On the accuracy of the probability method for quantifying beliefs about inflation

KOF

KOF Working Papers

On the Accuracy of the Probability Method
for Quantifying Beliefs about Inflation

Thomas Maag

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

KOF
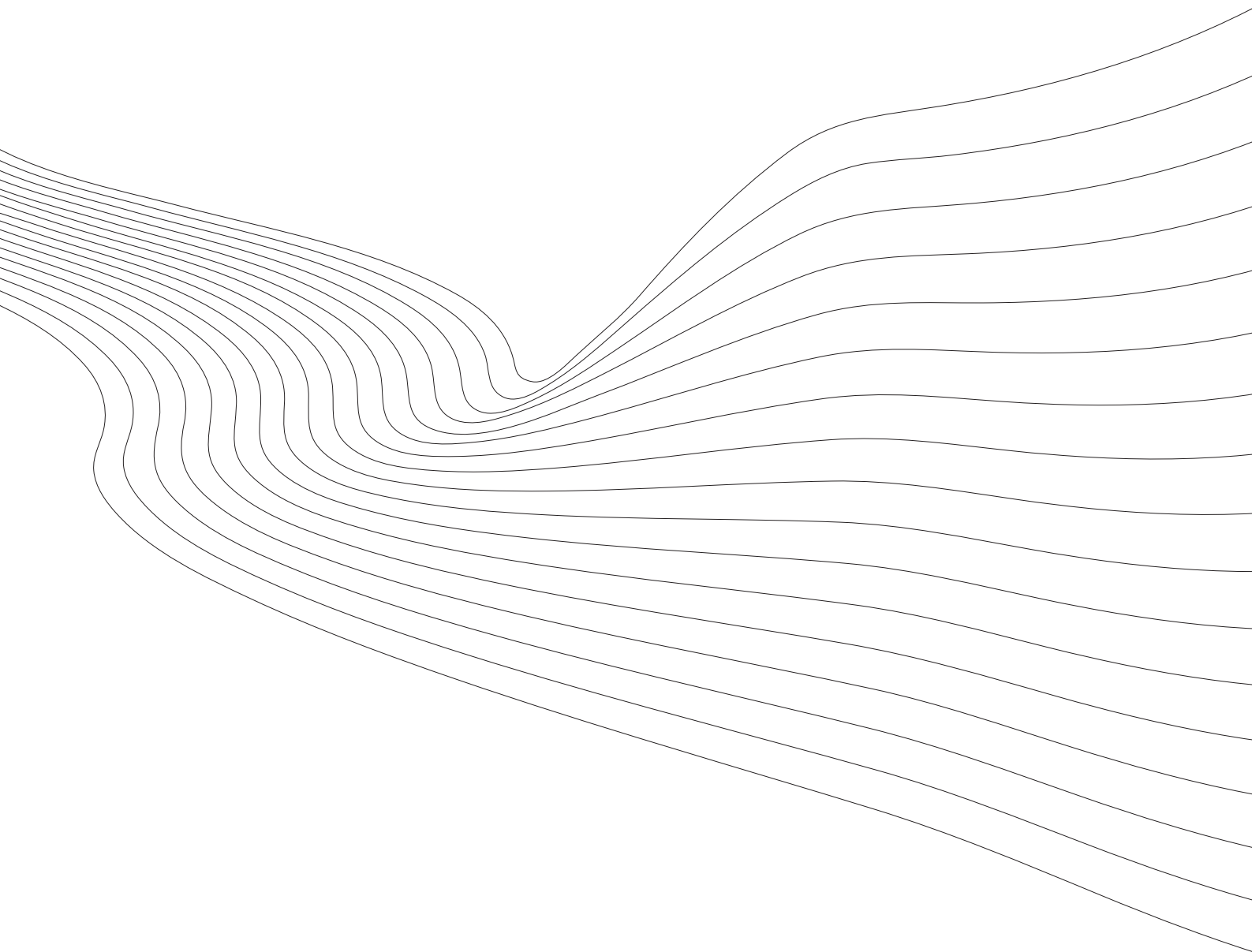
On the Accuracy of the Probability Method for Quantifying Beliefs about Inflation*

Thomas Maag[†]

July 2009

Abstract

This paper assesses the probability method for quantifying EU consumer survey data on perceived and expected inflation. Based on micro-data from the Swedish consumer survey that asks for both qualitative and quantitative responses, I find that the theoretical assumptions of the method do not hold. In particular, estimated models of response behavior indicate that qualitative inflation expectations are not ordered. Nevertheless, the probability method generates series that are highly correlated with the mean of actual quantitative beliefs. For quantifying the cross-sectional dispersion of beliefs, however, an index of qualitative variation outperforms the probability method.

JEL classification: C53, D84, E31

Keywords: quantification, inflation expectations, inflation perceptions, qualitative response data, belief formation

1   Introduction

Surveys of households and firms are often qualitative. Rather than giving a quantitative estimate of a particular variable, respondents are asked to indicate their belief on a qualitative scale. In the European Union (EU), inflation perceptions and expectations of households are surveyed as part of the Joint Harmonized EU Consumer Survey programme. Within this framework, harmonized qualitative surveys are conducted in all member states, covering a national sample size of roughly 1,500 households on a monthly basis. The EU consumer survey thus provides an extensive and consistent data-set on beliefs about inflation.[1] Consequently, the data has been investigated by a large literature. Only recently, the euro cash changeover and its effects on households' inflation perceptions has given rise to a new strand of research.[2] However, since the EU consumer survey is qualitative, most empirical applications rely on a method to quantify the qualitative response data in the first place. This paper assesses the validity of one particular method, the probability method for 5-category scales, and compares its accuracy to other quantification approaches.

Possibly the most widely used quantification method is the balance statistic proposed by Anderson (1952). It is originally defined as the difference between the share of respondents that perceive or expect positive inflation rates and the share of respondents that perceive or expect negative inflation rates. Theil (1952) rationalizes the balance statistic, demonstrating that it is an appropriate measure of the population mean if quantitative beliefs are uniformly distributed in the population. Furthermore, Theil (1952) suggests

---

[1]Currently, the Joint Harmonized EU Consumer Survey covers a monthly sample of roughly 40,000 consumers in 27 member states. The consumer survey consists of 15 qualitative questions about the household's financial situation, perceived economic conditions, and planned savings and spending. The questionnaire is translated into national languages and may include additional country specific questions, see European Commission (2007).

[2]This literature centers on the rise in perceived inflation coinciding with the euro cash changeover, as documented in ECB (2005). Several explanations are being discussed, including increased information processing requirements due to conversion rates, overreaction to prices of frequently bought items, and anchoring of perceptions to prior expectations. See, e.g., Ehrmann (2006), Aucremanne, Collin, and Stragier (2007), Doehring and Mordonu (2007), Dziuda and Mastrobuoni (2006), Aalto-Setälä (2006), and Fluch and Stix (2007). Abstracting from the euro cash changeover, other contributions investigate belief formation in general, see, e.g., Döpke, Dovern, Fritsche, and Slacalek (2008), Forsells and Kenny (2004), Lamla and Lein (2008), and Lein and Maag (2008).

that the distributional assumption may be relaxed by imposing a normal distribution on the quantitative beliefs instead. Combined with the assumption that respondents reply that they perceive or expect prices to be constant if their quantitative belief is within an indifference interval around 0% the mean and variance of the imposed distribution can be identified. The model of Theil (1952) has been rediscovered by Carlson and Parkin (1975) and is known today as the Carlson-Parkin method or the 3-category probability method.[3] Batchelor and Orr (1988) extend the probability method to response data on 5-category scales as it is available from the EU consumer survey. Taking into account the particular questioning in the EU consumer survey, Berk (1999) additionally suggests an identification scheme that links inflation expectations to inflation perceptions.

The goal of this paper is to assesses the 5-category probability method and to derive lessons for applied research. The analysis relies on joining qualitative and quantitative response data on household level which is available from the Swedish Consumer Tendency Survey from 01/1996 to 10/2008. A similar approach has been adopted, to the best of my knowledge, only by Defris and Williams (1979) and Batchelor (1986). Defris and Williams (1979) investigate a 5-year sample from an Australian consumer survey. They document that the balance statistic as well as the 3-category probability method generate series that are only weakly correlated with quantitative survey responses. Batchelor (1986) investigates micro-data from the University of Michigan Surveys of Consumers, 1979–1984. In line with Defris and Williams (1979), Batchelor (1986) finds that both the balance statistic and quantified expectations using the probability method track quantitative survey responses only inaccurately, in particular in the short term. This result is in contrast to my findings for Sweden.

This paper extends the existing literature in several respects. First, a detailed assessment of the theoretical assumptions of the 5-category probability method is provided. Exist-

---

[3]A less common quantification method is the regression approach of Pesaran (1987). The regression method extends the balance statistic, allowing for a non-linear relation between response shares and quantitative beliefs. The method is outlined in Section 4. Pesaran (1987) discusses the three-category probability method and the regression approach in detail. For a recent survey of quantification methods see Nardo (2003).

ing research focuses on the 3-category probability method and on testing distributional assumptions. Joining quantitative and qualitative data on household level allows to estimate unrestricted response schemes. The restrictions imposed by the 5-category probability method can then be tested using likelihood theory. Second, the accuracy of the 5-category probability method relative to the mean and cross-sectional standard deviation of quantitative survey responses is assessed in a long sample of 154 monthly observations. The discussion centers on comparing correlation coefficients employing the Fisher z-transformation and double block bootstrap confidence intervals. Accuracy is compared to a set of alternative quantification methods, including the 3-category probability method, the balance statistic, and the regression approach. For quantifying the cross-sectional heterogeneity of beliefs the set of alternatives includes the 3-category probability method, an index of qualitative variation, an index of ordinal variation, and the disconformity index. Third, I assess the probability method both for quantifying households' inflation expectations as well as inflation perceptions.

To anticipate a central result, the 5-category probability method performs well for quantifying the mean of beliefs, despite that its underlying assumptions are rejected. For quantifying the cross-sectional heterogeneity, however, an index of qualitative variation is most accurate.

The paper is structured as follows. Section 2 presents the data and discusses some key statistical properties. Section 3 assesses the assumptions of the 5-category probability method. Section 4 investigates the accuracy of the approach and contrasts it with alternative methods. Section 5 draws lessons for applied research. Section 6 concludes.

2 Data

Inflation perceptions and expectations of Swedish households are being surveyed on a monthly basis since 1973. Unlike most surveys in other countries, the Swedish Consumer Tendency Survey simultaneously records qualitative and quantitative beliefs about infla-

tion. This paper uses monthly household-level response data spanning 01/1996–10/2008.[4] During this period, the survey comprises a representative monthly sample of roughly 1,500 households which are interviewed by telephone.

The questionnaire of the Swedish Consumer Tendency Survey is based on the Joint Harmonized EU Consumer Survey questionnaire. Households' beliefs about inflation are recorded in two steps.[5] In a first step, the respondent is asked to report perceived inflation on a five-category ordinal scale. The question reads:

"Compared with 12 months ago, do you find that prices in general are...?" "Lower (S_1), about the same (S_2), a little higher (S_3), somewhat higher (S_4), a lot higher (S_5), don't know".

Following, S_1 through S_5 denote the qualitative response categories, while s_1 through s_5 are the fractions of answers in the respective qualitative response category.[6] In a second step, the respondent is asked for a direct quantitative estimate of the current inflation rate. The question reads: "How much higher/lower in percent do you think prices are now? (In other words, the present rate of inflation)". In a similar manner, expected inflation is first captured by asking:

"Compared to the situation today, do you think that in the next 12 months prices in general will...?" "Go down a little (S_1), stay more or less the same (S_2), go up more slowly (S_3), go up at the same rate (S_4), go up faster (S_5), don't know".

Quantitative estimates are recorded in a second step by asking: "Compared with today, how much in percent do you think prices will go up/down? (In other words, inflation/deflation 12 months from now)". As a result and in contrast to the Joint Harmonized EU Consumer Survey households report qualitative as well as quantitative inflation beliefs.

---

[4]The sample horizon is limited by data availability. Before 1996 qualitative responses were only recorded on a 3 option ordinal scale and quantitative beliefs were only surveyed on a quarterly basis.

[5]The exact procedure is outlined in the GfK (2002) survey manual. A schematic of the questioning can be found in Palmqvist and Strömberg (2004). Note that the English description of the response categories provided by GfK (2002) differs from the official terminology in European Commission (2007). In particular, European Commission (2007) labels the category S_4 for inflation perceptions "moderately higher".

[6]Response shares are computed excluding the "don't know" category, i.e. s_1 through s_5 sum up to 100%.

In line with the literature, the quantitative response data is adjusted for outliers. Responses outside the interval $[-30\%, 30\%]$ are omitted which reduces the sample size by 0.3%.[7] Moreover, only observations that simultaneously contain non-missing responses to qualitative and quantitative questions are considered. Dropping respondents that only answer the qualitative question on perceived (expected) inflation further reduces the sample size by 13% (15%). As will be discussed in the next section, a theoretical assumption of all quantification methods considered here is that a respondent needs to form a quantitative belief in the first place to to express a qualitative opinion at a later stage. The observed response pattern can be considered as evidence against this assumption. However, an alternative interpretation is that qualitative responses with missing quantitative responses are uninformed and should be attributed to the "don't know" category.[8]

The resulting sample comprises 154 monthly surveys spanning 01/1996–10/2008, which amounts to almost 200,000 observations. Throughout this paper the discussion centers on this sample. The appendix additionally presents estimation results for the shorter 01/2002–10/2008 sample. Considering this period accounts for a potential structural break due to a change in the surveying institution in 01/2002. As will be shown, results for both estimation periods are consistent, confirming the validity of the results for the 01/1996–10/2008 sample.[9]

As a measure of actual inflation I use the year-over-year percent change in the Harmonized Index of Consumer Prices (HICP) as published by Eurostat.[10]

---

[7]Over the entire sample period, 667 observations include inflation perceptions and 443 observations include inflation expectations that are classified as outliers.

[8]This view is supported by the distribution of missing quantitative answers by qualitative answer category. For inflation perceptions, about 70% of missing quantitative responses are assigned to respondents that opt for the qualitative response category S_3 ("a little higher"). For inflation expectations, 40% of missing quantitative responses are assigned to respondents that opt for the qualitative response category S_4 ("go up at the same rate").

[9]In 01/2002 the surveying institution has changed from Statistics Sweden to GfK Sweden. The change goes along with a decline in the share of missing quantitative responses. This might be partly due to differences in the questioning, as outlined by Palmqvist and Strömberg (2004). However, the share of missing observations rises again sharply in 2008 to levels before 2002. Hence, part of the initial decline in the share of missing quantitative responses appears to be pure coincidence.

[10]I have also considered the Consumer Price Index (CPI) and the Consumer Price Index excluding mortgage payments and indirect taxes (CPIX). In particular at the beginning of the sample period these indices might have obtained more attention by the Swedish public than the HICP. Employing these
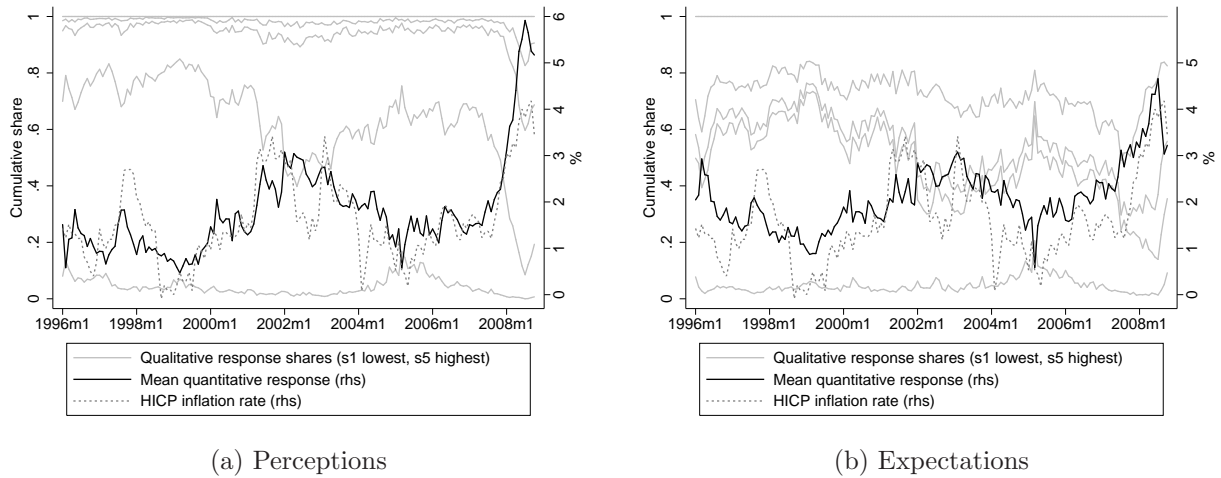
| (a) Perceptions | (b) Expectations |

Figure 1: Qualitative response shares and mean quantitative response

Notes: The lowest grey line shows the share s_1 of qualitative answers in the category S_1, the second-lowest grey line shows the cumulative share of answers in the categories S_1 and S_2, etc.

Figure 1 shows qualitative response shares, the mean of quantitative responses, and actual inflation over time. The figure indicates that quantitative inflation perceptions closely track actual inflation. The correlation coefficient between the two series is 0.78, the root mean squared error (RMSE) is 0.46%. The high correlation is particularly noteworthy as the consumer survey does not ask specifically about HICP inflation, but about the evolution of prices "in general". Moreover, the first panel of Table 1 shows that the overall mean of inflation perceptions is 1.79%, as opposed to an average HICP inflation rate of 1.61%. Hence, inflation perceptions of the Swedish public are only slightly biased, which is in line with earlier findings of Jonung and Laidler (1988). In light of quantitative survey results for the U.S. (Bryan and Ventaku, 2001a, 2001b) and for the U.K. (Driver and Windram, 2007) inflation perceptions of the Swedish public are relatively accurate.[11] Figure 1 shows that inflation expectations also follow actual inflation quite closely. The contemporaneous correlation of the mean expectation with actual inflation is 0.70, the RMSE is 0.66%.

_____

alternative indices does not significantly alter the results.

[11]Relying on a monthly household survey conducted by the Federal Reserve Bank of Cleveland, Bryan and Venkatu (2001a, 2001b) find that inflation perceptions (and expectations) are biased by several percentage points. For the U.K., Driver and Windram (2007) report a correlation coefficient between actual and perceived inflation of roughly 0.5 over a similar sample period.

Table 1: Summary statistics of quantitative responses

	Perceptions			Expectations		
	Overall	Min	Max	Overall	Min	Max
Observations	197,487	1,031	1,456	192,845	961	1,417
Number of months	154			154		
Mean	1.81	0.47	5.91	2.10	0.59	4.66
Median	0.00	0.00	5.00	1.00	0.00	4.00
Standard deviation	4.06	2.68	5.37	3.73	2.68	4.94
Share of integer answers	0.94	0.86	0.98	0.92	0.82	0.98
Share of focal point answers	0.81	0.50	0.92	0.72	0.56	0.86
Share of zero responses	0.59	0.08	0.80	0.44	0.12	0.70
Mean response, given S_1	-4.79	-20.00	-2.12	-3.89	-6.36	-2.53
Mean response, given S_2	0.03	-0.05	0.46	0.02	-0.03	0.24
Mean response, given S_3	4.72	3.98	5.95	4.06	3.04	5.55
Mean response, given S_4	7.94	5.22	10.72	4.77	3.91	6.06
Mean response, given S_5	9.58	4.25	16.00	3.98	3.12	6.14

Notes: The column *overall* presents results for the entire sample spanning 01/1996–10/2008. *Min* and *Max* are the monthly minimum and maximum of the respective statistic. All shares are relative to the overall number of observations.

Predictive power is relatively low, as the correlation with 12 months ahead inflation is only 0.28, with the RMSE increasing to 0.97%. Table 1 documents that the sample mean of inflation expectations lies somewhat higher at 2.05%. Figure 1 shows that inflation perceptions surge in 2008, exceeding actual inflation by roughly 2%. Meanwhile, the share of qualitative responses in the lowest two categories S_1 and S_2 ("lower" and "about the same") sharply declines. Finally, Figure 1 reveals a systematic difference between qualitative inflation perceptions and expectations. Qualitative inflation perceptions are concentrated in categories S_2 and S_3 ("about the same" and "a little higher"). In 1996–2008, 87% of respondents opt for these categories. In contrast, 70% of qualitative inflation expectations fall into answer categories S_2 and S_5 ("stay more or less the same" and "go up faster").

Table 1 highlights a number of interesting properties of the quantitative response data. First, panel 1 shows that inflation perceptions and expectations are highly heterogeneous. Despite the low mean of both variables, inflation perceptions and expectations exhibit a cross-sectional standard deviation of 4.06% and 3.73% respectively. Second, panel 2

indicates that more than 90% of all quantitative answers are integers. Third, these integer answers are concentrated at a few focal points.[12] Both for perceptions and expectations, the most frequently mentioned focal points are -5%, -2%, 0%, 2%, 5%, 10%.[13] As Table 1 indicates, the most important focal point both for perceptions and expectations is 0%, which accounts for more than half of all focal point responses. Towards the end of the sample horizon, the share of zero responses declines significantly, attaining a minimum of 8% for perceptions and 12% for expectations in 06/2008. The high share of zero responses also explains the low median of quantitative beliefs. The median of inflation perceptions is 0%, while the median of inflation expectations is 1%. Fourth, mean quantitative inflation perceptions are generally rising in the qualitative response category. This is shown by panel 3 of Table 1 which summarizes the mean of quantitative responses conditional on the qualitative response the respondent simultaneously opts for. However, this is not the case for inflation expectations. For expectations, the mean response given S_4 ("go up at the same rate") is higher than the mean response given S_5 ("go up faster"). Also, in comparison to inflation perceptions the differences between the cross-sectional mean of expectations given qualitative answers S_3, S_4, S_5 are only minor.[14] Fifth, the relation between quantitative and qualitative responses is time varying. The differences between mean, minimum, and maximum are considerable for most categories. The only exception is S_2 ("about the same"): given this qualitative answer the mean quantitative response is always close to 0%.

These initial results suggest that the relation between quantitative and qualitative inflation beliefs is complex. The response scheme, i.e. the formal relation between quantitative and qualitative responses, appears to be time varying. Moreover, the conditional mean of quantitative expectations is not monotonously rising in the order of the qualitative

---

[12]In line with Bryan and Palmqvist (2006) focal points are defined as integers that are mentioned more often than their neighboring integers. I have not found any evidence for important non-integer focal points.

[13]Of the remaining integers, 1, 3, and 4 obtain the highest response shares. This set accounts for 9% of quantitative inflation perceptions and for 15% of expectations.

[14]On a monthly basis the mean of inflation perceptions is not always strictly rising too, as indicated by the minima of monthly means in panel 3 of Table 1. But this only is the case in 27 months as opposed to 136 months in which the conditional mean of inflation expectations is not monotonously rising.

response categories. While the 5-category probability method allows for a time varying response scheme, it imposes a certain symmetry on the response scheme and requires ordered qualitative data. Regarding the distributional assumptions, the above results indicate that quantitative beliefs are not normally distributed. Since the mean exceeds the median the distributions of inflation perceptions and expectations are positively skewed. The concentration of answers at focal points and in particular at 0% raises doubt whether any of the common parametric distributions adequately describes the quantitative response data. The next section thus discusses in detail whether the assumptions of the probability method are consistent with the data.

3   Validity of the Probability Method

This section tests the main theoretical assumptions of the 5-category probability method for quantifying qualitative response data. Building on contributions of Theil (1952) and Carlson and Parkin (1975), the 5-category probability method has been introduced by Batchelor and Orr (1988). To begin with, this method is briefly outlined.

Assume that previous to answering the consumer survey, respondent i forms a quantitative belief π_{it}^e about inflation over the upcoming 12 months.[15] Respondent i then answers the qualitative survey question on expected inflation according to the following response scheme:

$$
\begin{aligned}
\pi_{it}^e < -\delta_t \quad &: \quad \textit{prices in general will } \text{go down a little } (S_1) \\
-\delta_t \leq \pi_{it}^e < \delta_t \quad &: \quad \text{stay more or less the same } (S_2) \\
\delta_t \leq \pi_{it}^e < \pi_t^r - \eta_t \quad &: \quad \text{go up more slowly } (S_3) \\
\pi_t^r - \eta_t \leq \pi_{it}^e < \pi_t^r + \eta_t \quad &: \quad \text{go up at the same rate } (S_4) \\
\pi_{it}^e \geq \pi_t^r + \eta_t \quad &: \quad \text{go up faster } (S_5)
\end{aligned}
\tag{1}
$$

The response scheme is defined by the parameters δ_t, η_t, π_t^r. In the following, π_t^r is

---

[15]The analogous approach for quantifying perceived inflation π_{it}^p and detailed derivations can be found in the Appendix.

called reference inflation. It is the inflation rate that people have in mind when opting for answer S_4 ("prices will go up at the same rate" and, for inflation perceptions, "prices are moderately higher"). A first key assumption of the probability method restricts the response scheme to be fully defined by three parameters:

Assumption 1: The response intervals are symmetric around 0% and around π_t^r.

The corresponding intervals $[-\delta_t, \delta_t)$ and $[\pi_t^r - \eta_t, \pi_t^r + \eta_t)$ correspond to qualitative responses S_2 and S_4 respectively. A second assumption imposes structural homogeneity on the response scheme:

Assumption 2: Threshold parameters δ_t and η_t and the reference inflation rate π_t^r are identical across all respondents.

Quantitative inflation expectations π_{it}^e will vary across respondents due to differences in information sets and information processing. To infer the mean quantitative inflation expectation from qualitative response shares the probability method imposes a distributional assumption on π_{it}^e. The standard assumption is that the cross-sectional distribution of quantitative beliefs is normal:

Assumption 3: The cross-sectional distribution of quantitative beliefs is normal, i.e. $\pi_{it}^e \sim N(\pi_t^e, \sigma_t^e)$.

The parameters of interest are the cross-sectional mean π_t^e and standard deviation σ_t^e of quantitative beliefs. As outlined in the Appendix, the above assumptions yield a system of 4 linearly independent equations with 5 unknowns (π_t^e σ_t^e, δ_t, η_t, π_t^r) which can be solved for π_t^e and σ_t^e. The solution for both parameters is equal to the product of reference inflation π_t^r and a function of the response shares $s_t^1, ..., s_t^5$.

The usual identification scheme restricts reference inflation π_t^r. For quantifying inflation expectations two choices of π_t^r are apparent. First, reference inflation can be set equal to some actual rate of inflation, assuming that the respondent knows the actual rate of inflation and answers the question relative to this value. Second, reference inflation can

be set equal to previously quantified perceived inflation π_t^p as suggested by Berk (1999). This approach is supported by empirical evidence that households are not necessarily well informed about actual inflation.[16] Identifying π_t^r is less obvious for inflation perceptions. Relying on Carlson and Parkin (1975) it is commonly assumed that inflation perceptions are unbiased over the sample horizon. This assumption can be imposed by restricting π_t^r to a constant accordingly.[17] The last assumption thus reads:

> Assumption 4: The reference rate of inflation π_t^r for quantifying inflation expectations is equal to some actual or quantified perceived rate of inflation. The reference rate of inflation for quantifying inflation perceptions is time invariant.

3.1   Symmetry of the Response Scheme

Assumption 1 restricts response intervals to be symmetric around 0% and around π_t^r. To test the validity of this assumption I estimate an unrestricted response scheme with 4 threshold parameters. Assume that respondent i answers the qualitative question according to the following scheme:[18]

$$
\begin{aligned}
\pi_{it}^e + \varepsilon_{it} < \mu_t^1 &\quad : \quad \text{\textit{prices in general will} go down a little } (S_1) \\
\mu_t^1 \leq \pi_{it}^e + \varepsilon_{it} < \mu_t^2 &\quad : \quad \text{stay more or less the same } (S_2) \\
\mu_t^2 \leq \pi_{it}^e + \varepsilon_{it} < \mu_t^3 &\quad : \quad \text{go up more slowly } (S_3) \\
\mu_t^3 \leq \pi_{it}^e + \varepsilon_{it} < \mu_t^4 &\quad : \quad \text{go up at the same rate } (S_4) \\
\pi_{it}^e + \varepsilon_{it} \geq \mu_t^4 &\quad : \quad \text{go up faster } (S_5)
\end{aligned}
\tag{2}
$$

The idiosyncratic component ε_{it} allows the response scheme to shift between individuals. Under the assumption that the idiosyncratic component represents the sum of various, independent individual specific factors it is reasonable to assume that ε_{it} is normally distributed. This reasoning is in line with the normality assumption commonly

---

[16]See, e.g., Bryan and Venkatu (2001a, 2001b) who document that inflation perceptions of U.S. households are significantly biased.

[17]The solution for π_t^r is given by Equation (A.8) in the Appendix.

[18]The identical scheme is applied to inflation perceptions, where π_{it}^e is replaced by π_{it}^p.

imposed by the probability method. One thus obtains an ordered probit model (Zavoina and McKelvey, 1975). Deviating from the usual identification scheme, I restrict the coefficient on the quantitative belief π_{it}^e to unity, while the variance of ε_{it} remains unrestricted. With $\varepsilon_{it} \sim N(0, \sigma_t)$ the following probabilities are obtained:

$$
\begin{aligned}
P(\text{lower}|\pi_{it}^e, \mu_t, \sigma_t) = P(S_1|\pi_{it}^e, \mu_t, \sigma_t) &= \Phi\left(\frac{-\pi_{it}^e + \mu_t^1}{\sigma_t}\right) \\
P(\text{about the same}|\pi_{it}^e, \mu_t, \sigma_t) = P(S_2|\pi_{it}^e, \mu_t, \sigma_t) &= \Phi\left(\frac{-\pi_{it}^e + \mu_t^2}{\sigma_t}\right) - \Phi\left(\frac{-\pi_{it}^e + \mu_t^1}{\sigma_t}\right) \\
P(\text{a little higher}|\pi_{it}^e, \mu_t, \sigma_t) = P(S_3|\pi_{it}^e, \mu_t, \sigma_t) &= \Phi\left(\frac{-\pi_{it}^e + \mu_t^3}{\sigma_t}\right) - \Phi\left(\frac{-\pi_{it}^e + \mu_t^2}{\sigma_t}\right) \\
P(\text{moderately higher}|\pi_{it}^e, \mu_t, \sigma_t) = P(S_4|\pi_{it}^e, \mu_t, \sigma_t) &= \Phi\left(\frac{-\pi_{it}^e + \mu_t^4}{\sigma_t}\right) - \Phi\left(\frac{-\pi_{it}^e + \mu_t^3}{\sigma_t}\right) \\
P(\text{a lot higher}|\pi_{it}^e, \mu_t, \sigma_t) = P(S_5|\pi_{it}^e, \mu_t, \sigma_t) &= 1 - \Phi\left(\frac{-\pi_{it}^e + \mu_t^4}{\sigma_t}\right)
\end{aligned}
\tag{3}
$$

where $\mu_t = \{\mu_t^1, ..., \mu_t^4\}$. Unlike in the probability method, μ_t^1, μ_t^2 and μ_t^3, μ_t^4 are not required to be symmetric around 0% and a reference rate of inflation respectively.

Table 2 presents the maximum likelihood estimates of this response scheme. By construction, the threshold parameters are rising in the qualitative response category. The model is confirmed by highly significant parameter estimates which are stable across subperiods. The width of the interval $[\mu_t^1, \mu_t^2)$ corresponding to qualitative response S_2 ("about the same") exceeds 8% both for perceptions and expectations. The estimated parameters suggest that this interval is not symmetric around 0%. Relying on maximum likelihood theory I test the restriction that $\mu_t^1 = -\mu_t^2$ with a likelihood ratio test.[19] The second panel of Table 2 shows that this test clearly rejects the null hypothesis of symmetry.

The estimates confirm the systematic differences between quantitative perceptions and expectations that have been noticed before. While the threshold parameters for inflation perceptions are increasing from $\mu_t^1 = -6.91\%$ to $\mu_t^4 = 13.89\%$, the thresholds for inflation

---

[19]The test statistic is given by $LR = -2\left(\log L_r - \log L_i\right) \rightarrow \chi^2(q)$, where $\log L_r$ is the log likelihood of the restricted model and $\log L_i$ is the log likelihood of the unrestricted model. The number of restrictions is given by $q = 1$.

Table 2: Estimated response scheme for perceived and expected inflation

| | 1996–2008 | | 2002–2008 | |
	Perceptions	Expectations	Perceptions	Expectations
μ_t^1	-6.909***	-7.404***	-7.595***	-7.883***
	(0.0300)	(0.0367)	(0.0457)	(0.0556)
μ_t^2	3.194***	1.739***	2.526***	0.617***
	(0.0138)	(0.0141)	(0.0188)	(0.0230)
μ_t^3	10.95***	2.797***	10.97***	2.005***
	(0.0267)	(0.0141)	(0.0362)	(0.0212)
μ_t^4	13.89***	5.609***	14.04***	5.682***
	(0.0388)	(0.0174)	(0.0520)	(0.0251)
σ_t	3.782***	4.427***	4.260***	5.083***
	(0.0118)	(0.0163)	(0.0181)	(0.0270)
N	197487	192845	110071	109782
Log L	-138263	-220521	-88738	-136562
Likelihood ratio tests				
H_0^1: Symmetry such that $\mu_t^2 = -\mu_t^1$ ($q = 1$)				
LR statistic	20978.94	35749.40	17427.61	25138.60
P-value	0.00	0.00	0.00	0.00

Notes: This table shows maximum likelihood estimates of the unrestricted response scheme (2). Monthly data, 01/1996–10/2008 and 01/2002–10/2008. N is the number of observations, log L is the log likelihood, standard errors in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level.

expectations range between $\mu_t^1 = -7.04\%$ and $\mu_t^4 = 5.61\%$. For inflation expectations, the threshold parameters that define the response intervals for S_3, S_4, S_5 are in a narrow range of 4% to 5%. This allows for two interpretations: either, the response intervals are indeed narrower for inflation expectations than for inflation perceptions, or the ordered model does not adequately describe the formation of qualitative inflation expectations in the positive region. The second interpretation is supported by the relatively high estimated standard deviation σ_t and a substantially lower log likelihood than for inflation perceptions (despite a lower number of observations). That qualitative expectations are not necessarily ordered is indeed suggested by the relative nature of the response categories. The qualitative response S_4 ("prices will go up at the same rate") anchors the qualitative expectation to the perception of current inflation. A respondent who has an identical quantitative

Table 3: Estimated relative response scheme for expected inflation

Expectations	Income groups (1st quartile lowest)				
	1st quartile	2nd quartile	3rd quartile	4th quartile	Overall
μ_t^1	-7.172***	-6.014***	-5.056***	-3.928***	-5.316***
	(0.184)	(0.116)	(0.0569)	(0.0132)	(0.0500)
μ_t^2	1.322***	1.059***	0.805***	0.575***	0.814***
	(0.0839)	(0.0550)	(0.0445)	(0.0104)	(0.0242)
σ_t	6.212***	4.956***	4.056***	3.182***	4.369***
	(0.139)	(0.0826)	(0.0062)	(0.0039)	(0.0347)
N	10046	15133	15584	19618	60381
Log L	-8759	-12591	-12436	-14921	-49217
Likelihood ratio tests				LR statistic	P-value
H_0^1: Identical threshold parameters μ_t^1, μ_t^2 ($q = 6$)				746.60	0.00
H_0^2: Identical standard deviation σ_t ($q = 3$)				907.96	0.00
H_0^3: Identical thresholds and standard deviation ($q = 9$)				1020.93	0.00

Notes: This table shows maximum likelihood estimates of the relative response scheme (4). Monthly data, 01/2002–10/2008. Standard errors in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level.

perception and expectation of inflation will always opt for qualitative response S_4 unless answers are inconsistent. Hence, the qualitative response S_4 is chosen independently of the level of expected inflation.

I therefore estimate a scheme for positive inflation expectations π_{it}^e that incorporates the relative nature of answers with respect to quantitative inflation perceptions π_{it}^p:

$$
\begin{aligned}
\pi_{it}^e + \varepsilon_{it} < \pi_{it}^p + \mu_t^1 \quad &: \quad \textit{prices in general will} \text{ go up more slowly } (S_3) \\
\pi_{it}^p + \mu_t^1 \leq \pi_{it}^e + \varepsilon_{it} < \pi_{it}^p + \mu_t^2 \quad &: \quad \text{go up at the same rate } (S_4) \\
\pi_{it}^e + \varepsilon_{it} \geq \pi_{it}^p + \mu_t^2 \quad &: \quad \text{go up faster } (S_5)
\end{aligned} \tag{4}
$$

where $\varepsilon_{it} \sim N(0, \sigma_t)$. This scheme assumes that the respondent will opt for S_4 if $\pi_{it}^e - \pi_{it}^p$ lies in the range $[\mu_t^1, \mu_t^2)$. The maximum likelihood estimates can be found in the last column of Table 3. All estimated parameters are highly significant, confirming that the qualitative response about expected inflation is indeed linked to the quantitative inflation perception. Moreover, the response interval is highly asymmetric: qualitative responses

are more responsive to an increase of quantitative expectations over perceptions than to a decrease. The significant estimates also reveal that, despite the lack of an unambiguous relation between quantitative and qualitative expectations, the 5-category qualitative survey contains more information than a 3-option survey that does not distinguish between S_3, S_4, and S_5.

In sum, the results indicate that Assumption 1 is not satisfied. Under the imposed normality assumption the estimated response interval is not symmetric around 0%, both for inflation perceptions and expectations.[20] Furthermore, for expectations the response interval is not symmetric around π_t^r. The estimations confirm that qualitative inflation expectations are formed relative to perceived inflation. This result suggests that the link between expectations and perceptions should be exploited in quantifying qualitative responses. Additional evidence on the relation of reference inflation and quantitative perceptions is provided in Section 3.4.

3.2   Homogeneity of the Response Scheme

Assumption 2 imposes that threshold parameters δ_t and η_t and reference inflation π_t^r do not systematically differ across respondents. Since the Swedish data-set only contains one observation per individual this assumption is tested by estimating the response scheme for different income groups.[21] Tables 4 and 5 summarize the estimation results for perceptions and expectations respectively. The tables show that the absolute values of threshold parameters tend to decline in income. The lower panel of Tables 4 and 5 present likelihood ratio tests of three restrictions. The null hypotheses considered are that threshold parameters are identical across income groups (H_0^1), that standard deviation is identical across income groups (H_0^2), and that both threshold parameters and standard deviation

---

[20]This finding is consistent with Henzel and Wollmershäuser (2005) who investigate data from a special edition of the ifo World Economic Survey that directly asks to quantitatively indicate the indifference interval. Henzel and Wollmershäuser (2005) report that the positive threshold parameter is larger in absolute terms than the negative parameter. As opposed to the Swedish survey, however, the ifo survey queries professional forecasters and answers are given on a 3-category ordinal scale.

[21]I have also considered education groups, with unchanged qualitative results.

Table 4: Estimated response scheme for perceived inflation by income groups

Perceptions	Income groups (1st quartile lowest)				
	1st quartile	2nd quartile	3rd quartile	4th quartile	Overall
μ_t^1	-9.074***	-8.201***	-6.657***	-5.833***	-7.308***
	(0.142)	(0.105)	(0.0821)	(0.0627)	(0.0459)
μ_t^2	2.411***	2.487***	2.405***	2.366***	2.479***
	(0.0571)	(0.0429)	(0.0352)	(0.0271)	(0.0192)
μ_t^3	11.55***	11.68***	10.63***	9.405***	10.79***
	(0.100)	(0.0824)	(0.0704)	(0.0541)	(0.0372)
μ_t^4	14.60***	15.07***	13.53***	12.16***	13.77***
	(0.136)	(0.119)	(0.104)	(0.0836)	(0.0535)
σ_t	5.232***	4.682***	3.759***	3.179***	4.120***
	(0.0583)	(0.0425)	(0.0326)	(0.0238)	(0.0183)
N	17092	24845	25482	32614	100033
Log L	-15584	-20718	-19346	-23163	-79851
Likelihood ratio tests				LR statistic	P-value
H_0^1: Identical threshold parameters $\mu_t^1,...,\mu_t^4$ ($q = 12$)				1868.21	0.00
H_0^2: Identical standard deviation σ_t ($q = 3$)				1115.68	0.00
H_0^3: Identical thresholds and standard deviation ($q = 15$)				2080.00	0.00

Notes: This table shows maximum likelihood estimates of the unrestricted response scheme (2). Monthly data, 01/2002–10/2008. Standard errors in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level.

are identical across income groups (H_0^3). All three hypotheses are clearly rejected.

Table 3 presents estimation results for the relative response scheme (4) that links expected inflation to perceived inflation. Again, all three hypotheses are clearly rejected. The estimates show the same pattern as above: the absolute values of threshold parameters are declining in income. Overall, these results suggest that the response scheme systematically differs across income-groups, which implies that Assumption 2 is violated.[22]

---

[22]Note that the mean perceptions and expectations of inflation are also systematically linked to socioeconomic characteristics. The cross-sectional mean of quantitative perceptions and expectations is declining in income. This is consistent with the response scheme estimates. These suggest that individuals in the highest income quartile experience any deviation of inflation from zero as more relevant in qualitative terms than individuals in lower income quartiles.

Table 5: Estimated response scheme for expected inflation by income groups

Expectations	Income groups (1st quartile lowest)				
	1st quartile	2nd quartile	3rd quartile	4th quartile	Overall
μ_t^1	-9.229***	-8.303***	-7.031***	-5.968***	-7.598***
	(0.165)	(0.124)	(0.102)	(0.0747)	-0.0561
μ_t^2	0.703***	0.440***	0.591***	0.783***	0.600***
	(0.0686)	(0.0527)	(0.0429)	(0.0315)	-0.0233
μ_t^3	2.452***	1.972***	1.816***	1.835***	1.950***
	(0.0633)	(0.0480)	(0.0394)	(0.0294)	-0.0215
μ_t^4	7.075***	6.164***	5.136***	4.407***	5.502***
	(0.0805)	(0.0575)	(0.0452)	(0.0328)	-0.0253
σ_t	6.089***	5.448***	4.488***	3.789***	4.905***
	(0.0827)	(0.0614)	(0.0484)	(0.0344)	-0.0271
N	17232	24757	25384	32528	99901
Log L	-21816	-31163	-31206	-39092	-124179
Likelihood ratio tests				LR statistic	P-value
H_0^1: Identical threshold parameters $\mu_t^1,...,\mu_t^4$ ($q = 12$)				1102.84	0.00
H_0^2: Identical standard deviation σ_t ($q = 3$)				1689.36	0.00
H_0^3: Identical thresholds and standard deviation ($q = 15$)				1804.00	0.00

Notes: This table shows maximum likelihood estimates of the unrestricted response scheme (2). Monthly data, 01/2002–10/2008. Standard errors in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level.

3.3   Normality of Quantitative Responses

Assumption 3 requires that the cross-sectional distribution of quantitative beliefs is normal. Normality has been tested and rejected both for inflation expectations of consumes (Batchelor and Dua, 1987) and professional forecasters (Carlson, 1975, Lahiri and Teigland, 1987). These studies generally find that quantitative beliefs are positively skewed and leptokurtic. Both properties are also present in the Swedish survey data, as panel 1 of Table 6 indicates. Inflation perceptions as well as inflation expectations exhibit a pronounced positive skewness and are leptokurtic. Not surprisingly, the Jarque-Bera test rejects the null hypothesis of normality in every single survey month, as panel 2 of Table 6 shows.

More generally, the probability method requires that beliefs follow some identifiable parametric distribution. Lahiri and Teigland (1987) suggest that a noncentral t distri-

Table 6: Tests for the distribution of quantitative responses

	Perceptions			Expectations		
	Overall	Min	Max	Overall	Min	Max
Skewness	4.04	0.05	8.50	3.64	-0.31	9.31
Kurtosis	40.51	9.96	117.20	43.96	10.85	151.95
Jarque-Bera test for normal distribution						
J-B statistic	6,099.24	771.61	22,361.00	5,837.78	543.04	27,213.38
P-value	0.00	0.00	0.00	0.00	0.00	0.00
Kolmogorov-Smirnov test for noncentral t distribution						
K-S statistic	1.03	0.12	0.41	0.24	0.14	0.37
P-value	0.00	0.00	0.00	0.00	0.00	0.00
μ	0.51	0.10	4.00	0.74	0.20	2.70
df	1.02	1	3	1.01	1	2

Notes: Monthly data, 01/1996–10/2008. *Overall* denotes the mean of monthly statistics, *Min* and *Max* are the monthly minimum and maximum of the respective statistic. The Jarque-Bera statistic is asymptotically χ^2_2 distributed. The approximate 1% critical values for the Kolmogorov-Smirnov are given by $1.52N^{-0.5}$, where N is the number of observations. The noncentral t distribution is defined by the noncentrality parameter μ and the degrees of freedom df. The table shows the parameters that minimize the Kolmogorov-Smirnov statistic.

bution as an alternative to the normal distribution. The noncentral t distribution allows for positive skewness and fat tails. I formally test whether quantitative responses follow a noncentral t distribution employing the Kolmogorov-Smirnov test.[23] Test results are summarized in panel 3 of Table 6 and show that the null hypothesis is rejected in all months.

These formal tests do not answer the question which parametric distribution produces the best results. The answer will also depend on the time period. In 1996–2007, the high share of zero responses cannot be reconciled with both the normal and noncentral t distribution. With the rise in inflation perceptions and expectations in 2008, the shape of the distribution becomes somewhat smoother and less skewed as the share of zero responses declines. In general, however, the results indicate that differences between the commonly considered parametric distributions are predominated by the high share of zero responses, which cannot be accounted for by any of these distributions. This conjecture is in line

---

[23]The Kolmogorov-Smirnov statistic is given by $D_n(F) = \sup_x |F_n(x) - F(x)|$, where $F_n(.)$ is the empirical distribution function. Note that the noncentral t distribution is equal to the t distribution if the noncentrality parameter μ is zero.

with previous literature which finds that the accuracy of the probability method does not significantly vary between any of the common parametric distributions, see Berk (1999), Dasgupta and Lahiri (1992), and Smith and McAleer (1995).

3.4 Defined Reference Rate of Inflation

Assumption 4 requires that the reference rate of inflation π_t^r that people have in mind when opting for the answer that prices are "somewhat/moderately higher" or are expected to "go up at the same rate" is equal to some defined value. This assumption is required to identify the system of equations that is generated by Assumptions 1 to 3. As outlined above, for identifying perceived inflation it is typically assumed that reference inflation is a constant such that perceptions are unbiased. For expected inflation, reference inflation is commonly assumed to be equal to actual inflation or to (previously quantified) perceived inflation.

To assess these assumptions, Figure 2 shows the mean of quantitative responses given by households that opt for qualitative answer S_4. The conditional mean of perceptions is highly volatile with a standard deviation of 1.15%. It averages at 7.93% but shows a declining tendency over time. The assumption that the moderate rate of inflation is constant over time is clearly at odds with this pattern.

The conditional mean given qualitative answer S_4 is less volatile for inflation expectations, with a standard deviation of only 0.53%. Figure 2 includes three candidate series that might correspond to the conditional mean: the mean quantitative inflation perception of those survey participants that expect prices to "go up at the same rate" (S_4), quantified inflation perceptions (based on the 5-category probability method under the unbiasedness assumption), and actual HICP inflation. Clearly, the conditional mean of inflation perceptions closely follows the conditional mean of inflation expectations. The correlation coefficient of the two series is 0.94, the mean difference is only 0.39%. This confirms the result of Section 3.2 that qualitative inflation expectations are linked to quantitative perceptions. Correlation of the conditional mean with quantified inflation perceptions and
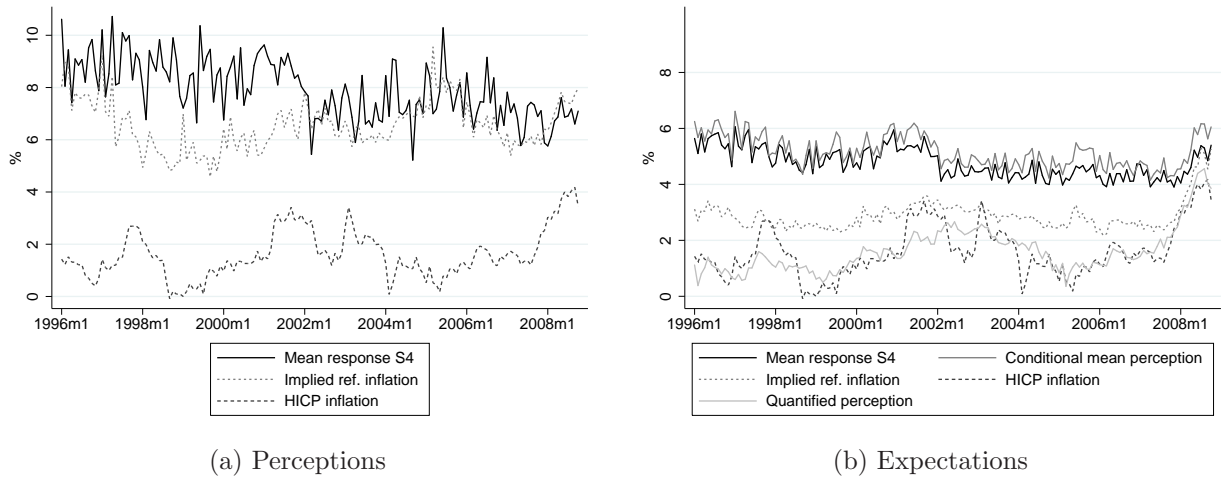
(a) Perceptions   (b) Expectations

Figure 2: Measures of reference inflation π_t^r

Notes: This figure shows alternative measures of π_t^r. *Mean response S4* is the mean quantitative belief of respondents that opt for qualitative answer S_4. *Conditional mean perception* is the mean quantitative inflation perception of respondents that opt for S_4 in the question about expected inflation. Implied reference inflation and quantified inflation perceptions are derived under the 5-category probability method. In quantifying perceptions the HICP unbiasedness condition is imposed.

actual HICP inflation is -0.15 and 0.06 respectively. Both series are nearly uncorrelated with the conditional mean of inflation perceptions. Hence, Assumption 4 that the reference rate of inflation is equal to actual inflation or quantified perceived inflation can be rejected. This of course raises doubt about the usual procedure to employ these measures in quantifying inflation expectations.

An alternative route to test Assumption 4 is to assess the implied level of moderate inflation by joining the mean of actual quantitative beliefs with Assumptions 1 to 3.[24] Comparing the implied level of moderate inflation with the conditional mean given qualitative answer S_4 amounts to a joint test of Assumptions 1 to 3 of the probability method. Figure 2 indicates that for inflation perceptions the implied reference inflation fluctuates around a similar level as the conditional mean. However, the correlation with the conditional mean is only 0.06. For inflation expectations, the implied reference inflation lies

---

[24]Given the mean π_t^e of quantitative inflation expectations, implied reference inflation can be obtained by rearranging Equation (A.4) to $\pi_t^r = \dfrac{\pi_t^e}{\frac{1}{T}\sum_{t=1}^{T}\frac{G_t^4 + G_t^3}{G_t^4 + G_t^3 - G_t^2 - G_t^1}}$.

about 2% below the conditional mean. The correlation of the two series is 0.31. Provided that the true reference rate of inflation is indeed equal to the conditional mean given qualitative answer S_4 these results suggests that Assumptions 1 to 3 can be jointly rejected. In light of these findings the next section assesses the joint validity of all 4 hypotheses in more detail.

3.5   Joint Assessment

While all four hypotheses can be individually rejected, this section investigates the joint validity of the assumptions. The focus does not lie on rejection/non-rejection but rather on the degree of overall validity. I proceed by quantifying the qualitative survey data employing the 5-category probability method.[25] This yields the threshold parameters δ_t and η_t which can be used to construct the implied response scheme on a monthly basis.

Figure 3 shows box plots of the distribution of monthly response shares. For each answer category, the fraction of quantitative beliefs that lie within the implied response interval is compared to the actual share of qualitative beliefs. Note that, by construction, the actual share of qualitative responses corresponds to the the predicted share of quantitative responses under the normality assumption. For inflation perceptions, the figure signals pronounced deviations of implied from actual response fractions in categories S_3 and S_5. The high share of quantitative responses in the implied range of S_5 is consistent with the previous finding that the distribution of responses is asymmetrical and leptokurtic. More importantly, the low fraction of quantitative responses in the implied range of S_3 appears to be a direct consequence from fitting the normal distribution to the high share of zero responses.

A similar pattern can be observed for inflation expectations. Figure 3 illustrates that the deviation of the implied from the actual response share is highest for categories S_3, S_4, and S_5. Similar to perceptions, the fraction of quantitative responses in the implied

---

[25]Inflation perceptions are quantified by imposing the HICP unbiasedness condition. For inflation expectations it is assumed that the reference rate of inflation is equal to quantified inflation perceptions. Detailed derivations are provided in the Appendix.
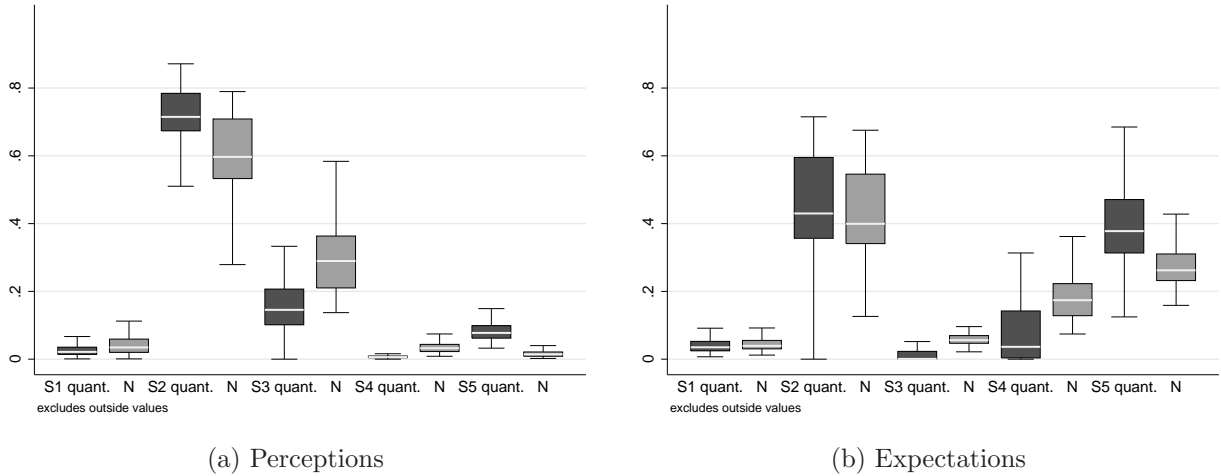
(a) Perceptions

(b) Expectations

Figure 3: Actual and theoretical response fractions

Notes: These figures show the fraction of quantitative answers within the implied response interval (*quant.*) and the actual share of qualitative responses (N). *S1* through *S5* are the qualitative response categories. Sample period 01/1996–10/2008. Perceptions are quantified using the 5-category probability method unbiased with respect to HICP inflation. Expectations are quantified using the 5-category probability method with quantified perceptions as reference inflation. Each box covers the range between the 25th and 75th percentile of monthly fractions and contains a median line. Upper (lower) adjacent values are given by the highest value not greater than the 75th (25th) percentile + (-) 3/2 of the interquartile range.

range of S_5 exceeds the actual share of qualitative responses. This pattern also relates to the finding of Section 3.4 that the mean quantitative answer of respondents opting for qualitative answer S_4 is significantly higher than actual inflation or quantified inflation perceptions. By consequence, a large fraction of these quantitative answers fall into the interval of the qualitative answer S_5.

Further insights can be gained by looking at the fraction of quantitative responses that lie below or above the implied response interval. Figure A.1 in the Appendix shows this fraction, relative to the number of responses in the respective qualitative response category. Both for perceptions and expectations, the figure reveals that the 5-category probability method best accommodates qualitative answer S_2. On average, 99% of quantitative responses associated with qualitative answer S_2 lie within the implied response interval. S_2 also is the most important qualitative response, accounting for roughly 59% of perceptions and 42% of expectations in 1996–2008. Regarding inflation perceptions, coverage for the

23

second most important category S_3, which obtains 30% of responses, is lower. Only about 30% of quantitative responses are within the implied response interval. A relatively large share of quantitative responses lies below the implied response interval, indicating that the interval around 0% is too wide. The worst coverage results for S_4, but only 4% of respondents opt for this qualitative category.

The pattern is different for inflation expectations. Figure A.1 indicates that only about 10% of quantitative beliefs fall into the implied response intervals for S_3 and S_4. Most quantitative responses are above the implied interval. This can be explained by the high share of on average 27% of responses in category S_5. Fitting this share leads to a downward shift of the lower response intervals. Also, Section 3.4 shows that quantified perceptions are significantly lower than reference inflation π^r. Hence, the implied response intervals linked to quantified perceptions will be too low. The resulting distortion might be notable as categories S_3 and S_4 account for 25% of qualitative responses.

The above findings also hold in the 01/2002–10/2008 subperiod, as Figures A.2 and A.3 in the Appendix confirm. In sum, the results suggest that Assumptions 1 through 4 are invalid. This leads to significant distortions primarily concerning the incorporation of information from positive categories S_3, S_4, S_5, which seem more pronounced for inflation expectations than perceptions. The next section assesses the implications for the accuracy of the probability method.

4   Accuracy of the Probability Method

4.1   Level and Dynamics of Beliefs

This section assesses the accuracy of the 5-category probability method relative to the mean of actual quantitative survey responses. Inflation perceptions and expectations are identified by imposing the usual restrictions. Inflation perceptions are assumed to be unbiased with respect to HICP inflation. Reference inflation for quantifying inflation expectations is set equal to HICP inflation or, following Berk (1999), to quantified perceived

inflation.[26] The 5-category probability method is compared to a set of alternative quantification methods. The first alternative is the 3-category probability method of Carlson and Parkin (1975).[27] The second alternative is the scaled balance statistic with mean and variance of actual inflation. In line with the literature, the 5-category balance statistic is computed as $s_5 + 0.5s_4 - 0.5s_2 - s_1$, while the 3-category balance statistic is given by $s_5 + s_4 + s_3 - s_1 = s_t^p - s_t^n$. Here, s_t^p and s_t^n are the fractions of respondents that report that inflation is rising or falling. The third alternative is the Pesaran (1987) regression approach for 3-category response data.[28]

The primary measure of accuracy I consider is the (Pearson) correlation coefficient between the quantified series and the mean quantitative answer. Unlike the mean absolute error (MAE) or root mean squared error (RMSE), the correlation coefficient is robust to a constant scaling of the involved series.[29] In particular, the correlation coefficient is unaffected by the average level of reference inflation π^r. Another advantage of employing the correlation coefficient is that its distributional properties have been explored. The Fisher z-transformation of the correlation coefficient results in an approximately normal random variable, provided the underlying data follows a bivariate normal distribution (Fisher, 1925). Relying on the Fisher z-transformation one can test the null hypothesis that two correlation coefficients are equal using the following statistic:

$$z = \tanh^{-1}(\rho_1) - \tanh^{-1}(\rho_2) \tag{5}$$

where $\tanh^{-1}(\rho_i) = 0.5\ln\left(\frac{1+\rho_i}{1-\rho_i}\right)$. This statistic is approximately normal with variance

---

[26]Perceived inflation is quantified using the 5-category probability method under the assumption of unbiasedness with respect to HICP inflation.

[27]Answer categories are aggregated following Berk (1999), see the Appendix for details.

[28]Unlike the early regression approaches suggested by Theil (1952) and Anderson (1952), the Pesaran (1987) approach allows for asymmetric response behavior in periods of rising and falling inflation. The Pesaran approach is based on nonlinear least squares estimation of the model $\pi_t = \frac{\beta_1 s_t^p - \beta_2 s_t^n}{1 - \beta_3 s_t^p} + \varepsilon_t$, where π_t denotes actual HICP inflation. Expected inflation is generated in a second step as a prediction of this model based on answering fractions about inflation expectations (where coefficient estimates are obtained in the first step using perceptions data). A measure of perceived inflation is computed as the prediction of the model using the perceptions data it has been estimated with.

[29]The squared correlation coefficient is equal to the coefficient of determination in a simple linear regression.

$\frac{1}{T_1-3} + \frac{1}{T_2-3}$. However, the approximation may be inaccurate in the present case because $|\rho_i|$ is high (Mudholkar, 2006) and the underlying series are serially dependent. I therefore assess significance based on double block bootstrap confidence intervals for the z statistic.[30]

Table 7 summarizes the results. The underlying series are plotted in Figures A.4 and A.5 in the Appendix. All statistics are provided for levels and first differences of the series. Since the quantified series and the mean quantitative beliefs are highly persistent the discussion focusses on results for first differences.[31] These results are not subject to spurious regression problems as the first differences are stationary. However, the results on correlation are broadly consistent among levels and first differences because the block bootstrap accounts for serial dependence.

Panel 1 of Table 7 indicates that, in terms of correlation with actual perceptions, all quantification methods perform well. The 5-category probability method and the 3-category method produce virtually identical results. Interestingly however, the simple balance statistics are more accurate, with correlation coefficients of 0.91 and 0.89 in first differences. The z-statistic indicates that the correlation coefficient for the 5-category balance statistic is significantly higher than the correlation coefficient for the 5-category probability method. The regression approach shows a similar performance as the probability methods.

For expectations, panel 2 of Table 7 shows that the accuracy of the 5-category probability method depends on the chosen reference rate of inflation. Employing quantified perceptions generates significantly better results in first differences than employing actual HICP inflation. The correlation coefficients are 0.80 and 0.50 respectively and the z-statistic is highly significant. The most accurate method for quantifying inflation expectations is again a balance statistic. Moreover, the 3-category regression approach is slightly more accurate than the 5-category probability method. These differences are not

---

[30]Matlab codes are available from the author. The double moving block bootstrap of the percentile confidence interval is based on 1,000 first level replications and 2,500 second level replications and a block size of 5. See Efron and Tibshirani (1993) for a description of the method.

[31]Employing an Augmented Dickey-Fuller test the null hypothesis of a unit root cannot be rejected on the 10% level for all actual and quantified mean series.

Table 7: Accuracy of quantified inflation perceptions and expectations

Perceptions	Level					First differences				
	Bias	MAE	RMSE	ρ	z	Bias	MAE	RMSE	ρ	z
P.HICP (5 cat.)	-0.17	0.22	0.32	0.97		0.00	0.12	0.15	0.86	
P.HICP (3 cat.)	-0.17	0.19	0.24	0.98	-0.24	0.00	0.11	0.15	0.83	0.09
Balance (5 cat.)	-0.20	0.20	0.25	0.99	-0.51*	0.00	0.11	0.13	0.91	-0.22**
Balance (3 cat.)	-0.20	0.22	0.31	0.97	0.04	0.00	0.10	0.14	0.89	-0.12
Pesaran (3 cat.)	-0.21	0.25	0.47	0.94	0.44	0.00	0.12	0.16	0.85	0.02

Expectations	Level					First differences				
	Bias	MAE	RMSE	ρ	z	Bias	MAE	RMSE	ρ	z
P.Perc. (5 cat.)	-0.85	0.85	0.89	0.94		0.00	0.13	0.17	0.80	
P.HICP (5 cat.)	-0.87	0.87	0.96	0.86	0.47	0.00	0.23	0.30	0.50	0.55**
P.HICP (3 cat.)	-0.49	0.49	0.24	0.95	-0.06	0.00	0.13	0.15	0.82	-0.05
Balance (5 cat.)	-0.49	0.56	0.25	0.89	0.35	0.00	0.17	0.13	0.82	-0.06
Balance (3 cat.)	-0.49	0.51	0.31	0.96	-0.18	0.00	0.14	0.14	0.84	-0.13
Pesaran (3 cat.)	0.12	0.22	0.29	0.95	-0.08	0.00	0.13	0.18	0.81	-0.01

Notes: Accuracy is measured relative to the mean of quantitative responses, 01/1996–10/2008. For perceptions, *P.HICP (5 cat.)* and *P.HICP (3 cat.)* denote the 5-category and 3-category probability method under the condition that perceptions are unbiased with respect to HICP inflation. For expectations, *P.Perc. (5 cat.)* and *P.HICP (5 cat.)* are the 5-category probability methods with reference inflation given by quantified perceptions and HICP inflation, respectively. *P.HICP (3 cat.)* is the 3-category probability method under the condition that expectations are unbiased. ρ is the correlation coefficient with the mean of quantitative responses. The last column in each panel shows the Fisher z statistic for testing the null hypothesis that the difference between the correlation coefficient in the first row of each panel (ρ_1) and the correlation coefficient in the corresponding row (ρ_2) is zero. *, **, and *** indicate statistical significance of z at the 10%, 5%, and 1% level based on double block bootstrap percentile confidence intervals.

27

statistically significant, however. Results for the subsample 01/2002–10/2008 in Table A.1 confirm these findings.

In sum, the 5-category balance statistic tracks actual quantitative perceptions most accurately. Still, the 5-category probability method as well as the other methods are highly correlated with mean quantitative perceptions. For expectations, none of the alternative methods performs significantly better than the 5-category probability method with reference inflation given by quantified perceptions. The reasonable performance of the probability method is in contrast to findings of Batchelor (1986) for the U.S.[32] However, the 5-category probability method may perform weakly to quantify expectations, depending on the chosen reference rate of inflation. Also, the similar accuracy of the 5-category probability method and the 3-category methods signals that the 5-category method is not fully efficient at incorporating information from positive answer categories.

4.2 Dispersion of Beliefs

The cross-sectional heterogeneity of beliefs is subject to increasing research in macroeconomics. This section investigates how to best infer cross-sectional heterogeneity, measured by the standard deviation of quantitative beliefs, from qualitative survey data. The 5-category probability method not only allows to identify the mean but also the implied standard deviation of the fitted normal distribution given by Equation (A.5). In addition, I consider four alternative measures of heterogeneity. The first alternative is implied standard deviation from the 3-category probability method given by Equation (A.11). The second alternative is an index of qualitative variation (IQV) based on the response shares s_1 through s_5:

$$IQV \;=\; \frac{K}{K-1}\left(1 - \sum_{i=1}^{K} s_i^2\right)$$

---

[32]Batchelor (1986) documents that the quantified series do not predict the direction of change in mean quantitative responses. In the present case, a comparison of signs confirms the high correlation in first differences. For inflation perceptions, the balance statistic and the probability method indicate the correct direction of change of the mean quantitative response in 131 and 129 out of 153 months, for expectations in 121 and 120 months.

28

where $K = 5$ is the number of response categories and s_i the fraction of answers in category i. The scaling factor $\frac{K}{K-1}$ ensures that $0 \leq q^2 \leq 1$. Unlike the probability method, the IQV does not account for the ordered nature of the data. The third alternative is the d^2-index of ordinal variation proposed by Lacy (2006).[33] This index is given by:

$$DSQ \;\; = \;\; \sum_{i=1}^{K-1} F_i(1 - F_i)$$

where $K = 5$ is the number of response categories and F_i the cumulative response share in category i, e.g., $F_3 = s_1 + s_2 + s_3$. As the IQV, the DSQ statistic attains its minimum of 0 if all answers lie in the same response category. But while the IQV is maximal when answers are uniformly distributed, the DSQ attains its maximum of 1 if the distribution is polarized, i.e. if $s_1 = s5 = 0.5$. The fourth alternative is the disconformity index of Theil (1955) defined as $DIS = s_p + s_n - (s_p - s_n)^2$.[34]

Table 8 summarizes the results on accuracy. Since the null hypothesis of a unit-root process can be rejected for the standard deviation of quantitative responses the discussion centers on results in levels.[35] The alternative measures of dispersion are plotted in Figures A.6 and A.7. Table 8 shows that both the 5-category probability method and the 3-category probability method considerably underestimate the heterogeneity of quantitative beliefs. The implied standard deviation lies 1.7% to 2.4% below the actual standard deviation of quantitative responses. This finding is consistent with earlier results of Defris and Williams (1979) and Batchelor (1986).

Regarding inflation perceptions, the first panel of Table 8 shows that implied standard deviation from the 5-category probability method traces actual heterogeneity only poorly. The correlation coefficient is 0.30. The 3-category probability method performs significantly better, as the z-statistic indicates. However, the qualitative measures of variation

---

[33]Lacy (2006) builds on earlier work of Blair and Lacy (1996, 2000).

[34]The disconformity index relies on the same theoretical assumptions as the 3-category balance statistic, see Batchelor (1986).

[35]Both for perceptions and expectations, the Augmented Dickey-Fuller test rejects the null hypothesis of a unit root for the standard deviation of actual quantitative responses and for the quantified series using the 5-category probability method linked to HICP inflation.

Table 8: Accuracy of measures of cross-sectional dispersion

Perceptions	Level					First differences				
	Bias	MAE	RMSE	ρ	z	Bias	MAE	RMSE	ρ	z
Implied SD (P.HICP, 5 cat.)	-1.76	1.76	1.83	0.30		0.00	0.29	0.36	0.31	
Implied SD (P.HICP, 3 cat.)	-1.83	1.83	1.88	0.68	-0.52*	0.00	0.28	0.36	0.36	-0.06
IQV (5 cat.)				0.83	-0.90**				0.48	-0.19**
DSQ (5 cat.)				0.87	-1.01**				0.47	-0.19**
DIS (3 cat.)				0.35	-0.05				0.30	0.01

Expectations	Level					First differences				
	Bias	MAE	RMSE	ρ	z	Bias	MAE	RMSE	ρ	z
Implied SD (P.Perc., 5 cat.)	-2.42	2.42	2.45	0.67		0.00	0.27	0.33	0.40	
Implied SD (P.HICP, 5 cat.)	-2.44	2.44	2.50	0.52	0.24**	0.00	0.33	0.42	0.20	0.22**
Implied SD (P.HICP, 3 cat.)	-1.95	1.95	1.98	0.71	-0.07	0.00	0.28	0.34	0.33	0.07
IQV (5 cat.)				0.80	-0.29**				0.28	0.13
DSQ (5 cat.)				0.45	0.32				0.18	0.24**
DIS (3 cat.)				0.30	0.49**				0.14	0.27**

Notes: Accuracy is measured relative to the cross-sectional standard deviation of quantitative responses, 01/1996–10/2008. For perceptions, *Implied SD (P.HICP, 5 cat.)* is the implied standard deviation from the 5-category probability method under the HICP unbiasedness condition. For expectations, *Implied SD (P.Perc., 5 cat.)* and *Implied SD (P.HICP, 5 cat.)* are implied standard deviations from the 5-category probability method with reference inflation given by quantified perceptions and actual HICP inflation, respectively. Both for perceptions and expectations, *Implied SD (P.HICP, 3 cat.)* is the implied standard deviation from the 3-category probability method under the HICP unbiasedness condition. *IQV* is the index of qualitative variation, *DSQ* is the d^2-index of ordinal variation proposed by Lacy (2006), and *DIS* is the disconformity statistic. ρ is the correlation coefficient with the standard deviation of quantitative responses. The last column in each panel shows the Fisher z statistic for testing the null hypothesis that the difference between the correlation coefficient in the first row of each panel (ρ_1) and the correlation coefficient in the corresponding row (ρ_2) is zero. *, **, and *** indicate statistical significance of z at the 10%, 5%, and 1% level based on double block bootstrap percentile confidence intervals.

are even more highly correlated with the standard deviation of quantitative beliefs. The correlation coefficients of the IQV and the DSQ are 0.83 and 0.87. The performance of the 3-category disconformity index is substantially lower, its correlation with actual standard deviation of quantitative responses is similar to the 5-category probability method.

The implied standard deviation from the 5-category probability method performs better for quantifying heterogeneity of inflation expectations, as the second panel of Table 8 indicates. Again, the correlation depends on the choice of the reference inflation. Employing actual HICP inflation instead of quantified perceptions reduces the correlation coefficient significantly from 0.67 to 0.52. The implied standard deviation from the 3-category approach is about as accurate as the 5-category probability method with quantified perceptions as reference inflation. The IQV most closely tracks actual heterogeneity of quantitative responses. The correlation with actual standard deviation is 0.80, which is significantly higher than the correlation of the 5-category probability method. Unlike for perceptions, the DSQ-statistic performs substantially worse with a correlation of only 0.45. The disconformity index is also only weakly correlated with actual heterogeneity, which is in line with earlier findings of Batchelor (1986). The results for first differences are broadly consistent, although the stationary series are over-differentiated. Moreover, results for the subsample 01/2002–10/2008 in Table A.2 confirm the above findings.[36]

In sum, these results suggest that while the probability method is relatively accurate in describing the central tendency, it is considerably less accurate in capturing the level and dynamics of cross-sectional heterogeneity. Consistent with findings of the previous section, the 3-category probability method performs better than the 5-category method.[37] The IQV, however, dominates the other methods in terms of correlation with the cross-sectional standard deviation of quantitative beliefs. The DSQ is only accurate for quantifying the

---

[36]Note, however, that the probability method gains relative accuracy for quantifying inflation expectations. I have also assessed accuracy of the square root of the index of qualitative variation, the square root of the DSQ-statistic, and the square root of the disconformity index. The correlations with actual standard deviation of survey responses do not significantly change, both in their absolute level and relative ordering.

[37]The reasonable performance of the 3-category probability method also reinforces the results of Dasgupta and Lahiri (1993) who show that implied dispersion from the 3-category method is useful for predicting business cycle turning points.

heterogeneity of inflation perceptions. This suggests that the IQV is less distorted by the unordered qualitative answers on expected inflation than the DSQ. Finally, the assessment of accuracy relies on correlation coefficients. Bias, MAE, and RMSE seem less informative since units for qualitative indices are not directly. Moreover, these measures are driven by the identifying assumptions.

5   Which Quantification Method Should Be Used?

The assumptions of the 5-category probability method can be individually and jointly rejected, as Section 3 shows. Moreover, the response scheme estimates indicate that three separate positive categories S_3, S_4, S_5 contain additional information over just one positive category. This is the case for inflation perceptions as well as for inflation expectations. For inflation expectations, however, the qualitative answers are not strictly increasing in quantitative expectations since positive responses are given relative to perceived inflation.

Despite that the theoretical assumptions are violated, the accuracy of the 5-category probability method for quantifying the mean of inflation perceptions is high. The relative performance of the 3-category probability method indicates, however, that the 5-category method does not efficiently incorporate the additional information from three positive response categories. The most accurate method is the 5-category balance statistic.

Regarding the mean of inflation expectations, the accuracy of the 5-category probability method largely depends on the chosen reference inflation π_t^r. Employing quantified inflation perceptions yields significantly more accurate results than employing actual HICP inflation. This appears to be inconsistent with the finding of Section 3.4 that quantified perceptions are uncorrelated with reference inflation. But as Figure 2 indicates, the relation between quantified perceptions and reference inflation becomes stronger towards the end of the sample period. The correlation coefficient of reference inflation and quantified perceptions is 0.46 in 2002–2008.[38] In particular, quantified perceptions surge in 2007 and 2008, while reference inflation increases to about 6%. This suggests that the 5-category

---

[38] As in Section 3.4 reference inflation is measured by the conditional mean of quantitative perceptions.

Table 9: Actual and imposed conditional means

	Perceptions			Expectations		
	Actual	Balance	P.HICP	Actual	Balance	P.Perc.
Mean response, given S_1	-4.79	-3.96	-4.94	-3.89	-2.87	-3.60
Mean response, given S_2	0.03	-0.23	0.37	0.02	-0.54	0.37
Mean response, given S_3	4.72	3.50	4.62	4.06	1.79	2.40
Mean response, given S_4	7.94	7.22	7.70	4.77	4.12	3.53
Mean response, given S_5	9.58	10.95	9.75	3.98	6.45	6.25

Notes: The column *Actual* presents results for actual quantitative survey responses, 01/1996–10/2008. The column *Balance* shows the mean values that the balance statistic attributes to qualitative survey categories. *P.HICP* denotes implied conditional means of the 5-category probability method unbiased with respect to HICP inflation. *P.Perc.* are the implied conditional means of the 5-category probability method with reference inflation given by quantified perceptions. The conditional means are scaled to match the mean and standard deviation of actual conditional means.

probability method with quantified perceptions as reference inflation gains relative accuracy once inflation perceptions diverge from actual inflation.[39]

It remains the question why the simple balance statistic traces mean beliefs at least as closely as the 5-category probability method. Table 9 provides a possible explanation. The table shows the mean of quantitative responses conditional on the qualitative response. Conditional means of actual quantitative responses are compared to the imposed conditional means of the balance statistic and of the 5-category probability method. The imposed conditional means are scaled to match the mean and variance of actual conditional means.[40] Table 9 reveals that both methods impose (scaled) conditional means that match actual conditional means quite closely. In other words, the conditional means imposed by the balance statistic are roughly proportional to the actual conditional means in the data. Not surprisingly, the fit is better for perceptions than for expectations. This also suggests that the balance statistic, unlike the probability method, will loose relative accuracy once the ratio of conditional means changes.

---

[39]This is the case, e.g., around the euro cash changeover in 2002 (ECB, 2005).

[40]The (unscaled) conditional means of the 5-category balance statistic are -1, 0.5, 0, 0.5, 1. The implied conditional means of the probability method are computed by numerical integration using the quantified parameters π_t^e, σ_t, δ_t, η_t, π_t^r. Perceptions are quantified employing the 5-category probability method unbiased with respect to HICP inflation. Expectations are quantified using the 5-category probability method with reference inflation given by quantified perceptions.

33

The accuracy of the 5-category probability method is low for quantifying the cross-sectional standard deviation of beliefs. Here, an index of qualitative variation is preferable. The index of qualitative variation also dominates the DSQ-index of ordinal variation for quantifying the heterogeneity of expectations. Similar to the findings on the choice of reference inflation, the IQV seems less distorted by the unordered qualitative answers than the DSQ.

In sum, these results are in favor of the 5-category probability method for quantifying the mean of beliefs. The index of qualitative variation is preferable for quantifying the cross-sectional standard deviation of beliefs. The findings also indicate, that the 5-category probability method with quantified perceptions as reference inflation might gain relative accuracy if inflation perceptions deviate from actual inflation. This, however, points to a general limitation of this study's results: findings on the quantification of expectations might differ from other countries as Swedish consumers are relatively well informed about actual HICP inflation.

6 Conclusion

This paper assesses the validity and accuracy of the 5-category probability method for quantifying inflation perceptions and expectations. The analysis capitalizes on jointly available qualitative and quantitative response data from the Swedish Consumer Tendency Survey. Relying on monthly data on household level, 01/1996–10/2008, the theoretical assumptions of the 5-category probability method are individually and jointly tested and rejected. Maximum likelihood estimations of unrestricted response schemes indicate that the actual response scheme is neither symmetric nor homogeneous across individuals. Moreover, it is shown that qualitative inflation expectations are formed relative to inflation perceptions, which is a direct result of the survey design. By consequence, qualitative inflation expectations are not monotonously rising in quantitative inflation expectations. Furthermore, quantitative beliefs are not normally distributed and cannot be reconciled with a noncentral t distribution either. Finally, the reference rate of inflation (the "moderate"

rate of inflation) is shown to be time varying for perceptions and significantly different from actual inflation for expectations. The joint assessment indicates that, by consequence, the 5-category method does not adequately accommodate the positive qualitative response categories S_3, S_4, S_5.

Nevertheless, the accuracy of the 5-category probability method in terms of correlation with the mean of actual quantitative beliefs is high. For quantifying inflation expectations the accuracy of the method strongly depends on the identifying restriction imposed by the choice of reference inflation. Relying on double block bootstrap confidence intervals for Fisher's z statistic it is shown that setting reference inflation equal to previously quantified inflation perceptions yields significantly better results than setting reference inflation equal to actual HICP inflation. However, the 5-category probability method is not more accurate than the balance statistic and the 3-category probability method. This suggests that the 5-category probability method does not efficiently incorporate information from positive qualitative response categories. In sum, however, the results are in favor of the 5-category probability method. In particular, the 5-category probability method with quantified perceptions as reference inflation might gain relative accuracy once inflation perceptions substantially deviate from actual inflation. The most accurate measure of the cross-sectional standard deviation of quantitative beliefs is the index of qualitative variation. This index performs significantly better than the 5-category probability method and other common approaches.

The findings for Sweden suggest a number of avenues for further research. To exploit the additional information from three positive response categories research is needed on how to choose the reference rate of inflation. Moreover, a non-parametric analysis will generate further insights that will help to improve the imposed response scheme and distribution. Looking ahead, implications for survey design should also be discussed in more depth. The results for Sweden indicate that it is difficult to efficiently handle the relative nature of the positive qualitative responses about inflation expectations. Obvious alternative survey designs include to adopt the same response scheme for expectations as currently in use for perceptions or to directly ask for quantitative responses in the first place.

References

AALTO-SETÄLÄ, V. (2006): "Why Do Consumers Believe that the Euro Raised Prices?," *International Journal of Consumer Studies*, 30(2), 202–206.

ANDERSON, O. (1952): "The Business Test of the IFO-Institute for Economic Research Munich, and its Theoretical Model," *Revue de l'Institut International de Statistique*, 20, 1–17.

AUCREMANNE, L., M. COLLIN, AND T. STRAGIER (2007): "Assessing the Gap Between Observed and Perceived Inflation in the Euro Area: Is the Credibility of the HICP at Stake?," *National Bank of Belgium Working Paper No. 112*.

BATCHELOR, R. A. (1982): "Aggregate Expectations Under the Stable Laws," *Journal of Econometrics*, 16(2), 199–210.

——— (1986): "Quantitative v. Qualitative Measures of Inflation Expectations," *Oxford Bulletin of Economics & Statistics*, 48(2), 99–120.

BATCHELOR, R. A., AND P. DUA (1987): "The Accuracy and Rationality of UK Inflation Expectations: Some Quantitative Evidence," *Applied Economics*, 19(6), 819–828.

BATCHELOR, R. A., AND A. B. ORR (1988): "Inflation Expectations Revisited," *Economica*, 55(219), 317–331.

BERK, J. M. (1999): "Measuring Inflation Expectations: A Survey Data Approach," *Applied Economics*, 31(11), 1467–1480.

BLAIR, J., AND M. G. LACY (1996): "Measures of Variation for Ordinal Data as Functions of the Cumulative Distribution," *Perceptual and Motor Skills*, 82(2), 411–418.

——— (2000): "Statistics of Ordinal Variation," *Sociological Methods & Research*, 28(3), 251–280.

BRYAN, M. F., AND S. PALMQVIST (2006): "Testing Near-Rationality Using Detailed Survey Data," *Sveriges Riksbank Research Paper Series No. 20.*

BRYAN, M. F., AND G. VENKATU (2001a): "The Curiously Different Inflation Perspectives of Men and Women," *Federal Reserve Bank of Cleveland Economic Commentary Series.*

——— (2001b): "The Demographics of Inflation Opinion Surveys," *Federal Reserve Bank of Cleveland Economic Commentary Series.*

CARLSON, J. A. (1975): "Are Price Expectations Normally Distributed?," *Journal of the American Statistical Association*, 352, 749–754.

CARLSON, J. A., AND M. PARKIN (1975): "Inflation Expectations," *Economica*, 42(166), 123–138.

DASGUPTA, S., AND K. LAHIRI (1992): "A Comparative Study of Alternative Methods of Quantifying Qualitative Survey Responses Using NAPM Data," *Journal of Business and Economic Statistics*, 10(4), 391–400.

——— (1993): "On the Use of Dispersion Measures from NAPM Surveys in Business Cycle Forecasting," *Journal of Forecasting*, 12(3-4), 239–253.

DEFRIS, L. V., AND R. A. WILLIAMS (1979): "Quantitative Versus Qualitative Measures of Price Expectations: The Evidence from Australian Consumer Surveys," *Economics Letters*, 2(2), 169–173.

DOEHRING, B., AND A. MORDONU (2007): "What Drives Inflation Perceptions? A Dynamic Panel Data Analysis," European Commission Economic Papers 284.

DÖPKE, J., J. DOVERN, U. FRITSCHE, AND J. SLACALEK (2008): "The Dynamics of European Inflation Expectations," *The B.E. Journal of Macroeconomics*, 8(1).

DRIVER, R., AND R. WINDRAM (2007): "Public Attitudes to Inflation and Interest Rates," *Bank of England Quarterly Bulletin*, 47(2), 208–223.

DZIUDA, W., AND G. MASTROBUONI (2006): "The Euro Changeover and its Effects on Price Transparency and Inflation," Working Paper 26, Collegio Carlo Alberto.

ECB (2005): "Consumers' Inflation Perceptions: Still at Odds With Official Statistics?," *ECB Monthly Bulletin*, (4), 30–32.

EFRON, B., AND R. J. TIBSHIRANI (1993): *An Introduction to the Bootstrap.* Chapman & Hall.

EHRMANN, M. (2006): "Rational Inattention, Inflation Developments and Perceptions After the Euro Cash Changeover," Working Paper Series 588, European Central Bank.

EUROPEAN COMMISSION (2007): *The Joint Harmonised EU Programme of Business and Consumer Surveys, User Guide,* European Commission, Directorate-General for Economic and Financial Affairs.

FISHER, R. A. (1925): *Statistical Methods for Research Workers.* Oliver and Boyd.

FLUCH, M., AND H. STIX (2007): "The Development of Euro Prices Subjective Perception and Empirical Facts," *Monetary Policy and the Economy (OeNB)*, 07(1), 55–84.

FORSELLS, M., AND G. KENNY (2004): "Survey Expectations, Rationality and the Dynamics of Euro Area Inflation," *Journal of Business Cycle Measurement and Analysis*, 1(1), 13–41.

GFK (2002): *Consumer Tendency Survey,* GfK Sweden, Stockholm, Sweden.

HENZEL, S., AND T. WOLLMERSHÄUSER (2005): "Quantifying Inflation Expectations with the Carlson-Parkin Method: A Survey-based Determination of the Just Noticeable Difference," *Journal of Business Cycle Measurement and Analysis*, 2(3).

JONUNG, L., AND D. LAIDLER (1988): "Are Perceptions of Inflation Rational? Some Evidence for Sweden," *The American Economic Review*, 78(5), 1080–1087.

LACY, M. G. (2006): "An Explained Variation Measure for Ordinal Response Models With Comparisons to Other Ordinal R^2 Measures," *Sociological Methods & Research*, 34(4), 469–520.

LAHIRI, K., AND C. TEIGLAND (1987): "On the Normality of Probability Distributions of Inflation and GNP Forecasts," *International Journal of Forecasting*, 3(2), 269–279.

LAMLA, M. J., AND S. M. LEIN (2008): "The Role of Media for Consumers' Inflation Expectation Formation," KOF Working Papers 201, ETH Zurich.

LEIN, S. M., AND T. MAAG (2008): "The Formation of Inflation Perceptions - Some Empirical Facts for European Countries," KOF Working Papers 204, ETH Zurich.

MUDHOLKAR, G. S. (2006): "Fisher's z-Transformation," in *Encyclopedia of Statistical Sciences*, ed. by S. Kotz, C. B. Read, N. Balakrishnan, and B. Vidakovic. Wiley.

NARDO, M. (2003): "The Quantification of Qualitative Survey Data: A Critical Assessment," *Journal of Economic Surveys*, 17, 645–668.

PALMQVIST, S., AND L. STRÖMBERG (2004): "Households Inflation Opinions - A Tale of Two Surveys," *Sveriges Riksbank Economic Review*, 4, 23–42.

PESARAN, M. H. (1987): *The Limits to Rational Expectations*. Blackwell.

SMITH, J., AND M. MCALEER (1995): "Alternative Procedures for Converting Qualitative Response Data to Quantitative Expectations: An Application to Australian Manufacturing," *Journal of Applied Econometrics*, 10(2), 165–185.

THEIL, H. (1952): "On the Shape of Microvariables and the Munich Business Test," *Revue de l'Institut International de Statistique*, 20, 105–120.

——— (1955): "Recent Experiences with the Munich Business Test: An Expository Article," *Econometrica*, 23(2), 184–192.

Zavoina, W., and R. D. McKelvey (1975): "A Statistical Model for the Analysis of Ordinal Level Dependent Variables," *Journal of Mathematical Sociology*, 4, 103–120.

Appendix

A.1 Derivation of the 5-Category Probability Method

This section derives the 5 category probability method based on the assumptions introduced in Section 3. The method has been originally proposed by Batchelor and Orr (1988) and relies on earlier contributions of Theil (1952) and Carlson and Parkin (1975). Again all derivations equally hold for quantifying inflation perceptions as well as inflation expectations. The response scheme for inflation perceptions is given by:

$$
\begin{aligned}
\pi_{it}^p < -\delta_t &\quad : \quad \textit{prices in general are } \text{lower } (S_1) \\
-\delta_t \leq \pi_{it}^p < \delta_t &\quad : \quad \text{about the same } (S_2) \\
\delta_t \leq \pi_{it}^p < \pi_t^r - \eta_t &\quad : \quad \text{a little higher } (S_3) \\
\pi_t^r - \eta_t \leq \pi_{it}^p < \pi_t^r + \eta_t &\quad : \quad \text{moderately higher } (S_4) \\
\pi_{it}^p \geq \pi_t^r + \eta_t &\quad : \quad \text{a lot higher } (S_5)
\end{aligned}
\tag{A.1}
$$

Under the assumptions introduced in Section 3 the relation between aggregate response shares and expected inflation π_t^e in period t is given by:

$$
\begin{aligned}
s_t^1 &= P(\pi_{it}^e < -\delta_t) = \Phi\left(\frac{-\delta_t - \pi_t^e}{\sigma_t}\right) \\
s_t^2 &= P(-\delta_t \leq \pi_{it}^e < \delta_t) = \Phi\left(\frac{\delta_t - \pi_t^e}{\sigma_t}\right) - \Phi\left(\frac{-\delta_t - \pi_t^e}{\sigma_t}\right) \\
s_t^3 &= P(\delta_t \leq \pi_{it}^e < \pi_t^r - \eta_t) = \Phi\left(\frac{\pi_t^r - \eta_t - \pi_t^e}{\sigma_t}\right) - \Phi\left(\frac{\delta_t - \pi_t^e}{\sigma_t}\right) \\
s_t^4 &= P(\pi_{it}^r - \eta_t \leq \pi_{it}^e < \pi_t^r + \eta_t) = \Phi\left(\frac{\pi_t^r + \eta_t - \pi_t^e}{\sigma_t}\right) - \Phi\left(\frac{\pi_t^r - \eta_t - \pi_t^e}{\sigma_t}\right) \\
s_t^5 &= P(\pi_{it}^r + \eta_t \leq \pi_{it}^e) = 1 - \Phi\left(\frac{\pi_t^r + \eta_t - \pi_t^e}{\sigma_t}\right)
\end{aligned}
\tag{A.2}
$$

where $\Phi(.)$ is the standard normal cumulative distribution function. The same holds for inflation perceptions, in which case π_t^e is substituted with π_t^p. System (A.2) can be rewritten

to get a system of 4 linearly independent equations with 5 unknowns $(\pi_t^e, \sigma_t, \delta_t, \eta_t, \pi_t^r)$:

$$
\begin{aligned}
G_t^1 &= \Phi^{-1}\left(s_t^1\right) = \frac{-\delta_t - \pi_t^e}{\sigma_t} \\
G_t^2 &= \Phi^{-1}\left(1 - s_t^5 - s_t^4 - s_t^3 - s_t^2\right) = \frac{-\delta_t - \pi_t^e}{\sigma_t} \\
G_t^3 &= \Phi^{-1}\left(1 - s_t^5 - s_t^4 - s_t^3\right) = \frac{\delta_t - \pi_t^e}{\sigma_t} \\
G_t^4 &= \Phi^{-1}\left(1 - s_t^5 - s_t^4\right) = \frac{\pi_t^r - \eta_t - \pi_t^e}{\sigma_t} \\
G_t^5 &= \Phi^{-1}\left(1 - s_t^5\right) = \frac{\pi_t^r + \eta_t - \pi_t^e}{\sigma_t}
\end{aligned}
\tag{A.3}
$$

System (A.3) can be solved to get the following expression for the mean π_t^e of expected inflation:

$$
\pi_t^e = \pi_t^r \frac{G_t^2 + G_t^3}{G_t^2 + G_t^3 - G_t^4 - G_t^5}
\tag{A.4}
$$

For simplicity π_t^e is called expected inflation (rather than mean of expected inflation). The remaining unknowns are given by:

$$
\sigma_t = \pi_t^r \frac{-2}{G_t^2 + G_t^3 - G_t^4 - G_t^5}
\tag{A.5}
$$

$$
\delta_t = \pi_t^r \frac{G_t^2 - G_t^3}{G_t^2 + G_t^3 - G_t^4 - G_t^5}
\tag{A.6}
$$

$$
\eta_t = \pi_t^r \frac{G_t^4 - G_t^5}{G_t^2 + G_t^3 - G_t^4 - G_t^5}
\tag{A.7}
$$

For quantifying inflation perceptions, π_t^r is commonly identified by restricting inflation perceptions to be unbiased over the sample period. Rearranging Equation (A.4) and imposing unbiasedness yields:

$$
\pi_t^r = \frac{\overline{\pi}}{\frac{1}{T}\sum_{t=1}^{T} \frac{G_t^2 + G_t^3}{G_t^2 + G_t^3 - G_t^4 - G_t^5}}
\tag{A.8}
$$

where T is the number of periods and $\overline{\pi}$ the average actual rate of inflation.

42

A.2 Derivation of the 3-Category Probability Method

Theil (1952) and Carlson and Parkin (1975) have originally developed the probability method for three-option ordinal scales. In line with Berk (1999), the EU consumer survey responses are aggregated to three categories by defining $s_n = s_1$, $s_s = s_2$, and $s_p = s_3 + s_4 + s_5$. The relation between response shares and expected inflation is then given by:

$$
\begin{aligned}
s_t^n &= P(\pi_{it}^e < -\delta) = \Phi\left(\frac{-\delta - \pi_t^e}{\sigma_t}\right) \\
s_t^s &= P(-\delta < \pi_{it}^e < \delta) = \Phi\left(\frac{\delta - \pi_t^e}{\sigma_t}\right) - \Phi\left(\frac{-\delta - \pi_t^e}{\sigma_t}\right) \\
s_t^p &= P(\delta < \pi_{it}^e) = 1 - \Phi\left(\frac{\delta - \pi_t^e}{\sigma_t}\right)
\end{aligned}
\tag{A.9}
$$

where $\Phi(.)$ is a standard normal cumulative distribution function. Hence at any t we have a system of 2 linearly independent equations with 3 unknowns $(\delta, \mu_t, \sigma_t)$. Solving yields:

$$
\pi_t^e = \delta \frac{\Phi^{-1}(s_t^n) + \Phi^{-1}(1 - s_t^p)}{\Phi^{-1}(s_t^n) - \Phi^{-1}(1 - s_t^p)}
\tag{A.10}
$$

$$
\sigma_t = \delta \frac{2}{\Phi^{-1}(1 - s_t^p) - \Phi^{-1}(s_t^n)}
\tag{A.11}
$$

Again the identical equations hold for perceived inflation π_t^p. I follow the existing literature and identify the system by imposing an unbiasedness assumption. δ can be recovered by equating the mean of expected inflation (perceived inflation) to average actual inflation in the sample period:

$$
\delta = \frac{\overline{\pi}}{\frac{1}{T}\sum_{t=1}^{T}\left[\frac{\Phi^{-1}(s_t^n) + \Phi^{-1}(1 - s_t^p)}{\Phi^{-1}(s_t^n) - \Phi^{-1}(1 - s_t^p)}\right]}
\tag{A.12}
$$

A.3 Additional Results

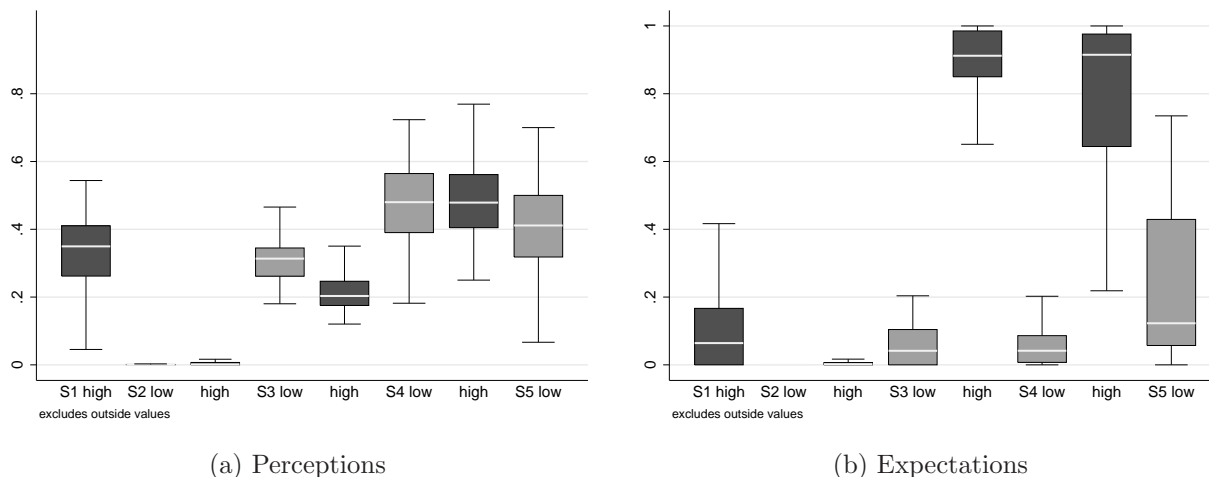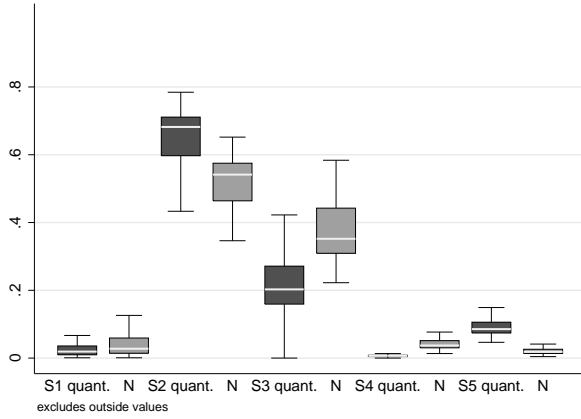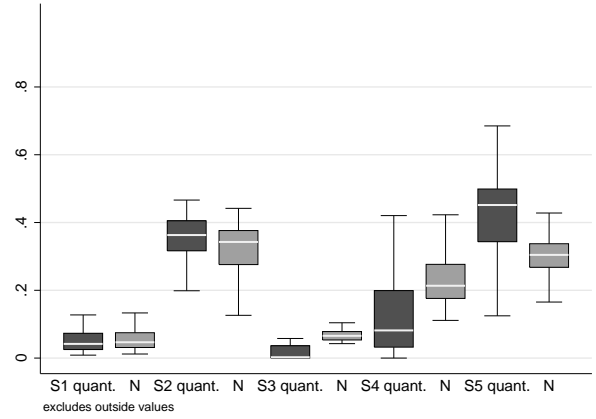(a) Perceptions

(b) Expectations

Figure A.1: Response fractions below (low) and above (high) the implied response interval

Notes: These figures show the fraction of quantitative responses that lie below or above the implied response interval (defined by the simultaneous qualitative response) relative to the sum of responses in the respective qualitative response category. Sample period 01/1996–10/2008. Quantified perceptions are based on the 5 category probability method unbiased with respect to HICP inflation. Expectations are quantified using the probability method with quantified perceptions as reference inflation. Each box covers the range between the 25th and 75th percentile of monthly fractions and contains a median line.
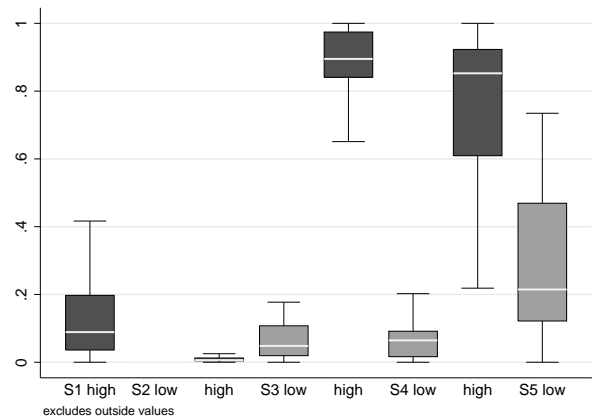
(a) Perceptions

(b) Expectations

Figure A.2: Actual and theoretical response fractions, 01/2002–10/2008

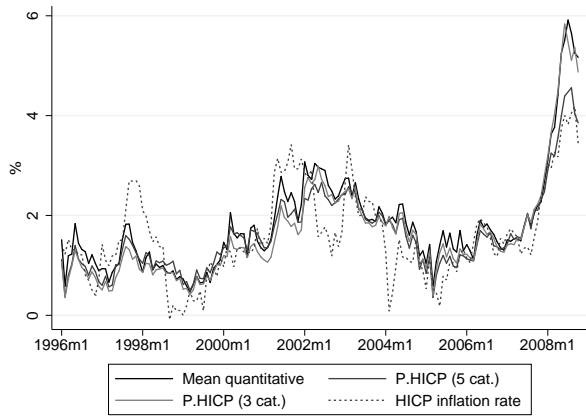*Notes:* See footnote of Figure 3 for a detailed description.
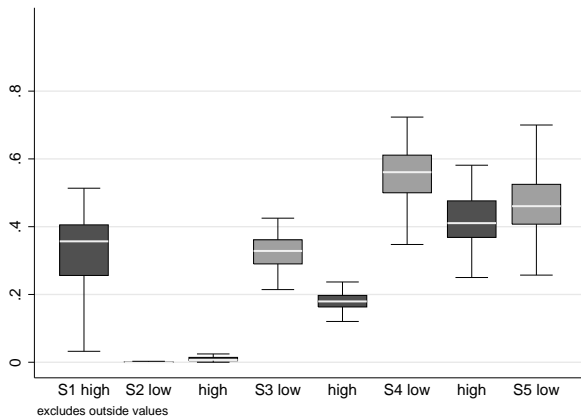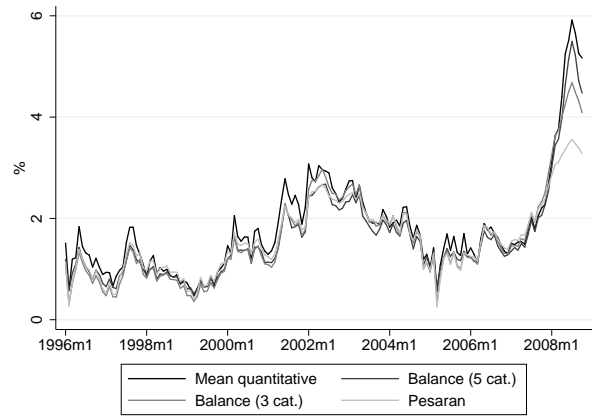

(a) Perceptions

(b) Expectations

Figure A.3: Response fractions below (low) and above (high) the implied response interval, 01/2002–10/2008

Notes: See footnote of Figure A.1 for a detailed description.
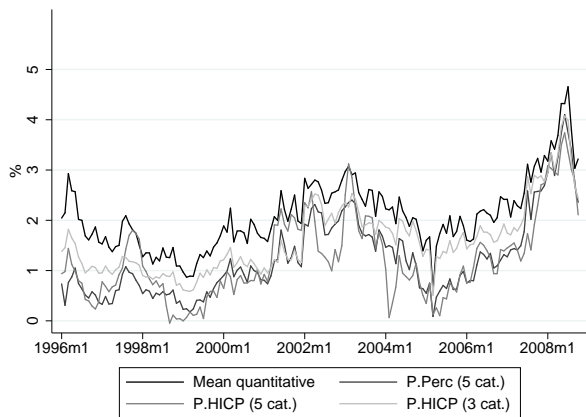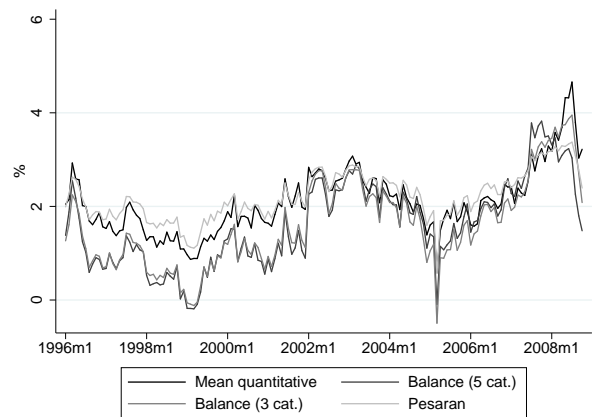
(a) Probability method

(b) Balance statistics and regression method

Figure A.4: Mean of inflation perceptions
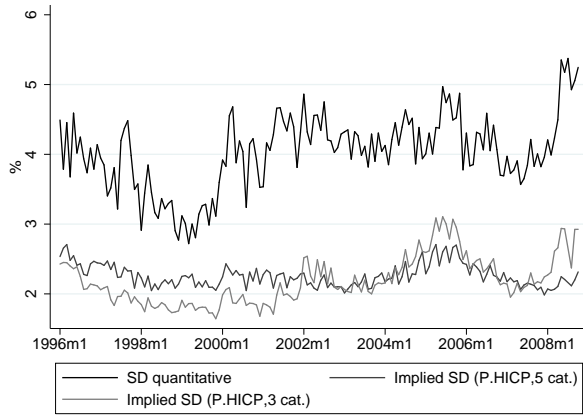

(a) Probability method

(b) Balance statistics and regression method

Figure A.5: Mean of inflation expectations

Table A.1: Accuracy of quantified inflation perceptions and expectations, 01/2002–10/2008

Perceptions	Level					First differences				
	Bias	MAE	RMSE	ρ	z	Bias	MAE	RMSE	ρ	z
P.HICP (5 cat.)	-0.26	0.27	0.40	0.97		0.00	0.12	0.15	0.85	
P.HICP (3 cat.)	-0.11	0.14	0.18	0.99	-0.56**	0.00	0.10	0.15	0.79	0.15
Balance (5 cat.)	-0.20	0.20	0.25	0.99	-0.79	0.00	0.10	0.12	0.89	-0.22**
Balance (3 cat.)	-0.15	0.19	0.33	0.97	0.02	0.00	0.10	0.14	0.86	-0.08
Pesaran (3 cat.)	-0.27	0.31	0.60	0.93	0.30	0.00	0.12	0.17	0.81	0.09

Expectations	Level					First differences				
	Bias	MAE	RMSE	ρ	z	Bias	MAE	RMSE	ρ	z
P.Perc. (5 cat.)	-0.76	0.76	0.80	0.97		0.00	0.15	0.20	0.79	
P.HICP (5 cat.)	-0.92	0.92	1.00	0.89	0.70**	0.00	0.26	0.35	0.51	0.50**
P.HICP (3 cat.)	-0.39	0.39	0.18	0.97	0.09	0.00	0.15	0.15	0.83	-0.08
Balance (5 cat.)	-0.22	0.35	0.25	0.82	0.99	0.00	0.17	0.12	0.84	-0.13
Balance (3 cat.)	-0.27	0.31	0.33	0.95	0.31	0.00	0.16	0.14	0.84	-0.13
Pesaran (3 cat.)	0.05	0.22	0.32	0.92	0.53	0.00	0.15	0.20	0.81	-0.03

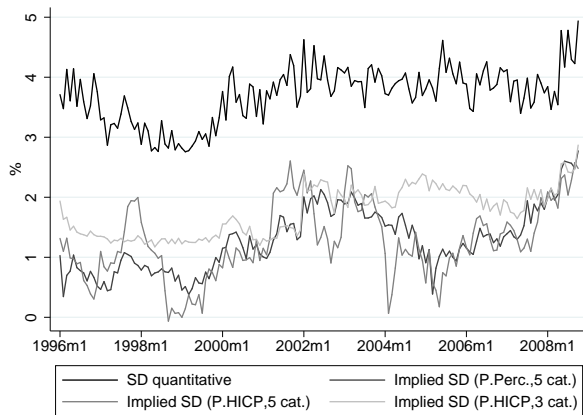*Notes*: See Table 7 for a detailed description.

(a) Probability method
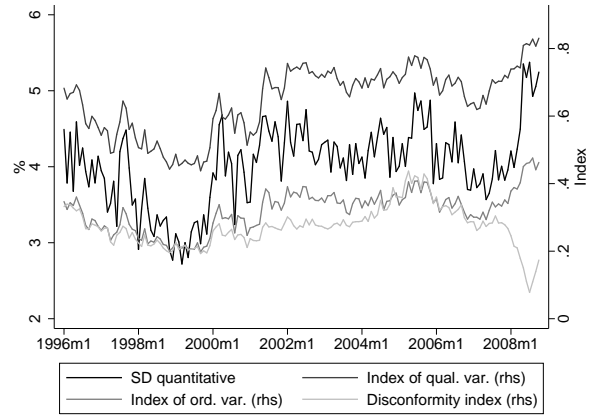
(b) Qualitative measures

Figure A.6: Dispersion of inflation perceptions

Notes: SD quantitative is the cross-sectional standard deviation of quantitative survey responses. Perceptions and 3-category probability method are restricted to be unbiased with respect to HICP inflation, reference inflation for quantifying expectations is given by quantified perceptions. *Index of ord. var.* is the d^2 statistic.
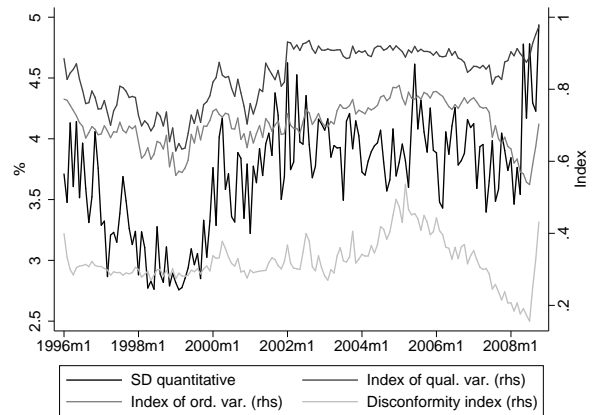

(a) Probability method

(b) Qualitative measures

Figure A.7: Dispersion of inflation expectations

Notes: SD quantitative is the cross-sectional standard deviation of quantitative survey responses. Perceptions and 3-category probability method are restricted to be unbiased with respect to HICP inflation, reference inflation for quantifying expectations is given by quantified perceptions. *Index of ord. var.* is the d^2 statistic.

Table A.2: Accuracy of measures of cross-sectional dispersion, 01/2002–10/2008

Perceptions

	Level					First differences				
	Bias	MAE	RMSE	ρ	z	Bias	MAE	RMSE	ρ	z
Implied SD (P.HICP, 5 cat.)	-2.00	2.00	2.04	0.27		0.00	0.28	0.34	0.20	
Implied SD (P.HICP, 3 cat.)	-1.86	1.86	1.88	0.65	-0.51	0.00	0.24	0.31	0.45	-0.27*
IQV (5 cat.)				0.80	-0.82*				0.47	-0.26**
DSQ (5 cat.)				0.79	-0.82				0.33	-0.13
DC (3 cat.)				-0.15	0.43				0.29	-0.09

Expectations

	Level					First differences				
	Bias	MAE	RMSE	ρ	z	Bias	MAE	RMSE	ρ	z
Implied SD (P.Perc, 5 cat.)	-2.35	2.35	2.40	0.30		0.00	0.28	0.36	0.27	
Implied SD (P.HICP, 5 cat.)	-2.51	2.51	2.57	0.30	0.01	0.00	0.33	0.42	0.27	-0.01
Implied SD (P.HICP, 3 cat.)	-1.88	1.88	1.90	0.42	-0.13	0.00	0.29	0.36	0.26	0.01
IQV (5 cat.)				0.33	-0.03				0.05	0.22
DSQ (5 cat.)				-0.09	0.41				0.10	0.17
DC (3 cat.)				0.09	0.23				0.13	0.14

Notes: See table 8 for a detailed description.

49