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Abstract

Understanding the variability of public transport travel times is essential for various reasons, e.g. for gaining knowledge of the deteriorations and ameliorations in daily traffic, for providing adequate (real time) information to customers and for optimizing transit schedules. This paper deals with this issue by modeling the day-to-day variability of travel times of urban buses based on planned and actual arisen arrival and departure time data of selected bus routes in Zurich by the aid of statistical distributions. For a detailed description of the variability, the travel times are decomposed in running and dwell times. The most appropriate distribution models for the running, dwell and travel times at different times of the day are identified, and their performance is evaluated. This is done under consideration of the issue of how the temporal aggregation (i.e. length of departure time window) as well as the spatial aggregation (i.e. number of considered bus stops or sections) influence the nature and shape of these distributions.

Keywords

Public transport, Travel time variability, Travel time distributions, Data aggregation,

1 Introduction

The travel times of public bus operations, as well as its components dwell and running time, are affected by various factors resulting in uncertainty. The stochastic nature of those influencing factors, which can be grouped into internal and external factors (van Oort *et al.,* 2015), lead to deviation between scheduled and real observed times, which is perceived by users as unreliability. This unreliability is highly inconvenient for users. It has been found that a reduction in travel time variability is even more valuable to users than a reduction in travel time (Bates *et al.,* 2001).

The variability of travel time (or the uncertainty in trip journey times) can be split into three distinct components: the day-to-day variability, the variability over the course of a day and the vehicle-to-vehicle variability (Noland and Polak, 2002). Research puts the most focus on the day-to-day variability, which describes the variability of the travel time of the same route made at the same time on different days. Day-to-day variability deteriorates the reliability of a public transport system by increasing in-vehicle times and waiting times of passengers (Mazloumi *et al.,* 2010).

Probabilistic distributions are capable of describing the nature and the pattern of travel time variability. Understanding the distributions of travel times and its components is a prerequisite for analyzing reliability. An appropriate choice of the travel time distribution is an essential input to effective microsimulations of transport and transit systems, as well as for travel time prediction and discrete choice modeling in route selections (Mazloumi *et al.,* 2010). Furthermore, having simple but meaningful distribution models allows performance increases for probabilistic delay forecasting.

Various studies put significant effort in fitting travel time distributions (e.g. Xue *et al.,* 2011, Kieu *et al.,* 2014a, Ma *et al.,* 2016). The findings are strongly influenced by the type and aggregation of the tested data. Therefore it is difficult to compare and contrast past findings. This work aims to start closing this gap in literature and thus specially focuses on aspects of temporal and spatial aggregation. It exploits the travel time variability, split into its components dwell and running time, of urban public buses which run on a fixed schedule under special consideration of data aggregation.

2 Travel Time Distributions Analysis

Understanding the distributions of public transport travel times is essential as they describe the variability in a statistical way. Optimally we gain a deep insight in the variability by knowing just few distribution parameters. Even though, the choice of an appropriate travel time distribution is essential, only little empirical research is conducted in the scope of public transport. For private vehicles on the other side, research started quite some decades ago. Initially symmetrical continuous distributions, in particular normal distribution, were proposed to characterize vehicle travel time on a link level. However,further research pointed out that travel time distributions are asymmetric and considerably right-skewed (Richardson and Taylor, 1978, Herman and Lam, 1974). Nowadays, the most recommended and applied time distribution is the log-normal distribution due to its good fit and relatively simplicity (Xue *et al.,* 2011).

The travel time of buses consists out of running time (i.e. time which bus takes to travel form station to station) and of dwell time (i.e. the sum of passenger service time and the time needed to open and close the bus doors (TRB, 2010)). The route-based travel time can be written as the following equation:

$$t_t = \sum_{n \in r} t_d + \sum_{n \in r} d_r \tag{1}$$

where t_t gives the travel time of a vehicle on bus route r, t_d represents the dwelling times at the stations of the route r, t_r represents the running time in all sections of the bus route. Note that on one route there is one less (intermediate) station as there are sections.

The research on fitting statistical distributions to travel time data of public buses began in the 1980s and was only intensified in the last decade, mainly because previously travel time data was not comprehensively available. Therefore, only a limited amount of studies exist. In an early work, Taylor (1982) collected data of successive daily travel times of the trip from home to work commencing 8:15 am each day. The conclusion of this study was that the bus travel time on sections follow a normal distribution. Kieu *et al.* (2014b) analyzed the distributions of public transport travel time by the aid of transit signal priority data and recommended the log-normal distribution as the best descriptor of bus travel time on urban corridors. Mazloumi *et al.* (2010) showed, using a GPS data set for a bus route in Melbourne, Australia, that travel time distributions are in most case best characterized by normal distributions. Also Xue *et al.* (2011) substantiated that the actual distribution of travel times is a highly-skewed distribution rather than a symmetric distribution like normal distribution. Furthermore, they verified that bus vehicle travel time in peak hours is best fitted by log-logistic distribution. Rahman *et al.* (2018) analyzed bus travel time distribution based on GPS data and concluded that log-normal and normal distributions provide the best fit. Durán-Hormazábal and Tirachini (2016) stated that in the majority of cases asymmetrical distributions and in particular log-logistic distributions describes the travel time of trips done by users well. Ma *et al.* (2016) explored travel time distribution using automatic vehicle location data collected on two typical bus routes over 6 months in Brisbane. Considering unimodal distributions, it was found that the normal, log-normal, logistic, log-logistic, and Gamma models have a relatively similar performance considering travel times on a route level. However, they suggest the use of multi-modal distributions and propose to use Gaussian mixture models. This due to the fact that Van Lint and Van Zuylen (2005) identified for private vehicles four phases that yielded distinctively different shapes of travel time variability. Ma *et al.* (2016) used up to three modes, which can be related to free flow, recurrent and non-recurrent traffic states, to describe the probability distributions. Also Chen and Sun (2017) fitted mixture models to travel time of buses in order to account for different states in traffic. The results indicate that during peak hours a model with four modes represents the data the best and during off-peak hours a model with two modes. However, it remains unclear how this service states can be described.

The presented empirical studies don't present a clear overall result. It remains unclear which statistical distribution should be used under what circumstances. Due to the fact that differing empirical data is used, it seems impossible to draw a superordinate conclusion out of this studies. The shape and nature of the resulting distributions seem to strongly depend on the used data. Leaving aside different evaluation approaches (i.e. how a distributions gets to be the best fit and which distributions were considered), the main limitation is the data aggregation. Table 1 presents selected studies on travel time distribution fitting. In this table information on temporal aggregation and spatial aggregation is given. The temporal aggregation is characterized through departure time widows (DTW).

It is known and widely agreed upon in private vehicle traffic, that the temporal aggregation of traffic data alters the stochastic characteristics of the traffic. A investigation by Vlahogianni and Karlaftis (2011) came to the conclusion that time series of traffic volume and occupancy in urban signalized arterials are strongly sensitive towards effects of temporal aggregation. Mazloumi *et al.* (2010) investigated the effects of the temporal aggregation of bus travel time data by aggregating the travel time data into departure time windows. In that way they explored the travel time distributions for different departure time windows at different times of the day. Their study shows that in narrower departure time windows, travel time distributions are best characterized by normal distributions. Also considering wider departure time windows, peak-hour travel times are well represented by normal distributions, while off-peak travel times seem to follow log-normal distributions. Also Ma *et al.* (2016) investigated temporal data aggregation influences on travel time distribution. They considered the following temporal aggregation attributes: weekday or weekend, period (peak, off-peak, inter-peak), 60 minutes, 30 minutes, 15

Table 1: Selected studies considering travel time distribution fitting for public buses

Source	Data Collection Method	Study Area	Time of the day	DTW (temporal aggreagation)	spatial aggregation	Proposed Distribution
Taylor (1982)	Manually	Paris, France	Morning Peak (8:15)	0 min (one specific bus)	Section	normal
Mazloumi *et al.* (2010)	GPS	Melbourne, Australia	6:30 - 18:30	whole period	Route	normal
			Morning / afternoon peak	whole period, 5 min		normal
			Interpeak / afternoon off-peak	whole period		log-normal
			Interpeak / afternoon off-peak	5 min		normal
Xue *et al.* (2011)	GPS data, smart card data, vehicle dispatching information	Nanning, China	Morning peak, afternoon peak	peaks together and separately	Route	log-logistic
					Section	normal
Kieu *et al.* (2014a)	Transit Signal Priority Sensor	Brisbane, Australia	all day	0 min (specific buses)	Section (2.2 - 3.1 km)	gamma, log-normal
Durán-Hormazábal and Tirachini (2016)	Smartcar + GPS and travel time survey	Santiago, Chile	all day	0 (?)	Journeys of passengers (avg length 5.6 km)	log-logistic
Ma *et al.* (2016)	AVL data	Brisbane, Australia	peak and off-peak	period, 60 min, 30 min, 15 min	Section, Route	gaussian mixture models
Rahman *et al.* (2018)	GPS	Calgary, Canada	Morning Peak (6:00-9:00)	-	Pseudo horizon	lognormal, normal

minutes, and 5 minutes. It was concluded that the increase of temporal aggregation of travel times tends to decrease the normality of distributions.

Furthermore, Ma *et al.* (2016) had a look on the influence of the spatial aggregation towards the travel time distribution. In order to do so, they looked at route level and link level distributions. It was found that link travel times are often multi-modal whereas route travel time seem to be rather unimodal. In other words, the multi-modality of the distribution on a link level seems to be broken up when they are aggregated to a route level. This is explained as when a vehicle drives comparatively slow in the first link, the driver can speed up in the succeeding link in order to catch up with the time table. If a bus drives along a bus-only corridor, this effect is intensified, as the driver has in this case relatively more flexibility to adjust speed to catch up with schedules in comparison with driving on a road in mixed-traffic.

When considering running times, we see similarities to private vehicle transport, as long as buses have the same free flow speed as private cars and cannot travel on designated bus lines. In other words, when buses share the same road space with cars and have similar characteristics, the statistical modeling of those means of transport should also be similar. For modeling of running times of bus routes mostly log-normal distributions are proposed (Uno *et al.,* 2009, Mazloumi *et al.,* 2010, Xue *et al.,* 2011). Some studies also suggest to use Gamma distributions, such as Jordan and Turnquist (1979), which studied the running time of buses in the morning peak in Chicago.

The dwell time is in literature often modeled as a function of the amount of passengers boarding and alighting as well as the crowding level on vehicle (one can also find studies with look at much more factors as e.g. bus type). Studies have shown that dwell time would increase significantly when the vehicles are crowded because it is hard for passengers to board and alight at that time. Only a handful of studies dealing with the modeling of statistical distributions of dwell times can be found. Jiang and Yang (2014) did video analysis of three bus stops in peak hours in Shanghai, China and proposed a Erlang distribution to fit the data best. Li *et al.* (2012) studied the dwell times in Changzhou, an other Chinese city, and proposed a log-normal distribution, consistently with Rajbhandari *et al.* (2003) who studies dwell times in New Jersey, USA. Khoo (2013) investigated the influence of some parameters to the dwell time distribution. It was found, that both for peak and off-peak conditions the Person 6 distributions fits the data the best. A further investigated parameter is the platform crowding level. The claim, that for less crowded situations the Weibull distribution fits the data the best, whereas for more crowded conditions the Person 6 distribution has the better fit. Furthermore, they showed that for both payment methods (Cash/Card System and Conductor System) the Pearson 6 distribution fits the data the best.

All proposed continuous distributions can only approximate the effective distributions of travel time and its components. The effective travel time distributions can not be perfectly modeled. This is due to fact that all distributions have certain limitations. Normal distributions exhibit negative values for travel times, which obviously doesn't conform to reality. Also log-normal distributions, in which negative values cannot arise, exhibit small values, whose appearance is physically not possible. This problem is true for all distributions which are not bound to the left. While fitting models, it is always possible to increase the likelihood by adding parameters, but doing so may result in over-fitting. The previously presented empirical studies aiming to statistically model travel time distributions give different results. A major reason for this inconsistency is the data used. The spatial and temporal aggregation of data as well as the evaluation approach seems to lead to quite different results and has to be looked at closely before drawing conclusions.

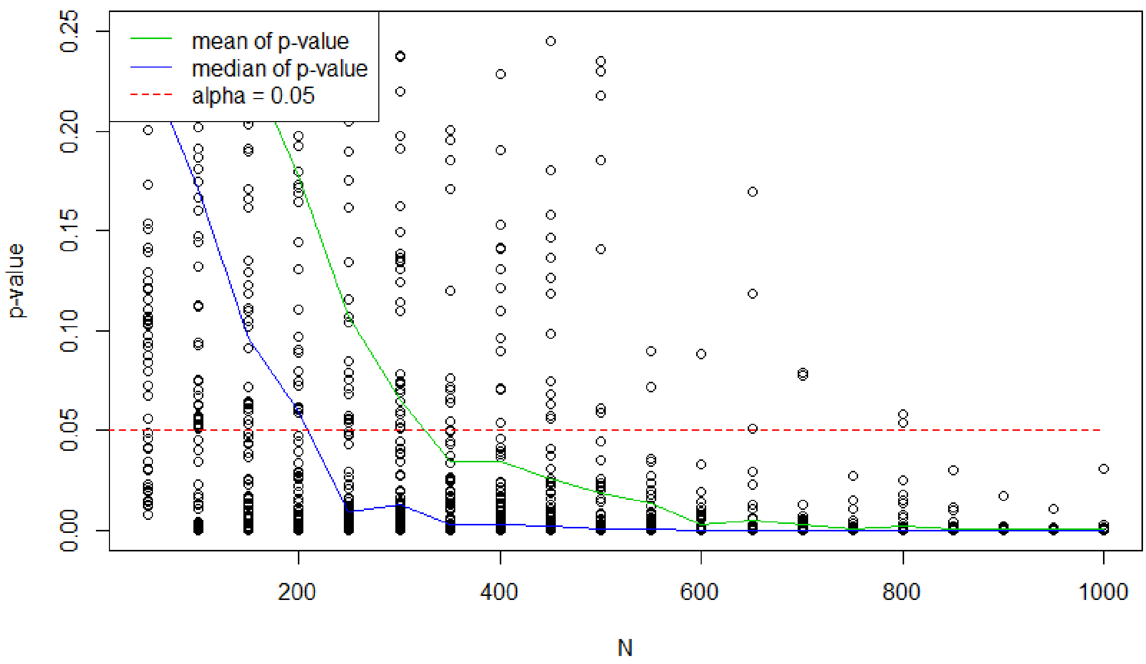
2.1 Distribution Fitting

When trying to find the most suitable statistical distribution to model travel times and its components, different methodologies are used in literature. It is common for studies on travel time distribution to evaluate the the distributions based on goodness-of fit tests as Anderson-Darling (Ma *et al.,* 2016), Kolmogorov-Smirnov (Mazloumi *et al.,* 2010) or chi-squared (Khoo, 2013). A description of the procedure of hypothesis tests can be found in Appendix A. These kind of tests have different limitations. A major limitation of Kolmogorov-Smirnov and Anderson-Darling tests are, that they are not valid if parameters are directly estimated from the tested data. A second major limitation is the interpretation of p-values. The often used threshold of $p < 0.05$ is not well founded. A conclusion is not automatically "true" on one side of the threshold and "false" on the other side (Wasserstein and Lazar, 2016). Secondly, given the above presented null hypothesis, the p-value depend on the sample size.

In order to underly this argument, the total dwell time along the route 32 (which will also be analyzed in the later chapters) is tested by the aid of the Kolmogorov-Smirnov test following a log-normal distribution. The null hypothesis is defined by: The data follows any other distribution than a log-normal distribution. The significance level, α, is assumed to be 0.05. The sample data consist out of many thousand observations. Out of these observations 100 random samples with sample size N are retained. In a next step the p-value of those samples were calculated. In Fig. 1 the p-values as a function of the sample size are shown. Additionally, two curve representing the means and medians of the p-values are shown. It can be seen that if the sample size exceeds a certain number, the p-value is smaller than the α-value, meaning the log-normal distribution should be rejected. Therefore a big sample size seems to have an effect in the p-value. Especially in the era of big data one have to use p-values carefully. It will often be possible to find a tiny difference between two results which is statistically significant, but there is no real world meaning in this difference.

There are different approaches to do deal with the aforementioned problems. It is possible to make use of the Bayesian information criterion (BIC, also known as Schwarz criterion), which measures the relative quality. It considers the number of parameters used and goodness-of-fit of the fitted distribution. The fitted distribution with the lowest BIC is considered the preferred

Figure 1: Influence of the number of observations to the p-Value



descriptor of the data. There is no hypothesis testing involved. Additionally, it is possible to examine the fitted distributions graphically. In order to do so, one can make use of probability plots, cumulative probability plots, probability-probability plots or quantile-quantile plots. By graphical means multiple distributions can be compared visually. However, the performance evaluation is subjective.

3 Data & Methodology

3.1 Data

The urban bus line 32 in Zurich, Switzerland belonging to the network of Verkehrsbetriebe Zurich (VBZ) was investigated. The line 32, which is a trolleybus line that crosses the city, as seen in Fig. 2 is chosen for the analysis. This bus line is round 11 km long and runs from Zurich, Stassenverkehrsamt (STRV) to Zürich, Holzerhurd (HOLZ). It serves Goldbrunnenplatz, where a number of buses depart to the agglomeration of western Zurich. At Kalkbreite / Bhf Wiedikon it connects to the tram lines 2 and 3 and to the train network. It passes Langstrasse, a very populated nightlife area and connects at Limmatplatz to tram 4, 13 and 17. At Bucheggplatz and at Glaubtenstrasse Süd it connects to bus lines. The line operates from 5 am to 1 am, with a minimal planned headway of 6 min. The service frequency varies from 10 bus/h (peak) to 6 bus/h (evening off-peak). This line is one of the most delayed line in the public transport network of Zurich.

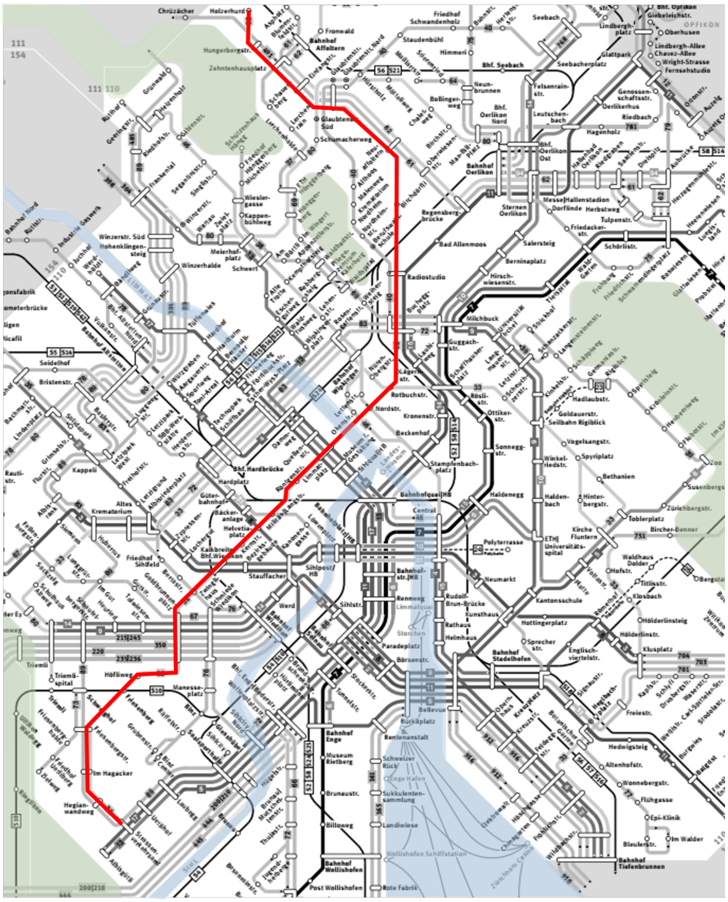
The VBZ issues all planned and real occurred arrival and departure times of the buses and trams at the stations of the network. This data is publicly available and can be found on the open data portal of the city of Zurich (https://data.stadt-zuerich.ch). The data is captured to the accuracy of seconds. For this study, the data of October and November 2017 is used. The raw data is filtered for regular trips on line 32, meaning special routes to and from depots / garages as well as trips that turn early or do a detour are neglected. It is important to note that do to this filter the occurrence of extreme values (i.e. high delays) need to be interpreted with care as buses with high delays are often subject to some dispatch measure.

3.2 Methodology

3.2.1 Data aggregation

The time of the day, in other words peak and off-peak hours, have an underlying effect towards travel time data. Therefore these periods are in this study never aggregated together. Temporal data aggregation is only done within a period of the day. Two two-hour periods of the day were investigated in particular afternoon peak (16:00-18:00) and off peak (11:00-13:00). For the temporal aggregations aggregation attributes between 2 h (= whole period) and 0 min (= single bus) were used. An aggregation with the attribute 30 min means that the variability of the travel time of buses departing in a time window of 30 min are considered. In order to consider always

Figure 2: Public transport network of Zurich with highlighted line 32 (red).



Source: https://www.stadt-zuerich.ch/vbz/de/index/fahrplan/liniennetzplaene.html (27.03.2018, adapted)

the same data at each aggregation level, the period is divided into time slots with the length of the aggregation attribute. Considering the aggregation attribute of 30 min, four ($2h/30min = 4$) evaluations are done independently. For all found characteristics the average is considered.

In order to account for the spatial aggregation, different aggregation attributes are considered. Here, successive sections are aggregated. Analogous to the temporal aggregation, the whole route of the bus is divided into parts with aggregation length. For example if the aggregation attribute is 3 sections and the route consists out of 24 sections in total, eight evaluations are done independently. Finally the average is considered.

Each combination of aggregation attributes leads to a test case. Each test case considers the same data and hence different test cases are comparable. For each test case travel time, running time and dwell times are analyzed.

3.2.2 Descriptive Statistics

Descriptive statistics, which characterize the description of the travel times are conducted for each test case. The skewness, a measure of asymmetry of a distributions, and the kurtosis, a measure for the weight of the tails in relation to the rest of the distributions, are used. Given a unimodal distribution, positive skew indicates that the tail on the right hand side of the probability density function is more distinct (i.e. longer or fatter) than the left side. Kurtosis gives information about the tails in comparison with the rest of the distribution and is hence a measure of whether the data are heavy-tailed or light-tailed in comparison with a normal distribution (DeCarlo, 1997). It is often stated that skewness and kurtosis values of more than twice the corresponding standard error[1] are sufficient to reject the value 0 for skewness and kurtosis (Wright and Herrington, 2011) . However, this has to be critically questioned as the value of these measures depend on the sample size [2].

3.2.3 Distribution Fitting and Evaluation

For each test case, distributions fitting is performed. The used distributions models are Weibull, normal, log-normal, log-logistic. Especially normal and log-normal distributions are commonly used in public transport studies. The parameters of the continuous density functions are estimated by utilizing the maximum likelihood method. No hypothesis tests, as Anderson-Darling or Komogorv-Smirnoff tests, were done to justify the fitted distributions, because of the limitations of the tests, which are pointed out in section 2.1 . However, the fitted distributions are tested with visual tests, by the aid of probability plots, cumulative probability plots, probability-probability plots and quantile-quantile plots. Furthermore, the BIC was used to identify the best fitting distributions. The model with the lowest BIC statistic exhibits the best fit. The distributions are ranked by an increasing BIC value.

When aggregating data, it can occur that different probability distributions exhibits the best fit. The aggregated best fit is defined as the distribution which is the most times the best fitting distribution. The second best fit is defined as the fit which occurs the most times among the individual two best fits while neglecting the best fitting distribution.

[1] The standard error of the skewness can roughly be estimated as $\sqrt{\frac{6}{N}}$ and for kurtosis as $\sqrt{\frac{24}{N}}$, where N is the sample size

[2] Additionally, this practice is only true, if the standard error estimate is accurate and the sampling distribution is approximately normal. (Wright and Herrington, 2011)

4 Results

4.1 Analysis of Travel Times Components

In a fist step the planned route level time components are compared with the realized travel time components. In Fig. 3 this is done for the direction HOLZ-STRV (Fig. 3(a)) and for STRV-HOLZ (Fig. 3(b)). The black lines represent the planned times, the observed time components are represented by dots. Additionally, the mean as well as the mean plus minus one standard deviation are presented as colored lines. The time represents the staring time at the first bus stop.

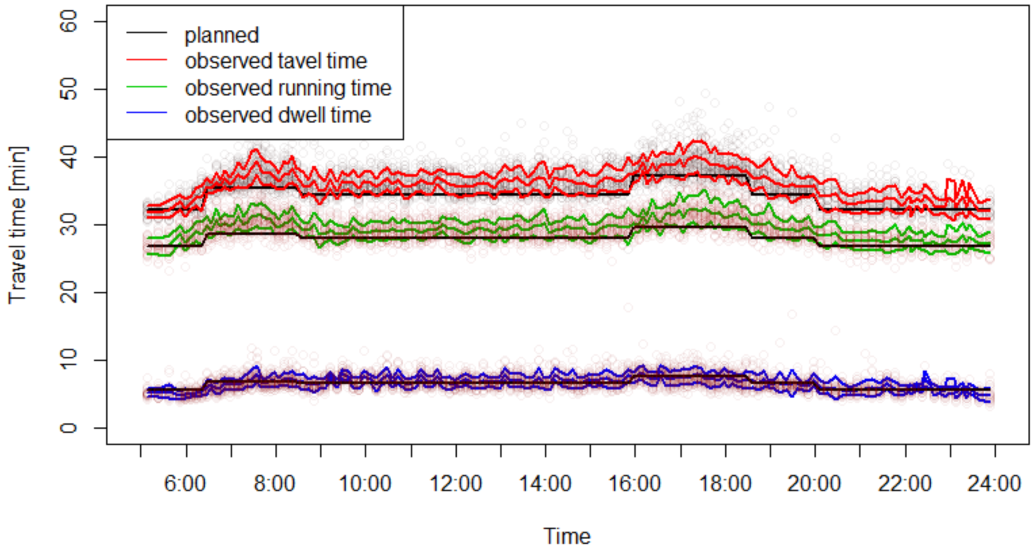
The major part of the travel time of line 32 is the running time, which is about 75% of the travel time. The dwell time contributes the other 25%. Direction HOLZ-STRV exhibits the morning peak (6:30 - 9:00) as well as during the afternoon peak (16:00 -18:30) slightly higher travel times. The real occurring as well as the planned travel times are higher than during other times of the day. In the other direction, there is no explicit peak in travel times in the morning hours. However, the travel times in the afternoon are clearly higher than during the rest of the day. Also the variation of the travel, running and dwell times increase clearly during these hours.

These travel time components on a route-level are in the next step visualized on a section level. Fig. 4 shows running, dwell and travel times for each station or section for both directions for peak and off-peak conditions. As peak a departure time on the first stop between 16:00 and 18:00 and as off-peak a departure time on the first stop between 11:00 and 13:00 was considered. The characteristics, such as road type, section length or amount of signalized intersections, of section along a bus route vary typically strongly. This different characteristics are the reason for different travel time distributions.

Fig. 5 shows the mean values of the travel times and its components at different sections of the bus lines 32 in the direction STRV-HOLZ during off-peak conditions (solid lines). Centered moving averages with periods of 3 and 5 are introduced to show the effect of spatial aggregation towards the mean of travel times and its components. It can be seen that a higher period for calculating the moving averages leads to less peaky lines. Taking averages and therefor also increasing spatial aggregation smooths out the lines. A similar figure could be obtained by looking at the variation (e.g. by considering the standard deviation) of travel times in every section.

Figure 3: Travel time variation over the course of one day for line 32. The travel time is split into running times and dwell times.

(a) Direction HOLZ-STRV



(b) Direction STRV-HOLZ

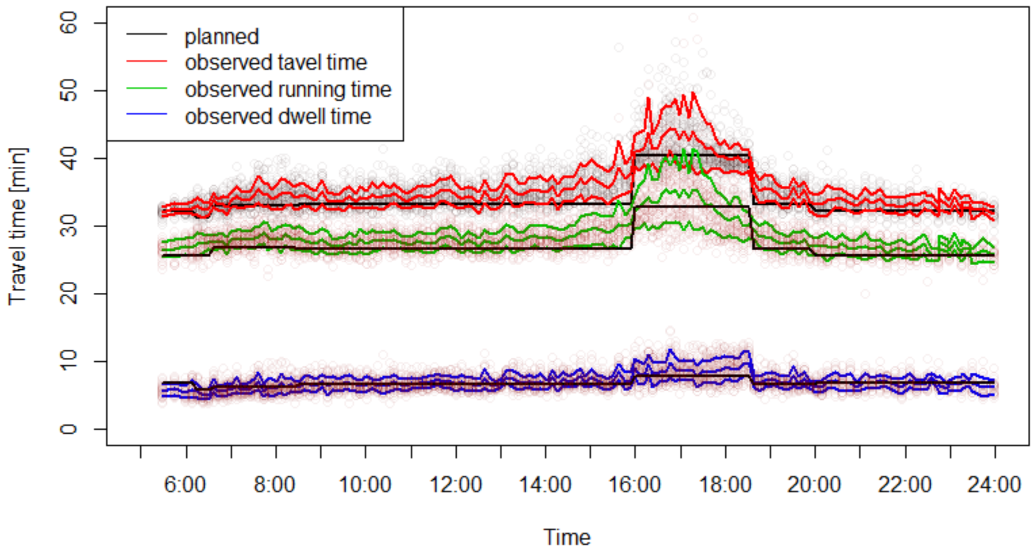
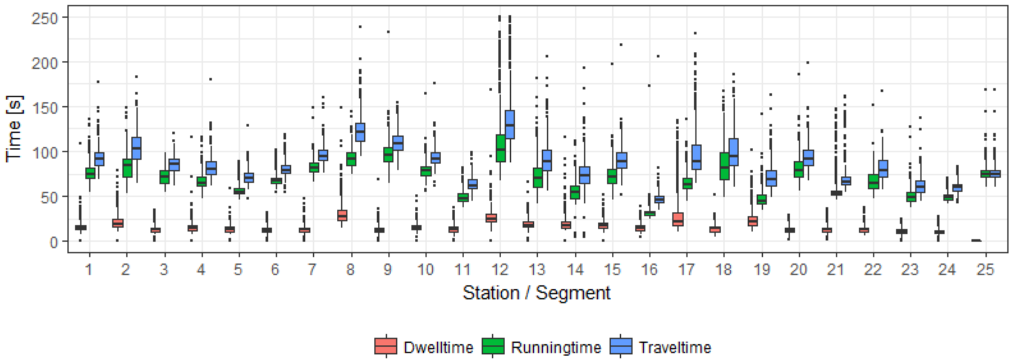
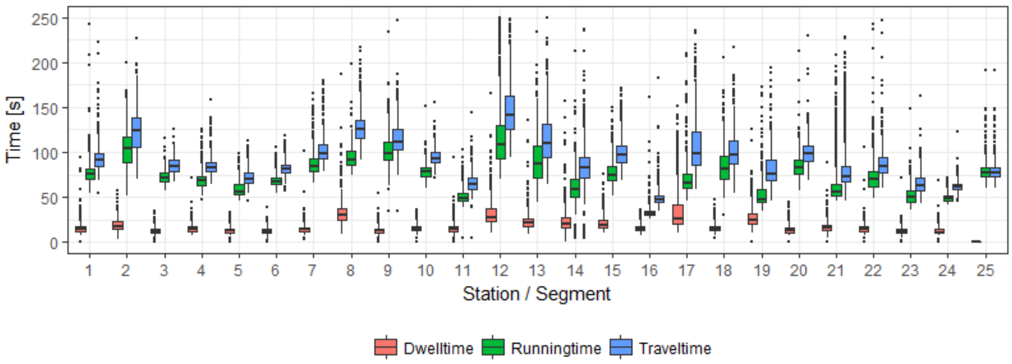


Figure 4: Boxplots for travel time and its components at peak and off-peak for direction HOLZ-STRV and direction STRV-HOLZ at a section level

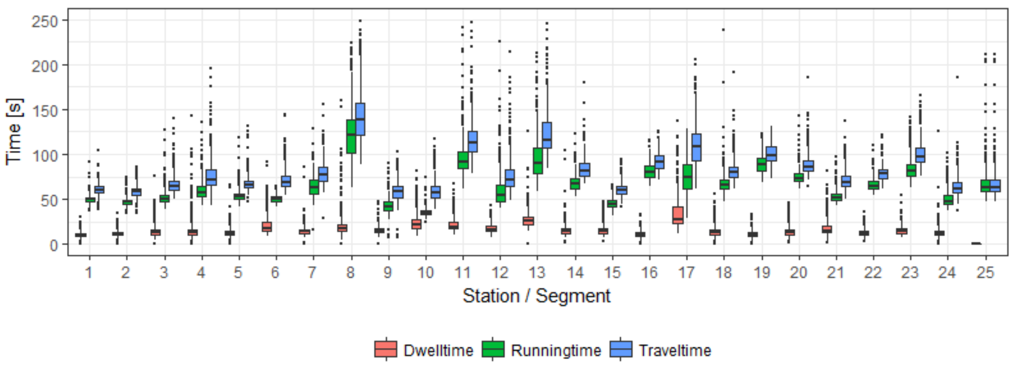
(a) Direction HOLZ-STRV, Off-Peak



(b) Direction HOLZ-STRV, Peak



(c) Direction STRV-HOLZ, Off-Peak



(d) Direction STRV-HOLZ, Peak

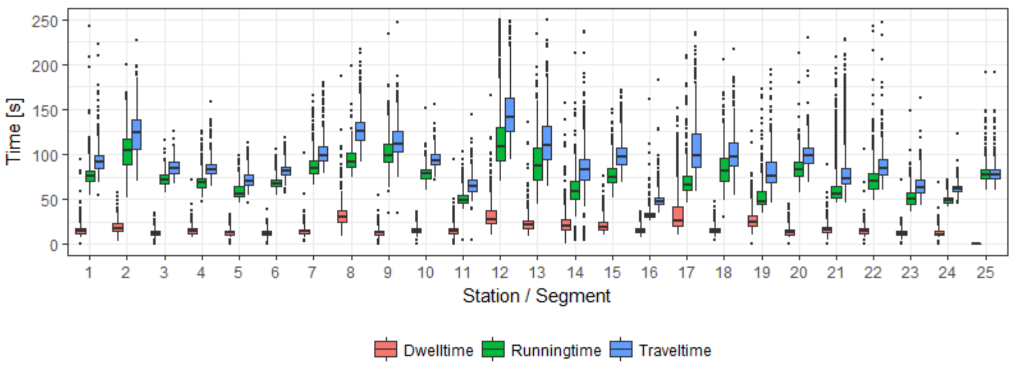
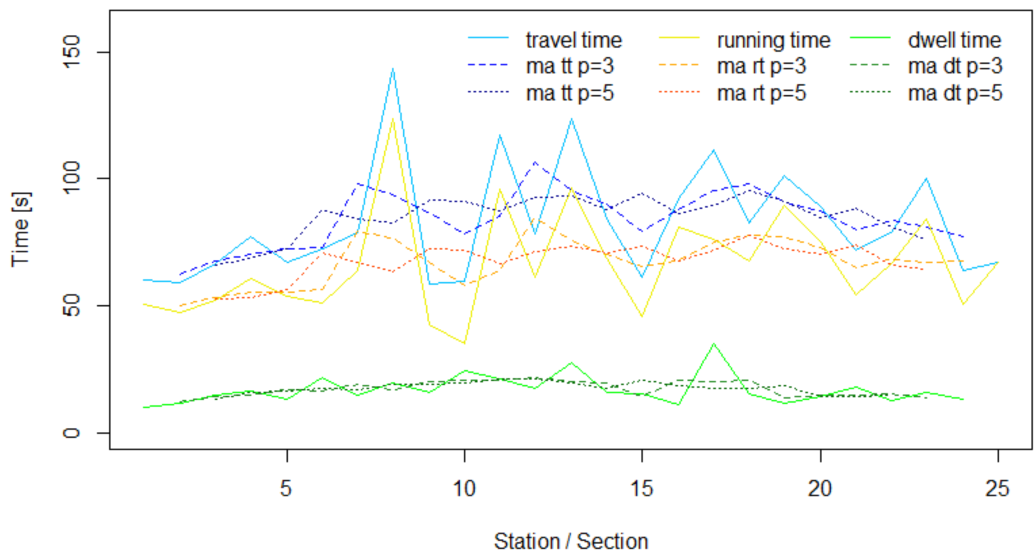


Figure 5: Mean values of travel times (tt), running times (rt) and dwell times (dt) at different sections of direction STRV-HOLZ during off-peak conditions. Additionally, centered moving averages (ma) with periods (p) of 3 and 5 si shown.



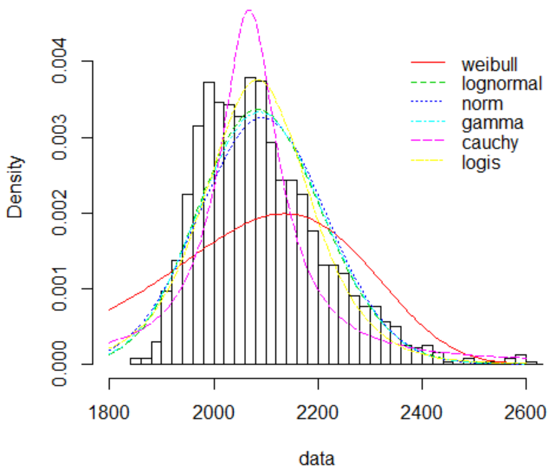
4.2 Distribution Fitting

The distribution fitting was in a fist step reviewed visually. Considering the distributions for travel time and its components visually, it sticks out that they are always skewed to the right. Often the log-normal, gamma, logis and the normal distributions seem to fit the data quite nicely. Visually, the shape of the Weibull and Cauchy distribution seem inadequate for travel times and its components. The very distinct peak of the Cauchy distribution as well as the extreme values of the Weibull doesn't represent the data well. It is not put emphasis towards the fitting of extreme values, as those values of the data are not meaningful.

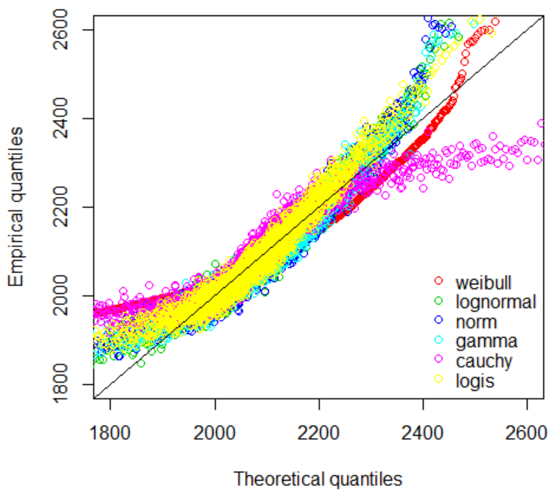
Fig. 6 shows the results of the distribution fitting for the total travel time of the route 32 direction STRV-HOLZ during the off-peak. From the comparison of the histogram with the theoretical densities (Fig. 6(a)), the Q-Q plot (Fig. 6(b)), the comparison of the empirical with the theoretical cumulative distribution function (Fig. 6(c)) and the P-P plot (Fig. 6(d)) it can be seen that log-normal, gamma, logis and the normal distributions, present appealing fits to the data. However, it is not possible to determine which one is the best.

Figure 6: Influence of the number of observations to the p-Value

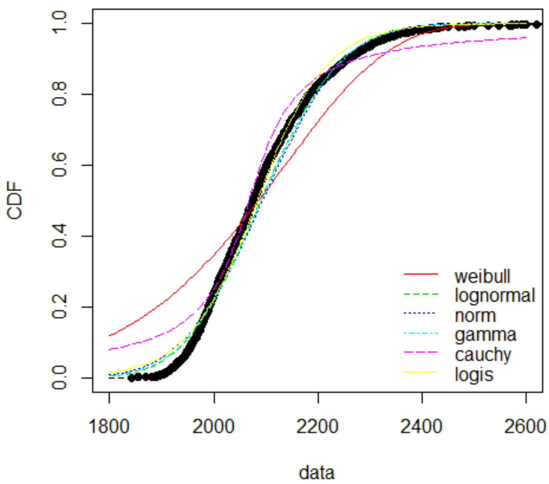
(a) Histogram and theoretical densities



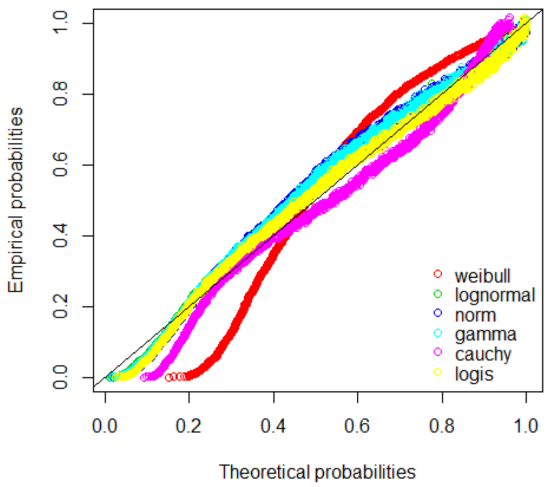
(b) Q-Q plot



(c) Empirical and theoretical CDFs



(d) P-P plot



4.3 Spatio-temporal Aggregation

In the following the descriptive statistics coefficient of variation (Fig. 7), skewness/se [3] (Fig. 8), kurtosis/se [4] (Fig. 9) of the bus travel time components (dwell - , running - and travel time) are presented. They are presented for combinations of different spatial and temporal aggregation levels for peak and off-peak conditions for the direction STRV-HOLZ. Temporal aggregation reaches form 0 min (single bus) to 120 min (duration of peak period). Considering the spatial aggregation, between 1 and 10 sections were considered.

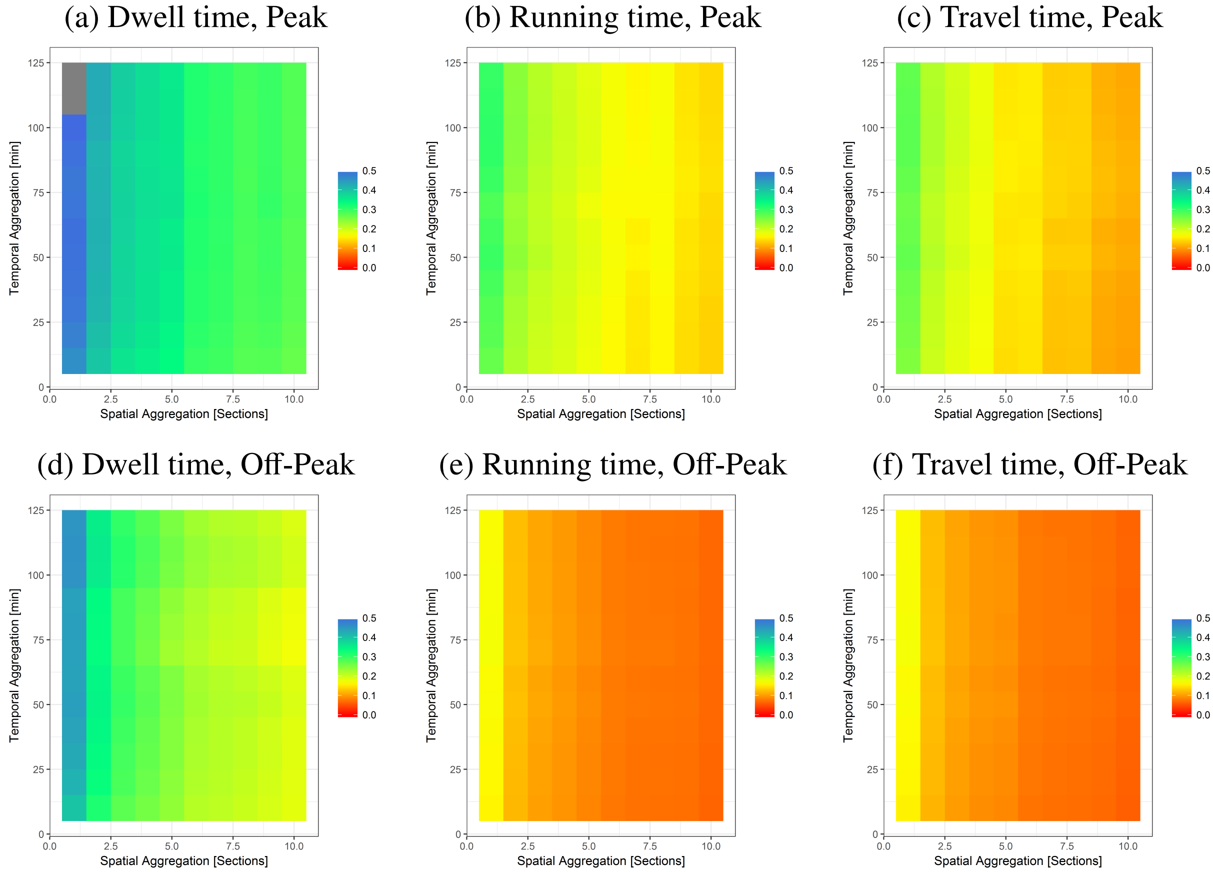
First, looking at the coefficient of variation, reducing spatial aggregation leads to higher coefficient of variation. This holds true for all travel time components and for both, peak and off-peak conditions. However, considering the temporal aggregation no clear trend is visible. This was also reported by Ma *et al.* (2016). The coefficient of variation of dwell times is slightly higher for big temporal aggregation intervals. Comparing peak and off-peak time periods for a same combination of temporal and spatial aggregation, the coefficients of variation of the peak period are larger than those of the off-peak period. This can be explained with the fact that traffic conditions at peak periods are more complicated, and operation is affected by high volume of car traffic on the lanes in mixed traffic. Dwell times have a distinguishably higher coefficient of variation comparing with running time, which oneself have slightly higher coefficients of variation that travel times. It appears meaningful that the travel time has the lowest and the dwell time the highest coefficient of variation. As in practice, buses which are ahead of schedule wait at certain stations until their scheduled departure time. The dwell time has a certain buffer and hence varies strongly. The travel time on the other hand is smoothed. Furthermore, the passenger volume is not regular and there is a massive difference in dwell time if just one or no passenger boards the vehicle at a stop.

Both, the skewness/se and the kurtosis/se values increase with increasing temporal aggregation intervals under all scenarios. This represents a more skewed and a less flat (more peaked) distribution. A similar behavior was reported by Ma *et al.* (2016). Comparing peak and off-peak time periods for a same combination of temporal and spatial aggregation, the skewness/se and kurtosis/se of the peak period are larger than those of the off-peak period. Dwell, running and travel times exhibit similar skew/se and kurtosis/se values.

The best and second best fitting distributions are presented in Fig. 10 and Fig. 11 respectively. For the dwell and the travel time the log-normal distribution exhibits the best fit, independent on the spatial and temporal aggregation. Also for the running time the log-normal distribution is in most cases superior to the other investigated distributions. But for few combination of

[3] skewness/se = skewess divided by the standard error of the skeweness
[4] kurtosis/se = kurtosis divided by the standard error of the kurtosis

Figure 7: Coefficient of variation of travel time components under different spatio-temporal aggregations for different time of the day (peak and off-peak)



spatial and temporal aggregation arguments the logis or the gamma distribution outperforms the log-normal distribution. The second best distribution of the dwell time is gamma distribution. Only for small spatial aggregation arguments the Cauchy distributions outperforms the gamma distribution. Also for running and travel time the gamma distribution in often the second best distribution. Especially for off-peak exhibits the logis distribution a quite good fit. It seems therefore reasonable to model all travel times components by log-normal distributions. This is promising result, as it is very convenient to just deal with one distribution. Furthermore, for the log-normal distribution exists the well-known rule of thumb that the sum of two log-normally distributed variables follows a log-normal distribution[5].

[5]Dufresne (2004) shows how the sum of two normal distributions can be mathematically approximated.

Figure 8: Skewness/se of travel time components under different spatio-temporal aggregations for different time of the day (peak and off-peak)

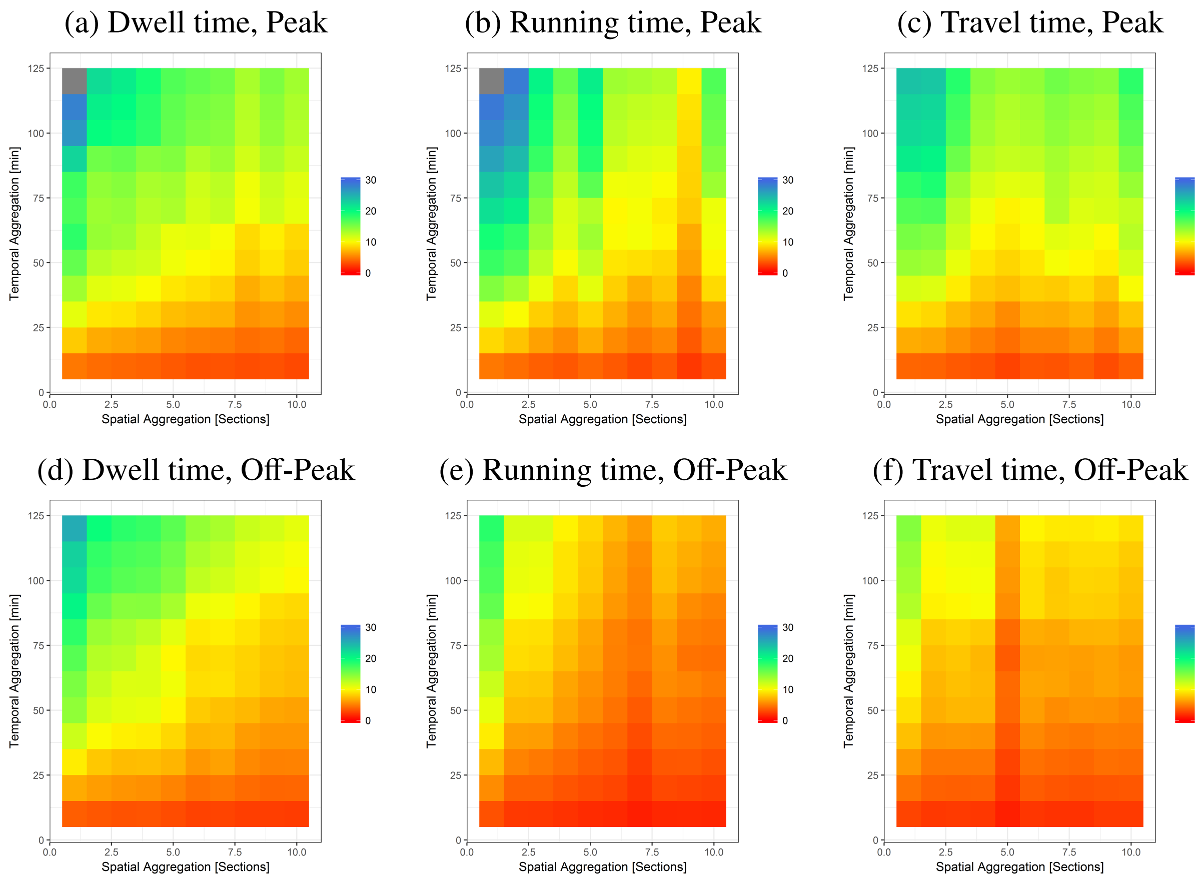


Figure 9: Kurtosis/se of travel time components under different spatio-temporal aggregations for different time of the day (peak and off-peak)

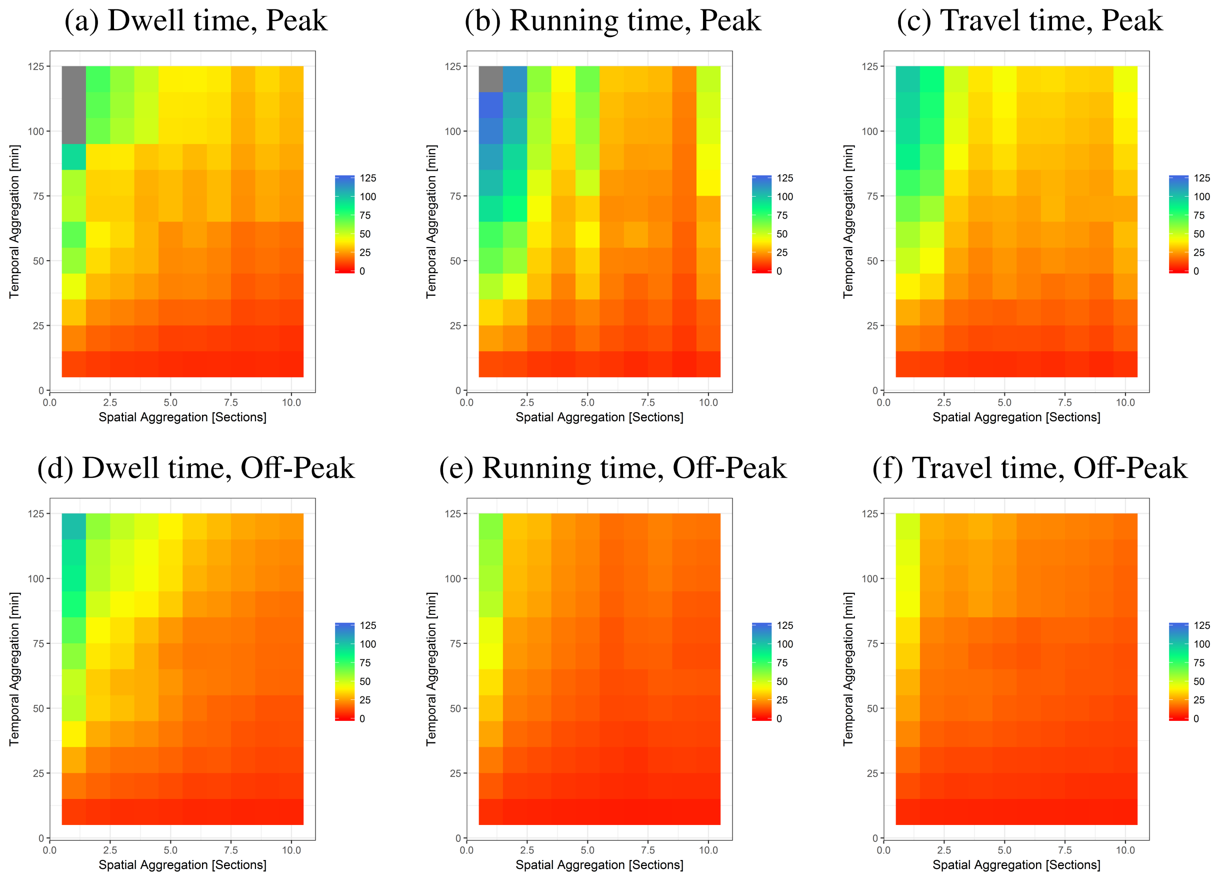


Figure 10: Best fitting distribution of travel time components under different spatio-temporal aggregations for different time of the day (peak and off-peak)

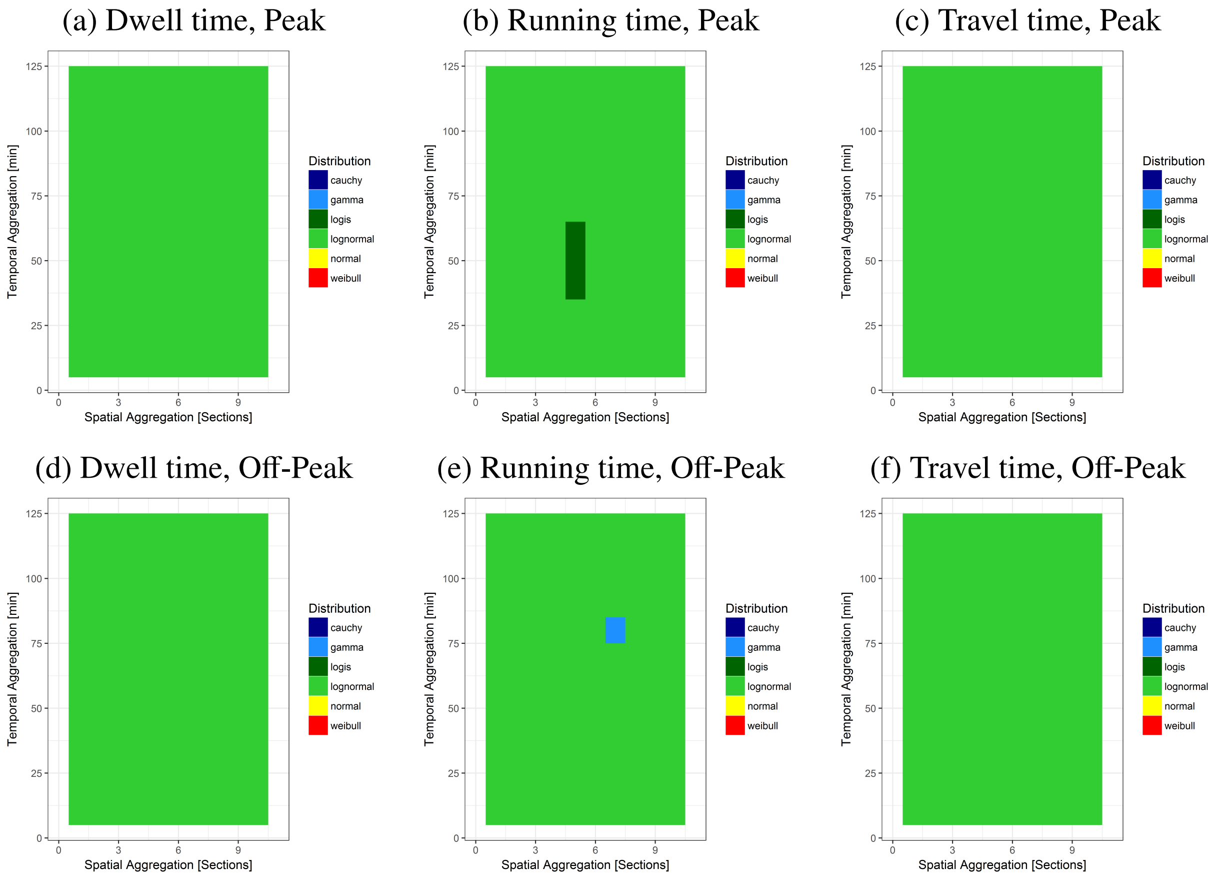
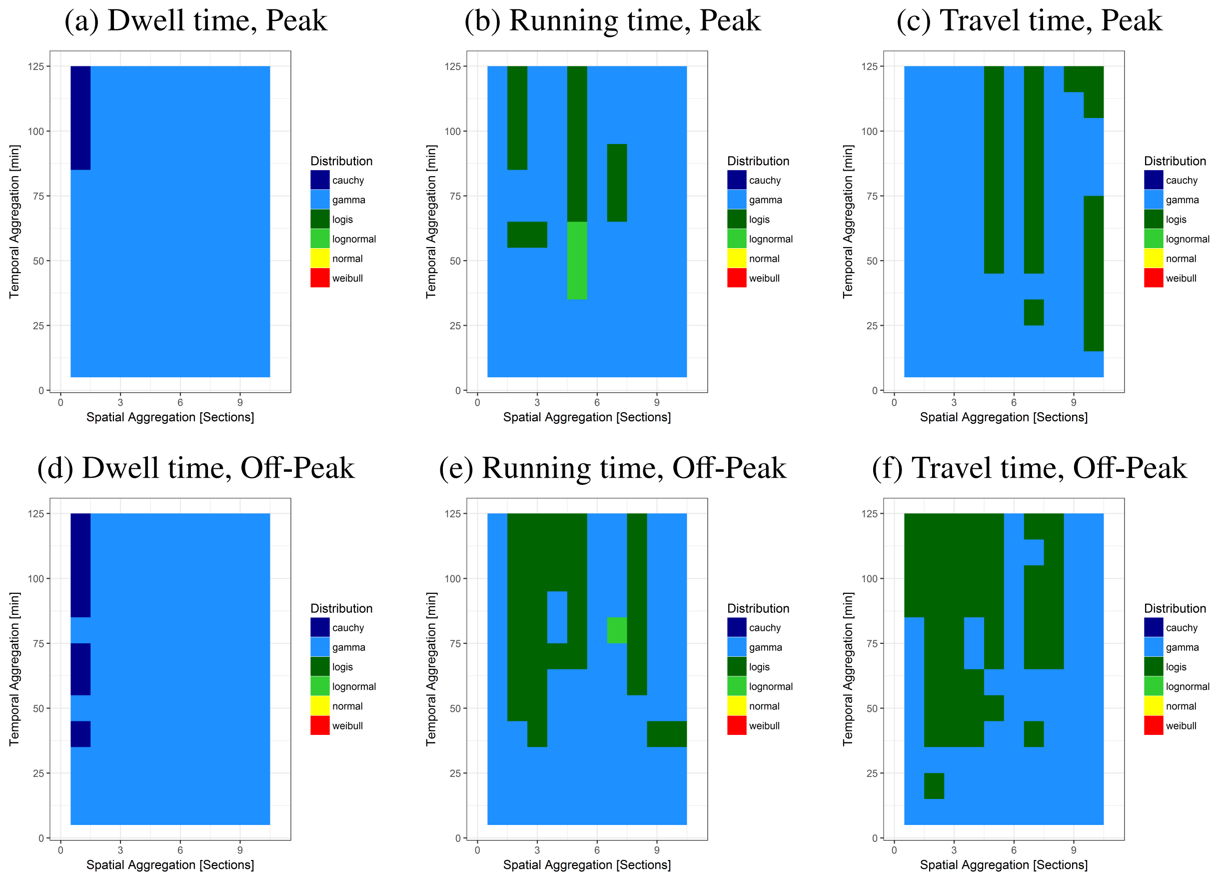


Figure 11: Second best fitting Distribution of travel time components under different spatio-temporal aggregations for different time of the day (peak and off-peak)



5 Conclusions

Understanding travel time distribution properties is critical in order to understand travel time variability. In this work travel times were split into running times and dwell times and analyzed in different spacial and temporal aggregations. The results underline that travel time distributions tend to towards normality for short departure time windows, which was described earlier by Mazloumi *et al.* (2010) and Ma *et al.* (2016).

In this work the influence of spatial aggregation, to which often only limited consideration was given, was taken into account. It was shown that the spatial aggregation has a major effect to the statistics of travel time components. Based on the findings of this work, earlier studies that aggregated data differently can be better compared.

Various unimodal continuous probability distributions were fit to the travel time data and its components. It was shown that the log-normal probability distribution is a good fit for dwell, running and travel times at peak and off-peak conditions. It goes without saying that the log-normal distributions is only a good approximation of the occurring distributions and by no means the true distribution. This work does not imply any statement on extreme values of travel times and its component. It seems promising for further analysis that all components of travel time can be well described by the same distribution type (log-normal distribution).

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A Distribution Fitting

First a probability distribution which should be tested is defined. In a next step the null hypothesis H0, that the observations comes from the alternative distributions, is defined. As shown before, there are different methods for statistical hypothesis testing. By the aid of those methods it should be elaborated whether a result has statistical significance. This is the case when a result is very unlikely to have occurred given the null hypothesis. The significance level, α, which is the probability of the study rejecting the null hypothesis, given that it were true, is often set to 0.05. The p-value is obtained by applying a statistical hypothesis test gives the probability of obtaining a result at least as extreme, given that the null hypothesis were true. When $p < \alpha$, the result is assumed to be statistically significant. Therefore, the assumed distribution model is rejected when the, trough hypothesis testing received p-value, is smaller than 0.05.