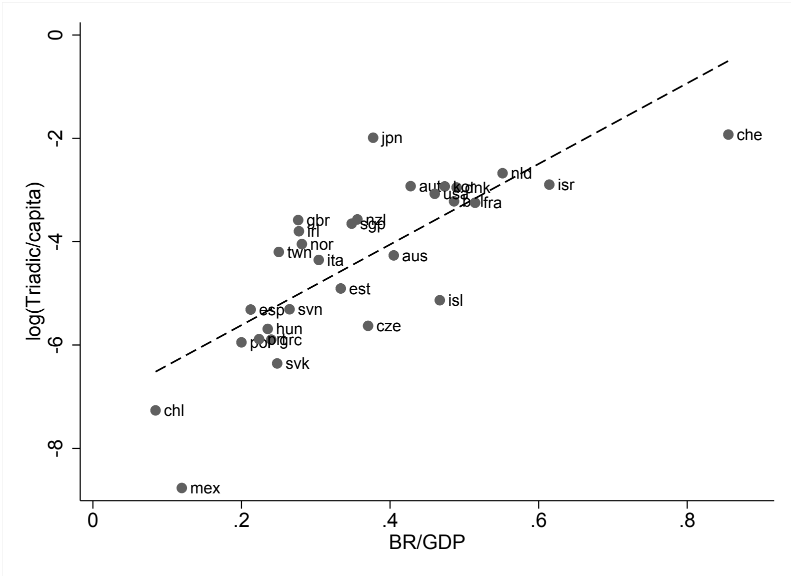
[7] Since the early studies by Mansfield (1980) and Link (1981), a series of empirical studies has shown that basic research has a significant positive effect on productivity and growth in manufacturing industries (Griliches, 1986; Adams, 1990; Guellec and Van Pottelsberghe de la Potterie, 2004; Luintel and Khan, 2011; Czarnitzki and Thorwarth, 2012; McKinsey Global Institute, 2012). Local effects from basic research are also consistent with the spatial dependence in the diffusion of knowledge, cf. Footnote 1.

[8] We use average past investments because basic research impacts the economy with time lags and because entitlement to foreign profits is built up gradually through past innovation. Ireland has been excluded from Figure 2.1 (b) as it is an outlier due to its tax policy. Note that the regression line would be more steeply upward sloping if it included Ireland.

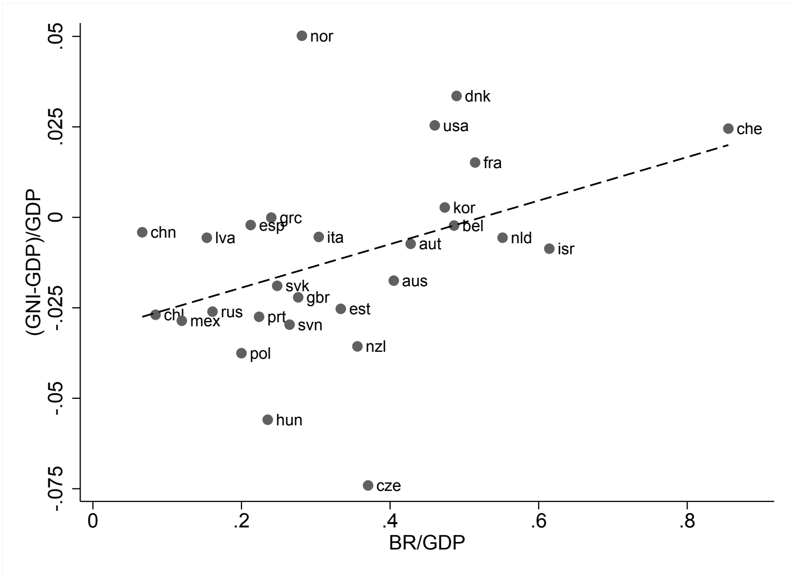
[9] A hierarchy of R&D activities is also the predominant view in the literature on basic research (Aghion and Howitt, 1996; Akcigit et al., 2013; Cozzi and Galli, 2014; Gersbach and Schneider, 2015).

Figure 2.1.: Basic Research and Innovation

(a) Patenting



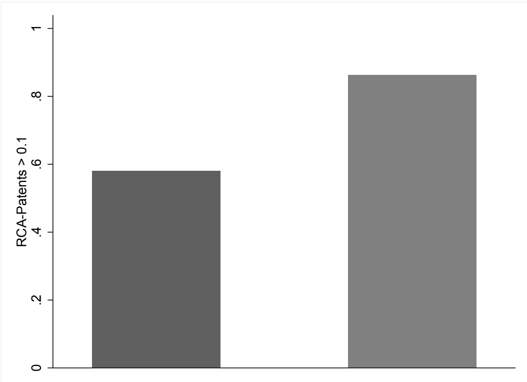
(b) Gains



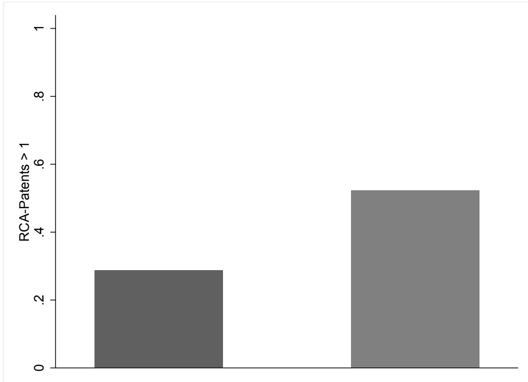
Notes: Data on basic research investments relative to GDP are taken from the OECD dataset "Main Science and Technology Indicators" (downloaded in December 2017) and refer to the 20-year average from 1995 to 2015. The variables on the ordinate are for the year 2015.
Figure (a): Own illustration. Data are taken from the OECD dataset "Main Science and Technology Indicators" (downloaded in December 2017). Triadic patents count the number of priority filings of triadic patent families by a country's inventors. We relate this number to the population size and take logs.
Figure (b): Own illustration. Data are taken from the OECD dataset "National Accounts" (downloaded in December 2017).

Figure 2.2.: Innovation and Production

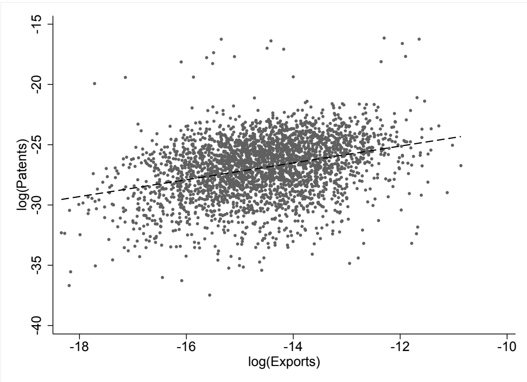
(a) Patenting with and without exporting (RCA 0.1)



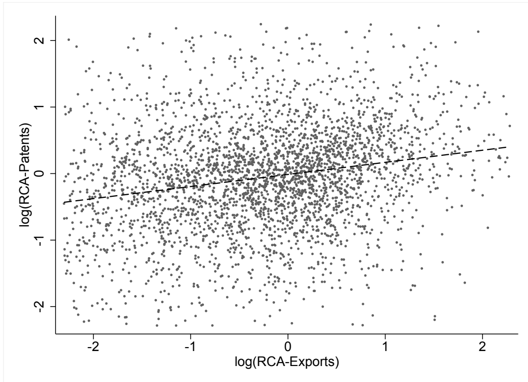
(b) Patenting with and without exporting (RCA 1)



(c) Exports and patents by industry



(d) RCA in exporting and patenting



Notes: Patents and exports are per country and per industry (ISIC Rev 3) in 2013. Exports are taken from CEPII BACI and converted from the HS6 classification system to the ISIC Rev 3 classification using the Worldbank's concordance tables. Patents are taken from the OECD "Patents by Technology" dataset and converted from the IPC 4 patent classification system to the ISIC Rev 3 classification using the ALP concordance tables (Lybbert and Zolas (2014)).
Figure (a): Own illustration. The dark (bright) bar shows the fraction of country-industry pairs with RCA in patenting greater than 0.1 when RCA in exporting is smaller (greater) than 0.1 in 2013.
Figure (b): Own illustration. The dark (bright) bar shows the fraction of country-industry pairs with RCA in patenting greater than 1 when RCA in exporting is smaller (greater) than 1 in 2013.
Figures (c) and (d): Own illustration. A dot refers to a country-industry pair. Outliers with an RCA in exporting or patenting of smaller than 0.1 or greater than 10 are excluded. In Figure (c) export and patent data are normed by a country's population and an industry's total global exports.

2.3. Model

Starting from the key characteristics of basic research, we will now develop a theory of a country's investment in basic research within the global economy. To that end, we embed a two-stage innovation process with public basic research and private applied research into a variant of the multi-country, multi-industry model of international trade developed in Schetter (2014, 2018). In this model, industrialized countries successfully export varieties in both simple and complex industries, while developing countries systematically specialize in simple industries, in line with what we observe from the data. It is therefore particularly well suited for our purposes.

We begin by describing the macroeconomic environment before introducing innovation.

2.3.1. Macroeconomic Environment

We consider a world with a continuum of countries of measure 1.[10] Countries differ in some parameter r. For sake of concreteness, we think of r as representing a country's *productive knowledge*, but we can allow different interpretations of the origins of r or of r itself.[11] We use $\underline{r}$ and $\bar{r}$ ($0 < \underline{r} < \bar{r} < 1$) to denote the smallest and largest elements in this set, respectively, i.e. the lowest and highest levels of productive knowledge in the world. Across countries, r is assumed to be distributed on $\mathcal{R} := [\underline{r}, \bar{r}] \subseteq (0, 1)$ according to some density function $f_r(r)$ with associated distribution function $F_r(r)$. For convenience, we will assume that $f_r(r)$ is atomless, allowing us to uniquely identify countries with their productive knowledge r.[12] The set of countries is identified with $\mathcal{R}$.

Country r is populated by $L^r > 0$ households. We assume that L^r when viewed as a density on $[\underline{r}, \bar{r}]$ is integrable. Each household is endowed with one unit of labor that it supplies inelastically. Labor is perfectly immobile across countries, but perfectly mobile across a finite set $\mathcal{I}$ of industries, indexed by $i \in \mathcal{I} = \left\{\underline{i}, ..., \bar{i}\right\}$ with $0 < \underline{i} < \bar{i}$. The index i identifies the industry and simultaneously characterizes the complexity of products in the industry, as detailed below.[13] $\underline{i}$ and $\bar{i}$ denote the lowest and highest complexity levels of industries in the world, respectively. Within each industry, there is a continuum of *horizontally* differentiated varieties, $j \in [0, N_i]$, where N_i is the (endogenous) measure of varieties in industry i. We can identify a given variety—henceforth called a product—by a pair (i, j). All products are final consumption goods. They can be offered in different qualities, as detailed below, and are freely traded across the world. We use $\mathcal{Q}_{i,j}$ to denote the set of qualities for which product (i, j) is offered.

[10] With a continuum of countries, individual basic research investment decisions do not impact other countries' decisions. This arguably provides the most realistic set-up for analyzing real-world basic research investments. We provide further discussions in Section 2.8.

[11] The variable r is a reduced-form parameter that can capture anything that contributes to a country's productive potential. It may include a country's infrastructure and institutions that foster complex, high-tech industries, for example, or simply the skill level of labor, which is assumed to be homogeneous within a country.

[12] At the expense of additional notational complexity, the analysis can be performed for distribution with mass points.

[13] Analogously to the productive knowledge of countries, we will assume that industries differ in their complexity such that there is a one-to-one mapping from an industry to its complexity. This is again for convenience only and not essential in any way.

Households and Consumption

Households derive utility from the quality and the quantity consumed of each of the available products, $(i, j) \in \mathcal{I} \times [0, N_i]$ according to the following nested CES-utility

$$U\left(\{c_{i,j,q}\}_{(i,j,q)\in\mathcal{I}\times[0,N_i]\times\mathcal{Q}_{i,j}}\right) = C \; , \tag{2.1}$$

where[14]

$$C := \left[\sum_{i\in\mathcal{I}}\left[\psi_i^{\frac{1}{\sigma_I-1}}\left(\int_0^{N_i}\left(\int_{q\in\mathcal{Q}_{i,j}} qc_{i,j,q}\,dq\right)^{\frac{\sigma_v-1}{\sigma_v}} dj\right)^{\frac{\sigma_v}{\sigma_v-1}}\right]^{\frac{\sigma_I-1}{\sigma_I}}\right]^{\frac{\sigma_I}{\sigma_I-1}} . \tag{2.2}$$

In the consumption basket defined in (2.2), $c_{i,j,q}$ is the consumed amount of product (i, j) at quality level q. The parameter ψ_i is an industry-specific demand shifter. With the above specification, higher qualities of a unit of product (i, j) are valued higher by the household, and different qualities of the product (i, j) are perfect substitutes.[15] The parameter σ_v describes the elasticity of substitution between varieties within a given industry, and the parameter σ_I describes the elasticity of substitution between different industries. We will assume that products are more substitutable within industries than across industries, and that both elasticities are greater than 1, i.e. $\sigma_v > \sigma_I > 1$.

Perfect substitutability between different qualities of the same product implies that all qualities of a product will be sold at the same quality-adjusted price. This quality-adjusted price is denoted by $\rho_{i,j} := \frac{p_{i,j,q}}{q}$, where $p_{i,j,q}$ denotes the globally prevailing price of product (i, j) of quality q.

Our economy admits a global representative household. While domestic production and consumption will matter for basic research policies of national governments, it suffices to consider this representative household to characterize the demand side of our economy. Let $c_{i,j} := \int_{\mathcal{Q}_{i,j}} qc_{i,j,q}dq$ denote total quality-adjusted consumption of product (i, j). The representative household maximizes (2.1) with respect to his budget constraint[16]

$$\sum_{i\in\mathcal{I}}\int_0^{N_i} \rho_{i,j}c_{i,j}\,dj \leq \int_{\underline{r}}^{\overline{r}} \left[w^r\left(L^r - L^r_{BR}\right) + \Pi^r\right] f_r(r)dr \; , \tag{2.3}$$

where w^r denote the wage of the representative household in country r, L^r_{BR} denotes labor employed in basic research in country r and Π^r aggregate profit income of the population

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[14] The equilibrium approach also works for a finite or discrete countable set of quality levels. In this case the inner integral is replaced by the corresponding sum.

[15] Note that perfect substitutability is conditioned on a variety within a given industry.

[16] Later we will incorporate basic research expenditures and taxation which, however, do not affect the structure of the demand function.

in country r as will be detailed below. It is well-known (Dixit and Stiglitz, 1977), that such an optimization problem yields the following demand for product (i,j)

$$c_{i,j} = \psi_i \left(\frac{P_i}{\rho_{i,j}}\right)^{\sigma_v} \left(\frac{P}{P_i}\right)^{\sigma_I} C \; ,$$

where $P_i = \left(\int_0^{N_i} \rho_{i,j}^{1-\sigma_v}\, dj\right)^{\frac{1}{1-\sigma_v}}$ and $P = \left(\sum_{i\in\mathcal{I}} \psi_i P_i^{1-\sigma_I}\right)^{\frac{1}{1-\sigma_I}}$ are the globally prevailing industry-specific and aggregate price indices.[17]

Production Technologies

Industries differ in their complexity i, which is the same for all varieties within a given industry. To model complexity of production, we follow Schetter (2018). Specifically, if a firm in industry i fabricates products of a quality q in country r and hires an amount of labor l_i^r, its expected output denoted by $\mathbb{E}[x_i]$ is given by

$$\mathbb{E}[x_i] = [r]^{iq^\lambda} l_i(r), \quad q \geq 1 \; , \tag{2.4}$$

where λ $(\lambda > 0)$ is a parameter and the lower bound of q is a minimum-quality functional requirement that is assumed to be 1 for all industries.[18] The rationale for the technology embodied in (2.4) is as follows. Production of a product with complexity i and quality q requires that a measure of tasks iq^λ is simultaneously performed successfully. We can think of i as representing the number of tasks involved in production, where quality q scales the intensity or overall difficulty of each task. The parameter λ measures the elasticity of this intensity with respect to quality. In the special case of $\lambda = 1$, this intensity is linear in quality. The higher the productive knowledge of a worker, r, the better he is at performing tasks. Specifically, $[r]^{iq^\lambda}$ is the probability of success of a worker with productive knowledge r producing a product of complexity i and quality q. Overall, the production technology implies that productive knowledge r is valuable in production, and more so for higher quality and more complex products.

There are constant returns to scale with respect to labor. Hence, we can apply the law of large numbers with regard to the amount of units that are produced successfully by a density of labor input equal to $l_i(r)$ and thus we dispense with the expectation operator in (2.4) and in the remainder of the thesis.[19]

[17] A derivation of the demand structure of a similar household optimization problem is shown in Appendix C.1. Though, the specific derivation belongs to the model of Part II of the thesis.

[18] Such requirements are product-intrinsic and arise from the necessary characteristics that a given product needs to satisfy in order to serve its intended purpose. Stricter requirements may also be introduced by law. Cf. Schetter (2018) for a detailed account of these requirements.

[19] Throughout this thesis, we follow the convention and apply an appropriate law of large numbers to a continuum of random variables.

Market Structure and Firm Optimization

There is a monopolist for each variety j of each industry i who owns a global patent to manufacture his variety. A patent covers all qualities of the respective variety. All firms within a given industry face the same optimization problem, independent of the specific variety $j \in [0, N_i]$. For convenience, we will henceforth use the index i to identify both an industry and a *representative* firm within this given industry that produces a product. Hence the complexity level, the representative firm and the representative product are indexed with i. The pair (i, j) is only used when there is a need to differentiate explicitly between varieties.

The representative firm i chooses a set of countries, where it is willing to open up production sites. This set is denoted by $\mathcal{R}_i$. Moreover, in each production site where it is operating, the firm selects a product quality level and chooses a globally prevailing quality adjusted price ρ_i. Finally, the firm chooses a distribution of output among the production sites to meet the demand for its variety which it takes as given. To produce the output in each production site, the firm demands the necessary amount of labor. The optimization problem of firm i is thus given as follows:

$$\begin{aligned} \max_{\mathcal{R}_i, \rho_i, \{q_i(r)\}_{r\in\mathcal{R}_i}, \{x_i(r)\}_{r\in\mathcal{R}_i}, \{l_i(r)\}_{r\in\mathcal{R}_i}} \quad & \int_{r\in\mathcal{R}_i} \left[\rho_i q_i(r) x_i(r) - l_i(r) w^r\right] dr \ , \qquad (2.5) \\ s.t. \quad & x_i(r) = [r]^{i q_i(r)^\lambda}\, l_i(r) \ , \\ & \int_{r\in\mathcal{R}_i} q_i(r) x_i(r) dr = c_{i,j} = \psi_i \left(\frac{P_i}{\rho_i}\right)^{\sigma_v} \left(\frac{P}{P_i}\right)^{\sigma_I} C \ , \\ & q_i(r) \geq 1 \ , \forall r \in \mathcal{R}_i \ , \\ & \mathcal{R}_i \subseteq \mathcal{R} \ . \end{aligned}$$

It is useful to introduce the notion of *effective output* of representative firm i,

$$\chi_i := \int_{r\in\mathcal{R}_i} q_i(r) x_i(r) dr \ .$$

With this notion, representative firm i's decision problem boils down to the following two sub-decisions:

(i) The choice of locations for production and associated qualities to minimize the cost per unit of effective output;

(ii) The choice of a quality-adjusted price, given the minimal costs per unit of effective output. Effective output and also the labor input are then determined by the size of the demand.

Note that a firm will open up production sites in two or more countries only if they share the minimal costs per unit of effective output, in which case the firm is indifferent as to the allocation of the production of its total effective output, χ_i, to these countries.

For each production site, firms will endogenously choose the quality which best complements the local skill level. In particular, they choose the quality that maximizes their productivity in quality-adjusted terms, $q\,[r]^{iq^\lambda}$. Taking derivatives and considering the minimum-quality constraint yields the optimal quality for the product of firm i in country r

$$q_i(r) = \max\left\{1, \left[-\frac{1}{\lambda i \ln(r)}\right]^{\frac{1}{\lambda}}\right\}, \qquad \forall (i,r) \in \mathcal{I} \times \mathcal{R}_i \,.$$

Whenever a firm is not constrained by the minimum-quality requirement, we have $q_i(r) = \left[-\frac{1}{\lambda i \ln(r)}\right]^{\frac{1}{\lambda}}$ and we will say that it is operating at *preferred quality*. It is useful to introduce notation for the boundary complexity and skill levels that just allow production at preferred quality. These boundaries are determined by the optimality of the minimum-quality

$$\tilde{i}(r) = -\frac{1}{\lambda \ln(r)} \quad \text{and} \quad \tilde{r}(i) = e^{-\frac{1}{\lambda i}} \,.$$

The value $\tilde{i}(r)$ denotes the highest complexity level that can be produced in country r without being constrained by the minimum-quality requirement ($q \geq 1$). In turn, $\tilde{r}(i)$ denotes the minimal skill level needed to have an unconstrained quality choice when producing complexity level i. Note that both $\tilde{i}(r)$ and $\tilde{r}(i)$ are strictly increasing. With this notation at hand, we make three assumptions with regard to the distribution of productive knowledge over countries: First, the most complex industry in the economy operates at a complexity level $\bar{i}$. Note that all countries with $r \geq \tilde{r}(\bar{i})$ are able to produce even in the most complex industry without being constrained by the minimum-quality requirement. We assume that there is always a set of countries of strictly positive measure for which this will be the case, i.e. $\bar{r} > \tilde{r}(\bar{i})$. Second, we assume that for each country there is an industry in which it can produce at preferred quality, i.e. $\underline{r} \geq \tilde{r}(\underline{i})$. Finally, we assume that not all countries can produce all products at preferred quality, i.e. there is always a set of countries of strictly positive measure for which this is not the case, $\tilde{i}(r) < \bar{i}$ for some $r > \underline{r}$.

Now, with the optimal choice of quality, the productivity of the representative firm i in producing *effective* output in country r is given by

$$z(i,r) := q_i(r)[r]^{iq_i(r)^\lambda} = \begin{cases} [-e\lambda i \ln(r)]^{-\frac{1}{\lambda}} & \text{if} \quad r \geq \tilde{r}(i)\,, \\ [r]^i & \text{otherwise}\,, \end{cases} \tag{2.6}$$

where e simply denotes Euler's number.

It will turn out (see Section 2.4.1) that the relative productivities in therms of effective output for any two countries r^h and r^l, $\frac{z(i,r^h)}{z(i,r^l)}$, is in equilibrium the same for all industries and thus independent of i. The representative firm will open up production sites in the subset of countries that share the minimum cost per unit of effective output

$$\mathcal{R}_i = \left\{ r \in \mathcal{R} : \frac{w^r}{z(i,r)} = MC_i \right\} \; ,$$
$$MC_i = \min_{r \in \mathcal{R}} \left\{ \frac{w^r}{z(i,r)} \right\} \; .$$

It will then set its price to charge the well-known constant mark-up over its marginal costs

$$\rho_i = \frac{\sigma_v}{\sigma_v - 1} MC_i \; .$$

2.3.2. Innovation

We introduce innovation into the framework. Thereby, the measure of varieties for each industry is endogenized. Our modeling choices for the innovation process are guided by the key characteristics of basic research, as detailed above. In particular, we consider a two-stage hierarchical innovation process: Governments invest into basic research in order to generate ideas for new varieties. Those ideas diffuse with spatial dependence, at first they only diffuse domestically, later they spill over to other countries, reflecting the local effects and international spillovers of public basic research. Ideas typically consist of new materials, methods, or discoveries. They have no commercial value by themselves, but can be taken up in applied research and commercialized. Applied research benefits from industry-specific production know-how, capturing the critical role of domestic manufacturing for innovation. Commercialization results in a blueprint for a new product.

We now elaborate on the two hierarchical stages of the innovation process, first for basic research then for applied research.

Basic Research

In each country, the government decides how many workers to employ in the basic research sector. We them call "scientists" or, equivalently, "researchers". These scientists undertake basic research and generate ideas that are later on turned into new varieties in a particular industry through applied research. Scientists' productivity is determined by their innate ability, denoted by a ($a \geq 0$), and a country-specific productivity shifter $\eta_1(r)$ satisfying $\eta_1(\underline{r}) > 0$ and $\eta_1'(\,\cdot\,) \geq 0$. Without loss of generality we define $\eta_1(\bar{r}) := 1$.

In particular, if the government in country r hires L_{BR}^r scientists with ability a, then they produce an amount of η^r ideas

$$\eta^r = \eta_1(r) a L_{BR}^r \,.$$

Hence, there are no congestion effects with respect to total employment in science, but as outlined below, ability for undertaking basic research is scarce.[20] In what follows, we will assume that basic research is at least as skill intense as production which requires[21]

$$\epsilon_{\eta_1} \geq -\frac{1}{\lambda \ln(r)} \,,$$

where ϵ_{η_1} denotes the elasticity of $\eta_1(r)$ with respect to r.

Households are perfectly mobile between becoming a scientist or working in production. They differ in their innate ability of being scientists but there are no additional utility components attached to being employed as scientists.[22] Abilities are distributed according to some strictly increasing and continuous distribution function $F_a(a)$ on $[\underline{a}, \infty)$ with $F_a(\underline{a}) = 0$ and $F_a'(a) > 0$, $\forall\, a \geq \underline{a}$, where $\underline{a}$ is the lowest innate ability level.[23] We assume that this distribution is the same for all countries.[24]

The government invests in basic research, financed via lump-sum taxes. It will hire the most talented scientists and pay them the equilibrium wage rate in production, i.e. a unique wage w^r will prevail in country r.[25] By investing BR^r in basic research, the

[20] We thus focus on limits on idea generation in basic research that are imposed by abilities and not by the size of the pool of potentially fruitful research endeavors.

[21] Observe from Equation (2.6) that $[-\ln(r)]^{-\frac{1}{\lambda}}$ governs cross-country differences in production efficiency for the case of an interior solution for quality. In particular, with an interior solution for quality, the elasticity of productivity in terms of effective output with respect to r is equal to $-\frac{1}{\lambda \ln(r)}$. $\epsilon_{\eta_1} > -\frac{1}{\lambda \ln(r)}$ is our model-counterpart of the view often found in the literature that a certain stock of technological knowledge is required to be able to effectively perform basic research. It will imply that basic research investments are non-decreasing in a country's skill level, in line with what we observe from the data.

[22] Such benefits can easily be incorporated and would lower the wages that need to be paid to scientists.

[23] We consider distributions of innate ability that are unbounded from above as they deliver the empirically attractive feature that most or all countries devote some, potentially very small, funds to scientific research (UNESCO, 2015). Introducing an upper bound for innate abilities $\overline{a}$ would not affect the essence of our analysis. It might imply that some countries find it optimal not to invest in basic research at all.

[24] However, note that countries differ in terms of their basic research productivity, related to differences in r, as detailed above.

[25] The household's innate ability may be private knowledge. In this case, the government can hire the most talented scientists at the prevailing equilibrium wage rate by conditioning wages on research outcomes. In particular, the government in country r can hire the L_{BR}^r most talented scientists by offering

$$w_{BR}^r \begin{cases} = w^r & \text{if } \eta^{r,h} \geq F_a^{-1}\left(1 - \frac{L_{BR}^r}{L^r}\right)\eta_1(r) \,, \\ < w^r & \text{otherwise,} \end{cases}$$

where $\eta^{r,h}$ denotes household h's research outcome in country r. This will induce the most productive households to become scientists.

government in country r will therefore generate an amount of η^r ideas

$$\eta^r = \eta_1(r) L^r \eta_2 \left(\frac{BR^r}{L^r w^r} \right) , \tag{2.7}$$

where

$$\eta_2 \left(\frac{BR^r}{L^r w^r} \right) := \int_0^{\frac{BR^r}{L^r w^r}} F_a^{-1}(1-x)\, dx . \tag{2.8}$$

$\eta_2(\,\cdot\,)$ satisfies $\eta_2(0) = 0$ and $\eta_2' \left(\frac{BR^r}{L^r w^r} \right) > 0$, $\eta_2'' \left(\frac{BR^r}{L^r w^r} \right) < 0$, as detailed in Appendix A.1.1. In what follows it will be convenient to use ξ^r to denote the share of the population in country r that is working as basic researchers, i.e. $\xi^r = \frac{BR^r}{L^r w^r}$.

Each idea belongs to one industry. There is a one-to-one mapping between an idea and a potential new variety in its industry.[26] Basic research is generally considered as being undirected. There may, however, be some room for targeting basic research investments to certain industries, for example.[27] We will allow such targeting in our framework. In particular, the government can decide to target its basic research investments to a subset of industries $\mathcal{I}_{BR}^r \subseteq \mathcal{I}$, if desired. Targeting will be successful with probability $\kappa \in [0, 1]$. With probability $(1 - \kappa)$, the targeting is not successful, and the basic research effort results in an idea that has equal chance to belong to any particular industry. Thus, the probability that such an idea belongs to industry i is $\frac{1-\kappa}{I}$.

We will use η_i^r to denote the amount of ideas in industry i that originates in country r

$$\eta_i^r \left(\xi^r, \mathcal{I}_{BR}^r \right) = \left[\frac{\kappa}{I_{BR}^r} \mathbb{1}_{[i \in \mathcal{I}_{BR}^r]} + \frac{1-\kappa}{I} \mathbb{1}_{[i \in \mathcal{I}]} \right] \eta_1(r) L^r \eta_2(\xi^r) , \tag{2.9}$$

where I denotes the total number of industries and I_{BR}^r the number of elements in $\mathcal{I}_{BR}^r$. Furthermore, $\mathbb{1}_{[i \in \mathcal{I}]} = 1$ for all industries and $\mathbb{1}_{[i \in \mathcal{I}_{BR}^r]} = 1$ for industries in subset $\mathcal{I}_{BR}^r$ only.

Applied Research

There is spatial dependence in the diffusion of ideas. In particular, we assume that there is a time span T ($T > 0$) during which an idea diffuses only locally within its country of origin. We can think of T as being the time of publication of the underlying research for an idea, i.e. the time of public dissemination of the results. Prior to that, domestic

[26] In reality, of course, insights from basic research may be valuable in many different contexts and important cross-industry spillovers exist. In fact, heterogeneous applications of insights from basic research and the associated lack of appropriability have been identified as a key reason for underinvestment in basic research in the decentralized equilibrium (Nelson, 1959; Arrow, 1962). Note that we do not impose any restrictions on how fundamental insights from basic research translate into ideas, and in particular that our set-up allows an interpretation where a given insight from basic research translates into many ideas in several (or all, for that matter) industries.

[27] Such targeting features prominently in policy debates. Cf. the discussion in Section 2.2.

agents—who are domestic households—learn about ideas through local interactions, e.g. via personal encounters with the scientists involved, in line with positive local effects of basic research as described in Section 2.2.[28] For the sake of concreteness, we assume that such encounters follow a Poisson process with arrival rate $\tilde{\theta}_D$.[29] For an analysis later on, only the probability that domestic applied researchers learn an idea prior to global dissemination matters. Since this probability enters as a structural parameter of our model, any other diffusion process can be assumed and integrated in our model.

Assumption 2.1 (Local Effects of Basic Research)
During an initial time span T, ideas are disseminated via personal encounters between scientists and domestic agents. For each idea, personal encounters follow a Poisson process with arrival rate $\tilde{\theta}_D$. At time T, ideas enter the public domain.

Once ideas enter the public domain, they become accessible to applied researchers in all other countries, following some arbitrary stochastic process, which we detail later.[30] We will assume without loss of generality that the local gains from ideas are negligible once they enter the public domain, and that there is no waste of ideas.[31] The diffusion of ideas will, however, impact the global distribution of the gains from innovation. We will get back to this in Section 2.5 where we discuss the properties of equilibrium investments in our economy.

There are positive spillovers from domestic production to commercialization, as documented in Section 2.2. To capture these, we assume that industry-specific tacit production know-how is a necessary condition for the successful commercialization of ideas. Such know-how is built up through production.

Assumption 2.2 (Applied Research and Manufacturing)
In every country $r \in \mathcal{R}$ ideas can only be commercialized in industries with domestic production.

As we will see in Section 2.4 below, in the equilibria of interest each country is compet-

[28] Cf. Arrow (1969) for an early account of the idea that the diffusion of tacit know-how requires personal contact.

[29] This arrival rate is independent of the number of scientists and the population size, reflecting the fact that the share of scientists in a population is generally small. To account for potential congestion effects, the arrival rate could be made dependent on the ratio of households to scientists. This would not qualitatively affect our results, as this effect would simply reinforce the concavity of $\eta_2(\xi^r)$.

[30] The empirical literature points to a rich pattern of spatial dependence of the diffusion of knowledge (Keller, 2002; Keller and Yeaple, 2013; Bahar et al., 2014). Note that the precise form of the diffusion process of ideas from the public domain will not matter for governments' basic research investment decisions neither in the decentralized solution nor in the social planner solution. However, the diffusion process will determine the distribution of profits across countries, as we discuss in Section 2.5.

[31] Note that introducing local gains from domestic ideas once they enter the public domain is isomorph to an increase in T, and that a waste of ideas is isomorph to a proportionate change in $\eta_1(r)$.

itive in all industries up to the country-specific threshold complexity level $\tilde{i}(r)$ which is strictly increasing in countries' productive knowledge, captured by the variable r, and no firm $i > \tilde{i}(r)$ is willing to produce in country r. Hence, only ideas in industries $i \leq \tilde{i}(r)$ can be commercialized in country r.[32,33]

Whenever an agent learns about an idea, he can decide to start commercializing the new product by investing υ in order to set up a research lab. Commercialization of an idea results in a global patent for the product. This patent is subsequently sold to the highest bidding firm. We assume many (at least two) bidding production firms and thus standard Bertrand competition reasoning implies that the price of the patent equals the ex-post profits of the representative production firm in industry i, which is denoted by π_i. Note that the product market profits π_i in industry i do not depend on the location of the inventor, as the subsequent production decisions are separated from the applied research process. Hence, all profits from production are transferred to patent holders. The profit from selling a patent in industry i for an applied researcher is denoted by $\pi_{AR,i}$ and thus given by

$$\pi_{AR,i} = \pi_i - \upsilon \ . \tag{2.10}$$

The described outcome in (2.10) presumes that there is no duplication of applied research efforts. This can be rationalized in a patent race in which one agent learns an idea first and sets up a research lab earlier than potential competitors. Then the first-mover can always deter entry by other R&D firms in a patent race, by choosing high enough applied research intensities, which renders the success of second-movers sufficiently unlikely.[34]

In what follows, we assume that υ is negligible, such that it is always profitable to commercialize an idea. In particular, we study $\pi_{AR,i}$ in the limit as υ goes to zero and therefore $\lim_{\upsilon\to 0} \pi_{AR,i} = \pi_i$. This simplifies the analysis and allows to focus on basic research investments alone.[35] Then, whether or not an agent in country r has a chance to develop a blueprint (i, j) can be summarized by the following indicator function

$$\mathbb{1}_{[i\leq\tilde{i}(r)]} \ ,$$

[32] The equilibrium will exhibit indifference in terms of location of production. We will assume that all countries have positive production in all industries for which they are competitive, in line with what we observe from the data.

[33] Domestic production know-how is a necessary condition for commercialization. As an alternative, we could assume that domestic production fosters the productivity of commercialization. This would not impair our main insights.

[34] Another rationale are small fixed entry costs into a patent race. Then, a second R&D firm does not enter the patent race once the first one has entered, since it anticipates that subsequent R&D efforts would match the profit π_i and the entry costs could not be recovered. However, duplication of research efforts could also be integrated into the model by explicitly accounting for these additional costs.

[35] Of course, costs of applied research can be deducted in all of the formulas. Moreover, if applied research costs are a substantial fraction of the industry profits, and thus the profits from patenting are dissipated, incentives of governments to invest in basic research will decline.

and the share of country r's ideas in industry i that are commercialized domestically is given by[36]

$$\begin{aligned}\theta^r_{D,i} &= \mathbb{1}_{[i \leq \tilde{i}(r)]}\left[1 - \exp(-\tilde{\theta}_D T)\right] \\ &= \mathbb{1}_{[i \leq \tilde{i}(r)]}\theta_D \ , \end{aligned} \tag{2.11}$$

where θ_D is the fraction of ideas that domestic households learn prior to global dissemination of the research.[37] With probability $(1 - \theta^r_{D,i})$, the idea enters the public domain and it is part of the *global pool of ideas*. The diffusion and commercialization of ideas imply that ideas are not forgotten, i.e. in equilibrium we have

$$N_i = \int_{\underline{r}}^{\overline{r}} \eta^r_i f_r(r) dr \ , \quad \forall\, i \in \mathcal{I} \ . \tag{2.12}$$

We note that Equation (2.12) expresses the conservation of commercialized ideas and (2.11) and the upcoming Equation (2.17) in Section 2.5 describe the distribution of applied research.

2.3.3. Sequence of Events

The sequence of events may be summarized as follows:

1. In all countries governments decide on how much basic research to provide.
2. Ideas diffuse throughout the economy and are turned into patented blueprints for new products by applied research.
3. Patents for new products are sold to production firms.
4. Production firms choose locations for production and supply the world market.

2.4. Equilibrium for Given Basic Research Investments

In this section, we analyze the equilibrium in our economy, taking government policies, ξ^r and $\mathcal{I}^r_{BR}$, as given. We start with its definition.

[36] Commercialization is random. Thus, we consider expected values and ignore the expectation operator. This follows from appropriately defining the set of countries and of varieties within an industry and from applying a law of large numbers to a particular constellation. Note, that households are risk-neutral with respect to their aggregate income and, hence, we could easily allow for uncertainty at the country level since country specific risks are fully diversified.

[37] An equally valid interpretation is one where basic researchers potentially engage in commercialization and where θ_D is the probability that this engagement will happen.

Definition 2.1 (Equilibrium)

An equilibrium for given basic research policies, ξ^r, $\mathcal{I}_{BR}^r$ $\forall r \in \mathcal{R}$, is

(i) an applied research firm for every idea j in each industry, $\{\eta_i^r\}_{(i,r)\in\mathcal{I}\times\mathcal{R}}$,

(ii) a set of countries $\mathcal{R}_i \subseteq \mathcal{R}$ for the representative firm of each industry i, where the firm is operating a production site,

(iii) for each production site of each representative firm i, a quality level $\{q_i(r)\}_{(i,r)\in\mathcal{I}\times\mathcal{R}_i}$, an effective output level $\{\chi_i(r)\}_{(i,r)\in\mathcal{I}\times\mathcal{R}_i}$, and a mass of labor employed, $\{l_i(r)\}_{(i,r)\in\mathcal{I}\times\mathcal{R}_i}$,[38]

(iv) a set of quality-adjusted consumption levels for the representative household for each representative product i, $\{c_i\}_{i\in\mathcal{I}}$,

(v) a quality-adjusted price for each representative product i, $\{\rho_i\}_{i\in\mathcal{I}}$,

(vi) a set of wage rates, $\{w^r\}_{r\in\mathcal{R}}$,

such that

(A) all ideas $\{\eta_i^r\}_{(i,r)\in\mathcal{I}\times\mathcal{R}}$ are commercialized according to (2.12),

(B) $\mathcal{R}_i$, $\{q_i(r)\}_{r\in\mathcal{R}_i}$, $\{\chi_i(r)\}_{r\in\mathcal{R}_i}$, $\{l_i(r)\}_{r\in\mathcal{R}_i}$, and ρ_i solve the representative firm i's profit maximization problem, $\forall i \in \mathcal{I}$,

(C) $\{c_i\}_{i\in\mathcal{I}}$ maximizes utility of the representative household, subject to his budget constraint, Equation (2.3),

(D) goods markets clear for all products,

(E) labor markets clear in all countries.

2.4.1. Equilibrium in the Labor Market

We begin by analyzing the equilibrium in the labor market. Basic research policies will have two effects on the labor market. There is a direct effect via tying up labor in basic research, L_{BR}^r, which is no longer available for production. The supply of labor for production, L_p^r, is given by

$$L_p^r = L^r - L_{BR}^r \; .$$

[38] Without further assumptions, it will remain indeterminate how much the firm i produces in each production site.

There is an indirect effect via the generation of varieties across industries through basic research, which in turn affects the demand for labor in each industry. The amount of varieties in industry i is given by

$$N_i = \int_{\underline{r}}^{\bar{r}} \eta_i^r \left(\xi^r, \mathcal{I}_{BR}^r\right) f_r(r) dr \ .$$

We will endogenize these effects later on. For now, we take L_p^r and N_i as given. Labor markets then are in equilibrium if firms take up all labor available for production in each country.

For all industries $i \in \mathcal{I}$ and for any two countries $r, r' \in \mathcal{R}$ with $r, r' \geq \tilde{r}(i)$, the relative productivities in terms of *effective* output is the same

$$\frac{z(i,r)}{z(i,r')} = \left[\frac{\ln(r')}{\ln(r)}\right]^{\frac{1}{\lambda}} \ , \quad \forall (i, r^h, r^l) \in \mathcal{I} \times [\tilde{r}(i), \bar{r}]^2 \ .$$

In a world with no minimum-quality requirements, the unique equilibrium wage would then be

$$w^r = \left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} \ , \tag{2.13}$$

where we choose $w^{\bar{r}} = 1$ to be the numéraire. As shown in Schetter (2018, Proposition 1), the unique equilibrium wage scheme is still given by (2.13), even with minimum-quality requirements, if there are *sufficient skills* in the economy. This logic also applies here and we next derive the sufficient skill condition in our economy where also basic research takes place.

Intuitively, the minimum-quality requirement, if binding, introduces inefficiency for production. Hence, with wages given by (2.13), the representative firm i is willing to operate in all countries $r \in \mathcal{R} : r \geq \tilde{r}(i)$. In turn, this implies that two conditions have to be satisfied in order for (2.13) to constitute the equilibrium wage scheme. First, the representative firm in every industry i must be able to satisfy its total demand for labor in countries with skill level $r \geq \tilde{r}(i)$. Second, the overall labor market must clear.

To formalize these conditions, note first that $\tilde{r}(i)$ is increasing in i, i.e. firms in less complex industries are willing to produce in all countries where firms of more complex industries are willing to produce, plus some additional countries with lower productive knowledge. Second, it is useful to introduce the notion of effective labor at the country and the firm level. Specifically, we define

$$\tilde{L}^r := L^r \left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} \ .$$

$\tilde{L}^r$ is called *effective labor* of country r and measures labor in country r in terms of its productivity relative to labor in the country with the highest productive knowledge, $\bar{r}$.[39] Next if a firm i can produce at *preferred quality*, $q_i(r) = \left[-\frac{1}{\lambda i \ln(r)}\right]^{\frac{1}{\lambda}}$, its demand for *effective labor* is independent of the skill level $r \geq \tilde{r}(i)$ it uses in production. This demand for effective labor of firm i is given by

$$\tilde{l}_i := \int_{\mathcal{R}_i} l_i(r) \left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} dr = [-e\lambda i \ln(\bar{r})]^{\frac{1}{\lambda}} \chi_i ,$$

and hence linearly depends on the firm's effective output, χ_i. With these notations, we can define sufficient skills as follows:

Definition 2.2 (Sufficient Skills Condition: SSC)

$$\int_{\tilde{r}(\hat{i})}^{\overline{r}} [L^r - L^r_{BR}] \left[\frac{\ln(\overline{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} dF_r(r) \geq \sum_{i \in \mathcal{I}: i \geq \hat{i}} N_i \tilde{l}_i , \quad \forall \hat{i} \in \mathcal{I} . \tag{SSC}$$

The Sufficient Skill Condition (SSC) guarantees that the supply of skills is always greater or equal to the demand for skills, such that the minimum-quality constraint will never be binding for any firm in any industry. In that sense we say that there are *sufficient skills* in the economy. Whenever this is the case, the wage scheme of Equation (2.13) must hold,[40] and if SSC holds with equality for $\hat{i} = \underline{i}$, where by assumption $\underline{i} \leq \tilde{i}(\underline{r})$, then the overall effective labor market clears and labor markets are in equilibrium.

Condition SSC depends on the endogenous demand for effective labor, $\tilde{l}_i$, and basic research policies, which enter both sides of SSC. In each country, labor available for production is reduced by the number of scientists. In addition, basic research policies impact the cross-industry distribution of the number of varieties N_i and, hence, the total demand for effective labor for production. In the end, Condition SSC translates into an assumption on parameter values, in particular the successfulness of basic research targeting (expressed by κ) and the distributions of productive knowledge, labor, complexities, and demand shifters.

From an economic perspective, SSC simply guarantees that the countries with the highest productive knowledge are not only active in the few most complex industries, but that country r will be competitive for all industries $i \leq \tilde{i}(r)$, i.e. we are in a situation where more developed economies are more diversified, in line with our motivating facts. This is our equilibrium of interest and we will henceforth limit attention to situations where SSC

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[39] With no minimum-quality requirements, $\left[\frac{\ln(\overline{r})}{\ln(r)}\right]^{\frac{1}{\lambda}}$ is the marginal rate of technical substitution of labor in country $\overline{r}$ for labor in country r.

[40] Any other constellation would violate labor market clearing.

is satisfied. It follows that wages are pinned down by international competition on goods markets and are independent of the exact basic research policies, which is economically attractive, given that in practice only a small share of the population is engaged in basic research, and this fraction is well below 1% of the labor force.[41] Note that we can always find parameter values such that SSC is indeed satisfied in both, a decentralized equilibrium of basic research investments and the global social planner solution considered below. We discuss these issues in Appendix A.2.

2.4.2. Equilibrium Values

With the equilibrium wage at hand, the derivations of Section 2.3, along with some straightforward algebra, allow to characterize the equilibrium for given basic research policies.

Proposition 2.1
Suppose that basic research policies $\xi^r, \mathcal{I}^r_{BR}$ are given and that Condition SSC holds. Then there exists a unique equilibrium with

(i) $N_i{}^\star = \int_{\underline{r}}^{\overline{r}} \eta_i^r\left(\xi^r, \mathcal{I}^r_{BR}\right) f_r(r)dr \quad \forall\, i \in \mathcal{I}$,

(ii) $w^{r\star} = \left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} \quad \forall\, r \in \mathcal{R}$,

(iii) $\mathcal{R}_i{}^\star \subseteq \{r \in \mathcal{R} \mid r \geq \tilde{r}(i)\} \quad \forall\, i \in \mathcal{I}$,

(iv) $q_i^\star(r) = \left[-\frac{1}{\lambda i \ln(r)}\right]^{\frac{1}{\lambda}} \quad \forall\, (i, r) \in \mathcal{I} \times \mathcal{R}_i^\star$,

(v) $\rho_i{}^\star = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i \ln(\bar{r})\right]^{\frac{1}{\lambda}} \quad \forall\, i \in \mathcal{I}$,
$P_i{}^\star = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i \ln(\bar{r})\right]^{\frac{1}{\lambda}} N_i{}^{\star\frac{1}{1-\sigma_v}} \quad \forall\, i \in \mathcal{I}$,
$P^\star = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda \ln(\bar{r})\right]^{\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} N_i{}^{\star\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{1-\sigma_I}}$,

(vi) $\tilde{l}_i^\star = \frac{\psi_i N_i{}^{\star\frac{\sigma_v - \sigma_I}{1-\sigma_v}} i^{\frac{1-\sigma_I}{\lambda}}}{\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} N_{\hat{i}}{}^{\star\frac{1-\sigma_I}{1-\sigma_v}} \hat{i}^{\frac{1-\sigma_I}{\lambda}}} \tilde{L}_p^\star$,

(vii) $\chi_i{}^\star = \left[-e\lambda i \ln(\bar{r})\right]^{-\frac{1}{\lambda}} \tilde{l}_i^\star$,

(viii) $\pi_i{}^\star = \frac{\tilde{l}_i^\star}{\sigma_v - 1}$,

(ix) $C^\star = \left[-e\lambda \ln(\bar{r})\right]^{-\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} N_i{}^{\star\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I - 1}} \tilde{L}_p^\star$ *and* $P^\star C^\star = \frac{\sigma_v}{\sigma_v - 1}\tilde{L}_p^\star$,

[41] Countries devote less than 1% of their GDP to basic research (cf. OECD (2016)). Also cf. Gersbach and Schneider (2015).

where $\tilde{L}_p^\star := \int_{\underline{r}}^{\bar{r}} L^r \left[1-\xi^r\right] \left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} f_r(r)dr$ *is aggregate supply of effective labor for production.*[42] *The values of the representative firm of each industry* i *hold for all firms* $j \in [0, N_i^\star]$ *in that industry.*

Note that all varieties in the economy are the result of basic and applied research efforts. In the remainder of the chapter, we will use the equilibrium of Proposition 2.1, and simplify the notation by disposing of superscript * in all expressions.

2.5. Decentralized Investment in Basic Research

In the previous sections, we have outlined our model and derived the equilibrium for given basic research policies. In this model environment, we will first analyze basic research investments in the decentralized equilibrium with investments undertaken by national governments. Then, we will confront this equilibrium with the solution of a global social planner. Henceforth, we will assume that basic research investments are such that the ensuing equilibrium is according to Proposition 2.1.[43]

Governments in all countries decide how much basic research to provide, ξ^r, and on which industries to target, $\mathcal{I}_{BR}^r$, in order to maximize the domestic gains from the associated innovations net of the costs of doing research. They anticipate the optimization behavior of all other governments. With a continuum of countries, however, they will take this behavior and all equilibrium values as given.

Among the set of industries where ideas can be commercialized domestically, governments will always target the industries where blueprints for new varieties are most valuable, i.e. those industries that yield the highest profits for the representative firm. In turn, this immediately implies that among industries that receive non-zero targeting of basic research, profits have to be non-decreasing in complexity in equilibrium.

Let i_{BR}^r denote the industry with highest profits among all industries $i \leq \tilde{i}(r)$, i.e. among all industries where ideas can be commercialized in country r. The government in country r will target this industry.[44] It chooses its level of basic research investments to maximize

[42] Note the differences between aggregate labor supply $\tilde{L}_p^\star$ and the labor input $\tilde{l}_i^\star$ for the representative firm in a given industry.

[43] The equilibrium of Proposition 2.1 exists and is unique if the implied distributions of labor supply across countries and labor demand across industries satisfy Condition SSC. As we discuss in Section 2.4.1, this will ultimately depend on the exogenous distributions of productive knowledge, labor, complexities, and demand shifters, as well as on the endogenous basic research policies. Importantly, we can always find exogenous parameter values such that SSC is necessarily satisfied in both, the decentralized equilibrium of basic research investments and the global social planner solution considered below (see Proposition A.1 in the Appendix A.2).

[44] In principle, $\mathcal{I}_{BR}^r$ may contain multiple industries. In such case, the government will be indifferent between targeting any of the industries in $\mathcal{I}_{BR}^r$, and we will assume that it targets any one of these, i.e. to simplify notation, we will consider the case of $\mathcal{I}_{BR}^r$ being a singleton, i.e. i_{BR}^r

the total domestic income from selling blueprints for new varieties, net of basic research investment,

$$\max_{\xi^r} \left\{ \eta_1(r) L^r \eta_2\left(\xi^r\right) \theta_D \left[\kappa \pi_{i^r_{BR}} + \frac{1-\kappa}{I} \sum_{i \in \mathcal{I}(r)} \pi_i \right] - \xi^r w^r L^r \right\} , \tag{2.14}$$

where $\mathcal{I}(r) := \{i \in \mathcal{I} : i \leq \tilde{i}(r)\}$ denotes the set of industries with domestic production.

The associated first order condition is

$$\eta_1(r) \eta_2'\left(\xi^r\right) \theta_D \left[\kappa \pi_{i^r_{BR}} + \frac{1-\kappa}{I} \sum_{i \in \mathcal{I}(r)} \pi_i \right] - w^r = 0 , \tag{2.15}$$

where governments consider $\{\pi_i\}_{i \in \mathcal{I}}$ as given since an individual country cannot affect these profits. In economic terms, Equation (2.15) simply requires that the marginal profit of an additional scientist equals her marginal costs.

In Section 2.3.2 we have established that for any distribution of innate abilities, $F_a(\,\cdot\,)$, with continuous support on $[\underline{a}, \infty)$ $\eta_2(\,\cdot\,)$ is strictly increasing and concave. Moreover, it satisfies $\eta_2'(\xi^r) = \tilde{a}(\xi^r) := F_a^{-1}\left(1 - \xi^r\right)$. The optimal level of basic research investment in country r is therefore the unique solution to the above first order condition,

$$\xi^r_E = 1 - F_a \left(\frac{w^{r*}}{\eta_1(r) \theta_D \left[\kappa \pi_{i^r_{BR}} + \frac{1-\kappa}{I} \sum_{i \in \mathcal{I}(r)} \pi_i \right]} \right) , \tag{2.16}$$

where here and below we use a subscript $_E$ to denote an optimal solution of a national government. We summarize our insights in the following proposition.

Proposition 2.2
In the decentralized equilibrium, the government in country r targets its basic research investments to industry $i^r_{BR} := \arg\max_{i \in \mathcal{I}(r)} \pi_i$ and its basic research intensity is given by (2.16).

Proposition 2.2 does not yet prove the existence of a decentralized equilibrium. We can show existence of a decentralized equilibrium by showing that the set of industries for targeting is unique and by imposing conditions such that SSC holds, which will also describe the macroeconomic environment in which Proposition 2.2 holds. This will be addressed in Appendix A.2.

Proposition 2.2 implies that countries with high productive knowledge conduct more basic and more applied research. To see this, note that (2.16) can be rewritten as

$$\tilde{a}\left(\xi^r\right) = \frac{w^r}{\eta_1(r)\theta_D\left[\kappa\pi_{i^r_{BR}} + \frac{1-\kappa}{I}\sum_{i\in\mathcal{I}(r)}\pi_i\right]} \ ,$$

where $\tilde{a}(\xi^r) = F_a^{-1}\left(1-\xi^r\right)$ is the innate ability of the marginal scientist. On the one hand, scientists in countries with higher productive knowledge r earn higher wages. On the other hand, they are more productive as researchers, $\eta_1'(\,\cdot\,) > 0$, and their economy is weakly more diversified,[45] which increases the chance that the scientists discover an idea that can be commercialized domestically. This also weakly increases the targeting potential for the government. Whether or not the basic research intensity will be increasing in r then depends on the magnitudes of the different effects. We consider the case of basic research being at least as skill intensive as production. Thus, $\frac{w^r}{\eta_1(r)}$ is weakly monotonously decreasing in r and $\theta_D\left[\kappa\pi_{i^r_{BR}} + \frac{1-\kappa}{I}\sum_{i\in\mathcal{I}(r)}\pi_i\right]$ non-continuously increasing. It follows that ξ^r_E is weakly monotonously increasing in r.

The fact that ξ^r_E is increasing in r also feeds back into applied research intensities in different countries. These, however, not only depend on a country's own basic research, but also on spillovers of ideas from the rest of the world. Hence, for the equilibrium distribution of applied research activities, the diffusion of ideas—once they have entered the public domain—will matter. Again, it is often argued that innovation is at least as skill intense as production.[46] In the context of our model, this suggests that applied researchers encounter ideas in the public domain with a probability that is proportionate to their endowment with effective labor, so we may assume that an idea from the public domain is commercialized in country $r \in \mathcal{R}$ with probability

$$\theta^r_{G,i} = \mathbb{1}_{[i\leq\tilde{i}(r)]} \cdot \frac{\tilde{L}^r}{\int_{\tilde{r}(i)}^{\bar{r}} \tilde{L}^r \, dF_r(r)} \ , \tag{2.17}$$

where $\int_{\tilde{r}(i)}^{\bar{r}} \tilde{L}^r \, dF_r(r)$ is the total effective labor in countries having a sufficiently high productive knowledge to commercialize ideas in industry i. Then applied research intensities and resulting product innovations are increasing in r. This is the case for two reasons: First, countries with a higher r invest more in basic research, and they are more productive in doing so, i.e. they generate more ideas. Second, they can commercialize a greater fraction of ideas due to their stronger manufacturing base. This applies to both, domestically generated ideas and ideas that spill over from other countries.

[45] For any pair of countries $r^h > r^l$, $\mathcal{I}(r^h)$ is a superset of $\mathcal{I}(r^l)$.

[46] Insofar that innovation is considered more skill intensive than production, the following spatial diffusion assumption can be regarded as fair-minded.

Corollary 2.1
A country's investments in basic (and applied) research are increasing in its productive knowledge.

While a rigorous test of our model is not possible, due to a lack of good data, note that these patterns of equilibrium research investments are consistent with salient features in the data. In particular, as documented in Figure 2.3, on balance, countries closer to the frontier devote larger shares of their GDP to both basic and applied research.[47]

These equilibrium outcomes also have important distributional consequences: As industrialized countries innovate more, they are able to appropriate a disproportionately large share of global profits. In particular, in Appendix B.1.1, we show that the ratio

$$\frac{GNI - GDP}{GDP}$$

is increasing in r.

Corollary 2.2
Countries with high productive knowledge appropriate a disproportionately large share of global profits.

We note that this result in Corollary 2.2 is consistent with Figure 2.1(b) in our motivating Section 2.2.

2.6. Social Planner Solution

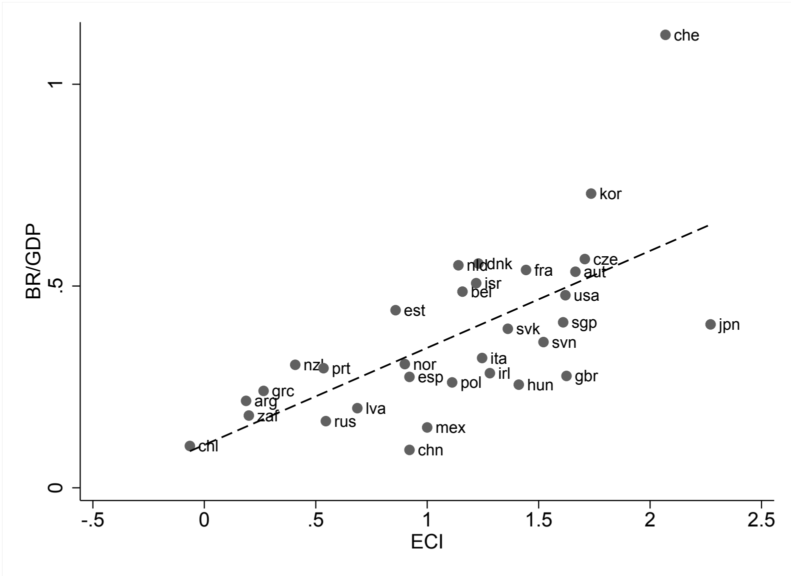
In this section, we analyze the optimal basic research investment of a global social planner. For the economic environment we are considering, it is well known that conditional on investments in basic research and their respective targeting, equilibrium outcomes according to Definition 2.1 will be efficient.[48] However, with endogenous innovation fueled by basic research various external effects emerge that may introduce inefficiencies. For instance, foreigners benefit from cross-border spillovers of ideas and a widening of the variety-base for consumption. Negative externalities arise from rent-seeking of governments (through increasing N) and the loss of profit-potential associated with a diminution of the labor force available for production.

[47] The pattern is robust to measuring countries' productive knowledge by their GDP per capita or their diversification, measured by the number of industries with strong exporting.

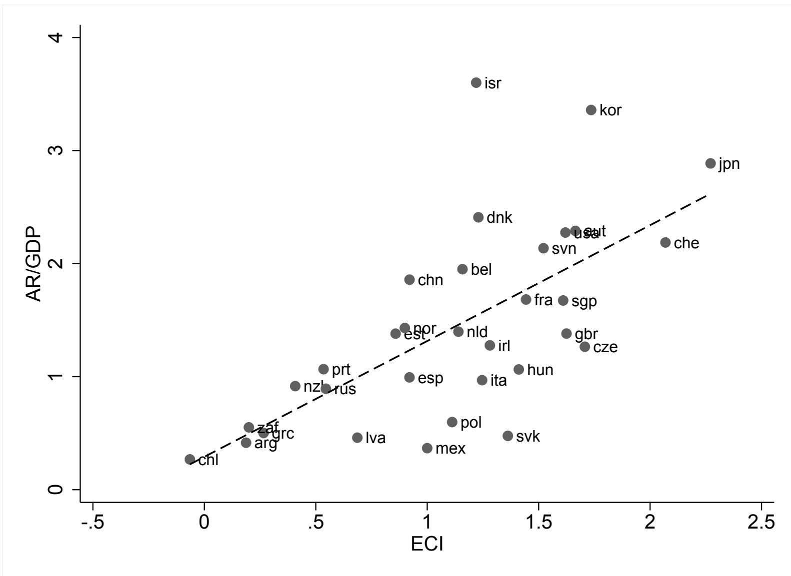
[48] Cf. e.g. Epifani and Gancia (2011). Note that in itself, this is not a limitation of our theoretical framework, given that our main focus of interest lies in comparing socially efficient (coordinated) basic research investment to decentralized investments.

Figure 2.3.: Productive Knowledge, Basic and Applied Research

(a) Basic Research



(b) Applied Research



Notes: Own illustration. ECI is the economic complexity index developed by Hausmann and Hidalgo (2011) and taken from their open database (downloaded in July 2016). Data on basic research and applied research is taken from the OECD dataset "Main Science and Technology Indicators" (downloaded in December 2017). Applied research is calculated by subtracting basic research from gross domestic expenditures on R&D. All data is averaged over a 5-year span with the last observation in 2015.

In contrast to the national governments, the global social planner takes these externalities into account. His decision problem boils down to choosing the level of basic research investment and targeting it for each country in the economy, such that he maximizes the utility of the global representative household in the implied equilibrium according to Proposition 2.1. He will not care about the distribution of burdens of basic research investment and associated benefits across the world. The optimization problem of the social planner is

$$\max_{\left\{\xi^r, i^r_{BR}\right\}_{r\in\mathcal{R}}} \quad C = \left[-e\lambda\ln(\bar{r})\right]^{-\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} N_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I - 1}} \left[\tilde{L} - \tilde{L}_{BR}\right] ,$$

$$\text{s.t.} \;\; N_i = \int_{\underline{r}}^{\bar{r}} \eta_i^r\left(\xi^r, i^r_{BR}\right) f_r(r)dr \quad \forall\, i \in \mathcal{I} ,$$

$$\tilde{L}_{BR} = \int_{\underline{r}}^{\bar{r}} L^r \xi^r \left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} f_r(r)dr \; .$$

It will be instructive to tackle this optimization problem in three steps. In particular, note that targeting impacts social welfare only via its effect on the distribution of varieties across industries, $\left\{n_i := \frac{N_i}{N}\right\}_{i\in\mathcal{I}}$. It will neither impact the total number of varieties, N, nor the cost of providing these varieties in terms of effective labor, $\tilde{L}_{BR}$. For any total investment in basic research, as reflected in N, and allocation of this investment across countries, the social planner will thus always seek to distribute varieties across industries to maximize total utility from consumption of these varieties. Second, conditional on N, allocation of basic research investments across countries will only impact the total cost of providing N, $\tilde{L}_{BR}$. A necessary condition for welfare maximization is therefore to minimize the cost of providing N which will determine allocation of basic research investment across countries. We will later provide explicit comparison of the social efficient distribution of basic research efforts across countries to the decentralized equilibrium. We will use $\tilde{L}_{BR,S}(N)$, to denote the optimal solution as a function of N, with a subscript $_S$ denoting the social planner solution from now on. The social planner problem then boils down to choosing the optimal level of N, given optimal targeting thereof, and taking into account its bearings on total cost of providing basic research $\tilde{L}_{BR,S}(N)$.
We next study each of these subproblems in turn.

2.6.1. Optimal Targeting

Industries differ in their *attractiveness*, reflected in the term $\psi_i i^{\frac{1-\sigma_I}{\lambda}}$. Ceteris paribus, they are more attractive if the industry-specific consumption bundle has a higher demand shifter (ψ_i higher) or if the industry is less complex (i lower), which, in turn, implies that productivity is higher. The social planner targets his basic research investments to exploit this cross-industry heterogeneity. As we detail in Appendix A.1.2, the associated decision problem boils down to the following

$$\max_{\{n_i\}_{i\in\mathcal{I}}} \quad \left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I-1}} ,$$
$$\text{s.t.} \;\; n_i \geq \frac{1-\kappa}{I} , \quad \forall\, i \in \mathcal{I} ,$$
$$\sum_{i\in\mathcal{I}} n_i = 1 .$$

The lower bound on n_i arises from the constraint that targeting must be non-negative. The objective is strictly increasing and concave in each of its arguments. It immediately follows that the social planner would ideally equate the marginal returns to n_i,

$$\frac{1-\sigma_I}{1-\sigma_v} \psi_i i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}} , \tag{2.18}$$

across industries. Note that this would imply that the share of varieties in industry i is related to its attractiveness. Equating the marginal returns to varieties across industries may, however, not always be feasible due to limited scope of targeting. In such a case, the social planner can do no better than hierarchically target industries in descending order of attractiveness to equate marginal returns to varieties among industries that receive positive targeting, up to the point where he has fully exploited his targeting opportunities. We formally characterize the resulting distribution of varieties across industries in Appendix A.1.2. For our subsequent analysis, it will be sufficient to note that irrespective of total investment in basic research and its distribution across countries, targeting will result in the same optimal value of the above objective.

We summarize these insights in the following lemma:

Lemma 2.1
Let industries be ranked by attractiveness $\psi_i i^{\frac{1-\sigma_I}{\lambda}}$, in descending order. The social planner will target the most attractive industries up to some threshold industry. Targeting is increasing in an industry's attractiveness and is such that the social returns to an additional variety are equal across all industries that receive strictly positive targeting. The optimal value of the above objective will henceforth be denoted by ω_S,

$$\omega_S := \left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} n_{i,S}^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I-1}} .$$

In the above expression, $n_{i,S}$ denotes the share of all varieties that fall into industry i in the social planner solution.

2.6.2. Optimal Basic Research Allocation

We now turn to the optimal allocation of basic research investments to countries. As argued above, for any desired level of total investment in basic research, as reflected in N, the social planner will allocate basic research investments such that he minimizes the cost in terms of effective labor. This allocation thus solves the following decision problem

$$\min_{\forall r \in \mathcal{R},\ 1 \geq \xi^r \geq 0} \quad \tilde{L}_{BR} = \int_{\underline{r}}^{\bar{r}} L^r \xi^r \left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} f_r(r) dr \ ,$$
$$\text{s.t.} \quad \int_{\underline{r}}^{\bar{r}} \eta(\xi^r) f_r(r) dr = N \ .$$

In principle, it may be optimal to choose a corner solution for ξ^r. With the assumptions made, $\xi^r = 0$ will never be optimal. To simplify the exposition, we will focus on the economically most meaningful scenario where the same holds true for $\xi^r = 1$.[49] The necessary and sufficient first order condition then requires that relative marginal costs of hiring additional scientists equate their relative marginal products across countries

$$\left[\frac{\ln(r')}{\ln(r)}\right]^{\frac{1}{\lambda}} = \frac{\eta_1(r)\eta_2'(\xi_S^r)}{\eta_1(r')\eta_2'(\xi_S^{r'})} \ , \quad \forall r, r' \in \mathcal{R} \ , \tag{2.19}$$

and we can infer, using $\eta_1(\bar{r}) := 1$, that

$$\xi_S^r = 1 - F_a\left(\left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} \frac{1}{\eta_1(r)} F_a^{-1}(1 - \xi_S^{\bar{r}})\right) \ , \tag{2.20}$$

and where $\xi_S^{\bar{r}}$ is the unique solution to[50]

$$N = \int_{\underline{r}}^{\bar{r}} \eta_1(r) L^r \eta_2\left(1 - F_a\left(\left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} \frac{1}{\eta_1(r)} F_a^{-1}(1 - \xi_S^{\bar{r}})\right)\right) f_r(r) dr \ . \tag{2.21}$$

---

[49] Cf. also Footnote 2.

[50] Note that the right hand side of (2.21) is strictly increasing in $\xi_S^{\bar{r}}$. Intuitively, the more basic research in the highest-skilled country, the more basic research there will be in all other countries according to the first order condition above and, hence, the higher N will be.

The associated cost of providing basic research are

$$\tilde{L}_{BR,S}(N) = \int_{\underline{r}}^{\bar{r}} L^r \xi_S^r \left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} f_r(r)dr$$
$$= \int_{\underline{r}}^{\bar{r}} L^r \left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} \left[1 - F_a\left(\left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} \frac{1}{\eta_1(r)} F_a^{-1}(1-\xi_S^{\bar{r}}(N))\right)\right] f_r(r)dr \ , \tag{2.22}$$

where the second equality follows from using (2.20) above and where $\xi_S^{\bar{r}}(N)$ is implicitly defined in (2.21). Note that ξ_S^r is continuous on $\mathcal{R}$, and that $\xi_S^{\bar{r}}$ is strictly increasing in N and, therefore, so is $\tilde{L}_{BR,S}(N)$.

2.6.3. Optimal Number of Varieties

From the above, the social planner's decision on the optimal number of varieties boils down to the following

$$\max_N \quad C = [-e\lambda \ln(\bar{r})]^{-\frac{1}{\lambda}} \omega_S N^{\frac{1}{\sigma_v - 1}} \left[\tilde{L} - \tilde{L}_{BR,S}(N)\right] \ . \tag{2.23}$$

The associated first order condition is

$$\frac{1}{\sigma_v - 1} = \frac{\partial \tilde{L}_{BR,S}(N)}{\partial N} \frac{N}{\tilde{L} - \tilde{L}_{BR,S}(N)} \ . \tag{2.24}$$

This condition is very intuitive: The CES aggregator is just aggregate productivity in welfare terms, $[-e\lambda \ln(\bar{r})]^{-\frac{1}{\lambda}} \omega_S N^{\frac{1}{\sigma_v - 1}}$, scaled by the effective labor available for production. In optimum, the social planner thus chooses N so as to equate the elasticities of these two with respect to the number of varieties. As we show in Proposition 2.3, the optimal number of varieties is then the unique solution to

$$\frac{1}{\sigma_v - 1} \frac{\tilde{L} - \tilde{L}_{BR,S}(N_S)}{N_S} = \frac{1}{F_a^{-1}(1 - \xi_S^{\bar{r}}(N_S))}$$
$$= \frac{1}{\tilde{a}^{\bar{r}}} \ , \tag{2.25}$$

where $\tilde{a}^{\bar{r}}$ is the ability in basic research of the marginal scientist in country $\bar{r}$, and $\frac{1}{\tilde{a}^{\bar{r}}}$ therefore corresponds to the marginal increase in effective labor for basic research needed in order to marginally increase N.

We summarize our key insights in the following proposition:

Proposition 2.3

(i) The social planner's optimal number of varieties is the unique solution to (2.25).

(ii) The social planner's basic research allocation function is implicitly defined in (2.20) *and* (2.21). *It is continuous on $\mathcal{R}$.*

(iii) The social planner's optimal targeting strategy is as characterized in Lemma 2.1.

Most elements of Proposition 2.3 follow from our discussions above. We prove the missing parts in Appendix B.1.2.

Note that neither the optimal number of varieties nor the allocation of required basic research investments depends on targeting of basic research. Intuitively, targeting impacts both, benefits arising from an increase in the number of varieties, and the costs in the form of tying up effective labor in basic research in the same way, i.e. it does not affect the trade-off between the two. Hence, the targeting problem can be entirely separated from the decision where and how much to invest in basic research.

We characterize globally efficient basic research investment in the following corollary:

Corollary 2.3

(i) The globally optimal allocation of basic research investment to countries depends only on the distribution of innate abilities, $F_a(\cdot)$, and the ratio of basic research to production productivities, $\eta_1(r)[\ln(r)]^{\frac{1}{\lambda}}$. It will be socially desirable to invest a larger share of GDP in basic research in higher skilled countries whenever $\epsilon_{\eta_1} > -\frac{1}{\lambda \ln(r)}$.

(ii) The optimal basic research intensity $\xi_S^{\bar{r}}$ is not affected by a proportional increase of the population in all countries, by an increase of skills in production or the innate abilities of households.[51] Ceteris paribus, it is higher the lower the substitution between varieties (σ_v lower) and the larger the elasticity ϵ_{η_1}.

The proof of Corollary 2.3 is provided in Appendix B.1.3.

2.7. Comparing the Decentralized Equilibrium to the Social Planner Solution

In the previous sections, we have characterized in detail both the competitive equilibrium of national investments in basic research and the optimal solution of a global social planner. One attractive feature of our theoretical approach is that it allows to compare the

[51] Of course, the optimal basic research intensity depends on the distribution $F_a(a)$.

two and to identify policy measures for improving global outcomes of the decentralized equilibrium. We will consider these issues next.

When deciding on their decentralized basic research investments, national governments take into consideration neither gains from innovation that accrue to foreign governments due to knowledge spillovers nor the positive effects of basic research on aggregate consumption. Of course, the magnitude of industry profits matters for basic research investment decisions, but these profits are the result of basic research investments by all countries. On the other hand, they engage in rent seeking and do not take the profit-potential that comes with every unit of labor used in production into consideration. It is a-priori not obvious how these different externalities play out in equilibrium, all the more as they also depend on a country's skill level r.

Our economic environment is one with many countries and industries. Naturally, we may then be concerned about global efficiency along three dimensions: Aggregate investments, allocation thereof to countries, and targeting thereof to industries. We will consider each of them in turn. We begin by analyzing targeting of basic research.

Targeting

Industries differ in terms of their ex-ante attractiveness, as determined by their complexity i and their demand shifter ψ_i. This attractiveness, along with the distribution of skills, will drive optimal basic research investments in our economy. In particular, industries with a greater attractiveness will ceteris paribus be associated with higher profits for the representative firm and hence attract more basic research investments. Such targeting, in turn, will tend to attenuate the ex-ante differences in attractiveness.

Among the set of industries with domestic production, national governments will always target the ones with highest profits. From Proposition 2.1, we know that for any pair of industries i, i', relative profits are

$$\frac{\pi_i}{\pi_{i'}} = \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}} N_i^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}}}{\psi_{i'} i'^{\frac{1-\sigma_I}{\lambda}} N_{i'}^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}}} .$$

Observe from (2.18) that this ratio is equal to the ratio of marginal social benefits from increasing the number of varieties in each of the industries. This is intuitive, as with CES preferences, ratios of profits of varieties reflect ratios of expenditures and revenues on these varieties between industries. The following Lemma 2.2 demonstrates this relation intuitively. Recall from our discussions in Section 2.6.1 and Appendix A.1.2 that the social planner's aggregate investment in basic research, and its allocation to countries, do not depend on targeting of ideas across industries. We will therefore consider the case of the social planner adopting the decentralized equilibrium targeting. Thereby we compare the marginal costs and benefits for a government and for the social planner at the decentralized solution.

Lemma 2.2

(i) Ceteris paribus, the marginal cost of hiring an additional basic researcher for a national government is $\frac{\sigma_v - 1}{\sigma_v}$ times the marginal cost to the social planner of hiring this researcher.

(ii) Ceteris paribus, the marginal benefit for a national government of a domestically commercialized variety is $\frac{\sigma_v - 1}{\sigma_v}$ times the corresponding marginal benefit of the social planner of producing this variety.

A proof is given in Appendix B.1.5. Lemma 2.2 shows that global gains from a new variety in any industry are just $\frac{\sigma_v}{\sigma_v - 1}$ times the national gains for the inventor of the new variety, i.e. real profits. In turn, this immediately implies that there can never be too much targeting of basic research towards complex industries. Governments with domestic production in all industries will, ceteris paribus, face the same trade-off as the social planner. And governments in lower-skilled countries never target complex industries, given that ideas cannot be commercialized domestically. The opposite is, however, not always true. Precisely because governments in less skilled countries will always target simpler industries, this may result in inefficiently many ideas being targeted towards these industries in the decentralized equilibrium.[52] We summarize these insights in the following proposition:

Proposition 2.4

Targeting in the decentralized equilibrium is either globally efficient or else inefficiently concentrated in industries with low complexity.

The proof of Proposition 2.4 is given in Appendix B.1.4.

Ceteris paribus, inefficient targeting is the more likely, the higher the (relative) gains from innovation in complex industries. In our static set-up, these gains depend only on industries' attractiveness as governed by the exogenous parameters ψ_i and i. More generally,

[52] The possibility of inefficient targeting can easily be illustrated for the limiting case where the social planner targets all basic research towards the most complex industry, and where low-skilled countries without production in this industry invest positive amounts in basic research. Suppose, for example, that there are only two countries $r_1 > r_2$ and industries $i_1 > i_2$, where $\tilde{i}(r_1) \geq i_1 > \tilde{i}(r_2) \geq i_2$. Let targeting be just efficient if every idea is targeted towards industry i_1, i.e.

$$\frac{1+\kappa}{1-\kappa} = \left[\frac{\psi_{i_1} i_1^{\frac{1-\sigma_I}{\lambda}}}{\psi_{i_2} i_2^{\frac{1-\sigma_I}{\lambda}}} \right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}} .$$

Then, for $\kappa > 0$, any positive investment in basic research in country r_2 will result in inefficient targeting.

however, the gains from innovation in different industries will also depend on the number of industry-specific varieties inherited from the past.[53] If complex, high-tech industries are relatively new, i.e. if new industries—or products, for that matter—are relatively complex, for example, then complex industries may have inherited fewer varieties from the past and gains from innovation will be particularly large in these industries.[54] In turn, this increases the share of global basic research investments that the social planner targets to complex industries and, therefore, makes it more likely that targeting is not efficient in the decentralized equilibrium.

Allocation to Countries

We next consider the cross-country distribution of basic research investments. In particular, we ask how the social planner would allocate total investment in basic research across countries in order to achieve the equilibrium number of varieties. Note that while targeting outcomes will matter for the distribution of basic research investments in the decentralized equilibrium, it will not affect the optimal allocation of the social planner as we have shown above.

Observe from (2.19) that the social planner allocates basic research investments so as to equate the marginal basic research productivity of effective labor across countries. In turn, this implies that the relative productivity of the marginal basic researcher in any pair of countries $r' > r$ satisfies

$$\begin{aligned} \frac{\tilde{a}_S^r}{\tilde{a}_S^{r'}} &= \left[\frac{\ln(r')}{\ln(r)}\right]^{\frac{1}{\lambda}} \frac{\eta_1(r')}{\eta_1(r)} \\ &= \frac{w^r}{w^{r'}} \frac{\eta_1(r')}{\eta_1(r)} \ . \end{aligned}$$

The second equality follows from using the equilibrium wage rate. Intuitively, the social planner will require the marginal scientist to have higher ability in country r compared to country r' if he is more expensive (w^r higher) or less productive ($\eta_1(r)$ lower).

As opposed to this, (2.15) implies

$$\frac{\tilde{a}_E^r}{\tilde{a}_E^{r'}} = \frac{w^r}{w^{r'}} \frac{\eta_1(r')}{\eta_1(r)} \frac{\kappa \pi_{i_{BR}^{r'}} + \frac{1-\kappa}{I} \sum_{i \in \mathcal{I}(r')} \pi_i}{\kappa \pi_{i_{BR}^{r}} + \frac{1-\kappa}{I} \sum_{i \in \mathcal{I}(r)} \pi_i} \geq \frac{w^r}{w^{r'}} \frac{\eta_1(r')}{\eta_1(r)} = \frac{\tilde{a}_S^r}{\tilde{a}_S^{r'}} \ . \tag{2.26}$$

[53] We discuss a dynamic extension of our model in Section 2.8.2.

[54] The assumption that new industries are relatively complex is similar in spirit to Acemoglu and Restrepo (2018), for example, who consider arrival of new tasks and assume that these are more complex than pre-existing ones. Relatively large gains from innovation in new, high-tech industries are also consistent with e.g. the fact that, as of 31 March 2018, the five most valuable companies in the world were all tech companies, namely Apple, Alphabet, Microsoft, Amazon, and Tencent (see https://www.pwc.com/gx/en/audit-services/assets/pdf/global-top-100-companies-2018-report.pdf, retrieved on 12 November 2018).

The inequality follows from the fact that the expected profits from commercialization of domestic ideas are non-decreasing in r. It is strict whenever a larger set of industries can be commercialized in country r', i.e. whenever $\mathcal{I}(r) \subsetneq \mathcal{I}(r')$, and holds with equality otherwise. The above inequality implies that in the decentralized equilibrium basic research investment are inefficiently concentrated in the high-skilled countries.

Proposition 2.5
To generate the same number of varieties as in the decentralized equilibrium, the social planner will allocate basic research investments such that $\xi_S^r > \xi_E^r$ for all $r < \tilde{r}_1$ and $\xi_S^r < \xi_E^r$ for all $r \geq \tilde{r}_2$ where $\underline{r} < \tilde{r}_1 \leq \tilde{r}_2 < \overline{r}$ and $\xi_S^r = \xi_E^r$ for all $\tilde{r}_1 \leq r < \tilde{r}_2$ in case of $\tilde{r}_1 < \tilde{r}_2$.

The proof of Proposition 2.5 is given in Appendix B.1.6.

Basic Research Investment

We finally turn to the analysis of total investment in basic research. As discussed above, decentralized investments are subject to several positive and negative externalities. It turns out that with CES preferences these externalities just offset each other, but for technological spillovers. Ideas originating in one country spill over to the rest of the world for two reasons: First, the domestic population may not become aware of the underlying research prior to public dissemination of the results ($\theta_D < 1$). And second, ideas may arise in industries that are not present in domestic production and thus cannot be commercialized domestically.
Now, suppose that the social planner is constrained to adopt the equilibrium allocation scheme of basic research investments across countries. In particular, while he can freely choose aggregate investments, $\tilde{L}_{BR}$, he is constrained to allocate these to countries, such that for every pair of countries r, r' it holds

$$\frac{\xi^r}{\xi^{r'}} = \frac{\xi_E^r}{\xi_E^{r'}} \, .$$

As we show in Appendix B.1.7, in such case, the technological spillovers imply the following result:

Proposition 2.6
Suppose the social planner is constrained to adopt the equilibrium allocation scheme of basic research investments across countries. Then, he will choose strictly higher aggregate investments in basic research compared to the decentralized equilibrium.

Note that the aggregate basic research investment decision of the social planner is unique for any given basic research allocation (see Appendix B.1.7). The allocation of basic research investments across countries is, however, socially inefficient, as shown in the previous section. In turn, this implies that when allocating these investments efficiently, the social planner can achieve a larger number of varieties with the same input, which generate higher aggregate income. This "income effect" may, in principle, induce him to invest less in basic research. As we show in Appendix B.1.8, with a Pareto distribution of abilities, this income effect and the associated substitution effect just offset each other, such that the optimal aggregate investments in basic research of the social planner are the same, irrespective of their allocation to countries. In turn, this immediately implies that in such case, aggregate investments in the decentralized equilibrium will be lower than in the social planner solution.[55]

Proposition 2.7
With a Pareto distribution of abilities, there is too little aggregate investment in basic research in the decentralized equilibrium.

Proposition 2.7 suggests that there are efficiency gains from coordinate increases of basic research investments in a subset or in all countries. Irrespective of the ability distribution, there will of course always be too little investment in basic research if knowledge spillovers are sufficiently large.[56]

While there is too little investment in basic research at the aggregate level, the inefficient allocation vis-à-vis the social planner solution implies that it may still happen that some countries, the highest-skilled ones, invest too much in the decentralized equilibrium. From Proposition 2.5 we know that this would always be the case if the aggregate number of varieties in the decentralized equilibrium was globally efficient. This, however, is not the case, and whether or not investments are too high in the highest-skilled countries will depend on parameter values, the strength of the local effect of basic research and the global distribution of skills, in particular.

Corollary 2.4
For some parameter values, the highest-skilled countries invest less in basic research than in the social planner solution and for some parameter values they invest more.

Corollary 2.4 follows immediately from considering limiting cases and those limiting cases provide insights whether over- or underinvestment in highest-skilled countries oc-

[55] Empirical distributions of economic variables often follow power laws (Newman, 2005; Gabaix, 2016). This is also roughly the case for the (upper tail of the) income distribution and, more to the point, for the (upper tail of the) distribution of citations of scientific papers (Newman, 2005).

[56] This follows immediately from considering the limiting case where $\theta_D \to 0$. Interestingly, Keller (2002, 2004) points to a "globalization of technology", i.e. strong technology spillovers.

curs: For $\underline{r} \to \tilde{r}(\bar{\imath})$, i.e. in an environment where all countries can produce all goods at preferred quality, the targeting of basic research and the allocation of these investments to countries are efficient, and all countries will invest less in basic research when compared to the social planner solution, as long as $\theta_D < 1$. Conversely, with a Pareto distribution of abilities and $\theta_D = 1$ and as long as $\underline{r} < \tilde{r}(\bar{\imath})$, the highest-skilled countries will invest more in basic research compared to the social planner solution. This will be the case because some low-skilled countries ($r < \tilde{r}(\bar{\imath})$) can only target few industries. Thus they invest less compared to the social planner solution. There will be less aggregate investment in basic research by Proposition 2.7, implying that the total number of varieties N will also be smaller. Expected profits for highest-skilled countries of domestically innovated varieties will be higher than in the social planner solution. Hence, these highest-skilled countries will invest more in basic research.

2.8. Complementary Policy Tools and Extensions

In this section we discuss complementary policy tools and extensions of the model. Besides basic research investments, the government may use further policy tools to delay or accelerate local commercialization of basic research output. As an example we discuss the Bayh-Dole Act below.[57] Moreover, the model allows several extensions which may provide useful frameworks for further policy analyses.

2.8.1. The Bayh-Dole Act

So far, we have treated the strength of the local effects of basic research (θ_D) as exogenously given. However this need not be the case in reality. For example, governments can more or less incentivize basic researchers to engage in the commercialization of their work. In fact, the desire to increase the domestic commercial gains from publicly funded basic research features very prominently in policy debates.[58]

One prominent policy intervention to stimulate such commercialization is the Bayh-Dole Act of 1980, allowing US universities to acquire patent rights over innovations from federally funded research. Arguably, this opportunity increases incentives for scientists to

[57] One could also think of subsidizing applied researchers in order to incentivize them to commercialize a disproportionate share of ideas in the pool of ideas that is globally available. We provide a brief discussion of this issue in Section 3.4.

[58] Canada, for example, intends to transform its National Research Council into a business driven, industry-relevant research and technology organization (National Research Council Canada, 2012). David and Metcalfe (2007, p. 22) even argue that *"... it is hard to find a policy document from government, business or university sources that does not call for greater, wider or deeper 'interactions' between private business firms and the universities"*.

contribute their tacit knowledge to applied research.[59] On the downside, it may undermine the Mertonian norms of science and divert scientists from truly basic to more applied research (Nelson, 2004).

University patenting and, closely related to it, upstream patenting is the subject of a large economic literature (Scotchmer, 1991; Heller and Eisenberg, 1998; Hopenhayn et al., 2006; Akcigit et al., 2013; Cozzi and Galli, 2014). This literature typically focuses on closed economies. Our work allows a new, global perspective on this issue. In particular, in the context of our model, we may think of university patenting as increasing θ_D at the cost of potentially lowering $\eta_1(r)$. From the perspective of a global social planner, this is, of course, a wasteful policy intervention, as the social planner is not concerned with θ_D. It may, however, be a feasible second-best solution if coordination of basic research investments is impossible. In particular, rational governments will only implement such policies if the domestic net effect is positive. In turn, such higher gains induce governments to invest more in basic research, thus closing the gap to the socially optimal level of investment. Depending on which effect dominates, the greater investment or the loss in basic research efficiency, an equilibrium with Bayh-Dole may be globally strictly more desirable.[60]

2.8.2. Extensions

Two extensions—dynamics and strategic investments in basic research—will bring the model closer to frameworks that may be connected to the empirical work on how varieties expand in the global market place (see e.g. Broda et al. (2017)).

Dynamics

We have considered a static environment. As long as governments only care about the benefits of their basic research investment decisions for the current generation of households, a static framework is appropriate. Our current static model can, however, also be directly embedded into a dynamic set-up with non-overlapping generations of households and corresponding governments. If governments only care about the generation they represent, all our analyses apply directly to this dynamic variant, with the sole change, that

[59] Thursby and Thursby (2002) suggest several reasons why additional incentives are needed. In particular, they argue that researchers may dislike being involved in commercialization because of delay-of-publication clauses in licensing agreements, or because they are unwilling to spend their time on applied research. Cf. also the discussion in Howitt (2013).

[60] Cf. Proposition 2.7. In a very different context, Akcigit et al. (2013) also present public basic research in combination with intellectual property rights as a feasible second-best solution. In their model, however, first-best would be to subsidize basic research by private firms which, they argue, may not be feasible due to asymmetric information, and intellectual property rights mitigate the "ivory tower property" of public basic research.

the number of varieties in the economy, N, is strictly positive, with zero aggregate investments in basic research, and that this number increases over time fueled by the efforts of each generation.

Yet, our main insights apply even when including forward-looking behavior of governments in such a dynamic set-up. In such a case, governments weigh the current costs of investments against discounted future benefits. Along a balanced growth path, discounted future profits are a constant multiple of per-period profits, i.e. the main trade-offs involved are qualitatively the same as in the static model we consider.

Strategic investment

We have considered a continuum of countries. The main implication of this assumption is that an individual government's basic research decision does not trigger a feedback via change of investment decisions by other countries and countries only care about the aggregate amount of basic research investments by other countries. Again, this is arguably one of the most relevant scenario for understanding real-world policies. Yet, the main mechanisms remain intact even when considering a finite number of countries and allowing for strategic interaction. In either case, such interaction does not concern the global social planner. Moreover, strategic interaction would typically not affect optimal targeting of basic research to industries by national governments, and, as long as the associated effects are not strongly biased in favor of developing countries, these investments would still be inefficiently concentrated in the industrialized countries. In the model we are considering, countries' investments in basic research are strategic substitutes and, depending on the distribution of innate abilities, aggregate investment would tend to be higher with strategic interaction. Yet, it would typically fall short of the globally efficient level, in particular if knowledge spillovers to the rest of the world are strong enough.

2.9. Conclusion

We have analyzed basic research policies in a general equilibrium framework with many countries, many industries, and international trade. We have shown that decentralized investments in basic research are inefficient along three dimensions: They may not be sufficiently directed to support innovation in complex high-tech industries, they are inefficiently concentrated in industrialized countries, and the aggregate level is typically too low for reasonable parameter assumptions. The latter finding further implies that regulations, such as the Bayh-Dole Act, that seek to stimulate technology transfers from universities to the domestic economy may yield welfare improvements.

Our work is a step towards a better understanding of innovation policies in a globalized world. Many related research questions deserve careful scrutiny in future work. Scientists and inventors are, for example, mobile internationally (Hunter et al., 2009; Stephan, 2012; Miguelez and Fink, 2013). If the most able scientists migrate to places with greatest investments in basic research, this will contribute to mitigating aggregate inefficiencies, yet possibly at the cost of reinforcing cross-country differences in innovation abilities and incomes. Carefully analyzing migration in this context and disentangling different effects is a promising avenue for future research.[61] More generally, it would be interesting to scrutinize the distributional effects of innovation in a globalized world.

[61] We provide a first simple framework that addresses this issue in Section 3.3.

3. Extensions and Discussions

In the previous chapter, we developed a unified theory of public basic research. We now examine topics of basic research that have been neglected up to now: Fixed costs in basic research, mobility of researchers and openness of research.

In our model of the previous chapter, every country invested in basic research. We will analyze in Section 3.2 the effects of fixed costs for building up the infrastructure needed for basic research on the distribution of basic research investments. Building up basic research comes with substantial costs, which we will call *"basic research infrastructure costs"*. These fixed costs can act as an entry barrier for the basic research involvement of countries.[1] We assume that these costs are independent from basic research investments. We find that such fixed costs can force some countries of low productive knowledge to *not* invest in basic research at all. Thus, fixed costs can lead to basic research investments that are more unequally distributed across countries. We also find that the countries of high productive knowledge may invest more or less in basic research than without such fixed costs. They invest *more* if fewer investments in basic research by the countries of low productive knowledge imply fewer varieties, higher profits, and thus more profitable basic research investments. But they invest *less* if the costs for the buildup are such that many countries still invest in basic research.

In Chapter 2, we assumed that workers and scientists are immobile. In Section 3.3, we will relax this assumption and introduce a *global* market for researchers. Researchers across the globe show a relatively high propensity to change domicile for work (Thorn and Holm-Nielsen, 2008). In other words, they are mobile. The migration of researchers or high-skilled individuals from less developed countries to developed countries is also called "brain drain". It was extensively analyzed (Bhagwati and Hamada, 1974; Beine et al., 2001, 2008; Thorn and Holm-Nielsen, 2008). Whereas the literature analyzes the benefits and costs of brain drain away from the less developed countries—where bene-

[1] Observe that an upper bound of ability in the ability distribution may also induce some countries of low productive knowledge to abstain from undertaking basic research, as noted in Footnote 23 of the previous chapter. However, in contrast to basic research infrastructure such an upper bound of ability does not induce additional costs.

fits origin from investments in human capital because of individual migration prospects and where costs stem from the actual migration—, we focus on the consequences for the distribution of basic research investments across countries when researchers' wages are determined globally. We analyze an extreme situation, in which all researchers of the economy belong to one market. Subsequently, governments employ the researchers given the researcher's wage rate, which is determined on the global market. We find that mobility of researchers with a globally prevailing wage rate implies that basic research investments are more unequally distributed across countries. For countries with high productive knowledge, researchers are cheaper compared to the case of immobile researchers, and the contrary holds for countries with low productive knowledge.[2]

In the previous chapter, governments could not influence the diffusion process of ideas, neither in basic research nor in applied research. In Section 3.4, we will discuss openness of research, i.e. how easily ideas diffuse and become absorbed, from a small open economy perspective.[3] We discuss that governments have an interest in securing and attracting ideas, where attracting ideas can be seen as a sharing and open research policy, whereas securing ideas stands for a less open research policy.[4] Previous theoretical models of basic research investment in open economies have focused on one or another notion of openness. The early literature emphasized the role of basic research as a global public good (Arrow, 1962; Nelson, 1959). Later, local effects and strategic interactions were introduced. Park (1998) analyzes a two-country model with local effects and spillovers, where domestic and foreign knowledge accumulation are independent. Openness refers to the degree of substitutability between the knowledge stocks of countries. In Gersbach and Schneider (2015), who study basic research investment in a small open economy, ideas created in basic research are only non-rival in the country of origin, and spillovers occur because of openness with respect to exports and foreign direct investment. In the model presented in Chapter 2, there were *local* gains from basic research and *global* spillovers. We will now discuss how governments decide, when they can invest to secure own ideas and absorb ideas from the public domain, i.e. ideas that were generated via basic research investments of other governments. In our model, openness is thus not related to trade, and we assume that trade is unimpeded by the governments' decisions on the inflow and outflow of ideas generated by basic research, but related to openness in the sense of Park

[2] We abstain form analyzing efficiency gains from the reallocation of researchers from countries with low productive knowledge to countries with high productive knowledge.

[3] Thus, openness of research does not refer to the Mertonian norms of basic research nor to the freedom of researchers in their research endeavors.

[4] In contrast to our discussion on the Bayh-Dole Act in Section 2.8.1 where we focused on the trade-off between θ_D and $\eta_1(r)$, we will discuss the trade-off between profits obtained through own basic research investments and those obtained though basic research investments of other countries.

(1998). We will not be able to conclude from our discussion for which levels of productive knowledge openness of research is of highest relevance, as we find that the effects depend on the parameter choices.

For the analysis of these extensions, we will simplify the model of Chapter 2 substantially. We present the simplified model in Section 3.1 and will use it in all following sections. We analyze basic research infrastructure costs in Section 3.2, the effects mobile researchers have on the distribution of basic research investments in Section 3.3, and openness of research in Section 3.4.

3.1. A Simplified Model

We make two simplifications. First, we assume that there is no targeting. The effects of the following extensions are present independent of whether governments can target their basic research investments to industries or not. Targeting shows potential inefficiencies of basic research investments across industries, yet, the effect of these inefficiencies is unidirectional and can only skew the distribution of basic research investments more towards the higher-skilled countries. This is because industries of high complexity are weakly more profitable than industries of low complexity. Targeting would not yield additional insights to our analysis of the following extensions, thus we assume that $\kappa := 0$, i.e. basic research cannot be targeted.

Second, we assume that abilities are Pareto distributed. This allows us to analytically solve each variable of the model in dependence of exogenous parameters. We assume that basic research production is characterized by a power function

$$\eta_2(\xi^r) := \zeta \xi^{r\alpha} ,$$

where $\alpha \in (0, 1)$ and ζ is a positive scaling parameter. Following (2.9), country r produces an amount of ideas, η^r, in *every* industry, corresponding to its choice of ξ^r, and thus

$$\begin{aligned} \eta^r =& \eta_i^r(\xi^r, \mathcal{I}_{BR}^r) \left[\frac{\kappa}{I_{BR}^r} \mathbb{1}_{[i \in \mathcal{I}_{BR}^r]} + \frac{1-\kappa}{I} \right]_{\kappa=0} \\ =& \eta_1(r)\eta_2(\xi^r)\frac{L^r}{I} \\ =& \eta_1(r)\zeta \left[\xi^r\right]^\alpha L^r , \end{aligned} \tag{3.1}$$

where we integrated I in ζ without loss of generality. The power function $\eta_2(\xi^r)$ fulfills the concavity assumption we need for existence and uniqueness of the equilibrium, given the parameter constraint on α and because $\xi^r \in (0,1)$. A small value of α will produce a small share of scientists in the labor force and reflect the fact that generating more half-ideas quickly becomes very difficult.[5] Following Chapter 2, we assume that congestion effects stem from the innate ability levels.[6] By using power function (3.1), we define the ability distribution, denoted by $F_a(a)$, and we show that $F_a(a)$ corresponds to a Pareto distribution. We know that $\eta_2(\xi^r) = \int_0^{\xi^r} F_a^{-1}(1-x)dx = \zeta \xi^{r\alpha}$ and thus the ability, a, of the marginal worker employed is $a(x) = F_a^{-1}(1-x) = \alpha\zeta x^{\alpha-1}$. We restrict the ability domain to cover $[\alpha\zeta, \infty)$ and it must hold that

$$\lim_{x\to 0} a(x) = \infty \qquad \text{and} \qquad \lim_{x\to 1} a(x) = \alpha\zeta \; .$$

Observe that the fewer scientists employed, the higher the ability of the marginal worker. Thus, the Pareto distribution of abilities,

$$F_a(a) = 1 - \left[\frac{\alpha\zeta}{a}\right]^{\frac{1}{1-\alpha}} \; ,$$

corresponds to the power function assumption for $\eta_2(\xi^r)$ above on the domain $a \in [\alpha\zeta, \infty)$. The lower α and ζ, the closer is the lowest ability to zero.

The rest of the innovation process is equal to the innovation process in Chapter 2, i.e. the diffusion process of an idea is governed by the probability $\theta^r_{D,i} = \mathbb{1}_{[i \leq \tilde{i}(r)]}\theta_D$ that an idea can be commercialized by the local domestic production and by the probability $\theta^r_{G,i}$ that domestic applied researchers encounter ideas in the public domain (see (2.11) and (2.17)). We next turn to the decentralized equilibrium.

3.1.1. Decentralized Investment in Basic Research

Optimization problems and equilibrium outcomes are as in Chapter 2. We assume that there are sufficient skills in the economy, and thus, we can make use of the equilibrium wage rate $w^r = \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}}$ and of the concept of *effective* labor.[7] Furthermore, we assume

[5] A small value of α also implies a high return on investment, $\frac{1-\alpha}{\alpha}$. Three features lead to this high return in the model: First, there are no fixed costs to undertake basic research. We analyze basic research infrastructure costs in Section 3.2. Second, and most importantly, the model is static, implying that the entire range of products available in the economy is generated instantaneously. If there is a stock of varieties available in the economy, the marginal return—and thus profits—of an additional variety would be smaller. Third, ideas might be indivisible, implying an additional fixed cost effect of basic research. Then, for profitable basic research investments, a fixed amount of indivisible ideas has to be generated.

[6] We could think of other causes for congestion effects, e.g. a ladder of knowledge creation or limitations of research endeavors. A ladder of knowledge creation can only be modeled if the framework encompasses dynamics.

[7] We show in Appendix A.2 that with Pareto distributed abilities, there are always parameters that guarantee sufficient skills in the economy.

that υ, the cost of setting up a research lab, is close to zero. The following system of equations then solves the model:

$$\xi^r(r) = \left[\zeta\alpha\theta_D \frac{\eta_1(r)}{w^r} \sum_{i\in\mathcal{I}(r)} \pi_i\right]^{\frac{1}{1-\alpha}} , \tag{3.2}$$

$$\pi_i = \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}}}{[\sigma_v - 1]\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}}} \frac{\tilde{L}_p}{\hat{N}} , \tag{3.3}$$

$$\hat{N} = \int_{\underline{r}}^{\bar{r}} \eta_1(r) L^r \zeta \xi^r(r)^\alpha f(r) dr , \tag{3.4}$$

$$\tilde{L}_p = \int_{\underline{r}}^{\bar{r}} L^r [1 - \xi^r(r)] \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r) dr , \tag{3.5}$$

where $\hat{N} = N_i \ \forall i \in \mathcal{I}$ denotes the number of varieties in every industry. Plug (3.3) into (3.2) and then first into (3.4) and solve for $\hat{N}$. Second, plug again (3.3) into (3.2) and then into (3.5) to obtain

$$\hat{N} = \zeta \left[\frac{\alpha\theta_D \tilde{L}_p}{\sigma_v - 1}\right]^\alpha \left[\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r) dr\right]^{1-\alpha} , \tag{3.6}$$

$$\tilde{L}_p = \tilde{L} - \left[\frac{\zeta\alpha\theta_D}{\sigma_v - 1} \frac{\tilde{L}_p}{\hat{N}}\right]^{\frac{1}{1-\alpha}} \int_{\underline{r}}^{\bar{r}} L^r w^r g(r)^{\frac{1}{1-\alpha}} f(r) dr , \tag{3.7}$$

where we define $g(r) := \frac{\eta_1(r)}{w^r} \sum_{i\in\mathcal{I}(r)} \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}}}{\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}}} := \frac{\eta_1(r)}{w^r} z(r)$ to ease notation.

We next plug (3.6) into (3.7). Some algebraic manipulations then yield total effective labor in production and basic research

$$\tilde{L}_p = \tilde{L} \frac{[\sigma_v - 1] \int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r) dr}{\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r) [\sigma_v - 1 + \alpha\theta_D z(r)] dr} ,$$

$$\tilde{L}_{BR} = \tilde{L} \frac{\alpha\theta_D \int_{\underline{r}}^{\bar{r}} w^r L^r g(r)^{\frac{1}{1-\alpha}} f(r) dr}{\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r) [\sigma_v - 1 + \alpha\theta_D z(r)] dr} ,$$

and the number of varieties in the industries is

$$\hat{N} = \frac{\zeta \left[\alpha\theta_D \tilde{L}\right]^\alpha \int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r) dr}{\left[\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r) [\sigma_v - 1 + \alpha\theta_D z(r)] dr\right]^\alpha} .$$

Note that the scaling parameter ζ linearly affects the number of varieties in the industries. However, it does not affect the amount of effective labor that is used in basic research. We know from Chapter 2 that the higher the productive knowledge of a country, the higher its share of scientists, ξ^r, if basic research production is more skill-intensive than goods production, i.e. if $\frac{\eta_1(r)}{w^r}$ is increasing in r.[8] We assume that this is the case. Furthermore, some countries are never able to produce in all industries, i.e. $z(r) < 1$ for these countries. Country $\bar{r}$, the country with the highest productive knowledge, decides to invest a share

$$\xi^r(\bar{r}) = \frac{\alpha\theta_D\tilde{L}}{\int_{\underline{r}}^{\bar{r}} L^r\eta_1(r)g(r)^{\frac{\alpha}{1-\alpha}}f(r)[\sigma_v - 1 + \alpha\theta_D z(r)]dr}$$

of its labor force in basic research. We can conclude that either α or θ_D has to be sufficiently small to guarantee that $\xi^r(\bar{r}) \ll 1$. For any country r, the share of scientists can be related to $\xi^r(\bar{r})$ by

$$\xi^r(r) = g(r)^{\frac{1}{1-\alpha}}\xi^r(\bar{r}) \ .$$

Observe that $g(r)$ is monotonically and non-continuously increasing in r and that $g(r) \leq 1$ on $[\underline{r}, \bar{r})$ under the assumptions made.
The sufficient skill condition is now analytically defined as

Definition 3.1 (SSC / Pareto: SSCP)

$$\int_{e^{-\frac{1}{\hat{i}\lambda}}}^{\bar{r}} \tilde{L}^r \left[1 - g(r)^{\frac{1}{1-\alpha}}\xi^r_{BR}(\bar{r})\right] dF_r(r) \geq$$
$$\sum_{i\in\mathcal{I}:i\geq\hat{i}} \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}}}{\sum_{\hat{i}\in\mathcal{I}}\psi_{\hat{i}}\hat{i}^{\frac{1-\sigma_I}{\lambda}}} \int_{\underline{r}}^{\bar{r}} \tilde{L}^r \left[1 - g(r)^{\frac{1}{1-\alpha}}\xi^r_{BR}(\bar{r})\right] dF_r(r) \ , \quad \forall \hat{i} \in \mathcal{I} \ . \quad \text{(SSCP)}$$

This is a special case of SSC presented in Section 2.4.1. Condition SSCP states that governmental basic research investments do not tighten labor strongly enough to distort the wage scheme. Thus, also the relation between the level of productive knowledge and the diversity of production remains unimpaired. Evidence in the data supports this relation, since basic research investments even in the countries with the highest level of productive knowledge is below 1% of GDP[9] and highly productive countries export a diverse range of products.[10] We assumed that $\underline{i} = \min\mathcal{I} < \tilde{i}(\underline{r})$, $\bar{i} = \max\mathcal{I} < \tilde{i}(\bar{r})$, and $\bar{i} > \tilde{i}(\underline{r})$, i.e. there are industries that can be mastered by all countries and the most complex industry is only present in countries where $r > \tilde{r}(\bar{i})$.

[8] In Chapter 2 we assumed that $\epsilon_{\eta_1} \geq -\frac{1}{\lambda\log(r)}$. If the inequality holds strictly, then $\frac{d}{dr}\frac{\eta_1(r)}{w^r} = \frac{\eta_1(r)}{w^r}\frac{1}{r}\left[\epsilon_{\eta_1} - \epsilon_{w^r}\right] > \frac{\eta_1(r)}{w^r}\frac{1}{r}\left[-\frac{1}{\lambda\log(r)} + \frac{1}{\lambda\log(r)}\right]$. If $\epsilon_{\eta_1} = -\frac{1}{\lambda\log(r)}$, then $\frac{d}{dr}\frac{\eta_1(r)}{w^r} = 0 \ \forall r \in \mathcal{R}$ and hence $\frac{\eta_1(r)}{w^r} = \frac{\eta_1(\bar{r})}{w^{\bar{r}}} = 1$.
[9] Cf. Footnote 41 in Chapter 2.
[10] See Hausmann and Hidalgo (2010), Hausmann et al. (2011), and Schetter (2014, 2018).

Let us revert to Assumption 2.1 and Assumption 2.2 of Chapter 2. Under these assumptions, an idea cannot be retained by country r if it has complexity $i > \tilde{i}(r)$ or if no encounter takes place between a scientist in basic research and a domestic agent who would be willing to set up an applied research lab. Then, ideas that are generated in one country, but are not absorbed by this country, flow into the "global pool of ideas". From this global pool, ideas flow back to countries. This diffusion process can be modeled differently. We assume again that the countries' effective labor force relative to the effective global labor force that is able to manage the complexity of said industry governs this diffusion process,[11]

$$\theta^r_{G,i} = \frac{\tilde{L}^r}{\int_{\tilde{r}(i)}^{\bar{r}} \tilde{L}^r f(r)dr} .$$

We can now derive the ratio of profits to total wage income in country r, which is

$$\begin{aligned}\frac{\Pi^r}{L^r w^r} =& \frac{\tilde{L}}{x} \sum_{i \in \mathcal{I}(r)} \frac{1}{\int_{\tilde{r}(i)}^{\bar{r}} L^r f(r)dr} \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}}}{\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}}} \left[\hat{N} - \theta_D \int_{\tilde{r}(i)}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{\alpha-1}} f(r)dr\right] \\ &+ \frac{\tilde{L}}{x}\theta_D \left[\frac{\eta_1(r)}{w^r} \sum_{i\in\mathcal{I}(r)} \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}}}{\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}}}\right]^{\frac{1}{1-\alpha}} ,\end{aligned}$$

where $x = \int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r)[\sigma_v - 1 + \alpha\theta_D z(r)]dr$. Both terms are increasing in r. The higher r, the more basic research a country undertakes and the more ideas it is able to absorb from the global pool of ideas. This result was also obtained in Corollary 2.2.

We next demonstrate the equilibrium in Proposition 3.1.

Proposition 3.1

Suppose that Condition SSCP holds. Then, there exists a unique equilibrium with

(i) $\hat{N}^\star = \frac{\zeta\left[\alpha\theta_D \tilde{L}\right]^\alpha \int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r)dr}{\left[\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r)[\sigma_v - 1 + \alpha\theta_D z(r)]dr\right]^\alpha}$,

(ii) $\eta^{r^\star} = \eta_1(r) L^r \zeta \xi^{r\star}(r)^\alpha$,

(iii) $\xi^{r\star}(\bar{r}) = \frac{\alpha\theta_D \tilde{L}}{\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r)[\sigma_v - 1 + \alpha\theta_D z(r)]dr}$,

(iv) $\xi^{r\star}(r) = g(r)^{\frac{1}{1-\alpha}} \xi^{r\star}(\bar{r})$,

(v) $w^{r^\star} = \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} \quad \forall r \in \mathcal{R}$,

(vi) $\mathcal{R}^\star_i \subseteq \{r \in \mathcal{R} \mid r \geq \tilde{r}(i)\} \quad \forall i \in \mathcal{I}$,

[11] Cf. Section 2.5.

(vii) $q_i^\star(r) = \left[-\frac{1}{\lambda i \log(r)}\right]^{\frac{1}{\lambda}} \quad \forall\, (i,r) \in \mathcal{I} \times \mathcal{R}_i^\star$,

(viii) $\rho_i^\star = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i \log(\bar{r})\right]^{\frac{1}{\lambda}} \quad \forall\, i \in \mathcal{I}$,
$P_i^\star = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i \log(\bar{r})\right]^{\frac{1}{\lambda}}\, \hat{N}^{\star\frac{1}{1-\sigma_v}} \quad \forall\, i \in \mathcal{I}$,
$P^\star = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda \log(\bar{r})\right]^{\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{1}{1-\sigma_I}} \hat{N}^{\star\frac{1}{1-\sigma_v}}$,

(ix) $\tilde{l}_i^\star = \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}}}{\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}}} \frac{\tilde{L}_p^\star}{\hat{N}^\star} \quad \forall\, i \in \mathcal{I}$,

(x) $\chi_i^\star = \left[-e\lambda i \log(\bar{r})\right]^{-\frac{1}{\lambda}}\, \tilde{l}_i^\star \quad \forall\, i \in \mathcal{I}$,

(xi) $\pi_i^\star = \frac{\tilde{l}_i^\star}{\sigma_v - 1} \quad \forall\, i \in \mathcal{I}$,

(xii) $\tilde{L}_p^\star = \tilde{L} \frac{[\sigma_v - 1]\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r) dr}{\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r)[\sigma_v - 1 + \alpha\theta_D z(r)] dr}$,

(xiii) $\tilde{L}_{BR}^\star = \tilde{L} \frac{\alpha\theta_D \int_{\underline{r}}^{\bar{r}} L^r w^r g(r)^{\frac{1}{1-\alpha}} f(r) dr}{\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r)[\sigma_v - 1 + \alpha\theta_D z(r)] dr}$,

(xiv) $C^\star = \left[-e\lambda \log(\bar{r})\right]^{-\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{1}{\sigma_I - 1}} \hat{N}^{\star\frac{1}{\sigma_v - 1}} \tilde{L}_p^\star$ *and* $P^\star C^\star = \frac{\sigma_v}{\sigma_v - 1} \tilde{L}_p^\star$,

(xv) $P^\star C^{r\star} = L^r w^{r\star}\left[1 - \xi^{r\star}(r)\right] + \Pi^{r\star} \quad \forall\, r \in \mathcal{R}$,

(xvi) $\Pi^{r\star} = \sum_{i\in\mathcal{I}(r)} \frac{\tilde{L}^r}{\int_{\tilde{r}(i)}^{\bar{r}} \tilde{L}^r f(r) dr} \left[\hat{N}^\star - \theta_D \int_{\tilde{r}(i)}^{\bar{r}} \eta^{r\star}(r) f(r) dr\right] \pi_i^\star + \eta^{r\star}\theta_D \sum_{i\in\mathcal{I}(r)} \pi_i^\star$,

where $\tilde{L}_p^\star := \int_{\underline{r}}^{\bar{r}} L^r \left[1 - \xi^{r\star}(r)\right] \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r) dr$ *is aggregate supply of effective labor. The values of the representative firm of each industry* i *hold for all firms producing variety* $j \in [0, N^\star]$ *in that industry.*

We next turn to the social planner solution.

3.1.2. Social Planner Solution

With a utilitarian approach and a linear utility function, the social planner simply maximizes aggregate consumption. We will use subscript S to denote variables and equilibrium outcomes chosen by the social planner. The social planner distributes basic research efforts across the countries, such that the marginal product and the marginal cost of basic research investments are equalized. This results in

$$\xi_S^r(r) = \left[\frac{\eta_1(r)}{w^r}\right]^{\frac{1}{1-\alpha}} \xi_S^{\bar{r}} \, . \tag{3.8}$$

Given the social planner's optimal allocation choice, he is left to choose aggregate basic research investments. His optimization problem then can be described by

$$\max_{\xi_S{}^{\bar{r}}} C(\hat{N}_S(\xi_S{}^{\bar{r}}), \tilde{L}_{BR,S}(\xi_{BR,S}{}^{\bar{r}})) \quad \text{s.t.}$$

$$\hat{N}_S = \zeta[\xi_S{}^{\bar{r}}]^{\alpha} \int_{\underline{r}}^{\bar{r}} L^r \eta_1(r)^{\frac{1}{1-\alpha}} w^{r\frac{\alpha}{\alpha-1}} f(r)dr \ ,$$

$$\tilde{L}_{BR,S} = \xi_S{}^{\bar{r}} \int_{\underline{r}}^{\bar{r}} L^r \eta_1(r)^{\frac{1}{1-\alpha}} w^{r\frac{\alpha}{\alpha-1}} f(r)dr \ .$$

The solution to the social planner's optimization equals

$$\xi_S{}^{\bar{r}} = \frac{\alpha}{\sigma_v - 1 + \alpha} \tilde{L} \left[\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r)^{\frac{1}{1-\alpha}} w^{r\frac{\alpha}{\alpha-1}} f(r)dr \right]^{-1} .$$

Using (3.2), it follows that labor is allocated between production and basic research according to,

$$\tilde{L}_{p,S} = \frac{\sigma_v - 1}{\sigma_v - 1 + \alpha} \tilde{L} \quad \text{and} \quad \tilde{L}_{BR,S} = \frac{\alpha}{\sigma_v - 1 + \alpha} \tilde{L} \ . \tag{3.9}$$

The number of varieties in the industries is

$$\hat{N}_S = \zeta \left[\frac{\alpha}{\sigma_v - 1 + \alpha} \tilde{L} \right]^{\alpha} \left[\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r)^{\frac{1}{1-\alpha}} w^{r\frac{\alpha}{\alpha-1}} f(r)dr \right]^{1-\alpha} .$$

Note that if $w^r = \eta_1(r)$, i.e. there are equal elasticities in production and basic research, the number of varieties can be expressed through

$$\hat{N}_S = \zeta \left[\frac{\alpha}{\sigma_v - 1 + \alpha} \right]^{\alpha} \tilde{L} \ . \tag{3.10}$$

Formula (3.10) has intuitive comparative static properties. A higher value of α increases $\hat{N}_S$, as it becomes less difficult to produce new ideas. A higher value of σ_v reduces $\hat{N}_S$, as utility gains from more varieties decline.

In order to ensure that the presented variables represent the equilibrium chosen by the social planner, again, there must be sufficient skills in the economy:

Definition 3.2 (SSC / Pareto / Social Planner: SSCP$_S$)

$$\int_{e^{-\frac{1}{\hat{i}\lambda}}}^{\bar{r}} \tilde{L}^r \left[1 - \left[\frac{\eta_1(r)}{w^r}\right]^{\frac{1}{1-\alpha}} {\xi_S}^{\bar{r}}\right] dF_r(r) \geq$$
$$\sum_{i \in \mathcal{I}: i \geq \hat{i}} \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}}}{\sum_{\hat{i} \in \mathcal{I}} \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}}} \int_{\underline{r}}^{\bar{r}} \tilde{L}^r \left[1 - \left[\frac{\eta_1(r)}{w^r}\right]^{\frac{1}{1-\alpha}} {\xi_S}^{\bar{r}}\right] dF_r(r) , \quad \forall \hat{i} \in \mathcal{I} . \tag{SSCP$_S$}$$

Whether condition SSCP or SSCP$_S$ is tighter crucially depends on $F_r(r)$.[12] We next compare the two equilibria.

3.1.3. Comparing the Decentralized Equilibrium to the Social Planner Solution

The share of scientists in country $\bar{r}$ in the two equilibria is,

$$\text{D.E.:} \qquad \xi^r(\bar{r}) = \frac{\alpha\theta_D}{\int_{\underline{r}}^{\bar{r}} \eta_1(r)^{\frac{1}{1-\alpha}} w^{r\,\frac{\alpha}{\alpha-1}} L^r z(r)^{\frac{\alpha}{1-\alpha}} f(r)[\sigma_v - 1 + \alpha\theta_D z(r)]dr} \tilde{L} ,$$
$$\text{S.P.:} \qquad {\xi_S}^{\bar{r}} = \frac{\alpha}{\sigma_v - 1 + \alpha} \left[\int_{\underline{r}}^{\bar{r}} \eta_1(r)^{\frac{1}{1-\alpha}} w^{r\,\frac{\alpha}{\alpha-1}} L^r f(r)dr\right]^{-1} \tilde{L} .$$

We observe that if $\theta_D = 0$, no country is willing to invest into basic research and there is no decentralized equilibrium with positive basic research, whereas the social planner solution is independent of θ_D.

If $\theta_D = 1$, we can show that $\xi^r(\bar{r}) > {\xi_S}^{\bar{r}}$ because $z(r) < 1$ for a measurable set of countries. Countries with low productive knowledge invest less in basic research compared to the social planner, because they cannot retain all ideas generated via their investments. Their productive knowledge is not high enough for firms of high-complexity industries to locate a production site in their country. Yet, by Assumption 2.2, domestic production is a necessary (and sufficient) precondition that ideas can be retained in a country. Because these countries invest less in basic research compared to the social planner, countries with

[12] Condition SSCP$_S$ can also be written as

$$\int_{e^{-\frac{1}{\hat{i}\lambda}}}^{\bar{r}} \tilde{L}^r - \tilde{L}_{BR,S} \frac{\left[\frac{\eta_1(r)}{w^r}\right]^{\frac{1}{1-\alpha}} \tilde{L}^r}{\int_{\underline{r}}^{\bar{r}} \left[\frac{\eta_1(r)}{w^r}\right]^{\frac{1}{1-\alpha}} \tilde{L}^r dF_r(r)} dF_r(r) \geq \sum_{i \in \mathcal{I}: i \geq \hat{i}} \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}}}{\sum_{\hat{i} \in \mathcal{I}} \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}}} \tilde{L}_{p,S} , \quad \forall \hat{i} \in \mathcal{I} .$$

where the right-hand side is simply the demand of the industries depending on total effective labor in production and the left-hand side subtracts the scientist from the labor supply for every skill level.

high productive knowledge invest more in basic research than the social planner.

We now compare the governments' scientist allocation decision (3.2) to the social planner's allocation function (3.8),

$$\text{D.E.:} \qquad \xi^r(r) = \left[\frac{\eta_1(r)}{w^r} z(r)\right]^{\frac{1}{1-\alpha}} \xi^r(\bar{r}) \ ,$$

$$\text{S.P.:} \qquad \xi^r_S(r) = \left[\frac{\eta_1(r)}{w^r}\right]^{\frac{1}{1-\alpha}} \xi_S{}^{\bar{r}} \ .$$

Observe that $z(r)$ is an increasing step function, and it holds that $\frac{\xi^r(r^h)}{\xi^r(r^l)} \geq \frac{\xi^r_S(r^h)}{\xi^r_S(r^l)}$, where $r^h > r^l$ and $r^h, r^l \in \mathcal{R}$. In that sense, the allocation scheme in the decentralized equilibrium is steeper along the productive knowledge dimension.

We compare total effective labor allocated to the basic research sector,

$$\text{D.E.:} \qquad \tilde{L}_{BR} = \frac{\alpha\theta_D \int_{\underline{r}}^{\bar{r}} L^r w^r g(r)^{\frac{1}{1-\alpha}} f(r) dr}{\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r)[\sigma_v - 1 + \alpha\theta_D z(r)] dr} \tilde{L} \ ,$$

$$\text{S.P.:} \qquad \tilde{L}_{BR,S} = \frac{\alpha}{\sigma_v - 1 + \alpha} \tilde{L} \ ,$$

and we infer the following proposition:

Proposition 3.2
There is more total effective labor in basic research in the social planner solution than in the decentralized equilibrium.

PROOF: We prove that $\tilde{L}_{BR,S} > \tilde{L}_{BR}$ for all $\theta_D \in [0, 1]$.

Suppose $\tilde{L}_{BR,S} < \tilde{L}_{BR}$, then[13]

$$\theta_D > \frac{\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r) dr}{\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} z(r) f(r) dr} > 1$$

as $z(r) < 1$ for a measurable set of countries. This contradicts the supposition, as $\theta_D \in [0, 1]$.

□

[13] This ratio is simply a rearrangement of the two aggregate basic research investments, $\tilde{L}_{BR,S}$ and $\tilde{L}_{BR}$.

The numbers of varieties in each industry in the two equilibria are

$$
\begin{aligned}
\text{D.E.:} \qquad \hat{N} =& \zeta \left[\alpha\theta_D \tilde{L}\right]^{\alpha} \frac{\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r) dr}{\left[\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r)[\sigma_v - 1 + \alpha\theta_D z(r)] dr\right]^{\alpha}} \\
=& \zeta \left[\xi^r(\bar{r})\right]^{\alpha} \int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r) dr \\
=& \zeta \left[\tilde{L}_{BR}\right]^{\alpha} \int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r) dr \left[\int_{\underline{r}}^{\bar{r}} L^r w^r g(r)^{\frac{1}{1-\alpha}} f(r) dr\right]^{-\alpha} , \\
\text{S.P.:} \qquad \hat{N}_S =& \zeta \left[\frac{\alpha}{\sigma_v - 1 + \alpha}\tilde{L}\right]^{\alpha} \left[\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r)^{\frac{1}{1-\alpha}} w^{r\,\frac{\alpha}{\alpha-1}} f(r) dr\right]^{1-\alpha} \\
=& \zeta \left[\xi_S{}^{\bar{r}}\right]^{\alpha} \int_{\underline{r}}^{\bar{r}} L^r \eta_1(r)^{\frac{1}{1-\alpha}} w^{r\,\frac{\alpha}{\alpha-1}} f(r) dr \\
=& \zeta \left[\tilde{L}_{BR,S}\right]^{\alpha} \left[\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r)^{\frac{1}{1-\alpha}} w^{r\,\frac{\alpha}{\alpha-1}} f(r) dr\right]^{1-\alpha} .
\end{aligned}
$$

Analyzing the numbers of varieties leads to Proposition 3.3.

Proposition 3.3

For the same amount of total effective labor in basic research, there are fewer varieties in the decentralized equilibrium than in the social planner equilibrium.

PROOF: We prove that $\hat{N}(\tilde{L}_{BR}) \leq \hat{N}_S(\tilde{L}_{BR})$ for any $\tilde{L}_{BR} > 0$, which implies

$$
\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r) dr \left[\int_{\underline{r}}^{\bar{r}} L^r w^r g(r)^{\frac{1}{1-\alpha}} f(r) dr\right]^{-\alpha} \leq \left[\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r)^{\frac{1}{1-\alpha}} w^{r\,\frac{\alpha}{\alpha-1}} f(r) dr\right]^{1-\alpha} .
$$

We rearrange to obtain

$$
\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r) dr \leq \left[\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r)^{\frac{1}{1-\alpha}} w^{r\,\frac{\alpha}{\alpha-1}} f(r) dr\right]^{1-\alpha} \left[\int_{\underline{r}}^{\bar{r}} L^r w^r g(r)^{\frac{1}{1-\alpha}} f(r) dr\right]^{\alpha} ,
$$

which we use in Hölder's Inequality, which states that

$$
\int_{\mathcal{S}} |h(x)t(x)|\, dx \leq \left[\int_{\mathcal{S}} |h(x)|^p\, dx\right]^{\frac{1}{p}} \left[\int_{\mathcal{S}} |t(x)|^q\, dx\right]^{\frac{1}{q}} ,
$$

where $\mathcal{S}$ is a measurable subset of $\mathcal{R}^n$and f and t are measurable real-valued or complex-valued functions on $\mathcal{S}$. Additionally $p, q \in (1, \infty)$ and $\frac{1}{p} + \frac{1}{q} = 1$.

We set $p := \frac{1}{1-\alpha}$, $q := \frac{1}{\alpha}$, $h(x) := L^{r\,1-\alpha}\eta_1(r) w^{r\,-\alpha} f(r)^{1-\alpha}$, $t(x) := L^{r\,\alpha} w^{r\,\alpha} g(r)^{\frac{\alpha}{1-\alpha}} f(r)^{\alpha}$ and $\mathcal{S} := [\underline{r}, \bar{r}]$. The functions $\eta_1(r)$, w^r and $z(r)$ are positive-valued and we can discard the absolute value and obtain

$$\begin{aligned}\int_S h(x)t(x)dx &= \int_{\underline{r}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} f(r)dr \\ &\leq \left[\int_S h(x)^p dx\right]^{\frac{1}{p}} \left[\int_S t(x)^q dx\right]^{\frac{1}{q}} \\ &= \left[\int_{\underline{r}}^{\bar{r}} \left[L^{r\,1-\alpha} \eta_1(r) w^{r\,-\alpha} f(r)^{1-\alpha}\right]^{\frac{1}{1-\alpha}} dr\right]^{1-\alpha} \\ &\quad \left[\int_{\underline{r}}^{\bar{r}} \left[L^{r\alpha} w^{r\alpha} g(r)^{\frac{\alpha}{1-\alpha}} f(r)^{\alpha}\right]^{\frac{1}{\alpha}} dr\right]^{\alpha} \\ &= \left[\int_{\underline{r}}^{\bar{r}} L^r \eta_1(r)^{\frac{1}{1-\alpha}} w^{r\,\frac{\alpha}{\alpha-1}} f(r)dr\right]^{1-\alpha} \\ &\quad \left[\int_{\underline{r}}^{\bar{r}} L^r w^r g(r)^{\frac{1}{1-\alpha}} f(r)dr\right]^{\alpha}.\end{aligned}$$

□

Not surprisingly, the social planner allocates scientists more efficiently across countries and generates a higher number of varieties in each industry given effective labor input. Note that this effective labor input equals total cost for this labor and thus, total basic research investment.

Suppose the social planner can only choose the socially optimal total amount of investments in basic research given the allocation decisions of governments. The social planner's choice of total effective labor is then unaffected by this constraint and he again chooses total effective labor in basic research equal to (3.9). In such case, there is an over-provision of scientists in countries with high levels of productive knowledge, because the decentralized allocation function is steeper than the one optimally chosen by the social planner, and thus there are fewer varieties than in the socially optimal equilibrium.[14]

We are now equipped with a simple and tractable model for the following extensions.

3.2. Basic Research Infrastructure

We now examine the impact of basic research infrastructure costs. Typically, generating ideas via basic research requires an infrastructure in the form of buildings, labs, equipments, materials and governance structures. If an adequate infrastructure is a prerequisite for basic research investments, which are investments in the form of labor, then infrastructure costs can prevent governments from investing in basic research. Specifically, we

[14] This result stems from the assumption of a Pareto distribution of abilities (c.f. Appendix B.1.8).

assume that building a basic research infrastructure requires fixed costs in terms of the consumption good, which is proportional to the effective labor force of country r. Thus, basic research infrastructure requires fixed costs $\Phi L^r w^r$ ($\Phi > 0$). We will assume that there are sufficient skills in the economy, and thus, $\Phi L^r w^r = \Phi \tilde{L}^r$, i.e. the share of the economy's total wage income, which is spent for the buildup of the basic research infrastructure, equals the share of the workforce that must be detached for the buildup.

Decision variables in this section are subscripted by Φ. The government's problem now includes the costs of the buildup,

$$\max_{\xi_\Phi^r} \left\{ \eta^r(\xi_\Phi^r)\theta_D \sum_{i \in \mathcal{I}(r)} \pi_i - \xi_\Phi^r L^r w^r - \Phi \tilde{L}^r \right\} .$$

The constant Φ does not influence the government's optimal functional form of its decision process with respect to labor allocation, provided that it wants to invest in basic research at all. Government r then only invests if

$$\Phi \tilde{L}^r \leq \eta^r(\xi_\Phi^r(r))\theta_D \sum_{i \in \mathcal{I}(r)} \pi_i - \xi_\Phi^r(r) L^r w^r , \tag{3.11}$$

where $\xi_\Phi^r(r)$ denotes the optimal labor allocation decision depending on r. Note that the profits in the equilibrium with infrastructure costs differ from those of an equilibrium without such costs. Hence, the amount of labor optimally allocated to basic research also differs. We denote the country with the minimal profit of a basic research infrastructure investment by $r^\square$ and it must hold that

$$r^\square = \arg\min_r \quad \left\{ \eta_1^r(r)\eta_2(\xi_\Phi^r(r))\theta_D \sum_{i \in \mathcal{I}(r)} \pi_i - \xi_\Phi^r(r) w^r - \Phi w^r \right\} \tag{3.12}$$
$$\text{s.t. (3.11) holds.}$$

The government must levy a consumption tax from each household to pay for the infrastructure.[15]

Total effective labor in basic research, in production, and in infrastructure must sum up to total effective labor $\tilde{L}$,

$$\tilde{L}_{p,\Phi} = \tilde{L} - \int_{r^\square}^{\bar{r}} \xi_\Phi^r(r)\tilde{L}^r f(r)dr - \Phi \int_{r^\square}^{\bar{r}} \tilde{L}^r f(r)dr . \tag{3.13}$$

[15] Consumption and wage taxes are equivalent, as labor linearly enters the production function of firms and utility linearly depends on consumption. It is thus convenient to assume that the government directly withdraws labor from the production process that is equivalent to the loss of consumption due to the infrastructure cost.

Only countries with a productive knowledge level $r > r^{\square}$ are involved in basic research. The number of varieties in an industry then is

$$\hat{N}_{\Phi} = \int_{r^{\square}}^{\bar{r}} \eta(\xi_{\Phi}^{r}(r))f(r)dr \ . \tag{3.14}$$

Equations (3.12), (3.13), and (3.14) represent a non-linear system of equations with three unknowns, $\hat{N}_{\Phi}$, $\tilde{L}_{p,\Phi}$, and $r^{\square}$. In dependence on $r^{\square}$—which depends on the magnitude of Φ—we can solve for total effective labor in production and basic research and the number of varieties,

$$\tilde{L}_{p,\Phi} = \left[\tilde{L} - \Phi \int_{r^{\square}}^{\bar{r}} \tilde{L}^{r} f(r)dr\right] \frac{[\sigma_v - 1]\int_{r^{\square}}^{\bar{r}} \eta_1(r)L^r g(r)^{\frac{\alpha}{1-\alpha}} f(r)dr}{\int_{r^{\square}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{\alpha-1}} f(r)[\sigma_v - 1 + \alpha\theta_D z(r)]dr} ,$$
$$\tilde{L}_{BR,\Phi} = \left[\tilde{L} - \Phi \int_{r^{\square}}^{\bar{r}} \tilde{L}^{r} f(r)dr\right] \frac{\alpha\theta_D \int_{r^{\square}}^{\bar{r}} w^r L^r g(r)^{\frac{1}{1-\alpha}} f(r)dr}{\int_{r^{\square}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{\alpha-1}} f(r)[\sigma_v - 1 + \alpha\theta_D z(r)]dr} ,$$
$$\hat{N}_{\Phi} = \left[\frac{\alpha\theta_D\left[\tilde{L} - \Phi \int_{r^{\square}}^{\bar{r}} \tilde{L}^{r} f(r)dr\right]}{\int_{r^{\square}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{\alpha-1}} f(r)[\sigma_v - 1 + \alpha\theta_D z(r)]dr}\right]^{\alpha} \zeta \int_{r^{\square}}^{\bar{r}} \eta_1(r)L^r g(r)^{\frac{\alpha}{1-\alpha}} f(r)dr \ .$$

There are two effects: On the one hand, building up basic research infrastructure absorbs labor, which diminishes total effective labor. On the other hand, some countries may decide to abstain from doing basic research. Then, fewer countries are involved in basic research. Countries still engaged in basic research may or may not allocate a higher share of their population to the basic research sector than in an equilibrium without basic research infrastructure costs. The country with the highest productive knowledge allocates a share

$$\xi_{\Phi}^{r}(\bar{r}) = \frac{\alpha\theta_D\left[\tilde{L} - \Phi \int_{r^{\square}}^{\bar{r}} \tilde{L}^{r} f(r)dr\right]}{\int_{r^{\square}}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{\alpha-1}} f(r)[\sigma_v - 1 + \alpha\theta_D z(r)]dr} \tag{3.15}$$

to the basic research sector. The higher Φ, the higher $r^{\square}$, and the more labor is allocated to basic research in country $\bar{r}$, $\xi_{\Phi}^{r}(\bar{r})$, as long as its government decides to invest in basic research at all. Thus, sufficiently high basic research infrastructure costs, Φ_h, force some countries to refrain from doing basic research, whereas sufficiently low basic research infrastructure costs, Φ_l, allow every country to undertake basic research. We now choose two equilibria associated with Φ_h and Φ_l and obtain

$$\xi_{BR,\Phi_h}^{r}(\bar{r}) > \xi_{BR}^{r}(\bar{r}) > \xi_{BR,\Phi_l}^{r}(\bar{r}) \ .$$

The first inequality is analyzed below. The second inequality is trivial. Labor is absorbed for the basic research infrastructure buildup, and thereby diminishes the total labor force.

Thus, any $\Phi > 0$ for which $r^{\square}(\Phi) \leq \underline{r}$ diminishes overall basic research investments.

To examine the first inequality, we define

$$h(r') = [1-\alpha]\,\theta_D g(r')^{\frac{1}{1-\alpha}} \frac{\tilde{L} - \Phi \int_{r'}^{\bar{r}} \tilde{L}^r f(r)dr}{x(r')} w^{r'} - \Phi w^{r'} ,$$

where $x(r') = \int_{r'}^{\bar{r}} L^r \eta_1(r) g(r)^{\frac{\alpha}{\alpha-1}} f(r)[\sigma_v - 1 + \alpha\theta_D z(r)]dr$. $h(r)$ is the objective function of the problem introduced in (3.12). Function $h(r)$ is strictly monotonically, non-continuously increasing in the productive knowledge of countries on $\mathcal{R}$. The country with the lowest productive knowledge still engaged in basic research is

$$r^{\square} = \arg\min_{r} \quad h(r) \quad \text{s.t. (3.11) holds} .$$

We define the residual of the objective function at $r^{\square}$ by $c(\Phi, r^{\square})$, and thus $h(r^{\square}) = c(\Phi, r^{\square})$. Whenever $c(\Phi, r^{\square}) = 0$ the relation between Φ and $r^{\square}$ is

$$\Phi = \frac{y(r^{\square})\tilde{L}}{x(r^{\square}) + y(r^{\square}) \int_{r^{\square}}^{\bar{r}} \tilde{L}^r f(r)dr} , \tag{3.16}$$

where $y(r^{\square}) = [1-\alpha]\,\theta_D g(r^{\square})^{\frac{1}{1-\alpha}}$. As $y(r^{\square})$ is a step-function, (3.16) cannot hold with equality for all values of Φ. Thus, whenever $y(r^{\square})$ jumps at a marginal increase in $r^{\square}$, the residual $c(\Phi, r^{\square})$ is positively valued. Whenever $c(\Phi, r^{\square}) > 0$, the residual declines with higher basic research infrastructure cost, while $r^{\square}$ stays constant, in accordance to

$$c(\Phi, r^{\square}) = \frac{y(r^{\square})\tilde{L} - \Phi\left[x(r^{\square}) + y(r^{\square}) \int_{r^{\square}}^{\bar{r}} \tilde{L}^r f(r)dr\right]}{x(r^{\square})} .$$

The higher the cost of the basic research infrastructure, Φ, the weakly higher the minimum productive knowledge needed, $r^{\square}$, to profitably undertake basic research, i.e. the relation is monotone. Profits are monotonically increasing along the productive knowledge domain. This is because of the weakly increasing set of industries with domestic production.[16] At these productive knowledge levels, the residual $c(\Phi, r^{\square})$ turns positive, and said levels stay the lowest productive knowledge level engaged in basic research when Φ increases, until $c(\Phi, r^{\square}) = 0$ again.[17]

We assume for now that Φ is in a range where $c(\Phi, r^{\square}) = 0$ and plug (3.16) into (3.15) to

---

[16] If at some $r^{\square}$, it holds that $\mathbf{n}[\mathcal{I}(r^{\square})] > \mathbf{n}[\mathcal{I}(r^{\square} - \epsilon)]$ for arbitrary small $\epsilon > 0$, then $r^{\square}$ locates a step. The intervals between two step locations are left-closed and right-opened.

[17] We can introduce a continuum of industries to obtain a continuous one-to-one mapping between Φ and $r^{\square}$. With such an environment the analysis would be substantially simplified.

obtain

$$\xi^r_\Phi(\bar{r}) = \frac{\alpha\theta_D \tilde{L}}{x(r^\square) + y(r^\square)\int_{r^\square}^{\bar{r}} \tilde{L}^r f(r)dr} .$$

The denominator strictly decreases in $r^\square$. Thus, a sufficiently large $r^\square$—as a result of an exogenously given high Φ—results in the above-mentioned inequality $\xi^r_{\Phi_h}(\bar{r}) > \xi^r(\bar{r})$. Such a case induces all governments that are willing to pay for the infrastructure to allocate more labor to the basic research sector, compared to a case without basic research infrastructure costs. This is, because the relation

$$\xi^r_\Phi(r) = g(r)^{\frac{1}{1-\alpha}} \xi^r_\Phi(\bar{r}) ,$$

remains valid for all $r \geq r^\square$.

To describe the labor market, we assume that no country is constrained by the basic research infrastructure costs in the choice how many scientists to employ, i.e. for any country r engaged in basic research, it must hold that

$$\xi^r_\Phi(r) + \Phi \ll 1 . \tag{3.17}$$

The country with the highest productive knowledge employs the most scientists. Thus, it suffices to analyze (3.17) for $\bar{r}$. We obtain the following parameter restriction that must be fulfilled:

$$\frac{\tilde{L}\left[\alpha\theta_D + y(r^\square)\right]}{x(r^\square) + y(r^\square)\int_{r^\square}^{\bar{r}} \tilde{L}^r f(r)dr} \ll 1 .$$

The restriction essentially states that every country has a producing sector.[18]

There is a non-trivial connection to the sufficient skill condition, SSCP_Φ, when in addition to basic research, some labor is also absorbed for the basic research buildup.

Definition 3.3 (SSC / Pareto / Infrastructure Cost Φ: SSCP_Φ)

$$\int_{e^{-\frac{1}{\hat{i}\lambda}}}^{\bar{r}} \tilde{L}^r \, dF_r(r) - \int_{\max\left\{e^{-\frac{1}{\hat{i}\lambda}}, r^\square\right\}}^{\bar{r}} g(r)^{\frac{1}{1-\alpha}} \xi^r_\Phi(\bar{r}) \tilde{L}^r \, dF_r(r) - \Phi \int_{\max\left\{e^{-\frac{1}{\hat{i}\lambda}}, r^\square\right\}}^{\bar{r}} \tilde{L}^r \, dF_r(r) \geq$$
$$\sum_{i \in \mathcal{I}: i \geq \hat{i}} z(i) \left[\tilde{L} - \int_{r^\square}^{\bar{r}} g(r)^{\frac{1}{1-\alpha}} \xi^r_\Phi(\bar{r}) \tilde{L}^r \, dF_r(r) - \Phi \int_{r^\square}^{\bar{r}} \tilde{L}^r \, dF_r(r)\right] , \quad \forall \hat{i} \in \mathcal{I} . \tag{SSCP_Φ}$$

[18] For any reasonable calibration this inequality must hold (cf. Footnote 41 in Chapter 2).

We assume that basic research infrastructure costs are such that some countries abstain from investing in basic research. We then observe that there is more labor left for production in the countries that abstain from undertaking basic research. However, this labor is of low productive knowledge. In contrast, high-skilled labor of the countries still engaged in basic research may be scarcer because in addition to the researchers, some labor is absorbed in the basic research infrastructure buildup. Depending on parameters, there can be more or less labor in basic research in countries with high productive knowledge.

Observe that basic research infrastructure costs prevent some highly able researchers from engaging in basic research, as the productive knowledge r of their home country is too low. Essentially, this means that immobility of researchers, together with high basic research infrastructure costs, can result in substantial inefficiencies.

The case of infrastructure costs in basic research is representative for other entry barriers that are costly to overcome. Examples are scientist networks and research communities, tacit knowledge, collaborations and knowledge retention. We showed that countries with high productive knowledge might invest more in basic research when basic research infrastructure costs are high, because in such a case some countries of low productive knowledge abstain from investing in basic research at all. Note that the countries without basic research might obtain profits from absorbing "foreign" ideas. However, they are unable to generate profit via own ideas generated in the basic research sector.

In the case in which only a subset of countries, i.e. countries $r^{\square}$ to $\bar{r}$, undertakes basic research, total consumption is

$$\begin{aligned} C_{\Phi}(r^{\square}) =& [-e\lambda \log(\bar{r})]^{-\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{1}{\sigma_I-1}} N_{\Phi}{}^{\frac{1}{\sigma_v-1}} \tilde{L}_{p,\Phi} \\ =& \mathcal{C}\left[x(r^{\square}) + y(r^{\square}) \int_{r^{\square}}^{\bar{r}} \tilde{L}^r \, dF_r(r)\right]^{\frac{1-\alpha}{\sigma_v-1}} \left[\frac{\int_{r^{\square}}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr}{x(r^{\square}) + y(r^{\square}) \int_{r^{\square}}^{\bar{r}} \tilde{L}^r \, dF_r(r)}\right]^{\frac{\sigma_v}{\sigma_v-1}}, \end{aligned}$$

where $\mathcal{C}$ is a constant and where the second equality only holds if $c(\Phi, r^{\square}) = 0$. We can now derive the following proposition:

Proposition 3.4
Total consumption $C_{\Phi}(r^{\square})$ strictly decreases in basic research infrastructure costs.

The proof is given in the Appendix B.2.1. The following analysis now only applies if relation (3.16) holds, i.e. if a marginal increase in basic research infrastructure costs, Φ, leads to a marginal increase in the lowest productive knowledge level that engages in basic research, $r^{\square}$, and thus $c(\Phi, r^{\square}) = 0$.

It follows from Proposition 3.4 that there is always less total consumption in an economy with basic research infrastructure costs compared to the same economy without such costs.[19]

Next we examine utility from consumption, i.e. the consumption basket, in the country with the highest productive knowledge, $\bar{r}$, when there are basic research infrastructure costs,

$$\begin{aligned} C_{\Phi}^{\bar{r}}(r^{\square}) =& \frac{\sigma_v - 1}{\sigma_v} L^{\bar{r}} \left[-e\lambda \log(\bar{r})\right]^{-\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{1}{\sigma_I - 1}} \\ & \left[\frac{\tilde{L}}{x(r^{\square}) + y(r^{\square}) \int_{r^{\square}}^{\bar{r}} \tilde{L}^r \, dF_r(r)}\right]^{\frac{\alpha+\sigma_v-1}{\sigma_v-1}} \left[\int_{r^{\square}}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr\right]^{\frac{1}{\sigma_v-1}} \\ & \left\{\sum_{i\in\mathcal{I}(\bar{r})} \frac{z(i)}{\int_{\tilde{r}(i)}^{\bar{r}} L^r f(r) dr} \left[\int_{r^{\square}}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr - \right.\right. \\ & \left. \theta_D \int_{max\{r^{\square}, \tilde{r}(i)\}}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr \right] + \\ & \left. \frac{x(r^{\square})}{\tilde{L}} + y(r^{\square}) \left[\frac{\int_{r^{\square}}^{\bar{r}} \tilde{L}^r \, dF_r(r)}{\tilde{L}} - 1\right] + [1-\alpha]\theta_D \right\} . \end{aligned} \tag{3.19}$$

We observe that for $r^{\square} = \underline{r}$ it must hold that $C_{\Phi}^{\bar{r}}(\underline{r}) < C^{\bar{r}}$.[20] However, the higher basic research costs, the more countries are excluded from the innovation process, the lower is the

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[19] We can show this result by taking the ratio of total consumption in the two economies,

$$\frac{C_{\Phi}(r^{\square})}{C} = \left[\frac{x(\underline{r})}{x(r^{\square}) + y(r^{\square}) \int_{r^{\square}}^{\bar{r}} \tilde{L}^r \, dF_r(r)}\right]^{\frac{\alpha+\sigma_v-1}{\sigma_v-1}} \left[\frac{\int_{r^{\square}}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr}{\int_{\underline{r}}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr}\right]^{\frac{\sigma_v}{\sigma_v-1}} . \tag{3.18}$$

At $r^{\square} = \underline{r}$, the first term is less than 1 and the second term in Equation (3.18) vanishes, resulting in $C_{\Phi}(\underline{r}) < C$. Then, also $C_{\Phi} < C$ for any $0 < \Phi < \Phi(\underline{r})$. Furthermore, we know that $C_{\Phi}(r^{\square})$ decreases in $r^{\square}$ from Proposition 3.4. Thus, the consumption ratio in (3.18) must also decrease in $r^{\square}$.

[20] If $r^{\square} = \underline{r}$, and hence $x(\underline{r}) < x(\underline{r}) + y(\underline{r})\tilde{L}$, then only the first term remains in the following ratio of consumption levels:

$$\begin{aligned} \frac{C_{\Phi}^{\bar{r}}}{C^{\bar{r}}} =& \left[\frac{x(\underline{r})}{x(r^{\square}) + y(r^{\square}) \int_{r^{\square}}^{\bar{r}} \tilde{L}^r \, dF_r(r)}\right]^{\frac{\alpha+\sigma_v-1}{\sigma_v-1}} \left[\frac{\int_{r^{\square}}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr}{\int_{\underline{r}}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr}\right]^{\frac{1}{\sigma_v-1}} \\ & \left\{ \frac{\sum_{i\in\mathcal{I}(\bar{r})} \frac{z(i)}{\int_{\tilde{r}(i)}^{\bar{r}} \tilde{L}^r f(r) dr} \left[\int_{r^{\square}}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr - \theta_D \int_{max\{r^{\square}, \tilde{r}(i)\}}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr\right] +}{\sum_{i\in\mathcal{I}(\bar{r})} \frac{z(i)}{\int_{\tilde{r}(i)}^{\bar{r}} \tilde{L}^r f(r) dr} \left[\int_{\underline{r}}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr - \theta_D \int_{\tilde{r}(i)}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr\right] +} \right. \\ & \left. \frac{\frac{x(r^{\square})}{\tilde{L}} + y(r^{\square}) \left[\frac{\int_{r^{\square}}^{\bar{r}} \tilde{L}^r \, dF_r(r)}{\tilde{L}} - 1\right] + [1-\alpha]\theta_D}{\frac{x(\underline{r})}{\tilde{L}} + [1-\alpha]\theta_D} \right\} . \end{aligned}$$

Hence, if $r^{\square} = \underline{r}$ then $C_{\Phi}^{\bar{r}} < C^{\bar{r}}$.

number of varieties, the greater is the workforce available for production, and the higher are the profits of single firms. Whether or not $C_{\Phi}^{\bar{r}}(r^{\square})$ can increases in $r^{\square}$ remains unclear. A sufficient condition for $C_{\Phi}^{\bar{r}}(r^{\square})$ to decrease under higher basic research infrastructure costs is given in Lemma 3.1.

Lemma 3.1
$C_{\Phi}^{\bar{r}}(r^{\square})$ *strictly decreases in higher basic research infrastructure costs if*

$$\sigma_v \leq 2 - \alpha \, . \tag{3.20}$$

The derivation of the condition is given in Appendix B.2.2. Note that if (3.20) does not hold, $C_{\Phi}^{\bar{r}}(r^{\square})$ must not necessarily increase. Then, it cannot be analytically demonstrated which effects are the strongest, i.e. whether the higher profits of country $\bar{r}$ overcompensate the low measure of varieties consumed and the high infrastructure costs. Ceteris paribus, the greater the preference for variety, the more a representative household prefers a broad range of countries engaged in basic research production.
The range of excluded countries relative to the basic research infrastructure costs crucially depends on the distribution of productive knowledge in the economy. The more skewed this distribution, the more countries are excluded for a certain infrastructure cost, and thus the higher the profits for those still engaged in basic research.[21]
It is unclear whether the social planner would choose a lower $r_S^{\square}$ compared to $r^{\square}$. This is because of two opposing effects. On the one hand, we showed in Section 3.1 that the social planner would reallocate investments in basic research more equally across countries, given he is constrained to take the decentralized aggregate investment in basic research, $\tilde{L}_{BR}$. Given (3.12), this implies that $r_S^{\square} < r^{\square}$. On the other hand, a lower $r_S^{\square}$ implies additional fixed costs $\Phi \int_{r_S^{\square}}^{r^{\square}} L^r f(r) dr$ for the social planner. Thus, the social planner may also decide on economizing on infrastructure costs by choosing $r_S^{\square} > r^{\square}$.

3.3. Global Market for Researchers

We now analyze the impact of mobility among researchers. There is a trade-off between efficiency and equality. On the one hand, high ability in research is most effective when paired with a high productive knowledge. On the other hand, the distributional effects of mobility point towards less equally distributed basic research investments and corre-

[21] In a dynamic model, in which basic research investments feed back into the productive knowledge of a country, we run into a poverty trap with respect to basic research investment when there are basic research infrastructure costs. In such a setting, basic research is distributed even less equally. Countries of high productive knowledge might benefit from higher basic research infrastructure costs and they might have an interest in sustaining this entry barrier.

sponding profits.

We analyze the scenario of a global market for researchers. We assume that in each country, there is an equal amount of $\bar{L}_{BR}$. All scientists from all countries enter a single pool from which they are drawn by governments. Scientists cannot be hired by the private sector—they are useless in production[22]—and a unique wage rate prevails in the market for scientists.

Global effective labor in basic research then is $\tilde{L}_{BR} = \bar{L}_{BR} \int_{\underline{r}}^{\bar{r}} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr$. In production, an amount of $\tilde{L}_p = \tilde{L} - \tilde{L}_{BR}$ remains. The wage scheme in the production sector is not affected by the withdrawal of the research workforce, as long as there are sufficient skills in the economy, which there are by assumption. The wage rate in the researcher market is endogenously determined and clears the market. Subsequently, we use an adapted basic research production function, closely related to (3.1), that is

$$\eta^r = \eta_1(r)\eta_2(\xi^r)\bar{L}_{BR} = \eta_1(r)\zeta\left[\xi^r\right]^\alpha \bar{L}_{BR} \ ,$$

where $\bar{L}_{BR}$ is the absolute number of scientists in the global pool. Note that the ability distribution is constant across countries, i.e. ζ can be re-interpreted as the average ability among all scientists in the pool divided by the number of industries. Again, we assume that the innovation process and the probabilities of local and global dissemination are as in Chapter 2. Presuming that governments cannot observe a single scientist's ability and only know the ability distribution, they simply choose their basic research investment based on the expected ability of researchers. The labor market of researchers must clear,

$$\int_{\underline{r}}^{\bar{r}} \xi^r f(r)dr = 1 \ . \tag{3.21}$$

We define w_R as the equilibrium wage rate of researchers. The optimization problem of a country r's government then is

$$\max_{\xi^r} \left\{ \eta_1(r)\zeta\left[\xi^r\right]^\alpha \theta_D \bar{L}_{BR} \sum_{i\in\mathcal{I}(r)} \pi_i - w_R \xi^r \bar{L}_{BR} \right\} \ ,$$

and a government of country r demands a share of

$$\xi^r = \left[\alpha\zeta\theta_D \frac{\eta_1(r)}{w_R} \sum_{i\in\mathcal{I}(r)} \pi_i\right]^{\frac{1}{1-\alpha}}$$

of all scientists in the global pool. Integrating over $\xi^r(r)$, using $\pi_i = \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}}}{\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}}} \frac{\tilde{L}_p}{\hat{N}[\sigma_v - 1]}$,

[22] We could also assume that scientists exhibit stark intrinsic motivation.

and solving for $\hat{N}$, the number of varieties in an industry, we obtain

$$\hat{N} = \zeta \left[\frac{\alpha\theta_D \tilde{L}_p}{[\sigma_v - 1]w_R}\right]^{\alpha} \left[\int_{\underline{r}}^{\bar{r}} \eta_1(r)^{\frac{1}{1-\alpha}} z(r)^{\frac{\alpha}{1-\alpha}} f(r)dr\right]^{1-\alpha} .$$

Using labor market clearing in the global pool of researchers (3.21), we are able to derive the equilibrium wage rate for scientists,

$$w_R = \frac{\alpha\theta_D \tilde{L}_p}{\sigma_v - 1} \frac{\int_{\underline{r}}^{\bar{r}} \eta_1(r)^{\frac{1}{1-\alpha}} z(r)^{\frac{1}{1-\alpha}} f(r)dr}{\int_{\underline{r}}^{\bar{r}} \eta_1(r)^{\frac{1}{1-\alpha}} z(r)^{\frac{\alpha}{1-\alpha}} f(r)dr} .$$

The lower the supply of researchers, $\bar{L}_{BR}$, and thus the higher $\tilde{L}_p$, the higher the prevailing wage in the pool of researchers. And the more countries have a small set of accessible industries—reflected in small $z(r)$—, the smaller w_R. A government employs scientists according to

$$L_{BR}^r = \xi^r \bar{L}_{BR} = \frac{\eta_1(r)^{\frac{1}{1-\alpha}} z(r)^{\frac{1}{1-\alpha}}}{\int_{\underline{r}}^{\bar{r}} \eta_1(r)^{\frac{1}{1-\alpha}} z(r)^{\frac{1}{1-\alpha}} f(r)dr} \bar{L}_{BR} .$$

Also observe that $\xi^r > 1 \, \forall \, r \geq \breve{r}$, where $\breve{r} = \arg\min \frac{\eta_1(r)^{\frac{1}{1-\alpha}} z(r)^{\frac{1}{1-\alpha}}}{\int_{\underline{r}}^{\bar{r}} \eta_1(r)^{\frac{1}{1-\alpha}} z(r)^{\frac{1}{1-\alpha}} f(r)dr} \geq 1$. Observe that such an allocation scheme is much steeper than the allocation schemes of Chapter 2 and the previous sections. We can compare the allocation scheme across countries (i) for the social planner, (ii) for the decentralized equilibrium, and (iii) for the decentralized equilibrium when researchers enter a global pool,

$$(i)\text{ S.P.:} \qquad \left[\frac{\eta_1(r)}{w^r}\right]^{\frac{1}{1-\alpha}} ,$$

$$(ii)\text{ D.E.:} \qquad \left[\frac{\eta_1(r)}{w^r} z(r)\right]^{\frac{1}{1-\alpha}} ,$$

$$(iii)\text{ G.P.:} \qquad [\eta_1(r) z(r)]^{\frac{1}{1-\alpha}} .$$

In this extreme case of mobility of researchers, basic research investments are heavily concentrated on the upper tail of the productive knowledge distribution.

3.4. Openness of Research

Openness of research is the extent to which a country exchanges its basic research findings within the international community. In our framework ideas are the sole origin of profits. However, a country obtains these profits either by undertaking basic research or by free-riding on other countries' basic research. Note that we still assume that there is frictionless trade and all products are globally available at the same price. The following expression then shows the country profits,

$$\Pi^r = \theta_D \eta^r(\xi^r) \sum_{i \in \mathcal{I}(r)} \pi(i) + \sum_{i \in \mathcal{I}(r)} \frac{\tilde{L}^r}{\int_{\tilde{r}(i)}^{\bar{r}} \tilde{L}^r f(r) dr} GP(i) \pi(i) \,, \tag{3.22}$$

where $GP(i) = \hat{N} - \theta_D \int_{\tilde{r}(i')}^{\bar{r}} \eta^r(\xi^r) f(r) dr$ stands for the "Global Pool" of ideas in industry i, i.e. the public domain where ideas enter if they are not absorbed by the country of origin in industry i. A detailed discussion of the global pool of ideas is shown in Appendix A.3. The first term in (3.22) describes profits from own research, whereas the second term stands for profits obtained from the global pool of ideas. Both terms increase more in r than the wage scheme w^r.[23] We use the two profit sources to analyze whether a policy of attracting ideas from abroad or a policy of securing domestic ideas is more important for a country with productive knowledge r.

Whenever countries are atomistic, securing ideas is a dominant strategy. Thus, we assume, that attracting and securing ideas are two costly policies, and that they are conflicting. We thus assume a trade-off: If a country r benefits more from the local effect of basic research than from the global pool of ideas, then a policy that secures ideas dominates attracting ideas from abroad and vice versa. The trade-off can be implemented in the framework by adding decision variables to the governments' optimization problem, e.g. θ_S for securing ideas and θ_A for attracting ideas, where the two are linked by some function that represents the trade-off. However, this is beyond the scope of this section and we limit ourselves to discussing domestic and global profits that emerge in our model.

We denote profits from domestic basic research investments by Π_D^r and profits from global basic research investments by Π_G^r. The ratio of these profits for country r is

$$\frac{\Pi_G^r}{\Pi_D^r} = \left[\frac{w^r}{\eta_1(r)}\right]^{\frac{1}{1-\alpha}} \theta_D^{-1} \left[\sum_{\check{i} \in \mathcal{I}(r)} z\left(\check{i}\right)\right]^{\frac{1}{\alpha-1}} \sum_{i \in \mathcal{I}(r)} \frac{z(i)}{\int_{\tilde{r}(i)}^{\bar{r}} \tilde{L}^r f(r) dr}$$
$$\left[\int_{\underline{r}}^{\bar{r}} \eta_1(r) L^r g(r)^{\frac{\alpha}{1-\alpha}} f(r) dr - \theta_D \int_{\tilde{r}(i)}^{\bar{r}} \eta_1(r) L^r g(r)^{\frac{\alpha}{1-\alpha}} f(r) dr\right] .$$

The smaller the parameter θ_D, the more dominant are profits from the global pool. Countries with high productive knowledge have a high absorptive capacity for ideas from the global pool.[24] Yet, whether the ratio is increasing in r cannot be determined. The relative size of the profits depends on the industry-composition, the distribution of productive knowledge, the parameters θ_D and α, the basic research elasticity in production, and—

[23] We have seen in Sections 2.5 and 3.1.1 that this results in an increasing $\frac{GNI-GDP}{GDP}$-ratio.

[24] Note that with $\theta_D = 1$, some ideas still seep into the global pool of ideas. This is the case because some countries have no domestic production in industries of high complexity.

most importantly—on the diffusion process. We assumed in Chapter 2 that ideas from the global pool of ideas diffuse according to

$$\frac{\tilde{L}^r}{\int_{\tilde{r}(i)}^{\bar{r}} \tilde{L}^r f(r) dr} .$$

Until now, this assumption was independent from the governments' optimization problem. However, if governments strive to obtain ideas from the global pool, the assumed diffusion process directly affects the governments optimization problem. In particular, the diffusion process is decisive in determining whether a government secures or attracts ideas. Depending on the elasticity of the diffusion process with respect to r, different results can be obtained. Whether there is a pattern between the diffusion of ideas and the institutional settings in countries promoting the openness of ideas is left to further research.

4. Conclusion

We studied basic research in a multi-country, multi-industry general equilibrium model. In Chapter 2, we formalized a unified theory of public basic research, in which the basic research investment decision of a government depends on the domestic basic research productivity and on the domestic production. An industry with domestic production is a prerequisite that an idea—generated by basic research—can be absorbed by the country. The more diverse domestic production in a country is, the higher the local benefits of basic research investments can be and the more industries a government can target. In the model, firms can freely locate their production sites across countries, i.e. their location decisions determine a country's domestic production.

Our main results are as follows: First, basic research may be inefficiently targeted towards industries of low complexity. The reason is that industries of high complexity require high productive knowledge. If the productive knowledge of a country is not high enough, there is no domestic production in industries of high complexity that can absorb the ideas generated by the government's basic research investments. Thus, some industries can only be targeted by a small set of countries. As a result, few varieties exist in these industries, what implies higher profits in these industries compared to industries of low complexity, which in turn are targeted extensively.

Second, basic research is inefficiently allocated across countries. Countries with high productive knowledge invest too much in basic research compared to countries with low productive knowledge. Countries with high productive knowledge have a domestic production that is able to absorb most ideas generated by basic research, whereas countries with low productive knowledge have a domestic production with limited ability to do so.

Third, there is typically too little aggregate basic research investment in the world. Under reasonable parameter assumptions—in particular about the strength of the local diffusion expressed in the parameter θ_D—, there is too little aggregate investment in basic research compared to the aggregate investments the social planner chooses.

In Chapter 3, we simplified the model by neglecting targeting and by assuming a Pareto distribution in abilities. These simplifications are useful when further issues are addressed in a multi-country, multi-industry model with respect to basic research investments, because they yield an explicit solution of the model. The analysis of basic research infrastructure shows that some countries with low productive knowledge are excluded from investing in basic research, because building up the infrastructure would be too costly. As a consequence, countries with high productive knowledge may invest more in basic research.

The analysis of a global market for researchers with a single global wage for scientists implies more unequal investments in basic research. The analysis shows that mobile researchers accentuate the inequality of investments in basic research. However, we did not analyze the efficiency gains of this reallocation, which arise because researchers of high ability are matched to countries with high productivity in basic research.

Last, we discussed openness of research. Which countries benefit the most from attracting (or securing) ideas depends on parameters. The extensions show promising paths for future research. They reveal potentially more unequally-distributed basic research investments compared to the simple model.

Further promising extensions would be to add further policy instruments or to relax the assumption of a continuum of countries. Then basic research investments could be considered as a strategic game played by countries, and coordination between countries could enhance efficiency. In such a setting it would be fruitful to analyze the reaction function of countries if there is one big first-mover. Another promising research path would be to implement our model into a dynamic setting, with basic research investments feeding back into the productive knowledge of countries.

The research issues opened by our multi-country, multi-industry basic research framework are numerous, challenging, and they have the potential for further research issues.

Part II.

Skills, Tasks, and Capital

5. Introduction

In the first part of the thesis, we focused on basic research, an important though often neglected source of technological change, and on its potential inefficiencies. We outlined the distributional implications of the incentives and mechanisms observed in a global economy that relies on basic research investment. In the second part of the thesis, we will now study wage inequality in a new task-based framework. Wage inequality has been studied extensively (Tinbergen, 1974; Katz and Murphy, 1992; Autor et al., 2003). We will focus on wage inequality—and other labor market dynamics—caused by technological change.[1]

Technological change affects the economic agents in many ways. It is responsible for large-scale distributional effects generating winners and losers (Brynjolfsson and McAfee, 2014). It can be strongly disruptive and it has also lifted humankind into economic abundance (Harari, 2014).[2] The disruptive forces of technological change are often connected to changing production processes. The agricultural revolution, i.e. the transition from hunting and gathering to settled agriculture, is an example of such a change in the main production process for food. The agricultural revolution prepared ensuing economic and cultural developments (Harari, 2014).

Changing production processes, thus, are sometimes labeled revolutions, when they are particularly disruptive. The industrial revolution started with the invention of new production processes based on new technologies, most notably the steam engine (Brynjolfsson and McAfee, 2014). It stirred up concerns about implications for labor markets and cultural developments. Yet, the expected permanent mass-unemployment and the emergence of a permanent working-poor class did not appear (Brynjolfsson and McAfee, 2014).

Interestingly, such revolutions were named after the sectors where production processes changed—agricultural revolution, industrial revolution—, whereas nowadays, they are named after the technology that takes over tasks formerly performed by humans—digital revolution, robotic revolution. This distinction might point to a difference in the impact

[1] Note that individuals are also affected by technological change in other areas, i.e. changing cultural and social norms, new communication methods and other disruptive innovations have a direct impact on their lives as well.

[2] John Maynard Keynes already noted in his essay "Economic Possibilities for Our Grandchildren" that the economic problem—the coverage of absolute needs—will soon, and for the first time for a species in the biological kingdom, not be the permanent problem of the human race (Keynes, 1931).

on workers: Whereas in the past, revolutions allowed workers to find employment in another sector or industry, current revolutions might render a worker's labor entirely useless. Moreover the new revolutions spread over the entire economy and affect all industries. As it seems, we are currently witnessing the beginning of such a new revolution, the so called "robotic revolution", also named "the second machine age" (Brynjolfsson and McAfee, 2014). Current technology shows the potential to take over tasks that require low and middle or even high skills from workers, and to greatly substitute for low-skilled workers (Frey and Osborne, 2017; McKinsey Global Institute, 2017).

This is the starting point of our analysis. We build a framework with a focus on the different production processes, the workers' employability, and the workers' wages. We analyze changing skill requirements when production processes change, i.e. the introduction of new tasks that require high-skilled workers, as well as the substitution between capital and workers. Furthermore, we will study the difference between today's production processes compared to earlier ones: During the industrial revolution, production processes were automated by *machines*, whereas in the robotic revolution, automation consists of intelligently designed *robots* taking over entire production processes.
Several studies depict the threatening potential of automation. Frey and Osborne (2017) estimate that 47% of all jobs are threatened through computerization. A report of the McKinsey Global Institute (2017) estimates the worldwide potential for automation at the *current* technological level to be equivalent to 1.1 billion employees or $15.8 trillion in wages. For the United States, the estimate is 60 million employees or $2.7 trillion in wages.
The sheer magnitude of these numbers requires a thorough investigation of the possible effects of technological change. Of course, such studies cannot predict what will happen, and many new jobs will be created, while technology will further advance and continue to automate production processes.[3]
It is uncertain whether the ongoing displacement of workers—the destruction and creation of jobs—that will follow from further automation, will entail labor market dynamics that resemble the ones of the industrial revolution. If history is an indicator for the future, the prospects should be quite reassuring, as job creation mostly paralleled or outrun job destruction and displacement in the medium run (see e.g. Brynjolfsson and McAfee (2014) and Autor (2015)). But today's production process automation could be more disruptive than the past ones, in the sense that winners and losers could be more systematically apportioned over the skill distribution and this apportionment might be more permanent.

[3] Note that the estimates of McKinsey Global Institute (2017) are based on the current technological level. Thus, the more technology is advancing, the more jobs are threatened (Autor et al., 2003; Frey and Osborne, 2017; Autor, 2015), neglecting countervailing forces such as job creation.

Brynjolfsson and McAfee (2014) explain it with the concept of the spread and the bounty. Where bounty stands for the general lift in income and for quality improvements of goods and services, the spread characterizes the parallel development how this bounty is distributed. Even if job creation could proceed at the same speed as job destruction, it is uncertain what will happen to wages, wage inequality, and to today's low-skilled and middle-skilled workers.

In the next three chapters we will provide new insights to this issue by focusing on the medium-term—a time frame in which workers' skill levels are largely given or only change slowly compared to the technological change. We will analyze a new skill-task-assignment using a model developed by Schetter (2014, 2018). The skill-task-assignment is determined by technology and is pivotal for our analyses. And we will examine what happens to wages and wage inequality under two different production modes that reflect a key difference between the industrial economy of the beginning of the 20^{st} century and today's economy.

We will proceed in three steps. In Chapter 6, we introduce our model with two different task-based and complexity-based production processes. In Chapter 7, we introduce a third industry, manufacturing, that produces capital that can replace routine work in the other two industries. We will call capital "machines" or "robots", depending on the production mode. This third industry itself will be subject to technological progress. In Chapter 8, we present two generalizations of the basic model, first by distinguishing three different types of tasks, and second by assuming that households exhibit non-homothetic preferences.

6. Task-complexity, Skills, and Wages

"In the nineteenth century the Industrial Revolution created a huge new class of urban proletariats [...]. In the twenty-first century we might witness the creation of a massive new unworking class: people devoid of any economic, political or even artistic value, who contribute nothing to the prosperity, power and glory of society. This useless class will not be merely unemployed—it will be unemployable."
(Harari, 2016, p. 379)

6.1. Introduction

The macroeconomic model presented in this chapter involves two different production processes and introduces *task-complexity*, which characterizes the difficulty of a production process. We develop a simple, tractable model to study the consequences of the assumption that an individual endowed with a particular skill level cannot perform a task in a production process that is above a certain complexity level. Thus, there is a link between skill and the complexity of a task—which we call "task-complexity"—that we exploit to capture skill requirements in production at the micro-level and to analyze the implied general equilibrium macro-level demand and supply dynamics. The model is built to study employment and unemployment, wages and wage inequality from comparative static exercises.

Our model is motivated by three observations. First, empirical evidence (Autor et al., 2003; Frey and Osborne, 2017), as well as anecdotal daily experience, suggest that a task's complexity determines on whether a worker with a given skill level can execute the task successfully or not.[1] Intuitively, not every individual in the economy can execute

[1] A task-complexity can also encompass a *series* of tasks in a production process that is *summarized* by a certain complexity level. In this interpretation, a range of tasks is associated with a complexity level, and not an individual task. We will abstain from this interpretation and focus on a single task. The results are not affected by the different interpretations of complexity. Yet, the debate on the definition and measurement of complexity of tasks is particularly important in the area of automation (Frey and Osborne, 2017).

high complexity tasks, such as working as a lawyer, an engineer, or a mathematician. Second, the O-ring theory of production (Kremer, 1993) offers an intriguing approach to modeling the advantages of higher skills in task-based production: Workers with higher skills not only have higher success probabilities to execute a task in a production process than lower-skilled workers, they can also execute a *set* of tasks that is larger and contains higher complexities.[2] Third, with technological change, workers are subjected to changes in wages, displacement, job creation and destruction (Katz and Murphy, 1992; Davis et al., 1998). The rate of job creation and destruction differs for different skill levels[3] and this can have stark implications on wage dynamics in our model. We will examine the introduction and destruction of tasks and discuss basic concepts of automation.[4]

Our main insights are as follows: The labor market can be integrated or separated, depending on parameters. A separated labor market leads to a wage premium paid to the workers able to perform high task-complexities. The wage premium is a monotonic increasing step-function along the skill dimension, featuring upward jumps whenever the labor market is separated. This happens if there is excessive demand for high-skilled workers. Thus, the wage premium balances out high-skilled worker demand and high-skilled worker supply. The introduction of high task-complexities directs the labor market towards (more) separation. An increase of the lowest task-complexity level, or the automation of the corresponding task, can create an unemployable class in the sense of the statement at the beginning of this chapter.

Our framework differs from the task-based model developed by Acemoglu and Autor (2011): There will be no continuum of tasks, instead there will be a continuum of skills. The assignment of tasks to skills will be micro-founded and we argue that the task-complexity, and hence technology, determines the skills that can be used in a production process. Therefore, technology co-determines labor market dynamics by determining which workers can earn wage premia, and which cannot.

The chapter is organized as follows: Section 6.2 presents the basic model and in Section 6.3 we determine the equilibria. In Section 6.4 we analyze the implications of a continuous skill distribution and introduce the concept of task life-cycles. Section 6.5 presents the empirical model that is implied by our model. In Section 6.6 we relate our model to the task-based model of Acemoglu and Autor (2011) and we conclude in Section 6.7.

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[2] The different sets of tasks workers of different skill levels are able to perform can lead, as we will see, to segregated labor markets. Skill segregation has also been documented by Kremer and Maskin (1996).

[3] Cf. Bauer and Bender (2004).

[4] A more detailed analysis of automation is provided in Chapter 7.

6.2. The Model

In this section we present a reinterpretation of the model introduced by Schetter (2014, 2018). We describe the macroeconomic environment, labor and skills, the product space and firms, the consumption choices, the production technologies and the firms' production decisions.

6.2.1. Macroeconomic Environment

Workers are endowed with individual skill levels. A worker has to fulfill a task in the production process. Tasks are characterized by their complexity, henceforth called "task-complexity". The task-complexity, denoted by i, is central to our analysis. It indicates the degree of difficulty to successfully complete a task, i.e. the higher the task-complexity, the more difficult the production process. The task has to be successfully accomplished by a worker for the production of one unit of output. For the sake of simplicity, we assume that there is a one-to-one mapping from tasks to industries.[5] Thus, the task-complexity also represents an industry, and we use the terms interchangeably. A higher task-complexity requires, ceteris paribus, higher skill levels from workers. The critical assumption is that at the micro level, workers of different skill levels are substitutable for a certain task, as long as they dispose of a high enough skill level to master the corresponding task-complexity level. In other words, either a worker is able to perform the task, in which case the worker is paid his marginal product, or the worker is not able to perform the task, in which case he is not hired at all.[6]

We introduce the model with a set of task-complexity levels, $\mathcal{I}$, that only comprises two elements, task-complexity i_m for *"manual"* tasks, and task-complexity i_a for *"abstract"* tasks, and $\mathcal{I} = \{i_m, i_a\}$.

6.2.2. Labor and Skills

There is a continuum of labor, also representing households and workers, each endowed with L units of labor. Labor is characterized by its skill level. Skill levels are assumed to be either low or high, denoted by r_l (low) and r_h (high) respectively, where $0 < r_l < r_h < 1$. The set of the two skill levels is denoted by $\mathcal{R} = \{r_l, r_h\}$. A share ϕ_{r_l} of the labor

[5] In Section 6.4, we allow for multiple industries and discuss the relaxation of the one-to-one mapping from tasks to industries and in Section 8.1, we solve the model for three task-complexities.

[6] This assumption stands in stark contrast to the often-used Constant Elasticity of Substitution (CES) production in the Skill-Biased Technological Change (SBTC) literature. However, Graetz and Feng (2015) also assume that at the task level, all factors of production are perfect substitutes. In that sense, if a skill level is too low for a certain task-complexity, then it can no longer be considered as a factor for production in a production process with this task-complexity.

force is low-skilled and a share ϕ_{r_h} is high-skilled.[7] The sum of the two shares, $\sum_{r\in\mathcal{R}}\phi_r$, equals 1, where ϕ_r is used to represent the share of either type. A *representative* worker is denoted by L^{r_l} and L^{r_h}, respectively.

6.2.3. The Product Space and Firms

The product space comprises two dimensions: An industry dimension and a variety dimension. Varieties are differentiated products within an industry. In the baseline model, there are two industries, representative of the two task-complexities introduced above, $\mathcal{I} = \{i_m, i_a\}$. The index i stands for both industry and task-complexity. In each industry there is an exogenously-given number of firms, n_{i_m} and n_{i_a}. A representative firm of either industry is simply called firm i, where i is either i_a or i_m, and also a representative product of each industry can be identified by i. A single firm is atomistic and takes the wage rate as given. Thus, i equally indicates a representative product, a representative firm, an industry and a task-complexity level. In the following, we will speak of "goods" or "services" interchangeably, which are produced in industry i. The distinction between goods and services is not important.[8]

We next turn to the households' consumption decisions.

6.2.4. Households and Consumption

Households derive utility from the consumption of products. Each product (i, j) is a variety j that belongs to industry i. The utility of household r is described by a nested CES-function

$$U^r\left(\left\{c^r_{i,j}\right\}_{(i,j)\in\mathcal{I}\times[0,n_i]}\right) = C^r, \tag{6.1}$$

where

$$C^r := \left[\sum_{i\in\mathcal{I}}\left[\psi_i^{\frac{1}{\sigma_I-1}}\left[\int_{n_i} c^{r\,\frac{\sigma_v-1}{\sigma_v}}_{i,j}\,dj\right]^{\frac{\sigma_v}{\sigma_v-1}}\right]^{\frac{\sigma_I-1}{\sigma_I}}\right]^{\frac{\sigma_I}{\sigma_I-1}}.$$

C^r equally represents utility and the consumption basket of a household. In the consumption basket, $c^r_{i,j}$ is the amount of product variety j of industry i consumed by household r. The parameter ψ_i is a demand shifter corresponding to industry i. The parameter σ_I describes the elasticity of substitution *between* industries and σ_v describes the elasticity of substitution between varieties *within* an industry. A central assumption to our framework is that $\sigma_I < \sigma_v$. Goods and services *within* an industry are closer substitutes than goods

[7] The analysis can be generalized to any skill distribution (see Section 6.4).

[8] In Chapter 7, when we introduce capital, we will explicitly distinguish between manufacturing and service industries.

and services *between* industries. We can formulate the budget constraint of household r

$$\sum_{i\in\mathcal{I}} \int_{n_i} p_{i,j} c^r_{i,j} dj \leq L^r w^r + \Pi^r \ , \tag{6.2}$$

where $p_{i,j}$ denotes the price of product (i,j). The wage of household r is denoted by w^r. Π^r describes profits obtained by household r.
The demand of household r for a product (i,j) is

$$\begin{aligned} c^r_{i,j} &= \psi_i \left[\frac{p_{i,j}}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C^r \ , \\ PC^r &= L^r w^r + \Pi^r \ , \end{aligned}$$

where $P_i = \left[\int_{n_i} p_{i,j}^{1-\sigma_v} dj\right]^{\frac{1}{1-\sigma_v}}$ and $P = \left[\sum_{i\in\mathcal{I}} \psi_i P_i^{1-\sigma_I}\right]^{\frac{1}{1-\sigma_I}}$. The derivation of household r's demand is presented in Appendix C.1. Firms face aggregate demand from households and total demand for the product (i,j) is given by

$$c_{i,j} = \phi_{r_h} c^{r_h}_{i,j} + \phi_{r_l} c^{r_l}_{i,j} = \psi_i \left[\frac{p_{i,j}}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C \ , \tag{6.3}$$

where $C = \phi_{r_h} C^{r_h} + \phi_{r_l} C^{r_l}$ is total consumption. The aggregate of the budget constraints can be written as

$$PC = \sum_{r\in\mathcal{R}} L^r w^r + \sum_{r\in\mathcal{R}} \Pi^r \ .$$

We now turn to the production technology and then to the firms' optimization problem.

6.2.5. Production Technology

A producing firm holds a patent to produce product (i,j). Henceforth we discard the subscript j and consider the case of a representative firm i in industry i. Firm i chooses (i) the labor it wants to employ, (ii) the quality of the production process, (iii) the price of the product, and thus total output.
The production function stems from Kremer (1993) and is based on the O-ring theory of economic development, which says that the production process fails, if one tasks of the production process is *not* executed successfully.[9] The functional form used in our model is based on Schetter (2018). We deviate from the traditional interpretation of the O-ring theory insofar as we assume, that the production process as an entity, is reflected in one task, characterized by its task-complexity. Firm i produces good x_i by hiring a measure of $l_i(r)$ workers with skill level r. We assume that skill levels are perfectly observable by

[9] Based on this theory Kremer (1993) explains assortative matching.

firms. For each skill level, the firm chooses a certain quality level q for the production process. The quality level demanded of an employed worker's labor by firm i can vary in $[1, \bar{q}_i]$. For now, we assume that $\bar{q}_i \to \infty$ for both $i \in \mathcal{I}$.[10] Note that quality then is bounded below, i.e. $q \geq 1$. This is a functional minimum-quality requirement that is normalized to 1 across task-complexities (industries). The impact of quality in the production process on output is twofold: On the one hand, quality linearly scales output. On the other hand, quality increases the complexity level of the production process. We explain this concept in more detail below.

Firm i's expected output denoted by $\mathbb{E}[x_i]$ is given by

$$\mathbb{E}[x_i] = \sum_{r \in \mathcal{R}} q[r]^{iq^\lambda} l_i(r) \ . \tag{6.4}$$

λ $(\lambda > 0)$ is a parameter we explain below. The production of representative product i, with task-complexity i and quality q using skill level r, is successful with probability $[r]^{iq^\lambda} \in (0, 1)$. We define $\zeta_i(q) := iq^\lambda$, and $\zeta_i(q)$ is called *"complexity"*. Complexity is determined by task-complexity i, the chosen quality q, and the parameter λ.

The higher the chosen quality of the production process the higher the complexity of production, $\zeta_i(q)$. Note that the chosen functional form of the complexity increase through quality, q^λ, can be convex or concave, depending on the value given to λ. Thus the parameter λ measures how much complexity rises when quality is increased.

Higher quality in the production process leads to greater output, conditional on the successful completion of the production process. Intuitively, on the one hand, higher quality increases the complexity of the production process and thus lowers the probability of its successful completion. On the other hand, higher quality increases the output of the production process, if successfully completed. The success probability of the production process is higher if the worker's skill level is higher. We make the following assumption:

Assumption 6.1 (Appropriate Skill Condition: ASC)
Labor of skill level r can only successfully perform at complexities for which it holds that

$$\zeta_i(q) \leq -\frac{1}{\lambda \log(r)} \ . \tag{ASC}$$

Assumption ASC states that for a complexity $\zeta_i(q)$, a minimum skill level r is required. Note that the right-hand side of ASC is a continuous, strictly increasing function of r. Intuitively, a firm chooses the labor it wants to employ, taking ASC into account, and accordingly chooses a certain quality for the production process, which must be greater

[10] In Appendix C.3 we briefly analyze an upper bound on process quality in addition to the minimum-quality requirement.

or equal to one. Subsequently, $q \geq 1$ is phrased as the *minimum-quality* constraint. Assumption ASC is needed to introduce potential strict cut-offs in the labor market into the framework, which makes the model analytically tractable. One valid interpretation of ASC is that it summarizes institutionalized knife-edge conditions in the education and labor market system, such as licenses needed for acting as a doctor, a lawyer, an engineer, a teacher or a translator. Observe that the ASC imposes a constraint on the complexity of the production process, $\zeta_i(q)$, and *not* on task-complexity i itself.

The skill levels that are appropriate for a given complexity are bounded from below. For firm i, this lower bound is determined by the task-complexity level, the quality chosen and the ASC. Complexity $\zeta_i(q)$ is minimized for $q = 1$, when the quality choice hits the minimum-quality constraint. Thus, the bounding complexity level for a worker of skill level $\tilde{r}$ is $\zeta_i(1) = -\frac{1}{\lambda \log(\tilde{r})}$. The skill level $\tilde{r}$ bounds the set of skill levels that can be used by firm i if it chooses quality $q \geq 1$ in the production process, namely the skill levels $r \geq \tilde{r}(i) = \exp(-\frac{1}{\lambda i})$.[11] In other words, the minimum complexity for task-complexity i is simply i ($i = \zeta_i(1)$).

We can apply the law of large numbers and thus dispense with the expectation operator in (6.4). Overall, the production technology implies that higher skill levels are always more valuable in the production process.

6.2.6. Firms

For every single firm j of the number of firms n_i in industry i, the optimization problem stated below is the same and we can thus again discard the subscript j. The profit maximization problem of a representative firm i of industry i is then

$$\max_{\mathcal{R}_i, p_i, \left\{ \begin{array}{c} \{q_i(r)\} \\ \{x_{i,q_i(r)}\} \\ \{l_i(r)\} \end{array} \right\}_{r \in \mathcal{R}_i}} \quad \sum_{r \in \mathcal{R}_i} \left[p_i x_i - l_i(r) w^r \right] , \tag{6.5}$$

$$\begin{aligned} s.t. \quad & x_{i,q_i(r)} = q_i(r) \left[r \right]^{i q_i(r)^\lambda} l_i(r) , \\ & x_i = \sum_{r \in \mathcal{R}_i} x_{i,q_i(r)} = \psi_i \left[\frac{p_i}{P_i} \right]^{-\sigma_v} \left[\frac{P_i}{P} \right]^{-\sigma_I} C , \\ & q_i(r) \geq 1 \quad \forall r \in \mathcal{R}_i , \\ & r \geq \exp\left(-\frac{1}{\lambda i} \right) \quad \text{Assumption (ASC)} , \\ & \mathcal{R}_i \subseteq \mathcal{R} . \end{aligned}$$

Firm i hires a set of skills in production, $\mathcal{R}_i$, and chooses an amount of labor input, $l_i(r)$,

[11] This derivation is shown below in more detail.

for each skill level in the set. This labor input produces quantities $x_{i,q_i(r)}$ at chosen quality $q_i(r)$. Moreover, the firm chooses the price p_i. Considering the skill set in production that the firm chooses, $\mathcal{R}_i$, total output of firm i then is $x_i := \sum_{r\in\mathcal{R}_i} q_i(r)\,[r]^{iq_i(r)^\lambda}\,l_i(r)$.
Firm i's decision problem is solved by dividing it into the following three sub-decisions:

(i) *Quality Choice:* The optimal quality in the production process is chosen for any skill level r, $q_i(r)$.

(ii) *Cost Minimization:* Given the optimal choice of quality, the firm chooses skill levels suitable for production (ASC), which minimize the cost per unit of output, $\mathcal{R}_i$.

(iii) *Profit Maximization:* Given the minimal costs per unit of output, the firm chooses a price, p_i.

The price, in turn, determines the output, $\{x_{i,q_i(r)}\}_{\mathcal{R}_i}$, as well as the labor input, $\{l_i(r)\}_{\mathcal{R}_i}$ of firm i.
Note that the costs per unit of output might be minimized for both skill inputs. Then a firm is indifferent as to the hiring scheme of skill levels to produce its output and hence, the two skill levels are perfect substitutes. We next study the three sub-decision problems of firm i each in detail.

(i) *Quality Choice*

Given a skill level r, the firm chooses a quality of the production process which best complements the skill level, by maximizing $q\,[r]^{iq^\lambda}$. The FOC of the quality choice, given r, solves

$$[r]^{iq^\lambda} = -\lambda q^\lambda i \log(r)[r]^{iq^\lambda} \,. \tag{6.6}$$

The optimality condition trades off higher production outcomes against a higher probability of failure in the production process.[12] From (6.6) and from the exogenously-given minimum-quality constraint ($q \geq 1$) we obtain a uniquely-determined cost-minimizing quality choice of the production process, defined by

$$q_i(r) = \max\left\{1, \left[-\frac{1}{\lambda i \log(r)}\right]^{\frac{1}{\lambda}}\right\}, \qquad \forall (i,r) \in \mathcal{I}\times\mathcal{R}_i \,. \tag{6.7}$$

Assumption ASC and (6.7) together determine boundary values on the maximum task-complexity that can be produced by a certain skill level and equivalently, on the minimum skill level that can be employed by firms in a certain industry that is able to successfully manage the corresponding task-complexity. Specifically, these boundary values are

$$\tilde{i}(r) = -\frac{1}{\lambda \log(r)} \qquad \text{and} \qquad \tilde{r}(i) = e^{-\frac{1}{\lambda i}} \,,$$

[12] Quality could also be understood as product quality, which the consumer values. This approach is pursued in Part I.

where the value $\tilde{i}(r)$ denotes the highest task-complexity that skill level r can master without violating the ASC.[13] In turn, $\tilde{r}(i)$ denotes the minimal skill level needed to produce with task-complexity i. We assume that $i_a < \tilde{i}(r_h)$. If this were not the case, i_a could not be produced.

(ii) *Cost Minimization*

Firm i minimizes costs per unit of output. Using (6.7), the cost per unit of output when labor of skill level r is employed is $\frac{w^r}{q_i(r)[r]^{iq_i(r)^\lambda}}$, where w^r denotes the wage of a worker with skill level r. Firm i chooses accordingly a subset of skills, $\mathcal{R}_i \subseteq \mathcal{R}$, that fulfills the following minimization problem:

$$\min_r \quad w^r \left[-e\lambda i \log(r)\right]^{\frac{1}{\lambda}} \qquad s.t. \quad r \geq e^{-\frac{1}{\lambda i}} \ .$$

(iii) *Profit Maximization*

Firm i chooses a price to solve its profit maximization problem in (6.5), given (i) the optimal quality choice and (ii) the optimal set of skill levels in production, $\mathcal{R}_i$. Without loss of generality, we can assume that all of firm i's production is performed by a single skill level $r \in \mathcal{R}_i$. Firm i's optimization problem then is

$$\begin{aligned} \max_{p_i} \qquad & p_i x_i - x_i w^r \left[-e\lambda i \log(r)\right]^{\frac{1}{\lambda}} \ , \\ s.t. \qquad & x_i = \psi_i \left[\frac{p_i}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C \ , \end{aligned}$$

with the well-known solution

$$p_i = \frac{\sigma_v}{\sigma_v - 1} w^r [-e\lambda i \log(r)]^{\frac{1}{\lambda}} \ . \tag{6.8}$$

The price equals the constant mark-up, $\frac{\sigma_v}{\sigma_v - 1}$, times the marginal costs, $w^r[-e\lambda i \log(r)]^{\frac{1}{\lambda}}$. Note that only the elasticity of substitution within a given industry is relevant for the price setting of a firm.[14] Knowing the firm's price decision, also the quantity produced and the labor employed are determined in equilibrium. We next solve for the wage scheme and establish the equilibrium.

6.3. Equilibrium

We start with the definition of the equilibrium.

[13] In Section 6.4, we discuss why we impose Assumption ASC in addition to the minimum-quality constraint. If we omit Assumption ASC, the model could not be solved analytically, while the results would not significantly change.

[14] For a calibration one could introduce industry-specific elasticities.

Definition 6.1 (Equilibrium)
An equilibrium is

(i) *a set of skill levels $\mathcal{R}_i \subseteq \mathcal{R}$ for each representative firm i, with $i \in \mathcal{I}$, that this firm is willing to employ and that fulfills Assumption ASC,*

(ii) *quality levels in the production process, $\{q_i(r)\}_{r\in\mathcal{R}_i}$, output levels, $\{x_{i,j,q}\}_{(i,j,q)\in\mathcal{I}\times[0,n_i]\times[1,\infty)}$, and labor, $\{l_i(r)\}_{(i,j,r)\in\mathcal{I}\times[0,n_i]\times\mathcal{R}_i}$, for each firm i,*

(iii) *a set of consumption levels, $\left\{c^r_{i,j}\right\}_{(i,j,r)\in\mathcal{I}\times[0,n_i]\times\mathcal{R}}$, for each household r's consumption of each product (i,j),*

(iv) *a set of goods prices, $\{p_{i,j}\}_{i\in\mathcal{I}\times[0,n_i]}$,*

(v) *a set of wage rates, $\{w^r\}_{r\in\mathcal{R}}$,*

such that

(A) *$r \in \mathcal{R}_i$, $\{q_i(r)\}_{r\in\mathcal{R}_i}$, $\{x_{i,q}\}_{q\in[1,\infty)}$, $\{l_i(r)\}_{r\in\mathcal{R}_i}$ and p_i solve the representative firm i's profit maximization problem in* (6.5)*, $\forall i \in \mathcal{I}$,*

(B) *$\left\{c^r_{i,j}\right\}_{i\in\mathcal{I}\times[0,n_i]}$ maximizes the utility of the household r in* (6.1)*, subject to this household's budget constraint in* (6.2)*, $\forall r \in \mathcal{R}$,*

(C) *goods markets clear for all products, and*

(D) *labor markets clear.*

Before solving the equilibrium, we relate labor of any skill level to the productivity of the highest skill level in the economy,

$$\tilde{l}_i = l_i(r)\left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} . \tag{6.9}$$

We call such labor that is normalized in productivity across skill levels "effective" labor. The relation (6.9) essentially shows how much labor input $\hat{l}_i(r)$ of skill level r is needed to achieve the same output compared to an amount $\hat{l}_i(r_h)$ of the highest-skilled worker employed, i.e. $\hat{l}_i(r_h) = \hat{\tilde{l}}_i = \hat{l}_i(r)\left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}}$. In equilibrium, the exact allocation of labor of a certain skill to firms may remain indeterminate. However, we can infer how much effective labor a firm is willing to use. As we will see, effective labor used by a firm can be a compound of the two skill levels.
We next turn to the equilibrium.

6.3.1. Equilibria and the Labor Market

We will henceforth call the composition of industries and their corresponding task-complexities $i \in \mathcal{I}$ the "industry-composition". Depending on the distribution of skills and the industry-composition, different equilibria may arise, which we will now analyze.

Table 6.1.: Skills and Task-complexities

Scenario		r_l		r_h
(i)		$r_l <$	$\tilde{r}(i_m) < \tilde{r}(i_a)$	$< r_h$
(ii)	$\tilde{r}(i_m) < \tilde{r}(i_a) <$	$r_l <$		$< r_h$
(iii)	$\tilde{r}(i_m)$	$r_l <$	$\tilde{r}(i_a)$	$< r_h$

Table 6.1 shows three possible scenarios. Scenario (i) states that r_l cannot be used in the production process, as it is too low-skilled even for the manual task. Hence there is superfluous labor in the economy. In this sense, labor of skill level r_l remains *unemployed* by construction, as the quotation at the beginning of the chapter suggests. In their seminal book "The Second Machine Age" Brynjolfsson and McAfee (2014) make the following statement (p. 179):

> *"If neither the worker nor any enrepreneur can think of a profitable task that requires that worker's skills and capabilities, then that worker will go unemployed indefinitely [...]. In other words, just as technology can create inequality, it can also create unemployment. And in theory, this can affect a large number of people, even a majority of the population, and even if the overall economic pie is growing."*

Thus, in our task-complexity model, firms cannot employ workers with skill level below $\tilde{r}(i_m)$, as the production process would break down by assumption ASC,[15] i.e. just as in the statement, the skill of these workers can no longer be used and technology creates inequality *and* unemployment. In Scenario (ii), low-skilled labor can be used in the production process of both industries (i_m and i_a). We will call this equilibrium "Parallel Equilibrium". In Scenario (iii), only high-skilled labor can be used in industry i_a and we speak of a "Triangle Equilibrium".

In the Parallel Equilibrium, both skill levels may work in parallel in both industries, whereas in the Triangle Equilibrium, working in parallel is only possible in industry i_m. In industry i_a only skill level r_h can be used. Next we take a closer look at the labor

[15] We excluded the case in which i_a is not produced because the task-complexity in this industry is too high for the skill levels prevailing in the economy. Consequently, we also exclude the case where neither of both types of skills can be used in production.

market clearing condition (LMCC). The LMCC basically indicates whether or not there is a shortage of skills in the economy, given a wage scheme w^r.

LMCC (Labor Market Clearing Condition)

$$\sum_{e^{-\frac{1}{\hat{i}\lambda}}}^{r_h} \phi_r L^r \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} \geq \sum_{i \in \mathcal{I}: i \geq \hat{i}} n_i \tilde{l}_i(w^r), \quad \forall\, \hat{i} \in \mathcal{I}\,. \tag{LMCC}$$

The left-hand side is the supply of effective labor that is able to at least manage the task-complexity of industry $\hat{i}$. The right-hand side is the demand by firms for effective labor that is able to at least manage the task-complexity of their corresponding industry. If condition LMCC holds for every industry $\hat{i} \in \mathcal{I}$ and with equality for the minimum element of the industry set $\mathcal{I}$, $\min \mathcal{I}$, which is i_m in the two-industries case, there are sufficient skills for every firm to be unconstrained in its labor choice. This leads to a wage scheme w^r, which is derived below. However, wage scheme w^r is disrupted if demand for high-skilled labor is greater than its supply under wage scheme w^r. Then, LMCC with wages scheme w^r can no longer hold, and the wage scheme adapts to some wage scheme $\hat{w}^r$, which ensures that labor markets clear again.[16]

Parallel Equilibrium. By using (6.8), we can derive that a firm i is indifferent between producing a product with skill levels r_h or r_l ($r_h > r_l \geq \tilde{r}(i)$) if and only if the wages satisfy

$$w^{r_l} = \left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}} w^{r_h}\,.$$

This essentially states that the relative productivity difference between r_h and r_l must be reflected in their respective wages. We assume the following wage scheme:

$$w^r = \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}}, \tag{6.10}$$

where we normalized the wage of skill r_h to unity without loss of generality. This is the unique wage scheme in an equilibrium with sufficient skills, as shown in Schetter (2018). Whenever the industry-composition and skills are such that a Parallel Equilibrium arises, there must be sufficient skills by definition. Firm i's labor demand depends on the choice of output x_i,

$$l_i(r) = \frac{x_i}{q_i(r) r^{i q_i(r)^{\lambda}}} = \left[-e\lambda i \log(r)\right]^{\frac{1}{\lambda}} x_i\,. \tag{6.11}$$

[16] Suppose we had many different additional task-complexities with corresponding industries i. If there is only one segregation in the entire labor market, i.e. if LMCC is separated only once, then labor market outcomes are the same as in the two-industry case. We discuss this case in Section 6.4.

Observe that wages and productivity are aligned, i.e. higher productivity is rewarded by the exact same amount of higher wages. In other words, *effective* labor, (6.9), equals the wage scheme, (6.10), and therefore $\tilde{l} = l(r)\left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} = l(r)w^r$. Note that in this case, firms are indeed indifferent as to which labor to use. Moreover, the characteristics of the Parallel Equilibrium are such that Assumption ASC remains superfluous.

We next derive the households' demand for representative product i, denoted by c_i. For this purpose, we use the price indices stemming from the firms' optimal price choice (6.8) and wage scheme (6.10) to obtain

$$\begin{aligned}
p_i =& \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i \log(r_h)\right]^{\frac{1}{\lambda}} \quad \forall\, i \in \mathcal{I}\,, \\
P_i =& \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i \log(r_h)\right]^{\frac{1}{\lambda}} n_i^{\frac{1}{1-\sigma_v}} \quad \forall\, i \in \mathcal{I}\,, \\
P =& \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda \log(r_h)\right]^{\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{1-\sigma_I}} .
\end{aligned}$$

Using (6.3) and the price indices, the households' demand for representative product i is derived,

$$c_i = \psi_i i^{-\frac{\sigma_I}{\lambda}} n_i^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} \left[\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}} n_{\hat{i}}^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{\sigma_I}{1-\sigma_I}} C\,.$$

Goods market clearing implies that $c_i = x_i$. We use (6.11) and transform labor demand into effective labor demand. Then the effective labor demand of representative firm i is

$$\tilde{l}_i = \psi_i \left[-e\lambda \log(r_h)\right]^{\frac{1}{\lambda}} i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} \left[\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}} n_{\hat{i}}^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{\sigma_I}{1-\sigma_I}} C\,,$$

which, ultimately, depends on total consumption C. Summing over all firms and imposing labor market clearing then yields total consumption

$$C = \left[-e\lambda \log(r_h)\right]^{-\frac{1}{\lambda}} \left[\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}} n_{\hat{i}}^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I - 1}} \tilde{L}\,,$$

where $\tilde{L}$ is the total effective labor in the economy. In this Parallel Equilibrium markets are integrated, i.e. both skill levels work side by side in both industries. The exact allocation remains indeterminate. LMCC must always hold, as from the production side, the two skill levels are equivalent, given their relative productivity and their wages, w^r. As a consequence, total effective labor is equal to total wages earned in the economy, denoted by TW,

$$\tilde{L} = L\left[\phi_{r_l}\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}} + \phi_{r_h}\right] = TW\ .$$

Triangle Equilibrium. We now analyze two cases of the Triangle Equilibrium. In a Triangle Equilibrium, the more complex industry is reliant upon the high-skilled workers. Each of the two Triangle Equilibria arises if either

(i) LMCC holds with (6.10) $\Rightarrow$ *Integrated labor market – ILM*,

(ii) or LMCC does not hold with (6.10) $\Rightarrow$ *Disintegrated labor market – DLM*.

The two equilibria are described in what follows:

(i) *Integrated labor market – ILM*

The firms of industry i_m employ both skill levels r_l and r_h, i.e. the aggregate demand for high-skilled labor by industry i_a does not exceed the supply of high-skilled labor, given wages that perfectly represent relative productivity in the production in industry i_m. The equilibrium wage scheme (6.10) arises, which we derived in the preceding paragraphs.

(ii) *Disintegrated labor market – DLM*

With wage scheme (6.10), labor markets would not clear, i.e. LMCC is violated. This essentially means that there is more demand for high-skill labor than there would be supply of such labor if wage scheme (6.10) is present. Thus, wages adjust to the forces of supply and demand, and the following wage scheme arises:

$$\hat{w}^r = \omega_i\left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}}\ , \tag{6.12}$$

where $\omega_i \geq 1$ is an endogenous function, henceforth called the *"wage premium"*.[17,18] The LMCC implies that

$$\sum_{e^{-\frac{1}{\tilde{i}\lambda}}}^{r_h} \phi_r L\left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} \geq \sum_{\hat{i}}^{I} n_i \tilde{l}_i(\hat{w}^r), \quad \forall \hat{i} \in \mathcal{I}\ ,$$

where the left-hand side (supply of skills scaled by their respective labor productivity) remains the same as before. However, with wage scheme (6.12), $\hat{w}^r$, the demand for labor of firms changes (right-hand side). Marginal costs increase by the factor ω_i in industry i

[17] What we call wage premium is simply a scaling factor. Whenever the scaling factor is greater one, we say that the worker obtains a wage premium.

[18] Later we will express the wage premium as a function of the skill level r, by taking advantage of $\tilde{i}(r)$.

and thus, firms in this industry set prices according to

$$p_i = \frac{\sigma_v}{\sigma_v - 1}\omega_i \left[-e\lambda i \log(r_h)\right]^{\frac{1}{\lambda}} \quad \forall\, i \in \mathcal{I} \, .$$

In the two-industries case, the function ω_i simplifies to $\omega_{i_m} = 1$ and $\omega_{i_a} = \omega > 1$, and the labor market is disintegrated insofar that the high-skilled labor (r_h) is employed in the more complex industry (i_a) only, whereas the low-skilled labor (r_l) is employed in the less complex industry (i_m). Prices chosen by the firms, price indices of the industries and the aggregate price index in the DLM Equilibrium are

$$\begin{aligned}
p_i =& \frac{\sigma_v}{\sigma_v - 1}\omega_i \left[-e\lambda i \log(r_h)\right]^{\frac{1}{\lambda}} \quad \forall\, i \in \mathcal{I} \, , \\
P_i =& \frac{\sigma_v}{\sigma_v - 1}\omega_i \left[-e\lambda i \log(r_h)\right]^{\frac{1}{\lambda}} n_i{}^{\frac{1}{1-\sigma_v}} \quad \forall\, i \in \mathcal{I} \, , \\
P =& \frac{\sigma_v}{\sigma_v - 1} \left[-e\lambda \log(r_h)\right]^{\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i \omega_i^{1-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i{}^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{1-\sigma_I}} \, .
\end{aligned}$$

Using the prices, we can infer the households' demand, which, in turn, leads to effective labor demand by firms that is equal to

$$\tilde{l}_i = \psi_i \left[-e\lambda \log(r_h)\right]^{\frac{1}{\lambda}} \omega_i^{-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i{}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}} \left[\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \omega_{\hat{i}}^{1-\sigma_I} \hat{i}^{\frac{1-\sigma_I}{\lambda}} n_{\hat{i}}{}^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{\sigma_I}{1-\sigma_I}} C \quad \forall\, i \in \mathcal{I} \, . \tag{6.13}$$

In the *parallel* equilibrium, the *total wages* paid, denoted by TW, are equal to the total effective labor in the economy, i.e. $TW = \tilde{L}$. This is the case because relative wages reflected relative productivity in production and thus, for every skill level r, wages and productivity were linearly coupled. However, with wages scheme (6.12), $\hat{w}^r$, this linear relation no longer holds, and in the aggregate, $TW \neq \tilde{L}$. Effective labor demand by representative firm i, (6.13), times the wage premium function, ω_i, yields the wages earned by workers in industry i. There is no savings decision at all, and thus, wages earned are entirely spent for consumption. We aggregate total wages earned in the economy by multiplying labor demand and the wage premium function. We rearrange to obtain total consumption

$$C = \left[-e\lambda \log(r_h)\right]^{-\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i \omega_i^{1-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i{}^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I-1}} TW \, ,$$

where $TW = L\left[\phi_{r_l}\hat{w}^{r_l} + \phi_{r_h}\hat{w}^{r_h}\right]$ and $\hat{w}^{r_l} = \left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$ and $\hat{w}^{r_h} = \omega$. The wage scheme (6.12) must ensure that demand and supply for high-skilled labor matches. Essentially, this leads to two *separated* labor markets. Each of them must clear in equilibrium

$$\begin{aligned}\phi_{r_h} L &= n_{i_a} \tilde{l}_{i_a}(\hat{w}^{r_h}) \ , \\ &= \psi_{i_a} \omega^{-\sigma_I} i_a^{\frac{1-\sigma_I}{\lambda}} n_{i_a}^{\frac{1-\sigma_I}{1-\sigma_v}} \left[\sum_{i \in \mathcal{I}} \psi_i \omega_i^{1-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}} \right]^{-1} TW \ ,\end{aligned}$$

$$\begin{aligned}\phi_{r_l} L \left[\frac{\log(r_h)}{\log(r_l)} \right]^{\frac{1}{\lambda}} &= n_{i_m} \tilde{l}_{i_m}(\hat{w}^{r_l}) \ , \\ &= \psi_{i_m} i_m^{\frac{1-\sigma_I}{\lambda}} n_{i_m}^{\frac{1-\sigma_I}{1-\sigma_v}} \left[\sum_{i \in \mathcal{I}} \psi_i \omega_i^{1-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}} \right]^{-1} TW \ ,\end{aligned}$$

and the labor supply of the two skill levels sums up to the total supply of effective labor in the economy, $\tilde{L}$. We can now solve for the wage premium, which is equal to the wages earned by high-skilled labor,

$$\omega = \left[\frac{\phi_{r_l}}{\phi_{r_h}} \frac{\psi_{i_a}}{\psi_{i_m}} \left[\frac{\log(r_h)}{\log(r_l)} \right]^{\frac{1}{\lambda}} \left[\frac{i_a}{i_m} \right]^{\frac{1-\sigma_I}{\lambda}} \left[\frac{n_{i_a}}{n_{i_m}} \right]^{\frac{1-\sigma_I}{1-\sigma_v}} \right]^{\frac{1}{\sigma_I}} . \tag{6.14}$$

Note that the wage earned by high-skilled labor is never below 1. Either high-skilled labor earns $w^{r_h} = 1$ and the wage premium has no impact, or it earns $\omega > 1$. In other words, if we can exclude the Parallel Equilibrium and $\omega > 1$, the economy is in the DLM Equilibrium.

Subsequently we denote wage schemes by w^r in all equilibria and we use ω to make an explicit distinction between them.

Definition 6.2 (Wage Scheme)
The wage scheme is $w^r := \omega(r) \left[\frac{\log(r_h)}{\log(r)} \right]^{\frac{1}{\lambda}} = \omega_i \left[\frac{\log(r_h)}{\log(r)} \right]^{\frac{1}{\lambda}}$*, where* $\omega(r)$ *is the wage premium as a function of the skill level and* ω_i *is the same wage premium as a function of the task-complexity level.*

Definition 6.2 relates the notations for the wage schemes. Either notation will be used whenever it is more useful.
We adapt Table 6.1 and show in Table 6.2 the different employment statuses and wages that arise in the different equilibria under the assumptions made up to now.
There are four possible equilibria defined in Table 6.2: (i) Unemployment Equilibrium U, (ii) Parallel Equilibrium PE, (iii) Integrated Labor Market Equilibrium ILM, and (iv) Disintegrated Labor Market Equilibrium DLM. The crucial differences between

Table 6.2.: Equilibria of the Task-complexity Model

eq	r_l		w^{r_l}	r_h	w^{r_h}
U	unempl.	$r_l < \tilde{r}(i_m)$	0	empl. in i_m & i_a	1
PE	empl. in i_m & i_a	$\tilde{r}(i_a) < r_l$	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	empl. in i_m & i_a	1
ILM	empl. in i_m	$\tilde{r}(i_m) < r_l < \tilde{r}(i_a)$	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	empl. in i_m & i_a	1
DLM	empl. in i_m	$\tilde{r}(i_m) < r_l < \tilde{r}(i_a)$	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	empl. in i_a	ω

Note that equilibria ILM and DLM are both Triangle Equilibria. Furthermore, it holds by assumption that $\tilde{r}(i_a) < r_h$.

the equilibria, $eq = \{U, PE, ILM, DLM\}$, are the wage scheme and the occupational possibilities.

Note that equilibria PE and ILM are equal in terms of wages and also in all other equilibrium variables, the sole exception being that in the integrated labor market equilibrium, ILM, low-skilled labor is only employed in industry i_m, whereas low-skilled labor is employed in both industries in the Parallel Equilibrium, PE. Thus the two equilibria differ with respect to their potential developments. In contrast to the Parallel Equilibrium, PE,[19] the integrated labor market equilibrium, ILM, always entails the potential labor market disintegration. Proposition 6.1 presents the equilibria.

Proposition 6.1

The four equilibria $eq = \{U, PE, ILM, DLM\}$ of interest are

$$(i)\quad w^{r^\star} = \omega_i \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} \quad \forall\, r \in \mathcal{R} \text{ and}$$

$$\begin{cases} \omega^\star_{i_m} = 0 \text{ and } \omega^\star_{i_a} = 1 \text{ if } eq = \{U\} \ , \\ \omega^\star_i = 1\ \forall i \in \mathcal{I} \text{ if } eq = \{PE, ILM\} \ , \\ \omega^\star_{i_m} = 1 \text{ and } \omega^\star_{i_a} = \omega^\star \text{ if } eq = \{DLM\} \ , \end{cases}$$

$$\text{where } \omega^\star = \left[\frac{\phi_{r_l}}{\phi_{r_h}} \frac{\psi_{i_a}}{\psi_{i_m}} \left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}} \left[\frac{i_m}{i_a}\right]^{\frac{\sigma_I - 1}{\lambda}} \left[\frac{n_{i_a}}{n_{i_m}}\right]^{\frac{\sigma_I - 1}{\sigma_v - 1}}\right]^{\frac{1}{\sigma_I}} \ ,$$

$$(ii)\quad \begin{cases} \mathcal{R}^\star_i \subseteq \{r \in \mathcal{R} \mid r \geq \tilde{r}(i)\}\ \forall\, i \in \mathcal{I} \text{ if } eq = \{U, PE\} \ , \\ \mathcal{R}^\star_{i_m} \subseteq \{r \in \mathcal{R} \mid r \geq \tilde{r}(i_m)\} \text{ and } \mathcal{R}^\star_{i_a} \subseteq \{r \in \mathcal{R} \mid r \geq \tilde{r}(i_a)\} \text{ if } eq = \{ILM\} \ , \\ \mathcal{R}^\star_{i_m} \subseteq \{r \in \mathcal{R} \mid \tilde{r}(i_a) > r \geq \tilde{r}(i_m)\} \text{ and } \mathcal{R}^\star_{i_a} \subseteq \{r \in \mathcal{R} \mid r \geq \tilde{r}(i_a)\} \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } eq = \{DLM\} \ , \end{cases}$$

[19] Only technological change that brings forth new task-complexities might transform the PE into a ILM or DLM. We discuss the emergence and transition of task-complexities in Section 6.4.

(iii) $q_i^\star(r) = \left[-\frac{1}{\lambda i \log(r)}\right]^{\frac{1}{\lambda}} \quad \forall\,(i,r) \in \mathcal{I} \times \mathcal{R}_i^\star\,,$

(iv) $p_i^\star = \frac{\sigma_v}{\sigma_v - 1}\omega_i^\star\,[-e\lambda i \log(r_h)]^{\frac{1}{\lambda}} \quad \forall\, i \in \mathcal{I}\,,$

$P_i^\star = \frac{\sigma_v}{\sigma_v - 1}\omega_i^\star\,[-e\lambda i \log(r_h)]^{\frac{1}{\lambda}}\, n_i^{\frac{1}{1-\sigma_v}} \quad \forall\, i \in \mathcal{I}\,,$

$P^\star = \frac{\sigma_v}{\sigma_v - 1}\,[-e\lambda \log(r_h)]^{\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i \omega_i^{\star\,1-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{1-\sigma_I}}\,,$

(v) $\tilde{l}_i^\star = \frac{\psi_i \omega_i^{\star\,-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}}}{\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \omega_{\hat{i}}^{\star\,1-\sigma_I} \hat{i}^{\frac{1-\sigma_I}{\lambda}} n_{\hat{i}}^{\frac{1-\sigma_I}{1-\sigma_v}}} TW^\star\,,$

(vi) $x_i^\star = [-e\lambda i \log(r_h)]^{-\frac{1}{\lambda}}\, \tilde{l}_i^\star\,,$

(vii) $\pi_i^\star = \frac{\tilde{l}_i^\star}{\sigma_v - 1}\,,$

(viii) $C^\star = [-e\lambda \log(r_h)]^{-\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i \omega_i^{\star\,1-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I - 1}} TW^\star\,,$

and $P^\star C^\star = \frac{\sigma_v}{\sigma_v - 1} TW^\star\,,$

(ix) $TW^\star = L \sum_{r\in\mathcal{R}} \phi_r w^{r\star}\,,$

where $\pi_i^\star$ *denotes the equilibrium profit of representative firm* i*.*

The different equilibria represent different labor market structures. They are unique up to the exact allocation of skill levels to industries in the Parallel Equilibrium, PE, and the integrated labor market equilibrium, ILM. Given an industry-composition, an economy equipped with pervasive high-skill levels is more likely *not* to run out of skill, and hence more likely to be in an integrated labor market equilibrium, whereas an economy with fewer high-skilled workers tends to a disintegrated labor market. However, observe from wage premium (6.14), that the relative headcount does not matters, but that the relative amount of effective labor crucially determines the labor market dynamics.

In which labor market equilibrium an economy is situated not only depends on the skill distribution—i.e. on the skill supply—but also on the industry-composition. The industry-composition, in the context of the model, encompasses the industries located in the economy, $i \in \mathcal{I}$, their respective amount, $\{n_i\}_{i\in\mathcal{I}}$, and their respective demand shifters $\{\psi_i\}_{i\in\mathcal{I}}$. The skill distribution and the industry-composition must match for the labor market to operate efficiently. In Section 6.4, we introduce a continuous skill distribution and many different industries. We demonstrate that the industry-composition affects the skill supply in the economy.

In Appendix C.3, we discuss the case of bounded process enhancement, i.e. a setting in which there is not only a *lower* quality constraint but also an *upper* quality constraint. In this environment, not only sufficient high-skilled workers, but also a sufficient amount of low-skilled workers are necessary that no wage premia arise. Thus, in such an environment, an efficient match between industry-composition and skill distribution is not guaranteed by a heavy-tailed skill distribution.

In general, a certain industry-composition can generate an efficient labor market if matched with one skill distribution, but it can also lead to inefficient labor market outcomes if it is matched with another.

We next turn to measuring the inequality prevalent in the different equilibria.

6.3.2. Wages, Inequality, and the Gini Coefficient

We introduce a measurement for wage inequality, the Gini Coefficient,[20]

$$\mathcal{G}_{eq} = 1 - \frac{\phi_{r_l}^2 w_{eq}^{r_l} + 2\phi_{r_l}\phi_{r_h} w_{eq}^{r_l} + \phi_{r_h}^2 w_{eq}^{r_h}}{TW_{eq}} L \ ,$$

where $eq \in \{U, PE/ILM, DLM\}$. In the following we compare different economies with equal labor supply, ϕ_{r_l} and ϕ_{r_h}, that are situated in different equilibria, i.e. the industry-composition is such that the equilibria $eq \in \{U, PE/ILM, DLM\}$ arise.

U **– Unemployment.** Only L^{r_h} is employed. This equilibrium is interesting with respect to dynamics in the task-complexities of the production processes. In Section 6.4, we will see that an increase in the task-complexity i_m can cause low-skilled labor to drop out of the production process completely. The Gini Coefficient of this equilibrium is

$$\mathcal{G}_U = 1 - \phi_{r_h} \ ,$$

and total wages earned are $TW_U = \phi_{r_h} L$.

PE/ILM **– Parallel Equilibrium & Integrated Labor Market.** The PE and the ILM Equilibrium are very similar. They only differ with respect to the employment possibilities of low-skilled workers. In the Parallel Equilibrium, labor of both skill levels can work in either industry, whereas in the ILM Equilibrium, only high-skilled labor can be employed in both industries. But in terms of inequality and total output, they are the same. Wages are $w_{PE/ILM}^{r_l} = \left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$ and $w_{PE/ILM}^{r_h} = 1$. And the corresponding Gini

[20] The Gini Coefficient measures the dispersion in the distribution under study, i.e. in our case the dispersion in the income of households. A Gini Coefficient of value 0 expresses perfect equality among the households, whereas a value of 1 expresses maximal inequality.

Coefficient is

$$\mathcal{G}_{PE/ILM} = 1 - \frac{\phi_{r_l}\left[\phi_{r_l} + 2\phi_{r_h}\right]\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}} + \phi_{r_h}^2}{\phi_{r_l}\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}} + \phi_{r_h}} .$$

DLM **– Disintegrated Labor Market.** In the DLM Equilibrium, the wage premium directly affects the Gini Coefficient. The wage for a low-skilled worker is $w_{DLM}^{r_l} = \left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$ and for a high-skilled worker $w_{DLM}^{r_h} = \omega$, respectively, resulting in a Gini Coefficient of

$$\mathcal{G}_{DLM} = 1 - \frac{\phi_{r_l}\left[\phi_{r_l} + 2\phi_{r_h}\right]\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}} + \phi_{r_h}^2\omega}{\phi_{r_l}\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}} + \phi_{r_h}\omega} ,$$

where ω is defined in (6.14).

The comparison of the Gini Coefficients of the equilibria results in the following Proposition 6.2.

Proposition 6.2
For any skill distribution, ϕ_{r_l} and ϕ_{r_h}, it holds that

$$\mathcal{G}_{PE,ILM} < \mathcal{G}_{DLM} < \mathcal{G}_U .$$

The proof is given in Appendix D.1.

6.4. Skill Distribution and the Task Life-cycle

We now extend the analysis to encompass a whole skill distribution. We assume that skill is distributed according to some density function $f(r)$ with support $[r_l, r_h]$, where $0 < r_l < r_h < 1$. Each household still provides L units of labor inelastically. Note that any skill level $r < \tilde{r}(i_m)$ remains unemployed. Unemployed labor does impact the model by narrowing the support of employable labor, i.e. there is less production. However, with the chosen linear utility specification the marginal benefit from consumption is simply linear accordingly. With any concave utility specification welfare results of downward redistribution would be strengthened. In order to neglect zero-consumption (unemployment), we assume for now that $\tilde{r}(i_m) < r_l$. We denote the set of "low-skilled" labor by $\mathcal{R}_L = [r_l, \tilde{r}(i_a))$ and the set of "high-skilled" labor by $\mathcal{R}_H = [\tilde{r}(i_a), r_h]$.
Observe that the *low-skilled* labor and *high-skilled* labor are pure labels intended to facilitate the understanding of the model. However, the cutoff task-complexities—here the task-complexity i_m, determining the lower bound of the low-skilled group, and the task-

complexity i_a, determining the upper bound of the low-skilled group and the lower bound of the high-skilled group—can vary in response to technological change. The supply of low-skilled and high-skilled workers is thus a function of the task-complexity levels. We will analyze this issue below.

The wage scheme then remains as in Definition 6.2, but the wage scheme now holds for the entire support of the skill distribution, i.e.

$$w^r = \omega(r) \left[\frac{\log(r_h)}{\log(r)} \right]^{\frac{1}{\lambda}} \quad \forall\, r \in [r_l, r_h] \ , \tag{6.15}$$

where $\omega(r) = \omega$ for all $r \in \mathcal{R}_H$ and $\omega(r) = 1$ for all $r \in \mathcal{R}_L$.

Observe that the task-complexity i_a affects the supply of skills in both sets, $\mathcal{R}_L$ and $\mathcal{R}_H$. The skill sets can be characterized as a function of their support and, in particular, as a function of the group-specific task-complexity levels determining their boundaries, i.e. $\mathcal{R}_L(i_a)$ and $\mathcal{R}_H(i_a)$ if $\tilde{r}(i_m) < r_l$. The higher i_a the smaller the set of labor able to manage this task-complexity, $\mathcal{R}_H(i_a)$, and the larger the set of labor able to manage lower task-complexities, $\mathcal{R}_L(i_a)$. We can generalize the concept of task-complexities to encompass more than two values. Then the more task-complexities there are, and the more they spread out across the complexity set, the more separated wage groups may emerge. In such case there may be multiple jumps in the wage scheme along the skill distribution.[21]

Analyzing our model with two task-complexities, we observe that shifts of either task-complexity, i_m or i_a, *directly* impact wage inequality via the wage premium, ω, and also, *indirectly*, the amount of labor suitable for a task, and hence, the labor supply. Thus, the labor supply is partially determined by *technology* in the framework. In most models that explain wages and, in particular, the wage premium, dynamics of labor supply are taken as exogenously-given (or modeled as an endogenous choice variable for workers). And often, only labor demand is expected to vary under the forces of technological change (thus the SBTC hypothesis). Our model in contrast allows for shifts in demand *and* supply of labor due to technology in the medium-term when a task-complexity level changes, when new task-complexities emerge, and when old ones become obsolete. Before continuing our analysis, let us discuss the necessity of Assumption ASC.

Discussion of Assumption ASC. In principle, Assumption ASC only emphasizes what the minimum-quality constraint already achieves. However, whereas ASC is an assumption, $\tilde{i}(r)$ results from maximizing production, given a task-complexity level that does not violate the minimum-quality constraint. Essentially, a producer could produce a task-complexity $i'' > \tilde{i}(r')$ also with skill level r' if we disregard Assumption ASC, and

[21] Cf. Section 8.1

only fulfill the minimum-quality constraint (i.e. $q = 1$). In that case, the production function would be $[r']^{i''}$. But then, a worker of skill level r' can only produce an amount per labor unit equal to $[r']^{i''} < [-e\lambda i'' \log(r')]^{-\frac{1}{\lambda}}$, and given a wage scheme $w^r = \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}}$, he could not compete against skill levels $r \geq r'' = \tilde{r}(i'')$.

Without ASC, we would run into non-trivial interactions if there are no sufficient skills in the economy that labor markets remain integrated, in particular if there is a continuous skill distribution. ASC allows us to obtain clear cutoffs in the wage scheme, whereas this would be impossible if firms only minimized costs under the minimum-quality constraint alone. In such an environment, however, the wage scheme remains a continuous function and there is a continuously increasing $\omega(r) \in [1, \omega]$ on $[\check{r}, \tilde{r}(i_a)]$, where for any $r \in [\check{r}, \tilde{r}(i_a)]$ it must hold that

$$\omega(r) \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} [r]^{-i_a} = \omega \left[-e\lambda i_a \log(r_h)\right]^{\frac{1}{\lambda}} \ .$$

Note that such a setting directs the model towards the task-based model developed by Acemoglu and Autor (2011), which we analyze in Section 6.6.1, in the sense that a continuously-increasing wage-premium function would allow that increases in the wage premium are accompanied by workers of skills $r \in [\check{r}, \tilde{r}(i_a))$ crowding into the industry i_a, in line with the rationale of Acemoglu and Autor (2011). However, the continuously-increasing section of $\omega(r)$ complicates the analysis without generating additional insights compared to the task-based model of Acemoglu and Autor (2011). Assumption ASC offers a simple solution to this problem. However, we believe that institutional settings of job admissions and private-sector job assessments do generate such strict cutoffs. Furthermore, even without the assumption, the implications of the model largely stay the same, yet, with less strict and potentially slightly smaller level differences in the wage premium function. We continue the analysis of continuously-distributed skills and then analyze the implications of changing task-complexities.

Labor Supply and Technology. We first perform two simple comparative static scenarios. For that we assume that $\tilde{r}(i_m) = r_l$ and $\tilde{r}(i_a) < r^h$, i.e. there is full employment. The labor supply of each group $\mathcal{R}_L(i_m, i_a)$ and $\mathcal{R}_H(i_a)$ is

$$\phi_L = \int_{\tilde{r}(i_m)}^{\tilde{r}(i_a)} f(r)dr \qquad\qquad \phi_H = \int_{\tilde{r}(i_a)}^{r_h} f(r)dr \ ,$$

and in effective labor terms, this is

$$\tilde{\phi}_L = \int_{\tilde{r}(i_m)}^{\tilde{r}(i_a)} \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr \qquad \tilde{\phi}_H = \int_{\tilde{r}(i_a)}^{r_h} \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr\ .$$

In such an economy, the wage premium reads

$$\omega = \left[\frac{\tilde{\phi}_L}{\tilde{\phi}_H}\frac{\psi_{i_a}}{\psi_{i_m}}\left[\frac{i_a}{i_m}\right]^{\frac{1-\sigma_I}{\lambda}}\left[\frac{n_{i_a}}{n_{i_m}}\right]^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I}}\ .$$

In the first scenario we marginally raise the complexity level i_m. The derivative of the wage premium with respect to i_m is[22,23]

$$\frac{\partial\omega}{\partial i_m} = \frac{1}{\sigma_I}\frac{\omega}{\lambda i_m}\left[\sigma_I - 1 - \frac{1}{\tilde{\phi}_L}\left[\frac{\log(r_h)}{\log\left(e^{-\frac{1}{\lambda i_m}}\right)}\right]^{\frac{1}{\lambda}} f\left(e^{-\frac{1}{\lambda i_m}}\right)\frac{e^{-\frac{1}{\lambda i_m}}}{i_m}\right]\ ,$$

which can be smaller or greater than zero. The term $\sigma_I - 1$ reflects the substitution effect due to the price increase of good i_m. The price increases in i_m because the complexity of the production process is characterized through the task-complexity level, and the more complex the production, the higher the probability that production fails and the lower expected output—and thus the higher the price charged by the firm. This effect raises the wage premium, as consumer substitute for good i_a, leading to a higher demand for high-skilled labor. The last term, which is a negative density, characterizes the marginally smaller labor supply of the low-skilled group in effective labor terms. This effect, in turn, decreases the wage premium and sends this density of workers into unemployment. Which of the two effects dominates cannot be determined.

In the second scenario, we marginally raise the complexity level i_a, which yields the following effect on the wage premium:

$$\frac{\partial\omega}{\partial i_a} = \frac{1}{\sigma_I}\frac{\omega}{\lambda i_a}\left[1 - \sigma_I + \frac{\tilde{\phi}_L + \tilde{\phi}_H}{\tilde{\phi}_L\tilde{\phi}_H}\left[\frac{\log(r_h)}{\log\left(e^{-\frac{1}{\lambda i_a}}\right)}\right]^{\frac{1}{\lambda}} f\left(e^{-\frac{1}{\lambda i_a}}\right)\frac{e^{-\frac{1}{\lambda i_a}}}{i_a}\right]\ . \tag{6.16}$$

Again, there is a substitution effect because of the price increase in industry i_a and an

[22] We use integration by substitution

$$\int_{\tilde{r}(i_m)}^{\tilde{r}(i_a)} \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr = \int_{i_m}^{i_a}\left[\frac{\log(r_h)}{\log\left(e^{-\frac{1}{\lambda i}}\right)}\right]^{\frac{1}{\lambda}} f\left(e^{-\frac{1}{\lambda i}}\right)\frac{e^{-\frac{1}{\lambda i}}}{\lambda i^2}di\ ,$$

where we used $r(i) = e^{-\frac{1}{\lambda i}}$.

[23] We assume that the demand shifter ψ_i and the firm number n_i are independent of i.

effect caused by the change in relative supply of low-skilled and high-skilled labor. The price effect now points into the opposite direction compared to the first scenario. In this second scenario, the price of good i_a increases, due to the same reasoning as before, and consumers substitute for good i_m. The second term shows the combined effect of fewer high-skilled workers and more low-skilled workers. Note that the skill structure of the economy has not changed. However, a shift in i_a re-categorizes some of the lowest-skilled high-skilled workers as the highest-skilled low-skilled workers.

Following the discussion of Assumption ASC, such a shift in i_a may be the result of tighter institutional regulations as to the admission of workers to certain jobs. An other explanation is technological change. Due to the increasing complexity of the production process, the skill requirement to do a certain task within the more complex production process might rise. There are plenty of examples where workers need to obtain further education to achieve the capabilities to manage the ongoing on-the-job changes. Some of the workers might not have the skill to succeed in this exercise.

However, technological change is a complex process and it might also work into the opposite direction. History is also full of tasks that could only be executed by the high-skilled workers in the labor force, but which got less and less complex to execute due to tailored tools generated through innovation and new knowledge, i.e. technological progress (Autor, 2015). Goldin and Katz (1998) analyze the simplification ot tasks in the nineteenth century via emerging manufacturing technologies, resulting in a skill-capital-substitutability. Frey and Osborne (2017) study the susceptibility of jobs to computerization. In their analysis the concept of tasks is more narrowly defined than in our analysis, in the sense that a job encompasses different tasks, of which some are more, and some are less, susceptible to automation. Accordingly, the skill required to do a job can fall, depending on which tasks of a job can be automated, and which tasks of the job remain to be mastered by the worker. If the task with the highest skill requirement within a job can be automated, then the task-complexity level—as defined in our model—falls.

Skill-biased technological innovation leads to the introduction of tasks with high task-complexities. The emergence of such tasks and their following descent to lower task-complexities we call *task life-cycle*. A task life-cycle can end in a fully automated production process.[24] The concept of a task life-cycle is already observed by Acemoglu and Restrepo (2016). They note that new tasks are typically of higher complexity and they find that new job titles—that are associated with new tasks—have higher skill requirements. Furthermore, they also observe *“a pattern of ‘mean reversion’ whereby average years of schooling in these occupations decline in each subsequent decade, most likely,*

[24] Examples are the tasks of typesetters (fully automated), or bank tellers and booksellers (at the end of the task life-cycle), or pilots and graphic designers (in the middle or in the beginning of the task life-cycle).

reflecting the fact that new job titles became more open to less skilled workers over time" (Acemoglu and Restrepo, 2016, p. 29).

Our model reacts to such task life-cycles in non-trivial ways. Suppose there are many task-complexities in each industry, i.e. $\mathcal{I}^m = \{i_m^1, i_m^2, ...i_m^n\}$ and $\mathcal{I}^a = \{i_a^1, i_a^2, ...i_a^n\}$, where $i_j^s < i_j^t$ for $s < t$ and $j \in \{m, a\}$. Assume that the labor market is separated between the two sets, i.e. there is enough skill within the manual task industry and there is enough skill within the abstract task industry, but there is a wage premium for the higher-skilled workers that are able to manage the abstract tasks.[25] If now technological change allows—through disruptive innovation—a task $i_a^z > i_a^1$ to be produced requiring only task-complexity i_m^x, where $i_m^x \ll i_a^z$, then this puts downward pressure on the wage premium. Firstly, the good i_m^x (formerly i_a^z) is now produced at a lower price, because of a lower task-complexity and lower wages—no wage premium has to be paid. Consumers shift their consumption towards the good. Secondly, the demand for low-skilled worker increases—i_m^x enters the set $\mathcal{I}^m$—, and the demand for high-skilled workers decreases—i_a^z drops out of set $\mathcal{I}^a$. Note that if i_a^1 becomes less complex to produce, then the lower bound of the task set $\mathcal{I}^a$ shifts. A marginal change in i_a^1 then has the effect that is described in (6.16). A re-categorization of i_a^1 to some i_m^x can have ambiguous effects, as it is unclear whether the labor market separation continues and if so, where the separation sets in. Observe that in the framework, the exogenous demand shifters ψ_i and firm numbers n_i have great impact on labor market outcomes, insofar as their corresponding task-complexities determine how the demand for goods translates into demand for workers with a certain skill. A re-categorization of these task-complexities with their corresponding demand shifter and firm number then directly affects the demand for skill.

An important feature of technological change is the introduction of new technologies and thus new tasks and jobs. Just as there were hardly any computer scientists some decades ago, let alone social media coordinators, or app developers, new tasks and jobs will emerge. These newly-emerging tasks typically require high-skilled workers, at least at the beginning of their task life-cycle, as noted above and the emergence of a new task i_a^n in the set $\mathcal{I}^a$ increases the demand for high-skilled workers. Wage inequality increases when the emergence of new tasks with high task-complexities, with the corresponding demand, is not matched by a descent from tasks of high task-complexities to tasks of low task-complexities or the emergence of new tasks with low task-complexities. Such an increase in wage inequality is even more pronounced, if low task-complexities become automated. Brynjolfsson and McAfee (2014) state: *"As old tasks get automated away,*

[25] Note that there is no inherent difference between task-complexities in set $\mathcal{I}_m$ and task-complexities in set $\mathcal{I}_a$, i.e. we could simply define the two sets without reference to manual or abstract tasks, and define the cut-off value wherever the labor market is separated—if the labor market is separated at all.

along with demand for their corresponding skills, the economy must invent new jobs and industries" (p. 214). Though, if these new jobs and industries require high-skilled labor, wage inequality further increases.

Note that we have not introduced the concept of automation yet. Within the model we analyze here, we see that automation processes, which will take over low and middle task-complexity jobs, put downward pressure on low-skilled wages, and thus, upward pressure on the wage premium. The more technological change brings forth automation processes that manage task-complexities at higher levels—i.e. earlier in a task life-cycle—, the more the downward pressure on wages for higher skill levels, and the greater the low-skilled group in our model. Also job-polarization (Autor et al., 2003) can be rationalized when low and middle task-complexities are automated.

In Chapter 7 we analyze the remaining question in this context: *Who produces the capital used for the automation processes?* I.e. can low-skilled workers be used in the production of the technology that replaces their labor elsewhere?

6.5. Empirical Model

We can use the model to explain the wage difference between high-skilled and low-skilled labor and compare the results to the original equation used to estimate wage differences (Katz and Murphy, 1992; Autor et al., 1998) based on the SBTC-hypothesis.

6.5.1. Estimation Equations

Average wage income in both groups, denoted by $\overline{w}_H$ and $\overline{w}_L$, is

$$\overline{w}_H = \frac{\omega\tilde{\phi}_H}{\phi_H} = \frac{\omega \int_{\tilde{r}(i_a)}^{\overline{r}} \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr}{\int_{\tilde{r}(i_a)}^{\overline{r}} f(r)dr} ,$$

$$\overline{w}_L = \frac{\tilde{\phi}_L}{\phi_L} = \frac{\int_{\tilde{r}(i_m)}^{\tilde{r}(i_a)} \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr}{\int_{\tilde{r}(i_m)}^{\tilde{r}(i_a)} f(r)dr} .$$

We can build a structural equation determining the relative difference of the average wages of each group:

$$\frac{\overline{w}_H}{\overline{w}_L} = \frac{\phi_L}{\phi_H}\left[\frac{\tilde{\phi}_H}{\tilde{\phi}_L}\right]^{\frac{\sigma_I-1}{\sigma_I}} \left[\frac{\psi_{i_a}}{\psi_{i_m}}\left[\frac{i_m}{i_a}\right]^{\frac{\sigma_v-1}{\lambda}}\left[\frac{n_{i_a}}{n_{i_m}}\right]^{\frac{\sigma_I-1}{\sigma_v-1}}\right]^{\frac{1}{\sigma_I}} .$$

Following convention, we take logs and first differences, and we index all variables by time. The notation Δ stands for the first difference, e.g. $\Delta\log\left(\frac{\overline{w}_H}{\overline{w}_L}\right) = \left\{\log\left(\frac{\overline{w}_H}{\overline{w}_L}\right)\right\}_t -$

$\left\{\log\left(\frac{\overline{w}_H}{\overline{w}_L}\right)\right\}_{t-1}$, where t is a time index. Neglecting unemployment, two regimes may arise. The first regime arises if labor markets are integrated, thus $\omega = 1$ applies,

$$\Delta \log\left(\frac{\overline{w}_H}{\overline{w}_L}\right) = \Delta \log\left(\frac{\tilde{\phi}_H}{\tilde{\phi}_L}\right) - \Delta \log\left(\frac{\phi_H}{\phi_L}\right) ,$$

and the second regime arises if labor markets are disintegrated, thus $\omega > 1$ applies,

$$\begin{aligned} \Delta \log\left(\frac{\overline{w}_H}{\overline{w}_L}\right) = &- \Delta \log\left(\frac{\phi_H}{\phi_L}\right) + \frac{\sigma_I - 1}{\sigma_I}\Delta \log\left(\frac{\tilde{\phi}_H}{\tilde{\phi}_L}\right) + & (i) \\ &\frac{1}{\sigma_I}\Delta \log\left(\frac{\psi_{i_a}}{\psi_{i_m}}\right) + \frac{\sigma_v - 1}{\sigma_I \lambda}\Delta \log\left(\frac{i_m}{i_a}\right) + \frac{\sigma_I - 1}{\sigma_I(\sigma_v - 1)}\Delta \log\left(\frac{n_{i_a}}{n_{i_m}}\right) . & (ii) \end{aligned} \tag{6.17}$$

Note that there is a distinction between the *absolute* amount of labor in each group, ϕ_H and ϕ_L, and the amount of *effective* labor in each group, $\tilde{\phi}_H$ and $\tilde{\phi}_L$. They have distinct influence on the wage difference between the two groups. An increase in the productivity of high-skilled workers may or may not increase the wage difference, depending on the elasticity of substitution between industries that can be observed from the second term in line (i). If $\sigma_I > 1$, higher productivity of high-skilled labor—assuming ϕ_H stays constant—leads to increased wage inequality. In fact, the wage premium decreases, albeit to a lesser extent than the increased productivity is reflected in higher wages. If $\sigma_I < 1$, however, higher productivity leads wage inequality to decrease. Again, higher productivity results in a lower wage premium, implying that products of industry i_a become less expensive and as σ_I is inelastic, consumers switch to the products of industry i_m.

We see that the skill distribution within each group has important effects on overall wage inequality. Assuming $\sigma_I > 1$, we observe that a redistribution of skills within the low-skilled group from higher to lower skills, thereby lowering $\tilde{\phi}_L$ while not affecting ϕ_L, increases wage inequality, but decreases the wage premium.[26] And so does a redistribution of skills within the high-skilled group from lower to higher skills.

Line (ii) shows the effects of the demand shifters, the complexity differential, and the amount of firms. The closer the complexity levels i_m and i_a are to each other, the higher the demand for high-skilled labor, ceteris paribus, from industry i_a. A higher task-complexity is reflected in higher prices, less demand for the product and hence, less labor demand, and vice-versa.

In the model, technology classifies skills into high skills and low skills. Thus, this classification must not be reflected by the often-used differentiation between college graduates

[26] Observe that we analyze the second regime, when labor markets are disintegrated. In the first regime, when labor markets are integrated, there is no wage premium and $\omega = 1$.

and high-school graduates. In principle, the cut-off could be at any other complexity level, if markets are segregated at all. This is even more true, if there are more than two task-complexity levels.[27] However, the differentiation between college graduates and high-school graduates is arguably a good approximation for the division of the skill distribution into low skills and high skills.

Whenever $\sigma_I > 1$, an increase in the amount of firms within an industry increases the corresponding labor demand.

We have shown above that $\omega = 1$ if the economy is in $eq = \{PE, ILM\}$ and $\omega > 1$ if the economy is in $eq = \{DLM\}$. Depending on the equilibrium, there is a different structural estimation equation for the wage premium. Furthermore, there is a within-group estimation and a between-group estimation, where a group refers to skill sets $\mathcal{R}_L$ and $\mathcal{R}_H$. The within-group estimation then measures the differences in productivity within a group, which is reflected in wages. The between-group estimation depends on whether the labor market is integrated or not.

6.5.2. Elasticity of Substitution

The elasticity of substitution between industries, σ_I, plays a core role in our framework. σ_I is presented as a preference parameter (see (6.1)). Yet, we could also model a single consumption good produced by a competitive firm which produces a final good by assembling intermediate inputs from the industries. In such a context, σ_I—as part of the production function—reflects a technological parameter.

Observe that σ_I differs from estimates usually used in labor market contexts to analyze wage dynamics, which indicate the elasticity of substitution between high-skilled and low-skilled workers, and are based on the relative supply and wage differentials of college graduates and high-school graduates (Katz and Murphy, 1992; Acemoglu and Autor, 2011). In these studies it is assumed that there is an aggregate CES-production function. The estimated elasticities lie between 1.4 and 2 (Freeman, 1986; Heckman et al., 1998; Acemoglu and Autor, 2011). In our context, any two workers are perfectly substitutable under the condition that both are able to do the task they are hired for and given that labor markets are integrated. In the aggregate, however, labor markets might disintegrate. As long as labor markets are *not* disintegrated, the elasticity of substitution between industries does not affect the single labor market. The equilibrium in such an economy is

[27] On the basis of measures on task-complexity on the job, more diligent subdivisions into a multitude of skill groups could be constructed. An interesting case arises if there are subgroups in the upper tail of the skill distribution, where only small measures of high-skilled labor are able to perform certain task. Superstar-effects in the sense of Rosen (1981) can emerge. Note that we have not provided a clear-cut definition of the skill level variable. The catch-all variable r can also encompass relations to influential people or the possibility to obtain high-class education.

indeterminate with respect to the exact allocation of workers to firms.[28]
Observe from (6.17) that in our model,

$$\frac{\partial \log\left(\frac{\overline{w}_H}{\overline{w}_L}\right)}{\partial \log\left(\frac{\phi_H}{\phi_L}\right)} = -1 + \frac{\sigma_I - 1}{\sigma_I} \frac{\partial \log\left(\frac{\tilde{\phi}_H}{\tilde{\phi}_L}\right)}{\partial \log\left(\frac{\phi_H}{\phi_L}\right)} .$$

The relative amount of effective labor is always greater than the relative absolute (head-count) of labor, i.e. $\frac{\tilde{\phi}_H}{\tilde{\phi}_L} > \frac{\phi_H}{\phi_L}$.[29] We use variable η to denote the factor by which relative effective labor increases more than relative absolute labor, i.e.

$$\eta(F_r, \phi_H, \phi_L) = \frac{\partial \log\left(\frac{\tilde{\phi}_H}{\tilde{\phi}_L}\right)}{\partial \log\left(\frac{\phi_H}{\phi_L}\right)} > 1 ,$$

where F_r is an object representing the skill distribution. Denoting $\hat{\sigma}$ as an estimate for the elasticity of substitution between high-skilled and low-skilled labor in the canonical model, we can back out an estimate for σ_I from the estimates found in the literature, by using

$$\hat{\sigma}_I = \frac{\eta}{\eta - \frac{\hat{\sigma}-1}{\hat{\sigma}}} ,$$

and it immediately follows that $\hat{\sigma}_I \in (1, \hat{\sigma})$ for $\eta > 1$ and $\hat{\sigma} > 1$. Thus, our model suggests that σ_I is lower than the estimates found in the literature, but not below 1. However, this reasoning is based on the assumption that the division into high-school graduates and college graduates, on which the estimates of $\hat{\sigma}$ in the empirical literature are based, is a valid measure division of labor into the skill groups L and H.

6.6. Relation to the Task-based Model

The model presented in the previous sections is related to the task-based models pioneered by Acemoglu and Zilibotti (2001), Autor et al. (2003), and principally Acemoglu and Autor (2011). Such task-based models differ from earlier models developed to understand wage inequality by analyzing demand and supply of skills, insofar as they incorporate the assignments from skills to tasks within the model. Following Acemoglu and Autor (2011), we speak of the *canonical model* whenever referring to the first models which formally analyzed the demand and supply of skills and their effects on wage dynamics (Tinbergen

[28] Naturally, workers with high-skills more likely fulfill high-complex tasks and workers of low skills more likely perform low-complex tasks that the labor market clears.
[29] Effective labor is computed by scaling each worker by his productivity, which is greater for higher-skilled worker than for lower-skilled ones.

(1974, 1975); Welch (1973); Katz and Murphy (1992); Card and Lemieux (2001a,b)). Acemoglu and Autor (2011) present a task-based model with comparative advantage for certain skill groups as to certain tasks. We show that a special case of our framework leads to a closely related model of theirs. We will call their model the *"A/A-model"* and our model the *"task-complexity model"*.

6.6.1. Replicating the A/A-Model

Assume we have two skill groups, H and C, standing for high-school graduates and college graduates respectively. Each skill group incorporates a skill distribution F_H and F_C over the skill domain $\mathcal{R}$, which may or may not be overlapping. The tasks i are now uniformly distributed over the interval $(0, 1]$ and also represent firms. Consumers have CES-preferences over the products of these firms, with an elasticity of substitution denoted by σ. Furthermore, we adapt our production function and introduce an ad-hoc comparative advantage, in the sense of Acemoglu and Autor (2011),

$$\mathbb{E}[x_i] = A_z \sum_{r \in \mathcal{R}(z)} q[r]^{\frac{iq^{\lambda}}{\alpha_z(i)}} l_i(r, z) \ ,$$

where A_z is the productivity factor and $\alpha_z(i)$ is the task productivity schedule of skill group $z \in \{C, H\}$ in performing task $i \in (0, 1]$, and $l_i(r, z)$ denotes the labor employed by firm i of skill group z and skill level r. The firm's profit maximization is

$$\max_{\mathcal{R}(z), p_i(z), z, \left\{ \begin{array}{c} \{q_i(r,z)\} \\ \{x_{i,q_i(r,z),z}\} \\ \{l_i(r,z)\} \end{array} \right\}_{r \in \mathcal{R}(z)}} \sum_{r \in \mathcal{R}(z)} \left[p_i(z) x_{i,z} - l_i(r, z) w_z^r \right] \ ,$$

$$\begin{aligned} s.t. \qquad & x_{i,q_i(r,z),z} = A_z q_i(r, z) \, [r]^{\frac{iq_i(r,z)^{\lambda}}{\alpha_z(i)}} \, l_i(r, z) \ , \\ & x_{i,z} = \sum_{r \in \mathcal{R}(z)} x_{i,q_i(r,z)} dr = \psi_i \left[\frac{p_i(z)}{P_i} \right]^{-\sigma_v} \left[\frac{P_i}{P} \right]^{-\sigma_I} C \ . \end{aligned}$$

Representative firm i first chooses optimal quality. There is no minimum-quality constraint. Optimal quality incorporates the employee's skill level and the employee's group affiliation,

$$q_i(r, z) = \max \left\{ 1, \left[-\frac{\alpha_z(i)}{\lambda i \log(r)} \right]^{\frac{1}{\lambda}} \right\}, \qquad \forall (i, r, z) \in \mathcal{I} \times \mathcal{R}(z) \times \{C, H\} \ .$$

Given optimal quality and wages, the firm must decide which skill group $z \in \{C, H\}$ it will employ. The price it chooses is simply the mark-up times the chosen group's marginal costs,

$$p_i(z) = \frac{\sigma}{\sigma - 1} \frac{w_z^r}{A_z} [-e\lambda \frac{i}{\alpha_z(i)} \log(r)]^{\frac{1}{\lambda}} \; .$$

Note that the wage level depends on the group affiliation and the skill level of the employed worker. Because there is no minimum-quality constraint, every skill level is able to do any task. Thus, the firms are indifferent about the skill level, but not about the skill group affiliation. By definition, we then also have sufficient skills in the economy and every worker is paid his marginal product, i.e.

$$w_z^r = \omega_z \left[\frac{\log(r_h)}{\log(r)} \right]^{\frac{1}{\lambda}} \; , \tag{6.18}$$

where r_h is the highest skill prevalent in the economy.[30] Following Acemoglu and Autor (2011), we impose the following assumption:

Assumption 6.2 (Comparative Advantage)
$\alpha_C(i)/\alpha_H(i)$ *is continuously differentiable and strictly increasing.*

Firm $\hat{\imath}$ is indifferent between employing workers from skill group C or H if the following holds:

$$\left[\frac{\alpha_H \left(\hat{\imath} \right)}{\alpha_C \left(\hat{\imath} \right)} \right]^{\frac{1}{\lambda}} \frac{A_H}{A_C} \frac{\omega_C}{\omega_H} = 1 \; .$$

Wage scheme (6.18) and Assumption 6.2 together imply that prices are set according to

$$\begin{aligned} p_i =& \frac{\sigma}{\sigma - 1} \frac{\omega_C}{A_C} [-e\lambda \frac{i}{\alpha_C(i)} \log(r_h)]^{\frac{1}{\lambda}} && \text{for } i > \hat{\imath} \; , \\ p_i =& \frac{\sigma}{\sigma - 1} \frac{\omega_H}{A_H} [-e\lambda \frac{i}{\alpha_H(i)} \log(r_h)]^{\frac{1}{\lambda}} && \text{for } i < \hat{\imath} \; . \end{aligned}$$

Note that labor of skill group H is used in all tasks $i < \hat{\imath}$, whereas labor of skill group C is used in tasks $i > \hat{\imath}$. Thus, labor market clearing implies that

$$\int_0^{\hat{\imath}} \tilde{l}_i(H) di = \tilde{L}_H = \int_H \tilde{l} f_H(r) dr \qquad \text{and} \qquad \int_{\hat{\imath}}^1 \tilde{l}_i(C) di = \tilde{L}_C = \int_C \tilde{l} f_C(r) dr \; .$$

[30] It does not matter whether or not this skill level is present in both skill groups.

Next, we construct the price index in the economy, using the price setting derived above,

$$
\begin{aligned}
P &= \left[\int_0^1 p_i^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} , \\
&= \left[\int_0^{\hat{i}} p_i^{1-\sigma} di + \int_{\hat{i}}^1 p_i^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} , \\
&= \frac{\sigma}{\sigma-1}[-e\lambda \log(r_h)]^{\frac{1}{\lambda}} \mathcal{Q}^{\frac{1}{1-\sigma}} ,
\end{aligned}
$$

where $\mathcal{Q} = \left[\frac{\omega_C}{A_C}\right]^{1-\sigma} \int_0^{\hat{i}} \left[\frac{i}{\alpha_C(i)}\right]^{1-\sigma} di + \left[\frac{\omega_H}{A_H}\right]^{1-\sigma} \int_{\hat{i}}^1 \left[\frac{i}{\alpha_H(i)}\right]^{1-\sigma} di$. A household of skill level r belonging to skill group z demands of good i an amount

$$
c_z^r(i) = \left[\frac{p_i}{P}\right]^{-\sigma} C_z^r ,
$$

where C_z^r denotes the household's total consumption. Firms take households' demand as given. Assuming that goods market clear, firm i employs an amount of $\tilde{l}_i(z)$ effective labor, from the chosen skill group, that is equal to

$$
\begin{aligned}
\tilde{l}_i(C) &= A_C^{-1}[-e\lambda \frac{i}{\alpha_C(i)} \log(r_h)]^{\frac{1}{\lambda}} x_i , \\
&= A_C^{-1}[-e\lambda \frac{i}{\alpha_C(i)} \log(r_h)]^{\frac{1}{\lambda}} \left[\frac{p_i}{P}\right]^{-\sigma} C , \\
&= A_C^{\sigma-1} \omega_C^{-\sigma} [-e\lambda \log(r_h)]^{\frac{1}{\lambda}} \left[\frac{i}{\alpha_C(i)}\right]^{\frac{1-\sigma}{\lambda}} \mathcal{Q}^{\frac{\sigma}{1-\sigma}} C \qquad \text{for } i > \hat{i} , \\
\tilde{l}_i(H) &= A_H^{\sigma-1} \omega_H^{-\sigma} [-e\lambda \log(r_h)]^{\frac{1}{\lambda}} \left[\frac{i}{\alpha_H(i)}\right]^{\frac{1-\sigma}{\lambda}} \mathcal{Q}^{\frac{\sigma}{1-\sigma}} C \qquad \text{for } i < \hat{i} ,
\end{aligned}
$$

where $C = \int_H c_H^r(i) f_H(r) dr + \int_C c_C^r(i) f_C(r) dr$ is aggregate consumption in the economy.

Labor market clearing balances demand and supply of each skill group. Using the ratio of the two equalities, the following equilibrium condition is derived:

$$
\frac{\tilde{L}_C}{\tilde{L}_H} = \left[\frac{\omega_H}{\omega_C} \frac{A_C}{A_H}\right]^{\sigma} \frac{A_H \int_{\hat{i}}^1 \left[\frac{i}{\alpha_C(i)}\right]^{\frac{1-\sigma}{\lambda}}}{A_C \int_0^{\hat{i}} \left[\frac{i}{\alpha_H(i)}\right]^{\frac{1-\sigma}{\lambda}}} .
$$

Defining $\omega_H \equiv 1$ as the numeraire, the following system of equations can be solved for the two unknowns $\hat{\imath}$ and ω_C:

$$\frac{\omega_C}{\omega_H} = \frac{A_C}{A_H}\left[\frac{A_H \tilde{L}_H}{A_C \tilde{L}_C}\frac{\int_{\hat{\imath}}^{1}\left[\frac{i}{\alpha_C(i)}\right]^{\frac{1-\sigma}{\lambda}}}{\int_0^{\hat{\imath}}\left[\frac{i}{\alpha_H(i)}\right]^{\frac{1-\sigma}{\lambda}}}\right]^{\frac{1}{\sigma}}, \tag{6.19}$$

$$\frac{\omega_C}{\omega_H} = \frac{A_C}{A_H}\left[\frac{\alpha_C(\hat{\imath})}{\alpha_H(\hat{\imath})}\right]^{\frac{1}{\lambda}}. \tag{6.20}$$

Observe that the right-hand-side of (6.19) is decreasing in $\hat{\imath}$, whereas the right-hand-side of (6.20) is increasing in $\hat{\imath}$ by assumption. Both equations are strictly positive, thus there is a unique solution for $\hat{\imath}$. This is largely the model by Acemoglu and Autor (2011) in our model environment.

6.6.2. The A/A-Model and the Task-complexity Model

One might ask why we developed a new model to explain the evolution of wage inequality: The canonical models of Tinbergen (1974, 1975), Welch (1973), and Katz and Murphy (1992) do not incorporate the assignment of skills to tasks. The A/A-model of Acemoglu and Autor (2011) introduces an assignment of skills to tasks and explains wage dynamics by assuming that there are threshold tasks that adapt to supply and demand of skills. Depending on supply and demand of these skills, one group is more efficient than other groups in the execution of a task. Thus, the A/A-model relies on separate skill groups, each endowed with an exogenous factor-augmenting technology. Therefore, the A/A-model remains factor-augmenting, supplemented with comparative advantages in performing certain tasks. There is, however, no micro-foundation of the comparative advantages. In contrast to the A/A-model, the assignment of tasks to skills is determined by technology in our task-complexity model. This new assignment is the novelty of our approach and it describes technological change from a new perspective.

There are three main differences between the A/A-model and the task-complexity model. First, the task-complexity model departs from the assumption of different skill groups, each with a factor-augmenting technology. There is only one skill distribution and no factor-augmenting technology.[31]

[31] In principle, factor-augmenting technologies could be added to our model. The model then is assimilated to the factor-augmenting models and looses parts of its distinguishing features. However, in the form presented here, we abstain from factor-augmenting technologies. But there can emerge new task-complexities that are biased to have high complexity (and that are not biased to a skill group per se). The execution of high task-complexities is only possible for the high-skilled and demand for the high-skilled increases, when such new task-complexities emerge. The difference to factor-augmenting technology is subtle but important. According to our model there is only one factor that can be augmented, which is labor, and high-skilled and low-skilled are mere labels for the two groups when the labor market is separated at one point. Thus, even without factor-augmenting technologies, we can have skill-biased

Second, the A/A-model assumes that any skill level is able to perform any task, however, skills have comparative advantages in performing certain tasks. In the task-complexity framework, all skills are perfect substitutes as long as they are able to perform a certain task-complexity, and as long as the labor market does not disintegrate. Thus, the micro-level task-complexity is decisive as to whether a worker can perform a certain task-complexity. Wage dynamics are then determined by supply and demand. On the aggregate level, there is always a twofold comparative advantage of high-skilled labor: First, high-skilled labor is more productive and earns higher wages. Second, only high-skilled labor can receive the wage premium.[32] We must stress that a *task*—in the A/A-model—and a *task-complexity*—in the task-complexity model—are not entirely comparable, although they reflect similar concepts. Thus, in the A/A-model the labor market is separated at an *endogenous* task $\hat{\imath}$, where there is no comparative advantage between two neighbouring skill groups, given their respective wages. The task $\hat{\imath}$ is therefore endogenously-determined and reflects exogenous developments in supply and demand of skills, as well as developments in the factor-augmenting technologies. In contrast, the task-complexity model assumes that the assignment of skills to task-complexities is technologically given, i.e. whether or not a skill is sufficiently high to perform a certain task-complexity cannot change through dynamics in supply and demand for skills. Thus, no factor-augmenting technology is needed in the task-complexity model, and the wage premium is solely determined by technological constraints on the micro level and aggregate supply and demand dynamics, which, in turn, result from the skill distribution and the industry-composition in an economy.

Third, the task-complexity model allows the analysis of changing task-complexity levels, the emergence of new task-complexities and the labor market implications of the match between the industry-composition and the skill distribution. In the A/A-model, the continuum of tasks causes the endogenous displacement of workers of certain skill groups in response to exogenous skill-group-specific, factor-augmenting technological developments. However, unemployment is not possible, as workers can always switch to tasks previously performed by another skill group, if they accept lower wages.[33] In contrast, the task-complexity model is also able to give a micro-founded explanation—based on the concept of task-complexity—for the predicted "useless" and "unemployable class" described by Harari (2016).[34]

technological change through the emergence of new task-complexities of high complexity. We discussed the introduction of new technologies, and in particular what we call the task life-cycle, in Section 6.4.

[32] This must not be the case in the extension we present in Appendix C.3.

[33] Technically, with a subsistence level of consumption, there can also be unemployment in the A/A-model.

[34] Cf. the quote of Harari (2016) at the beginning of the chapter.

The A/A-model is very useful to understand wage dynamics that can explain the empirical patterns of wage inequality and the recently observed wage polarization (Acemoglu and Autor, 2011). Yet, our model allows a different view on wage dynamics that are caused by technological change, that evades a catch-all factor-augmenting technology and a comparative advantage concept. The task-complexity model features a micro-foundation of the production function for each task and skill.[35] The production function, together with the simple question whether or not a worker is able to do a certain task, yields non-trivial dynamics. E.g. skill-biased technological innovation leads to the emergence of new task-complexities, that require high-skilled labor. In that case, growing wage inequality is a feature of technological change that is biased to *innovating* task-complexities of high complexity and *automating* low task-complexities.

The implications of automation for certain tasks-complexities or tasks is very different in the two models. In th A/A-model, workers who lose their job because of automation simply take over tasks previously performed by higher-skilled workers. In the task-complexity model, such an upward shift is not possible if low-skilled workers are unable to produce higher task-complexities. Such a barrier to the reallocation of low-skilled workers, together with the occurring automation of low task-complexities, can greatly aggravate inequality.[36]

We believe that the two frameworks complement each other and should both be considered when analyzing the implications of a given policy.

6.7. Conclusion

Our framework shows rich patterns with respect to labor market outcomes based on the skill-task-assignment that is co-determined by technology. Thus, the interaction between demand and supply of skills and task-complexities—i.e. the concept of the production process complexity that is assigned to a task and that requires a minimum skill level—can lead to labor market separation. We analyzed the equilibria that can arise in this framework and introduced the concept of the task life-cycle.

Further research has to be done to endogenize the task life-cycle and associated skill-

[35] In Appendix C.2 we discuss whether or not our proposed production function that relies on the O-ring theory can be regarded as a micro-foundation of the skill-task-assignment. We show, that the same results could be obtained with a simpler production function, which is, though, less based on economic reasoning. Thus, we argue that the production function used in our framework indeed is a convenient representation of micro-level characteristics in production that are important with regard to wage inequality.

[36] Not only automation on low task-complexities may aggravate inequality, but also automation in the middle range of task-complexities—and even on some high task-complexities. The industry-composition of the model—the workers' task-complexity-composition, i.e. the task-complexities that remain being executed by workers—is decisive for the evolution of inequality in combination with the skill distribution.

biased technological innovation. It would be worth to study the task-complexity model within an endogenous growth model, where technological change introduces new task-complexities and new products within the industries, and spurs overall productivity.
We ignored the role of capital and, in particular, the role of capital as a substitution technology for labor. In the next chapter, we analyze uneven technological change by introducing capital that is increasingly productive and that can be used as an input in the production relying on low task-complexities. We then perform comparative statics.[37]

[37] This is an exemption in the technological change literature, in which endogenous growth frameworks are mostly used to address technological change. We thus regard this study of our task-complexity model, in which technological change can operate through different channels, as a starting point for more elaborate models.

7. Who Produces Capital?

> *"We are being afflicted with a new disease of which some readers may not yet have heard the name, but of which they will hear a great deal in the years to come—namely, technological unemployment. This means unemployment due to our discovery of means of economising the use of labour outrunning the pace at which we can find new uses for labour."*
> (Keynes, 1931, p. 325)

7.1. Introduction

In this chapter, we enrich our model by four features. First, we make a sharper distinction between two sets of task-complexities. Following Autor et al. (2003) we call them *"routine"* and *"non-routine"* task-complexities.[1] Routine task-complexities are representative for production processes that follow explicit programmed rules, whereas non-routine task-complexities represent production processes that cannot be specified by such programmed rules. Routine task-complexities can be executed by workers of any skill level, while non-routine task-complexities always require high-skilled workers.[2] Second, capital can substitute routine work to some extent. Third, there is a third production process that generates capital. As we motivate below in detail, we distinguish two modes to produce capital: in one mode, capital is produced with routine task-complexity and in the other mode, it is produced with non-routine task-complexity. Fourth, we will explore what happens with wages and wage inequality when there is technological progress in the third production process, i.e. when it becomes easier to produce capital.

[1] Graetz and Feng (2015) divide the task space into *training-intensive* and *innate ability* tasks, whereas each dimension is further differentiated by complexity. They observe that the division in routine and non-routine is insufficient to describe automation processes, when firms are allowed to choose which tasks they want to automate, and to show endogenized job polarization. In our model, we could introduce refined subdivisions of the task-complexities. However, for our main results the subdivision into routine and non-routine task-complexities is already insightful.

[2] In contrast to the previous chapter, we focus on Triangle Equilibra (Integrated and Disintegrated Labor Market Equilibria), neglecting the case of Parallel Equilibrium and Unemployment Equilibrium. See Chapter 6 for a detailed discussion of possible equilibria.

It is a convenient and significant abstraction to call the three industries as follows:

- Manufacturing (RM or NM): This industry produces capital. We further distinguish the two production modes in manufacturing:
  - Manufacturing (RM) that has a production process based on *routine* task-complexity (Section 7.3);
  - Manufacturing (NM) that has a production process based on *non-routine* task-complexity (Section 7.4);
- Non-routine Services (NS): This industry produces services that are based on non-routine task-complexity;
- Routine Services (RS): This industry produces services that are based on routine task-complexity.

The motivation for this set-up is as follows. Regarding services, many industries involve routine task-complexities such as retail and transportation. However, a significant fraction of industries involve non-routine task-complexities such as consulting, auditing, design or software development. Typically, manufacturing, since the industrial revolution, involved a large share of routine task-complexities and enabled low-skilled workers to work in factories on production and assembly lines (Goldin and Katz, 1998; Autor et al., 2003). In the following we will call capital generated by routine task-complexity "machines".
Frey and Osborne (2017) state that *"over the past decades, industrial robots have taken on the routine tasks of most operatives in manufacturing"* (p. 260).[3] The production of such robots involves a large amount of non-routine task-complexities (Acemoglu and Restrepo, 2016; Frey and Osborne, 2017). Accordingly, we will call capital generated by non-routine task-complexity "robots". Hence we will investigate two variants of the economy:

- *Routine* manufacturing (RM) and the service industries (NS, RS)—henceforth called the *"industrial"* economy—in Section 7.3;
- *Non-routine* manufacturing (NM) and the service industries (NS, RS)—henceforth called the *"robotic"* economy—in Section 7.4.

Thus, in context of the industrial economy we will speak of capital as machines, that are substitutes for routine labor, and in context of the robotic economy we will speak of capital as robots, that are substitutes for routine labor. The substitution of labor through

[3] Frey and Osborne (2017) also note that robots are gaining the ability to perform also some non-routine tasks. Such developments could be implemented in our model, however, as long as the dominant development of robots is to perform routine tasks, this observation does not impair our results.

capital is called "automation", and thus encompasses machines *and* robots in our model. We will show, that the robotic economy shares key features with the predictions of Frey and Osborne (2017) about the future of employment and automation potential.[4] Thus, in contrast to models which assume that capital is generated from the final good in the economy, we specifically focus on who can be used for the production of capital.

Throughout the chapter we remain analyzing technological change, wages and wage inequality, in the context of the *task-complexity* model outlined in the previous chapter.

Our main insights are as follows: In an industrial economy, the substitution of workers in the routine service industry through machines that can be produced by the same workers, leads to an integrated labor market when there is uneven technological progress in manufacturing. In contrast, in a robotic economy, the substitution of workers in the routine service industry through robots that can only be produced with workers of high skill levels, leads to disintegrated labor markets. Thus, the analysis shows the importance of the production mode when there is uneven technological change.

The chapter is organized as follows: Section 7.2 gives a summary of the relevant literature. Section 7.3 details the industrial economy. Section 7.4 details the robotic economy. In Section 7.5, we discuss the results and compare the two models. Section 7.6 concludes.

7.2. Relation to the Literature

In this section, we relate to the literature on uneven productivity growth and automation. The question how productivity improvements in one industry affect employment in other industries and in the economy as a whole is a long-standing issue in economics. One of the first contributor is Baumol (1967), who examined the employment consequences of uneven technological progress, and Baumol et al. (1985), who further explore uneven technological progress. In the 1990ies, a large literature explored how uneven (or even) productivity improvements affect unemployment in the presence of labor market frictions (Cohen et al., 1994; Aghion and Howitt, 1994, 1998; Peretto, 2011). Overall, the literature found ambiguous results.

Since, theory on labor-replacing technologies developed greatly: Peretto and Seater (2013) and Benzell et al. (2015) build dynamic models of factor-eliminating technical change. Acemoglu and Restrepo (2016) and Hémous and Olsen (2016) develop growth models that involve automation and horizontal innovations, producing a rich dynamic pattern, of

[4] Frey and Osborne (2017) focus on the destruction effect of technology, neglecting the capitalization effect that causes firms to expand production and employment because of increased productivity. We will see, that in our framework focusing only on the destruction effect of technology, i.e. the substitution of workers, will not suffice to understand wage inequality and the difference between an industrial economy and a robotic economy.

how wages and wage differentials of low-skilled and high-skilled workers develop.

Our paper complements the growth approach to automation. We consider how productivity improvements in the industry that produces the capital—i.e. machines or robots, depending on the production mode—impact the wages of low-skilled and high-skilled workers.

Brynjolfsson and McAfee (2014) document, how robots can be produced with increased ease, and how this will become a central element of the future economy. We focus on this aspect. We adopt a medium-run perspective and do not specify all elements of productivity improvements in the industry that produces the robots. For our qualitative results, the fact that such productivity improvements take place at all is important, but their magnitude is not. The magnitude of such productivity improvements should be rationalized in an endogenous growth set-up.[5]

There is a large literature on wage and income inequality—what has happened over the last decades and how it can be explained. Acemoglu and Autor (2011) or Hémous and Olsen (2016) offer detailed discussions of this literature. The following result of this literature is important for our exercise: In relative terms, middle-skill wages have been declining in the US since the mid-1980's and low-skill wages in the period before. The hypothesis is that many low-skilled tasks have already been automated and routine tasks such as storing, processing and gathering information performed by middle-skill workers are now undergoing an automation process (Spitz-Oener, 2006; Goos et al., 2009; Autor and Dorn, 2013; Hémous and Olsen, 2016).

As to the impact of productivity improvements on automation in an industrial economy and in a robotic economy, we will take the elasticity of substitution between machines and labor performing routine tasks as given. Arguably, increasing automation may also impact the elasticities of substitution directly.

7.3. Industrial Economy – Capital from Routine Labor

In this section we analyze an economy where capital can be produced by low-skilled labor in the manufacturing industry. The presentation of the model is kept short whenever appropriate, as many building blocks have been introduced in the previous chapter.

[5] Taking a long-run perspective, also the response of the labor supply to changes in wages and adjustments in the capital stock would have to be considered.

7.3.1. Macroeconomic Environment

The macroeconomic environment is closely related to the one in Section 6.2. The principal distinction is the apposition of a third industry to the economy—manufacturing—which produces capital through labor input. Furthermore, we exclude the exogenous demand shifters from the model.

There is a continuum of households, each endowed with L units of labor. Each household is characterized by a skill level r, reflecting the household's productivity when employed as a worker in production. We will use the terms household r, worker r and labor of skill level r interchangeably. We assume that the skill level is distributed according to some density function $f(r)$ with support $\mathcal{R} = [\underline{r}, \bar{r}]$, where $0 < \underline{r} < \bar{r} < 1$.

The households optimize on their utility by maximizing their consumption basket (6.1).

7.3.2. Industries, Firms and Households

We assume that there are two distinctive task-complexities, i_R and i_N, standing for *"routine"* and *"non-routine"*.[6] These two task-complexities also represent two industries, which we call industry i_R and industry i_N—routine and non-routine industry—, respectively. Both of these industries produce services, while the manufacturing industry produces capital. We use the terms capital and machines interchangeably in the industrial economy. The production of machines is based on a production process involving the routine task-complexity i_R.

Industries. In accordance with the previous chapter, the production function for non-routine service of firm (i_N, j) is

$$\mathbb{E}[x_{i_N,j}] = \sum_{r \in \mathcal{R}_{i_N,j}} q[r]^{i_N q^{\lambda}} l_{i_N,j}(r) \; . \tag{7.1}$$

The production function for routine service for firm (i_R, j) is[7]

$$\mathbb{E}[x_{i_R,j}] = \left[\left[\sum_{r \in \mathcal{R}_{i_R,j}} q[r]^{i_R q^{\lambda}} l_{i_R,j}(r) \right]^{\frac{\sigma_R - 1}{\sigma_R}} + [k_{i_R,j}]^{\frac{\sigma_R - 1}{\sigma_R}} \right]^{\frac{\sigma_R}{\sigma_R - 1}} , \tag{7.2}$$

where, in contrast to the non-routine service industry, capital k can be used as a substi-

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[6] For a detailed discussion of task-complexities, see Section 6.2 and for a discussion of the implication of more than two task-complexities, see Section 6.4 and Section 8.2.

[7] The differences to the previous chapter are twofold: First, the routine industry is assumed to provide services exclusively, in contrast to the produced goods and services in the simple task-complexity model. Second, labor can be substituted by capital.

tute for labor and σ_R denotes the elasticity of substitution between labor and capital. We assume $\sigma_R > \max\{\sigma_I, 1\}$, in line with our previous assumption of high substitution elasticities between factors of production. Note that we assumed perfect substitution between labor of different skill levels, that are able to perform the task-complexity of a production process. This implies that factor prices are all the more important.

The third industry, manufacturing, produces capital k—machines—usable in the production of the routine service. For simplicity we assume that this industry is competitive. The production function is

$$k = A_k \sum_{r \in \mathcal{R}_k} q\,[r]^{i_R q^\lambda}\, l_k(r)\ , \tag{7.3}$$

where A_k is an exogenous technological parameter. The production process relies on labor able to perform task-complexity i_R. Manipulating A_k will be central to our comparative statics analysis. We assume that the manufacturing industry is represented by a single firm.

Increases in A_k can be interpreted in two ways. First, the production of machines itself becomes more efficient, i.e. more capital can be produced with the same labor input. This is the intuitive interpretation when considering the functional form chosen. Second, machines become more effective in the production of routine services for the same price, i.e. capital becomes more and more effective in the production of routine services. This second interpretation is more in line with the historical record. We analyze the firms' decision problems.

Firms. We still assume that Assumption ASC holds (see p. 88), in particular, that labor of skill level r cannot perform any complexity, $\zeta_i(q)$, higher than $-\frac{1}{\lambda \log(r)}$.

The quality choice of the production process in all industries remains the same optimization problem as in Section 6.2, i.e.

$$q_i(r) = \max\left\{1, \left[-\frac{1}{\lambda i \log(r)}\right]^{\frac{1}{\lambda}}\right\}, \qquad \forall (i,r) \in \mathcal{I} \times \mathcal{R}_i\ .$$

From the previous chapter we know that the wage scheme, factoring in the productivity differences of different skill levels and aggregate supply and demand, is

$$w^r = \begin{cases} \omega \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} & \text{if } r \geq \tilde{r}(i_N)\ , \\ \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} & \text{if } r < \tilde{r}(i_N)\ , \end{cases}$$

where $\omega = 1$ if the labor market is integrated and $\omega > 1$ if the labor market is disintegrated. ω is called the "wage premium" and depends on aggregate demand and supply for skills able to perform non-routine task-complexities. The wage of the highest-skilled worker, $\bar{r}$, in the *integrated* labor market equilibrium is normalized to one.

We next relate labor of skill level r to the productivity of the highest-skilled worker $\bar{r}$ in the economy. We express labor as *effective* labor, $\tilde{l} = l(r)\left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}}$. The higher the skill level of the worker, the less absolute labor input is needed to produce one unit of output, yet, it takes the same amount of effective labor.[8]

Then the optimal choice between capital and labor, obtained by equating the relative marginal products to relative marginal cost of the two inputs, capital and effective labor, is reflected in the ratio

$$\frac{\tilde{l}_{i_R,j}}{k_{i_R,j}} = p_k^{\sigma_R}\left[-e\lambda i_R \log(\bar{r})\right]^{\frac{1-\sigma_R}{\lambda}} ,$$

where p_k denotes the price of capital. The manufacturing industry is competitive. Thus the price of capital equals its marginal production costs, $p_k = A_k^{-1}\left[-e\lambda i_R \log(\bar{r})\right]^{\frac{1}{\lambda}}$ and relative inputs optimally chosen by firm (i_R, j) result in[9]

$$\frac{k_{i_R,j}}{\tilde{l}_{i_R,j}} = A_k^{\sigma_R}\left[-e\lambda i_R \log(\bar{r})\right]^{-\frac{1}{\lambda}} .$$

Using their relative inputs determined by the factor prices, we can compute the marginal costs of firm (i_R, j). The marginal costs are equal to costs per unit of output and average costs, because the production function is linear once optimal quality is chosen and labor is converted to effective labor, i.e.

$$mc_{i_R,j} = \left[-e\lambda i_R \log(\bar{r})\right]^{\frac{1}{\lambda}}\left[1 + A_k^{\sigma_R - 1}\right]^{\frac{1}{1-\sigma_R}} .$$

The marginal costs for firm (i_N, j) are

$$mc_{i_N,j} = \omega\left[-e\lambda i_N \log(\bar{r})\right]^{\frac{1}{\lambda}} .$$

Note that the marginal costs in the non-routine industry linearly depend on the wage premium ω.

Symmetry of the production technology across firms within an industry leads to the fol-

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[8] To give an example, an amount of labor $l_i(r) = \left[\frac{\log(\bar{r})}{\log(r)}\right]^{-\frac{1}{\lambda}} l_i(\bar{r})$ of skill level r is needed to achieve the same output as $l_i(\bar{r}) = \tilde{l}_i$.

[9] For the derivation of the marginal cost given the production function (7.3) see Chapter 6.

lowing prices and price aggregators:

$$p_{i_R,j} = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i_R \log(\bar{r})\right]^{\frac{1}{\lambda}} \hat{\theta}(A_k)^{\frac{1}{1-\sigma_R}} \quad \text{and} \quad p_{i_N,j} = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i_N \log(\bar{r})\right]^{\frac{1}{\lambda}} \omega \,,$$
$$P_{i_R} = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i_R \log(\bar{r})\right]^{\frac{1}{\lambda}} \hat{\theta}(A_k)^{\frac{1}{1-\sigma_R}} n_R^{\frac{1}{1-\sigma_v}} \quad \text{and} \quad P_{i_N} = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i_N \log(\bar{r})\right]^{\frac{1}{\lambda}} \omega n_N^{\frac{1}{1-\sigma_v}} \,,$$

where $\hat{\theta}(A_k) = 1 + A_k^{\sigma_R - 1}$, and n_{i_R} and n_{i_N} denote the amount of firms in each industry. We then aggregate to obtain the ideal price index

$$P = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda \log(\bar{r})\right]^{\frac{1}{\lambda}} \hat{\mathcal{M}}^{\frac{1}{1-\sigma_I}} \,,$$

where $\hat{\mathcal{M}} = n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}} \omega^{1-\sigma_I} + n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}} \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}$.

Next we incorporate into the production function of industry i_R the effective labor needed to produce k as an input,

$$x_{i_R,j} = \left[-e\lambda i_R \log(\bar{r})\right]^{-\frac{1}{\lambda}} \tilde{l}_{i_R,j} \hat{\theta}(A_k)^{\frac{\sigma_R}{\sigma_R - 1}} \,.$$

Households. The aggregate household demand faced by a firm of each industry is

$$\begin{aligned} c_{i_N,j} &= n_N^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} i_N^{-\frac{\sigma_I}{\lambda}} \omega^{-\sigma_I} \hat{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C \,, \\ c_{i_R,j} &= n_R^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} i_R^{-\frac{\sigma_I}{\lambda}} \hat{\theta}(A_k)^{\frac{\sigma_I}{\sigma_R - 1}} \hat{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C \,. \end{aligned} \tag{7.4}$$

Goods market clearing implies that total demand for the service of firm (i_R, j), $c_{i_R,j}$, must equal production, $x_{i_R,j}$, and we obtain

$$\left[-e\lambda i_R \log(\bar{r})\right]^{-\frac{1}{\lambda}} \tilde{l}_{i_R,j} \hat{\theta}(A_k)^{\frac{\sigma_R}{\sigma_R - 1}} = n_R^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} i_R^{-\frac{\sigma_I}{\lambda}} \hat{\theta}(A_k)^{\frac{\sigma_I}{\sigma_R - 1}} \hat{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C \,.$$

Solving for labor in industry i_R yields

$$\tilde{l}_{i_R,j} = \left[-e\lambda i_R \log(\bar{r})\right]^{\frac{1}{\lambda}} n_R^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} i_R^{-\frac{\sigma_I}{\lambda}} \hat{\theta}(A_k)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}} \hat{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C \,. \tag{7.5}$$

Analogously, applying goods market clearing in industry i_N and solving for labor we obtain

$$\tilde{l}_{i_N,j} = \left[-e\lambda i_N \log(\bar{r})\right]^{\frac{1}{\lambda}} n_N^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} i_N^{-\frac{\sigma_I}{\lambda}} \omega^{-\sigma_v} \hat{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C \,. \tag{7.6}$$

The manufacturing industry is competitive. Thus, total revenues are paid to the single input in this industry: workers. Total effective labor in manufacturing is $\tilde{L}_k$. Effective labor in an industry performing task-complexity i_R always equals wages, i.e. relative productivity is mirrored in relative wages. Therefore we know that $n_{i_R} p_k k = n_{i_R} A_k^{\sigma_R - 1} \tilde{l}_{i_R} = \tilde{L}_k$, i.e. revenues equal total effective labor and equal total costs. Labor market clearing yields

$$\begin{aligned}\tilde{L} =& \tilde{L}_k + n_{i_R}\tilde{l}_{i_R,j} + n_{i_N}\tilde{l}_{i_N,j} \\ =& n_{i_R}\tilde{l}_{i_R,j}\hat{\theta}(A_k) + n_{i_N}\tilde{l}_{i_N,j} \ ,\end{aligned}$$

and total wages paid are

$$\begin{aligned}TW =& \tilde{L}_k + n_{i_R}\tilde{l}_{i_R,j} + \omega n_{i_N}\tilde{l}_{i_N,j} \\ =& n_{i_R}\tilde{l}_{i_R,j}\hat{\theta}(A_k) + \omega n_{i_N}\tilde{l}_{i_N,j} \ .\end{aligned}$$

Total wages paid and total consumption are linked through the following equation:

$$C = \left[-e\lambda \log(\bar{r})\right]^{-\frac{1}{\lambda}} \hat{\mathcal{M}}^{\frac{1}{\sigma_I - 1}} TW \ . \tag{7.7}$$

We next analyze the two equilibria of interest.

7.3.3. Equilibrium

The two equilibria, the ILM Equilibrium and the DLM Equilibrium, are as follows:

ILM Equilibrium. If there are sufficient skills in the economy, the labor demand of the i_N industry does not surpass labor supply of skills with $r \geq \tilde{r}(i_N)$. High-skilled labor may be employed in all of the three industries prevalent in the economy: manufacturing, routine services, and non-routine services. The main difference to Proposition 6.1 is the additional manufacturing industry that demands labor. The manufacturing industry is reflected in $\hat{\theta}(A_k)$ and hidden in the term $\hat{\mathcal{M}}$. There exists a unique equilibrium wage scheme and a unique equilibrium, up to the allocation of labor and skills to industries. The principal variables of the ILM Equilibrium are as follows:

(i) $w^{r\star} = \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} \quad \forall r \in \mathcal{R}$,

(ii) $P^\star = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda \log(\bar{r})\right]^{\frac{1}{\lambda}} \hat{\mathcal{M}}^{\frac{1}{1-\sigma_I}}$,

(iii) $C^\star = \left[-e\lambda \log(\bar{r})\right]^{-\frac{1}{\lambda}} \hat{\mathcal{M}}^{\frac{1}{\sigma_I - 1}} \tilde{L}$ and $P^\star C^\star = \frac{\sigma_v}{\sigma_v - 1}\tilde{L}$.

For a detailed derivation of the equilibrium see Chapter 6.

DLM Equilibrium. We now examine the DLM Equilibrium, i.e. there are no sufficient skills in the economy for the labor markets to remain integrated. Labor markets are thus separated because of the skill requirement for the production of non-routine work, which is $r \geq \tilde{r}(i_N)$. All workers able to perform the non-routine task-complexity are employed in the corresponding non-routine industry. Wages balance demand for skills and supply of skills, resulting in a wage premium for labor able to perform the non-routine task-complexity. Thus, in the DLM Equilibrium, the wage premium, $\omega > 1$, must be such that labor markets clear. We next define the relative supply in effective routine and effective non-routine labor,

$$\tilde{\phi}_R = \int_{\underline{r}}^{\tilde{r}(i_N)} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr \qquad \tilde{\phi}_N = \int_{\tilde{r}(i_N)}^{\bar{r}} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr \ ,$$

where $\tilde{L} = \tilde{\phi}_R L + \tilde{\phi}_N L$. Using (7.5) and (7.6) we obtain,

$$\begin{aligned} L\tilde{\phi}_N &= n_N \tilde{l}_{i_N,j} \\ &= [-e\lambda \log(\bar{r})]^{\frac{1}{\lambda}} n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}} \omega^{-\sigma_I} \hat{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C \ , \\ L\tilde{\phi}_R &= n_R \tilde{l}_{i_R,j} + \tilde{L}_k \\ &= [-e\lambda \log(\bar{r})]^{\frac{1}{\lambda}} n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}} \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}} \hat{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C \ . \end{aligned}$$

We determine the wage premium by taking the ratio of the two equalities above and obtain

$$\omega = \left[\frac{\tilde{\phi}_R}{\tilde{\phi}_N} \left[\frac{i_N}{i_R}\right]^{\frac{1-\sigma_I}{\lambda}} \left[\frac{n_N}{n_R}\right]^{\frac{1-\sigma_I}{1-\sigma_v}} \hat{\theta}(A_k)^{\frac{1-\sigma_I}{\sigma_R-1}}\right]^{\frac{1}{\sigma_I}} \ . \tag{7.8}$$

Using Equation (7.8), we can solve for total consumption[10]

$$\begin{aligned} C &= [-e\lambda \log(\bar{r})]^{-\frac{1}{\lambda}} \hat{\mathcal{W}}^{-1} \hat{\mathcal{M}}^{\frac{\sigma_I}{\sigma_I-1}} \tilde{L} \\ &= [-e\lambda i_R \log(\bar{r})]^{-\frac{1}{\lambda}} n_R^{\frac{1}{\sigma_v-1}} \hat{\theta}(A_k)^{\frac{1}{\sigma_R-1}} \left[\frac{\tilde{\phi}_N}{\tilde{\phi}_R} + 1\right]^{-1} \left[\frac{\omega\tilde{\phi}_N}{\tilde{\phi}_R} + 1\right]^{\frac{\sigma_I}{\sigma_I-1}} \tilde{L} \ , \end{aligned}$$

[10] We use the following equivalence:

$$\begin{aligned} \hat{\mathcal{M}} &= n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}} \omega^{1-\sigma_I} + n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}} \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}} \\ &= n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}} \omega \left[\frac{\tilde{\phi}_R}{\tilde{\phi}_N} \left[\frac{i_N}{i_R}\right]^{\frac{1-\sigma_I}{\lambda}} \left[\frac{n_N}{n_R}\right]^{\frac{1-\sigma_I}{1-\sigma_v}} \hat{\theta}(A_k)^{\frac{1-\sigma_I}{\sigma_R-1}}\right]^{-1} + n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}} \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}} \\ &= n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}} \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}} \left[\frac{\omega\tilde{\phi}_N}{\tilde{\phi}_R} + 1\right] \ . \end{aligned}$$

where $\hat{\mathcal{W}} = n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}} \omega^{-\sigma_I} + n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}} \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}$.
Total wages, TW, are related to total effective labor through $TW = \hat{\mathcal{W}}^{-1}\hat{\mathcal{M}}\tilde{L}$.

7.3.4. Technological Change

This section relates technological progress to wage inequality and aggregate consumption in an industrial economy. We call "technological progress" the increase in the productivity parameter A_k assigned to the production function of the manufacturing industry. Thus, the technological advancement in our analysis is characterized by the uneven impact on the industries, i.e. it affects only the manufacturing industry directly.

ILM Equilibrium. As long as the economy is situated in an ILM Equilibrium, the wage scheme does not change, and technological progress has primarily an income effect, and aggregate consumption increases in A_k. Nevertheless, relative industry price indices change, i.e. $^{P_{i_R}}/_{P_{i_N}}$ falls, and thus relative consumption also changes. Thereby, technological progress might tighten (i.e. increase the relative demand for high-skilled labor) or loosen (i.e. decrease the relative demand for high-skilled labor) the labor market in an ILM Equilibrium. Whether or not the labor market tightens and thereby drifts towards a DLM Equilibrium depends on σ_I. (i) If $\sigma_I > 1$, households consume more of the services from industry i_R, as prices in this industry fall with technological progress. (ii) If, however, $\sigma_I < 1$, households shift their consumption towards services of industry i_N, and the integrated labor market tightens until it disintegrates. This pattern can be shown by taking derivatives of the total effective labor demand of industry i_N, denoted by $\tilde{L}^d_{i_N}$,

$$\begin{aligned}\frac{\partial \tilde{L}^d_{i_N}}{\partial A_k} &= [-e\lambda \log(\bar{r})]^{\frac{1}{\lambda}}\, n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}} \tilde{L} \frac{\partial}{\partial A_k} \hat{\mathcal{M}}^{-1} \\ &= [1-\sigma_I]\, [-e\lambda \log(\bar{r})]^{\frac{1}{\lambda}}\, n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}} \frac{\tilde{L}}{\hat{\mathcal{M}}^2} n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}} \hat{\theta}(A_k)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}} A_k^{\sigma_R-2}\,.\end{aligned}$$

Recall that high-skilled labor is also employed in the routine service industry i_R, as well as in the manufacturing industry. In an ILM Equilibrium, all workers benefit equally from the efficiency gains and, thus, lower prices in industry i_R.

DLM Equilibrium. We assume that labor markets are disintegrated. Now, the wage premium is also a function of technological progress, as can be seen in (7.8). The following proposition presents the derivative of the wage premium with respect to technological productivity level A_k, i.e. technological progress, in the manufacturing industry.

Proposition 7.1

The elasticity of the wage premium with respect to A_k, $\sigma_{A_k,\omega}$, is[11]

$$\sigma_{A_k,\omega} = \frac{\partial \omega}{\partial A_k}\frac{A_k}{\omega} = \frac{1-\sigma_I}{\sigma_I}\frac{A_k^{\sigma_R-1}}{\theta(A_k)} \begin{cases} < 0 & \text{if } \sigma_I > 1 , \\ = 0 & \text{if } \sigma_I = 1 , \\ > 0 & \text{if } \sigma_I < 1 . \end{cases} \tag{7.9}$$

Technological progress decreases the skill premium if the elasticity of substitution between industries is greater than one ($\sigma_I > 1$). In this case, routine labor is substituted in the routine service industry by capital that is produced in the manufacturing industry through increasingly productive routine labor, i.e. routine labor is substituted and reemployed in the manufacturing industry, where it is more productive than before. Thus, wage inequality decreases, as long as $\sigma_I > 1$. The scarcer high-skilled effective labor, the higher the wage premium.[12]

The derivative of total consumption with respect to A_k is

$$\frac{\partial C}{\partial A_k} = [-e\lambda \log(\bar{r})]^{-\frac{1}{\lambda}} n_R^{\frac{1}{\sigma_v - 1}} \hat{\theta}(A_k)^{\frac{2-\sigma_R}{\sigma_R-1}} A_k^{\sigma_R-2} \tilde{L} \left[\frac{\omega\tilde{\phi}_N}{\tilde{\phi}_R} + 1\right]^{\frac{1}{\sigma_I-1}} \frac{\tilde{\phi}_R}{\tilde{\phi}_N + \tilde{\phi}_R} > 0 .$$

We denote the real wage of a worker by $\tilde{w}^r$ and the real wage scheme is

$$\tilde{w}^r = \begin{cases} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} \frac{\omega}{P} & \text{if } r \geq \tilde{r}(i_N) , \\ \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} \frac{1}{P} & \text{if } r < \tilde{r}(i_N) , \end{cases}$$

[11] The result is derived by simply taking derivatives,

$$\frac{\partial \omega}{\partial A_k} = \frac{1}{\sigma_R - 1}\frac{1-\sigma_I}{\sigma_I}\frac{\omega}{\theta(A_k)}\frac{\partial \hat{\theta}(A_k)}{\partial A_k} \lesseqgtr 0 ,$$
$$\frac{\partial \hat{\theta}(A_k)}{\partial A_k} = [\sigma_R - 1]\, A_k^{\sigma_R-2} > 0 .$$

[12] This result was already derived in Chapter 6. Now, the effect of a change in the ratio of effective labor able to perform non-routine task-complexities over those who are not depends also on A_k,

$$\frac{\partial \omega}{\partial \frac{\tilde{\phi}_N}{\tilde{\phi}_R}} = -\frac{1}{\sigma_I}\left[\frac{\tilde{\phi}_N}{\tilde{\phi}_R}\right]^{-\frac{1}{\sigma_I}-1} \left[\frac{i_R}{i_N}\right]^{\frac{\sigma_I-1}{\lambda\sigma_I}} \left[\frac{n_R}{n_N}\right]^{\frac{1}{1-\sigma_v}\frac{\sigma_I-1}{\sigma_I}} \hat{\theta}(A_k)^{\frac{1}{\sigma_R-1}\frac{1-\sigma_I}{\sigma_I}} < 0 ,$$

but only quantitatively. Naturally, the direction of the effect remains unaffected.

where $\left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} \frac{1}{P} = \frac{\sigma_v - 1}{\sigma_v} \left[-e\lambda \log(r)\right]^{-\frac{1}{\lambda}} \hat{\mathcal{M}}^{\frac{1}{\sigma_I - 1}}$. Thus, for skill levels $r \geq \tilde{r}(i_N)$,

$$\frac{\partial \tilde{w}^r}{\partial A_k} = \text{II}(r)\sigma_I^{-1} \frac{\omega \tilde{\phi}_R}{\omega \tilde{\phi}_N + \tilde{\phi}_R} > 0 \ ,$$

where $\text{II}(r) = \frac{\sigma_v - 1}{\sigma_v} \left[-e\lambda i_R \log(r)\right]^{-\frac{1}{\lambda}} n_R^{\frac{1}{\sigma_v - 1}} \hat{\theta}(A_k)^{\frac{2-\sigma_R}{\sigma_R - 1}} A_k^{\sigma_R - 2} \left[\frac{\omega \tilde{\phi}_N}{\tilde{\phi}_R} + 1\right]^{\frac{1}{\sigma_I - 1}}$, and real wages for non-routine labor always increase with technological progress in the manufacturing industry. High-skilled labor benefits from a positive income effect, as prices of the routine services lower. Whereas for skill levels $r < \tilde{r}(i_N)$,

$$\frac{\partial \tilde{w}^r}{\partial A_k} = \text{II}(r) \left[1 - \sigma_I^{-1} \frac{\omega \tilde{\phi}_N}{\omega \tilde{\phi}_N + \tilde{\phi}_R}\right]$$

can be greater or smaller zero. Per effective labor, the average change in wages plus the corresponding change in profits—expressed in the constant markup $\frac{\sigma_v}{\sigma_v - 1}$—must equal per effective labor average change in consumption, which is equal to aggregate change in consumption, and thus[13]

$$\frac{\partial C}{\partial A_k} = \left[\tilde{\phi}_N + \tilde{\phi}_R\right]^{-1} \frac{\sigma_v}{\sigma_v - 1} \left[\int_{\underline{r}}^{\tilde{r}(i_N)} \frac{\partial \tilde{w}^r}{\partial A_k} f(r) dr + \int_{\tilde{r}(i_N)}^{\bar{r}} \frac{\partial \tilde{w}^r}{\partial A_k} f(r) dr\right] .$$

For the sake of simplicity, we next assume that each worker obtains the profits that emerge from his own work, i.e. every worker earns $\frac{\sigma_v}{\sigma_v - 1} \tilde{w}^r$.[14]
We integrate over the wage changes. Then the change in per-capita-consumption from technological progress, $\frac{\partial C}{\partial A_k} \left[\tilde{\phi}_N + \tilde{\phi}_R\right]$, is shared between routine and non-routine labor according to

$$\frac{\sigma_v}{\sigma_v - 1} \int_{\tilde{r}(i_N)}^{\bar{r}} \frac{\partial \tilde{w}^r}{\partial A_k} f(r) dr = \frac{\partial C}{\partial A_k} \left[\tilde{\phi}_N + \tilde{\phi}_R\right] \sigma_I^{-1} \frac{\omega \tilde{\phi}_N}{\omega \tilde{\phi}_N + \tilde{\phi}_R} \ , \tag{7.10}$$

$$\frac{\sigma_v}{\sigma_v - 1} \int_{\underline{r}}^{\tilde{r}(i_N)} \frac{\partial \tilde{w}^r}{\partial A_k} f(r) dr = \frac{\partial C}{\partial A_k} \left[\tilde{\phi}_N + \tilde{\phi}_R\right] \left[1 - \sigma_I^{-1} \frac{\omega \tilde{\phi}_N}{\omega \tilde{\phi}_N + \tilde{\phi}_R}\right] \ , \tag{7.11}$$

where total non-routine labor obtains $\sigma_I^{-1} \frac{\omega \tilde{\phi}_N}{\omega \tilde{\phi}_N + \tilde{\phi}_R}$ and total routine labor obtains $1 -$

[13] Equivalently, the expression can also be written as

$$\frac{\partial C}{\partial A_k} = \frac{\tilde{\phi}_R}{\tilde{\phi}_N + \tilde{\phi}_R} \frac{\sigma_v}{\sigma_v - 1} \frac{\int_{\underline{r}}^{\tilde{r}(i_N)} \frac{\partial \tilde{w}^r}{\partial A_k} f(r) dr}{\tilde{\phi}_R} + \frac{\tilde{\phi}_N}{\tilde{\phi}_N + \tilde{\phi}_R} \frac{\sigma_v}{\sigma_v - 1} \frac{\int_{\tilde{r}(i_N)}^{\bar{r}} \frac{\partial \tilde{w}^r}{\partial A_k} f(r) dr}{\tilde{\phi}_N} ,$$

i.e. the change in consumption equals the sum of the wage changes per effective labor of both skill groups, weighted by their respective share in total effective labor times $\frac{\sigma_v}{\sigma_v - 1}$.

[14] A rationale for this assumption is that each worker runs his own firm and pays himself a wage, being his own employee, and earns profits from his own firm.

$\sigma_I^{-1}\frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}$ from the per-capita-change in consumption.
We can distinguish three cases on the real wage dynamics of low-skilled labor:

(i) $\sigma_I > 1$: The income effect and the substitution effect are aligned. Both effects lead to increasing consumption of services from industry i_R. Thus, real wages of low-skilled workers increase (and the increase is higher than the increase of the real wages of high-skilled labor. See the upcoming Corollary 7.1).

(ii) $\sigma_I \in \left[\frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}, 1\right]$: The income effect is stronger than (or equal to) the substitution effect. The substitution effect leads households to shift their consumption towards services from industry i_N. The income effect leads households to consume more from both industries. The demand for routine services increases—the income effect remains stronger than the substitution effect—, leading to an increase in real wages for low-skilled labor, albeit not as high as the increase in real wages for high-skilled labor (Corollary 7.1).

(iii) $\sigma_I \in \left(0, \frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}\right)$: The substitution effect dominates the income effect and households shift their consumption towards services from industry i_N. The increased demand after i_N-services increases the wage premium and thereby lowers the real wage of low-skilled workers.

In the ILM Equilibrium, technological progress benefits workers irrespective of their skill level. In contrast, the DLM Equilibrium features different wage developments in response to technological progress in the manufacturing industry.

Proposition 7.2
The real wage of the non-routine labor always increases in A_k. The real wage of routine labor increases if $\sigma_I > \frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}$.

We rewrite Equations (7.10) and (7.11) to isolate the income and substitution effects,

$$\begin{aligned}\frac{\sigma_v}{\sigma_v-1}\int_{\tilde{r}(i_N)}^{\bar{r}}\frac{\partial\tilde{w}^r}{\partial A_k}f(r)dr =& \frac{\partial C}{\partial A_k}\left[\tilde{\phi}_N+\tilde{\phi}_R\right]\left[\frac{1-\sigma_I}{\sigma_I}\frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}+\frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}\right],\\ \frac{\sigma_v}{\sigma_v-1}\int_{\underline{r}}^{\tilde{r}(i_N)}\frac{\partial\tilde{w}^r}{\partial A_k}f(r)dr =& \frac{\partial C}{\partial A_k}\left[\tilde{\phi}_N+\tilde{\phi}_R\right]\left[\frac{\sigma_I-1}{\sigma_I}\frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}+\frac{\tilde{\phi}_R}{\omega\tilde{\phi}_N+\tilde{\phi}_R}\right],\end{aligned} \tag{7.12}$$

where the first term in the last bracket denotes the substitution effect and the second term the income effect respectively. We now can isolate the income effect by assuming that $\sigma_I = 1$ (Cobb-Douglas Utility), i.e. there is no substitution effect. Then each of the group obtains a share from per capita consumption gains according to each groups' share in total wages. This share in total wages stays constant if $\sigma_I = 1$.[15] If now σ_I is *not* equal to unity, then the substitution effect requires that due to technological progress, either

- more of service i_R is consumed (if $\sigma_I > 1$), which lowers demand for non-routine labor and puts downward pressure on the wage premium,
- or more of service i_N is consumed (if $\sigma_I < 1$), which raises demand for non-routine labor and puts upward pressure on the wage premium.

Naturally, the substitution effect shifts the wage shares earned by the two groups through changes in the wage premium. Thereby the income effect is either shifted towards non-routine labor (if $\sigma_I < 1$) or towards routine labor (if $\sigma_I > 1$).

Note that $\frac{\omega\tilde{\phi}_N}{\tilde{\phi}_R}$ not only represents relative wages, but also relative revenues and thus aggregate expenditures of households for services of the two industries. The substitution effect is then the combination of the direction, $\frac{\sigma_I - 1}{\sigma_I}$, and the magnitude, $\frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N + \tilde{\phi}_R}$, of the effect.

In Appendix D.2.6 we derive the result of Proposition 7.2 by analyzing changes in the consumption bundle of routine labor in response to technological progress A_k. We can now infer the following two corollaries from Proposition 7.1 and 7.2.

Corollary 7.1

The real wage of non-routine labor always increases less (more) in A_k than the real wage of routine labor if $\sigma_I > 1$ ($\sigma_I < 1$).

Corollary 7.2

The real wage of routine labor more likely decreases in A_k if $\sigma_I < 1$, the higher the current wage premium.

Corollary 7.2 follows from the derivative $\frac{\partial}{\partial\omega}\frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N + \tilde{\phi}_R} > 0$. Essentially Corollary 7.2 states that the boundary $\frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N + \tilde{\phi}_R}$ increases whenever $\sigma_I < 1$, because in such case the wage premium must increase (Proposition 7.1). This shift in the strength of the income effect and substitution effect implies the following corollary:

Corollary 7.3

If $\sigma_I < 1$ then the real wage of routine labor eventually decreases when A_k grows large.

Endogenous growth models are often based on expanding varieties. In our model the benefits of a marginal expansion of varieties—or equivalently the productivity increase if we model a production function which aggregates intermediate inputs into a single final consumption good—do not have to be the same for the two industries, which in itself, adds

[15] In Proposition 7.1, we show that in such case, the wage premium stays constant

a further dimension to variety expansion and growth. This must be examined. Models of endogenous growth often build on profit incentives that lead research firms to innovate in certain industries. This is a strong assumption insofar as it supposes that innovators' sole driver are profit incentives and as it excludes other drivers, such as technological possibilities, opportunity, luck, technological feasibility, and market structures. Furthermore, the assumption implies that firms innovate in such a way that expected profits equate.[16] However, the technological terrain in one industry may be very fertile for innovation even if profits are low in that industry. Even if all firms have equal profits (in expectation), the marginal utility of a new variety can be different for high-skilled and low-skilled labor, as wage inequality is also affected by marginal shifts in the ratio of varieties (in a DLM Equilibrium). Analyzing again (7.8) reveals that if $\sigma_I > 1$, then

$$\frac{\partial \omega}{\partial \frac{n_N}{n_R}} = \frac{1}{\sigma_v - 1} \frac{\sigma_I - 1}{\sigma_I} \omega \frac{n_R}{n_N} > 0 \, ,$$

i.e. an increase in the ratio of non-routine varieties to routine varieties increases inequality, as demand for high-skilled workers increases. It is thus also a question in which industry the development of new varieties is more likely.

A further effect that might play a crucial role is the task-complexity i_N—and possibly also i_R—that may change with technological progress and structural change. Eventually, the technological value i_N acts as a cut-off value for the current skill segregation. The derivative of the wage premium with respect to i_N, assuming $\sigma_I > 1$, is[17]

$$\frac{\partial \omega}{\partial i_N} = \frac{\omega}{\sigma_I \lambda i_N} \left[\left[\frac{\log(\bar{r})}{\log(\tilde{r}(i_N))} \right]^{\frac{1}{\lambda}} f(\tilde{r}(i_N)) \frac{\tilde{r}(i_N)}{i_N} \frac{\tilde{\phi}_N + \tilde{\phi}_R}{\tilde{\phi}_N \tilde{\phi}_R} + 1 - \sigma_I \right] \lesseqgtr 0 \, .$$

This essentially means that if the task-complexity increases, on the one hand, the requirements to fulfill the task increase, and there is thus less labor that is able to master such a task. On the other hand, for any worker able to do the task, the output per worker decreases, as the very task itself is now more difficult to produce. Thus the price of the service also increases. The second effect is reflected in the term $1 - \sigma_I$.[18]

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[16] In Part I, we have analyzed basic research in a multi-country and multi-industry framework. One result we obtained was that profits can differ across industries in a decentralized equilibrium.

[17] The derivative is obtained by using integration by substitution (see Footnote 22 in Chapter 6).

[18] In Section 6.4 we analyzed varying task-complexity levels in more detail.

Marginal utilities from increasing either variety are

$$\begin{aligned}\frac{\partial C}{\partial n_R} =& C\frac{1}{\sigma_v - 1}\frac{\tilde{\phi}_R}{\tilde{\phi}_R + \omega\tilde{\phi}_N}n_R^{-1} > 0 \ , \\ \frac{\partial C}{\partial n_N} =& C\frac{1}{\sigma_v - 1}\frac{\omega\tilde{\phi}_N}{\tilde{\phi}_R + \omega\tilde{\phi}_N}n_N^{-1} > 0 \ .\end{aligned}$$

We observe that $\frac{\partial C}{\partial n_R} > \frac{\partial C}{\partial n_N}$ whenever $\frac{\tilde{\phi}_R}{n_R} > \frac{\omega\tilde{\phi}_N}{n_N}$, i.e. when production costs per variety are higher in industry i_R compared to industry i_N. The fraction $\frac{\omega\tilde{\phi}_N}{\tilde{\phi}_R}$ is proportional to total wages paid, and to total expenditures and to total revenues in the two industries. The households optimally have the same real cost per variety in both industries.

7.3.5. Summary

Table 7.1 summarizes the occurring effects in the ILM Equilibrium and the DLM Equilibrium. It presents the main variables of the model and illustrates whether or not they are affected by technological progress in the manufacturing industry. Unaffected variables, i.e. those that stay constant, are marked with a 'c'. Variables that increase are marked with a '$+$' and variables that decrease are marked with a '$-$'.[19]

For the consumption decisions of households, we must know how profits are distributed in the economy. We again assume that income through profits is proportional to households' wage income. There is no savings decision. Thus every household simply consumes all off its income. Under this assumption, the real wage of a household r, $\tilde{w}^r$, times the factor $\frac{\sigma_v}{\sigma_v - 1}$ must equal household r's total consumption, C^r,

$$C^r = \frac{\sigma_v}{\sigma_v - 1}\tilde{w}^r \ . \tag{7.13}$$

ILM Equilibrium. The effects of rising productivity in the manufacturing industry within an ILM Equilibrium are presented on the left side of Table 7.1. The wage premium stays equal to unity and the aggregate price index unambiguously decreases due to technological progress. Thereby, real wages increase for every worker in the economy. Because of lower prices, demand for service i_R increases, leading to higher production in this industry. Depending on whether industries are substitutes $(\sigma_I > 1)$ or complements $(\sigma_I < 1)$, consumers shift part of their consumption away from or towards services of industry i_N. This pattern is the same for low-skilled and high-skilled households.

[19] We stated in Section 7.2 that we are interested in the qualitative effects of the medium-term, i.e. about the direction rather than the magnitude of effects, where the latter would have to be analyzed in an endogenous growth framework.

Table 7.1.: Effects of Rising Productivity in Manufacturing—Industrial Economy

Variable		ILM			DLM		
		Aggregate	$r < \tilde{r}(i_N)$	$r > \tilde{r}(i_N)$	Aggregate	$r < \tilde{r}(i_N)$	$r > \tilde{r}(i_N)$
1. Wage Premium	ω	c			$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$		
2. Price Index	P	$-$			$-$ $\left(\sigma_I > \frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}\right)$ $+$ $\left(\sigma_I < \frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}\right)$		
3. Real Wage	$\tilde{w}^r$		$+$	$+$		$-\left(\sigma_I < \frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}\right)$ $+\left(\sigma_I > \frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}\right)$	$+$
4. Service i_R	$n_{i_R}x_{i_R}$	$+$			$+$		
Service i_N	$n_{i_N}x_{i_N}$	$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$			c		
5. Consumption i_R	$c^r_{i_R}$		$+$	$+$		$-\left(\sigma_I < \frac{\omega\tilde{\phi}_N}{2\omega\tilde{\phi}_N+\tilde{\phi}_R}\right)$ $+\left(\sigma_I > \frac{\omega\tilde{\phi}_N}{2\omega\tilde{\phi}_N+\tilde{\phi}_R}\right)$	$+$
Consumption i_N	$c^r_{i_N}$		$-(\sigma_I > 1)$ $+(\sigma_I < 1)$	$-(\sigma_I > 1)$ $+(\sigma_I < 1)$		$-(\sigma_I < 1)$ $+(\sigma_I > 1)$	$-(\sigma_I > 1)$ $+(\sigma_I < 1)$

DLM Equilibrium. The effects of rising productivity in the manufacturing industry within a DLM Equilibrium are illustrated on the right side of Table 7.1, and differ substantially from the effects within an ILM Equilibrium previously discussed. First, note that the DLM Equilibrium only exists for $\omega > 1$. The wage premium then falls with technological progress in the manufacturing industry whenever industries are substitutes $(\sigma_I > 1)$.

An increase in A_k leads to lower prices for services i_R. Thus if services (industries) are substitutes, households shift their consumption towards service i_R. Thereby, demand for services i_N decreases. The shift in demand leads to less labor demand of industry i_N. As only industry i_N employs high-skilled labor, less demand for labor of this industry immediately leads to a decrease in the wage premium. Furthermore, as long as all high-skilled labor is used in the production of services i_N, i.e. whenever the economy is in a DLM Equilibrium, output in this industry cannot vary. Thus the production of this industry stays constant (see Table 7.1). So, the wage premium adjusts in order to balance demand for high-skilled labor and the constant output level.

The aggregate price index falls if $\sigma_I > \frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}$ and thus, the real wage of the low-skilled increases. If now $\sigma_I < \frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}$, the wage premium increases sufficiently, due to higher demand for high-skilled labor, to overcompensate the efficiency gains from lower prices in industry i_R on the aggregate price level. Intuitively, when $\sigma_I < 1$, households wish to consume more of services i_N. The production of this service, however, is restricted by the supply of high-skilled labor, leading to an increase in the wage premium to balance demand and supply. Now, whenever σ_I is lower than total wages of the high-skilled workers relative to total wages of all workers, the increased demand of all households for services i_N, due to technological progress, results in an increase of the wage premium, and thereby also of the industry price index P_{i_N}, which makes the low-skilled worse off. Because the services are strong complements, the efficiency gain in production of service i_R (with the help of machines) does not suffice to compensate the higher prices for service i_N for the low-skilled. In contrast, the high-skilled benefit from lower prices for service i_R and higher wages.

The low-skilled also consume less of service i_R if $\sigma_I < \frac{\omega\tilde{\phi}_N}{2\omega\tilde{\phi}_N+\tilde{\phi}_R}$, i.e. if their demand for services is strongly inelastic. Whenever this is the case, they consume less of both services.[20] All results not derived in the main text are given in Appendix D.2.5.

Note that hitherto, we assumed that $\sigma_I > 1$. In this case, the wage premium decreases,

[20] We can assume that $\sigma_I = \frac{\omega\tilde{\phi}_N}{2\omega\tilde{\phi}_N+\tilde{\phi}_R}$. This is equal to $\frac{\sigma_I-1}{\sigma_I} = -\frac{\omega\tilde{\phi}_N+\tilde{\phi}_R}{\omega\tilde{\phi}_N}$. We can use this second equality in Equation (7.12) to observe that the substitution effect for the low-skilled is exactly as strong to both mirror the substitution effect of the high-skilled *and* to reverse the income effect of the low-skilled.

and the real wage of the low-skilled increases, even more than the real wage of the high-skilled (see Corollary 7.1). As we mentioned above, the production of service i_N stays constant in a DLM Equilibrium. Therefore the changes in consumption decisions of the low-skilled and the high-skilled must mirror each other, and thus

$$\int_{\tilde{r}(i_N)}^{\bar{r}} \frac{\partial C_{i_N}^r}{\partial A_k} f(r)dr = -\int_{\underline{r}}^{\tilde{r}(i_N)} \frac{\partial C_{i_N}^r}{\partial A_k} f(r)dr \ .$$

The proof is given in Appendix D.2.7, showing that this equivalence must indeed hold.

7.3.6. Increased Wage Inequality and More College Graduates

There is a large literature explaining the simultaneous increase in the wage premium for college graduates—compared to high-school graduates—and the increased supply of these graduates (Tinbergen, 1974; Katz and Murphy, 1992). The explanations mostly argue that the demand for high skills outpaced the increased supply of such skills. Further, there is a complementarity of high skills and technology, which is summarized under the term Skill-Biased Technological Change (SBTC). In our framework, such dynamics can have different causes. In the following, they are listed dependent on the possible equilibria. We will denote college graduates with a subscripted C and high school graduates with a subscripted H.

Note first that $\frac{\phi_N}{\phi_R}$ must not equal $\frac{\tilde{\phi}_N}{\tilde{\phi}_R}$, where $\frac{\phi_N}{\phi_R}$ denotes the ratio of the shares in the population with skill levels above and below the critical threshold $\tilde{r}(i_N)$.[21] Subsequently we denote the lowest skill level able to graduate from college by r^C. We discuss in Section 6.4 and 8.1 that the wage premium function can have various steps. It is not entirely clear, despite its intuitive appeal, whether the division of the labor force into college graduates and high-school graduates is optimal to describe wage dynamics.

We denote by $\frac{\hat{\phi}_C}{\hat{\phi}_H}$ the observed ratio of college graduates to high-school graduates. In our framework, an increase in $\frac{\hat{\phi}_C}{\hat{\phi}_H}$ must not imply an increase in $\frac{\phi_N}{\phi_R}$ nor must an increase in $\frac{\phi_N}{\phi_R}$ imply an increase in $\frac{\tilde{\phi}_N}{\tilde{\phi}_R}$, and $\frac{\tilde{\phi}_N}{\tilde{\phi}_R}$ matters most regarding the wage premium. Measuring the number of graduates is arguably a good approximation for skill. Nevertheless, educational quality and/or the education system might change. Therefore, changes in $\frac{\hat{\phi}_C}{\hat{\phi}_H}$ must not necessarily reflect equivalent or even approximate changes in $\frac{\tilde{\phi}_N}{\tilde{\phi}_R}$.[22]

[21] This threshold value can be located anywhere along the task-complexity dimension and thus along the skill dimension if there are many task-complexities.

[22] E.g. in a DLM Equilibrium, if $\frac{\hat{\phi}_C}{\hat{\phi}_H}$ rises, but $\frac{\tilde{\phi}_N}{\tilde{\phi}_R}$ falls, wage inequality still rises. Such a pattern could be explained by poor universities, that award easily attainable diplomas.

ILM Equilibrium. If the labor market is integrated, our framework implies that the wage inequality between college and high-school graduates, measured as the ratio in average wages, is

$$\frac{\overline{w}_C}{\overline{w}_H} = \frac{\tilde{\phi}_C}{\phi_C}\frac{\phi_H}{\tilde{\phi}_H} .$$

The simultaneous rise in wage inequality and relative supply of college graduates is expressed in a simultaneous increase in the ratio $\frac{\overline{w}_C}{\overline{w}_H}$ and ϕ_C. This occurs if

- college graduates become more productive, i.e. $\tilde{\phi}_C$ increases even more than ϕ_C,
- high school graduates become less productive, i.e. $\tilde{\phi}_H$ decreases more than $1-\phi_C$,
- there are unemployed workers, who enter the labor market (potentially because i_R decreases) and increase ϕ_H.[23]

Technological progress in manufacturing, however, has no influence on relative wages.

DLM Equilibrium. If the labor market is disintegrated, the average wage ratio must consider whether or not r^c is equal to $\tilde{r}(i_N)$. We can distinguish three cases.

Case (i): We assume that some college graduates are employed in the routine industry, $\tilde{r}(i_N) > r^c$. The average wages of college and high-school graduates are

$$\overline{w}_C = \phi_C^{-1}\left[\int_{r^c}^{\tilde{r}(i_N)}\left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr + \int_{\tilde{r}(i_N)}^{\bar{r}} \omega \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr\right] ,$$
$$\overline{w}_H = \frac{\tilde{\phi}_H}{\phi_H} ,$$

implying that

$$\frac{\overline{w}_C}{\overline{w}_H} == \frac{\int_{r^c}^{\tilde{r}(i_N)}\left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr}{\tilde{\phi}_H}\frac{\phi_H}{\phi_C} + \frac{\int_{\tilde{r}(i_N)}^{\bar{r}}\left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr}{\tilde{\phi}_H}\frac{\phi_H}{\phi_C}\left[\frac{\tilde{\phi}_R}{\tilde{\phi}_N}\left[\frac{i_N}{i_R}\right]^{\frac{1-\sigma_I}{\lambda}}\left[\frac{n_N}{n_R}\right]^{\frac{1-\sigma_I}{1-\sigma_v}}\hat{\theta}(A_k)^{\frac{1-\sigma_I}{\sigma_R-1}}\right]^{\frac{1}{\sigma_I}} .$$

Case (ii): We assume that $r^c = \tilde{r}(i_N)$ and thus $\phi_N = \phi_C$ and $\phi_R = \phi_H$,

$$\frac{\overline{w}_C}{\overline{w}_H} = \frac{\phi_H}{\phi_C}\left[\frac{\tilde{\phi}_C}{\tilde{\phi}_H}\right]^{\frac{\sigma_I-1}{\sigma_I}}\left[\frac{i_R}{i_N}\right]^{\frac{\sigma_I-1}{\lambda\sigma_I}}\left[\frac{n_R}{n_N}\right]^{\frac{1}{1-\sigma_v}\frac{\sigma_I-1}{\sigma_I}}\hat{\theta}(A_k)^{\frac{1}{\sigma_R-1}\frac{1-\sigma_I}{\sigma_I}} .$$

[23] An example is the increase in labor force participation of women starting after the second world war.

Case (iii): In this last case we assume that some high-school graduates conduct non-routine work, $\tilde{r}(i_N) < r^c$, i.e.

$$\overline{w}_C = \frac{\omega\tilde{\phi}_C}{\phi_C} \,,$$
$$\overline{w}_H = \phi_H^{-1}\left[\int_{\underline{r}}^{\tilde{r}(i_N)} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr + \int_{\tilde{r}(i_N)}^{r^c} \omega \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr\right] .$$

implying that

$$\frac{\overline{w}_C}{\overline{w}_H} = \tilde{\phi}_C \left[\frac{\tilde{\phi}_R}{\tilde{\phi}_N}\left[\frac{i_N}{i_R}\right]^{\frac{1-\sigma_I}{\lambda}}\left[\frac{n_N}{n_R}\right]^{\frac{1-\sigma_I}{1-\sigma_v}}\hat{\theta}(A_k)^{\frac{1-\sigma_I}{\sigma_R-1}}\right]^{\frac{1}{\sigma_I}}\left[\int_{\underline{r}}^{\tilde{r}(i_N)}\left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr + \right.$$
$$\left.\left[\frac{\tilde{\phi}_R}{\tilde{\phi}_N}\left[\frac{i_N}{i_R}\right]^{\frac{1-\sigma_I}{\lambda}}\left[\frac{n_N}{n_R}\right]^{\frac{1-\sigma_I}{1-\sigma_v}}\hat{\theta}(A_k)^{\frac{1-\sigma_I}{\sigma_R-1}}\right]^{\frac{1}{\sigma_I}}\int_{\tilde{r}(i_N)}^{r^c}\left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr\right]^{-1}\frac{\phi_H}{\phi_C} .$$

In Case (i) and (iii) the magnitude of ω is underestimated when productivities of workers are perfectly estimated.
The empirical observation under study, the simultaneous rise in $\frac{\overline{w}_C}{\overline{w}_H}$ and ϕ_C can have different causes in our model.

- Emerging new firms and varieties in the non-routine service industry—or fewer firms/varieties in the routine service industry—, i.e. an increase in $\frac{n_N}{n_R}$, may have caused an increase in demand for high-skilled workers. This reasoning is closely related to the SBTC-hypothesis.
- An increase in i_N may or may not increase the wage premium, depending on $\tilde{r}(i_N) \lesseqgtr r^C$ and on the skill distribution.
- Shifts in the skill distribution within the two groups, can increase the wage inequality (the average wage ratio), while decreasing the wage premium and vice versa.[24]

Higher productivity in manufacturing would lead to an increase in the wage premium only if we assume that $\sigma_I < 1$. The framework provides some explanations for the simultaneous increase in wage premium and supply of college graduates. Note that we have entirely neglected industry-specific or skill-group-specific technological advancements.

We summarize our insights. We assumed that machines are produced by an increasingly productive manufacturing industry. This structure generates an industry i_R that needs

[24] We, however, abstain from this reasoning, as we merely analyze the medium-term, where we assumed that skill remains fixed compared to technological change.

fewer workers, but at the same time workers—able to perform task-complexity i_R—are needed in the manufacturing industry. Thus, workers move between industries, become increasingly productive and prices fall. Depending on σ_I, this productivity increase in the manufacturing industry results in a tightened/loosened labor market. However, σ_I is generally assumed to be greater than one. Thus, this framework cannot offer new explanations for the increase in wage inequality despite the rise in college graduates.

In the next section, we analyze the same framework with the sole difference that the manufacturing industry is characterized by a production process with task-complexity i_N. The implications of this change are strong.

7.4. Robotic Economy – Capital from Non-routine Labor

In the previous section, we assumed that low-skilled (and high-skilled) labor can be used in the manufacturing industry to produce capital (machines). Suppose the economy has developed to a more advanced, more automated state.[25] The manufacturing industry now produces capital that we call robots. These robots are able to substitute low-skilled labor in industry i_R, just as machines in the previous section. However, their design and production requires non-routine labor, as we will emphasize below. The economy captures the ongoing automation processes in production described in Brynjolfsson and McAfee (2014). Note that the underlying structure of the industrial economy of the previous section and the robotic economy differs. Nonetheless, we will compare their respective reaction to technological change in manufacturing.

The production function in (7.3) shows that the task-complexity for the production of machines is i_R. This assumption is critical and represents the industrial capital production (machines) and not the production of robots. We now want to model the production of robots, thus we must assume that the production of these robots requires task-complexity i_N, that represents the high complexity of robot development, design and production. Thus, only labor able to perform non-routine tasks can be used to produce robots, which are usable in the production of routine services. In this sense, the manufacturing industry is also representative for high-tech industries. We assume that manufacturing is competitive and we keep the outline of the model as simple as possible. The macroeconomic environment remains the same as in the previous sections (except for manufacturing).

[25] We have not included any total factor productivity parameter.

7.4.1. Industries, Firms and Households

Industries. The production function in manufacturing differs from (7.3) solely in its dependence on i_N (rather than on i_R) and is

$$k = A_k \sum_{r \in \mathcal{R}} q\,[r]^{i_N q^\lambda}\, l_k(r)\ ,$$

where A_k is again an exogenous technological parameter. The industry can be represented by a single firm. The production functions of industry i_N and industry i_R are given in (7.1) and in (7.2), respectively.

Firms. All firms choose optimal quality levels for the labor they employ, given ASC. The manufacturing industry is competitive and the price of robots equals marginal costs: $p_k = \frac{\omega}{A_k}\left[-e\lambda i_N \log(\bar{r})\right]^{\frac{1}{\lambda}}$. Note that the price of robots linearly depends on the wage premium. Optimal relative inputs chosen by a routine firm (i_R, j) are

$$\frac{k_j}{\tilde{l}_{i_R,j}} = \left[\frac{A_k}{\omega}\right]^{\sigma_R} \left[-e\lambda \log(\bar{r})\right]^{-\frac{1}{\lambda}}\, i_N^{-\frac{\sigma_R}{\lambda}}\, i_R^{\frac{\sigma_R - 1}{\lambda}}\ ,$$

where we transformed labor into effective labor.[26] Marginal costs of firm (i_R, j) then are

$$mc_{i_R,j} = \left[-e\lambda i_R \log(\bar{r})\right]^{\frac{1}{\lambda}} \left[1 + \left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1}\right]^{\frac{1}{1-\sigma_R}}\ .$$

We define that $\tilde{\theta}(\omega, A_k) := 1 + \left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1}$.[27] The term $\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1}$ is the cost of capital optimally used as an input relative to one unit of effective routine labor. The higher the productivity of the robots, i.e. the higher A_k, the more of this input firms in industry i_R would like to use. The contrary holds true for ω. Prices and industry price aggregators are

$$\begin{aligned}
p_{i_R,j} &= \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i_R \log(\bar{r})\right]^{\frac{1}{\lambda}} \tilde{\theta}(\omega, A_k)^{\frac{1}{1-\sigma_R}}\ ,\\
P_{i_R} &= \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i_R \log(\bar{r})\right]^{\frac{1}{\lambda}} n_R^{\frac{1}{1-\sigma_v}} \tilde{\theta}(\omega, A_k)^{\frac{1}{1-\sigma_R}}\ ,\\
p_{i_N,j} &= \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i_N \log(\bar{r})\right]^{\frac{1}{\lambda}} \omega\ ,\\
P_{i_N} &= \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i_N \log(\bar{r})\right]^{\frac{1}{\lambda}} n_N^{\frac{1}{1-\sigma_v}} \omega\ ,
\end{aligned}$$

[26] See the previous section for the concept of effective labor (p. 125).

[27] Note the difference to $\hat{\theta}(A_k) = 1 + A_k^{\sigma_R - 1}$, which we defined in the previous section.

and the aggregate price index is

$$P = \frac{\sigma_v}{\sigma_v - 1} \left[-e\lambda \log(\bar{r})\right]^{\frac{1}{\lambda}} \tilde{\mathcal{M}}^{\frac{1}{1-\sigma_I}} ,$$

where $\tilde{\mathcal{M}} = i_N^{\frac{1-\sigma_I}{\lambda}} n_N^{\frac{1-\sigma_I}{1-\sigma_v}} \omega^{1-\sigma_I} + i_R^{\frac{1-\sigma_I}{\lambda}} n_R^{\frac{1-\sigma_I}{1-\sigma_v}} \tilde{\theta}(\omega, A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}$.

The production function in the routine industry with optimal quality choice and optimal relative input choice is

$$x_{i_R,j} = [-e\lambda i_R \log(\bar{r})]^{-\frac{1}{\lambda}} \tilde{l}_{i_R,j} \tilde{\theta}(\omega, A_k)^{\frac{\sigma_R}{\sigma_R - 1}} .$$

Households. Total household demand faced by firms of industries i_R and i_N is

$$\begin{aligned} c_{i_R,j} =& n_R^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} i_R^{-\frac{\sigma_I}{\lambda}} \tilde{\theta}(\omega, A_k)^{\frac{\sigma_I}{\sigma_R - 1}} \tilde{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C , \\ c_{i_N,j} =& n_N^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} i_N^{-\frac{\sigma_I}{\lambda}} \omega^{-\sigma_I} \tilde{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C , \end{aligned} \tag{7.14}$$

and routine firms' input demand for k results in a demand for non-routine labor, denoted by $\tilde{L}_{k,i_N}$. As the manufacturing industry is competitive, all revenues are paid to workers, i.e. $n_{i_R} p_k k = \omega \tilde{L}_{k,i_N}$, and thus

$$\omega \tilde{L}_{k,i_N} = n_{i_R} \tilde{l}_{i_R,j} \left[\frac{A_k}{\omega} \left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}} \right]^{\sigma_R - 1} .$$

The rationale for this equality is the following: Per unit of effective routine labor input, industry i_R demands capital at cost $\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}$. The capital industry is competitive and all revenues are paid to the factor inputs, which is non-routine labor in the robotic economy.

We impose goods market clearing and solve for the effective labor demand from the routine industry, and obtain

$$\tilde{l}_{i_R,j} = [-e\lambda i_R \log(\bar{r})]^{\frac{1}{\lambda}} n_R^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} i_R^{-\frac{\sigma_I}{\lambda}} \tilde{\theta}(\omega, A_k)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}} \tilde{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C .$$

Analogously we apply goods market clearing to industry i_N and a firm (i_N, j)'s demand for effective labor is $\tilde{l}_{i_N,j} = [-e\lambda i_N \log(\bar{r})]^{\frac{1}{\lambda}} n_I^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} i_I^{-\frac{\sigma_I}{\lambda}} \omega^{-\sigma_I} \tilde{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C$. Labor market clearing implies that

$$\tilde{L} = \tilde{L}_{k,i_N} + n_{i_R} \tilde{l}_{i_R,j} + n_{i_N} \tilde{l}_{i_N,j} ,$$

and total wages paid are

$$\begin{aligned} TW &= \omega \tilde{L}_{k,i_N} + n_{i_R}\tilde{l}_{i_R,j} + \omega n_{i_N}\tilde{l}_{i_N,j} \\ &= n_{i_R}\tilde{l}_{i_R,j}\tilde{\theta}(\omega, A_k) + \omega n_{i_N}\tilde{l}_{i_N,j} \ . \end{aligned}$$

Aggregate consumption then is a function of total wages paid to workers,

$$C = \left[-e\lambda \log(\bar{r})\right]^{-\frac{1}{\lambda}} \tilde{\mathcal{M}}^{\frac{1}{\sigma_I - 1}} TW \ . \tag{7.15}$$

Depending on the skill demand and skill supply, distinct equilibria may arise. In the following, we analyze the two cases of an integrated and a disintegrated labor market again.

7.4.2. Equilibrium

ILM Equilibrium. The ILM Equilibrium arises if there are sufficient skills in the economy for labor markets to remain integrated, implying $\omega = 1$ and $\tilde{L} = TW$. The demand for high-skilled labor from industry i_N and from the manufacturing industry is smaller than the high-skilled labor supply. Thus high-skilled labor is also employed in the routine industry i_R. This ILM Equilibrium is described by

(i) $w^{r\star} = \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} \quad \forall r \in \mathcal{R}$,

(ii) $P^\star = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda \log(\bar{r})\right]^{\frac{1}{\lambda}} \tilde{\mathcal{M}}^{\frac{1}{1-\sigma_I}}$,

(iii) $C^\star = \left[-e\lambda \log(\bar{r})\right]^{-\frac{1}{\lambda}} \tilde{\mathcal{M}}^{\frac{1}{\sigma_I - 1}} \tilde{L}$ and $P^\star C^\star = \frac{\sigma_v}{\sigma_v - 1}\tilde{L}$.

DLM Equilibrium. If labor markets are separated, the economy is in a DLM Equilibrium (i.e. $\omega > 1$). High-skilled labor is no longer employed in the routine industry. Firms in the non-routine industry i_N and in manufacturing are willing to pay a wage premium for the scarce skill levels. The wage premium ω then clears the labor markets,

$$\begin{aligned} L\tilde{\phi}_N &= n_N\tilde{l}_{i_N,j} + \tilde{L}_{k,i_N} \\ &= \left[-e\lambda \log(\bar{r})\right]^{\frac{1}{\lambda}} n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}} \omega^{-\sigma_I} \tilde{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C + \\ &\quad \left[-e\lambda \log(\bar{r})\right]^{\frac{1}{\lambda}} n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}} \tilde{\theta}(\omega, A_k)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}} \omega^{-\sigma_R} \left[A_k \left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1} \tilde{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C \ , \\ L\tilde{\phi}_R &= n_R\tilde{l}_{i_R,j} \\ &= \left[-e\lambda \log(\bar{r})\right]^{\frac{1}{\lambda}} n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}} \tilde{\theta}(\omega, A_k)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}} \tilde{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C \ , \end{aligned}$$

where again $\tilde{\phi}_N = \int_{\tilde{r}(i_N)}^{\bar{r}} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr$ and $\tilde{\phi}_R = \int_{\underline{r}}^{\tilde{r}(i_N)} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr$.
We can now implicitly determine the dynamics of the wage premium by taking the ratio of the two equalities above.[28] We define

$$\mathcal{F} = \tilde{X}\tilde{\theta}(\omega, A_k)^{\frac{\sigma_I - \sigma_R}{1-\sigma_R}} \omega^{-\sigma_I} - \frac{\tilde{\phi}_N}{\tilde{\phi}_R} [1 - z] = 0 \;, \tag{7.16}$$

where $z = \frac{\tilde{\phi}_R}{\omega \tilde{\phi}_N} \left[\frac{A_k}{\omega} \left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1}$ denotes the fraction of the non-routine labor force demanded through the routine-task industry's need for capital. For notational convenience, $\tilde{X} = \left[\frac{i_N}{i_R}\right]^{\frac{1-\sigma_I}{\lambda}} \left[\frac{n_N}{n_R}\right]^{\frac{1-\sigma_I}{1-\sigma_v}}$. We still assume that $\sigma_R \geq \max\{1, \sigma_I\}$. We stressed that a high value of σ_R coincides with the chosen modeling assumptions, namely the assumption of high (or even perfect) substitutability among different factors able to manage a certain task-complexity. In principle, assumption $\sigma_R > \sigma_I$ is stronger than assumption $\sigma_R > 1$ only if we assume that $\sigma_I > 1$. However, with a high substitutability among factors, both assumptions are fulfilled with slack.[29] We take the partial derivative of (7.16) with respect to ω and obtain

$$\begin{aligned}\frac{\partial \mathcal{F}}{\partial \omega} =& \tilde{X}\tilde{\theta}(\omega, A_k)^{\frac{\sigma_I - \sigma_R}{1-\sigma_R}} \omega^{-\sigma_I - 1} \left[[\sigma_I - \sigma_R]\, \tilde{\theta}(\omega, A_k)^{-1} \left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1} - \sigma_I \right] \\ & - \sigma_R \omega^{-\sigma_R - 1} \left[A_k \left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1} < 0 \;.\end{aligned}$$

Thus, for $\omega \geq 1$, there is a unique solution $\omega^\star$, as $\frac{\partial \mathcal{F}(\omega, A_k)}{\partial \omega}$ is strictly negative. Using $\omega^\star$, we can solve for all other equilibrium variables.

7.4.3. Technological Change

We now analyze the effects of technological progress on wage inequality and on aggregate consumption in a robotic economy. Again, we define technological progress as the increase in productivity in the manufacturing industry.

ILM Equilibrium. We know that in an ILM Equilibrium, the wage scheme is unaffected by technological progress. Technological progress then leads to higher productivity and lower prices of robots, and benefits all workers in the economy equally. The real wage for any worker is $\tilde{w}^r = \frac{w^r}{P}$, where P decreases when productivity increases. Nevertheless,

[28] Note that the two equations only hold with equality in the DLM Equilibrium.
[29] We discussed estimates for σ_I in Section 6.5.2. The estimates for σ_I are typically in the elastic range, i.e. greater than one, but only little.

the following result shows that the bounty of increased productivity in manufacturing is only temporarily given to all workers in a robotic economy.

Proposition 7.3
If $\sigma_R \geq \max\{1, \sigma_I\}$, *technological progress guides the economy unambiguously towards a DLM Equilibrium, i.e.*

$$\frac{\partial L_{i_n}^d}{\partial A_k} > 0\,.$$

Again $L_{i_n}^d$ denotes the non-routine labor demanded by the industries. The derivation is shown in Appendix D.2.1.

DLM Equilibrium. A DLM Equilibrium features a disintegrated labor market. In order to analyze wage dynamics, we first derive the partial derivative of $\mathcal{F}$ with respect to technological progress A_k,

$$\frac{\partial \mathcal{F}}{\partial A_k} = \frac{1}{A_k}\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1}\left[\tilde{X}\left[\sigma_R - \sigma_I\right]\tilde{\theta}(\omega, A_k)^{\frac{\sigma_I - 1}{1-\sigma_R}}\omega^{-\sigma_I} + \left[\sigma_R - 1\right]\omega^{-1}\right] > 0\,,$$

which is strictly greater zero for $\sigma_R > \max\{1, \sigma_I\}$. Note that $\partial\mathcal{F}/\partial\frac{\tilde{\phi}_R}{\tilde{\phi}_N} = 1 > 0$, i.e. $\mathcal{F}$ increases the scarcer effective non-routine labor is. Assuming $\sigma_I > 1$ we obtain

$$\frac{\partial \mathcal{F}}{\partial \frac{n_N}{n_R}} = \frac{\sigma_I - 1}{\sigma_v - 1}\tilde{X}\tilde{\theta}(\omega, A_k)^{\frac{\sigma_I - \sigma_R}{1-\sigma_R}}\omega^{-\sigma_I}\frac{n_R}{n_N} > 0\,.$$

In equilibrium it must always hold that

$$1 > z = \frac{\tilde{\phi}_R}{\omega\tilde{\phi}_N}\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1}\,. \tag{7.17}$$

The demand for non-routine labor originating from the routine industry cannot be greater than total non-routine labor supply, as otherwise the wage premium ω rises sufficiently to restore the inequality in (7.17).[30]
In the following proposition, the dynamics of the wage premium with respect to exogenous parameter changes in A_k are analyzed.

Proposition 7.4
Assuming that $\sigma_R > \max\{1, \sigma_I\}$, *the elasticity of the wage premium* ω *with respect to*

[30] For the limiting case, when $A_k \to \infty$, (7.17) holds with equality if $\sigma_I > 1$. This result will be derived in Corollary 7.5

A_k, $\sigma_{A_k,\omega}$, is positive,

$$\sigma_{A_k,\omega} = \frac{\partial \omega}{\partial A_k}\frac{A_k}{\omega} = -\frac{\frac{\partial \mathcal{F}}{\partial A_k}}{\frac{\partial \mathcal{F}}{\partial \omega}}\frac{A_k}{\omega} = \frac{1}{1 + \frac{z+[1-z]\sigma_I}{z[\sigma_R - 1]+[1-z][\sigma_R-\sigma_I]\mathcal{C}}} > 0 \,, \tag{7.18}$$

where $\mathcal{C} = \left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1}\tilde{\theta}(\omega, A_k)^{-1}$.

Observe that for the result in Proposition 7.4, the parameter σ_I is not directly restricted. Technological progress in the manufacturing industry that produces robots with high-skilled labor leads to an increase in the wage premium. Intuitively, high-skilled labor produces robots at increasingly lower costs, and the robots can be used as a substitute for low-skilled labor. Low-skilled labor is threatened by this development. The term $\mathcal{C}$ denotes the share of total costs in the routine industry allocated to the capital input, and therefore indirectly paid to non-routine labor.

The wage premium increases the scarcer high-skilled labor is

$$\frac{\partial \omega}{\partial \frac{\tilde{\phi}_R}{\tilde{\phi}_N}} = -\left. \frac{\partial \mathcal{F}}{\partial \frac{\tilde{\phi}_R}{\tilde{\phi}_N}} \middle/ \frac{\partial \mathcal{F}}{\partial \omega} \right. > 0\,.$$

Assuming that $\sigma_I > 1$, the wage premium increases in the ratio of non-routine varieties to routine varieties

$$\frac{d\omega}{d\frac{n_N}{n_R}} = -\left. \frac{\partial \mathcal{F}}{\partial \frac{n_N}{n_R}} \middle/ \frac{\partial \mathcal{F}}{\partial \omega} \right. > 0\,.$$

We now examine the real wage dynamics. For $r < \tilde{r}(i_N)$ the derivative of the real wage $\tilde{w}^r$ with respect to technology is[31]

$$\begin{aligned}
\frac{\partial \tilde{w}^r}{\partial A_k} &= \frac{\sigma_v - 1}{\sigma_v(\sigma_I - 1)}\left[-e\lambda \log(r)\right]^{-\frac{1}{\lambda}}\tilde{\mathcal{M}}^{\frac{1}{\sigma_I - 1}-1}\frac{\partial \tilde{\mathcal{M}}}{\partial A_k} \\
&= \frac{\sigma_v - 1}{\sigma_v}\left[-e\lambda \log(r)\right]^{-\frac{1}{\lambda}}\tilde{\mathcal{M}}^{\frac{1}{\sigma_I - 1}-1} \times \left[i_R^{\frac{1-\sigma_I}{\lambda}} n_R^{\frac{1-\sigma_I}{1-\sigma_v}}\tilde{\theta}(\omega, A_k)^{\frac{\sigma_R - \sigma_I}{1-\sigma_R}}\omega^{1-\sigma_R}\frac{\mu(A_k)}{A_k} - \right. \\
&\left. \left[i_N^{\frac{1-\sigma_I}{\lambda}} n_N^{\frac{1-\sigma_I}{1-\sigma_v}}\omega^{-\sigma_I} + i_R^{\frac{1-\sigma_I}{\lambda}} n_R^{\frac{1-\sigma_I}{1-\sigma_v}}\tilde{\theta}(\omega, A_k)^{\frac{\sigma_R - \sigma_I}{1-\sigma_R}}\omega^{-\sigma_R}\mu(A_k)\right]\frac{\partial \omega}{\partial A_k}\right]\,,
\end{aligned}$$

where $\mu(A_k) = \left[A_k\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1}$. The derivative can be positive or negative, depending on parameters and on the elasticity of the wage premium with respect to technological progress. Note that the dynamics of the real wages of low-skilled labor equal the inverse dynamics of the aggregate price index.

[31] Remember that $\tilde{\mathcal{M}} = i_N^{\frac{1-\sigma_I}{\lambda}} n_N^{\frac{1-\sigma_I}{1-\sigma_v}}\omega^{1-\sigma_I} + i_R^{\frac{1-\sigma_I}{\lambda}} n_R^{\frac{1-\sigma_I}{1-\sigma_v}}\tilde{\theta}(\omega, A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}$.

Corollary 7.4

The real wage of low-skilled labor decreases (increases), $\frac{\partial \tilde{w}^r}{\partial A_k} < 0 \ (> 0)$*, if*

$$\sigma_{A_k,\omega} > z \ (< z) \ .$$

The derivation is given in Appendix D.2.2. Observe that z also denotes the share of the non-routine labor's total wages, $\omega\tilde{\phi}_N$, that are paid through the demand for robots. An increase in ω leads to a one-to-one increase of the prices in industry i_N. A share z that is smaller than the elasticity of the wage premium with respect to A_k then means that the price increases in industry i_N are not sufficiently counteracted by the lower prices in industry i_R trough the productivity gains in the robot production.[32] The following condition demonstrates whether or not $\sigma_{A_k,\omega}$ is greater or smaller than z.

Condition DRWC-LS (Decreasing real wage condition for the low-skilled)

$$[\sigma_R - \sigma_I] \frac{1 + \omega \frac{\tilde{\phi}_N}{\tilde{\phi}_R}}{1 + z\omega \frac{\tilde{\phi}_N}{\tilde{\phi}_R}} > \frac{1}{1-z} \ \left(< \frac{1}{1-z}\right) \ . \qquad \text{(DRWC-LS)}$$

The derivation is shown in Appendix D.2.3. If Condition DRWC-LS holds, the real wage of low-skilled workers decreases. It is apriori not clear whether z increases or decreases at ever higher levels of A_k, i.e. whether the efficiency increase through technological progress in manufacturing overcompensates the demand for non-routine skill in production of robots. Taking derivatives, it can be shown that $\frac{\partial z}{\partial A_k} > 0$ if the elasticity of substitution between the industries, σ_I, is large enough, leading to the following corollary.

Corollary 7.5

The share of non-routine labor demanded to produce the robots increases in A_k *(i.e.* $\frac{\partial z}{\partial A_k} > 0$*) if* $\frac{\sigma_R - 1}{\sigma_R} > \sigma_{A_k,\omega}$*, which is true whenever*

$$\sigma_I > \frac{\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1}}{\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1} + \frac{\sigma_R - 1}{\sigma_R}} \ . \qquad (7.19)$$

[32] Suppose that $\sigma_{A_k,\omega} = 0.5$. Then a $g_{A_k} = 1\%$ growth in A_k leads to a $g_\omega = 0.5\%$ increase in ω. If now $z < 0.5$, fewer high-skilled workers are used in the production of robots than in the production of service i_N. In such a case, the productivity gains in manufacturing lead to prices for routine services that are not lowered enough compared to the increased prices for non-routine services to make the low-skilled better off due to technological progress. This effect can also be shown by merely analyzing the dynamics within the high-skilled labor force. Both technological progress and the wage premium affect the cost-productivity ratio of the high-skilled. Then $z\,[g_\omega - g_{A_k}] + [1-z]g_\omega > 0$ (and equals 0 if $z = 0.5$), i.e. on average, non-routine labor becomes more expensive despite the productivity increase.

This always holds if $\sigma_I \geq 1$.

The derivation is shown in Appendix D.2.4. Note first that (7.19) depends on the technological level. The term $\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1}$ always increases in A_k. The wage premium cannot grow faster than technological progress, i.e. $\sigma_{A_k,\omega} < 1$ always (see also Appendix D.2.4 for the derivation). The more advanced A_k, the more restrictive (7.19) becomes. However, (7.19) always holds for $\sigma_I > 1$, independent of the level of A_k. Intuitively, if the elasticity of substitution between industries, σ_I, is greater than one, households shift their consumption towards routine services i_R in relative terms, because prices in this industry always decline, no matter how much of the production is performed by high-skilled labor. Thus, technological progress in manufacturing leads to more efficient robot production, and the resulting price decrease in this industry is not outweighed by the wage premium increase.

If industries are complements ($\sigma_I < 1$), lower prices for routine services have an income effect and a substitution effect. The income effect lets households consume more from both industries. The substitution effect lets households shift their consumption towards services i_N. If this substitution effect is strong enough, the wage premium rises sufficiently to dominate the productivity increase, i.e. less high-skilled labor is employed to produce the robots which are needed in the routine industry.

In the following, we assume that $\sigma_I > 1$. Then, z always increases in A_k, i.e. the routine industry expands relative to the non-routine industry in demanding non-routine labor for the production of robots, and the right-hand side of DRWC-LS strictly increases in A_k. In contrast, the left-hand side of DRWC-LS can increase or decrease. Therefore, real wages of the low-skilled can increase or decrease as a reaction to technological progress.

Proposition 7.5

(i) *Assume σ_R is bounded. Then if A_k is large enough, the real wage of low-skilled labor always increases.*

(ii) *When A_k large, the elasticity of the wage premium with respect ot A_k, $\sigma_{A_k,\omega}$, converges to $\frac{\sigma_R - 1}{\sigma_R}$.*

We briefly prove the two statements in Proposition 7.5.

PROOF:

(i) Whenever $\sigma_I > 1$, the share z strictly increases in A_k (see Corollary 7.5) and $z_{\lim A_k \to \infty} = 1$, as the routine service is provided through robots—produced with high-skilled labor—at cost approaching zero. The price of robots, p_k, declines because the elasticity of the wage premium with respect to A_k is always smaller than one, $\sigma_{A_k,\omega} < 1$ (see Appendix D.2.4).

Then, the left-hand side of DRWC-LS is bounded by the assumption that σ_R is bounded, while the right-hand side eventually increases enough to become greater than the left-hand side when z converges to 1. Using Condition DRWC-LS, we show statement (i) by contradiction: Suppose Condition DRWC-LS is true for large A_k and the parameter restrictions ($\sigma_I > 1$ and σ_R bounded). Then

$$\lim_{A_k\to\infty}[\sigma_R - \sigma_I]\frac{1+\omega\frac{\tilde{\phi}_N}{\tilde{\phi}_R}}{1+z\omega\frac{\tilde{\phi}_N}{\tilde{\phi}_R}} > \lim_{A_k\to\infty}\frac{1}{1-z}$$

$$[\sigma_R - \sigma_I]\frac{1+\lim_{A_k\to\infty}\omega\frac{\tilde{\phi}_N}{\tilde{\phi}_R}}{1+\lim_{A_k\to\infty} z\lim_{A_k\to\infty}\omega\frac{\tilde{\phi}_N}{\tilde{\phi}_R}} > \frac{1}{1-\lim_{A_k\to\infty} z}$$

$$[\sigma_R - \sigma_I] > \infty\ ,$$

which contradicts the supposition.

□

(ii) We know that $z_{\lim A_k\to\infty} = 1$ and for the share of total costs in the routine industry allocated to the capital input it holds that $\mathcal{C}_{\lim A_k\to\infty} = 1$. We use this in Proposition 7.4,

$$\lim_{A_k\to\infty}\frac{1}{1+\frac{z+[1-z]\sigma_I}{z[\sigma_R-1]+[1-z][\sigma_R-\sigma_I]\mathcal{C}}} = \frac{1}{1+\frac{\lim_{A_k\to\infty} z+[1-\lim_{A_k\to\infty} z]\sigma_I}{\lim_{A_k\to\infty} z[\sigma_R-1]+[1-\lim_{A_k\to\infty} z][\sigma_R-\sigma_I]\lim_{A_k\to\infty}\mathcal{C}}} = \frac{\sigma_R - 1}{\sigma_R}\ .$$

□

We now study the case of perfect substitutability between labor and robots in industry i_R.

Proposition 7.6

Assume $\sigma_R \to \infty$. Then the real wage of the low-skilled workers decreases in A_k.

PROOF: Suppose A_k has finite value. First, we show that ω has finite value too. Second, we show that this implies $z < 1$ and thus, that the right-hand side of DRWC-LS is bounded. Third, assuming $\sigma_R \to \infty$ we show that the left-hand side of DRWC-LS is unbounded.

(i) Observe from (7.16) that ω must be of finite value, whenever A_k is of finite value. We show this by contradiction. Thus, suppose that $\omega \to \infty$. Then $z_{\lim_{\omega\to\infty}} = 0$, and

$\tilde{\theta}(\omega, A_k)_{\lim_{\omega\to\infty}} = 1$, and

$$\lim_{\omega\to\infty} \mathcal{F} = \lim_{\omega\to\infty} \tilde{X}\tilde{\theta}(\omega, A_k)^{\frac{\sigma_I-\sigma_R}{1-\sigma_R}} \omega^{-\sigma_I} - \lim_{\omega\to\infty} \frac{\tilde{\phi}_N}{\tilde{\phi}_R} [1-z]$$
$$= -\frac{\tilde{\phi}_N}{\tilde{\phi}_R} < 0 \,.$$

This contradicts the definition of $\mathcal{F}$. Thus, for A_k of finite value, also ω must be of finite value.

(ii) If both A_k and ω are finite, then z cannot converge to 1 because of (7.16), i.e. $\lim_{\sigma_R\to\infty} \tilde{X}\tilde{\theta}(\omega, A_k)^{\frac{\sigma_I-\sigma_R}{1-\sigma_R}} \omega^{-\sigma_I} > 0$ and thus $\lim_{\sigma_R\to\infty} z < 1$.

(iii) We reexamine DRWC-LS and obtain

$$\lim_{\sigma_R\to\infty} [\sigma_R - \sigma_I] \frac{1+\omega\frac{\tilde{\phi}_N}{\tilde{\phi}_R}}{1+z\omega\frac{\tilde{\phi}_N}{\tilde{\phi}_R}} > \lim_{\sigma_R\to\infty} \frac{1}{1-z}$$
$$\lim_{\sigma_R\to\infty} [\sigma_R - \sigma_I] \frac{1+\frac{1}{z}}{2} > \lim_{\sigma_R\to\infty} \frac{1}{1-z}$$
$$\infty > \lim_{\sigma_R\to\infty} \frac{1}{1-z} \,,$$

and thus, the condition for decreasing real wages for the low-skilled is fulfilled, what was to be shown.

□

Intuitively, when firms of industry i_R are indifferent between producing with labor or robots, they choose the cheaper input factor for the production. The cost of effective labor is always 1. Thus, whenever the price of robots, p_k, falls below this threshold, firms of industry i_R solely want to produce with robots. Real wages of the low-skilled must then fall and the wage premium rises to keep up with the technological advancement of robots. Thus, ω must be such that the price of robots equals the price of labor, i.e. $p_k = 1$.[33]

To analyze the conditions under which real wages for the low-skilled decrease, it is helpful to study the process of labor market separation. When the labor market is still integrated ($\omega = 1$), then $z = \frac{\tilde{\phi}_R}{\tilde{\phi}_N}\mu(A_k)$. Thus, everything else equal, there exists a technological level at which the labor market separation starts, i.e. where $\omega = 1$ and a marginal increase in A_k separates the market. This technological level is denoted by A_k^s, where the superscript

[33] Note that we abstain from analyzing $A_k \to \infty$ in conjunction with $\sigma_R \to \infty$.

s stands for "separation". Then A_k^s solves the following equation

$$\mathcal{F}^s = \tilde{X}\left[1+\mu(A_k^s)\right]^{\frac{\sigma_I-\sigma_R}{1-\sigma_R}} - \frac{\tilde{\phi}_N}{\tilde{\phi}_R} + \mu(A_k^s) = 0 \; .$$

$\mathcal{F}^s$ is strictly increasing in A_k^s, and there is a unique solution. We now analyze condition DRWC-LS in more detail when the technological level is A_k^s. At the point of labor market separation, the right-hand side of DRWC-LS is minimized. If we can show that there exist parameters that result in

$$[\sigma_R - \sigma_I]\,\frac{1+\frac{\tilde{\phi}_N}{\tilde{\phi}_R}}{1+\mu(A_k^s)} > \frac{1}{1-\frac{\tilde{\phi}_R}{\tilde{\phi}_N}\mu(A_k^s)} \; , \tag{7.20}$$

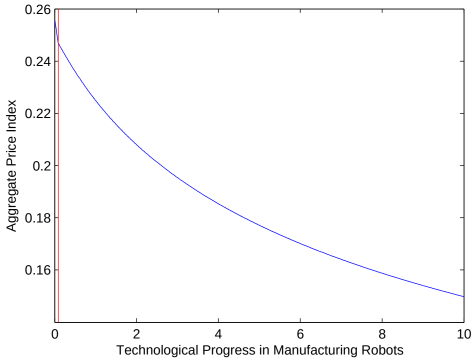
we know that there are some stretches along the evolution of A_k where routine labor looses even in real terms.[34] If A_k grows sufficient large, real wages of the low-skilled must increase, as the right-hand side of DRWC-LS grows to infinity, whereas the left-hand side is always finite, given finite σ_R (see also Proposition 7.5).

Inequality (7.20) indeed holds if parameters are such that $\sigma_R - \sigma_I \gg 1$ (σ_R large), and $\frac{\tilde{\phi}_R}{\tilde{\phi}_N}\mu(A_k^s)$ small, then the real wage of routine workers decreases at first, when the labor market separates. Intuitively, a marginal increase in the technological factor has a higher effect on the replacement of low-skilled labor, the higher σ_R and the smaller the level of A_k. The higher A_k, the smaller the replacement effect of low-skilled labor due to a marginal increase in A_k and thus the efficiency gain through technology (lower prices) surpasses the effect of replacement.

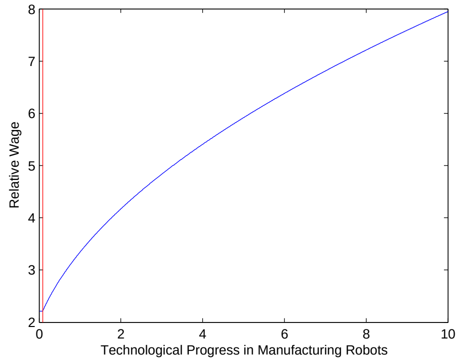
We cannot determine the exact evolution of the left-hand side of DRWC-LS, but as we noted above, the left-hand side of DRWC-LS is always bounded. Figure (7.1) demonstrates the evolution of the price index and relative wages, $\frac{w^{r^h}}{w^{r^l}}$, with respect to different parameter choices of σ_R and for an exogenous evolution of productivity parameter A_k. Figure (7.1a) demonstrates a situation where low-skilled workers benefit from technological progress right from the start of labor market separation. In this case, the productivity effect dominates the replacement effect. However, evidence in the literature suggests high replacement effects of robots (Acemoglu and Restrepo, 2017). Thus, in line with our assumption of a high substitution elasticity among input factors, a high parameter choice of σ_R seems appropriate. Parameter estimates in the literature range from the lower-bound estimate 1.9 (DeCanio, 2016) up to 4 (Hémous and Olsen, 2016).[35] Figures (7.1c), (7.1d), (7.1e) and (7.1f) present the dynamics when σ_R is in a higher range. In these cases, the real wage of low-skilled workers falls first. For high substitution elasticity, σ_R, technol-

[34] This was already the case in Proposition 7.6 under strong assumptions.
[35] In Section 7.5.1 we discuss this parameter.

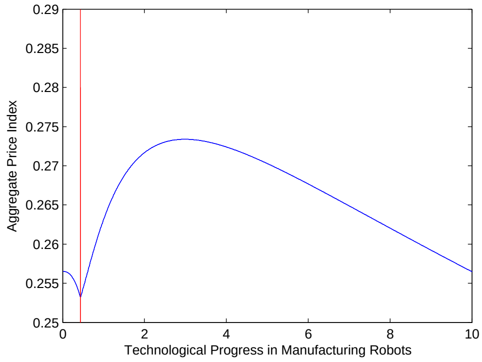
Figure 7.1.: Aggregate Price Index and Relative Wages



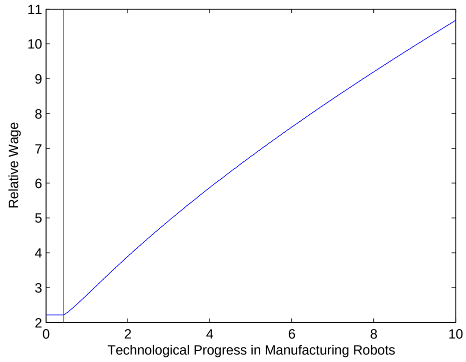
(a) Price index ($\sigma_R = 2$)



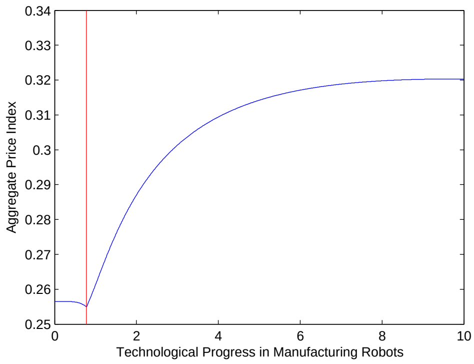
(b) Relative wage ($\sigma_R = 2$)



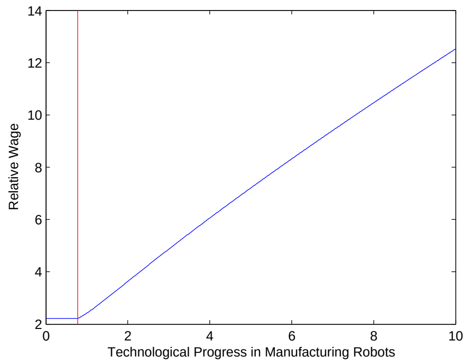
(c) Price index ($\sigma_R = 3.5$)



(d) Relative wage ($\sigma_R = 3.5$)



(e) Price index ($\sigma_R = 6$)



(f) Relative wage ($\sigma_R = 6$)

ogy needs to improve much more than with low σ_R after labor market separation, until low-skilled workers profit again from the technological evolution.

The remaining parameters are $i_R = 0.5$, $i_N = 1$, $n_R = 2$, $n_N = 1$, $\bar{r} = 0.9$, $\lambda = 2$, $\sigma_I = 1.6$, $\sigma_v = 2.3$, $\tilde{\phi}_N = 0.4$, and $\tilde{\phi}_R = 0.6$. Figure (7.1) compares three levels of σ_R[36]

- (7.1a) and (7.1b) with $\sigma_R = 2$, and $A_k^s \approxeq 0.088$,

[36] Note that the technological level of labor market separation, A_k^s, increases in higher values of σ_R. This is because non-routine labor must become sufficiently productive that firms increasingly produce with robots, when robots and low-skilled labor are close substitutes.

- (7.1c) and (7.1d) with $\sigma_R = 3.5$, and $A_k^s \approxeq 0.437$,
- (7.1e) and (7.1f) with $\sigma_R = 6$, and $A_k^s \approxeq 0.779$.

For skill levels $r > \tilde{r}(i_N)$, it can be shown that the real wage always increases,

$$\frac{\partial \tilde{w}^r}{\partial A_k} = \frac{\sigma_v - 1}{\sigma_v} \left[-e\lambda \log(r)\right]^{-\frac{1}{\lambda}} \tilde{\mathcal{M}}^{\frac{1}{\sigma_I - 1}} \left[\frac{1}{\sigma_I - 1} \tilde{\mathcal{M}}^{-1} \frac{\partial \tilde{\mathcal{M}}}{\partial A_k} + \frac{\partial \omega}{\partial A_k}\right] > 0.$$

This leads to the following proposition:

Proposition 7.7
The real wage of the non-routine labor always increases in A_k. The real wage of routine labor might increase or decrease in A_k.

Because of the additional wage premium in the wages of high-skilled labor, we can show that the following corollary also holds true.

Corollary 7.6
The real wage of the non-routine labor always increases more in A_k than the real wage of routine labor.

A related result is shown by Hémous and Olsen (2016). They show in their endogenous growth model that the growth rate of the real wage of high-skilled labor is always greater than the growth rate of the real wage of low-skilled labor.

It is obvious that assumption $\sigma_R > \max\{1, \sigma_I\}$ is crucial for the results obtained. Recall that σ_I is a preference parameter, whereas σ_R is a technological, institutional and cultural parameter, indicating how easily the producers substitute between labor and robots, depending on their respective prices.[37] We argued in the discussion above that σ_R is likely to be relatively high, compared to σ_I. In line with our arguments, Frey and Osborne (2017) unveil that already the current technological level shows great potential for automation. There is no reason to doubt that future technological level do convey even greater potential.

7.4.4. Summary

Table 7.2 summarizes the effects of rising technological progress A_k in both states of the economy, i.e. in an ILM Equilibrium and in a DLM Equilibrium. The dynamics of

[37] As noted before, σ_I can also be interpreted as a technological parameter, by reformulating the utility function so that it solely depends on a final good (in a linear way), and by redefining services to be intermediate goods that are aggregated into a single final good.

aggregate variables are indicated, as well as the dynamics of variables at the household level.

The remaining proofs of these effects are given in Appendix D.2.8, in particular the effects of A_k on consumption in the DLM Equilibrium. An increase of a variable in response to a rise in A_k is indicated by a '$+$', and a decrease is indicated by a '$-$'. A variable that remains constant is simply presented with a 'c'. If an increase or a decrease can occur, then a condition under which the increase/decrease occurs is presented. Again we assume that profits are distributed proportional to real wages, and thus (7.13) denotes a household r's consumption.

ILM Equilibrium. In the ILM Equilibrium, the wage premium remains unaffected by technological progress. However, through the technological progress, the price index falls, thereby raising real wages of all workers in the economy equally. Services i_R are in higher demand because of lower prices. If $\sigma_I > 1$, the lower prices in the i_R-industry induce households to shift their consumption away from the i_N-industry. If, however, the industries are complements, i.e. $\sigma_I < 1$, then the lower prices in the i_R-industry enable households to consume more of the i_N-services. All households are equally affected by the aggregate dynamics.

DLM Equilibrium. In the DLM Equilibrium, the wage premium always rises in response to technological progress. In conjunction with this rise, the wages spread, i.e. the wage of a high-skilled worker always increases more than the wage of a low-skilled worker. If condition DRWC-LS is fulfilled, the real wage of the low-skilled even falls in response to technological progress. More services i_R are demanded, as their prices fall in the wake of the productivity increase caused by technological progress. For the i_N-industry, there are countervailing effects: Technological progress leads to more demand of services from industry i_R because of lower prices. The rising demand for these services exerts upward pressure on the wage premium, because high-skilled workers produce the robots needed in the i_R industry. In parallel, the services from industry i_N become more expensive to purchase because of higher production costs (wage premium). Therefore, even if $\sigma_I < 1$, i.e. when industries are complements, the diametrically opposed directions of the price dynamics induce households to further shift their consumption towards services of industry i_R. Note that in Table 7.2, $\kappa_{A_k,\omega}$, the critical value of σ_I, indicating regime change, is always less than unity. $\kappa_{A_k,\omega}$ is equal to the term in Corollary 7.5, i.e. higher demand for high-skilled labor to produce robots mechanically diminishes high-skilled labor demand in industry i_N, and thus directly infers less output in this industry. The derivation of $\kappa_{A_k,\omega}$ is presented in Appendix D.2.4.

We study the households' consumption decisions in relation to their skill level. The high-skilled essentially receive a higher real wage through lower prices for services i_R and through the wage premium they earn. The aggregate price level might increase, however, the increase in the wage premium overcompensates high-skilled workers for a potential higher aggregate price index.
We know already that low-skilled workers may have a decreasing or an increasing real wage, depending on DRWC-LS. A low-skilled household consumes less of service i_R if $\sigma_I < \kappa_{i_R,R}$. The value of $\kappa_{i_R,R}$ is given in Table 7.2 and is derived in Appendix D.2.8. $\kappa_{i_R,R}$ is always smaller than unity. This essentially means that if a household endowed with a low skill level consumes more of services i_N in response to technological progress, services must be stronger complements than if a household of a high skill level consumes more of services i_N. In other words, high-skilled households always consume more of service i_N when A_k increases, if the two services are complements. For low-skilled households, this must not be the case. The reason is that for low-skilled households, the service i_R becomes cheaper through technological progress, while the service i_N becomes more expensive through the increase in the wage premium.
Whenever a low-skilled worker's real wage decreases, he consumes less of i_N-services. If his real wage increases, he may still consume less of service i_N if $\sigma_I > \kappa_{i_N,R}$. The term $\kappa_{i_N,R}$ can be found in Table 7.2 and the derivation is shown in Appendix D.2.8.
Now, if DRWC-LS holds, there are values for σ_I for which the low-skilled decrease their consumption in both services. This is the case when $\kappa_{i_R,R} > \sigma_I > 0 > \kappa_{i_N,R}$, i.e. whenever the real wage of low-skilled decreases and $\sigma_I \in (0, \kappa_{i_R,R})$.

Table 7.2.: Effects of Rising Productivity in Manufacturing—Robotic Economy

Variable		ILM Equilibrium			DLM Equilibrium		
		Aggregate	$r < \tilde{r}(i_N)$	$r > \tilde{r}(i_N)$	Aggregate	$r < \tilde{r}(i_N)$	$r > \tilde{r}(i_N)$
1. Wage Premium	ω	c			$+$		
2. Price Index	P	$-$			$-/+$ DRWC-LS		
3. Real Wage	$\tilde{w}^r$		$+$	$+$		$-/+$ DRWC-LS	$+$
4. Service i_R	$n_{i_R}x_{i_R}$	$+$			$+$		
Service i_N	$n_{i_N}x_{i_N}$	$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$			$-$ $(\sigma_I > \kappa_{A_k,\omega})$ $+$ $(\sigma_I < \kappa_{A_k,\omega})$		
5. Consumption i_R	$c^r_{i_R}$		$+$	$+$		$-$ $(\sigma_I < \kappa_{i_R,R})^\dagger$ $+$ $(\sigma_I > \kappa_{i_R,R})$	$+$
Consumption i_N	$c^r_{i_N}$		$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$	$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$		$-$ $(\sigma_I > \kappa_{i_N,R})^\star$ $+$ $(\sigma_I < \kappa_{i_N,R})$	$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$

where $\kappa_{A_k,\omega} = \frac{\frac{\mu(A_k)}{\omega^{\sigma_R - 1}}}{\frac{\mu(A_k)}{\omega^{\sigma_R-1}} + \frac{\sigma_R - 1}{\sigma_R}} (< 1)$, and $\kappa_{i_N,R} = \frac{\frac{\mu(A_k)}{\omega^{\sigma_R-1}} z^{-1}\left[z - \sigma_{A_k,\omega}\right]}{\sigma_{A_k,\omega} + \frac{\mu(A_k)}{\omega^{\sigma_R-1}}} (< 1)$, and $\kappa_{i_R,R} = \frac{\sigma_{A_k,\omega} - z}{1-z} \frac{1 + \frac{\mu(A_k)}{\omega^{\sigma_R-1}}}{\sigma_{A_k,\omega} + \frac{\mu(A_k)}{\omega^{\sigma_R-1}}}$.

† If DRWC-LS holds, then consumption of service i_R by the low-skilled may or may not decrease. Otherwise, it always increases.

* If DRWC-LS holds, then consumption of service i_N by the low-skilled always decreases. However, the contrary is not necessarily true.

7.4.5. Increased Wage Inequality and More College Graduates

We saw in this framework that when the substitution technology for routine labor is produced by non-routine labor, technological progress in manufacturing leads to labor market separation and increases wage inequality. Again we confront our framework with the empirical observation of an increasing wage premium of college graduates and the simultaneous rise in their supply in the US (Tinbergen, 1974; Katz and Murphy, 1992).

ILM Equilibrium. When the labor market is integrated, technological progress leads unambiguously to labor market separation (Proposition 7.3). The empirical observation then can be explained by a change in the skill distribution and the analysis remains equal to Section 7.3.6.

DLM Equilibrium. Suppose the labor market is disintegrated. For our model the value of r^c, the lowest skill level that attends college, is relevant to understand the empirical observation. Again we can distinguish the three cases presented in Section 7.3.6, where in Case (i) $r^c < \tilde{r}(i_N)$, in Case (ii) $r^c = \tilde{r}(i_N)$ and in Case (iii) $r^c > \tilde{r}(i_N)$. We observe, that a decrease in r^c can imply the empirical observation, when the economy is in Case (iii). In the robotic economy ω unambiguously increases in A_k (Proposition 7.4) and so must the wage premium of college graduates. Thus, although the simultaneous rise in $\frac{\overline{w}_C}{\overline{w}_H}$ and ϕ_C can have different causes in our model, most importantly, an increasingly productive manufacturing industry that produces robots leads to a more unequally distributed wage scheme and can explain the empirical observation. If we assume that $\sigma_I > 1$, the empirical observation in a robotic economy can also have the same causes as in an industrial economy (see Section 7.3.6).

We summarize our insights: We assumed that robots are produced in the manufacturing industry and technological progress leads to higher productivity in this industry. Further, we assumed that workers able to perform task-complexity i_N are needed in the production of robots. Industry i_R uses the robots as an input factor and the demand for routine worker falls when robots become more productive (or cheaper). Thus, technological progress implies that routine labor is substituted by non-routine labor through the usage of robots, thereby wage inequality is accentuated. If this development is strong enough, it can surpass the increased supply of college graduates and the wages of college graduates relative to the wages of high-school graduates increase.

7.5. Discussion

7.5.1. Elasticity of Substitution between Capital and Labor

DeCanio (2016) estimates that the elasticity of substitution between robots and humans is greater than about 1.9. Similarly to our approach, he uses a production function with three factors, robotic capital, ordinary capital and humans, to analyze condition under which the expansion of the robotic input leads to declining human wages. However, he does not distinguish different skill levels. Thus, his estimate can be used as a lower bound for the substitutability of routine labor and robots in our model. Furthermore, DeCanio (2016) analyzes the short-run dynamics of substitution. The medium-term elasticity of substitution is likely to be higher, as technologies get refined and the adoption of new technologies is more likely to happen. Hémous and Olsen (2016) use a value of 4 for the elasticity of substitution between capital and low-skilled labor in their model.
We discussed estimates of the elasticity of substitution between industries, σ_I, in Section 6.5.2, which are significantly lower than 1.9, when tailored to our model. Thus, our parameter assumption of $\sigma_R > \sigma_I$ is supported by the data.

7.5.2. From Machines to Robots

We analyzed two stylized economies, the industrial economy and the robotic economy. The former equipped with a manufacturing industry that used routine labor input to produce machines and the latter equipped with a manufacturing industry that used non-routine labor input to produce robots. Table 7.3 shows the effects of both economies in the DLM Equilibrium. In the ILM Equilibrium, both economies show the same wage scheme and all households in the economy benefit equally from technological progress. However, in the DLM Equilibrium the differences are strong. The comparison of the two economies is founded on the following observation: The industrial economy resembles the economy of the first part of the 20^{st} century, and the robotic economy resembles the economy of today. We recall that the robotic economy has much stronger separating forces compared to the industrial economy. These separating forces unambiguously lead to higher wage inequality when the production of robots becomes cheaper.
The transition between the two states of the economy, and how this transition affects wages, cannot be inferred from the results obtained so far. We believe that the shift from routine labor input to non-routine labor input in manufacturing and the simultaneous rise in productivity in this industry, leads to a gradual shift from the industrial economy towards the robotic economy, that increases wage inequality, in accordance with the empirical observations.

Thus, we have come up with a new explanation for the increasing wage inequality observed between skill levels, that relies on uneven technological change and a rigid skill-task-assignment. Note that in this chapter, task-complexities remain constant. In Chapter 6, we introduced the task live-cycle. Changing task-complexities and the emergence of new ones can profoundly change wage inequality, because it directly affects the match between the industry-composition and the skill distribution. Thus, further research should be conducted to understand how uneven technological change and changing task-complexities interact.

Our results are supported by the empirical analysis of Acemoglu and Restrepo (2017), who find large negative and robust effects of robots on employment and wages. In their assessment these effects, however, not only apply to routine or low-skilled labor. DeCanio (2016), who analyzes the elasticity between labor and robots, notes that the[38] *"Expansion of AIs' skill sets (which in the terminology of the paper entails increases in the elasticity of substitution between AIs and humans) is likely to depress wages over time. This will increase measured inequality unless the returns to robotic assets are broadly spread across the population"* (p. 289).

Frey and Osborne (2017) note that more and more non-routine task can be executed by robots. Thus, more research has to be done to further understand the substitution possibilities in production of increasingly sophisticated robots and their implications for the labor market and for wage inequality.

[38] AI: Artificial Intelligence. DeCanio (2016) refers to systems equipped with AI, the technology that can match or surpass human capabilities in his definition, as robots.

Table 7.3.: Effects of Rising Productivity in Manufacturing

Variable		DLM Equilibrium – *Industrial* Economy			DLM Equilibrium – *Robotic* Economy		
		Aggregate	$r < \tilde{r}(i_N)$	$r > \tilde{r}(i_N)$	Aggregate	$r < \tilde{r}(i_N)$	$r > \tilde{r}(i_N)$
1. Wage Premium	ω	$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$			$+$		
2. Price Index	P	$-$ $\left(\sigma_I > \frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}\right)$ $+$ $\left(\sigma_I < \frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}\right)$			$-$ DRWC-LS $+$ (otherwise)		
3. Real Wage	$\tilde{w}^r$		$-\left(\sigma_I < \frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}\right)$ $+\left(\sigma_I > \frac{\omega\tilde{\phi}_N}{\omega\tilde{\phi}_N+\tilde{\phi}_R}\right)$	$+$		$-$ DRWC-LS $+$ (otherwise)	$+$
4. Service i_R	$n_{i_R}x_{i_R}$	$+$			$+$		
Service i_N	$n_{i_N}x_{i_N}$	c			$-$ $(\sigma_I > \kappa_{A_k,\omega})$ $+$ $(\sigma_I < \kappa_{A_k,\omega})$		
5. Consumption i_R	$c^r_{i_R}$		$-\left(\sigma_I < \frac{\omega\tilde{\phi}_N}{2\omega\tilde{\phi}_N+\tilde{\phi}_R}\right)$ $+\left(\sigma_I > \frac{\omega\tilde{\phi}_N}{2\omega\tilde{\phi}_N+\tilde{\phi}_R}\right)$	$+$		$-(\sigma_I < \kappa_{i_R,R})$ $+(\sigma_I > \kappa_{i_R,R})$	$+$
Consumption i_N	$c^r_{i_N}$		$-(\sigma_I < 1)$ $+(\sigma_I > 1)$	$-(\sigma_I > 1)$ $+(\sigma_I < 1)$		$-(\sigma_I > \kappa_{i_N,R})$ $+(\sigma_I < \kappa_{i_N,R})$	$-(\sigma_I > 1)$ $+(\sigma_I < 1)$

where $\kappa_{A_k,\omega} = \frac{\frac{\mu(A_k)}{\omega^{\sigma_R-1}}}{\frac{\mu(A_k)}{\omega^{\sigma_R-1}}+\frac{\sigma_R-1}{\sigma_R}}(<1)$, and $\kappa_{i_N,R} = \frac{\frac{\mu(A_k)}{\omega^{\sigma_R-1}}z^{-1}\left[z-\sigma_{A_k,\omega}\right]}{\sigma_{A_k,\omega}+\frac{\mu(A_k)}{\omega^{\sigma_R-1}}}(<1)$, and $\kappa_{i_R,R} = \frac{\sigma_{A_k,\omega}-z}{1-z}\frac{1+\frac{\mu(A_k)}{\omega^{\sigma_R-1}}}{\sigma_{A_k,\omega}+\frac{\mu(A_k)}{\omega^{\sigma_R-1}}}$.

7.6. Conclusion

The fundamental assumption of our model is that the skill-task-assignment is determined by technology, in particular by the minimum skill requirement of every production process. The minimum skill requirement is rationalized by a minimum-quality constraint imposed on the production processes in a technological environment of O-ring production. Thus, if not every skill level is usable in every production process, the assignment of skills to tasks co-determines labor market dynamics.

We showed that in an industrial economy, where low-skilled labor is substituted by capital that can be produced by low-skilled labor, technological progress in manufacturing has equaling effects on the wage scheme. In contrast the robotic economy, where low-skilled labor is substituted by capital that can only be produced by high-skilled labor, technological progress in manufacturing reveals strong tendencies towards a diverging wage scheme. Thus, we assume that only routine work can be replaced by capital. Graetz and Feng (2015) loosen this rigid assumption.[39] They abandon the categorization of routine and non-routine tasks and give an incentive-based explanation of which tasks firms automate. They can thereby explain job-polarization. We could enlarge the set of task-complexities in our model,[40] and patterns of job-polarization could be rationalized. However, firms cannot choose which tasks they want to automate, because there is a one-to-one mapping of firms to tasks. To loosen this one-to-one mapping further improvements of the model have to be done.

Further research building on the task-complexity model with uneven technological change would entail the introduction of capital accumulation, endogenous task-automation, and endogenous growth.

[39] Frey and Osborne (2017) note that also non-routine tasks are more and more subjected to automation.
[40] A discussion is provided in Section 6.4.

8. Extensions

In this chapter, we generalize the models of the previous chapters. In Section 8.1, we take the model of Chapter 6 and we assume that there are three task-complexity levels, i.e. a manual, a routine, and an abstract task-complexity. In Section 8.2, the models of Chapters 6 and 7 are rebuilt and we assume that households exhibit non-homothetic preferences.

8.1. Three Task-complexities

We extend the model of Chapter 6. We incorporate a third task-complexity level. The three task-complexities—with corresponding skill requirements—are

(i) manual task-complexity i_1,

(ii) routine task-complexity i_2, and

(iii) abstract task-complexity i_3,

where $i_1 < i_2 < i_3$. We keep the analysis simple by assuming again a one-to-one mapping form task-complexities to industries. Yet, we could extend the analysis to incorporate a range of industries in every type of task, and each type would represent a set of industries.[1] Furthermore, we assume that there are three skill levels in the economy,[2] which we denote by r_l, r_m, and r_h, where $r_l < r_m < r_h$. The skill subscripts (l, m, h) refer to low, middle and high skills.

Assumption ASC states that a task i can only be accomplished by skill levels $r \geq \tilde{r}(i)$. Table 8.1 gives an overview of all equilibria, without considering unemployment. Productivity is again expressed in terms of the highest-skilled labor in the economy, r_h. The wage of skill level r_h, divided by its wage premium, is taken as the numeraire.

Whenever we state that the Labor Market Clearing Condition (LMCC) is biding, we mean that labor markets cannot clear when the wage scheme represents relative productivities in industry i_1. Thus, LMCC is binding for industry i_2, i_3 or both i_2 and i_3 whenever there is a step in the wage-premia function ω from industry i_{n-1} to industry i_n, where $n \in \{2, 3\}$.

[1] The only requirement would be that the sets are non-overlapping.
[2] As in Section 6.4, we could assume any skill distribution.

Table 8.1.: Equilibria of the Task-complexity Model with Three Task-complexities

Equilibrium		Wages			Employment Status	Wage Premium		
eq	sub	w^{r_l}	w^{r_m}	w^{r_h}		ω_{i_1}	ω_{i_2}	ω_{i_3}
PE		$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	$\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}$	1	$i_1 < i_2 < i_3 < \tilde{i}(r_l) < \tilde{i}(r_m) < \tilde{i}(r_h)$	1	1	1
PE/TE-1	a	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	$\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}$	1	$i_1 < i_2 < \tilde{i}(r_l) < i_3 < \tilde{i}(r_m) < \tilde{i}(r_h)$	1	1	1
	b	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	$\omega\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}$	ω		1	ω	ω
PE/TE-2	a	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	$\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}$	1	$i_1 < \tilde{i}(r_l) < i_2 < i_3 < \tilde{i}(r_m) < \tilde{i}(r_h)$	1	1	1
	b	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	$\omega\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}$	ω		1	ω	ω
PE/TE-3	a	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	$\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}$	1	$i_1 < i_2 < \tilde{i}(r_l) < \tilde{i}(r_m) < i_3 < \tilde{i}(r_h)$	1	1	1
	b	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	$\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}$	ω		1	1	ω
PE/TE-4	a	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	$\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}$	1	$i_1 < \tilde{i}(r_l) < \tilde{i}(r_m) < i_2 < i_3 < \tilde{i}(r_h)$	1	1	1
	b	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	$\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}$	ω		1	1	ω
TE	**a**	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	$\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}$	1	$i_1 < \tilde{i}(r_l) < i_2 < \tilde{i}(r_m) < i_3 < \tilde{i}(r_h)$	1	1	1
	b	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	$\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}$	ω		1	1	ω
	c	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	$\omega\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}$	ω		1	ω	ω
	d	$\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}$	$\omega_{i_2}\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}$	ω_{i_3}		1	ω_{i_2}	ω_{i_3}

We focus on equilibrium TE—Triangle Equilibrium—, as it incorporates all possible wage premia. The other equilibria are essentially versions of the equilibria analyzed in Chapter 6, where the wage premium function reveals one single step at most.

For a more convenient exposition of the terms, we use the wage premium notation of Table 8.1 in the following. Then ω_i denotes the wage premium paid in industry i.

(a) ***TE*** – *sub* **a** – Integrated Labor Market (ILM Equilibrium)

LMCC is fulfilled with the wage scheme $w^{r^\star}$ of the following corollary.

Corollary 8.1

If LMCC holds for every industry $\hat{i} \in \mathcal{I}$, the following equilibrium is unique, up to the exact allocation of skill levels to firms:

(i) $w^{r^\star} = \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} \quad \forall\, r \in \mathcal{R}$,

(ii) $\mathcal{R}_i^\star \subseteq \{r \in \mathcal{R} \mid r \geq \tilde{r}(i)\} \quad \forall\, i \in \mathcal{I}$,

(iii) $q_i^\star(r) = \left[-\frac{1}{\lambda i \log(r)}\right]^{\frac{1}{\lambda}} \quad \forall\, (i, r) \in \mathcal{I} \times \mathcal{R}_i^\star$,

(iv) $p_i^\star = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i \log(r_h)\right]^{\frac{1}{\lambda}} \quad \forall\, i \in \mathcal{I}$,
$P_i^\star = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i \log(r_h)\right]^{\frac{1}{\lambda}} n_i^{\frac{1}{1-\sigma_v}} \quad \forall\, i \in \mathcal{I}$,
$P^\star = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda \log(r_h)\right]^{\frac{1}{\lambda}} \left[\sum_{i \in \mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{1-\sigma_I}}$,

(v) $\tilde{l}_i^\star = \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}}}{\sum_{\hat{i} \in \mathcal{I}} \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}} n_{\hat{i}}^{\frac{1-\sigma_I}{1-\sigma_v}}} TW^\star$,

(vi) $x_i^\star = \left[-e\lambda i \log(r_h)\right]^{-\frac{1}{\lambda}} \tilde{l}_i^\star$,

(vii) $\pi_i^\star = \frac{\tilde{l}_i^\star}{\sigma_v - 1}$,

(viii) $C^\star = \left[-e\lambda \log(r_h)\right]^{-\frac{1}{\lambda}} \left[\sum_{i \in \mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I - 1}} TW^\star$
and $P^\star C^\star = \frac{\sigma_v}{\sigma_v - 1} TW^\star$,

(ix) $TW^\star = L \sum_{r \in \mathcal{R}} \phi_r w^{r\star}$.

Next, we look at disintegrated labor markets.

(b) ***TE*** – *sub* **b** – Disintegrated Labor Market (DLM Equilibrium)

There is no sufficient supply of skill level r_h if there is no wage premium. Low-skilled workers and middle-skilled workers provide their labor within an integrated labor market, and the high-skilled workers' labor market is separated.

Corollary 8.2

If LMCC is binding for industry $i_3 \in \mathcal{I}$, the following equilibrium is unique, up to the exact allocation of skill levels to firms:

(i) $w^{r\star} = \omega_i^\star \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} \quad \forall\, r \in \mathcal{R}$, *with*

$$\omega_i^\star = \begin{cases} 1 \text{ for } i \in \{i_1, i_2\} \ , \\ \omega^\star \text{ for } i = i_3 \ , \text{ with } \omega^\star > 1 \ , \end{cases}$$

$$\text{where } \omega^\star = \left[\frac{\sum_{r\in\mathcal{R}/r_h} \phi_r \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}}}{\phi_{r_h}} \frac{\psi_{i_3} i_3^{\frac{1-\sigma_I}{\lambda}} n_{i_3}^{\frac{1-\sigma_I}{1-\sigma_v}}}{\sum_{i\in\mathcal{I}/i_3} \psi_i i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}} \right]^{\frac{1}{\sigma_I}} ,$$

(ii) $$\mathcal{R}_i^\star \subseteq \begin{cases} \{r \in \mathcal{R} \mid r_h > r \geq \tilde{r}(i)\} \text{ for } i \in \{i_1, i_2\} \ , \\ \{r \in \mathcal{R} \mid r \geq \tilde{r}(i)\} \text{ for } i = i_3 \ , \end{cases}$$

(iii) $q_i^\star(r) = \left[-\frac{1}{\lambda i \log(r)}\right]^{\frac{1}{\lambda}} \quad \forall\,(i,r) \in \mathcal{I} \times \mathcal{R}_i^\star$,

(iv) $p_i^\star = \frac{\sigma_v}{\sigma_v - 1}\omega_i^\star \left[-e\lambda i \log(r_h)\right]^{\frac{1}{\lambda}} \quad \forall\, i \in \mathcal{I}$,
$P_i^\star = \frac{\sigma_v}{\sigma_v - 1}\omega_i^\star \left[-e\lambda i \log(r_h)\right]^{\frac{1}{\lambda}} n_i^{\frac{1}{1-\sigma_v}} \quad \forall\, i \in \mathcal{I}$,
$P^\star = \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda \log(r_h)\right]^{\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i \omega_i^{\star 1-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{1-\sigma_I}}$,

(v) $\tilde{l}_i^\star = \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}}}{\sum_{\hat{i}\in\mathcal{I}} \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}} n_{\hat{i}}^{\frac{1-\sigma_I}{1-\sigma_v}}} TW^\star$,

(vi) $x_i^\star = \left[-e\lambda i \log(r_h)\right]^{-\frac{1}{\lambda}} \tilde{l}_i^\star$,

(vii) $\pi_i^\star = \frac{\tilde{l}_i^\star}{\sigma_v - 1}$,

(viii) $C^\star = \left[-e\lambda \log(r_h)\right]^{-\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i \omega_i^{\star 1-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I - 1}} TW^\star$
and $P^\star C^\star = \frac{\sigma_v}{\sigma_v - 1} TW^\star$,

(ix) $TW^\star = L \sum_{r\in\mathcal{R}} \phi_r w^{r\star}$.

Note that the wage scheme in (i) suggests that different wages are paid to the same skill in different industries. This, however, is not true because labor markets are separated and only sills $r \geq \tilde{r}(i_3)$ can earn the wage premium $\omega = \omega_{i_3}$. This can be seen in the skill sets, (ii), that firms choose to use in production.

(c) ***TE*** – *sub* – **c** Disintegrated Labor Market (DLM Equilibrium – M/RA)

There would be no sufficient supply of skill levels r_m and r_h without wage premium. Thus, low-skilled workers are separated from the higher-skilled workers, whose labor market is integrated.

Corollary 8.3

If LMCC is binding for industry $i_2 \in \mathcal{I}$, the following equilibrium is unique, up to the exact allocation of skill levels to firms:

(i) $w^{r^\star} = \omega_i^\star \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} \quad \forall\, r \in \mathcal{R}$, *with*

$$\omega_i^\star = \begin{cases} 1 \textit{ for } i = i_1 \ , \\ \omega^\star \textit{ for } i \in \{i_2, i_3\} \ , \ \textit{ with } \omega^\star > 1 \ , \end{cases}$$

$$\textit{where } \omega^\star = \left[\frac{\phi_{r_l}\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}}{\sum_{r\in\mathcal{R}/r_l} \phi_r \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}}} \frac{\sum_{i\in\mathcal{I}/i_1} \psi_i i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}}{\psi_{i_1} i_1^{\frac{1-\sigma_I}{\lambda}} n_{i_1}^{\frac{1-\sigma_I}{1-\sigma_v}}} \right]^{\frac{1}{\sigma_I}} ,$$

$$\textit{(ii)}\ \ \mathcal{R}_i^\star \subseteq \begin{cases} \{r \in \mathcal{R} \mid r_m > r \geq \tilde{r}(i)\} \textit{ for } i = i_1 \ , \\ \{r \in \mathcal{R} \mid r \geq \tilde{r}(i)\} \textit{ for } i \in \{i_2, i_3\} \ , \end{cases}$$

(iii) – (ix) as in Corollary 8.2.

(d) ***TE** – sub* **d** – Fully Disintegrated Labor Market (FDLM Equilibrium – M/R/A)

Every skill level is only present in one industry, i.e. labor markets are fully disintegrated.

Corollary 8.4

If LMCC is binding for industries $i_2, i_3 \in \mathcal{I}$, the following equilibrium is unique, up to the exact allocation of skill levels to firms:

(i) $w^{r^\star} = \omega_i^\star \left[\frac{\log(r_h)}{\log(r)}\right]^{\frac{1}{\lambda}} \quad \forall\, r \in \mathcal{R}$, *with*

$$\omega_i^\star = \begin{cases} 1 \textit{ for } i = i_1 \ , \\ \omega_{i_2}^\star \textit{ for } i = i_2 \ , \ \textit{ with } \omega_{i_2}^\star > 1 \ , \\ \omega_{i_3}^\star \textit{ for } i = i_3 \ , \ \textit{ with } \omega_{i_3}^\star > \omega_{i_2}^\star \ , \end{cases}$$

$$\textit{where } \omega_{i_2}^\star(\omega_{i_3}^\star) = \left[\frac{\sum_{r\in\mathcal{R}/r_m} \phi_r w^{r^\star}(\omega_{i_3}^\star)}{\phi_{r_m}\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}} \frac{\psi_{i_2} i_2^{\frac{1-\sigma_I}{\lambda}} n_{i_2}^{\frac{1-\sigma_I}{1-\sigma_v}}}{\sum_{i\in\mathcal{I}/i_2} \psi_i \omega_i^\star(\omega_{i_3}^\star)^{1-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}} \right]^{\frac{1}{\sigma_I}} ,$$

$$\textit{and } \omega_{i_3}^\star(\omega_{i_2}^\star) = \left[\frac{\sum_{r\in\mathcal{R}/r_h} \phi_r w^{r^\star}(\omega_{i_2}^\star)}{\phi_{r_h}} \frac{\psi_{i_3} i_3^{\frac{1-\sigma_I}{\lambda}} n_{i_3}^{\frac{1-\sigma_I}{1-\sigma_v}}}{\sum_{i\in\mathcal{I}/i_3} \psi_i \omega_i^\star(\omega_{i_2}^\star)^{1-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}} \right]^{\frac{1}{\sigma_I}} ,$$

$$\textit{(ii)}\ \ \mathcal{R}_i^\star \subseteq \begin{cases} \{r \in \mathcal{R} \mid r_m > r \geq \tilde{r}(i)\} \textit{ for } i = i_1 \ , \\ \{r \in \mathcal{R} \mid r_h > r \geq \tilde{r}(i)\} \textit{ for } i = i_2 \ , \\ \{r \in \mathcal{R} \mid r \geq \tilde{r}(i)\} \textit{ for } i = i_3 \ , \end{cases}$$

(iii) – (ix) as in Corollary 8.2.

We can show that the FDLM Equilibrium is unique if $\sigma_I > 1$.

Lemma 8.1
If $\sigma_I > 1$, the FDLM Equilibrium in Corollary 8.4 is unique.

PROOF: We assume that $\sigma_I > 1$. Then, we use $\omega^\star_{i_2}(\omega^\star_{i_3})$ in $\omega^\star_{i_3}(\omega^\star_{i_2})$ and obtain

$$\omega^\star_{i_3} = \left[\left[\frac{\psi_{r_l}\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}}{\psi_{r_h}} + \frac{\psi_{r_m}\left[\frac{\log(r_h)}{\log(r_m)}\right]^{\frac{1}{\lambda}}}{\psi_{r_h}}\mathcal{K}(\omega^\star_{i_3})^{\frac{1}{\sigma_I}}\right]\frac{\mathcal{Y}_3}{\mathcal{Y}_1 + \mathcal{Y}_2\mathcal{K}(\omega^\star_{i_3})^{\frac{1-\sigma_I}{\sigma_I}}}\right]^{\frac{1}{\sigma_I}}, \quad (8.1)$$

where $\mathcal{Y}_i = \psi_i i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\ \forall i \in \mathcal{I}$ and $\mathcal{K}(\omega^\star_{i_3}) = \frac{\sum_{r\in\mathcal{R}/r_m}\phi_r w^{r^\star}(\omega^\star_{i_3})}{\phi_{r_h}} \frac{\psi_{i_2} i_2^{\frac{1-\sigma_I}{\lambda}} n_{i_2}^{\frac{1-\sigma_I}{1-\sigma_v}}}{\sum_{i\in\mathcal{I}/i_2}\psi_i\omega^\star_i(\omega^\star_{i_3})^{1-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}}$.[3]
Observe that $\frac{\partial\mathcal{K}(\omega^\star_{i_3})}{\partial\omega^\star_{i_3}} > 0$ on $\omega^\star_{i_3} \geq 1$ and thus, there is a unique solution to Equation (8.1).

□

With higher numbers of task-complexities the evolution of the wage premium/wage premia rapidly becomes difficult to analyze, in particular when other forces, such as uneven technological change as studied in Chapter 7, also affect the economy. Thus, more research is needed to understand the task-complexity model in more complex environments.

8.2. Non-homothetic Preferences

In this section, we analyze the macroeconomic environment of Chapters 6 and 7 with a different preference relation of households. In particular we use the "Price-independent Generalized Linearity" (PIGL) utility developed by Muellbauer (1975). Empirical analyses of consumer behavior reject homotheticity (Houthakker, 1957; Deaton and Muellbauer, 1980). Other empirical contributions rejecting homotheticity can be found in the trade literature (Hunter, 1991; Francois and Kaplan, 1996; Dalgin et al., 2008).

Thus, the analysis of uneven technological change—which affects prices and the consumers' relative income—when consumers exhibit non-homothetic preference relations, is crucial for the conclusions we can draw from our analyses, because non-homothetic behavior amplifies the separating effects and thus, wage inequality. Thus, we show in this section that the results obtained in Chapter 7 can be interpreted as a lower bound of the forces that separate labor markets when technological progress occurs in the manufacturing industry.

[3] Note that $\sum_{i\in\mathcal{I}/i_2}\psi_i\omega^\star_i(\omega^\star_{i_3})^{1-\sigma_I} i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}} = \psi_1 i_1^{\frac{1-\sigma_I}{\lambda}} n_{i_1}^{\frac{1-\sigma_I}{1-\sigma_v}} + \psi_{i_3}{\omega^\star_{i_3}}^{1-\sigma_I} i_3^{\frac{1-\sigma_I}{\lambda}} n_{i_3}^{\frac{1-\sigma_I}{1-\sigma_v}}$.

8.2.1. Price-independent Generalized Linearity

Muellbauer (1975) shows that the aggregate demand equations are only consistent with the demand equations of a specific micro-level agent—i.e. in our case the representative household, which we will denote by RA, standing for "Representative Agent"—when relative prices vary, if price-independent generalized linearity holds.

Price-independent generalized linearity can be expressed through the following Indirect Utility Function:

$$V(P, e^r) = \frac{1}{\epsilon}\left[\frac{e^r}{a(P)}\right]^{\epsilon} - b(P)\ , \tag{8.2}$$

where e^r is the expenditure level of household r, $a(P)$ is a linearly homogeneous function, $b(P)$ is homogeneous of degree zero and P is a price vector. The parameter ϵ indicates the degree of non-homotheticity. We adapt (8.2) to our two-industries model. Following Boppart (2014), we choose $a(p) = P_{i_N}$ and $b(P) = \frac{\beta}{\gamma}\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}$.[4] The chosen functional forms of $a(P)$ and $b(P)$ imply (i) higher expenditure shares of industry i_N with higher expenditure levels, and (ii) substitution effects between industries due to price changes. The indirect utility function used in the following then is

$$V(P_{i_R}, P_{i_N}, e^r) = \frac{1}{\epsilon}\left[\frac{e^r}{P_{i_N}}\right]^{\epsilon} - \frac{\beta}{\gamma}\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}\ , \tag{8.3}$$

where P_{i_R} and P_{i_N} are again the price aggregators of the routine and the non-routine industry, respectively. The parameters ϵ, γ and β together determine the characteristics of the preference relation:

- ϵ determines the degree of non-homotheticity. The higher ϵ, the more non-homothetic is the preference relation.

- γ (among other parameters) shapes the elasticity of substitution between the two industries, $\sigma^r_{N,R}$, of a household, which depends on the household's expenditure level. Ceteris paribus, the higher γ, the more inelastic is $\sigma^r_{N,R}$.

- β is an industry weight.[5]

[4] In contrast to Chapters 6 and 7, the price index of the consumption basket of a particular household does not have to be equal to the price index of another household, as with non-homothetic preferences, the expenditure shares allocated to the two industries vary across households of different skill levels. Thus, we cannot denominate the households' expenditures with an aggregate price index.

[5] In Chapter 6, we had industry-specific demand shifters ψ_i, which could be used to discuss shifts in the taste of households. With the non-homothetic preference relation, we have endogenized the households' demand shifts with respect to technological progress. Then β can be interpreted as the relative industry-weight, $\frac{\psi_{i_R}}{\psi_{i_N}}$, which remains constant.

The representative agent, RA, in the Muellbauer sense is the household that fulfills the following condition:

$$e_{RA} = \left[\frac{\int_{\underline{r}}^{\bar{r}} [e^r]^{1-\epsilon} f(r) dr}{\int_{\underline{r}}^{\bar{r}} e^r f(r) dr} \right]^{-\frac{1}{\epsilon}} . \tag{8.4}$$

The macro-dynamics of the model then correspond to the dynamics of the specific household, whose expenditure level is characterized by e_{RA} in (8.4) (see Muellbauer (1975) for a detailed derivation of PIGL preferences).

In the context of our framework, we will consider the non-routine service as the luxury service and the routine service as the basic service (Buera and Kaboski, 2012). We make the following assumption, which guarantees that the preference relation in (8.3) describes a valid indirect utility function:

Assumption 8.1 (Positive Consumption Assumption (PCA))
We assume that for the lowest skill level, $\underline{r}$, the following condition holds:

$$\left[\frac{e^{\underline{r}}}{P_{i_N}} \right]^{\epsilon} > \left[\frac{1-\epsilon}{1-\gamma} \right] \beta \left[\frac{P_{i_R}}{P_{i_n}} \right]^{\gamma} . \tag{PCA}$$

Assumption PCA essentially guarantees that all households consume positive amounts of both industries. In addition, the assumption implies that the elasticity of substitution between the industries is always greater than zero for any household, as we will see below. Note that in our model, P_{i_N} increases one-to-one with the wage premium. Thus, whenever ω grows, both sides of Assumption PCA decrease. PCA then tightens for household $\underline{r}$ whenever $\epsilon > \gamma$. In the following, we will assume that parameters are such that $e^{\underline{r}}$ is high enough for Assumption PCA to hold. In Sections 8.2.3 and 8.2.4, we analyze technological progress in manufacturing that affects P_{i_N} *and* P_{i_R}. We will then discuss the consequences for Assumption PCA. Following Boppart (2014), we first state the following lemma:

Lemma 8.2
Function (8.3) *represents a non-homothetic preference relation with an expenditure elasticity of demand that is greater than one for non-routine services and smaller than one for routine services if and only if* $0 < \epsilon < 1$ *and* $\gamma < 1$ *and Assumption PCA holds.*

If $\epsilon = 0$, the preference relation becomes homothetic. If $\gamma = \epsilon = 0$, we have Cobb-Douglas preferences, and with $\beta = 0$, there is only one industry and households exhibit CRRA preferences.

The elasticity of substitution between industry i_R and industry i_N depends on the expenditure level of each household and thus on skill level r. The expenditure level, in turn, depends on wages and profits, $e^r = w^r + \pi^r$. We will later introduce a profit distribution. The profits of a household will also depend on the household's skill level. This assumption simplifies the model and allows us to obtain analytical results. Applying the Allen-Uzawa Formula, we can derive the elasticity of substitution between services from the routine and the non-routine industry, and obtain[6]

$$\sigma^r_{N,R} = 1 - \gamma - [\gamma - \epsilon] \frac{\beta \left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}}{\left[\frac{e^r}{P_{i_N}}\right]^{\epsilon} - \beta \left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}} . \tag{8.5}$$

The elasticity of substitution between the industries is always greater 0 by Assumption PCA and by the parameter restrictions.

If we assume that $\epsilon \leq \gamma < 1$, this leads to an elasticity of substitution between the industries which is always smaller than one.[7] With such a parameter assumption, structural change, relative price dynamics of luxury and basic goods, and income effects that match empirical observations in the U.S. since World War II can be explained (Boppart, 2014). However, this parameter assumption is chosen to examine the long-run dynamics of an economy with structural change. It would stand in stark contrast to the assumptions made in the previous chapters. Our analysis addresses medium-term technological progress, where relative prices are subjected to shifts due to uneven technological change, but where expenditure growth is less important. In Section 6.5.2 we argued that σ_I, the elasticity of substitution between industries with CES-utility, lies somewhere between 1 and the usual estimates in the literature, which are located in the range between 1.4 and 2 (Acemoglu and Autor, 2011). We thus continue assuming that aggregate elasticity of substitution between industries is in the elastic range. Note that at the micro-level, there can exist households with an inelastic elasticity even if the aggregate elasticity of substitution is elastic. Whenever $\gamma < 0$, the elasticities of substitution between industries are greater than unity for every household. Observe that $\sigma^r_{N,R}$ is only equal for all households if $\gamma = \epsilon$ or $\epsilon = 0$. We will denote the aggregate elasticity of substitution—the elasticity of substitution of the representative household—simply by $\sigma_{N,R}$.

We next analyze the task-complexity model from Chapter 6.

[6] See Boppart (2014) for a detailed derivation.
[7] This implies that the industry that experiences a relative price decrease also experiences a decrease in expenditure shares.

8.2.2. The Task-complexity Model and Non-homotheticity

At the exception of the consumer side, the building blocks of the following model are from Chapter 6. Thus, we will not show the production function of firms and their optimization problems or the concept of effective labor again.

Then the wage scheme is as in (6.15). The consumption decision of a household r is derived by applying Roy's Identity to (8.3) and we obtain

$$\begin{aligned} c^r_{i_R} =& \beta \frac{P^{\epsilon}_{i_N}}{P_{i_R}} \left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma} [e^r]^{1-\epsilon} \ , \\ c^r_{i_N} =& \frac{e^r}{P_{i_N}} \left[1 - \beta \left[\frac{P_{i_N}}{e^r}\right]^{\epsilon} \left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}\right] \ , \end{aligned}$$

where $c^r_{i_R}$ and $c^r_{i_N}$ denote the consumption bundles of industry i_R and industry i_N of household r, respectively. We still assume that the consumption bundle within each industry is a CES-aggregate of the n_{i_R} and n_{i_N} firms in each industry. Thus, firms almost face the same optimization problem as in the previous chapters, but now relative aggregate demand for the industries' services shift with expenditure dynamics that follow productivity gains through technological change.

Observe that the expenditure elasticity of demand for services of industry i_R is $1-\epsilon$, i.e. that with higher expenditure levels, relatively less of this industry is demanded. We can better demonstrate this relation by deriving the expenditure shares of household r for the industries,

$$\begin{aligned} s^r_{i_R} =& \beta \left[\frac{P_{i_N}}{e^r}\right]^{\epsilon} \left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma} \ , \\ s^r_{i_N} =& 1 - \beta \left[\frac{P_{i_N}}{e^r}\right]^{\epsilon} \left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma} \ , \end{aligned} \tag{8.6}$$

where $\frac{\partial s^r_{i_R}}{\partial e^r} < 0$ and $\frac{\partial s^r_{i_N}}{\partial e^r} > 0$ under the parameter assumptions. We will use s_{i_R} and s_{i_N} to denote the aggregate expenditure shares, which—by definition—also correspond to the expenditure shares of the representative household RA.

As in the preceding chapters, the prices and price aggregators are

$$\begin{aligned} p_i =& \frac{\sigma_v}{\sigma_v - 1} \omega_i \left[-e\lambda i \log(\bar{r})\right]^{\frac{1}{\lambda}} \quad \forall\, i \in \mathcal{I} \ , \\ P_i =& \frac{\sigma_v}{\sigma_v - 1} \omega_i \left[-e\lambda i \log(\bar{r})\right]^{\frac{1}{\lambda}} n_i^{\frac{1}{1-\sigma_v}} \quad \forall\, i \in \mathcal{I} \ . \end{aligned}$$

(8.4) implies that $\int_{\underline{r}}^{\bar{r}} [e^r]^{1-\epsilon} f(r) dr = \frac{E}{L} e^{-\epsilon}_{RA}$, where $L \int_{\underline{r}}^{\bar{r}} e^r f(r) dr = E$ and E denotes the aggregate expenditure level. The demand for the industries' consumption baskets are

$$
\begin{aligned}
C_{i_R} =& L\int_{\underline{r}}^{\bar{r}} c_{i_R}^r f(r)dr \\
=& L\beta P_{i_N}^{\epsilon-\gamma} P_{i_R}^{\gamma-1} \int_{\underline{r}}^{\bar{r}} [e^r]^{1-\epsilon} f(r)dr \\
=& \beta P_{i_N}^{\epsilon-\gamma} P_{i_R}^{\gamma-1} E e_{RA}^{-\epsilon} \\
=& \beta \left[\frac{\sigma_v}{\sigma_v - 1} [-e\lambda \log(\bar{r})]^{\frac{1}{\lambda}}\right]^{\epsilon-1} \left[n_{i_R}{}^{\frac{1}{1-\sigma_v}} i_R^{\frac{1}{\lambda}}\right]^{-1} \nu(\omega) E e_{RA}^{-\epsilon} ,
\end{aligned}
$$

where $\nu(\omega) = \left[n_{i_R}{}^{\frac{1}{1-\sigma_v}} i_R^{\frac{1}{\lambda}}\right]^{\gamma} \left[n_{i_N}{}^{\frac{1}{1-\sigma_v}} i_N^{\frac{1}{\lambda}}\omega\right]^{\epsilon-\gamma}$, and

$$
\begin{aligned}
C_{i_N} =& L\int_{\underline{r}}^{\bar{r}} c_{i_N}^r f(r)dr \\
=& L\int_{\underline{r}}^{\bar{r}} \frac{e^r}{P_{i_N}} \left[1 - \beta P_{i_N}^{\epsilon-\gamma} P_{i_R}^{\gamma} [e^r]^{-\epsilon}\right] f(r)dr \\
=& \frac{L}{P_{i_N}} \int_{\underline{r}}^{\bar{r}} e^r f(r)dr - L\beta P_{i_N}^{\epsilon-1-\gamma} P_{i_R}^{\gamma} \int_{\underline{r}}^{\bar{r}} [e^r]^{1-\epsilon} f(r)dr \\
=& \frac{E}{P_{i_N}} - \beta P_{i_N}^{\epsilon-1-\gamma} P_{i_R}^{\gamma} E e_{RA}^{-\epsilon} \\
=& \frac{\sigma_v - 1}{\omega\sigma_v} [-e\lambda i_N \log(\bar{r})]^{-\frac{1}{\lambda}} n_{i_N}{}^{\frac{1}{\sigma_v - 1}} E - \\
& \beta \left[\frac{\sigma_v}{\sigma_v - 1} [-e\lambda \log(\bar{r})]^{\frac{1}{\lambda}}\right]^{\epsilon-1} \left[n_{i_N}{}^{\frac{1}{1-\sigma_v}} i_N^{\frac{1}{\lambda}}\omega\right]^{-1} \nu(\omega) E e_{RA}^{-\epsilon} .
\end{aligned}
$$

We can verify that $P_{i_R}C_{i_R} + P_{i_N}C_{i_N} = E$.

Total production must equal total demand in each industry. We use the symmetry of firms within an industry and impose goods market clearing, and thus

$$
C_i = \left[\int_{n_i} c_{i,j}^{\frac{\sigma_v - 1}{\sigma_v}} dj\right]^{\frac{\sigma_v}{\sigma_v - 1}} = n_i^{\frac{\sigma_v}{\sigma_v - 1}} c_i = n_i^{\frac{\sigma_v}{\sigma_v - 1}} [-e\lambda i \log(\bar{r})]^{-\frac{1}{\lambda}} \tilde{l}_i \qquad \forall i \in \mathcal{I} .
$$

Total demand for labor within an industry, given total demand C_{i_R} and C_{i_N}, yields

$$
\begin{aligned}
n_{i_R}\tilde{l}_{i_R} =& [-e\lambda i_R \log(\bar{r})]^{\frac{1}{\lambda}} n_{i_R}^{\frac{1}{1-\sigma_v}} C_{i_R} \\
=& [-e\lambda \log(\bar{r})]^{\frac{1}{\lambda}} \beta \left[\frac{\sigma_v}{\sigma_v - 1} [-e\lambda \log(\bar{r})]^{\frac{1}{\lambda}}\right]^{\epsilon-1} \nu(\omega) E e_{RA}^{-\epsilon} , \\
n_{i_N}\tilde{l}_{i_N} =& [-e\lambda i_N \log(\bar{r})]^{\frac{1}{\lambda}} n_{i_N}^{\frac{1}{1-\sigma_v}} C_{i_N} \\
=& [-e\lambda \log(\bar{r})]^{\frac{1}{\lambda}} \omega^{-1} \left[\frac{\sigma_v - 1}{\sigma_v} [-e\lambda \log(\bar{r})]^{-\frac{1}{\lambda}} E - \right. \\
& \left. \beta \left[\frac{\sigma_v}{\sigma_v - 1} [-e\lambda \log(\bar{r})]^{\frac{1}{\lambda}}\right]^{\epsilon-1} \nu(\omega) E e_{RA}^{-\epsilon}\right] .
\end{aligned}
$$

ILM Equilibrium. Using labor market clearing, the model can be solved. Whenever there are sufficient skills in the economy—see ILM Equilibrium, in Chapter 6—, i.e. the labor market is integrated ($\omega = 1$), labor market clearing yields

$$\begin{aligned}\tilde{L} =& n_{i_R}\tilde{l}_{i_R} + n_{i_N}\tilde{l}_{i_N} \\ =& \left[-e\lambda\log(\bar{r})\right]^{\frac{1}{\lambda}} \beta \left[\frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda\log(\bar{r})\right]^{\frac{1}{\lambda}}\right]^{\epsilon-1} \nu(1) E e_{RA}^{-\epsilon} + \\ & \left[-e\lambda\log(\bar{r})\right]^{\frac{1}{\lambda}} \left[\frac{\sigma_v - 1}{\sigma_v}\left[-e\lambda\log(\bar{r})\right]^{-\frac{1}{\lambda}} E - \beta\left[\frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda\log(\bar{r})\right]^{\frac{1}{\lambda}}\right]^{\epsilon-1} \nu(1) E e_{RA}^{-\epsilon}\right] \\ =& \frac{\sigma_v - 1}{\sigma_v} E \, .\end{aligned}$$

Monopolistic competition implies that aggregate profits are $\Pi = \frac{1}{\sigma_v - 1} TW$ and we know that aggregate wage income is simply $TW = \tilde{L}$. Thus, total expenditures equals $E = \frac{\sigma_v}{\sigma_v - 1}\tilde{L}$, or stated equivalently,

$$\frac{\sigma_v - 1}{\sigma_v} E = \frac{\sigma_v - 1}{\sigma_v}\left[\Pi + TW\right] = \tilde{L} \, .$$

DLM Equilibrium. Next we analyze the economy when there are *no* sufficient skills—see DLM Equilibrium, in Chapter 6—, i.e. the labor markets are disintegrated ($\omega > 1$). We define

$$\tilde{\phi}_R = \int_{\underline{r}}^{\tilde{r}(i_N)} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr \quad \text{and} \quad \tilde{\phi}_N = \int_{\tilde{r}(i_N)}^{\bar{r}} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}} f(r)dr \, ,$$

and

$$\tilde{\psi}_R = \int_{\underline{r}}^{\tilde{r}(i_N)} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1-\epsilon}{\lambda}} f(r)dr \quad \text{and} \quad \tilde{\psi}_N = \int_{\tilde{r}(i_N)}^{\bar{r}} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1-\epsilon}{\lambda}} f(r)dr \, .$$

Total demand for effective labor from each industry and the effective labor supply that fulfills the industries' skill requirements must equate. Thus, labor market clearing implies $n_{i_R}\tilde{l}_{i_R} = \int_{\underline{r}}^{\tilde{r}(i_N)} \tilde{L}^r f(r)dr = \tilde{\phi}_R L$ and $n_{i_N}\tilde{l}_{i_N} = \int_{\tilde{r}(i_N)}^{\bar{r}} \tilde{L}^r f(r)dr = \tilde{\phi}_N L$. Using the expressions and the firms' demand for labor, we obtain

$$\tilde{\phi}_R = \beta\left[\frac{\sigma_v}{\sigma_v - 1}\right]^{\epsilon-1} \left[-e\lambda\log(\bar{r})\right]^{\frac{\epsilon}{\lambda}} \nu(\omega)\frac{E}{L} e_{RA}^{-\epsilon} \, , \tag{8.7}$$

$$\omega\tilde{\phi}_N = \frac{\sigma_v - 1}{\sigma_v}\frac{E}{L} - \beta\left[\frac{\sigma_v}{\sigma_v - 1}\right]^{\epsilon-1} \left[-e\lambda\log(\bar{r})\right]^{\frac{\epsilon}{\lambda}} \nu(\omega)\frac{E}{L} e_{RA}^{-\epsilon} \, . \tag{8.8}$$

Total expenditures naturally depend on the wage premium ω, i.e. how strongly markets are separated, and are

$$\begin{aligned} E =& \Pi + TW \\ =& \Pi + \tilde{L}_R + \omega \tilde{L}_N \\ =& \frac{\sigma_v}{\sigma_v - 1} L \left[\tilde{\phi}_R + \omega \tilde{\phi}_N \right] . \end{aligned}$$

Using this equivalence, we can see that (8.7) and (8.8) are equivalent. Using (8.4) and the assumption of profits that are proportional to household wages, we can derive

$$e_{RA} = \frac{\sigma_v}{\sigma_v - 1} \left[\frac{\tilde{\psi}_R + \omega^{1-\epsilon} \tilde{\psi}_N}{\tilde{\phi}_R + \omega \tilde{\phi}_N} \right]^{-\frac{1}{\epsilon}} . \tag{8.9}$$

Using (8.9), we obtain $\frac{E}{L} e_{RA}^{-\epsilon} = \left[\frac{\sigma_v}{\sigma_v - 1} \right]^{1-\epsilon} \left[\tilde{\psi}_R + \omega^{1-\epsilon} \tilde{\psi}_N \right]$. We use (8.7) to define the following function $\mathcal{G}$:

$$\mathcal{G} := \left[-e\lambda \log(\bar{r}) \right]^{-\frac{\epsilon}{\lambda}} \frac{\tilde{\phi}_R}{\beta} \nu(\omega)^{-1} - \tilde{\psi}_R - \omega^{1-\epsilon} \tilde{\psi}_N = 0 . \tag{8.10}$$

We can derive the following lemma:

Lemma 8.3
There is a unique solution for ω if Assumption PCA is fulfilled.

Lemma 8.3 is obtained by showing that the derivative $\frac{\partial \mathcal{G}}{\partial \omega}$ is always negative when Assumption PCA is fulfilled. The proof is given in Appendix D.3.1.

In contrast to Chapters 6 and 7, where the utility was equal to the consumption basket, we must now analyze households' utility through their indirect utility function. Using the price indices, we can rewrite (8.3) as

$$V(P_{i_R}, P_{i_N}, e^r) = \frac{1}{\epsilon} \left[\frac{e^r}{\frac{\sigma_v}{\sigma_v - 1} \omega \left[-e\lambda i_N \log(\bar{r}) \right]^{\frac{1}{\lambda}} n_{i_N}^{\frac{1}{1-\sigma_v}}} \right]^{\epsilon} - \frac{\beta}{\gamma} \left[\left[\frac{i_R}{i_N} \right]^{\frac{1}{\lambda}} \left[\frac{n_R}{n_N} \right]^{\frac{1}{1-\sigma_v}} \omega^{-1} \right]^{\gamma} .$$

Taking the derivative of the indirect utility function with respect to the wage premium yields

$$\frac{\partial V(e^r)}{\partial \omega} = \frac{1}{\omega} \left[\frac{e^r}{P_{i_N}} \right]^{\epsilon} \left[\frac{\partial e^r}{\partial \omega} \frac{\omega}{e^r} - 1 \right] + \frac{\beta}{\omega} \left[\frac{P_{i_R}}{P_{i_N}} \right]^{\gamma} \lesseqgtr 0 .$$

The important term is the elasticity of expenditure with respect to the wage premium, $\frac{\partial e^r}{\partial \omega} \frac{\omega}{e^r}$. We have to analyze the households within each labor market separately, because only non-routine labor earns the wage premium, and thus

$$\frac{\partial e^r}{\partial \omega} = \begin{cases} \left[\frac{\log(\tilde{r})}{\log(r)}\right]^{\frac{1}{\lambda}} + \frac{\partial \pi^r}{\partial \omega} & \text{for } r \geq \tilde{r}(i_N) \\ \frac{\partial \pi^r}{\partial \omega} & \text{for } r < \tilde{r}(i_N) \end{cases} .$$

To obtain results and to determine the expenditure elasticity with respect to the wage premium for every skill level, we must assume some profit distribution.[8] As noted above, we can again assume that each household is self-employed and earns, in addition to the wage, the profits from its own labor input, i.e. that

$$e^r = \frac{\sigma_v}{\sigma_v - 1} w^r . \tag{8.11}$$

This can be regarded as a conservative assumption with respect to the profit distribution, as wealth—and thus profit income—is typically less equally distributed than wage income (Piketty, 2014). Then the derivative of the expenditure level of households equals

$$\frac{\partial e^r}{\partial \omega} = \begin{cases} \frac{\sigma_v}{\sigma_v - 1} \left[\frac{\log(\tilde{r})}{\log(r)}\right]^{\frac{1}{\lambda}} & \text{for } r \geq \tilde{r}(i_N) , \\ 0 & \text{for } r < \tilde{r}(i_N) . \end{cases} \tag{8.12}$$

Not surprisingly, for skill levels $r < \tilde{r}(i_N)$, utility decreases when the wage premium increases, i.e.

$$\frac{\partial V(e^r)}{\partial \omega} = -\frac{1}{\omega}\left[\frac{e^r}{P_{i_N}}\right]^{\epsilon} + \frac{\beta}{\omega}\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma} < 0 .$$

The inequality is strict because of Assumption PCA, which essentially states that the household with lowest expenditures also consumes a positive amount from the non-routine consumption basket—the luxury services—, and thus $\frac{c^r_{i_N} P_{i_N}}{e^r} = 1 - \beta \left[\frac{P_{i_N}}{e^r}\right]^{\epsilon} \left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma} > 0$. For skill levels $r > \tilde{r}(i_N)$, utility increases

$$\frac{\partial V(e^r)}{\partial \omega} = \frac{\beta}{\omega}\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma} > 0 .$$

We now turn back to uneven technological progress. We assume that there is a third industry—manufacturing—that produces capital, i.e. we repeat the analyses conducted in Chapter 7 and study the case of capital as a substitute for routine labor. We distinguish the case of an industrial economy, where capital represents machines, that is produced with a routine task-complexity, from the case of a robotic economy, where capital represents

[8] Labor is supplied inelastically. So we can assume any integrable profits distribution. This implies that the same profits per skill level are required.

robots, that is produced with a non-routine task-complexity. We focus on the implications of uneven technological progress in manufacturing. The only difference to the previous analyses in Chapter 7 is the utility function.

8.2.3. Industrial Economy with Non-homothetic Preferences

The following is based on the analysis in Section 7.3. The production technologies in industries i_R and i_N lead to the following industry-specific price indices

$$P_{i_R} = \frac{\sigma_v}{\sigma_v - 1} \left[-e\lambda i_R \log(\bar{r})\right]^{\frac{1}{\lambda}} n_R^{\frac{1}{1-\sigma_v}} \hat{\theta}(A_k)^{\frac{1}{1-\sigma_R}} ,$$
$$P_{i_N} = \frac{\sigma_v}{\sigma_v - 1} \left[-e\lambda i_N \log(\bar{r})\right]^{\frac{1}{\lambda}} n_N^{\frac{1}{1-\sigma_v}} \omega ,$$

where again $\hat{\theta}(A_k) = \left[1 + A_k^{\sigma_R - 1}\right]$. Given prices, the demand for routine services from households is

$$\begin{aligned} C_{i_R} =& \beta P_{i_N}^{\epsilon-\gamma} P_{i_R}^{\gamma-1} E e_{RA}^{-\epsilon} \\ =& \beta \left[\frac{\sigma_v}{\sigma_v - 1} \left[-e\lambda \log(\bar{r})\right]^{\frac{1}{\lambda}}\right]^{\epsilon-1} \left[n_{i_R}{}^{\frac{1}{1-\sigma_v}} i_R^{\frac{1}{\lambda}}\right]^{-1} \nu(\omega) \hat{\theta}(A_k)^{\frac{\gamma-1}{1-\sigma_R}} E e_{RA}^{-\epsilon} , \end{aligned}$$

and the demand for non-routine services is

$$\begin{aligned} C_{i_N} =& \frac{E}{P_{i_N}} - \beta P_{i_N}^{\epsilon-1-\gamma} P_{i_R}^{\gamma} E e_{RA}^{-\epsilon} \\ =& \frac{\sigma_v - 1}{\omega\sigma_v} \left[-e\lambda i_N \log(\bar{r})\right]^{-\frac{1}{\lambda}} n_{i_N}{}^{\frac{1}{\sigma_v - 1}} E - \\ & \beta \left[\frac{\sigma_v}{\sigma_v - 1} \left[-e\lambda \log(\bar{r})\right]^{\frac{1}{\lambda}}\right]^{\epsilon-1} \left[n_{i_N}{}^{\frac{1}{1-\sigma_v}} i_N^{\frac{1}{\lambda}} \omega\right]^{-1} \nu(\omega) \hat{\theta}(A_k)^{\frac{\gamma}{1-\sigma_R}} E e_{RA}^{-\epsilon} . \end{aligned}$$

Note that $n_{i_R} \tilde{l}_{i_R} + \tilde{L}_{k,i_R} = n_{i_R} \tilde{l}_{i_R} \hat{\theta}(A_k)$ denotes the total demand for effective labor that can master task-complexity i_R from both the routine industry and the manufacturing industry. Implying goods market clearing, we obtain total effective labor demand within each industry,

$$\begin{aligned} n_{i_R} \tilde{l}_{i_R} \tilde{\theta}(A_k) =& \left[-e\lambda i_R \log(\bar{r})\right]^{\frac{1}{\lambda}} n_{i_R}^{\frac{1}{1-\sigma_v}} \hat{\theta}(A_k)^{\frac{1}{1-\sigma_R}} C_{i_R} \\ =& \left[-e\lambda \log(\bar{r})\right]^{\frac{\epsilon}{\lambda}} \beta \left[\frac{\sigma_v}{\sigma_v - 1}\right]^{\epsilon-1} \nu(\omega) \hat{\theta}(A_k)^{\frac{\gamma}{1-\sigma_R}} E e_{RA}^{-\epsilon} , \end{aligned}$$

and

$$
\begin{aligned}
n_{i_N}\tilde{l}_{i_N} &= [-e\lambda i_N \log(\bar{r})]^{\frac{1}{\lambda}} n_{i_N}^{\frac{1}{1-\sigma_v}} C_{i_N} \\
&= \frac{\sigma_v - 1}{\omega\sigma_v} E - \\
&\quad [-e\lambda \log(\bar{r})]^{\frac{\epsilon}{\lambda}} \frac{\beta}{\omega} \left[\frac{\sigma_v}{\sigma_v - 1}\right]^{\epsilon-1} \nu(\omega)\hat{\theta}(A_k)^{\frac{\gamma}{1-\sigma_R}} E e_{RA}^{-\epsilon} .
\end{aligned}
$$

When $\omega = 1$ and the supply of effective labor that is able to master task-complexity i_N is greater than the demand for such labor, there are sufficient skills in the economy, and labor markets are integrated. In such an ILM Equilibrium, total expenditures are $E = \frac{\sigma_v}{\sigma_v-1}\tilde{L}$ and $n_{i_N}\tilde{l}_{i_N} + n_{i_R}\tilde{l}_{i_R} + \tilde{L}_{k,i_R} = \tilde{L}$.

If, on the other hand, there are *no* sufficient skills for labor markets to remain integrated and thus labor markets are disintegrated—DLM Equilibrium—and $\omega > 1$, then

$$
\begin{aligned}
\tilde{\phi}_R &= \beta \left[\frac{\sigma_v}{\sigma_v - 1}\right]^{\epsilon-1} [-e\lambda \log(\bar{r})]^{\frac{\epsilon}{\lambda}} \nu(\omega)\hat{\theta}(A_k)^{\frac{\gamma}{1-\sigma_R}} \frac{E}{L} e_{RA}^{-\epsilon} , \\
\omega\tilde{\phi}_N &= \frac{\sigma_v - 1}{\sigma_v}\frac{E}{L} - \beta \left[\frac{\sigma_v}{\sigma_v - 1}\right]^{\epsilon-1} [-e\lambda \log(\bar{r})]^{\frac{\epsilon}{\lambda}} \nu(\omega)\hat{\theta}(A_k)^{\frac{\gamma}{1-\sigma_R}} \frac{E}{L} e_{RA}^{-\epsilon} .
\end{aligned}
\tag{8.13}
$$

We use $\frac{E}{L}e_{RA}^{-\epsilon} = \left[\frac{\sigma_v}{\sigma_v-1}\right]^{1-\epsilon}\left[\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N\right]$, derived from (8.9). We further use (8.13) to define the following function $\breve{\mathcal{G}}$:

$$
\breve{\mathcal{G}} := [-e\lambda \log(\bar{r})]^{-\frac{\epsilon}{\lambda}} \frac{\tilde{\phi}_R}{\beta} \nu(\omega)^{-1}\hat{\theta}(A_k)^{\frac{\gamma}{\sigma_R-1}} - \tilde{\psi}_R - \omega^{1-\epsilon}\tilde{\psi}_N = 0 . \tag{8.14}
$$

Lemma 8.3 also applies to (8.14), i.e. the solution ω is unique under Assumption PCA. We take the derivatives with respect to the wage premium, ω, and the technological level, A_k, and obtain

$$
\frac{\partial\breve{\mathcal{G}}}{\partial A_k} = \gamma\, [-e\lambda \log(\bar{r})]^{-\frac{\epsilon}{\lambda}} \frac{\tilde{\phi}_R}{\beta} \nu(\omega)^{-1}\hat{\theta}(A_k)^{\frac{\gamma}{\sigma_R-1}-1} A_k^{\sigma_R-2} > 0 , \tag{8.15}
$$

$$
\frac{\partial\breve{\mathcal{G}}}{\partial \omega} = [\gamma - \epsilon]\, [-e\lambda \log(\bar{r})]^{-\frac{\epsilon}{\lambda}} \frac{\tilde{\phi}_R}{\beta} \nu(\omega)^{-1}\hat{\theta}(A_k)^{\frac{\gamma}{\sigma_R-1}} \omega^{-1} - [1-\epsilon]\,\omega^{-\epsilon}\tilde{\psi}_N \lesseqgtr 0 . \tag{8.16}
$$

The partial derivatives, (8.15) and (8.16), and the function in (8.14) imply the following proposition:

Proposition 8.1

The elasticity of the wage premium with respect to A_k,

$$\sigma_{A_k,\omega} = \frac{\partial \omega}{\partial A_k}\frac{A_k}{\omega} = -\frac{\frac{\partial \breve{\mathcal{G}}}{\partial A_k}}{\frac{\partial \breve{\mathcal{G}}}{\partial \omega}}\frac{A_k}{\omega} = -\frac{\gamma \frac{A_k^{\sigma_R - 1}}{1 + A_k^{\sigma_R - 1}}}{\gamma - \epsilon - [1-\epsilon]\frac{\omega^{1-\epsilon}\tilde{\psi}_N}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N}}\,, \tag{8.17}$$

(i) is negative, if $\gamma < 0$ *,*

(ii) is positive, if $\gamma \in (0, \epsilon)$ *,*

(iii) is positive, if $\gamma \in \left[\epsilon, \epsilon + [1-\epsilon]\left[1 - \frac{\tilde{\phi}_R\left[\frac{\sigma_v - 1}{\sigma_v}e^{\underline{r}}\right]^{-\epsilon}}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N}\right]\right)$ *,*

(iv) violates Assumption PCA, if $\gamma \in \left[\epsilon + [1-\epsilon]\left[1 - \frac{\tilde{\phi}_R\left[\frac{\sigma_v - 1}{\sigma_v}e^{\underline{r}}\right]^{-\epsilon}}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N}\right], 1\right)$ *.*

We now analyze the Cases $(i) - (iv)$ each separately.

Case (i) describes an environment in which $\sigma^r_{N,R} > 1 \; \forall \, r \in \mathcal{R}$. Thus, the result conforms to Proposition 7.1.

Case (ii) describes an environment in which parameters are such that the wage premium increases in A_k. We can further distinguish two sub-cases.

(a) If $\gamma \in \left[0, \epsilon \frac{\omega \tilde{\phi}_N}{\tilde{\phi}_R + \omega \tilde{\phi}_N}\right]$, then $\sigma_{N,R} \geq 1$, and if $\gamma \in \left[0, \epsilon\left[1 - \frac{\tilde{\phi}_R\left[\frac{\sigma_v - 1}{\sigma_v}e^{\underline{r}}\right]^{-\epsilon}}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N}\right]\right]$, then $\sigma^r_{N,R} \geq 1 \; \forall \, r \in \mathcal{R}$. Thus, there exists a range for γ where aggregate elasticity of substitution between industries is greater than 1 and for which the wage premium still increases in A_k.

Proposition 8.2

If $\gamma \in \left(0, \epsilon\left[1 - \frac{\tilde{\phi}_R\left[\frac{\sigma_v - 1}{\sigma_v}e^{\underline{r}}\right]^{-\epsilon}}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N}\right]\right)$*, the wage premium increases in* A_k *and* $\sigma^r_{N,R} > 1 \; \forall \, r \in \mathcal{R}$ *and if* $\gamma \in \left(0, \epsilon \frac{\omega \tilde{\phi}_N}{\tilde{\phi}_R + \omega \tilde{\phi}_N}\right]$*, aggregate elasticity of substitution* $\sigma_{N,R} \geq 1$ *and the scaling factor increases in* A_k*.*

Note that $\epsilon\left[1 - \frac{\tilde{\phi}_R\left[\frac{\sigma_v - 1}{\sigma_v}e^{\underline{r}}\right]^{-\epsilon}}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N}\right]$ is the lower bound of a range where some households have inelastic preferences, while others have elastic preferences. Proposition 8.2 shows the impact of the non-homothetic preference relation compared to Proposition 7.1. Even with an elasticity of substitution between industries that is greater than one for all households, the wage premium can increase. This is a major difference to the analysis of Chapter 7.

(b) If $\gamma \in \left(\epsilon \frac{\omega\tilde{\phi}_N}{\tilde{\phi}_R+\omega\tilde{\phi}_N}, \epsilon\right)$, the wage premium still increases. However, the aggregate elasticity of substitution is

Case (iii) describes an environment in which households shift their consumption towards services i_N because of both the income effect and the substitution effect.

Case (iv) describes an environment that contradicts the assumptions made.

The proof is given in Appendix D.3.3.

Note that (8.17) is ceteris paribus higher the greater γ—i.e. the more inelastic households preferences are between the industries—, and the greater ϵ—i.e. the higher the non-homotheticity of the preference relation. We can also observe that when preferences are homothetic, $\epsilon = 0$, and aggregate elasticity of substitution $\sigma_{N,R}$ equals 1, the $\sigma_{A_k,\omega} = 0$.[9]

We next analyze the indirect utility function of the households,

$$V(P_{i_R}, P_{i_N}, e^r) = \frac{1}{\epsilon}\left[\frac{e^r}{\frac{\sigma_v}{\sigma_v-1}\omega\left[-e\lambda i_N \log(\bar{r})\right]^{\frac{1}{\lambda}} n_{i_N}^{\frac{1}{1-\sigma_v}}}\right]^{\epsilon} - \frac{\beta}{\gamma}\left[\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\left[\frac{n_R}{n_N}\right]^{\frac{1}{1-\sigma_v}} \frac{\hat{\theta}(A_k)^{\frac{1}{1-\sigma_R}}}{\omega}\right]^{\gamma} .$$

Taking the derivative of the indirect utility function with respect to the technological factor, A_k, yields

$$\frac{\partial V(e^r)}{\partial A_k} = \frac{1}{A_k}\left[\left[\frac{e^r}{P_{i_N}}\right]^{\epsilon}\left[\frac{\partial e^r}{\partial A_k}\frac{A_k}{e^r} - \sigma_{A_k,\omega}\right] + \beta\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}\left[\frac{A_k^{\sigma_R-1}}{\hat{\theta}(A_k)} + \sigma_{A_k,\omega}\right]\right] \lesseqgtr 0 .$$

We still assume that $e^r = \frac{\sigma_v}{\sigma_v-1}w^r$—see (8.11) for a brief discussion. We analyze the low-skilled first. Thus, for skill levels $r < \tilde{r}(i_N)$, utility might decrease or increase,

$$\frac{\partial V(e^r)}{\partial A_k} = -\frac{1}{A_k}\left[\left[\frac{e^r}{P_{i_N}}\right]^{\epsilon} - \beta\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}\right]\sigma_{A_k,\omega} + \beta\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}\frac{A_k^{\sigma_R-2}}{\hat{\theta}(A_k)} \lesseqgtr 0 . \tag{8.18}$$

The total effect depends on whether the wage premium increases if the machines become more efficient and on how great the increase is, if there is one. If the wage premium decreases with technological progress, i.e. if $\gamma < 0$, low-skilled workers always benefit

[9] We set $\sigma_{N,R} = 1$ and $\epsilon = 0$ and obtain

$$1 = 1 - \gamma - \gamma\frac{\beta\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}}{\left[\frac{e_{RA}}{P_{i_N}}\right]^{\epsilon} - \beta\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}} = 1 - \frac{\gamma}{s_{i_N}} ,$$

which implies that $\gamma = 0$ and we immediately see that $\sigma_{A_k,\omega} = 0$.

from such technological progress. Using (8.17) in (8.18), we obtain

$$\begin{aligned}\frac{\partial V(e^r)}{\partial A_k} =& \frac{1}{A_k}\left[\frac{e^r}{P_{i_N}}\right]^{\epsilon}\left[1-\beta\left[\frac{e^r}{P_{i_N}}\right]^{-\epsilon}\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}\right]\frac{\gamma\frac{A_k^{\sigma_R-1}}{\hat{\theta}(A_k)}}{\gamma-\epsilon-[1-\epsilon]\frac{\omega^{1-\epsilon}\tilde{\psi}_N}{\tilde{\psi}_R+\omega^{1-\epsilon}\tilde{\psi}_N}}+\beta\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}\frac{A_k^{\sigma_R-2}}{\hat{\theta}(A_k)}\\ =& \frac{1}{A_k}\left[\frac{e^r}{P_{i_N}}\right]^{\epsilon}\frac{A_k^{\sigma_R-1}}{\hat{\theta}(A_k)}\left[s^r_{i_N}\frac{\gamma}{\gamma-\epsilon-[1-\epsilon]\frac{\omega^{1-\epsilon}\tilde{\psi}_N}{\tilde{\psi}_R+\omega^{1-\epsilon}\tilde{\psi}_N}}+s^r_{i_R}\right],\end{aligned}$$

where $s^r_{i_R}$ and $s^r_{i_N}$ denote the expenditure share of household r, and we can derive the following corollary.

Corollary 8.5
The utility of low-skilled household r increases in A_k if

$$\frac{\gamma}{\gamma-\epsilon-[1-\epsilon]\frac{\omega^{1-\epsilon}\tilde{\psi}_N}{\tilde{\psi}_R+\omega^{1-\epsilon}\tilde{\psi}_N}} \geq -\frac{s^r_{i_R}}{s^r_{i_N}}. \tag{8.19}$$

Note that the left-hand side of (8.19) depends on the aggregate expenditure shares, whereas the right-hand side denotes household r's negative ratio of expenditure shares. Furthermore, the right-hand side increases in higher skill levels of households, i.e. $-\partial\frac{s^r_{i_R}}{s^r_{i_N}}/\partial r > 0$. Corollary 8.5 implies that utility can increase for some households within the routine labor force, while for some, utility decreases. This means that in contrast to Chapter 7 the group-affiliation to routine labor does not uniquely determine whether technological progress is beneficial for a household.

Corollary 8.6
If there exists a household $r^c < \tilde{r}(i_N)$ for which

$$\frac{\gamma}{\gamma-\epsilon-[1-\epsilon]\frac{\omega^{1-\epsilon}\tilde{\psi}_N}{\tilde{\psi}_R+\omega^{1-\epsilon}\tilde{\psi}_N}} = -\frac{s^{r^c}_{i_R}}{s^{r^c}_{i_N}},$$

all households with skill level $r < r^c$ experience utility decreases when A_k increases.

Corollary 8.6 directly follows from Corollary 8.5 and is caused by the non-homotheticity of the preference relation, i.e. the higher the expenditure, which is increasing in r, the lower the share $s^r_{i_R}$ devoted to the routine industry and the higher the share $s^r_{i_N}$ devoted to the non-routine industry.
Skill levels $r > \tilde{r}(i_N)$ have expenditure levels equal to $e^r = \frac{\sigma_v}{\sigma_v-1}\omega\left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1}{\lambda}}$. In the special case of profits that are proportional to wages, the expenditure elasticity with respect to A_k is equal to the elasticity of the wage premium with respect to A_k, i.e. $\frac{\partial e^r}{\partial A_k}\frac{A_k}{e^r} = \sigma_{A_k,\omega}$.

Then, for all skill levels that earn a wage premium, utility increases unambiguously in A_k,

$$\frac{\partial V(e^r)}{\partial A_k} = \frac{\beta}{A_k}\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}\left[\frac{A_k^{\sigma_R-1}}{\hat{\theta}(A_k)} + \sigma_{A_k,\omega}\right] > 0 \, ,$$

where we used (8.17). So we can infer that high-skilled labor always benefits from advances in A_k, whereas the benefits of low-skilled labor depend on the degree of non-homotheticity of utility and even more so on the elasticity of substitution between the industries. The change in the marginal utility of a low-skilled household depends on the aggregate expenditure shares, that determine the wage premium, and its own expenditure shares, that determine how much expenditure is allocated to each industry.

8.2.4. Robotic Economy with Non-homothetic Preferences

We now analyze the model presented in Section 7.4, augmented with the non-homothetic preference relation introduced at the beginning of this chapter. Again we omit the steps leading to the industry price indices,

$$\begin{aligned} P_{i_R} &= \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i_R \log(\bar{r})\right]^{\frac{1}{\lambda}} n_R^{\frac{1}{1-\sigma_v}} \tilde{\theta}(\omega, A_k)^{\frac{1}{1-\sigma_R}} \, , \\ P_{i_N} &= \frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda i_N \log(\bar{r})\right]^{\frac{1}{\lambda}} n_N^{\frac{1}{1-\sigma_v}} \omega \, , \end{aligned}$$

where $\tilde{\theta}(\omega, A_k) = 1 + \left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}$. Applying Roy's Identity to the Indirect Utility Function (8.3) and aggregating across households yields aggregate demand for routine services

$$\begin{aligned} C_{i_R} =& \beta P_{i_N}^{\epsilon-\gamma} P_{i_R}^{\gamma-1} E e_{RA}^{-\epsilon} \\ =& \beta\left[\frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda \log(\bar{r})\right]^{\frac{1}{\lambda}}\right]^{\epsilon-1}\left[n_{i_R}{}^{\frac{1}{1-\sigma_v}} i_R^{\frac{1}{\lambda}}\right]^{-1} \nu(\omega)\tilde{\theta}(\omega, A_k)^{\frac{\gamma-1}{1-\sigma_R}} E e_{RA}^{-\epsilon} \, , \end{aligned}$$

and aggregate demand for non-routine services

$$\begin{aligned} C_{i_N} =& \frac{E}{P_{i_N}} - \beta P_{i_N}^{\epsilon-1-\gamma} P_{i_R}^{\gamma} E e_{RA}^{-\epsilon} \\ =& \frac{\sigma_v - 1}{\omega\sigma_v}\left[-e\lambda i_N \log(\bar{r})\right]^{-\frac{1}{\lambda}} n_{i_N}{}^{\frac{1}{\sigma_v-1}} E - \\ & \beta\left[\frac{\sigma_v}{\sigma_v - 1}\left[-e\lambda \log(\bar{r})\right]^{\frac{1}{\lambda}}\right]^{\epsilon-1}\left[n_{i_N}{}^{\frac{1}{1-\sigma_v}} i_N^{\frac{1}{\lambda}}\omega\right]^{-1} \nu(\omega)\tilde{\theta}(\omega, A_k)^{\frac{\gamma}{1-\sigma_R}} E e_{RA}^{-\epsilon} \, . \end{aligned}$$

Applying the firm's optimization solution shown in Chapter 6—and further augmented to include the choice of optimal capital (robots) input in the routine service industry in

Chapter 7—implies that $\omega \tilde{L}_{k,i_N} = n_{i_R}\tilde{l}_{i_R,j}\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}$, where we assume again that manufacturing is a competitive industry. Thus, total wages paid to non-routine labor producing robots must equal total revenues in this industry. We next derive demand for effective labor given the households' demand for routine services

$$n_{i_R}\tilde{l}_{i_R} = [-e\lambda i_R \log(\bar{r})]^{\frac{1}{\lambda}}\, n_{i_R}^{\frac{1}{1-\sigma_v}}\tilde{\theta}(\omega, A_k)^{\frac{\sigma_R}{1-\sigma_R}} C_{i_R}\ ,$$

and given the households' demand for non-routine services

$$\begin{aligned} n_{i_N}\tilde{l}_{i_N} + \tilde{L}_{k,i_N} =& [-e\lambda i_N \log(\bar{r})]^{\frac{1}{\lambda}}\, n_{i_N}^{\frac{1}{1-\sigma_v}} C_{i_N} + \\ & [-e\lambda i_R \log(\bar{r})]^{\frac{1}{\lambda}}\, n_{i_R}^{\frac{1}{1-\sigma_v}}\tilde{\theta}(\omega, A_k)^{\frac{\sigma_R}{1-\sigma_R}}\omega^{-1}\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1} C_{i_R}\ , \end{aligned}$$

where goods market clearing implies that C_{i_R} equals total production of services in industry i_R and C_{i_N} equals total production of services in industry i_N. Using the households' demand, we obtain

$$\begin{aligned} n_{i_R}\tilde{l}_{i_R} =& [-e\lambda \log(\bar{r})]^{\frac{\epsilon}{\lambda}}\,\beta\left[\frac{\sigma_v}{\sigma_v-1}\right]^{\epsilon-1}\nu(\omega)\tilde{\theta}(\omega, A_k)^{\frac{\gamma+\sigma_R-1}{1-\sigma_R}} E e_{RA}^{-\epsilon}\ , \\ n_{i_N}\tilde{l}_{i_N} + \tilde{L}_{k,i_N} =& \frac{\sigma_v-1}{\omega\sigma_v}E - \\ & [-e\lambda \log(\bar{r})]^{\frac{\epsilon}{\lambda}}\,\frac{\beta}{\omega}\left[\frac{\sigma_v}{\sigma_v-1}\right]^{\epsilon-1}\nu(\omega)\tilde{\theta}(\omega, A_k)^{\frac{\gamma+\sigma_R-1}{1-\sigma_R}} E e_{RA}^{-\epsilon}\ . \end{aligned}$$

The ILM Equilibrium again arises when there are sufficient skills for the labor markets to remain integrated and $\omega = 1$. In such an environment, total expenditure is $E = \frac{\sigma_v}{\sigma_v-1}\tilde{L}$ and $n_{i_R}\tilde{l}_{i_R} + n_{i_N}\tilde{l}_{i_N} + \tilde{L}_{k,i_N} = \tilde{L}$.

If there are *no* sufficient skills when $\omega = 1$ and thus labor markets are disintegrated—DLM Equilibrium—, then $\omega > 1$ and the total demand for effective labor able to master the non-routine task-complexity i_N equals the total supply for such labor, $n_{i_N}\tilde{l}_{i_N} + \tilde{L}_{k,i_N} = \tilde{L}_N$. Labor market clearing implies

$$\begin{aligned} \tilde{\phi}_R =& [-e\lambda \log(\bar{r})]^{\frac{\epsilon}{\lambda}}\,\beta\left[\frac{\sigma_v}{\sigma_v-1}\right]^{\epsilon-1}\nu(\omega)\tilde{\theta}(\omega, A_k)^{\frac{\gamma+\sigma_R-1}{1-\sigma_R}}\frac{E}{L}e_{RA}^{-\epsilon}\ , \\ \omega\tilde{\phi}_N =& \frac{\sigma_v-1}{\sigma_v}\frac{E}{L} - [-e\lambda \log(\bar{r})]^{\frac{\epsilon}{\lambda}}\,\beta\left[\frac{\sigma_v}{\sigma_v-1}\right]^{\epsilon-1}\nu(\omega)\tilde{\theta}(\omega, A_k)^{\frac{\gamma+\sigma_R-1}{1-\sigma_R}}\frac{E}{L}e_{RA}^{-\epsilon}\ . \end{aligned} \tag{8.20}$$

We use again $\frac{E}{L}e_{RA}^{-\epsilon} = \left[\frac{\sigma_v}{\sigma_v-1}\right]^{1-\epsilon}\left[\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N\right]$, derived from (8.9). Using (8.20), we define the following function $\check{\mathcal{G}}$:

$$\check{\mathcal{G}} := [-e\lambda \log(\bar{r})]^{-\frac{\epsilon}{\lambda}} \frac{\tilde{\phi}_R}{\beta} \nu(\omega)^{-1} \tilde{\theta}(\omega, A_k)^{\frac{\gamma+\sigma_R-1}{\sigma_R-1}} - \tilde{\psi}_R - \omega^{1-\epsilon}\tilde{\psi}_N = 0 \, . \tag{8.21}$$

In line with the arguments developed so far, we assume that firms are sufficiently willing to substitute labor for robots and that $\gamma + \sigma_R - 1 > 0$ always holds.[10] Taking derivatives with respect to A_k and ω, we obtain

$$\frac{\partial \check{\mathcal{G}}}{\partial A_k} = [\gamma + \sigma_R - 1] \, [-e\lambda \log(\bar{r})]^{-\frac{\epsilon}{\lambda}} \frac{\tilde{\phi}_R}{\beta} \nu(\omega)^{-1} \tilde{\theta}(\omega, A_k)^{\frac{\gamma}{\sigma_R-1}} \left[\frac{A_k}{\omega} \left[\frac{i_R}{i_N} \right]^{\frac{1}{\lambda}} \right]^{\sigma_R-1} \frac{1}{A_k} > 0 \, ,$$

$$\frac{\partial \check{\mathcal{G}}}{\partial \omega} = [-e\lambda \log(\bar{r})]^{-\frac{\epsilon}{\lambda}} \frac{\tilde{\phi}_R}{\beta} \nu(\omega)^{-1} \tilde{\theta}(\omega, A_k)^{\frac{\gamma+\sigma_R-1}{\sigma_R-1}} \omega^{-1}$$
$$[\gamma - \epsilon - [\gamma + \sigma_R - 1] \, \mathcal{C}] - [1 - \epsilon] \, \omega^{-\epsilon} \tilde{\psi}_N < 0 \, ,$$

where $\mathcal{C} = \left[\frac{A_k}{\omega} \left[\frac{i_R}{i_N} \right]^{\frac{1}{\lambda}} \right]^{\sigma_R-1} \tilde{\theta}(\omega, A_k)^{-1}$ denotes the share of total costs in the routine industry allocated to the capital input.[11] Using the implicit function theorem, we can derive the following proposition:

Proposition 8.3
The elasticity of substitution of the wage premium with respect to A_k, is

$$\sigma_{A_k,\omega} = \frac{\partial \omega}{\partial A_k} \frac{A_k}{\omega} = \frac{1}{1 - \frac{\gamma - \epsilon - [1-\epsilon] \frac{\omega^{1-\epsilon}\tilde{\psi}_N}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N}}{[\gamma + \sigma_R - 1]\mathcal{C}}} > 0 \, . \tag{8.23}$$

The elasticity is greater than 0, as by Assumption PCA $\gamma - \epsilon - [1-\epsilon] \frac{\omega^{1-\epsilon}\tilde{\psi}_N}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N} < 0$ always holds (compare Equation (D.3)) and as $\sigma_R - 1 + \gamma$ is greater than 0 by the assumption

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[10] This parameter space follows from our assumption that σ_R ($\sigma_R > \max\{\sigma_I, 1\}$) is greater than the elasticity of substitution among industries we introduced in Chapters 6 and 7. Then, by assumption,

$$\sigma_R - \sigma_{N,R} = \sigma_R - 1 + \gamma + [\gamma - \epsilon] \frac{s_{i_R}}{s_{i_N}} > 0,$$

which implies $\sigma_R - 1 + \frac{\gamma}{s_{i_N}} > 0$ (see Appendix D.3.3 for a derivation of $\sigma_{N,R}$). Now if (i) $\gamma > 0$, and we know that $\sigma_R > 1$, then $\gamma + \sigma_R - 1 > 0$, too. And if (ii) $\gamma < 0$, then $\sigma_R - 1 + \gamma > \sigma_R - 1 + \frac{\gamma}{s_{i_N}} > 0$. Thus, the parameter space $\sigma_R - 1 + \gamma > 0$ is directly implied by the assumption that $\sigma_R - \sigma_{N,R} > 0$.

[11] We use (8.21) to substitute for the first term in $\frac{\partial \check{\mathcal{G}}}{\partial \omega}$. Then, ignoring the negative term $-[\gamma + \sigma_R - 1] \, \mathcal{C}$, and rearranging yields

$$\left[\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N \right] [\gamma - \epsilon] - [1 - \epsilon] \, \omega^{1-\epsilon}\tilde{\psi}_N < 0 \, . \tag{8.22}$$

This implies that the derivative $\frac{\partial \check{\mathcal{G}}}{\partial \omega}$ is negative. Using (D.3), we can show that (8.22) must be negative under Assumption PCA.

that $\sigma_R - \sigma_{N,R} > 0$.
we next analyze the utility of the households, using the price indices,

$$V(P_{i_R}, P_{i_N}, e^r) = \frac{1}{\epsilon}\left[\frac{e^r}{\frac{\sigma_v}{\sigma_v - 1}\omega\left[-e\lambda i_N \log(\bar{r})\right]^{\frac{1}{\lambda}} n_{i_N}^{\frac{1}{1-\sigma_v}}}\right]^{\epsilon} - \frac{\beta}{\gamma}\left[\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\left[\frac{n_R}{n_N}\right]^{\frac{1}{1-\sigma_v}}\frac{\tilde{\theta}(\omega, A_k)^{\frac{1}{1-\sigma_R}}}{\omega}\right]^{\gamma}.$$

Taking the derivative of the indirect utility function with respect to the technological factor, A_k, yields

$$\frac{\partial V(e^r)}{\partial A_k} = \frac{1}{A_k}\left[\frac{e^r}{P_{i_N}}\right]^{\epsilon}\left[\frac{\partial e^r}{\partial A_k}\frac{A_k}{e^r} - \sigma_{A_k,\omega}\right] + \frac{\beta}{A_k}\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}\frac{1}{\tilde{\theta}(\omega, A_k)}\left[\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1} + \sigma_{A_k,\omega}\right] \lesseqgtr 0\,.$$

We use the assumed profit distribution described in (8.11). Using (8.12), we derive that for $r > \tilde{r}(i_N)$, utility always increases in A_k,

$$\frac{\partial V(e^r)}{\partial A_k} = \frac{\beta}{A_k}\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}\frac{1}{\tilde{\theta}(\omega, A_k)}\left[\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1} + \sigma_{A_k,\omega}\right] > 0\,,$$

and that for $r < \tilde{r}(i_N)$, the derivative is

$$\frac{\partial V(e^r)}{\partial A_k} = -\frac{1}{A_k}\left[\frac{e^r}{P_{i_N}}\right]^{\epsilon}\sigma_{A_k,\omega} + \frac{\beta}{A_k}\left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}\frac{1}{\tilde{\theta}(\omega, A_k)}\left[\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1} + \sigma_{A_k,\omega}\right] \lesseqgtr 0\,. \tag{8.24}$$

This directly implies the following proposition.

Proposition 8.4
The higher e^r within the range of low-skilled workers ($r < \tilde{r}(i_N)$), the lower the utility increase due to A_k.

The statement in Proposition 8.4 can be observed directly in Derivation (8.24), as e^r is the only household-specific variable. Utility might even decrease when capital becomes more productive. The higher e^r, the higher the share of expenditures allocated to the non-routine service industry. Thus, for this household, the marginal benefits from lower prices in industry i_R, P_{i_R}, are weighted by less and the marginal costs of higher prices in industry

i_N, P_{i_N}, are weighted by more, compared to a household with lower expenditures.
For households of lowest skill, the inequality in Assumption PCA is tightest. The expenditure share of these households allocated to the routine service industry is the largest. Thus, within the routine labor force, they benefit the most from efficiency increases in the production of robots which then increase the efficiency of services from industry i_R. In contrast, households with skill levels $r = \tilde{r}(i_N) - \varepsilon$, where ε is small, do not obtain the wage premium for their labor and have relatively high expenditures for the non-routine services, which become more expensive through increases in the wage premium. Thereby, their marginal utility increases less than the marginal utility for lower-skilled households, because their consumption shares differ.

8.2.5. Comparison

We can now compare our analysis of the non-homothetic preference relation to our analysis of Chapter 7. By setting $\epsilon = 0$, we show that our principal results of Chapter 7 are special cases of our analysis in Sections 8.2.3 and 8.2.4.

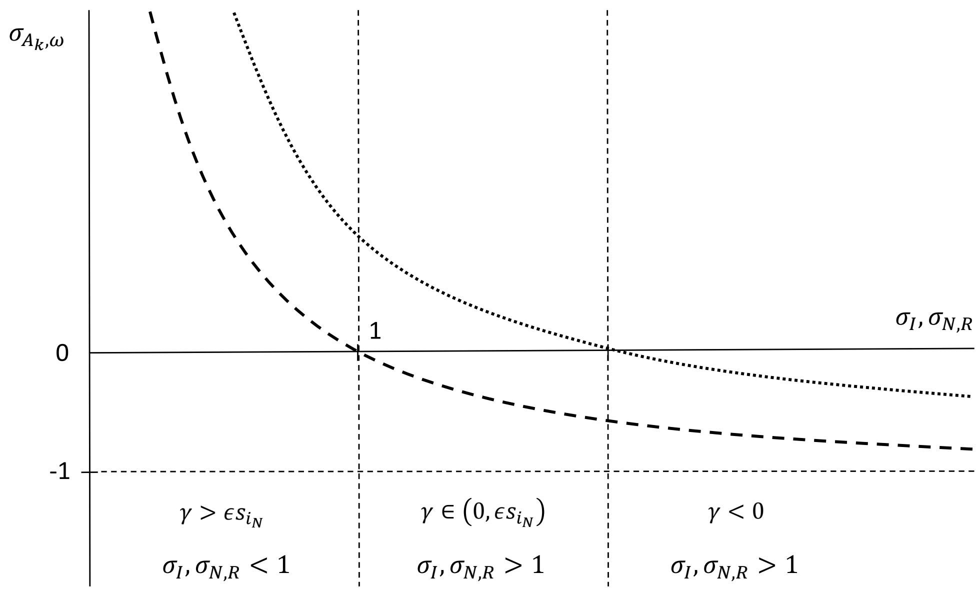
Industrial Economy. In an industrial economy, $\frac{\sigma_v}{\sigma_v - 1}\tilde{\phi}_R = s_{i_R}E$ and $\frac{\sigma_v}{\sigma_v - 1}\omega\tilde{\phi}_N = s_{i_N}E$, i.e. the expenditure shares are equal to the shares of earnings of the respective labor group.[12] We equate the elasticity of substitution between the two industries in the two models, setting $\sigma_I = \sigma_{N,R}$, where $\sigma_{N,R} = 1 - \frac{\gamma}{s_{i_N}}$—see (8.5) and (8.6)—, because of the assumed homotheticity, $\epsilon = 0$. Importantly, under homotheticity it holds that $\tilde{\psi}_N = \tilde{\phi}_N$ and $\tilde{\psi}_R = \tilde{\phi}_R$.
Using (7.9) from Proposition 7.1, we derive

$$\begin{aligned}\sigma_{A_k,\omega} &= \frac{1-\sigma_I}{\sigma_I}\frac{A_k^{\sigma_R - 1}}{\theta(A_k)}\\ &= \frac{\gamma}{s_{i_N} - \gamma}\frac{A_k^{\sigma_R - 1}}{\theta(A_k)}\\ &= -\frac{\gamma\frac{A_k^{\sigma_R-1}}{1+A_k^{\sigma_R-1}}}{\gamma - \frac{\omega\tilde{\psi}_N}{\tilde{\psi}_R + \omega\tilde{\psi}_N}},\end{aligned}$$

where the last equality equals (8.17) in Proposition 8.1 evaluated at $\epsilon = 0$.
We know from Proposition 8.2 that with a non-homothetic preference relation $\sigma_{A_k,\omega}$ can be positive when the elasticity of substitution between the industries is greater than 1. Figure 8.1 demonstrates $\sigma_{A_k,\omega}$ in dependence of the aggregate elasticity of substitution when utility is homothetic—the dashed curve (Section 7.3 or 8.2.3 with $\epsilon = 0$)—, and

[12] Of course, this critically depends on our assumption on profit distribution in our economy.

Figure 8.1.: Industrial Economy: $\sigma_{A_k,\omega}$ in Dependence on $\sigma_I, \sigma_{N,R}$



when utility reflects non-homotheticity—the dotted curve (Section 8.2.3 and $\epsilon > 0$). The elasticity of substitution between industries, σ_I and $\sigma_{N,R}$, and the elasticity of the wage premium with respect to A_k, $\sigma_{A_k,\omega}$, can only both be greater than one if the preference relation is non-homothetic ($\epsilon > 0$).

Proposition 8.5

Given an aggregate elasticity of substitution between industries in an industrial economy, the elasticity of the wage premium with respect to A_k, $\sigma_{A_k,\omega}$, is the greater the more non-homothetic the preference relation is.

The proof is given in Appendix D.3.4.

Robotic Economy. In contrast to the industrial economy, total wages and profits of routine labor equals total expenditures allocated to industry i_R *minus* the costs of the robots,

$$\frac{\sigma_v}{\sigma_v - 1}\tilde{\phi}_R L = E s_{i_R} [1 - \mathcal{C}] = E s_{i_R} \tilde{\theta}(\omega, A_k)^{-1}, \tag{8.25}$$

and total wages and profits of non-routine labor equals total expenditures allocated to industry i_N *plus* the cost of producing the robots,

$$\frac{\sigma_v}{\sigma_v - 1}\omega\tilde{\phi}_N L = E\left[s_{i_N} + s_{i_R}\mathcal{C}\right] \ . \tag{8.26}$$

Equivalences (8.25) and (8.26) directly imply that

$$\frac{\omega\tilde{\phi}_N}{\tilde{\phi}_R + \omega\tilde{\phi}_N} = s_{i_N} + s_{i_R}\mathcal{C} \ .$$

We can now relate $\sigma_{A_k,\omega}$ of the robotic economies of Section 7.4 and Section 8.2.4 and in particular Proposition 7.4 and Proposition 8.3. Using (8.25) and (8.26), we derive that

$$z = \frac{\tilde{\phi}_R}{\omega\tilde{\phi}_N}\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1} = \frac{s_{i_R}\mathcal{C}}{s_{i_N} + s_{i_R}\mathcal{C}} \ .$$

Next, we assume homotheticity in the preference relation, i.e. $\epsilon = 0$, and therefore $\sigma_{N,R} = 1 - \frac{\gamma}{s_{i_N}}$—see (8.5) and (8.6). Using $\sigma_{A_k,\omega}$ of (7.18) and replacing z and setting $\sigma_I = \sigma_{N,R} = 1 - \frac{\gamma}{s_{i_N}}$ yields

$$\begin{aligned}
\sigma_{A_k,\omega} =& \frac{1}{1 + \frac{z + [1-z]\sigma_I}{z[\sigma_R - 1] + [1-z][\sigma_R - \sigma_I]\mathcal{C}}} \\
=& \frac{1}{1 + \frac{\frac{s_{i_R}\mathcal{C}}{s_{i_N}+s_{i_R}\mathcal{C}} + \frac{s_{i_N}}{s_{i_N}+s_{i_R}\mathcal{C}}\left[1 - \frac{\gamma}{s_{i_N}}\right]}{\frac{s_{i_R}\mathcal{C}}{s_{i_N}+s_{i_R}\mathcal{C}}[\sigma_R - 1] + \frac{s_{i_N}}{s_{i_N}+s_{i_R}\mathcal{C}}[\sigma_R - 1 + \frac{\gamma}{s_{i_N}}]\mathcal{C}}} \\
=& \frac{1}{1 - \frac{\gamma - s_{i_N} - s_{i_R}\mathcal{C}}{[\sigma_R - 1 + \gamma]\mathcal{C}}} \ .
\end{aligned}$$

The last equality expresses (8.23) in Proposition 8.3 if $\epsilon = 0$.[13] Our analysis from Section 8.2.4 using the non-homothetic PIGL preference relation thus encompasses the case of CES-utility studied in Section 7.4.

Proposition 8.6
Given an aggregate elasticity of substitution between industries in a robotic economy, $\sigma_{A_k,\omega}$ is the greater the more non-homothetic the preference relation is.

The proof is given in Appendix D.3.5. Note that this is the same result as for the industrial economy.

[13] Under homotheticity it holds that $\tilde{\psi}_N = \tilde{\phi}_N$ and $\tilde{\psi}_R = \tilde{\phi}_R$.

8.2.6. Conclusion

We showed that non-homotheticity in preferences amplifies the trend towards inequality that uneven technological progress in the manufacturing industry has on the wage scheme, when the manufacturing industry produces capital which can be used as a substitute in the production processes reliant on routine task-complexities. This is the case in both the industrial and the robotic economy.

Our model allowed us to analyze the medium-term. Thus, we focused merely on the directions of the trends. It would now be fruitful to analyze long-term implications to determine the magnitude of the effects, and in particular the fraction of the effects that is caused by non-homotheticity. Also technological progress and capital accumulation should be endogenized. In such a long-term model, expenditure levels grow and the shifts in expenditure shares should be more pronounced, while the PCA would be a lesser concern.[14] Thus, our model opens issues for future research.

[14] Technological change resulting in price decreases in the non-routine industry would also imply less pressure on the PCA.

9. Conclusion

In this second part of the thesis, we focused on labor market dynamics in the context of uneven technological change. In Chapter 6, we developed a new task-based model—the task-complexity model—to study the role of complexity in the production process for labor market outcomes. We provided a new perspective on how the task of a production process and the skills of workers are assigned. In particular, we assumed that the complexity of a production process requires a certain skill level for successful production. This micro-level skill requirement can cause macro-level dynamics. It may divide the labor market, so that the higher-skilled workers obtain a wage premium.

In Chapter 7, we studied uneven technological change, focusing on automation—the substitution of capital for routine labor. We provided two frameworks. In the first, *routine* labor produces machines in the manufacturing industry, which can replace routine labor in the routine industry. In the second, *non-routine* labor produce robots in the manufacturing industry, which can replace routine labor in the routine industry. Our analyses showed that the robotic economy displays a strong tendency towards labor market separation and thus a strong upward pressure on the wage premium.

In Chapter 8 we presented two extensions. In a first extension we analyzed the model of Chapter 6, extended to three task-complexities (Section 8.1). We introduced a potential path for models with many task-complexities and (potentially) many steps in the wage premium function. In a second extension, we extended the analysis of Chapter 7, by adding non-homotheticity to the preference relation (Section 8.2). We showed that the tendency for labor market separation and the upward pressure on the wage premium are strictly more pronounced when preferences are non-homothetic and when there is technological progress in manufacturing.

The medium-term, an important, yet often neglected time span, comprises variables on the household level that are fixed—such as a household's skill level—and variables at the macro-level that are dynamic. Thus, the households have limited ability to cope with macroeconomic dynamics. In such an environment, uneven technological progress can exert stark disruptive forces.

Our analysis offers new concepts and a new framework that can set the structural basis for further research on the effects of technological change on wages. Additional research should be conducted to understand the effects of complementarity in production between skills and substitution technologies, which can, as our analysis has shown, further accentuate the separation dynamics of technological progress. The skill-task-assignment and the evolution of complexity in production should now be tested empirically. It would be fruitful to augment our model with additional and endogenous forces of technological change and innovation, such as the task life-cycle or total factor productivity, and with long-term dynamics.

Part III.

Appendix

A. Comments for Part I

A.1. Detailed Derivations

A.1.1. Production Function for Ideas

In this appendix, we provide details on the production function for ideas (2.7) and (2.8). With an ability distribution $F_a(a)$, the government employs the workers with highest ability as scientists, i.e. all workers with ability $a \geq \tilde{a}^r$ for some cutoff $\tilde{a}^r$

$$\eta^r := \eta_1(r) L^r \int_{\tilde{a}^r}^{\infty} a f_a(a) da \ .$$

The costs of employing the scientists are

$$BR^r = w^r L^r \int_{\tilde{a}^r}^{\infty} f_a(a) da \ ,$$

and therefore $\tilde{a}^r := F_a^{-1}\left(1 - \frac{BR^r}{L^r w^r}\right)$. Using this expression yields

$$\eta^r = \eta_1(r) L^r \eta_2\left(\frac{BR^r}{L^r w^r}\right) = \eta_1(r) L^r \int_{F_a^{-1}\left(1-\frac{BR^r}{L^r w^r}\right)}^{\infty} a f_a(a) da \ ,$$

and finally using integration by substitution results in the expression shown in the main text

$$\eta_2\left(\frac{BR^r}{L^r w^r}\right) = \int_0^{\frac{BR^r}{w^r L^r}} F_a^{-1}(1-x)\, dx \ .$$

Note that $\eta_2(0) = 0$. Moreover, using Leibniz's Rule we obtain

$$\eta_2'\left(\frac{BR^r}{L^r w^r}\right) = F_a^{-1}\left(1 - \frac{BR^r}{L^r w^r}\right) > 0$$

and

$$\eta_2''\left(\frac{BR^r}{L^r w^r}\right) = -\frac{1}{f_a\left(1 - \frac{BR^r}{L^r w^r}\right)} < 0 \ ,$$

i.e. $\eta_2(\,\cdot\,)$ is indeed strictly increasing and concave.

A.1.2. Details on the Optimal Targeting Problem of the Social Planner

In this appendix, we provide details on the optimal targeting problem of the social planner. We begin by showing that this problem can indeed be reduced to the problem analyzed in Section 2.6.1.

Optimal Targeting Problem

Using

$$\eta_i^r\left(\xi^r, i_{BR}^r\right) = \begin{cases} \eta^r\left(\xi^r\right) \frac{1-\kappa+\kappa I}{I} & \text{if } i = i_{BR}^r \\ \eta^r\left(\xi^r\right) \frac{1-\kappa}{I} & \text{otherwise,} \end{cases}$$

we obtain the number of varieties that fall in industry i

$$\begin{aligned} N_i &= \int_{\underline{r}}^{\overline{r}} \eta^r\left(\xi^r\right) \left[\frac{1-\kappa}{I} + \mathbb{1}\left[i = i_{BR}^r\right]\kappa\right] f_r(r) dr \\ &= \left[\frac{1-\kappa}{I} + \kappa s_i\right] \int_{\underline{r}}^{\overline{r}} \eta^r\left(\xi^r\right) f_r(r) dr \ , \end{aligned}$$

where

$$s_i := \frac{\int_{\underline{r}}^{\overline{r}} \eta^r\left(\xi^r\right) \mathbb{1}\left[i = i_{BR}^r\right] f_r(r) dr}{\int_{\underline{r}}^{\overline{r}} \eta^r\left(\xi^r\right) f_r(r) dr}$$

denotes the share of ideas targeted towards industry i.[1] We can thus rewrite the set of constraints (2.6) as

$$\begin{aligned} n_i &= \frac{1-\kappa}{I} + \kappa s_i \ , \quad \forall\, i \in \mathcal{I} \ , \\ N &= \int_{\underline{r}}^{\overline{r}} \eta^r\left(\xi^r\right) f_r(r) dr \ . \end{aligned}$$

We note that $\sum_{i\in\mathcal{I}} n_i = 1$ since $n_i := \frac{N_i}{N}\ \forall i \in \mathcal{I}$. Further, using the definition of n_i, we can rewrite the objective as

$$C = \left[-e\lambda \ln(\bar{r})\right]^{-\frac{1}{\lambda}} N^{\frac{1}{\sigma_v - 1}} \left[\tilde{L} - \tilde{L}_{BR}\right] \left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I - 1}} \ ,$$

[1] We assume that targeting is made such that $\eta^r\left(\xi^r\right) \mathbb{1}\left[i = i_{BR}^r\right]$ is integrable.

implying first that targeting can be reduced to the choice of $\{s_i\}_{i\in\mathcal{I}}$, and second that it will enter the social planner problem only via its impact on the term

$$\left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I-1}} . \tag{A.1}$$

The objective is always positive, and the social planner will thus target basic research to maximize (A.1), irrespective of N and $\tilde{L}_{BR}$, i.e. targeting can be separated from the choices of $\{\xi^r\}_{r\in\mathcal{R}}$, as argued in the main body of the text. Taking into account that targeting must be non-negative, the problem reduces to the following

$$\begin{aligned} \max_{\{n_i\}_{i\in\mathcal{I}}} \quad & \left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I-1}} , \\ \text{s.t.} \quad & n_i \geq \frac{1-\kappa}{I} , \quad \forall\, i \in \mathcal{I} , \\ & \sum_{i\in\mathcal{I}} n_i = 1 . \end{aligned}$$

This is the optimal targeting problem studied in Section 2.6.1.

Details on the Optimal Distribution of Varieties Across Industries

The marginal return to n_i is

$$\frac{1-\sigma_I}{1-\sigma_v} \psi_i i^{\frac{1-\sigma_I}{\lambda}} n_i^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}} .$$

First note that this marginal return depends only on n_i, and in particular does not depend on the distribution of varieties across all other industries. Note further that in the absence of targeting, the marginal return to n_i is strictly increasing in the industry's attractiveness. Finally, note that n_i is the same across all industries that do not receive any targeting, and that targeting one industry $\hat{i}$ will increase $n_{\hat{i}}$ at the expense of decreasing n_i in all other industries, where this decrease is the same for all industries.

Now, let industries be ranked in descending order of attractiveness, $\{i_1, i_2, ..., i_I\}$, such that $\forall\, m, n \in \{1, 2, ..., I\}, m < n$, it holds $\psi_{i_m} i_m^{\frac{1-\sigma_I}{\lambda}} \geq \psi_{i_n} i_n^{\frac{1-\sigma_I}{\lambda}}$. The social planner will target the industries that yield the highest returns. Starting from a situation without targeting, he will thus start targeting the most attractive industry first. This will increase n_{i_1} and decrease n_i in all other industries. Targeting more and more basic research to i_1, he will eventually reach a point where

$$\psi_{i_1} i_1^{\frac{1-\sigma_I}{\lambda}} n_{i_1}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}} = \psi_{i_2} i_2^{\frac{1-\sigma_I}{\lambda}} n_{i_2}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}} ,$$

at which point he will start to jointly target these industries. He will continue in the same manner until he has targeted all of his basic research investments. This will result in a situation where all industries up to some rank t receive positive targeting. For each of these industries, marginal return to n_i will be equal to the marginal return to increasing n_{i_1}. All other industries will receive zero targeting. This will give rise to the following distribution of n_i across industries:

$$n_{i_m} = \begin{cases} \left[\frac{\psi_{i_m} i_m^{\frac{1-\sigma_I}{\lambda}}}{\psi_{i_1} i_1^{\frac{1-\sigma_I}{\lambda}}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}} n_{i_1}(t) & \text{if } m \le t \\ \frac{1-\kappa}{I} & \text{otherwise,} \end{cases}$$

where m denotes the ranking of the industry according to their attractiveness. Using these industry shares in the constraint that

$$\sum_{i \in \mathcal{I}} n_i = 1$$

and solving for $n_{i_1}(t)$ yields:

$$n_{i_1}(t) = \frac{1 - \sum_{m>t} \frac{1-\kappa}{I}}{\sum_{m \le t} \left[\frac{\psi_{i_m} i_m^{\frac{1-\sigma_I}{\lambda}}}{\psi_{i_1} i_1^{\frac{1-\sigma_I}{\lambda}}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}}} = \frac{1 - (I - t) \frac{1-\kappa}{I}}{\sum_{m \le t} \left[\frac{\psi_{i_m} i_m^{\frac{1-\sigma_I}{\lambda}}}{\psi_{i_1} i_1^{\frac{1-\sigma_I}{\lambda}}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}}},$$

and t can be solved as the highest rank of attractiveness for which non-negative targeting is unconstrained optimal if only industries along the ranking up to and including this industry are targeted:

$$i_t := \max \left\{ i_m \in \mathcal{I} : \frac{1-\kappa}{I} \le \left[\frac{\psi_{i_m} i_m^{\frac{1-\sigma_I}{\lambda}}}{\psi_{i_1} i_1^{\frac{1-\sigma_I}{\lambda}}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}} n_{i_1}(m) \right\}.$$

A.2. Further Considerations on the Sufficient Skill Condition and Uniqueness of Targeting

In the main body of our paper, we focused on economies with *sufficient skills* in equilibrium. In such an equilibrium, it will be the case that there is systematically more (effective) labor available in countries with high productive knowledge than needed in production of the complex goods, implying that some of this labor will need to be employed in the less complex industries, where labor in less developed countries can also operate at preferred quality. This puts downward pressure on wages for labor in industrialized countries, and implies that in equilibrium, every country will be competitive in all industries, up to some threshold complexity level denoted by $\tilde{i}(r)$, which is strictly increasing in r.

While this equilibrium exhibits the empirically attractive features that more developed countries are more diversified in international trade and that countries' exports tend to be nested,[2] it is not obvious if and when the underlying *Sufficient Skills Condition* will be satisfied in equilibrium with decentralized basic research policy decisions or in the optimal solution of the social planner as there are non-trivial interactions with basic research investment policies. In particular, targeting of basic research will impact the cross-industry distribution of labor demand while allocation of basic research to countries will impact the skill distribution of labor available for production. We recall the definition of SSC:

$$\int_{\tilde{r}(\hat{i})}^{\overline{r}} [L^r - L^r_{BR}] \left[\frac{\ln(\overline{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} dF_r(r) \geq \sum_{i \in \mathcal{I}: i \geq \hat{i}} N_i \tilde{l}_i \,, \quad \forall \hat{i} \in \mathcal{I}\,. \tag{SSC}$$

There are sets of parameter specifications for which Condition SSC will be satisfied both in the decentralized equilibrium and in the social planner solution. In this appendix, we discuss one such set of parameter specifications for which Condition SSC holds. The basic argument is summarized in Figure A.1. In this figure, the dashed line shows for every industries i the total global supply of effective labor for production that can operate in industry i at preferred quality, $\int_{r \geq \tilde{r}(i)}^{\overline{r}} \tilde{L}^r_p f_r(r) dr$. The solid line shows total demand for effective labor in industries with complexity i or higher, $\sum_{\hat{i} \in \mathcal{I}: \hat{i} \geq i} N_{\hat{i}} \tilde{l}_{\hat{i}}$. Condition SSC is satisfied if the dashed line is above the solid line everywhere. In what follows, for every i we will derive a lower bound on the share of aggregate effective labor in production that has skill level $\tilde{r}(i)$ or higher, and an upper bound on the share of aggregate effective labor in production that is employed in industries with complexity of at least i. We then derive conditions such that these bounds satisfy SSC as illustrated in Figure A.1 (red lines).

To derive such conditions, it will be convenient to consider the case of a Pareto distribution of basic research abilities

$$F_a(a) = 1 - \left(\frac{a}{\underline{a}}\right)^{-\tilde{\alpha}} ,$$

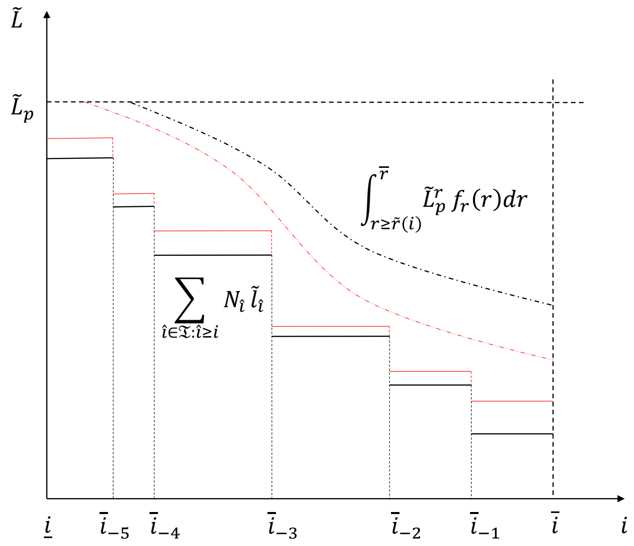
where $a \geq \underline{a} > 0$ and $\tilde{\alpha} > 1$ and which implies that

$$\eta_2(\xi) = \zeta \xi^{\alpha},$$

where $\zeta := \frac{\tilde{\alpha}}{\tilde{\alpha}-1}\underline{a}$ and $\alpha := \frac{\tilde{\alpha}-1}{\tilde{\alpha}}$. Further, to save notation we will assume that it is technically feasible to target basic research investments such that profits of the representative

[2] Cf. Hausmann and Hidalgo (2011), Bustos et al. (2012), and Schetter (2018).

Figure A.1.: Sufficient Criteria for the Sufficient Skill Condition (SSC)



firm in industry i (denoted by π_i) are equal across industries. The assumption of equal profits across industries is an assumption on parameters $\{\psi_i\}_{i\in\mathcal{I}}, \lambda, \kappa, \sigma_v, \sigma_I$ and restricts cross-industry differences in attractiveness when targeting is not perfect. We next introduce a formal assumption on the heterogeneity of attractiveness and the probability of success in targeting that allows equalization of profits.

Assumption A.1

$$\frac{\left[\psi_{\hat{\imath}}\hat{\imath}^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v-1}{\sigma_v-\sigma_I}}}{\sum_{i\in\mathcal{I}}\left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v-1}{\sigma_v-\sigma_I}}} \geq \frac{1-\kappa}{I}, \quad \forall \hat{\imath}\in\mathcal{I}.$$

Lemma A.1

Suppose that Assumption A.1 and SSC hold. Then, it is feasible to target basic research investments such that profits of the representative firms are equal across industries in a decentralized equilibrium.

The proof of Lemma A.1 is given in Appendix B.1.9. Intuitively, whether or not it is feasible to have equal profits in all industries will depend on the ex-ante attractiveness of industries and the probability of success in targeting basic research. With perfect targeting, $\kappa = 1$, this will be possible for arbitrary cross-industry differences in their attractiveness. On the contrary, for $\kappa = 0$, basic research cannot be targeted at all, and profits of the respective representative firm can only be equal across industries if there is no ex-ante heterogeneity in terms of attractiveness. Assumption A.1 restricts cross-industry differences in terms of ex-ante attractiveness accordingly. Note that neither Assumption A.1 nor Lemma A.1 is based on the case of a Pareto distribution of basic research abilities.

Intuitively, Lemma A.1 implies that as long as there are sufficient skills, targeting by the social planner and in a decentralized equilibrium will always be such, that profits are equal across industries. Any alternative targeting would imply that some industry with positive targeting will have lower profits than some other industry and, hence, both the social planner and a government in a country with $r \geq \tilde{r}(\bar{\imath})$ will benefit from retargeting their investments.

The fact that a government in country $r \geq \tilde{r}(\bar{\imath})$ would benefit from retargeting its investments follows immediately from the discussion of the optimal targeting in Section 2.5. The marginal benefit of a new variety in industry $\hat{\imath}$ for the social planner is

$$\begin{aligned}\frac{\partial C}{\partial N_{\hat{\imath}}} &= [-e\lambda \ln(\bar{r})]^{-\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} N_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I-1}} \tilde{L}_p \frac{1}{\sigma_v-1} \frac{\psi_{\hat{\imath}} \hat{\imath}^{\frac{1-\sigma_I}{\lambda}} N_{\hat{\imath}}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}}}{\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} N_i^{\frac{1-\sigma_I}{1-\sigma_v}}} \\ &= \frac{\sigma_v}{\sigma_v-1}\frac{\pi_{\hat{\imath}}}{P},\end{aligned}$$

where the first equality follows from differentiating C and rearranging terms, and the second equality follows form using the definitions of P and $\pi_{\hat{\imath}}$ which implies that indeed the social planner would also benefit from retargeting his investments.

Now, as demonstrated in the main body of this paper, the share of the population employed in basic research is weakly monotonously increasing in r in both the decentralized equilibrium and the social planner solution. Further, countries' basic research investments are strategic substitutes.[3] In the proof of Lemma A.2, we will make use of these observations to show that countries' basic research investments ξ_E^r and ξ_S^r are bounded from above.

Lemma A.2

With sufficient skills ξ_E^r and ξ_S^r are bounded from above by

$$\gamma := \frac{\eta_1(\bar{r})^{\tilde{\alpha}} \tilde{L}(\tilde{\alpha}-1)}{\tilde{\alpha}[\sigma_v-1] \int_{\tilde{r}(\bar{\imath})}^{\bar{r}} \eta_1(r)^{\tilde{\alpha}} \left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1-\tilde{\alpha}}{\lambda}} L^r f_r(r) dr}.$$

The proof of Lemma A.2 is given in Appendix B.1.10. Note that $\eta_1(\bar{r}) = 1$.

Lemma A.2 provides an upper bound on investments in basic research if there are suf-

[3] Observe from Proposition 2.1 that profits in industry i are decreasing in the number of varieties in any other industry $\hat{\imath}$ and in labor employed in basic research.

ficient skills. We use this bound in the proof of Proposition A.1 to derive a condition such that there are always sufficient skills in both the decentralkzed equilibrium and the solution of the global social planner.[4]

Assumption A.2

$$\frac{\int_{\tilde{r}(\hat{i})}^{\overline{r}}[1-\gamma]\,[-\ln(r)]^{-\frac{1}{\lambda}}\,L^r\,dF_r(r)}{\int_{\underline{r}}^{\tilde{r}(\hat{i})}[-\ln(r)]^{-\frac{1}{\lambda}}\,L^r\,dF_r(r)+\int_{\tilde{r}(\hat{i})}^{\overline{r}}[1-\gamma]\,[-\ln(r)]^{-\frac{1}{\lambda}}\,L^r\,dF_r(r)} \geq \frac{\sum_{i\in\mathcal{I}:i\geq\hat{i}}\left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_V-1}{\sigma_V-\sigma_I}}}{\sum_{i\in\mathcal{I}}\left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_V-1}{\sigma_V-\sigma_I}}}\,, \quad \forall\,\hat{i}\in\mathcal{I}\,.$$

Proposition A.1
Let abilities be Pareto distributed and Assumptions A.1 and A.2 be satisfied. Then Condition SSC is satisfied in both the equilibrium with decentralized investments in basic research and in the optimal solution of the global social planner.

The proof of Proposition A.1 is given in Appendix B.1.11. Note that the smaller γ is the less restrictive Assumption A.2. Considering that basic research activities account for a small fraction of the labor force and of the GDP therefore suggests that there will be sufficient skills for a broad set of parameter specifications indeed.[5]

A.3. The Global Pool of Ideas

We demonstrate that the global pool (GP) of ideas is mirrored by global profits. For this exercise we assume w.l.o.g. that $\kappa = 0$. This restriction enables us to show the equivalence in a simple way.[6] From (3.1) we know that country r produces an amount of $\eta^r(\xi^r)$ ideas in each industry i. Of this amount

(i) θ_D is absorbed by the country r itself in industries $i \leq \tilde{i}(r)$, and

(ii) $1-\theta_D$ enter the GP of ideas in the same industries.

[4] Note that in principle the social planner could still find it optimal to opt for basic research investments, such that Condition SSC is violated. This, however, will not be the case as our analysis implies that the social planner would not opt for the corresponding investment even if ignoring the additional inefficiencies arising from a violation of SSC. Hence, he will certainly not decide to do so when taking these inefficiencies into account.

[5] Cf. Footnote 41 in Chapter 2.

[6] If $\kappa > 0$ we allow for targeting. In equilibrium the set $\mathcal{I}^r_{BR}$ of targeted industries remains indeterminate. Therefore, to demonstrate the equivalence, we would have to allow for many cases, without obtaining further insights.

(iii) The remaining ideas enter the GP, i.e. $\eta^r \mathbb{1}_{[i>\tilde{i}(r)]}$.[7]

The GP of ideas then amounts to

$$GP = \int_{\underline{r}}^{\bar{r}} \eta^r(\xi^r)[1-\theta_D] \sum_{i \in \mathcal{I}} \mathbb{1}_{[i \leq \tilde{i}(r)]} f(r)dr + \int_{\underline{r}}^{\bar{r}} \eta^r(\xi^r) \sum_{i \in \mathcal{I}} \mathbb{1}_{[i>\tilde{i}(r)]} f(r)dr \ . \tag{A.2}$$

Countries can receive ideas from the GP that are compatible with their manufacturing base, i.e. country r' is able to receive ideas from the GP up to industry $\tilde{i}(r')$. Ideas in the GP that could potentially be realized in country r' are

$$\begin{aligned} GP(r') = &\int_{\underline{r}}^{\bar{r}} \eta^r(\xi^r)[1-\theta_D] \sum_{i \in \mathcal{I}(r')} \mathbb{1}_{[i \leq \tilde{i}(r)]} f(r)dr + \int_{\underline{r}}^{r'} \eta^r(\xi^r) \sum_{i \in \mathcal{I}(r')} \mathbb{1}_{[i>\tilde{i}(r)]} f(r)dr \\ = &\int_{\underline{r}}^{r'} \eta^r(\xi^r)[1-\theta_D]\mathbf{n}\left[\mathcal{I}(r)\right] f(r)dr + \int_{r'}^{\bar{r}} \eta^r(\xi^r)[1-\theta_D]\mathbf{n}\left[\mathcal{I}(r')\right] f(r)dr + \\ &\int_{\underline{r}}^{r'} \eta^r(\xi^r)\left[\mathbf{n}\left[\mathcal{I}(r')\right] - \mathbf{n}\left[\mathcal{I}(r)\right]\right] f(r)dr \end{aligned}$$

where $\mathbf{n}\left[\mathcal{I}(r)\right]$ denotes the cardinality of the set $\mathcal{I}(r)$. Note that the cardinality of the set comprising all industries is simply denoted by I. More conveniently, there is a measure of $GP(i')$ commercialize-able ideas in the GP for industry i'

$$\begin{aligned} GP(i') = &\int_{\tilde{r}(i')}^{\bar{r}} \eta^r(\xi^r)[1-\theta_D] f(r)dr + \int_{\underline{r}}^{\tilde{r}(i')} \eta^r(\xi^r) f(r)dr \\ = &\int_{\underline{r}}^{\bar{r}} \eta^r(\xi^r) f(r)dr - \theta_D \int_{\tilde{r}(i')}^{\bar{r}} \eta^r(\xi^r) f(r)dr \\ = &\hat{N} - \theta_D \hat{N}(\tilde{r}(i')). \end{aligned}$$

where $\hat{N}(\tilde{r}(i')) := \int_{\tilde{r}(i')}^{\bar{r}} \eta^r(\xi^r) f(r)dr$.[8] The probability of obtaining a certain idea from the GP is equal to the ratio of the effective labor force of a country to the remaining global economy's effective labor force able to potentially commercialize the idea. Following (2.17), the effective labor force dependent probability of country r to commercialize some idea (i', j^{GP}) is

$$\theta^r_{G,i} = \mathbb{1}_{[i' \leq \tilde{i}(r)]} \frac{\tilde{L}^r}{\int_{\tilde{r}(i')}^{\bar{r}} \tilde{L}^r f(r)dr} \ . \tag{A.3}$$

Note that $GP(\bar{i}) = \hat{N} - \theta_D \hat{N}(\tilde{r}(\bar{i})) < \hat{N}$ because $\tilde{r}(\bar{i}) < \bar{r}$ and therefore $\hat{N}(\tilde{r}(\bar{i})) > 0$. The amount of ideas in the GP the country with the highest skill level, $\bar{r}$, has access to

[7] We could also implement the waste of ideas by assuming that θ_W is the probability that an idea never actually gets commercialized. Then only $\eta^r[1-\theta_W]\theta_D$ are absorbed by the country, and only $\eta^r[1-\theta_W][1-\theta_D]$ on $i \leq \tilde{i}(r)$ and $[1-\theta_W]\eta^r \mathbb{1}_{[i \geq \tilde{i}(r)]}$ enter the GP. Up to the factor, θ_W, the analysis does not change.

[8] By definition then $\hat{N} := \int_{\underline{r}}^{\bar{r}} \eta^r(\xi^r) f(r)dr$

must be equal to the sum of the amounts of ideas over all industries. This equivalence is shown in the following:

$$\begin{aligned}
GP =&GP(\bar{r}) \\
=&\int_{\underline{r}}^{\bar{r}} \eta^r(\xi^r)[1-\theta_D]\mathbf{n}\left[\mathcal{I}(r)\right] f(r)dr + \int_{\underline{r}}^{\bar{r}} \eta^r(\xi^r)\left[\mathbf{n}\left[\mathcal{I}\right] - \mathbf{n}\left[\mathcal{I}(r)\right]\right] f(r)dr \\
=&\mathbf{n}\left[\mathcal{I}\right] \int_{\underline{r}}^{\bar{r}} \eta^r(\xi^r) f(r)dr - \theta_D \int_{\underline{r}}^{\bar{r}} \eta^r(\xi^r)\mathbf{n}\left[\mathcal{I}(r)\right] f(r)dr \\
=&\mathbf{n}\left[\mathcal{I}\right] \int_{\underline{r}}^{\bar{r}} \eta^r(\xi^r) f(r)dr - \theta_D \sum_{i\in\mathcal{I}} \int_{\tilde{r}(i)}^{\bar{r}} \eta^r(\xi^r) f(r)dr \\
=&\sum_{i\in\mathcal{I}} GP(i)\ .
\end{aligned}$$

Using industry dependent profits $\pi(i)$ we compute the profits country r receives from foreign basic research efforts. This simply amounts to the ideas country r receives from the GP of ideas summed over all industries compatible with the manufacturing base of country r multiplied by the respective profits,

$$\Pi_G^r = \sum_{i\in\mathcal{I}(r)} \frac{\tilde{L}^r}{\int_{\tilde{r}(i)}^{\bar{r}} \tilde{L}^r f(r)dr} GP(i)\pi(i)\ .$$

Total profits of country r are

$$\Pi^r = \Pi_G^r + \eta^r(\xi^r)\theta_D \sum_{i\in\mathcal{I}(r)} \pi(i)\ , \tag{A.4}$$

where the first term denotes the profit due to foreign investments in basic research and the second term denotes profits due to the own investments in basic research. We show that countries' profits sum up to the total profit in the economy

$$\begin{aligned}
\int_{\underline{r}}^{\bar{r}} \Pi^r f(r)dr =& \int_{\underline{r}}^{\bar{r}} \sum_{i\in\mathcal{I}(r)} \frac{\tilde{L}^r}{\int_{\tilde{r}(i)}^{\bar{r}} \tilde{L}^r f(r)dr} GP(i)\pi(i) f(r)dr + \int_{\underline{r}}^{\bar{r}} \eta^r(\xi^r)\theta_D \sum_{i\in\mathcal{I}(r)} \pi(i) f(r)dr \\
=& \sum_{i\in\mathcal{I}} GP(i)\pi(i) \int_{\tilde{r}(i)}^{\bar{r}} \frac{\tilde{L}^r}{\int_{\tilde{r}(i)}^{\bar{r}} \tilde{L}^r f(r)dr} f(r)dr + \sum_{i\in\mathcal{I}} \pi(i) \int_{\tilde{r}(i)}^{\bar{r}} \eta^r(\xi^r)\theta_D f(r)dr \\
=& \sum_{i\in\mathcal{I}} GP(i)\pi(i) + \sum_{i\in\mathcal{I}} \pi(i) \int_{\tilde{r}(i)}^{\bar{r}} \eta^r(\xi^r)\theta_D f(r)dr \\
=& \sum_{i\in\mathcal{I}} \left[\hat{N} - \theta_D \hat{N}(\tilde{r(i)})\right] \pi(i) + \sum_{i\in\mathcal{I}} \pi(i)\theta_D N(\tilde{r(i)}) \\
=& \hat{N} \sum_{i\in\mathcal{I}} \pi(i) \\
=& \frac{\tilde{L}_p}{\sigma_v - 1}\ .
\end{aligned}$$

From (A.4) we can already infer that countries with a high productive knowledge, r, have access to more complex industries in which they also have a higher probability to obtain an idea from the GP due to the decreasing amount of countries able to compete on high complexity levels. Therefore, high productive knowledge countries will commercialize more products in industries of high complexity relative to industries of low complexity. Furthermore, high productive knowledge allows countries to exploit ideas brought forth by the own basic research investment to a higher degree.

B. Proofs for Part I

B.1. Proofs of Chapter 2

B.1.1. Proof of Corollary 2.2

Note first that GDP in country r is labor income of production workers plus profits arising form these activities,

$$GDP^r = \frac{\sigma_v}{\sigma_v - 1} w^r L^r [1 - \xi_E^r] ,$$

while GNI is labor income of production workers plus profits appropriated by the domestic population, i.e. the total value of all domestic inventions which we denote by Π^r

$$GNI^r = w^r L^r [1 - \xi_E^r] + \Pi^r .$$[1]

Combining the previous two, we obtain

$$\frac{GNI - GDP}{GDP} = \frac{[\sigma_v - 1]\Pi^r}{\sigma_v w^r L^r [1 - \xi_E^r]} - \frac{1}{\sigma_v} ,$$

and we need to show that $\frac{\Pi^r}{w^r L^r [1-\xi_E^r]}$ is increasing in r. Now, total profits accruing to the population of country r are the sum of profits earned through commercialization of domestic ideas plus commercialization of ideas from the global pool. Profits from commercialization of domestic ideas are:

$$\Pi_D^r = \eta^r \theta_D \left[\kappa \left(\pi_{i_{BR}^r} \right) + \frac{1-\kappa}{I} \sum_{i \in \mathcal{I}(r)} \pi_i \right] ,$$

implying that

$$\frac{\Pi_D^r}{w^r L^r [1 - \xi_E^r]} = \frac{\eta_1(r) L^r}{w^r L^r [1 - \xi_E^r]} \eta_2(\xi_E^r) \theta_D \left[\kappa \left(\pi_{i_{BR}^r} \right) + \frac{1-\kappa}{I} \sum_{i \in \mathcal{I}(r)} \pi_i \right] ,$$

[1] We note that Π^r captures the profits due to own basic research investments as expressed in Equation (2.14) and due to commercialization of ideas generated by basic research investments of other countries.

which is increasing in r since each factor is increasing in r. Let $\eta_{G,i}$ denote the amount of ideas for industry i in the public domain. Profits from the commercialization of these ideas that accrue to the population in country r are then

$$\Pi_G^r = \sum_{i \leq \tilde{i}(r)} \eta_{G,i} \pi_i \frac{\tilde{L}^r}{\int_{\tilde{r}(i)}^{\bar{r}} \tilde{L}^{r'}\, dF_r(r')} \ ,$$

and hence $\frac{\Pi_G^r}{w^r L^r [1-\xi_E^r]}$ is increasing in r as well, which shows the desired result.

□

B.1.2. Proof of Proposition 2.3

Parts (ii) and (iii) have been shown in the main body of the text. It remains to show that the first order Condition (2.24) can be rewritten as in (2.25), and that this equation has a unique solution which corresponds to a global maximum.

Applying the chain rule, we obtain

$$\frac{\partial \tilde{L}_{BR,S}(N)}{\partial N} = \frac{\partial \tilde{L}_{BR,S}(N)}{\partial \xi_S^{\bar{r}}} \frac{\partial \xi_S^{\bar{r}}}{\partial N} \ . \tag{B.1}$$

Differentiating (2.22) with respect to $\xi_S^{\bar{r}}$ yields

$$\begin{aligned} \frac{\partial \tilde{L}_{BR,S}}{\partial \xi_S^{\bar{r}}} =& F_a{}^{-1\prime}\left(1-\xi_S^{\bar{r}}\right) \\ & \int_{\underline{r}}^{\bar{r}} L^r \left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{2}{\lambda}} \frac{1}{\eta_1(r)} F_a{}' \left(\left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} \frac{1}{\eta_1(r)} F_a^{-1}(1-\xi_S^{\bar{r}})\right) f_r(r) dr \ . \end{aligned} \tag{B.2}$$

Differentiating (2.21) with respect to $\xi_S^{\bar{r}}$ yields

$$\begin{aligned} \frac{\partial N}{\partial \xi_S^{\bar{r}}} = & \int_{\underline{r}}^{\bar{r}} \eta_1(r) L^r \eta_2' \left(1 - F_a\left(\left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} \frac{1}{\eta_1(r)} F_a^{-1}(1-\xi_S^{\bar{r}})\right)\right) \\ & F_a{}'\left(\left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} \frac{1}{\eta_1(r)} F_a^{-1}(1-\xi_S^{\bar{r}})\right) \\ & \left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} \frac{1}{\eta_1(r)} F_a{}^{-1\prime}\left(1-\xi_S^{\bar{r}}\right) f_r(r) dr \ . \end{aligned} \tag{B.3}$$

Now, using

$$\eta_2'\left(1-F_a\left(\left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}}\frac{1}{\eta_1(r)}F_a^{-1}(1-\xi_S^{\bar{r}})\right)\right)$$
$$=$$
$$F_a^{-1}\left(F_a\left(\left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}}\frac{1}{\eta_1(r)}F_a^{-1}(1-\xi_S^{\bar{r}})\right)\right)$$
$$=$$
$$\left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}}\frac{1}{\eta_1(r)}F_a^{-1}(1-\xi_S^{\bar{r}})\ ,$$

where the last equality follows from F_a being injective, we can simplify (B.3) to

$$\frac{\partial N_{SP}}{\partial \xi_S^{\bar{r}}} = F_a^{-1}(1-\xi_S^{\bar{r}})\frac{\partial \tilde{L}_{BR,S}}{\partial \xi_S^{\bar{r}}} > 0\ . \tag{B.4}$$

Using (B.1), (B.2), and (B.4) in (2.24) yields (2.25).

It remains to show that Equation (2.25) indeed has a unique solution that corresponds to a global maximum. To show this, we use (2.21) and (2.22) in (2.25) to obtain

$$\frac{\int_{\underline{r}}^{\bar{r}} L^r \left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} F_a\left(\left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}}\frac{1}{\eta_1(r)}F_a^{-1}(1-\xi_S^{\bar{r}})\right) f_r(r)dr}{[\sigma_v-1]\int_{\underline{r}}^{\bar{r}} \eta_1(r)L^r\eta_2\left(1-F_a\left(\left[\frac{\ln(\bar{r})}{\ln(r)}\right]^{\frac{1}{\lambda}}\frac{1}{\eta_1(r)}F_a^{-1}(1-\xi_S^{\bar{r}})\right)\right) f_r(r)dr} = \frac{1}{F_a^{-1}(1-\xi_S^{\bar{r}})}\ , \tag{B.5}$$

and the result follows from noting that the left-hand side of the above equation is strictly decreasing and continuous in $\xi_S^{\bar{r}}$, while the right-hand side is continuously increasing, i.e. there exists a unique solution $\xi_S^{\bar{r}} \in (0,1)$,[2] which, in turn, implies a unique N_S from (2.21). Finally, the same reasoning also implies that the elasticity

$$\frac{\partial \tilde{L}_{BR,S}(N)}{\partial N}\frac{N}{\tilde{L}-\tilde{L}_{BR,S}}$$

[2] We consider the economically interesting case where cross-country heterogeneity in basic research productivities is small enough, such that there is always an interior solution for $\xi_S^{\bar{r}} \in (0,1)$. For example, in the limiting case where $\ln(r)^{\frac{1}{\lambda}}\eta_1(r)$ is constant over r, the left-hand side (LHS) is decreasing with boundaries $\lim_{\xi_S^{\bar{r}}\to 0} LHS = \infty$ and $\lim_{\xi_S^{\bar{r}}\to 1} LHS = 0$, while the right-hand side (RHS) is increasing with boundaries $\lim_{\xi_S^{\bar{r}}\to 0} RHS = 0$ and $\lim_{\xi_S^{\bar{r}}\to 1} RHS = \frac{1}{\underline{a}}$, implying that there is an interior solution indeed.

is strictly increasing in N, i.e. the solution corresponds to a global maximum.

□

B.1.3. Proof of Corollary 2.3

We show each part in turn.
(i) The first statement in part (i) follows immediately from (2.20). The second statement follows from observing that ξ_S^r is increasing in r whenever $\eta_1(r)(\ln(r))^{\frac{1}{\lambda}}$ is increasing in r, which is the case if basic research productivity is more elastic in r than productivity in production.
(ii) The optimal basic research intensity $\xi_S^{\bar{r}}$ is pinned down by (B.5). This equation is neither affected by a poportionate increase of the population, $\hat{L}^r = \mu L^r$, nor by a proportionate increase of skills for production, $\hat{r} = r^{\mu}$, nor by a proportionate increase of the innate ability of each household, $\hat{a} = \mu a$. Moreover, proportionately increasing $\eta_1(r)$ is equivalent to proportionately increasing the ability of each household, which proves the first statement. A decrease in σ_v shifts the left-hand side of (B.5) upward, and is thus reflected in a higher $\xi_S^{\bar{r}}$. Finally, given that a proportionate change in $\eta_1(r)$ does not impact the optimal choice of $\xi_S^{\bar{r}}$, we consider increasing all $\eta_1(r)$, holding constant $\eta_1(\bar{r}) = 1$. For a given $\xi_S^{\bar{r}}$, this will decrease the left-hand side of (B.5) while leaving the right-hand side unaffected, i.e. this must be associated with a lower $\xi_S^{\bar{r}}$.

□

B.1.4. Proof of Proposition 2.4

We proceed in two steps: In step 1 we show that whenever $n_{\hat{i},E} > n_{\hat{i},S}$ for some $\hat{i}$, it must be that $n_{i,E} \geq n_{i,S}$ for all $i \leq \hat{i}$. In words: whenever a larger share of varieties arises from targeting industry $\hat{i}$ in the decentralized equilibrium, compared to the solution of the global social planner, (weakly) more varieties are targeted to all less complex industries in the decentralized equilibrium. In turn, this implies that the social planner must target a larger share of varieties to more complex industries $i > \hat{i}$,[3] i.e. there can never be too many varieties arising from targeting complex industries in the decentralized equilibrium. In step 2 we show by means of two examples that both, efficient targeting and insufficient targeting to complex industries are possible.

Step 1 Suppose that $n_{\hat{i},E} > n_{\hat{i},S}$ for some $\hat{i}$. Then, it must be that some varieties are targeted to industry $\hat{i}$ in the decentralized equilibrium. Recall that the government in

[3] Recall that both in the decentralized equilibrium and in the solution of the global social planner it must hold that $\sum_{i \in \mathcal{I}} n_i = 1$.

country r will always target the industry $i \leq \tilde{i}(r)$ with highest profits. Proposition 2.2 therefore implies that

$$\psi_i i^{\frac{1-\sigma_I}{\lambda}} n_{i,E}{}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}} \leq \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}} n_{\hat{i},E}{}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}} , \quad \forall i \leq \hat{i} .$$

Rearranging yields

$$n_{i,E}{}^{\frac{\sigma_v-\sigma_I}{\sigma_v-1}} \geq \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}}}{\psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}}} n_{\hat{i},E}{}^{\frac{\sigma_v-\sigma_I}{\sigma_v-1}} , \quad \forall i \leq \hat{i} . \tag{B.6}$$

Now, if the social planner does not target any varieties to industry $i \leq \hat{i}$, it trivially holds that $n_{i,E} \geq n_{i,S}$. It remains to be shown that the same is true if the social planner targets industries $i \leq \hat{i}$. We distinguish two cases, depending on whether or not the global social planer targets some varieties to industry $\hat{i}$.

(i) Suppose the social planner targets a set of varieties with positive measure to industry $\hat{i}$. Condition (2.18) then implies that

$$n_{i,S}{}^{\frac{\sigma_v-\sigma_I}{\sigma_v-1}} \geq \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}}}{\psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}}} n_{\hat{i},S}{}^{\frac{\sigma_v-\sigma_I}{\sigma_v-1}} , \quad \forall i \leq \hat{i} . \tag{B.7}$$

Note that (B.7) holds with equality for all industries i that the social planner targets. (B.6), (B.7), and the fact that $n_{\hat{i},E} > n_{\hat{i},S}$ therefore imply that $n_{i,E} > n_{i,S}$.

(ii) Suppose the social planner targets no varieties to industry $\hat{i}$. Then, we must have

$$n_{i,S}{}^{\frac{\sigma_v-\sigma_I}{\sigma_v-1}} \leq \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}}}{\psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}}} n_{\hat{i},S}{}^{\frac{\sigma_v-\sigma_I}{\sigma_v-1}} , \quad \forall i \leq \hat{i} \tag{B.8}$$

for all industries i that the social planner targets. (B.6), (B.8), and the fact that $n_{\hat{i},E} > n_{\hat{i},S}$ imply that $n_{i,E} > n_{i,S}$, which proves the desired result.

Step 2 We first provide an example for inefficient targeting and then one for efficient targeting. In doing so, it will be convenient to consider a world with only two types of countries $r_1 > r_2$ and two industries $i_1 > i_2$, where $\tilde{i}(r_1) \geq i_1 > \tilde{i}(r_2) \geq i_2$.

(i) Let targeting be just efficient if every idea is targeted towards industry i_1, i.e.

$$\frac{1+\kappa}{1-\kappa} = \left[\frac{\psi_{i_1} i_1^{\frac{1-\sigma_I}{\lambda}}}{\psi_{i_2} i_2^{\frac{1-\sigma_I}{\lambda}}} \right]^{\frac{\sigma_v-1}{\sigma_v-\sigma_I}} .$$

Then, for $\kappa > 0$, any positive investment in basic research in countries r_2 will result in inefficient targeting.

(ii) Let both industries have same attractiveness, i.e.

$$\psi_{i_1} i_1^{\frac{1-\sigma_I}{\lambda}} = \psi_{i_2} i_2^{\frac{1-\sigma_I}{\lambda}} ,$$

and countries r_1 account for at least 50% of the world population. Then, for any κ targeting will be efficient by Proposition 2.2, Corollary 2.1, and the fact that $\eta_1(\,\cdot\,)$ is increasing.

□

B.1.5. Proof of Lemma 2.2

We show each part in turn.

(i) The marginal cost to the government in country r of hiring an additional basic researcher is just this researcher's wage w^r divided by the ideal price index P. The marginal cost to the social planner is

$$\begin{aligned} -\frac{\partial C}{\partial L^r_{BR,S}} &= -\frac{\partial C}{\partial \tilde{L}_p} \frac{\partial \tilde{L}_p}{\partial L^r_{BR,S}} \\ &= [-e\lambda \ln(\bar{r})]^{-\frac{1}{\lambda}} \left[\sum_{i \in \mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} N_i^{\frac{1-\sigma_I}{1-\sigma_v}} \right]^{\frac{1}{\sigma_I - 1}} \left[\frac{\ln(\bar{r})}{\ln(r)} \right]^{\frac{1}{\lambda}} \\ &= \frac{\sigma_v}{\sigma_v - 1} \frac{w^r}{P} , \end{aligned}$$

where the second equality follows from using the equilibrium value for C along with the definition of effective labor, and where the last equality follows from observing that aggregate income is $\frac{\sigma_v}{\sigma_v-1}\tilde{L}_p$.

(ii) The marginal benefit for the government in country r of a domestically commercialized variety is just associated profits, π_i, divided by the ideal price index P. The marginal benefit for the social planner is

$$\begin{aligned} \frac{\partial C}{\partial N_i} &= [-e\lambda \ln(\bar{r})]^{-\frac{1}{\lambda}} \left[\sum_{i \in \mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} N_i^{\frac{1-\sigma_I}{1-\sigma_v}} \right]^{\frac{1}{\sigma_I - 1}} \tilde{L}_p \frac{1}{\sigma_v - 1} \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}} N_i^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}}}{\sum_{i \in \mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} N_i^{\frac{1-\sigma_I}{1-\sigma_v}}} \\ &= \frac{\sigma_v}{\sigma_v - 1} \frac{\pi_i}{P} , \end{aligned}$$

where the first equality follows from differentiating C and rearranging terms, and the second equality follows form using the definitions of P and π_i.

□

B.1.6. Proof of Proposition 2.5

Suppose the social planner wants to generate the same number of varieties as in the decentralized equilibrium. The solutions in the decentralized equilibrium are denoted by ξ_E^r and $\tilde{a}_E^r$ and by $\xi_{\tilde{S}}^r$ and $\tilde{a}_{\tilde{S}}^r$ the constrained social planner solutions we consider in this proof. Then because basic research is equally productive in both cases it cannot be that $\tilde{a}_{\tilde{S}}^r \geq (\leq)\tilde{a}_E^r$ for all r with the inequality being strict for some measurable set of countries, i.e. either investment patterns are identical almost everywhere or it must be that $\tilde{a}_{\tilde{S}}^r > \tilde{a}_E^r$ for some measurable set of countries and $\tilde{a}_{\tilde{S}}^r < \tilde{a}_E^r$ for some disjoint measurable set of countries. In turn, $\tilde{a}_{\tilde{S}}^r > (<)\tilde{a}_E^r$ implies that $\xi_{\tilde{S}}^r < (>)\xi_E^r$. Now, investment patterns cannot be identical by (2.26) in combination with sufficient skills and the fact that $\tilde{i}(r) < \bar{i}$ for some $r > \underline{r}$. What is more, $\tilde{a}_{\tilde{S}}^r > (<)\tilde{a}_E^r$ for some r implies that $\tilde{a}_{\tilde{S}}^{r'} > (<)\tilde{a}_E^{r'}$ for all $r' \geq (\leq)r$ by (2.26). The result then follows from noting that we may have $\xi_{\tilde{S}}^r = \xi_E^r$ for all $r \in [\tilde{r}_1, \tilde{r}_2)$, where $\tilde{r}_1 = \tilde{r}(i_k)$ and $\tilde{r}_2 = \tilde{r}(i_{k+1})$ for some industry $\underline{i} < i_k < \bar{i}$.

□

B.1.7. Proof of Proposition 2.6

For the purpose of this and the following proofs, it will be useful to introduce the following notation. We will say that there is a fixed allocation scheme $\varphi = \{\varphi^r\}_{r\in[\underline{r},\bar{r}]}$ of basic research investments to countries if, for any desired aggregate investment in basic research, $\tilde{L}_{BR}$, and every country $r \in [\underline{r}, \bar{r}]$, we have

$$\xi^r(\tilde{L}_{BR}; \varphi) = \frac{\tilde{L}_{BR}}{\int_{\underline{r}}^{\bar{r}} \varphi^r \tilde{L}^r f_r(r) dr} \varphi^r .$$

In other words, a fixed allocation scheme is characterized by $\frac{\xi^r}{\xi^{r'}} = \frac{\varphi^r}{\varphi^{r'}} = \text{constant}$ $\forall r, r' \in [\underline{r}, \bar{r}]$. (2.12) and (2.9) imply that for any such allocation scheme, the total number of varieties in the economy is strictly increasing in $\tilde{L}_{BR}$. Allocation scheme φ is thus associated with a strictly increasing function $\tilde{L}_{BR}(N; \varphi)$ that defines the required total amount of effective labor in basic research for every desired number of varieties N and, equivalently, with a strictly increasing function $N(\tilde{L}_{BR}; \varphi)$. We will use φ_E to denote the targeting scheme prevailing in the decentralized equilibrium and, without loss of generality, choose the normalization $\varphi_E^r = \xi_{BR,DE}^r$ for all r.

In Section 2.6 we have shown that the optimal number of varieties of the social planner is independent from the targeting of basic research investments to industries. By the same reasoning, the optimal aggregate investment in basic research is also independent from targeting when confronted with a fixed allocation scheme φ, and hence we are allowed to choose an arbitrary targeting as long as it satisfies Condition SSC. Now, suppose that the social planner adopts the equilibrium targeting of each country. With a fixed allocation scheme, the social planner's decision problem then boils down to the following:

$$\max_{\tilde{L}_{BR}} \quad C = [-e\lambda \ln(\bar{r})]^{-\frac{1}{\lambda}} \left[\sum_{i \in \mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}} N_i^{\frac{1-\sigma_I}{1-\sigma_v}} \right]^{\frac{1}{\sigma_I - 1}} \left[\tilde{L} - \tilde{L}_{BR} \right] ,$$
$$\text{s.t.} \quad N_i = \int_{\underline{r}}^{\overline{r}} \left[\mathbb{1}_{[i=i^r_{BR}]} \kappa + \frac{1-\kappa}{I} \right] \eta_1(r) \eta_2 \left(\varphi^r_E \frac{\tilde{L}_{BR}}{\tilde{L}_{BR,E}} \right) L^r f_r(r) dr \ .$$

The optimal $\tilde{L}_{BR}$ is then the unique solution to the associated first order condition, which after some straightforward modifications reads

$$\frac{C}{\tilde{L} - \tilde{L}_{BR}} = \frac{\sigma_v}{\sigma_v - 1} \frac{1}{P} \frac{1}{\tilde{L}_{BR,E}} \sum_{i \in \mathcal{I}} \pi_i \int_{\underline{r}}^{\overline{r}} \left[\mathbb{1}_{[i=i^r_{BR}]} \kappa + \frac{1-\kappa}{I} \right] \eta_1(r) \eta_2' \left(\varphi^r_E \frac{\tilde{L}_{BR}}{\tilde{L}_{BR,E}} \right) \varphi^r_E L^r f_r(r) dr \ . \quad \text{(B.9)}$$

Uniqueness and existence follow from the fact that when increasing $\tilde{L}_{BR}$ the social planner has to hire researchers that are less able, which, in turn, implies that the right-hand-side of (B.9) is strictly decreasing in $\tilde{L}_{BR}$. To see this we note that $\eta_2'(.)$ is a decreasing function of $\tilde{L}_{BR}$ and that π_i is also decreasing in $\tilde{L}_{BR}$ (and N_i).[4] In what follows, we will show that when evaluated at $\tilde{L}_{BR} = \tilde{L}_{BR,E}$, the right-hand-side must be strictly larger than the left-hand side, implying that the social planner will invest strictly more compared to the decentralized equilibrium.

The left-hand-side of (B.9) is the social planner's marginal cost of increasing $\tilde{L}_{BR}$. Using the budget constraint of the representative household and the fact that aggregate profits are a constant fraction $\frac{1}{\sigma_v}$ of aggregate revenues, this can be rewritten as

$$\frac{C}{\tilde{L} - \tilde{L}_{BR}} = \frac{\sigma_v}{\sigma_v - 1} \frac{1}{P} ,$$

i.e. the marginal cost of the social planner is just $\frac{\sigma_v}{\sigma_v - 1}$ times the real wage per unit of effective labor. In turn, this implies that the marginal cost for the social planner is just $\frac{\sigma_v}{\sigma_v - 1}$ times the total marginal costs of national governments for the corresponding increase in $\tilde{L}^r_{BR}$,

$$\frac{C}{\tilde{L} - \tilde{L}_{BR}} = \frac{\sigma_v}{\sigma_v - 1} \frac{1}{P} \int_{\underline{r}}^{\overline{r}} \frac{\xi^r_E \tilde{L}^r}{\tilde{L}_{BR,E}} f_r(r) dr ,$$

where the equality follows from the fact that $\int_{\underline{r}}^{\overline{r}} \xi^r_E \tilde{L}^r f_r(r) dr = \tilde{L}_{BR,E}$.

[4] Cf. the proof of Proposition 2.3 for the detailed argument for the case of the optimal allocation scheme across countries.

The right-hand-side of (B.9) is the marginal benefit of the social planner. Evaluating at $\tilde{L}_{BR} = \tilde{L}_{BR,E}$ and rearranging terms yields

$$\frac{\sigma_v}{\sigma_v - 1}\frac{1}{P}\frac{1}{\tilde{L}_{BR,E}}\sum_{i\in\mathcal{I}}\pi_i\int_{\underline{r}}^{\overline{r}}\left[\mathbb{1}_{[i=i^r_{BR}]}\kappa + \frac{1-\kappa}{I}\right]\eta_1(r)\eta_2'\left(\xi^r_E\right)\xi^r_E L^r f_r(r)dr$$

$$=$$

$$\frac{\sigma_v}{\sigma_v - 1}\frac{1}{P}\int_{\underline{r}}^{\overline{r}}\frac{\xi^r_E}{\tilde{L}_{BR,E}}\eta_1(r)\eta_2'\left(\xi^r_E\right)\left[\kappa\pi_{i^r_{BR}} + \sum_{i\in\mathcal{I}}\frac{1-\kappa}{I}\pi_i\right]L^r f_r(r)dr$$

$$>$$

$$\frac{\sigma_v}{\sigma_v - 1}\frac{1}{P}\int_{\underline{r}}^{\overline{r}}\frac{\xi^r_E}{\tilde{L}_{BR,E}}\eta_1(r)\eta_2'\left(\xi^r_E\right)\theta_D\left[\kappa\pi_{i^r_{BR}} + \sum_{i\in\mathcal{I}(r)}\frac{1-\kappa}{I}\pi_i\right]L^r f_r(r)dr\ .$$

The inequality follows from $\theta_D \leq 1$ and the fact that $\tilde{i}(\underline{r}) < \bar{i}$. The last term is just $\frac{\sigma_v}{\sigma_v-1}$ times the total marginal benefits of national governments associated with the respective increase in $\tilde{L}^r_{BR}$. By Condition (2.15) these are just equal to the corresponding total marginal costs, i.e. to the left-hand-side of (B.9), which proves the desired result.

□

B.1.8. Proof of Proposition 2.7

Recall that for any given targeting (Appendix A.1.2) and a fixed allocation scheme of basic research investments (Appendix B.1.7), the social planner's optimal investment is the unique solution to

$$\frac{1}{\sigma_v - 1} = \frac{\partial\tilde{L}_{BR}(N;\varphi)}{\partial N}\frac{N}{\tilde{L} - \tilde{L}_{BR}(N;\varphi)}\ . \tag{B.10}$$

For a given allocation scheme φ, aggregate investments $\tilde{L}_{BR}$ yield

$$N(\tilde{L}_{BR};\varphi) = \int_{\underline{r}}^{\overline{r}}\int_{\tilde{a}^r(\tilde{L}_{BR};\varphi)}^{\infty} a dF_a(a)\tilde{g}(r)dr \tag{B.11}$$

varieties, where $\tilde{g}(r) := \eta_1(r)L^r f_r(r)$ and where $\tilde{a}^r(\tilde{L}_{BR};\varphi)$ denotes the research ability of the marginal scientist in country r if aggregate investments are $\tilde{L}_{BR}$ with an allocation scheme φ. Differentiating with respect to $\tilde{L}_{BR}$ yields

$$\begin{aligned}\frac{\partial N}{\partial\tilde{L}_{BR}} &= \int_{\underline{r}}^{\overline{r}}\tilde{a}^r(\tilde{L}_{BR};\varphi)\frac{\varphi^r}{\int_{\underline{r}}^{\overline{r}}\varphi^r\tilde{L}^r f_r(r)dr}\tilde{g}(r)dr\\ &= \int_{\underline{r}}^{\overline{r}}\tilde{a}^r(\tilde{L}_{BR};\varphi)\frac{[1 - F_a(\tilde{a}^r(\tilde{L}_{BR}))]}{\tilde{L}_{BR}}\tilde{g}(r)dr\ ,\end{aligned} \tag{B.12}$$

where the second equality follows from $\frac{\varphi^r}{\int_{\underline{r}}^{\overline{r}} \varphi^r \tilde{L}^r f_r(r) dr} = \frac{\xi^r(\tilde{L}_{BR};\varphi)}{\tilde{L}_{BR}}$. Using (B.11) and (B.12) in (B.10), we obtain

$$\frac{1}{\sigma_v - 1} = \frac{\int_{\underline{r}}^{\overline{r}} \int_{\tilde{a}^r(\tilde{L}_{BR};\varphi)}^{\infty} a dF_a(a) \tilde{g}(r) dr}{\int_{\underline{r}}^{\overline{r}} \left[1 - F_a(\tilde{a}^r(\tilde{L}_{BR};\varphi))\right] \tilde{a}^r(\tilde{L}_{BR};\varphi) \tilde{g}(r) dr} \frac{\tilde{L}_{BR}}{\tilde{L} - \tilde{L}_{BR}} . \tag{B.13}$$

When confronted with allocation scheme φ, the social planner chooses aggregate investments in basic research to satisfy (B.13). The desired result then follows from noting that with a Pareto distribution of abilities $F_a(a) = 1 - \left(\frac{a}{\underline{a}}\right)^{-\tilde{\alpha}}$,

$$\frac{\int_{\underline{r}}^{\overline{r}} \int_{\tilde{a}^r(\tilde{L}_{BR};\varphi)}^{\infty} a dF_a(a) \tilde{g}(r) dr}{\int_{\underline{r}}^{\overline{r}} \left[1 - F_a(\tilde{a}^r(\tilde{L}_{BR};\varphi))\right] \tilde{a}^r(\tilde{L}_{BR};\varphi) \tilde{g}(r) dr} = \frac{\tilde{\alpha}}{\tilde{\alpha} - 1}$$

is constant, which proves that the optimal $\tilde{L}_{BR}$ is the same, irrespective of the allocation scheme. This implies that aggregate investments in the decentralized equilibrium are lower than in the social planner's solution, since $\theta_D < 1$, and thus the domestic population cannot retain every domestically generated idea through basic research, and $\mathcal{I}(r) \subset \mathcal{I}$ for a measurable set of countries, i.e. for these countries some ideas cannot be commercialized domestically.

□

B.1.9. Proof of Lemma A.1

Suppose there are sufficient skills. Then, by Proposition 2.1, we have for any $\hat{\imath}, i \in \mathcal{I}$:

$$\frac{\pi_{\hat{\imath}}}{\pi_i} = \frac{\psi_{\hat{\imath}} N_{\hat{\imath}}^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} \hat{\imath}^{\frac{1-\sigma_I}{\lambda}}}{\psi_i N_i^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} i^{\frac{1-\sigma_I}{\lambda}}} .$$

Hence, the profits of the respective representative firm are the same in industries i and $\hat{\imath}$ if and only if

$$N_i = \left[\frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}}}{\psi_{\hat{\imath}} \hat{\imath}^{\frac{1-\sigma_I}{\lambda}}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}} N_{\hat{\imath}} .$$

Summing over all industries and rearranging terms yields

$$n_{\hat{\imath}} = \frac{\left[\psi_{\hat{\imath}} \hat{\imath}^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}}}{\sum_{i \in \mathcal{I}} \left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}}} , \tag{B.14}$$

where, as before n_i, denotes the share of all varieties that exist in industry i. Further, remember that s_i denotes the share of all ideas that is targeted to industry i. From the discussion in Appendix A.1.2, we know that

$$s_i = n_i \frac{1}{\kappa} - \frac{[1-\kappa]}{\kappa} \frac{1}{I} \; . \tag{B.15}$$

(B.15) characterizes the share of all ideas that need to be targeted to industry i, such that the share of industry-i varieties in all varieties is n_i. Combining (B.14) and (B.15), we obtain

$$s_i = \frac{\left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}}}{\sum_{i \in \mathcal{I}} \left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}}} \frac{1}{\kappa} - \frac{[1-\kappa]}{\kappa} \frac{1}{I} \; .$$

By Assumption A.1, s_i is non-negative for all i and hence, it is feasible.

□

B.1.10. Proof of Lemma A.2

We consider a scenario where only countries with highest productive knowledge $r \geq \tilde{r}(\bar{\imath})$ invest in basic research and we show that their investments are bounded by γ in that case. As countries' investments are strategic substitutes, the same bound also applies if we allow investment by lower-skilled countries.

From Appendix A.2, we know that the highest-skilled countries would adopt the social planner's targeting if they were the only countries to invest in basic research. In such case, expected profits from a new variety are

$$\begin{aligned} \mathbb{E}[\pi] &= \sum_{i \in \mathcal{I}} n_i \pi_i \\ &= \frac{\tilde{L}_p}{N[\sigma_v - 1]} \; . \end{aligned}$$

With a Pareto distribution of abilities, this implies

$$\begin{aligned} \xi_E^r &= \left(\frac{\tilde{a}^r}{\underline{a}}\right)^{-\tilde{\alpha}} \\ &= \left(\frac{\underline{a}\eta_1(r)\theta_D \tilde{L}_p}{N[\sigma_v - 1] w^r}\right)^{\tilde{\alpha}} \; , \end{aligned} \tag{B.16}$$

for the optimal level of basic research investment in country $r \geq \tilde{r}(\bar{\imath})$.[5] For the aggregate

[5] See Equation (2.16).

number of varieties we obtain

$$N = \int_{\tilde{r}(\bar{i})}^{\overline{r}} \eta_1(r) \frac{\tilde{\alpha}}{\tilde{\alpha}-1} \underline{a} \xi_E^{r\,\frac{\tilde{\alpha}-1}{\tilde{\alpha}}} L^r f_r(r) dr$$

$$= \int_{\tilde{r}(\bar{i})}^{\overline{r}} \eta_1(r)^{\tilde{\alpha}} \frac{\tilde{\alpha}}{\tilde{\alpha}-1} \underline{a}^{\tilde{\alpha}} \left(\frac{\theta_D \tilde{L}_p}{N[\sigma_v - 1] w^r} \right)^{\tilde{\alpha}-1} L^r f_r(r) dr \; .$$

Solving for $N^{\tilde{\alpha}}$, plugging into (B.16), and substituting in the equilibrium value for w^r yields:

$$\xi_E^{\overline{r}} = \frac{[\tilde{\alpha}-1]\eta_1(\overline{r})^{\tilde{\alpha}} \theta_D \tilde{L}_p}{\tilde{\alpha}[\sigma_v - 1] \int_{\tilde{r}(\bar{i})}^{\overline{r}} \eta_1(r)^{\tilde{\alpha}} \left[\frac{\ln(\overline{r})}{\ln(r)}\right]^{\frac{1-\tilde{\alpha}}{\lambda}} L^r f_r(r) dr}$$

$$= \gamma \theta_D \frac{\tilde{L}_p}{\tilde{L}} < \gamma \; ,$$

where the inequality follows from $\theta_D \leq 1$ and $\tilde{L}_p < \tilde{L}$ with positive basic research. This proves that ξ_E^r is bounded from above by γ. $\xi_S^r < \gamma$ follows from the fact that for the case considered, the social planner will just choose investments according to (B.16), but with $\theta_D = 1$.

□

B.1.11. Proof of Proposition A.1

A necessary condition for labor market clearing is that total demand for effective labor equals total supply:

$$\int_{\underline{r}}^{\overline{r}} [L^r - L_{BR}^r] \left[\frac{\ln(\overline{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} dF_r(r) = \sum_{i \in \mathcal{I}} N_i \tilde{l}_i \; .$$

Normalizing SSC by the above equation, we obtain

$$\frac{\int_{\tilde{r}(\hat{i})}^{\overline{r}} [L^r - L_{BR}^r] \left[\frac{\ln(\overline{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} dF_r(r)}{\int_{\underline{r}}^{\overline{r}} [L^r - L_{BR}^r] \left[\frac{\ln(\overline{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} dF_r(r)} \geq \frac{\sum_{i \in \mathcal{I}: i \geq \hat{i}} N_i \tilde{l}_i}{\sum_{i \in \mathcal{I}} N_i \tilde{l}_i} \; , \quad \forall \hat{i} \in \mathcal{I} \; , \tag{SSC2}$$

i.e. there will be sufficient skills if for any industry $\hat{i}$, the share of total effective labor available for production in countries with productive knowledge $r \geq \tilde{r}(\hat{i})$ is at least as high as the share of total effective labor available for production that is demanded by industries $i \geq \hat{i}$.

As shown in Lemma A.2, ξ_E^r and ξ_S^r are bounded from above by γ. It follows that for any industry $\hat{i} \in \mathcal{I}$, the LHS of SSC2 is bounded from below by the LHS of Assumption A.2:

$$\frac{\int_{\tilde{r}(\hat{i})}^{\overline{r}} \left[L^r - L^r_{BR}\right] \left[\frac{\ln(\overline{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} dF_r(r)}{\int_{\underline{r}}^{\overline{r}} \left[L^r - L^r_{BR}\right] \left[\frac{\ln(\overline{r})}{\ln(r)}\right]^{\frac{1}{\lambda}} dF_r(r)} \geq \frac{\int_{\tilde{r}(\hat{i})}^{\overline{r}} [1-\gamma]\left[-\ln(r)\right]^{-\frac{1}{\lambda}} L^r\, dF_r(r)}{\int_{\underline{r}}^{\tilde{r}(\hat{i})} \left[-\ln(r)\right]^{-\frac{1}{\lambda}} L^r\, dF_r(r) + \int_{\tilde{r}(\hat{i})}^{\overline{r}} [1-\gamma]\left[-\ln(r)\right]^{-\frac{1}{\lambda}} L^r\, dF_r(r)}\,, \quad \forall \hat{i} \in \mathcal{I}\,. \tag{B.17}$$

Moreover, as argued in the main body of the text (see also Assumption A.1 and Lemma A.1 in Appendix A.2), the social planner will always target basic research investments such that profits are equal across industries. Then, from Proposition 2.1, we know that all production firms will demand the same amount of effective labor, irrespective of their industry. The share of industry i in total effective labor, $\frac{\tilde{L}_i}{\tilde{L}_p} = \frac{N_i \tilde{l}_i}{\tilde{L}_p}$, is then equal to its share in the total number of varieties, $\frac{N_i}{N}$:

$$\frac{\tilde{L}_{\hat{i}}}{\tilde{L}_p} = \frac{\left[\psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}}}{\sum_{i \in \mathcal{I}} \left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}}}\,. \tag{B.18}$$

(B.17), (B.18), and Assumption A.2 imply that ξ^r_S indeed satisfies condition SSC.

Finally, to prove that ξ^r_E also satisfies condition SSC we show that in the decentralized equilibrium, $\frac{\sum_{i \in \mathcal{I}: i \geq \hat{i}} N_i \tilde{l}_i}{\sum_{i \in \mathcal{I}} N_i \tilde{l}_i}$ is bounded from above by

$$\frac{\sum_{i \in \mathcal{I}: i \geq \hat{i}} N_i \tilde{l}_i}{\sum_{i \in \mathcal{I}} N_i \tilde{l}_i} \leq \frac{\sum_{i \in \mathcal{I}: i \geq \hat{i}} \left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}}}{\sum_{i \in \mathcal{I}} \left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}}}\,, \quad \forall \hat{i} \in \mathcal{I}\,,$$

and the result then follows from (B.17) and Assumption A.2.

We proceed by contradiction. Suppose, by contradiction, that

$$\frac{\sum_{i \in \mathcal{I}: i \geq \hat{i}} N_i \tilde{l}_i}{\sum_{i \in \mathcal{I}} N_i \tilde{l}_i} > \frac{\sum_{i \in \mathcal{I}: i \geq \hat{i}} \left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}}}{\sum_{i \in \mathcal{I}} \left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}}}\,,$$

for some $\hat{i} \in \mathcal{I}$. Then it must hold that

$$\frac{N_{i^h} \tilde{l}_{i^h}}{\sum_{i \in \mathcal{I}} N_i \tilde{l}_i} > \frac{\left[\psi_{i^h} {i^h}^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}}}{\sum_{i \in \mathcal{I}} \left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v - 1}{\sigma_v - \sigma_I}}}\,, \tag{B.19}$$

for some $i^h \geq \hat{i}$. Similarly, it must hold that

$$\frac{\sum_{i\in\mathcal{I}:i<\hat{i}} N_i \tilde{l}_i}{\sum_{i\in\mathcal{I}} N_i \tilde{l}_i} < \frac{\sum_{i\in\mathcal{I}:i<\hat{i}} \left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v-1}{\sigma_v-\sigma_I}}}{\sum_{i\in\mathcal{I}} \left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v-1}{\sigma_v-\sigma_I}}} ,$$

and thus

$$\frac{N_{i^l} \tilde{l}_{i^l}}{\sum_{i\in\mathcal{I}} N_i \tilde{l}_i} < \frac{\left[\psi_{i^l} {i^l}^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v-1}{\sigma_v-\sigma_I}}}{\sum_{i\in\mathcal{I}} \left[\psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{\sigma_v-1}{\sigma_v-\sigma_I}}} , \qquad \text{(B.20)}$$

for some $i^l < \hat{i}$. Combining (B.19) and (B.20) and rearranging terms implies

$$\frac{\psi_{i^l} {i^l}^{\frac{1-\sigma_I}{\lambda}} N_{i^l}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}}}{\psi_{i^h} {i^h}^{\frac{1-\sigma_I}{\lambda}} N_{i^h}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}}} \left[\frac{\tilde{l}_{i^l}}{\tilde{l}_{i^h}}\right]^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}} > 1 .$$

Proposition 2.1 and simple algebra then imply

$$\frac{\pi_{i^l}}{\pi_{i^h}} > 1 ,$$

a contradiction to equilibrium targeting which requires that profits in an industry that receives positive targeting are weakly higher than in any less complex industry.[6]

□

B.2. Proofs of Chapter 3

B.2.1. Proof of Proposition 3.4

We show that Proposition 3.4 holds for **(a)** $c(\Phi, r^\square) = 0$ and **(b)** $c(\Phi, r^\square) > 0$.

(a) First we rewrite total consumption using $k(r) = \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r)$ to obtain

$$C_\Phi(r^\square) = \mathcal{C} \left[\frac{\int_{r^\square}^{\bar{r}} k(r)dr}{x(r^\square) + y(r^\square) \int_{r^\square}^{\bar{r}} \tilde{L}^r \, dF_r(r)}\right]^{\frac{\sigma_v+\alpha-1}{\sigma_v-1}} \left[\int_{r^\square}^{\bar{r}} k(r)dr\right]^{\frac{1-\alpha}{\sigma_v-1}} .$$

[6] Under Assumption A.1 we have the stronger result that profits are equal.

Note that the last term above always decreases in $r^{\Box}$. We thus have to show that

$$\begin{aligned}&\frac{\int_{r^{\Box}}^{\bar{r}} k(r)dr}{x(r^{\Box}) + y(r^{\Box}) \int_{r^{\Box}}^{\bar{r}} \tilde{L}^r \, dF_r(r)}\\ =&\frac{\int_{r^{\Box}}^{\bar{r}} k(r)dr}{[\sigma_v - 1] \int_{r^{\Box}}^{\bar{r}} k(r)dr + \alpha\theta_D \int_{r^{\Box}}^{\bar{r}} k(r)z(r)dr + [1-\alpha]\theta_D g(r^{\Box})^{\frac{1}{1-\alpha}} \int_{r^{\Box}}^{\bar{r}} L^r w^r f(r)dr} \ ,\end{aligned}$$

decreases in $r^{\Box}$.

Note that whenever $r^{\Box}$ equals some $\tilde{r}(i)\ \forall i \in \mathcal{I}$, then **(b)** applies. Thus, it suffices to show that the derivative is negative everywhere else.[7]

We define

$$\begin{aligned}f &:= \int_{r^{\Box}}^{\bar{r}} k(r)dr \ ,\\ p &:= x(r^{\Box}) = [\sigma_v - 1] \int_{r^{\Box}}^{\bar{r}} k(r)dr + \alpha\theta_D \int_{r^{\Box}}^{\bar{r}} k(r)z(r)dr \ ,\\ q &:= y(r^{\Box}) \int_{r^{\Box}}^{\bar{r}} L^r w^r f(r)dr = [1-\alpha]\theta_D g(r^{\Box})^{\frac{1}{1-\alpha}} \int_{r^{\Box}}^{\bar{r}} L^r w^r f(r)dr \ .\end{aligned}$$

Then, the derivative can be rewritten as

$$\left(\frac{f}{p+q}\right)' = \frac{1}{[p+q]^2}\left[f'p - fp' + f'q - fq'\right] \ .$$

Thus, it is sufficient to show that the ratios $\frac{f}{p}$ and $\frac{f}{q}$ are monotonically decreasing separately:

- We show that $\frac{f}{p}$ decreases monotonically. We differentiate with respect to $r^{\Box}$,

$$\begin{aligned}\frac{\partial \frac{f}{p}}{\partial r^{\Box}} =&\frac{\partial \frac{\int_{r^{\Box}}^{\bar{r}} k(r)dr}{x(r^{\Box})}}{\partial r^{\Box}}\\ =&\frac{1}{x(r^{\Box})^2}\left[-h(r^{\Box})\left[[\sigma_v - 1]\int_{r^{\Box}}^{\bar{r}} k(r)dr + \alpha\theta_D \int_{r^{\Box}}^{\bar{r}} k(r)z(r)dr\right] + \right.\\ &\left.\int_{r^{\Box}}^{\bar{r}} k(r)dr\,[[\sigma_v - 1]h(r^{\Box}) + \alpha\theta_D h(r^{\Box})z(r^{\Box})]\right]\\ =&\alpha\theta_D\frac{h(r^{\Box})}{x(r^{\Box})^2}\left[z(r^{\Box})\int_{r^{\Box}}^{\bar{r}} k(r)dr - \int_{r^{\Box}}^{\bar{r}} k(r)z(r)dr\right] \leq 0\end{aligned}$$

  since $z(r) \geq z(r^{\Box})\ \forall r \geq r^{\Box}$.

- We show that $\frac{f}{q}$ decreases monotonically. Suppose $r_2^{\Box} > r_1^{\Box}$, where $r_1^{\Box}, r_2^{\Box} \in \mathcal{R}$, are two arbitrary points. We use Δ to denote the difference between a variable

[7] When $c(\Phi, r^{\Box}) = 0$ again, **(a)** applies, and the derivative is valid, because $C_{\Phi}(r^{\Box})$ is an upper semi-continuous function.

evaluated at $r_2^\square$ and $r_1^\square$, and then

$$\begin{aligned}\Delta\left[\frac{f}{q}\right] =&\frac{f(r_2^\square)}{q(r_2^\square)} - \frac{f(r_1^\square)}{q(r_1^\square)}\\ =&\frac{f(r_2^\square)q(r_1^\square) - f(r_1^\square)q(r_2^\square) + f(r_1^\square)q(r_1^\square) - f(r_1^\square)q(r_1^\square)}{q(r_2^\square)q(r_1^\square)}\\ =&\frac{1}{q(r_2^\square)}\left[f(r_2^\square) - f(r_1^\square) + \frac{f(r_1^\square)}{q(r_1^\square)}\Big(q(r_1^\square) - q(r_2^\square)\Big)\right]\end{aligned}$$

We know that $f, q > 0$, thus $\Delta\left(\frac{f}{q}\right)$ has the same sign as $\Delta f - \frac{f(r_1^\square)}{q(r_1^\square)}\Delta q$, and we can focus on the term in brackets.

Now note that in our case $\frac{f}{q} > 1$, for all $r^\square \in [\underline{r}, \overline{r}]$, because

$$f = \int_{r^\square}^{\overline{r}} g(r)^{\frac{1}{1-\alpha}}\frac{1}{z(r)}L^r w^r f(r)dr > [1-\alpha]\theta_D g(r^\square)^{\frac{1}{1-\alpha}}\int_{r^\square}^{\overline{r}} L^r w^r f(r)dr\ .$$

To show $\Delta\left(\frac{f}{q}\right) \leq 0$, it is sufficient to show $\Delta q \geq \Delta f$, or more conveniently $-\Delta f \geq -\Delta q$,

$$\begin{aligned}f(r_1^\square) - f(r_2^\square) =& \int_{r_1^\square}^{r_2^\square} k(r)dr\\ =&\int_{r_1^\square}^{r_2^\square} g(r)^{\frac{1}{1-\alpha}}\frac{1}{z(r)}L^r w^r f(r)dr\\ \geq&[1-\alpha]\theta_D g(r_1^\square)^{\frac{1}{1-\alpha}}\int_{r_1^\square}^{r_2^\square} L^r w^r f(r)dr\\ =&y(r_1^\square)\int_{r_1^\square}^{r_2^\square} L^r w^r f(r)dr\\ \geq&y(r_1^\square)\int_{r_1^\square}^{\overline{r}} L^r w^r f(r)dr - y(r_2^\square)\int_{r_2^\square}^{\overline{r}} L^r w^r f(r)dr\\ =&q(r_1^\square) - q(r_2^\square)\end{aligned}$$

since $g(r) \geq g(r_1^\square)\ \forall r \geq r_1^\square$, and $z(r) \leq 1$.

Thus we have shown that $C_\Phi(r^\square)$ decreases in $r^\square$.

(b) If now $c(\Phi, r^\square) > 0$, then an increase in basic research infrastructure costs simply removes more of consumption from the economy by absorbing labor for the buildups, without affecting $r^\square$ and consumption must decrease.

□

B.2.2. Proof of Lemma 3.1

We rewrite equation (3.19) without any changes,

$$\begin{aligned}
C_{\Phi}^{\bar{r}}(r^{\square}) =& \frac{\sigma_v - 1}{\sigma_v} L^{\bar{r}} \left[-e\lambda \log(\bar{r})\right]^{-\frac{1}{\lambda}} \left[\sum_{i\in\mathcal{I}} \psi_i i^{\frac{1-\sigma_I}{\lambda}}\right]^{\frac{1}{\sigma_I - 1}} \\
& \left[\frac{\tilde{L}}{x(r^{\square}) + y(r^{\square}) \int_{r^{\square}}^{\bar{r}} \tilde{L}^r \, dF_r(r)}\right]^{\frac{\alpha+\sigma_v-1}{\sigma_v-1}} \left[\int_{r^{\square}}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr\right]^{\frac{1}{\sigma_v-1}} \\
& \left\{\sum_{i\in\mathcal{I}(\bar{r})} \frac{z(i)}{\int_{\tilde{r}(i)}^{\bar{r}} L^r f(r) dr} \left[\int_{r^{\square}}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr - \right.\right. \\
& \left. \theta_D \int_{max\{r^{\square}, \tilde{r}(i)\}}^{\bar{r}} \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r) dr \right] + \\
& \left. \frac{x(r^{\square})}{\tilde{L}} + y(r^{\square}) \left[\frac{\int_{r^{\square}}^{\bar{r}} \tilde{L}^r \, dF_r(r)}{\tilde{L}} - 1\right] + [1-\alpha]\theta_D \right\} . \qquad \text{(B.21)}
\end{aligned}$$

Note that only the *first* term of the *second* line of (B.21) increases in $r^{\square}$. Thus, we focus on the second line and omitting constant terms and using $k(r) = \eta_1(r) g(r)^{\frac{\alpha}{1-\alpha}} L^r f(r)$ we rearrange to obtain

$$\left[\frac{\int_{r^{\square}}^{\bar{r}} k(r) dr}{x(r^{\square}) + y(r^{\square}) \int_{r^{\square}}^{\bar{r}} \tilde{L}^r \, dF_r(r)}\right]^{\frac{\alpha+\sigma_v-1}{\sigma_v-1}} \left[\int_{r^{\square}}^{\bar{r}} k(r) dr\right]^{\frac{2-\alpha-\sigma_v}{\sigma_v-1}} . \qquad \text{(B.22)}$$

We have shown in Appendix B.2.1 that the first term in (B.22) decreases in $r^{\square}$.
If now $2 - \alpha - \sigma_v > 0$, then also the second term in (B.22) decreases in $r^{\square}$, and we have a sufficient condition that $C_{\Phi}^{\bar{r}}(r^{\square})$ always decreases when basic research infrastructure costs increase.

□

C. Comments for Part II

C.1. Micro-foundation of Household's Demand

C.1.1. Lagrangian Derivation

To derive the demand of a household r, we solve the household's optimization problem by setting up the Lagrangian

$$\begin{aligned}\mathcal{L} = &\left[\sum_{\mathcal{I}}\left[\psi_i^{\frac{1}{\sigma_I-1}}\left[\int_{n_i} c_{i,j}^{r\ \frac{\sigma_v-1}{\sigma_v}}dj\right]^{\frac{\sigma_v}{\sigma_v-1}}\right]^{\frac{\sigma_I-1}{\sigma_I}}\right]^{\frac{\sigma_I}{\sigma_I-1}}\\ &-\lambda\left[\sum_{i\in\mathcal{I}}\int_{n_i} p_{i,j}c_{i,j}^r dj - L^r w^r - \Pi^r\right] .\end{aligned}$$

We take the derivative with respect to consumption of a single good (i,j)

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial c_{i,j}^r} =& \frac{\sigma_I}{\sigma_I-1}\left[\sum_{\mathcal{I}}\left[\psi_i^{\frac{1}{\sigma_I-1}}\left[\int_{n_i} c_{i,j}^{r\ \frac{\sigma_v-1}{\sigma_v}}dj\right]^{\frac{\sigma_v}{\sigma_v-1}}\right]^{\frac{\sigma_I-1}{\sigma_I}}\right]^{\frac{1}{\sigma_I-1}}\\ &\frac{\sigma_I-1}{\sigma_I}\left[\psi_i^{\frac{1}{\sigma_I-1}}\left[\int_{n_i} c_{i,j}^{r\ \frac{\sigma_v-1}{\sigma_v}}dj\right]^{\frac{\sigma_v}{\sigma_v-1}}\right]^{-\frac{1}{\sigma_I}}\\ &\frac{\sigma_v}{\sigma_v-1}\psi_i^{\frac{1}{\sigma_I-1}}\left[\int_{n_i} c_{i,j}^{r\ \frac{\sigma_v-1}{\sigma_v}}dj\right]^{\frac{1}{\sigma_v-1}}\\ &\frac{\sigma_v-1}{\sigma_v}c_{i,j}^{r\ -\frac{1}{\sigma_v}} - \lambda p_{i,j} = 0 .\end{aligned}$$

The ratio of the marginal utility of consumption of good $c_{i,j}^r$ and of good $c_{i',j'}^r$ must be proportional to their respective prices

$$\frac{p_{i,j}}{p_{i',j'}} = \left[\frac{c_i^r}{c_{i'}^r}\right]^{\frac{\sigma_I-\sigma_v}{\sigma_I\sigma_v}}\left[\frac{c_{i,j}^r}{c_{i',j'}^r}\right]^{-\frac{1}{\sigma_v}}\left[\frac{\psi_i}{\psi_{i'}}\right]^{\frac{1}{\sigma_I}} , \tag{C.1}$$

where $c_i^r := \left[\int_{n_i} c_{i,j}^{r\ \frac{\sigma_v-1}{\sigma_v}}dj\right]^{\frac{\sigma_v}{\sigma_v-1}}$. Using Equation (C.1) and the budget constraint in (6.2), we can derive the following condition

$$c^r_{i',j'} = p^{-\sigma_v}_{i',j'} c^{r\,\frac{\sigma_I-\sigma_v}{\sigma_I}}_{i'} \psi^{\frac{\sigma_v}{\sigma_I}}_{i'} \left[\sum_{\mathcal{I}} \psi^{\frac{\sigma_v}{\sigma_I}}_i \int_{n_i} p^{1-\sigma_v}_{i,j} c^{r\,\frac{\sigma_I-\sigma_v}{\sigma_I}}_{i,j} dj\right]^{-1} [L^r w^r + \Pi^r] \ , \tag{C.2}$$

and the consumption choice of the single good $c^r_{i',j'}$ depends on its price, $p_{i',j'}$, and on the choice of consumption of the industry basket $c^r_{i'}$. We multiply optimality condition (C.2) with $p_{i',j'}$ and integrate over the variety domain. Using the industry price index $P_{i'} = \left[\int_{n_{i'}} p^{1-\sigma_v}_{i',j} dj\right]^{\frac{1}{1-\sigma_v}}$, and industry-specific expenditures $P_{i'} c^r_{i'} = \int_{n_{i'}} p_{i',j} c^r_{i',j} dj$, we rearrange and obtain

$$c^r_{i'} = P^{-\sigma_I}_{i'} \psi_{i'} \left[\sum_{\mathcal{I}} \psi^{\frac{\sigma_v}{\sigma_I}}_i \int_{n_i} p^{1-\sigma_v}_{i,j} c^{r\,\frac{\sigma_I-\sigma_v}{\sigma_I}}_{i,j} dj\right]^{-\frac{\sigma_I}{\sigma_v}} [L^r w^r + \Pi^r]^{\frac{\sigma_I}{\sigma_v}} \ . \tag{C.3}$$

Next we multiply optimality condition (C.3) with the price index of industry i', $P_{i'}$, and sum over the industry set. The overall price index is $P = \left[\sum_{\mathcal{I}} \psi_i P^{1-\sigma_I}_i\right]^{\frac{1}{1-\sigma_I}}$ and the budget constraint is binding, i.e. $L^r w^r + \Pi^r = \sum_{\mathcal{I}} P_i c^r_i$, and we obtain

$$c^r_{i'} = \psi_{i'} \left[\frac{p_{i'}}{P}\right]^{-\sigma_I} \frac{L^r w^r + \Pi^r}{P} \ . \tag{C.4}$$

Using conditions (C.2), (C.3) and (C.4), the following allocation choice for any $(i,j) \in \mathcal{I} \times n_i$ is derived:

$$c^r_{i,j} = \psi_i \left[\frac{p_{i,j}}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} \frac{L^r w^r + \Pi^r}{P} \ .$$

The latter condition can then be substituted into the definition of C^r to obtain

$$\sum_{\mathcal{I}} \int_{n_i} p_{i,j} c^r_{i,j} dj = P C^r = L^r w^r + \Pi^r \ ,$$

and the optimal consumption allocation is subsumed in the condition stated in the main text

$$c^r_{i,j} = \psi_i \left[\frac{p_{i,j}}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C^r \ .$$

C.1.2. Intuitive Derivation

A more intuitive, though less rigorously derived, approach to obtain the optimal consumption allocation function is shortly outlined. The utility function (6.1) can be rewritten and rearranged into two separate CES-utilities, which are

$$U^r := \left[\sum_{\mathcal{I}} \psi^{\frac{1}{\sigma_I}}_i c^{r\,\frac{\sigma_I-1}{\sigma_I}}_i\right]^{\frac{\sigma_I}{\sigma_I-1}}$$

and

$$c_i^r := \left[\int_{n_i} {c_{i,j}^r}^{\frac{\sigma_v - 1}{\sigma_v}} dj \right]^{\frac{\sigma_v}{\sigma_v - 1}} .$$

We now proceed in two steps. First we derive household r's demand for industry i's products, c_i^r, given some demand C^r. Second we derive the demand for a single product (i, j), $c_{i,j}^r$, given some demand c_i^r. Prices are given. The following equations show the consumption decision of household r across industries,

$$c_i^r = \psi_i \left[\frac{P_i}{P} \right]^{-\sigma_I} C^r ,$$

and the consumption decision within an industry for a variety,

$$c_{i,j}^r = \left[\frac{p_{i,j}}{P_i} \right]^{-\sigma_v} c_i^r ,$$

given the household r's total consumption C^r and prices $p_{i,j}$, P_i and P. As the two CES-utilities are nested, household r's demand for the single product (i, j) is

$$c_{i,j}^r = \psi_i \left[\frac{p_{i,j}}{P_i} \right]^{-\sigma_v} \left[\frac{P_i}{P} \right]^{-\sigma_I} C^r .$$

C.2. Simple Production Function

The production function in (6.4) is based on the O-ring theory of economic development. In our setup, a task is linked to the overall complexity of the production process through the task-complexity. In this regard, we deviate from the traditional production function based on the O-ring theory that assumes a continuum of tasks that all have to be performed successfully in order to obtain some output.

The quality choice of firms in (6.4) opens up an additional dimension along which firms must optimize. In principal, the model yields the same results if we assume a production function

$$x_i(r) = f(i, r) l_i(r) ,$$

where $f(i_R, r) > f(i_N, r)$ and $f_r(i, r) > 0$, and $\frac{f(i, r^h)}{f(i, r^l)}$ is independent of i (e.g. multiplicative separable), together with Assumption ASC, which relates critical levels of skills usable in production to task-complexities. With such a production function, workers also earn their marginal product and prices in the non-routine industry are higher. However, a function $f(i, r)$ lacks of economic interpretation. The production function in (6.4), with the endogenous choice of quality, yields the following convenient economic interpretations in the model:

- An intuitive rationale for services and goods having a high output when they are produced at high production quality for a given task-complexity, given the production process is successful. Thus, high skill levels have a comparative advantage in producing at high quality levels.

- A task's complexity—the task-complexity—is reflected in the prices of the goods and services because the probability of an unsuccessful production is the higher the more complex the task.

- Quality specifies ranges within certain skills have comparative advantages. A certain skill level has a comparative advantage whenever the production process can be executed at preferred quality by this skill level. On the one side of the range, there is the minimum-quality requirement, which is an intuitive explanation for the lower bound on this range. On the other side, as discussed in Appendix C.3, there is also an intuitive explanation for an upper bounds on production process enhancement. In such an environment the match between skill distribution and the industry-composition is crucial for efficient labor markets.

- Assumption ASC only matters for achieving analytical results, but is not necessary for the model dynamics. Thus, we could neglect Assumption ASC and the minimum-quality requirement would imply a smooth transition of the wage premium from 1 to ω along the skill distribution (see Section 6.4). Minimum-quality constraint in production processes are widely acknowledged. They come in the form of minimum standards (e.g. food), minimum functional requirement (e.g. machines), safety standards (e.g. cars), educational achievements (e.g. certificates) and official accreditations (e.g. notary).

- Workers of different skill levels are perfectly substitutable if their relative wages reflect their relative productivities and both are able to perform the task-complexity. Thus, production function (6.4) provides a rationale for quality differentiation in the production process.

Thus, we believe that production function (6.4) can be regarded as a useful attempt to provide a micro-foundation of the firms' employment choice underlying the skill-task-assignment in the framework.

C.3. Upper Bound of Process Enhancement

We assume that there is an upper bound on the possibility to enhance the production process through the production quality choice, denoted by $\bar{q}_i$. Note that in contrast to the lower bound of skill requirement defined by the ASC, high-skilled labor can be employed in industry i_m anytime, but the highest possible quality that can be chosen is $\bar{q}_{i_m}$. Therefore, if there is a large supply of high-skilled labor, then wages of the high-skilled might fall below 1. The quality choice of firms is

$$q_i(r) = \begin{cases} 1 & \text{if } \left[-\frac{1}{\lambda i \log(r)}\right]^{\frac{1}{\lambda}} < 1 \,, \\ \left[-\frac{1}{\lambda i \log(r)}\right]^{\frac{1}{\lambda}} & \text{if } \left[-\frac{1}{\lambda i \log(r)}\right]^{\frac{1}{\lambda}} \in [1, \bar{q}_i] \,, \\ \bar{q}_i & \text{if } \left[-\frac{1}{\lambda i \log(r)}\right]^{\frac{1}{\lambda}} > \bar{q}_i \,. \end{cases}$$

Sufficient skill now has an additional meaning. There must not only be enough high-skilled workers in the economy, but there must also be a right amount of low-skilled workers, in order that firms have an unconstrained quality choice.

In the following we assume that $\bar{q}_i = \bar{q}$. Thus, the constraint is always more binding for industry i_m, given some constraint skill level r. Table C.1 summarizes the four equilibria that can arise in such an environment. We neglect any equilibria where some type of labor is not used in production. This implies some skill allocation constraints.

Table C.1.: Quality Choice in the Production Process

Eq:	**Parallel**		**Triangle - (a)**	
Industry	i_m	i_a	i_m	i_a
r_h	$\left[-\frac{1}{\lambda i_m \log(r_h)}\right]^{\frac{1}{\lambda}}$	$\left[-\frac{1}{\lambda i_a \log(r_h)}\right]^{\frac{1}{\lambda}}$	$\left[-\frac{1}{\lambda i_m \log(r_h)}\right]^{\frac{1}{\lambda}}$	$\left[-\frac{1}{\lambda i_a \log(r_h)}\right]^{\frac{1}{\lambda}}$
r_l	$\left[-\frac{1}{\lambda i_m \log(r_l)}\right]^{\frac{1}{\lambda}}$	$\left[-\frac{1}{\lambda i_a \log(r_l)}\right]^{\frac{1}{\lambda}}$	$\left[-\frac{1}{\lambda i_m \log(r_l)}\right]^{\frac{1}{\lambda}}$	1
Eq:	**Triangle - (b)**		**Diagonal**	
r_h	$\bar{q}$	$\left[-\frac{1}{\lambda i_a \log(r_h)}\right]^{\frac{1}{\lambda}}$	$\bar{q}$	$\left[-\frac{1}{\lambda i_a \log(r_h)}\right]^{\frac{1}{\lambda}}$
r_l	$\left[-\frac{1}{\lambda i_m \log(r_l)}\right]^{\frac{1}{\lambda}}$	$\left[-\frac{1}{\lambda i_a \log(r_l)}\right]^{\frac{1}{\lambda}}$	$\left[-\frac{1}{\lambda i_m \log(r_l)}\right]^{\frac{1}{\lambda}}$	1

In the equilibria ‘Triangle - (a)’, ‘Triangle - (b)’ and ‘Diagonal’ the possibility of wage premia arises. We analyzed the equilibrium ‘Triangle - (a)’ extensively in the main text. Equilibrium ‘Triangle - (b)’ separates the workers according to their skill level to the two industries and thereby the equilibrium can reveal a wage premium for the low-skilled workers. Finally, the 'Diagonal'-equilibrium allows for either wage premia. The wage distribution is therefore generated by the interaction of the skill supply, the industry-composition (task-complexity-composition) of the economy and the demand. We disregard cases in which a skill level is constraint in both industries.

D. Proofs for Part II

D.1. Proof of Proposition 6.2

We show that $\mathcal{G}_{PE,ILM} < \mathcal{G}_{DLM} < \mathcal{G}_U$, each inequality in turn. Note first that there need to be different industry-compositions behind the different equilibrium outcomes. Otherwise, we could not compare the inequality within the different equilibria as we do in the following. Thus, the proof relies on the existence of different industry-compositions across economies.

- Suppose $\mathcal{G}_{PE,ILM} > \mathcal{G}_{DLM}$ then

$$\frac{\phi_{r_l}\left[\phi_{r_l} + 2\phi_{r_h}\right]\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}} + \phi_{r_h}^2}{\phi_{r_l}\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}} + \phi_{r_h}} < \frac{\phi_{r_l}\left[\phi_{r_l} + 2\phi_{r_h}\right]\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}} + \phi_{r_h}^2\omega}{\phi_{r_l}\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}} + \phi_{r_h}\omega}$$

  and multiplying out yields $[\omega - 1]\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}}\phi_{r_l}\phi_{r_h} < 0$. We know that $\omega > 1$. This contradiction implies that $\mathcal{G}_{PE,ILM} < \mathcal{G}_{DLM}$.

$\square$

- Suppose $\mathcal{G}_{DLM} > \mathcal{G}_U$ then

$$\frac{\phi_{r_l}\left[\phi_{r_l} + 2\phi_{r_h}\right]\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}} + \phi_{r_h}^2\omega}{\phi_{r_l}\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}} + \phi_{r_h}\omega} < \phi_{r_h}$$

  and multiplying out yields $\phi_{r_l} + \phi_{r_h}\left[\frac{\log(r_h)}{\log(r_l)}\right]^{\frac{1}{\lambda}} < 0$. This contradiction implies that $\mathcal{G}_{DLM} < \mathcal{G}_U$.

$\square$

D.2. Proofs for Chapter 7

D.2.1. Proof of Proposition 7.3

We derive that $\frac{\partial L_{in}^d}{\partial A_k} > 0$ for $\sigma_R > \max\{1, \sigma_I\}$. For notational convenience we define $j_N = n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}}$, and $j_R = n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}}$, and $j_A = [\sigma_R - 1]\left[\frac{i_R}{i_N}\right]^{\frac{\sigma_R-1}{\lambda}} A_k^{\sigma_R-2}$. Note that when $\omega = 1$, then $\tilde{\mathcal{M}} = j_N + j_R\tilde{\theta}(1, A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}$ and that $1 - \left[A_k\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}\tilde{\theta}(1, A_k)^{-1} = \tilde{\theta}(1, A_k)^{-1}$. Then we take the partial derivative of labor demand from the non-routine industry with respect to the technological factor A_k and obtain

$$
\begin{aligned}
\frac{\partial L_{in}^d}{\partial A_k} =& \frac{\partial}{\partial A_k}\left\{j_N + j_R\tilde{\theta}(1, A_k)^{\frac{\sigma_I-\sigma_R}{\sigma_R-1}}\left[A_k\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}\right\}\frac{\tilde{L}}{\tilde{\mathcal{M}}} \\
=& j_N j_R j_A\frac{1-\sigma_I}{\sigma_R-1}\tilde{\theta}(1, A_k)^{\frac{\sigma_I-\sigma_R}{\sigma_R-1}}\frac{\tilde{L}}{\tilde{\mathcal{M}}^2} + j_R j_A\frac{\sigma_I-\sigma_R}{\sigma_R-1}\tilde{\theta}(1, A_k)^{\frac{\sigma_I-\sigma_R}{\sigma_R-1}-1}\left[A_k\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}\frac{\tilde{L}}{\tilde{\mathcal{M}}} + \\
& j_R^2 j_A\frac{1-\sigma_I}{\sigma_R-1}\tilde{\theta}(1, A_k)^{2\frac{\sigma_I-\sigma_R}{\sigma_R-1}}\left[A_k\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}\frac{\tilde{L}}{\tilde{\mathcal{M}}^2} + j_R j_A\tilde{\theta}(1, A_k)^{\frac{\sigma_I-\sigma_R}{\sigma_R-1}}\frac{\tilde{L}}{\tilde{\mathcal{M}}} \\
=& \frac{\tilde{L}}{\tilde{\mathcal{M}}^2}\frac{j_A j_R}{\sigma_R-1}\tilde{\theta}(1, A_k)^{\frac{\sigma_I-\sigma_R}{\sigma_R-1}}\left\{[1-\sigma_I]\,j_N + [\sigma_I-\sigma_R]\frac{\tilde{\mathcal{M}}}{\tilde{\theta}(1, A_k)}\left[A_k\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1} + \right. \\
& \left. [1-\sigma_I]\,j_R\tilde{\theta}(1, A_k)^{\frac{\sigma_I-\sigma_R}{\sigma_R-1}}\left[A_k\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1} + [\sigma_R-1]\,\tilde{\mathcal{M}}\right\} \\
=& \frac{\tilde{L}}{\tilde{\mathcal{M}}^2}\frac{j_A j_R}{\sigma_R-1}\tilde{\theta}(1, A_k)^{\frac{\sigma_I-\sigma_R}{\sigma_R-1}}\left\{[1-\sigma_I]\,j_N + [\sigma_I-\sigma_R]\frac{j_N}{\tilde{\theta}(1, A_k)}\left[A_k\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1} + \right. \\
& [\sigma_I-\sigma_R]\,j_R\tilde{\theta}(1, A_k)^{\frac{\sigma_I-\sigma_R}{\sigma_R-1}}\left[A_k\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1} + \\
& \left. [1-\sigma_I]\,j_R\tilde{\theta}(1, A_k)^{\frac{\sigma_I-\sigma_R}{\sigma_R-1}}\left[A_k\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1} + [\sigma_R-1]\,j_N + [\sigma_R-1]\,j_R\tilde{\theta}(1, A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}\right\} \\
=& \frac{\tilde{L}}{\tilde{\mathcal{M}}^2}\frac{j_A j_R}{\sigma_R-1}\tilde{\theta}(1, A_k)^{\frac{\sigma_I-\sigma_R}{\sigma_R-1}}\left\{[\sigma_R-\sigma_I]\,j_N + [\sigma_I-\sigma_R]\frac{j_N}{\tilde{\theta}(1, A_k)}\left[A_k\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1} + \right. \\
& \left. [1-\sigma_R]\,j_R\tilde{\theta}(1, A_k)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}}\left[A_k\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1} + [\sigma_R-1]\,j_R\tilde{\theta}(1, A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}\right\} \\
=& \frac{\tilde{L}}{\tilde{\mathcal{M}}^2}\frac{j_A j_R}{\sigma_R-1}\tilde{\theta}(1, A_k)^{\frac{\sigma_I-\sigma_R}{\sigma_R-1}}\left\{[\sigma_R-\sigma_I]\,j_N\tilde{\theta}(1, A_k)^{-1} + [\sigma_R-1]\,j_R\tilde{\theta}(1, A_k)^{\frac{\sigma_I-\sigma_R}{\sigma_R-1}}\right\} > 0\,.
\end{aligned}
$$

We have shown that for $\sigma_R > \max\{1, \sigma_I\}$ it holds that $\frac{\partial L_{in}^d}{\partial A_k} > 0$.

□

D.2.2. Proof of Corollary 7.4

For notational convenience we use $j_N = n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}}$, and $j_R = n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}}$, and $\mu(A_k) = \left[A_k \left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R - 1}$. From the main text we know that

$$\frac{\partial \tilde{w}^r}{\partial A_k} = \frac{\sigma_v - 1}{\sigma_v} \left[-e\lambda \log(r)\right]^{-\frac{1}{\lambda}} \tilde{\mathcal{M}}^{\frac{1}{\sigma_I - 1} - 1} \times$$
$$\left[j_R \tilde{\theta}(\omega, A_k)^{\frac{\sigma_R - \sigma_I}{1-\sigma_R}} \omega^{1-\sigma_R} \frac{\mu(A_k)}{A_k} - \left[j_N \omega^{-\sigma_I} + j_R \tilde{\theta}(\omega, A_k)^{\frac{\sigma_R - \sigma_I}{1-\sigma_R}} \omega^{-\sigma_R} \mu(A_k) \right] \frac{\partial \omega}{\partial A_k} \right] .$$

We focus on the second line of the derivative above, which determines the sign, and we divide by $j_R \tilde{\theta}(\omega, A_k)^{\frac{\sigma_R - \sigma_I}{1-\sigma_R}} \omega^{-\sigma_R} \mu(A_k) \frac{\partial \omega}{\partial A_k}$. Suppose now the following term is negative

$$\frac{\omega}{A_k} \frac{\partial A_k}{\partial \omega} - \tilde{X} \omega^{\sigma_R - \sigma_I} \tilde{\theta}(\omega, A_k)^{\frac{\sigma_R - \sigma_I}{\sigma_R - 1}} \mu(A_k)^{-1} - 1 < 0 \, , \tag{D.1}$$

where $\tilde{X} = \frac{j_N}{j_R}$. From function $\mathcal{F}$ in (7.16), we know that $\tilde{X}\tilde{\theta}(\omega, A_k)^{\frac{\sigma_R - \sigma_I}{\sigma_R - 1}} \omega^{-\sigma_I} = \frac{\tilde{\phi}_N}{\tilde{\phi}_R} [1 - z]$. We further use the equivalence $\omega^{\sigma_R} \mu(A_k)^{-1} = \frac{\tilde{\phi}_R}{\tilde{\phi}_N} z^{-1}$ and obtain for (D.1)

$$\frac{\omega}{A_k} \frac{\partial A_k}{\partial \omega} - \frac{1}{z} < 0 \, .$$

The partial derivative of the real wage of a low-skilled worker with respect to technological change in a DLM equilibrium of a robotic economy is negative whenever

$$\sigma_{A_k, \omega} > z \, .$$

and positive whenever the inequality is reversed.

□

D.2.3. Proof of Condition DRWC-LS

We use the Implicit Function Theorem to rewrite the condition presented in Corollary 7.4 and Appendix D.2.2, i.e.

$$\frac{\partial \mathcal{F}}{\partial A_k} > -z \frac{\partial \mathcal{F}}{\partial \omega} \frac{\omega}{A_k} \, .$$

Using again $\tilde{X}\tilde{\theta}(\omega, A_k)^{\frac{\sigma_R - \sigma_I}{\sigma_R - 1}} \omega^{-\sigma_I} = \frac{\tilde{\phi}_N}{\tilde{\phi}_R} [1 - z]$, and noting that $\tilde{\theta}(\omega, A_k) = 1 + \omega^{1-\sigma_R} \mu(A_k)$, and that $\omega^{1-\sigma_R}\mu(A_k) = z\omega\frac{\tilde{\phi}_N}{\tilde{\phi}_R}$, we arrive after some algebraic manipulations at

$$[\sigma_R - \sigma_I]\,\frac{1+\omega\frac{\tilde{\phi}_N}{\tilde{\phi}_R}}{1+z\omega\frac{\tilde{\phi}_N}{\tilde{\phi}_R}} > \frac{1}{1-z}\,,$$

which depicts the DRWC-LS in the main text, i.e. the condition under which the real wage of low-skilled workers decreases with technological progress in the manufacturing industry.

□

D.2.4. Proof of Corollary 7.5

The partial derivative of z with respect to A_k, $\frac{\partial z}{\partial A_k}$, is greater than zero if $\sigma_{A_k,\omega} < \frac{\sigma_R - 1}{\sigma_R}$. Using again the Implicit Function Theorem the inequality can be rewritten as

$$\frac{1-\sigma_R}{\sigma_R}\frac{\partial \mathcal{F}}{\partial \omega} > \frac{A_k}{\omega}\frac{\partial \mathcal{F}}{\partial A_k}\,.$$

Further manipulations yield

$$\begin{aligned}
&\frac{1-\sigma_R}{\sigma_R}\tilde{X}\tilde{\theta}(\omega,A_k)^{\frac{\sigma_I-\sigma_R}{1-\sigma_R}}\omega^{-\sigma_I}\left[[\sigma_I-\sigma_R]\,\tilde{\theta}(\omega,A_k)^{-1}\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}-\sigma_I\right] \\
&-[1-\sigma_R]\,\omega^{-\sigma_R}\left[A_k\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1} \\
&>\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}\left[\tilde{X}\,[\sigma_R-\sigma_I]\,\tilde{\theta}(\omega,A_k)^{\frac{\sigma_I-1}{1-\sigma_R}}\omega^{-\sigma_I}+[\sigma_R-1]\,\omega^{-1}\right] \\
\Longleftrightarrow\quad &\frac{1-\sigma_R}{\sigma_R}\tilde{X}\tilde{\theta}(\omega,A_k)^{\frac{\sigma_I-\sigma_R}{1-\sigma_R}}\omega^{-\sigma_I}\left[[\sigma_I-\sigma_R]\,\tilde{\theta}(\omega,A_k)^{-1}\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}-\sigma_I\right] \\
&>\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}\tilde{X}\,[\sigma_R-\sigma_I]\,\tilde{\theta}(\omega,A_k)^{\frac{\sigma_I-1}{1-\sigma_R}}\omega^{-\sigma_I} \\
\Longleftrightarrow\quad &\frac{1-\sigma_R}{\sigma_R}\tilde{\theta}(\omega,A_k)\left[[\sigma_I-\sigma_R]\,\tilde{\theta}(\omega,A_k)^{-1}\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}-\sigma_I\right]+[\sigma_I-\sigma_R]\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}>0 \\
\Longleftrightarrow\quad &[1-\sigma_R]\left[[\sigma_I-\sigma_R]\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}-\sigma_I\tilde{\theta}(\omega,A_k)\right]+\sigma_R\,[\sigma_I-\sigma_R]\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}>0 \qquad (\star) \\
\Longleftrightarrow\quad &\frac{\sigma_R-1}{\sigma_R}+\frac{\sigma_I-1}{\sigma_I}\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}>0\,.
\end{aligned}$$

This is equivalent to the expression stated in Corollary 7.5. Whenever $\frac{\sigma_R-1}{\sigma_R}+\frac{\sigma_I-1}{\sigma_I}\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1} > 0$ the share of high-skilled workers demanded in the production of robots increases with the productivity gains in the manufacturing industry.

□

Repeating the manipulations starting at $\sigma_{A_k,\omega} < 1$, we can show that this inequality indeed must be true. Intuitively, the elasticity of the wage premium with respect to technological progress can never exceed 1, i.e. the wage premium cannot increase faster than the productivity gains from an increase in A_k. We start at the line indicated by $(\star)$ in the above derivation, divide by σ_R, and replace the resulting factor $\frac{1-\sigma_R}{\sigma_R}$ with (-1) and obtain

$$\begin{aligned} 0 <& - \left[[\sigma_I - \sigma_R] \left[\frac{A_k}{\omega} \left[\frac{i_R}{i_N} \right]^{\frac{1}{\lambda}} \right]^{\sigma_R - 1} - \sigma_I \tilde{\theta}(\omega, A_k) \right] + [\sigma_I - \sigma_R] \left[\frac{A_k}{\omega} \left[\frac{i_R}{i_N} \right]^{\frac{1}{\lambda}} \right]^{\sigma_R - 1} \\ =& \sigma_I \tilde{\theta}(\omega, A_k) \ . \end{aligned}$$

□

D.2.5. Proofs for Table 7.1

For the analysis of a single household's consumption we must know how profits are distributed in the economy. For sake of simplicity we assumed in the main text that profits are distributed proportional to the wage distribution. Therefore total income of every household is $\frac{\sigma_v}{\sigma_v - 1}\omega(r)w^r$. For notational convenience we use $j_N = n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}}$, and $j_R = n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}}$, and $\hat{\theta}(A_k) = 1 + A_k^{\sigma_R - 1}$. A household's demand (7.4) together with the assumption about the profit distribution lets us obtain a single household's total consumption

$$C^r = [-e\lambda \log(\bar{r})]^{-\frac{1}{\lambda}} \hat{M}^{\frac{1}{\sigma_I - 1}} \omega(r) L^r w^r \ .$$

Households cannot save, therefore a single household's total income $\frac{\sigma_v}{\sigma_v - 1}\omega(r)w^r$ deflated by the aggregate price index P is equal to this household's total consumption C^r. Note that $\omega = 1$ for all low-skilled households. Integrating over all households yields total consumption in the economy, (7.7). We can now obtain all consumption decisions of households.

- Consumption of service i_N by a high-skilled household $r^h > \tilde{r}(i_N)$:

$$\begin{aligned}\frac{\partial c_{i_N}^{r^h}}{\partial A_k} =& \frac{\partial}{\partial A_k} n_N^{-1} j_N \left[-e\lambda i_N \log(\bar{r})\right]^{-\frac{1}{\lambda}} \omega^{1-\sigma_I} \hat{\mathcal{M}}^{-1} L^r w^r \\ =& \left[1-\sigma_I\right] n_N^{-1} j_N \left[-e\lambda i_N \log(\bar{r})\right]^{-\frac{1}{\lambda}} \omega^{1-\sigma_I} \hat{\mathcal{M}}^{-2} \hat{\theta}(A_k)^{-1} A_k^{\sigma_R-2} L^r w^r \times \\ & \left[\frac{1-\sigma_I}{\sigma_I}\hat{\mathcal{M}} - \frac{1-\sigma_I}{\sigma_I} j_N \omega^{1-\sigma_I} + j_R \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}\right] \\ =& \left[1-\sigma_I\right] n_N^{-1} j_N \left[-e\lambda i_N \log(\bar{r})\right]^{-\frac{1}{\lambda}} \omega^{1-\sigma_I} \hat{\mathcal{M}}^{-2} \hat{\theta}(A_k)^{-1} A_k^{\sigma_R-2} L^r w^r \times \\ & \frac{j_R}{\sigma_I} \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}} ,\end{aligned}$$

which is > 0, if $\sigma_I < 1$ (and vice-versa).

□

- Consumption of service i_N by a low-skilled household $r^l < \tilde{r}(i_N)$:

$$\begin{aligned}\frac{\partial c_{i_N}^{r^l}}{\partial A_k} =& \frac{\partial}{\partial A_k} n_N^{-1} j_N \left[-e\lambda i_N \log(\bar{r})\right]^{-\frac{1}{\lambda}} \omega^{-\sigma_I} \hat{\mathcal{M}}^{-1} L^r w^r \\ =& \left[1-\sigma_I\right] n_N^{-1} j_N \left[-e\lambda i_N \log(\bar{r})\right]^{-\frac{1}{\lambda}} \omega^{-\sigma_I} \hat{\mathcal{M}}^{-2} \hat{\theta}(A_k)^{-1} A_k^{\sigma_R-2} L^r w^r \times \\ & \left[-\hat{\mathcal{M}} - \frac{1-\sigma_I}{\sigma_I} j_N \omega^{1-\sigma_I} + j_R \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}\right] \\ =& \left[1-\sigma_I\right] n_N^{-1} j_N \left[-e\lambda i_N \log(\bar{r})\right]^{-\frac{1}{\lambda}} \omega^{-\sigma_I} \hat{\mathcal{M}}^{-2} \hat{\theta}(A_k)^{-1} A_k^{\sigma_R-2} L^r w^r \times \\ & (-1) \frac{j_N}{\sigma_I} \omega^{1-\sigma_I} ,\end{aligned}$$

which is > 0, if $\sigma_I > 1$ (and vice-versa).

□

- Consumption of service i_R by a high-skilled household $r^h > \tilde{r}(i_N)$:

$$\begin{aligned}\frac{\partial c_{i_R}^{r^h}}{\partial A_k} =& \frac{\partial}{\partial A_k} n_R^{-1} j_R \left[-e\lambda i_R \log(\bar{r})\right]^{-\frac{1}{\lambda}} \hat{\theta}(A_k)^{\frac{\sigma_I}{\sigma_R-1}} \omega \hat{\mathcal{M}}^{-1} L^r w^r \\ =& n_R^{-1} j_R \left[-e\lambda i_R \log(\bar{r})\right]^{-\frac{1}{\lambda}} \hat{\theta}(A_k)^{\frac{\sigma_I}{\sigma_R-1}-1} \omega \hat{\mathcal{M}}^{-2} A_k^{\sigma_R-2} L^r w^r \times \\ & \left[\left[\frac{\left[1-\sigma_I\right]^2}{\sigma_I} + 1\right] \hat{\mathcal{M}} - \frac{\left[1-\sigma_I\right]^2}{\sigma_I} j_N \omega^{1-\sigma_I} + \left[1-\sigma_I\right] j_R \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}\right] \\ =& n_R^{-1} j_R \left[-e\lambda i_R \log(\bar{r})\right]^{-\frac{1}{\lambda}} \hat{\theta}(A_k)^{\frac{\sigma_I}{\sigma_R-1}-1} \omega \hat{\mathcal{M}}^{-2} A_k^{\sigma_R-2} L^r w^r \times \\ & \left[\hat{\mathcal{M}} + \frac{1-\sigma_I}{\sigma_I} j_R \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}\right] ,\end{aligned}$$

which is > 0, always.

□

- Consumption of service i_R by a low-skilled household $r^l < \tilde{r}(i_N)$:

$$\begin{aligned}\frac{\partial c_{i_R}^{r^l}}{\partial A_k} =& \frac{\partial}{\partial A_k} n_R^{-1} j_R \left[-e\lambda i_R \log(\bar{r})\right]^{-\frac{1}{\lambda}} \hat{\theta}(A_k)^{\frac{\sigma_I}{\sigma_R - 1}} \hat{\mathcal{M}}^{-1} L^r w^r \\ =& n_R^{-1} j_R \left[-e\lambda i_R \log(\bar{r})\right]^{-\frac{1}{\lambda}} \hat{\theta}(A_k)^{\frac{\sigma_I}{\sigma_R - 1} - 1} \hat{\mathcal{M}}^{-2} A_k^{\sigma_R - 2} L^r w^r \times \\ & \left[\sigma_I \hat{\mathcal{M}} - \frac{\left[1-\sigma_I\right]^2}{\sigma_I} j_N \omega^{1-\sigma_I} + \left[1-\sigma_I\right] j_R \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}\right] \\ =& n_R^{-1} j_R \left[-e\lambda i_R \log(\bar{r})\right]^{-\frac{1}{\lambda}} \hat{\theta}(A_k)^{\frac{\sigma_I}{\sigma_R - 1} - 1} \hat{\mathcal{M}}^{-2} A_k^{\sigma_R - 2} L^r w^r \times \\ & \left[\hat{\mathcal{M}} - \frac{1-\sigma_I}{\sigma_I} j_N \omega^{1-\sigma_I}\right] ,\end{aligned}$$

which > 0, whenever $\sigma_I > \frac{\omega \tilde{\phi}_N}{2\omega \tilde{\phi}_N + \tilde{\phi}_R}$ (and vice-versa).

□

D.2.6. Proof of Equivalence between Consumption Changes and Proposition 7.2

First recall that a household's r consumption bundle (and utility) is

$$C^r := \left[\sum_{i \in \mathcal{I}} \left[\left[\int_{n_i} c_{i,j}^{r\ \frac{\sigma_v - 1}{\sigma_v}} dj\right]^{\frac{\sigma_v}{\sigma_v - 1}}\right]^{\frac{\sigma_I - 1}{\sigma_I}}\right]^{\frac{\sigma_I}{\sigma_I - 1}} .$$

We use households' demand (7.4) to derive the following results. Symmetry among services within an industry implies that $C_{i_R}^r = n_R^{\frac{\sigma_v}{\sigma_v - 1}} c_{i_R}^r$, where $c_{i_R}^r$ denotes a variety and where $C_{i_R}^r$ denotes the industry aggregate, both consumed by household r. In the following we will analyze the change in consumption of a low-skilled household, $r^l < \tilde{r}(i_N)$. But first we derive the consumption ratio of the two industry consumption bundles for household r^l, which we will use in the subsequent analysis,

$$\begin{aligned}\left[\frac{C_{i_R}^{r^l}}{C_{i_N}^{r^l}}\right]^{-\frac{1}{\sigma_I}} &= \left[\frac{n_R^{\frac{\sigma_v}{\sigma_v - 1}} c_{i_R}^r}{n_N^{\frac{\sigma_v}{\sigma_v - 1}} c_{i_N}^r}\right]^{-\frac{1}{\sigma_I}} \\ &= \left[\frac{n_R^{\frac{\sigma_v}{\sigma_v - 1}} n_R^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} i_R^{-\frac{\sigma_I}{\lambda}} \hat{\theta}(A_k)^{\frac{\sigma_I}{\sigma_R - 1}} \hat{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C^{r^l}}{n_N^{\frac{\sigma_v}{\sigma_v - 1}} n_N^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} i_N^{-\frac{\sigma_I}{\lambda}} \omega^{-\sigma_I} \hat{\mathcal{M}}^{\frac{\sigma_I}{1-\sigma_I}} C^{r^l}}\right]^{-\frac{1}{\sigma_I}} \\ &= \left[\frac{n_R}{n_N}\right]^{\frac{1}{1-\sigma_v}} \left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}} \hat{\theta}(A_k)^{\frac{1}{1-\sigma_R}} \omega^{-1} .\end{aligned}$$

The adjustments in the consumption bundle, C^r, have to be equal to the changes in the real wage. Thus, we analyze under what conditions consumption (and thus utility) increases or decreases. Results must match our derivation of the changes in the real wage shown in Proposition 7.2. Using the derived ratio of industry consumption bundles above and partial derivatives presented in Appendix D.2.5 we infer that

$$
\begin{aligned}
\frac{\partial C^{r^l}}{\partial A_k} = \quad & C^{r^l \frac{1}{\sigma_I}} \left[C_{i_R}^{r^l \, -\frac{1}{\sigma_I}} n_R^{\frac{1}{\sigma_v - 1}} \frac{\partial c_{i_R}^{r^l}}{\partial A_k} + C_{i_N}^{r^l \, -\frac{1}{\sigma_I}} n_N^{\frac{1}{\sigma_v - 1}} \frac{\partial c_{i_N}^{r^l}}{\partial A_k} \right] > 0 \\
\Longleftrightarrow \quad & \left[\frac{C_{i_R}^{r^l}}{C_{i_N}^{r^l}} \right]^{-\frac{1}{\sigma_I}} n_R^{\frac{1}{\sigma_v - 1}} j_R \left[-e\lambda i_R \log(\bar{r}) \right]^{-\frac{1}{\lambda}} \hat{\theta}(A_k)^{\frac{\sigma_I}{\sigma_R - 1} - 1} \frac{A_k^{\sigma_R - 2}}{\hat{\mathcal{M}}^2} L^r w^r \left[\hat{\mathcal{M}} - \frac{1 - \sigma_I}{\sigma_I} j_N \omega^{1-\sigma_I} \right] + \\
& \frac{1 - \sigma_I}{\sigma_I} n_N^{\frac{1}{\sigma_v - 1}} j_N \left[-e\lambda i_N \log(\bar{r}) \right]^{-\frac{1}{\lambda}} \omega^{-\sigma_I} \hat{\theta}(A_k)^{-1} \frac{A_k^{\sigma_R - 2}}{\hat{\mathcal{M}}^2} L^r w^r (-1) j_N \omega^{1-\sigma_I} > 0 \\
\Longleftrightarrow \quad & \left[\frac{n_R}{n_N} \right]^{\frac{1}{1-\sigma_v}} \left[\frac{i_R}{i_N} \right]^{\frac{1}{\lambda}} \hat{\theta}(A_k)^{\frac{1}{1-\sigma_R}} \omega^{-1} \times n_R^{\frac{1}{\sigma_v - 1}} j_R i_R^{-\frac{1}{\lambda}} \hat{\theta}(A_k)^{\frac{\sigma_I}{\sigma_R - 1}} \left[\hat{\mathcal{M}} - \frac{1 - \sigma_I}{\sigma_I} j_N \omega^{1-\sigma_I} \right] - \\
& \frac{1 - \sigma_I}{\sigma_I} n_N^{\frac{1}{\sigma_v - 1}} i_N^{-\frac{1}{\lambda}} \omega^{-\sigma_I} j_N^2 \omega^{1-\sigma_I} > 0 \\
\Longleftrightarrow \quad & \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}} \omega^{-1} j_R \left[\hat{\mathcal{M}} - \frac{1 - \sigma_I}{\sigma_I} j_N \omega^{1-\sigma_I} \right] - \frac{1 - \sigma_I}{\sigma_I} \omega^{-\sigma_I} j_N^2 \omega^{1-\sigma_I} > 0 \\
\Longleftrightarrow \quad & \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}} \frac{j_R}{j_N} \left[j_N \omega^{1-\sigma_I} + j_R \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}} - \frac{1 - \sigma_I}{\sigma_I} j_N \omega^{1-\sigma_I} \right] - \frac{1 - \sigma_I}{\sigma_I} j_N \omega^{2[1-\sigma_I]} > 0 \\
\Longleftrightarrow \quad & \omega^{-\sigma_I} \frac{\tilde{\phi}_R}{\tilde{\phi}_N} \left[1 + \frac{j_R}{j_N} \omega^{\sigma_I - 1} \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}} - \frac{1 - \sigma_I}{\sigma_I} \right] - \frac{1 - \sigma_I}{\sigma_I} \omega^{1-\sigma_I} > 0 \\
\Longleftrightarrow \quad & \frac{\tilde{\phi}_R}{\omega \tilde{\phi}_N} \left[1 + \frac{\omega \tilde{\phi}_N}{\tilde{\phi}_R} - \frac{1 - \sigma_I}{\sigma_I} \right] - \frac{1 - \sigma_I}{\sigma_I} > 0 \, ,
\end{aligned}
$$

whenever $\sigma_I > \frac{\omega \tilde{\phi}_N}{\omega \tilde{\phi}_N + \tilde{\phi}_R}$. Consumption of household r^l only increases if $\sigma_I > \frac{\omega \tilde{\phi}_N}{\omega \tilde{\phi}_N + \tilde{\phi}_R}$, which is the condition for a real wage increase for routine labor. The result is equal to our analysis of the real wage for $r < \tilde{r}(i_N)$ in Proposition 7.2.

□

D.2.7. Proof of Constant i_N-Service Production

The i_N-industry adjusts its labor input in response to technological progress according to

$$
\begin{aligned}
\frac{\partial \tilde{L}_{i_N}^d}{\partial A_k} = & \frac{\partial}{\partial A_k} j_N \omega^{-\sigma_I} \hat{\mathcal{W}}^{-1} \tilde{L} \\
= & \left[\sigma_I - 1 \right] j_N \omega^{-\sigma_I} \hat{\mathcal{W}}^{-2} \hat{\theta}(A_k)^{-1} A_k^{\sigma_R - 2} \tilde{L} \left[\hat{\mathcal{W}} - j_N \omega^{-\sigma_I} + j_R \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}} \right] \\
= & 0 \, ,
\end{aligned}
$$

i.e. the labor input remains constant. The supply of high-skilled labor is constraint. As long as there are no sufficient skills in the economy, technological progress does not affect labor input in industry i_N. However, technological progress affects the wage premium.

This intuitive result can also be shown in consumption terms. If there is equal labor input in industry i_N and technological progress only occurs in the manufacturing industry, then it must hold that consumption changes of the two skill groups in response to technological progress must balance each other,

$$\int_{\tilde{r}(i_N)}^{\bar{r}} \frac{\partial C^r_{i_N}}{\partial A_k} f(r)dr = -\int_{\underline{r}}^{\tilde{r}(i_N)} \frac{\partial C^r_{i_N}}{\partial A_k} f(r)dr$$

$$\int_{\tilde{r}(i_N)}^{\bar{r}} L^r w^r j_R \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}} f(r)dr = \int_{\underline{r}}^{\tilde{r}(i_N)} L^r w^r j_N \omega^{-\sigma_I} f(r)dr$$

$$\tilde{\phi}_N j_R \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}} = \tilde{\phi}_R j_N \omega^{-\sigma_I}$$

$$\omega = \left[\frac{\tilde{\phi}_R}{\tilde{\phi}_N} \left[\frac{i_N}{i_R}\right]^{\frac{1-\sigma_I}{\lambda}} \left[\frac{n_N}{n_R}\right]^{\frac{1-\sigma_I}{1-\sigma_v}} \hat{\theta}(A_k)^{\frac{1-\sigma_I}{\sigma_R-1}} \right]^{\frac{1}{\sigma_I}} ,$$

where the last equality is the equilibrium outcome of the wage premium (7.8), and thereby the equivalence is shown.

□

D.2.8. Proofs for Table 7.2

We analyze household consumption. We assumed in the main text that profits are distributed proportional to the wage distribution, i.e. total income of every household is $\frac{\sigma_v}{\sigma_v-1}\omega(r)w^r$. For notational convenience we use again $j_N = n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}}$, and $j_R = n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}}$, and $\mu(A_k) = \left[A_k \left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}$. Note that $\frac{j_N}{j_R} = \tilde{X}$. We use household demand (7.14) and the assumption about the profit distribution to obtain a household's total consumption:

$$C^r = [-e\lambda \log(\bar{r})]^{-\frac{1}{\lambda}} \tilde{M}^{\frac{1}{\sigma_I-1}} \omega(r) L^r w^r .$$

This is essentially a household's total income $\frac{\sigma_v}{\sigma_v-1}\omega(r)w^r$ deflated by the aggregate price index P, that must equal a household's consumption C^r. Integrating over all households yields total consumption in the economy, (7.15). We can now obtain all consumption decisions of households.

- Consumption of service i_N by a high-skilled household $r^h > \tilde{r}(i_N)$:

$$\begin{aligned}\frac{\partial c_{i_N}^{r^h}}{\partial A_k} =& \frac{\partial}{\partial A_k} n_N^{-1} j_N \left[-e\lambda i_N \log(\bar{r})\right]^{-\frac{1}{\lambda}} \omega^{1-\sigma_I} \tilde{\mathcal{M}}^{-1} L^r w^r \\ =& \left[1-\sigma_I\right] n_N^{-1} j_N \left[-e\lambda i_N \log(\bar{r})\right]^{-\frac{1}{\lambda}} \omega^{1-\sigma_I} \tilde{\mathcal{M}}^{-2} A_k^{-1} L^r w^r \times \\ & \left[\tilde{\mathcal{M}} \sigma_{A_k,\omega} - j_N \omega^{1-\sigma_I} \sigma_{A_k,\omega} - j_R \tilde{\theta}(A_k,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}} \frac{\mu(A_k)}{\omega^{\sigma_R-1}} \left[\sigma_{A_k,\omega} - 1\right]\right] \\ =& \left[1-\sigma_I\right] n_N^{-1} j_N \left[-e\lambda i_N \log(\bar{r})\right]^{-\frac{1}{\lambda}} \omega^{1-\sigma_I} \tilde{\mathcal{M}}^{-2} A_k^{-1} L^r w^r \times \\ & \left[j_R \tilde{\theta}(A_k,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}} \left[\sigma_{A_k,\omega} + \frac{\mu(A_k)}{\omega^{\sigma_R-1}}\right]\right],\end{aligned}$$

which is > 0, if $\sigma_I < 1$ (and vice-versa).

□

- Consumption of service i_N by a low-skilled household $r^l < \tilde{r}(i_N)$:

$$\begin{aligned}\frac{\partial c_{i_N}^{r^l}}{\partial A_k} =& \frac{\partial}{\partial A_k} n_N^{-1} j_N \left[-e\lambda i_N \log(\bar{r})\right]^{-\frac{1}{\lambda}} \omega^{-\sigma_I} \tilde{\mathcal{M}}^{-1} L^r w^r \\ =& n_N^{-1} j_N \left[-e\lambda i_N \log(\bar{r})\right]^{-\frac{1}{\lambda}} \omega^{-\sigma_I} \tilde{\mathcal{M}}^{-2} A_k^{-1} L^r w^r \times \\ & \left[-\sigma_I \tilde{\mathcal{M}} \sigma_{A_k,\omega} + \left[\sigma_I - 1\right] j_N \omega^{1-\sigma_I} \sigma_{A_k,\omega} + \left[\sigma_I - 1\right] j_R \tilde{\theta}(A_k,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}} \frac{\mu(A_k)}{\omega^{\sigma_R-1}} \left[\sigma_{A_k,\omega} - 1\right]\right] \\ =& n_N^{-1} j_N \left[-e\lambda i_N \log(\bar{r})\right]^{-\frac{1}{\lambda}} \omega^{-\sigma_I} \tilde{\mathcal{M}}^{-2} A_k^{-1} L^r w^r \times \\ & \left[\left[1-\sigma_I\right] j_R \tilde{\theta}(A_k,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}} \left[\sigma_{A_k,\omega} + \frac{\mu(A_k)}{\omega^{\sigma_R-1}}\right] - \tilde{\mathcal{M}} \sigma_{A_k,\omega}\right],\end{aligned}$$

which is > 0, if $\sigma_I < \frac{\frac{\mu(A_k)}{\omega^{\sigma_R-1}}\left[1-\frac{\sigma_{A_k,\omega}}{z}\right]}{\frac{\mu(A_k)}{\omega^{\sigma_R-1}}+\sigma_{A_k,\omega}} := \kappa_{i_N,R}$ (and vice-versa). Observe that $\kappa_{i_N,R} < 1$ always. Furthermore, whenever DRWC-LS holds, then $\sigma_{A_k,\omega} > z$ and σ_I would have to be negative, for the consumption of services i_N to increase. This is not possible, as $\sigma_I > 0$ by assumption. Therefore, consumption of services i_n by the low-skilled increases with A_k if $\sigma_I < \kappa_{i_N,R}$ and always decreases if their real wage decreases.

□

- Consumption of service i_R by a high-skilled household $r^h > \tilde{r}(i_N)$:
$c_{i_R}^{r^h}$ must always increase in A_k. Firstly, the prices of services i_R decreases in A_k. And secondly, high-skilled workers always earn higher real wages when A_k increases. Thus, the derivative of $c_{i_R}^{r^h}$ with respect to A_k must always be positive.

□

- Consumption of service i_R by a low-skilled household $r^l < \tilde{r}(i_N)$:

$$
\begin{aligned}
\frac{\partial c_{i_R}^{r^l}}{\partial A_k} =& \frac{\partial}{\partial A_k} n_R^{-1} j_R \left[-e\lambda i_R \log(\bar{r})\right]^{-\frac{1}{\lambda}} \tilde{\theta}(A_k,\omega)^{\frac{\sigma_I}{\sigma_R-1}} \tilde{\mathcal{M}}^{-1} L^r w^r \\
=& n_R^{-1} j_R \left[-e\lambda i_R \log(\bar{r})\right]^{-\frac{1}{\lambda}} \tilde{\theta}(A_k,\omega)^{\frac{\sigma_I}{\sigma_R-1}-1} \tilde{\mathcal{M}}^{-2} A_k^{-1} L^r w^r \times \\
& \left[\sigma_I \tilde{\mathcal{M}} \frac{\mu(A_k)}{\omega^{\sigma_R-1}} \left[1-\sigma_{A_k,\omega}\right] - \left[1-\sigma_I\right] \tilde{\theta} \left[j_N \omega^{1-\sigma_I} \sigma_{A_k,\omega} + j_R \tilde{\theta}(A_k,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}} \frac{\mu(A_k)}{\omega^{\sigma_R-1}} \left[\sigma_{A_k,\omega}-1\right]\right]\right] ,
\end{aligned}
$$

which is > 0, if $\sigma_I > \frac{\sigma_{A_k,\omega}-z}{1-z} \frac{1+\frac{\mu(A_k)}{\omega^{\sigma_R-1}}}{\sigma_{A_k,\omega}+\frac{\mu(A_k)}{\omega^{\sigma_R-1}}} := \kappa_{i_R,R}$ (and vice-versa). Note that only when DRWC-LS holds, i.e. $\sigma_{A_k,\omega} > z$, there are potential parameter values for σ_I, in particular $\sigma_I \in (0, \kappa_{i_R,R})$, for which the consumption of services i_R decreases. Whenever the real wage of low-skilled workers increases, also their consumption of service i_R increases.

□

D.3. Proofs for Chapter 8

D.3.1. Proof of Uniqueness

$s_{i_R}^{\underline{r}}$ and $s_{i_N}^{\underline{r}}$ denote the share of expenditure of household $\underline{r}$ allocated to the respective industry. Then the PCA requires that

$$
1-\gamma > [1-\epsilon]\, \beta \left[\frac{P_{i_N}}{e^{\underline{r}}}\right]^{\epsilon} \left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma} = [1-\epsilon]\, s_{i_R}^{\underline{r}} \,.
$$

We next derive an explicit expression for $s_{i_R}^{\underline{r}}$,

$$
\begin{aligned}
s_{i_R}^{\underline{r}} =& \beta \left[\frac{P_{i_N}}{e^{\underline{r}}}\right]^{\epsilon} \left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma} \\
=& \beta \left[\frac{\sigma_v}{\sigma_v-1}\right]^{\epsilon} \left[-e\lambda \log(\bar{r})\right]^{\frac{\epsilon}{\lambda}} \nu(\omega) [e^{\underline{r}}]^{-\epsilon} \,.
\end{aligned}
$$

We use (8.10) and rearrange, i.e., $\beta \left[-e\lambda \log(\bar{r})\right]^{\frac{\epsilon}{\lambda}} \nu(\omega) = \frac{\tilde{\phi}_R}{\tilde{\psi}_R+\omega^{1-\epsilon}\tilde{\psi}_N}$ to obtain

$$
s_{i_R}^{\underline{r}} = \frac{\tilde{\phi}_R \left[\frac{\sigma_v-1}{\sigma_v} e^{\underline{r}}\right]^{-\epsilon}}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N} \,.
$$

Then, we rewrite Assumption PCA which now requires that

$$1-\gamma > [1-\epsilon]\,\frac{\tilde{\phi}_R\left[\frac{\sigma_v-1}{\sigma_v}e^{\underline{r}}\right]^{-\epsilon}}{\tilde{\psi}_R+\omega^{1-\epsilon}\tilde{\psi}_N}\ . \tag{D.2}$$

Note that $\frac{\sigma_v-1}{\sigma_v}e^{\underline{r}}$ is the labor income of household $\underline{r}$ which is a direct implication of the assumption that the profits of each household are proportional to its wage and that this relation is equal across households. Then, total relative wages paid to non-routine and routine labor also correspond to the total relative income obtained by the respective labor groups. Thus, total expenditure allocated to industry i_R is paid in the form of wages and profits to the labor employed in this industry, i.e., $s_{i_R} = \frac{\tilde{\phi}_R}{\tilde{\phi}_R+\omega\tilde{\phi}_N}$ (see Appendix D.3.2). We show that

$$\frac{\partial\mathcal{G}}{\partial\omega} = -\frac{\sigma_v-1}{\sigma_v}\,[-e\lambda\log(\bar{r})]^{-\frac{\epsilon}{\lambda}}\,\frac{\tilde{\phi}_R}{\beta}\nu(\omega)^{-1}\omega^{-1}[\gamma-\epsilon]-[1-\epsilon]\omega^{-\epsilon}\tilde{\psi}_N < 0\ .$$

Using (8.10), i.e., $[-e\lambda\log(\bar{r})]^{-\frac{\epsilon}{\lambda}}\,\frac{\tilde{\phi}_r}{\beta}\nu(\omega)^{-1} = \tilde{\psi}_R+\omega^{1-\epsilon}\tilde{\psi}_N$, the inequality above is equivalent to

$$[\gamma-\epsilon]\tilde{\psi}_R+[\gamma-1]\omega^{1-\epsilon}\tilde{\psi}_N < 0\ .$$

Expanding the left-hand side with $\tilde{\psi}_R-\tilde{\psi}_R$, we derive that

$$1-\gamma > [1-\epsilon]\,\frac{\tilde{\psi}_R}{\tilde{\psi}_R+\omega^{1-\epsilon}\tilde{\psi}_N}\ . \tag{D.3}$$

Thus, $\frac{\partial\mathcal{G}}{\partial\omega} < 0$ whenever (D.3) holds. We next compare (D.2) and (D.3) to obtain

$$\tilde{\phi}_R\left[\frac{\sigma_v-1}{\sigma_v}e^{\underline{r}}\right]^{-\epsilon} \geq \tilde{\psi}_R\ .$$

The inequality must hold because $\epsilon \in (0,1)$. Therefore, under Assumption PCA, (D.3) must hold which implies $\frac{\partial\mathcal{G}}{\partial\omega} < 0$.

□

D.3.2. Proof of Equivalence between the RA and the Aggregate

We derive that $s_{i_R}^{RA} = \frac{\tilde{\phi}_R}{\tilde{\phi}_R + \omega\tilde{\phi}_N}$.

$$\begin{aligned} s_{i_R}^{RA} =& \beta \left[\frac{P_{i_N}}{e_{RA}}\right]^{\epsilon} \left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma} \\ =& \beta \left[\frac{\sigma_v}{\sigma_v - 1}\right]^{\epsilon} [-e\lambda \log(\bar{r})]^{\frac{\epsilon}{\lambda}} \nu(\omega) e_{RA}^{-\epsilon} \\ =& \beta \frac{\sigma_v}{\sigma_v - 1} [-e\lambda \log(\bar{r})]^{\frac{\epsilon}{\lambda}} \nu(\omega) \frac{L}{E} \left[\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N\right] \ , \end{aligned}$$

where we used $\frac{E}{L} e_{RA}^{-\epsilon} = \left[\frac{\sigma_v}{\sigma_v - 1}\right]^{1-\epsilon} \left[\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N\right]$. We next use (8.10) and rearrange, i.e., $\beta [-e\lambda \log(\bar{r})]^{\frac{\epsilon}{\lambda}} \nu(\omega) = \frac{\tilde{\phi}_R}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N}$, and make use of the equivalence $\frac{E}{L} = \frac{\sigma_v}{\sigma_v - 1} \left[\tilde{\phi}_R + \omega\tilde{\phi}_N\right]$ to obtain

$$s_{i_R} = s_{i_R}^{RA} = \frac{\tilde{\phi}_R}{\tilde{\phi}_R + \omega\tilde{\phi}_N} \ , \tag{D.4}$$

i.e., the aggregate economy behaves just like the representative agent defined in (8.4).

□

D.3.3. Proof of Proposition 8.1

Case (ii) The elasticity of substitution for household r, (8.5), is

$$\begin{aligned} \sigma_{N,R}^{r} =& 1 - \gamma - [\gamma - \epsilon] \frac{\beta \left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}}{\left[\frac{e^r}{P_{i_N}}\right]^{\epsilon} - \beta \left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}} \\ =& 1 - \gamma - [\gamma - \epsilon] \frac{\beta \left[\frac{e^r}{P_{i_N}}\right]^{-\epsilon} \left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}}{1 - \beta \left[\frac{e^r}{P_{i_N}}\right]^{-\epsilon} \left[\frac{P_{i_R}}{P_{i_N}}\right]^{\gamma}} \\ =& 1 - \gamma - [\gamma - \epsilon] \frac{s_{i_R}^r}{1 - s_{i_R}^r} \ . \end{aligned}$$

Then $s_{i_R}^r = \beta \left[\frac{\sigma_v}{\sigma_v - 1}\right]^{\epsilon} [-e\lambda \log(\bar{r})]^{\frac{\epsilon}{\lambda}} \nu(\omega) \hat{\theta}(A_k)^{\frac{\gamma - 1}{1 - \sigma_R}} [e^r]^{-\epsilon}$ and we can use (8.14) to obtain

$$s_{i_R}^r = \frac{\tilde{\phi}_R \left[\frac{\sigma_v - 1}{\sigma_v} e^r\right]^{-\epsilon}}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N} \ . \tag{D.5}$$

Using (D.4), we obtain

$$\begin{aligned}\sigma^{RA}_{N,R} =&1-\gamma-[\gamma-\epsilon]\,\frac{s^{RA}_{i_R}}{1-s^{RA}_{i_R}}\\ =&1-\gamma-[\gamma-\epsilon]\,\frac{s_{i_R}}{1-s_{i_R}}\\ =&1-\gamma-[\gamma-\epsilon]\,\frac{\tilde{\phi}_{\underline{r}}}{\omega\tilde{\phi}_{\bar{r}}}\\ =&\sigma_{N,R}\end{aligned} \tag{D.6}$$

Assume that $\sigma_{N,R} \geq 1$. Then $\sigma_{N,R} \geq 1$ implies $\gamma \leq \epsilon\frac{\omega\tilde{\phi}_N}{\tilde{\phi}_R+\omega\tilde{\phi}_N}$.

Assume that $\sigma^{r}_{N,R} \geq 1$. Then $\sigma^{r}_{N,R} \geq 1$ implies $\gamma \leq \epsilon\left[1-\frac{\tilde{\phi}_R\left[\frac{\sigma_v-1}{\sigma_v}e_{\underline{r}}\right]^{-\epsilon}}{\tilde{\psi}_R+\omega^{1-\epsilon}\tilde{\psi}_N}\right]$.

Note that $1-\frac{\tilde{\phi}_R\left[\frac{\sigma_v-1}{\sigma_v}e_{\underline{r}}\right]^{-\epsilon}}{\tilde{\psi}_R+\omega^{1-\epsilon}\tilde{\psi}_N} < \frac{\omega\tilde{\phi}_N}{\tilde{\phi}_R+\omega\tilde{\phi}_N} = 1-\frac{\tilde{\phi}_R\left[\frac{\sigma_v-1}{\sigma_v}e_{RA}\right]^{-\epsilon}}{\tilde{\psi}_R+\omega^{1-\epsilon}\tilde{\psi}_N}$.

Case (iv) Assumption PCA implies (D.2). We rearrange (D.2) to obtain

$$\gamma < 1-[1-\epsilon]\,\frac{\tilde{\phi}_R\left[\frac{\sigma_v-1}{\sigma_v}e_{\underline{r}}\right]^{-\epsilon}}{\tilde{\psi}_R+\omega^{1-\epsilon}\tilde{\psi}_N}\,.$$

We expand the right-hand side with $\epsilon-\epsilon$ and rearrange to obtain

$$\gamma <\epsilon+[1-\epsilon]\left[1-\frac{\tilde{\phi}_R\left[\frac{\sigma_v-1}{\sigma_v}e_{\underline{r}}\right]^{-\epsilon}}{\tilde{\psi}_R+\omega^{1-\epsilon}\tilde{\psi}_N}\right]\,.$$

We know from Appendix D.3.1 that under Assumption PCA, Inequality (D.3.3) has to be fulfilled.

□

D.3.4. Proof of Proposition 8.5

We take the total differential of the aggregate elasticity of substitution between the industries, (D.6), given parameter γ,

$$d\sigma_{N,R} = 0 = \frac{s_{i_R}}{s_{i_N}}d\epsilon-[\gamma-\epsilon]\frac{1}{s_{i_N}}ds_{i_R}-[\gamma-\epsilon]\frac{s_{i_R}}{s^2_{i_N}}ds_{i_R}\,,$$

which yields $ds_{i_R} = \frac{s_{i_R} s_{i_N}}{\gamma - \epsilon} d\epsilon$. Next we take $\sigma_{A_k,\omega}$ of (8.17) and rearrange,

$$\sigma_{A_k,\omega} = \frac{\gamma}{-\gamma + \epsilon \frac{\tilde{\psi}_R}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N} + \frac{\omega^{1-\epsilon}\tilde{\psi}_N}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N}} \frac{A_k^{\sigma_R - 1}}{\theta(A_k)} \,. \tag{D.7}$$

We use $s_{i_R}^{RA} = \frac{\tilde{\phi}_R \left[\frac{\sigma_v - 1}{\sigma_v} e_{RA}\right]^{-\epsilon}}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N}$. Thus, we rearrange and obtain

$$\frac{\tilde{\psi}_R}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N} = s_{i_R} \left[\frac{\sigma_v - 1}{\sigma_v} e_{RA}\right]^{\epsilon} \frac{\tilde{\psi}_R}{\tilde{\phi}_R}$$

We can now rewrite the denominator of (D.7), $-\gamma + \epsilon \frac{\tilde{\psi}_R}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N} + \frac{\omega^{1-\epsilon}\tilde{\psi}_N}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N}$, as

$$1 - \gamma + [\epsilon - 1] s_{i_R} \left[\frac{\sigma_v - 1}{\sigma_v} e_{RA}\right]^{\epsilon} \frac{\tilde{\psi}_R}{\tilde{\phi}_R} \,.$$

We next define $e_{RA,R} := \frac{\sigma_v}{\sigma_v - 1} \left[\frac{\tilde{\psi}_R}{\tilde{\phi}_R}\right]^{-\frac{1}{\epsilon}}$ as the representative agent within the routine labor group. Using this definition, we obtain

$$1 - \gamma + [\epsilon - 1] s_{i_R} \left[\frac{e_{RA}}{e_{RA,R}}\right]^{\epsilon} \,.$$

We take the total differential obtain

$$\left[\frac{e_{RA}}{e_{RA,R}}\right]^{\epsilon} [s_{i_R} d\epsilon + \epsilon ds_{i_R} - ds_{i_R}] + [\epsilon - 1] s_{i_R} \frac{\partial}{\partial \epsilon} \left[\frac{e_{RA}}{e_{RA,R}}\right]^{\epsilon} d\epsilon$$

The term $\frac{\partial}{\partial \epsilon} \left[\frac{e_{RA}}{e_{RA,R}}\right]^{\epsilon} d\epsilon$ is positive. Thus, it remains to show that $s_{i_R} d\epsilon + \epsilon ds_{i_R} - ds_{i_R}$ is negative. Then,

$$\begin{aligned} s_{i_R} d\epsilon + \epsilon ds_{i_R} - ds_{i_R} &= s_{i_R} d\epsilon + [\epsilon - 1] \frac{s_{i_R} s_{i_N}}{\gamma - \epsilon} d\epsilon \\ &= \left[1 + [\epsilon - 1] \frac{s_{i_N}}{\gamma - \epsilon}\right] s_{i_R} d\epsilon \,, \end{aligned}$$

which, by Assumption PCA, is negative. Thus, given aggregate elasticity of substitution between industries, $\sigma_{N,R}$, and given γ, $\sigma_{A_k,\omega}$ is the higher the more non-homothetic the preference relation is.

□

D.3.5. Proof of Proposition 8.6

We prove Proposition 8.6 analogously to the proof of Proposition 8.5 (see Appendix D.3.4). We take $\sigma_{A_k,\omega}$ of (8.23) and rearrange,

$$\sigma_{A_k,\omega} = \frac{1}{1 - \frac{\gamma - \epsilon - [1-\epsilon]\frac{\omega^{1-\epsilon}\tilde{\psi}_N}{\tilde{\psi}_R + \omega^{1-\epsilon}\tilde{\psi}_N}}{[\gamma + \sigma_R - 1]\mathcal{C}}} = \frac{1}{1 + \frac{1 - \gamma + [\epsilon - 1] s_{i_R} \left[\frac{\sigma_v - 1}{\sigma_v} e_{RA}\right]^{-\epsilon} \frac{\tilde{\psi}_R}{\tilde{\phi}_R}}{[\gamma + \sigma_R - 1]\mathcal{C}}} \,.$$

Observe that the term $1-\gamma+[\epsilon-1]s_{i_R}\left[\frac{\sigma_v-1}{\sigma_v}e_{RA}\right]^{\epsilon}\frac{\tilde{\psi}_R}{\tilde{\phi}_R}$ is just as in the Proof for Proposition 8.5. Thus, the same logic applied, we conclude that given aggregate elasticity of substitution between industries, $\sigma_{N,R}$, and given γ, $\sigma_{A_k,\omega}$ is the higher the more non-homothetic the preference relation is.

□

E. List of Notations

E.1. Basic Research

Any symbol with an additional subscript S denotes the social planner's equilibrium allocation decision.

Symbol	***Meaning***
$r \in \mathcal{R}$	Productive knowledge of country r, the skill level of country r's workforce, a country and its government
$\mathcal{R}$	Domain of productive knowledge $\mathcal{R} := [\underline{r}, \bar{r}]$, where $0 < \underline{r} < \bar{r} < 1$
$F_r(r)$	Distribution of r over $\mathcal{R}$ with corresponding density $f_r(r)$
$\mathcal{I}$	Industry set
I	Number of elements in $\mathcal{I}$
$i \in \mathcal{I}$	An industry, a complexity, a representative firm and a representative product, where $\underline{i} = \min \mathcal{I}$ and $\bar{i} = \max \mathcal{I}$
$\mathcal{I}(r)$	Set of industries where country r has domestic production, i.e. $i \leq \tilde{i}(r)$
σ_I	Elasticity of substitution between industries
N	Total number of varieties
N_i	Total number of varieties in industry i
$j \in [0, N_i]$	Variety in industry i
σ_v	Elasticity of substitution between varieties
$\mathcal{Q}_{i,j}$	Set of qualities the product (i, j) is offered
$q \in \mathcal{Q}_{i,j}$	Quality of a product (i, j)
$q_i(r)$	Chosen quality of firm i in country r
λ	Output elasticity of quality changes in production
$Pat(i, j)$	Global patent protecting a single product (i, j)
$l_i(r)$	Labor demand of the representative firm i for skill r
$\tilde{l}_i$	Effective labor demand of firm i
x_i	Output of representative firm i
χ_i	Effective output of representative firm i
$z(i, r)$	Productivity of representative firm i in producing effective output in country r

$\mathcal{R}_i$	Set of countries where firm i has lowest marginal cost
MC_i	Marginal cost of representative firm i for countries in $\mathcal{R}_i$
$\tilde{r}(i)$	Minimum skill level for representative firm i to have an unconstrained quality choice in production
$\tilde{i}(r)$	Highest complexity that is produced in country r without being constrained by the minimum-quality requirement
$L^r/\tilde{L}^r$	Total / effective labor force of a country r
$L^r_p/\tilde{L}^r_p$	Total / effective labor force of country r employed in production
$L^r_{BR}/\tilde{L}^r_{BR}$	Total / effective labor force of country r employed in basic research
$\tilde{L}_p$	Total effective labor force employed in production
$\tilde{L}_{BR}$	Total effective labor force employed in basic research
$L/\tilde{L}$	Total / effective labor force
$c_{i,j,q}$	Consumption of a product (i,j) of quality q by the representative household
$c_{i,j}$	Quality-adjusted consumption of product i and variety j by the representative household
c_i	Quality-adjusted consumption of product i by the representative household
C	Quality-adjusted global consumption basket
$p_{i,j,q}$	Price of product (i,j) with quality q
$\rho_{i,j}$	Quality-adjusted price of product (i,j)
ρ_i	Quality-adjusted price of the representative product i
P_i	Quality-adjusted price index in industry i
P	Quality-adjusted price index
w^r	Wage in country r
$\pi_{i,j}$	Ex-post profits of a single firm (i,j)
π_i	Ex-post profits of the representative firm i
$\pi_{AR,i}$	Profits of a representative applied-research firm i
υ	Fix costs for any applied-research firm for innovation
Π^r	Aggregate profits accruing to the representative household of country r
Π	Global profits
ψ_i	Industry-specific demand shifter
η^r_i	Amount of ideas from basic research in industry i in country r
$\eta_1(r)$	Country-specific productivity in basic research
ϵ_{η_1}	Elasticity of $\eta_1(r)$
$\eta_2(\xi^r)$	Function that captures congestion effects in basic research
BR^r	Basic research investment by government r
ξ^r	Basic research investment relative to total wages paid by government r and

	the share of the population employed in the basic research sector
ξ_E^r/ξ_S^r	Decentralized / social planner equilibrium outcome of ξ^r
a	Innate ability to do basic research
$\underline{a}$	Lowest innate ability to do basic research
$\tilde{a}^r$	Lowest ability employed by the government as a scientist in country r
$F_a(a)$	Distribution of innate ability within each country on $[\underline{a}, \infty)$
κ	Probability of successful targeting
ζ	Scaling parameter for Pareto distribution in abilities
α	Shape parameter for Pareto distribution in abilities
$\mathcal{I}_{BR}^r$	Set of industries manageable by productive knowledge r with highest profits
I_{BR}^r	Number of elements in $\mathcal{I}_{BR}^r$
$\mathbb{1}_{[i \in \mathcal{I}_{BR}^r]}$	Indicator function representing 1 if $i \in \mathcal{I}_{BR}^r$ and otherwise 0
$\pi_{i_{BR}^r}$	Profits of an industry in the set $\mathcal{I}_{BR}^r$
T	Initial time-span for local dissemination of ideas
$\tilde{\theta}_D$	Poisson arrival rate for personal encounters
θ_D	Fraction of ideas that domestic households learn prior to global dissemination
$\theta_{D,i}^r$	Probability that ideas remain in the country of its origin
$\theta_{G,i}^r$	Probability that an idea i of the global pool of ideas diffuses to country r
n_i	Relative amount of varieties in industry i
ω_S	Optimal distribution of targeting across industries by the social planner
GP	Measure of commercialize-able ideas not absorbed by the countries of origin (GP standing for Global Pool)
$GP(r)$	Measure of commercialize-able ideas in the GP that can potentially be realized in country r
$GP(i)$	Measure of commercialize-able ideas in the GP for industry i

Extensions and Discussions (Chapter 3):

ζ	Positive scaling parameter for the basic research production function
α	Exponent for the basic research production function with $\alpha \in (0,1)$
η^r	Amount of ideas from basic research in every industry in country r
$\hat{N}$	Number of varieties in every industry
Φ	Relative basic research infrastructure costs
$g(r)$	$:= \frac{\eta_1(r)}{w^r} z(r)$
$z(r)$	$:= \sum_{i \in \mathcal{I}(r)} z(i)$
$z(i)$	$:= \frac{\psi_i i^{\frac{1-\sigma_I}{\lambda}}}{\sum_{\hat{i} \in \mathcal{I}} \psi_{\hat{i}} \hat{i}^{\frac{1-\sigma_I}{\lambda}}}$

$x(r')$	$:= \int_{r'}^{\bar{r}} \eta_1(r)^{\frac{1}{1-\alpha}} w^{r\frac{\alpha}{\alpha-1}} L^r z(r)^{\frac{\alpha}{1-\alpha}} f_r(r)[\sigma_v - 1 + \alpha\theta_D z(r)]dr$
$y(r)$	$:= [1-\alpha]\, \eta_1(r)^{\frac{1}{1-\alpha}} w^{r\frac{\alpha}{\alpha-1}} z(r)^{\frac{1}{1-\alpha}} \theta_D$
ξ_Φ^r	Decentralized equilibrium outcome of ξ^r when there are basic research infrastructure costs Φ
$r^\square$	Country with lowest productive knowledge still engaged in basic research
$\overline{L}_{BR}$	Amount of scientists in every country
w_R	Equilibrium wage of researchers
Π_G^r	Profits of country r obtained through global investments in basic research
Π_D^r	Profits of country r obtained through own investments in basic research

E.2. Skills, Tasks, and Capital

First we list the notation of the model presented in Sections 6.2 and 6.3. Then the additional notation of the subsequent sections and chapters follows.

Symbol	***Meaning***
$r \in \mathcal{R}$	Skill level of worker / household r
r_l, r_h	Low and high skill level, with $0 < r_l < r_h < 1$
$\mathcal{R}$	Set (later domain) of skill levels $\mathcal{R} := \{r_l, r_h\}$
ϕ_r	Share of workers with skill level r
$\mathcal{I}$	Industry set
L^r	Representative worker of skill level r
$i \in \mathcal{I}$	A task-complexity, an industry, a representative firm and a representative product
i_m, i_a	Manual and abstract task-complexity
σ_I	Elasticity of substitution between industries
n_i	Number of varieties in industry i
$j \in [0, n_i]$	Variety in industry i
σ_v	Elasticity of substitution between varieties
$\bar{q}_i$	Upper quality bound of the representative product i
$q \in [1, \bar{q}_i]$	Quality range of product (i, j)
$q_i(r)$	Chosen quality level for a production process by firm i using skill level r
λ	Output elasticity of quality changes in production
$\zeta_i(q)$	Complexity of a production process ($\zeta_i(q) := iq^\lambda$)
$\mathcal{R}_i$	Set of skill levels firm i is willing and able to employ
$l_i(r)$	Labor demand of the representative firm i for skill r
$\tilde{l}_i(w^r)$	Effective labor demand of firm i in dependence of the wage scheme
x_i	Output of firm i
$L/\tilde{L}$	Total / effective labor force
TW	Total wages paid
$c^r_{i,j}$	Consumption of product (i, j) by household r
c^r_i	Consumption of the representative product i by household r
C^r	Consumption basket of household r
C_i	Total consumption of industry i
C	Total consumption
$p_{i,j}$	Price of product (i, j)
p_i	Price of the representative product i
P_i	Price index in industry i

P	Price index
w^r	Wage scheme / wage of worker r
$\omega,\ \omega_i,\ \omega(r)$	Wage premium
π_i	Profits of the representative firm i
ψ_i	Industry-specific demand shifter
Π^r	Profits of household r

Equilibrium (Section 6.3)

U	Unemployment Equilibrium
PE	Parallel Equilibrium
TE	Triangle Equilibrium
ILM	Integrated Labor Market Equilibrium
DLM	Disintegrated Labor Market Equilibrium
$\mathcal{G}$	Gini Coefficient

Skill Distribution and the Task Life-cycle (Section 6.4)

$\mathcal{R}$	Domain of skill levels $\mathcal{R} := [\underline{r}, \overline{r}]$, where $0 < \underline{r} < \bar{r} < 1$
$F(r)$	Distribution of r over $\mathcal{R}$, with corresponding density function $f(r)$
$\mathcal{R}_L(i_a), \mathcal{R}_H(i_a)$	Set of skill levels unable, $\mathcal{R}_L(i_a)$, and able, $\mathcal{R}_H(i_a)$, to master task-complexity i_a; the sets are also called skill groups
ϕ_L, ϕ_H	Measure of labor in skill set $\mathcal{R}_L(i_a)$ and $\mathcal{R}_H(i_a)$
$\tilde{\phi}_L, \tilde{\phi}_H$	Measure of effective labor in skill set $\mathcal{R}_L(i_a)$ and $\mathcal{R}_H(i_a)$
$\mathcal{I}^m,\ \mathcal{I}^a$	Sets of task-complexities

Empirical Model (Section 6.5):

$\overline{w}_L, \overline{w}_H$	Average wage in skill group $\mathcal{R}_L(i_a)$ and $\mathcal{R}_H(i_a)$
η	Factor of the change in effective labor relative to absolute labor
$\hat{\sigma}$	Estimated elasticity of substitution between consumption goods
$\hat{\sigma}_I$	Estimated elasticity of substitution between industries

Relation to the Task-based Model (Section 6.6):

$i \in (0, 1]$	Uniformly distributed tasks i
$z \in \{C, H\}$	Skill group H (high-school) and C (college)
A_z	Skill group-specific productivity factor
$\alpha_z(i)$	Task productivity schedule of skill group C and H in performing

	task $i \in (0,1]$
$l_i(r,z)$	Labor demand of firm i for skill r of skill group z
$\tilde{l}_i(z)$	Effective labor demand of firm i for skill group z
ω_z	Skill group specific wage premium
$c_z^r(i)$	Consumption of product i by household r of skill group z
C_z^r	Consumption by household r of skill group z
σ	Elasticity of substitution between consumption goods

Who Produces Capital? (Chapter 7)

Capital through Routine Work (Section 7.3):

i_R, i_N	Routine and non-routine task-complexity / industry
n_R, n_N	Number of varieties in each industry
k	Capital (Machines)
A_k	Productivity of the capital production
σ_R	Elasticity of substitution between capital and labor
mc_i	Marginal cost in industry i
$\sigma_{A_k,\omega}$	Elasticity of the wage premium with respect to A_k
p_k	Price of capital, substitution technology
$\tilde{w}^r$	Real wage of worker r
ϕ_R, ϕ_N	Measure of routine and non-routine labor
$\tilde{\phi}_R, \tilde{\phi}_N$	Measure of effective routine and non-routine labor
$\overline{w}_C,\ \overline{w}_H$	Average wages of college graduates and high-school graduates
r^c	Lowest skill level able to graduate from college
$\tilde{L}_{i_N}^d$	Effective labor demand from industry i_N
$\hat{\theta}(A_k)$	$:= 1 + A_k^{\sigma_R - 1}$
$\hat{\mathcal{M}}$	$:= n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}} \omega^{1-\sigma_I} + n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}} \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}$
$\hat{\mathcal{W}}$	$:= n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}} \omega^{-\sigma_I} + n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}} \hat{\theta}(A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}$
$\tilde{X}$	$:= \left[\frac{i_N}{i_R}\right]^{\frac{1-\sigma_I}{\lambda}} \left[\frac{n_N}{n_R}\right]^{\frac{1-\sigma_I}{1-\sigma_v}}$
$\Pi(r)$	$:= \frac{\sigma_v - 1}{\sigma_v} \left[-e\lambda i_R \log(r)\right]^{-\frac{1}{\lambda}} n_R^{\frac{1}{\sigma_v - 1}} \hat{\theta}(A_k)^{\frac{2-\sigma_R}{\sigma_R - 1}} A_k^{\sigma_R - 2} \left[\frac{\omega\tilde{\phi}_N}{\tilde{\phi}_R} + 1\right]^{\frac{1}{\sigma_I - 1}}$

Capital through Non-routine Work (Section 7.4):

k	Capital (Robots)
A_k^s	Technological level of the capital production where labor market separation starts
$L_{i_N}^d$	Labor demand from industry i_N

$\mu(A_k)$	$:= \left[A_k \left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}$
$\tilde{\theta}(\omega, A_k)$	$:= 1 + \left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}$
$\tilde{\mathcal{M}}$	$:= i_N^{\frac{1-\sigma_I}{\lambda}} n_N^{\frac{1-\sigma_I}{1-\sigma_v}} \omega^{1-\sigma_I} + i_R^{\frac{1-\sigma_I}{\lambda}} n_R^{\frac{1-\sigma_I}{1-\sigma_v}} \tilde{\theta}(\omega, A_k)^{\frac{1-\sigma_I}{1-\sigma_R}}$
$\tilde{X}$	$:= \left[\frac{i_R}{i_N}\right]^{\frac{\sigma_I-1}{\lambda}} \left[\frac{n_R}{n_N}\right]^{\frac{\sigma_I-1}{1-\sigma_v}}$
z	$:= \frac{\tilde{\phi}_R}{\omega\tilde{\phi}_N}\left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1}$
$\mathcal{C}$	$:= \left[\frac{A_k}{\omega}\left[\frac{i_R}{i_N}\right]^{\frac{1}{\lambda}}\right]^{\sigma_R-1} \tilde{\theta}(\omega, A_k)^{-1}$
$\kappa_{A_k,\omega}$	$:= \frac{\frac{\mu(A_k)}{\omega^{\sigma_R-1}}}{\frac{\mu(A_k)}{\omega^{\sigma_R-1}} + \frac{\sigma_R-1}{\sigma_R}}$
$\kappa_{i_N,R}$	$:= \frac{\frac{\mu(A_k)}{\omega^{\sigma_R-1}} z^{-1}\left[z - \sigma_{A_k,\omega}\right]}{\sigma_{A_k,\omega} + \frac{\mu(A_k)}{\omega^{\sigma_R-1}}}$
$\kappa_{i_R,R}$	$:= \frac{\sigma_{A_k,\omega} - z}{1-z} \frac{1 + \frac{\mu(A_k)}{\omega^{\sigma_R-1}}}{\sigma_{A_k,\omega} + \frac{\mu(A_k)}{\omega^{\sigma_R-1}}}$

Extensions (Chapter 8)

Three Types of Tasks—Three Industries (Section 8.1):

i_1, i_2, i_3	Manual, routine and abstract task-complexity
r_l, r_m, r_h	Low, middle and high skill level

Non-homothetic Preferences (Section 8.2):

ϵ	Degree of non-homotheticity
γ	Determines (among other parameters) the elasticity of substitution
β	Relative industry weight
e^r	Expenditure of household r
e_{RA}	Representative agent in the muellbauer sense
E	Total expenditures
$\sigma^r_{N,R}$	Elasticity of substitution between sector N and R of household r, with expenditure level e^r
$\sigma^{RA}_{N,R}$	Elasticity of substitution between sector N and R of the representative household
$\sigma_{N,R}$	Aggregate elasticity of substitution between sector N and R
s^r_i	Expenditure share of household r allocated to industry i
s^{RA}_i	Expenditure share of the representative household allocated to industry i
s_i	Aggregate expenditure share allocated to industry i
$\mathcal{G}, \breve{\mathcal{G}}, \check{\mathcal{G}}$	Implicit functions

$\tilde{\psi}_R$	$:= \int_{\underline{r}}^{\tilde{r}(i_N)} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1-\epsilon}{\lambda}} f(r)dr$
$\tilde{\psi}_N$	$:= \int_{\tilde{r}(i_N)}^{\bar{r}} \left[\frac{\log(\bar{r})}{\log(r)}\right]^{\frac{1-\epsilon}{\lambda}} f(r)dr$
$\nu(\omega)$	$:= \left[n_{i_R}{}^{\frac{1}{1-\sigma v}} i_R^{\frac{1}{\lambda}}\right]^{\gamma} \left[n_{i_N}{}^{\frac{1}{1-\sigma v}} i_N^{\frac{1}{\lambda}} \omega\right]^{\epsilon-\gamma}$

E.3. Appendix

Symbol	***Meaning***

Appendix (Chapter A)

s_i	Share of ideas targeted towards industry i
$\tilde{\alpha}$	Exponent of the Pareto distribution of abilities with $\tilde{\alpha} = \frac{1}{\alpha-1}$

Appendix (Chapter B)

φ	Basic research allocation scheme $\varphi = \{\varphi^r\}_{r\in[\underline{r},\overline{r}]}$
$\tilde{g}(r)$	$:= \eta_1(r) L^r f_r(r)$

Appendix (Chapter D)

j_N	$:= n_N^{\frac{1-\sigma_I}{1-\sigma_v}} i_N^{\frac{1-\sigma_I}{\lambda}}$
j_R	$:= n_R^{\frac{1-\sigma_I}{1-\sigma_v}} i_R^{\frac{1-\sigma_I}{\lambda}}$
j_A	$:= [\sigma_R - 1] \left[\frac{i_R}{i_N}\right]^{\frac{\sigma_R-1}{\lambda}} A_k^{\sigma_R-2}$

F. Glossary*

F.1. Basic Research

Applied Research: The process of commercializing ideas (see ideas).

Basic Research: The process of generating ideas (see ideas).

Bayh-Dole Act: Patent and Trademark Law Amendments Act that allowed the ownership of inventions made with federal funding.

Complexity: The difficulty of production. Complexity is homogeneous within an industry, and thus there is a one-to-one map between them.

CES: Constant elasticity of substitution.

Global Market for Researchers: A global market for every researcher in the world with a single wage rate.

Global Pool of Ideas: All ideas (see ideas), that diffuse across the border of the country of origin first enter the global pool of ideas. Thus it is the public domain of ideas.

Government: Sovereign that aims at maximizing the welfare of the households of its country by deciding on basic research investments.

GDP: Gross Domestic Product.

GNI: Gross National Income.

Households: Economic agents that maximize their consumption.

Ideas: Ideas typically consist of new materials, methods, or discoveries. They have no commercial value if they are not further processed through applied research.

Industry: An industry is characterized by its difficulty in production (see complexity). Products within an industry are more substitutable than across industries.

Infrastructure Cost: The cost for building up the infrastructure necessary for basic research.

Manufacturing Base: All industries with domestic production add up to the manufacturing base of a country.

Openness of Research: The decision of governments about how easily ideas diffuse to other countries and are taken up by the households they represent.

* The terms are defined in the context of this thesis. Different definitions of the same terms can exist in a context beyond this thesis.

Productive Knowledge: Captures anything that contributes to a country's productive potential, such as infrastructure, regulations, and institutions, or simply the overall skill level of the country.

Public Good: A good or service, the consumption of which is non-excludable and non-rivalrous.

RCA: Revealed comparative advantage.

Representative Household: Economic agent representative of the households of all countries.

Social Planner: Aims at maximizing the welfare of the households of all countries (the representative household) by deciding on basic research investments.

F.2. Skills, Tasks, and Capital

A/A-Model: The task-based model developed by Acemoglu and Autor (2011).
Abstract: See task-complexity.
AI: Artificial Intelligence.
Capital: Can be used in the production process of the routine task-complexity. Capital is a substitution technology for routine labor.
CES: Constant elasticity of substitution.
Complexity: Denotes the overall difficulty of a production process. Summarizes the task-complexity and the quality (see quality) of a production process.
Machines: Capital that can be produced with the routine task-complexity.
Manual: See task-complexity.
Manufacturing: The industry producing capital.
Non-routine: See task-complexity.
Robots: Capital that can be produced with the non-routine task-complexity.
Routine: See task-complexity.
SBTC: Skill Biased Technological Change. SBTC summarizes changes in the production process that favors skilled over unskilled workers by increasing the productivity of the skilled workers more than the productivity of the unskilled worker.
Skill Level: Captures anything that contributes to a worker's productive potential.
PIGL: Price-independent Generalized Linearity. A preference relation that exhibits non-homotheticity and is based on Muellbauer (1975).
Quality: Denotes the chosen worker-specific quality of a production process.
Task-complexity: Denotes the inherent difficulty of a production process, that requires high enough skill for the possibly successful completion of a production process. The task-complexity are characterized to represent manual and abstract tasks in Chapter 6, and routine and non-routine tasks in Chapters 7 and 8.
Task Live-cycle: The emergence of new task-complexities and the process until they are automated.
Technological Progress: The advancement of productivity in manufacturing (see manufacturing).
Uneven Technological Progress: See technological progress.
Wage Premium: Factor greater than one that indicates additional income to the skill group in short supply.

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