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Online Bayesian Identification of Non-Smooth Systems

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Abstract

The robustness of online Bayesian Identification algorithms has been illustrated for a wide range of physical problems. The successful convergence of such algorithms for problems of highly nonlinear nature is tied to the precision of the approximation of the observed system via the employed state-space model. More sophisticated approximations, result in an increase of both the convergence rate and the associated computational cost. Nonetheless, the latter is a price worth paying for ensuring the former in the case of highly nonlinear problems. The assumption placed by most Bayesian filtering algorithms is that the parameters to be estimated are identifiable at each updating step. This however is a property that does not necessarily hold for systems involving non-smooth nonlinearities, i.e., systems whose state-space or measurement equations are not differentiable. Such systems are linked to the modelling of damage-related phenomena such as plasticity, impact and sliding amongst other. Hence, a separate approach is proposed herein, namely the modification of algorithms to account for the lack of identifiability encountered for parameters of a non-smooth system at a specific step. This modification is termed by the authors as, the Discontinuous, $D-$ modification and relies on the idea that unidentifiable parameters should remain invariant in the corresponding updating steps. This work will illustrate the benefits of the $D-$ modification on the convergence of the Unscented Kalman Filter for non-smooth problems. An example from the dynamics of rocking bodies will be used to demonstrate the advantages of the method.

Keywords: identifiability; non-smooth systems; UKF; Bayesian identification; Discontinuous modification; Rocking

1. Introduction

A broad rage of engineered systems are described by behaviors that do not comply with simplified assumptions, such as linear, time invariant and deterministic traits. This pertains to systems featuring mechanized components, often met in the energy production and aerospace industries (wind turbines, shafts, aircrafts), as well as systems exposed to high and/or cyclic loading (wind, waves, earthquake). In ensuring an efficient and safe operation of these systems, monitoring technologies and the associated diagnostic modules have in recent years gained ground across various domains of engineering serving for condition assessment and control. Reliable and robust diagnostics

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however rely on the premise of i) sufficient computation speed, oftentimes required as near-real or hard-real time, and ii) adequate description of the system throughout its response cycle. This is particularly challenging for a class of systems that beyond nonlinearity, exhibit behavior that is non-smooth. Non-smooth systems, i.e., systems described by non-differentiable state-space equations, are typically linked to kinematic constraints or physical effects like plasticity, fracture friction, impacts or backlash [1]. The dynamics of rocking bodies also lead to non-smooth models, as the problem can be perceived as a combination of several of the previous behaviors including impact and sliding [2,3].

For the online monitoring and diagnostics of such systems, which may comprise of multiple smooth branches, it is essential to establish system identification techniques, which may follow the switching of the system within each of the branches. A major challenge lies in the "partial" observability of the system, manifested in the course of this switching behavior [4]. As only a subset of the system model's parameters is identifiable within each branch, system identification tools perform subpar, exhibiting divergence for unobservable states or parameters [5], which is more pronounced when nonlinearity is involved.

In order to overcome this hurdle and for rendering robust system identification tools, this work exploits the knowledge base offered in [4], where a set of tools is overviewed for determining the observability of a system. This information may then be seeded into time-domain system identification tools, such as Bayesian filters [6,7], allowing of a redefinition of the parameters of the system to be identified at different time instances. This work deals with the problem of identification of non-smooth systems in its full-blown form, assuming uncertain system parameters (to be identified), nonlinear and non smooth state-space equations, and noisy measurements of a subset only of the complete state vector. Joint state and parameter identification algorithms are adopted to this end, and a discontinuous formulation is delivered for the well established Unscented Kalman Filter (UKF) [8–10].

The discontinuous variant proposed herein, designated as the $D-$ modification, relaxes the original assumption of Bayesian filters, i.e., the premise that the states and time-invariant parameters of a dynamic system are persistently observable [6,11,12] and identifiable [13]. Following the work introduced in [14], a modified version of the standard UKF is herein described, evaluating observability within each time step, shifting between nested branches, and updating of the observable subset alone. The method, which proves superior to the standard implementation of the UKF is herein exemplified on the challenging of problem of identifying the properties of a rocking interface.

2. Non-Smooth Dynamical Systems

A nonlinear and non-smooth dynamical system is described by the following system of equations:

$$\dot{\mathbf{x}}_t = F(\mathbf{x}_t, \theta, \mathbf{u}), \qquad \dot{\theta} = 0, \qquad \mathbf{y} = H(\mathbf{x}_t, \theta, \mathbf{u}) \tag{1}$$

where $\mathbf{x}_t$ is the system's state vector, θ denotes the time-invariant parameter vector, $\mathbf{u}$ is the input to the system (loads), while $\mathbf{y}$ designates the set of observations drawn from the system's response (output). Functions F and H comprise the nonlinear state-space and measurement functions, respectively. As a standard step to joint state and parameter identification, the state vector is augmented as $\mathbf{x} = [\mathbf{x}_t, \theta]$, with updated system and measurement equations:

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}), \qquad \mathbf{y} = h(\mathbf{x}, \mathbf{u}) \tag{2}$$

The non-smooth systems treated as part of this work are continuous but not differentiable systems, whose state-space equations may be decomposed into smooth branches, i.e.

$$\dot{\mathbf{x}} = \mathscr{B}_1(\mathbf{x}), \text{ when } \mathbf{x} \in R_1^n \qquad \cdots \qquad \dot{\mathbf{x}} = \mathscr{B}_k(\mathbf{x}), \text{ when } \mathbf{x} \in R_k^n \tag{3}$$

where $\mathscr{B}_\rangle(\mathbf{x})$ is an analytic set of functions within branch $R_\imath^n$. As the system evolves dynamically over time, and depending on the imposed loads, it switches branches.

2.1. Observability of Non-Smooth Dynamical Systems

Chatzis et al. [4] outline a set of methods for the evaluation of observability in non-smooth dynamical systems. Following the separation of the system into smooth branches, the observability of the states within each branch is assessed. For the systems investigated as part of this work, a further assumption is placed on the state vector $\mathbf{x}$ being straightforwardly separable into observable and unobservable components, denoted as $\mathbf{x}^{oi}$ and $\mathbf{x}^{ui}$ for each branch i. As a result, each component of the state vector $\mathbf{x}$ ought to be identified only when the system moves within the smooth branch where that component lies in the observable subset, in this way improving estimation convergence. It will further be assumed that the unobservable states correspond to unidentifiable parameters. An implied assumption of observability tools is that the excitation is rich enough to excite the system in all branches of each response. For nonlinear systems, this correspondingly calls for load levels, which force the system to move beyond linearity.

3. The Unscented Kalman Filter and its Discontinuous $D-$ modification

Let us assume a dynamical system whose discrete state-space and measurement equations are written as:

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{w}_{k-1}, \qquad \mathbf{y}_k = h(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{v}_k \tag{4}$$

where $\mathbf{w}_k$ is the process noise and $\mathbf{v}_k$ is the observation noise, both of which are considered to be white Gaussian noise processes with covariance matrices $\mathbf{Q}$ and $\mathbf{P}$ respectively.

A non-linear Kalman Filter, such as the Unscented, UKF operates at any time instance on all of the augmented states. During a specific step only the observable states and the identifiable parameters can be updated in a meaningful way. If the algorithm is let to operate on the unidentifiable parameters then the occurring update is non-optimal. In that sense we can describe the behavior of the algorithms as divergence. To eliminate the periods of divergence it is suggested that the unidentifiable parameters are retained invariant during the corresponding intervals. This is termed by the authors as the Discontinuous $D-$ modification, which is described in the table below for nonlinear Kalman Filters:

Table 1. Nonlinear Kalman Filters versus their Discontinuous $D-$ modification

NL Kalman Filter	Discontinuous $D-$ Modification
Time Update	
$\hat{\mathbf{x}}_{k\|k-1} = \mathbb{F}(\hat{\mathbf{x}}_{k-1\|k-1}, \mathbf{P}_{k-1\|k-1}, \mathbf{u}_{k-1}, \mathbf{w}_{k-1})$ $\mathbf{P}_{k\|k-1} = \mathbb{G}(\hat{\mathbf{x}}_{k-1\|k-1}, \mathbf{P}_{k-1\|k-1}, \mathbf{u}_{k-1}, \mathbf{w}_{k-1})$	Observable part: $\hat{\mathbf{x}}^o_{k\|k-1} = \mathbb{F}(\hat{\mathbf{x}}^o_{k-1\|k-1}, \mathbf{P}^{oo}_{k-1\|k-1}, \mathbf{u}_{k-1}, \mathbf{w}_{k-1})$ $\mathbf{P}^{oo}_{k\|k-1} = \mathbb{G}(\hat{\mathbf{x}}^o_{k-1\|k-1}, \mathbf{P}^{oo}_{k-1\|k-1}, \mathbf{u}_{k-1}, \mathbf{w}_{k-1})$ Unobservable part: $\hat{\mathbf{x}}^u_{k\|k-1} = \hat{\mathbf{x}}^u_{k-1\|k-1}$ $\mathbf{P}^{uu}_{k\|k-1} = \mathbf{P}^{uu}_{k-1\|k-1}$ Cross terms: $\mathbf{P}^{uo}_{k\|k-1} = G_{KS}(\mathbf{P}^{uo}_{k-1\|k-1})$
Measurement Estimates	
$\hat{\mathbf{y}}_{k\|k-1} = \mathbb{H}(\hat{\mathbf{x}}_{k\|k-1}, \mathbf{P}_{k\|k-1}, \mathbf{u}_{k-1}, \mathbf{v}_{k-1})$ $\mathbf{P}^{yy}_k = \mathbb{I}(\hat{\mathbf{x}}_{k\|k-1}, \mathbf{P}_{k\|k-1}, \mathbf{u}_{k-1}, \mathbf{v}_{k-1})$ $\mathbf{P}^{xy}_k = \mathbb{J}(\hat{\mathbf{x}}_{k\|k-1}, \mathbf{P}_{k\|k-1}, \hat{\mathbf{y}}_{k\|k-1}, \mathbf{P}^{yy}_k)$	
Kalman Update	
$K = (\mathbf{P}^{xy}_k)^{-1}\, \mathbf{P}^{yy}_k$ $\hat{\mathbf{x}}_{k\|k} = \hat{\mathbf{x}}_{k\|k-1} + K\,(\tilde{\mathbf{y}}_k - \hat{\mathbf{y}}_{k\|k-1})$ $\mathbf{P}_{k\|k} = \mathbf{P}_{k\|k-1} - K\,\mathbf{P}^{yy}_k\, K^T$	$K^o = (\mathbf{P}^{xyo}_k)^{-1}\, \mathbf{P}^{yy}_k$ Observable Part: $\hat{\mathbf{x}}^o_{k\|k} = \hat{\mathbf{x}}^o_{k\|k-1} + K^o\,(\tilde{\mathbf{y}}_k - \hat{\mathbf{y}}_{k\|k-1})$ $\mathbf{P}^{oo}_{k\|k} = \mathbf{P}^{oo}_{k\|k-1} - K^o\,\mathbf{P}^{yy}_k\,(K^o)^T$ Unobservable Part: $\hat{\mathbf{x}}^u_{k\|k} = \hat{\mathbf{x}}^u_{k\|k-1}$ $\mathbf{P}^{uu}_{k\|k} = \mathbf{P}^{uu}_{k\|k-1}$ Cross Terms: $\mathbf{P}^{uo}_{k\|k} = \mathbf{P}^{uo}_{k\|k-1} - K^o\,\mathbf{P}^{xy}_k$

The functions $\mathbb{F}, \mathbb{G}, \mathbb{H}, \mathbb{I}, \mathbb{J}$ appearing in Table 1 correspond to the mean and variance propagation functions employed by the Filter, for the system and measurement update steps. The Discontinuous algorithms apply the procedures of their standard counterpart only on the observable part of the state vector and covariance matrix, $\mathbf{x}^o$ and $\mathbf{P}^{oo}$. The unobservable parts, $\mathbf{x}^u$ and $\mathbf{P}^{uu}$ remain invariant, while the cross covariance terms, $\mathbf{P}^{uo}$, are updated following the equations of the Schmidt-Kalman Filter ([15,16]). The application of the $D-$ modification the EKF, leads to the $DEKF$ described in [14]. This paper will focus on the UKF [8,17] and the corresponding $D-$ modification, $DUKF$.

The previous process requires an estimate of the smooth branch the dynamic system is in, so as to separate the state into observable and unobservable. In $DUKF$ this is conditioned on the estimate of the mean $\hat{\mathbf{x}}^u_{k|k-1}$.

4. Identification of a Planar Rocking Body

In this example the problem of identifying the properties a free-standing body on a deformable support medium is studied. When subjected to ground excitations such a body may experience a rocking response, including instances of sliding and free-flight. The behavior of the body can be modeled using the models suggested in [2,3]. It will further be assumed that the response of the body is planar, an assumption which is justifiable if the out of plane dimension of the body is comparable to its height h and larger than the width b. If it is further assumed the body has attached feet at the corners, the 2D CSM suggested in [2] can be used for its smulaion. The model is described in the following Figure 1.

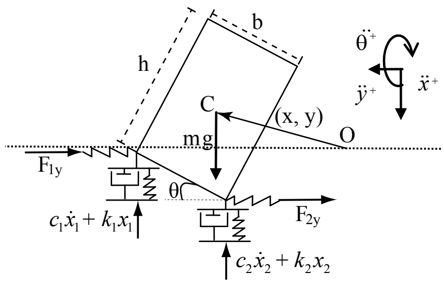


Fig. 1. *CSM* model for rocking bodies as suggested in [2].

The generalized coordinates used to describe the body are the vertical and horizontal coefficients (x, y) of a vector measuring measured from a point O on the undeformed surface to the center of mass C and the rotation of the body θ. The positive conventions for the three generalized coordinates are shown in Figure 1. As the body is assumed to be rigid, the location of any point i the body (x_i, y_i) relative to O can be described in terms of the three generalized coordinates. It should be noted that when a point is embedded in the support medium $x_i > 0$.

Each of the corner points, denoted as points 1 and 2 in Figure 1, is connected to a set of a vertical contact spring, k_i, and damper, c_i, that are active only when the point is embedded in the ground. Furthermore, for a corner point embedded in the ground a frictional force, F_{iy}. is active. This force is provided by the equation $F_{y_i} = k_{y_i} y_{el_i} + c_{y_i}\dot{y}_i$, where y_{el_i} is the elastic displacement, while k_{y_i} and c_{y_i} are the stiffness and damping constants of of the frictional spring of corner i. The frictional springs and dampers are active only when the corresponding point is embedded in the ground. The evolution of this elastic displacement is given by the equation $\dot{y}_{el_i} = \dot{y}_i$. However a return mapping scheme modifies the value of y_{el_i} so that the Coulomb friction constraint $\|F_{y_i}\| \leq \mu\|k_i x_i + c_i \dot{x}_i\|$ is satisfied, with μ being the coefficient of friction. When the constraint is activated the corresponding corner point is sliding. The frictional model used, as well as the occurring equations of motion, are described in greater detail in [2,3].

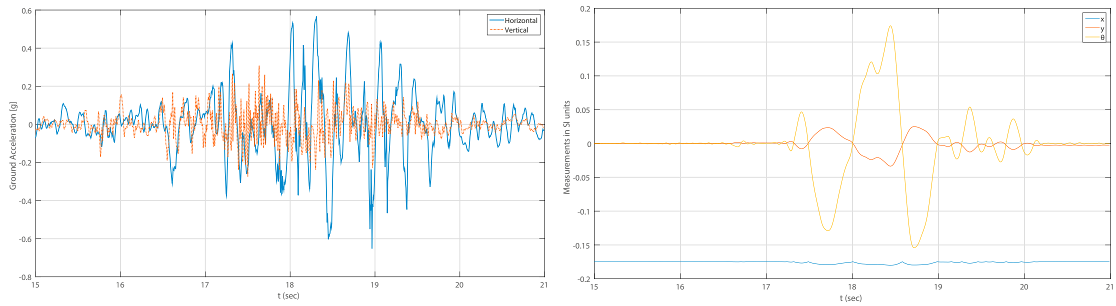
In this work it will be assumed that all springs and dampers have common properties: $k_i = k_{y_i} = k$, $c_i = c_{y_i} = c$. It should be noted that unlike the Inverted Pendulum model suggested by Housner [18] the CSM is continuous in terms of the states. The state-space equations are non-smooth but can be broken down into smooth branches. After studying the identifiability of the model in each of the branches, for measuring (x, y, θ) the following cases can be gathered:

1. if none of the corner points is embedded in the ground then k, c and μ are unidentifiable
2. if any of the corner points is embedded in the ground then k and c are identifiable, and:
   (a) If any of the embedded corners experiences sliding, then μ is identifiable.

(b) If none of the embedded corners experiences sliding, then μ is unidentifiable.

The previous points fully describe the identifiability of the parameters for any of the smooth branches the system may lie in. The dynamic states are for all cases observable.

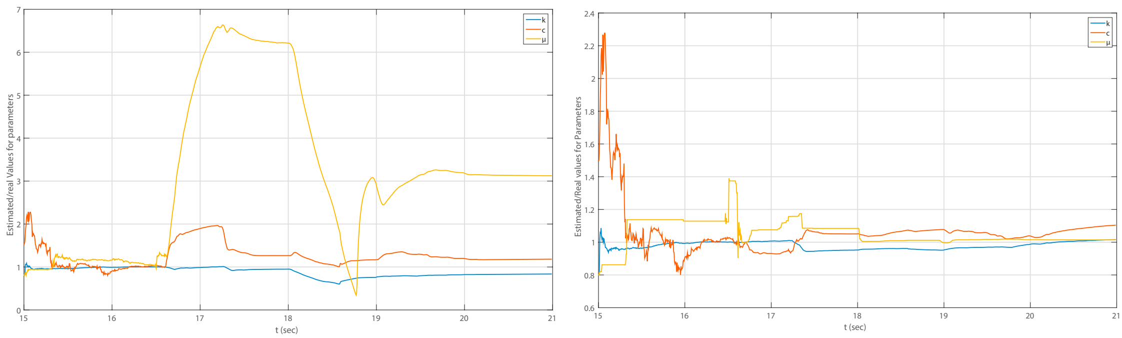
To generate the measurements, a body with $b = 0.09\,[m]$ and $h = 0.35\,[m]$ is placed on a foundation described by the normalized over mass, stiffness $k = 66667\,[1/sec^2]$ and damping $c = 267\,[1/sec^2]$,and a coefficient of friction $\mu = 0.25$. The body is initially at rest and is subjected to a horizontal and vertical ground acceleration shown in the following Figure 2(a). The measurements extracted for identification are x, y and θ, as shown in Figure 2(b).



(a) Input (b) Measurements

Fig. 2. (a) Ground accelerations and (b) Measured response.

The augmented state of the system to be identified is: $[x, y, \theta, y_{el_1}, y_{el_2}, k, c, \mu]$. The input and measurement signals are contaminated with white noise corresponding to roughly a 1% noise to signal rms ratio. As x has a non-zero mean the standard deviation is used for the ratio instead of the rms. The input and measurement signals are then sent to the UKF and $DUKF$ algorithms. Both algorithms use the same assumed initial estimates for the initial conditions $k_0 = 53333[1/sec^2]$, $c_0 = 400[1/sec]$, $\mu = 0.2$, $x_0 = g/k_0 - h/2$, while the rest of the dynamic states are assumed to be zero. The assumed variance of the process and measurement noises for the filters is taken to correspond to the variance of the real noise vectors.



(a) UKF (b) $DUKF$

Fig. 3. The estimated/real values of the parameters using (a) UKF and (b) $DUKF$.

As observed in the previous Figure 3, the $DUKF$ offers an improved estimate of the parameters over the UKF. This is attributed to the fact that the $DUKF$ does not update the value of the coefficient of friction μ when no sliding occurs, in which case μ would be unidentifiable. The isolated instances when such an update takes place are very few (see Figure 3(b)). On the other hand, the UKF will generally tend to update the values of μ, which results in divergent estimates during these unidentifiability windows.

5. Discussion and Conclusions

This paper studies the effect of the special observability and identifiability properties of non-smooth systems on a nonlinear Bayesian filter, namely the UKF. Subsets of the parameters of such systems become unidentifiable in different response branches, causing the estimation to diverge. The suggestion of the authors is to retain such parameters invariant during unidentifiable time windows. The resulting treatment, termed the Discontinuous, $D-$ modification, is applied in this work for the case of the robust UKF.

In the presented example, the identification of a rocking body is studied. The non-smoothness of the model occurs from the combination of a contact spring together with a frictional spring. It is shown that the $DUKF$ succeeds in identifying the properties of the rocking interface for a body subjected to ground motions. The $DUKF$ outperforms the UKF in the example studied, that is mainly attributed to the divergence of the latter during the periods of unidentifiability for the coefficient of friction μ. This paper illustrates the benefits of using the Discontinuous modifications in problems involving non-smoothness. Additionally, it is shown that such algorithms can be used effectively for highly non-smooth problems, such as the Rocking problem. In future work, the $DUKF$ will be implemented on experimental rocking data gathered via video-grammetry methods, as in [19].

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