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Codes on Graphs, Trellises and Spatial Coupling: another look at self-concatenated convolutional codes

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***Abstract*—We present recent results on spatially coupled turbo-like codes and point out a connection to the self-concatenated trellis construction proposed by Loeliger in 1997 [1].**

I. Codes on graphs with trellis constraints

We use the term turbo-like codes for generalized LDPC codes with convolutional component codes. In the corresponding factor graphs, the factor nodes represent trellis constraints instead of single parity-check equations. To efficiently describe the structure of different ensembles, similarly to protographs, we use a compact graph representation, illustrated in Fig. 1.

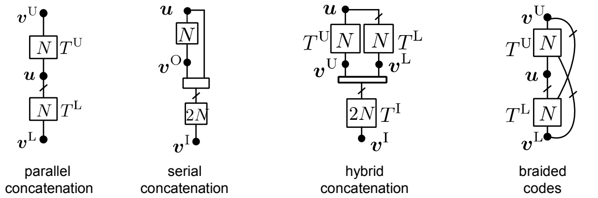

Fig. 1. A compact graph representation of turbo-like code ensembles [2].

II. Spatial coupling and threshold saturation

Spatially coupled (SC) turbo-like code ensembles can then be obtained from a sequence of compact graphs, analogously to SC-LDPC codes. The graphs are coupled by connecting the variable nodes at position t to factor nodes at positions $t, \ldots, t+m$, where m denotes the coupling memory. For the BEC, it can be shown using exact density evolution (DE) recursions that with spatial coupling the decoding threshold of an iterative BP decoder can be improved to the threshold of an optimal MAP decoder (threshold saturation) [2] [3]. For the AWGN channel, where exact DE recursions are not available, the thresholds can be estimated with Monte Carlo methods or predicted from the BEC thresholds [4]. For the ensembles shown in Fig. 1, the predicted thresholds are given in Table I.

TABLE I
PREDICTED AWGN CHANNEL THRESHOLDS (E_b/N_0[dB]) FOR DIFFERENT TURBO-LIKE CODE ENSEMBLES OF RATE $R = 1/3$.

	BP	MAP	$\text{BP}_{\text{SC}}^{m=1}$	$\text{BP}_{\text{SC}}^{m=3}$	$\text{BP}_{\text{SC}}^{m=5}$
Parallel	−0.1052	−0.3070	−0.3070	−0.3070	−0.3070
Serial	1.4023	−0.4740	−0.1196	−0.4673	−0.4740
Braided	1.2139	−0.4723	−0.4690	−0.4723	−0.4723
Hybrid	3.8846	−0.4941	0.2809	−0.4706	−0.4941

Observe that, while the classical turbo codes (parallel concatenation) have the best BP threshold, their MAP threshold is actually the worst. Vice versa, hybrid concatenated and braided codes have poor BP thresholds without spatial coupling. On the other hand, hybrid codes have the best MAP threshold, and braided codes are best for small coupling memory $m = 1$.

III. Self-concatenated convolutional codes

The idea of self-concatenated codes goes back to 1997 and has been proposed independently (at the same conference) by Loeliger [1] and by Divsalar / Pollara [5]. Compared to LDPC codes, a characteristic feature of turbo-like codes is that their factor graph contains a small number of long trellis constraints instead of a large number of short constraints. But what is the advantage of having more than one trellis? In [6] we have presented a unified self-concatenated ensemble, shown in Fig. 2, which contains all the ensembles in Fig. 1 as special cases. A novel element in this ensemble is the feedback of parity bits, which is required for representing the serial, hybrid, and braided ensembles. The various instances of the ensemble differ in the amount of feedback, puncturing and the structure of the permutations. Spatial coupling can be achieved by imposing a causality condition on the permutation matrices.

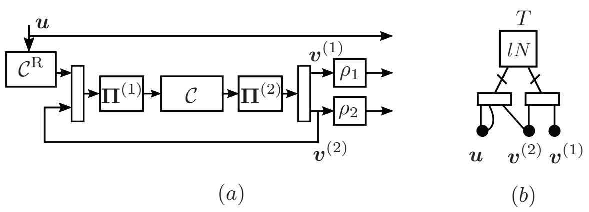

Fig. 2. A unified ensemble based on a single trellis (a) encoder block diagram (b) compact graph.

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