Strings on NS-NS backgrounds as integrable deformations

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Strings on NS-NS backgrounds as integrable deformations

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We consider the world-sheet S matrix of superstrings on an $AdS_3 \times S^3 \times T^4$ NS-NS background in uniform light-cone gauge. We argue that scattering is given by a CDD factor that is nontrivial only between opposite-chirality particles, yielding a spin-chain-like Bethe ansatz. Our proposal reproduces the spectrum of nonprotected states computed from the Wess-Zumino-Witten description and the perturbative tree-level S matrix. This suggests that the model is an integrable deformation of a free theory similar to those arising from the $T\bar{T}$ composite operator.

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I. INTRODUCTION

The study of the AdS/CFT correspondence [1–3] gave new energy to the search for exactly solvable string backgrounds. For a general string background only, few observables may be computed exactly, usually owing to supersymmetry. Notable exceptions are plane-wave backgrounds [4–6] where the light-cone Hamiltonian is free, Wess-Zumino-Witten (WZW) models [7–10] where current algebras can be used to solve the theory, and integrable backgrounds [11–13] where the world-sheet scattering in light-cone gauge factorizes along the lines of Ref. [14]. The best-understood integrable AdS background is $AdS_5 \times S^5$ supported by Ramond-Ramond (R-R) five-form fluxes. The string equations of motion are integrable [15] and the factorized S matrix can be computed from symmetry considerations [16–18]. The spectrum follows from imposing periodic boundary conditions and accounting for finite-size "wrapping" effects [19], leading eventually to a quantum spectral curve [20]. Remarkably, the integrability approach to $AdS_5 \times S^5$ superstrings can be extended to three- [21] and higher-point [22,23] correlation functions, and even to nonplanar corrections [24,25], though the treatment of wrapping corrections is less thoroughly understood in that context. Another important class of integrable backgrounds is given by $AdS_3 \times S^3 \times T^4$ and $AdS_3 \times S^3 \times S^3 \times S^1$ geometries supported by R-R and Neveu-Schwarz-Neveu-Schwarz (NS-NS) three-form fluxes. They are also classically integrable [26,27] and their S matrix can be fixed by symmetries [28–30], even for mixed background fluxes [31,32]. The purely R-R backgrounds resemble $AdS_5 \times S^5$: the S matrix has a complicated scalar factor [33,34] and the dispersion relation is periodic [28] suggesting a dual spin-chain interpretation similar to Ref. [35]. Instead the NS-NS flux yields a linear contribution to the dispersion [31,36]. The pure-NS-NS model corresponds to a supersymmetric WZW model and the S matrix should simplify drastically there. The analysis of light-cone gauge symmetries of Ref. [31], valid for generic mixtures of R-R and NS-NS fluxes, is insufficient to determine the S matrix at the WZW point. In this article, we analyze the $AdS_3 \times S^3 \times T^4$ WZW model in uniform light-cone gauge [37–39], considering its classical bosonic Hamiltonian, its spectrum and its S matrix. We observe that shifting the gauge parameter has a similar effect to a $T\bar{T}$ deformation [40–42] and that the T^4 sector of the theory is free in a suitable gauge. This motivates us to further investigate the spectrum to determine if it can be related to that of a free theory. We find that in the "spectrally unflowed" sector [10] the energies of nonprotected states can be reproduced from the Bethe-Yang equations by adding a CDD factor [43] to a free theory. The resulting S matrix coincides at tree level with the known perturbative result [44] in a suitable gauge. Moreover, we argue that, due to supersymmetry, wrapping corrections cancel out similarly to what happened for protected states in Ref. [45], so that the Bethe-Yang equations are exact. The CDD factor appearing in our construction is exactly that of a $T\bar{T}$ deformation when we restrict to T^4 [46]; in general however it differs from it due to the presence of an additional u(1) current. The simple form of the S matrix makes the study of this NS-NS background almost as straightforward as that of a plane-wave one, paving the way to a wealth of explicit computations. We conclude this article by detailing additional checks of our proposal, which we intend to present in an

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upcoming publication [47], as well as by commenting on several possible future directions.

II. WZW MODEL IN LIGHT-CONE GAUGE

The Green-Schwarz action for $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{T}^4$ with mixed NS-NS and R-R three-form fluxes has been analyzed in some detail in Ref. [31]. We restrict to the pure NS-NS case corresponding to the WZW model, and find the bosonic action

$$\mathbf{S}=-\frac{k}{4\pi}\int_{-\infty}^{+\infty}\mathrm{d}\tau\int_0^R\mathrm{d}\sigma(\gamma^{\alpha\beta}G_{\mu\nu}+\epsilon^{\alpha\beta}B_{\mu\nu})\partial_\alpha X^\mu\partial_\beta X^\nu,\quad(1)$$

where k is the WZW level, $\gamma^{\alpha\beta}$ is the world-sheet metric with $|\gamma|=-1$, $G_{\mu\nu}$ is the $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{T}^4$ metric and $B_{\mu\nu}$ is the Kalb-Ramond field. We write the line element as

$$\mathrm{d}s^2=-(1+|z|^2)\mathrm{d}t^2+(1-|y|^2)\mathrm{d}\phi^2+\mathrm{d}x^j\mathrm{d}x^j+\left(\delta_{ij}-\frac{z_iz_j}{1+|z|^2}\right)\mathrm{d}z^i\mathrm{d}z^j+\left(\delta_{ij}+\frac{y_iy_j}{1-|y|^2}\right)\mathrm{d}y^i\mathrm{d}y^j,\quad(2)$$

where t, z_1, z_2 are in AdS_3, ϕ, y^3, y^4 describe S^3 and $x^5,\dots x^8$ give T^4. The Kalb-Ramond field is given by $B=\epsilon^{ij}z_idz_j\wedge dt-\epsilon^{ij}y_idy_j\wedge d\phi$. Fermions couple to $H=dB$, see Refs. [31,48,49] for explicit formulas. To fix uniform light-cone gauge [37–39], we introduce, for $0\le a\le 1$,

$$x^+=(1-a)t+a\phi,\qquad x^-=\phi-t,\quad(3)$$

and following, e.g., Ref. [11], we introduce conjugate momenta $p_\mu=\delta\mathbf{S}/\delta(\partial_0X^\mu)$ and fix

$$x^+=\tau,\qquad p_-=(1-a)p_\phi-ap_t=1.\quad(4)$$

This breaks conformal invariance and, in particular, fixes the world-sheet size R

$$R=(1-a)\int_0^R\mathrm{d}\sigma p_\phi-a\int_0^R\mathrm{d}\sigma p_t=J+a(E-J),\quad(5)$$

where E is the string energy and J its angular momentum. The light-cone Hamiltonian is

$$\mathbf{H}=-\int_0^R\mathrm{d}\sigma p_+=E-J,\quad(6)$$

and the $\mathrm{AdS}_3\times\mathrm{S}^3$ BPS bound guarantees $\mathbf{H}\ge 0$. Notice that $-p_+$ is a-dependent, and gauge invariance dictates

$$\frac{\mathrm{d}}{\mathrm{d}a}\int_0^{R(a)}\mathrm{d}\sigma p_+(a)=0.\quad(7)$$

The density $p_+(a)$ can be easily found as in Ref. [31] by solving the Virasoro constraints. Truncating it to T^4 modes and setting $s=(a-1/2)$, we find

$$\mathbf{H}|_{\mathrm{T}^4}=\int_0^{R(s)}\mathrm{d}\sigma\frac{1-\sqrt{1-4sH_{\mathrm{free}}+4s^2(\frac{k}{2\pi}p_j\acute{x}^j)^2}}{2s},\quad(8)$$

with $H_{\mathrm{free}}=\frac{1}{2}p_jp^j+\frac{1}{2}(\frac{k}{2\pi})^2\acute{x}^j\acute{x}_j$. Equation (8) reduces to a free Hamiltonian at $s=0$, i.e., at $a=1/2$. In view of Eqs. (5)–(8), we conclude that the T^4 modes can be equivalently represented as a free system with state-dependent world-sheet length $R=J+\mathbf{H}/2$ (for $a=1/2$) or as an interacting one with fixed length $R=J$ (for $a=0$).

III. RELATION TO $T\bar{T}$ DEFORMATIONS

The form of Eq. (8) is that of a $T\bar{T}$ deformation of free bosons [40–42]. To understand why, let us review and in fact slightly generalize the construction of such deformations. Given two conserved local currents j_I^α, $I=1,2$ the limit

$$j_1j_2(x)=\lim_{y\to x}j_1^\alpha(x)j_2^\beta(y)\epsilon_{\alpha\beta},\quad(9)$$

is well defined owing to the arguments of Ref. [40], and

$$\langle j_1j_2\rangle=\langle j_1^\alpha\rangle\langle j_2^\beta\rangle\epsilon_{\alpha\beta}.\quad(10)$$

Notice that we do not require any of the currents j_I^α to be chiral. A $T\bar{T}$ deformation corresponds to the case $j_I^\alpha=T^{\alpha I}$. Coupling $T^{\alpha I}$ to a u(1) current yields deformations of the type considered in Ref. [50]; "$J\bar{J}$" deformations fall in this class too, by taking a current and its (conserved) Hodge dual. For such special choices of j_I^α the deformation has a simple effect on the spectrum [40,50], and, in particular, for a $T\bar{T}$ deformation of parameter α, we have

$$\partial_\alpha H_n=-H_n\partial_RH_n,\quad(11)$$

for a state $|n\rangle$ of energy H_n and zero momentum [40]. From this it follows that $H_n(R,\alpha)=H_n(R-\alpha H_n,0)$: the deformation amounts to a state-dependent shift of the length, which can be described as a CDD factor [41,42]. This is also the effect induced on p_+ by a-gauge transformations, cf. Eq. (7), which explains the form of Eq. (8). Gauge transformations and $T\bar{T}$ deformations should not be confused however: the former leave the spectrum invariant, while the latter correspond to changing p_+ while leaving R fixed or vice versa. Indeed the differential equation for a-gauge transformations is Eq. (7) rather than Eq. (11). Hence, our observation that the "flat" subsector of classical $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{T}^4$ strings is simply related to a free theory is unsurprising in view of Refs. [41,42]. It is remarkable, as we will see, that this extends to the full quantum level, not only for T^4 modes but for the whole superstring background.

IV. WZW SPECTRUM

The spectrum of the light-cone Hamiltonian can be constructed from the left and right Kač-Moody currents [10,51–55], as we very briefly review here. In the left sector, we have $sl(2)_{k+2}$ currents $L^{0,\pm}_{-n}$, $su(2)_{k-2}$ currents $J^{3,\mp}_{-n}$, torus modes α^r_{-n} as well as their fermionic superpartners. They act on a vacuum $|\ell_0, j_0\rangle$ given by a lowest (highest) weight state of sl(2), respectively, su(2). A generic state is obtained by acting with positive energy modes of the currents ($n \geq 0$) on the vacuum, e.g.,

$$\prod_{i=1}^{\ell^+} L^+_{-n_i} \prod_{i=1}^{\ell^-} L^-_{-n_i} \prod_{i=1}^{j^+} J^+_{-n_i} \prod_{i=1}^{j^-} J^-_{-n_i} \prod_{i=1}^{M_T} \alpha^{r_i}_{-n_i} |\ell_0, j_0\rangle. \quad (12)$$

Such a state has (left) energy $\ell = \ell_0 + \delta\ell$ and (left) angular momentum $j = j_0 - \delta j$ with $\delta\ell = \ell^+ - \ell^-$ and $\delta j = j^- - j^+$. Fermions can be added in a similar way and the usual subtleties arise depending on their boundary conditions; see e.g., Refs. [55,56] for details. Physical states are subject to restrictions, the most important being the mass-shell condition

$$-\frac{\ell_0(\ell_0 - 1)}{k} + \frac{j_0(j_0 + 1)}{k} + N_{\text{eff}} = 0, \quad (13)$$

where $N_{\text{eff}} = \sum_j n_j$ is the total mode number (which in the NS sector is shifted by $-1/2$). More general sectors of the spectrum can be described by spectral flow [10], though we will not consider them in this article. Similar expressions hold in the right sector, which we label with tildes. Imposing level-matching $N_{\text{eff}} = \tilde{N}_{\text{eff}}$ and $j_0 = \tilde{j}_0$, we finally get

$$E - J = \sqrt{(2j_0 + 1)^2 + 4kN_{\text{eff}}} - (2j_0 + 1) + \delta, \quad (14)$$

where $\delta = \delta\ell + \delta\tilde{\ell} + \delta j + \delta\tilde{j} + 2$ and we solved Eq. (13). Notice that for BPS states $E = J$ and $N_{\text{eff}} = \delta = 0$ [57].

V. FREE-THEORY INTERPRETATION

The spectrum of excitations over the BPS "vacuum" is simply related to a $T\bar{T}$-like deformation of a free theory. Consider a theory of eight free bosons with dispersion

$$H(p) = \left|\frac{k}{2\pi} p + \mu\right|, \qquad \mu \in \{0, 0, +1, -1\}^{\oplus 2}. \quad (15)$$

This coincides [58] with the plane-wave dispersion of our string background [6,56,59] which for the NS-NS background is exact [31,36]. Supersymmetry can be realized by adding eight fermions with the same masses μ. Imposing boundary conditions on a circle of size R_{eff}, we have

$$\mathbf{H} = \sum_i H\left(\frac{2\pi n_i}{R_{\text{eff}}}\right) = \frac{k(N + \tilde{N})}{R_{\text{eff}}} + \sum_i \mu_i \text{sgn}(n_i), \quad (16)$$

where we split left- and right-movers. Notice that to remove the absolute value we have assumed that $R_{\text{eff}} \leq k$; we will see shortly why this is the case. If we now postulate the state-dependent length

$$R_{\text{eff}} = R_0 + \frac{\mathbf{H} - \mathbf{m}}{2}, \qquad \mathbf{m} = \sum_i \mu_i \, \text{sgn}(n_i), \quad (17)$$

and solve Eq. (16), we precisely reproduce the WZW light-cone energy (14) with the following identifications. First, $\mathbf{H} = E - J$ as in Eq. (6). Next $R_0 = 2j_0 + 1$ is the J-charge of the BPS state in the R-R sector corresponding to the middle of the T^4 Hodge diamond. Notice that taking R_0 to be the charge of a reference vacuum rather than the J-charge of the state itself mimics the dual spin-chain construction for $AdS_5 \times S^5$ [35,60]. Finally $\mathbf{m} = \delta$. Let us justify this. Notice that when no excitations on $AdS_3 \times S^3$ are present, $\mu = 0$ and Eq. (17) precisely describes a $T\bar{T}$ deformation [40–42]. Consider now a state with some T^4 excitations over the BPS vacuum and a single S^3 mode, say $J^{\pm}_{-n}$. For the charges to match, this should correspond to a boson with $p = 2\pi n / R_{\text{eff}} \geq 0$ and $\mu = \mp 1$; conversely, $\tilde{J}^{\pm}_{-\tilde{n}}$ gives a boson with $p = -2\pi\tilde{n}/R_{\text{eff}} \leq 0$ and $\mu = \pm 1$ [61]. This matches the identification of μ with the su(2)-spin of S^3 excitations in the plane-wave limit [31]. The other bosons as well as the fermions can be similarly described and will be presented elsewhere [47]. Finally, notice that $R_{\text{eff}} = \ell_0 + j_0$ with our identifications. The condition $R_{\text{eff}} \leq k$ which we used to remove the absolute values in Eq. (16) follows from the unitarity bounds of the WZW model [10]. Sectors with larger values of R_{eff} should arise from spectral flow, see also Ref. [56] for a discussion of this fact in the plane-wave limit.

VI. S MATRIX AND BETHE-YANG EQUATIONS

An energy-dependent shift of the length can be described as a CDD factor [43] to the S matrix, see Ref. [41]. This is also the case for the shift of Eq. (17) which corresponds to a CDD factor whose phase is

$$\Phi_{jk} = p_j E_k - p_k E_j - p_j m_k + p_k m_j, \quad (18)$$

where $m_j = \mu_j \text{sgn}(k p_j + 2\pi\mu_j)$. Starting from a free theory, we get a *diagonal* S matrix with elements $S_{jk} = \exp(\frac{i}{2}\Phi_{jk})$. The Bethe-Yang equations follow immediately,

$$1 = \exp(ip_k R_0) \prod_{j\neq k} S_{kj} = \exp(ip_k R_{\rm eff}), \qquad (19)$$

where in the last equation we used the level-matching condition $\sum p_j = 0$. Given that H and m distinguish between left- and right-movers, it is convenient to treat such modes separately. We introduce labels "±" for particles having $\partial H/\partial p = \pm k/2\pi$, yielding four cases for the S matrix. We get

$$S^{++}_{jk} = S^{--}_{jk} = 1, \quad S^{-+}_{jk} = \exp\left(i\frac{k}{2\pi}p_j p_k\right) = \frac{1}{S^{+-}_{kj}}. \qquad (20)$$

This illustrates the role of $\mathbf{m}$: it makes the left-left and right-right scattering trivial, as we would expect in a theory where particles move at the speed of light. Notice that such scattering is much simpler than the one arising in Refs. [62,63], where nondiagonal and nonperturbative left-left and right-right S matrices appear. These expressions match the perturbative tree-level result for $S^{\pm\mp}_{ij}$ of Ref. [44]. To compare our expressions, we should firstly take the results of Ref. [44] in the $a = 0$ gauge; in that case, the length in the Bethe-Yang equations is the J-charge of the state—in contrast to our conventions, in which it is the J-charge of the BPS vacuum. Accounting for these different conventions is akin to going from the string-frame to the spin-chain frame [18,28,29]. With these identifications, the left-right and right-left S matrices match with Refs. [44,64]. Based on the integrability treatment of strings in flat space [65] it may appear surprising that our analysis relies solely on the Bethe-Yang equations (19) and does not require the mirror thermodynamic Bethe ansatz to account for finite-size effect, cf. Refs. [19,66]; this is all the more concerning given that this background features gapless excitations that usually lead to severe wrapping effects [67]. This simplification is due to supersymmetry: as the scattering is diagonal, wrapping corrections [68–70] to a state with momenta $p_1, \ldots p_M$ take a simple form

$$\int d\rho e^{-\varepsilon(\rho)L} \sum_X (-1)^{F_X} \prod_{j=1}^{M} S_{Xj}(\rho, p_j), \qquad (21)$$

where X is any virtual particle. Regardless of the details, here bosons and fermions come in pairs with identical dispersion and scattering, so that the integrand vanishes; this is the same argument that guarantees that BPS states are immune from wrapping corrections in Ref. [45].

VII. TOWARDS A DEFORMATION OF THE FULL ACTION

A formula for the action of $T\bar{T}$ deformations of scalar field theories is known [42,71,72]. We briefly discuss two subtleties arising when applying such an approach here: firstly, our transformation involves the current $\mathbf{m}$; secondly, our free action has the μ-dependent dispersion (15). Naïvely we would use Eq. (9) with one of the currents given by j^α such that $\int d\sigma j^0 = \mathbf{m}$; unfortunately, while such a conserved current exists in a free theory, it is nonlocal and Zamolodchikov's arguments [40] do not apply [73]. Alternatively we can ask whether the gauge-fixed WZW action is the $T\bar{T}$ deformation of some simpler theory; this is also quite subtle. In the presence of several so(2) symmetries such as the ones rotating $z_{1,2}$ and $y_{3,4}$ the stress-energy tensor is not uniquely defined. To be concrete, we truncate our theory to the S^3 modes and introduce complex coordinates y, $\bar{y}$. The dispersion (15) can be reproduced by coupling y, $\bar{y}$ to a constant u(1) background gauge field A^α. The Noether stress-energy tensor $T^{\alpha\beta}_{\rm N}$ is not gauge-invariant; adding improvement terms yields the Hilbert stress-energy tensor $T^{\alpha\beta}_{\rm H}$. The difference of the two $T\bar{T}$ operators is also of the form (9),

$$T_{\rm H}\bar{T}_{\rm H} - T_{\rm N}\bar{T}_{\rm N} = \epsilon_{\alpha\beta}\epsilon_{IJ} j^{\alpha;I} T^{\beta J}, \qquad (22)$$

where the two currents $j^{\alpha;I}$ are related to the components of the constant gauge field

$$j^{\alpha;I} = iA^I(p^\alpha \bar{y} - \bar{p}^\alpha y). \qquad (23)$$

Hence, we have at least two inequivalent $T\bar{T}$ deformations. *A priori* it is unclear which one is more natural; interestingly, a Hilbert-$T\bar{T}$ deformation relates the gauge-fixed GS action to a simple sigma model action for the sphere fields[74],

$$\mathbf{S}|_{S^3} = \frac{k}{2\pi}\int_{-\infty}^{+\infty} d\tau \int_0^R d\sigma \eta^{\alpha\beta} \frac{D_\alpha y D_\beta \bar{y}}{1 - y\bar{y}}, \qquad (24)$$

with background gauge field $A^\alpha = g^{-1}\partial^\alpha g$, $g = \exp[i\sigma]$. The integrability of the classical $AdS_3 \times S^3 \times T^4$ action suggests that (24) is classically integrable too.

VIII. CONCLUSIONS AND OUTLOOK

We have found evidence that superstrings on $AdS_3 \times S^3 \times T^4$ with NS-NS three-form flux are described by a simple integrable theory of eight relativistic bosons and fermions with dispersion (15) and S matrix (20) given by a CDD factor. For such a theory wrapping corrections cancel and the Bethe-Yang equations are exact, rather than asymptotic. While this description is strongly reminiscent of a $T\bar{T}$ deformation, constructing the appropriate perturbing operator is quite subtle. A number of questions immediately arise. Our construction here was limited to "unflowed" sector of the WZW model, corresponding to $R_{\rm eff} \leq k$ in Eq. (16). It would be interesting to extend this to the w-th spectrally flowed sector corresponding to $wk < R_{\rm eff} \leq (w+1)k$, see also Ref. [56] for a discussion of this in the plane-wave limit; notice that when the inequality is saturated the mass-gap in Eq. (16) vanishes and new gapless modes appear. We also restricted to states with vanishing total momentum (i.e., $N = \tilde{N}$). In light-cone gauge, winding sectors should also be included [11,37], which would modify our analysis and in particular Eq. (19). Finally, it is intriguing that the gauge-fixed WZW action is related to Eq. (24) and it would be worth exploring more such a sigma model. We will return to these questions in an upcoming publication [47]. It would also be worth extending this analysis to $AdS_3 \times S^3 \times S^3 \times S^1$ backgrounds, whose integrability [27,32,45] and WZW [10,75] descriptions are well-established, as well as more general supersymmetric theories with diagonal scattering, where wrapping corrections are also expected to cancel. It looks less likely that this scenario might hold for mixed R-R and NS-NS backgrounds, as the S matrix is nontrivial in that case [31,44], though one might hope that the first correction in the R-R flux is captured by Eqs. (16) and (17) with the exact mixed-flux dispersion [31,36] instead of Eq. (15). Such mixed-flux dynamics is particularly interesting as it captures a large part of the moduli space [76]. Describing strings on NS-NS backgrounds as simple integrable theories would have a number of interesting applications. As our description depends parametrically on the WZW level k we could apply it to, e.g., the semiclassical limit $k \gg 1$ as well as to special cases such as the $k = 1$ theory which was recently related to a symmetric-product orbifold CFT [77,78]. Interestingly, our dispersion (15) at $k = 1$ precisely describes the single-excitation spectrum of the symmetric-product orbifold CFT of T^4 [79]. This might help us find an integrability description for symmetric-product orbifold CFTs, cf. also Ref. [80]. It would also be interesting to extend this map beyond the spectrum: recently integrability techniques have been developed to compute three- [21] and higher-point [22,23] functions, and even nonplanar corrections [24,25]. In $AdS_5 \times S^5$ Lüscher-like wrapping effects make such computations very hard, while we have seen in Eq. (21) that those cancel here, at least for two-point functions. This, together with the wealth of data available might make NS-NS background an ideal playground for the hexagon bootstrap program [21–25].

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