Diss. ETH No. 24903

Designing Forward Guidance at the Zero Lower Bound

A thesis submitted to attain the degree of

DOCTOR OF SCIENCES of ETH ZURICH

(Dr. sc. ETH Zurich)

presented by

MARTIN TISCHHAUSER

M.A. in Quantitative Economics and Finance, University of St. Gallen

born on June 25, 1989

citizen of Switzerland

accepted on the recommendation of

Prof. Dr. Hans Gersbach (ETH Zurich), examiner
Prof. Dr. Jan-Egbert Sturm (ETH Zurich), co-examiner

2018

Acknowledgments

This dissertation was written during my time as a Doctoral Assistant at the Chair of Macroeconomics: Innovation and Policy at ETH Zurich. Developing a dissertation in a stimulating environment is a privilege and I would like to express my gratitude to everyone involved. Part of my thesis is based on joint research with Professor Hans Gersbach and Yulin Liu. I truly appreciate their support during the writing of the thesis.

First and foremost, I am thankful to my supervisor, Professor Hans Gersbach, for his steady guidance and his support throughout my doctoral studies. His help was very motivating and—due to his profound knowledge and experience—greatly beneficial. His advice helped me to continuously improve the thesis and his support to work independently enriched my skill set to approach problems. This experience will be a valuable asset for my future work.

Further, I dedicate a special thank to Yulin Liu for our constructive collaboration. His expertise and mentoring was crucial in the process of developing the main project.

I am grateful to my co-supervisor, Professor Jan-Egbert Sturm, for his availability and his support.

During my time as a Doctoral Student, I got to know many colleagues. I appreciated all the encounters and discussions and I would like to thank Afsoon, Akaki, Anastasia, Aurore, Elias, Ewelina, Evgenij, Felix, Florian, Jürg, Kamali, Marie, Markus, Moritz, Oriol, Philippe, Salomon, Samuel, Stelios, Svenja, Volker, and Yulin. In particular, I enjoyed the coffee breaks with Afsoon, Aurore, Elias, Markus, and Philippe and the pleasant experience of sharing an office with Samuel. Moreover, I am grateful to the people sharing their programming knowledge on the Internet. Their volunteer work supports research in so many aspects but they rarely get the appreciation they deserve.

Finally, I wish to express my gratitude to all my family and friends who were part of my journey of the last few years. Very special thanks go to my sister Manuela for all the little and big ways she has pitched in.

Zürich, December 2017

Abstract

We examine how forward guidance can be designed when an economy faces a negative natural real interest-rate shock and a subsequent supply shock. We use a standard New Keynesian Framework with a scrupulous central banker to study promising designs.

The thesis starts with an introductory chapter that summarizes the state of research on forward guidance up to today and defines the research questions. In Chapter 2, we develop the underlying framework and introduce two flexible designs: escaping and switching. With *escaping* forward guidance, the central banker commits to low interest rates in the presence of negative natural real interest-rate shocks contingent on a self-chosen inflation rate threshold. That is, once the inflation surpasses the threshold, the central banker regains full flexibility to react to a supply shock. With *switching* forward guidance, the central banker can switch from interest-rate forecasts to inflation forecasts in order to stabilize the supply shock. We show that for small and large natural real interest-rate shocks, *escaping* forward guidance is preferred to any of the other approaches, while *switching* forward guidance is optimal for intermediate natural real interest-rate shocks.

In Chapter 3, we extend the basic model and introduce heterogeneous beliefs. Economic agents receive a public signal from the central banker about future interest rates, i.e. the central banker conducts forward guidance. Additionally, the agents receive a private signal which is idiosyncratic to each of them. We show that flexible forward guidance designs lose some of their effectiveness but stay attractive compared to standard forward guidance and a discretionary central banker. When the economic agents are pessimistic about the future, *escaping* forward guidance proves to be the most welfare improving design for all natural real interest-rate shock sizes.

In Chapter 4, we test the robustness of our findings with respect to parameter uncertainty and identify those structural parameters with the greatest effect on our results. We use polynomial chaos expansion to build a surrogate model that enables us to calculate robust social loss functions efficiently. Furthermore, with this technique, we can calculate Sobol' Indices that can be used to rank the structural parameters according to their importance. Finally, we show that our findings are globally robust to parameter uncertainty.

Zusammenfassung

Wir untersuchen, wie man "Forward Guidance", i.e. die Orientierung über die zukünftige Ausrichtung der Geldpolitik, optimal gestalten könnte, wenn sich die Wirtschaft einem negativen natürlichen realen Zinssatz ausgesetzt sieht und die Möglichkeit eines nachfolgenden Angebotsschocks besteht. Zur Beantwortung dieser Fragestellung benützen wir ein New Keynesian Framework mit einem *gewissenhaften* Zentralbankers.

Das Einführungskapitel gibt einen Überblick über die aktuellen Erfahrungen mit Forward Guidance und erläutert die Forschungsfragen dieser Dissertation. In Kapitel 2 erarbeiten wir den Modellrahmen, der der Analyse unterliegt und führen zwei flexible Forward Guidance Designs ein: *Escaping* – für Entkommen – und *Switching* – für Wechsel. Unter *Escaping* Forward Guidance verpflichtet sich der Zentralbanker, die Zinsen im Umfeld von negativen realen Zinsen tief zu halten, vorrausgesetzt die Inflation bleibt unter einem selbst definierten Schwellenwert. Der Zentralbanker gewinnt seine uneingeschränkte Flexibilität zurück, sobald die Inflation den Schwellenwert überschreitet. Unter *Switching* Forward Guidance wechselt der Zentralbanker von Zinsvorhersagen zu Inflationsprognosen, um einen Angebotsschock zu stabilisieren. Wir zeigen, dass in der Umgebung von kleinen und grossen Zinsschocks *Escaping* Forward Guidance allen anderen Designs vorgezogen wird. *Switching* Forward Guidance ist die optimale Wahl für mittlere Zinsschocks.

In Kapitel 3 erweitern wir das Grundmodell und führen heterogene Erwartungen ein. Die Marktteilnehmer erhalten ein öffentliches Signal der Zentralbank, was Forward Guidance entspricht, und ein privates, idiosynkratisches Signal. Wir zeigen, dass die flexiblen Forward Guidance Designs einen Teil ihrer Effektivität einbüssen, aber wir bestätigen, dass sie immernoch wohlfahrtsfördernd sind, verglichen mit einer starren Form von Forward Guidance und einem Zentralbanker, der nach freiem Ermessen entscheiden kann. Werden die Marktteilnehmer zunehmend pessimistisch dominiert *Escaping* Forward Guidance über alle Grössen eines Zinsschocks.

Schliesslich testen wir in Kapitel 4, ob die Resultate aus Kapitel 2 robust sind, wenn wir die Werte für die strukturellen Parameter nicht kennen. Gleichzeitig identifizieren wir die Parameter, welche den grössten Einfluss auf die Wohlfahrtsfunktion haben. Um eine effiziente Berechnung der Sensitivitätsanalyse zu gewährleisten, approximieren wir die

ursprüngliche Wohlfahrtsfunktion mit der Technik "Polynomial Chaos Expansion". Das erlaubt uns, Sobol' Indices zu berechnen, um die wichtigsten Parameter zu klassifizieren und zu zeigen, dass unsere Resultate robust sind.

Contents

List of Figures

List of Tables

1 Introduction

1.1 Motivation

In the last decade, major central banks have announced unconventional monetary policy actions to ease the zero lower bound (henceforth ZLB) constraint[1] they faced in the aftermath of the financial crisis of 2007-2008. Together with balance sheet expansions, forward guidance was the most prominent unconventional policy action taken. Forward guidance is the attempt by central banks to affect expectations about the path of inflation and output by announcing the interest-rate policy they will follow in the near future. Various forms of forward guidance have been applied by central banks. This leads to the question which form is particularly suited to obtain the desired effects without unnecessary adverse implications in the long-run. The versatile forward guidance designs we discuss in this thesis are flexible forms of forward guidance, which address this specific question. We build on the foundation of Liu (2016). While he addresses the implementation of forward guidance in a liquidity trap with contracts, this dissertation analyses new versatile forward guidance designs in a more complex sequence of events, including supply shocks in normal times, and without relying on contracts.

Forward Guidance before the Financial Crisis

In this introductory chapter, we provide an overview of the role of forward guidance in economic theory and its impact in practice. Then we outline the contribution to the existing literature and outline the structure of this thesis.

A tendency towards more transparent monetary policy already emerged before the financial crisis (Woodford, 2012). Some central banks were rather cautious and used implicit guidance by putting specific code words into their statements.[2] Others became more explicit in their attempts to guide expectations, by providing information about their policy rates, starting in the late 1990s. There never was, however, any explicit commitment to specific projected policy rates. In particular, the Reserve Bank of New Zealand (starting

---

[1] The ZLB, i.e. the positivity constraint on nominal interest rates, occurs because the possibility of holding cash guarantees a nominal interest rate of zero.

[2] The Federal Reserve (FED), for example, used words such as *"bias towards"* or *"balance of risks"* to indicate the likely direction of future policy decisions (Olsen, 2017).

in 1997) and the Norges Bank (starting in 2005) communicated the banks' own policy interest-rate forecasts. Although there was no commitment involved, Mirkov and Natvik (2016) found that these noncommittal announcements in one period had an influence on the interest-rate-setting policy in a later period. Later, the Swedish Riksbank, the Central Bank of Iceland, the Czech National Bank, and the Bank of Israel followed and started to publish policy interest-rate forecasts. Moreover, the FED's Federal Open Market Committee's (FOMC) forecasts are publicly available. Bianchi and Melosi (2016) found that the FED's announcements reduce policy uncertainty and anchor inflationary beliefs, and that such greater transparency helps to increase welfare. Svensson (2015) elaborated on several reasons why forward guidance in the form of published policy-rate paths *"may be considered a natural part of a monetary policy"* (p. 24).

Campbell et al. (2012) coin the expression *"Delphic"* forward guidance to describe a central banker who reduces private decision-makers' uncertainty by publicly announcing forecasts of macroeconomic variables and announcing probable or intended monetary policy actions. From an outsider's perspective, however, it is not perfectly clear what a central bank wants to achieve by issuing such a forecast. Is the purpose to reduce uncertainty through an informational advantage that the central bank has—whatever the reason for such an advantage? Or are these forecasts the values that the central banker hopes to achieve in the future? It is undisputed that nowadays, no central bank promises anything and then reneges on this promise to simply act discretionarily. Discretion means that the central bank acts in the moment, as past events do not influence its current policy, and that it starts from the assertion that it cannot influence future policy decisions.

Forward Guidance after the Financial Crisis

At the ZLB, central banks are unable to change their *current* policy rates. By communicating how they intend to set *future* policy rates, they add a new tool to their policy toolkit, which still allows them to change current rates. Contrary to what they had done before the crisis, central banks began to insinuate commitment in their statements. Campbell et al. (2012) use the term *"Odyssean"* forward guidance to describe a policymaker that commits himself to a particular course of action he will follow in the future, *"[...] just as Odysseus committed himself to staying on his ship by having himself bound to the mast"* (Campbell et al., 2012, p. 3).[3] There is a consensus in macroeconomics that today's expectations are a key determinant for decisions. Odyssean forward guidance has the goal to affect these expectations and thus to influence current actions. Optimally, this should lead to a welfare gain today—in terms of increased growth and/or higher inflation—, with the

---

[3] The difference between Odyssean forward guidance and Delphic forward guidance is that the former commits the central bank to a particular future action, while the latter merely forecasts those monetary policy actions that it is *likely* to implement.

bill for such gain being presented tomorrow, by a reduced flexibility of monetary policy. A key feature of Odyssean forward guidance is that it only works if the central bank's announcement is credible. The seminal work of Kydland and Prescott (1977) shows that a precommited (Odyssean) policymaker is preferable to a policymaker who chooses his policy discretionarily. Moreover, they highlight the problem inherent to pre-commitment, which they referred to as "time-inconsistency", i.e. to feel the urge to abandon promises and re-optimize opportunistically. Hence, although a perfectly committed Ramsey-type central banker who specifies a predetermined path for the interest rate is desirable, it is not realistic to expect any central banker to behave this way. Naturally the question emerges: How does a central banker commit himself in a time-consistent manner? One way to address this time-inconsistency problem is by rule-based behavior in monetary policy, i.e. setting up a framework announcing in a clear and simple way how the central banker intends to conduct monetary policy. An example is an explicit Taylor-rule, i.e. a rule that connects the nominal interest rate to changes in inflation and output.[4] This enables the public to hold the central banker accountable. We tackle the time-inconsistency problem by assuming a scrupulous central banker who incurs some kind of costs from deviating from his commitment. This commitment is still imperfect, though. We also briefly motivate the introduction of a scrupulous central banker. Moreover, we introduce forward guidance designs that are in the spirit of a rule-based policy approach tailored to extraordinary economic circumstances, i.e. to a ZLB constraint. In particular, the designs we examine commit the central banker to a clear succession of steps for policy actions. Moreover, they feature a simple and clear communication rule for announcements, such that the central bank can be measured by its performance.

So far, three types of forward guidance have been pursued in practice when facing the ZLB: *open-end announcements*, *time-contingent announcements*, and *state-contingent announcements*.[5] As to state-contingent announcements, the period for which an announcement is supposed to hold depends on macroeconomic conditions such as the inflation or the unemployment rate. An overview of major central banks engaging in forward guidance is given in Appendix D.

Experience with forward guidance yields three insights. First, forward guidance can influence the market participants' expectations indeed. Gürkaynak et al. (2005) use intra-day data to investigate whether FOMC statements about the future policy path have an effect on bond yields and stock prices. They test the effects of the statements against the hy-

---

[4] For an overview of a variety of rule-based policymaking approaches that enhance commitment, see Plosser (2016).

[5] Open-end announcements specify how monetary policy will be conducted *for an extended period of time*, without any details on the duration. Time-contingent or calendar-based announcements include an indication of *how long* monetary policy will be conducted in a particular way.

pothesis that only changes in the federal funds rate target have an effect and conclude that FOMC statements have a significant effect on long-term yields. This *"[...] suggests that the FOMC may be able to credibly commit to paths for the federal funds rate"* (Gürkaynak et al., 2005, p. 57) and stimulate growth. Campbell et al. (2012) verify the results of Gürkaynak et al. (2005) for an extended time period and also come to the conclusion that by communicating its intentions, the FOMC is able to guide expectations into the desired direction. Moreover, they show that their findings remain valid for FOMC statements made after the crisis of 2007-2008. The approach used in these two studies, however, does not allow to identify *how* FOMC statements change expectations. As Woodford (2012) says, *"[...] do forecasts of the future funds rate change because beliefs about the FOMC's reaction function change as a result of the statement, or because forecasts of future economic conditions that are expected to determine FOMC policy change, as a result of inferences that are made about information that must be available to the FOMC?"*(p. 10). As to forward guidance, its aim is to change expectations about the central banker's reaction function.

Woodford (2012) investigated a central bank's ability to change the public's expectations about its monetary policy. He focused on public central bank statements that deliberately attempted to send a particular message about future policy. He showed that the Bank of Canada's statement, *"[c]onditional on the outlook for inflation, the target overnight rate can be expected to remain at its current level until the end of the second quarter of 2010 in order to achieve the inflation target"* (Bank of Canada, 2009), affected market expectations about the future path of the policy rate immediately. To show this, Woodford (2012) used the falling and flattening of the overnight interest-rate swap yield curve as evidence.

Del Negro et al. (2015) found that FOMC announcements had, on average, positive and meaningful effects on output expectations and inflation expectations. At the same time, however, they showed that theoretical macroeconomic models generally overestimate the impact of forward guidance, naming this divergence of theoretical prediction and empirical observation the "Forward Guidance Puzzle". In their work they point out that the problem mostly lies in an over-reaction of long term bond rates. The success of forward guidance is put into perspective by den Haan (2013), for example. He states that forward guidance affects the markets' expectations to some extent, but no form *"[...] of forward guidance has managed to closely align market expectations with the policymakers' intentions"* (den Haan, 2013, p. 16). Moreover, forward guidance sometimes failed to modify the public's expectation. Svensson (2015), for example, describes the experience of the Swedish Riksbank in 2011 which showed that it does not work to lean against the wind: The Riksbank published an interest rate path that lacked credibility—it announced a *rise*

of 75 basis points over the next six quarters while the market expected a *fall* of 75 basis points—and later had to adapt the path *it* expected to the path the *market* expected.

Second, when announcements are made in vague terms or indicate "measures" that are hard to verify, the central bank can abandon these announcements rather easily, and the impact of such forward guidance may be negligible. An example of vague terms is the announcement made by the Bank of England (BoE) in August 2013, stating that the bank rate would stay at 0.5% *"at least until the Labor Force Survey headline measure of unemployment rate [...]"* (Carney, 2013) would fall below 7%. Simultaneously, three criteria were set that enabled the bank to break this commitment: (i) if the consumer price index (CPI) inflation eighteen to twenty-four months ahead was, in the Monetary Policy Committee's view *"more likely than not to be"* 0.5% above the 2% inflation target, (ii) if inflation expectations seemed *"poorly anchored"*, and (iii) if the policy imposed *"potential threats"* on financial stability (Bank of England, 2013). All these criteria require interpretation and thus permit discretion as to their application. And so it came: In May 2014, the Bank of England continued its low bank rate policy, despite the fact that the unemployment rate had fallen below the announced threshold of 7%. The continuation of the low bank rate policy led to strong public criticism of the BoE.

Third, when central banks engage in state-contingent forward guidance with objectively measurable criteria such as the unemployment rate, for instance, they can modify their commitment over time. An example is the FED. In December 2012, it made its low policy rate dependent on the level of unemployment and set a critical threshold of 6.5% for it. In December 2013, it rephrased its own statement and announced it would keep the federal funds rate low as long as the projected inflation rate stayed below 2%, even if the unemployment rate fell below 6.5%.

Issues and Limitations of Forward Guidance

Besides the time-inconsistency problem discussed above, there are other open issues regarding forward guidance, both from a practical and a theoretical point of view.

Bernanke (2003) introduced the term *constrained discretion* to define a monetary policy guided by two principles: First, the central bank has a strong commitment to keeping inflation low and stable. Second, the central bank should seek to moderate cyclical swings in resource utilization, subject to the first principle. Bernanke described constrained discretion as the approach that increasingly characterized contemporary FED policymaking. If a central bank devotes itself to these two principles, it may find it difficult to seem credible when committing to inflation in the future. It becomes even more difficult to communicate how the central bank intends to behave, as it may be in an unprecedented situation. Hence, when engaging in forward guidance at the ZLB, central banks have to

be credible in the sense that they have to act against the strong expectation that they will fight inflation. How, then, should a central bank implement forward guidance, such that it is institutionalized to make market participants understand its intention *and* believe that it will deliver? The forward guidance design has to combine simple communication and credibility to be effective. This is the starting point of this thesis: we intend to contribute to the discussion about promising forward guidance designs.

From a theoretical perspective, the New Keynesian Framework, which is the standard structure to analyze forward guidance, has some innate weaknesses. Eggertsson and Krugman (2012) and Diba and Loisel (2017) provide an overview of these shortcomings, such as the *Paradox of Thrift*, the *Paradox of Toil*, the *Paradox of Flexibility*, *explosive fiscal multiplier effects*, and the *Forward Guidance Puzzle*.[6] The Forward Guidance Puzzle, which is addressed in this thesis, described in Del Negro et al. (2015) states that the effectiveness of forward guidance is grossly overestimated by theoretical analysis. There exist several contributions that propose a remedy for this problem. Del Negro et al. (2015) introduced a perpetual youth structure into their model, McKay et al. (2016b) featured incomplete markets and borrowing constraints, Gabaix (2016) and Farhi and Werning (2017b) used bounded rationality, and Angeletos and Lian (2016) imposed informational frictions, for example. We will attenuate the effects of forward guidance in two ways. First, we assume a scrupulous central banker that does not follow a rigid interest rate peg, which is different to all the literature discussed above. Second, we introduce heterogeneous beliefs due to informational frictions. The latter leads to a discounted New Keynesian Framework which is structurally similar to the discounted IS Curve and Phillips Curve in Gabaix (2016) which provide a resolution of the Forward Guidance Puzzle. Another distinguishing feature of our approach is the stochastic escape from the ZLB constraint. Particular assumptions in our set-up ensure that the Paradox of Thrift, the Paradox of Toil, and fiscal multiplier effects do not occur in our model, while the Paradox of Flexibility is excluded by the assumption of constant price rigidity.

From a practical point of view, forward guidance is incomplete, as pointed out by Barwell and Chadha (2013), for example, incompleteness meaning that only the first move of an interest-rate adjustment is defined. Steps following an initial adjustment, i.e. how fast interest rates will be raised and how energetically potential inflation will be addressed, are omitted. Another point is that forward guidance can also be used to announce how

---

[6] The effect that an increase in savings at the ZLB leads to a contraction in the economy is named the Paradox of Thrift (Eggertsson and Krugman, 2012). The contractionary effects of a positive supply shock at the ZLB are called the Paradox of Toil and was first addressed by Eggertsson (2010). The implication that in the environment of an adverse demand shock, increased price and wage flexibility lead to a decline in output is the Paradox of Flexibility (Eggertsson and Krugman, 2012). Farhi and Werning (2017a) describe the explosive effects of government spending on consumption when the duration of an interest-rate peg increases.

the central banks' balance sheets will be reduced. The blow-up of the balance sheets is unprecedented, and some guidance how central banks intend to tackle this issue may help to mitigate uncertainty in the economy.

1.2 Research Questions

We will focus on Odyssean forward guidance with an economy constrained by a ZLB. A central banker conducts monetary policy by explicit forms of forward guidance, with the aim to manage expectations at the ZLB. The thesis is divided into three parts: *(i)* The introduction of two flexible forward guidance designs, i.e. the versatile *escaping* and *switching* forward guidance designs, in a standard New Keynesian Framework with a *scrupulous central banker*; *(ii)* an assessment of forward guidance in an extended New Keynesian Framework that features *heterogeneous beliefs*; and *(iii)* an extensive *robustness analysis* of our findings in *(i)*.

We will address the following research questions:

Q1 What are simple yet effective forward guidance designs that improve social welfare over a discretionary policy design and a standard forward guidance design?

Q2 How does the answer to Q1 change when we abandon rational expectations?

Q3 Do our results still hold under parameter uncertainty? How sensitive are the results to the choice of the structural parameters?

1.3 Structure of the Thesis

Chapter 2: Versatile Forward Guidance

In this chapter, we develop the basic New Keynesian Framework to address Q1: *What are simple yet effective forward guidance designs that improve social welfare over a discretionary policy design and a standard forward guidance design?* The central banker faces an economic downturn, i.e. a negative natural real interest rate, and thus a ZLB constraint on the nominal interest rate. The economy stochastically returns to a positive natural real interest rate and is subsequently exposed to a potential supply shock. We propose two versatile designs, *escaping* and *switching* forward guidance, that allow the scrupulous central banker to react to this change in shocks. Both designs improve welfare over a discretionary policy. The reason is that they allow the central banker to commit to his interest-rate forecasts, which balances the losses in periods of downturn and in periods of supply shocks. Also, *escaping* and *switching* forward guidance allow the central

banker to react more flexibly than with standard forward guidance and to avoid unduly high welfare losses caused by the interest-rate forecast in downturns.

Chapter 3: Versatile Forward Guidance without Common Beliefs

The goal of forward guidance is to manage the agents' expectations in an economy. The properties of a standard New Keynesian Model allow a central banker to greatly improve welfare when he applies forward guidance within that framework. In this chapter we address Q2: *How does the answer to Q1 change when we abandon rational expectations?* We relax the rational expectation assumption in the sense that agents are exposed to idiosyncratic information and higher-order beliefs, which makes it harder for the central banker to shift expectations. Despite the decreased impact of the announcements, we find that forward guidance remains welfare improving in various designs. In fact, while the absolute gain of applying forward guidance attenuates—i.e. the so-called "Forward Guidance Puzzle" is less pronounced—, it becomes attractive for the central banker to apply forward guidance earlier in reaction to less severe natural real interest-rate shocks.

Chapter 4: Global Sensitivity Analysis

In this chapter, we answer Q3: *Do our results still hold under parameter uncertainty? How sensitive are the results to the choice of the structural parameters?* We use a surrogate model in the form of a polynomial chaos expansion to perform the analysis efficiently. We calculate Sobol' Indices, a variance-based sensitivity method, to identify those structural parameters that have the greatest effect on the results, and analyze how their effects change the results. In the New Keynesian Model augmented by a scrupulous central banker, typically, the slope of the New Keynesian Phillips Curve turns out to be the parameter to which social losses react the most. Most importantly, the analysis indicates that the areas for which *escaping* forward guidance and *switching* forward guidance dominate other monetary policy approaches are robust to parameter uncertainty. We also show that the principal gains of applying forward guidance will materialize even for those central bankers with a low degree of scrupulosity.

Chapter 5: Extensions to Versatile Forward Guidance

In this chapter, we provide extensions to the model built in Chapter 2 and test the robustness of the results of that model. For the sake of simplicity, we assumed an exogenously fixed interest-rate forecast in Chapter 2. We now relax the assumption and allow the central banker to issue an optimal interest-rate forecast that minimizes his loss function. In a second extension, we account for recent experiences that policy rates are not constrained

by a ZLB but by an effective lower bound (ELB). We show that flexible forward guidance designs are welfare improving under an optimal interest-rate setting and an ELB.

2 Versatile Forward Guidance*

2.1 Introduction

"[...] the most logical way to make such commitment achievable and credible is by publicly stating the commitment, in a way that is sufficiently unambiguous to make it embarrassing for policymakers to simply ignore the existence of the commitment when making decisions at a later time." (Woodford, 2012, p. 7)

Motivation

The fundamental problem of forward guidance is to make credible, time-consistent announcements while retaining elbow room for reacting to new shocks. While the literature reviewed in Subsection 2.1.1 examines optimal forward guidance in the presence of a particular type of shock, in this chapter we examine how forward guidance can be designed when the economy is hit by a sequence of different shocks, i.e. first a negative natural real interest-rate shock and then a supply shock. We compare two promising designs for forward guidance: escaping and switching.

With *escaping* forward guidance, the central banker commits to low future interest rates after a negative natural real interest-rate shock. At the same time, he announces a threshold inflation rate. As soon as inflation oversteps this threshold, the central banker is freed from his commitment to low interest rates and regains flexibility. With *switching* forward guidance, the central banker switches from interest-rate forecasts to inflation forecasts when the supply shock hits the economy. That is, he switches from a commitment to *interest rates* to a commitment to *inflation rates*.

Structure

This chapter is organized as follows: The model under standard forward guidance is presented in the next section. In Section 2.3 we investigate *escaping* forward guidance. In Section 2.4 we introduce *switching* forward guidance and study its welfare implications. Finally, a discussion and the conclusion make up Section 2.5.

---

* This chapter is based on joint research with Hans Gersbach and Yulin Liu.

2.1.1 Relation to the Literature

The literature on forward guidance and the degree of scrupulosity of central bankers can be divided into three parts. First, there is a considerable body of literature on the pros and cons of forward guidance. In a recent article, Svensson (2015) finds that applying forward guidance in the form of a published policy-rate path for the countries Sweden, New Zealand, and the U.S. has met with mixed success. Gersbach and Hahn (2011), Woodford (2012), and the survey by Moessner et al. (2017) provide detailed accounts both of what forward guidance can achieve and of its limitations.

Second, the potential and limitations of forward guidance have been analytically and numerically assessed for specific shock scenarios in Eggertsson and Woodford (2003), Rudebusch and Williams (2008), Gersbach and Hahn (2014), Gersbach et al. (2015), and Liu (2016), with Boneva et al. (2015) and Florez-Jimenez and Parra-Polania (2016) focusing on threshold-based forward guidance or forward guidance with an escape clause. In this chapter, we examine how forward guidance should be performed when the economy is hit by a series of different shocks. Moreover, we introduce and compare *escaping* and *switching* as promising approaches to forward guidance in such circumstances.

Third, the way in which central banks can increase the commitment power of their announcements—or equivalently, deviations from announcements generate material or immaterial costs for central bankers—has been discussed for several approaches to forward guidance. In this paper, we adopt the view that the central banker displays a certain degree of scrupulosity and will face intrinsic costs if he deviates from his previous announcements.[1] Such costs have been specified in various cases. Svensson (2009) reports from experience as a Deputy Governor at the Sveriges Riksbank: *"[...] any signal might pre-commit some members and distort the final decision [...]"* (p. 24). The former Bank of England Governor Mervyn King faced such costs when handling the Northern Rock bailout. His initial announcement not to bail out the bank and the subsequent reversal of this announcement led to public attacks.

At a deeper level, there are three potential causes for the costs of broken promises.[2] First, a central banker may have reputational concerns: breaking a promise may harm future payoffs (see Blinder (2000)). The anecdotal evidence discussed above can be seen under the heading of reputational concerns. Second, a central banker who makes a promise wants to avoid guilt after disappointing the expectations he has generated.[3] Third, the

---

[1] Other interesting approaches focus on inertia in revising plans of central bankers. Roberds (1987) ("stochastic replanning"), Schaumburg and Tambalotti (2007) ("quasi-commitment"), and Debortoli and Lakdawala (2016) ("loose commitment") introduce the concept of a central banker who revises his previously announced plans with a certain probability. The latter authors estimate the Federal Reserve's probability of fulfilling the announcement to be 80%.

[2] See Ederer and Stremitzer (2016).

[3] Ederer and Stremitzer (2016) provide a detailed account of the forces for keeping promises and experi-

promisor may have a preference for keeping his word, no matter what expectations others have. The latter two sources of costs from breaking promises have been documented by experimental research.[4] To sum up, anecdotal, empirical, and experimental evidence supports the assumption that central bankers face intrinsic costs when they break promises in the context of forward guidance. This, in turn, creates some—albeit weak—commitment to stick to the announcements. In this chapter, we assume that central bankers have some degree of scrupulosity and thus face intrinsic costs if they deviate from announcements. We also note that such costs could be actively generated by governments or central banks themselves by appointing scrupulous central bankers, as discussed in Gersbach and Hahn (2013), or with incentive pay or particular asset holdings for central banks, as discussed in Gersbach et al. (2015). In this article, we take an agnostic view on generating costs for central bankers when they deviate from announcements.

2.1.2 Main Results

We examine *escaping* and *switching* forward guidance and compare them to discretionary monetary policy, to standard unconditional interest-rate forward guidance, and to each other. We perform these analyses and comparisons in the New Keynesian Framework, with negative natural real interest-rate shocks and a subsequent supply shock. Monetary policy is performed by a scrupulous central banker who faces intrinsic losses if he deviates from his own forecasts.[5]

Our main results are as follows: First, a scrupulous central banker only applies standard forward guidance with zero interest-rate forecasts in a severe downturn. Otherwise, he uses either *escaping* or *switching* forward guidance.

Second, with *escaping* forward guidance, announcing zero interest-rate forecasts in the downturn becomes attractive for any negative natural real interest-rate shock. It matches or lowers social losses at any natural real interest-rate shock level, compared to social losses under standard forward guidance or without forward guidance. The reason is that this avoids the risk of having an excessively low interest rate connected with high inflation in a subsequent major boom. The inflation threshold above which the central banker can abandon his commitment without facing costs increases with the severity of the negative natural real interest-rate shock.

Third, *switching* forward guidance offers a further prospect for decreasing social losses,

---

mental evidence for expectation-based promise-keeping.

[4] See Charness and Dufwenberg (2006), Vanberg (2008), Charness and Dufwenberg (2010), Ellingsen et al. (2010), and Ederer and Stremitzer (2016), etc.,for expectation-based forces for keeping promises. See Ostrom et al. (1992), Ellingsen and Johannesson (2004), Vanberg (2008), or Ismayilov and Potters (2012), etc., for the commitment-based force.

[5] We provide a detailed discussion of the origins of scrupulosity in Subsection 2.1.1.

and it dominates *escaping* forward guidance for medium-sized negative natural real interest-rate shocks. The reason is that in a medium range, *switching* forward guidance is better at balancing gains and costs from committing to low future interest rates through the switch to inflation forecasts, since such forecasts moderate inflation immediately when positive supply shocks occur. This does not work efficiently for small natural real interest-rate shocks, since the excessive inflation expectations created by the zero interest-rate forecast in downturns will not be lowered sufficiently by inflation forecasts in normal times. This leads to higher expected losses in downturns under *switching* forward guidance compared to *escaping* forward guidance. Furthermore, for large natural real interest-rate shocks, *switching* forward guidance is unable to elevate inflation expectations as strongly as *escaping* forward guidance due to the moderating effect of inflation forecasts. Under *escaping* forward guidance, the central banker simply chooses a threshold which signals that he will never escape without cost, which in our setting maximally increases expectations. To sum up, *escaping* forward guidance provides desirable levels of inflation expectation in downturns for every level of the natural real interest-rate shock, since an adequate inflation threshold will be chosen. *Switching* forward guidance yields the same inflation expectation in downturns for different natural real interest-rate shocks and then moderates adverse supply shock effects in all remaining periods. This is beneficial only for the medium range of natural real interest-rate shocks. Compared to *escaping* forward guidance for this range of shocks, *switching* forward guidance leads to similar losses in downturns but lower losses in normal times due to the use of inflation forecasting in normal times.

Our results allow for two broader conclusions. First, *escaping* forward guidance is a promising policy approach, and our results rationalize recent attempts to apply this type of forward guidance: On December 12, 2012, the Federal Reserve announced *"[...] to keep the target range for the federal funds rate at 0 to 1/4 percent and [it] currently anticipates that this exceptionally low range for the federal funds rate will be appropriate at least as long as [...] inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee's 2 percent longer run goal [...]"*(Federal Reserve, 2012). Second, *switching* forward guidance tends to be superior for natural real interest-rate shocks of an intermediate size. Thus central banks might want to use the switching approach, as it enables them to switch credibly from one type of forward guidance to another.

2.2 The Model

2.2.1 The Macroeconomic Environment

We start from the standard New Keynesian Framework as described in Clarida et al. (1999). The dynamics of the economy are governed by the IS Curve and the Phillips Curve. The IS Curve is

$$x_t = \mathbb{E}_t[x_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t), \tag{2.1}$$

where x_t is the output gap in period t, $\mathbb{E}_t[\pi_{t+1}]$ and $\mathbb{E}_t[x_{t+1}]$ are the inflation rate and the output gap in period $t + 1$ expected in period t. i_t is the nominal interest rate set by the central banker, and r_t is the natural real interest rate. $\sigma > 0$ denotes the inverse inter-temporal elasticity of substitution.

The Phillips Curve is

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t[\pi_{t+1}] + \xi_t, \tag{2.2}$$

where $\kappa > 0$ and the discount factor is $\beta \in (0, 1)$. ξ_t denotes the supply shock at time t, which follows the AR(1) process

$$\xi_t = \rho \xi_{t-1} + \epsilon_t, \tag{2.3}$$

where $\rho \in [0, 1)$ and ϵ_t represents i.i.d. disturbance with zero mean.

We consider a sequence of shocks, first a negative natural real interest-rate shock and second, upon recovery, a supply shock. The earlier shock causes a ZLB problem, as due to the constraint $i_t \geq 0$ the central bank cannot do enough to counteract this shock.[6] The later shock may cause inflation and involves standard output/inflation trade-offs. More specifically, as in Eggertsson (2003) and Gersbach et al. (2015), the economy starts in a downturn (Phase D) with a negative natural real interest rate $r_t = r_D < 0$, and ends up in normal times (Phase H), where $r_t = r_H > 0$. In each period in the downturn, there is a certain probability $1 - \delta \in (0, 1)$ that the economy will extricate itself from this downturn. Once the economy reverts to normal times, the natural real interest rate will stay at r_H forever. However, the supply shock occurs after the economy has recovered.[7]

---

[6] The ZLB problem has been addressed in many papers. Several articles are relevant for our purpose. Eggertsson (2003) has outlined a convenient framework for assessing the optimal dynamic linkages between policies in the downturn and upon recovery. Adam (2007) shows that the existence of the ZLB on nominal interest rates makes it beneficial to have a central banker acting under commitment rater than a discretionary central banker. Orphanides and Wieland (2000) find that monetary policy should be asymmetric and that central banks should embark on a more aggressive and expansionary path when inflation declines and when they face a ZLB problem.

[7] For the sake of simplicity, we assume that the supply shock is zero in the downturn, since in the presence

The initial size of the shock is evenly distributed within the range $[-\xi, \xi]$. The sequence of events is illustrated in Figure 2.1.
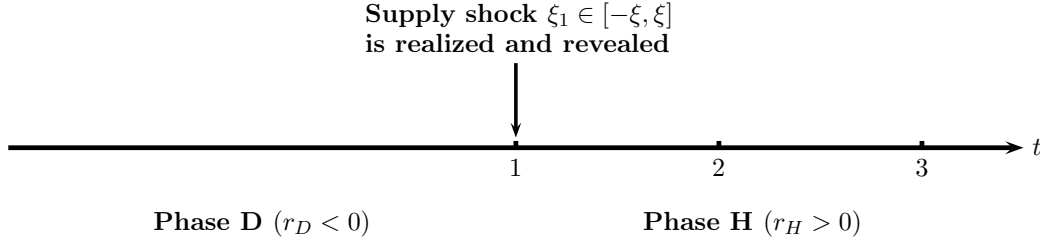
Figure 2.1: Sequence of events.

It is convenient to start the time index $t = 1$ with the period in which the economy enters Phase H. The stochastic return of the natural real interest rate to $r_H > 0$ during the downturn means that the situation in the downturn is identical in each period. Therefore we denote variables in a typical period in Phase D by subscript "D" without specifying for how many periods the economy has already been trapped in the downturn.

The instantaneous social loss function in period $t = D, 1, 2, ...$ is

$$l_t = \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right), \tag{2.4}$$

where $\lambda > 0$ and future losses are discounted by β.

We consider a scrupulous central banker who is reluctant to deviate from the forecasts—interest-rate or inflation forecast—he made in the previous period, if any. Therefore, apart from the social loss in Equation (2.4), the central banker incurs an additional intrinsic loss if he deviates from the forecast. Accordingly, the central banker's instantaneous loss function in period t is

$$\tilde{l}_t = \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right) + \frac{1}{2}b(q_t - q_t^f)^2, \tag{2.5}$$

where q_t is either the interest rate, i.e. $q_t = i_t$ (correspondingly, $q_t^f = i_t^f$ is the interest-rate forecast), or the inflation rate, i.e. $q_t = \pi_t$ (correspondingly, $q_t^f = \pi_t^f$ is the inflation forecast). If no forecasts were made in the previous period, the central banker would have the same loss function as the society. We will compare different types of forecasts, including the absence of forecasts, in the remainder of the paper. Parameter b measures the intrinsic costs the central banker incurs when he deviates from his forecasts, relative to the social losses. b thus stands for the central banker's degree of scrupulosity. With a

of a natural real interest-rate shock, the impact of the supply shock on the economy is only secondary.

larger value of b, the central banker has a higher willingness to stick to his forecast. We focus on values $b > 0$ and perform a robustness analysis for the range of values of b in $(0, 1]$. The polar case $b = 0$ stands for a central banker who acts in a purely discretionary manner in each period.

It is useful to introduce specific loss functions. The instantaneous social loss in a period in Phase D is given by

$$l_D = \frac{1}{2}(\pi_D^2 + \lambda x_D^2).$$

Once the natural real interest rate returns to r_H and the supply shock manifests itself, the latter follows the dynamic process in Equation (2.3), and we denote variables in Phase H by the respective time-subscript "$t = 1, 2, 3, ...$". The expected inter-temporal social losses in Phase H are denoted by an "H" subscript and are defined as

$$l_H = \frac{1}{2}\sum_{t=1}^{\infty}\beta^{t-1}\int_{-\rho^{t-1}\xi}^{\rho^{t-1}\xi}(\pi_t^2 + \lambda x_t^2)ds.$$

Since in the downturn there is a certain probability δ in each period that the economy will stay in this downturn, and the future losses are discounted by β, the expected cumulative social loss in a particular period in the downturn is

$$L = l_D + \beta(\delta l_D + (1-\delta)l_H)\sum_{t=0}^{\infty}\delta^t\beta^t \tag{2.6}$$

$$= l_D \sum_{t=0}^{\infty}\delta^t\beta^t + \beta(1-\delta)l_H \sum_{t=0}^{\infty}\delta^t\beta^t \tag{2.7}$$

$$= \frac{l_D + \beta(1-\delta)l_H}{1 - \beta\delta}. \tag{2.8}$$

Analogously, the central banker's expected cumulative loss in a particular period in the downturn is

$$\tilde{L} = \frac{\tilde{l}_D + \beta(1-\delta)\tilde{l}_H}{1 - \beta\delta}, \tag{2.9}$$

where[8]

$$\tilde{l}_D = \frac{1}{2}(\pi_D^2 + \lambda x_D^2 + b(q_D - q_D^f)^2),$$

$$\tilde{l}_H = \frac{1}{2}\sum_{t=1}^{\infty}\beta^{t-1}\int_{-\rho^{t-1}\xi}^{\rho^{t-1}\xi}(\pi_t^2 + \lambda x_t^2 + b(q_t - q_t^f)^2)ds.$$

In Subsection 2.2.2, we first consider a design where the scrupulous central banker does

---

[8] Since the situation is identical in all periods of the downturn, we assume that the central banker's behavior is the same in every period of Phase D.

not make any forecast at all, either in Phase D or in Phase H. We denote variables and loss functions in this discretionary design by superscript "N" and refer to it as no forward guidance (NFG). In Subsection 2.2.3, we consider a scenario where in the downturn the scrupulous central banker makes interest-rate forecasts only. We denote variables in this rigid forward guidance design by superscript "F" and refer to it as standard forward guidance or interest-rate forward guidance (IFG) in the remainder of the chapter.[9] At the end of this section, we compare the two designs and establish the central banker's optimal behavior. This will then serve as a basis for examining *escaping* and *switching* forward guidance.

Throughout the chapter we illustrate the designs' properties by calibrating them. We follow Woodford (2003) and use the quarterly values of Table 2.1 for the structural parameters λ, κ, and σ. The parameter values β and ρ are taken from Gersbach and Hahn (2014). We assume the probability of staying in the downturn to be $\delta = 0.5$. The natural real interest rate is assumed to be $r_H = 0.02$ in Phase H and $r_D \in (-\infty, 0)$ in the downturn. For the supply shock, we assume a symmetric range around zero $\xi_1 \in [-0.004, 0.004]$, where the lower bound is chosen in such a way that, when the economy enters Phase H, the ZLB is no longer binding. We set the degree of scrupulosity to $b = 1$, which is a comparatively high level. With $b = 1$, the central banker is indifferent between deviating from his forecast by one percentage point and incurring one percent inflation. We also explore the robustness of our findings subject to the central banker's scrupulosity $b \in [0, 1]$.

$\beta = 0.99$	Discount factor
$\lambda = 0.003$	Weight of output gap in social loss function
$\kappa = 0.024$	Slope of Phillips Curve
$\sigma = 0.16$	Inverse inter-temporal elasticity of substitution
$r_H = 0.02$	Natural real interest rate in Phase H
$r_D \in (-\infty, 0)$	Natural real interest rate in Phase D
$\xi_1 \in [-0.004, 0.004]$	Supply shock in Phase H, $t = 1$
$\delta = 0.5$	Probability of being trapped in Phase D
$\rho = 0.9$	Persistence of supply shock
$b = 1$	Scrupulous central banker's intrinsic weight on his forecast

Table 2.1: Quarterly parameter values used in the calibration.

---

[9] The abbreviation SFG is reserved for *switching* forward guidance.

2.2.2 The Discretionary Central Banker

In this design the central banker does not make any forecasts. Thus, his loss function in Equation (2.5) coincides with the social loss function in Equation (2.4). We derive the variables of interest by backward induction.[10] First, we consider Phase H. The central banker selects i_t optimally to minimize the loss function in Equation (2.4) in each period subject to the IS Curve (2.1) and the Phillips Curve (2.2). Inflation and the output gap in Phase H evolve according to

$$\pi_t^N = \frac{\lambda}{\lambda(1 - \rho\beta) + \kappa^2} \xi_t, \tag{2.10}$$

$$x_t^N = -\frac{\kappa}{\lambda(1 - \rho\beta) + \kappa^2} \xi_t. \tag{2.11}$$

Inserting Equations (2.10) and (2.11) into the IS Curve (2.1) yields[11]

$$i_t^N = r_H + \frac{\sigma\kappa(1 - \rho) + \lambda\rho}{\lambda(1 - \beta\rho) + \kappa^2} \xi_t. \tag{2.12}$$

We observe that in Phase H, inflation and the output gap merely depend on the supply shock and have opposite signs. Because of the inflationary (deflationary) pressure induced by a positive (negative) supply shock, the nominal interest rate is set above (below) the natural real interest rate r_H.

In a second step, we derive the dynamics in Phase D. Note that since the size of the supply shock in the first period of Phase H is symmetrically distributed, i.e. $\xi_1 \in [-\xi, \xi]$, expected inflation and the output gap in the downturn are

$$\mathbb{E}_D[\pi_{t+1}^N] = \delta\pi_D^N + (1 - \delta)\mathbb{E}_D[\pi_1^N] = \delta\pi_D^N \tag{2.13}$$

and

$$\mathbb{E}_D[x_{t+1}^N] = \delta x_D^N + (1 - \delta)\mathbb{E}_D[x_1^N] = \delta x_D^N. \tag{2.14}$$

Combining Equations (2.1), (2.2), (2.13), (2.14), and using $i_D^N = 0$ yields

$$\pi_D^N = \frac{\kappa}{h} r_D < 0 \tag{2.15}$$

and

$$x_D^N = \frac{1 - \beta\delta}{h} r_D < 0, \tag{2.16}$$

---

[10] The detailed derivation of the economic dynamics can be found in Appendix A.1.

[11] Note that the forward-looking property of the IS Curve yields $\mathbb{E}_t[\pi_{t+1}^N] = \frac{\lambda}{\lambda(1-\rho\beta)+\kappa^2}\mathbb{E}_t[\xi_{t+1}]$ and $\mathbb{E}_t[x_{t+1}^N] = -\frac{\kappa}{\lambda(1-\rho\beta)+\kappa^2}\mathbb{E}_t[\xi_{t+1}]$, where $\mathbb{E}_t[\xi_{t+1}] = \rho\xi_t$.

where

$$h := \sigma(1 - \delta)(1 - \beta\delta) - \kappa\delta > 0. \tag{2.17}$$

That is, in the downturn, the central banker lowers the nominal interest rate to the ZLB, and the economy incurs deflation and an output collapse.

2.2.3 Standard Forward Guidance

In the presence of a negative natural real interest-rate shock, the central banker's policy tool is constrained by the ZLB of the nominal interest rate as the previous subsection has illustrated. In this subsection, we consider the situation where, in each period of the downturn, the central banker makes a zero interest-rate forecast[12] for the next period to create inflationary expectations and stops making zero interest-rate forecasts in Phase H.[13] Again, we use backward induction.[14] In a first step, we derive the dynamics in Phase H. In periods $t \geq 2$, the dynamics of π_t, x_t, and i_t are the same as in Equations (2.10), (2.11), and (2.12), since the central banker does not make any zero interest-rate forecasts in Phase H. Hence, inflation and output-gap expectations in the first period of Phase H are

$$\mathbb{E}_1[\pi_2^F] = \frac{\lambda\rho}{\lambda(1 - \rho\beta) + \kappa^2}\xi_1, \tag{2.18}$$

$$\mathbb{E}_1[x_2^F] = -\frac{\kappa\rho}{\lambda(1 - \rho\beta) + \kappa^2}\xi_1. \tag{2.19}$$

In $t = 1$, the central banker is still constrained by the zero interest-rate forecast made in the downturn. He thus minimizes his loss function (2.5) by appropriately selecting the nominal interest rate i_1^F subject to the zero interest-rate forecast $i_1^f = 0$ and the IS Curve (2.1) and the Phillips Curve (2.2). In Appendix A.2, we calculate the interest rate, the inflation, and the output gap in Phase H, $t = 1$ and obtain

$$i_1^F = \frac{\lambda + \kappa^2}{\lambda + \kappa^2 + b\sigma^2}r_H + g_1(b)\xi_1 \leq i_1^N, \tag{2.20}$$

$$\pi_1^F = \frac{b\kappa\sigma}{\lambda + \kappa^2 + b\sigma^2}r_H + g_2(b)\xi_1 \geq \pi_1^N, \tag{2.21}$$

$$x_1^F = \frac{b\sigma}{\lambda + \kappa^2 + b\sigma^2}r_H + g_3(b)\xi_1 \geq x_1^N, \tag{2.22}$$

---

[12] Forecasting a positive interest rate would dampen the economic variables of interest, π_D and x_D, and would generate additional losses because there is a certain probability that the economy will remain in the downturn in the next period. The Federal Reserve, for instance, has adopted zero interest-rate forward guidance since 2008.

[13] In the Chapter 5 we analyze how the results change when we relax the assumptions of a zero interest-rate forecast and when we replace the ZLB with an effective lower bound.

[14] The detailed derivation of the economic dynamics is given in Appendix A.2.

where $g_1(b)$, $g_2(b)$, and $g_3(b)$ are functions of b and given in Appendix A.2. We note that when $b = 0$, Equations (2.20), (2.21), and (2.22) are the same as Equations (2.10), (2.11), and (2.12) for $t = 1$. That is, if the central banker does not incur an intrinsic loss from deviating from his own forecasts, he will ignore the zero interest-rate forecast and set an interest rate that minimizes the social loss function (2.4).

Also, we note that Equation (2.20) strictly decreases with b. In other words, the higher the central banker's utility loss from the forecast deviation—i.e. the larger the value of b or the higher the central banker's degree of scrupulosity—, the closer the interest rate is set to the zero forecast. This accommodative monetary policy stance leads to higher inflation and a higher output gap, i.e. $\pi_1^F \geq \pi_1^N$ and $x_1^F \geq x_1^N$.

Next, we derive the dynamics of the economy in the downturn. Expected inflation and output gap in the downturn are

$$\mathbb{E}_D[\pi_{t+1}^F] = \delta\pi_D^F + (1 - \delta)\mathbb{E}_D[\pi_1^F] \tag{2.23}$$

and

$$\mathbb{E}_D[x_{t+1}^F] = \delta x_D^F + (1 - \delta)\mathbb{E}_D[x_1^F]. \tag{2.24}$$

Note that due to the zero interest-rate forecast made by the scrupulous central banker, the expectations about inflation and the output gap in the initial period of Phase H, i.e. $\mathbb{E}_D[\pi_1^F]$ and $\mathbb{E}_D[x_1^F]$, are no longer zero as in Equations (2.13) and (2.14). This shows how forward guidance affects the economy both in the downturn and in normal times: Forward guidance lowers the real interest rate in a downturn at the expense of inflation and an output boom in normal times.

Using Equations (2.1), (2.2), (2.21), (2.22), (2.23), (2.24) and $\mathbb{E}_D[\xi_1] = 0$, we obtain

$$\pi_D^F = \frac{(1 - \delta)b\kappa\sigma(\sigma + \kappa + \sigma\beta(1 - \delta))}{(\lambda + \kappa^2 + b\sigma^2)h}r_H + \frac{\kappa}{h}(r_D - i_D^F) \geq \pi_D^N, \tag{2.25}$$

$$x_D^F = \frac{(1 - \delta)b\sigma(\sigma(1 - \beta\delta) + \kappa)}{(\lambda + \kappa^2 + b\sigma^2)h}r_H + \frac{1 - \beta\delta}{h}(r_D - i_D^F) \geq x_D^N. \tag{2.26}$$

From these dynamics, we obtain the following lemma:

Lemma 2.1

With zero interest-rate forecasts in the downturn, the inflation and the output gap in the downturn are higher than without forecasts, and they increase with the central banker's degree of scrupulosity, i.e. π_D^F and x_D^F increase with b.

Note that when $b = 0$, Equations (2.25) and (2.26) are the same as Equations (2.15) and (2.16). A zero interest-rate forecast in such circumstances has no impact on public

expectations, since it will be abandoned without cost once the economy returns to normal. Lemma 2.1 shows that the high inflation and output-gap expectations induced by the zero interest-rate forecast lower the real interest rate in the downturn and thus alleviate deflation and output collapse.

Using Equations (2.1), (2.2), and the zero interest-rate forecast $i_D^f = 0$, the first-order condition of the loss function (2.5) with respect to the nominal interest rate i_D^F yields

$$\kappa \pi_D^F + \lambda x_D^F - b\sigma i_D^F = 0. \tag{2.27}$$

Combining Equations (2.25), (2.26) and (2.27) leads to the following proposition:

Proposition 2.1

Under a zero interest-rate forecast in the downturn, the central banker will set the interest rate at zero in the downturn if

$$r_D \leq r_D^c, \tag{2.28}$$

where

$$r_D^c := -\frac{b\sigma(1-\delta)[\kappa^2(\kappa + \sigma(1 + \beta(1-\delta))) + \lambda(\kappa + \sigma(1 - \beta\delta))]}{(\lambda + \kappa^2 + b\sigma^2)(\kappa^2 + \lambda(1 - \beta\delta))} r_H. \tag{2.29}$$

Otherwise, he will set the interest rate to

$$i_D^F = \frac{r_D - r_D^c}{1 + \frac{b\sigma h}{\kappa^2 + \lambda(1 - \beta\delta)}} > 0. \tag{2.30}$$

Equation (2.30) implies that the central banker will set the nominal interest rate above zero in the downturn when the natural real interest rate is larger than r_D^c. The reason is that in the presence of a small natural real interest-rate shock, the zero interest-rate forecast will generate excessive inflation expectations. This may even lead to inflation and an output boom in the downturn.

Comparison to NFG

In Figure 2.2 we show the expected social losses and the expected central banker's losses in both NFG (black line) and IFG (red lines) as a function of the size of the natural real interest-rate shock. The expected social losses are represented by solid lines, the expected central banker's losses by the dotted line. Note that for NFG, the central banker and the society incur the same loss. For large shocks, the red dotted line is below the black line, i.e. the central banker incurs lower losses under IFG. Thus, the central banker will make zero interest-rate forecast in such circumstances. At $r_D = -0.64\%$, $\tilde{L}^N$ and $\tilde{L}^F$
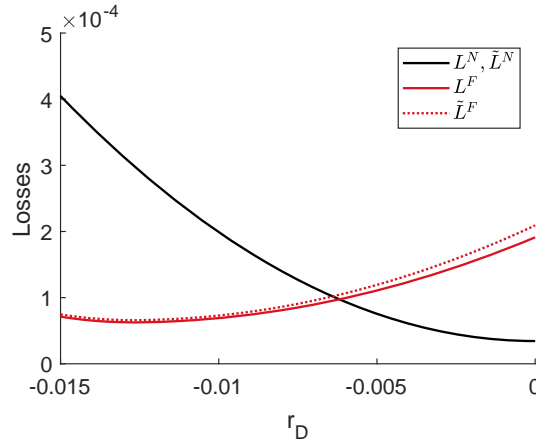
Figure 2.2: Expected social losses and central banker's losses.

intersect, i.e. the central banker is indifferent between NFG and IFG at this point. For $r_D > -0.64\%$, it is more beneficial for the central banker to abstain from forecasting. Thus, given that the central banker can decide between applying forward guidance in the downturn or abstaining from forecasting, the realized social loss is represented by the red solid line for $r_D < -0.64\%$ and jumps to the solid black line for $r_D \geq 0.64\%$.

We observe that without forecasts the expected social losses, i.e. L^N, increase with the size of the r_D shock. However, under zero interest-rate forecasts, i.e. L^F, the expected social losses first decrease, then increase with r_D. The reason is that with the zero interest-rate forecast, the inflation expectations are excessively high for small natural real interest-rate shocks. These excessively high inflation expectations lead to an output boom and push up inflation in the downturn. Thus, forecasting is costly both in the downturn and in normal times. Therefore, for small shocks it is socially desirable to abstain from forecasts in the downturn, and the central banker will not make any forecasts in such circumstances. As the size of the shock increases, high inflationary expectations induced by the zero interest-rate forecast become more and more socially beneficial, since they alleviate deflation and output decline in the downturn.

Note that the social desirability and the central banker's preference for making forecasts are not aligned, since the intersections of L^N with L^F and L^N with $\tilde{L}^F$ are different due to the additional deviation costs $b(i_t - i_t^f)^2$. The central banker starts making zero interest-rate forecasts when the shock is more severe ($r_D = -0.64\%$) than socially desirable (at $r_D = -0.62\%$, where the two solid lines intersect).

We summarize the figure in the following observation:

Numerical Finding 2.1

A zero interest-rate forecast in the downturn is only socially beneficial if the natural real interest-rate shock is sufficiently severe.

The Effect of *b*

In the benchmark case, we assumed $b = 1$, which represents a scrupulous central banker who puts equal weight on forecast precision and the inflation target. In this subsection, we study a less scrupulous central banker, i.e. $b \in [0, 1]$.

Figure 2.3 displays the difference in social losses between a discretionary central banker and standard forward guidance ($L^N - L^F$). A positive value for this difference indicates a benefit for providing a forecast. The contour lines in the right part of the figure show the same in a two dimensional plane. The area above the zero-contour line shows positive values, hence, combinations of b and r_D for which it is beneficial to apply IFG. For shocks that are severe enough, it is always socially beneficial to make a forecast, despite higher losses in Phase H due to $i_1^F < i_1^N$. For values $r_D < -0.62\%$, the shock is severe enough to make it beneficial to introduce standard forward guidance for any size of $b \in [0, 1]$. As the shock size decreases and b increases, the zero interest-rate forecast entails losses in Phase H that outweigh potential benefits in Phase D.

Contour lines that are close to each other indicate a steep increase. Therefore, from a shock size that is roughly around $r_D < -0.01$ to lower values, it is particularly helpful to make forecasts because even low b values of up to 0.2 allow the realization of the majority of the utility gains.
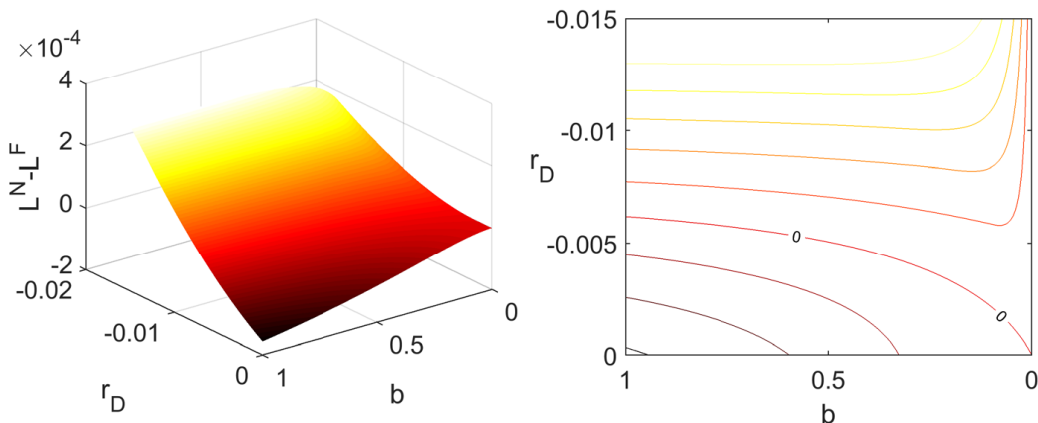
Figure 2.3: Expected social losses as a function of b and r_D.

In our analysis, we assumed a scrupulous central banker, which implies an intrinsically fixed value for b. What if b is set exogenously, as in Gersbach et al. (2015), for example? As a last exercise, we suppose that r_D is observed and $b = b^*$ is set optimally from a social point of view ($\tilde{b}^*$ from a central banker's point of view). Then, making a forecast is always beneficial for $r_D < 0$, as displayed in the left graph of Figure 2.4. We plot the optimal values of b from a social and a central banker's perspective in the right graph. For severe shocks, optimal b values substantially outreach the values we consider.[15] This implies that a stronger commitment to stick to the forecast in the next period is desirable. As the shock gets closer to zero, less commitment is needed to raise inflation expectations.
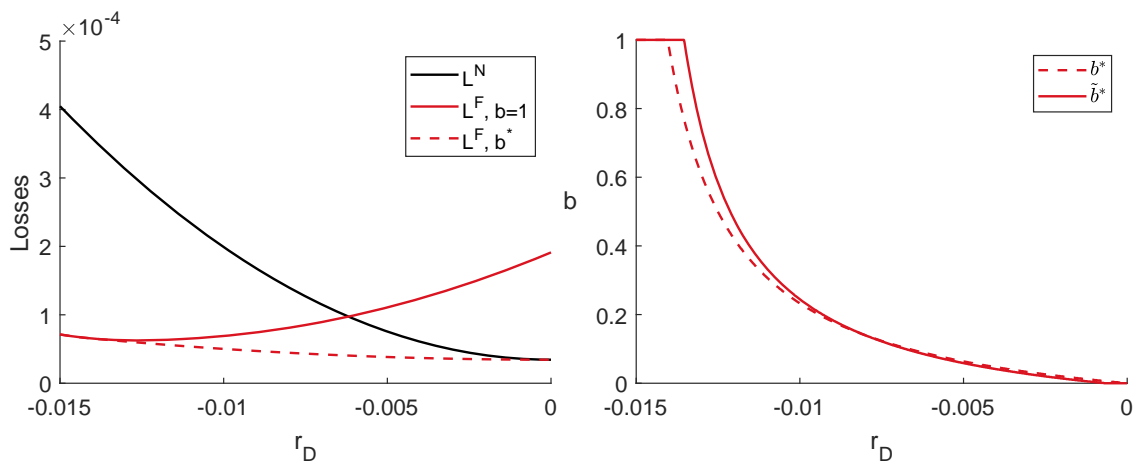
Figure 2.4: Expected social losses with optimal b and the optimal b as a function of r_D.

In the next two sections, we introduce two more sophisticated designs of forward guidance and explore whether they can further improve social welfare.

2.3 Escaping Forward Guidance

We introduce forward guidance with a self-chosen escaping clause. In the downturn, the central banker promises to keep the interest rate at zero in the next period, as long as the inflation in that period remains below a critical threshold π^c chosen by the central banker himself.[16] The idea is to find a way for central bankers to abandon their forecasts without

---

[15] A typical case discussed in the literature is the one of perfect commitment, i.e. $b = \infty$. When we let b go to infinity, the shape of the red curve in the left graph of Figure 2.4 stays qualitatively similar, but is tilted such that the losses around large r_D values become smaller, whereas losses close to zero become greater.

[16] One could use a critical interest rate as a threshold, i.e. the central banker could discard the zero interest-rate forecast if the interest rate is above the critical interest rate he has chosen in the downturn. A critical interest rate is more easily observable and verifiable than a critical inflation rate. However, with a critical

cost when due to the supply shock inflation is particularly high after recovery, and when stabilizing such a shock has high priority. The subtleties are twofold. First, the escape has to be chosen endogenously by the central banker, which, in turn, may encourage strategic interest-rate setting once the threshold π^c has been established.[17] Second, *escaping* forward guidance should allow inflationary pressure for a positive supply shock to be alleviated without waiving the necessity of future booms lifting output and inflation in the downturn.

We call the *escaping* forward guidance design "EFG" for short and denote variables by the superscript "E".

We use backward induction to study the dynamics of the economy under EFG and thus start in Phase H. In $t \geq 2$, the dynamics of the economy follow Equations (2.10), (2.11), and (2.12) since there are no interest-rate forecasts. In $t = 1$, the central banker's loss function is

$$\tilde{l}_1^E = \frac{1}{2}[(\pi_1^E)^2 + \lambda(x_1^E)^2 + b(i_1^E)^2], \tag{2.31}$$

if $\pi_1^E < \pi^c$, and

$$\tilde{l}_1^E = \frac{1}{2}[(\pi_1^E)^2 + \lambda(x_1^E)^2], \tag{2.32}$$

otherwise. If the inflation realized in the first period of Phase H is below the critical threshold π^c, the central banker will still be subject to the zero interest-rate forecast and will thus bear the additional loss $b(i_1^E)^2$. Otherwise, the central banker can discard the zero interest-rate forecast made in the previous period without incurring any deviation costs. In other words, depending on the inflation realized in $t = 1$, there are three regimes under EFG:

- $\pi_1 < \pi^c$, the central banker behaves as under IFG, i.e. $i_1^E = i_1^F, \pi_1^E = \pi_1^F$ and $x_1^E = x_1^F$;

- $\pi_1 > \pi^c$, the central banker behaves as under NFG, i.e. $i_1^E = i_1^N, \pi_1^E = \pi_1^N$ and $x_1^E = x_1^N$;

- $\pi_1 = \pi^c$, the central banker behaves as follows:

---

interest rate, the central banker has incentives to strategically raise the interest rate above the critical threshold to avoid deviation costs. This strategical interest rate hike induces deflation expectation in the downturn and thus worsens the situation.

[17] Note that in our setup setting a critical output gap is equivalent to setting a critical inflation rate. We take π^c as the critical threshold because inflation is easily measurable and quickly available. The output gap is often revised, sometimes long after its first publication, and it is only available on a quarterly or yearly basis. In addition, there are many uncertainties about the measurement of potential output.

Proposition 2.2

Under EFG with a self-chosen critical inflation π^c, the interest rate that just allows the central banker to escape is[18]

$$i_1^c = r_H - \frac{\sigma}{\kappa}\pi^c + \frac{\lambda(\kappa\rho + \sigma) + \sigma\kappa^2(1-\rho)}{\kappa[\lambda(1-\rho\beta) + \kappa^2]}\xi_1, \tag{2.33}$$

and the corresponding output gap is

$$x_1^c = \frac{\pi^c - \frac{\lambda+\kappa^2}{\lambda(1-\rho\beta)+\kappa^2}\xi_1}{\kappa}. \tag{2.34}$$

The formulas in Proposition 2.2 are obtained by using $\pi_1 = \pi^c$, Equations (2.1), (2.2), (2.10), and (2.11).

We note that i_1^c increases with the size of the supply shock and decreases with the choice of π^c. Hence, by his choice of π^c the central banker can influence how he will act when the economy enters Phase H.

The dynamics of π_D^E, x_D^E, and i_D^E in the downturn can be derived similarly to Equations (2.25), (2.26) and (2.30) (see Appendix A.3).

For a given natural real interest-rate shock, the central banker chooses the critical threshold π^c to minimize his expected losses (2.9). The respective loss functions $\tilde{l}_D^E$ and $\tilde{l}_H^E$ are given by

$$\tilde{l}_D^E = \frac{1}{2}[(\pi_D^E)^2 + \lambda(x_D^E)^2 + b(i_D^E)^2] \tag{2.35}$$

and

$$\tilde{l}_H^E = \tilde{l}_1^E + \sum_{t=2}^{\infty}\beta^{t-1}l_t^N. \tag{2.36}$$

We now provide a more intuitive discussion of EFG's properties.

Figures 2.5 and 2.6 plot the variables of interest in a single period $t = 1$ in Phase H for the whole range of $\xi_1 \in [-\xi, \xi]$ with (red lines) and without (black lines) forward guidance. The interest rates i_1^N and i_1^F are plotted as functions of the supply shock in Figure 2.5 and confirm Inequality (2.20), which states $i_1^F \leq i_1^N$. Lower interest rates under IFG lead to higher inflation and a higher output gap (see Figure 2.6), i.e. $\pi_1^F \geq \pi_1^N$ and $x_1^F \geq x_1^N$ (see Inequalities (2.21) and (2.22)).

Furthermore, Figures 2.5 and 2.6 illustrate the working of EFG for the polar case $\pi^c = 0$. The interest rate in Phase H, $t = 1$, set by the central banker under EFG with $\pi^c = 0$, is represented by the segmented solid lines in Figure 2.5 and the inflation and the output gap in Figure 2.6. In the presence of a large negative supply shock, the central banker

---

[18] We use the superscript "c" in combination with the time subscript to denote the critical values in the first period of Phase H.
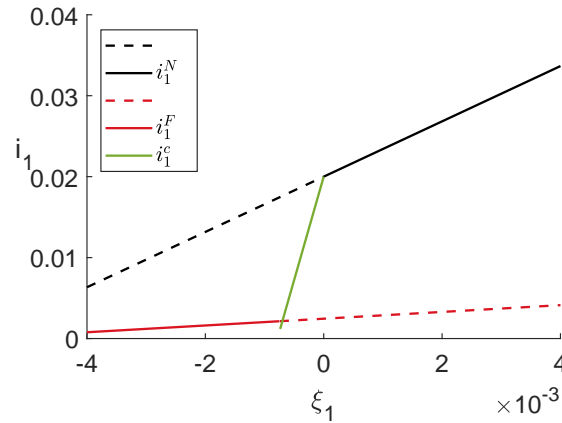
Figure 2.5: The interest rate under NFG, IFG, and EFG in $t = 1$.
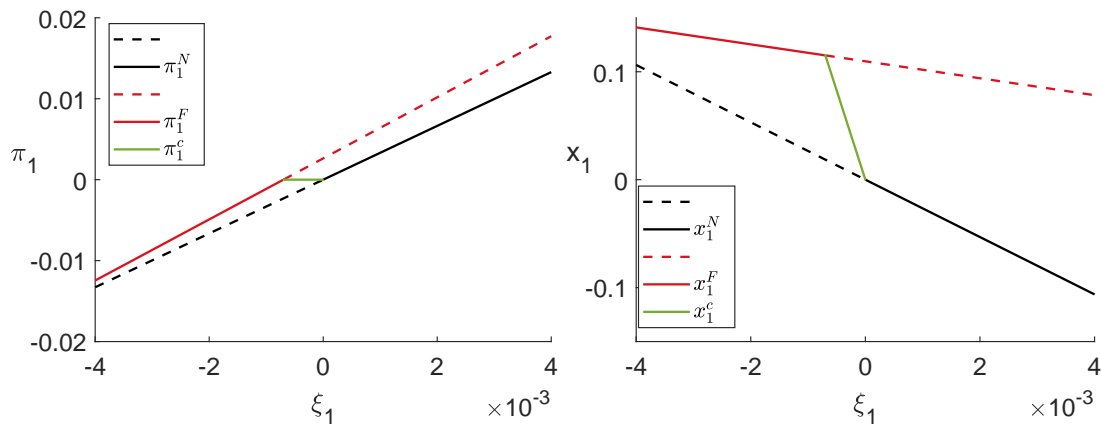
Figure 2.6: Inflation and the output gap under NFG, IFG, and EFG in $t = 1$.

sets the interest rate as he would set it under IFG (solid red line). When the supply shock is large and positive, the central banker sets the interest rate discretionarily (solid black line), which leads to inflation above the critical threshold $\pi^c = 0$, as shown in Figure 2.6. For a supply shock of a size close to zero, the central banker sets the interest rate higher than he would set it under IFG and lower than he would set it discretionarily (see the green line in Figure 2.5), so that inflation is maintained at π^c (see the green line in Figure 2.6). Thus inflation arrives at $\pi^c = 0$ and the central banker's loss function becomes Equation (2.32). Note that in the illustrative example in Figure 2.5, for supply shocks around $\xi_1 = -0.0007$, the central banker will even find it beneficial to lower the interest rate strategically below the value of i_1^F to escape and avoid the term $b(i_1^E)^2$ in his loss function (2.31).

Figure 2.7 depicts the central banker's losses in $t = 1$ under designs NFG, IFG, and EFG as a function of the size of the supply shock. The solid and dashed red (black) curves represent the central banker's loss if he is (is not) subject to the zero interest-rate forecast.
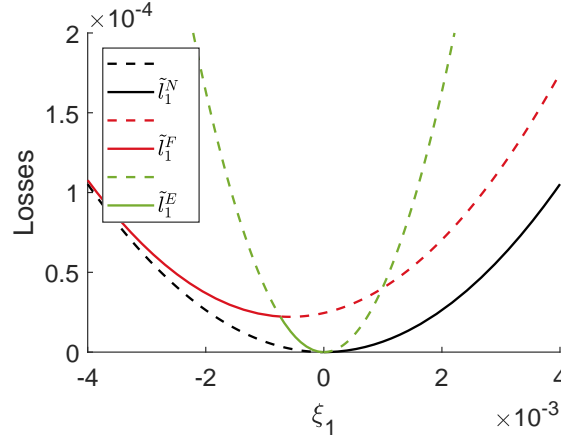
Figure 2.7: The central banker's losses under NFG, IFG, and EFG in $t = 1$.

The solid and dashed green curves represent the central banker's loss if he keeps inflation at $\pi^c = 0$. Figure 2.7 displays three situations the central banker faces when he engages in EFG, indicated by the three colors of the solid lines. For a supply shock within the range $\xi_1 \in [-0.004, -0.0007)$, it is too costly for the central banker to escape, and he remains fettered by the forecast made in the downturn. From $\xi_1 = -0.0007$ to $\xi_1 = 0$, the central banker escapes strategically by setting the interest rate at i_1^c so that the realized inflation reaches the critical threshold, i.e. $\pi_1^E = \pi^c$. In the range $\xi_1 \in [-0.004, 0)$, if the interest rate was set optimally according to a discretionary approach, the inflation rate would not reach the critical threshold (see the dashed black line in Figure 2.6). For all positive supply shocks, the central banker can escape his own forward guidance commitment without acting strategically. Hence, the segmented solid lines in Figure 2.7 depict the central banker's losses in Phase H, $t = 1$ under EFG with $\pi^c = 0$.

To sum up, under EFG with $\pi^c = 0$, the central banker faces three cases in $t = 1$, depending on the size of the supply shock:

(i) *No escape*: It is not beneficial to escape—thus we obtain $i_1^E = i_1^F$ and $\pi_1^E = \pi_1^F$.

(ii) *Strategic escape*: It is beneficial to escape by setting the interest rate strategically, just low enough to escape—i.e. $i_1^E = i_1^c$ and $\pi_1^E = \pi^c$. Hence, the central banker does not suffer a utility loss from the deviation of the zero interest-rate forecast, i.e. $b(i_1^E)^2$ falls off.

(iii) *Unconstrained escape*: Escaping by setting the interest rate as at the banker's full discretion is beneficial—$i_1^E = i_1^N$ and $\pi_1^E = \pi_1^N$.

We note that over the whole range of supply shocks, the discretionary solution dominates the other two. EFG, however, has a favorable impact on the economy in the downturn. This, together with the optimal choice of π^c, will be addressed in the remainder of this subsection.

Comparison to NFG and IFG

Figure 2.8 confirms the intuition that NFG and IFG are two polar cases of EFG.[19] That is, NFG (IFG) corresponds to EFG with very low (high) value of π^c such that the central banker will always (never) escape from the zero interest-rate forecast. Hence, the expected social losses under EFG dominate those under NFG and IFG, since the central banker can tune the desired levels of inflation expectation by setting the critical threshold π^c at proper levels. In the presence of small shocks, the central banker would set a low critical threshold so that when the economy reverts to normal times there is a large chance that the central banker will be able to discard the zero interest-rate forecast and act discretionarily. In such circumstances this yields a lower but welcome level of inflation expectation in downturns. Analogously for large r_D shocks, π^c is set to a large value.
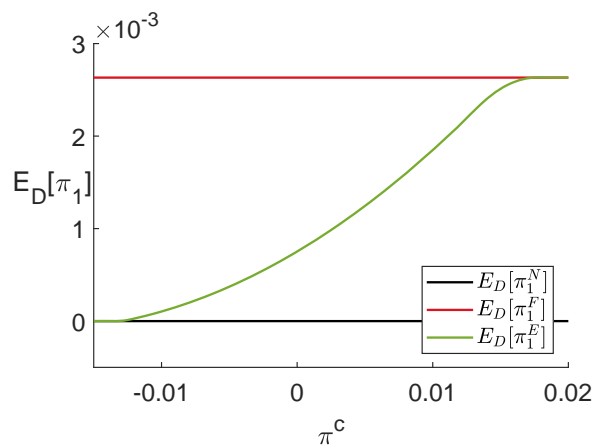
Figure 2.8: Inflation in $t = 1$ expected in Phase D under NFG, IFG, and EFG.

Figure 2.9 plots expected cumulative social losses of the two polar cases *(i)* always escape, i.e. L^N (black solid line) and *(ii)* never escape, i.e. L^F (red solid line), and *escaping* forward guidance, i.e. L^E with $\pi^c = \pi^c_{opt}$ (green solid line). L^E with π^c_{opt} constitutes the lower bound among the designs NFG, IFG, and EFG. Intuitively, this finding is straight forward because EFG encompasses NFG and IFG.

We summarize the numerical findings as follows:

Numerical Finding 2.2

It is socially beneficial to make a zero interest-rate forecast with a self-chosen escaping clause in the downturn. The critical threshold chosen by the central banker increases with the size of the natural real interest-rate shock.

---

[19] Note that from a society's point of view the case $\mathbb{E}_D[\pi_1^E] > \mathbb{E}_D[\pi_1^F]$ exists. Appendix A.3 provides an analysis of these cases.
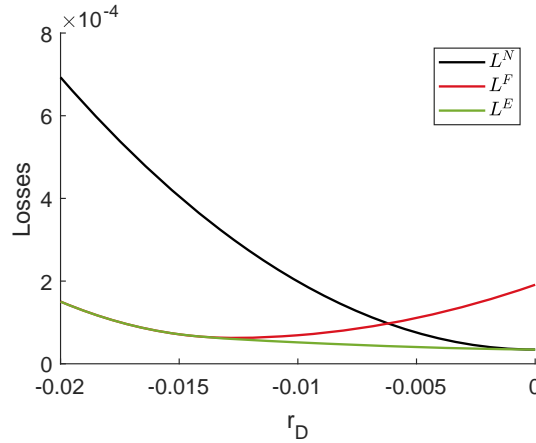
Figure 2.9: Expected social and central banker's losses.

The Effects of *b* and π^c

To complete the picture of EFG, we now discuss the role of b and π^c. Both influence the central banker's commitment to his zero interest-rate forecast, i.e. a higher b value and a higher π^c lead to an increase in inflation and output-gap expectations in downturns. Thus, contingent on the size of the natural real interest-rate shock, the central banker can manage the expected inflation in the downturn by choosing the critical threshold π^c in addition to making the zero interest-rate forecast.

We first turn our attention to the optimal choice of π^c by the central banker under EFG. The intuition is that with a higher π^c, there is a smaller probability that the central banker can escape and set the interest rate discretionarily. That is, a higher critical threshold chosen by the central banker will lead to a higher inflation expectation in the downturn. Figure 2.8 displays this property for the inflation in $t = 1$ expected in the downturn. The output gap exhibits a similar pattern. The optimal value of $\pi^c = \pi^c_{opt}$ is a decreasing function in r_D, as shown in Figure 2.10. For $r_D > -0.0042$ π^c_{opt} becomes negative.

Next, we examine under which circumstances a society wants the central banker to use EFG and abstain from the flexibility of NFG. The plots in Figure 2.11 show the differences in L^N and L^E for various r_D-π^c-combinations, given a fixed value of b. For positive z-axis values, it is socially beneficial to introduce an interest-rate forecast with an escape clause. When the difference is zero, the society is indifferent between EFG and NFG. Negative differences indicate that the society prefers to abstain from forecasting. In the upper part of Figure 2.11, b is set to 1 in the middle part to 0.5 and in the lower part to 0.1. At an x-axis value of $\pi^c = -0.015$, the central bank can always escape unconstrained. Thus, the forecast is never binding, which implies $L^N = L^E$. From a threshold of $\pi^c > 0.0175$, the central banker never escapes, which implies the forecast is always binding and $L^F = L^E$. Only the combinations of small natural real interest-rate shocks and a high threshold
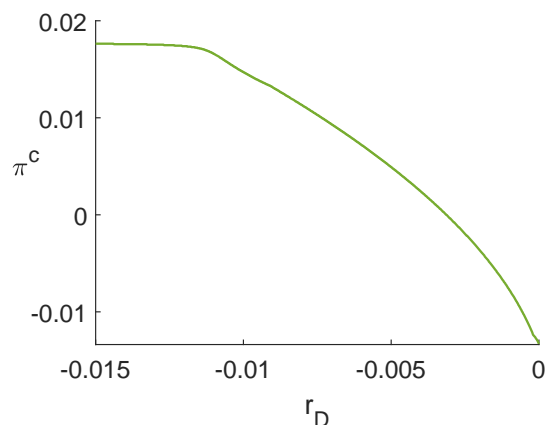
Figure 2.10: Optimal critical threshold π^c.

π^c result in a superiority of NFG. The majority of the combinations yields social gains, although the inflation threshold is not set optimally.

Given $b = 1$, the contour lines to the right side of the respective graph show that for a shock $r_D < -0.62\%$, it is always optimal to make a forecast, independent of how restrictive the escape clause is—i.e. in the area to the right of the zero-contour line, it is preferable to forecast. On the one hand, the smaller b becomes, the greater the area is in which it is optimal to introduce EFG. On the other hand, EFG becomes less effective at the same time, which is characterized by smaller difference in the minimum and maximum value of $(L^N - L^E)$. To sum up, Figure 2.11 shows that the finding in Figure 2.9—EFG is socially desirable to NFG—is robust to different values of b and to thresholds π^c that are not set optimally, but that are close enough to the optimal threshold.

Figure 2.12 displays $(L^N - L^E)$ as a function of b and π^c, with r_D fixed at -0.005 and -0.015, respectively. For a small shock, the combination of a large b value and a large π^c value yields utility losses compared to abstaining from forecasts. For $b < 0.76$, it is beneficial to forecast even for large π^c values. Furthermore, given a small shock, the optimal value of π^c depends on the value of b. In a sense, a sufficient degree of partial commitment can either be reached through high scrupulosity b and a low threshold π^c, a middle value of b and a high π^c, or mid sized values of both. As b grows, the incentive of the central banker to keep i_1^E low increases and, therefore, inflationary expectations in downturns increase. Once inflationary expectations are high enough, the central banker can counter the negative effects of a large b value by decreasing π^c, which makes an escape more probable, and inflation expectations decrease. Given a large shock, it is always socially preferable to engage in forward guidance. In fact, in the illustrative example of $r_D = -0.015$, the most restrictive b-π^c-combination of $b = 1$ and $\pi^c = 0.02$, which raises inflation expectations in downturns as much as possible, is the most welfare improving
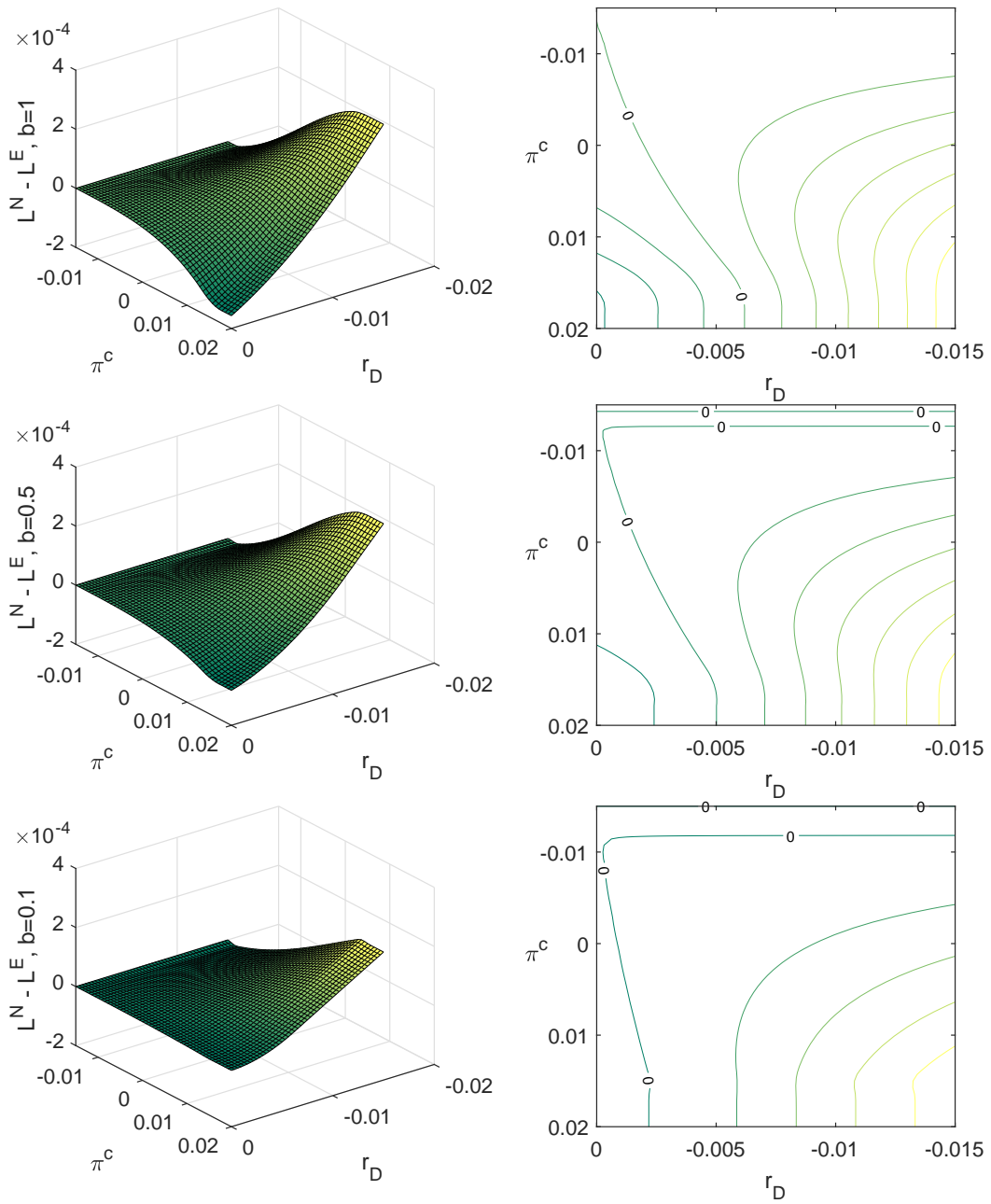
Figure 2.11: Utility gains in applying EFG compared to NFG for $b = 1,\ 0.5,\ 0.1$. Positive values indicate a preference for zero interest-rate forecast with escape clause.
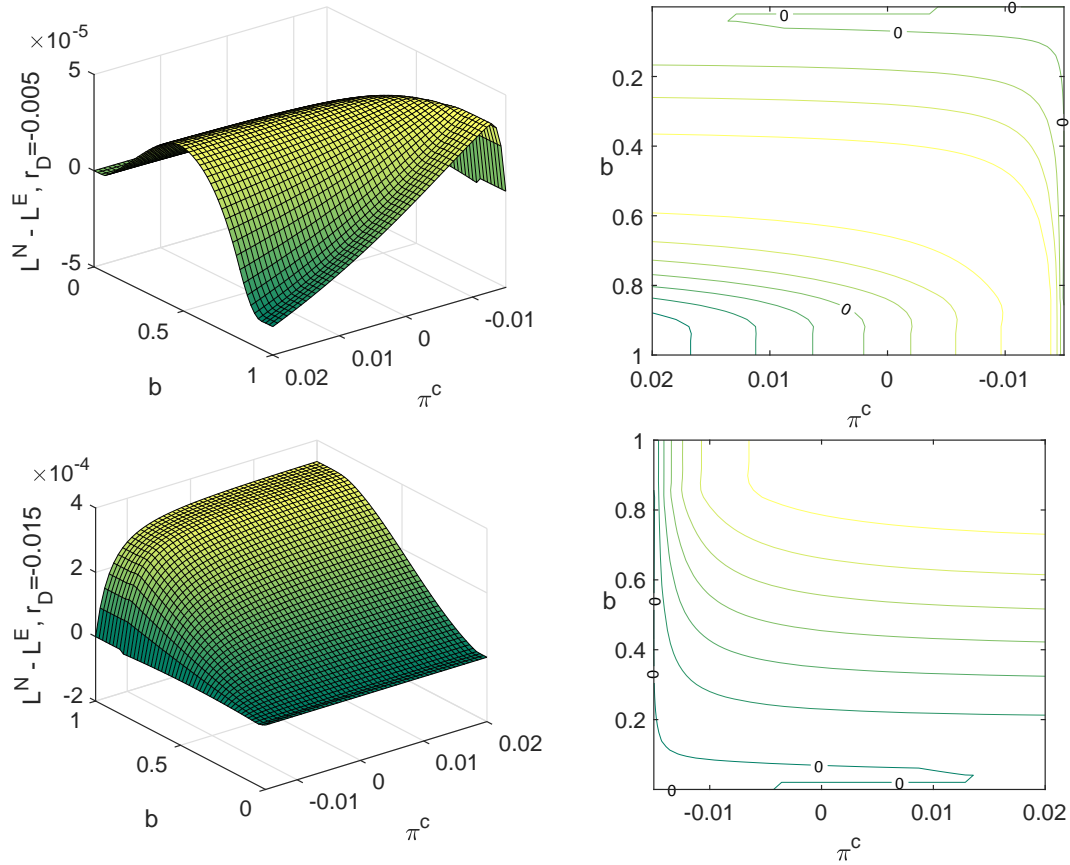
Figure 2.12: Utility gains in applying EFG compared to NFG for $r_D = -0.005$ and $r_D = -0.015$. Positive values indicate a preference for zero interest-rate forecast with an escape clause.

one. Figure 2.12 demonstrates that for a large set of b-π^c-combinations EFG is superior to NFG. Therefore, the Numerical Finding 2.2 is robust to a selection of values of b with $0 < b < 1$ values and a π^c threshold that is sufficiently close to its optimal value.

2.4 Switching Forward Guidance

In this section, we study the alternative design *switching* forward guidance (SFG). In the downturn, the central banker makes a zero interest-rate forecast without an escaping clause.[20] Once the economy recovers, i.e. in the initial period of Phase H, the central banker switches to issuing an inflation forecast. That is, in $t = 1$, the central banker is still subject to the zero interest-rate forecast made in the downturn. At the same time, the central banker makes an inflation forecast for the next period to anchor the inflation in the current period. In other words, the central banker makes zero interest-rate forecasts in the downturn to raise inflation expectations and switches to inflation forecasts in normal times to anchor the inflation. We denote variables in this section by superscript "S". The idea of SFG is to let the economy create a short-lived inflation and output boom as soon as the economy returns to normal both to lift the economy in the downturn and also to start fighting inflation by inflation forecasts once the supply shock is seen to be pushing up inflation strongly.

We first examine the dynamics of the economy under SFG in Phase H. In $t = 1$, the central banker is still subject to the zero interest-rate forecast made in the previous period. Thus, his instantaneous loss function is

$$\tilde{l}_1^S = \frac{1}{2}[(\pi_1^S)^2 + \lambda(x_1^S)^2 + b_1(i_1^S)^2].$$ (2.37)

Note that we use b_1 as the measure of scrupulosity with respect to interest-rate forecasts. The first-order condition with respect to i_1^S subject to the IS Curve (2.1) and the Phillips Curve (2.2) is

$$\kappa\pi_1^S + \lambda x_1^S - b\sigma i_1^S = 0.$$ (2.38)

Apart from setting the nominal interest rate i_1^S, the central banker also forecasts the next period's inflation π_2^f. The inflation forecast π_2^f affects the current inflation expectation, as the precision of the inflation forecast enters the central banker's loss function in $t = 2$.

In $t \geq 2$, the central banker's loss function is

$$\tilde{l}_t^S = \frac{1}{2}[(\pi_t^S)^2 + \lambda(x_t^S)^2 + b_2(\pi_t^S - \pi_t^f)^2],$$ (2.39)

where π_t^f is the inflation forecast made in $t - 1$ and b_2 is the measure of scrupulosity with respect to inflation forecasts.

---

[20] We do not intend to introduce more sophisticated designs of forward guidance, such as *switching* forward guidance with escaping clauses. The reasons are as follows: On the one hand, our intention is to study simple designs of forward guidance. On the other, sophisticated forward guidance comes at the cost of clarity, which may undermine its efficacy. Nevertheless, with the setup described in this chapter, one could indeed study different combinations of *escaping* and *switching* forward guidance.

Thus, in each period $t \geq 2$, the central banker sets i_t^S and π_{t+1}^f to minimize

$$\mathbb{E}_t\left[\sum_{j=t}^{\infty} \beta^{j-t} \tilde{l}_j^S\right]. \tag{2.40}$$

Given the central banker's own policy path $\{i_t, \pi_{t+1}^f\}_{t=2}^{\infty}$, in $t = 1$, he will set the nominal interest rate i_1^S and the inflation forecast π_2^f to minimize the expected inter-temporal loss function

$$\mathbb{E}_1\left[\sum_{j=1}^{\infty} \beta^{j-1} \tilde{l}_j^S\right]. \tag{2.41}$$

We use the algorithm presented by Söderlind (1999) to compute the solution.[21]
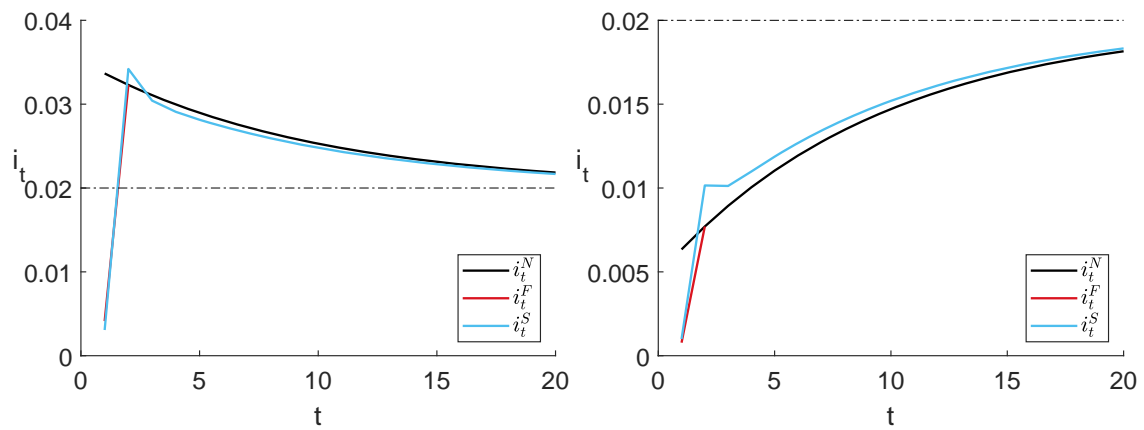
Figure 2.13: The dynamics of the interest rate in Phase H under NFG, IFG, and SFG when $\xi_1 = \xi$ (left) and $\xi_1 = -\xi$ (right).
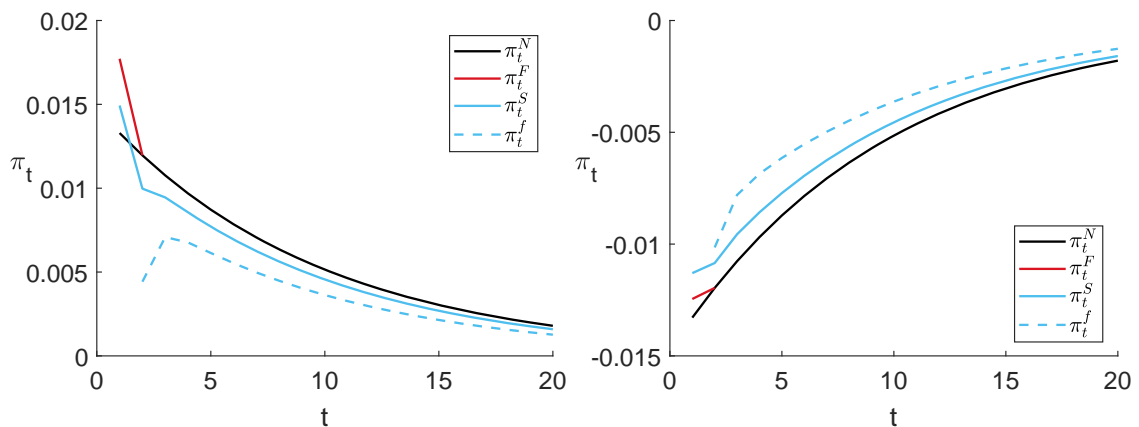
Figure 2.14: The dynamics of inflation in Phase H under NFG, IFG, and SFG when $\xi_1 = \xi$ (left) and $\xi_1 = -\xi$ (right).

---

[21] Details are provided in Appendix A.4.

Figures 2.13 and 2.14 plot the dynamics of the economy under NFG, IFG, and SFG in Phase H for two polar cases $\xi_1 = \xi$ (left panels) and $\xi_1 = -\xi$ (right panels).

Figure 2.13 shows the dynamics of the interest rates i_t^N and i_t^F in Phase H when $\xi_1 = \xi$ and $\xi_1 = -\xi$. In $t = 1$, the interest rate i_1^F is set closer to zero due to the zero interest-rate forecast, which leads to a higher inflation rate (see the red line in the left panel of Figure 2.14). In $t \geq 2$, the interest rate is set equal to i_t^N, since no zero interest-rate forecast is made in Phase H.

Figure 2.14 shows that inflation π_t^N converges to zero as the supply shock dies out. The dynamics of the output gap show a similar pattern. Further, Figure 2.14 shows that π_t^F is higher than π_t^N in the initial period of Phase H—since $i_1^F < i_1^N$—and then the two coincide.

Under SFG, the central banker is still subject to the zero interest-rate forecast in $t = 1$. This is why he sets interest rates lower than he would set them without any forecast, i.e. $i_1^S < i_1^N$. Note that the nominal interest rate under SFG is even lower than i_1^F when $\xi_1 = \xi$. The reason is that under SFG, the central banker can forecast low inflation π_2^f to lower the inflation expectations. The strong commitment to low future inflation is illustrated by the low π_2^f value at the origin of the blue dashed line in the left panel of Figure 2.14. Lower inflation expectations lead to lower current inflation $\pi_1^S < \pi_1^F$ and output gap $x_1^S < x_1^F$, although $i_1^S < i_1^F$. The former two inequalities suggest that the central banker can balance out the harmful effects of a low interest rate i_1^S. Together with Equation (2.38), the inequalities imply that the central banker will set a lower interest rate, i.e. $i_1^S < i_1^F$. In $t = 2$, due to the low inflation forecast π_2^f made in the previous period, the central banker will set a relatively high interest rate to achieve a low inflation rate π_2^S—see the peak of the blue line in the left panel in Figure 2.13.

In the presence of a negative supply shock, e.g. $\xi_1 = -\xi$, the central banker will set the interest rate i_1^S slightly higher than i_1^F. The inflation forecast π_2^f yields increased inflation expectations $\mathbb{E}_1[\pi_2^S] > \mathbb{E}_1[\pi_2^F]$, which leads to higher current inflation and output gap, i.e. $\pi_1^S > \pi_1^F$ and $x_1^S > x_1^F$. These two inequalities, together with Equation (2.38), imply $i_1^S > i_1^F$.

Comparison to NFG, IFG, and EFG

Compared to NFG, IFG, and EFG, *switching* forward guidance manages expectations in all periods of Phase D and Phase H. We briefly discuss the effect of inflation forecasts on the instantaneous social losses in Phase H presented in Figure 2.15. All four designs have distinct loss functions as functions of ξ_1 in Phase H, $t = 1$, due to the different characteristics of the designs. For illustrative purposes, we set $\pi^c = 0$. For positive and small negative supply shocks, l_1^N is the lower bound for social losses as the discretionary

central banker does not bear the effects of inter-temporal distribution of losses. For large negative shocks inflation forecasts prove to be loss minimizing, since π_2^f helps to increase inflationary expectations and, thus, brings inflation and output gap closer to zero. In
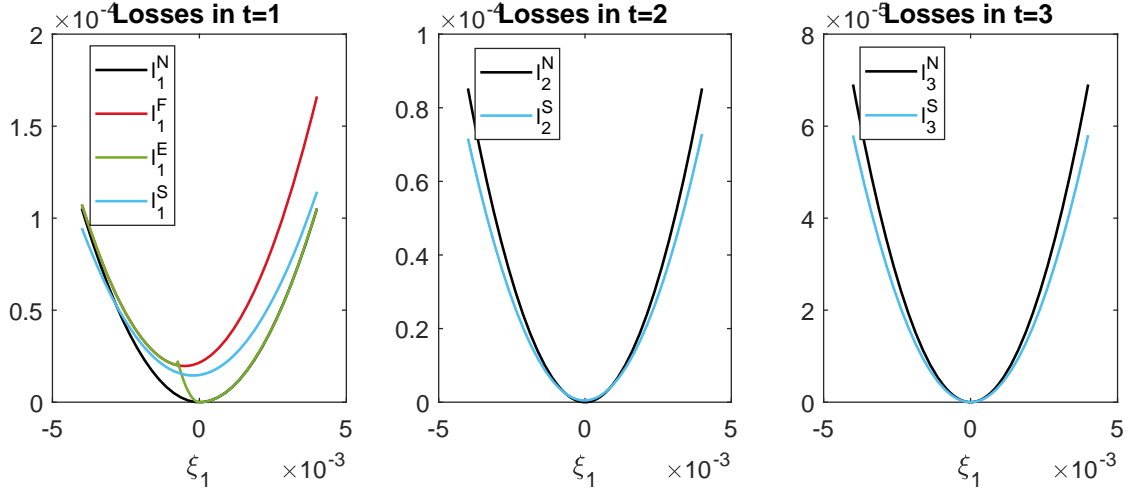
Figure 2.15: Social losses in Phase H, $t = 1, 2, 3$, under NFG, IFG, EFG, and SFG. We set $b_1 = b_2 = 1$ and $\pi^c = 0$.

periods $t \geq 2$, NFG, IFG, and EFG yield the same losses, because all designs return to a discretionary central banker ($l_{t\geq2}^N = l_{t\geq2}^F = l_{t\geq2}^E$). SFG, however, forces the central banker to make inflation forecasts for all future periods. As can be inferred from Figure 2.14, inflation forecasts help to contract $\pi_{t\geq2}^S$ in a closer interval around zero compared to the discretionary case. On the other hand, $x_{t\geq2}^S$ is pushed outwards compared to the discretionary case, but due to the smaller weight in the loss function, this negative effect does not outweigh the gains of lower inflation. In particular, large supply shocks make it socially desirable to forecast inflation.
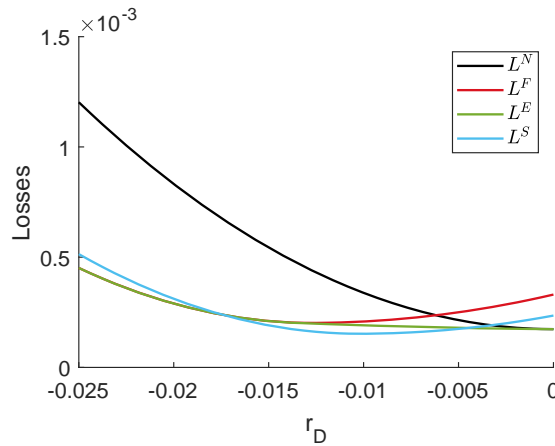
Figure 2.16: Expected social losses under NFG, IFG, EFG, and SFG.

We now compare the expected cumulative social losses under all designs. The losses are plotted in Figure 2.16. One main feature of SFG is that it reduces the expected social losses in Phase H since the central banker can effectively manage the public's inflation expectations. However, the ability to manage inflation expectations weakens the power of the zero interest-rate forecast in the downturn. Thus, we have

Numerical Finding 2.3

Given the parameter values in Table 2.1, switching forward guidance is socially more beneficial than all other forward guidance designs for the natural real interest-rate range $r_D \in (-1.74\%, -0.46\%)$.

The intuition of this finding is as follows: Unlike EFG, the inflation expectation created by SFG in downturns is at a fixed level, as for situations under NFG and IFG (see the black and red lines in Figure 2.8). For small natural real interest-rate shocks, the inflation expectations under SFG are excessively high, while under EFG, the central banker can choose the inflation expectation at the desired levels. For large natural real interest-rate shocks, SFG is not as effective as EFG in raising inflation expectations or as standard forward guidance because it reduces inflation expectations in $t = 1$ through inflation forecasts. Thus, for large and small natural real interest-rate shocks, EFG is preferable to SFG. Nevertheless, when the size of the shock lies within a medium range, the inflation expectation created under SFG is at an adequate level, and the unduly large social losses caused by supply shocks in normal times can be reduced significantly by inflation forecasts. This explains why SFG is socially more beneficial in the medium range of natural real interest-rate shocks.

The Effects of b_1 and b_2

Analogously to parameter $b(= b_1)$, b_2 is a parameter that indicates the central banker's degree of scrupulosity with respect to his inflation forecasts. In a downturn, a higher b_2 leads to a downward shift of π_D^S, x_D^S, and i_D^S. In Phase H, a higher b_2 value drags π_t^S closer to zero and i_t^S to r_H, respectively, at the cost of an output boom or collapse depending on the sign of the supply shock. Increasing b_2 results in a gradually downward shift of L^S up to r_D shocks of around -0.016 as displayed in Figure 2.17. Hence, for not too severe r_D shocks, excessive inflation expectations can be omitted by stronger partial commitment to the inflation forecasts. This helps to effectively reduce deviations from the steady state due to supply shocks in Phase H. For the same reason, a higher b_2 value leads to augmented losses for larger r_D shocks.

Next we examine the relation of IFG and SFG. Figure 2.18 shows the differences in $(L^F - L^S)$ as a function of r_D and b_2, given $b_1 = 1$, 0.5, and 0. For $b_2 = 0$, we have
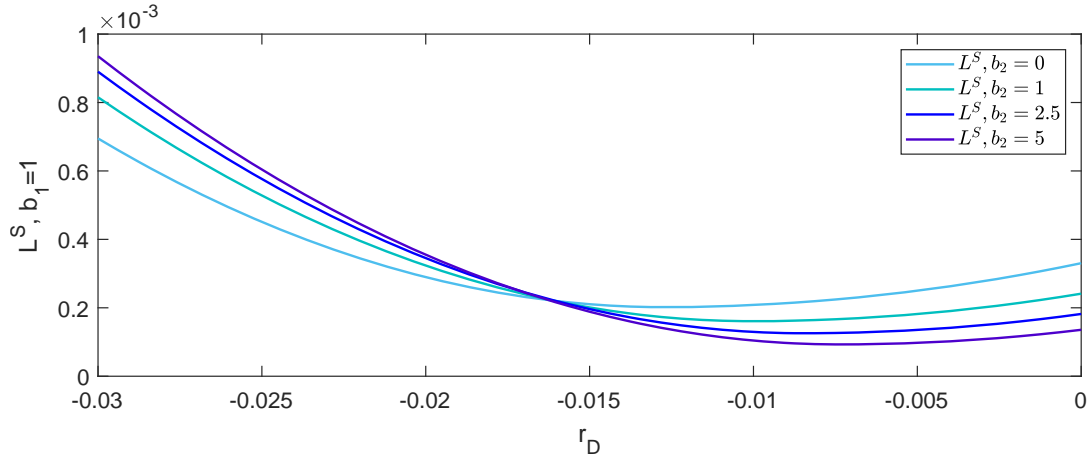
Figure 2.17: Expected social losses under SFG for $b_2 = 0,\ 1,\ 2.5,\ 5$.

$L^F = L^S$ as inflation forecasts have no influence on the central banker's loss function. Thus, the difference at $b_2 = 0$ is always equal to zero. For $b_1 = 0$, we in fact compare L^N to L^S. In the scenarios where $b_1 = 1$ and $b_1 = 0.5$, we see that it is beneficial for all values of b_2 to make inflation forecasts given $r_D > -0.016$ is not too severe. For larger shocks, the introduction of inflation forecasts is socially detrimental for all values of b_2. In the scenario where $b_1 = 0$, however, inflation forecasts are always beneficial. That is because there are no trade-offs between losses in Phase D and losses in Phase H, and the differences in L^N and L^S solely result from Phase H. Inflation forecasts manage to bring π_t closer to the steady state value than the discretionary approach, by influencing expectations, the cumulative expected losses under SFG are consistently lower than under NFG.

Among NFG, IFG, and EFG, the latter forms a lower bound for losses, given π^c is chosen optimally. This result is implied by Figures 2.12 and 2.16. Therefore, it is instructive to compare the two flexible designs EFG and SFG. Figure 2.19 displays graphically the differences $(L^E - L^S)$ as a function of r_D and b_2, given $b_1 = 1$ and 0.5, and $\pi^c = \pi^c_{opt}$. We neglect the case where $b_1 = 0$, as this yields the same graph as on the bottom of Figure 2.18.

In both instances we see that given r_D reaches a critical severity, EFG is superior to SFG for all values of b_2. This is due to the fact that severe r_D shocks lead to high losses in a downturn, relative to expected losses in normal times. Under EFG, the central banker can counteract negative effects of severe r_D shocks more effectively than under SFG. The central banker does that by selecting the inflation threshold such that he never escapes and thus raises inflation expectations as much as possible. For $b_2 > 0$, inflation expectations decrease in a downturn because it is anticipated that the central banker moderates inflation in normal times. In the other extreme case, with r_D close to zero, and b_2 not too large,
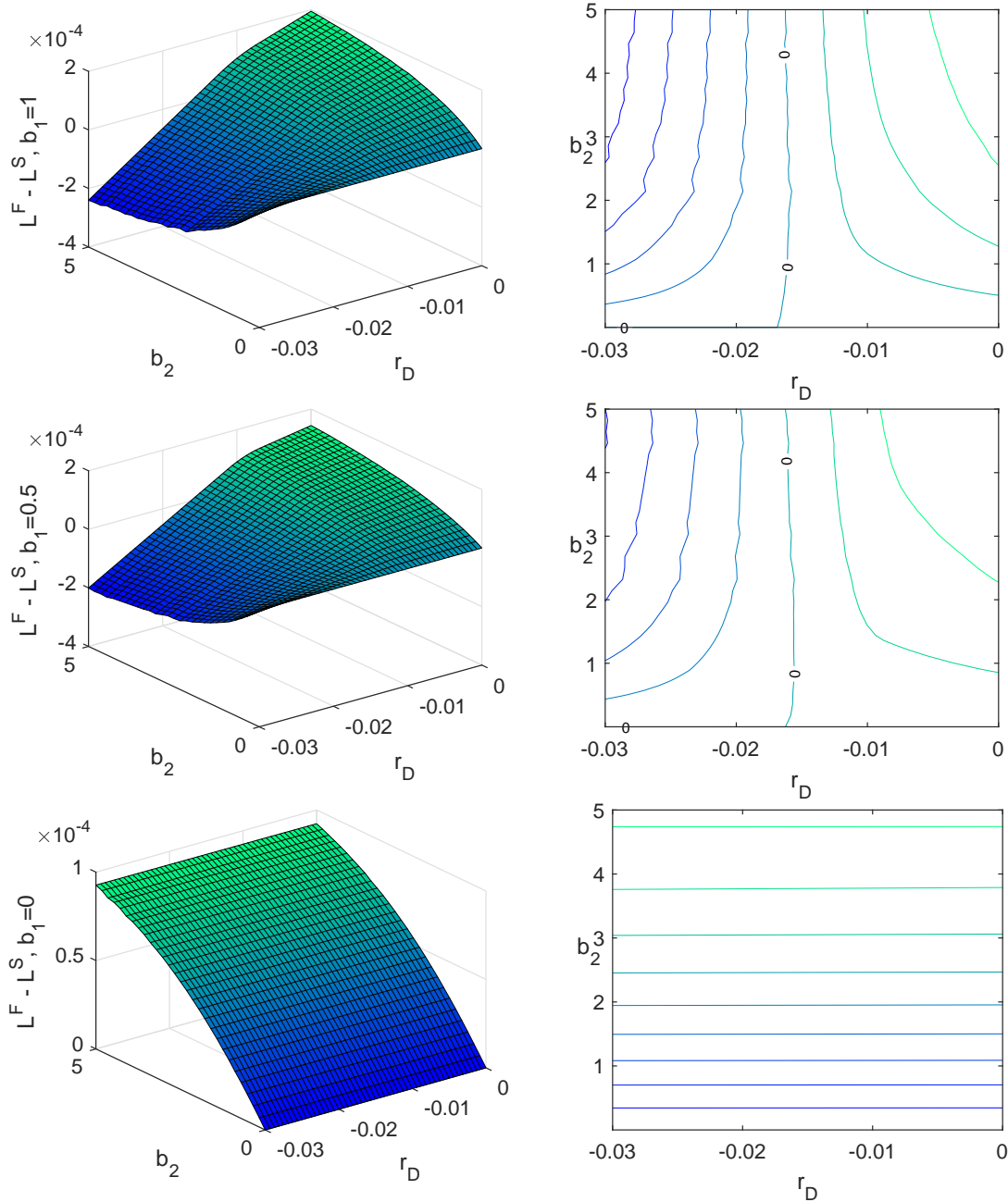
Figure 2.18: Utility gains in applying SFG compared to IFG and NFG.

EFG is superior to SFG as well. With EFG, the central banker can mitigate the zero interest-rate forecasts' negative effects by setting π^c in a way that allows him to escape easily. SFG balances the zero interest-rate forecasts' effects in $t = 1$ by inflation forecasts which only become effective at b_2 values large enough.[22] Hence, for "low r_D"-"high b_2"-combinations, SFG manages to improve, relative to EFG. In a middle range of r_D, SFG

---

[22] Depending on the interpretation of b_1 and b_2, it may be difficult to argue why these values can differ. A central banker with a strong reputation for fighting inflation may be a reason for $b_1 < b_2$.

dominates EFG for all values of b_2. Moreover, the greater b_2 in these ranges, the higher the gains of using inflation forecasts. The shape of the plane shows that the finding in Figure 2.16 is robust to the selection of b_2 and to a selection of $b_1 = 0.5$.
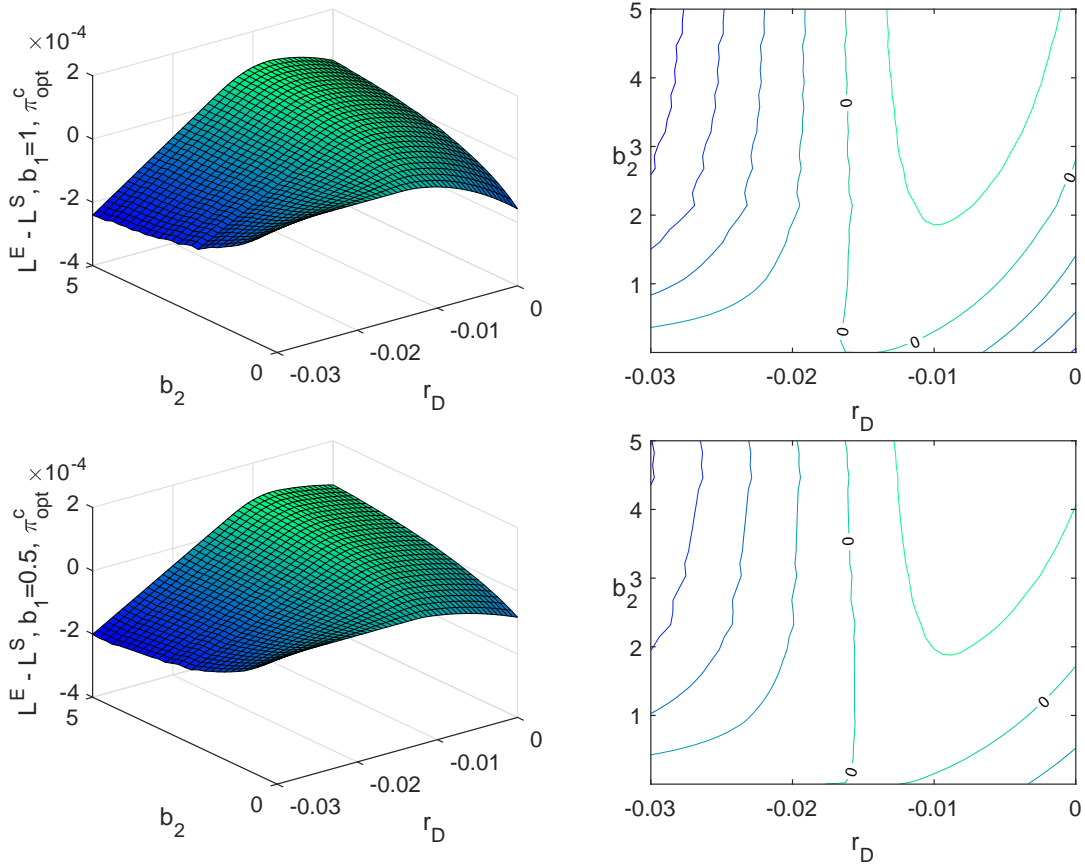
Figure 2.19: Utility gains in applying SFG compared to EFG.

2.5 Conclusions

We studied two promising forward guidance designs in the presence of sequential shocks—a natural real interest-rate shock followed by a supply shock of unknown size. We demonstrated that *escaping* forward guidance is preferable to discretionary monetary policy and standard forward guidance, while *switching* forward guidance further reduces welfare losses for medium-sized natural real interest-rate shocks. In this particular range, *switching* forward guidance is better at balancing the marginal benefits and costs of creating inflation expectations in the downturn and suppressing unduly high social losses (caused by the zero interest-rate forecast) in the presence of the supply shock.

3 Versatile Forward Guidance without Common Beliefs

3.1 Introduction

Major central banks introduced different forms of communication to conduct forward guidance to address the ZLB problem. The Bank of England, for example, relied on a state-contingent forward guidance strategy. In August 2013, it announced that it would keep its policy rate low until the unemployment rate fell below a predefined threshold, except if one criterion out of a set of exit criteria were to be fulfilled. If one of these criteria was fulfilled the Bank of England would be able to raise its rate even if the threshold unemployment rate was not reached yet. The European Central Bank uses an open-ended approach and has announced it would keep interest rates low for an extended period of time. The Federal Reserve used several strategies, starting with an open-ended announcement that it would keep the federal funds rate low, followed by a more precise time-contingent announcement to keep the federal funds rate low at least until a specified date to finally make a state-contingent announcement of the same type as the Bank of England's.

All these strategies aim at informing the economic agents about future behavior of the central bank to spur inflation and stimulate output. Thus expectations play a key role in the concept of forward guidance. Typically, a standard New Keynesian Framework with rational expectations is used to investigate the impact and properties of different forward guidance concepts.

This modeling approach is often criticized, as its results over-estimate the impact of forward guidance. Del Negro et al. (2015) call this over-estimation the "Forward Guidance Puzzle". What is more, rational expectations impose a strong assumption on the model. Phelps and Cagan (1984) analyze this in detail: *"[...] according to the hypothesis, each agent uses the modeler's own model to make the forecasts of the endogenous variables [...]"* (p. 31). In this chapter we address both aspects. We relax the rational expectation assumption in the New Keynesian Framework, which allows us to attenuate the impact of forward guidance. In our model, economic agents have heterogeneous beliefs about the future, based on dispersed information that they receive before forming expectations.

One piece of information stems from the central banker, who sends a signal to the agents. We call it the "public signal". This signal is the central banker's tool for managing expectations. The second piece of information is idiosyncratic to each agent. We call it the "private signal". The central banker then aggregates the agents' beliefs to obtain an average of the economy-wide beliefs about future inflation and output. Based on this aggregate information, the central banker conducts monetary policy. We investigate the welfare effects of heterogeneous beliefs in three different designs of forward guidance, which are introduced and discussed in Chapter 2, i.e. *standard* forward guidance, *escaping* forward guidance, and *switching* forward guidance.

In a broader perspective, this chapter is related to models dealing with informational frictions that lead to a shift in expectations, as in Mankiw and Reis (2002), Lorenzoni (2009) or Angeletos and La'O (2013). We extend their work by the possibility of a change in shock type, when the economy shifts from a demand shock—implying a ZLB constraint—to a supply shock. Coibion and Gorodnichenko (2012) provided empirical evidence that noisy information models are best able to model expectation formation. They showed that mean expectations do not completely adjust to new shocks. Furthermore, Coibion and Gorodnichenko (2015) showed that standard rational expectations adapt towards the direction predicted by models of information rigidities. We build our expectation formation process according to these empirical findings.

We make three further contributions to the literature. First, we derive a New Keynesian Model that relaxes the assumption of rational expectations and introduces heterogeneous beliefs. Andrade et al. (2016), for example, pursued a similar approach. They introduced a model with optimists and pessimists, based on the notion that a central banker's announcement to keep interest rates low can have two effects. It can be perceived as good news, as low interest rates stand for an expansionary future policy, or as bad news, because of a weak future macroeconomic outlook. Andrade et al. (2016) also gave empirical evidence of the existence of different perceptions of an interest-rate announcement based on the survey of professional forecaster. In our model, we do not distinguish between a group of optimists and a group of pessimists, but introduce a parameter that describes in a more general way how optimistic/pessimistic the agents' beliefs are about the economic development. The agents' individual beliefs, however, can have values in a continuous range from pessimistic to optimistic. Wiederholt (2015) introduced information dispersion on the household side of the economy. We enrich his model by additional dispersion on the firm's side as well. Gabaix (2016) used an attention parameter to construct a behavioral New Keynesian Model. The new parameter he introduced *"[...] quantifies how poorly agents understand future policy and its impact"* (Gabaix, 2016, p. 1). His model set-up resulted in a discounted New Keynesian Model, i.e. expectations in the IS Curve

and Phillips Curve are discounted more strongly than in the standard model, which structurally resembles the model we derive in this chapter. How agents form expectations, however, is different in our set-up. Morris and Shin (2002) characterized the approach we use to describe how agents form expectations. They showed that a combination of public and (independent) private information leads to an ambiguous welfare effect if additional public information is disclosed. This mechanism is also used by Angeletos and Lian (2016) in a monetary policy framework.

We also present a model that attenuates the power of forward guidance. Del Negro et al. (2015) found that compared to their empirical findings, standard medium scaled DSGE models *"grossly overestimate"* (p. 52) the effects of forward guidance. They attributed this result to a lack of discounting of future economic outcomes, a result they called Forward Guidance Puzzle. To address this issue they proposed perpetually young households. In every period, a new cohort is born and some old households die with a certain probability. McKay et al. (2016b) presented another solution. They assumed incomplete markets, which allows for uninsurable income risk and borrowing constraints. The rationale is that households might be unable to borrow against future income streams, which makes forward guidance less effective. In a related paper, McKay et al. (2016a) argued that their richer model with heterogeneous agents discussed in McKay et al. (2016b) can be approximated with a representative agent model with a discounted Euler Equation. This means that future income is more strongly discounted compared, to the standard New Keynesian Framework. Gabaix (2016) varied the attention parameter in his model to address the puzzle in *"a natural way"* (p. 2). His model structurally resembles the discounted model in McKay et al. (2016a) and the model we micro-found in this chapter. Angeletos and Lian (2016) and Andrade et al. (2016) used the relaxation of the rational expectation assumption to lessen the Forward Guidance Puzzle. This is the track we will follow as well.

We analyze two flexible forward guidance designs that are easy to understand and yield welfare improving results under rational expectations—i.e. *escaping* forward guidance and *switching* forward guidance. Angeletos and Lian (2016) restricted monetary policy in all periods but one, and the number of periods during which the ZLB is binding is known, i.e. the central banker manages expectations by pegging the interest rate in this single period, which is the first period in which the ZLB is not binding anymore. In our model, the central bank's loss-minimizing behavior is endogenous, and we do not have to impose more explicit assumptions about monetary policy. Andrade et al. (2016) used a central bank that follows a rule inspired by the Taylor Principle, and extended it to a central banker that maximizes social welfare, so that the ability to commit is simply assumed. We use a distinctive central banker loss function that enables the central banker

to partially commit to a certain behavior.

The chapter is structured as follows: In Section 3.2 we set the theoretical basis by providing the IS Curve and Phillips Curve with heterogeneous beliefs, and we describe how agents form their beliefs and how the central banker aggregates the agents' beliefs. In Section 3.3 we explain the different designs of forward guidance. In Section 3.4 we compare these designs to each other in a welfare analysis. Section 3.5 concludes.

3.2 The Model

Next, we develop the equations that represent the economy. In contrast to the standard New Keynesian micro-foundation approach, we neither assume a representative household when the IS Curve is developed nor firms that set the same prices when the Phillips Curve is derived.[1] The central banker aggregates over all individual firms and households to make monetary policy decisions. The notation and the basic set-up follow Chapter 3 of Galí (2008).

3.2.1 Phillips Curve

We start by describing the behavior of a continuum of monopolistically competitive firms, indexed by $j \in [0, 1]$, which maximize the discounted sum of expected future profits. Firms produce output Y_t^j according to the constant return to scales production function

$$Y_t^j = A_t N_t^j, \tag{3.1}$$

with labor input N_t^j and an identical technology A_t, which is given exogenously.

We use a Calvo Pricing Mechanism following the standard in the literature. Thus, each firm resets its price with probability $1 - \alpha$ in every period, no matter when it updated its price the last time. Suppose we are in period t and firm j sets a price P_t^j which remains effective in period $t + k$ with probability α^k. The firm faces the profit-maximization problem

$$\max_{P_t^j} \sum_{k=0}^{\infty} \alpha^k \mathbb{E}_t^j [Q_{t,t+k}(P_t^j Y_{t+k|t}^j - N_{t+k}^j W_{t+k})], \tag{3.2}$$

---

[1] Branch and McGough (2009) and Massaro (2013) introduce similar micro-foundations. In contrast to their contributions, we do not need to specify the proportion of agents who form expectations rationally—we could, however, introduce this feature at a later stage. The largest share of heterogeneous agents in Branch and McGough (2009)'s illustrations is 20% and in Massaro (2013)'s 60%.

subject to the demand constraint[2]

$$Y_{t+k|t}^j = \left(\frac{P_t^j}{P_{t+k}}\right)^{-\omega} Y_{t+k}. \tag{3.3}$$

$Y_{t+k|t}^j$ is the output in period $t+k$ for a firm that last reset its price in period t, and Y_{t+k}^j is the aggregate production in $t+k$. W_t is the nominal wage in period t, P_{t+k} is the aggregate price index in period $t+k$, and $\omega > 1$ is the elasticity of substitution between differentiated goods. $Q_{t,t+k}$ is the stochastic discount factor and is defined as

$$Q_{t,t+k} = \mathbb{E}_t\left[\prod_{m=0}^{k-1} \frac{1}{1+i_{t+m}}\right], \tag{3.4}$$

where i_{t+m} is the nominal interest rate in period $t+m$.

The first-order condition of the firm's profit maximization problem with respect to P_t^j is

$$\sum_{k=0}^{\infty} \alpha^k \mathbb{E}_t^j\left[Q_{t,t+k} Y_{t+k|t}^j\left(P_t^j - \mathcal{M}\frac{W_{t+k}}{A_{t+k}}\right)\right] = 0, \tag{3.5}$$

where $\mathcal{M}$ is the desired mark-up a firm claims in a frictionless economy, i.e. an economy with fully flexible prices.[3] $\frac{W_{t+k}}{A_{t+k}}$ is the nominal marginal cost. Slightly rewriting this first-order condition yields

$$\sum_{k=0}^{\infty} \alpha^k \mathbb{E}_t^j\left[Q_{t,t+k} Y_{t+k|t}^j\left(\frac{P_t^j}{P_{t-1}} - \mathcal{M}\frac{W_{t+k}}{A_{t+k}P_{t+k}}\frac{P_{t+k}}{P_{t-1}}\right)\right] = 0. \tag{3.6}$$

We denote by $MC_{t+k} := \frac{W_{t+k}}{A_{t+k}P_{t+k}}$ the real marginal cost in period $t+k$. As shown in Galí (2008), the first-order Taylor Approximation around the zero inflation steady state yields

$$p_t^j - p_{t-1} = (1-\beta\alpha)\sum_{k=0}^{\infty}(\beta\alpha)^k \mathbb{E}_t^j[\hat{mc}_{t+k} + (p_{t+k} - p_{t-1})]. \tag{3.7}$$

Lower case letters (i.e. p_t and mc_{t+k}) denote log-values of variables in capital letters. β is the discount factor and $\hat{mc}_{t+k}$ denotes the log-deviation of marginal costs from the steady state value. Further rearrangement yields

$$p_t^j - p_{t-1} = (1-\beta\alpha)\sum_{k=0}^{\infty}(\beta\alpha)^k \mathbb{E}_t^j[\hat{mc}_{t+k}] + \sum_{k=0}^{\infty}(\beta\alpha)^k \mathbb{E}_t^j[p_{t+k} - p_{t-1}], \tag{3.8}$$

---

[2] See p. 44 in Galí (2008).
[3] The detailed derivation is provided in Appendix B.1.

$$p_t^j - p_{t-1} = (1 - \beta\alpha)\hat{mc}_t + (p_t - p_{t-1}) \tag{3.9}$$
$$+ (1 - \beta\alpha) \sum_{k=1}^{\infty} (\beta\alpha)^k \mathbb{E}_t^j[\hat{mc}_{t+k}] + \sum_{k=1}^{\infty} (\beta\alpha)^k \mathbb{E}_t^j[p_{t+k} - p_{t-1}].$$

Throughout the paper, we assume that current values are publicly known. Hence, the expectation of the aggregate price level difference $\mathbb{E}_t^j[p_t - p_{t-1}]$ becomes $p_t - p_{t-1}$ in period t, without an individual j-superscript. The same holds for the current marginal costs $\mathbb{E}_t^j[\hat{mc}_t] = \hat{mc}_t$ in period t, which do not depend on the firms' individual expectations. The inflation rate in a particular period π_{t+k} is the log-difference of prices $\pi_{t+k} := p_{t+k} - p_{t+k-1}$. Again, in period t, $\pi_t = p_t - p_{t-1}$ is known and $\mathbb{E}_t^j[\pi_{t+k}]$ is the expectation of firm j in period t. We rewrite Equation (3.9) as

$$p_t^j - p_{t-1} = (1 - \beta\alpha)\hat{mc}_t + \pi_t + (1 - \beta\alpha) \sum_{k=1}^{\infty} (\beta\alpha)^k \mathbb{E}_t^j[\hat{mc}_{t+k}] + \sum_{k=1}^{\infty} (\beta\alpha)^k \mathbb{E}_t^j[\pi_{t+k}]. \tag{3.10}$$

Furthermore, Equation (3.10) can be rewritten in a first-order difference equation form

$$p_t^j - p_{t-1} = (1 - \beta\alpha)\hat{mc}_t + \pi_t +$$
$$\beta\alpha \left[(1 - \beta\alpha) \sum_{k=0}^{\infty} (\beta\alpha)^k \mathbb{E}_t^j[\hat{mc}_{t+1+k}] + \sum_{k=0}^{\infty} (\beta\alpha)^k \mathbb{E}_t^j[\pi_{t+1+k}] \right].$$

The expression in the square brackets on the right hand side of the equation, is structurally equal to the right hand side of Equation (3.8), with a different time subscript. Thus we can write

$$p_t^j - p_{t-1} = (1 - \beta\alpha)\hat{mc}_t + \pi_t + \beta\alpha \mathbb{E}_t^j[p_{t+1}^j - p_t]. \tag{3.11}$$

Aggregating over individual firms by taking the average yields

$$\bar{p}_t - p_{t-1} = (1 - \beta\alpha)\hat{mc}_t + \pi_t + \beta\alpha \overline{\mathbb{E}}_t^F[p_{t+1}^j - p_t], \tag{3.12}$$

where $\bar{p}_t$ is the average of the newly-set prices in period t and $\overline{\mathbb{E}}_t^F[p_{t+1}^j]$ is the average expectation of firms of newly set price in $t + 1$. That is

$$\overline{\mathbb{E}}_t^F[p_{t+1}] = \int_0^1 \mathbb{E}_t^j[p_{t+1}^j]dj. \tag{3.13}$$

Finally, from the relation $\pi_t = (1 - \alpha)(\bar{p}_t - p_{t-1})$, which is an adapted version of the

aggregate price dynamic described by Galí (2008, p. 43)[4], we obtain

$$\frac{\pi_t}{1 - \alpha} = (1 - \beta\alpha)\hat{mc}_t + \pi_t + \beta\alpha\frac{\overline{\mathbb{E}}_t^F[\pi_{t+1}]}{1 - \alpha} \tag{3.16}$$

$$\pi_t = \frac{(1 - \beta\alpha)(1 - \alpha)}{\alpha}\hat{mc}_t + \beta\overline{\mathbb{E}}_t^F[\pi_{t+1}]. \tag{3.17}$$

Along the lines of Galí (2008, p. 48) a relation can be derived between real marginal costs and a measure of economic activity, i.e. aggregate output gap x_t[5]

$$\hat{mc}_t = (\sigma + \phi)x_t, \tag{3.18}$$

where σ is the inverse inter-temporal elasticity of substitution and ϕ is the inverse Frisch Elasticity of labor supply. The two parameters are introduced in the next subsection with the households' utility functions. Therefore, the Phillips Curve that links aggregate output and aggregate inflation in (3.17) becomes

$$\pi_t = \kappa x_t + \beta\overline{\mathbb{E}}_t^F[\pi_{t+1}], \tag{3.19}$$

where $\kappa := \frac{(1-\beta\alpha)(1-\alpha)}{\alpha}(\sigma + \phi)$. We assume that an exogenous supply shock might hit the economy and, therefore, the Phillips Curve we use in our model has the form

$$\pi_t = \kappa x_t + \beta\overline{\mathbb{E}}_t^F[\pi_{t+1}] + \xi_t. \tag{3.20}$$

ξ_t follows an AR(1)-process

$$\xi_t = \rho\xi_{t-1} + \epsilon_t, \tag{3.21}$$

with $\rho \in [0, 1)$ and ϵ_t is i.i.d. with a mean of zero.

---

[4] A share of α firms leaves prices unchanged, and a share $(1 - \alpha)$ changes them. The average new price is $\overline{P}_t$. Therefore, the aggregate price dynamics are

$$P_t = [\alpha P_{t-1}^{1-\omega} + (1 - \alpha)\overline{P}_t^{1-\omega}]^{\frac{1}{1-\omega}}. \tag{3.14}$$

Dividing both sides by P_{t-1} and log-linearizing around the zero inflation steady state yields

$$\pi_t = (1 - \alpha)(\overline{p}_t - p_{t-1}). \tag{3.15}$$

[5] In Appendix B.3 we derive the relation in Equation (3.18).

3.2.2 IS Curve

Next, we derive the IS Curve that describes the households' consumption and labor supply. Household i wants to maximize expected lifetime utility

$$U_i = \mathbb{E}^i\left[\sum_{t=0}^{\infty} \beta^t u(C_t^i, N_t^i)\right] = \mathbb{E}^i\left[\sum_{t=0}^{\infty} \beta^t\left(\frac{(C_t^i)^{1-\sigma}}{1-\sigma} - \frac{(N_t^i)^{1+\phi}}{1+\phi}\right)\right], \qquad (3.22)$$

where C_t^i is consumption, σ is the inverse inter-temporal elasticity of substitution (equivalently the relative risk-aversion coefficient), $0 < \beta < 1$ is the discount factor, N_t^i is the labor supply, and ϕ denotes the inverse Frisch Elasticity of labor supply. The budget constraint is given by

$$P_t C_t^i + B_t^i \leq (1 + i_{t-1})B_{t-1}^i + N_t^i W_t + \int_0^1 Z_t^j dj - P_t T_t, \qquad (3.23)$$

where j is a firm index and Z_t^j are the firm's profits.[6] B_t^i is a one-period bond held by household i in period t, i_t is the nominal interest rate in period t, and T_t is a lump-sum tax. Note that the aggregate price index P_t is the same for every household and, thus, is not indexed by i. The budget constraint in (3.23) comprises three sources of income generation—wages, profits, and bond investments—which are used to pay taxes, investments in new bonds, and to buy a basket of consumption goods. The Lagrangian of the households' optimization problem in time t is

$$\begin{aligned}
\mathcal{L} = \mathbb{E}_t^i\Bigg[\sum_{n=0}^{\infty} \beta^n\Bigg\{& u(C_{t+n}^i, N_{t+n}^i) - \lambda_{t+n}\Bigg(P_{t+n}C_{t+n}^i + B_{t+n}^i \\
& - (1 + i_{t+n-1})B_{t+n-1}^i - N_{t+n}^i W_{t+n} - \int_0^1 Z_{t+n}^j dj + P_{t+n}T_{t+n}\Bigg)\Bigg\}\Bigg].
\end{aligned} \qquad (3.24)$$

Forming first-order conditions of the Lagrangian in (3.24) w.r.t. C_t^i, C_{t+1}^i, N_t^i and B_t^i yields

$$\frac{\partial U_i}{\partial C_t^i} = \beta^t((C_t^i)^{-\sigma} - \lambda_t P_t) = 0, \qquad (3.25)$$

$$\frac{\partial U_i}{\partial C_{t+1}^i} = \mathbb{E}_t^i[\beta^{t+1}((C_{t+1}^i)^{-\sigma} - \lambda_{t+1}P_{t+1})] = 0, \qquad (3.26)$$

$$\frac{\partial U_i}{\partial N_t^i} = \beta^t(-(N_t^i)^{\phi} + \lambda_t W_t) = 0, \qquad (3.27)$$

$$\frac{\partial U_i}{\partial B_t^i} = \lambda_t - \lambda_{t+1}\beta(1 + i_t) = 0. \qquad (3.28)$$

---

[6] The derivation of $P_t C_t^i = \int_0^1 P_t(j)C_t^i(j)dj$ is skipped. See Galí (2008, p. 42), for example.

λ_t is the Lagrange Multiplier associated with the budget constraint in period t.

Combining Equations (3.25), (3.26), and (3.28) yields the Euler Equation

$$\frac{(C_t^i)^{-\sigma}}{P_t} = \beta(1 + i_t)\mathbb{E}_t^i\left[\frac{(C_{t+1}^i)^{-\sigma}}{P_{t+1}}\right]. \tag{3.29}$$

Taking the logarithm and rearranging yields

$$c_t^i = \mathbb{E}_t^i[c_{t+1}^i] - \frac{1}{\sigma}(i_t - \mathbb{E}_t^i[\pi_{t+1}] - \rho), \tag{3.30}$$

where $\rho = -ln(\beta)$, $i_t \approx ln(1 + i_t)$, and $\mathbb{E}_t^i[\pi_{t+1}] = \mathbb{E}_t^i[p_{t+1}] - p_t$ is household i's inflation expectation. Different beliefs represented by $\mathbb{E}_t^i[.]$ can lead to individual consumption paths c_t^i. In the steady state, however, the individual consumption paths are equal. Aggregate consumption is calculated as

$$\int_0^1 c_t^i di = \mathbb{E}_t^i\left[\int_0^1 c_{t+1}^i di\right] - \frac{1}{\sigma}(i_t - \mathbb{E}_t^i[\pi_{t+1}] - \rho), \tag{3.31}$$

$$c_t = \mathbb{E}_t^i[c_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t^i[\pi_{t+1}] - \rho), \tag{3.32}$$

from an individual's perspective. Averaging over the individuals describes the aggregate household behavior

$$c_t = \overline{\mathbb{E}}_t^H[c_{t+1}] - \frac{1}{\sigma}(i_t - \overline{\mathbb{E}}_t^H[\pi_{t+1}] - \rho). \tag{3.33}$$

Again, $\overline{\mathbb{E}}_t^H[\pi_{t+1}]$ is the average expectation of the inflation π_{t+1} expected by households. The goods market clearing condition implies that aggregate output equals aggregate consumption $y_t = c_t$. Therefore, we can write

$$y_t = \overline{\mathbb{E}}_t^H[y_{t+1}] - \frac{1}{\sigma}(i_t - \overline{\mathbb{E}}_t^H[\pi_{t+1}] - \rho). \tag{3.34}$$

We next define the natural real interest rate, i.e. the interest rate accompanied by zero inflation, as

$$r_t = \sigma(\mathbb{E}_t[y_{t+1}^p] - y_t^p) + \rho, \tag{3.35}$$

where y_t^p is the potential output, i.e. the output reached with fully flexible prices.[7] Equation (3.35) implies $r_t - \sigma(\mathbb{E}_t[y_{t+1}^p] - y_t^p) = \rho$. This equation also holds for heterogeneous beliefs about future potential output $\overline{\mathbb{E}}_t^H[y_{t+1}^p]$. The important assumption, however, is that expected inflation is zero, $\overline{\mathbb{E}}_t^H[\pi_{t+1}] = 0$. Plugging the expression for ρ into Equation

---

[7] The derivation of y_t^p is provided in Appendix B.2.

(3.34) allows us to rewrite the IS Curve in terms of the (log) output gap $y_t - y_t^p$,

$$x_t = \overline{\mathbb{E}}_t^H[x_{t+1}] - \frac{1}{\sigma}(i_t - \overline{\mathbb{E}}_t^H[\pi_{t+1}] - r_t). \tag{3.36}$$

3.2.3 Sequence of Events

Before we specify how the central banker conducts monetary policy, we have to clarify the sequence of events. Following Eggertsson (2003), our economy starts in a downturn, which means that the natural real interest rate is negative, $r_D < 0$. In every period in the downturn (Phase D), the economy returns to normal times (Phase H), with a positive natural real interest rate $r_H > 0$, with probability $(1 - \delta)$, and stays in Phase D with probability $0 < \delta < 1$. After returning to normal times, a supply shock $\xi_1 \in [-\xi, \xi]$ may manifest itself.

In Phase D, the central banker observes $r_D < 0$ and decides whether he will publish an interest-rate forecast $i^f = 0$ or not. If he makes a forecast, four subsequent sub-steps follow, as displayed in Figure 3.1.

i) First, by publishing $i^f = 0$, the central banker announces he will keep interest rates low in the next period—i.e. sends a public signal to allow an augmented level of inflation and output gap in the first period after the economy has returned to normal times. In fact the central banker explicitly publishes the inflation and output under rational expectations associated with $i^f = 0$.

ii) The agents receive the public signal $i^f = 0$ and the associated rational expectations, and additionally observe individual private signals about inflation and the output gap. The private signals are only observed by the agent who receives them.

iii) Each agent forms an individual expectation about future inflation and output gap, based on the combination of public and private signals.

iv) The central banker then averages over individual expectations of inflation and output gap.

These four steps show three key points of the model. *i)* implies that the central banker can influence expectations with announcements, *ii)* and *iii)* describe why expectations about inflation and output gap differ among individuals, and *iv)* means that the central banker is aware of the fact that households have different expectations and describes how he aggregates information.

Once the central banker has an estimate of the agents' inflation and output-gap expectations, he sets his policy instrument i_t optimally. With probability δ, the economy stays in

the downturn and steps *i)-iv)* are repeated. With probability $(1 - \delta)$, the economy enters Phase H and possibly faces a supply shock. Depending on the forward guidance design, the central banker behaves discretionarily from then on or sends a new public signal along the schedule outlined in steps *i)-iv)*.

If the central banker does not make any announcement, expectations form without dispersed beliefs. This corresponds to the standard New Keynesian Framework with a single representative household and firms which set the same price when they re-optimize their prices. The beliefs only start to differ once the central banker starts providing additional information to the economic agents.

Phase D $(r_D < 0)$ **Phase H** $(r_H > 0)$
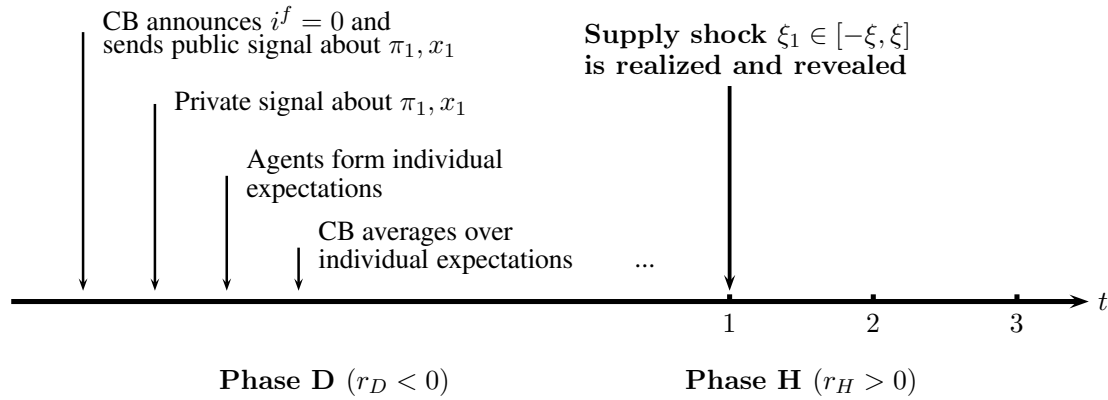
Figure 3.1: Sequence of events.

3.2.4 Monetary Policy

Monetary policy is conducted by a scrupulous central banker who minimizes his personal loss function by setting the policy instrument—the nominal interest rates—appropriately. The central banker's loss function consists of two parts. First, it encompasses the instantaneous social loss function

$$l_t = \frac{1}{2} \left(\pi_t^2 + \lambda x_t^2 \right), \tag{3.37}$$

where $\lambda > 0$. Second, an additional term measures the central banker's scrupulosity

$$\frac{1}{2} b (q_t - q_t^f)^2, \tag{3.38}$$

where q represents the variable about which the central banker makes a forecast, i.e. q^f. The parameter b measures the intrinsic costs the central banker incurs when he deviates from his forecast, relative to the social losses. b thus stands for the central banker's scrupulosity. In the polar case of $b = 0$ the central banker acts fully discretionarily and in the case of $b = \infty$, the central banker is fully committed to his forecast. Adding the two parts

yields the central banker's instantaneous loss function

$$\tilde{l}_t = \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right) + \frac{1}{2}b(q_t - q_t^f)^2. \tag{3.39}$$

When the central banker abstains from forecasting, the additional term $\frac{1}{2}b(q_t - q_t^f)^2$ falls off, and the social loss coincides with the central banker's loss. By his forecasts, the central banker shifts losses from today to the future. Hence, to obtain a complete picture of the effects of forward guidance we not only have to consider the instantaneous loss, but also future losses as well. Therefore, we construct the expected cumulative losses in a downturn. First, in any period of a downturn the loss is given by

$$\tilde{l}_D = \frac{1}{2}(\pi_D^2 + \lambda x_D^2) + \frac{1}{2}b(q_D - q_D^f)^2,$$

because from a central banker's perspective—but also from the perspective of all other agents in the economy—, every period poses the same optimization problem in a downturn. Thus, π_D, x_D, and the losses are equal in every period and can be marked by the same D-subscript. As the economy returns to normal, a supply shock might hit the economy. This shock decays according to an AR(1)-process and, therefore, losses in each period of Phase H vary. We distinguish periods by a time subscript $t = 1, 2, 3, \ldots$. Furthermore, depending on the forward guidance design chosen by the central banker, losses in the first period upon return vary as well. The following equation represents the cumulative central bank's losses in Phase H:

$$\tilde{l}_H = \frac{1}{2}\sum_{t=1}^{\infty}\beta^{t-1}\int_{-\rho^{t-1}\xi}^{\rho^{t-1}\xi}(\pi_t^2 + \lambda x_t^2 + b(q_t - q_t^f)^2)ds. \tag{3.40}$$

The exact form of the specific loss functions $\tilde{l}_D$ and $\tilde{l}_H$ will be developed later, when we discuss each forward guidance design separately.

Since in the downturn, there is a certain probability δ in each period that the economy stays in this downturn and a future loss is discounted by β, the expected cumulative central banker loss in a particular period in the downturn is

$$\tilde{L} = \tilde{l}_D + \beta(\delta\tilde{l}_D + (1-\delta)\tilde{l}_H)\sum_{t=0}^{\infty}\delta^t\beta^t \tag{3.41}$$

$$= \tilde{l}_D\sum_{t=0}^{\infty}\delta^t\beta^t + \beta(1-\delta)\tilde{l}_H\sum_{t=0}^{\infty}\delta^t\beta^t \tag{3.42}$$

$$= \frac{\tilde{l}_D + \beta(1-\delta)\tilde{l}_H}{1 - \beta\delta}. \tag{3.43}$$

Analogously, the expected cumulative social loss in a particular period in the downturn is

$$L = \frac{l_D + \beta(1-\delta)l_H}{1 - \beta\delta}. \tag{3.44}$$

3.2.5 Expectation Formation

Let us describe how information is spread within the model in more detail and, therefore how expectations are formed. The framework we use is based on Morris and Shin (2002) and DeGroot (2004).[8] As a first action, the central banker publishes the interest-rate forecast $i^f = 0$. With this forecast, he signals he will keep the interest rate i_1 below what is optimal, once the economy is out of the downturn. Low interest rates imply inflation π_1 and an output boom x_1 in the first period after the downturn. There is, however, uncertainty about π_1 and x_1 because these are future values which will ultimately depend on the size of the supply shock. The central banker makes assumptions about π_1 and x_1, of the following form:

$$\pi_1 \sim N(\pi_1^{RE}, \sigma_{\eta_\pi}^2), \tag{3.45}$$

$$x_1 \sim N(x_1^{RE}, \sigma_{\eta_x}^2), \tag{3.46}$$

where π_1^{RE} and x_1^{RE} are the expected values under rational expectations given $i^f = 0$. We denote rational expectations values by an RE-superscript. By publishing $i^f = 0$, the central banker intends to augment inflation and output-gap expectations to π_1^{RE} and x_1^{RE}, which he communicates to the public. The values π_1^{RE} and x_1^{RE} are publicly known. Additionally, each agent observes private signals about inflation and the output gap. We assume the private signals are

$$\pi_1^i = \tau_\pi \pi_1 + \epsilon_{\pi,i} \quad \text{where } \epsilon_{\pi,i} \sim N(0, \sigma_{\epsilon_\pi}^2), \tag{3.47}$$

$$x_1^i = \tau_x x_1 + \epsilon_{x,i} \quad \text{where } \epsilon_{x,i} \sim N(0, \sigma_{\epsilon_x}^2). \tag{3.48}$$

Each agent i observes π_1^{RE}, x_1^{RE}, π_1^i, and x_1^i. Yet, note that π_1 and x_1 are unknown. Parameters τ_π and τ_x can be interpreted in several ways.[9] We use the intuitive notion that τ_π and τ_x are parameters that measure the pessimism that is present in the economy. More explicitly, pessimism means that $\tau_\pi, \tau_x \in [0, 1)$ in Phase D, and $\tau_{H,\pi}, \tau_{H,x} \in (1, 1.1]$

---

[8] Other expectation-formation processes can be assumed. Examples are the mechanisms proposed by Carroll (2003) and Branch (2004). Carroll (2003) proposes a weighted average of a current rational forecast and last periods expectation and Branch (2004) constructs three methods with varying degrees of sophistication.

[9] Loosely speaking, τ_π and τ_x around 1 can be interpreted as an inertia parameter. This means that in Phase D, a value $\tau_\pi, \tau_x < 1$ anchors expectations. In Phase H, $\tau_{H,\pi}, \tau_{H,x} > 1$ augments expectations in the direction of the current values.

in Phase H in our model. A more technical description on the features of τ_π and τ_x is provided in the discussion of Equations (3.51) and (3.52).

In Bayesian terms, the public signal forms the prior and the private signal is the likelihood function. Therefore, after observing the prior and the likelihood, agents form the individual posterior beliefs which have the following mean values (see DeGroot (2004)):

$$\mathbb{E}_D^i[\pi_1] = \frac{\sigma_{\epsilon_\pi}^2 \pi_1^{RE} + \sigma_{\eta_\pi}^2 \pi_1^i}{\sigma_{\epsilon_\pi}^2 + \sigma_{\eta_\pi}^2}, \tag{3.49}$$

$$\mathbb{E}_D^i[x_1] = \frac{\sigma_{\epsilon_x}^2 x_1^{RE} + \sigma_{\eta_x}^2 x_1^i}{\sigma_{\epsilon_x}^2 + \sigma_{\eta_x}^2}. \tag{3.50}$$

In a next step, the central banker averages over all individual expectations, which we call "posterior expectations":

$$\overline{\mathbb{E}}_D^{CB}[\pi_1] := \int_0^1 \mathbb{E}_D^i[\pi_1]di = \frac{\sigma_{\epsilon_\pi}^2 \pi_1^{RE} + \sigma_{\eta_\pi}^2 \tau_\pi \mathbb{E}_D^{CB}[\pi_1]}{\sigma_{\epsilon_\pi}^2 + \sigma_{\eta_\pi}^2} = \frac{\sigma_{\epsilon_\pi}^2 + \tau_\pi \sigma_{\eta_\pi}^2}{\sigma_{\epsilon_\pi}^2 + \sigma_{\eta_\pi}^2} \pi_1^{RE}, \tag{3.51}$$

$$\overline{\mathbb{E}}_D^{CB}[x_1] := \int_0^1 \mathbb{E}_D^i[x_1]di = \frac{\sigma_{\epsilon_x}^2 x_1^{RE} + \sigma_{\eta_x}^2 \tau_x \mathbb{E}_D^{CB}[x_1]}{\sigma_{\epsilon_x}^2 + \sigma_{\eta_x}^2} = \frac{\sigma_{\epsilon_x}^2 + \tau_x \sigma_{\eta_x}^2}{\sigma_{\epsilon_x}^2 + \sigma_{\eta_x}^2} x_1^{RE}. \tag{3.52}$$

The CB-superscript denotes that the central banker forms expectations. Note that when the central banker aggregates, he sets $\mathbb{E}_D^{CB}[\pi_1] = \pi_1^{RE}$ and $\mathbb{E}_D^{CB}[x_1] = x_1^{RE}$. This is a simplifying assumption.

If agents expect that the central banker takes their expectations into account, they will adapt their expectations accordingly. In the limit, these higher-order expectations lead to an expectation of $c_\pi(\tau_\pi)\pi_1^{RE}$, $c_x(\tau_x)x_1^{RE}$, respectively. $c(\tau)$ is a function of τ, for which we obtain $0 \leq c(\tau_i) \leq 1$ for $\tau \leq 1$ and $c(\tau) > 1$ for $1 < \tau \leq 1.1$. The derivation is outlined in Appendix B.4.

Equations (3.51) and (3.52) show that for $\tau_\pi \in [0, 1)$ and $\tau_x \in [0, 1)$, an increase in $\sigma_{\eta_\pi}^2$ or $\sigma_{\eta_x}^2$ decreases the respective term before the rational expectation variable, whereby a ceteris paribus increase in $\sigma_{\epsilon_\pi}^2$ or $\sigma_{\epsilon_x}^2$ increases the terms before π_1^{RE} and x_1^{RE}, respectively. For $\tau_{H,\pi}, \tau_{H,x} > 1$, an increase in $\sigma_{\eta_\pi}^2$ or $\sigma_{\eta_x}^2$ increases, and an increase in $\sigma_{\epsilon_\pi}^2$ or $\sigma_{\epsilon_x}^2$ decreases the terms. To sum up, an increase in uncertainty of the public signal ($\sigma_\eta \uparrow$) results in a reduced ability to guide expectations in the desired direction for the central banker. Higher uncertainty in the private signal ($\sigma_\epsilon \uparrow$) increases the power of the central banker's guidance capabilities relative to the private information. Furthermore, if $\sigma_{\eta_\pi}^2$ or $\sigma_{\eta_x}^2$ dominate, i.e. if $\sigma_\eta^2 \gg \sigma_\epsilon^2$, the expectation $\overline{\mathbb{E}}_D^{CB}[\pi_1]$ ($\overline{\mathbb{E}}_D^{CB}[x_1]$) converges to $\tau_\pi \pi_1^{RE}$ ($\tau_x x_1^{RE}$). In the limiting case, when the private signal is received without any noise, i.e. $\sigma_\epsilon^2 = 0$, the expectations reduce to $\overline{\mathbb{E}}_D^{CB}[\pi_1] = \tau_\pi \pi_1^{RE}$ and $\overline{\mathbb{E}}_D^{CB}[x_1] = \tau_x x_1^{RE}$. When $\sigma_\epsilon^2 \gg \sigma_\eta^2$ the expectation $\overline{\mathbb{E}}_D^{CB}[\pi_1]$ ($\overline{\mathbb{E}}_D^{CB}[x_1]$) converges to π_1^{RE} (x_1^{RE}). Thus, if $\sigma_\eta^2 = 0$

then expectations collapse to $\overline{\overline{\mathbb{E}}}_D^{CB}[\pi_1] = \pi_1^{RE}$ and $\overline{\overline{\mathbb{E}}}_D^{CB}[x_1] = x_1^{RE}$.

The aggregated inflation expectation in (3.51) and the aggregated output-gap expectation in (3.52) are used by the central banker when he selects the optimal interest rate.

3.3 Monetary Policy Models

In this section we include the expectation formation process that we described in Subsection 3.2.5 into a New Keynesian Framework. Heterogeneous beliefs address the feature mentioned by Phelps and Cagan (1984), i.e. that economic agents do not need to know the exact model of the economy and do not have to agree on the dynamics of the economy. We present the results with numerical illustrations. The parameterization of the models is given in Table 3.1 and follows Chapter 2. Compared to the standard New Keynesian

$\beta = 0.99$	Discount factor
$\lambda = 0.003$	Weight of output gap in social loss function
$\kappa = 0.024$	Slope of Phillips Curve
$\sigma = 0.16$	Inverse inter-temporal elasticity of substitution
$r_H = 0.02$	Natural real interest rate in Phase H
$r_D \in (-\infty, 0)$	Natural real interest rate in Phase D
$\xi_1 \in [-0.004, 0.004]$	Supply shock in Phase H, $t = 1$
$\delta = 0.5$	Probability of being trapped in Phase D
$\rho = 0.9$	Persistence of the supply shock
$b = 1$	Scrupulous central banker's intrinsic weight on his forecast
τ_π	Measure of pessimism for private inflation expectations
τ_x	Measure of pessimism for private output-gap expectations
$\sigma_{\eta_\pi} = 0.0151$	Standard deviation of public signal about π
$\sigma_{\eta_x} = 0.0314$	Standard deviation of public signal about x
$\sigma_{\epsilon_\pi} = 0.0151$	Standard deviation of private signal about π
$\sigma_{\epsilon_x} = 0.0314$	Standard deviation of private signal about x

Table 3.1: Quarterly parameter values used in the calibration.

Framework, we introduce six new parameters. First, we use the pessimism parameters τ_π and τ_x which, for simplicity, are set at $\tau_\pi = \tau_x = \tau$ in Phase D and at $\tau_{H,\pi} = \tau_{H,x} = \tau_H$ in Phase H. Pessimism means that we set $0 < \tau < 1$ and $1 < \tau_H < 1.1$. Intuitively, values that are smaller than 1 lead to a decrease in average expectations and values larger than 1 to an increases in average expectations. Thus, if inflation is low today, agents will think that inflation will tend to be lower tomorrow and, vice versa: if it is high today it will be higher tomorrow.

Furthermore, four values that describe the uncertainty about different signals have to be specified. In the next subsection we show that given standard forward guidance with rational expectations and supply shocks of sizes $\xi_1 = -0.004$, 0, and 0.004, the realized inflation rates in the first period of Phase H are -0.0125, 0.0026, and 0.0177. Output gaps in the same period for the same supply shocks are 0.1477, 0.113, and 0.0783, respectively. We choose $\sigma_{\eta_\pi} = \sigma_{\epsilon_\pi}$ in such a way that the difference between realized inflation for $\xi_1 = 0$ and $\xi_1 = -0.004$ (and 0.004) is one standard deviation. In the same way we select the standard deviation $\sigma_{\eta_x} = \sigma_{\epsilon_x}$ for the output gap. Variances are used to weight the public and the private signal. Therefore, the relative size is important and not the absolute size. By setting $\sigma_{\eta_\pi} = \sigma_{\epsilon_\pi}$ and $\sigma_{\eta_x} = \sigma_{\epsilon_x}$ we deliberately push the expected value neither to rational expectations nor to the average private signal.

In the remainder of this section, we derive the dynamics of the variables of interest. We limit ourselves to sketching the derivations, since a more detailed version can be found in Chapter 2.

3.3.1 Standard Forward Guidance

First, we turn to the derivation of the dynamics of inflation, π^F, output gap, x^F, and the interest rate, i^F, when the central banker makes interest-rate forecasts in a downturn. We introduce an F-superscript to indicate that the central banker does make forecasts. Because the central banker does not announce any forecasts in Phase H, the dynamics in periods $t \geq 2$ are given by the discretionary dynamics:

$$i_t^N = r_H + \frac{\sigma\kappa(1-\rho) + \lambda\rho}{\lambda(1-\beta\rho) + \kappa^2}\rho^{t-1}\xi_1, \tag{3.53}$$

$$\pi_t^N = \frac{\lambda}{\lambda(1-\beta\rho) + \kappa^2}\rho^{t-1}\xi_1, \tag{3.54}$$

$$x_t^N = -\frac{\kappa}{\lambda(1-\beta\rho) + \kappa^2}\rho^{t-1}\xi_1. \tag{3.55}$$

The dynamics are derived in Appendix A. The N-superscript stands for "no forecast". In the first period of Phase H, the central banker incurs losses which arise from the forecast $i^f = 0$ published in the previous period. Therefore, the central banker minimizes

$$\tilde{l}_1^F = \frac{1}{2}[(\pi_1^F)^2 + \lambda(x_1^F)^2 + b(i_1^F)^2], \tag{3.56}$$

such that the IS Curve and Phillips Curve hold.

This yields the dynamics

$$i_1^F = \frac{\lambda + \kappa^2}{\lambda + \kappa^2 + b\sigma^2} r_H + g_1(b)\xi_1, \tag{3.57}$$

$$\pi_1^F = \frac{b\kappa\sigma}{\lambda + \kappa^2 + b\sigma^2} r_H + g_2(b)\xi_1, \tag{3.58}$$

$$x_1^F = \frac{b\sigma}{\lambda + \kappa^2 + b\sigma^2} r_H + g_3(b)\xi_1, \tag{3.59}$$

where

$$g_1(b) = \frac{(\kappa^2 + \lambda)[\sigma\kappa(1-\rho) + \rho\lambda]}{(\lambda + \kappa^2 + b\sigma^2)(\lambda(1-\beta\rho) + \kappa^2)},$$

$$g_2(b) = \frac{b\kappa^2\sigma^2(1-\rho) + b\sigma\lambda(\sigma + \kappa\rho) + \lambda(\kappa^2 + \lambda)}{(\lambda + \kappa^2 + b\sigma^2)(\lambda(1-\beta\rho) + \kappa^2)},$$

$$g_3(b) = \frac{b\sigma\rho(\lambda - \sigma\kappa) - \kappa(\kappa^2 + \lambda)}{(\lambda + \kappa^2 + b\sigma^2)(\lambda(1-\beta\rho) + \kappa^2)}.$$

For $b = 0$, we obtain the dynamics in $t = 1$ if no forecast was made in the downturn. In the downturn, the central banker has to form expectations about future values. With probability δ, the downturn value will realize again, and with probability $(1 - \delta)$, the economy will then be out of the downturn. In Subsection 3.2.5, and specifically in Equations (3.51) and (3.52), we described how the central banker forms expectations. Thus, we have the central bank's expectations

$$\mathbb{E}_D^{F,CB}[\pi_{t+1}] = \delta\pi_D^F + (1-\delta)\overline{\mathbb{E}}_D^{CB}[\pi_1^F] = \delta\pi_D^F + (1-\delta)\frac{\sigma_{\epsilon_\pi}^2 + \tau_\pi\sigma_{\eta_\pi}^2}{\sigma_{\epsilon_\pi}^2 + \sigma_{\eta_\pi}^2}\pi_1^{F,RE} \tag{3.60}$$

and

$$\mathbb{E}_D^{F,CB}[x_{t+1}] = \delta x_D^F + (1-\delta)\overline{\mathbb{E}}_D^{CB}[x_1^F] = \delta x_D^F + (1-\delta)\frac{\sigma_{\epsilon_x}^2 + \tau_x\sigma_{\eta_x}^2}{\sigma_{\epsilon_x}^2 + \sigma_{\eta_x}^2}x_1^{F,RE}. \tag{3.61}$$

Finally, using the first-order condition of the central banker loss function in a downturn, $\tilde{l}_D^F = \frac{1}{2}((\pi_D^F)^2 + \lambda(x_D^F)^2) + \frac{1}{2}b(i_D^F)^2$, with respect to the interest rate i_D^F

$$\frac{\partial \tilde{l}_D^F}{\partial i_D^F} = \kappa\pi_D^F + \lambda x_D^F - b\sigma i_D^F = 0, \tag{3.62}$$

in combination with the IS Curve and Phillips Curve, yields

$$\pi_D^F = \frac{\sigma\kappa(1-\delta)\theta_x x_1^{F,RE} + (1-\delta)(\kappa + \sigma\beta - \sigma\beta\delta)\theta_\pi \pi_1^{F,RE} + \kappa(r_D - i_D^F)}{h}, \quad (3.63)$$

$$x_D^F = \frac{\sigma(1-\delta)(1-\beta\delta)\theta_x x_1^{F,RE} + (1-\delta)\theta_\pi \pi_1^{F,RE} + (1-\beta\delta)(r_D - i_D^F)}{h}, \quad (3.64)$$

$$i_D^F = \max\left\{0, \frac{\kappa\pi_D^F + \lambda x_D^F}{b\sigma}\right\}, \quad (3.65)$$

and we define $\theta_\pi := \frac{\sigma_{\epsilon_\pi}^2 + \tau_\pi \sigma_{\eta_\pi}^2}{\sigma_{\epsilon_\pi}^2 + \sigma_{\eta_\pi}^2}$, $\theta_x := \frac{\sigma_{\epsilon_x}^2 + \tau_x \sigma_{\eta_x}^2}{\sigma_{\epsilon_x}^2 + \sigma_{\eta_x}^2}$, and

$$h := \sigma(1-\delta)(1-\beta\delta) - \kappa\delta > 0. \quad (3.66)$$

If the central banker behaves discretionarily the ZLB is always binding, and the dynamics of inflation and output in the downturn reduce to

$$\pi_D^N = \frac{\kappa}{h} r_D < 0 \quad (3.67)$$

and

$$x_D^N = \frac{1-\beta\delta}{h} r_D < 0. \quad (3.68)$$

Introducing disperse beliefs lowers the central banker's power to increase π_D^F and x_D^F due to decreased expectations about the intended inflation and output boom in the first period of Phase H. Figure 3.2 shows the downward shift of the inflation (left graph) and output (right graph) for different values of the pessimism parameter τ. For illustrative purposes, we plot inflation and output for $\tau = 1$—which implies $\theta_\pi = \theta_x = 1$ and, hence, the rational expectation case—, $\tau = 0.9$, and $\tau = 0.1$. The same decrease in τ shifts the curve by the same amount, independent of the absolute size of τ. Figure 3.2 shows that a forecast $i^f = 0$ can create inflation and an output boom in a downturn if the r_D-shock is not too severe. This is due to the expected expansionary policy in Phase H, $t = 1$.

Note that even for a τ value of zero—which is equivalent to stating that the average private inflation and output expectations are equal to the discretionary expectations of $\mathbb{E}_D[\pi_1^N] = \mathbb{E}_D[x_1^N] = 0$—and despite dispersed beliefs, the central banker's signal leads to a noticeable upward shift of π_D^F and x_D^F. To reach $\pi_D^F = \pi_D^N$ and $x_D^F = x_D^N$, one has to set the pessimism parameter at $\tau = -1$. Hence, the central banker's signal and the private signal have to be exactly diametric to each other.

Less optimism about future inflation and output boom leads to a ZLB problem at smaller r_D shocks. This is displayed in Figure 3.3.
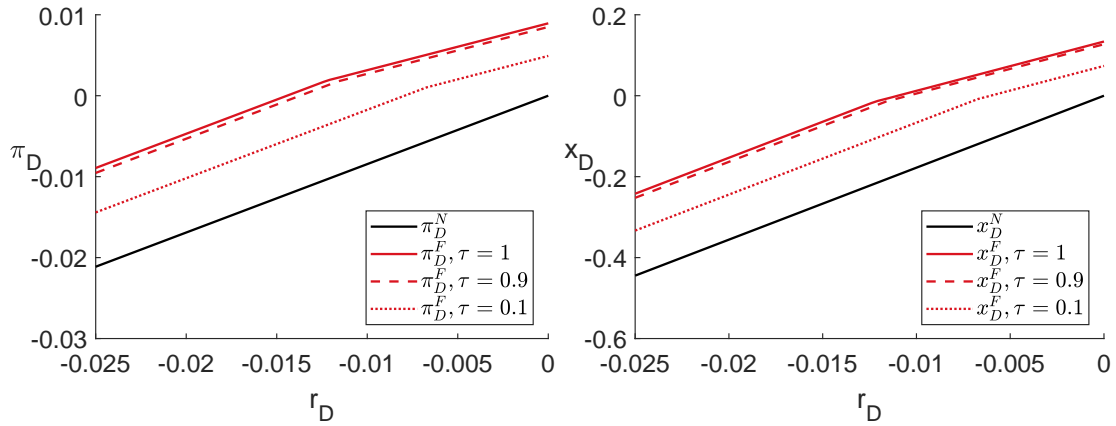
Figure 3.2: Inflation and output gap under standard forward guidance in Phase D.
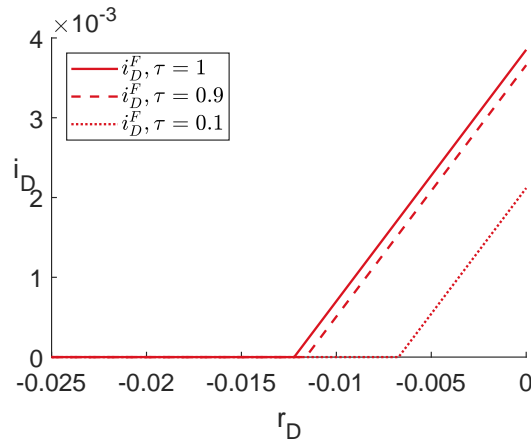
Figure 3.3: Interest rate under standard forward guidance in Phase D.

3.3.2 Escaping Forward Guidance

Now, we investigate the dynamics of π^E, x^E, and i^E under escaping forward guidance. We use an E-superscript to indicate escaping. As under standard forward guidance, dynamics in Phase H are not affected by disperse beliefs because the central banker returns to discretionary behavior once the economy is out of the downturn.[10]

The economy starts in a downturn and the central banker can choose between publishing an interest-rate forecast or not. If he makes a forecast, he specifies a threshold inflation rate π^c at the same time, which allows him to discard the forecast and engage in a discretionary policy, setting in $t = 1$ once the threshold is surpassed. Hence, in $t = 1$, the central banker's loss function is

$$\tilde{l}_1^E = \frac{1}{2}[(\pi_1^E)^2 + \lambda(x_1^E)^2 + b(i_1^E)^2], \tag{3.69}$$

---

[10] For a more detailed introduction we refer to Chapter 2.

if $\pi_1^E < \pi^c$, and

$$\tilde{l}_1^E = \frac{1}{2}[(\pi_1^E)^2 + \lambda(x_1^E)^2], \tag{3.70}$$

otherwise.

If the realized inflation in the first period of Phase H is below the critical threshold π^c, the central banker is still subject to the zero interest-rate forecast and thus bears the additional loss $b(i_1^E)^2$. Otherwise, the central banker can discard the zero interest-rate forecast made in the previous period without incurring any deviation costs.

It is helpful to introduce some auxiliary variables at this point. First, we derive the output gap at the critical inflation. We set $\pi^c = \pi_1^E$ and use the Phillips Curve to obtain[11]

$$x^c = \frac{1}{\kappa}(\pi^c - \beta\mathbb{E}_1[\pi_2^E] - \xi_1). \tag{3.71}$$

Note that $\mathbb{E}_1[\pi_2^E] = \mathbb{E}_1[\pi_2^N]$, as the central banker does not make any forecasts in Phase H. Thus, inserting the dynamics in Equation (3.54) yields the critical output gap

$$x_1^c = \frac{\pi^c - \frac{\lambda+\kappa^2}{\lambda(1-\rho\beta)+\kappa^2}\xi_1}{\kappa}. \tag{3.72}$$

Second, the interest rate which allows the central banker to just escape from the zero interest-rate forward guidance in $t = 1$. Rearranging the IS Curve yields

$$i^c = \sigma(\mathbb{E}_1[x_2^E] - x^c) + \mathbb{E}_2[\pi_2^E] + r_H. \tag{3.73}$$

By inserting Equations (3.54), (3.55), and (3.72), we can calculate

$$i_1^c = r_H - \frac{\sigma}{\kappa}\pi^c + \frac{\lambda(\kappa\rho + \sigma) + \sigma\kappa^2(1 - \rho)}{\kappa[\lambda(1 - \rho\beta) + \kappa^2]}\xi_1. \tag{3.74}$$

We note that i_1^c increases with the size of the supply shock and decreases with the choice of π^c. The corresponding loss function is

$$\tilde{l}_1^C = \frac{1}{2}[(\pi^c)^2 + \lambda(x_1^c)^2]. \tag{3.75}$$

With the help of the loss functions $\tilde{l}_1^C$, $\tilde{l}_1^N$, and $\tilde{l}_1^F$ we can define the inflation and the output gap in $t = 1$ that are expected in the downturn for a given value of the inflation threshold π^c.

The expectations are formed as follows: In a first step, we define two auxiliary functions $\underline{\xi}_1$ and $\bar{\xi}_1$. $\underline{\xi}_1$ is the value of ξ_1 at which the central banker's loss functions $\tilde{l}_1^F$ and $\tilde{l}_1^C$

---

[11] We use the superscript "c" in combination with the time subscript to denote the critical values in the first period of Phase H.

intersect. $\bar{\xi}_1$ is the value of ξ_1 at which the central banker's loss functions $\tilde{l}_1^N$ and $\tilde{l}_1^C$ intersect. That is, if the realized supply shock in $t = 1$ is lower (higher) than $\underline{\xi}_1$ ($\bar{\xi}_1$), the central banker will set $i_1^E = i_1^F$ ($i_1^E = i_1^N$), and the realized inflation and the output gap will be $\pi_1^E = \pi_1^F$ ($\pi_1^E = \pi_1^N$) and $x_1^E = x_1^F$ ($x_1^E = x_1^N$). If $\xi_1 \in [\underline{\xi}_1, \bar{\xi}_1]$, the central banker will set $i_1^E = i_1^c$, and inflation and output gap will be π^c and x_1^c. Therefore, for a supply shock $\xi_1 \in [-\xi, \xi]$ we have

$$
\pi_1^{E,RE} = \begin{cases}
\int_{-\xi}^{\xi} \frac{\pi_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } \bar{\xi}_1 < -\xi, \\
\frac{(\bar{\xi}_1+\xi)\pi^c}{2\xi} + \int_{\bar{\xi}_1}^{\xi} \frac{\pi_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } \underline{\xi}_1 < -\xi \le \bar{\xi}_1 < \xi, \\
\int_{-\xi}^{\underline{\xi}_1} \frac{\pi_1^F(\xi_1)}{2\xi} d\xi_1 + \frac{(\bar{\xi}_1-\underline{\xi}_1)\pi^c}{2\xi} + \int_{\bar{\xi}_1}^{\xi} \frac{\pi_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } -\xi \le \underline{\xi}_1 < \bar{\xi}_1 < \xi, \\
\int_{-\xi}^{\underline{\xi}_1} \frac{\pi_1^F(\xi_1)}{2\xi} d\xi_1 + \frac{(\xi-\underline{\xi}_1)\pi^c}{2\xi} & \text{if } -\xi < \underline{\xi}_1 < \xi \le \bar{\xi}_1, \\
\int_{-\xi}^{\xi} \frac{\pi_1^F(\xi_1)}{2\xi} d\xi_1 & \text{if } \xi \le \underline{\xi}_1,
\end{cases}
\tag{3.76}
$$

and

$$
x_1^{E,RE} = \begin{cases}
\int_{-\xi}^{\xi} \frac{x_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } \bar{\xi}_1 < -\xi, \\
\int_{-\xi}^{\bar{\xi}_1} \frac{x_1^c(\xi_1)}{2\xi} d\xi_1 + \int_{\bar{\xi}_1}^{\xi} \frac{x_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } \underline{\xi}_1 < -\xi \le \bar{\xi}_1 < \xi, \\
\int_{-\xi}^{\underline{\xi}_1} \frac{x_1^F(\xi_1)}{2\xi} d\xi_1 + \int_{\underline{\xi}_1}^{\bar{\xi}_1} \frac{x_1^c(\xi_1)}{2\xi} d\xi_1 + \int_{\bar{\xi}_1}^{\xi} \frac{x_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } -\xi \le \underline{\xi}_1 < \bar{\xi}_1 < \xi, \\
\int_{-\xi}^{\underline{\xi}_1} \frac{x_1^F(\xi_1)}{2\xi} d\xi_1 + \int_{\underline{\xi}_1}^{\xi} \frac{x_1^c(\xi_1)}{2\xi} d\xi_1 & \text{if } -\xi < \underline{\xi}_1 < \xi \le \bar{\xi}_1, \\
\int_{-\xi}^{\xi} \frac{x_1^F(\xi_1)}{2\xi} d\xi_1 & \text{if } \xi \le \underline{\xi}_1.
\end{cases}
\tag{3.77}
$$

Thus, expectations of a central banker in the downturn are

$$
\mathbb{E}_D^{E,CB}[\pi_{t+1}] = \delta \pi_D^E + (1-\delta)\overline{\mathbb{E}}_D^{CB}[\pi_1^E] = \delta \pi_D^E + (1-\delta)\frac{\sigma_{\epsilon_\pi}^2 + \tau_\pi \sigma_{\eta_\pi}^2}{\sigma_{\epsilon_\pi}^2 + \sigma_{\eta_\pi}^2} \pi_1^{E,RE}
$$

and

$$
\mathbb{E}_D^{E,CB}[x_{t+1}] = \delta x_D^E + (1-\delta)\overline{\mathbb{E}}_D^{CB}[x_1^E] = \delta x_D^E + (1-\delta)\frac{\sigma_{\epsilon_x}^2 + \tau_x \sigma_{\eta_x}^2}{\sigma_{\epsilon_x}^2 + \sigma_{\eta_x}^2} x_1^{E,RE}.
$$

Once we know these expectations, we can calculate the downturn values π_D^E, x_D^E, and i_D^E. Plugging the expectations into the IS Curve and Phillips Curve yields

$$
\pi_D^E = \frac{\sigma \kappa (1-\delta)\theta_x x_1^{E,RE} + (1-\delta)(\kappa + \sigma\beta - \sigma\beta\delta)\theta_\pi \pi_1^{E,RE} + \kappa(r_D - i_D^E)}{h},
\tag{3.78}
$$

$$
x_D^E = \frac{\sigma(1-\delta)(1-\beta\delta)\theta_x x_1^{E,RE} + (1-\delta)\theta_\pi \pi_1^{E,RE} + (1-\beta\delta)(r_D - i_D^E)}{h}.
\tag{3.79}
$$

The central banker's loss function in the downturn is

$$\tilde{l}_D^E = \frac{1}{2}[(\pi_D^E)^2 + \lambda(x_D^E)^2 + b(i_D^E)^2]. \tag{3.80}$$

Taking the first-order condition with respect to i_D^E yields $\kappa\pi_D^E + \lambda x_D^E - \sigma b i_D^E = 0$. We rearrange the condition to $i_D^E = \frac{\kappa\pi_D^E + \lambda x_D^E}{\sigma b}$ and use Equations (3.78) and (3.79) to obtain

$$i_D^E = \max\{0, \hat{i}_D^E\}, \tag{3.81}$$

where

$$\hat{i}_D^E = \frac{(1-\delta)\sigma(\kappa^2 + \lambda(1-\beta\delta))\theta_x x_1^{E,RE} + (1-\delta)(\kappa(\kappa + \sigma\beta - \sigma\beta\delta) + \lambda)\theta_\pi \pi_1^{E,RE}}{b\sigma h + \kappa^2 + \lambda(1-\beta\delta)}$$
$$+ \frac{(\kappa^2 + \lambda(1-\beta\delta))r_D}{b\sigma h + \kappa^2 + \lambda(1-\beta\delta)}.$$

The equations show that the values in a downturn are functions of the natural real interest-rate shock r_D and the expectations $\theta_x x_1^{E,RE}$ and $\theta_\pi \pi_1^{E,RE}$. The central banker has two instruments to manage expectations in the downturn. On the one hand, he has the interest-rate forecast i^f, which is fixed to keep things simple. On the other, he has the inflation threshold π^c, which is flexible. Therefore, a lower τ value can be counteracted by a higher π^c value.

Corollary 3.1

If pessimism becomes more pronounced in the economy and expectations about future economic outcomes decrease, the central banker can counteract this development by increasing π^c.

Figure 3.4 illustrates the result that a lower τ—i.e. lower inflation and output expectations—are met by a more aggressive increase in the optimally set π^c.

In the environment with less optimism, i.e. low τ values, the escaping clause enables the central banker to announce higher inflation thresholds and bring up inflation expectations due to his stronger commitment. In our setting with a symmetric supply shock and a maximum value of $\xi_1 = 0.004$, a threshold inflation of $\pi^c = 1.75\%$ leads to a maximal increase in inflation expectations. Figure 3.5 displays how π_D^E and x_D^E evolve, depending on the realized natural real interest-rate shock. Note that π^c is chosen in such a way that i_D^E is always equal to zero. For the range of r_D shocks where a simple $i^f = 0$ forecast creates excessive inflation expectations, the additional flexibility introduced by π^c allows the central banker to anchor π_D^E around 0. Hence, the possibility of escaping in $t = 1$ avoids particularly high inflation realizations in $t = 1$ and brings down current inflation.
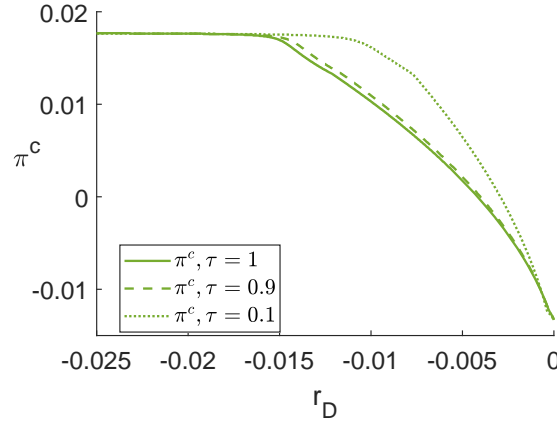
Figure 3.4: Optimal critical threshold π^c for different τ.

A long the same line of argument, x_D^E can be drawn closer to 0 as well.
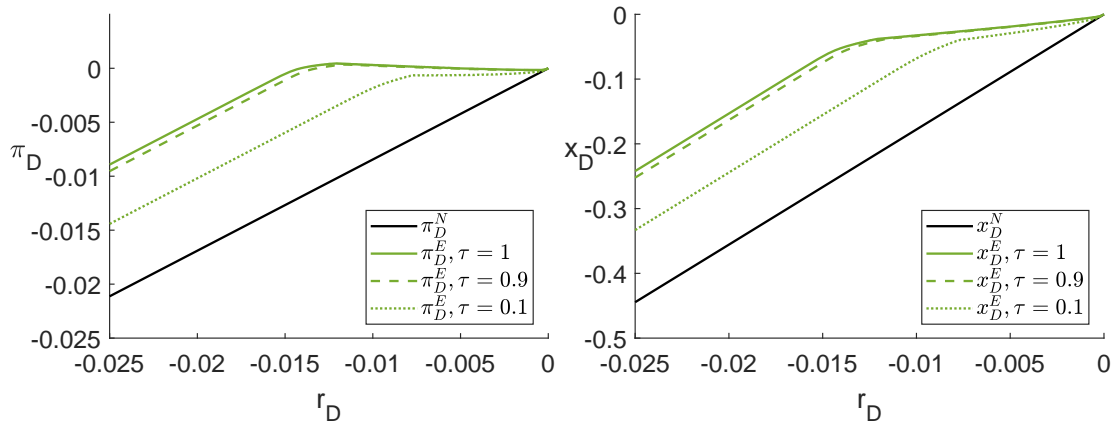
Figure 3.5: Inflation and output gap under escaping forward guidance in Period D.

3.3.3 Switching Forward Guidance

We now briefly outline how to derive the dynamics of the economy in Phase D and Phase H under switching forward guidance, with the S-superscript for switching. As before, we proceed by backward induction. We first derive the dynamics of the economy in $t \geq 2$ for each value of ξ_2 and an initial value of π_2^f. Given the dynamics of the economy in $t \geq 2$, we can derive the central banker's optimal inflation forecast π_2^f in $t = 1$ for each realized ξ_1. Then, with inflation and output gap in $t = 1$ for each supply shock, we can derive inflation, the output gap and the interest rate in the downturn.

Using the notation of Söderlind (1999), we have $y_t := (\xi_t, \pi_t^f, r_H, \pi_t, x_t)'$, where $y_{1,t} := (\xi_t, \pi_t^f, r_H)'$ are predetermined and $y_{2,t} := (\pi_t, x_t)'$ are non-predetermined entries for $t \geq 2$. The vector of policy instruments is $u_t := (i_t, \pi_{t+1}^f)'$.

Note that we use rational expectations in the algorithm, which leads to the notation

$$\overline{\mathbb{E}}[\pi_{t+1}^S] = \theta_{\pi,H}\pi_{t+1}^{S,RE} = \frac{1}{\beta}(\pi_t^S - x_t^S - \xi_t), \tag{3.82}$$

$$\overline{\mathbb{E}}[x_{t+1}^S] = \theta_{x,H}x_{t+1}^{S,RE} = \left(1 + \frac{\kappa}{\beta\sigma}\right)x_t^S + \frac{1}{\beta\sigma}\xi_t - \frac{1}{\sigma}r_H - \frac{1}{\beta\sigma}\pi_t^S + \frac{1}{\sigma}i_t^S. \tag{3.83}$$

where $\theta_{\pi,H} = \frac{\sigma_{\epsilon_\pi}^2 + \tau_{\pi,H}\sigma_{\eta_\pi}^2}{\sigma_{\epsilon_\pi}^2 + \sigma_{\eta_\pi}^2}$, $\theta_{x,H} = \frac{\sigma_{\epsilon_x}^2 + \tau_{x,H}\sigma_{\eta_x}^2}{\sigma_{\epsilon_x}^2 + \sigma_{\eta_x}^2}$. The dynamics of y_t for $t \geq 2$ can be written in the more compact form

$$\begin{pmatrix} y_{1,t+1} \\ \mathbb{E}_t[y_{2,t+1}] \end{pmatrix} = A \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} + Bu_t, \tag{3.84}$$

where

$$A := \begin{pmatrix} \rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -\frac{1}{\theta_{\pi,H}\beta} & 0 & 0 & \frac{1}{\theta_{\pi,H}\beta} & -\frac{\kappa}{\theta_{\pi,H}\beta} \\ \frac{1}{\theta_{x,H}\beta\sigma} & 0 & -\frac{1}{\theta_{x,H}\sigma} & -\frac{1}{\theta_{x,H}\beta\sigma} & \frac{1}{\theta_{x,H}}\left(1 + \frac{\kappa}{\beta\sigma}\right) \end{pmatrix}, B := \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\theta_{x,H}\sigma} & 0 \end{pmatrix}. \tag{3.85}$$

The central banker's loss function is

$$\tilde{l}_t^S = \frac{1}{2}[(\pi_t^S)^2 + \lambda(x_t^S)^2 + b(\pi_t^S - \pi_t^f)^2] \tag{3.86}$$

and, rewritten in the form used by Söderlind, it is

$$\tilde{l}_t^S = y_t'Qy_t + 2y_t'Uu_t + u_t'Ru_t, \tag{3.87}$$

with

$$Q := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{b}{2} & 0 & -\frac{b}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{b}{2} & 0 & \frac{1+b}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{\lambda}{2} \end{pmatrix}, U := \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, R := \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{3.88}$$

The central banker's optimization problem can then be formulated in a Bellman Equation with the value function $y_{1,t}'V_ty_{1,t} + v_t$. The value function incorporates the predetermined variables $y_{1,t}$, the matrix V_t, which is assumed to be independent of exogenous shocks,

and the term v_t that includes shocks:

$$y'_{1,t}V_t y_{1,t} + v_t = \min_{u_t}\left\{ y'_t Q y_t + 2y'_t U u_t + u'_t R u_t + \beta \mathbb{E}_t[y'_{1,t+1}V_{t+1}y_{1,t+1} + v_{t+1}] \right\}, \quad (3.89)$$

such that non-predetermined variables in $t+1$ are linear functions of the predetermined variables in this period, $\mathbb{E}_t[y_{2,t+1}] = C_{t+1}\mathbb{E}_t[y_{1,t+1}]$, Equation (3.84) holds, and given $y_{1,t}$. The Matlab algorithm provided by Söderlind (1999) recursively solves this Bellman Equation. We use a grid of ξ_2 and π_2^f values to obtain the dynamics of the economy for $t \geq 2$ for each ξ_2-π_2^f-combination. The central banker's loss function in $t = 1$ is

$$\tilde{l}_1^S = \frac{1}{2}[(\pi_1^S)^2 + \lambda(x_1^S)^2 + b(i_1^S)^2]. \quad (3.90)$$

Hence, the first-order condition with respect to i_1^S such that the IS Curve in (3.36) and Phillips Curve in (3.20) hold is

$$\kappa\pi_1^S + \lambda x_1^S - b\sigma i_1^S = 0. \quad (3.91)$$

Again, using the IS Curve and Phillips Curve, we obtain the dynamics in $t = 1$:

$$x_1^S = \frac{b\sigma^2 \overline{\mathbb{E}}_1^{CB}[x_2^S] - (\kappa\beta - b\sigma)\overline{\mathbb{E}}_1^{CB}[\pi_2^S] - \kappa\xi_1 + b\sigma r_H}{b\sigma^2 + \kappa^2 + \lambda}, \quad (3.92)$$

$$\pi_1^S = \frac{(\beta(b\sigma^2 + \lambda) + \kappa\sigma b)\overline{\mathbb{E}}_1^{CB}[\pi_2^S] + \kappa\sigma^2 b\overline{\mathbb{E}}_1^{CB}[x_2^S] + (b\sigma^2 + \lambda)\xi_1 + \kappa\sigma b r_H}{b\sigma^2 + \kappa^2 + \lambda}, \quad (3.93)$$

$$i_1^S = \frac{(\kappa^2 + \beta\kappa\sigma + \lambda)\overline{\mathbb{E}}_1^{CB}[\pi_2^S] + \sigma(\kappa^2 + \lambda)\overline{\mathbb{E}}_1^{CB}[x_2^S] + \sigma\kappa\xi_1 + (\kappa^2 + \lambda)r_H}{b\sigma^2 + \kappa^2 + \lambda}. \quad (3.94)$$

$\overline{\mathbb{E}}_1^{CB}[.]$ varies with the choice of π_2^f. Hence, once ξ_1 is realized, the central banker chooses π_2^f such that his cumulative loss function $\sum_{t=1}^{\infty}\beta^{t-1}\tilde{l}_t^S$ is minimized. Similar to Equations (3.78) – (3.81), we can obtain the inflation, output gap, and interest rate under switching forward guidance in the downturn.

In addition to the standard deviations of the policy signal and the private signal in downturns, $\sigma_{\epsilon_\pi}, \sigma_{\epsilon_x}$ and $\sigma_{\eta_\pi}, \sigma_{\eta_x}$, respectively, variations of the same in Phase H have to be specified. Standard deviations are assumed to be $\sigma_{\epsilon_{\pi,H}} = \sigma_{\eta_{\pi,H}} = 0.002$ and $\sigma_{\epsilon_{x,H}} = \sigma_{\eta_{x,H}} = 0.0125$. As stated in the introduction to this section, we choose τ such that we introduce pessimism. In a downturn, this means $0 < \tau < 1$, and in normal times, $1 < \tau_H < 1.1$. $\tau_H > 1$ implies that the reversion to the steady state is slower as the curves are pushed away from it. This is shown in Figure 3.6 for the inflation dynamics, with $\xi_1 = 0.004$ on the left graph and $\xi_1 = -0.004$ on the right graph. For rational expectations, i.e. $\tau_H = 1$, inflation can be shifted towards the steady state with the help

of inflation forecasts. The exception is the first period of Phase H and $\xi_1 = 0.004$, when the interest-rate forecast is still constraining the central banker's decision. Furthermore, the curves show that for $\tau = 1.1$ and $t \geq 3$, π_t^S is roughly the same as π_t^N. Therefore, the introduction of inflation forecasts—which in our model simultaneously leads to the emergence of heterogeneous beliefs—does not produce lower inflation compared to the discretionary case if the agents are pessimistic enough. It depends on the value of τ_H whether the additional communication can reduce current inflation—as well as it depends on the relative sizes of σ_η and σ_ϵ. Furthermore, Figure 3.7 shows the output gap for different τ_H values and Figure 3.8 the interest rates.
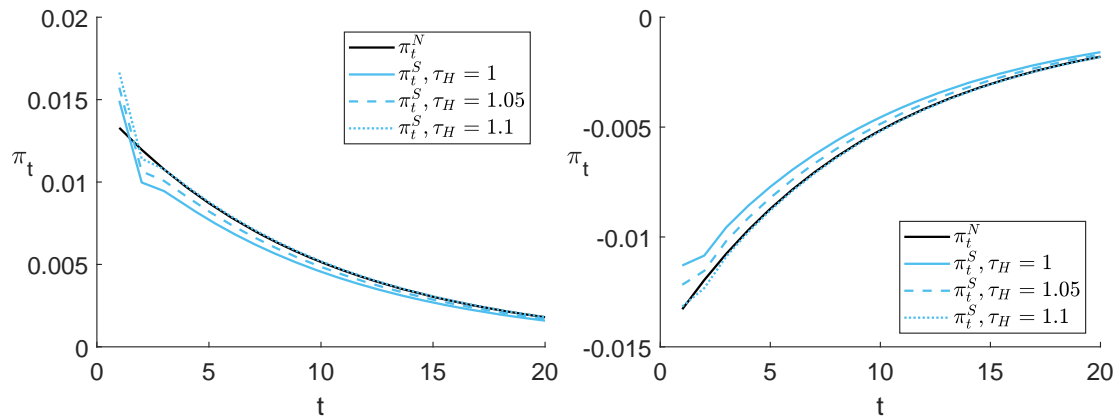
Figure 3.6: Inflation under SFG in Period H, with $\xi_1 = 0.004$ in the left graph and $\xi_1 = -0.004$ in the right graph.
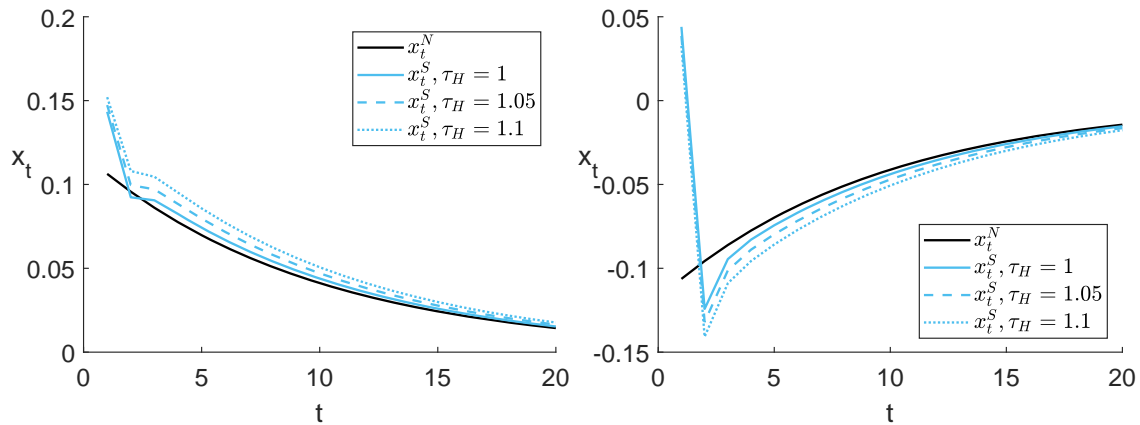
Figure 3.7: Output gap under SFG in Period H, with $\xi_1 = 0.004$ in the left graph and $\xi_1 = -0.004$ in the right graph.
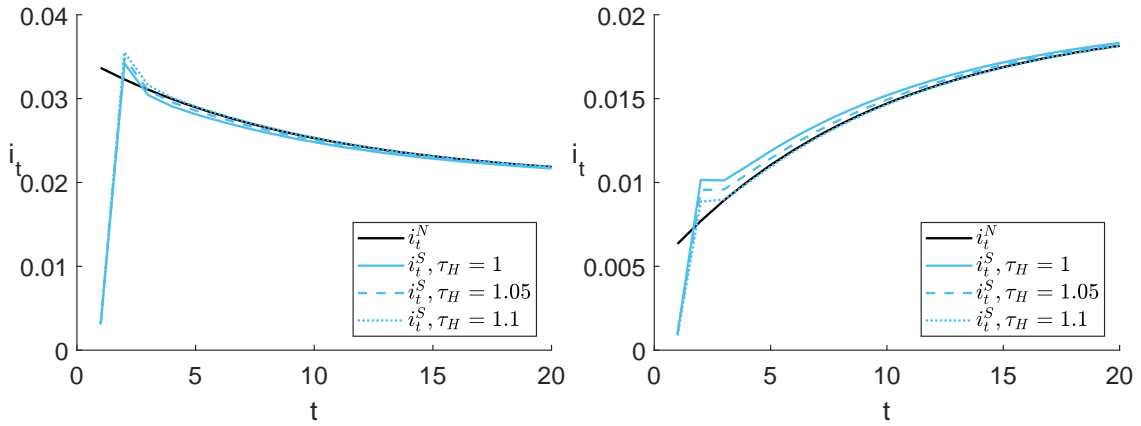
Figure 3.8: Interest rate under SFG in Period H, with $\xi_1 = 0.004$ in the left graph and $\xi_1 = -0.004$ in the right graph.

Similar to the interpretation in Andrade et al. (2016), inflation forecasts in Phase H can be perceived in two ways: First, it can be perceived as a negative sign for the economy, that pushes expectations away from the steady state values or, second, as a positive sign that anchor expectations closer to the steady state.

The dynamics in Phase H show that for $t = 1, 2$, the switching forward guidance values vary substantially from one period to the next and then smoothly approach the steady state values over time. This is due to the forecast $i^f = 0$ that lowers i_1^S and drives up π_1^S and x_1^S. π_2^f is selected so that it smooths the effect of i^f and, thus, lowers π_2^S and x_2^S. From a central banker's perspective, in Phase H, $\tilde{l}_{t\geq 3}^S \approx l_{t\geq 3}^N$ when $\tau_H = 1.05$. This is shown visually in Appendix B.5 in Figure B.1.

Corollary 3.2

In the environment of a pessimistic economy and supply shocks, inflation forecasts are detrimental to welfare.

3.4 Results

This section presents the results of the loss function analysis and compares the different forward guidance designs. As before, the discretionary case will serve as a benchmark.

3.4.1 Welfare Comparison

It is beneficial for the central banker to engage in standard forward guidance for large r_D shocks as displayed in Figure 3.9. For mild r_D shocks, the central banker prefers to abstain from forecasting due to excessive inflation and output creation. Introducing

heterogeneous beliefs to standard forward guidance lowers its power by lowering agents' expectations. This reduces the absolute difference of the loss functions L^N and $\tilde{L}^F$. Moreover, less power to raise expectations makes standard forward guidance more attractive in the environment of small r_D shocks. The left graph of Figure 3.9 shows that without common beliefs, standard forward guidance extends the range for which forward guidance is beneficial from $r_D = -0.64\%$ (rational expectations) to $r_D = -0.62\%$ ($\tau = 0.9$), and $r_D = -0.46\%$ ($\tau = 0.1$).

Under escaping forward guidance, providing forecasts is always beneficial. Heterogeneous beliefs attenuate the power of forward guidance, which becomes more apparent with increasing r_D shock size. The results are plotted in the right graph of Figure 3.9.

The characteristic results under switching forward guidance are the same as for standard forward guidance. This means; disperse beliefs make forward guidance attractive for a wider range of r_D shocks, while the power of forward guidance is lower and abstaining from forecasts is preferred by the central banker for mild r_D shocks.
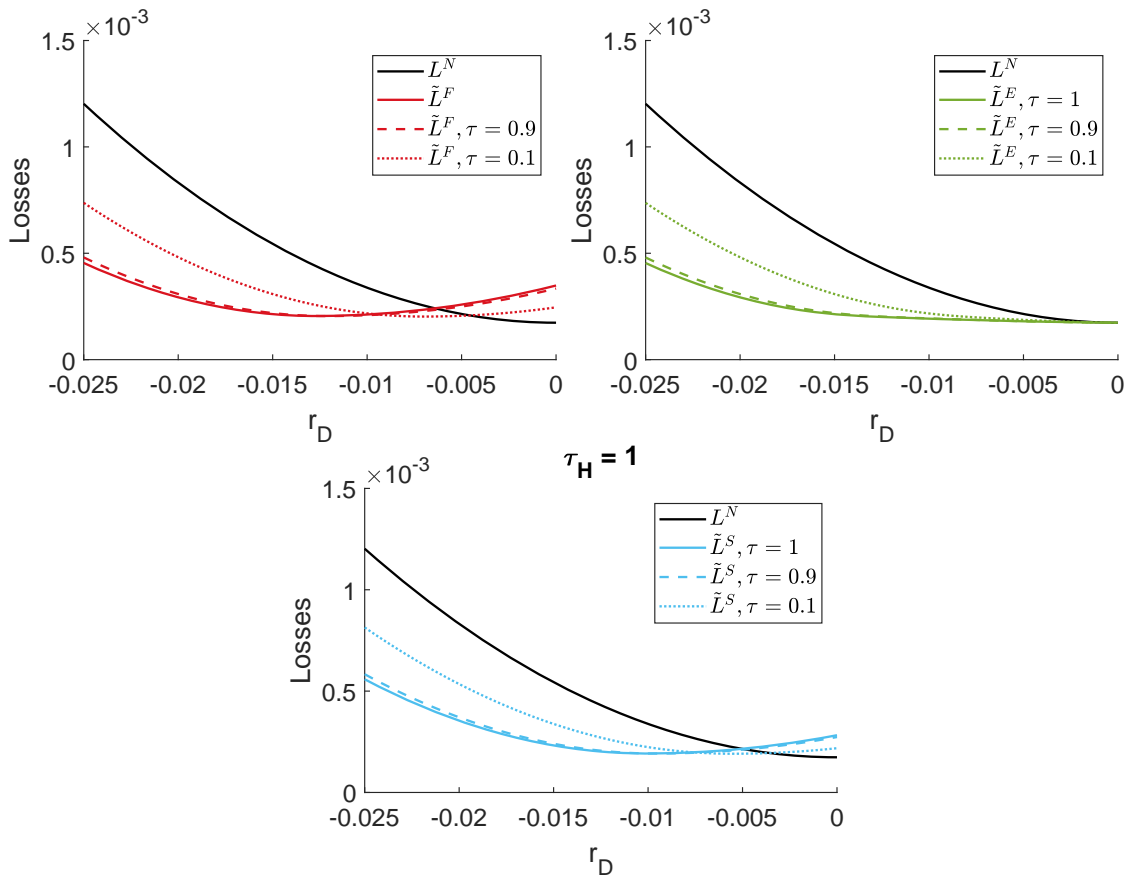
Figure 3.9: Expected central bank losses under standard, escaping, and switching forward guidance.

Numerical Finding 3.1

Heterogeneous beliefs and the accompanying lower power to shift expectations lessen the impact of forward guidance and make it attractive to apply forward guidance at less severely negative natural real interest rates.

Note that in the bottom graph of Figure 3.9, we assume that $\tau_H = 1$. To verify the robustness of the result, we plot $\tilde{L}^S$ for increasing values of τ_H in Figure 3.10. An increasing value of τ_H shifts the losses upwards, and the range for which switching is beneficial compared to abstaining from forecasts shrinks. The reason is that pessimism pushes $\pi_{t\geq 1}^S$ and $x_{t\geq 1}^S$ outwards, due to increased/decreased expectations of the respective variables. We already illustrated this result in Figures 3.6 and 3.7.
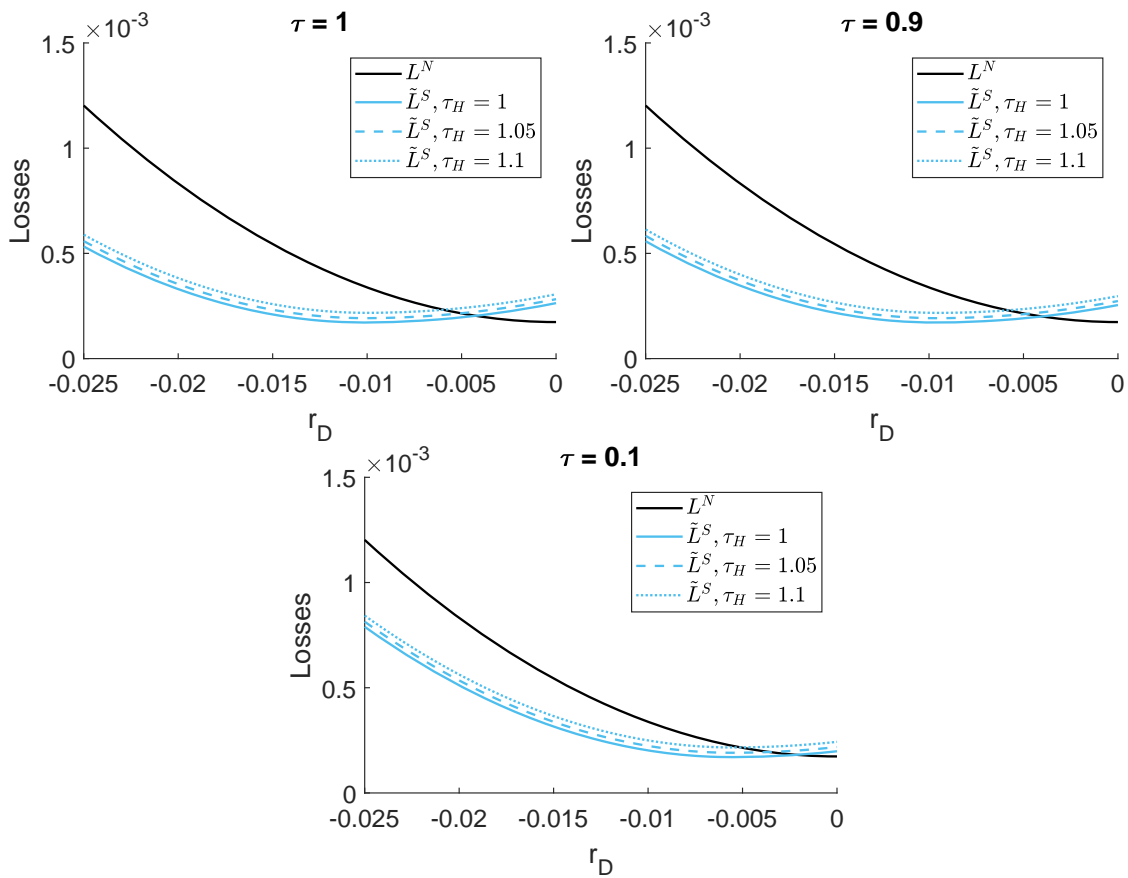
Figure 3.10: Expected central bank losses for different degrees of pessimism.

Further, we compare the two flexible forward guidance designs with each other and the case without forward guidance. Figure 3.11 shows a comparison between the designs. We focus on three scenarios: First, we show rational expectations with $\tau = \tau_H = 1$ in the left graph of Figure 3.11. Second, we depict an environment where agents exhibit mild pessimism with $\tau = 0.9$, $\tau_H = 1.05$ in the right graph. Third, we show severe pessimism with $\tau = 0.1$ and $\tau_H = 1.1$ in the bottom graph. All nine combinations are shown in

the Appendix B.5. We find that for rational expectations, switching forward guidance is preferred over escaping and over no forward guidance for a mid-sized r_D shock. This is also true in a more general case where rational expectations prevail in Phase H but not in Phase D. For small- and large-sized r_D shocks, escaping forward guidance minimizes expected losses. With the introduction of pessimism, escaping forward guidance gradually becomes the dominant design for all shock sizes. Hence, the loss of switching forward guidance's ability to improve welfare in Phase H—caused by the increase of τ_H—leads to a general loss of attractiveness of switching compared to escaping. For severe r_D shocks switching forward guidance still dominates the discretionary case, although pessimism causes a surplus in losses in Phase H.[12]
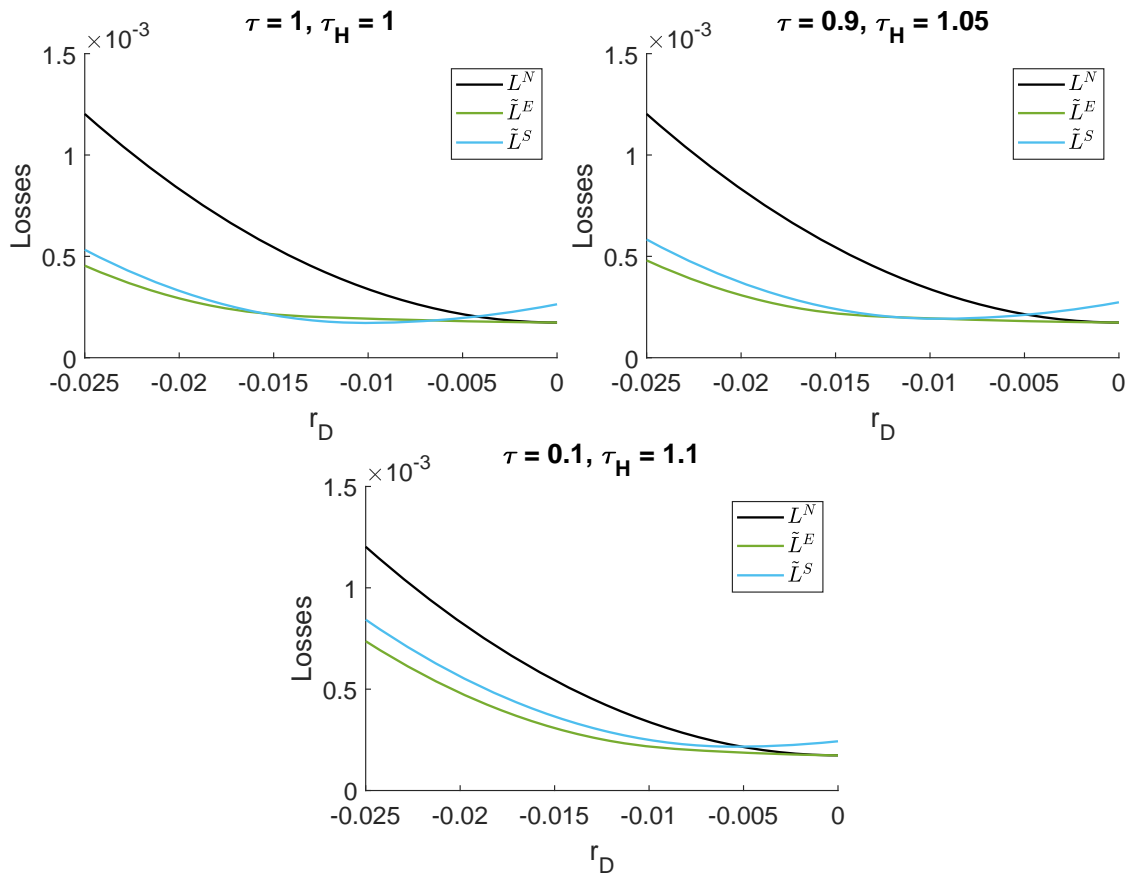
Figure 3.11: Expected central bank losses under escaping and switching forward guidance.

Numerical Finding 3.2

In the presence of either mild or severe pessimism in the economy, escaping forward guidance is the dominant design for all realizations of the negative natural real interest rate.

---

[12] Note that in this section, we analyze the central banker's losses. Social losses qualitatively produce the same results.

3.4.2 Robustness to Degree of Scrupulosity b

In this subsection, we investigate parameter b's influence on our results. If we let b go to zero, all forward guidance designs become ineffective, as the central banker incurs no loss from deviating from his forecast. Ultimately, all designs collapse to the discretionary central banker design.

If b approaches infinity, the central banker is fully committed to his forecast. Figure 3.12 shows the functions of a fully committed central banker. Standard forward guidance becomes unattractive for an extended range of r_D shocks compared to the benchmark case with $b = 1$. Again, this is due to the zero interest-rate forecast, which is enforced in Phase H, $t = 1$, under full commitment and, thus creates excessive inflation and output-boom expectations for small r_D shocks. Escaping still dominates no forward guidance, due to the fact that it encompasses no forward guidance for small r_D shocks. In a full commitment case, switching forward guidance can reduce the expected losses in such a way that it is beneficial to apply it for all r_D shock sizes. As b goes to infinity, τ_H becomes less important. Changing τ_H to 1.05 or 1.1 results in only very slight shifts in the loss functions. The substantial reduction in expected losses can be attributed to the advantage of having the possibility to manage expectations in all periods of Phase H and to perfectly commit to the forecasts.
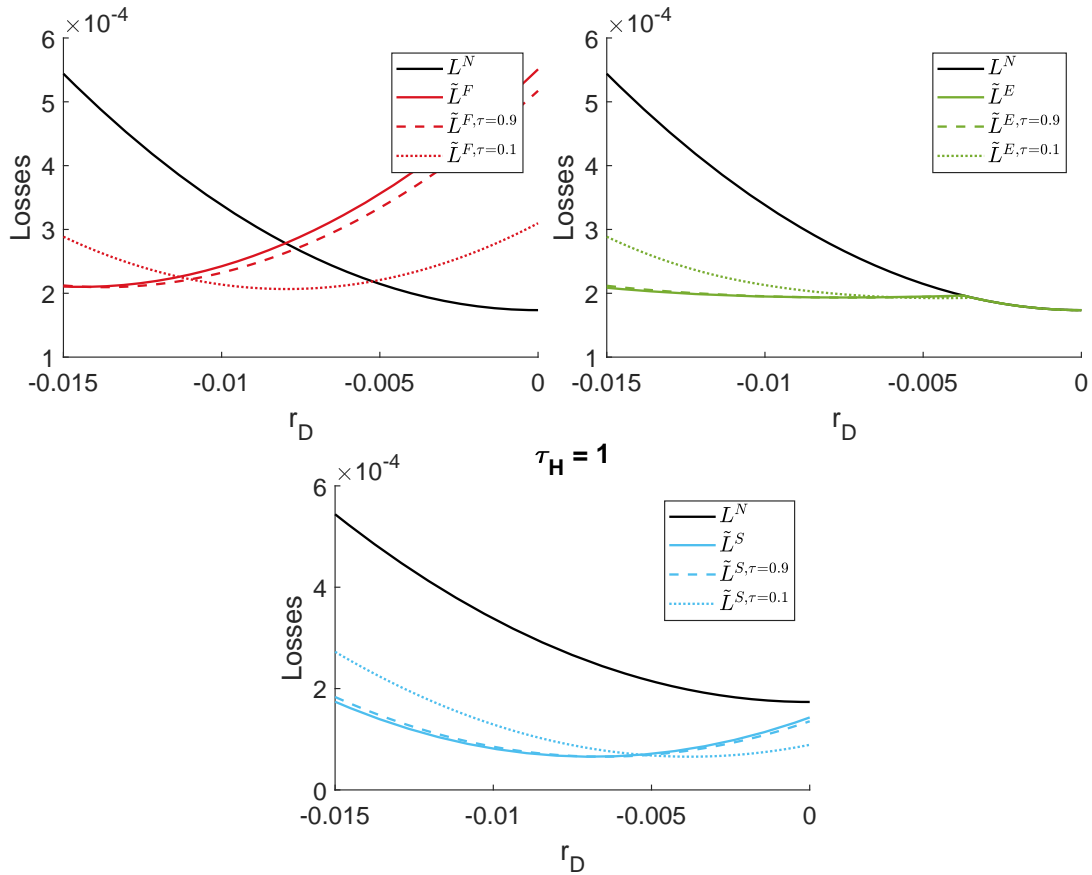
Figure 3.12: Expected central bank losses under standard, escaping, and switching forward guidance with full commitment.

3.5 Conclusions

In this chapter we relaxed the assumption of rational expectations in a New Keynesian Framework with nominal rigidities. We provided a general approach to include heterogeneous expectations in the model where, in principle, a variety of expectation forming process can be implemented. More specifically, we introduced informational frictions into the expectation formation process. Our model is set up in such a way that the rational expectation model is encompassed. We used this modified framework to analyze the welfare effects of standard forward guidance and the two flexible designs escaping forward guidance and switching forward guidance.

Introducing heterogeneous beliefs to forward guidance has two main effects. First, the central banker's power to raise inflation and the output-gap expectations is reduced. Hence, the absolute welfare difference between a discretionary central banker and one who applies forward guidance diminishes. This leads to a less pronounced Forward Guidance Puzzle. Although—or rather because—the forward guidance designs are not as effective

anymore, standard and switching forward guidance become attractive for a wider range of natural real interest-rate shocks. These lower expectations are due to the fact that the average agent is pessimistic when the economy is in a downturn. Then, the central banker cannot generate expansionary expectations as easily as under rational expectations.

Second, under the rational expectations model, switching forward guidance is the most beneficial design for a medium-sized r_D shock. When pessimism is introduced into the model, escaping forward guidance manages to achieve the lowest welfare loss among all designs for all r_D shock sizes. The reason is that inflation forecasts in Phase H do not help to reduce welfare losses at mild levels of pessimism. Escaping forward guidance abstains from forecasting and, hence, avoids the emergence of heterogeneous beliefs through additional communication in Phase H.

4 Global Sensitivity Analysis*

4.1 Introduction

In Chapter 2 we have followed Rotemberg and Woodford (1997) and Woodford (2003) in setting the parameter values (see Table 2.1). We now examine whether changes in the parameter values qualitatively affect our results, and if so, which specific parameters have a strong impact on our analysis. In other words, we examine whether our findings regarding the relative desirability of EFG and SFG are robust to parameter uncertainty.
In this chapter we use Sobol' Indices and the polynomial chaos expansion methodology (henceforth PCE) to assess the global robustness of the results we obtained in Chapter 2. This is a method borrowed from engineering (see Sudret (2008)), introduced to economics by Ratto (2008) and Harenberg et al. (2017), and goes beyond standard local sensitivity analysis. Given plausible parameter spaces it enables us to draw a more complete picture of the sensitivity of a model. In particular, PCE helps to efficiently identify those structural parameters that contribute most to the variance of the model's output, in our setup, expected social losses. The application of this method to the New Keynesian Model with a scrupulous central banker reveals that, typically, the slope of the Phillips Curve turns out to be the parameter to which social losses react the most. Most importantly, the analysis reveals that the areas for which *escaping* forward guidance and *switching* forward guidance dominate other monetary policy approaches are robust to parameter uncertainty. That is, *escaping* forward guidance remains the optimal approach for substantial or small negative natural real interest-rate shocks, while *switching* forward guidance is preferred for intermediate negative natural real interest-rate shocks under parameter uncertainty. We also show that the principal gains of applying forward guidance will materialize even for central bankers with a low degree of scrupulosity.

Motivation

In economics, robustness analyses are usually addressed by means of comparative statics routines—one value is varied while all others remain constant—or by a robustness check, where different parameter scenarios are considered, i.e. parameter constellations that can

---

Parts of this chapter have been used in the discussion paper Gersbach et al. (2018).

be either optimistic or pessimistic.[1] But, such approaches produce only local results and thus do not allow for non-linearities and interactions of the parameters in the entire parameter space. Moreover, the results also depend on the chosen combination of parameters. Therefore, these measures generally yield an incomplete picture.

Instead of using a traditional local approach, we apply a global sensitivity analysis (henceforth GSA) in the form of Sobol' Indices. Sobol' Indices break down the variance of the output variable into variance contributions from each input parameter and the interactions of these input parameters. Using the Sobol' Indices, we can thus rank the parameters according to their importance. This enables us, first, to identify the parameters that contribute most to the variance of the output variable and should, therefore, receive particular attention. Second, the method enables us to determine the non-influential input parameters that can be fixed at a constant value without significantly affecting model output, i.e. the expected social losses in our model. Third, the GSA is independent of the chosen evaluation points and disentangles the direct effect of a parameter from its interactions with other parameters. Fourth, through the investigation of the interaction of the input parameters the method deepens our understanding of the drivers of model output.

In a methodological paper, Ratto (2008) first discusses Sobol' Indices as an analytical tool to analyze the properties of the structural parameters in a DSGE model. In the context of an RBC model, Harenberg et al. (2017) conclude that the sensitivity measures typically used in economics can be highly misleading and that the findings in the respective analysis vary substantially, depending on the values chosen for the analysis. They advocate the application of a GSA in the form of Sobol' Indices in combination with PCE.

Sobol' Indices are calculated by Monte Carlo Simulations with sample draws from underlying input parameter distributions. At each draw, the model is evaluated, which makes the analysis extremely power- and time-consuming due to the slow convergence properties of the Monte Carlo Method. Hence the main drawback is the cost of analysis if the model is expensive to run (Saltelli et al., 2008). In the engineering sciences, computationally efficient methods have been developed that lower the computational burden significantly. We use the PCE proposed by Sudret (2008) to address this particular problem. In Section 4.2.2, the PCE is introduced, and the derivation of Sobol' Indices from PCE is explained in Section 4.2.3.

---

[1] In the monetary policy literature, Giannoni (2007), for example, tackles the problem of uncertainty about structural parameters in such a *pessimistic*-scenario-based approach. Giannoni specifies a vector of structural parameters which lie in a given compact set. The structural parameter values are the ones that maximize social losses, given the central bank's optimal behavior. Then, subject to this worst-case parameter constellation the central bank sets the nominal interest rate to minimize social losses.

Structure

This chapter is organized as follows: An introduction to Sobol' Indices and PCE is presented in the next section. In Section 4.3 we apply Sobol' Indices to the different forward guidance designs. In Section 4.4 we analyze the univariate effects of each parameter in more detail. Finally, a discussion and the conclusion make up Section 4.5.

4.2 Theory

Sobol' Indices belong to the class of analysis of variance techniques (ANOVA) that aim to decompose the variance of the output variable—in our case, the expected social losses L^p with $p \in \{N, F, E, S\}$—into a sum of variances of each input variable and the variances of interaction terms of these input variables. Sobol' Indices allow to identify to what extent each input parameter and the interaction of the input parameters contribute to the variation of the output variable. Sobol' Indices are calculated by using Monte Carlo Simulation. This turns out to be a drawback if the underlying model requires a lot of computational effort for the calculation of these indices. For this reason, we use the so-called PCE Method. We outline the concept of Sobol' Indices in Section 4.2.1. The theoretical foundation behind PCE and its connection to Sobol' Indices is provided by Sudret (2008) and will be addressed in Sections 4.2.2 and 4.2.3. For more detailed information about this global sensitivity approach the reader is referred to Le Gratiet et al. (2016) and Harenberg et al. (2017).

4.2.1 Sobol' Indices

Consider a computational model $\mathcal{M} : \mathbf{x} \in \mathcal{D}_{\mathbf{X}} \subset \mathbb{R}^d \mapsto Y = \mathcal{M}(\mathbf{x}) \in \mathbb{R}$, where $\mathbf{x}$ is a d-dimensional vector containing the input parameters. Denote by $w(\mathbf{x})$ the density function of the input $\mathbf{x}$. The Sobol' decomposition then reads

$$
\begin{aligned}
\mathcal{M}(\mathbf{x}) =& \mathcal{M}_0 + \sum_{i=1}^{d} \mathcal{M}_i(x_i) + \sum_{1 \leq i < j \leq d} \mathcal{M}_{ij}(x_i, x_j) + ... \\
&+ \sum_{1 \leq i_1 < ... < i_s \leq d} \mathcal{M}_{i_1...i_s}(x_{i_1}, ..., x_{i_s}) + ... + \mathcal{M}_{12 \cdots d}(\mathbf{x}),
\end{aligned}
\tag{4.1}
$$

where we have the mean of the function

$$
\mathcal{M}_0 = \int_{\mathcal{D}_{\mathbf{X}}} \mathcal{M}(\mathbf{x}) w(\mathbf{x}) d\mathbf{x} = \mathbb{E}[\mathcal{M}(\mathbf{X})],
$$

the univariate functions[2]

$$\mathcal{M}_i(x_i) = \int_{\mathcal{D}_{\mathbf{X}_{\sim i}}} \mathcal{M}(\mathbf{x})w(\mathbf{x})d\mathbf{x} - \mathcal{M}_0 = \mathbb{E}[\mathcal{M}(\mathbf{X})|X_i = x_i] - \mathcal{M}_0,$$

the bivariate functions

$$\mathcal{M}_{ij}(x_i, x_j) = \int_{\mathcal{D}_{\mathbf{X}_{\sim ij}}} \mathcal{M}(\mathbf{x})w(\mathbf{x})d\mathbf{x} - \mathcal{M}_i(x_i) - \mathcal{M}_j(x_j) - \mathcal{M}_0$$
$$= \mathbb{E}[\mathcal{M}(\mathbf{X})|X_i, X_j = x_i, x_j] - \mathcal{M}_i(x_i) - \mathcal{M}_j(x_j) - \mathcal{M}_0,$$

and so on.

Using the set notation $A \equiv \{i_1, ..., i_s\} \subset \{1, ..., d\}$, we can rewrite Equation (4.1) as

$$\mathcal{M}(\mathbf{x}) = \mathcal{M}_0 + \sum_{\substack{A \subset \{1,...,d\} \\ A \neq \emptyset}} \mathcal{M}_A(\mathbf{x}_A), \tag{4.2}$$

where $\mathbf{x}_A$ is a subvector of $\mathbf{x}$ and only contains parameters that belong to the index set A. Due to the orthogonality property of the summands, the total variance of the model output Y can be written as

$$Var[Y] = \sum_{\substack{A \subset \{1,...,d\} \\ A \neq \emptyset}} \mathbb{E}[\mathcal{M}_A^2(\mathbf{x}_A)]. \tag{4.3}$$

Therefore we can define the partial variance associated to $\{i_1, ..., i_s\}$ by

$$V_{i_1...i_s} := \int \mathcal{M}_{i_1...i_s}^2(x_{i_1}, ..., x_{i_s})w(x_{i_1}, ..., x_{i_s})dx_{i_1}...dx_{i_s}. \tag{4.4}$$

The total variance of the model output (4.3) can thus be split up into parts:

$$Var[Y] = V = \sum_{i=1}^{d} V_i + \sum_{1 \leq i < j \leq d} V_{ij} + ... + V_{12...d}. \tag{4.5}$$

Finally, the s-order Sobol' Indices are defined as

$$S_{i_1...i_s}^s = \frac{V_{i_1...i_s}}{V}. \tag{4.6}$$

The total Sobol' Index of a given parameter x_i is defined as

$$S_i^{tot} := S_i + \sum_{j=1,j \neq i}^{d} S_{ij} + \sum_{1 \leq j < k \leq d, j,k \neq i} S_{ijk} + ... + S_{12...d}. \tag{4.7}$$

---

[2] We denote by X_i a given component of $\mathbf{X}$ and by $\mathbf{X}_{\sim i}$ the remaining components, such that $\mathbf{X} \equiv (X_i, \mathbf{X}_{\sim i})$.

The total Sobol' Index quantifies the total effect of parameter x_i on the variance of output variable.

In our setting, Sobol' Indices mainly serve two purposes (see Saltelli et al. (2008)). First, they can be used for *factor prioritization*. Total Sobol' Indices allow an importance ranking of the input variables and subsequently indicate which input parameters are worth investigating. In other words, total Sobol' Indices tell us which undetermined parameter leaves the largest variance in the output if all other parameters are fixed. Second, they are used for *factor fixing*. We can identify those parameters that do not significantly contribute to output variation. That is, we can set these parameters at deterministic values. In general, if $S_i^{tot} < 1\%$, the respective parameter x_i can be set to a constant value within the respective range.

4.2.2 Polynomial Chaos Expansion

As in the previous section, we consider a model[3] $\mathcal{M} : x \in D_X \subset \mathbb{R} \to \mathbb{R}$, where $\mathcal{M}(x)$ has a finite variance: $\mathbb{E}[\mathcal{M}^2(x)] < \infty$ and the input variable x with support D_X has a probability density function $w(x)$. Using PCE, $\mathcal{M}(x)$ can be represented by a series expansion (*metamodel*)

$$\mathcal{M}(x) = \sum_{k=0}^{\infty} y_k \psi_k(x), \tag{4.8}$$

where $\{\psi_k\}_{k=0}^{\infty}$ forms the orthonormal polynomials basis of a suitable space and $\{y_k\}_{k=0}^{\infty}$ is the set of coordinates of $\mathcal{M}(x)$ in this basis. These are the PCE coefficients to be computed. Let us now see how we can construct the orthonormal polynomials basis and determine the coefficients.

Construction of the Orthonormal Basis

We first explain how the orthonormal basis $\{\psi_k\}_{k=0}^{\infty}$ is constructed in PCE.

For any two functions $\phi_k, \phi_l : x \in D_X \to \mathbb{R}$, we define the functional inner product as

$$< \phi_k, \phi_l >_w \equiv \mathbb{E}[\phi_k(x)\phi_l(x)] = \int_{D_X} \phi_k(x)\phi_l(x)w(x)dx. \tag{4.9}$$

Moreover, ϕ_k and ϕ_l are said to be orthogonal if their inner product is zero.

Given the notation above and some algebra[4], we can define the sequence of *orthogonal*

---

[3] Note that for illustrative reasons we consider the univariate case. We later apply a multivariate version where we assume the input variables to be statistically independent.

[4] For example, one can use the Gram-Schmit Orthogonalization Procedure of $\{1, x, x^2, \cdots\}$ to build a family of orthogonal polynomials. For the generic method of constructing an orthonormal basis we refer the reader to Abramowitz and Stegun (1970).

polynomials $\{P_k, k \in \mathbb{N}\}$ with respect to a weight function w as

$$< P_k, P_l >_w = a_k \delta_{kl}, \tag{4.10}$$

where δ_{kl} is the Kronecker Symbol, i.e. $\delta_{kl} = 0$ if $k \neq l$ and $\delta_{kl} = 1$ if $k = l$, and k is the degree of polynomial P_k. Hence, the functional inner product $< P_k, P_l >_w$ is equal to the squared norm $a_k \equiv ||P_k||^2$ if $k = l$ and zero otherwise. The polynomials can then be normalized:

$$\psi_k = \frac{P_k}{\sqrt{a_k}}.$$

Classical families of orthonormal polynomials are known analytically. For instance, Legendre Polynomials are the orthonormal basis for the uniform distribution over $[-1, 1]$ and Hermite Polynomials are the orthonormal basis for the Gaussian distribution.

To exactly replicate the model $\mathcal{M}(x)$ in Equation (4.8) we need a metamodel made up of an infinite series. In practice, we truncate the series to a reasonable number of terms, so that it is possible to estimate the PCE coefficients. By using polynomials up to degree p, we achieve the approximation

$$\hat{\mathcal{M}}(x) = \sum_{k=0}^{p} y_k \psi_k(x). \tag{4.11}$$

The corresponding approximation error is

$$\mathcal{M}(x) - \hat{\mathcal{M}}(x) = \sum_{k=p+1}^{\infty} y_k \psi_k(x). \tag{4.12}$$

Later, we will want to use multivariate functions and thus are interested in multivariate orthonormal polynomials that can be constructed from the univariate orthonormal polynomials using tensor products. We define $\alpha \in \mathbb{N}^d$, which are ordered lists of integers $\alpha = (\alpha_1, ..., \alpha_d)$, where d is the number of input variables. Then we can write the multivariate polynomial $\boldsymbol{\Psi}_\alpha$ to a multi-index α as

$$\boldsymbol{\Psi}_\alpha(\mathbf{x}) \equiv \prod_{i=1}^{d} \psi_{\alpha_i}^{(i)}(x_i), \tag{4.13}$$

where $\psi_{\alpha_i}^{(i)}(x_i)$ is the univariate polynomial of degree α_i from the orthonormal family associated with variable x_i.

It can be proved that the set of all multivariate polynomials in the random input vector x

form a basis of the Hilbert space in which $Y = \mathcal{M}(\mathbf{x})$ is represented[5] by

$$Y = \sum_{\alpha \in \mathbb{N}^d} y_\alpha \boldsymbol{\Psi}_\alpha(\mathbf{x}). \tag{4.14}$$

Computation of PCE Coefficients by Least-square Minimization

In the preceding subsection we derived the orthonormal basis $\boldsymbol{\Psi}_\alpha$. We now turn to the computation of the PCE coefficients y_α. As shown above, we need to introduce a *truncation scheme* $\mathcal{A}$ to reduce the infinite series to a finite number of terms, so that the coefficients can be computed. Once the truncation scheme is selected[6], a variety of approaches to computing the coefficients is available in the literature. Following Harenberg et al. (2017) and Le Gratiet et al. (2016), we use the least-square estimation, which we address in the following.

Equation (4.14) in a truncated form is

$$Y = \mathcal{M}(\mathbf{x}) = \sum_{\alpha \in \mathcal{A}} y_\alpha \boldsymbol{\Psi}_\alpha(\mathbf{x}) + \epsilon, \tag{4.15}$$

where ϵ is the residual that contains all the PCE polynomials not in the truncation set $\mathcal{A}$ (see Equation (4.12) for the univariate case). The set of coefficients $\mathbf{y} = \{y_\alpha, \alpha \in \mathcal{A}\}$ is selected such that the mean squared error $\mathbb{E}[\epsilon^2]$ is minimized:

$$\mathbf{y} = \arg \min_{\mathbf{y} \in \mathbb{R}^{card\mathcal{A}}} \mathbb{E}\left[\left(\mathcal{M}(\mathbf{x}) - \sum_{\alpha \in \mathcal{A}} y_\alpha \boldsymbol{\Psi}_\alpha(\mathbf{x})\right)^2\right]. \tag{4.16}$$

To obtain an estimate of $\mathbf{y}$, we draw N samples[7] of the input parameters $\mathcal{X}_{ed} = \{\mathbf{X}^{(i)}, i = 1, ..., N\}$, where ed stands for *experimental design*. The above equation can then be written as

$$\hat{\mathbf{y}} = \arg \min_{\hat{\mathbf{y}} \in \mathbb{R}^{card\mathcal{A}}} \frac{1}{N} \sum_{i=1}^{N} \left(\mathcal{M}(\mathbf{X}^{(i)}) - \sum_{\alpha \in \mathcal{A}} \hat{y}_\alpha \boldsymbol{\Psi}_\alpha(\mathbf{X}^{(i)})\right)^2. \tag{4.17}$$

Hence, in a first step, the model $\mathcal{M}$ has to be run N times to obtain a vector $\mathcal{Y} = \{\mathcal{M}(\mathbf{X}^{(1)}), ..., \mathcal{M}(\mathbf{X}^{(N)})\}^T$. Second, the basis polynomials have to be evaluated at each point of the experimental design to create the information matrix $\mathbf{A}$:

$$\mathbf{A} = \{\mathbf{A}_{ij} \equiv \boldsymbol{\Psi}_j(\mathbf{X}^{(i)}), \quad i = 1, ..., N, \quad j = 1, ..., card\mathcal{A}\}. \tag{4.18}$$

---

[5] See Soize and Ghanem (2004).

[6] The detailed truncation scheme is presented in Section 4.2.2.

[7] N should be larger than the number of unknowns ($card\mathcal{A}$), but at the same time not too large, so that the model $\mathcal{M}(\mathbf{x})$ does not have to be evaluated too many times. Le Gratiet et al. (2016) propose a rule of thumb $N \approx 2\binom{d+p}{p}$ or $3\binom{d+p}{p}$, where p represents the total degree of the truncation scheme, i.e. $|\alpha| = \alpha_1 + \cdots + \alpha_d = p$.

Therefore the mean-square error can be written as

$$\Delta = \sum_{i=1}^{N} \epsilon_i^2 = (\mathcal{Y} - \mathbf{A}\hat{\mathbf{y}})^T \cdot (\mathcal{Y} - \mathbf{A}\hat{\mathbf{y}}). \tag{4.19}$$

The error is minimized when

$$\frac{\partial \Delta}{\partial \hat{\mathbf{y}}^{\mathbf{T}}} = -2\mathbf{A}^T \mathcal{Y} + 2(\mathbf{A}^T \mathbf{A})\hat{\mathbf{y}} = \mathbf{0}. \tag{4.20}$$

Thus the solution to the minimization problem takes the form

$$\hat{\mathbf{y}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathcal{Y}. \tag{4.21}$$

Finally, the truncated PCE can be written as

$$Y^{PC} = \mathcal{M}^{PC}(\mathbf{x}) = \sum_{\alpha \in \mathcal{A}} \hat{y}_\alpha \boldsymbol{\Psi}_\alpha(\mathbf{x}). \tag{4.22}$$

To avoid over-fitting, we reduce the number of PCE coefficients by employing the least-angle regression (LAR) algorithm to select only the significant coefficients in the PCE. That is, we apply the LAR algorithm to the candidate basis $\mathcal{A}$, which contains all possible coefficients, i.e. $\binom{d+p}{p}$, and we select the most significant coefficients to form a sparse basis.[8]
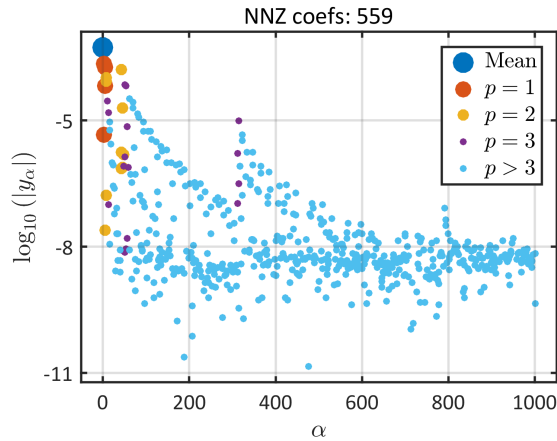
Figure 4.1: Sparse basis selected using LAR for the model L_{PCE}^N.

Figure 4.1 enumerates the ordered lists of integers α on the x-axis and shows the size of

---

[8] This algorithm was initially proposed by Efron et al. (2004). Blatman and Sudret (2011) introduced LAR into the PCE literature. For a formal discussion of the algorithm, we refer to these papers. Note that the LAR algorithm is only defined for non-constant regressors. Hence, after selecting the sparse basis, we perform an ordinary least-square regression, which includes a constant regressor, and calculate the coefficients we ultimately use in the PCE. Marelli and Sudret (2015) call this approach *"hybrid LAR"*.

the estimated PCE coefficients y_α on the y-axis for the PCE approximation of L^N with $p = 10$ and $d = 4$. The mean value and the coefficients of the linear polynomial terms are largest in size and are hence the most influential. As the degree of the polynomial terms increases, the size of the coefficients decreases. The total candidate basis consists of $\binom{d+p}{p} = 1001$ coefficients and is reduced to the sparse basis of 559 non-zero coefficients (NNZ) using the LAR algorithm.

Truncation Scheme of PCE Coefficients

We now turn to the question of how the truncation set should be chosen. Once the coefficients are obtained (see Equation (4.21)), the approximation error of the truncated PCE can be computed ex post by comparing the fitted output responses (see Equation (4.22)) to the true output response (see Equation (4.15)). As suggested in Le Gratiet et al. (2016), we use the *leave-one-out* (LOO) error estimator to find an appropriate degree of truncation that yields an accurate approximation. A brief sketch of the procedure follows. First, an experimental design $\mathcal{X}_{ed} \setminus X^{(i)} \equiv \{X^1, ..., X^{(i-1)}, X^{(i+1)}, ..., X^N\}$ is set up. Second, a PCE model $\mathcal{M}^{PC \setminus i}$ is estimated and the error at the point that was left out is computed. Then the average over the sum of the squared errors is calculated:

$$err_{LOO} \equiv \frac{1}{N} \sum_{i=1}^{N} \left(\mathcal{M}(\mathbf{X}^{(i)}) - \mathcal{M}^{PC \setminus i}(\mathbf{X}^{(i)}) \right)^2. \tag{4.23}$$

Le Gratiet et al. (2016) show that, after some algebra, this expression reduces to

$$err_{LOO} = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{\mathcal{M}(\mathbf{X}^{(i)}) - \mathcal{M}^{PC}(\mathbf{X}^{(i)})}{1 - h_i} \right)^2, \tag{4.24}$$

where h_i is the i-th diagonal term of $\mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$ and $\mathcal{M}^{PC}$ is the PCE model of the full experimental design $\mathcal{X}_{ed}$. We follow Harenberg et al. (2017) and assume that an $err_{LOO} \leq 10^{-2}$ yields sufficient accuracy for a sensitivity analysis.

For example, for the expected social losses L^N_{PCE} and L^F_{PCE} in NFG and IFG, we truncate the PCE at $p = 10$ and use 3100 samples in the experimental design. The corresponding errors are of a magnitude $err_{LOO} \approx 10^{-7}$ and $err_{LOO} \approx 10^{-8}$, respectively. Due to the computationally more involved approaches in EFG and SFG, we truncate the PCE at $p = 7$ and use 990 sample draws. The resulting errors are of a magnitude $err_{LOO} \approx 10^{-5}$ and $err_{LOO} \approx 10^{-5}$, respectively. Figure 4.2 shows how well the metamodel approximates the true model by plotting the true model response ($Y = L^N$) on the x-axis and the truncated PCE model response on the y-axis ($Y^{PC} = L^N_{PCE}$). The figure shows that all 10'000 sample points lie on a straight 45-degree line. Therefore the PCE model of degree 10 manages to capture the true model very well.
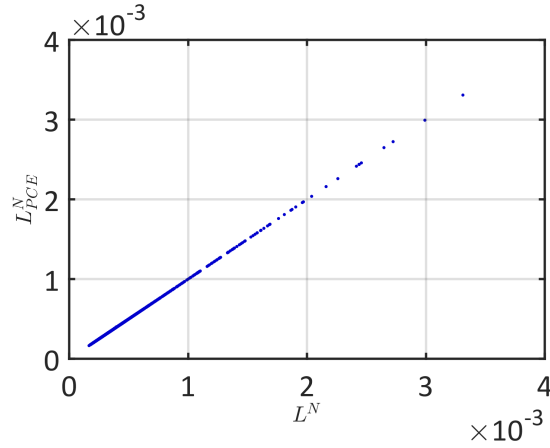
Figure 4.2: Metamodel output ($Y^{PC} = L_{PCE}^{N}$) vs. true model output ($Y = L^{N}$).

4.2.3 Sobol' Indices and PCE

Sudret (2008) shows that the sum of orthogonal functions in the Sobol' decomposition in Equation (4.2) can be analytically derived from the sum of orthogonal functions in the truncated PCE in Equation (4.22). First, note that due to the orthogonality[9] of the PCE basis, mean and variance of the output variable read

$$\mathbb{E}[\hat{Y}] = \mathbb{E}\left[\sum_{\alpha \in \mathcal{A}} \hat{y}_{\alpha} \Psi_{\alpha}(\mathbf{x})\right] = \hat{y}_{0}, \tag{4.25}$$

$$Var[\hat{Y}] = \mathbb{E}\left[(\hat{Y} - \hat{y}_{0})^{2}\right] = \sum_{\substack{\alpha \in \mathcal{A} \\ \alpha \neq 0}} \hat{y}_{\alpha}^{2}. \tag{4.26}$$

That is, the mean value is the first coefficient of the series, and the variance is the sum of the squares of the remaining coefficients.

Second, given the PCE coefficients, Sobol' Indices of any order may be obtained by combining the squares of the respective coefficients, i.e. first-order Sobol' Indices read

$$\hat{S}_{i}^{1} = \frac{\sum_{\alpha \in \mathcal{A}_{i}} \hat{y}_{\alpha}^{2}}{\sum_{\alpha \in \mathcal{A}, \alpha \neq 0} \hat{y}_{\alpha}^{2}}, \quad \text{where} \quad \mathcal{A}_{i} = \{\alpha \in \mathcal{A} : \alpha_{i} > 0, \alpha_{j \neq i} = 0\}. \tag{4.27}$$

In general, the polynomials can be gathered according to the parameters they depend on, and the Sobol' Indices are written as

$$\hat{S}_{i_{1}...i_{s}}^{s} = \frac{\sum_{\alpha \in \mathcal{A}_{i_{1}...i_{s}}} \hat{y}_{\alpha}^{2}}{\sum_{\alpha \in \mathcal{A}, \alpha \neq 0} \hat{y}_{\alpha}^{2}}, \quad \text{where} \quad \mathcal{A}_{i_{1},...,i_{s}} = \{\alpha \in \mathcal{A} : \alpha_{k} > 0, \text{ iff } k \in \{i_{1}, ..., i_{s}\}\}. \tag{4.28}$$

---

[9] The orthogonality property of the polynomial basis implies that $\mathbb{E}[\Psi_{\alpha}(\mathbf{x})] = 0$ and $\mathbb{E}[\Psi_{\alpha}(\mathbf{x})\Psi_{\beta}(\mathbf{x})] = \delta_{\alpha\beta}$.

The PCE-based total Sobol' Indices read

$$\hat{S}_i^{tot} = \frac{\sum_{\alpha \in \mathcal{A}_i^{tot}} \hat{y}_\alpha^2}{\sum_{\alpha \in \mathcal{A}, \alpha \neq 0} \hat{y}_\alpha^2}, \quad \text{where} \quad \mathcal{A}_i^{tot} = \{\alpha \in \mathcal{A} : \alpha_i > 0\}. \tag{4.29}$$

Note that the univariate function of parameter x_i can be written as

$$\mathcal{M}_i(x_i) = \mathbb{E}[\mathcal{M}(\mathbf{X})|X_i = x_i] - \mathcal{M}_0 = \sum_{\alpha \in \mathcal{A}_i} y_\alpha \Psi_\alpha(\mathbf{x}). \tag{4.30}$$

$\mathcal{M}_i(x_i)$ represents the deviation of the model's expected output for a given x_i from the mean value of the model's output. While the total Sobol' Indices in Equation (4.29) can be used to identify the importance ranking of input parameters, univariate function provides further information about the parameter's impact on the output variable. For example, whether the parameter's effect on the output variable is positive or negative, whether the relationship is linear or non-linear, and the regions of the parameter space in which output sensitivity is most pronounced.

4.2.4 Parameters

To apply the method, we assume that the four standard New Keynesian structural parameters, β, λ, κ, and σ, are uniformly distributed. This distributional assumption is consistent with the maximum entropy principle.[10] In other words *"[...] we express complete ignorance by assigning a uniform prior probability density [...]"* (Jaynes, 2003, p. 377). We choose two specifications for the uniform distribution. First, a "narrow" one that includes values most commonly used in the literature and second, a "wide" one that comprises parameter-outliers from the literature. Table 4.1 displays the respective distributions.

We make this distinction between the two scenarios as a robustness check. By selecting less conservative parameter ranges, we can detect to what extent the GSA results are driven by the parameter ranges.[11] A drawback of applying the wide parameter scenario is that the approximation accuracy of the surrogate model decreases significantly.

---

[10] A compact overview of maximum entropy distributions is given in Park and Bera (2009).

[11] Most notably, λ is almost exclusively set to 0.003, at the exception of one estimate of 0.007 in Adam (2007) and an outlier of 0.25 in Evans et al. (2015). This means the importance of the output gap in the loss function is increased by factor 36.

Another important change is that the σ range is extended to values up to 2. Thus, the inter-temporal elasticity of substitution is in the range 0.5 to 6.25. This implies the response of output growth to interest rate changes, a key mechanism in monetary policy, is largely decreased. A study by Hall (1988) even suggests that the inter-temporal elasticity of substitution is probably not greater than 0.1.

A quarterly discount factor β of 0.99 implies an annual interest rate of 4%. A β of 0.95 thus has an annual interest rate of approximately 22%, which is extremely high.

Note that the PCE is very sensitive to κ, i.e., when κ is set to 0.08, predictions turn very imprecise. We investigate this sensitivity property at particular parameter values in more detail in Section 4.4.

Narrow	Wide
$\beta \sim \mathcal{U}(0.99, 0.995)$	$\beta \sim \mathcal{U}(0.95, 0.995)$
$\lambda \sim \mathcal{U}(0.003, 0.007)$	$\lambda \sim \mathcal{U}(0.003, 0.25)$
$\kappa \sim \mathcal{U}(0.024, 0.057)$	$\kappa \sim \mathcal{U}(0.014, 0.057)$
$\sigma \sim \mathcal{U}(0.16, 0.26)$	$\sigma \sim \mathcal{U}(0.157, 2)$

Table 4.1: Narrow and wide parameter ranges in the global sensitivity analysis.

4.3 Sensitivity of the Models

In this section, we perform a GSA of the forward guidance designs NFG, IFG, EFG, and SFG introduced in Chapter 2.[12] We investigate how much the parameters listed in Table 4.1 and their interactions contribute to the variances of the output variables, i.e. we derive Sobol' Indices calculated with PCE of the expected social losses L^p, where $p \in \{N, F, E, S\}$. We examine the models in isolation and in comparison to each other as well. Finally, we apply the GSA to the canonical New Keynesian Model and to the canonical model constrained by a ZLB.

4.3.1 Forward Guidance Models

Throughout this subsection, we choose $r_D = -0.015$ as an illustrative example and discuss the results for this specification in detail. At the end of this subsection, we provide a more aggregate view on the designs for different r_D sizes. We abbreviate the total Sobol' Indices to S_i^{tot}, where i indicates the parameter we are referring to and all other indices to S_i^j, where j denotes the order.

Global Sensitivity Analysis of NFG

The results of the GSA of L^N are summarized in Figure 4.3 and Table 4.2. We use polynomials up to a degree of $p = 10$ and an experimental design of $N = 3100$. The total Sobol' Indices order the importance of the parameters in the narrow and wide range scenarios in the same way: $\sigma > \kappa > \lambda > \beta$. The relative sizes among the parameters vary, though. S_β^{tot} is smaller than 1% in both instances and therefore can be set to a constant within the wide range without heavy consequences on the results. In the narrow scenario, the dominating total Sobol' Indices are S_κ^{tot} and S_σ^{tot} which are mainly influenced by the single effects of κ, σ, and the interaction of these two. A first-order index measures the linear, additive influence of a specific parameter. A second-order index, i.e. $S_{\kappa\sigma}^2$, is the joint effect of κ and σ and, with a size of 25%, is relatively important. The size of

---

[12] We use the Matlab-based software UQLab for the calculations. The software is available on http://www.uqlab.com and Marelli et al. (2017) provide a user manual and introduce of the methods.

this interaction term implies that the underlying model is non-additive and non-linear in the input parameters. Intuitively, two parameters interact when their effect on the output variable cannot be expressed as a sum of their single effects (Saltelli et al., 2008). In the wide scenario, σ dominates, as its single effect is by far the greatest one. κ and λ mainly influence output variation in interaction with σ.

Higher-order indices, i.e. S_i^3 and S_i^4 indices, are either of minor importance (narrow scenario) or the PCE approximation is not accurate enough to make precise statements (wide scenario). We therefore neglect them in the analysis.
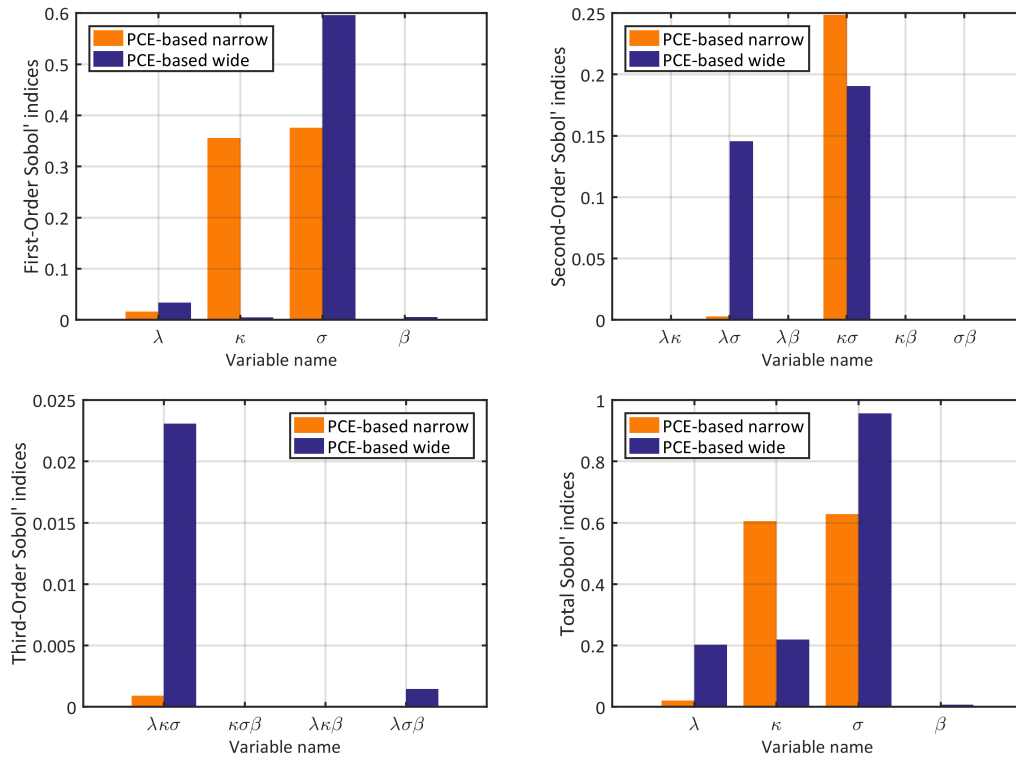
Figure 4.3: PCE-based Sobol' Indices of L^N with $p = 10$, $N = 3100$.

Accuracy of PCE

The scatter plot of L^N and L^N_{PCE} in Figure 4.4 visually displays the accuracy of the PCE. In the narrow scenario, the PCE captures the output variation of the original model closely with an $err_{LOO} = 10^{-7}$. The candidate basis of 1001 coefficients is reduced to a sparse basis of 597 coefficients. In the wider scenario, the variation of the original model is only captured to a sufficient degree with an $err_{LOO} = 8.86 * 10^{-2}$. At the lower end of the wide parameter evaluations of L^N, PCE predicts negative values, which is not possible due to the quadratic nature of the loss function. The candidate basis is greatly reduced to 38 coefficients.

	λ	κ	σ	β
S^{tot}_{narrow}	0.0204	0.6054	0.6276	0.0002
S^{tot}_{wide}	0.2024	0.2200	0.9563	0.0069
S^{1}_{narrow}	0.0163	0.3555	0.3755	0
S^{1}_{wide}	0.0338	0.0045	0.5957	0.0054

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^{2}_{narrow}	0.0005	0.0027	0	0.2485	0	0
S^{2}_{wide}	0	0.1455	0	0.1906	0	0

Narrow – Mean: 0.0011, SD: 0.0009, err_{LOO}: $3.98 * 10^{-7}$
Wide – Mean: 0.0014, SD: 0.0027, err_{LOO}: $8.86 * 10^{-2}$

Table 4.2: First- and second-order Sobol' Indices for L^{N}_{PCE} with $p = 10$, $N = 3100$.
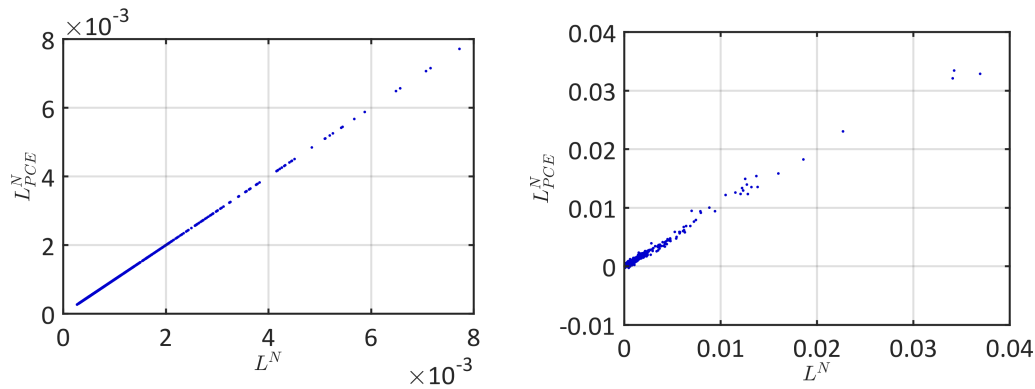
Figure 4.4: L^{N} surrogate model accuracy with $p = 10$, $N = 3100$.

Relation to the Benchmark

In Figure 4.5, we plot output values of the PCE-model for 100'000 random draws of λ, κ, σ, and β. This provides an insight into where our benchmark parameterization is located, compared to all possible parameter constellations. On the x-axis, the output values are displayed and on the y-axis, the number of occurrences of the respective values are displayed. In each plot, the mean value (Mean), the standard deviation (SD), and the value of the benchmark parameterization (BM) are provided. Given the narrow ranges our benchmark is close to the mode of the empirical distribution but only roughly half the size of the mean as the distribution is right skewed. Given wide ranges, the histogram shows some negative values. The benchmark is again close to the mode of the distribution but only one third of the mean value. Moreover, the standard deviation increases dramatically.
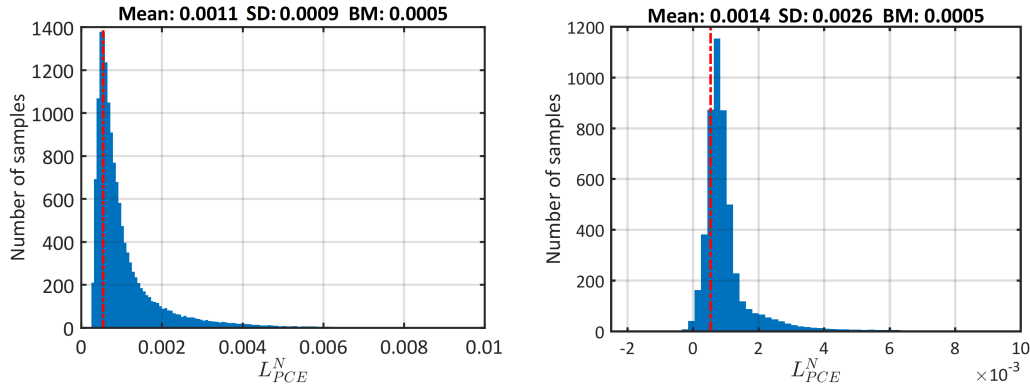
Figure 4.5: Histogram of L_{PCE}^N with n=100'000, and $r_D = -0.015$. The dotted red line provides the benchmark $\lambda = 0.003$, $\kappa = 0.024$, $\sigma = 0.16$, $\beta = 0.99$.

Global Sensitivity Analysis of IFG

We now investigate the design in which a central bank makes interest-rate forecasts in downturns and partially commits to these forecasts. Thus we analyze L^F. Computationally, the model is not particularly expensive, hence, we set $p = 10$ and $N = 3100$ to achieve high accuracy. We use the benchmark case with $b = 1$.[13] The results of the GSA are summarized in Figure 4.6 and Table 4.3.

Compared to the NFG analysis, the ranking of the parameters changes to $\kappa > \lambda > \sigma > \beta$ in the narrow scenario. Again, S_i^1 plays the most important role. Therefore, the variation in output can be mainly attributed to the variation in single parameters. In the wide scenario, σ stays the most important parameter but now, λ follows as the second-to-most important parameter, with ranking $\sigma > \lambda > \kappa > \beta$. λ's and κ's influence exerts itself mostly through the interaction with σ. σ's main contribution, however, can be attributed to the first-order effect of σ on the loss function. In line with the analysis up to now, β plays only a minor role and can be set to a constant in the proposed range, without influencing the results too greatly. In general, higher-order Sobol' Indices, i.e. S_i^3 and S_i^4, are unimportant, except $S_{\lambda\kappa\sigma}^3$ which contributes 4.1% to the total variation.

---

[13] If b is determined endogenously and set to its optimal value $b = b^*$, the results of the GSA stay roughly the same and only vary in a magnitude of 10^{-3}.
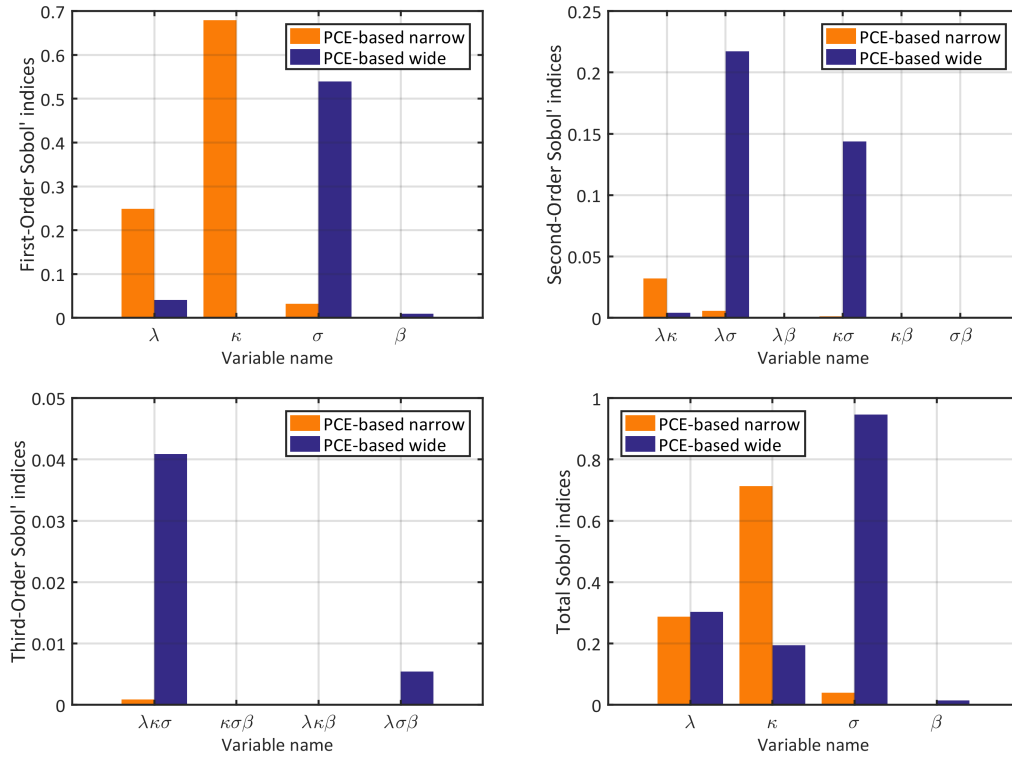
Figure 4.6: PCE-based Sobol' Indices of L^F with $p = 10$, $N = 3100$, and $b = 1$.

	λ	κ	σ	β
S^{tot}_{narrow}	0.2872	0.7131	0.0396	0.0010
S^{tot}_{wide}	0.3027	0.1941	0.9463	0.0146
S^1_{narrow}	0.2489	0.6787	0.0320	0.0006
S^1_{wide}	0.0404	0	0.5391	0.0092

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^2_{narrow}	0.0320	0.0056	0	0.0012	0.0003	0
S^2_{wide}	0.0041	0.2173	0	0.1437	0	0

Narrow – Mean: 0.0002, SD: 0.0001, err_{LOO}: $2.77 * 10^{-8}$
Wide – Mean: 0.0010, SD: 0.0019, err_{LOO}: $8.58 * 10^{-2}$

Table 4.3: First- and second-order Sobol' Indices for L^F_{PCE} with $p = 10$, $N = 3100$, and $b = 1$.

Accuracy of PCE

The surrogate model is very accurate in the narrow scenario, as displayed in Figure 4.7. In the wide scenario, the dots in the scatter plot are set closely around a 45 degree line with some L_{PCE}^F values in the negative territory. The model evaluations are mainly concentrated at the origin, with some outliers to the right. The err_{LOO}s are $2.77 * 10^{-8}$ in the narrow range and $8.58 * 10^{-2}$ in the wide range, respectively.
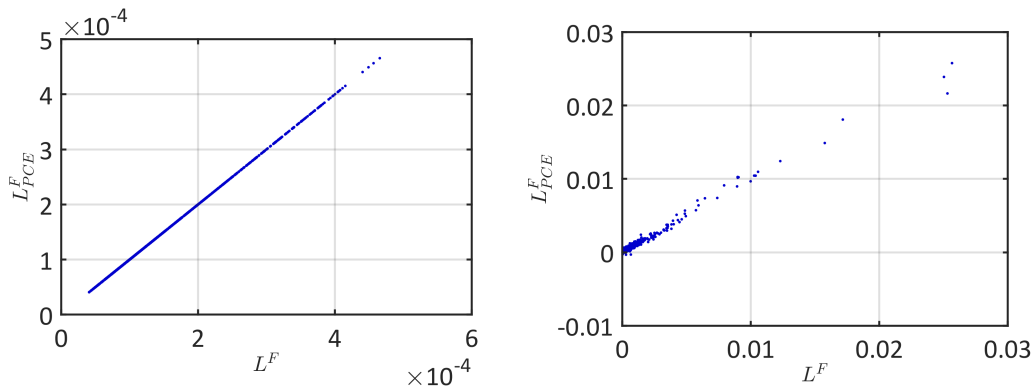
Figure 4.7: L^F surrogate model accuracy with $p = 10$, $N = 3100$.

Relation to the Benchmark

The histograms in Figure 4.8 show that the mean loss under IFG is reduced by a factor 10 compared to NFG in the narrow parameter range. Furthermore, in contrast to NFG, the benchmark parameterization yields a larger loss than the average loss. For wide parameter ranges, the mean loss is only slightly reduced compared to NFG. Further, in contrast to the narrow parameter ranges, the benchmark yields losses that are about five times lower than the average loss and lie to the right of the median.
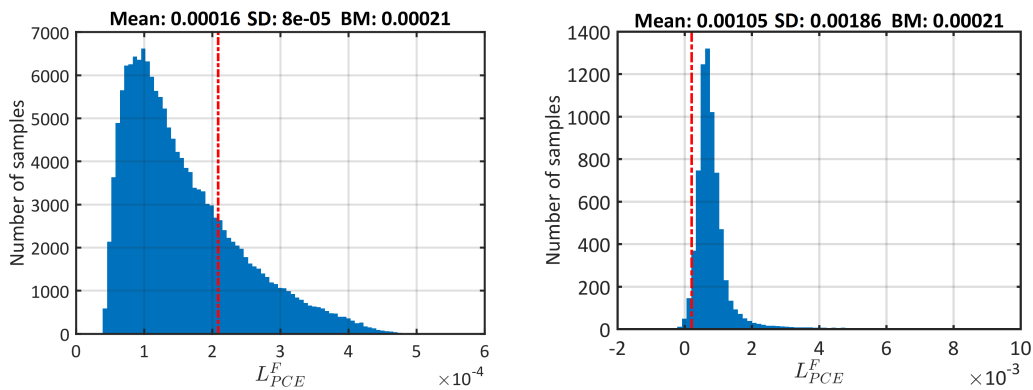
Figure 4.8: Histogram of L_{PCE}^F with n=100'000, and $r_D = -0.015$. The dotted red line provides the benchmark $\lambda = 0.003$, $\kappa = 0.024$, $\sigma = 0.16$, $\beta = 0.99$.

Global Sensitivity Analysis of EFG

Next we focus on the case where the central banker can engage in interest-rate forecasts and set a threshold value, π^c, at the same time. Thus, we investigate L^E with $b = 1$ and $\pi^c = \pi^c_{opt}$. Due to the larger computational burden, we reduce the polynomial degree to $p = 7$, which allows us to decrease N to 990. The results are summarized in Figure 4.9 and Table 4.4.

S_i^{tot} shows the same ranking as under IFG—in the narrow ranges: $\kappa > \lambda > \sigma > \beta$ and in the wide ranges: $\sigma > \lambda > \kappa > \beta$—with roughly the same indices' sizes. Furthermore, all the other results stay approximately the same compared to IFG, with only minor deviations in the size of the indices. This indicates that IFG and EFG coincide at $r_D = -0.015$ for the majority of parameter constellations.
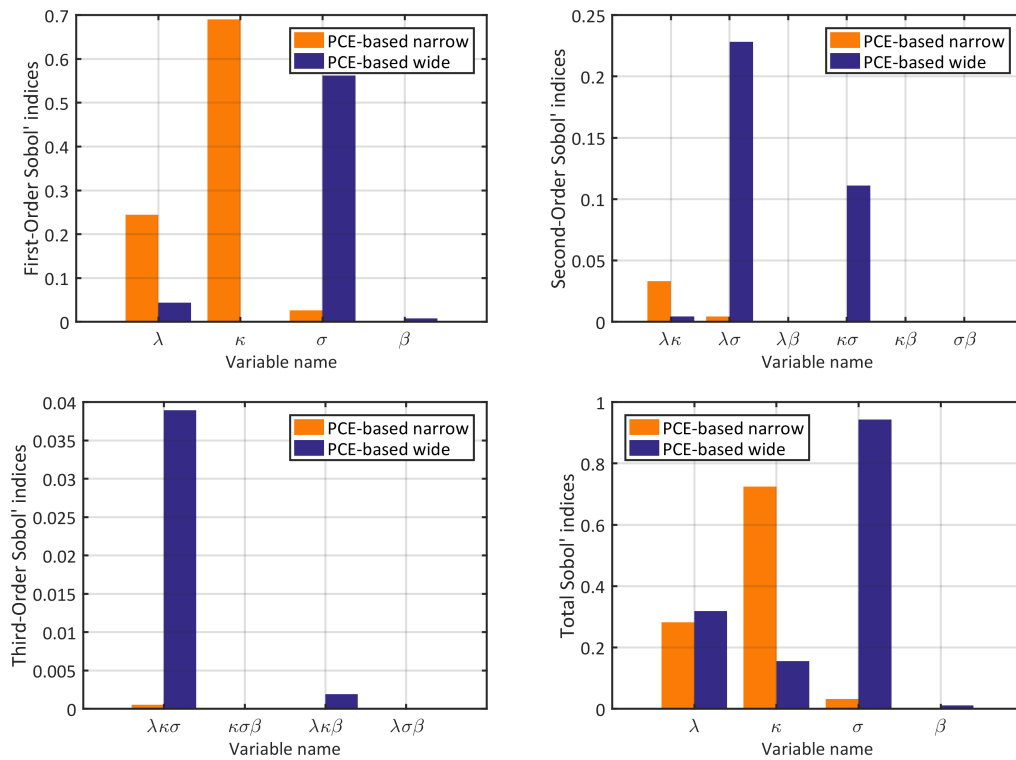
Figure 4.9: PCE-based Sobol' Indices of L^E with $p = 7$, $N = 990$, and $\pi^c = \pi^c_{opt}$.

Accuracy of PCE

The surrogate model roughly performs as under the IFG design, with a majority of the evaluations at the origin and some outliers to the right. The PCE with narrow parameter ranges achieves a high accuracy, with an err_{LOO} of $1.14 * 10^{-5}$. Under the wide parameter ranges the surrogate model barely reaches the accuracy threshold with an err_{LOO} of $9.54 * 10^{-2}$.

	λ	κ	σ	β		
S^{tot}_{narrow}	0.2823	0.7245	0.0319	0.0010		
S^{tot}_{wide}	0.3184	0.1554	0.9427	0.0109		
S^{1}_{narrow}	0.2442	0.6897	0.0264	0.0006		
S^{1}_{wide}	0.0438	0.0006	0.5623	0.0077		
	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^{2}_{narrow}	0.0332	0.0043	0	0.0007	0.0003	0
S^{2}_{wide}	0.0044	0.2280	0.0007	0.1109	0	0.0001

Narrow – Mean: 0.0002, SD: 0.0001, err_{LOO}: $1.14 * 10^{-5}$
Wide – Mean: 0.0010, SD: 0.0018, err_{LOO}: $9.54 * 10^{-2}$

Table 4.4: First- and second-order Sobol' Indices for L^{E}_{PCE} with $p = 7$, $N = 990$, $\pi^c = \pi^c_{opt}$.
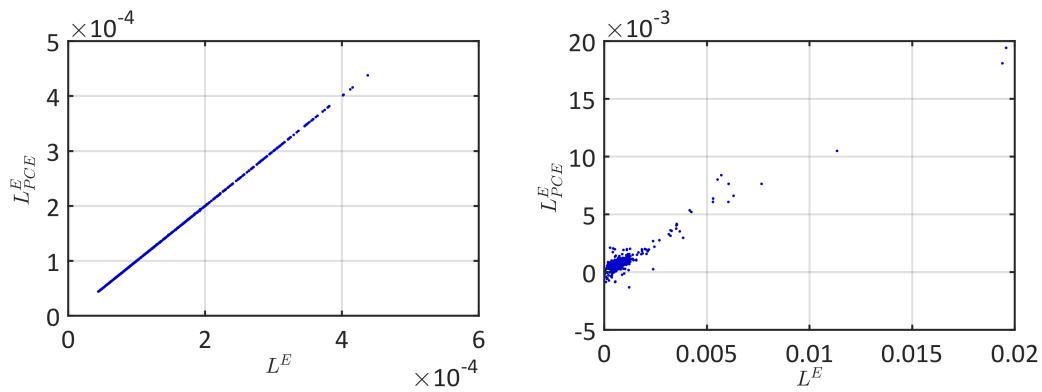
Figure 4.10: L^E surrogate model accuracy with $p = 7$, $N = 990$.

Relation to the Benchmark

Again, the histograms of EFG provide evidence that at $r_D = -0.015$, the inflation threshold is chosen such that the central banker never escapes under the majority of parameter constellations. Hence the statistics from the simulation are almost identical to the ones under the IFG design.
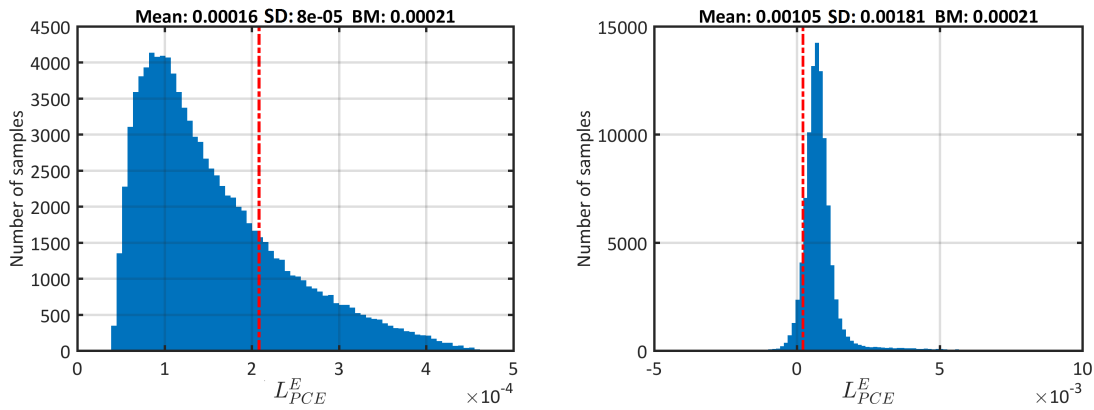
Figure 4.11: Histogram of L^E_{PCE} with n=100'000, $b = 1$, $\pi^c = \pi^c_{opt}$, and $r_D = -0.015$. The dotted red line provides the benchmark $\lambda = 0.003$, $\kappa = 0.024$, $\sigma = 0.16$, $\beta = 0.99$.

Global Sensitivity Analysis of SFG

Finally, we analyze SFG, i.e. L^S with $b_1 = b_2 = 1$. To minimize the computational burden, we use $p = 7$, $N = 990$ for the narrow range and only increase the polynomial degree for the wide range, with $p = 9$, $N = 1500$. The results are summarized in Figure 4.12 and Table 4.5.

The total Sobol' Indices rank the parameters $\sigma > \kappa > \lambda > \beta$ with narrow parameter ranges and $\sigma > \lambda > \kappa > \beta$ with wide parameter ranges. As before, β is unimportant in both instances. For narrow parameter ranges, first-order Sobol' Indices contribute the major part to the variation. Additionally, $S^2_{\kappa\sigma}$ has a significant effect. In the wide ranges S^1_σ and interactions with σ, i.e. $S^2_{\lambda\sigma}$ and $S^2_{\kappa\sigma}$, are of main importance. λ's first-order effect is the only other effect that contributes to the variation significantly. In both ranges, higher-order indices do not play an important role, except for $S^3_{\lambda\kappa\sigma}$ in the wide range with a value of 2.2%.
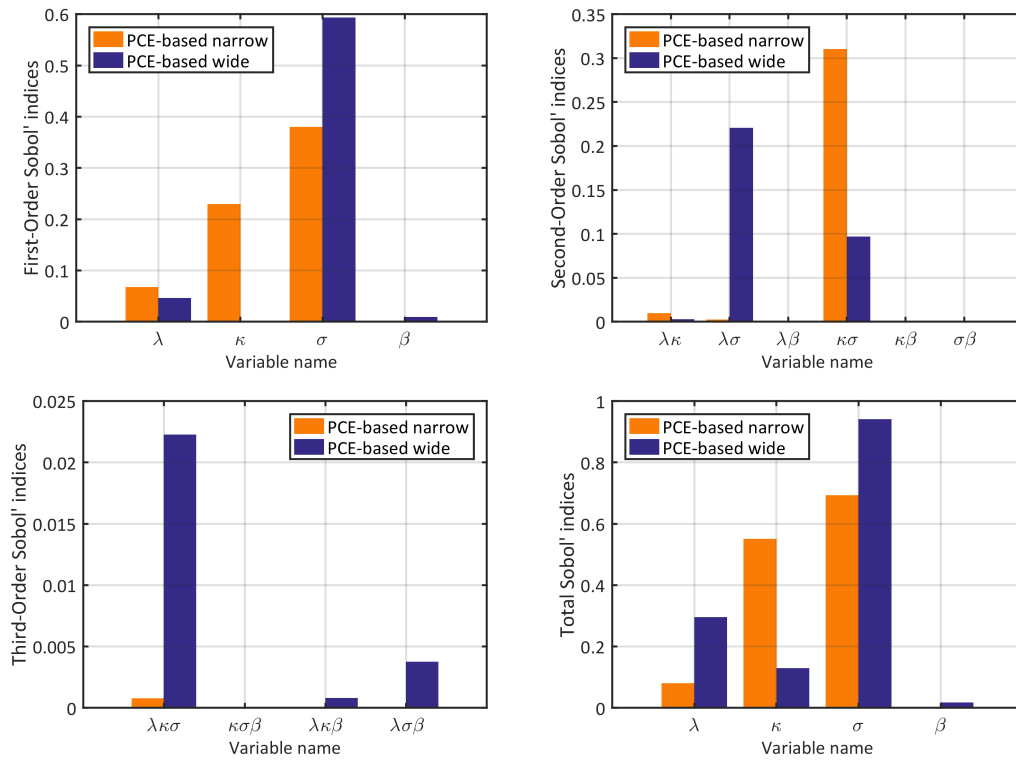
Figure 4.12: PCE-based Sobol' Indices of L^S with $p = 7$ and $N = 990$ for narrow ranges, $p = 9$ and $N = 1500$ for wide ranges, and $b_1 = b_2 = 1$.

	λ	κ	σ	β		
S^{tot}_{narrow}	0.0804	0.5503	0.6931	0.0003		
S^{tot}_{wide}	0.2951	0.1289	0.9406	0.0170		
S^1_{narrow}	0.0674	0.2293	0.3798	0.0002		
S^1_{wide}	0.0460	0.0006	0.5936	0.0089		

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^2_{narrow}	0.0099	0.0023	0	0.3102	0	0
S^2_{wide}	0.0028	0.2207	0.0005	0.0970	0.0005	0.0006

Narrow – Mean: 0.0002, SD: 0.0001, Coef. of Var.: 61%, err_{LOO}: $5.24 * 10^{-5}$
Wide – Mean: 0.0010, SD: 0.0017, Coef. of Var.: 170%, err_{LOO}: $4.05 * 10^{-2}$

Table 4.5: First- and second-order Sobol' Indices for L^S_{PCE} with $p = 7$ and $N = 990$ for narrow ranges, $p = 9$ and $N = 1500$ for wide ranges, and $b_1 = b_2 = 1$.

Accuracy of PCE

As in the analyses we made before, the surrogate model yields accurate results for narrow parameter distributions. The approximation in the wide parameter ranges is just sufficiently close. Figure 4.13 provides a visual assessment.
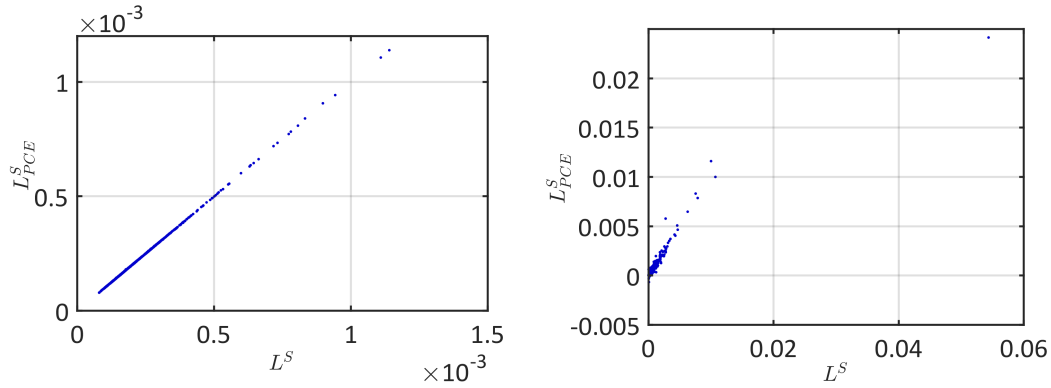
Figure 4.13: L^S surrogate model accuracy with $p = 7$, $N = 990$ for narrow ranges, $p = 9$ and $N = 1500$ for wide ranges, and $b_1 = b_2 = 1$.

Relation to the Benchmark

In Figure 4.14, we see that the benchmark parameterization is close to the mode and at the mean of the distribution in the narrow ranges. In the wide ranges, the benchmark is located in the left tail of the distribution.
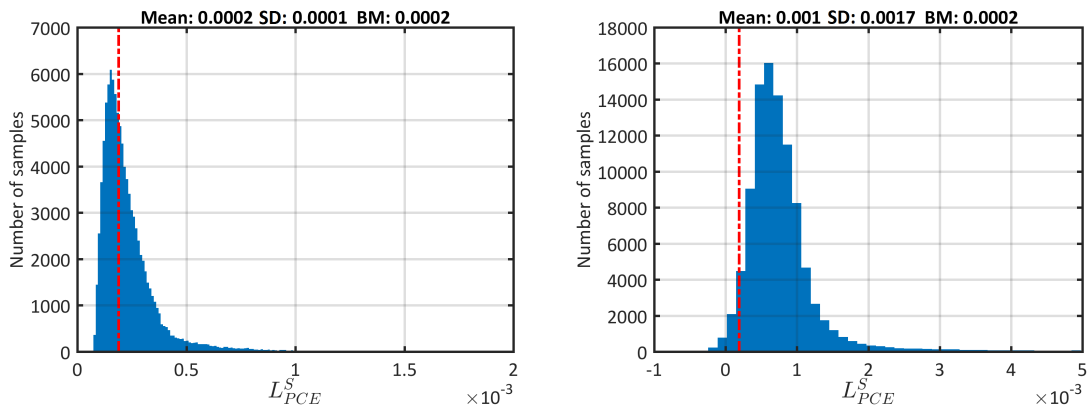
Figure 4.14: Histogram of L^S_{PCE} with n=100'000, $b_1 = b_2 = 1$, and $r_D = -0.015$. The dotted red line provides the benchmark $\lambda = 0.003$, $\kappa = 0.024$, $\sigma = 0.16$, $\beta = 0.99$.

Summary of GSA Results and Robustness

Narrow Parameter Range

In Table 4.6, we summarize the total Sobol' Indices for the narrow parameter range at three r_D sizes. A main insight we gain from the analysis above, at $r_D = -0.015$ is that output variation is mainly driven by the first-order Sobol' Indices. Second-order indices play a minor role and none surpasses 4%, except $\kappa\sigma$ in NFG and SFG at $r_D = -0.015$. Hence, the contributions of the parameters to the output variation are mostly linear and additive. However, given a large negative r_D value, i.e. $r_D = -0.03$, the second-order Sobol' Index of $\kappa\sigma$ is important in all forward guidance designs with a size of roughly 24%. Given a small r_D value, i.e. $r_D = -0.005$, the interaction of $\lambda\kappa$ becomes important in EFG (5.3%), SFG (5.6%), IFG (15%), and NFG (25%).

Furthermore, β is an unimportant parameter with a total Sobol' Index of below 1% for all shock sizes r_D. λ's importance changes from being close to negligible for severely negative r_D values to being the second-to-most important parameter given mildly negative r_D values. The importance of σ depends on the size of r_D and the specific design under consideration. At small shocks and under flexible designs, σ is unimportant but S_σ^{tot} increases for more severe shocks. For all designs and shock sizes, κ is either the most important or second-to-most important parameter.

To sum up, the importance ranking of the parameters depends on the size of r_D. When the natural real interest-rate shock becomes very severe, the ranking of S_i^{tot} indices converges under all designs to κ ($\approx 64\%$) $> \sigma$ ($\approx 59\%$) $> \lambda$ ($\approx 1\%$) $> \beta$ ($\approx 0\%$). When r_D approaches zero, the ranking and the size of total indices become more heterogeneous. In general, we can state that for the designs NFG, EFG, and SFG, the ranking is $\kappa > \lambda > \sigma > \beta$, where contributions of σ and β to the output variation are negligible. For IFG the ranking is $\kappa > \lambda > \sigma > \beta$. Here only β's contribution is negligible.

Wide Parameter Range

In Table 4.7 we summarize total Sobol' Indices for the wide parameter range at the three sizes of r_D. Given $r_D = -0.005$, the first-order Sobol' Indices cover the most important effects. Only $S_{\lambda\sigma}^2 = 9\%$ and $S_{\kappa\sigma}^2 = 12\%$ under NFG contribute crucially to the output variation. At $r_D = -0.015$ and $r_D = -0.03$, the interaction terms $\lambda\sigma$ and $\kappa\sigma$ become relevant for all designs and are in the range $15\% - 23\%$ and $10\% - 19\%$, respectively. As r_D becomes severe, β is not important and can be set to a constant as under the narrow parameter ranges. It is only under mild shocks, β is ranked as the most important factor. σ, on the other hand, is the most important factor for severe r_D shocks and the least important for small values (however, still relevant at $r_D = -0.005$ but not at $r_D = -0.001$ in all examples, for instance). λ and κ keep their ordering and also, roughly, their size.

Compared to the narrow range, the ranking of the parameters is more uniform over r_D sizes, especially among the forward guidance designs. As r_D gets closer to zero, the ranking $\beta \, (\approx 50\%) > \lambda \, (\approx 40\%) > \kappa \, (\approx 15\%) > \sigma \, (\approx 0\%)$ emerges for all designs. For r_D more and more negative the ranking converges to $\sigma \, (\approx 97\%) > \kappa \, (\approx 22\%) > \lambda \, (\approx 20\%) > \beta \, (\approx 0\%)$.

		Narrow			
$r_D = -0.005$	S_{NFG}^{tot}	σ	κ	λ	β
		(0.64)	(0.45)	(0.18)	(0)
	S_{IFG}^{tot}	κ	λ	σ	β
		(0.67)	(0.38)	(0.12)	(0)
	S_{EFG}^{tot}	κ	λ	σ	β
		(0.84)	(0.21)	(0)	(0)
	S_{SFG}^{tot}	κ	λ	σ	β
		(0.81)	(0.24)	(0)	(0)
$r_D = -0.015$	S_{NFG}^{tot}	σ	κ	λ	β
		(0.63)	(0.61)	(0.02)	(0)
	S_{IFG}^{tot}	κ	λ	σ	β
		(0.71)	(0.29)	(0.04)	(0)
	S_{EFG}^{tot}	κ	λ	σ	β
		(0.72)	(0.28)	(0.03)	(0)
	S_{SFG}^{tot}	σ	κ	λ	β
		(0.69)	(0.55)	(0.08)	(0)
$r_D = -0.03$	S_{NFG}^{tot}	κ	σ	λ	β
		(0.63)	(0.60)	(0.01)	(0)
	S_{IFG}^{tot}	σ	κ	λ	β
		(0.67)	(0.54)	(0.05)	(0)
	S_{EFG}^{tot}	σ	κ	λ	β
		(0.65)	(0.54)	(0.06)	(0)
	S_{SFG}^{tot}	κ	σ	λ	β
		(0.64)	(0.58)	(0.01)	(0)

Table 4.6: Total Sobol' Indices for L_{PCE}^N, L_{PCE}^F, L_{PCE}^E, and L_{PCE}^S and narrow parameter ranges.

Wide					
$r_D = -0.005$	S_{NFG}^{tot}	σ	κ	β	κ
		(0.60)	(0.31)	(0.19)	(0.17)
	S_{IFG}^{tot}	β	λ	κ	σ
		(0.43)	(0.43)	(0.14)	(0.13)
	S_{EFG}^{tot}	β	λ	κ	σ
		(0.43)	(0.41)	(0.15)	(0.13)
	S_{SFG}^{tot}	β	λ	κ	σ
		(0.45)	(0.42)	(0.14)	(0.10)
$r_D = -0.015$	S_{NFG}^{tot}	σ	κ	λ	β
		(0.96)	(0.22)	(0.20)	(0.01)
	S_{IFG}^{tot}	σ	λ	κ	β
		(0.95)	(0.30)	(0.19)	(0.01)
	S_{EFG}^{tot}	σ	λ	κ	β
		(0.94)	(0.32)	(0.16)	(0.01)
	S_{SFG}^{tot}	σ	λ	κ	β
		(0.94)	(0.30)	(0.13)	(0.02)
$r_D = -0.03$	S_{NFG}^{tot}	σ	κ	λ	β
		(0.97)	(0.23)	(0.20)	(0)
	S_{IFG}^{tot}	σ	λ	κ	β
		(0.97)	(0.23)	(0.21)	(0)
	S_{EFG}^{tot}	σ	λ	κ	β
		(0.96)	(0.26)	(0.17)	(0.01)
	S_{SFG}^{tot}	σ	λ	κ	β
		(0.97)	(0.25)	(0.18)	(0)

Table 4.7: Total Sobol' Indices for L_{PCE}^N, L_{PCE}^F, L_{PCE}^E, and L_{PCE}^S and wide parameter ranges.

4.3.2 Welfare Gains

Gains Compared to NFG

In Figure 2.16 we, show that for a large natural real interest-rate shock—at $r_D = -0.015$ for example—the loss function of NFG constitutes the upper bound and SFG the lower bound, given the menu of different designs. Yet which parameters influence the differences in the loss functions, in particular? To answer this question, we investigate

$$[L^N(\lambda, \kappa, \sigma, \beta, r_D) - L^p(\lambda, \kappa, \sigma, \beta, r_D)|r_D = r_{D,i}]_{PCE} \quad \text{where } p \in \{F, E, S\}, \quad (4.31)$$

with $b = 1$ and when r_D has three values $r_D \in \{-0.005, -0.015, -0.03\}$.

Narrow Parameter Range

The results in Table 4.8 show that within the narrow parameter ranges, the ranking of the total Sobol' Indices stays the same for all r_D sizes, i.e. $\kappa > \sigma > \lambda > \beta$ have roughly the same index sizes. The exception is $S^{tot}_{NFG-IFG}$, with $r_D = -0.005$, where σ exerts a larger influence on the difference than under increasingly negative r_D values. In all cases, β's total index is smaller than 1% and can be set to a constant within the range. λ is often close to the 1% threshold and could be set to a constant in the narrow range, as it only contributes a negligible part to total variation. The variation in the differences is driven by $\kappa \approx 63 - 67\%$ and $\sigma \approx 54 - 62\%$. Therefore, the slope of the Phillips Curve and the inter-temporal elasticity of substitution are the crucial parameters when calibrating the designs.

Wide Parameter Ranges

For the wide ranges, the ranking is $\sigma > \kappa > \lambda > \beta$, and it is consistent through all three shock sizes. As before, β's effect is smaller than 1% and thus has only a negligible effect on the total variation. $\lambda \approx 5 - 12\%$ and $\kappa \approx 23 - 28\%$ contribute a minor part to the variation. The inter-temporal elasticity of substitution σ is the dominating parameter with total Sobol' Indices of $95 - 98\%$.

In general, the results imply that σ and κ are crucial for the determination of the differences in expected loss functions.

Narrow					
$r_D = -0.005$:	$S^{tot}_{NFG-IFG}$	σ	κ	λ	β
		(0.78)	(0.57)	(0.09)	(0)
	$S^{tot}_{NFG-EFG}$	κ	σ	λ	β
		(0.63)	(0.59)	(0.02)	(0)
	$S^{tot}_{NFG-SFG}$	κ	σ	λ	β
		(0.64)	(0.62)	(0.02)	(0)
$r_D = -0.015$:	$S^{tot}_{NFG-IFG}$	κ	σ	λ	β
		(0.65)	(0.58)	(0.01)	(0)
	$S^{tot}_{NFG-EFG}$	κ	σ	λ	β
		(0.66)	(0.56)	(0.01)	(0)
	$S^{tot}_{NFG-SFG}$	κ	σ	λ	β
		(0.63)	(0.57)	(0.02)	(0)
$r_D = -0.03$:	$S^{tot}_{NFG-IFG}$	κ	σ	λ	β
		(0.67)	(0.54)	(0.01)	(0)
	$S^{tot}_{NFG-EFG}$	κ	σ	λ	β
		(0.67)	(0.54)	(0.01)	(0)
	$S^{tot}_{NFG-SFG}$	κ	σ	λ	β
		(0.63)	(0.57)	(0.02)	(0)
Wide					
$r_D = -0.005$:	$S^{tot}_{NFG-IFG}$	σ	κ	λ	β
		(0.98)	(0.28)	(0.12)	(0)
	$S^{tot}_{NFG-EFG}$	σ	κ	λ	β
		(0.97)	(0.27)	(0.14)	(0)
	$S^{tot}_{NFG-SFG}$	σ	κ	λ	β
		(0.97)	(0.23)	(0.14)	(0.01)
$r_D = -0.015$:	$S^{tot}_{NFG-IFG}$	σ	κ	λ	β
		(0.96)	(0.27)	(0.07)	(0.01)
	$S^{tot}_{NFG-EFG}$	σ	κ	λ	β
		(0.96)	(0.27)	(0.07)	(0.01)
	$S^{tot}_{NFG-SFG}$	σ	κ	λ	β
		(0.95)	(0.24)	(0.10)	(0)
$r_D = -0.03$:	$S^{tot}_{NFG-IFG}$	σ	κ	λ	β
		(0.95)	(0.27)	(0.06)	(0.01)
	$S^{tot}_{NFG-EFG}$	σ	κ	λ	β
		(0.96)	(0.28)	(0.05)	(0.01)
	$S^{tot}_{NFG-SFG}$	σ	κ	λ	β
		(0.96)	(0.25)	(0.06)	(0.01)

Table 4.8: Total Sobol' Indices for $[L^N - L^F | r_D]_{PCE}$, $[L^N - L^E | r_D]_{PCE}$, and $[L^N - L^S | r_D]_{PCE}$.

Relation to Benchmark

To provide a more complete picture, we now plot the histograms of the differences between the loss functions. Formally, we analyze

$$[L^N(\lambda, \kappa, \sigma, \beta, r_D) - L^p(\lambda, \kappa, \sigma, \beta, r_D)|r_D = r_{D,i}]_{PCE} \quad \text{where } p \in \{F, E, S\}, \quad (4.32)$$

at $r_D = -0.015$. This provides an insight if there exist parameter constellations that lead to a reversal of the results in Figure 2.16.

The histograms in Figure 4.15 are the empirical distributions, at $r_D = -0.015$ with narrow parameter ranges for 100'000 sample draws from the parameter distributions. The red vertical line marks the benchmark results. Besides confirming that all means are positive, the histograms show that all parameter constellations of all forward guidance designs yield lower losses than NFG. Larger r_D shocks do not change the shape of the distributions but shift it to the right. At smaller r_D shocks, i.e. at $r_D = -0.005$, the characteristic shape of the histograms stays the same as well, except for $[L^N - L^F|r_D = -0.005]_{PCE}$ where the mode and the mean of the histogram shift to negative territory.
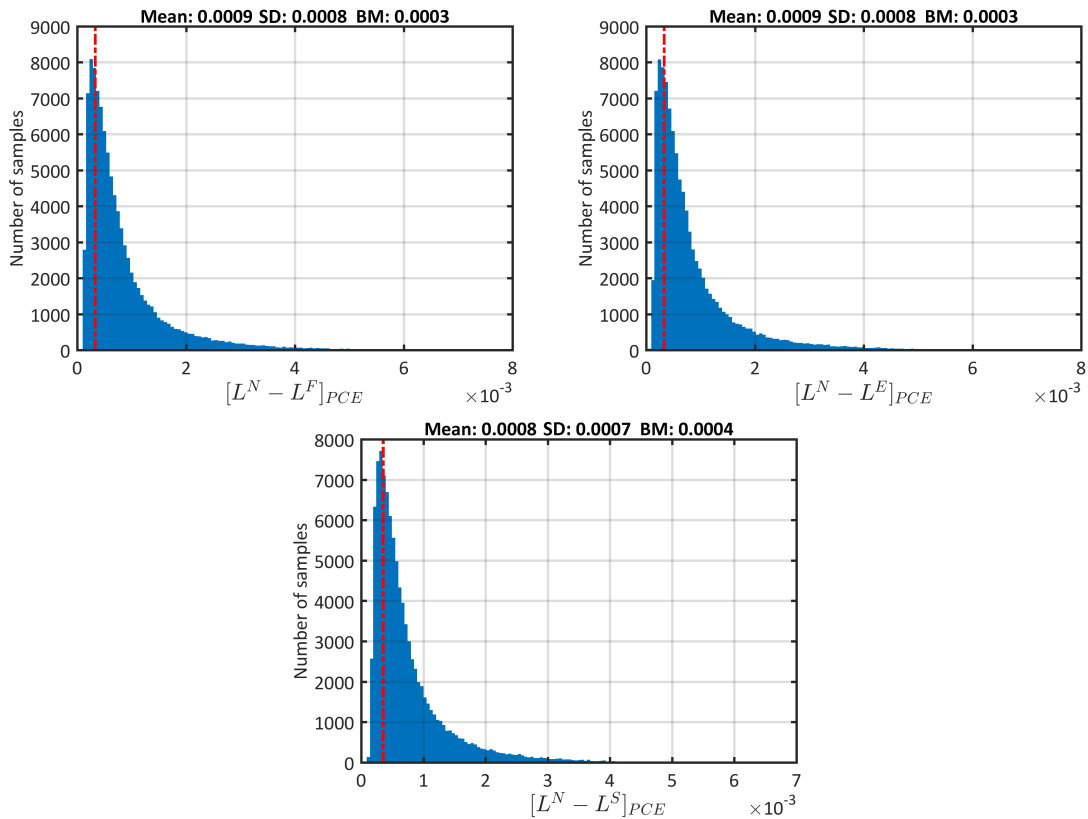
Figure 4.15: Histograms of $[L^N - L^F|r_D = -0.015]_{PCE}$, $[L^N - L^E|r_D = -0.015]_{PCE}$, and $[L^N - L^S|r_D = -0.015]_{PCE}$ for narrow parameter ranges.

In the wide parameter ranges, some differences are negative, which indicates that for some particular parameter constellations, the discretionary approach is superior, even for a shock size of $r_D = -0.015$. The results are displayed in Figure 4.16. The tendency is that small λ values, small κ values, and some particular σ values lead to the negative differences. Small σ, in particular, are never accompanied by a negative difference. Figures C.9 to C.11 in the Appendix provide an indicative graphical illustration of these findings. The results are robust to larger r_D shocks. At smaller shock sizes, the mode and mean of $[L^N - L^F|r_D]_{PCE}$ and $[L^N - L^S|r_D]_{PCE}$ shift to negative values.
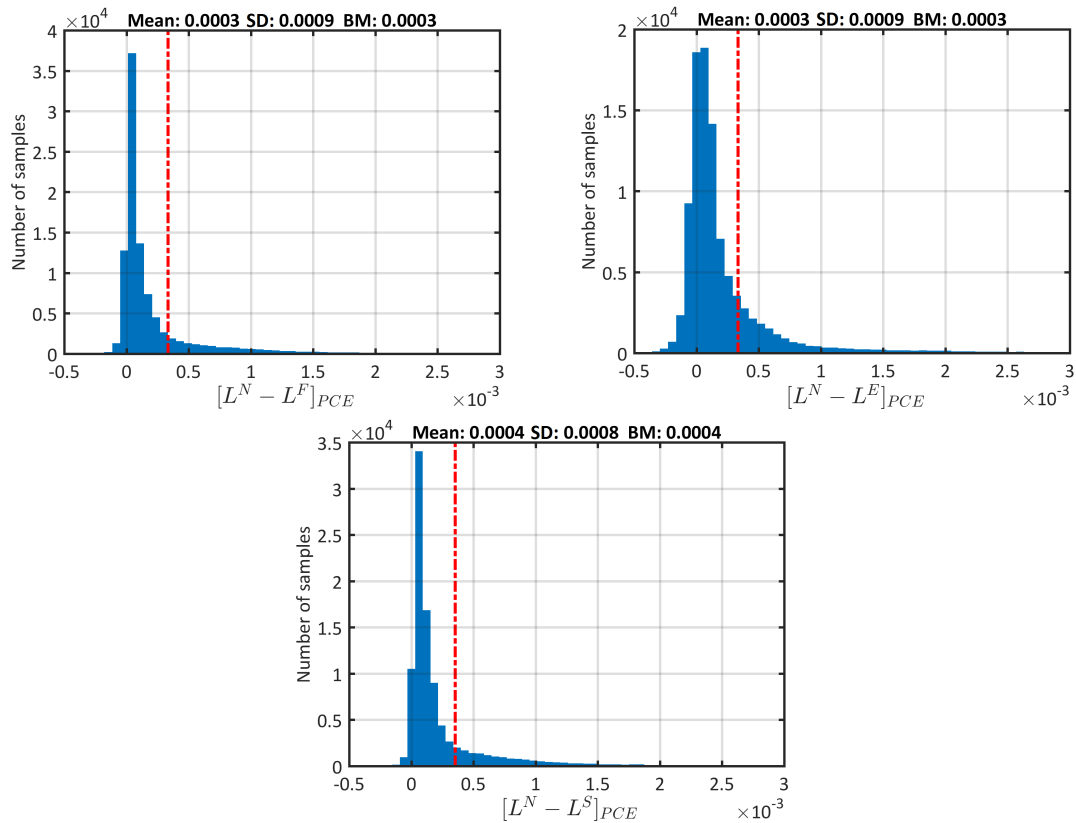
Figure 4.16: Histograms of $[L^N - L^F|r_D = -0.015]_{PCE}$, $[L^N - L^E|r_D = -0.015]_{PCE}$, and $[L^N - L^S|r_D = -0.015]_{PCE}$ for wide parameter ranges.

Gains Compared to IFG

In the beginning of Subsection 4.3.2, we compared forward guidance designs to the discretionary central banker. Next we compare the flexible designs EFG and SFG to the more rigid design IFG. Formally, we analyze

$$[L^F(\lambda, \kappa, \sigma, \beta, r_D) - L^p(\lambda, \kappa, \sigma, \beta, r_D)|r_D = r_{D,i}]_{PCE} \quad \text{where } p \in \{E, S\}, \quad (4.33)$$

with $b = 1$ and $r_D \in \{-0.005, -0.015, -0.03\}$. One remark is in order at this stage. In the environment of larger r_D values, PCE does not approximate the true model well under a reasonable truncation set, which allows a GSA in a reasonable computation time. The results are summarized in Table 4.9.

Narrow Parameter Range

Under the narrow parameter range, $[L^F - L^E|r_D = -0.015, -0.03]_{PCE}$ scores err_{LOO}s of the magnitude $5 * 10^{-1}$ and $3 * 10^{-1}$, respectively. This may be due to the fact that the two designs start to overlap as soon as a critical shock size is reached. The importance rankings that pass the accuracy threshold yield a ranking $\kappa > \sigma > \lambda > \beta$, where β is unimportant. λ becomes less important as the shock size increases. The slope of the Phillips Curve, in particular, and the inter-temporal elasticity of substitution are important in determining the difference of IFG and EFG—for small shocks where IFG and EFG differ—and IFG and SFG over all shock sizes.

Wide Parameter Range

Using wide parameter ranges and given $r_D = -0.03$, the PCE approximation of $L^F - L^E$ scores an err_{LOO} of $1 * 10^{-1}$ and $L^F - L^S$ does not return any results, which prohibits a valid analysis. Therefore, the respective results in Table 4.9 have to be treated with appropriate caution. Some general statements can still be made for smaller shocks. First, β plays a minor role and is ranked least important of all variations. λ's importance decreases as the shock size increases. κ's importance, on the other hand, increases with the shock size. σ is the most important parameter in the calibration.

		Narrow			
$r_D = -0.005$:	$S^{tot}_{IFG-EFG}$	κ	σ	λ	β
		(0.81)	(0.19)	(0.04)	(0)
	$S^{tot}_{IFG-SFG}$	κ	σ	λ	β
		(0.82)	(0.18)	(0.06)	(0)
$r_D = -0.015$:	$S^{tot}_{IFG-EFG}$	σ	λ	κ	β
		(0.81)	(0.79)	(0.73)	(0.04)
	$S^{tot}_{IFG-SFG}$	κ	σ	λ	β
		(0.76)	(0.50)	(0)	(0)
$r_D = -0.03$:	$S^{tot}_{IFG-EFG}$	σ	λ	κ	β
		(0.79)	(0.78)	(0.76)	(0.01)
	$S^{tot}_{IFG-SFG}$	κ	σ	λ	β
		(0.76)	(0.48)	(0)	(0)
		Wide			
$r_D = -0.005$:	$S^{tot}_{IFG-EFG}$	σ	λ	κ	β
		(0.94)	(0.55)	(0.10)	(0.03)
	$S^{tot}_{IFG-SFG}$	σ	λ	κ	β
		(0.70)	(0.45)	(0.31)	(0.08)
$r_D = -0.015$:	$S^{tot}_{IFG-EFG}$	σ	κ	λ	β
		(0.98)	(0.70)	(0.26)	(0)
	$S^{tot}_{IFG-SFG}$	σ	κ	λ	β
		(0.97)	(0.59)	(0.28)	(0.05)
$r_D = -0.03$:	$S^{tot}_{IFG-EFG}$	σ	κ	λ	β
		(0.97)	(0.70)	(0.21)	(0.04)
	$S^{tot}_{IFG-SFG}$				
		(NaN)	(NaN)	(NaN)	(NaN)

Table 4.9: Total Sobol' Indices for $[L^F - L^E|r_D]_{PCE}$ and $[L^F - L^S|r_D]_{PCE}$.

Relation to the Benchmark

Next, we analyze the histograms of

$$[L^F(\lambda, \kappa, \sigma, \beta, r_D) - L^p(\lambda, \kappa, \sigma, \beta, r_D)|r_D = r_{D,i}]_{PCE} \quad \text{where } p \in \{E, S\} \quad (4.34)$$

at $r_D = -0.015$ and $b = 1$.

Figure 4.17 shows the histograms of $[L^F - L^E|r_D = -0.015]_{PCE}$ and $[L^F - L^S|r_D = -0.015]_{PCE}$ of 100'000 evaluations of random parameter draws for the narrow parameter ranges. $[L^F - L^E|r_D = -0.015]_{PCE}$ values are distributed closely around zero in the range of 10^{-7}. L^F is the upper bound of L^E by construction, hence negative values originate from inaccuracies in the estimation process—because the positive values are of

the same magnitude, this is likely also true for them. Indeed, the $err_{LOO} = 10^{-1}$ is above the accuracy threshold we use. The histogram of $[L^F - L^S | r_D = -0.015]_{PCE}$ shows that the differences' magnitude of 10^{-4} is significant and there is a cut-off very close to zero suggesting that it is beneficial to provide an unconstrained forecast in most cases. Given $[L^F - L^E | r_D = -0.005]_{PCE}$ and $[L^F - L^S | r_D = -0.005]_{PCE}$ and narrow parameter ranges, all evaluations are positive and our benchmark calibration is roughly at the mode of the distributions. At $r_D = -0.03$ only $[L^F - L^S]_{PCE}$ yields a sufficiently accurate model approximation. The distribution is heavily left-skewed with a cut-off at zero, as in Figure 4.17.
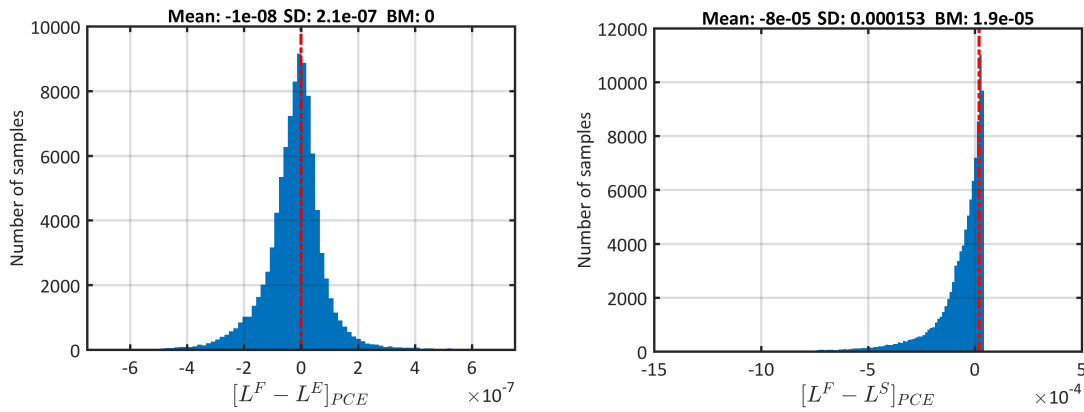
Figure 4.17: Histograms of $[L^F - L^E | r_D = -0.015]_{PCE}$ and $[L^F - L^S | r_D = -0.015]_{PCE}$.

For the wide parameter ranges, we do not obtain accurate results with surrogate models that are calculated in a reasonable time span, except for $[L^F - L^E | r_D = -0.005]_{PCE}$ and $[L^F - L^S | r_D = -0.005]_{PCE}$. In both instances, the benchmark lies in the right tail of the distribution. Mean and mode are positive, but more conservative in favoring the flexible forward guidance designs.

Gains in Applying EFG vs. SFG

Finally, we directly compare the two flexible forward guidance designs EFG and SFG

$$[L^E(\lambda, \kappa, \sigma, \beta, r_D) - L^S(\lambda, \kappa, \sigma, \beta, r_D) | r_D = r_{D,i}]_{PCE}, \qquad (4.35)$$

with $b = 1$ and $r_D \in \{-0.005, -0.015, -0.03\}$.

Narrow Parameter Range

In the narrow parameter ranges, β and λ are not particularly important for determining the difference of the two designs. κ and σ, on the other hand, play the major roles and

heavily impact the gap between the two designs. $S_\kappa^{tot} \approx 75\%$ for all shock sizes, while S_σ^{tot} varies from 24% at small shocks to 50% for larger shocks.

Wide Parameter Range

Again, we have to note that when we use the wide parameter ranges, the PCE approximation does not work well for reasonable truncation sets. To be more precise, at $r_D = -0.005$ we obtain an err_{LOO} of $4*10^{-1}$, at $r_D = -0.015$ of $2*10^{-1}$, and for $r_D = -0.03$ of $4*10^{-1}$. In the wide ranges, the results suggest that σ is the most important parameter for all r_D sizes. λ is important for r_D close to zero and becomes unimportant as r_D decreases. In contrast, κ is crucial for severe r_D shocks and becomes unimportant as r_D approaches zero. Finally, β is mostly least important parameter.

		Narrow			
$r_D = -0.005$:	$S_{EFG-SFG}^{tot}$	κ	σ	λ	β
		(0.75)	(0.24)	(0.05)	(0)
$r_D = -0.015$:	$S_{EFG-SFG}^{tot}$	κ	σ	λ	β
		(0.76)	(0.50)	(0)	(0)
$r_D = -0.03$:	$S_{EFG-SFG}^{tot}$	κ	σ	λ	β
		(0.75)	(0.48)	(0)	(0)
		Wide			
$r_D = -0.005$:	$S_{EFG-SFG}^{tot}$	σ	λ	κ	β
		(0.84)	(0.46)	(0.29)	(0.09)
$r_D = -0.015$:	$S_{EFG-SFG}^{tot}$	σ	κ	β	λ
		(0.97)	(0.53)	(0.14)	(0.09)
$r_D = -0.03$:	$S_{EFG-SFG}^{tot}$	σ	κ	λ	β
		(0.98)	(0.60)	(0.03)	(0.02)

Table 4.10: Total Sobol' Indices for $[L^E - L^S|r_D]_{PCE}$.

Relation to Benchmark

In a last robustness exercise, we compare the two flexible forward guidance designs EFG and SFG. Formally, we analyze

$$[L^E(\lambda, \kappa, \sigma, \beta, r_D) - L^S(\lambda, \kappa, \sigma, \beta, r_D)|r_D = r_{D,i}]_{PCE}. \tag{4.36}$$

We focus on the narrow parameter ranges because the PCE approximation does not work well with wide parameter ranges and practicable truncation sets. The histogram in Figure

4.18 again provides a more detailed perspective at $r_D = -0.015$. The benchmark calibration suggests that SFG should be preferred to EFG. However, drawing random values from the narrow ranges shows that the distribution is highly left-skewed, and the main part lies in negative territory, indicating that EFG yields lower losses than SFG. Because L^E is equal to L^F at a shock size of $r_D = -0.015$, the histograms in Figure 4.17 and 4.18 are almost identical. The shape of the histogram is robust with respect to larger shock sizes but the distribution shifts to the right. For shocks of $r_D = -0.005$, the distribution has a negative mode, a positive mean, and a strict cut-off in the left tail of the distribution.
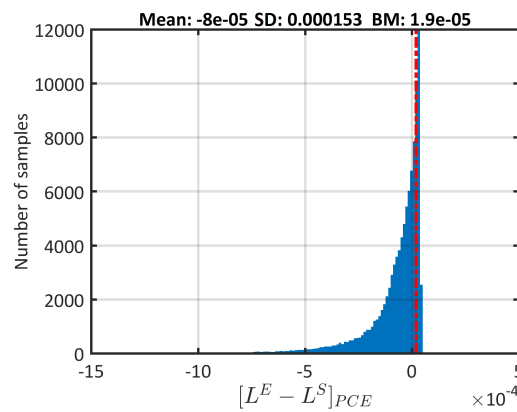
Figure 4.18: Histogram of $[L^E - L^S | r_D = -0.015]_{PCE}$.

4.3.3 Standard New Keynesian Model

Global Sensitivity Analysis on Canonical New Keynesian Model

The New Keynesian Model is a standard framework in monetary policy. Hence, it is instructive to perform a GSA of the New Keynesian Model usually used in the literature. Specifically we analyze the model with a supply shock but without a ZLB constraint. More formally, we investigate the loss function l_H^N, which is stated in Equation (2.4) and depicts the loss function of a discretionary central banker in Phase H. Note that $l_H^N(\lambda, \kappa, \beta)$ is not a function of σ, as both inflation π_t^N in Equation (A.8) and output gap x_t^N in Equation (A.9) only contain λ, κ, and β.[14] We truncate the PCE at $p = 5$ and use $N = 378$ samples in the experimental design. The results are summarized in Table 4.11 and Figure 4.19. The approximation works for narrow and wide ranges, with an err_{LOO} of $1.91 * 10^{-6}$ and $1.3 * 10^{-3}$, respectively. Figure 4.20 displays the accuracy.

Narrow Parameter Range

A first look at the higher-order Sobol' Indices shows that S_i^3 and S_i^4 are of negligible size, as they are of magnitudes 10^{-5} – 10^{-9}. Second-order indices are of magnitude 10^{-4} – 10^{-9}, and thus are negligible as well. $S_{\lambda\kappa}^2$ is an exception, with a contribution of 3.9% to the total variation. The analysis shows that the first-order Sobol' Indices S_λ^1 and S_κ^1 play the major role in contributing to the output variability. 80% of the total variability can be attributed to κ and 15% to λ. Accordingly, the total impact of κ—measured by S_κ^{tot}—dominates and is followed by λ. S_λ^{tot} is approximately four times smaller than the effect of κ. σ and β do not visibly contribute to the total variability. Hence, they can be set to constants within the parameter range we specified above without greatly impacting the loss function. Selecting κ and λ is more delicate. S_i^1 and S_i^{tot} are very similar, which implies that interaction terms are unimportant and that the model is close to additive. This result can already be inferred by Equations (A.8) and (A.9), which make up the terms in the loss function. In these equations, κ and λ are dominant but not strongly entangled.

Wide Parameter Range

When we use wide parameter ranges, β becomes an important factor. In fact, the importance ranking is $\lambda > \beta > \kappa > \sigma$. The focus shifts from the slope of the Phillips Curve to the weight of output in the loss function and the discount factor. First-order indices remain the dominant force but interactions do play a minor role as well with $S_{\lambda\kappa}^2 = 3.9\%$, $S_{\lambda\beta}^2 = 1.8\%$, and $S_{\kappa\beta}^2 = 1.1\%$.

---

[14] To determine Equations (A.8) and (A.9), we only need the first order condition of the loss function (A.1) and the Phillips Curve (2.2). Because the ZLB is not binding, x_t^N can be thought of as being directly
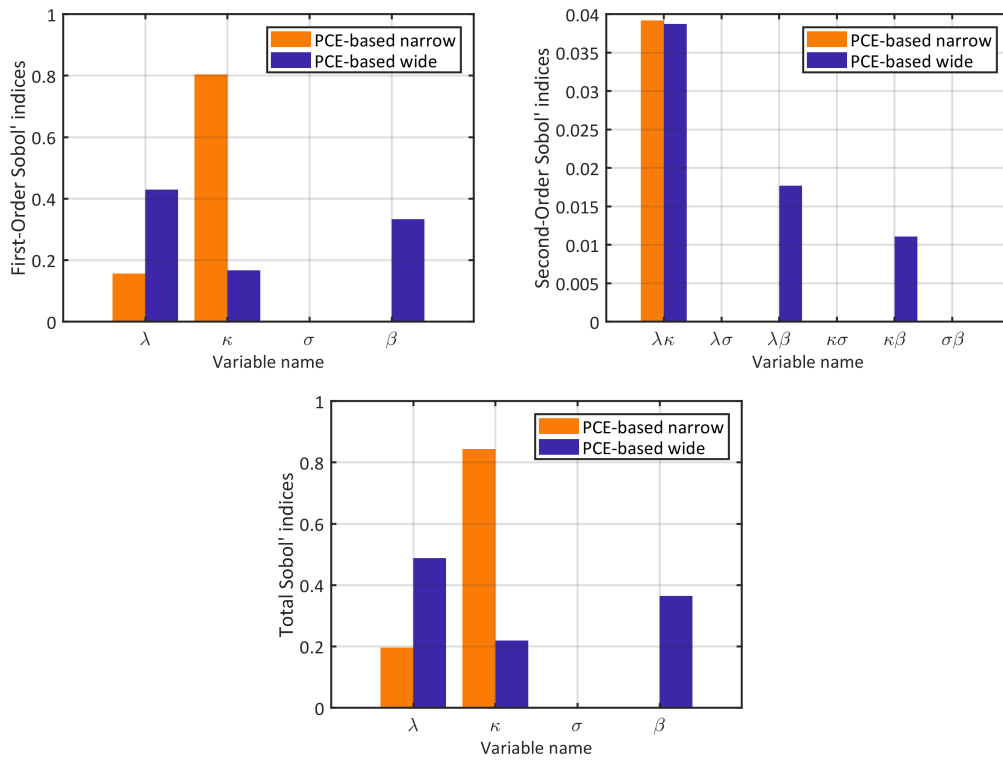
Figure 4.19: PCE-based Sobol' Indices of l_H^N with $p = 5$, $N = 378$.
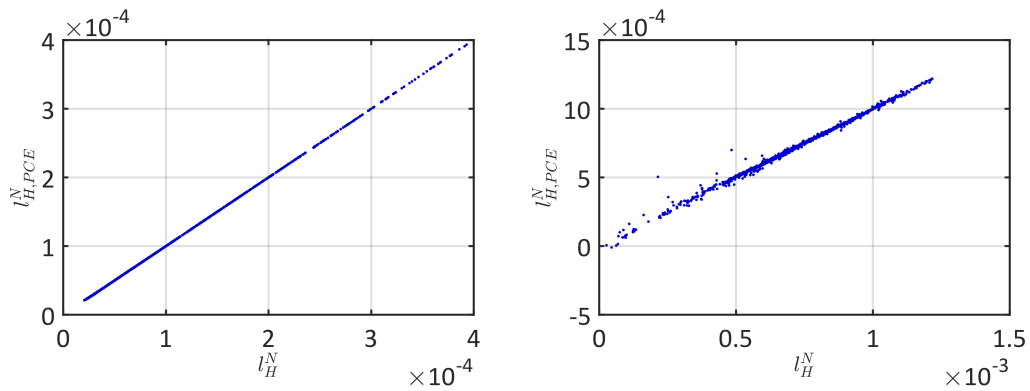
Figure 4.20: l_H^N surrogate model accuracy with $p = 5$, $N = 378$.

controllable by the central bank. Hence the IS Curve, which introduces σ, is not relevant.

	λ	κ	σ	β
S^{tot}_{narrow}	0.1959	0.8431	0	0.0003
S^{tot}_{wide}	0.4878	0.2192	0.0006	0.3644
S^{1}_{narrow}	0.1567	0.8038	0	0.0001
S^{1}_{wide}	0.4292	0.1674	0	0.3336

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^{2}_{narrow}	0.0392	0	0	0	0.0001	0
S^{2}_{wide}	0.0387	0	0.0177	0	0.0111	0.0001

Narrow – Mean: 0.0001, SD: 0.0001, Coef. of Var.: 67%, err_{LOO}: $1.91 * 10^{-6}$

Wide – Mean: 0.0007, SD: 0.0002, Coef. of Var.: 30%, err_{LOO}: $1.3 * 10^{-3}$

Table 4.11: First- and second-order Sobol' Indices for l^{N}_{H} with $p = 5$, $N = 378$.

Global Sensitivity Analysis on a New Keynesian Model with ZLB

To complete the picture of the sensitivity properties of the New Keynesian Model with respect to its structural parameters, we conduct a GSA of the benchmark model in the environment of a natural real interest-rate shock, a binding ZLB, and neglect the possibility of supply shocks. We investigate l^{N}_{D} with $r_D = -0.015$. The behavior of the inflation and the output gap are determined by Equations (2.15) and (2.16), which are functions of κ, σ, and β. Therefore, l^{N}_{D} is a function of all structural parameters we want to investigate. We truncate the PCE at $p = 5$ and use $N = 378$ in the experimental design for the narrow parameter range and increase the polynomial to $p = 7$ and $N = 990$ for the wide ranges. Again, the approximation of the truncated PCE works well, as can be seen in Figure 4.22 and an $err_{LOO} = 6.34*10^{-4}$ and $err_{LOO} = 8.61*10^{-2}$, respectively. For some l^{N}_{D} values, the surrogate model yields negative values, which is not possible in the true model. The results are summarized in Table 4.12 and Figure 4.21 and are robust to the size of r_D.

Narrow Parameter Ranges

Higher-order indices S^{3}_{i} and S^{4}_{i} are of magnitudes $10^{-3} - 10^{-5}$ and thus neglected in the further analysis. Compared to the absence of a ZLB, second-order effects become more important, i.e. $S^{2}_{\kappa\sigma} = 21.3\%$. As before, first-order effects are dominating, in particular $S^{1}_{\kappa} = 42.8\%$ and $S^{1}_{\sigma} = 34.6\%$. The importance ranking yields $\kappa > \sigma > \lambda > \beta$, where λ and β are essentially unimportant.

Wide Parameter Ranges

In the wide ranges, the parameter σ dominates the sensitivity results of the model. Mainly through its first-order impact $S_\sigma^1 = 64\%$, and to a lesser extent, but still significantly through second- and third-orders $S_{\lambda\sigma}^2 = 15.2\%$, $S_{\kappa\sigma}^2 = 15\%$, and $S_{\lambda\kappa\sigma}^3 = 2.2\%$. The first-order impacts of all other parameters are of minor ($S_\lambda^1 = 2.2\%$) or of no importance ($S_\kappa^1 = 0.7\%$, $S_\beta^1 = 0\%$). The importance ranking yields $\sigma > \lambda > \kappa > \beta$.
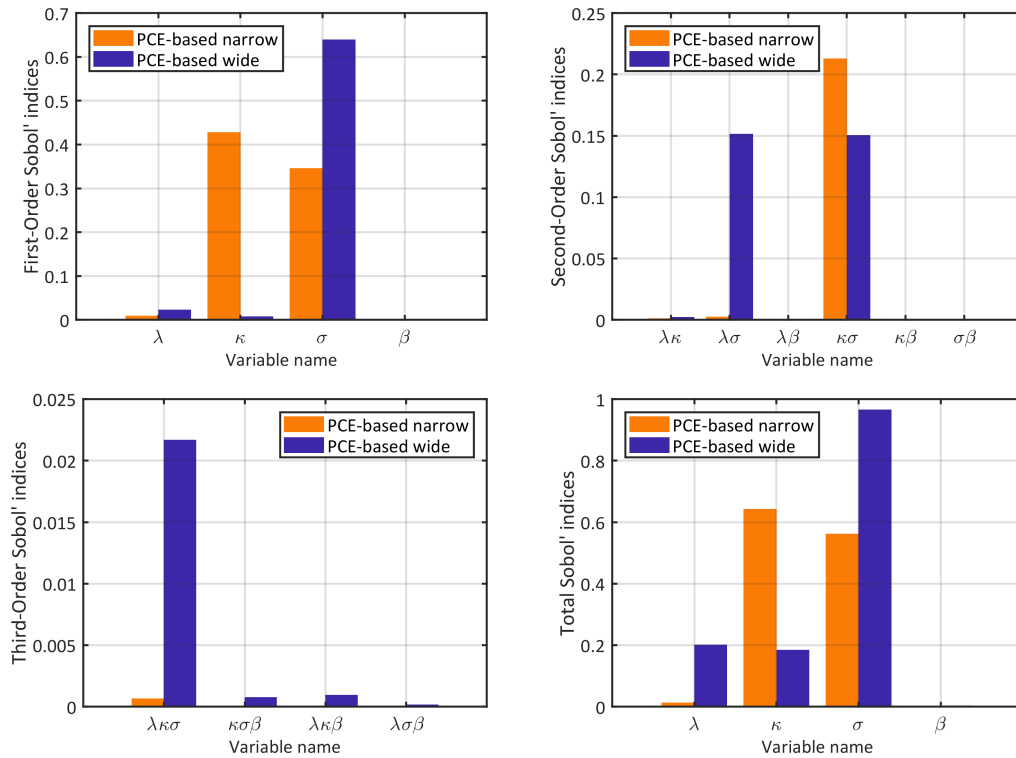
Figure 4.21: PCE-based Sobol' Indices of l_D^N with $p = 5$, $N = 378$ for narrow ranges and $p = 7$, $N = 990$ for wide ranges.
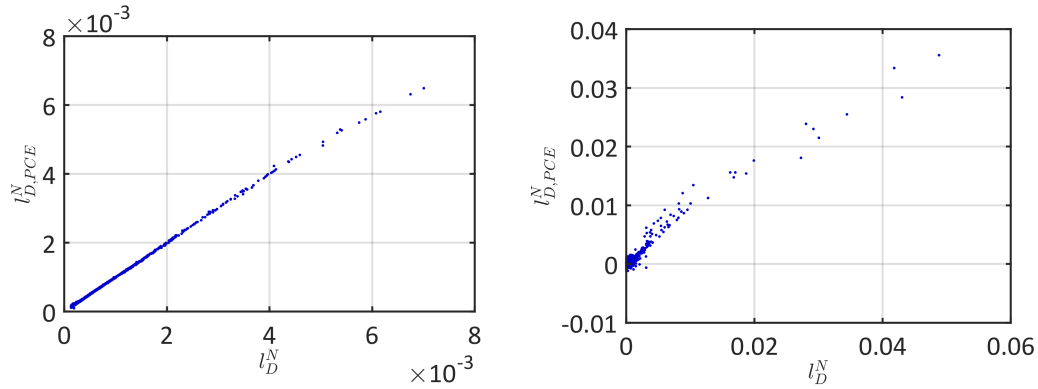
Figure 4.22: l_D^N surrogate model accuracy with $p = 5$, $N = 378$ for narrow ranges and $p = 7$, $N = 990$ for wide ranges.

	λ	κ	σ	β
S_{narrow}^{tot}	0.0134	0.6427	0.5620	0.0001
S_{wide}^{tot}	0.2018	0.1848	0.9664	0.0038
S_{narrow}^{1}	0.0090	0.4278	0.3458	0
S_{wide}^{1}	0.0229	0.0077	0.6397	0

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S_{narrow}^{2}	0.0011	0.0024	0	0.2130	0	0
S_{wide}^{2}	0.0023	0.1515	0	0.1504	0	0.0001

Narrow – Mean: 0.0009, SD: 0.0008, Coef. of Var.: 94%, err_{LOO}: $6.34 * 10^{-4}$

Wide – Mean: 0.0007, SD: 0.0025, Coef. of Var.: 337%, err_{LOO}: $8.61 * 10^{-2}$

Table 4.12: First- and second-order Sobol' Indices for l_D^N with $p = 5$, $N = 378$ for narrow ranges and $p = 7$, $N = 990$ for wide ranges.

4.4 Univariate Effects

The analysis in Section 4.3 provides information on the importance of the structural parameters. Based on the insights provided by Sobol' Indices, we can set parameters to constants within particular ranges without significant effects on the loss function, or single out to which parameters the model results react sensitively. Beyond that contribution, the surrogate model allows us to study the univariate effects of each input parameter. Specifically, we analyze *(i)* whether the effect on the output variable of changing a parameter is positive or negative, *(ii)* whether the relationship is linear or non-linear, and *(iii)* for which regions of the parameter space the output sensitivity is most pronounced.

We follow Harenberg et al. (2017) and Deman et al. (2016) and use univariate effects of each input parameter to elaborate on the questions above. Univariate effects can be formally defined as a univariate function of the parameter of interest,

$$\mathcal{M}_i(x_i) = \mathbb{E}[\mathcal{M}(\mathbf{X}|X_i = x_i)] - \mathcal{M}_0. \qquad (4.37)$$

From the discussion in Subsection 4.2.3, we know that the PCE terms are closely related to the terms in the Sobol' decomposition in Equation (4.1). Hence, we have an analytical solution for the conditional expectation minus the unconditional expectation in (4.37),

$$\mathcal{M}_i(x_i) = \sum_{\alpha \in \mathcal{A}_i} y_\alpha \Psi_\alpha(\mathbf{X}), \quad \mathcal{A}_i = \{\alpha \in \mathcal{A} : \alpha_i > 0, \alpha_{i \neq j} = 0\}. \qquad (4.38)$$

$\mathcal{M}_i(x_i)$ is a univariate function of the parameter under investigation, x_i, and indicates how a structural parameter moves the conditional mean from its unconditional mean level. The deviation from the mean and the plots we discuss cross the zero line where zero indicates at what parameter value the unconditional mean $\mathbb{E}[\mathcal{M}(\mathbf{X})]$ is located.

We present the univariate effects under different designs and use a shock size of $r_D = -0.015$ for illustrative purposes. A discussion of the robustness of the results with respect to the shock size will conclude this section.

Univariate Effects – Discretionary Central Banker

The left column of Figure 4.23 shows the univariate effects of the narrow parameter ranges and the right column the effects of the wide parameter ranges. If a parameter's line plot is flat or is small in magnitude on the y-axis relative to the other parameters, then it has a negligible effect on the average loss. Notably, β, for example, has relatively small effects of the magnitudes 10^{-5} (narrow) and 10^{-4} (wide). This confirms our finding that the β parameter is of minor importance. λ has a positive and linear (narrow), or slightly concave (wide) slope, κ has a non-linear positive, and σ a non-linear negative effect. The non-linearities show that the conditionally-expected social loss $\mathbb{E}[L^N|\kappa = \kappa_i]$ is very sensitive to values of $\kappa > 0.06$ and $\mathbb{E}[L^N|\sigma = \sigma_i]$ is very sensitive to $\sigma < 0.015$. The steep slope of these curves indicates that a small shift in the parameter value will lead to large variations in the output.

A negative value on the y-axis indicates that if this particular parameter is selected, it reduces the average loss and from a positive value follows an increase compared to the average loss. Therefore, $\lambda = 0.003$, $\kappa = 0.024$, and $\beta = 0.99$ have a negative effect on the average loss, given narrow parameter ranges. Hence, within their ranges, these parameter selections lead to a relatively low absolute loss. On the contrary, $\sigma = 0.16$ has

a positive effect and leads to an increase over average losses.
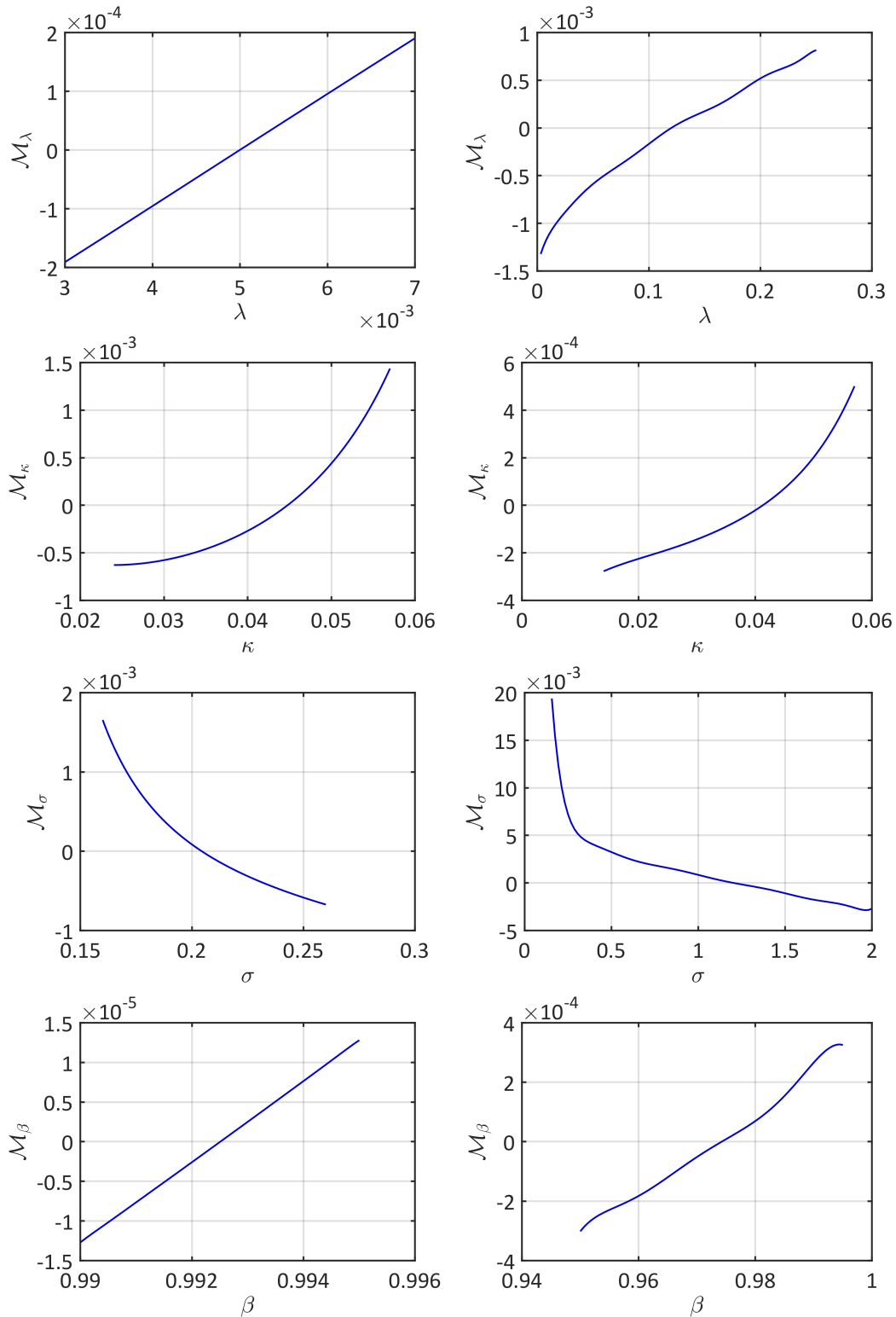
Figure 4.23: Univariate effects of the structural parameters under NFG. Narrow parameter ranges on the left and wide ranges on the right at $r_D = -0.015$.

Univariate Effects – Forward Guidance

We summarize the results of the univariate effects for all forward guidance designs in one section because the findings are relatively homogeneous among the designs. Figures 4.24 – 4.26 show the results graphically. In all cases the linear or close to linear effect of β is of negligible size as it is of a magnitude $10^1 - 10^2$ smaller than the other parameters' effect. λ has a linear (narrow) or slightly concave (wide) effect on the respective average losses. The slope of $\mathcal{M}_\lambda$ under IFG, EFG, and SFG is roughly half the NFG's slope size. Hence, the discretionary central banker reacts stronger to changes in λ. The effect of σ on average losses is negative and convex (narrow) or L-shaped (wide) under all designs. The L-shape suggest a high sensitivity for small σ values and a low sensitivity for $\sigma > 0.5$. Loosely speaking, the literature is divided into two strands: One that sets σ around the high sensitivity part at 0.16, which means slight changes in σ lead to strong reactions of the absolute losses, and one that sets σ in the low sensitivity part $\sigma > 1$.

Let us now focus on the differences in sensitivity between the models. First, in the narrow parameter range $\mathcal{M}_\sigma$ magnitudes' differ (NFG 10^{-3}, SFG 10^{-4}, IFG and EFG 10^{-5}). Second, at a shock size of $r_D = -0.015$, the reactions to changes in κ are different, although they display the same magnitude. The effect for the narrow parameter range is positive convex (NFG), negative convex (IFG, EFG), or U-shaped (SFG). For the wide parameter range, κ's effect is positive convex (NFG, IFG, EFG) or positive linear (SFG). Note that these univariate effects are only a snapshot at $r_D = -0.015$. Thus we need a robustness check with regard to r_D.
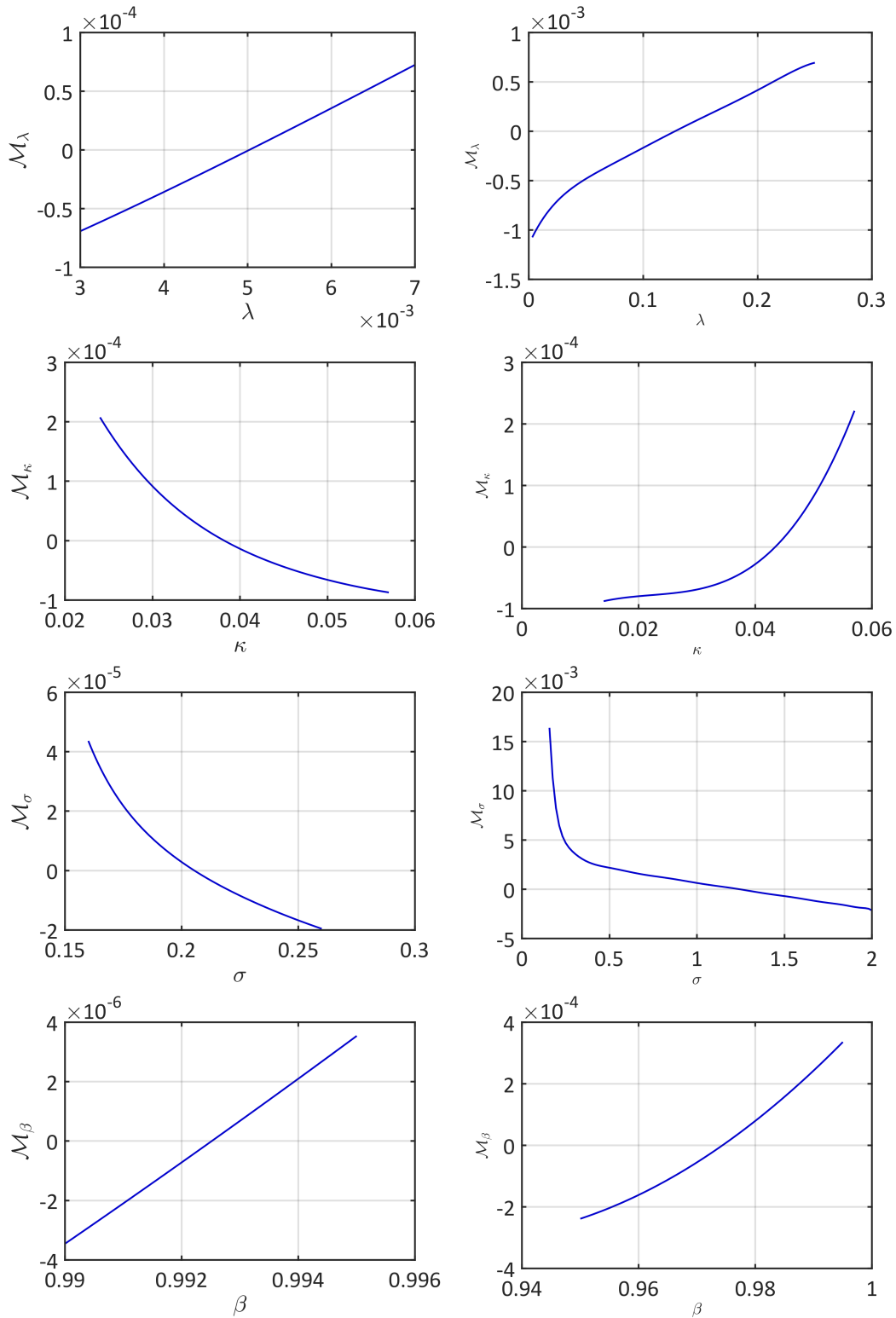
Figure 4.24: Univariate effects of the structural parameters under IFG. Narrow parameter ranges on the left and wide ranges on the right at $r_D = -0.015$.
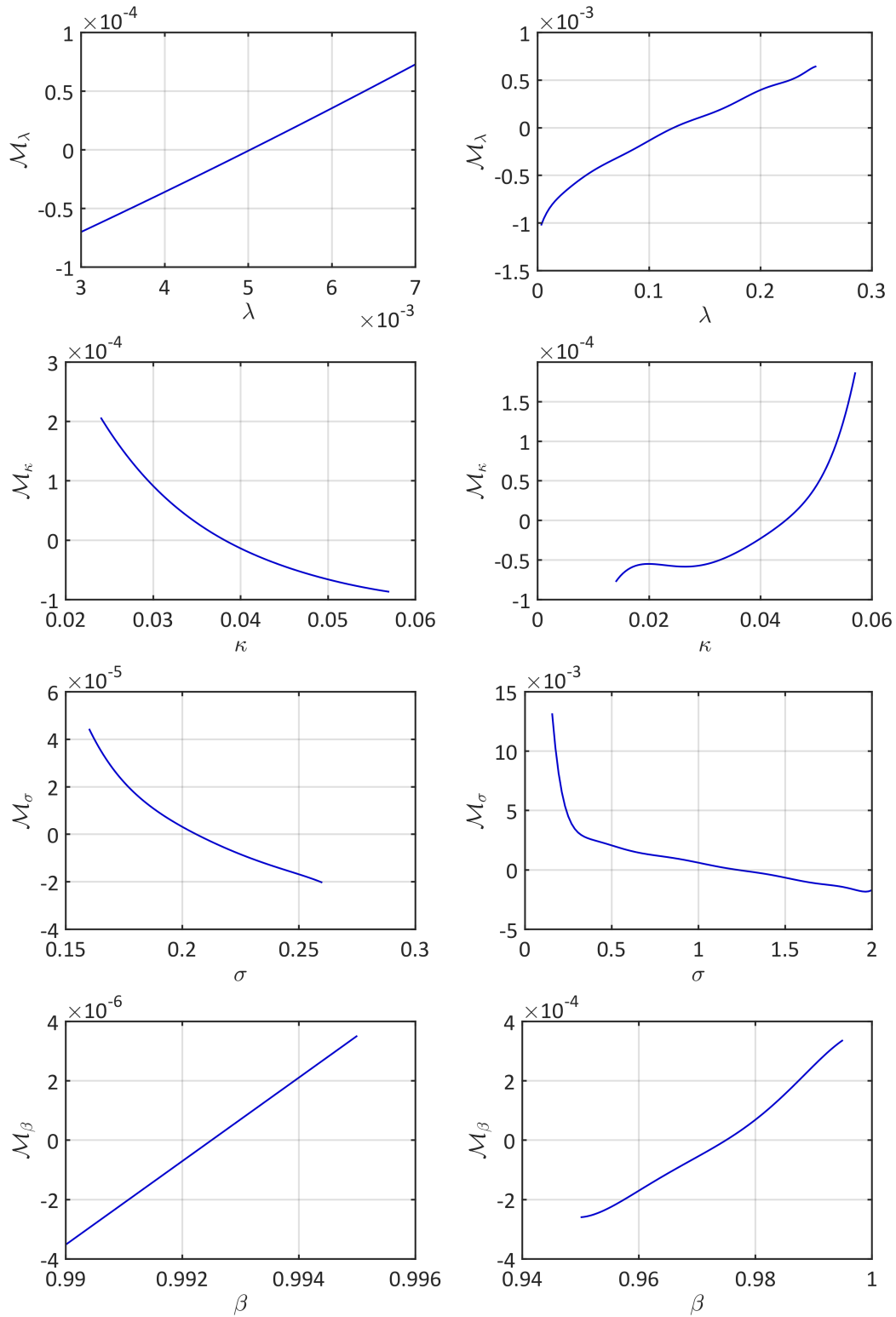
Figure 4.25: Univariate effects of the structural parameters under EFG. Narrow parameter ranges on the left and the wide ranges on the right at $r_D = -0.015$.
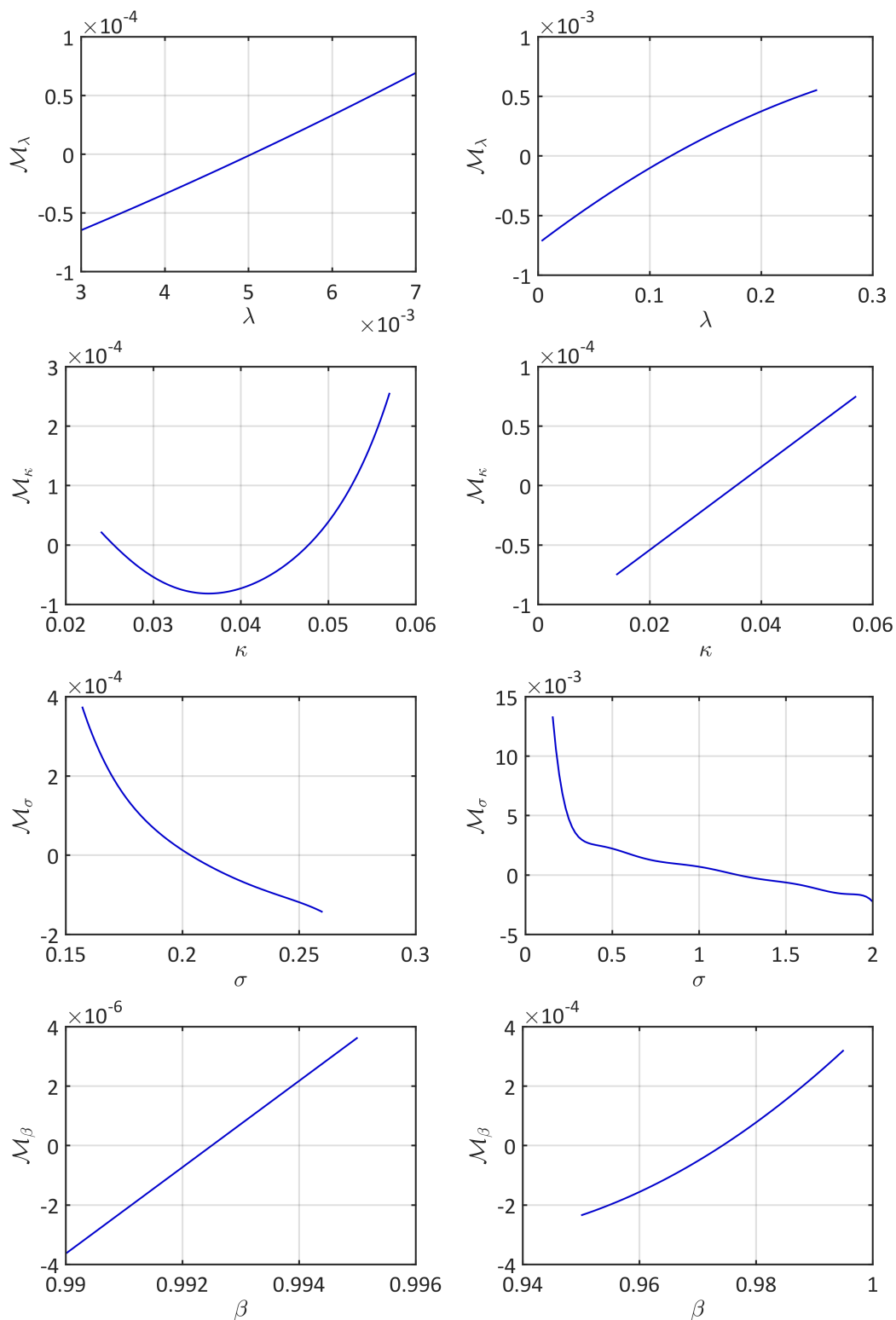
Figure 4.26: Univariate effects of the structural parameters under SFG. Narrow parameter ranges on the left and the wide ranges on the right at $r_D = -0.015$.

Univariate Effects – Robustness Check

To verify whether the univariate effects in Figures 4.23 – 4.26 are robust with regard to sizes of r_D, we evaluate the effects at a mild shock size $r_D = -0.005$ and a more severe shock size $r_D = -0.03$. The figures of the results can be found in the Appendix C.2. β's effect is consistently positive and linear (narrow) or slightly convex (wide). Moreover, β affects the average loss by a smaller magnitude than all the other parameters. σ's effect in the narrow range is always negative and either slightly non-linear or linear. In the wide range, the L-shape persists in all cases, indicating a high sensitivity of the losses for small σ values and lower for larger σ values throughout all sizes of r_D. λ has a positive linear effect (narrow) or slightly non-linear effect (wide) at $r_D = -0.005$. At $r_D = -0.03$, λ's influence turns linear in both parameter ranges. Thus, for β, λ, and σ, the qualitative results remain the same for all r_D sizes. κ's influence is more heterogeneous and depends on two characteristics: *(i)* on the size of r_D and *(ii)* on the parameter range. First, for a mild shock, i.e. $r_D = -0.005$, and a narrow parameter range, the slope is negative and either convex or U-shaped. Second, for a mild shock and a wide parameter range, the slope is linear and negative. Third, as we have demonstrated above, for a mid-sized shock, i.e. $r_D - 0.015$, κ's effects are heterogeneous and depend on the designs and their abilities to increase expectations and on the parameter ranges. Fourth, for a severe shock, i.e. $r_D = -0.03$, the effects align and are positive and convex throughout the designs. Hence, analogously to the importance rankings in Tables 4.6 and 4.7, where the rankings converge as shocks become severe, univariate effects converge as well. Designs with a higher ability to raise inflation in the downturn approach the typical pattern of univariate effects only for more negative r_D values.

Univariate Effect of Parameter b

We extend the univariate analysis to parameter b. As a default value, we assume a scrupulous central banker with an intrinsic value of $b = 1$.[15] We now perform a robust analysis on the effect of b, given that we do not know the exact values of the structural parameters. Formally, we analyze:

$$\mathbb{E}[\mathcal{M}(\lambda, \ \kappa, \ \sigma, \ \beta, \ b)|b = b_i] - \mathcal{M}_0. \tag{4.39}$$

Figure 4.27 shows the univariate effects of parameter b for designs IFG, EFG, and SFG in the narrow parameter ranges (top) and wide parameter ranges (bottom). For all designs, the effects of b are almost identical at $r_D = -0.015$. The magnitudes are nearly identical,

---

[15] A different interpretation of b is used in Gersbach et al. (2015) and Liu (2016). In their framework, b can be used as a policy parameter and is contractually specified by the government.
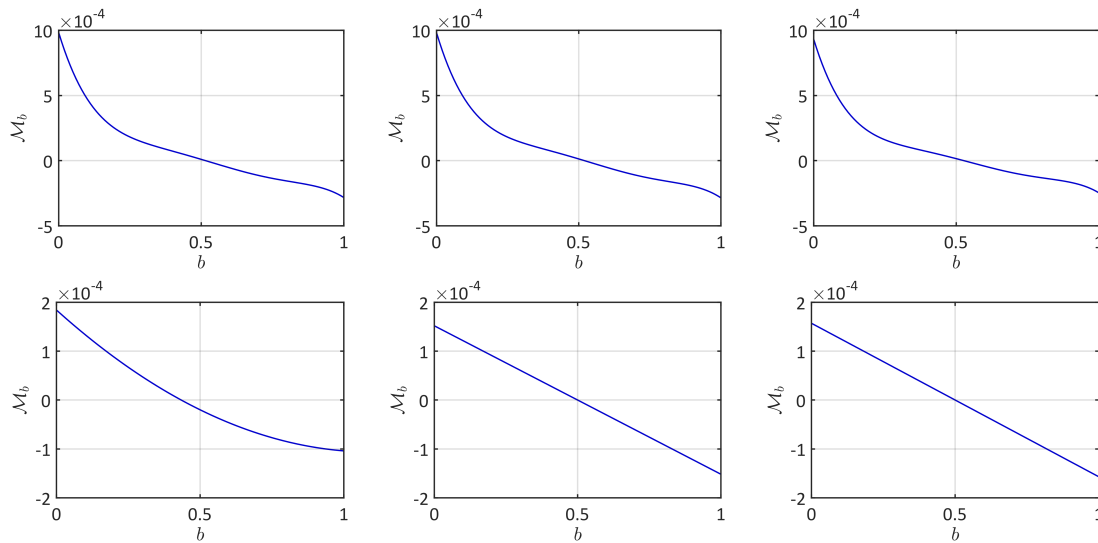
Figure 4.27: Robust analysis on the effect of b at $r_D = -0.015$. From left to right: IFG, EFG, SFG.

ranging from roughly $-3 * 10^{-4}$ to 10^{-3} (narrow) and -10^{-4} to $2 * 10^{-4}$ (wide), respectively. This implies that a particular value of b will roughly yield the same improvement in all designs. None of the designs, however, turns out to be particularly efficient in converting small b values (and therefore small extra losses for the central banker) into lower social losses. All functions are strictly downward-sloping in b. This means that introducing forward guidance is an improvement over NFG, which is represented by $b = 0$. Furthermore, the steep slope for small values of b indicates that a small degree of scrupulosity will already generate a large part of the gains achieved through forward guidance, which is in line with the findings in Chapter 2.

Given $r_D = -0.005$, the two flexible designs, EFG and SFG, have decreasing L-shaped $\mathcal{M}_b$ functions in the narrow parameter range and linear ones in the wide range. The more rigid IFG design has a V-shape, with a minimum around $b = 0.1$ in the narrow and a U-shape with a minimum around $b = 0.7$ in the wide parameter range.

For $r_D = -0.03$, the univariate effects of b are either convex (narrow) or linear (wide) with a negative slope for all designs.

Robust Expected Loss Functions

In this subsection, we address the most important question, i.e. whether our results regarding the social desirability of EFG and SFG are robust over a range of r_D shocks when we only know the parameter distributions of Table 4.1. That is, we reproduce Figure 2.16 under parameter uncertainty. We can calculate the conditionally-expected loss function

given an r_D shock of a specific size,

$$\mathbb{E}[L^p(\lambda, \kappa, \sigma, \beta, r_D)|r_D = r_{D,i}], \quad \text{where } p \in \{N, F, E, S\}. \quad (4.40)$$

Narrow Parameter Range

Figure 4.28 shows the conditionally-expected social losses, given the benchmark parameterization indicated by the subscript BM, and the social losses, given the narrow parameter distributions of Table 4.1 indicated by the subscript GSA. When we compare designs, not only the absolute level, but also the point of intersection is important. In the upper left corner of Figure 4.28, we observe that the intersection point of L_{GSA}^N and L_{GSA}^F—the intersection of the two solid lines—is to the right of the intersection point of L_{BM}^N and L_{BM}^F—the intersection of the two dashed lines. The same holds for the comparison between NFG and SFG in Figure 4.28. Hence, by using the benchmark parameters, we might underestimate the range where applying forward guidance is beneficial.
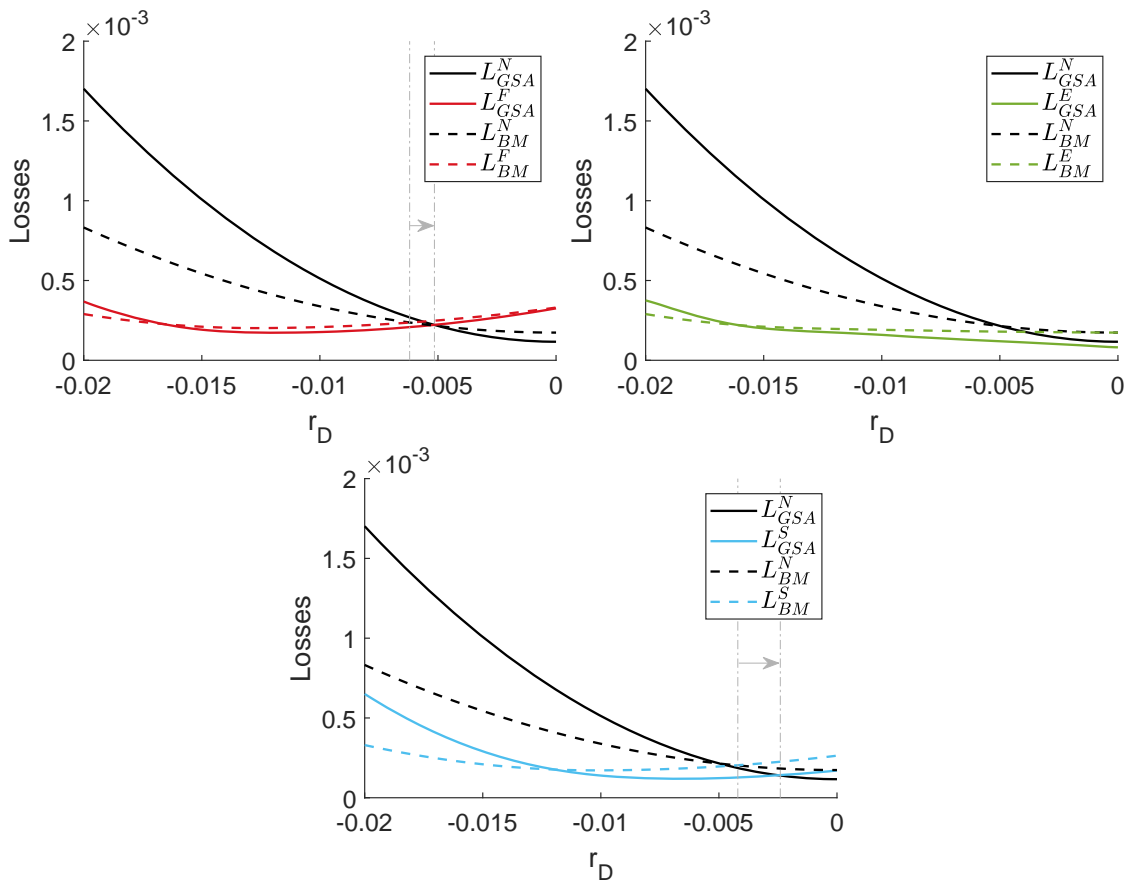
Figure 4.28: Robust loss analysis of conditionally-expected social losses and narrow parameter ranges.

Wide Parameter Range

Figure 4.29 shows the conditionally-expected social losses, given the wide parameter ranges. In absolute terms, the levels shift upwards. Under IFG and SFG, the intersections with NFG move to the right and make the two designs socially beneficial for smaller r_D shocks. Note that NFG and EFG intersect at $r_D < 0$, although the two designs should be identical as r_D approaches zero. This may be due to the lower accuracy of the surrogate model when we use the wide parameter ranges.
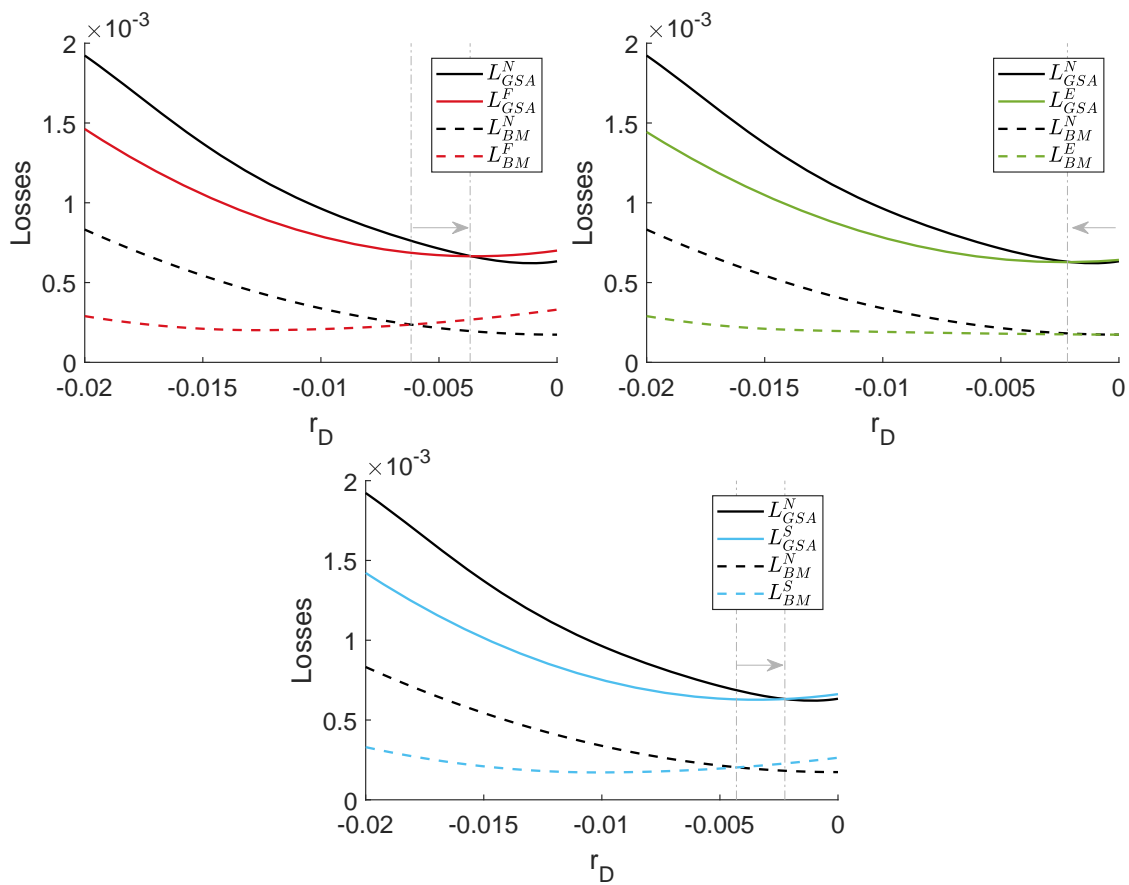
Figure 4.29: Robust loss analysis of conditionally-expected social losses and wide parameter ranges.

Figure 4.30 combines the social losses under NFG, IFG, EFG, and SFG, given the two uncertainty calibrations. The left graph shows that the qualitative results remain unchanged compared to the benchmark calibration in Figure 2.16 in the narrow ranges. EFG dominates for small and large natural real interest-rate shocks, while in the intermediate range, SFG yields lower expected social losses. However, the range for which SFG dominates EFG is narrower than under the benchmark calibration in Figure 2.16. Under the benchmark calibration, SFG dominates for $r_D \in (-1.74\%, -0.46\%)$, while under the uncer-

tainty calibration, SFG dominates for $r_D \in (-1.17\%, -0.55\%)$.

When we use the wide parameter ranges the results change significantly, although the low accuracy of the surrogate models only allows rough conclusions. Conditionally-expected losses under SFG are slightly lower than under all other designs for $r_D < -0.26\%$. For the small range close to zero, EFG and the discretionary central banker are welfare improving compared to SFG.
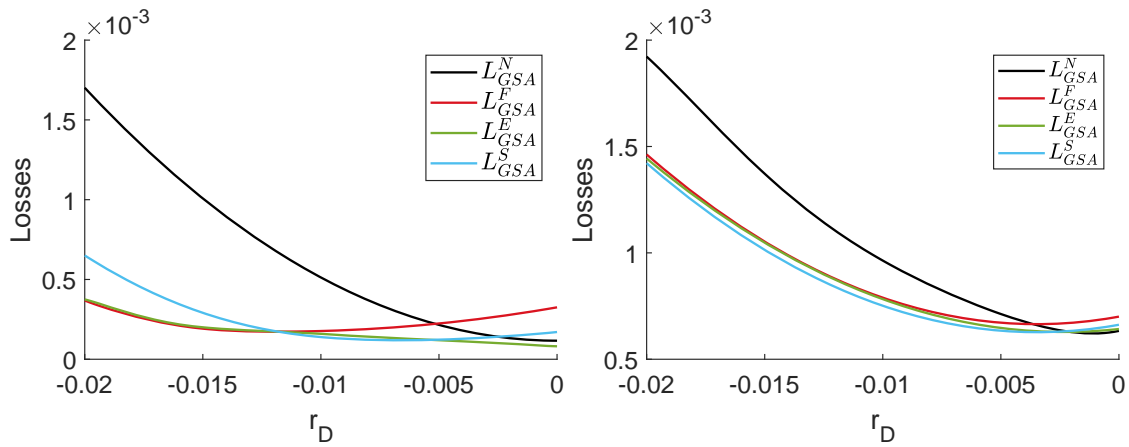
Figure 4.30: Robust loss analysis of conditionally-expected social losses.

4.5 Conclusions

We subjected the forward guidance designs to a global sensitivity analysis and we distinguished two scenarios of narrow parameter ranges and wide parameter ranges.

The first scenario yielded accurate results, and it turned out that for small r_D shocks, the slope parameter in the Phillips Curve (κ) and the weight on the output gap in the loss function (λ) are crucial for calibrating the model. The inverse inter-temporal elasticity of substitution (σ) and the discount factor (β) proved to be of negligible importance. As r_D shocks become more severe, κ and σ contribute most to the output variation and the effects of λ and β are negligible. The influence of the parameters on the expected losses are predominantly linear and additive. The second-order effect of $S^2_{\kappa\sigma}$ under NFG and SFG and the effect of $S^2_{\lambda\kappa}$ under IFG and EFG, however, indicate that we deal with non-linear and non-additive effects as well. Moreover, we showed that substantial benefits from applying versatile forward guidance already materialize if central bankers have a small degree of scrupulosity.

In the latter scenario of wide parameter ranges, it turned out that the surrogate model does not yield very accurate results. Thus we focused on the total Sobol' Indices and the first-order effects. For small r_D shocks β, λ, and κ are key parameters, whereas σ is unimportant. As r_D becomes more severe, σ turns out to be the determining parameter. Furthermore, we showed that κ and λ contribute a substantial part to the output variance as well, and β's contribution is negligible. We also showed that applying versatile forward guidance is socially beneficial for all degrees of scrupulosity.

Finally, our global sensitivity analysis demonstrated that the conclusion that both escaping forward guidance and switching forward guidance are socially welfare improving is robust to parameter uncertainty.

5 Extensions to Versatile Forward Guidance

We examine two auxiliary assumptions stated in Chapter 2 and test the robustness of our results with respect to these assumptions. First, we allow the central banker to publish an optimal interest-rate forecast and abolish the assumption that the interest-rate forecast is always equal to zero. Second, we use an effective lower bound (ELB) instead of a ZLB.

5.1 Optimal Interest-rate Forecast

To simplify the analysis, we assumed interest-rate forecasts to be zero in downturns in Chapter 2. We now show how to dispense with the assumption on the forecast. Otherwise the set-up remains exactly the same as in Chapter 2. We start by deriving expected social losses under standard forward guidance, continue with *escaping* forward guidance and *switching* forward guidance, and finally compare the designs.

5.1.1 Standard Forward Guidance

We proceed analogously to Chapter 2 and use backward induction to receive the dynamics in Phase H, $t = 1$. Following the calculations in Appendix A.2 and using the new first-order condition $-\frac{1}{\sigma}[\kappa\pi_1^F + \lambda x_1^F - b\sigma(i_1^F - i^f)] = 0$ which replaces Equation (A.21) yields

$$i_1^F = \frac{(\kappa^2 + \lambda)[\sigma\kappa(1-\rho) + \rho\lambda]}{(\kappa^2 + \lambda + b\sigma^2)(\lambda(1-\rho\beta) + \kappa^2)}\xi_1 + \frac{\kappa^2 + \lambda}{\kappa^2 + \lambda + b\sigma^2}r_H + \frac{b\sigma^2}{\kappa^2 + \lambda + b\sigma^2}i^f, \quad (5.1)$$

$$\pi_1^F = \frac{b\kappa\sigma\rho(\lambda - \sigma\kappa) + (\lambda + b\sigma^2)(\kappa^2 + \lambda)}{(\kappa^2 + \lambda + b\sigma^2)(\lambda(1-\rho\beta) + \kappa^2)}\xi_1 + \frac{b\kappa\sigma}{\kappa^2 + \lambda + b\sigma^2}(r_H - i^f), \quad (5.2)$$

$$x_1^F = \frac{b\sigma\rho(\lambda - \sigma\kappa) - \kappa(\kappa^2 + \lambda)}{(\kappa^2 + \lambda + b\sigma^2)(\lambda(1-\rho\beta) + \kappa^2)}\xi_1 + \frac{b\sigma}{\kappa^2 + \lambda + b\sigma^2}(r_H - i^f). \quad (5.3)$$

By applying the same steps to calculate Equations (A.31), (A.32), and (A.33) we obtain inflation, output gap, and nominal interest rate in Phase D:

$$\pi_D^F = \frac{(1-\delta)\kappa\sigma b(\sigma\beta(1-\delta)+\kappa+\sigma)}{h(\delta)(\kappa^2+\lambda+b\sigma^2)}(r_H - i^f) - \frac{\kappa}{h(\delta)}(i_D^F - r_D), \tag{5.4}$$

$$x_D^F = \frac{(1-\delta)b\sigma[\sigma(1-\beta\delta)+\kappa]}{h(\delta)(\kappa^2+\lambda+b\sigma^2)}(r_H - i^f) - \frac{(1-\beta\delta)}{h(\delta)}(i_D^F - r_D), \tag{5.5}$$

$$i_D^F = \frac{b\sigma[\kappa^2(\sigma\beta(1-\delta)+\kappa+\sigma)+\lambda(\sigma(1-\beta\delta)+\kappa)]}{(\kappa^2+\lambda+b\sigma^2)(b\sigma h(\delta)+\kappa^2+\lambda(1-\beta\delta))}r_H \tag{5.6}$$

$$+ \frac{\kappa^2+\lambda(1-\beta\delta)}{b\sigma h(\delta)+\kappa^2+\lambda(1-\beta\delta)}r_D + \frac{b\sigma[h(\delta)b\sigma^2-\kappa((1-\delta)\sigma\beta+\kappa)-\lambda\kappa]}{(\kappa^2+\lambda+b\sigma^2)(b\sigma h(\delta)+\kappa^2+\lambda(1-\beta\delta))}i^f. \tag{5.7}$$

The central banker chooses the interest-rate forecast that minimizes his cumulative expected losses, i.e.

$$\min_{i^f}\left\{\tilde{L}^F\right\} \tag{5.8}$$

where the structure of $\tilde{L}^F$ is given by Equation (2.9). Figure 5.1 shows the social loss function of the discretionary central banker (black solid line), of IFG with an optimal interest-rate forecast (solid red), and of IFG with $i^f = 0$ (dashed red). In the environment of small shocks, a zero interest-rate forecast is very constricting and therefore elevates expectations too strongly. An optimal interest-rate forecast that deviates from zero helps avoiding unnecessarily high losses. The optimal interest-rate forecast is zero for $r_D < -0.0137$ and it linearly increases to $r_H = 0.02$ as the shock approaches zero. Note that the ZLB starts binding at $r_D = -0.0123$. Hence, although the central banker sets $i_D^F = 0$, he optimally forecasts a positive interest rate. This is due to the fact that with probability $(1-\delta)$, the natural real interest rate reverts back to its high state. IFG with optimal interest-rate forecast dominates NFG for shocks $r_D < -0.0015$.

5.1.2 Escaping Forward Guidance

Next, we relax the assumption of a zero interest-rate forecast under EFG. In a first step, we provide an illustrative example with $\pi^c = 0$ and in a second step we extend the approach in such a way that the central banker can choose the π^c-i^f-combination that minimizes $\tilde{L}^E$.

The numerical procedure to calculate expected values in Phase H stay the same as in Chapter 2 and are described in Appendix A.3 in Equations (A.34) and (A.35). The only difference is that we use the equations derived in Section 5.1.1 to calculate the losses $\tilde{l}_1^F$. Because the dynamics in Phase D, i.e. Equations (A.38) and (A.39), are derived in a very
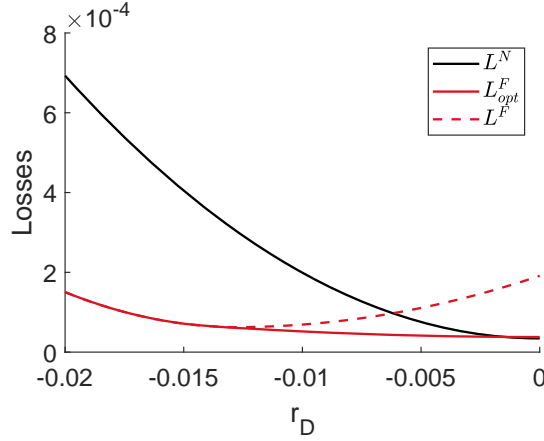
Figure 5.1: Optimal inflation forecast under IFG with $b = 1$.

general form, we can still use the same formulas. The formula for i_D^E has to be slightly extended since i^f is not set to zero. We thus obtain

$$\pi_D^E = \frac{\sigma\kappa(1-\delta)\mathbb{E}_D[x_1^E] + (1-\delta)(\kappa + \sigma\beta - \sigma\beta\delta)\mathbb{E}_D[\pi_1^E] + \kappa(r_D - i_D^E)}{h(\delta)}, \qquad (5.9)$$

$$x_D^E = \frac{\sigma(1-\delta)(1-\beta\delta)\mathbb{E}_D[x_1^E] + (1-\delta)\mathbb{E}_D[\pi_1^E] + (1-\beta\delta)(r_D - i_D^E)}{h(\delta)}, \qquad (5.10)$$

$$i_D^E = \max\left[0, \frac{e_1\mathbb{E}_D[x_1^E] + e_2\mathbb{E}_D[\pi_1^E] + e_3 r_D + b\sigma h(\delta)i^f}{b\sigma h(\delta) + \kappa^2 + \lambda(1-\beta\delta)}\right], \qquad (5.11)$$

where

$$e_1 = (1-\delta)\sigma(\kappa^2 + \lambda(1-\beta\delta)) > 0,$$
$$e_2 = (1-\delta)(\kappa(\kappa + \sigma\beta - \sigma\beta\delta) + \lambda) > 0,$$
$$e_3 = (\kappa^2 + \lambda(1-\beta\delta)) > 0.$$

The expectations $\mathbb{E}_D[x_1^E]$ and $\mathbb{E}_D[\pi_1^E]$—with π_1^F and x_1^F derived in Section 5.1.1—are given by expectations formulated in (A.34) and (A.35), respectively.

Figure 5.2 plots expected social loss functions for $\pi^c = 0$ and an optimally chosen i^f (solid green) and expected social losses for $\pi^c = 0$ and $i^f = 0$ (dashed green). The central banker forecasts a zero interest rate for $r_D < -0.0039$ and positive values otherwise. Furthermore, for small shocks, i.e. $r_D > -0.0015$, not to provide a forecast dominates EFG with $\pi^c = 0$. Thus, a threshold of zero creates expectations that are too high for small shocks, despite a positive interest-rate forecast.

Let us now relax the assumption $\pi^c = 0$. The central banker chooses the π^c-i^f-combination that minimizes $\tilde{L}^E$, which we denote by $\tilde{L}_{opt}^E$. Figure 5.3 shows expected social losses of the latter procedure and adds expected losses under $\pi^c = 0$ and $i^f = 0$ as a reference. The
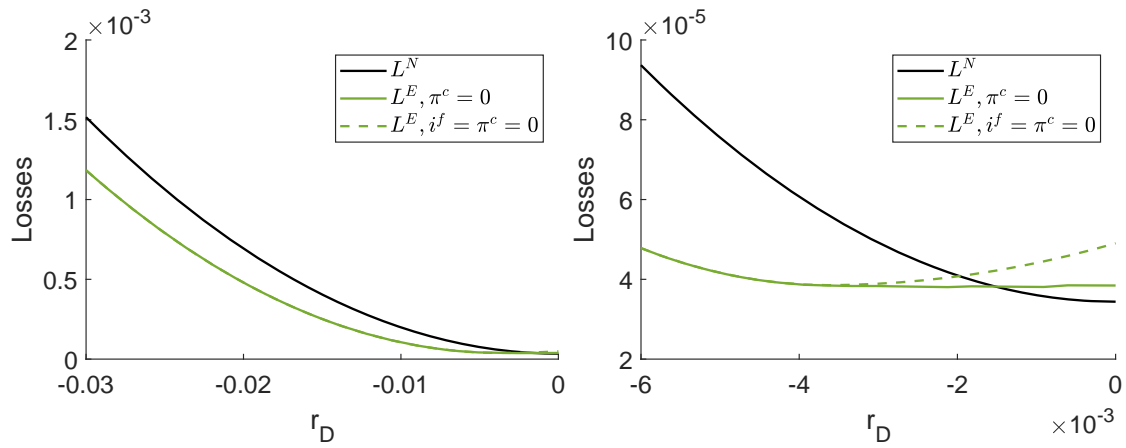
Figure 5.2: Optimal interest-rate forecast under EFG with $b = 1$, $\pi^c = 0$ and $i^f = 0$.

possibility to set π^c optimally enables the central banker to increase or decrease inflation expectations in Phase D. The optimal forecast is always set to zero. Hence, the central banker mainly uses π^c as an instrument to manage expectations and leaves i^f fixed.
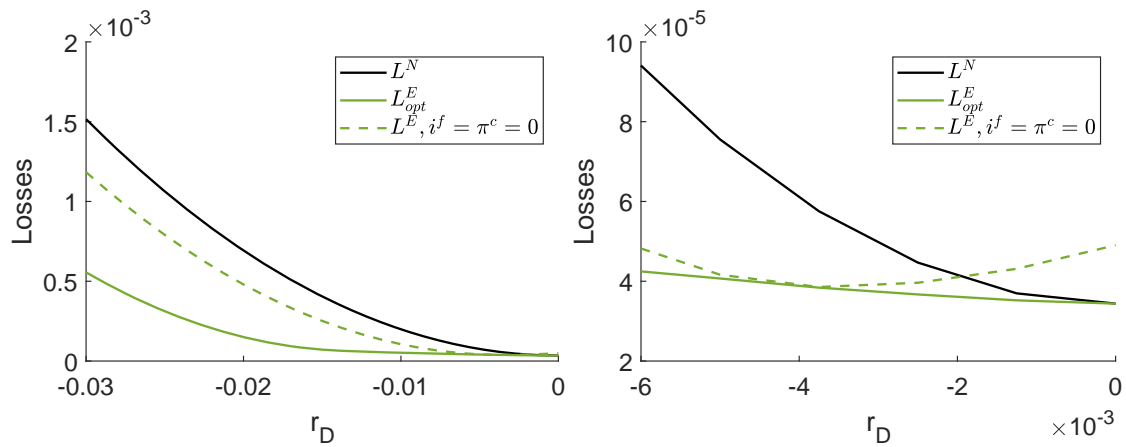
Figure 5.3: Optimal interest-rate forecast under EFG with $b = 1$ and $\pi^c = \pi^c_{opt}$.

5.1.3 Switching Forward Guidance

Determining the expected losses under SFG can be described as a procedure in three steps. The first step is to calculate values in Phase H, $t \geq 2$, with the algorithm from Söderlind (1999). In the second step, we extended the Equations in (A.47), (A.48), and (A.49) such that they include interest-rate forecasts:

$$x_1^S = \frac{b_1\sigma^2\mathbb{E}_1[x_2^S] - (\kappa\beta - b_1\sigma)\mathbb{E}_1[\pi_2^S] - \kappa\xi_1 + b_1\sigma(r_H - i^f)}{b_1\sigma^2 + \kappa^2 + \lambda}, \tag{5.12}$$

$$\pi_1^S = \frac{\kappa\sigma^2 b_1\mathbb{E}_1[x_2^S] + (\beta(b_1\sigma^2 + \lambda) + \kappa\sigma b_1)\mathbb{E}_1[\pi_2^S] + (b_1\sigma^2 + \lambda)\xi_1 + \kappa\sigma b_1(r_H - i^f)}{b_1\sigma^2 + \kappa^2 + \lambda}, \tag{5.13}$$

$$i_1^S = \frac{(\kappa\beta\sigma + \kappa^2 + \lambda)\mathbb{E}_1[\pi_2^S] + \sigma(\kappa^2 + \lambda)\mathbb{E}_1[x_2^S] + \kappa\sigma\xi_1 + (\kappa^2 + \lambda)r_H + \sigma^2 i^f}{b_1\sigma^2 + \kappa^2 + \lambda}. \tag{5.14}$$

By substituting the results of the first step into the second, we can numerically derive the expected values in Phase D. Finally, we can apply the same formulas as under EFG in (5.9), (5.10), and (5.11), together with expectations obtained under step one and two in Phase D.

The left graph of Figure 5.4 plots expected social losses under optimal interest-rate forecasts in solid blue and under fixed forecasts $i^f = 0$ in dashed blue. SFG is socially beneficial for all shock sizes when the interest-rate forecast is set optimally. The right graph shows cumulative losses for all designs, i.e. all periods in Phase H. SFG dominates for small shock sizes $r_D > -0.0175$. EFG and IFG dominate at $r_D < -0.0175$, as at these shock sizes, the two designs intersect.
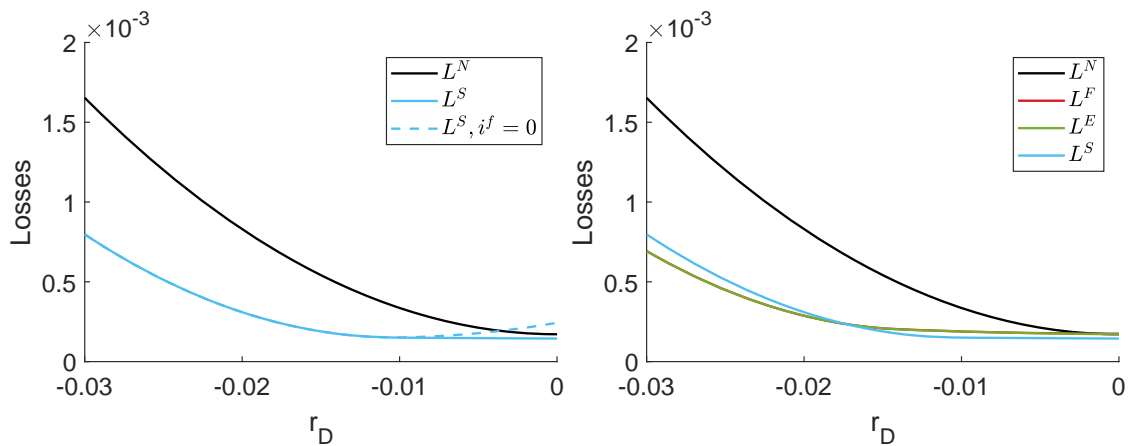
Figure 5.4: Optimal interest-rate forecast under SFG with $b_1 = b_2 = 1$.

5.2 Zero Lower Bound vs. Effective Lower Bound

Recent experiences—as observed in Switzerland, the EU, and Sweden, for instance—suggests that it is more realistic to constrain the central banker by an ELB than by a strict ZLB. It is thus interesting to verify how the results of the preceding sections change under an ELB constraint. Calculations underlying the analysis are given at the end of this section.

5.2.1 Social Welfare Under an Effective Lower Bound

First, the exact same set-up as in Chapter 2 is applied, i.e. the central banker issues a predefined forecast i^f. In the benchmark case, we used $i^f = ZLB = 0$. Now, we set $i^f = ELB = -0.0075$.[1] The results are depicted in Figure 5.5. Qualitatively the graph resembles the one in Figure 2.16. Additionally, there now exists a range $r_D \in (-0.0075, 0)$ for which providing no forecast dominates any other forward guidance design. In the range $r_D \in (-0.0221, -0.0121)$ SFG dominates. In all other circumstances, EFG is the design that minimizes social losses.
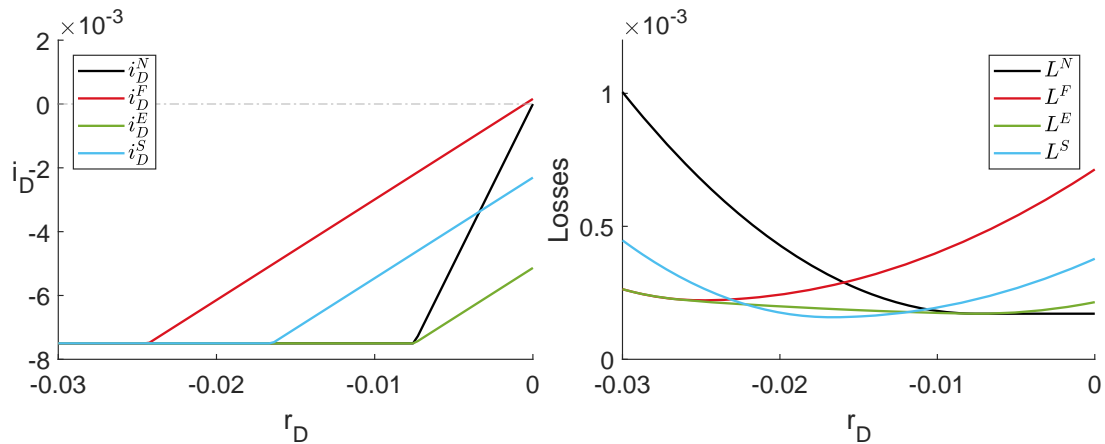
Figure 5.5: Interest rates and expected social losses under an ELB with $i^f = ELB$.

This set-up, however, incorporates the strong assumption of forcing the central banker to set the forecast to the ELB even for r_D values close to zero. This is very restrictive and unfavorable by construction for the forward guidance designs. Furthermore, a central banker who sets his actual policy rate above the forecast might not be credible, given the sequence of events. The central banker either expects to stay in the same economic environment in the next period, i.e. r_D is held constant, or to be in a situation where he

---

[1] The rate -0.0075 corresponds to the interest rate the Swiss National Bank charges on deposits above a threshold of CHF 10m Schweizerische Nationalbank (2015).

would like to increase the policy rate. A forecast below the actual policy rate in Phase D is thus contrary to the central banker's interests. Therefore, we introduce another set-up which grants the central banker more freedom in setting i^f. The central banker can either abstain from forecasts or set the forecast equal to the current interest rate, $i^f = \{ELB, \tilde{i}_D^p\}$, where $p \in \{N, F, E, S\}$ and $\tilde{i}_D^p$ is the interest rate the central banker would choose if he had full flexibility to set the interest rate. Figure 5.6 shows that under such a set-up, the qualitative results change slightly, as SFG dominates the other designs for r_D sizes down to $r_D = -0.028$. This is due to the fact that SFG is the only design that manages to decrease losses in Phase H, $t \geq 2$. Note that IFG does not equal NFG and EFG at $r_D = 0$, since the central banker issues a forecast $i^f = 0$ for next period, but the economy enters Phase H with probability $(1 - \delta)$. For values $r_D < -0.028$, EFG and IFG both dominate, as for such r_D sizes, these two designs coincide.
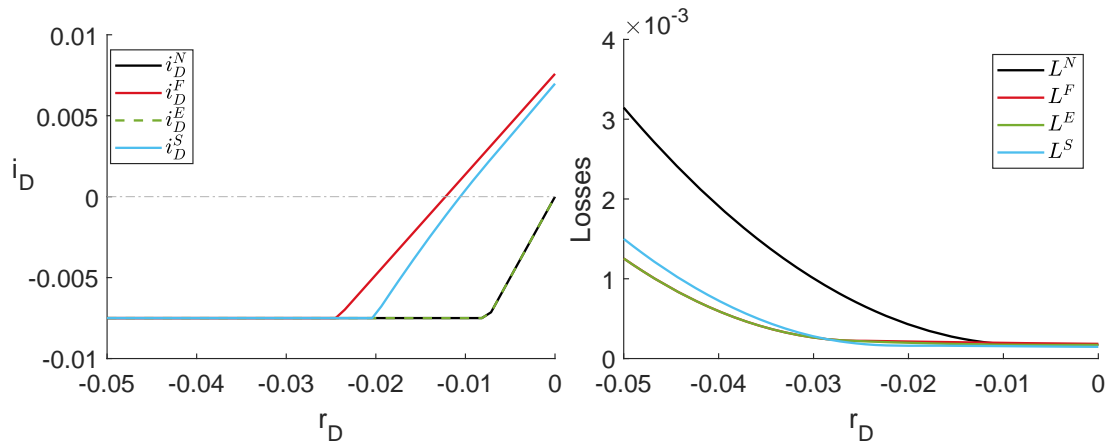
Figure 5.6: Interest rates and expected social losses under an ELB with $i^f = \max\{ELB, \tilde{i}_D^p\}$.

5.2.2 Calculations Under the Effective Lower Bound

Discretionary Central Banker

The assumption of a ZLB constraint of the central banker is replaced by an ELB constraint with ELB $\leq$ ZLB, which enables the central banker to set a negative nominal interest rate. In practice, the Swiss National Bank, for example, sets its current policy rate at -0.75%.[2] In the discretionary case, the dynamics of the variables in Phase H are still given by Equations (A.11), (A.12), and (A.13). In downturns, the dynamics in Equations (A.16) and (A.17) have to be modified slightly to

$$\pi_D^N = \frac{\kappa}{h(\delta)}(r_D - \max\{ELB, i_D^N\}) \tag{5.15}$$

---

[2] Date: 21. Nov. 2017

and

$$x_D^N = \frac{1 - \beta\delta}{h(\delta)}(r_D - \max\{ELB,\ i_D^N\}), \tag{5.16}$$

where ELB represents the ELB at -0.75%. From (A.1) it follows that the central banker sets $i_D^N = r_D$ as long as the ELB is not crossed from above. The same formulas apply to both set-ups discussed in Section 5.2.1.

Standard Forward Guidance

In contrast to the case with a discretionary central banker, we have to distinguish the two set-ups under standard forward guidance. Let us first outline the derivation of the formulas for $i^f = ELB$ and then $i^f = \max\{ELB,\ \tilde{i}_D^F\}$.

$i^f = ELB$:

The main idea of the derivation follows the description given in Section 5.1.1. We use backward induction to derive the dynamics. In the periods in Phase H, $t \geq 2$ remain unchanged, as in the discretionary solution. Hence, we can use Equations (5.1), (5.2), and (5.3) with $i^f = ELB$ to derive the dynamics in Phase H, $t = 1$. Furthermore, we can reuse Equations (5.4), (5.5), and (5.6) with $i^f = ELB$ to describe Phase D.

$i^f = \max\{ELB,\ \tilde{i}_D^F\}$:

The derivation is divided into two parts. We first compute i_D^F as a function of r_D and r_H and then derive the remaining formulas. For this purpose, we use Equations (2.1), (2.2), and the FOC: $b\sigma(i_D^F - i^f) = \kappa\pi_D^F + \lambda x_D^F$ to derive

$$\tilde{i}_D^F = \frac{e_1\mathbb{E}_D[x_1^F] + e_2\mathbb{E}_D[x_1^F] + e_3 r_D + b\sigma h(\delta)i^f}{b\sigma h(\delta) + f_3}, \tag{5.17}$$

where $e_1 > 0$, $e_2 > 0$ and $e_3 > 0$. Setting $i^f = i_D^F$ and applying the previously derived dynamics, i.e. plugging in the expected values of Equations (5.1) and (5.2), and solving for i_D^F yields

$$i_D^F = \max\left[ELB,\ \frac{b\sigma(f_1 + \kappa f_2)r_H + (\kappa^2 + \lambda + b\sigma^2)f_3 r_D}{b\sigma(f_1 + \kappa f_2) + (\kappa^2 + \lambda + b\sigma^2)f_3}\right]. \tag{5.18}$$

We are now in a position to write the variables of interest as functions of r_D, r_H, and ξ_1. Given the expected values of Equations (5.1) and (5.2), the natural real interest rate r_D,

and i_D^F, we can derive

$$\pi_D^F = \frac{\sigma\kappa(1-\delta)\mathbb{E}_D[x_1^F] + (1-\delta)(\kappa + \sigma\beta - \sigma\beta\delta)\mathbb{E}_D[\pi_1^F] + \kappa(r_D - i_D^F)}{h(\delta)}, \quad (5.19)$$

$$x_D^F = \frac{\sigma(1-\delta)(1-\beta\delta)\mathbb{E}_D[x_1^F] + (1-\delta)\mathbb{E}_D[\pi_1^F] + (1-\beta\delta)(r_D - i_D^F)}{h(\delta)}. \quad (5.20)$$

Furthermore, i_1^F, π_1^F, and x_1^F in 5.1, 5.2, and 5.3 can be written as functions of ξ_1 and i_D^f:

$$i_1^F = \frac{(\kappa^2 + \lambda)[\sigma\kappa(1-\rho) + \rho\lambda]}{(\kappa^2 + \lambda + b\sigma^2)(\lambda(1-\rho\beta) + \kappa^2)}\xi_1 + \frac{\kappa^2 + \lambda}{\kappa^2 + \lambda + b\sigma^2}r_H + \frac{b\sigma^2}{\kappa^2 + \lambda + b\sigma^2}i_D^F,$$
$$(5.21)$$

$$\pi_1^F = \frac{b\kappa\sigma\rho(\lambda - \sigma\kappa) + (\lambda + b\sigma^2)(\kappa^2 + \lambda)}{(\kappa^2 + \lambda + b\sigma^2)(\lambda(1-\rho\beta) + \kappa^2)}\xi_1 + \frac{b\kappa\sigma}{\kappa^2 + \lambda + b\sigma^2}(r_H - i_D^F), \quad (5.22)$$

$$x_1^F = \frac{b\sigma\rho(\lambda - \sigma\kappa) - \kappa(\kappa^2 + \lambda)}{(\kappa^2 + \lambda + b\sigma^2)(\lambda(1-\rho\beta) + \kappa^2)}\xi_1 + \frac{b\sigma}{\kappa^2 + \lambda + b\sigma^2}(r_H - i_D^F). \quad (5.23)$$

Escaping Forward Guidance

Again we derive the two set-ups separately.

$i^f = ELB$:

We apply the same reasoning as in Section 5.1.2. The formulas in Equations (5.9), (5.10), and (5.11) with $i^f = ELB$ describe the dynamics in Phase D. To calculate $\mathbb{E}_D[x_1^E]$ and $\mathbb{E}_D[\pi_1^E]$, we use Expectations (A.34) and (A.35), where the functions $\pi_1^F(q)$ and $x_1^F(q)$ are defined in Equations (5.2) and (5.3), respectively.

$i^f = \max\{ELB, \tilde{i}_D^E\}$:

Our aim is to express i_D^E as a function of r_D. Given Equation (5.11) and $i^f = i_D^E$, we obtain:

$$i_D^E = \max\left[ELB, \frac{e_1\mathbb{E}_D[x_1^E] + e_2\mathbb{E}_D[\pi_1^E] + e_3 r_D}{\kappa^2 + \lambda(1-\beta\delta)}\right]. \quad (5.24)$$

Next, we determine $\mathbb{E}_D[x_1^E]$ and $\mathbb{E}_D[\pi_1^E]$ as functions of r_D. As above, we use Equations (A.34) and (A.35), where the functions $\pi_1^F(q)$ and $x_1^F(q)$ are defined in Equations (5.2) and (5.3) with $i^f = i_D^F$. Given these expectations, we can determine π_D^E, x_D^E, and i_D^E with (5.9), (5.10), and (5.11) and the expected losses L^E as a function of r_D.

Switching Forward Guidance

As before, we first consider the set-up $i^f = ELB$ and then describe the set-up $i^f = \max\{\tilde{i}_D^S, \; ELB\}$.

$i^f = ELB$:

Equations (5.12) and (5.13), the assumption $i^f = ELB$, and the algorithm in Söderlind (1999) provide the values of interest in Phase H, $t = 1$. We use these expressions to calculate the expected values of inflation and the output gap, and plug them into Equations (A.47), (A.48), and (A.49) to obtain inflation, the output gap, and the interest rate in Phase D.

$i^f = \max\{ELB \; \tilde{i}_D^S\}$:

In a first step, we write i_D^S as a function of π_2^S, x_2^S, ξ_1, r_H, and r_D. To achieve that, we use the IS Curve (2.1), the Phillips Curve (2.2), and the FOC in downturns to obtain

$$i_D^S = \max\left[ELB, \; \frac{e_1\mathbb{E}_D[x_1^S] + e_2\mathbb{E}_D[\pi_1^S] + e_3 r_D + b\sigma h(\delta)i^f}{b\sigma h(\delta) + \kappa^2 + \lambda(1 - \beta\delta)}\right]. \tag{5.25}$$

Setting $i_D^S = i^f$ and plugging in the expressions in Equations (5.12) and (5.13) yields

$$i_D^S = \max\left[\frac{\sigma^2 b_1[e_1 + \kappa e_2]\mathbb{E}_D[x_2^S] + [e_2(\beta(b_1\sigma^2 + \lambda) + \kappa\sigma b_1) - e_1(\kappa\beta - b_1\sigma)]\mathbb{E}_D[\pi_2^S]}{(\kappa^2 + \lambda(1 - \beta\delta))(b_1\sigma^2 + \kappa^2 + \lambda) + b_1\sigma(e_1 + e_2\kappa)}\right.$$

$$\left. + \frac{[e_2(b_1\sigma^2 + \lambda) - e_1\kappa]\xi_1 + b_1\sigma[e_1 + \kappa e_2]r_H + e_3(b_1\sigma^2 + \kappa^2 + \lambda)r_D}{(\kappa^2 + \lambda(1 - \beta\delta))(b_1\sigma^2 + \kappa^2 + \lambda) + b_1\sigma(e_1 + e_2\kappa)}, \; ELB\right]. \tag{5.26}$$

This expression for $i_D^S = i^f$ in combination with Equations (5.12), (5.13), and (5.14) can be used to write variables in Phase H, $t = 1$ as functions of π_2^S, x_2^S, ξ_1, r_H, and r_D:

$$x_1^S = \frac{b_1\sigma^2\mathbb{E}_1[x_2^S] - (\kappa\beta - b_1\sigma)\mathbb{E}_1[\pi_2^S] - \kappa\xi_1 + b_1\sigma r_H - b_1\sigma i_D^S}{b_1\sigma^2 + \kappa^2 + \lambda}, \tag{5.27}$$

$$\pi_1^S = \frac{\kappa\sigma^2 b_1\mathbb{E}_1[x_2^S] + (\beta(b_1\sigma^2 + \lambda) + \kappa\sigma b_1)\mathbb{E}_1[\pi_2^S] + (b_1\sigma^2 + \lambda)\xi_1 + \kappa\sigma b_1 r_H - \kappa\sigma b_1 i_D^S}{b_1\sigma^2 + \kappa^2 + \lambda}, \tag{5.28}$$

$$i_1^S = \frac{(\beta(\lambda + \sigma^2 b_1) + \kappa\sigma b_1)\mathbb{E}_1[\pi_2^S] + \kappa\sigma^2 b_1\mathbb{E}_1[x_2^S] + \kappa\sigma b_1 r_H + (\lambda + \sigma^2 b_1)\xi_1 - \kappa\sigma b_1 i_D^S}{b_1\sigma^2 + \kappa^2 + \lambda}. \tag{5.29}$$

Now that we have these functional forms, we can use backward induction. In a preliminary step the algorithm in Söderlind (1999) can be used to calculate $\mathbb{E}_1[x_2^S]$ and $\mathbb{E}_1[\pi_2^S]$ for a given supply shock ξ_1. Once we pinned down $\mathbb{E}_1[x_2^S]$ and $\mathbb{E}_1[\pi_2^S]$ for all possible ξ_1

we can determine $\mathbb{E}_D[x_1^S]$ and $\mathbb{E}_D[x_1^S]$. Finally, these expectations can be used to calculate the dynamics in downturns:

$$x_D^S = \frac{\sigma(1-\delta)(1-\beta\delta)\mathbb{E}_D[x_1^S] + (1-\delta)\mathbb{E}_D[\pi_1^S] + (1-\beta\delta)(r_D - i_D^S)}{h(\delta)}, \qquad (5.30)$$

$$\pi_D^S = \frac{\sigma\kappa(1-\delta)\mathbb{E}_D[x_1^S] + (1-\delta)(\kappa + \sigma\beta - \sigma\beta\delta)\mathbb{E}_D[\pi_1^S] + \kappa(r_D - i_D^S)}{h(\delta)}, \qquad (5.31)$$

$$i_D^S = \max\left\{ELB, \ \frac{e_1\mathbb{E}_D[x_1^S] + e_2\mathbb{E}_D[\pi_1^S] + e_3 r_D}{\kappa^2 + \lambda(1-\beta\delta)}, \right\}. \qquad (5.32)$$

5.3 Conclusions

In this supplementary chapter, we relaxed two auxiliary assumptions from the benchmark model, namely the zero interest-rate forecast and the ZLB constraint. In both instances the flexible forward guidance designs continue to be welfare improving. The most notable change is that preferences for the designs are not divided into three partitions along the shock size r_D anymore, but into two. SFG dominates for smaller r_D shocks, i.e. for $r_D > -0.0175$ when i^f is set optimally and for $r_D > -0.028$ with an ELB at -0.75%. EFG dominates for large shocks. In fact, shocks are so severe that EFG and IFG coincide.

A Appendix to Chapter 2

A.1 No Forecast in the Downturn

We derive the variables of interest by backward induction and thus first consider Phase H. The central banker selects i_t optimally to minimize the loss function in (2.4) in each period subject to the IS Curve and Phillips Curve:

$$\frac{\partial l_t}{\partial i_t} = \frac{1}{2}\frac{\partial \pi_t^2}{\partial i_t} + \frac{\lambda}{2}\frac{\partial x_t^2}{\partial i_t}, \text{ s. t.} \quad x_t = \mathbb{E}_t[x_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_H),$$

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t[\pi_{t+1}] + \xi_t.$$

The first-order condition of the loss function with respect to i_t yields

$$\kappa \pi_t + \lambda x_t = 0. \tag{A.1}$$

Using Equation (A.1) to eliminate x_t in the Phillips Curve (2.2) yields

$$\pi_t^N = \frac{\lambda\beta}{\lambda + \kappa^2}\mathbb{E}_t[\pi_{t+1}^N] + \frac{\lambda}{\lambda + \kappa^2}\xi_t. \tag{A.2}$$

Inserting Equation (A.2) into itself recursively forms the sum

$$\pi_t^N = \left[\frac{\lambda\beta}{\lambda + \kappa^2}\right]^2\mathbb{E}_t[\pi_{t+2}^N] + \beta\left[\frac{\lambda}{\lambda + \kappa^2}\right]^2\mathbb{E}_t[\xi_{t+1}] + \frac{\lambda}{\lambda + \kappa^2}\xi_t \tag{A.3}$$

$$= ... \tag{A.4}$$

$$= \left[\frac{\lambda\beta}{\lambda + \kappa^2}\right]^n\mathbb{E}_t[\pi_{t+n}^N] + \beta^{n-1}\left[\frac{\lambda}{\lambda + \kappa^2}\right]^n\mathbb{E}_t[\xi_{t+n-1}] +$$

$$\beta^{n-2}\left[\frac{\lambda}{\lambda + \kappa^2}\right]^{n-1}\mathbb{E}_t[\xi_{t+n-2}] + ... + \frac{\lambda}{\lambda + \kappa^2}\xi_t. \tag{A.5}$$

After noting that $\mathbb{E}_t[\xi_{t+1}] = \rho\xi_t$ and letting $n \to \infty$, we obtain

$$\pi_t^N = \left[1 + \rho\beta\frac{\lambda}{\lambda + \kappa^2} + \rho^2\beta^2\left(\frac{\lambda}{\lambda + \kappa^2}\right)^2 + \ldots\right]\frac{\lambda}{\lambda + \kappa^2}\xi_t \tag{A.6}$$

$$= \frac{1}{1 - \rho\beta\frac{\lambda}{\lambda+\kappa^2}}\frac{\lambda}{\lambda + \kappa^2}\xi_t \tag{A.7}$$

$$= \frac{\lambda}{\lambda(1 - \rho\beta) + \kappa^2}\xi_t. \tag{A.8}$$

Substituting Equation (A.8) into Equation (A.1), we obtain

$$x_t^N = -\frac{\kappa}{\lambda(1 - \rho\beta) + \kappa^2}\xi_t. \tag{A.9}$$

Combining Equations (A.8) and (A.9) with the IS Curve in (2.1), we conclude that

$$i_t^N = r_H + \frac{\sigma\kappa(1 - \rho) + \lambda\rho}{\lambda(1 - \beta\rho) + \kappa^2}\xi_t. \tag{A.10}$$

By definition, we can write $\mathbb{E}_1[\xi_t] = \rho^{t-1}\xi_1$, which results in

$$i_t^N = r_H + \frac{\sigma\kappa(1 - \rho) + \lambda\rho}{\lambda(1 - \beta\rho) + \kappa^2}\xi_1\rho^{t-1}, \tag{A.11}$$

$$\pi_t^N = \frac{\lambda\xi_1}{\lambda(1 - \beta\rho) + \kappa^2}\rho^{t-1}, \tag{A.12}$$

$$x_t^N = -\frac{\kappa\xi_1}{\lambda(1 - \beta\rho) + \kappa^2}\rho^{t-1}. \tag{A.13}$$

In a second step, we derive the dynamics in Phase D. Note that if the economy is in downturn in two subsequent periods, the central banker will face the same optimization problem in both instances. Therefore we drop the time subscript and mark variables in downturn by a D subscript. We combine the IS Curve (2.1) and the Phillips Curve (2.2) and resolve the forward-looking property of these equations by noting that, since $\xi_1 \in [-\xi, \xi]$ is a symmetric supply shock,

$$\mathbb{E}_D^N[\pi_{t+1}] = \delta\pi_D^N + (1 - \delta)\mathbb{E}_D[\pi_1^N] = \delta\pi_D^N \tag{A.14}$$

and

$$\mathbb{E}_D^N[x_{t+1}] = \delta x_D^N + (1 - \delta)\mathbb{E}_D[x_1^N] = \delta x_D^N. \tag{A.15}$$

Combining Equations (2.1), (2.2), (A.14), and (A.15) and using $\xi_D = 0$ yields

$$\pi_D^N = \frac{\kappa}{h}(r_D - i_D^N) \tag{A.16}$$

and

$$x_D^N = \frac{1 - \beta\delta}{h}(r_D - i_D^N), \tag{A.17}$$

where[1]

$$h := \sigma(1 - \delta)(1 - \beta\delta) - \kappa\delta > 0. \tag{A.18}$$

As $r_D < 0$, the optimal interest rate in the downturn is constrained at the ZLB, i.e. $i_D^N = 0$.

A.2 Interest-Rate Forecast in the Downturn

We now turn to the case where the central banker makes a zero interest-rate forecast in Phase D. As in the case without interest-rate forecasts, we use backward induction and, in a first step, derive the dynamics in Phase H. In periods $t \geq 2$, the dynamics of i_t, π_t, and x_t are the same as in Equations (A.11), (A.12), and (A.13), since the central banker does not provide interest-rate forecasts in Phase H. For this reason, inflation and output-gap expectations in the first period of Phase H are

$$\mathbb{E}_1[\pi_2^F] = \mathbb{E}_1[\pi_2^N] = \frac{\lambda\rho}{\lambda(1 - \rho\beta) + \kappa^2}\xi_1, \tag{A.19}$$

$$\mathbb{E}_1[x_2^F] = \mathbb{E}_1[x_2^N] = -\frac{\kappa\rho}{\lambda(1 - \rho\beta) + \kappa^2}\xi_1. \tag{A.20}$$

In $t = 1$, the central banker is still subject to the zero interest-rate forecast made in the downturn. The optimal interest rate in the first period can be attained by using the first-order condition:

$$\frac{\partial \tilde{l}_1}{\partial i_1^F} = \frac{1}{2}\frac{\partial(\pi_1^F)^2}{\partial i_1^F} + \frac{\lambda}{2}\frac{\partial(x_1^F)^2}{\partial i_1^F} + \frac{b}{2}\frac{\partial(i_1^F)^2}{\partial i_1^F}, \text{ s. t. } x_1^F = \mathbb{E}_1[x_2^N] - \frac{1}{\sigma}(i_1^F - \mathbb{E}_1[\pi_2^N] - r_H),$$

$$\pi_1^F = \kappa x_1^F + \beta\mathbb{E}_1[\pi_2^N] + \xi_1.$$

Simplifying the first-order condition yields

$$-\frac{1}{\sigma}\left[\kappa\pi_1^F + \lambda x_1^F - b\sigma i_1^F\right] = 0. \tag{A.21}$$

---

[1] In our calibration the inequality is satisfied for $\delta < 0.68$.

Combining Equations (2.1), (2.2), (A.19), (A.20), and (A.21), we obtain

$$i_1^F = \frac{\lambda + \kappa^2}{\lambda + \kappa^2 + b\sigma^2} r_H + g_1(b)\xi_1, \tag{A.22}$$

$$\pi_1^F = \frac{b\kappa\sigma}{\lambda + \kappa^2 + b\sigma^2} r_H + g_2(b)\xi_1, \tag{A.23}$$

$$x_1^F = \frac{b\sigma}{\lambda + \kappa^2 + b\sigma^2} r_H + g_3(b)\xi_1, \tag{A.24}$$

where

$$g_1(b) = \frac{(\kappa^2 + \lambda)[\sigma\kappa(1-\rho) + \rho\lambda]}{(\lambda + \kappa^2 + b\sigma^2)(\lambda(1-\beta\rho) + \kappa^2)},$$

$$g_2(b) = \frac{b\kappa^2\sigma^2(1-\rho) + b\sigma\lambda(\sigma + \kappa\rho) + \lambda(\kappa^2 + \lambda)}{(\lambda + \kappa^2 + b\sigma^2)(\lambda(1-\beta\rho) + \kappa^2)},$$

$$g_3(b) = \frac{b\sigma\rho(\lambda - \sigma\kappa) - \kappa(\kappa^2 + \lambda)}{(\lambda + \kappa^2 + b\sigma^2)(\lambda(1-\beta\rho) + \kappa^2)}.$$

We now proceed to derive the dynamics in the downturn. The central banker minimizes his loss function in (2.5) subject to the IS Curve (2.1) and the Phillips Curve (2.2) by choosing i_D^F appropriately. The first-order condition has the same form as (A.21):

$$\frac{\partial \tilde{l}_D^F}{\partial i_D^F} = -\frac{1}{\sigma}[\kappa\pi_D^F + \lambda x_D^F - b\sigma i_D^F] = 0. \tag{A.25}$$

Combining Equation (A.25) with (2.1) and (2.2) and using the forward-looking nature of these equations with

$$\mathbb{E}_D^F[\pi_{t+1}] = \delta\pi_D^F + (1-\delta)\mathbb{E}_D[\pi_1^F] \tag{A.26}$$

and

$$\mathbb{E}_D^F[x_{t+1}] = \delta x_D^F + (1-\delta)\mathbb{E}_D[x_1^F], \tag{A.27}$$

we obtain in an intermediary step

$$\pi_D^F = \frac{\sigma\kappa(1-\delta)\mathbb{E}_D[x_1^F] + (1-\delta)(\kappa + \sigma\beta - \sigma\beta\delta)\mathbb{E}_D[\pi_1^F] + \kappa(r_D - i_D^F)}{h}, \tag{A.28}$$

$$x_D^F = \frac{\sigma(1-\delta)(1-\beta\delta)\mathbb{E}_D[x_1^F] + (1-\delta)\mathbb{E}_D[\pi_1^F] + (1-\beta\delta)(r_D - i_D^F)}{h}, \tag{A.29}$$

$$i_D^F = \frac{\kappa\pi_D^F + \lambda x_D^F}{b\sigma}. \tag{A.30}$$

Using Equations (A.23) and (A.24), and $\mathbb{E}_D[\xi_1] = 0$ yields

$$\pi_D^F = \frac{(1-\delta)b\kappa\sigma(\sigma + \kappa + \sigma\beta(1-\delta))}{(\lambda + \kappa^2 + b\sigma^2)h}r_H + \frac{\kappa}{h}(r_D - i_D^F), \tag{A.31}$$

$$x_D^F = \frac{(1-\delta)b\sigma(\sigma(1-\beta\delta) + \kappa)}{(\lambda + \kappa^2 + b\sigma^2)h}r_H + \frac{1-\beta\delta}{h}(r_D - i_D^F), \tag{A.32}$$

$$i_D^F = \max\Bigg\{0, \tag{A.33}$$

$$\frac{(1-\delta)b\sigma[(\kappa^2 + \lambda)(1-\beta\delta) + \kappa^4 + \kappa^2\sigma\beta + \kappa\lambda]}{(\lambda + \kappa^2 + b\sigma^2)(b\sigma h + \kappa^2 + \lambda(1-\beta\delta))}r_H + \frac{\kappa^2 + \lambda(1-\beta\delta)}{b\sigma h + \kappa^2 + \lambda(1-\delta\beta)}r_D\Bigg\}.$$

A.3 Escaping Forward Guidance

For *escaping* forward guidance, we first derive the inflation and the output gap in $t = 1$ that are expected in the downturn for a given value of the inflation threshold π^c.

In a first step, we define two auxiliary functions $\underline{\xi}_1$ and $\bar{\xi}_1$. $\underline{\xi}_1$ is the value of ξ_1 at which the central banker's loss functions $\tilde{l}_1^F$ and $\tilde{l}_1^C$ intersect[2]—see the intersection point of the solid green and solid red lines in Figure 2.7. $\bar{\xi}_1$ is the value of ξ_1 at which the central banker's loss functions $\tilde{l}_1^N$ and $\tilde{l}_1^C$ intersect—see the intersection point of the solid green and solid black lines in Figure 2.7.[3] That is, if the realized supply shock in $t = 1$ is lower (higher) than $\underline{\xi}_1$ ($\bar{\xi}_1$), the central banker will set $i_1^E = i_1^F$ ($i_1^E = i_1^N$) and the realized inflation and the output gap will be $\pi_1^E = \pi_1^F$ ($\pi_1^E = \pi_1^N$), and $x_1^E = x_1^F$ ($x_1^E = x_1^N$). If $\xi_1 \in [\underline{\xi}_1, \bar{\xi}_1]$, the central banker will set $i_1^E = i_1^c$, and the inflation and the output gap will be π^c and x_1^c. Therefore we have

$$\mathbb{E}_D[\pi_1^E] = \begin{cases} \int_{-\xi}^{\xi} \frac{\pi_1^N(\xi_1)}{2\xi}d\xi_1 & \text{if } \bar{\xi}_1 < -\xi \\ \frac{(\bar{\xi}_1+\xi)\pi^c}{2\xi} + \int_{\bar{\xi}_1}^{\xi} \frac{\pi_1^N(\xi_1)}{2\xi}d\xi_1 & \text{if } \underline{\xi}_1 < -\xi \leq \bar{\xi}_1 < \xi \\ \int_{-\xi}^{\underline{\xi}_1} \frac{\pi_1^F(\xi_1)}{2\xi}d\xi_1 + \frac{(\bar{\xi}_1-\underline{\xi}_1)\pi^c}{2\xi} + \int_{\bar{\xi}_1}^{\xi} \frac{\pi_1^N(\xi_1)}{2\xi}d\xi_1 & \text{if } -\xi \leq \underline{\xi}_1 < \bar{\xi}_1 < \xi \\ \int_{-\xi}^{\underline{\xi}_1} \frac{\pi_1^F(\xi_1)}{2\xi}d\xi_1 + \frac{(\xi-\underline{\xi}_1)\pi^c}{2\xi} & \text{if } -\xi < \underline{\xi}_1 < \xi \leq \bar{\xi}_1 \\ \int_{-\xi}^{\xi} \frac{\pi_1^F(\xi_1)}{2\xi}d\xi_1 & \text{if } \xi \leq \underline{\xi}_1 \end{cases} \tag{A.34}$$

---

[2] $\tilde{l}_1^C = \frac{1}{2}[(\pi_1^c)^2 + \lambda(x_1^c)^2]$ denotes the central banker's loss when inflation in $t = 1$ just reaches the critical threshold, i.e. when $i_1 = i_1^c$, $\pi_1 = \pi_1^c$ and $x_1 = x_1^c$.
[3] Note that $\underline{\xi}_1$ and $\bar{\xi}_1$ depend on the choice of π^c.

and

$$
\mathbb{E}_D[x_1^E] = \begin{cases}
\int_{-\xi}^{\xi} \frac{x_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } \bar{\xi}_1 < -\xi \\
\int_{-\xi}^{\bar{\xi}_1} \frac{x_1^c(\xi_1)}{2\xi} d\xi_1 + \int_{\bar{\xi}_1}^{\xi} \frac{x_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } \underline{\xi}_1 < -\xi \leq \bar{\xi}_1 < \xi \\
\int_{-\xi}^{\underline{\xi}_1} \frac{x_1^F(\xi_1)}{2\xi} d\xi_1 + \int_{\underline{\xi}_1}^{\bar{\xi}_1} \frac{x_1^c(\xi_1)}{2\xi} d\xi_1 + \int_{\bar{\xi}_1}^{\xi} \frac{x_1^N(\xi_1)}{2\xi} d\xi_1 & \text{if } -\xi \leq \underline{\xi}_1 < \bar{\xi}_1 < \xi \\
\int_{-\xi}^{\underline{\xi}_1} \frac{x_1^F(\xi_1)}{2\xi} d\xi_1 + \int_{\underline{\xi}_1}^{\xi} \frac{x_1^c(\xi_1)}{2\xi} d\xi_1 & \text{if } -\xi < \underline{\xi}_1 < \xi \leq \bar{\xi}_1 \\
\int_{-\xi}^{\xi} \frac{x_1^F(\xi_1)}{2\xi} d\xi_1 & \text{if } \xi \leq \underline{\xi}_1.
\end{cases}
\tag{A.35}
$$

Note that, as in Equations (2.23) and (2.24), we have

$$
\mathbb{E}_D[\pi_{t+1}^E] = \delta \pi_D^E + (1-\delta)\mathbb{E}_D[\pi_1^E]
\tag{A.36}
$$

and

$$
\mathbb{E}_D[x_{t+1}^E] = \delta x_D^E + (1-\delta)\mathbb{E}_D[x_1^E].
\tag{A.37}
$$

Hence, using Equations (2.1), (2.2), (A.36), (A.37), and (A.21)—which also applies to EFG,—we obtain the inflation, the output gap, and the interest rate in the downturn

$$
\pi_D^E = \frac{\sigma\kappa(1-\delta)\mathbb{E}_D[x_1^E] + (1-\delta)(\kappa + \sigma\beta - \sigma\beta\delta)\mathbb{E}_D[\pi_1^E] + \kappa(r_D - i_D^E)}{h},
\tag{A.38}
$$

$$
x_D^E = \frac{\sigma(1-\delta)(1-\beta\delta)\mathbb{E}_D[x_1^E] + (1-\delta)\mathbb{E}_D[\pi_1^E] + (1-\beta\delta)(r_D - i_D^E)}{h},
\tag{A.39}
$$

$$
i_D^E = \max\{0, \hat{i}_D^E\},
\tag{A.40}
$$

where

$$
\hat{i}_D^E = \frac{(1-\delta)\sigma(\kappa^2 + \lambda(1-\beta\delta))\mathbb{E}_D[x_1^E]}{b\sigma h + \kappa^2 + \lambda(1-\beta\delta)}
$$
$$
+ \frac{(1-\delta)(\kappa(\kappa + \sigma\beta - \sigma\beta\delta) + \lambda)\mathbb{E}_D[\pi_1^E] + (\kappa^2 + \lambda(1-\beta\delta))r_D}{b\sigma h + \kappa^2 + \lambda(1-\beta\delta)}.
$$

Expectations in a Downturn

Under EFG, the situation may emerge that $\mathbb{E}_D[\pi_1^E] > \mathbb{E}_D[\pi_1^F]$ and $\mathbb{E}_D[x_1^E] > \mathbb{E}_D[x_1^F]$. It is thus interesting to see how this is possible. Assume π^c is chosen in such a way that the central banker strategically escapes only at supply shocks close to the upper bound ξ. Then, the only viable options for the central banker are (i) no escape, with $i_1^E = i_1^F$, and (ii) strategic escape, with $i_1^E < i_1^F$. The probability that the central banker will face the cases $i_1^E > i_1^F$ is equal to zero. For this reason, $i_1^E \leq i_1^F$ holds for all supply shock realizations, which implies $\mathbb{E}_D[i_1^E] \leq \mathbb{E}_D[i_1^F]$. A low interest rate promotes inflation and

it follows that $\mathbb{E}_D[\pi_1^E] \geq \mathbb{E}_D[\pi_1^F]$ and $\mathbb{E}_D[x_1^E] \geq \mathbb{E}_D[x_1^F]$.[4]
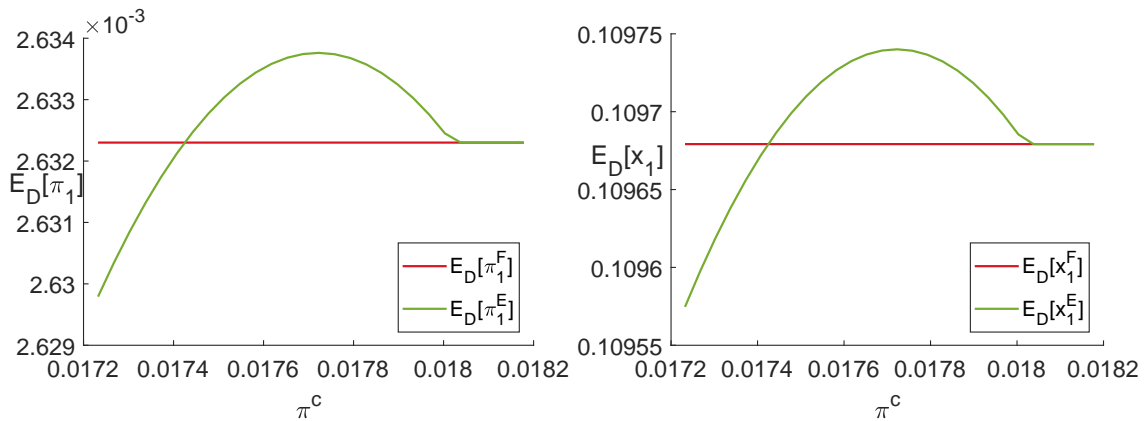
Figure A.1: Expected inflation and output gap in Phase H, t=1, with π^c close to the case of never escaping.

Due to the higher expected values in π_1 and x_1, the expected social losses are higher than the alleged upper bound that is set by $\mathbb{E}_D[l_1^F]$. However, from a central banker's perspective, $\mathbb{E}_D[\tilde{l}_1^E]$ gradually approaches $\mathbb{E}_D[\tilde{l}_1^F]$. He balances out the losses from escaping—comprising the terms $\pi_1^E > \pi_1^F$ and $x_1^E > x_1^F$, but not the extra term $b(i_1^E)^2$—and the losses from not escaping—comprising the terms $\pi_1^E = \pi_1^F$, $x_1^E = x_1^F$, and $b(i_1^E)^2$. A graphical representation is displayed in Figure A.2.
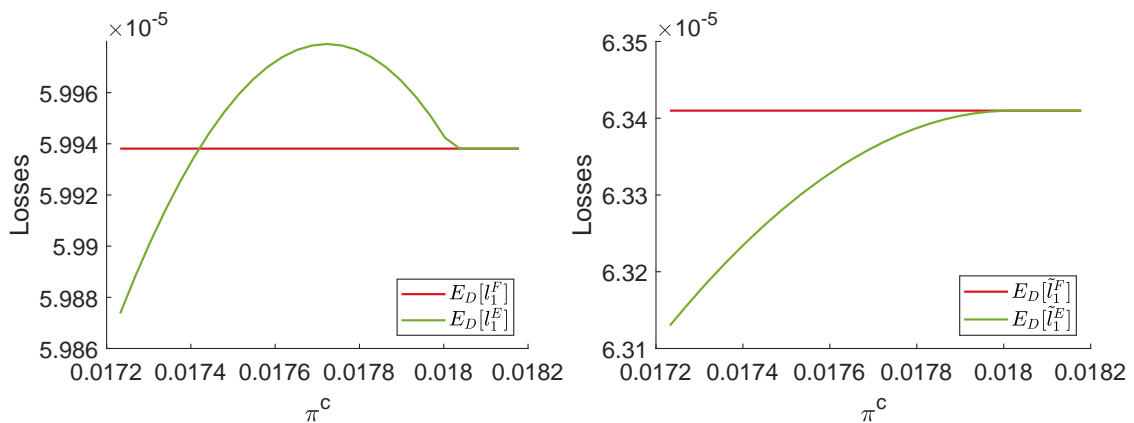
Figure A.2: Expected social losses and expected central banker's losses in Phase H, t=1, with π^c close to the case of never escaping.

---

[4] For a range of ξ_1 close to the upper bound ξ, there may be scenarios where the central banker sets $i_1^E > i_1^F$, but this range is too narrow to offset the effects of $i_1^E < i_1^F$. Thus, it still holds that $\mathbb{E}_D[i_1^E] \leq \mathbb{E}_D[i_1^F]$.

A.4 Switching Forward Guidance

We now show how to derive the dynamics of the economy in the downturn and Phase H under SFG. As before, we proceed by backward induction. We first derive the dynamics of the economy in $t \geq 2$ for each given ξ_2 and π_2^f. Given the dynamics of the economy in $t \geq 2$, we can then derive the central banker's optimal inflation forecast π_2^f in $t = 1$ for each realized ξ_1. In the last step, with the inflation and the output gap in $t = 1$ for each supply shock, we can derive the inflation, the output gap, and the interest rate in the downturn.

Using the notation of Söderlind (1999), we have $y_t := (\xi_t, \pi_t^f, r_H, \pi_t, x_t)'$, where $y_{1,t} := (\xi_t, \pi_t^f, r_H)'$ are predetermined and $y_{2,t} := (\pi_t, x_t)'$ are non-predetermined entries. The vector of policy instruments is $u_t := (i_t, \pi_{t+1}^f)'$.

In our model, the dynamics of y_t for $t \geq 2$ are given by

$$\begin{pmatrix} y_{1,t+1} \\ \mathbb{E}_t[y_{2,t+1}] \end{pmatrix} = A \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} + Bu_t, \tag{A.41}$$

where

$$A := \begin{pmatrix} \rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -\frac{1}{\beta} & 0 & 0 & \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ \frac{1}{\beta\sigma} & 0 & -\frac{1}{\sigma} & -\frac{1}{\beta\sigma} & 1+\frac{\kappa}{\beta\sigma} \end{pmatrix}, B := \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\sigma} & 0 \end{pmatrix}. \tag{A.42}$$

The central banker's loss function

$$\tilde{l}_t^S = \frac{1}{2}[(\pi_t^S)^2 + \lambda(x_t^S)^2 + b(\pi_t^S - \pi_t^f)^2] \tag{A.43}$$

can be written in the form used by Söderlind:

$$\tilde{l}_t^S = y_t'Qy_t + 2y_t'Uu_t + u_t'Ru_t, \tag{A.44}$$

with

$$Q := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{b}{2} & 0 & -\frac{b}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{b}{2} & 0 & \frac{1+b}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{\lambda}{2} \end{pmatrix}, U := \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, R := \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{A.45}$$

As in Söderlind (1999), the central banker's optimization problem can then be formulated in a Bellman Equation with the value function $v(y_t) = y_{1,t}'V_t y_{1,t} + v_t$. The value func-

tion incorporates the predetermined variables $y_{1,t}$, the matrix V_t, which is assumed to be independent of exogenous shocks, and the term v_t that includes shocks:

$$v(y_t) = \min_{u_t} \left\{ y_t' Q y_t + 2 y_t' U u_t + u_t' R u_t + \beta \mathbb{E}_t[v(y_{t+1})] \right\}, \tag{A.46}$$

s.t. $\mathbb{E}_t[y_{2,t+1}] = C_{t+1} \mathbb{E}_t[y_{1,t+1}]$, (A.41), and given $y_{1,t}$, where C_{t+1} is a 2×3-matrix. We use the Matlab algorithm provided by Söderlind (1999) to recursively solve this Bellman Equation. Thus, for each given ξ_2 and π_2^f, we obtain the dynamics of the economy for $t \geq 2$. Using Equations (2.1), (2.2) and (2.38), we obtain the dynamics in $t = 1$:

$$x_1^S = \frac{b\sigma^2 \mathbb{E}_1[x_2^S] - (\kappa\beta - b\sigma)\mathbb{E}_1[\pi_2^S] - \kappa\xi_1 + b\sigma r_H}{b\sigma^2 + \kappa^2 + \lambda}, \tag{A.47}$$

$$\pi_1^S = \frac{(\beta(b\sigma^2 + \lambda) + \kappa\sigma b)\mathbb{E}_1[\pi_2^S] + \kappa\sigma^2 b\mathbb{E}_1[x_2^S] + (b\sigma^2 + \lambda)\xi_1 + \kappa\sigma b r_H}{b\sigma^2 + \kappa^2 + \lambda}, \tag{A.48}$$

$$i_1^S = \frac{(\kappa^2 + \beta\kappa\sigma + \lambda)\mathbb{E}_1[\pi_2^S] + \sigma(\kappa^2 + \lambda)\mathbb{E}_1[x_2^S] + \sigma\kappa\xi_1 + (\kappa^2 + \lambda)r_H}{b\sigma^2 + \kappa^2 + \lambda}. \tag{A.49}$$

Therefore, the dynamics of the economy for a given π_2^f in normal times are determined, and the central banker chooses the inflation forecast π_2^f so that the loss function in (2.41) is minimized. Similar to Equations (A.38) – (A.40), we could obtain the inflation, output gap, and interest rate under SFG in the downturn.

B Appendix to Chapter 3

B.1 Firm's Profit Maximization Problem

The first-order condition of the maximization problem in (3.2) subject to the demand constraint in (3.3) is

$$
\frac{\partial\{\sum_{k=0}^{\infty} \alpha^k \mathbb{E}_t^j [Q_{t,t+k} P_t^j Y_{t+k|t}^j - Q_{t,t+k} N_{t+k}^j W_{t+k}]\}}{\partial P_t^j}
$$

$$
= \frac{\partial\{\sum_{k=0}^{\infty} \alpha^k \mathbb{E}_t^j [Q_{t,t+k} P_t^j Y_{t+k|t}^j - Q_{t,t+k} Y_{t+k|t}^j A_{t+k}^{-1} W_{t+k}]\}}{\partial P_t^j},
$$

$$
= \frac{\partial\{\sum_{k=0}^{\infty} \alpha^k \mathbb{E}_t^j [Q_{t,t+k} P_t^j \left(\frac{P_t^j}{P_{t+k}}\right)^{-\omega} Y_{t+k} - Q_{t,t+k} \left(\frac{P_t^j}{P_{t+k}}\right)^{-\omega} Y_{t+k} A_{t+k}^{-1} W_{t+k}]\}}{\partial P_t^j},
$$

$$
= \sum_{k=0}^{\infty} \alpha^k \mathbb{E}_t^j \Bigg[Q_{t,t+k} \left(\frac{P_t^j}{P_{t+k}}\right)^{-\omega} Y_{t+k}
$$

$$
- \omega Q_{t,t+k} \frac{P_t^j}{P_{t+k}} \left(\frac{P_t^j}{P_{t+k}}\right)^{-\omega-1} Y_{t+k} + \omega Q_{t,t+k} \left(\frac{P_t^j}{P_{t+k}}\right)^{-\omega-1} Y_{t+k} \frac{W_{t+k}}{P_{t+k} A_{t+k}} \Bigg],
$$

$$
= \sum_{k=0}^{\infty} \alpha^k \mathbb{E}_t^j \Bigg[Q_{t,t+k} Y_{t+k|t}^j - \omega Q_{t,t+k} Y_{t+k|t}^j + \omega Q_{t,t+k} Y_{t+k|t}^j \frac{W_{t+k}}{P_t^j A_{t+k}} \Bigg],
$$

$$
= \sum_{k=0}^{\infty} \alpha^k \mathbb{E}_t^j \Bigg[Q_{t,t+k} Y_{t+k|t}^j \left(P_t^j - \frac{\omega}{\omega - 1} \frac{W_{t+k}}{A_{t+k}} \right) \Bigg] = 0.
$$

B.2 Potential Output

For fully flexible prices (i.e. $\alpha = 0$), the first-order condition of the maximization problem in (3.2) subject to the demand constraint in (3.3) is

$$
P_t^j - \frac{\omega}{\omega - 1} \frac{W_t}{A_t} = 0. \tag{B.1}
$$

Note that all firms face the same maximization problem and thus the j superscript can be dropped. By using the first-order condition in (3.27)

$$P_t(N_t^i)^\phi(C_t^i)^\sigma = W_t = P_t(N_t)^\phi(C_t)^\sigma,$$

we can rewrite Equation (B.1) as

$$P_t - \frac{\omega}{\omega - 1}\frac{P_t N_t^\phi C_t^\sigma}{A_t} = 0. \tag{B.2}$$

We define $\mathcal{M} := \frac{\omega}{\omega - 1}$ and use the production function to write Equation (B.2) as

$$1 = \mathcal{M}\frac{Y_t^\phi C_t^\sigma}{A_t^{1+\phi}}.$$

Taking the logarithm yields

$$0 = \mu + \phi y_t + \sigma c_t - (1+\phi)a_t.$$

Market clearing implies $y_t = c_t$. Hence, output under fully flexible prices, which we denote as y_t^p, is

$$y_t^p = \frac{1+\phi}{\phi + \sigma}a_t - \frac{\mu}{\phi + \sigma}. \tag{B.3}$$

B.3 Output Gap

Real marginal costs in period t are defined in Subsection 3.2.1 as $MC_t := \frac{W_t}{A_t P_t}$. Plugging in the first-order condition in (3.27) to replace W_t, taking logarithms, and noting that market clearing implies $c_t = y_t$ yields

$$mc_t = (\sigma + \phi)y_t - (1+\phi)a_t. \tag{B.4}$$

Under flexible prices, real marginal costs are

$$mc = (\sigma + \phi)y_t^p - (1+\phi)a_t, \tag{B.5}$$

where y_t^p is derived in Appendix B.2. Subtracting (B.5) from (B.4) yields

$$\hat{mc}_t = (\sigma + \phi)(y_t - y_t^p), \tag{B.6}$$

and we denote the output gap $(y_t - y_t^p)$ as x_t.

B.4 Iterated Expectations

In the model, we use a mechanism where the central banker sends a public signal first. After receiving this signal, the agents receive a private signal and form expectations on the basis of these two pieces of information. The central banker is aware of the private signals and takes them into account. Then the private agents expect the central banker to take their decision into consideration and adapt their expectations accordingly. In turn, the central banker adapts his expectations again. We do the iterative calculations for π, but every step is analogous for x. If we continue iteration, we obtain

$$\mathbb{E}^i[\overline{\mathbb{E}}[\pi_1]] = \frac{\sigma_{\epsilon_\pi}^2 \pi_1^{RE} + \sigma_{\eta_\pi}^2 \tau \left(\frac{\sigma_{\epsilon_\pi}^2 \pi_1^{RE} + \sigma_{\eta_\pi}^2 \pi_1^i}{\sigma_{\epsilon_\pi}^2 + \sigma_{\eta_\pi}^2} \right)}{\sigma_{\epsilon_\pi}^2 + \sigma_{\eta_\pi}^2}, \tag{B.7}$$

$$= \left[1 - \frac{(\sigma_{\eta_\pi}^2)^2}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^2} - (1 - \tau_\pi) \frac{\sigma_{\eta_\pi}^2 \sigma_{\epsilon_\pi}^2}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^2} \right] \pi_1^{RE} + \tau_\pi \frac{(\sigma_{\eta_\pi}^2)^2}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^2} \pi_1^i, \tag{B.8}$$

$$\overline{\mathbb{E}}[\mathbb{E}^i[\overline{\mathbb{E}}[\pi_1]]] = \left[1 - \frac{(\sigma_{\eta_\pi}^2)^2}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^2} - (1 - \tau_\pi) \frac{\sigma_{\eta_\pi}^2 \sigma_{\epsilon_\pi}^2}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^2} \right] \pi_1^{RE} + \tau_\pi^2 \frac{(\sigma_{\eta_\pi}^2)^2}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^2} \pi_1, \tag{B.9}$$

...

$$\mathbb{E}^i[\overline{\mathbb{E}}^n[\pi_1]] = \left[1 - \frac{(\sigma_{\eta_\pi}^2)^n}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^n} - \frac{\sigma_{\eta_\pi}^2 \sigma_{\epsilon_\pi}^2}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^2} \left(\sum_{j=0}^{n-2} \frac{(\sigma_{\eta_\pi}^2)^j}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^j} - \tau_\pi \sum_{j=0}^{n-2} \frac{(\tau_\pi \sigma_{\eta_\pi}^2)^j}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^j} \right) \right]$$
$$+ \frac{(\sigma_{\eta_\pi}^2)^n \tau^{n-1}}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^n}, \tag{B.10}$$

$$\overline{\mathbb{E}}^n[\pi_1] = \left[1 - \frac{(\sigma_{\eta_\pi}^2)^n}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^n} - \frac{\sigma_{\eta_\pi}^2 \sigma_{\epsilon_\pi}^2}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^2} \left(\sum_{j=0}^{n-2} \frac{(\sigma_{\eta_\pi}^2)^j}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^j} - \tau_\pi \sum_{j=0}^{n-2} \frac{(\tau_\pi \sigma_{\eta_\pi}^2)^j}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^j} \right) \right]$$
$$+ \frac{(\sigma_{\eta_\pi}^2)^n \tau^n}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)^n}. \tag{B.11}$$

Letting $n \to \infty$, we can simplify the expression

$$\mathbb{E}^i[\overline{\mathbb{E}}^n[\pi_1]] = \left[1 - \frac{(\sigma_{\eta_\pi}^2)}{(\sigma_{\eta_\pi}^2 + \sigma_{\epsilon_\pi}^2)} \left(1 - \frac{\tau_\pi \sigma_{\epsilon_\pi}^2}{\sigma_{\epsilon_\pi}^2 + (1 - \tau_\pi)\sigma_{\eta_\pi}^2} \right) \right] \pi_1^{RE} = \overline{\mathbb{E}}^n[\pi_1]. \tag{B.12}$$

Morris and Shin (2002) discuss the case $\tau = 1$ and state that expectations approach the public value. This is merely a special case in our setting. If $\tau \leq 1$, the expectations are $c(\tau_\pi)\pi_1^{RE}$, where $0 \leq c(\tau_\pi) \leq 1$ is a function depending on τ_π. For our parameterization, we obtain $c(\tau_\pi) > 1$ for $1 < \tau < 2$.

B.5 Variations in the Pessimism Parameter

Figure B.1 illustrates how under switching and without forward guidance, welfare losses depend on the supply shock. The graph on the left outlines the feature that interest-rate forecasts shift losses from the downturn to normal times. The interest-rate forecast leads to an upward shift in losses compared to the discretionary case in $t = 1$. After a transition period in $t = 2$, where $i^f = 0$ still exerts influence on economic outputs through π_2^f, we see in the right graph that depending on the value of τ_H, inflation forecasts can be both beneficial or detrimental to welfare in $t \geq 3$.
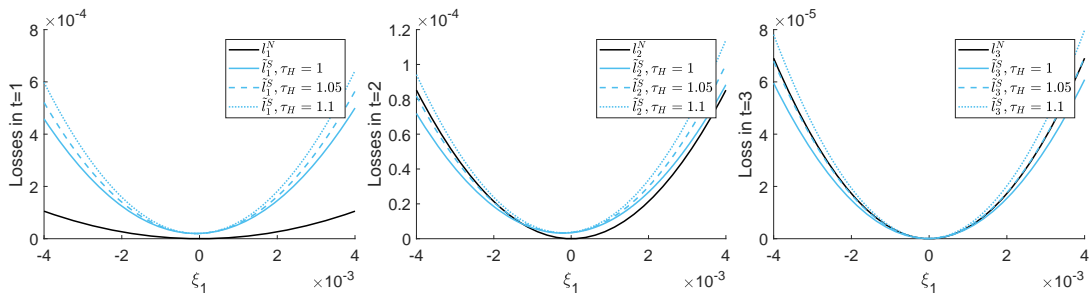
Figure B.1: Central banker losses under switching forward guidance in Phase H, t=1, 2, 3.

Figures B.2, B.3, and B.4 show different combinations of pessimism in the economy. The left graph in each figure shows the scenario when agents in the economy are only pessimistic in a downturn but not in normal times. In such circumstances, switching forward guidance remains attractive for medium-sized r_D shocks, and escaping forward guidance is dominant otherwise. A soon as pessimism—in terms of returning to the steady state quickly—is strong enough in Phase H, escaping forward guidance dominates for all r_D values.
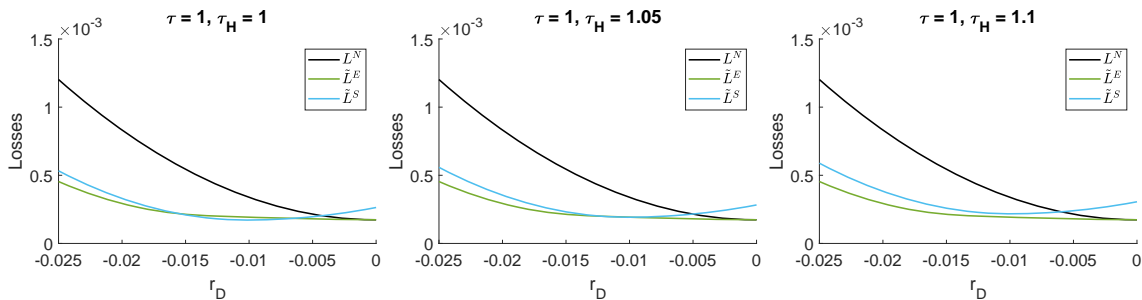
Figure B.2: Central banker losses under escaping and switching forward guidance.
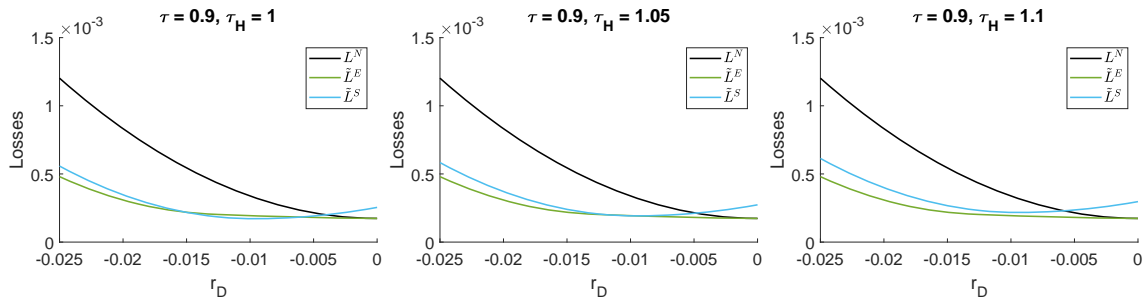
Figure B.3: Central banker losses under escaping and switching forward guidance.
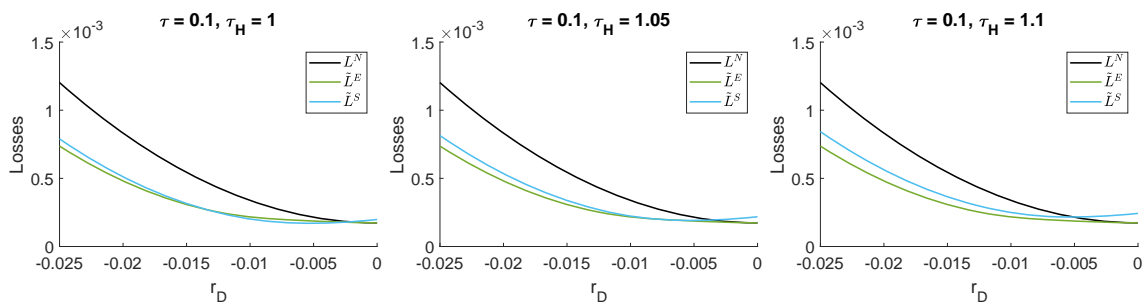
Figure B.4: Central banker losses under escaping and switching forward guidance.

C Appendix to Chapter 4

C.1 Welfare Differences

In this section, we provide more detailed results which extend and support the analysis in Section 4.3.2.

Gains Compared to NFG

	λ	κ	σ	β		
S^{tot}_{narrow}	0.087	0.5708	0.7844	0.0001		
S^{tot}_{wide}	0.1232	0.2750	0.9777	0.0039		
S^{1}_{narrow}	0.0488	0.1494	0.3636	0		
S^{1}_{wide}	0.0095	0.0128	0.6146	0		
	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^{2}_{narrow}	0.0173	0.0168	0	0.3999	0	0
S^{2}_{wide}	0	0.1008	0	0.2454	0	0

Narrow – Mean: -0.0000, SD: 0.0001, Coef. of Var.: 415%, err_{LOO}: $4.23 * 10^{-7}$
Wide – Mean: 0.0000, SD: 0.0002, Coef. of Var.: 646%, err_{LOO}: $7.46 * 10^{-2}$

Table C.1: First- and second-order Sobol' Indices for $L^N - L^F$ with $p = 10$, $N = 3100$, and $r_D = -0.005$.

	λ	κ	σ	β
S^{tot}_{narrow}	0.0100	0.6507	0.5793	0.0001
S^{tot}_{wide}	0.0741	0.2701	0.9573	0.0074
S^{1}_{narrow}	0.0063	0.4132	0.3410	0
S^{1}_{wide}	0.0181	0.0227	0.6664	0

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^{2}_{narrow}	0.0011	0.0020	0	0.2356	0	0
S^{2}_{wide}	0.0011	0.0438	0	0.2331	0.0003	0.0009

Narrow – Mean: 0.0009, SD: 0.0009, Coef. of Var.: 102%, err_{LOO}: $4.78 * 10^{-7}$
Wide – Mean: 0.0003, SD: 0.0008, Coef. of Var.: 257%, err_{LOO}: $1.86 * 10^{-2}$

Table C.2: First- and second-order Sobol' Indices for $L^N - L^F$ with $p = 10$, $N = 3100$, and $r_D = -0.015$.

	λ	κ	σ	β
S^{tot}_{narrow}	0.0060	0.6733	0.5411	0.0001
S^{tot}_{wide}	0.0625	0.2706	0.9547	0.0065
S^{1}_{narrow}	0.0043	0.4541	0.3214	0
S^{1}_{wide}	0.0191	0.0245	0.6752	0.0001

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^{2}_{narrow}	0.0005	0.0009	0	0.2184	0	0
S^{2}_{wide}	0.0009	0.0335	0	0.2346	0.0002	0.0009

Narrow – Mean: 0.0025, SD: 0.0024, Coef. of Var.: 97%, err_{LOO}: $1.78 * 10^{-7}$
Wide – Mean: 0.0008, SD: 0.0018, Coef. of Var.: 232%, err_{LOO}: $1.86 * 10^{-2}$

Table C.3: First- and second-order Sobol' Indices for $L^N - L^F$ with $p = 10$, $N = 3100$, and $r_D = -0.03$.

	λ	κ	σ	β
S^{tot}_{narrow}	0.0188	0.6325	0.5922	0.0001
S^{tot}_{wide}	0.1372	0.2657	0.9677	0.0044
S^1_{narrow}	0.0127	0.3923	0.3523	0
S^1_{wide}	0.0136	0.0152	0.6231	0

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^2_{narrow}	0.0012	0.0019	0	0.2199	0	0
S^2_{wide}	0.0022	0.0961	0.0003	0.2244	0	0

Narrow – Mean: 0.0001, SD: 0.0001, Coef. of Var.: 103%, err_{LOO}: $2.30*10^{-5}$
Wide – Mean: 0.0001, SD: 0.0002, Coef. of Var.: 371%, err_{LOO}: $7.00*10^{-2}$

Table C.4: First- and second-order Sobol' Indices for $L^N - L^E$ with $p = 7$, $N = 990$, and $r_D = -0.005$.

	λ	κ	σ	β
S^{tot}_{narrow}	0.0106	0.6575	0.5561	0.0001
S^{tot}_{wide}	0.0662	0.2694	0.9627	0.0071
S^1_{narrow}	0.0069	0.4358	0.3336	0
S^1_{wide}	0.0151	0.0210	0.6738	0

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^2_{narrow}	0.0012	0.0019	0	0.2199	0	0
S^2_{wide}	0.0005	0.0395	0.0004	0.2383	0	0

Narrow – Mean: 0.0009, SD: 0.0008, Coef. of Var.: 98%, err_{LOO}: $4.73*10^{-7}$
Wide – Mean: 0.0003, SD: 0.0008, Coef. of Var.: 262%, err_{LOO}: $5.75*10^{-2}$

Table C.5: First- and second-order Sobol' Indices for $L^N - L^E$ with $p = 7$, $N = 990$, and $r_D = -0.015$.

	λ	κ	σ	β
S^{tot}_{narrow}	0.0060	0.6730	0.5408	0.0001
S^{tot}_{wide}	0.0541	0.2785	0.9598	0.0078
S^1_{narrow}	0.0043	0.4544	0.3217	0
S^1_{wide}	0.0160	0.0229	0.6757	0

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^2	0	0.0001	0	0.2178	0	0
S^2_{wide}	0.0003	0.0277	0.0003	0.2463	0	0

Narrow – Mean: 0.0025, SD: 0.0024, Coef. of Var.: 96%, err_{LOO}: $2.16*10^{-5}$
Wide – Mean: 0.0008, SD: 0.0018, Coef. of Var.: 239%, err_{LOO}: $5.64*10^{-2}$

Table C.6: First- and second-order Sobol' Indices for $L^N - L^E$ with $p = 7$, $N = 990$, and $r_D = -0.003$.

	λ	κ	σ	β
S^{tot}_{narrow}	0.0190	0.6361	0.6186	0.0001
S^{tot}_{wide}	0.1409	0.2346	0.9718	0.0056
S^1_{narrow}	0.0124	0.3667	0.3480	0
S^1_{wide}	0.0088	0.0155	0.6466	0.0005

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^2_{narrow}	0.0023	0.0034	0	0.2662	0	0
S^2_{wide}	0.0022	0.1082	0.0003	0.1959	0	0.0001

Narrow – Mean: 0.0001, SD: 0.0001, Coef. of Var.: 98%, err_{LOO}: $2.40*10^{-5}$
Wide – Mean: 0.0001, SD: 0.0002, Coef. of Var.: 333%, err_{LOO}: $6.11*10^{-2}$

Table C.7: First- and second-order Sobol' Indices for $L^N - L^S$ with $p = 7$, $N = 990$, and $r_D = -0.005$.

	λ	κ	σ	β
S^{tot}_{narrow}	0.0165	0.6308	0.5688	0.0001
S^{tot}_{wide}	0.0953	0.2420	0.9541	0.0045
S^{1}_{narrow}	0.0115	0.4181	0.3549	0
S^{1}_{wide}	0.0220	0.0215	0.6791	0

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^{2}_{narrow}	0.0015	0.0027	0	0.2104	0	0
S^{2}_{wide}	0.0018	0.0554	0	0.2014	0.0003	0.0007

Narrow – Mean: 0.0008, SD: 0.0007, Coef. of Var.: 88%, err_{LOO}: $1.76 * 10^{-5}$
Wide – Mean: 0.0003, SD: 0.0008, Coef. of Var.: 231%, err_{LOO}: $1.99 * 10^{-2}$

Table C.8: First- and second-order Sobol' Indices for $L^N - L^S$ with $p = 7$ and $N = 990$ (narrow), $p = 9$ and $N = 2150$ (wide), and $r_D = -0.015$.

	λ	κ	σ	β
S^{tot}_{narrow}	0.0172	0.6268	0.5661	0.0002
S^{tot}_{wide}	0.0623	0.2543	0.9584	0.0056
S^{1}_{narrow}	0.0122	0.4201	0.3582	0.0001
S^{1}_{wide}	0.0191	0.0216	0.6910	0

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^{2}_{narrow}	0.0015	0.0027	0	0.2044	0	0
S^{2}_{wide}	0.0001	0.0341	0.0002	0.2240	0	0.0002

Narrow – Mean: 0.0019, SD: 0.0016, Coef. of Var.: 87%, err_{LOO}: $1.69 * 10^{-5}$
Wide – Mean: 0.0008, SD: 0.0017, Coef. of Var.: 223%, err_{LOO}: $4.93 * 10^{-2}$

Table C.9: First- and second-order Sobol' Indices for $L^N - L^S$ with $p = 7$, $N = 990$, and $r_D = -0.03$.

Gains Compared to IFG

Note that we do not have accurate results of $L^F - L^E$ with $r_D = -0.03$ in the narrow and wide parameter ranges, and $L^F - L^S$ with $r_D = -0.015, \; -0.03$ in the wide parameter ranges.

	λ	κ	σ	β		
S^{tot}_{narrow}	0.0354	0.8060	0.1932	0.0003		
S^{tot}_{wide}	0.5519	0.1047	0.9367	0.0327		
S^1_{narrow}	0.0127	0.7840	0.1701	0.0002		
S^1_{wide}	0.0321	0.0183	0.3519	0.0101		
	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^2_{narrow}	0.0098	0.0109	0	0.0102	0	0
S^2_{wide}	0.0011	0.4839	0.0012	0.0580	0.0003	0.0066

Narrow – Mean: 0.0001, SD: 0.0000, Coef. of Var.: 37%, err_{LOO}: $1.09 * 10^{-7}$
Wide – Mean: 0.0000, SD: 0.0000, Coef. of Var.: 80%, err_{LOO}: $6.54 * 10^{-2}$

Table C.10: First- and second-order Sobol' Indices for $L^F - L^E$ with $p = 10$, $N = 3000$, and $r_D = -0.005$.

	λ	κ	σ	β
S^{tot}_{narrow}	0.8155	0.7759	0.8207	0.0030
S^{tot}_{wide}	0.2776	0.7000	0.9755	0.0005
S^{1}_{narrow}	0.0298	0.0314	0.0590	0
S^{1}_{wide}	0.0021	0.0155	0.2299	0

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^{2}_{narrow}	0.1180	0.1347	0	0.0938	0	0
S^{2}_{wide}	0.0069	0.0680	0	0.4769	0	0

Narrow – Mean: 0.0000, SD: 0.0000, Coef. of Var.: 637%, err_{LOO}: $2.86 * 10^{-2}$
Wide – Mean: 0.0000, SD: 0.0000, Coef. of Var.: 1190%, err_{LOO}: $1.44 * 10^{-3}$

Table C.11: First- and second-order Sobol' Indices for $L^F - L^E$ with $p = 20$, $N = 32000$, and $r_D = -0.015$.

	λ	κ	σ	β
S^{tot}_{narrow}	0.0578	0.8158	0.1804	0.0004
S^{tot}_{wide}	0.4453	0.3061	0.7006	0.0786
S^{1}_{narrow}	0.0228	0.7757	0.1490	0.0005
S^{1}_{wide}	0.0263	0.1987	0.2864	0.0131

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^{2}_{narrow}	0.0206	0.0119	0	0.0169	0	0
S^{2}_{wide}	0.0071	0.3350	0.0272	0.0307	0.0223	0.0019

Narrow – Mean: 0.0001, SD: 0.0000, Coef. of Var.: 31%, err_{LOO}: $5.19 * 10^{-6}$
Wide – Mean: 0.0000, SD: 0.0000, Coef. of Var.: 29%, err_{LOO}: $3.94 * 10^{-2}$

Table C.12: First- and second-order Sobol' Indices for $L^F - L^S$ with $p = 7$, $N = 990$, and $r_D = -0.005$.

	λ	κ	σ	β
S^{tot}_{narrow}	0.0013	0.7608	0.4961	0
S^{tot}_{wide}	0.2836	0.5914	0.9705	0.0484
S^{1}_{narrow}	0.0007	0.5029	0.2384	0
S^{1}_{wide}	0	0.0089	0.3472	0

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^{2}_{narrow}	0.0206	0.0119	0	0.0169	0	0
S^{2}_{wide}	0.0003	0.0001	0	0.2574	0	0

Narrow – Mean: -0.0001, SD: 0.0002, Coef. of Var.: 192%, err_{LOO}: $7.02 * 10^{-6}$
Wide – Mean: 0.0000, SD: 0.0000, Coef. of Var.: 60%, err_{LOO}: $2.12 * 10^{-1}$

Table C.13: First- and second-order Sobol' Indices for $L^F - L^S$ with $p = 10$, $N = 2000$, and $r_D = -0.015$.

	λ	κ	σ	β
S^{tot}_{narrow}	0.0018	0.7591	0.4811	0.0001
S^{tot}_{wide}				
S^{1}_{narrow}	0.0011	0.5176	0.2394	0
S^{1}_{wide}				

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^{2}_{narrow}	0.0002	0.0003	0	0.2412	0	0
S^{2}_{wide}						

Narrow – Mean: -0.0006, SD: 0.0008, Coef. of Var.: 127%, err_{LOO}: $2.22 * 10^{-5}$
Wide – Mean: 0.0000, SD: 0.0000, Coef. of Var.: , err_{LOO}:

Table C.14: First- and second-order Sobol' Indices for $L^F - L^S$ with $p = 7$, $N = 990$, and $r_D = -0.03$.

Gains in Applying EFG vs. SFG

Note that we do not have accurate results of $L^E - L^S$ in the wide parameter ranges and for all shock sizes.

	λ	κ	σ	β		
S^{tot}_{narrow}	0.0461	0.7518	0.2423	0.0011		
S^{tot}_{wide}						
S^1_{narrow}	0.0111	0.7178	0.2309	0.0003		
S^1_{wide}						

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^2_{narrow}	0.0277	0.0058	0.0001	0.0.0042	0.0007	0
S^2_{wide}						

Narrow – Mean: 0.0000, SD: 0.0000, Coef. of Var.: 4831%, err_{LOO}: $2.22 * 10^{-5}$
Wide – Mean: 0.0000, SD: 0.0000, Coef. of Var.: , err_{LOO}: $3.66 * 10^{-1}$

Table C.15: First- and second-order Sobol' Indices for $L^E - L^S$ with $p = 7$, $N = 990$, and $r_D = -0.005$.

	λ	κ	σ	β
S^{tot}_{narrow} S^{tot}_{wide}	0.0013	0.7606	0.4960	0
S^{1}_{narrow} S^{1}_{wide}	0.0007	0.5030	0.2386	0

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^{2}_{narrow} S^{2}_{wide}	0.0003	0.0001	0	0.2571	0	0

Narrow – Mean: -0.0001, SD: 0.0002, Coef. of Var.: 192%, err_{LOO}: $3.48 * 10^{-5}$
Wide – Mean: 0.0000, SD: 0.0000, Coef. of Var.: , err_{LOO}: $1.86 * 10^{-1}$

Table C.16: First- and second-order Sobol' Indices for $L^E - L^S$ with $p = 7$, $N = 990$, and $r_D = -0.015$.

	λ	κ	σ	β
S^{tot}_{narrow} S^{tot}_{wide}	0.0017	0.7587	0.4813	0.0001
S^{1}_{narrow} S^{1}_{wide}	0.0011	0.5174	0.2399	0

	$\lambda\kappa$	$\lambda\sigma$	$\lambda\beta$	$\kappa\sigma$	$\kappa\beta$	$\sigma\beta$
S^{2}_{narrow} S^{2}_{wide}	0.0002	0.0003	0	0.2409	0	0

Narrow – Mean: -0.0006, SD: 0.0008, Coef. of Var.: 127%, err_{LOO}: $3.28 * 10^{-5}$
Wide – Mean: 0.0000, SD: 0.0000, Coef. of Var.: , err_{LOO}: $3.63 * 10^{-1}$

Table C.17: First- and second-order Sobol' Indices for $L^E - L^S$ with $p = 7$, $N = 990$, and $r_D = -0.03$.

C.2 Univariate Effects – Robustness Check

In this subsection, we provide the graphical illustration of the univariate effects of the structural parameters for the natural real interest-rate shocks $r_D = -0.005$ and $r_D = -0.03$. Figures C.1 – C.4 display the case of $r_D = -0.005$ and Figures C.5 – C.8 the case of $r_D = -0.03$.
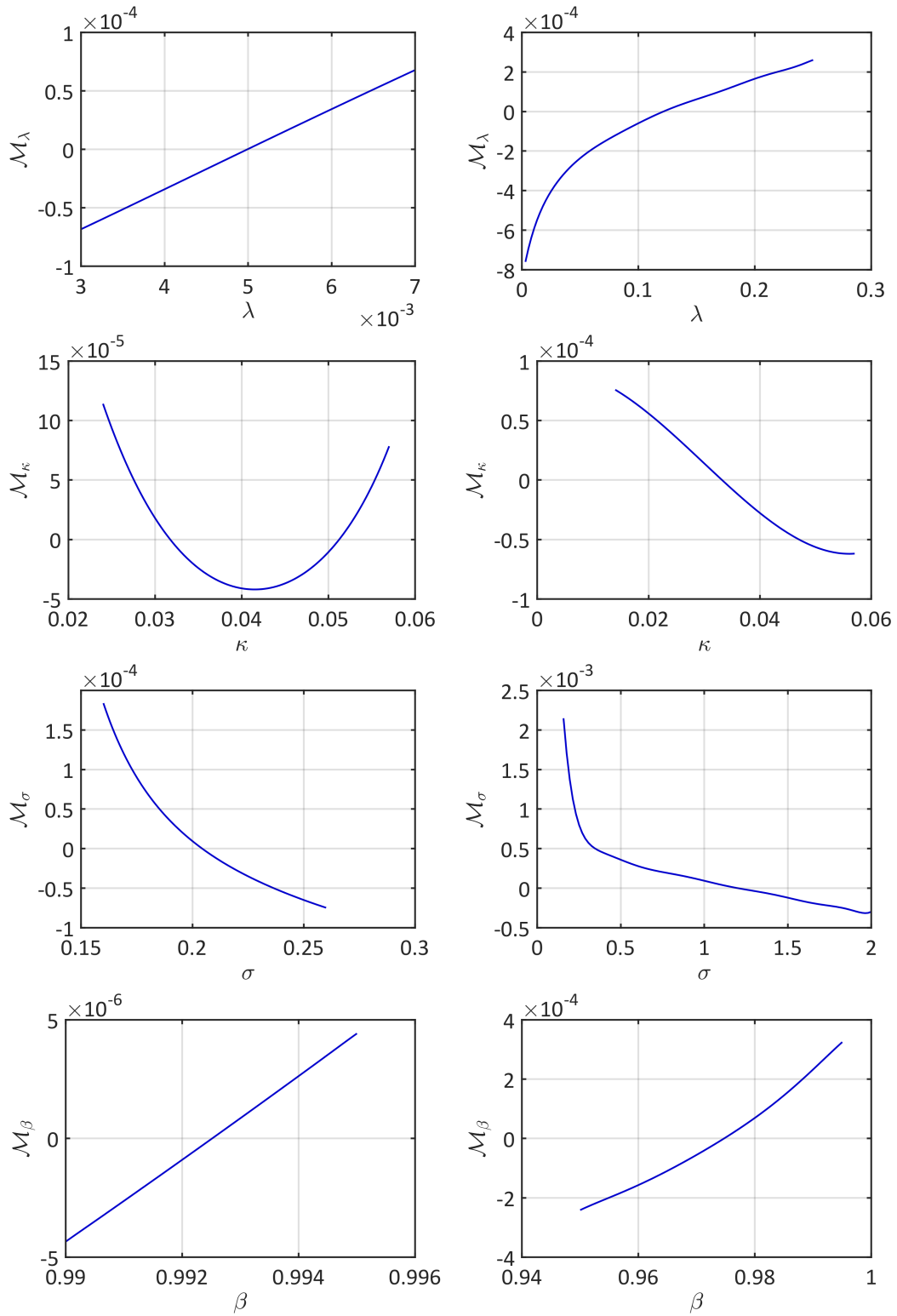
Figure C.1: Univariate effects under NFG with $r_D = -0.005$.
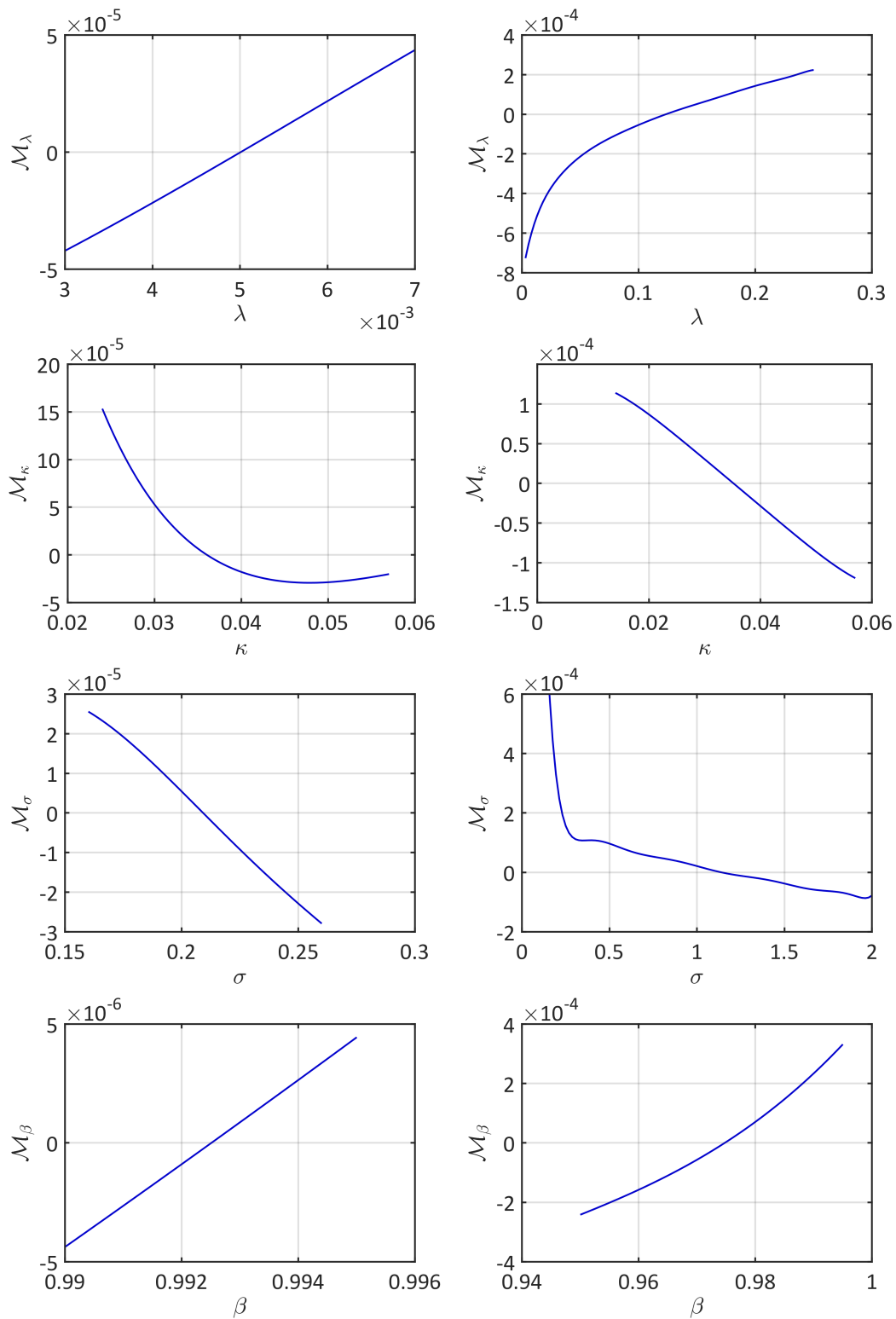
Figure C.2: Univariate effects under IFG with $r_D = -0.005$.
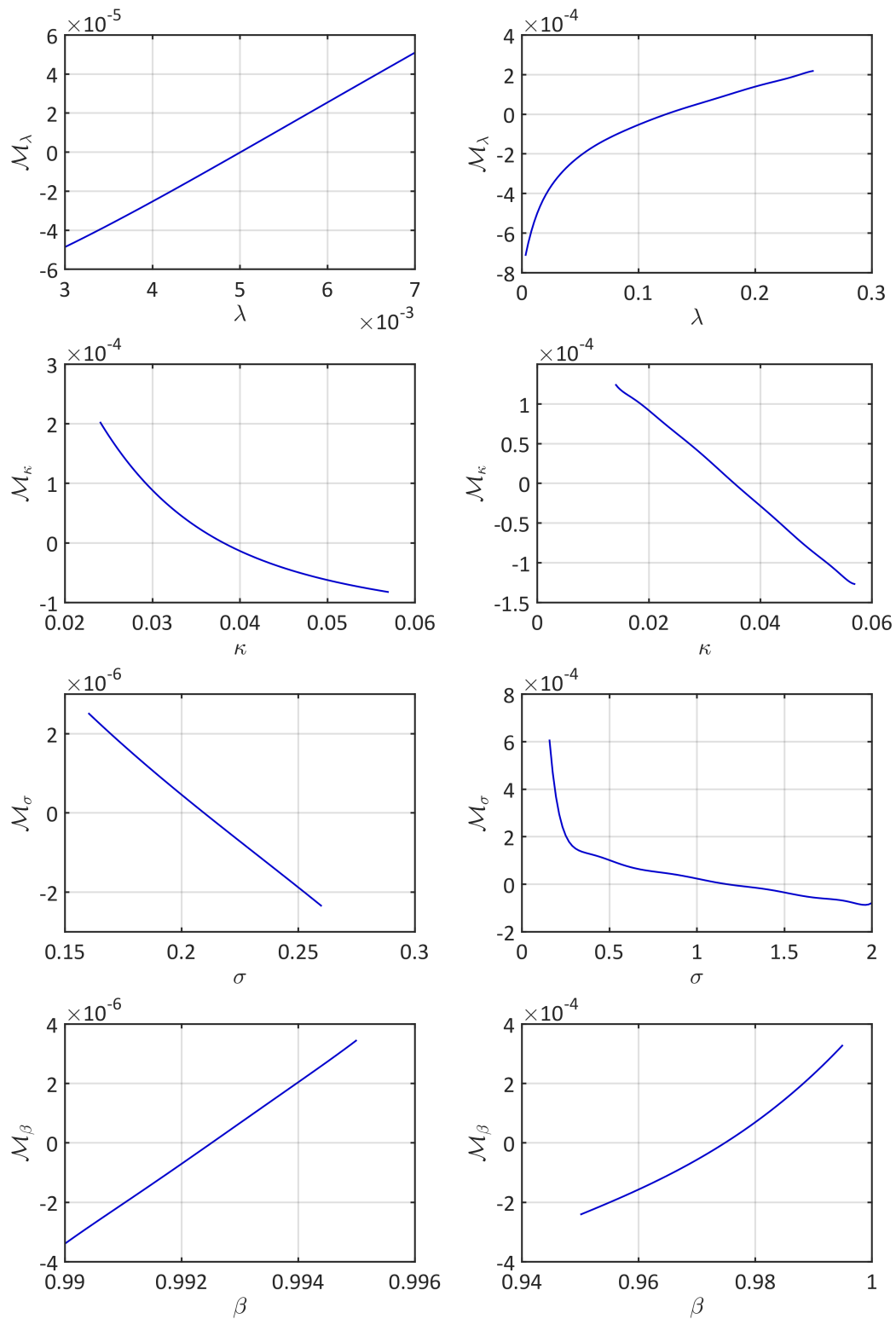
Figure C.3: Univariate effects under EFG with $r_D = -0.005$.
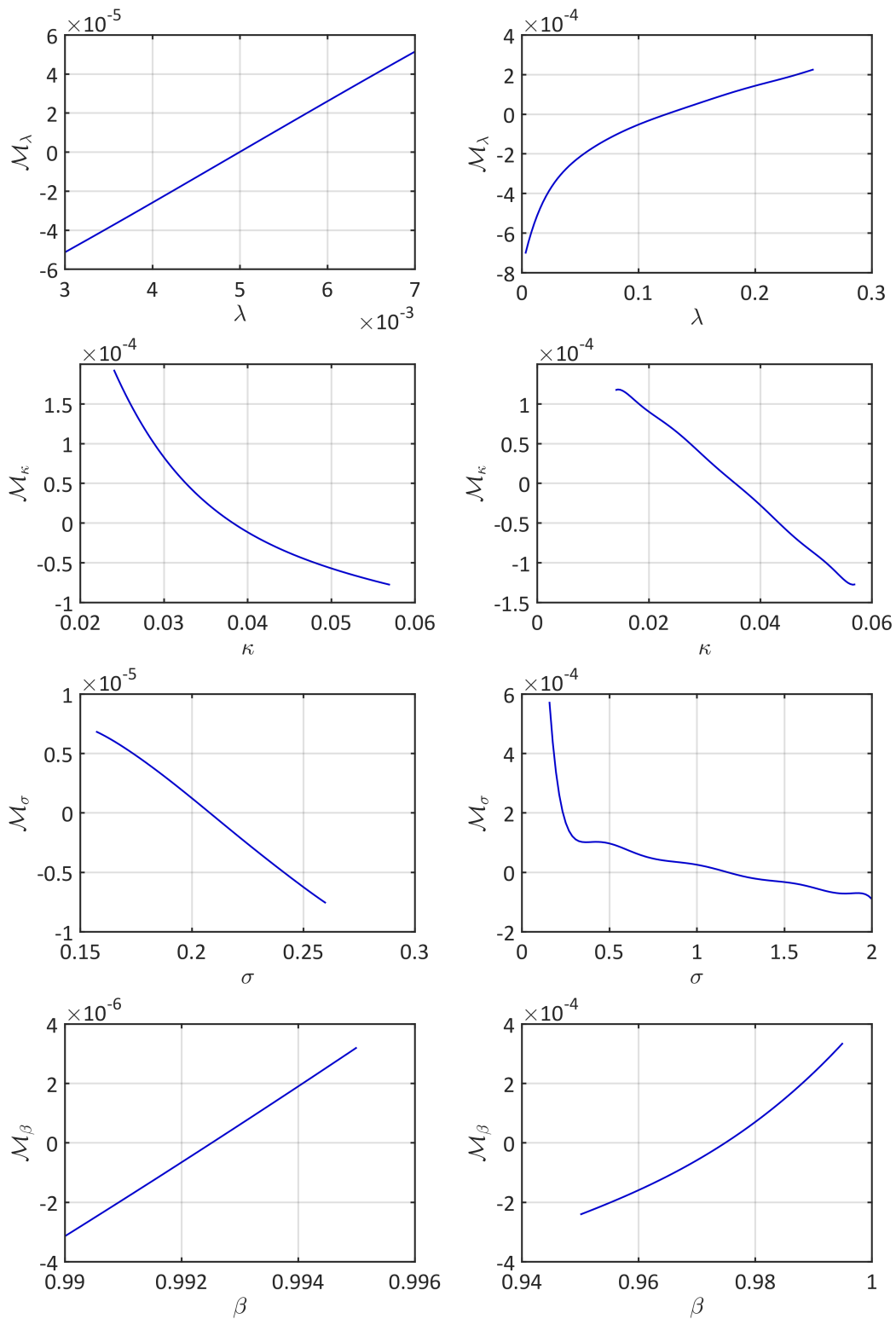
Figure C.4: Univariate effects under SFG with $r_D = -0.005$.
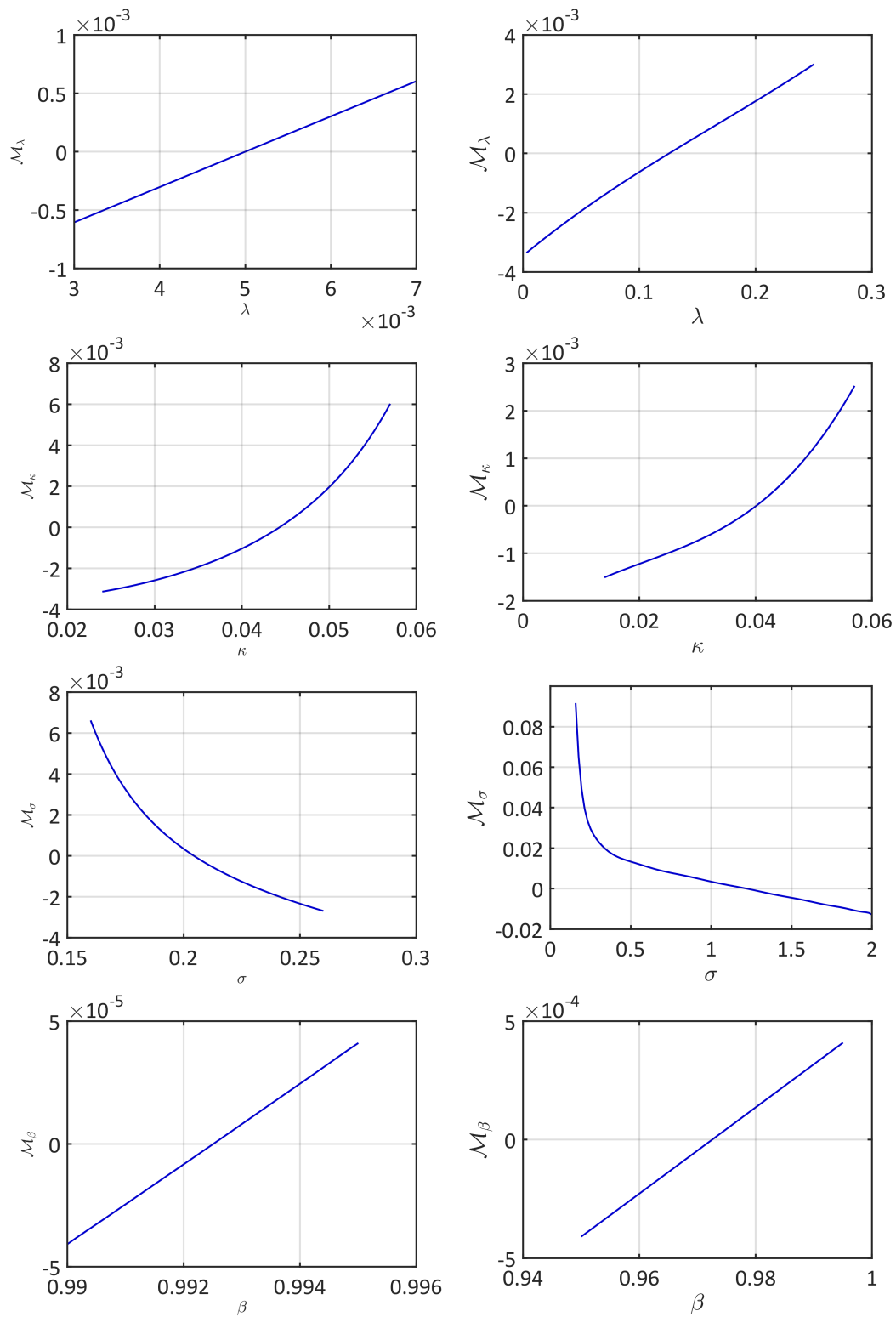
Figure C.5: Univariate effects under NFG with $r_D = -0.03$.
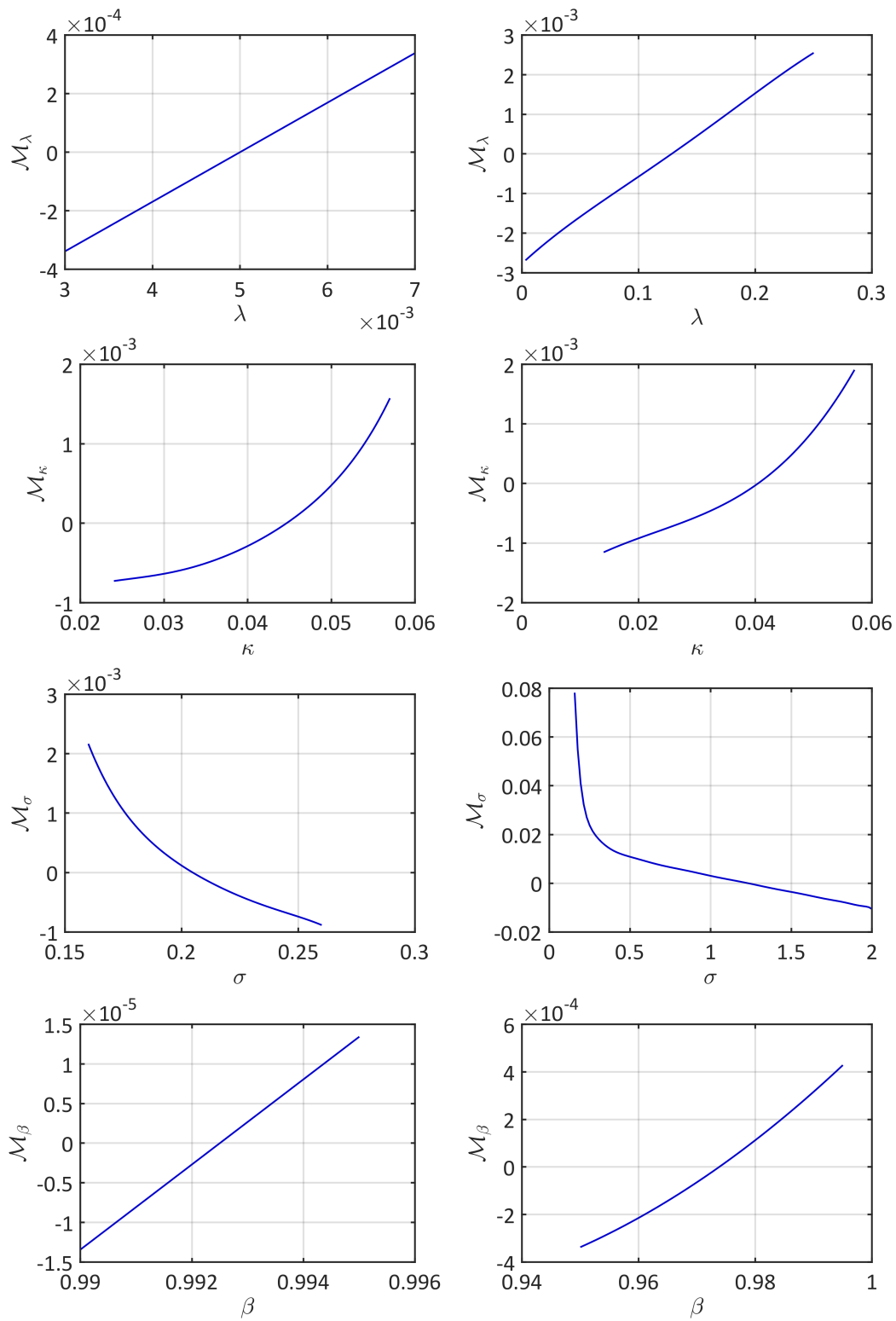
Figure C.6: Univariate effects under IFG with $r_D = -0.03$.
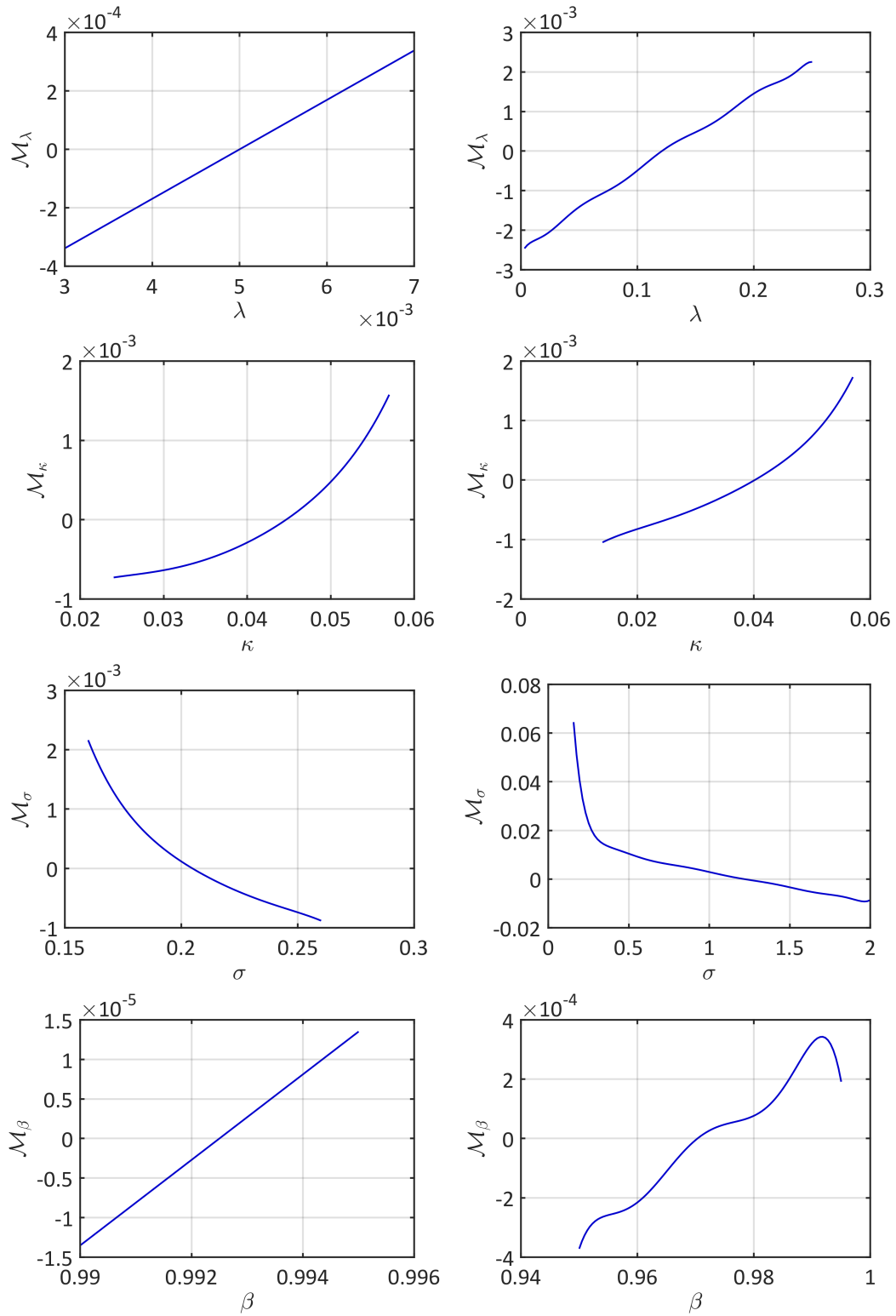
Figure C.7: Univariate effects under EFG with $r_D = -0.03$.
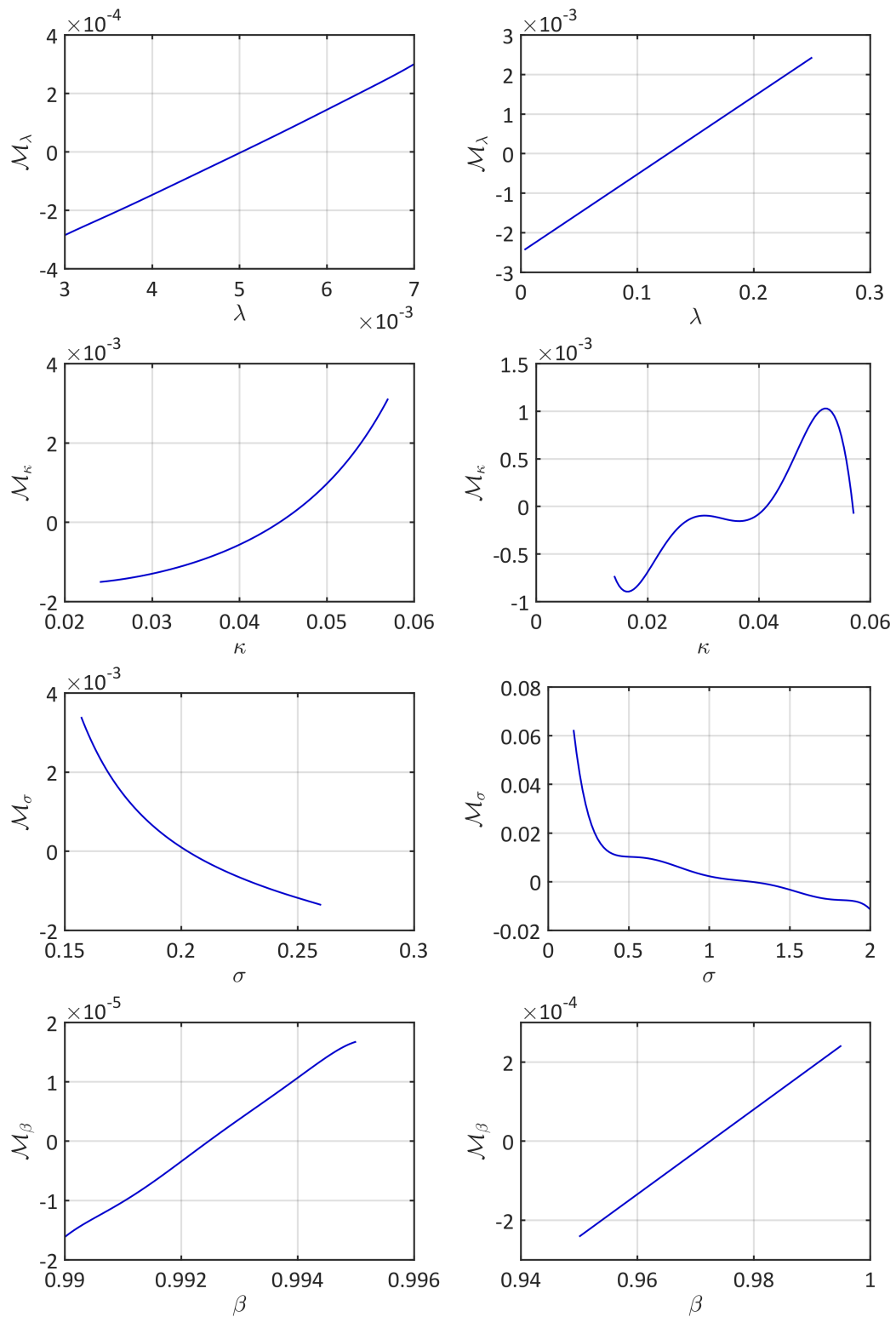
Figure C.8: Univariate effects under SFG with $r_D = -0.03$.

C.3 Parameter-Histograms of $L^N - L^p < 0$

In Section 4.3.2, we point out that the differences in $L^N - L^p$, where $p \in \{F, E, S\}$, are prone to be negative under some parameter constellations. The histograms in Figures C.9 – C.11 plot evidence which parameter values lead to negative differences. We show the histograms of the parameters at negative differences and the original random draws from the uniform distribution.
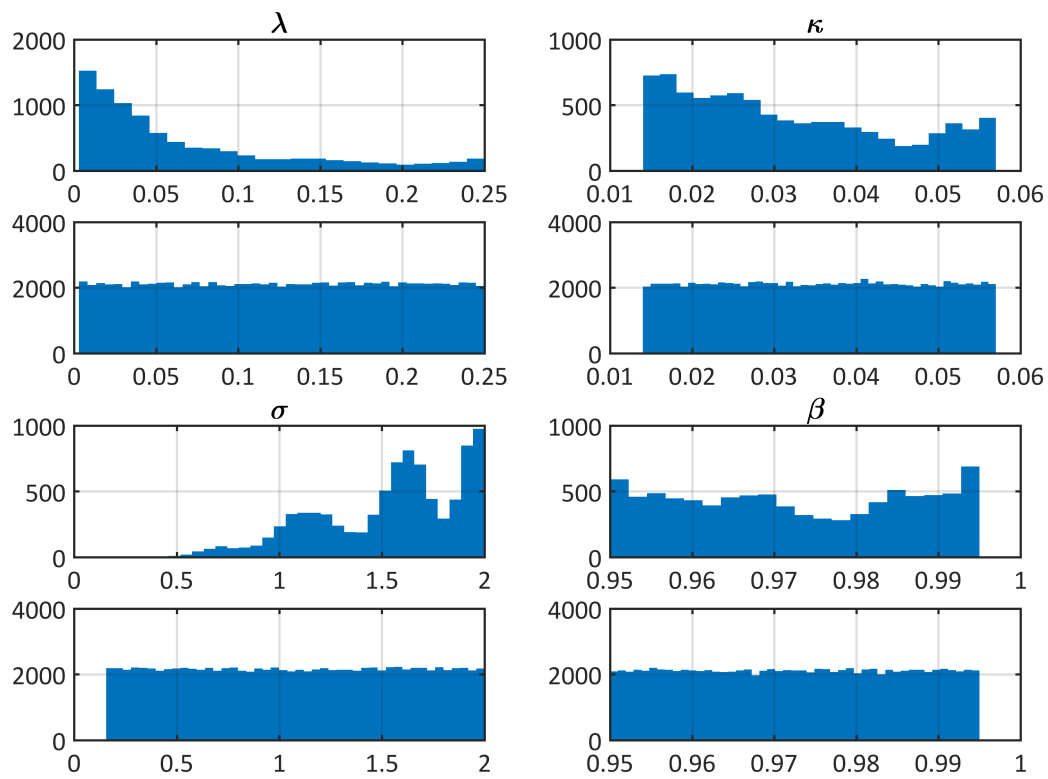
Figure C.9: Parameter-histograms of $L^N - L^F < 0$.
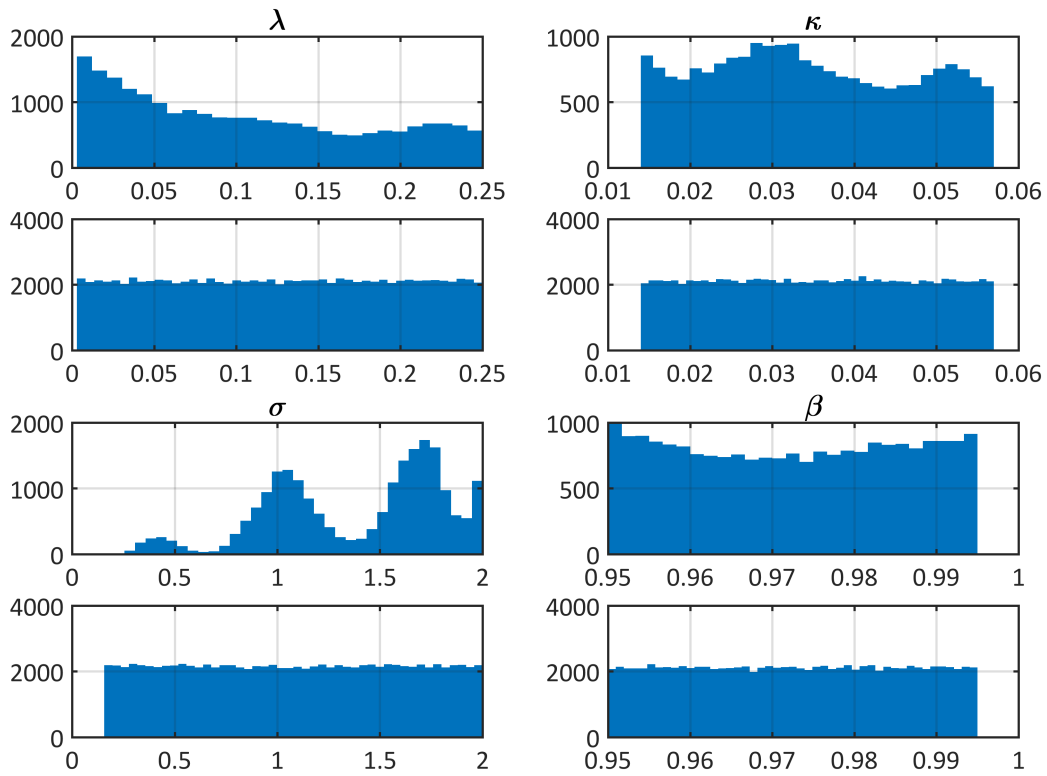
Figure C.10: Parameter-histograms of $L^N - L^E < 0$.
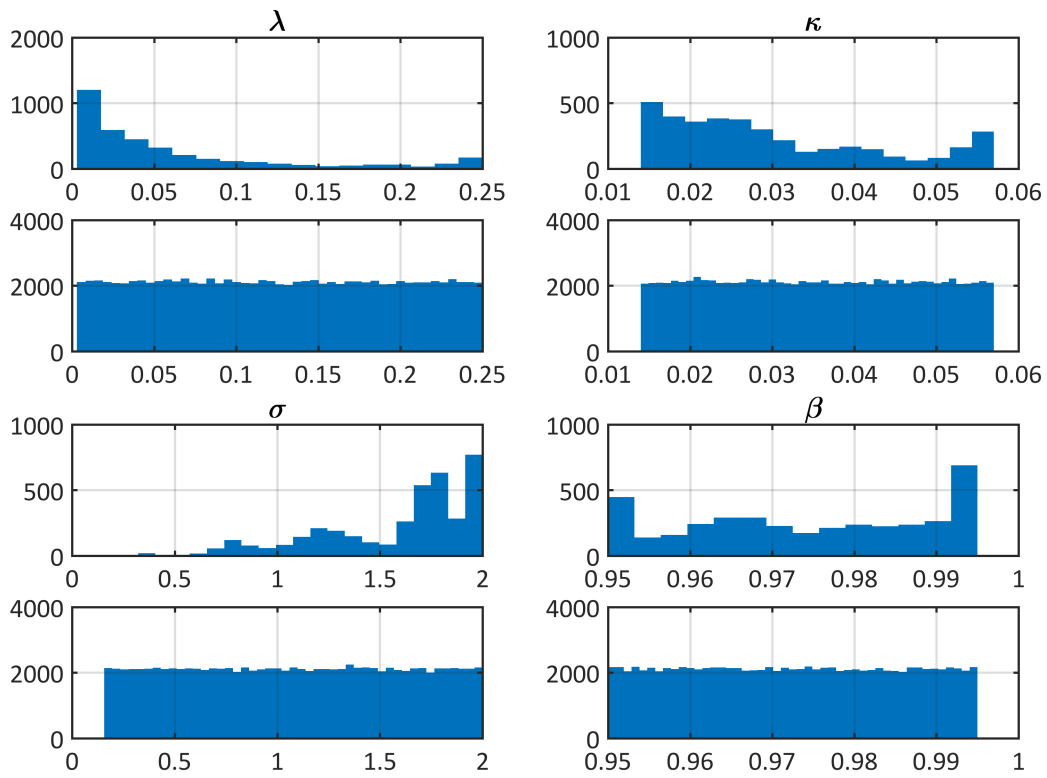
Figure C.11: Parameter-histograms of $L^N - L^S < 0$.

C.4 GSA Including ρ

As a robustness check of our results in Section 4.4, we include the supply-shock persistence parameter ρ in the sensitivity analysis. Table C.18 specifies the parameter ranges and sets $\rho \sim \mathcal{U}[0.75, 0.95]$. Figure C.12 shows that the results qualitatively remain the same as in Figure 4.30. In the narrow parameter ranges, the area for which SFG dominates EFG reduces further to $r_D \in (-1.07\%, -0.65\%)$ compared to the benchmark, $r_D \in (-1.74\%, -0.46\%)$, and the GSA in Chapter 4, $r_D \in (-1.17\%, -0.55\%)$. The intuition is that a lower ρ value leads to a faster reversion to the steady state in Phase H. Thus, the benefit of SFG, i.e. that it is able to manage expectations in normal times, diminishes.

In wide parameter ranges, the accuracy of the surrogate model decreases to just below the threshold. All forward guidance designs are close to each other and do not allow for a distinct evaluation of the designs compared to each other.

	Narrow	Wide
	$\beta \sim \mathcal{U}(0.99, 0.995)$	$\beta \sim \mathcal{U}(0.95, 0.995)$
	$\lambda \sim \mathcal{U}(0.003, 0.007)$	$\lambda \sim \mathcal{U}(0.003, 0.25)$
	$\kappa \sim \mathcal{U}(0.024, 0.057)$	$\kappa \sim \mathcal{U}(0.014, 0.057)$
	$\sigma \sim \mathcal{U}(0.16, 0.26)$	$\sigma \sim \mathcal{U}(0.157, 2)$
	$\rho \sim \mathcal{U}(0.75, 0.95)$	$\rho \sim \mathcal{U}(0.75, 0.95)$

Table C.18: Narrow and wide parameter ranges in the extended global sensitivity analysis.
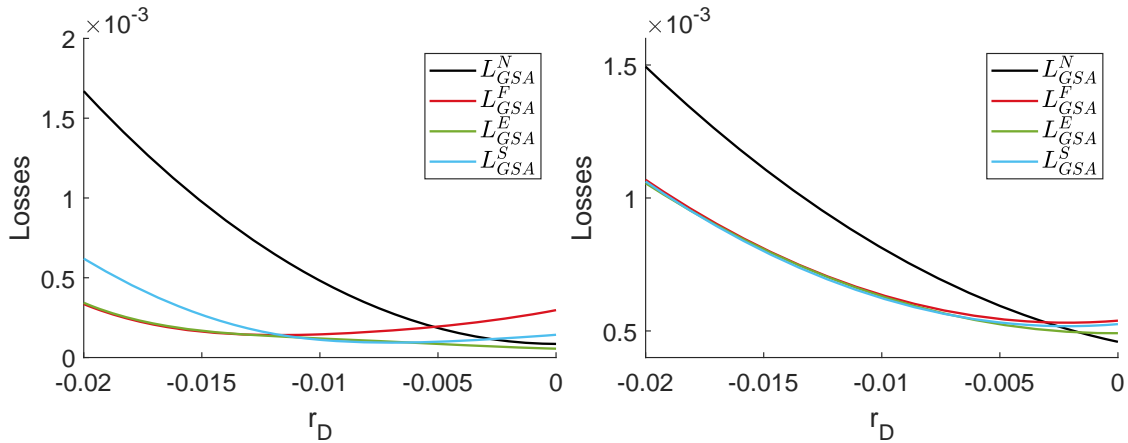
Figure C.12: Robust loss analysis on conditionally-expected losses.

D Forward Guidance at the Zero Lower Bound

Central Bank	Guidance	Statement	Comment
BoE	State contingent	*"[...] agreed its intention not to raise Bank Rate from its current level of 0.5%* **at least until** *[...] the* **unemployment rate has fallen to a threshold** *of 7% [...]"* (Aug 2013)[1]	BoE defined knockout criteria: CPI Inflation 0.5% above target of 2%, inflation expectations are not well anchored, policy imposes potential threats to financial stability.
BoE	State contingent	*"All members agreed that, in the absence of other inflationary pressures, it would be necessary to see* **more evidence of slack reducing before an increase in Bank Rate** *would be warranted."* (May 2014)[1]	Low rates remained in place although unemployment fell below threshold.
FED	Open ended	*"[T]he Committee anticipates that weak economic conditions are likely to warrant* **exceptionally low levels of the federal funds rate for some time***."* (Dec 2008)[1]	FOMC introduced forward guidance about interest rates as an unconventional monetary policy tool at the ZLB.
FED	Open ended	*"For some time"* was replaced by *"for an extended period"*. (Mar 2009)[1]	FOMC changed formulation.
FED	Time contingent	Exceptionally low rates expected to last *"[...] at least through mid-2013."* (Aug 2011)[1]	FED changed from open ended guidance to a more specific time span of low policy rates.
FED	Time contingent	*"[...] at least through late 2014."* (Jan 2012)[1]	Extension of the time span.
FED	Time contingent	*"[...] at least through mid-2015"* (Sep 2012)[1]	Extension of the time span.
FED	State contingent	Exceptionally low level of the federal funds rate would *"[...] be appropriate* **at least as long as** *the unemployment rate remains above 6-1/2 percent, inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee's 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored."* (Dec 2012)[1]	FED switched its forward guidance rhetoric to state contingent forward guidance with exit criteria similar to the BoE's criteria.
FED	State contingent	Addition: *"The Committee now anticipates [...] that it likely will be appropriate to maintain the current target range for the federal funds rate* **well past the time that the unemployment rate declines below 6-1/2 percent**, *especially if projected inflation continues to run below the Committee's 2 percent longer-run goal."* (Dec 2013)[1]	Additional commitment to low policy rates with a stronger focus on inflation.

FED	State contingent (open ended)	*"The Committee continues to anticipate [...] that it likely will be appropriate to maintain the current target range for the federal funds rate for a considerable time after the asset purchase program ends, especially if projected inflation continues to run below the Committee's 2 percent longer-run goal, and provided that longer-term inflation expectations remain well anchored. [...]* The Committee currently anticipates that, even after employment and inflation are near mandate-consistent levels,* **economic conditions** *may, for some time, warrant keeping the target federal funds rate below levels the Committee views as normal in the longer run."* (Mar 2014)[1]	Unemployment and inflation thresholds were replaced by a more vague notion of *"economic conditions"* to indicate how long the policy rate stays low.
FED	State contingent (open ended)	Addition: *"However, if incoming information indicates faster progress toward the Committee's employment and inflation objectives than the Committee now expects, then increases in the target range for the federal funds rate are likely to occur* **sooner than currently anticipated**. *Conversely, if progress proves slower than expected, then increases in the target range are likely to occur later than currently anticipated."* (Oct 2014)[1]	FOMC mentioned the possibility of an early return the first time.
FED	State contingent (open ended)	*"The Committee judges that it can be patient in beginning to normalize the stance of monetary policy."* (Jan 2015)[1]	The FOMC returned to a more open ended formula.
FED	State contingent (open ended)	*"The Committee judges that an increase in the target range for the federal funds rate remains unlikely at the April FOMC meeting. The Committee anticipates that it will be appropriate to raise the target range for the federal funds rate* **when it has seen further improvement in the labor market** *and is reasonably confident that inflation will move back to its 2 percent objective over the medium term."* (Mar 2015)[1]	
BoJ	Open ended	The BoJ indicated that it would maintain the zero interest rate policy until deflationary concerns were dispelled. (Apr 1999)[2]	
BoJ	State contingent	*"[M]onetary easing would continue to be in place* **until the core CPI registers stably zero percent** *or an increase year on year."* (Mar 2001)[3]	Forward guidance with a clear target was introduced, in combination with Quantitative Easing.
BoJ	State contingent	*"[...] refined the expression so that (1) not only that the most recently published core CPI should register zero percent or above,* **but also that such tendency should be confirmed over a few months**, *and (2) the prospective core CPI will not be expected to register below zero percent."* (Oct 2003)[3]	The formulation of targets was refined.
BoJ	State contingent	*"[...] the Bank will maintain the virtually zero interest rate policy* **until price stability is in sight** *on the basis of the 'understanding of medium- to long-term price stability,' on the condition that no serious risk factors were identified."* (Oct 2010)[3]	Reformulation of the criteria.

BoJ	State contingent	*"For the time being, the Bank will pursue powerful monetary easing by conducting its virtually zero interest rate policy and by implementing the Asset Purchase Program [...] with the aim of achieving the goal of 1 percent. The Bank will continue pursuing the powerful easing until it judges that the 1 percent goal is in sight, on the condition that no significant risk factors were identified."* (Feb 2012)[3]	The core CPI target was raised from *"not be expected to register below zero percent"* to one percent.
BoJ	State contingent	The bank would maintain its near zero interest-rate policy conditional on a variation in the CPI of 1% in 2012 and 2% in 2013. (Oct 2013)[2]	
BoJ	State contingent	*"The Bank will continue with 'QQE with a Negative Interest Rate,' aiming to achieve the **price stability target of 2 percent**, as long as it is necessary for maintaining that target in a stable manner. It will examine risks to economic activity and prices [...]"* (Jan 2016)[4]	BoJ reformulated their notion of price stability.
BoC	State contingent	The BoC indicated that the target rate would remain at its current level until a specific date, conditional on the outlook for inflation. (Apr 2009)[2]	
ECB	Open ended	The ECB indicated that the policy rate would be maintained at present or lower levels for an extended period of time. (Jul 2013)[2]	
RBS	Time contingent	The RBS made a statement that indicates that the policy rate was expected to remain low until autumn 2010. (Apr 2009)[2]	
RBS	Time contingent	The RBS provided guidance regarding the timing of the policy rate evolution. (Feb 2013)[2]	

Table D.1: [1]: Table adapted from Moessner et al. (2017), [2]: Charbonneau and Rennison (2015), [3]: Shirai (2013), and [4]: Bank of Japan (2016).
BoE: Bank of England, FED: Federal Reserve, BoJ: Bank of Japan, BoC: Bank of Canada, ECB: European Central Bank, RBS: Riksbank Sweden

E Parameters in the Literature

Paper	λ-value	κ-value	β-value	σ-value	Source
Florez-Jimenez and Parra-Polania (2016)	0.003	0.024	0.9913	0.16	Bodenstein et al. (2012) which refer to Woodford (2003) which refers to **Rotemberg and Woodford (1997)**. Quarterly data.
Keen et al. (2016)			0.9957	0.167	**Own calculations**. Quarterly data.
Lu et al. (2016)	0.017	0.17	0.995		Parameters are chosen to be consistent with Galí (2008) and 2% inflation bias. Quarterly data.
Boneva et al. (2015)	0.0031	0.024	0.9913	0.16	Adam and Billi (2006) which refer to Woodford (2003) which refers to **Rotemberg and Woodford (1997)**. Quarterly data.
Evans et al. (2015)	0.25	0.02	0.995	2	Adam and Billi (2007) which refer to **Rotemberg and Woodford (1997)**. No source for λ. Quarterly data.
Gersbach and Hahn (2014)	0.03	0.3	11		Clarida et al. (2000) which refer to Woodford (1996) and **Roberts (1995)**. Quarterly data.
Bodenstein et al. (2012)	0.003	0.024	0.9913	0.16	Woodford (2003) which refers to **Rotemberg and Woodford (1997)**. Quarterly data.
Adam and Padula (2011)		0.026	1.031		**Own calculations**. Quarterly data.
Nakov (2008)	0.003	0.024	0.993	4	**Rotemberg and Woodford (1997)**. No source for σ, but corresponds to CRRA of 4, see Eggertsson and Woodford (2003). Quarterly data.
Eggertsson (2008)		0.1822	11 (1.8-14.8)	1 (0.74-7.14)	Refer to values as *"Relatively standard values from the literature"* (p. 1493) and **own calculations**. κ is annualized. Yearly data.
Henzel and Wollmershäuser (2008)		0.14 (US) -0.1 (EU) 0.03 (France) -0.12 (DE)	0.99 (fixed) 0.99 0.99 0.99		**Own calculations**. Quarterly data.

Adam and Billi (2007)	0.003	0.05 (Italy) 0.22 (GB)	0.024	0.99 0.99	0.16	**Rotemberg and Woodford (1997)**. Quarterly data.
	0.007		0.057	0.9913 0.9913	1	**Adam and Billi (2006)**. Quarterly data.
Giannoni (2007)	0.048		0.0238 [0.0168, 0.0308]	0.99	0.1571 [0.0915, 0.2227]	**Rotemberg and Woodford (1997)**. Quarterly data.
Fuhrer (2006)			0.055			**Own calculations**. Quarterly data.
Eggertsson (2005)			0.02		2	Eggertsson and Woodford (2003), **Rotemberg and Woodford (1997)**. Quarterly data.
Jung et al. (2005)	0.003		0.024	0.99	0.157	**Rotemberg and Woodford (1997)**. Quarterly data.
Rudd and Whelan (2005)			-0.014	1.014		**Own calculations**. Quarterly data.
Eggertsson and Woodford (2003)			0.02	0.99	2	**Rotemberg and Woodford (1997)**. Want to bias the effect of interest rates on output gap contraction downwards, to avoid an exaggerated contraction at the ZLB. Quarterly data.
Amato and Laubach (2003)				0.99	0.26	**Own calculations**. Quarterly data.
Woodford (2003)	0.048		0.024	0.99	0.16	**Rotemberg and Woodford (1997)**. Quarterly data. λ is annualized.
Galí et al. (2001)			-0.021 US / -0.003 EU	1.012 US/0.990 EU		**Own calculations**. Quarterly data.
Kim (2000)				0.999	14.22	**Own calculations**. Quarterly data.
Gersbach and Hahn (2011)	0.03		0.3	11	1	Cecchetti and Krause (2002) which refer to **Cecchetti and Ehrmann (1999)**. σ is from Clarida et al. (2000). Quarterly data.
Clarida et al. (2000)			0.3		1	Woodford (1996) which refers to **Roberts (1995)**. Quarterly data.
Cecchetti and Ehrmann (1999)	0.25 - 0.59					**Own calculations**. λ is the relative weight. Quarterly data.
Cecchetti (1998)	0.075-0.12					**Own calculations**. λ is the relative weight. Monthly data.
Rotemberg and Woodford (1997)			0.024	0.99	0.16	**Own calculations**. Quarterly data.
Woodford (1996)			0.3		1	**Roberts (1995)**. Annual data.
Roberts (1995)			0.249-0.355			**Own calculations**. Annual data.

F List of Notations

Symbol	Meaning
π_t	Inflation in period t
x_t	Output gap in period t
i_t	Nominal interest rate in period t
β	Households' discount factor
κ	Slope of Phillips Curve
σ	Inverse inter-temporal elasticity of substitution
δ	Probability of being trapped in the downturn
λ	Weight of the output gap in social loss function
ϕ	Inverse Frisch elasticity of labor supply
ξ	Supply shock
r_D	Natural real interest rate in downturn
r_H	Natural real interest rate in normal times
ρ	Persistence of supply shock
i^f	Central banker's interest-rate forecast
π_t^f	Central banker's inflation forecast
π^c	Critical inflation to escape
x_1^c, i_1^c, l_1^c	Critical output gap, interest rate, and social loss when $\pi_1^E = \pi^c$
b	Scrupulous central banker's intrinsic weight on forecast
b_1	Scrupulous central banker's intrinsic weight on interest-rate forecast
b_2	Scrupulous central banker's intrinsic weight on inflation forecast
π_t^N, x_t^N, i_t^N	Inflation, output gap, interest rate in t with NFG
π_t^F, x_t^F, i_t^F	Inflation, output gap, interest rate in t with IFG
π_t^E, x_t^E, i_t^E	Inflation, output gap, interest rate in t with EFG
π_t^S, x_t^S, i_t^S	Inflation, output gap, interest rate in t with SFG
l_t	Instantaneous social loss function
$\tilde{l}_t$	Instantaneous central banker's loss function
L	Expected cumulative social loss function
$\tilde{L}$	Expected cumulative central banker's loss function
r^c	Critical natural real interest rate below ZLB is binding

τ_π	Measure of pessimism for private inflation expectations
τ_x	Measure of pessimism for private output-gap expectations
τ_H	Measure of pessimism in Phase H
σ_{η_π}	Standard deviation of public signal about inflation
σ_{η_x}	Standard deviation of public signal about output gap
σ_{ϵ_π}	Standard deviation of private signal about inflation
σ_{ϵ_x}	Standard deviation of private signal about output gap
π_1^{RE}	Inflation expectation under rational expectations
x_1^{RE}	Output-gap expectation under rational expectations
θ_x	Weighting term of x_1^{RE} under heterogeneous beliefs
θ_π	Weighting term of θ_π under heterogeneous beliefs
Phase H	Denotes periods during a supply shock
Phase D	Denotes periods during a real natural interest-rate shock
Y_t^j	Output of firm j in period t
$Y_{t+k|t}^j$	Output of firm j in period $t+k$ with price set in period t
A_t	Technology in period t
N_t^j	Labor input of firm j in period t
P_t^j	Price of firm j in period t
P_t	Aggregate price index in period t
$Q_{t,t+k}$	Stochastic discount factor from period t to $t+k$
W_t	Nominal wage in period t
B_t^i	One-period bond held by household i
Z_t^j	Firm j's profit
T_t	Lump-sum tax
y_t^p	Logarithm of potential output
$(1-\alpha)$	Probability of each firm to reset price
ω	Elasticity of substitution between differentiated goods
$\mathcal{M}$	Firm's mark-up in frictionless economy
MC_t	Real marginal cost in period t
$\overline{\mathbb{E}}^F[.]$	Average expectation of firms
$\overline{\mathbb{E}}^H[.]$	Average expectation of households
$\overline{\mathbb{E}}^{CB}[.]$	Posterior expectation of the central banker
S_i^{tot}	Total Sobol' Index of parameter i
S_i^j	j-th order Sobol' Index of parameter i
p	Polynomial degree of PCE
N	Size of experimental design

Function	*Meaning*
h	Auxiliary function to determine π_D^N, x_D^N
g_1, g_2, g_3	Auxiliary functions to determine π_1^F, x_1^F, i_1^F
e_1, e_2, e_3	Auxiliary functions to determine π_1^E, x_1^E, i_1^E
$\bar{\xi}$	Auxiliary function with output ξ_1, such that $l_1^C(\xi_1) = l_1^N(\xi_1)$
$\underline{\xi}$	Auxiliary function with output ξ_1, such that $l_1^C(\xi_1) = l_1^F(\xi_1)$

G List of Abbreviations

Abbreviation	*Meaning*
BoC	Bank of Canada
BoE	Bank of England
BoJ	Bank of Japan
ECB	European Central Bank
FED	Federal Reserve Bank
FOMC	Federal Open Market Committee
RBS	Riksbank Sweden
SNB	Swiss National Bank
NFG	No Forward Guidance, Discretionary Central Banker
IFG	Interest-rate Forward Guidance, Standard Forward Guidance
EFG	Escaping Forward Guidance
SFG	Switching Forward Guidance
CPI	Consumer Price Index
ELB	Effective Lower Bound
GSA	Global Sensitivity Analysis
PCE	Polynomial Chaos Expansion
ZLB	Zero Lower Bound

Bibliography

Abramowitz, M. and Stegun, I. (1970). *Handbook of Mathematical Functions*. Dover Publications, New York, NY, 9th edition.

Adam, K. (2007). Optimal monetary policy with imperfect common knowledge. *Journal of Monetary Economics*, 54(2):267–301.

Adam, K. and Billi, R. M. (2006). Optimal monetary policy under commitment with a zero bound on nominal interest rates. *Journal of Money Credit and Banking*, 38(7):1877–1905.

Adam, K. and Billi, R. M. (2007). Discretionary monetary policy and the zero lower bound on nominal interest rates. *Journal of Monetary Economics*, 54(3):728–752.

Adam, K. and Padula, M. (2011). Inflation dynamics and subjective expectations in the United States. *Economic Inquiry*, 49(1):13–25.

Amato, J. D. and Laubach, T. (2003). Estimation and control of an optimization-based model with sticky prices and wages. *Journal of Economic Dynamics and Control*, 27(7):1181–1215.

Andrade, P., Gaballo, G., Mengus, E., and Mojon, B. (2016). Forward guidance and heterogenous beliefs. École des Hautes Études Commerciales de Paris, Research Paper No. ECO/SCD-2017-1192.

Angeletos, G.-M. and La'O, J. (2013). Sentiments. *Econometrica*, 81(2):739–779.

Angeletos, G.-M. and Lian, C. (2016). Forward guidance without common knowledge. NBER Working Paper No. 22785.

Bank of Canada (2009). Bank of Canada lowers overnight rate target by 1/4 percentage point to 1/4 per cent and, conditional on the inflation outlook, commits to hold current policy rate until the end of the second quarter of 2010. Press release, Bank of Canada. http://www.bankofcanada.ca/2009/04/fad-press-release-2009-04-21/ (retrieved October 03, 2017).

Bank of England (2013). Forward guidance. Monetary policy committee statement, Bank of England. http://www.bankofengland.co.uk/monetarypolicy/Pages/forward-guidance.aspx (retrieved March 29, 2017).

Bank of Japan (2016). Introduction of "quantitative and qualitative monetary easing with negative interest rate". Statements on monetary policy, Bank of Japan. https://www.boj.or.jp/en/announcements/release _2016/k160129a.pdf (retrieved September 22, 2017).

Barwell, R. and Chadha, J. S. (2013). Complete forward guidance. In den Haan, W., editor, *Forward Guidance: Perspectives from Central Bankers, Scholars and Market Participants*, pages 125–130. Centre for Economic Policy Research, London.

Bernanke, B. S. (2003). "Constrained discretion" and monetary policy. https://www.federalreserve.gov/boarddocs/speeches/2003/20030203/default.htm (retrieved August 17, 2017).

Bianchi, F. and Melosi, L. (2016). Constrained discretion and central bank transparency. Federal Reserve Bank of Chicago, WP-2016-15.

Blatman, G. and Sudret, B. (2011). Adaptive sparse polynomial chaos expansion based on least angle regression. *Journal of Computational Physics*, 230:2345–2367.

Blinder, A. S. (2000). Central-bank credibility: Why do we care? How do we build it? *American Economic Review*, 90(5):1421–1431.

Bodenstein, M., Hebden, J., and Nunes, R. (2012). Imperfect credibility and the zero lower bound. *Journal of Monetary Economics*, 59(2):135–149.

Boneva, L., Harrison, R., and Waldron, M. (2015). Threshold-based forward guidance: Hedging the zero bound. Bank of England, Staff Working Paper No. 561.

Branch, W. A. (2004). The theory of rational heterogeneous expectations: Evidence from survey data on inflation expectations. *Economic Journal*, 114(497):592–621.

Branch, W. A. and McGough, B. (2009). A New Keynesian model with heterogeneous expectations. *Journal of Economic Dynamics and Control*, 33(5):1036–1051.

Campbell, J. R., Evans, C. L., Fisher, J. D., and Justiniano, A. (2012). Macroeconomic effects of Federal Reserve forward guidance. *Brookings Papers on Economic Activity*, 43(1):1–80.

Carney, M. (2013). Letter from the Governor to the Chancellor, August 2013, Bank of England. https://www.bankofengland.co.uk/-/media/boe/files/letter/2013/governor-letter-070813.pdf (retrieved December 07, 2017).

Carroll, C. D. (2003). Macroeconomic expectations of households and professional forecasters. *Quarterly Journal of Economics*, 118(1):269–298.

Cecchetti, S. G. (1998). Polcy rules and targets: Framing the central banker's problem. *Federal Bank of New York Economic Policy Review*, 4(2):1–14.

Cecchetti, S. G. and Ehrmann, M. (1999). Does inflation targeting increase output volatility? An international comparison of policymakers' preferences and outcomes. NBER Working Paper No. 7426.

Cecchetti, S. G. and Krause, S. (2002). Central bank structure, policy efficiency, and macroeconomic performance: Exploring empirical relationships. *Federal Reserve Bank of St. Louis Review*, 84(4):47–60.

Charbonneau, K. and Rennison, L. (2015). Forward guidance at the effective lower bound: International experience. Bank of Canada, Staff Discussion Paper 2015-15.

Charness, G. and Dufwenberg, M. (2006). Promises and partnership. *Econometrica*, 74(6):1579–1601.

Charness, G. and Dufwenberg, M. (2010). Bare promises: An experiment. *Economic Letters*, 107:281–283.

Clarida, R., Galí, J., and Gertler, M. (1999). The science of monetary policy: A New Keynesian perspective. *Journal of Economic Literature*, 37(4):1661–1707.

Clarida, R., Galí, J., and Gertler, M. (2000). Monetary policy rules and macroeconomic stability: Evidence and some theory. *Quarterly Journal of Economics*, 115(1):147–180.

Coibion, O. and Gorodnichenko, Y. (2012). What can survey forecasts tell us about information rigidities? *Journal of Political Economy*, 120(1):116–159.

Coibion, O. and Gorodnichenko, Y. (2015). Information rigidity and the expectation formation process: A simple framework and new facts. *American Economic Review*, 105(8):2644–2678.

Debortoli, D. and Lakdawala, A. (2016). How credible is the Federal Reserve? A structural estimation of policy re-optimizations. *American Economic Journal: Macroeconomics*, 8(3):42–76.

DeGroot, M. H. (2004). *Optimal Statistical Decisions*. Wiley, Hoboken, NJ.

Del Negro, M., Giannoni, M., and Patterson, C. (2015). The forward guidance puzzle. Federal Reserve Bank of New York, Staff Reports No. 574.

Deman, G., Konakli, K., Kerrou, B. S. S., Perrochet, P., and Benabderrahmane, H. (2016). Using sparse polynomial chaos expansion for global sensitivity analysis of groundwater lifetime expectancy in a multi-layered hydrogeological model. *Reliability Engineering and System Safety*, 147:156–169.

den Haan, W. (2013). *Forward Guidance: Perspective from Central Banker, Scholars and Market Participants*. Centre for Economic Policy Research, London.

Diba, B. and Loisel, O. (2017). Pegging the interest rate on bank reserves: A resolution of new keynesian puzzles and paradoxes. mimeo.

Ederer, F. and Stremitzer, A. (2016). Promises and expectations. Cowles Foundation, Discussion Paper No. 1931.

Efron, B., Hastie, T., Johnstone, I., and Tibshirani, R. (2004). Least angle regression. *Annals of Statistics*, 32(2):407–499.

Eggertsson, G. B. (2003). How to fight deflation in a liquidity trap: Committing to being irresponsible. International Monetary Fund, Working Paper 03/64.

Eggertsson, G. B. (2005). Optimal monetary and fiscal policy under discretion in the New Keynesian model: A technical appendix to "great expectations and the end of the depression". Federal Reserve Bank of New York, Staff Reports No. 235.

Eggertsson, G. B. (2008). Great expectations and the end of the depression. *American Economic Review*, 98(4):1476–1516.

Eggertsson, G. B. (2010). The Paradox of Toil. Federal Reserve Bank of New York, Staff Report No. 433.

Eggertsson, G. B. and Krugman, P. (2012). Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach. *Quarterly Journal of Economics*, 127(3):1469–1513.

Eggertsson, G. B. and Woodford, M. (2003). The zero bound on interest rates and optimal monetary policy. *Brookings Papers on Economic Activity*, 34(1):139–235.

Ellingsen, T. and Johannesson, M. (2004). Promises, threats and fairness. *Economic Journal*, 114(495):397–420.

Ellingsen, T., Johannesson, M., Tjotta, S., and Torsvik, G. (2010). Testing guilt aversion. *Games and Economic Behavior*, 68:95–107.

Evans, C., Fisher, J., Gourio, F., and Krane, S. (2015). Risk management for monetary policy near the zero lower bound. *Brookings Papers on Economic Activity*, 46(1):141–219.

Farhi, E. and Werning, I. (2017a). Fiscal multipliers: Liquidity traps and currency unions. *Handbook of Macroeconomics*, 2:2417–2492.

Farhi, E. and Werning, I. (2017b). Monetary policy, bounded rationality, and incomplete markets. mimeo.

Federal Reserve (2012). Monetary policy releases. press release. http://www.federalreserve.gov/newsevents /press/monetary/20121212a.htm (retrieved February 08, 2016).

Florez-Jimenez, M. L. and Parra-Polania, J. A. (2016). Forward guidance with an escape clause: When half a promise is better than a full one. *Applied Economics*, 48(15):1372–1381.

Fuhrer, J. C. (2006). Intrinsic and inherited inflation persistence. *International Journal of Central Banking*, 2(3):49–86.

Gabaix, X. (2016). A behavioral New Keynesian model. NBER Working Paper No. 22954.

Galí, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press, Princeton, NJ.

Galí, J., Gertler, M., and Lopez-Salido, D. (2001). European inflation dynamics. *European Economic Review*, 45(7):1237–1270.

Gersbach, H. and Hahn, V. (2011). Monetary policy inclinations. *Journal of Money, Credit and Banking*, 43(8):1707–1717.

Gersbach, H. and Hahn, V. (2013). Effective forward guidance: Scrupulous central bankers and forecast contracts. In den Haan, W., editor, *Perspectives from Central Bankers, Scholars and Market Participants*. Centre for Economic Policy Research, London.

Gersbach, H. and Hahn, V. (2014). Inflation forecast contracts. *Journal of Economic Dynamics and Control*, 48:26–40.

Gersbach, H., Hahn, V., and Liu, Y. (2015). Forward guidance contracts. CESifo Working Paper No. 5375.

Gersbach, H., Liu, Y., and Tischhauser, M. (2018). Versatile forward guidance: Escaping or switching. CEPR Discussion Paper DP12559.

Giannoni, M. P. (2007). Robust optimal monetary policy in a forward-looking model with parameter and shock uncertainty. *Journal of Applied Econometrics*, 22:179–213.

Gürkaynak, R. S., Sack, B., and Swanson, E. T. (2005). Do actions speak louder than words? The response of asset proces to monetary policy actions and statements. *International Journal of Central Banking*, 1(1):55–93.

Hall, R. E. (1988). Intertemporal substitution in consumption. *Journal of Political Economy*, 96(2):339–357.

Harenberg, D., Marelli, S., Sudret, B., and Winschel, V. (2017). Uncertainty quantification and global sensitivity analysis for economic models. SSRN eLibrary Working Paper 2903994, available at: https://ssrn.com/abstract=2903994.

Henzel, S. and Wollmershäuser, T. (2008). The New Keynesian Phillips Curve and the role of expectations: Evidence from the CESifo World Economic Survey. *Economic Modelling*, 25(5).

Ismayilov, H. and Potters, J. (2012). Promises as commitments. CentER Discussion Paper No. 2012-064.

Jaynes, E. T. (2003). *Probability Theory: The Logic of Science*. Cambridge University Press, Cambridge.

Jung, T., Teranishi, Y., and Watanabe, T. (2005). Optimal monetary policy at the zero-interest-rate bound. *Journal of Money, Credit and Banking*, 37(5):813–835.

Keen, D. B., Richter, A. W., and Throckmorton, N. A. (2016). Forward guidance and the state economy. Federal Reserve Bank of Dallas, Working Paper 1612.

Kim, J. (2000). Constructing and estimating a realistic optimizing model of monetary policy. *Journal of Monetary Economics*, 45(2):329–359.

Kydland, F. E. and Prescott, E. C. (1977). Rules rather than discretion: The inconsistency of optimal plans. *Journal of Political Economy*, 85(3):473–492.

Le Gratiet, L., Marelli, S., and Sudret, B. (2016). Metamodel-based sensitivity analysis: Polynomial chaos expansions and Gaussian processes. In Ghanem, R., Higdon, D., and Owhadi, H., editors, *Handbook of Uncertainty Quantification*, pages 1–37. Springer, Cham.

Liu, Y. (2016). *An analysis of three variants of forward guidance contracts*. Doctoral thesis Nr. 23449, ETH Zürich.

Lorenzoni, G. (2009). A theory of demand shocks. *American Economic Review*, 99(5):2050–2084.

Lu, Y. K., King, R. G., and Pasten, E. (2016). Optimal reputation building in the New Keynesian model. *Journal of Monetary Economics*, 84:233–249.

Mankiw, N. G. and Reis, R. (2002). Sticky information versus sticky prices: A proposal to replace the New Keynesian Phillips Curve. *Quarterly Journal of Economics*, 117(4):1295–1328.

Marelli, S., Lamas, C., Sudret, B., and Konakli, K. (2017). Uqlab user manual – sensitivity analysis. Report uqlab-v1.0-106. Technical report, Chair of Risk, Safety & Uncertainty Quantification, ETH Zurich.

Marelli, S. and Sudret, B. (2015). Uqlab user manual – polynomial chaos expansions. Report uqlab-v0.9-104. Technical report, Chair of Risk, Safety & Uncertainty Quantification, ETH Zurich.

Massaro, D. (2013). Heterogeneous expectations in monetary DSGE models. *Journal of Economic Dynamics and Control*, 37(3):680–692.

McKay, A., Nakamura, E., and Steinsson, J. (2016a). The discounted Euler Equation: A note. NBER Working Paper 22129.

McKay, A., Nakamura, E., and Steinsson, J. (2016b). The power of forward guidance revisited. *American Economic Review*, 106(10):3133–3158.

Mirkov, N. and Natvik, G. J. (2016). Announcements of interest rate forecasts: Do policymakers stick to them? *Journal of Money, Credit and Banking*, 48(5):901–920.

Moessner, R., Jansen, D.-J., and Haan, J. D. (2017). Communication about future policy rates in theory and practice: A survey. *Journal of Economic Surveys*, 31(3):678–711.

Morris, S. and Shin, H. S. (2002). Social value of public information. *American Economic Review*, 92(5):1521–1534.

Nakov, A. (2008). Optimal and simple monetary policy rules with zero floor on the nominal interest rate. *International Journal of Central Banking*, 4(2):73–127.

Olsen, Ø. (2017). A decade of forward guidance in Norway. Speech, Norges Bank. http://www.norges-bank.no/en/Published/Speeches/2014/8-April-Oystein-Olsen-New-York (retrieved October 10, 2017).

Orphanides, A. and Wieland, V. (2000). Efficient monetary policy design near price stability. *Journal of the Japanese and International Economies*, 14(4):327–365.

Ostrom, E., Walker, J., and Gardner, R. (1992). Covenants with and without a sword: Self-governance is possible. *American Political Science Review*, 86(2):404–417.

Park, S. Y. and Bera, A. K. (2009). Maximum entropy autoregressive conditional heteroskedasticity model. *Journal of Econometrics*, 150:219–230.

Phelps, E. and Cagan, P. (1984). The trouble with "rational expectations" and the problem of inflation stabilization. In Frydman, R. and Phelps, E., editors, *Individual Forecasting and Aggregate Outcomes: 'Rational Expectations' Examined*, pages 31–46. Cambridge University Press, Cambridge.

Plosser, C. I. (2016). Commitment, rules, and discretion. *Cato Journal*, 36(2):251–259.

Ratto, M. (2008). Analysing DSGE models with global sensitivity analysis. *Computational Economics*, 31:115–139.

Roberds, W. (1987). Models of policy under stochastic replanning. *International Economic Review*, 28(3):731–755.

Roberts, J. M. (1995). New Keynesian economics and the Phillips Curve. *Journal of Money, Credit and Banking*, 27(4):975–984.

Rotemberg, J. J. and Woodford, M. (1997). An optimization-based econometric model for the evaluation of monetary policy. *NBER Macroeconomics Annual*, 12:297–346.

Rudd, J. and Whelan, K. (2005). New tests of the new-Keynesian Phillips Curve. *Journal of Monetary Economics*, 52(6):1167–1181.

Rudebusch, G. D. and Williams, J. C. (2008). Revealing the secrets of the temple: The value of publishing central bank interest rate projections. In Campbell, J. Y., editor, *Asset Prices and Monetary Policy*, pages 247–289. University of Chicago Press, Chicago, IL.

Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D., Saisana, M., and Tarantola, S. (2008). *Global Sensitivity Analysis. The Primer*. Wiley, Chichester.

Schaumburg, E. and Tambalotti, A. (2007). An investigation of the gains from commitment in monetary policy. *Journal of Monetary Economics*, 54(2):302 – 324.

Schweizerische Nationalbank (2015). Merkblatt zu Negativzinsen auf Girokontoguthaben. http://www.snb.ch/de/mmr/reference/mbnz_20150501/source/mbnz_2015 0501.de.pdf (retrieved November 17, 2016).

Shirai, S. (2013). Monetary policy and forward guidance in Japan. Speeches at the International Monetary Fund (September 19, 2013) and the Board of Governors of the Federal Reserve System (September 20, 2013) held in Washington, D.C., Bank of Japan.

Söderlind, P. (1999). Solution and estimation of RE macromodels with optimal policy. *European Economic Review*, 43(4-6):813 – 823.

Soize, C. and Ghanem, R. (2004). Physical systems with random uncertainties: Chaos representation with arbitrary probability measure. *SIAM Journal on Scientific Computing*, 26(2):395–410.

Sudret, B. (2008). Global sensitivity analysis using polynomial chaso expansions. *Reliability Engineering and System Safety*, 93:964–979.

Svensson, L. E. O. (2009). Transparency under flexible inflation targeting: Experiences and challenges. *Sveriges Riksbank Economic Review*, 2009(1):5–44.

Svensson, L. E. O. (2015). Forward guidance. *International Journal of Central Banking*, 11(S1):19 – 64.

Vanberg, C. (2008). Why do people keep their promises? An experimental test of two explanations. *Econometrica*, 76(6):1467–1480.

Wiederholt, M. (2015). Empirical properties of inflation expectations and the zero lower bound. mimeo.

Woodford, M. (1996). Control of the public debt: A requirement for price stability? NBER Working Paper No. 5684.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton, NJ.

Woodford, M. (2012). Methods of policy accommodation at the interest-rate lower bound. In Federal Reserve Bank of Kansas City, editor, *The Changing Policy Landscape*, pages 185–288. Federal Reserve Bank of Kansas City, Kansas City, MO. A Symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, WY. August 30-September 1, 2012.