DISS. ETH NO. 24299

INSIDE MONEY CREATION OUT OF THIN AIR: A GENERAL EQUILIBRIUM PERSPECTIVE

A thesis submitted to attain the degree of
DOCTOR OF SCIENCES of ETH ZURICH
(Dr. sc. ETH Zurich)

presented by

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2017

Abstract

We study inside money creation by commercial banks in today's monetary architecture and examine the impact of monetary policy and capital regulation in a general equilibrium setting. In our model, there are two production sectors, financial intermediation, aggregate shocks, safe deposits, and two types of money created: bank deposits created when banks grant loans to firms or to other banks, and central bank money created when the central bank grants loans to private banks. We demonstrate that in the baseline model, equilibria yield the first-best level of money creation and lending when prices are flexible, regardless of monetary policy or capital regulation. Under rigid prices, we identify the circumstances in which money creation is excessive or breaks down, and the ones in which an adequate combination of monetary policy and capital regulation can restore efficiency. Under normal economic conditions, an adequate combination of monetary policy and capital regulation can restore the existence of equilibria and efficiency. A slump in money creation and lending can only be avoided if economic conditions are sufficiently favorable. Our main findings can be extended to various changes to our assumptions, such as the denomination of bonds and of profit maximization, the extent of the macroeconomic shock, the form of the production functions in the two sectors we defined, additional states of the world, the households' risk aversion, and the introduction of savings decisions, for example. Moreover, if the real deposit rates do not adjust to the macroeconomic shock, we demonstrate that only capital requirements that are sufficiently high can establish the existence and uniqueness of efficient equilibria with banks. If banks incur costs in real terms for the issuance of their equity, we prove that governmental authorities can use monetary policy and capital requirements to implement any welfare that is arbitrarily close to the first-best welfare. We show that reserve requirements coupled with haircuts are equivalent to capital requirements as to their effect on banks' money creation. We also show that inefficient asymmetric equilibria may also arise when prices are flexible, but that these inefficient equilibria are eliminated if capital requirements are sufficiently high, so that only the efficient equilibria with banks persist. We find that deposit insurance increases welfare in our setting. If there are financial frictions at bank level and if these frictions are not too intense, we demonstrate that the second-best allocation can only be implemented by a combination of monetary policy and capital regulation. Finally, we outline alternative monetary architectures in which money is solely created by the central bank, and we discuss some of their properties.

Résumé

Nous étudions la création monétaire par les banques commerciales et nous examinons l'influence de la politique monétaire et des régulations en matière de capitaux propres sur cette création monétaire dans le cadre d'un modèle d'équilibre général fondé sur la structure du système bancaire actuel. Notre modèle est composé de deux secteurs de production, d'une intermédiation financière, d'un choc qui affecte toutes les entreprises d'un des secteurs de production, de l'assurance des dépôts par le gouvernement, ainsi que de deux monnaies: les dépôts bancaires créés au moment où les banques privées octroient des prêts et la monnaie de banque centrale émise lors d'un emprunt d'une banque commerciale à la banque centrale. Nous démontrons que, dans le modèle de référence, les équilibres sont associés au niveau optimal de création monétaire et de crédit lorsque les prix sont flexibles, et ceci indépendamment de la politique monétaire choisie et des régulations en matière de capitaux propres. Lorsque les prix sont rigides, nous identifions les circonstances macroéconomiques pour lesquelles la création monétaire est excessive ou insuffisante, ainsi que celles pour lesquelles une politique monétaire adéquatement combinée avec une réglementation des fonds propres rétablit l'existence d'équilibres monétaires et la meilleure allocation des ressources: dans des conditions macroéconomiques normales, une politique monétaire et une réglementation des fonds propres appropriées peuvent offrir un cadre propice à l'existence d'équilibres et à la performance économique. Cependant, un effondrement de la création monétaire et des crédits ne peut être évité dans des conditions macroéconomiques trop défavorables. Nos conclusions principales restent valables lorsque les hypothèses suivantes sont modifiées: quand le choc macroéconomique est global, quand la forme des fonctions de production est généralisée, quand d'autres états macroéconomiques sont ajoutés, quand les ménages ont une aversion pour le risque, et quand les ménages doivent consommer et investir en même temps. De plus, si les taux d'intérêts sur les comptes courants en termes réels ne s'ajustent pas au choc macroéconomique, nous prouvons que seule une réglementation des fonds propres assez stricte peut établir l'existence et l'unicité d'équilibres optimaux. Lorsque l'émission des fonds propres engendre des coûts pour les banques, nous démontrons que les autorités gouvernementales peuvent coordonner la politique monétaire et la réglementation des fonds propres pour atteindre tout niveau de bien-être social arbitrairement proche du niveau optimal. Nous démontrons que la réglementation des réserves, associée à une décote sur les garanties, a un effet équivalent à la régulation en matière de capitaux propres sur la création monétaire. Nous prouvons que des équilibres asymétriques peuvent aussi apparaître lorsque les prix sont flexibles et que certains de ces équilibres asymétriques sont associés à une affectation inefficace des ressources. Ces équilibres peuvent être éliminés par l'instauration d'une réglementation des fonds propres assez stricte, de telle manière que seuls les équilibres impliquant une affectation efficace des ressources subsistent. Nous découvrons aussi que, dans notre modèle, la réglementation d'assurance des dépôts a un effet positif sur le bien-être social. Nous démontrons que, lorsque des frictions financières sont introduites au niveau bancaire, la deuxième meilleure affectation des ressources peut être implémentée par une combinaison adéquate de politique monétaire et de réglementation sur les capitaux propres. Pour conclure, nous donnons un aperçu d'autres architectures monétaires pour lesquelles la monnaie serait seulement émise par la banque centrale et nous discutons certaines de leurs propriétés.

Acknowledgments

My first and foremost acknowledgments for my doctoral thesis are undoubtedly addressed to Professor Hans Gersbach, to whom I am very grateful for his guidance, his support, and his example. I am also greatly indebted to Professor Antoine Bommier for his advice and his availability as a co-examiner during his sabbatical leave as well as to Professor Didier Sornette for immediately accepting to chair my PhD defense.

I greatly benefited from discussions with my colleague Dr. Volker Britz and particularly from his expertise in general equilibrium theory. His comments and insights helped me substantially improve the mathematical structure of my main results.

I received financial support from the Risk Center at ETH Zurich, where I was a doctoral student member within the "ETH48 Systemic Risk – Systemic Solutions" project, from the two projects of the Swiss National Science Foundation "Monetary Policy and Banking Regulation in Normal Times and Crises" (SNF project no. 100018_137570) and "Money Creation by Banks, Monetary Policy, and Regulation" (SNF project no. 100018_165491/1), as well as from the Chair of Macroeconomics: Innovation and Policy at ETH Zürich, which is led by Professor Hans Gersbach. I thank all these institutions and all persons involved.

I received very insightful suggestions and remarks about Chapter 2, which I had the opportunity to present at the Risk Center of ETH Zurich, in the Astute Modeling Seminar at ETH Zurich, and in a research seminar at the Swiss National Bank. For their thorough understanding and their inspiring comments, I thank in particular

Professor Cyril Monnet, Professor Hans Haller, Professor Clive Bell, Professor Frank Schweitzer, Dr. Michael Kumhof, Dr. Leonardo Melosi, and Dr. Stephan Imhof. I really appreciated the opportunity given by Professor Cyril Monnet to present my work at the Swiss National Bank.

I also thank Professor Dirk Niepelt, Professor Martin Brown, Dr. Toni Beutler, Dr. Mathieu Grobéty, Dr. Adriel Jost, Dr. Christoph Basten, Antonios Koumbarakis, and Damien Klossner for interesting conversations.

Of course, I express my gratefulness to all my former and current colleagues for the constructive team work and the very pleasant atmosphere, with special gratitude to Professor Maik Schneider, 刘玉林博士 (Dr. Yulin Liu), Stylianos Papageorgiou, Aschkan Mery, and 朱致远 (Zhiyuan Zhu) for detailed discussions on my work. I am particularly indebted to Stylianos Papageorgiou and Aschkan Mery for their very careful and very helpful proofreading. Outside my workplace, my horizon was enlarged by fascinating ideas connected to monetary architectures in the area of law discussed with Adrien Clinard and with Simon Janin in the area of cryptocurrencies.

J'aimerais aussi remercier chaleureusement tous mes amis, qui m'ont soutenu et se sont même parfois intéressés au sujet de cette thèse. Ceux qui liront ces lignes se reconnaîtront. J'exprime aussi ma sincère et profonde gratitude à Anne Buechi et au professeur Jean-Paul Thommen pour m'avoir sauvé d'une situation très pénible au moment crucial d'écriture de la thèse.

Maman, Papa, André, Laumer, et Aljosha,
Je vous remercierai du fond du cœur pour tout
Tant que je le pourrai. L'Alpha et l'Oméga,
C'est vous. Je n'en serais pas venu seul à bout.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Motivation

> *"The essence of the contemporary monetary system is creation of money, out of nothing, by private banks' often foolish lending."*
> Martin Wolf, 9 November 2010[1]

Since the 2008 financial crisis and the ensuing Great Recession, the financial system has been under tight scrutiny: The United States Congress passed the Dodd–Frank Wall Street Reform and Consumer Protection Act[2] that represents the most significant financial regulation overhaul in the United States since the 1930s.[3] The Basel Committee on Banking Supervision established a comprehensive new international regulatory standard known as Basel III, with more stringent capital requirements.[4] Some countries like Switzerland unilaterally took a step further in the level of capital requirements for systematically relevant institutions,[5] and many prominent economists are calling for even higher capital requirements.[6] These new

[1] See Wolf (2010).

[2] See Senate and House of Representatives of the United States of America in Congress (2010).

[3] See Financial Times Lexicon (no date a).

[4] See Basel Committee on Banking Supervision (2010).

[5] On 21 October 2015, the Swiss Federal Council decided to implement a new set of capital adequacy standards for systematically important banks. See Mathys (2015).

[6] One example is given by Admati and Hellwig (2013).

regulatory measures are likely to enhance the resilience of the banking system and reduce the moral hazard problem caused by bank bail-outs.[7] Still, most of the literature that provides foundations for banking regulation has not addressed a core feature of the monetary architecture in most developed countries yet, namely the creation of money by commercial banks.[8]

The current monetary architecture works as follows: When commercial banks grant loans to firms or households, they create deposits out of thin air: They add two items to their balance sheets: First, they add a loan, which is an asset and represents the promise of the borrower to repay the principal plus interest, typically at a given maturity. Second, the banks add a liability, which is a promise that the bank has to pay the principal to the borrower on demand, in terms of banknotes created by the central bank, for example.[9]

To illustrate the money creation process and the ensuing transactions with balance sheets,[10] let us assume that there are two firms—Firm B (the Buyer) and Firm S (the Seller)—and two banks—Bank b^B and Bank b^S. Firm B would like to buy a good $\mathbf{G}$ at price p_G from Firm S. However, Firm B has no money for this purchase and it thus borrows some amount of money $L_0 = D_0 \geq p_G$ from Bank b^B. We assume that Firm B borrows more than p_G, as it may want to buy other goods at a later stage. We thus obtain $D_0 = L_0 > p_G$. Once Bank b^B has granted a loan to Firm B, we obtain the balance sheets given in Table 1.1.

In the balance sheets, apart from Firm S's ownership of Good $\mathbf{G}$, which we denote by E_S, we do not consider any other banks' and firms' balance sheet item for the sake of simplicity, and we only consider the case where Firm B does not transfer its deposits to another bank. Firm B now buys the good from Firm S. Two different situations may occur: Either Firm S opens a deposit account at Bank b^B to receive the money or it opens an account at another bank, say Bank b^S. In

[7]For example, Gersbach (2013) shows that higher equity ratio requirements mitigate moral hazard at bank level.

[8]Hicks (1967) defines money by its functions: *"Money is what money does"*. Money typically serves as a medium of exchange, a unit of account, and a store of value. Bank deposits at a bank that is perceived as safe can thus be considered as money.

[9]There are other ways how money is created in our economy: Money is also created when the private banking sector buys assets from the non-bank private sector.

[10]The process of money creation is described in McLeay et al. (2014), for instance.

Firm B		Firm S		Bank b^B	
D_0	L_0	**G**	E_S	L_0	D_0

Table 1.1: Balance sheets after Bank b^B grants a loan to Firm B. Source: Own illustration.

the former case, the balance sheets are given in Table 1.2, where the deposits of Firm B are denoted by $D_B = D_0 - p_G$ and the deposits of Firm S are denoted by $D_S = p_G$.

Firm B		Firm S		Bank b^B	
D_B	L_0	D_S	E_S	L_0	D_B
G					D_S

Table 1.2: Balance sheets after Firm B has purchased the good from Firm S in the case where Firm S opens an account at Bank b^B. Source: Own illustration.

In the case where Firm S opens an account at Bank b^S, Bank b^B either has to pay Bank b^S to transfer the deposits or Bank b^S has to grant Bank b^B a loan, if the two banks agree on such a loan contract. In the former case, as Bank b^B does not have any central bank money—in the form of banknotes for example—to pay Bank b^S and as it cannot pay with the loan it has granted to Firm B, it has no other choice but to apply for a loan from the central bank.[11] When Bank b^B borrows from the central bank, the balance sheets are given in Table 1.3.

Now Bank b^B is able to pay Bank b^S the amount p_G, and we obtain the balance sheets given in Table 1.4. Alternatively, Bank b^S may grant a loan to Bank b^B. Then the balance sheets are given in Table 1.5, where $L_{IB} = p_G$.

[11] In practice, central banks mostly grant loans only against some collateral, whose value has to exceed the principal of the loan plus a so-called "haircut" that depends on the quality of the collateral and is normally given as a percentage of its value.

Bank b^B	
L_0	D_B
D_{CB}	L_{CB}

Central Bank	
L_{CB}	D_{CB}

Table 1.3: Balance sheets of Bank b^B and the central bank in the case where Firm S opens an account at Bank b^S and Bank b^B borrows from the central bank an amount $L_{CB} = D_{CB} = p_G$. Source: Own illustration.

Bank b^B	
L_0	D_B
	L_{CB}

Bank b^S	
D_{CB}	D_S

Central Bank	
L_{CB}	D_{CB}

Table 1.4: Balance sheets after Firm B has purchased the good from Firm S in the case where Firm S opens an account at Bank b^S and Bank b^B settles its liability to Bank b^S with central bank money, which it has obtained via a loan. Source: Own illustration.

Bank b^B	
L_0	D_B
	L_{IB}

Bank b^S	
L_{IB}	D_S

Table 1.5: Balance sheets after Firm B has purchased the good from Firm S in the case where Firm S opens an account at Bank b^S and Bank b^S grants a loan to Bank b^B. Source: Own illustration.

From this simple stylized example, we can infer some observations. First, if we use the terminology of Gurley and Shaw (1960),[12] money in the form of bank deposits

[12] See Lagos (2006) for a detailed discussion of the definition of inside and outside money

or central bank deposits is called "inside money" in this simple example and there is no "outside money".[13]

Second, a bank does not need central bank money *before* granting a loan. It first grants a loan and then *refinances* the part of the deposits that the borrower uses to make payments. However, banking models in real terms[14] use an approach to banking according to which households first provide banks with physical goods against deposit or equity contracts. Banks then lend these physical goods to firms or to other households. This approach—thereafter called the "loanable-funds approach"[15]—clearly does not represent the actual simultaneous process of lending and money creation: Loans are not created by deposits, but deposits are created by loans. As the more realistic approach—thereafter called the "money creation approach"—is fundamentally different from the loanable-funds approach, it is a priori not clear if using the money creation approach to banking would affect the conclusions of banking models based on the loanable-funds approach. Jakab and Kumhof (2015) use a DSGE model framework to argue that the two approaches imply very different results.

Third, unless new loans are granted, loans are repaid or deposits are withdrawn in the form of banknotes and coins,[16] the amount of deposits in the economy is constant. If no additional banknote or coin is put into circulation, the amount

by Gurley and Shaw (1960). An alternative definition used in the literature is given by White (1983), for instance. According to this definition, outside money is the money created by the central bank, which comprises banknotes, coins, and central bank reserves. Inside money is *"bank liabilities such as deposits transferable by check, usually privately produced, whose value derives from their being redeemable for basic cash".*

[13]The definition of outside money by Gurley and Shaw (1960) given in Lagos (2006) is as follows: Outside money is a medium of exchange *"of a fiat nature (unbacked) or backed by some asset that is not in zero net supply within the private sector of the economy. The qualifier* outside *is short for* (coming from) outside the private sector. Inside money *is an asset representing, or backed by, any form of private credit that circulates as a medium of exchange. Since it is one private agent's liability and at the same time some other agent's asset, inside money is in zero supply within the private sector. The qualifier* inside *is short for* (backed by debt from) inside the private sector.*"*

[14]Examples of such banking models in real terms include Gersbach (2013), Gersbach et al. (2015b), and Gersbach et al. (2015c).

[15]This terminology is inspired by Jakab and Kumhof (2015), who call a model of banking based on this approach an "intermediation of loanable-funds (ILF) model of banking".

[16]In this case, the bank would ask the central bank for a loan and would exchange the digital central bank money against banknotes and coins.

of deposits in the economy is only influenced by banks' lending and firms' and households' repayment of loans.

Fourth, if a bank grants loans, part of the deposits it creates will flow to other banks, part of the deposits will eventually flow back to the bank, and finally, part of the deposits will be kept at the banks, which implies that the bank that initially granted loans does not necessarily need to borrow the full amount of the loans from the central bank or from other banks to meet its obligations to the depositors.

Fifth, as long as households do not withdraw their deposits to obtain money in physical form, there is no need for banknotes and coins. Moreover, as long as in addition, banks exclusively lend money to each other, there is no need for banks to borrow from the central bank or to hold central bank money at all, assuming that there is no reserve requirement and that there is no outside money.

From these last observations, we may wonder which proportion of money is created by banks and which proportion is created by the central bank in practice. In Switzerland, in December 2016, notes and coins represented 15% of the monetary aggregate M1, defined by the Swiss National Bank as being sight deposits, deposits in transaction accounts, and notes and coins.[17] In the UK, in February 2017, notes and coins represented 5% of the monetary aggregate M1, which the Bank of England defines as being the outstanding amount of monetary financial institutions' liabilities to private and public sectors.[18] Money is thus mainly created by commercial banks, when they grant loans to households or firms.[19] It is interesting to note that among the population, the understanding of money creation by commercial banks is limited. For instance, Nietlisbach (2015) indicates that according to a survey performed in Switzerland in 2015, only 13% of the respondents were aware that most of the money is created by commercial banks.

In the light of these observations, the following research questions arise.

[17]Own calculations. Source: http://www.snb.ch/en/ (retrieved on 5 March 2017).
[18]Own calculations. Source: http://www.bankofengland.co.uk (retrieved on 5 March 2017).
[19]Note that commercial banks can also create money in a similar way when they buy assets.

1.2 Research Questions

In all subsequent questions, we use the abbreviated expression "money creation" for "inside money creation by commercial banks".

Q1: How is money creation controlled?

Q2: Which combinations of policy rates and capital requirements lead to socially efficient levels of money creation, which intermediate the investment goods from the households to the production sectors optimally?

Q3: How do reserve requirements and haircuts impact money creation?

Q4: How do monetary policy, capital requirements, and reserve requirements coupled with haircuts impact banking stability?

Q5: Which role does deposit insurance play with regard to money creation?

Q6: How do financial frictions influence money creation?

Q7: Which sets of policies can help stimulate the economy in a downturn when the central bank interest rate is already at the zero lower bound?

1.3 Approach

As all our research questions require a detailed understanding of agents' incentives as well as their influence on each other, a sequential general equilibrium approach with four types of agents—households, firms, banks, and government authorities—is necessary to study money creation. We depart from the literature modeling outside money[20] by exclusively considering endogenously created inside monies and the incentives for their creation.[21] Endogenous inside money creation and its control (question **Q1**) can only be studied properly in a model that replicates

[20]Shubik and Wilson (1977) and Dubey and Geanakoplos (1992, 2003a,b) show that outside money can have positive value in a finite-horizon model when, first, there are sufficiently large penalties when debt to governments—such as tax liabilities—is not paid and, second, when there are sufficiently large gains from using and trading money.

[21]A simple explanation why central bank money in our model is also inside money is that the central bank creates an asset that is in zero net supply within the private sector (cf. definition given in Lagos (2006)).

the structure of today's monetary architecture. In particular, we build the two-tier structure with privately created and publicly created monies in our general equilibrium model.

More specifically, we consider a two-period general equilibrium model with two production sectors and one investment good. In Period $t = 0$, investment takes place in both sectors. In one sector, firms can obtain direct financing from the bond market and thus from households. In the second sector, firms can only be financed by bank loans. At the beginning of Period $t = 1$, the production technologies transform the investment good into a *consumption good*. The gross rates of return are impacted by a macroeconomic shock. At the end of Period $t = 1$, households consume the consumption good.

Banks grant loans to firms in one sector, thereby creating money (privately created money) in the form of deposits, which enable them later to buy the investment goods and thus serve as a means of payment and as a store of value.[22] Households, who are initially endowed with the investment good, sell some amount of it to the latter firms in exchange for firms' deposits enabling households to invest in bank equity and bank deposits. Households then directly provide the remaining amount of the investment good to the firms in the other sector in exchange for bonds promising the delivery of some amount of consumption good after production in the next period.

The payment processes are supported by a central bank that sets the policy rate and creates reserves (publicly created money) when it grants loans to commercial banks. Banks facing an outflow of deposits to other banks that is higher than the inflow—and hence net debt against other banks—can refinance themselves at the central bank freely at the policy rate. These banks can fulfill the claims of other banks by paying with their reserves, which are claims against the central bank. The publicly created money is thus often called "central bank money". Banks that

[22]We directly impose a deposit-in-advance constraint, i.e. households can only buy physical goods with bank deposits. Such constraints—usually in the form of cash-in-advance constraints—have been introduced by Clower (1967) and Lucas (1982). For a discussion of their microfoundations, see Shi (2002). Microfounded approaches to monetary policy include Lagos and Wright (2005). Their model, however, only considers exogenously created outside money. We build on these microfoundations, but we then depart from this approach by concentrating on the process of inside money creation and the banks' incentives for its endogenous creation.

have net claims against other banks will thus receive reserves at the central bank and interest payments according to the policy rate. Therefore, the central bank policy rates will steer the banks' money creation (question **Q2**).

Governmental authorities impose either a minimum equity ratio requirement or a minimum reserve requirement coupled with haircuts on borrowing from the central bank, which banks have to comply with at the end of Period $t = 0$. In a general equilibrium perspective, these constraints at the *end* of Period $t = 0$ will be taken into account in the banks' lending decisions at the *beginning* of Period $t = 0$ (questions **Q2** and **Q3**). We consider both the case where banks maximize the shareholders' value and the case where there are financial frictions at bank level, for example, in the sense of Holmström and Tirole (1997). Both cases are very different in terms of the banks' incentive to create money, and we can then examine how the introduction of financial frictions influences this money creation (question **Q6**).

Figure 1.1 summarizes the agents' interactions during Period $t = 0$.

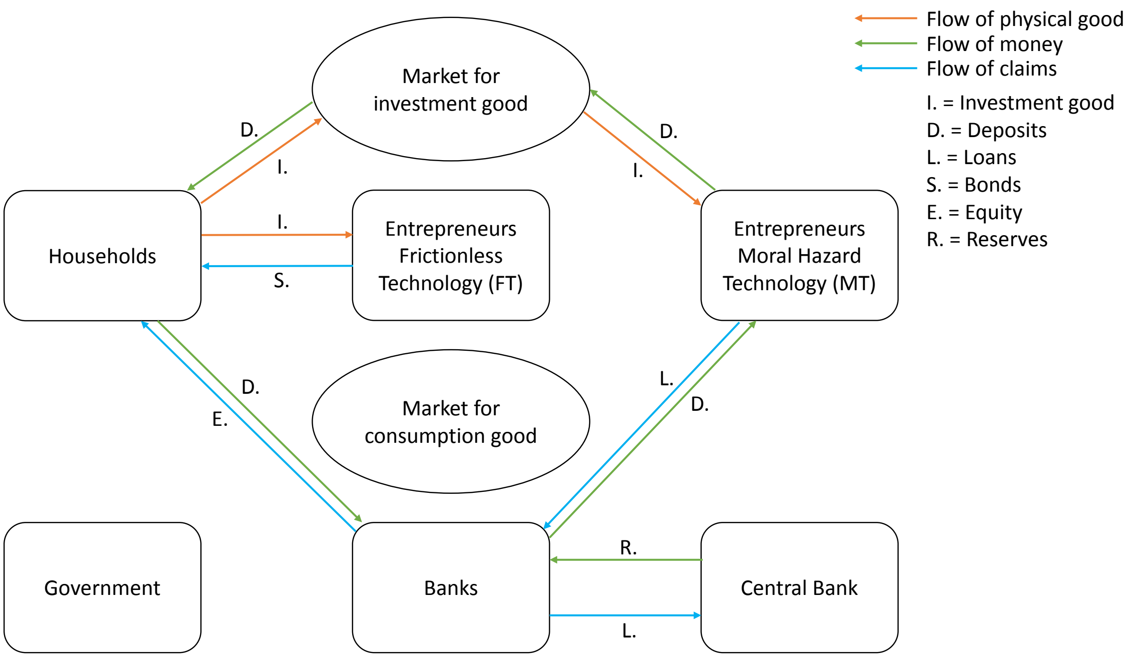

Figure 1.1: Flows and interactions during Period $t = 0$. Source: Own illustration.

At the beginning of Period $t = 1$, a macroeconomic shock occurs and affects the output from production. The firms' production technologies transform the amount of investment good acquired in the previous period into some amount of consumption good. An economy in a downturn can then be easily modeled as being an economy for which the rate of production of the consumption good in terms of the investment good is sufficiently low in all states of the economy. We can then study how this parameter influences the banks' money creation (question **Q7**).

The firms directly financed by bonds repay them by delivering the amount of consumption good due, which means that bonds are real assets. The other firms sell the amount of produced consumption good to households in exchange for deposits. These latter firms use the deposits to repay bank loans. When borrowers pay back loans, the deposits originally created during Period $t = 0$ are destroyed. Since repayments by borrowers depend on the macroeconomic shock, the items in the banks' balance sheets are risky. As banks' shareholders are protected by limited liability, some banks may default on depositors. As monetary policy, capital requirements, and reserve requirements together with haircuts will have an impact on the banks' lending decisions, they will also impact the banks' stability (question **Q4**). The households' deposits may be either fully insured by government authorities or not insured, which enables to investigate the role of deposit insurance with regard to money creation (question **Q5**). To guarantee the value of deposits, the government resorts to lump-sum taxation if some banks default and households' deposits are indeed protected by deposit insurance. The dividends of non-defaulting banks are paid to households in the form of deposits. At the end of Period $t = 1$, households consume the consumption good. We focus on a complete market setting in the sense that all contracts can be conditioned on macroeconomic events[23] and we consider both the case where prices are flexible, i.e. where they can adjust to the macroeconomic shock, and the cases where prices are rigid, which means that they do not react to macroeconomic conditions.

Figure 1.2 summarizes the agents' interactions during Period $t = 1$.

[23] The market setting is incomplete in two other respects: Payments must be made with bank deposits, and households cannot invest in all firms directly. The firms in one sector rely on financial intermediation.

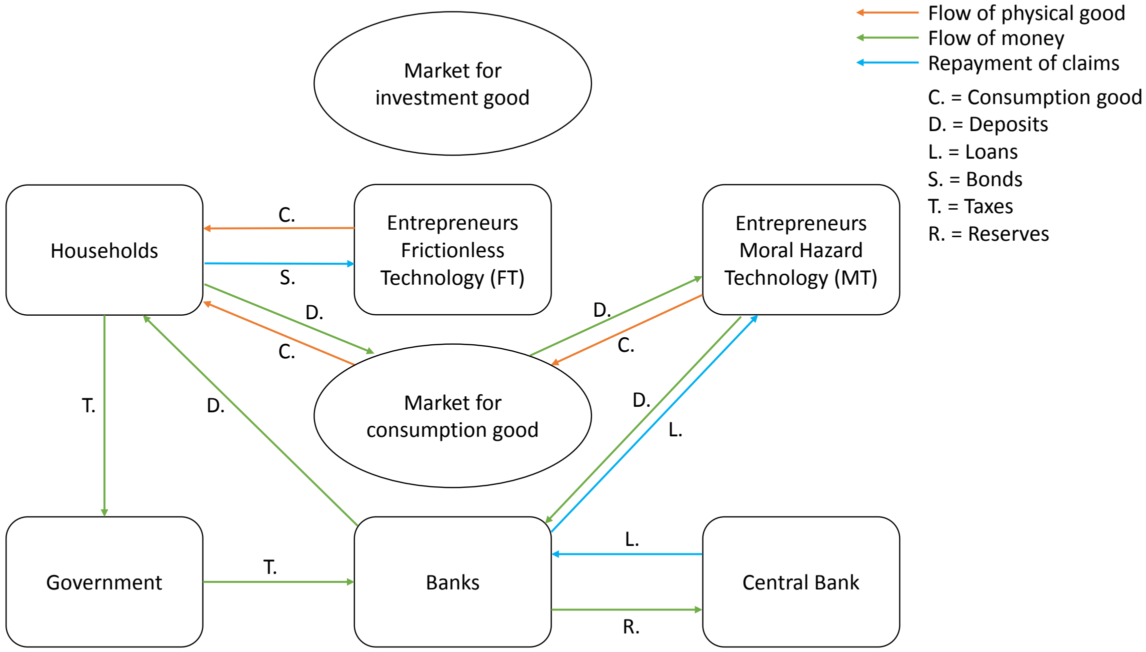

Figure 1.2: Flows and interactions during Period $t = 1$. Source: Own illustration.

It is a priori not clear how some of our assumptions affect our results, and we perform several robustness checks in Chapters 3 to 4, using the model of Chapter 2 as a baseline model.

Finally, one important remark is in order. The features of our model entail results of knife-edge type. For instance, money creation is either at optimal level, explodes, or collapses to zero. This has the advantage of illustrating in the simplest and most transparent way both the forces at work and the appropriate monetary policy and capital regulation.

1.4 Results

1.4.1 Money Creation in a General Equilibrium Model (Chapter 2)

The analysis of the model of Chapter 2 produces three main insights. First, with perfectly flexible prices, i.e. prices adjusting perfectly to macroeconomic conditions, equilibria with money creation are associated with the first-best allocation, regardless of the central bank's monetary policy. If prices are rigid, there is a central bank policy that induces socially efficient money creation and lending. But for all other central bank policies, money creation collapses or explodes, there is no financial intermediation, and the allocation is inefficient. Capital regulation in the form of a minimum equity ratio requirement that is sufficiently strict can supplement some of the latter central bank policies and restore socially efficient money creation and lending. Second, with price rigidities and the zero lower bound, there may not even exist a feasible central bank monetary policy inducing socially efficient money creation and lending. Again, a capital regulation that is sufficiently strict together with an appropriate monetary policy can limit money creation and—under normal economic conditions—can implement the first-best allocation. Third, with rigid prices and the zero lower bound, capital regulation and monetary policy can merely avoid a slump in money creation and lending if economic conditions are sufficiently favorable.[24]

1.4.2 Generalizations and Variations of the Model with Money Creation (Chapter 3)

In the absence of moral hazard, firms and banks maximize the shareholders' value, which is expressed in real terms. However, in the model of Chapter 2, we assume that agents maximize their profits in nominal terms. We show that in our setting, these two formulations are equivalent. For the remainder of the thesis, we

[24]Formally, this means that there is a positive probability that the real interest rate is above zero.

will assume that firms and banks maximize profits in real terms unless otherwise specified.

In the model of Chapter 2, bonds are denominated in real terms. In a formulation where the bonds are denominated in nominal terms, the payment processes are more complex, as they may involve several rounds of payments—which, however, do not involve conceptual changes—, and these payment processes have to be rewritten. Moreover, additional constraints on equilibria with banks must hold to ensure that in equilibria with banks, firms do not default. Under these new no-default conditions, we show that this formulation is equivalent in the sense that it implies qualitative results that are identical to the formulation with denomination of bonds in real terms.

We also demonstrate that the assumption that the macroeconomic shock only impacts the firms financed by banks is not critical for our conclusions and that we could allow the shock to also impact firms financed by households. Again, this would only require that a no-default condition is added to the definition of equilibria with banks. Similarly, replacing the linear production function of the technology financed by banks by a general concave function does not change our findings qualitatively, as long as some new assumptions about this function hold. Yet, it requires no-default conditions to hold in any equilibrium with banks. We also generalize our results to $N > 2$ states of the world.

When the real deposit rates cannot be written contingently on the state of the world, no equilibrium with banks exists if no capital requirement is imposed. A capital requirement, however, can restore the existence of equilibria with banks. For low capital requirements, there are only inefficient equilibria with banks and with sufficiently high capital requirements, only efficient equilibria with banks exist.

When banks incur costs in real terms for the issuance of their equity, the banks' incentive to create money is impacted by these costs. If prices are flexible and there is no capital requirement, there is no equilibrium with banks when the costs of equity issuance are positive but small enough, and there is a set of inefficient equilibria with banks when the costs of equity issuance are sufficiently high. In the latter case, the government authorities are thus unable to implement the second-best allocation. However, when prices are rigid and there is a minimum equity ratio, the government authorities can implement any welfare arbitrarily close even to the first-best welfare.

Finally, we prove that reserve requirements and haircuts are equivalent to a capital requirement in the sense that they have a similar effect on banks' money creation and that they thus do not impact any equilibrium variable differently.

1.4.3 More Sophisticated Households' Problems in the Model with Money Creation (Chapter 4)

In Chapter 2, households are risk-neutral. We demonstrate in Chapter 4 that for flexible prices, our main result still holds when households are risk-averse for an arbitrary concave utility function, i.e. we prove that the allocation is first-best for any central bank policy rates and we illustrate this result with a particular concave utility function, namely the quadratic utility function, and a linear production function, from which closed-form solutions can be derived and interpreted.

In the macroeconomic model of Chapter 2, no investment-consumption decision is taken by households. In a model with an initial investment-savings decision by households, we demonstrate that the first-best allocation is still implemented in a competitive equilibrium.

1.4.4 Changes in Critical Features of the Model with Money Creation (Chapter 5)

In Chapter 2, we only consider symmetric equilibria with banks. We demonstrate that when prices are flexible, there are inefficient asymmetric equilibria with banks, along with efficient equilibria, and we show that these inefficient equilibria are eliminated by sufficiently high capital requirements. Moreover, when prices are rigid and the central bank policy rates are not equal to the real gross rates of return, there is no symmetric equilibrium with banks, and inefficient asymmetric equilibria with banks may appear. In the latter case, only capital requirements can establish the existence of efficient equilibria with banks.

In the absence of deposit insurance, we prove that there never exists an inefficient equilibrium with banks and we show that when prices are rigid, deposit insurance eliminates inefficient equilibria with banks, thus potentially leaving only the inefficient equilibria without banks. As equilibria without banks implement a lower welfare than inefficient equilibria with banks, we can thus conclude that deposit insurance increases welfare in our setting.

When there are financial frictions at bank level, there only exist equilibria with banks when a capital requirement is imposed. Moreover, there only exist equilibria with banks that implement the second-best allocation when the intensity of financial frictions is sufficiently low.

Finally, we describe alternative monetary architectures, for which money is created by the central bank alone. In a *Decentralized Deposit System*, the banks' default against households is still possible. However, the banks' incentive to grant loans may differ according to the existence of a rationing scheme. The important consequence is that in the presence of a rationing scheme, money creation is limited and that there may thus be inefficient equilibria with banks when prices are flexible. In a *Centralized Deposit System*, banks cannot default on households, and either there is no equilibrium with banks or the prevailing equilibria with banks are efficient.

Chapter 2

Money Creation in a General Equilibrium Model

Abstract[1]

We study inside money creation by commercial banks in today's monetary architecture and examine the impact of monetary policy and capital regulation in a general equilibrium setting. In our model, there are two production sectors, financial intermediation, aggregate shocks, safe deposits, and two types of money created: bank deposits created when banks grant loans to firms or to other banks and central bank money created when the central bank grants loans to private banks. We demonstrate that equilibria yield the first-best level of money creation and lending when prices are flexible, regardless of monetary policy or capital regulation. Under rigid prices, we identify the circumstances in which money creation is excessive or breaks down, and the ones in which an adequate combination of monetary policy and capital regulation can restore efficiency. Under normal economic conditions, an adequate combination of monetary policy and capital regulation can restore the existence of equilibria and efficiency. A slump in money creation and lending can only be avoided if economic conditions are sufficiently favorable.

[1]This chapter is joint work with Hans Gersbach and was published as "Money Creation and Destruction", *Center for Financial Studies Working Paper* 555, (2016) as well as "Money

2.1 Introduction

2.1.1 Motivation and Approach

Money is predominantly held by the public in the form of bank deposit contracts.[2] These deposits are typically created by the banks' lending decisions. How is such money creation controlled, and how can it be steered towards socially desirable levels? These long-standing questions are the focus of our research.[3]

For several reasons the constraints on asset and money creation in the commercial banking system in today's architectures have received renewed attention recently (see McLeay et al. (2014)). First, the price of reserves, i.e. the short-term interest rate, has widely replaced traditional quantity instruments in the form of reserve requirements, which do not restrict lending directly.[4] Moreover, at exceptional times some central banks purchase securities or lend to banks at low and even negative interest rates. Whether such policies trigger corresponding money creation and foster economic activities is unclear.

In Chapter 2 we develop a sequential general equilibrium model to study these issues. Bank deposits are essential to buy physical goods, and these deposits are created in the lending process by banks for firms that can only obtain funds through monitored lending. The central bank sets an interest rate (or policy rate) at which

Creation, Monetary Policy, and Capital Regulation", *Centre for Economic Policy Research Discussion Paper* 11368, (2016) (cited as Faure and Gersbach (2016a) and Faure and Gersbach (2016b), respectively). The research on which this chapter is based was supported by both the SNF project no. 100018_137570 "Monetary Policy and Banking Regulation in Normal Times and Crises" and the Risk Center project "ETH48 Systemic Risk – Systemic Solutions".

[2]The use of banknotes and coins in daily transactions today is low. For instance Bennett et al. (2014) estimate the share of the volume of payments made in cash in the US at 14%.

[3]Gurley and Shaw (1960) and Tobin (1963) are well-known contributions. For instance, Tobin (1963) established the so-called "new view" by stressing that there are natural economic limits to the amount of assets and liabilities the commercial banking industry can create.

[4]Based on a 2010 IMF survey of 121 central banks, Gray (2011) describes the main purposes of reserve requirements and points out that nine countries do not have any reserve requirements, including the United Kingdom, Australia, Mexico, and Canada. Similarly, Carpenter and Demiralp (2012) show that the standard money multiplier model cannot explain the relationship between reserves and money. For instance, they point out that reserve balances held at the Fed increased dramatically—by a factor of at least 50—from July 2007 to December 2008 and that no similar increase in any measure of money could be observed during this time frame.

banks are able to refinance themselves and which they can earn by holding reserves at the central bank. Households sell their endowment of investment goods to firms and choose a portfolio of bank equity, bank deposits, and bonds. Consumption goods are produced by firms and sold to and consumed by households. With the proceeds, banks and firms pay dividends and reimburse bonds and loans.

2.1.2 Relation to the Literature

Our research is inspired by the long-standing issue of the limits on money creation by commercial banks in a world where money is fiat. Independently of this work, Jakab and Kumhof (2015) construct a DSGE model in which a bank can create money. They show quantitatively that shocks have larger effects on bank lending and on the real economy than in the corresponding loanable-funds model in which banks are constrained by resources provided by depositors. We focus on the welfare properties of general equilibrium models when private banks compete with regard to money creation—both in the absence and presence of price rigidities.

Conceptually, our research is connected to three further strands of the literature.

First, one important line of reasoning and the corresponding models show that fiat money can have positive value in a finite-horizon model when, first, there are sufficiently large penalties when debts to governments—such as tax liabilities—are not paid and, second, there are sufficiently large gains from using and trading money (Dubey and Geanakoplos, 1992, 2003a,b; Shubik and Wilson, 1977).[5] To this literature we add the two-tier structure with privately and publicly created monies. Commercial banks create bank deposits (privately created money) when they grant loans to firms enabling them to buy investment goods. Bank deposits will be used later by households to buy consumption goods.[6] The central bank creates reserves (publicly created money) when it grants loans to commercial banks

[5]There are various important approaches to constructing general equilibrium models with money to which we cannot do justice in our model. We refer to Huber et al. (2014) for a summary of the reasons why the value of fiat money can be positive in finite and infinite horizon models.

[6]We will directly impose a deposit-in-advance constraint, i.e. households can only buy physical goods with bank deposits. Such constraints—usually in the form of cash-in-advance constraints—have been introduced by Clower (1967) and Lucas (1982). For a discussion of their foundations, see Shi (2002).

enabling them to settle claims on privately created money among banks. The publicly created money is often called “central bank money”.

Second, beside their role in money creation, the existence of banks in our model is justified by their role as delegated monitors.[7] In this respect, our research builds on the seminal work by Diamond (1984), whose rationale for the existence of financial intermediaries relies on economies of scale in monitoring borrowers under moral hazard. Furthermore, Boot and Thakor (1997) provide a rationale explaining why financial markets and banks can coexist. They show that high-quality firms can borrow directly from the financial markets and that the moral hazard problem can be alleviated by banks' monitoring activities. Similarly, Bolton and Freixas (2000) develop a model based on asymmetric information with equity and bond issues as well as bank loans. They show that safe firms borrow from the bond market, whereas riskier firms are financed by banks. Based on these insights we construct our model on the assumption that there are two different types of firms. The first type encompasses small and opaque firms, which are risky and need to be monitored by banks to get financing. The second type assembles large firms, which are safe and can obtain financing directly from households through bond issues.

Third, a large body of literature on banks in partial or general equilibrium has provided important insights on how appropriate capital regulation may reduce excessive risk-taking and stabilize credit cycles.[8] We examine the role of capital regulation with regard to money creation.

2.1.3 Main Insights

The analysis of our model produces three main insights. First, with perfectly flexible prices, i.e. prices adjusting perfectly to macroeconomic conditions, equilibria

[7] For a complete account of the role of banks as delegated monitors, see Freixas and Rochet (2008).

[8] Recent general equilibrium models are developed by Gersbach and Rochet (2017) to provide a foundation for counter-cyclical capital regulation and Gersbach et al. (2015b) on the role of capital regulation as an equilibrium selection device. Cao and Illing (2015) model banks' incentives to overinvest in illiquid assets and provide a rationale for ex ante liquidity coverage requirements.

with money creation are associated with the first-best allocation, regardless of the central bank's monetary policy. If prices are rigid, there exist central bank policies for which money creation collapses or explodes. In the only equilibrium possible, in these cases, there is no financial intermediation, and an inefficient allocation occurs. Appropriate central bank policy can restore socially efficient money creation and lending. Second, with price rigidities and the zero lower bound, there may not exist a feasible central bank monetary policy inducing socially efficient money creation and lending. Capital regulation in the form of a minimum equity ratio and monetary policy can jointly limit money creation and under normal economic conditions restore the existence of equilibria with socially efficient money creation and lending. Third, when prices are rigid, the central bank's choice of zero interest rates[9] and appropriate capital regulation can only avoid a slump in money creation and lending if economic conditions are sufficiently favorable.[10]

One important remark is in order. The features of our model entail results of the knife-edge type. For instance, money creation is either at optimal level, or explodes, or collapses to zero. This has the advantage of illustrating in the simplest and most transparent way both the forces at work and appropriate monetary policy and capital regulation.

2.1.4 Structure

The set-up of the model is outlined in Section 2.2. Section 2.3 derives the resulting equilibria and their welfare properties and the role of capital regulation when prices are perfectly rigid and the central bank policy rate is constrained by the zero lower bound is analyzed in Section 2.4.

[9] Since the central bank chooses its interest rate before the shock is realized, such monetary policy commitment can be called Forward Guidance.

[10] Formally, this means that there is a positive probability that the real interest rate is above zero.

2.2 Model

The economic activities of the four types of agent—entrepreneurs, bankers, households, and the government—are described in Subsections 2.2.1 and 2.2.3. Subsection 2.2.2 describes the macroeconomic shock. The institutional set-up is given in Subsection 2.2.3. The sequence of decisions by the agents and the markets across the two periods ($t = 0, 1$), including all payment processes, are detailed in Subsection 2.2.4. Subsection 2.2.5 defines the notion of equilibrium.

2.2.1 Agents

Entrepreneurs

Two different technologies are employed by firms to transform the investment good into a consumption good. These firms are run by entrepreneurs, who only play a passive role and simply maximize the value of shareholders.

There is a moral hazard technology called hereafter Sector MT or simply MT. Entrepreneurs running the firms employing this technology are subject to moral hazard and need to be monitored.[11] We use[12] $\mathbf{K_M} \in [\mathbf{0}, \mathbf{W}]$ to denote the aggregate amount of investment good invested in MT, where $\mathbf{W} > 0$ denotes the total amount of the investment good in the economy. An investment of $\mathbf{K_M}$ produces $\mathbf{K_M R_M}$ units of the consumption good, where $\mathbf{R_M} > 0$ denotes the real gross rate of return.[13]

There is a frictionless technology referred to hereafter as Sector FT or simply FT. Entrepreneurs running the firms employing this technology are not subject to any

[11] Typically, Sector MT comprises small or opaque firms that cannot obtain direct financing.

[12] To differentiate nominal from real variables—investment or consumption goods—we express the latter in bold characters. Furthermore, to distinguish individual quantities from aggregate quantities, the former are denoted by small letters, the latter by capitals.

[13] We define a real gross rate of return—also called hereafter real gross rate or simply gross rate—as being the amount of the consumption good produced by investing one unit of the investment good. Similarly, we define a nominal gross rate of return—also called hereafter nominal gross rate or simply gross rate—as being the amount of money which has to be repaid to the creditor by the debtor per unit of nominal investment.

moral hazard problem.[14] We use $\mathbf{K_F} \in [\mathbf{0}, \mathbf{W}]$ to denote the aggregate amount of investment good invested in FT and $\mathbf{f}(\mathbf{K_F})$ to denote the amount of consumption good produced by FT. We assume $\mathbf{f}' > 0$ and $\mathbf{f}'' < 0$ as well as the following conditions:

Assumption 1

$$\mathbf{f}'(\mathbf{W}) < \mathbf{R_M} < \mathbf{f}'(\mathbf{0}).$$

In words, the above assumption ensures that the expected total production can never be maximized by allocating the entire amount of the investment good to one sector of production.

Firms in MT and FT are owned by households, and as long as they are positive, the resulting profits from both technologies, denoted by Π_M and Π_F, are paid to owners as dividends. The shareholders' values in nominal terms are given by $\max(\Pi_M, 0)$ and $\max(\Pi_F, 0)$, respectively.

Bankers

There is a continuum of banks labeled $b \in [0, 1]$ and operated by shareholders' value-maximizing bankers.[15] At the very beginning, banks are only labels or indices and offer equity contracts. We assume that each bank receives the same amount of equity financing, denoted by e_B. The aggregate amount is denoted by E_B. As the measure of banks is 1, the aggregate amount is numerically identical to the individual amount e_B. For the time being, we will concentrate on constellations with $E_B > 0$ and thus on circumstances in which banks are founded[16] and can engage in money creation and lending activities.[17] For simplicity, we assume that

[14] Typically, these entrepreneurs run well-established firms that do not need to be monitored for repayment after having borrowed money.

[15] Without loss of generality we assume that bankers maximize the shareholders' value in nominal terms, as this assumption eases the presentation of our results in Chapter 2. In Chapter 3 in Section 3.1, we show that the same model with shareholders' value maximization in real terms is equivalent. However, the maximization of shareholders' value in real terms is more appropriate, as shareholders' utility only depends on real and not nominal variables, and we use this formulation generally in Chapters 3 to 5 unless otherwise specified.

[16] Typically, banks need to have some minimal equity to obtain a banking license.

[17] The case $E_B = 0$ will be discussed in Subsection 2.2.4.

banks can perfectly alleviate the moral hazard problem when investing in MT by monitoring borrowers and enforcing contractual obligations. Moreover, we assume that monitoring costs are zero. Banks provide (nominal) loans to firms in Sector MT at a lending gross rate R_L. The individual and aggregate amounts of loans are denoted by l_M^b and L_M, respectively. We can express the ratio of individual lending by Bank b to average lending by banks as $\alpha_M^b := \frac{l_M^b}{L_M}$.[18]

By granting loans to firms in MT, Bank b simultaneously creates deposits $d_M^b = l_M^b$. We use $D_M = L_M$ to denote aggregate private deposits. d_M^b (or α_M^b) is the distribution of MT firms' deposits across banks. In the course of economic activities, these deposits will be transferred to households that will keep them to buy some amount of the consumption good. We assume that households keep deposits evenly distributed across all banks at all times. For example, they never transfer money from their account at one bank to another bank. Bank owners are protected by limited liability, and as long as they are positive, the resulting profits of Bank b, denoted by Π_B^b, are paid as dividends to owners. The bank shareholders' value and the gross rate of return on equity are given by $\max(\Pi_B^b, 0)$ and $\frac{\max(\Pi_B^b,0)}{E_B}$, respectively.

Households

There is a continuum of identical and risk-neutral[19] households represented by $[0, 1]$. They are the only consuming agents in the economy. We can focus on a representative household initially endowed with $\mathbf{W}$ units of the investment good and ownership of all firms in the economy. It sells a part of its endowment of the investment good to firms in MT against bank deposits. Then it chooses a portfolio of bank equity and bank deposits and lends the remaining endowment of the investment good directly to firms in FT against bonds.[20] The dividends

[18]As the continuum of banks is of a measure equal to one, the aggregate lending L_M can also be interpreted as the average lending per bank and α_M^b as the ratio of individual lending to average lending.

[19]Household risk aversion would require more elaborate portfolio decisions. A detailed analysis is given in Chapter 4 in Section 4.1.

[20]Alternatively, we could assume that firms in FT are only financed by equity. Since households are the only agents financing firms in FT and financing is frictionless, they are indifferent between different capital structures, and this would not affect our results.

from firm ownership and bank equity investment as well as the repayments from bonds and bank deposits are used to buy the consumption good. The details of this process are set out in Subsection 2.2.4.

2.2.2 Macroeconomic Shock

A macroeconomic shock $s = l, h$ occurs at the beginning of Period $t = 1$ after the investment good has been allocated to the two technologies during Period $t = 0$. It affects the real gross rate of return from production in Sector MT.[21] Specifically, an investment of $\mathbf{K_M}$ in MT produces $\mathbf{K_M R_M^h}$ and $\mathbf{K_M R_M^l}$ with probability σ in the good state and $1 - \sigma$ in the bad state of the world, respectively $(0 < \sigma < 1)$,[22] where $\mathbf{R_M^s}$ is the real gross rate of return in State s $(s = l, h)$. We assume that $0 < \mathbf{R_M^l} < \mathbf{R_M^h}$.

Banks monitor entrepreneurs running firms in MT and plagued by moral hazard (see Subsection 2.2.1) and offer state-contingent loans with nominal lending gross rates $(R_L^s)_{s=l,h}$. The lending interest rates are given by $(R_L^s - 1)_{s=l,h}$.

In general, we assume that in all contracts during Period $t = 0$ all nominal gross rates to be repaid during Period $t = 1$ can be written contingently on the outcome of the macroeconomic shock. This reflects our assumption of complete markets. As the output in FT is not stochastic, the real gross rate of return on bonds $\mathbf{R_F}$ is risk-free.

We will use interchangeably the notations $\mathbb{E}[X]$ and $\overline{X}$ to denote the expected value of some real or nominal variable X. Finally, taking into account the occurrence of a macroeconomic shock, we restate Assumption 1 as follows:

$$\mathbf{f'(W)} < \overline{\mathbf{R}}_\mathbf{M} < \mathbf{f'(0)}.$$

[21]Letting the macroeconomic shock impact Sector FT would not change our results qualitatively but would complicate the analysis. Details are given in Chapter 3 in Section 3.3.

[22]Throughout the thesis, we also use the notation σ^s to denote the probability that State s occurs.

2.2.3 Institutional Set-up

We purposely impose favorable conditions on the working of the monetary architecture and the public authorities involved.

Monies and Interbank Market

Two types of money (privately created and publicly created monies) and three forms of money creation are representative of the modern money architecture.[23] A first type of money is privately created by commercial banks through loans to firms, held at banks in the form of deposits by households or firms and destroyed when households buy bank equity and when firms repay loans. This type of money can also be privately created by commercial banks when they grant loans to other banks. It is held at the former banks by the latter in the form of deposits. We call the first type of money "private deposits". A second type of money is publicly created by the central bank via loans to banks. It is held at the central bank in the form of deposits by banks. We call this second type of money "CB deposits".

The essential rules linking publicly created and privately created monies are illustrated as follows. We impose a deposit-in-advance constraint, which means that all physical goods traded in markets have to be paid for with private bank deposits. When households use private deposits to make payments, these deposits typically move from one bank (account of buyer, say b_j) to another bank (account of seller, say b_i). To settle the transfer of private deposits, Bank b_j becomes liable to b_i. These banks now have two options. Either b_j obtains a loan from Bank b_i, or it refinances itself at the central bank and transfers the central bank money received, CB deposits, to Bank b_i. The institutional rule is that one unit of central bank money settles one unit of liabilities of privately created money, and both types of money have the same unit. This fixes the "exchange rate" between central bank money and privately created money at 1.[24] Finally, we assume that there are no transaction costs for paying with private or CB deposits.

[23] We abstract from banknotes and coins, as they are not used in our economy and all payments are done by transferring deposits.

[24] In principle, this exchange rate could be fixed at any other level.

The prices of the investment and the consumption goods in units of both privately created and publicly created monies are denoted by p_I and $(p_C^s)_{s=l,h}$, respectively.

We integrate an interbank market. In this market banks can lend to and borrow from each other at the same gross rate. This lending / borrowing gross rate and the amount of lending can be conditioned on the macroeconomic shock. Banks cannot discriminate between deposits owned by households and deposits owned by other banks. As a consequence, the gross rate at which banks can lend to, and borrow from, each other is equal to the households' deposit gross rate, which we denote by $(R_D^s)_{s=l,h}$. The interbank market works as follows: At any time, banks can repay their central bank liabilities[25] by using their deposits at other banks, repay their interbank liabilities by using CB deposits, and require their debtor banks to repay their interbank liabilities with CB deposits.[26] Accordingly, as long as banks can refinance themselves at the central bank, interbank borrowing is not associated with default risk. Moreover, we assume that no bank participating in the interbank market makes any loss by doing so. Finally, the following tie-breaking rule simplifies the analysis: If banks are indifferent between lending to other banks and depositing money at the central bank, they will choose the latter.

Role of Public Authorities

Two public authorities—a central bank and a government—ensure the functioning of the monetary architecture. These authorities fulfill three roles.

First, banks can obtain loans from the central bank and can thus acquire CB deposits at the same policy gross rates $(R_{CB}^s)_{s=l,h}$ at any stage of economic activities, where $(R_{CB}^s - 1)_{s=l,h}$ are the central bank interest rates. This assumption implies that banks do not have to worry about the exact flow of funds at any particular stage. Only their net position at the final stage matters.[27] Banks can also borrow from, or deposit at, the central bank contingently on the state of the world s.

Second, the government impose heavy penalties on those bankers whose bank

[25] Liabilities against the central bank are called "CB liabilities" in the remainder of the thesis.

[26] The mechanisms by which banks become liable to other banks or hold assets against them are explained in detail in Appendix 2.D.

[27] Note that this assumption also rules out the possibility of bank runs.

defaults on obligations to any public authority.[28] As a consequence, no bank will default on its liabilities against the central bank in any state of the economy. Moreover, we assume that the central bank ensures the repayment of interbank loans by taking them on its balance sheet if the counterparty bank were to default. By this assumption, heavy penalties on bankers whose bank defaults against the central bank translate directly into heavy penalties on bankers whose bank defaults on other banks. A bank, however, may default on households' deposits.

In such cases, the government has a third role. It makes deposits safe by levying lump-sum taxes on households to bail out banks that default on households' deposits. In practice, making deposits safe is a necessary condition for their use as money. Later we will introduce a third public authority, i.e. bank regulators, and bank regulation in the form of a capital requirement.

We explore equilibrium outcomes for different policies—the central bank policy gross rate and the capital requirement—and for each combination of these outcomes we determine the associated level of welfare expressed in terms of household consumption. We assume that the central bank and the bank regulators aim at maximizing the welfare of households.

2.2.4 Timeline of Events

An overview of the timeline of events is given in Figure 2.1.

We next describe the timeline of events in detail. For this purpose we divide each period into several stages.

Period $\mathbf{t = 0}$

It is convenient to describe the sequence of economic activities via the balance sheets of households and banks. The economy starts with the following balance sheets:

[28] As banks are able to borrow from the central bank at any time, it is sufficient to assume that heavy penalties are imposed on those bankers whose bank defaults on obligations to the central bank.

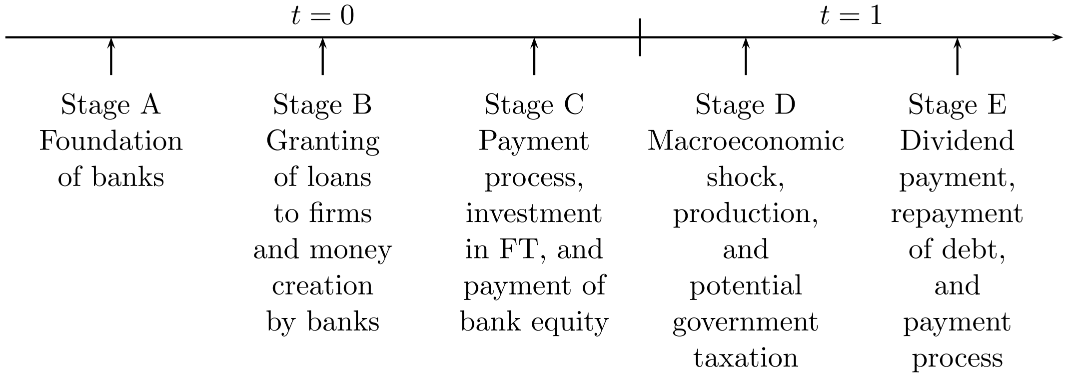

Figure 2.1: Timeline of events. Source: Own illustration.

Households		Bank b	
W	E_H	0	0

Table 2.1: Balance sheets at the beginning of Period $t = 0$. Source: Own illustration.

E_H denotes the households' equity, which represents the ownership of the investment good and both production technologies at the beginning of Period $t = 0$.[29]

Stage A: Foundation of Banks Either banks are not founded because no household invests in bank equity and the only possible allocation is given in Subsection 2.2.4, or households found banks by pledging to convert a predefined share $\varphi \in (0, 1]$ of their initial deposits, which we denote by D_M, into an amount $E_B = \varphi D_M$ of bank equity before production in Stage C. When banks are founded, the gross rate of return on equity is equal to the shareholders' value per unit of equity, and it is denoted by $R_E^{b,s} = \frac{\max(\Pi_B^{b,s}, 0)}{e_B}$. In the remainder of Subsection 2.2.4, we focus on the case where banks are founded (unless specified otherwise).

[29]Note that households also own firms in Sectors MT and FT and may receive dividends from firms' profits.

Stage B: Granting of Loans by Banks Bank b grants loans $l^b_M = \alpha^b_M L_M$ to firms in MT at the contingent lending gross rates $(R^s_L)_s$, which simultaneously creates d^b_M private deposits at Bank b and aggregate private deposits D_M. The resulting balance sheets are given in Table 2.2.

Households		Bank b	
W	E_H	l^b_M	d^b_M

Table 2.2: Balance sheets at the end of Stage B. Source: Own illustration.

Stage C: Payment Process, Investment in FT, and Payment of Bank Equity Households sell an amount of the investment good to firms in MT. Then they invest in FT by buying S_F bonds denominated in real terms at the gross rate of return $\mathbf{R_F}$, meaning that such a bond costs one unit of investment good and promises the delivery of $\mathbf{R_F}$ units of the consumption good once production has occurred.[30] Finally, at the end of Period $t = 0$, households pay for the equity E_B pledged in Stage A with deposits, which reduces the amount of deposits in the economy. The resulting amount of deposits is denoted by d_H for an individual bank and $D_H = L_M - E_B$ for the aggregate banking system. At the end of Stage C and depending on their lending decisions, some banks labeled b_i have claims $d^{b_i}_{CB}$, and the other banks have liabilities $l^{b_j}_{CB}$ against the central bank. These processes are detailed in Appendix 2.A. The balance sheets are displayed in Table 2.3.

A summary of the agents' interactions during Period $t = 0$ is given in Figure 2.2.

Period $\mathbf{t = 1}$

In Period $t = 1$ we distinguish between two cases, when either no bank is founded by households, or banks are founded by households. The latter case can again be divided into two subcases: Either no bank defaults, or some banks default.

[30] In practice, such bonds are called "inflation-indexed bonds". Using bonds denominated in nominal terms does not change the results qualitatively but complicates the analysis, as one has to verify that firms do not default. Details are given in Chapter 3 in Section 3.2.

Households		Bank b_i		Bank b_j	
S_F		$d^{b_i}_{CB}$			$l^{b_j}_{CB}$
D_H	E_H	$l^{b_i}_M$	d_H	$l^{b_j}_M$	d_H
E_B			e_B		e_B

Table 2.3: Balance sheets at the end of Stage C. Source: Own illustration.

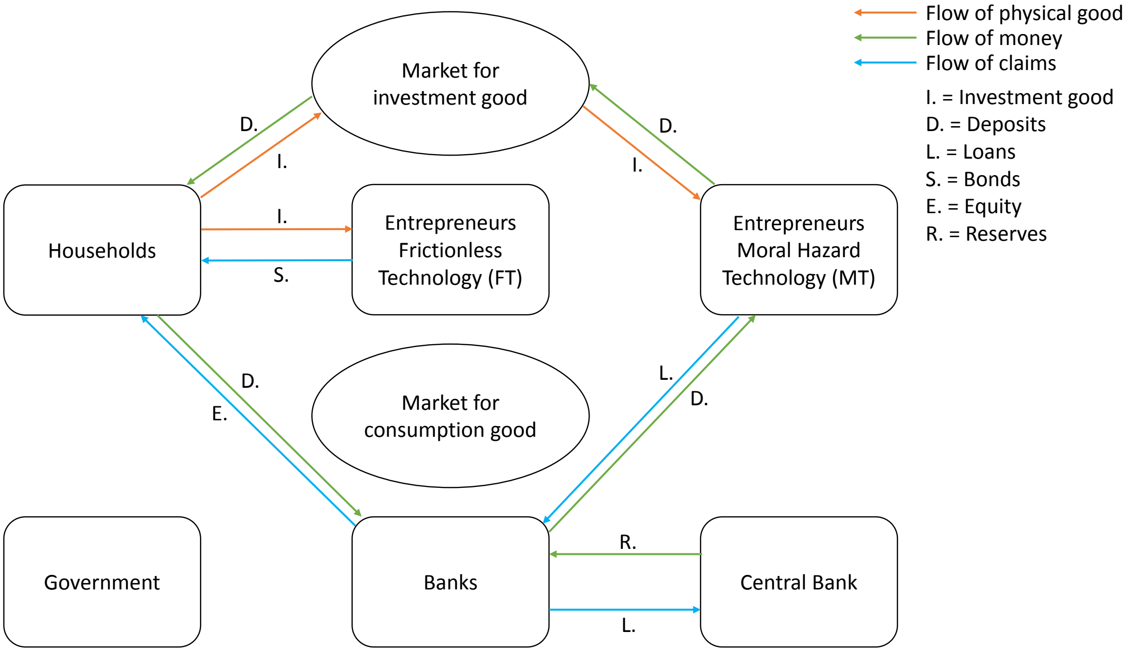

Figure 2.2: Flows and interactions during Period $t = 0$. Source: Own illustration.

Case I: No Bank Is Founded When no bank is founded, we have $E_B = 0$. This could constitute an equilibrium, as no household can found a bank individually. We call this an *equilibrium without banks*. In such circumstances, no money creation takes place, the central bank is inactive, no investment in MT is possible, and the investment good is allocated entirely to Sector FT, which leads to the following allocation:[31]

$$\mathbf{K_M^*} = \mathbf{0},$$
$$\mathbf{K_F^*} = \mathbf{W},$$

where * denotes equilibrium variables. This is an inefficient allocation, as households are risk-neutral and Assumption 1 stipulates that $\mathbf{f}'(\mathbf{W}) < \overline{\mathbf{R}}_\mathbf{M}$.

Case II: Banks are Founded When banks are founded, they grant loans to firms in MT, and we can considerably simplify the description of Period $t = 1$ by making the observation given by Lemma 1:[32]

Lemma 1
An equilibrium with banks and hence with positive lending to Sector MT requires

$$\mathbf{R_M^s} p_C^s = R_L^s p_I$$

and implies $\Pi_M^s = 0$ *for* $s = l, h$.

Lemma 1 is a direct consequence of the MT technology. If for some state s $\mathbf{R_M^s} p_C^s > R_L^s p_I$, firms in MT would demand an infinite amount of loan, as their shareholders' value per loan unit would be positive in one state, be at least zero in the other state,[33] and scale with the level of borrowing. If $\mathbf{R_M^s} p_C^s < R_L^s p_I$ for both states of the world, firms would forgo borrowing from banks.[34]

Subcase II.a: No Bank Defaults. Suppose next that no bank defaults. Then the following stages occur:

[31]Note that no bank deposits are needed to buy the output from Sector FT, as bonds are in real terms and are repaid in terms of the output.

[32]This observation enables us to rule out considerations in which firms in MT would make positive profits or go bankrupt.

[33]Since entrepreneurs running firms in Sector MT do not have any wealth, they have zero profit if they cannot repay and thus default against banks.

[34]Other arguments could be used to derive the zero profit condition in Sector MT. As banks monitor entrepreneurs running firms in Sector MT, they can offer them state-contingent repayment gross rates of return, and are thus able to extract the entrepreneurs' entire surplus.

Stage D: Production. The macroeconomic state s is realized. Firms produce and repayments contingent on s fall due. Using bank balance sheets in Table 2.3 as well as the expression of the net position of Bank b against the central bank given by Equation (2.11) in Appendix 2.A, we derive the expression of Bank b's profits as follows:[35]

$$\begin{aligned}\Pi_B^{b,s} &= (1-\alpha_M^b)L_M R_{CB}^s + \alpha_M^b L_M R_L^s - d_H R_D^s \\ &= (1-\alpha_M^b)L_M R_{CB}^s + \alpha_M^b L_M R_L^s - (L_M - E_B)R_D^s \\ &= \alpha_M^b L_M (R_L^s - R_{CB}^s) + L_M(R_{CB}^s - R_D^s) + E_B R_D^s. \end{aligned} \tag{2.1}$$

Profits from firms in the real sector are given by

$$\begin{aligned}\Pi_M^s &= \mathbf{K_M}(\mathbf{R_M^s} p_C^s - R_L^s p_I), \\ \Pi_F^s &= (\mathbf{f}(\mathbf{K_F}) - \mathbf{K_F}\mathbf{R_F})p_C^s.\end{aligned}$$

The balance sheets are given in Table 2.4, where R_H^s denotes the resulting gross rate of return on household ownership of the investment good and of both production technologies.

Stage E: Dividend Payment, Repayment of Debt, and Payment Process. Households obtain dividends from their equity investment[36] and buy the amount of consumption good produced. All debts are paid back. These processes are detailed in Appendix 2.B. The resulting balance sheets are given in Table 2.5.

Subcase II.b: Some Banks Default. Finally, we consider the scenario where some banks default. In this case, Stages D and E have to be modified as follows:

Stage D: Production and Government Taxation. The macroeconomic state s is realized. Firms produce, and repayments fall due. Two cases can occur. First, if $-d_H R_D^s \leq \Pi_B^{b,s} < 0$, Bank b defaults on households but not on the central

[35]Note that profits are non-negative here, as we have assumed that banks do not default. In the case of default by Bank b, $\Pi_B^{b,s}$ will be negative, but the shareholders' value will be equal to zero, and bank shareholders will not be affected by the magnitude of $\Pi_B^{b,s}$, as they are protected by limited liability.

[36]Banks pay dividends to households in anticipation of the repayment of loans by firms in Sector MT.

Households	
$S_F\mathbf{R_F}$	$E_H R_H^s$
$D_H R_D^s$	
$E_B R_E^s$	
Π_F^s	

Bank b_i	
$d_{CB}^{b_i} R_{CB}^s$	
$l_M^{b_i} R_L^s$	$d_H R_D^s$
	$e_B R_E^{b_i,s}$

Bank b_j	
	$l_{CB}^{b_j} R_{CB}^s$
$l_M^{b_j} R_L^s$	$d_H R_D^s$
	$e_B R_E^{b_j,s}$

Table 2.4: Balance sheets at the end of Stage D if no bank defaults. Source: Own illustration.

Households	
$\mathbf{K_M R_M^s}$	$E_H R_H^s$
$\mathbf{f}(\mathbf{K_F})$	

Bank b	
0	0

Table 2.5: Balance sheets at the end of Stage E if no bank defaults. Source: Own illustration.

bank. Second, if $\Pi_B^{b,s} > 0$, Bank b does not default. We note that the case $\Pi_B^{b,s} < -d_H R_D^s < 0$ cannot occur, as banks would default on households and the central bank. Due to the heavy penalties incurred for default against governmental authorities banks will avoid the latter case under all circumstances.

Consider now a non-defaulting bank b. If $R_{CB}^s > R_L^s$ for some state s, there then exists an upper bound on α_M^b given by

$$\alpha_M^b \leq \alpha_{DH}^s := \frac{R_{CB}^s - (1-\varphi)R_D^s}{R_{CB}^s - R_L^s},$$

such that this bank does not default on households in State s. α_{DH}^s is the critical amount of money creation at which a bank is just able to pay back depositors in State s. α^s_{DH} is obtained from Equation (2.1) by setting $\Pi^{b,s}_B = 0$ and using $\varphi = \frac{E_B}{L_M}$. From now on, consider a defaulting bank b. If $R^s_{CB} > R^s_L$ for some state s, there exist a lower bound α^s_{DH} and an upper bound α^s_{DCB} for α^b_M given by

$$\alpha^s_{DH} < \alpha^b_M \leq \alpha^s_{DCB} := \frac{R^s_{CB}}{R^s_{CB} - R^s_L},$$

which mark two default points. For $\alpha^b_M \in (\alpha^s_{DH}, \alpha^s_{DCB}]$, Bank b defaults against households but not against the central bank in State s. For $\alpha^b_M > \alpha^s_{DCB}$, the bank would default against households *and* the central bank in State s. α^s_{DCB} is the critical amount of money creation at which a bank is just able to pay back the central bank in State s. α^s_{DCB} is obtained from Equation (2.1) by setting $\Pi^{b,s}_B = -D_H R^s_D$. The lump-sum tax levied to bail out Bank b in State s is denoted by $t^{b,s}$. Aggregate tax payments in State s by households are then given by

$$T^s = \int_{b \in [0,1]} t^{b,s} \mathrm{d}b.$$

Furthermore, we use $\Pi^{+,s}_B$ to denote the aggregate profits of non-defaulting banks in State s. The balance sheets possible are given in Table 2.6.

Households		Bank $b_{i'}$		Bank $b_{j'}$	
$S_F \mathbf{R_F}$	$E_H R^s_H$	$d^{b_{i'}}_{CBT}$			$l^{b_{j'}}_{CBT}$
$D_H R^s_D - T^s$		$l^{b_{i'}}_M R^s_L$	$d_H R^s_D - T^s$	$l^{b_{j'}}_M R^s_L$	$d_H R^s_D - T^s$
$\Pi^{+,s}_B$			$\Pi^{b_{i'},s}_B + t^{b_{i'},s}$		$\Pi^{b_{j'},s}_B + t^{b_{j'},s}$
Π^s_F					

Table 2.6: Balance sheets at the end of Stage D if some banks default. Source: Own illustration.

In Table 2.6, the labels $b_{i'}$ and $b_{j'}$ denote banks with a non-negative and negative net position against the central bank, respectively. The exact expressions of $d^{b_{i'}}_{CBT}$ and $l^{b_{j'}}_{CBT}$ are not needed for the subsequent analysis, but for completeness they are given in Appendix 2.C. We note that the balance sheets in Table 2.6 are structurally identical to the ones in Subcase II.a of Subsection 2.2.4. Therefore, the description of Stage E is similar to the one laid out in Appendix 2.B.

A summary of the agents' interactions during Period $t = 1$ is given in Figure 2.3.

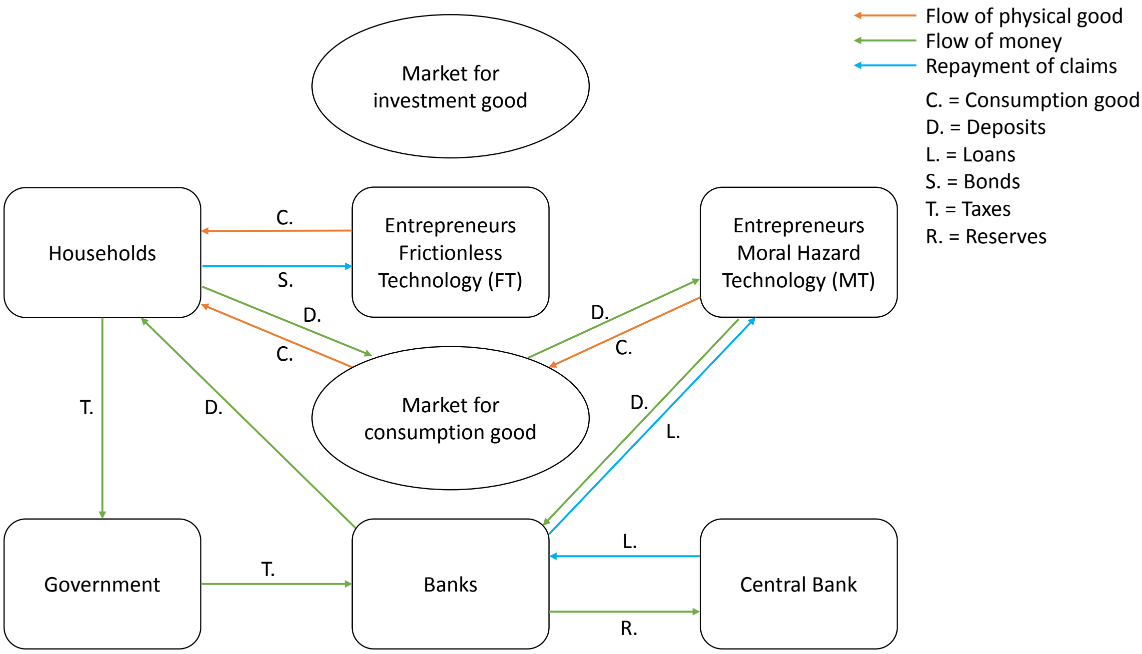

Figure 2.3: Flows and interactions during Period $t = 1$. Source: Own illustration.

2.2.5 Definition of an Equilibrium with Banks

We look for symmetric equilibria with banks in the sequential market process described in Subsection 2.2.4. In a symmetric equilibrium with banks, all banks take the same decision regarding money creation and lending and thus have identical balance sheets in equilibrium. Moreover, the policy gross rates $(R^s_{CB})_s$ are set by the central bank, so equilibria with banks are dependent on this choice.

Definition 1

Given the central bank policy gross rates $(R^s_{CB})_s$*, a symmetric equilibrium with* *banks in the sequential market process described in Subsection 2.2.4 is defined as a tuple*

$$\mathcal{E} := \Big((R_E^s)_s, (R_D^s)_s, (R_L^s)_s, \mathbf{R_F}, \\ p_I, (p_C^s)_s, \\ E_B, D_H, (\tilde{D}_H^s)_s, L_M, S_F, \\ \mathbf{K_M}, \mathbf{K_F} \Big)$$

consisting of positive and finite gross rates of return, prices, savings, bank deposits D_H *at the end of Stage C of Period* $t = 0$*, bank deposits* $(\widetilde{D}_H^s)_s$ *in Stage E of Period* $t = 1$*, and the corresponding physical investment allocation, such that*

- *households hold some private deposits* $D_H > 0$ *at the end of Stage C,*[37]
- *households maximize their expected utility*

$$\max_{\{D_H, E_B, S_F\}} \left\{ E_B \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] + D_H \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] + \mathbf{f}(S_F) \right\}$$
$$s.t. \quad E_B + D_H + p_I S_F = p_I \mathbf{W},$$

  taking gross rates of return $(R_E^s)_s$ *and* $(R_D^s)_s$ *as well as prices* p_I *and* $(p_C^s)_s$ *as given,*

- *firms in MT and FT as well as each bank* $b \in [0, 1]$ *maximize their expected shareholders' value,*[38] *given respectively by*

$$\max_{\mathbf{K_M} \in [\mathbf{0}, \mathbf{W}]} \{\mathbb{E}[\max(\mathbf{K_M}(\mathbf{R_M^s} p_C^s - R_L^s p_I), 0)]\},$$
$$s.t.\ \mathbf{R_M^s} p_C^s = R_L^s p_I \text{ for } s = l, h,$$
$$\max_{\mathbf{K_F} \in [\mathbf{0}, \mathbf{W}]} \{\mathbb{E}[\max((\mathbf{f}(\mathbf{K_F}) - \mathbf{K_F} \mathbf{R_F}) p_C^s, 0)]\},$$
$$\text{and} \quad \max_{\alpha_M^b \geq 0} \{\mathbb{E}[\max(\alpha_M^b L_M (R_L^s - R_{CB}^s) + L_M (R_{CB}^s - R_D^s) + E_B R_D^s, 0)]\},$$

[37] As deposits are the only means of payment, we rule out knife-edge equilibria with banks in which private money creation at the end of Period $t = 0$ is zero.

[38] In our setting the maximization of profits in nominal terms by firms and by banks is qualitatively equivalent to the maximization of profits in real terms. Details are given in Chapter 3 in Section 3.1.

taking gross rates of return $(R_D^s)_s$, $(R_L^s)_s$, *and* $\mathbf{R_F}$ *as well as prices* p_I *and* $(p_C^s)_s$ *as given,*

- *all banks choose the same level of money creation, and*
- *markets for investment and consumption goods clear in each state.*

In the remainder of this thesis we will use Superscript * to denote equilibrium variables. Henceforth, for ease of presentation, an equilibrium with banks given $(R_{CB}^s)_s$ is a symmetric equilibrium with banks given $(R_{CB}^s)_s$ in the sense of Definition 1.

2.3 Equilibria with Banks

2.3.1 Individually Optimal Choices

In this subsection we prepare the characterization of equilibria with banks by determining the individually optimal choices of banks, households, and firms. We first establish the way in which deposit gross rates are related to policy gross rates. Since banks can grant loans to, or borrow from, other banks, we obtain

Lemma 2
In any equilibrium with banks, the nominal lending gross rates on the interbank market satisfy

$$R_D^{s*} = R_{CB}^s \quad \textit{for all states } s = l, h.$$

The proof of Lemma 2 can be found in Appendix 2.G. It is based on a simple arbitrage argument: Any differential in the gross rates could be used in the interbank market by borrowing or lending to infinitely increase the expected shareholders' value.

We next investigate the optimal choice of money creation by an individual bank. For convenience, we denote circumstances in which no finite amount of money creation is optimal by "∞". Then we obtain

Proposition 1
If $R_D^s = R_{CB}^s$ in all states $s = l, h$, the privately optimal amounts of money creation and lending by an individual bank are represented by a correspondence denoted by[39] $\hat{\alpha}_M : \mathbb{R}^4_{++} \times (0,1) \to \mathcal{P}(\mathbb{R} \cup \{+\infty\})$ *and given by*

$$\hat{\alpha}_M\big((R_L^s)_s, (R_{CB}^s)_s, \varphi\big) =$$
$$\begin{cases} \{+\infty\} & \text{if } R_L^s \geq R_{CB}^s \text{ for all states } s = l,h \\ & \text{with at least one strict inequality,} \\ \{\alpha_{DCB}^l\} & \text{if } (\overline{R}_L \geq \overline{R}_{CB},\ R_L^l < R_{CB}^l, \text{ and } R_{CB}^h < R_L^h) \text{ or} \\ & \text{if } (\overline{R}_L < \overline{R}_{CB},\ R_{CB}^h < R_L^h, \text{ and } \varphi < \left(\frac{\sigma}{1-\sigma}\right)\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}), \\ \{\alpha_{DCB}^h\} & \text{if } (\overline{R}_L \geq \overline{R}_{CB},\ R_L^h < R_{CB}^h, \text{ and } R_{CB}^l < R_L^l) \text{ or} \\ & \text{if } (\overline{R}_L < \overline{R}_{CB},\ R_{CB}^l < R_L^l, \text{ and } \varphi < \left(\frac{1-\sigma}{\sigma}\right)\frac{R_L^l - R_{CB}^l}{R_{CB}^h - R_L^h}), \\ [0,+\infty) & \text{if } R_L^s = R_{CB}^s \text{ for all states } s = l,h, \\ \{0, \alpha_{DCB}^l\} & \text{if } \overline{R}_L < \overline{R}_{CB},\ R_{CB}^h < R_L^h, \text{ and } \varphi = \left(\frac{\sigma}{1-\sigma}\right)\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}, \\ \{0, \alpha_{DCB}^h\} & \text{if } \overline{R}_L < \overline{R}_{CB},\ R_{CB}^l < R_L^l, \text{ and } \varphi = \left(\frac{1-\sigma}{\sigma}\right)\frac{R_L^l - R_{CB}^l}{R_{CB}^h - R_L^h}, \\ \{0\} & \text{if } (R_L^s \leq R_{CB}^s \text{ for all states } s = l,h \\ & \text{with at least one strict inequality) or} \\ & \text{if } (\overline{R}_L < \overline{R}_{CB},\ R_{CB}^h < R_L^h, \text{ and } \left(\frac{\sigma}{1-\sigma}\right)\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l} < \varphi) \text{ or} \\ & \text{if } (\overline{R}_L < \overline{R}_{CB},\ R_{CB}^l < R_L^l, \text{ and } \left(\frac{1-\sigma}{\sigma}\right)\frac{R_L^l - R_{CB}^l}{R_{CB}^h - R_L^h} < \varphi). \end{cases}$$

The proof of Proposition 1 can be found in Appendix 2.G. There are several observations to make. First, the banks' behavior depends only on $(R_L^s - R_{CB}^s)_{s=l,h}$, which is the intermediation margin, on average lending by banks, and on their capital structure φ. If the intermediation margin is zero in all states, it is obvious that banks are indifferent between all lending levels. For positive intermediation margins in all states, banks would like to grant as many loans as possible. For negative intermediation margins, banks are not willing to grant any loans. Finally, if the intermediation margin is positive in one state and negative in the other state, banks can use shareholders' limited liability and depositors' bail-out by the government to maximize their expected gross rate of return on equity by defaulting against households in one state and by making large profits in the other. This

[39] If X denotes a set, we use $\mathcal{P}(X)$ to denote the power set of X.

strategy is only profitable in the following two cases: (i) when the expected intermediation margin is non-negative, meaning that banks can weakly increase their expected shareholders' value even if they do not use limited liability and depositors' bail-out by the government, (ii) when the expected intermediation margin is negative and banks can sufficiently leverage on limited liability, which occurs when the banks' equity ratio is sufficiently low. Next we turn to the households' investment behavior. We obtain

Lemma 3
The representative household's optimal portfolio choice depends solely on the comparison of expected real gross rates of return $\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]$, $\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]$, $\frac{\mathbf{f}'(0)}{p_I}$, *and* $\frac{\mathbf{f}'(\mathbf{W})}{p_I}$ *when choosing* E_B, D_H, *and* S_F.

The correspondences representing households' optimal choices for different constellations of these expected real gross rates of return are given in Lemma 6 in Appendix 2.E.

We next turn to the firms' behavior.

Lemma 4
Demands for the investment good by firms in MT and FT are represented by two correspondences denoted by $\hat{\mathbf{K}}_{\mathbf{M}} \in \mathcal{P}(\mathbb{R} \cup \{+\infty\})$ *and* $\hat{\mathbf{K}}_{\mathbf{F}} : \mathbb{R}_{++} \to \mathcal{P}([\mathbf{0}, \mathbf{W}])$, *respectively and given by*

$$\hat{\mathbf{K}}_{\mathbf{M}} = [\mathbf{0}, \mathbf{W}]$$

$$\textit{and} \quad \hat{\mathbf{K}}_{\mathbf{F}}(\mathbf{R}_{\mathbf{F}}) = \begin{cases} \{\mathbf{0}\} & \textit{if } \mathbf{f}'(\mathbf{0}) \leq \mathbf{R}_{\mathbf{F}}, \\ \{\mathbf{W}\} & \textit{if } \mathbf{R}_{\mathbf{F}} \leq \mathbf{f}'(\mathbf{W}), \\ \{\mathbf{f}'^{-1}(\mathbf{R}_{\mathbf{F}})\} & \textit{otherwise.} \end{cases}$$

The proof of Lemma 4 can be found in Appendix 2.G. We note that in Sector MT, firms are indifferent between any investment level $\mathbf{K}_{\mathbf{M}}$, as the condition in Lemma 1, $\mathbf{R}^{\mathbf{s}}_{\mathbf{M}} p_C^s = R_L^s p_I$ for $s = l, h$, implies that these firms make zero profits at any level of $\mathbf{K_M}$.

2.3.2 Characterization of Equilibria with Banks

The preceding lemmata enable us to characterize all equilibria with banks.

Theorem 1
Given the policy gross rates $(R^s_{CB})_{s=l,h}$, all equilibria with banks take the following form:

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^s, \quad \mathbf{R_F^*} = \overline{\mathbf{R}}_\mathbf{M}, \tag{2.2}$$

$$p_I^* = p, \quad p_C^{s*} = p\frac{R_{CB}^s}{\mathbf{R_M^s}}, \tag{2.3}$$

$$E_B^* = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big), \quad D_H^* = (1-\varphi^*)p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big), \tag{2.4}$$

$$\tilde{D}_H^{s*} = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big)R_{CB}^s, \tag{2.5}$$

$$L_M^* = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big), \quad S_F^* = \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big), \tag{2.6}$$

$$\mathbf{K_M^*} = \mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big), \quad \mathbf{K_F^*} = \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big), \tag{2.7}$$

where the price of the investment good denoted by $p \in (0, +\infty)$ and the aggregate equity ratio $\varphi^* \in (0,1)$ are arbitrary. The equilibrium profits of firms and banks are given by

$$\Pi_M^{s*} = 0, \quad \Pi_F^{s*} = p\frac{R_{CB}^s}{\mathbf{R_M^s}}\left(\mathbf{f}\Big(\mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big) - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\overline{\mathbf{R}}_\mathbf{M}\right), \tag{2.8}$$

$$\Pi_B^{s*} = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big)R_{CB}^s. \tag{2.9}$$

The proof of Theorem 1 can be found in Appendix 2.G.

We now look at the equilibrium conditions in detail. First, all nominal gross rates are equal to the policy gross rates set by the central bank, as expressed in (2.2). The equilibrium with banks is unique in real terms, i.e. the physical investments in both sectors expressed in (2.7), and thus with respect to the real values of lending and savings expressed in (2.6), where we divide L_M^* by p.

As expressed in (2.4) the initial split of investments in banks into deposits and equity is indeterminate. In fact, in an equilibrium with banks any capital structure of banks can occur. Equation (2.5) reflects macroeconomic uncertainty, as the dividends and the deposit gross rates depend on the state of the world. Equations (2.8) and (2.9) represent the profits of firms and banks. The representative firm's profits in Sector FT are paid in terms of the consumption good, while banks' dividends are paid in the form of bank deposits.

Finally, the second equation in (2.3) relates the prices of the consumption good in different states to the price of the investment good. The latter is not determinate. The economic system is nominally anchored by the price of the investment good and by the central bank interest rate. While these parameters determine prices and interest rates, the asset structure and the payment processes are additionally determined by the capital structure of banks.

Here, more remarks are in order. First, no bank defaults in equilibrium. Indeed, the profits of any bank in State s are given by $\varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\Big) R^s_{CB}$ and are thus positive. The reason is twofold. On the one hand, loan interest rates equal deposit interest rates in each state of the world. On the other hand, low gross rates of return $\mathbf{R}^{\mathbf{l}}_{\mathbf{M}}$ trigger a high price p^{l*}_C for the consumption good, which enables firms in Sector MT to pay back their loans, which, in turn, enables banks to pay back depositors.

Second, the theorem shows that in any equilibrium with banks, private money creation is naturally limited. Since $R^{s*}_L = R^s_{CB}$ in both states $s = l, h$, banks have no incentive to increase money creation, as they would be forced to refinance themselves at the gross rates $(R^s_{CB})_s$ to cover additional money creation.

Third, the capital structure of banks has no impact on the physical investment allocation, so there is no need to regulate bank equity capital. Fourth, the physical investment allocation is independent of the central bank's policy gross rates. Monetary policy is neutral.

There are important implications and a variety of further consequences of Theorem 1, which we summarize in the next subsection.

2.3.3 Welfare Properties and Implications

We start with the characterization of the optimal investment allocation. The social planner's problem is given by

$$\begin{aligned} &\max_{(\mathbf{K_M},\mathbf{K_F})} \mathbb{E}[\mathbf{K_M}\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F})] \\ \text{s.t.} \quad &\mathbf{K_M} + \mathbf{K_F} = \mathbf{W}. \end{aligned}$$

It is clear that household utility is maximized at $\mathbf{K_F^{FB}} := \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})$. From Theorem 1, we immediately obtain

Corollary 1
Given any policy gross rates $(R^s_{CB})_s$, the equilibria with banks yield the first-best allocation.

As a direct consequence, the central bank is indifferent between policy gross rates $(R^s_{CB})_{s=l,h}$, as they all implement the first-best allocation. Essentially, Theorem 1 is a first welfare theorem for an economy with private money creation. It is a benchmark for the results we derive in the next section.

We stress that the welfare theorem does not depend on whether the policy gross rates—and as a consequence, all nominal interest rates—depend on the state of the world. Indeed, another immediate consequence is given by

Corollary 2
Suppose that R^s_{CB} is the same in both states s of the world. Then the nominal lending and deposit gross rates are not contingent on the states of the world, and the resulting allocation is first-best.

The corollary implies that the nominal gross rate of return on deposits does not need to depend on the macroeconomic shock to guarantee the first-best allocation. The reason is that in the event of a negative macroeconomic shock, firms in Sector MT compensate for lower real production gross rates of return by higher prices for the consumption good, thereby avoiding default against banks and rendering non-contingent deposits safe even without government intervention.[40] The reason why the prices of the consumption good increase when a negative macroeconomic shock occurs is detailed below.

The equilibria with banks described in Theorem 1 are indeterminate in two respects, with regard to (a) the price of the investment good and (b) the capital structure of banks. Regarding the former, it simply represents a price normalization problem, and we can set $p_I = 1$ without loss of generality. The indeterminacy of the capital structure in equilibrium is a macroeconomic manifestation of the Modigliani-Miller Theorem. As banks do not default in equilibrium and the gross rates of return on equity and deposits are the same, households are indifferent between equity and deposits. Moreover, different capital structures of banks have no impact on money creation and lending by banks. Finally, we note in the following corollary that with price normalization $p_I = 1$ and some capital structure choice φ^*, all equilibrium values are uniquely determined.

Corollary 3
Given $p_I = 1$ and some $\varphi^* \in (0, 1)$, all equilibrium values are uniquely determined when the central bank sets the policy gross rates $(R^s_{CB})_s$.

The relationship between the policy gross rates and the prices of the consumption good in different states of the world is contained in the following corollary:

Corollary 4

(i) If R^s_{CB} does not depend on the state s of the economy, i.e. if $R^l_{CB} = R^h_{CB}$,

$$\text{then} \quad p^h_C < p^l_C \quad \text{and} \quad \frac{p^l_C}{p^h_C} = \frac{\mathbf{R^h_M}}{\mathbf{R^l_M}}.$$

(ii) For central bank policy gross rates $(R^s_{CB})_s$ characterized by

$$\frac{R^h_{CB}}{R^l_{CB}} = \frac{\mathbf{R^h_M}}{\mathbf{R^l_M}},$$

[40] The conclusion would not hold if the real deposit gross rates of return were independent of the state of the world and thus if deposit interest rates were inflation-linked. This is addressed in Chapter 3 in Section 3.6.

the price of the consumption good is independent of the state of the world ($p^h_C = p^l_C$).

We note that central bank policy gross rates described in (*ii*) imply $R^l_{CB} < R^h_{CB}$. Corollary 4 stems from the equilibrium condition in (2.3) and is based on the following intuition: If R^s_{CB} is independent of the state of the world and State l occurs, the households possess a comparatively large amount of deposits in Period $t = 1$ when production has occurred, which causes the price of the consumption good to rise, as its supply is low. When the central bank chooses lower interest rates in bad states, the amount of privately created money declines in line with the supply of the consumption good. As a consequence, the price of the consumption good remains constant across states.

In the next section we explore potential cases of friction that may move allocations away from the first-best allocation and may even cause a collapse of the monetary system. We also explore whether monetary policy or capital regulation might help to restore efficiency. We note that the explosion of money creation and lending could not happen in a banking model that only comprises a real sector, as in such models lending is constrained by the funding of banks with the investment good.

2.4 Price Rigidities and Capital Requirements

2.4.1 Absence of Capital Requirements

In Section 2.4 we explore what happens when money creation is affected by price rigidities and the zero lower bound. In such a setting we also examine how a capital requirement can improve the possible equilibrium allocations. For this purpose, it is useful to introduce three types of situation:

(*i*) Money creation is positive and limited, but aggregate investment is distorted between sectors,

(*ii*) money creation is zero, and physical investment occurs only in Sector FT, and (*iii*) money creation explodes without limit, the monetary system collapses, and physical investment remains viable in Sector FT only.[41]

In Section 2.4, without loss of generality, we normalize the price of the investment good to $p_I = 1$. We assume in this section that nominal prices are perfectly rigid in the sense that they do not depend on the state of the world, and we assume that they are equal to some value p_C, which for convenience we set to 1.[42]

From Corollary 4, we obtain that when

$$R^s_{CB} = \mathbf{R^s_M}, \tag{2.10}$$

which means that the central bank chooses the real gross rates of return as its policy gross rates, we recover the first-best equilibria with banks in Theorem 1. We next investigate circumstances where the central bank does not or cannot choose the policy gross rates according to (2.10). This occurs, for example, if $\mathbf{R^l_M} < 1$, i.e. the real gross rate of return in the bad macroeconomic state is sufficiently low, since due to the zero lower bound the policy gross rate R^l_{CB} cannot be set smaller than one.[43] It could also occur if the central bank—for example because of uncertainty about the underlying real gross rates of return—does not or cannot choose the policy gross rates according to (2.10). From Proposition 1, we immediately obtain

Proposition 2

Suppose prices are rigid and $R^s_{CB} \neq \mathbf{R^s_M}$ for some state s of the world. Then either there is no money creation, or it explodes.[44] *In both cases all investments are channeled to Sector FT.*

[41]Essentially, no equilibrium with banks with finite money creation exists. However, there exists an equilibrium in which no household offers equity to banks, all investment goods are channeled to Sector FT, and no lending to Sector MT occurs.

[42]Of course, this is a strong assumption. The results could be extended to models with multiple consumption goods, where a subset of firms would face such rigidities in the sense of Calvo (1983). Throughout Section 2.4 the concept of price rigidities refers to $p^s_C = p_I = 1$ for both states $s = l, h$.

[43]In practice, banks can exchange central bank deposits for banknotes and coins. By storing cash, banks could in principle bypass negative central bank policy interest rates. The same possibility protects depositors from negative interest rates. Accordingly, the presence of banknotes and coins is essential in rationalizing the zero lower bound. In our model we assume that the central bank is constrained by the zero lower bound by the threat of private agents to withdraw deposits and store banknotes, but we do not explicitly model banknotes and coins.

[44]We say that there is no money creation when $\hat{\alpha}_M\big((\mathbf{R^s_M})_s, (R^s_{CB})_s, \varphi\big)$ is a set that

We note that the equilibrium allocation of Proposition 2 is inefficient, as expected output is maximized only when investment is channeled to both sectors. Expected loss in output is given by

$$\big(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_{\mathbf{M}})\big)\overline{\mathbf{R}}_{\mathbf{M}} + \mathbf{f}\big(\mathbf{f}'^{-1}(\overline{\mathbf{R}}_{\mathbf{M}})\big) - \mathbf{f}(\mathbf{W}).$$

	$R^l_{CB} < \mathbf{R^l_M}$	$R^l_{CB} = \mathbf{R^l_M}$	$R^l_{CB} > \mathbf{R^l_M}$
$R^h_{CB} < \mathbf{R^h_M}$	**Money Explosion**	**Money Explosion**	**Money Crunch or Money Explosion**
$R^h_{CB} = \mathbf{R^h_M}$	**Money Explosion**	**Efficient Equilibrium**	**Money Crunch, No Banking**
$R^h_{CB} > \mathbf{R^h_M}$	**Money Crunch or Money Explosion**	**Money Crunch, No Banking**	**Money Crunch, No Banking**

Table 2.7: Possible constellations with price rigidities. Source: Own illustration.

The possible constellations with price rigidities are depicted in Table 2.7.

2.4.2 Capital Requirements

We next investigate the extent to which whenever there is a difference between R^s_{CB} and $\mathbf{R^s_M}$ for some state s a capital requirement can restore both the existence of an equilibrium with banks in the sense of Theorem 1 as well as efficiency. A capital requirement is defined as follows:

Definition 2

A minimum bank equity ratio φ^{reg} ($\varphi^{reg} \in (0,1)$) requires each bank to hold more equity at the end of Period $t = 0$ than the fraction φ^{reg} of its total assets. In other words, the realized equity ratio of each bank b, which we denote by φ^b, has to be larger than φ^{reg}.

is only constituted of elements smaller than 1 and that money creation explodes when $\hat{\alpha}_M\big((\mathbf{R^s_M})_s, (R^s_{CB})_s, \varphi\big)$ is a set that is only constituted of elements larger than 1. Finally, we say that either there is no money creation, or it explodes if $1 \notin \hat{\alpha}_M\big((\mathbf{R^s_M})_s, (R^s_{CB})_s, \varphi\big)$.

We first establish a lemma describing how a capital requirement impacts money creation by an individual bank.

Lemma 5

Suppose the average capital structure in the economy is φ and $\varphi^{reg} \leq \varphi$. Then the capital requirement φ^{reg} imposes an upper bound on individual money creation:

$$\alpha_M^b \leq \frac{\varphi}{\varphi^{reg}} \quad \textit{for all banks } b.$$

The proof of Lemma 5 can be found in Appendix 2.G. We next determine the optimal money creation choice by banks when the government sets a capital requirement. When $R_{CB}^s \neq \mathbf{R_M^s}$ for some state s of the economy, money creation is either limited by the threat of default against the central bank, by the capital requirement, or it is not profitable. The detailed characterization of the correspondence describing these three situations is given in Lemma 7 in Appendix 2.F. We use Lemma 7 to derive general conditions under which equilibria with banks exist when $R_{CB}^s \neq \mathbf{R_M^s}$ for some state s of the world.

Proposition 3

Suppose that prices are rigid and $R_{CB}^s \neq \mathbf{R_M^s}$ for some state s. Then there exists an equilibrium with banks if the central bank policy gross rates $(R_{CB}^s)_s$ and the capital requirement level φ^{reg} are set as follows:

(i) $\overline{R}_{CB} = \overline{\mathbf{R}}_\mathbf{M}$ *and* $\max\left(\frac{R_{CB}^h - \mathbf{R_M^h}}{R_{CB}^h}, \frac{R_{CB}^l - \mathbf{R_M^l}}{R_{CB}^l}\right) \leq \varphi^{reg}$.

(ii) $\overline{R}_{CB} > \overline{\mathbf{R}}_\mathbf{M}$ *and* $0 < \varphi^{reg} = \max\left(\frac{1-\sigma}{\sigma}\frac{\mathbf{R_M^l} - R_{CB}^l}{R_{CB}^h}, \frac{\sigma}{1-\sigma}\frac{\mathbf{R_M^h} - R_{CB}^h}{R_{CB}^l}\right) < 1$.

The proof of Proposition 3 is given in Appendix 2.G. From Proposition 3 and its proof, we can derive the welfare properties of equilibria with banks when a capital requirement is imposed. These welfare properties are summarized in the following corollary:

Corollary 5

Suppose that prices are rigid and $R_{CB}^s \neq \mathbf{R_M^s}$ for some state s. Then the central bank policy gross rates $(R_{CB}^s)_s$ and the capital requirement level φ^{reg} implement a *socially efficient equilibrium with banks if and only if*

$$\overline{R}_{CB} = \overline{\mathbf{R}}_{\mathbf{M}} \quad and \quad \max\left(\frac{R^h_{CB} - \mathbf{R^h_M}}{R^h_{CB}}, \frac{R^l_{CB} - \mathbf{R^l_M}}{R^l_{CB}}\right) \leq \varphi^{reg}.$$

The intuition for Proposition 3 and Corollary 5 runs as follows: If in some state s, $R^s_{CB} < \mathbf{R^s_M}$, banks would like to expand money creation to high, if not infinite, levels because potential losses in the other state $s' \neq s$ would be bounded, due to limited shareholder liability. In such cases, the capital requirement constrains money creation. Two cases may occur.

When $\overline{R}_{CB} = \overline{\mathbf{R}}_{\mathbf{M}}$, no bank has any incentive to push money creation above average, since first, losses in some state s' exactly offset gains from money creation in the other state $s \neq s'$, and second, the minimum capital requirement is set at a level that prevents banks from defaulting against depositors and thus from leveraging on limited shareholder liability. By preventing default against depositors, such a minimum capital requirement induces socially efficient money creation and lending.

When $\overline{R}_{CB} > \overline{\mathbf{R}}_{\mathbf{M}}$, banks would expand money creation above average in the absence of a capital requirement, since for an increasing money creation level, the shareholders' value increases in some state s, while it stays at zero in the other state s'. Thus, the capital requirement directly limits money creation by preventing banks from granting any above-average amount of loans.

In this case, even though such a minimum capital requirement restores a potential equilibrium with banks, it does not implement a socially efficient allocation. The inefficiency results from banks' default against depositors. When they make their investment decision, households do not take into account the impact of banks' default on the lump-sum taxes levied to bail them out. From the proof of Proposition 3, it is straightforward that the equilibria with banks' default can be ranked in terms of welfare according to the capital requirement level φ^{reg}. The intuition runs as follows: A larger equity ratio reduces the amount of taxes levied to bail out banks, which in turn improves households' investment decision making. Therefore, the intensity of the inefficiency associated with banks' default declines in the capital requirement level φ^{reg}.

2.4.3 The Zero Lower Bound and Capital Requirements

We next explore the case where the central bank is constrained by the zero lower bound and prices are assumed to be rigid, i.e. when $p_C^{s*} = p_I^* = 1$ for all states $s = l, h$. From Corollary 5, we obtain

Corollary 6
Suppose that prices are rigid, $\mathbf{R_M^l} < 1 \leq \overline{\mathbf{R}}_\mathbf{M}$, and the central bank is constrained by the zero lower bound ($R_{CB}^s \geq 1$ for all states $s = l, h$). Then there exist central bank policy gross rates $(R_{CB}^s)_s$ and capital requirement levels φ^{reg} such that the allocation of the resulting equilibrium with banks is socially efficient.

(i) The central bank policy gross rates have to satisfy $\overline{R}_{CB} = \overline{\mathbf{R}}_\mathbf{M}$. One example is

$$R_{CB}^l = 1, \quad R_{CB}^h = \frac{\overline{\mathbf{R}}_\mathbf{M} - (1 - \sigma)}{\sigma}.$$

(ii) The regulatory capital requirement levels φ^{reg} have to satisfy

$$\varphi^{reg} \geq \frac{R_{CB}^l - \mathbf{R_M^l}}{R_{CB}^l}.$$

The proof of Corollary 6 can be found in Appendix 2.G. Corollary 6 shows that price rigidities and the zero lower bound can be countered by a suitable combination of monetary policy and capital regulation. The capital requirement ensures that money creation is sufficiently constrained for no individual bank to default. The central bank policy gross rates $R_{CB}^l = 1$, $R_{CB}^h = \frac{\overline{\mathbf{R}}_\mathbf{M} - (1-\sigma)}{\sigma}$ ensure that in the good state gains from money creation are sufficiently high to offset losses in the bad state. In other words, setting $R_{CB}^h < \mathbf{R_M^h}$ generates sufficient incentives for banks to lend and to create money. The capital requirement, in turn, ensures that money creation does not become excessive. We note that any monetary policy that satisfies $\overline{R}_{CB} = \overline{\mathbf{R}}_\mathbf{M}$ achieves the same purpose and induces a socially efficient allocation. In Appendix 2.H we illustrate our results with a simple numerical example.

From Corollary 5 and the proof of Proposition 3, we also immediately obtain

Proposition 4

Suppose that prices are rigid, $\overline{\mathbf{R}}_{\mathbf{M}} < 1$*, and the central bank is constrained by the zero lower bound (*$R^s_{CB} \geq 1$ *for all states* $s = l, h$*). Then there exist no central bank policy gross rates* $(R^s_{CB})_s$ *and capital requirement level* φ^{reg} *making the allocation of the resulting equilibrium with banks socially efficient. We derive two cases:*

- *If* $1 < \mathbf{R}^{\mathbf{h}}_{\mathbf{M}}$*, there exist central bank policy gross rates* $(R^s_{CB})_s$ *and a capital requirement level* φ^{reg} *implementing equilibria with banks.*
  - *(a) The central bank policy gross rates have to satisfy* $R^h_{CB} < \mathbf{R}^{\mathbf{h}}_{\mathbf{M}}$*. An example is*

$$R^l_{CB} = R^h_{CB} = 1.$$

  - *(b) The regulatory capital requirement level* φ^{reg} *has to satisfy*

$$\varphi^{reg} = \frac{\sigma}{1-\sigma} \frac{\mathbf{R}^{\mathbf{h}}_{\mathbf{M}} - R^h_{CB}}{R^l_{CB}}.$$

- *If* $\mathbf{R}^{\mathbf{h}}_{\mathbf{M}} \leq 1$*, there are no central bank policy gross rates* $(R^s_{CB})_s$ *and capital requirement level* φ^{reg} *implementing an equilibrium with banks.*

Proposition 4 states that in a depressed economy characterized by $\overline{\mathbf{R}}_{\mathbf{M}} < 1$, where prices are rigid and the central bank is constrained by the zero lower bound, money creation can only be induced by a suitable combination of monetary policy and capital regulation if $\mathbf{R}^{\mathbf{h}}_{\mathbf{M}} > 1$.

If $\mathbf{R}^{\mathbf{h}}_{\mathbf{M}} \leq 1$, the only possible equilibrium is the equilibrium without banks, which is inefficient, as all investments are channeled to FT.[45] The reason is that under any feasible monetary policy and even with no capital requirement, money creation and lending are not profitable in such cases.

If $\overline{\mathbf{R}}_{\mathbf{M}} < 1$ but $1 < \mathbf{R}^{\mathbf{h}}_{\mathbf{M}}$, the central bank and the bank regulators can only make banking profitable and thus trigger money creation and lending by inducing profits in the good state and letting them default against depositors in the bad state. From the proof of Proposition 3, we deduce that the policy gross rates

[45] In such a case, other kinds of policies such as Quantitative Easing might be useful to stimulate money creation and lending. We leave this to future research.

inducing the equilibrium with banks with highest welfare are given by $R^s_{CB} = 1$ for $s = l, h$. Moreover, a capital requirement has to be imposed on banks to prevent money creation from exploding.

The equilibria associated with the policy gross rates $R^s_{CB} = 1$ for $s = l, h$ in the case $1 < \mathbf{R_M^h}$ are inefficient. Hence, the central bank and the bank regulators will implement such a policy only if the welfare induced by the policy described in Proposition 4, (a) and (b) is higher than the welfare associated to the equilibrium without banks. A sufficient condition for this is $\mathbf{f}(\mathbf{W}) < \overline{\mathbf{R}}_\mathbf{M}\mathbf{W}$.

The above result in the cases $\overline{\mathbf{R}}_\mathbf{M} < 1$ and $\mathbf{R_M^h} > 1$ can be interpreted in terms of Forward Guidance.[46] The central bank announces that it will set the policy gross rates at 1 in both states of the world, even if the real gross rate $\mathbf{R_M^h}$ is larger than one. This announcement means that banks can expect positive profits in the good state of the world, thereby making money creation and lending profitable. This stimulates money creation and lending at the zero lower bound. However, in the bad state of the world money creation is associated with bank failures, so expected social welfare is lower than in the first-best allocation.

[46] In our two-period model the central bank does not face a time-inconsistent problem regarding such announcements. For the implementation of Forward Guidance at the zero lower bound, see e.g. Woodford (2013), Gersbach et al. (2015a), and Liu (2016).

Appendix

2.A Stage C

We examine the detailed payment process, investment in FT, and payment of bank equity in Stage C through a series of substages. For this purpose, we index all variables changing in some substage by an integer starting from 1.

Stage C, Substage 1: Borrowing of Banks from the Central Bank

To have enough CB deposits to guarantee payments using bank deposits, Bank b borrows from the central bank[47] the amount of[48]

$$l^b_{CB_1} := l^b_M = \alpha^b_M D_M.$$

As a result, bank-specific CB deposits amounting to $d^b_{CB_1} := l^b_{CB_1}$ as well as an aggregate amount of CB deposits amounting to $D_{CB_1} := D_M > 0$ are created. The balance sheets of banks and households are given in Table 2.A.1.

[47] As the description of the interbank lending process is formally identical to that of depositing at and borrowing from the central bank, we limit the description to the case where all banks deposit at, and borrow exclusively from, the central bank.

[48] In this first substage, banks do not need to borrow that much to guarantee payments in subsequent substages, as banks will obtain deposits back from households when firms make payments with their deposits. The amount Bank b needs to borrow from the central bank is given by $\max\big((1-\alpha^b_M)L_M, 0\big)$. This result will be demonstrated in the subsequent substages.

Households	
$\mathbf{W}$	E_H

Bank b	
$d^b_{CB_1}$	$l^b_{CB_1}$
l^b_M	d^b_M

Table 2.A.1: Balance sheets at the end of Stage C, Substage 1. Source: Own illustration.

Stage C, Substage 2: Sale of an Amount of Investment Good to MT

We assume that firms in MT buy the largest possible amount of investment good they can afford and do not hold deposits in the production stage D:[49]

$$\mathbf{K_M} = \frac{L_M}{p_I}.$$

To settle these payments, each bank b transfers $d^b_M = \alpha^b_M D_M$ to other banks and receives the same amount $d_{H_1} := D_M$ from other banks in the form of CB deposits. We note that d_{H_1} does not depend on the individual bank b, due to our assumption that households keep deposits evenly distributed across all banks at all times. The corresponding aggregate amount is denoted by D_{H_1}. This transaction impacts the CB deposits of Bank b as follows:

$$d^b_{CB_2} := d^b_{CB_1} - \alpha^b_M D_M + D_M = D_M.$$

The balance sheets of banks and households are given in Table 2.A.2.

[49]Note that relaxing this assumption would not change the equilibrium allocation of the investment good, as firms would not be able to improve the shareholders' value in equilibrium by holding deposits.

Households		Bank b	
$\mathbf{K_F}$	E_H	$d^b_{CB_2}$	$l^b_{CB_1}$
D_{H_1}		l^b_M	d_{H_1}

Table 2.A.2: Balance sheets at the end of Stage C, Substage 2. Source: Own illustration.

Stage C, Substage 3: Investment in FT

When buying S_F bonds from firms in FT, households deliver $\mathbf{K_F} = S_F$ units of the investment good against the promise to obtain $\mathbf{K_F R_F}$ units of the consumption good from FT after production has taken place. The balance sheets of banks and households are given in Table 2.A.3.

Households		Bank b	
S_F	E_H	$d^b_{CB_2}$	$l^b_{CB_1}$
D_{H_1}		l^b_M	d_{H_1}

Table 2.A.3: Balance sheets at the end of Stage C, Substage 3. Source: Own illustration.

Stage C, Substage 4: Netting of CB Deposits and CB Liabilities

Now banks can net their CB deposits and CB liabilities, as no further payment has to be made before production. We use

$$\delta^b := d^b_{CB_2} - l^b_{CB_1} = (1 - \alpha^b_M)L_M \tag{2.11}$$

to denote the net position of Bank b against the central bank. We distinguish banks with claims against the central bank and banks that are debtors of the central bank:

$$\begin{aligned} B_I &:= \{b_i \in [0,1] \quad \text{s.t.} \quad \delta^{b_i} \geq 0\} \\ \text{and} \quad B_J &:= \{b_j \in [0,1] \quad \text{s.t.} \quad \delta^{b_j} < 0\}. \end{aligned}$$

Net claims against the central bank are denoted by $d_{CB}^{b_i} := \delta^{b_i}$ for all $b_i \in B_I$, and net liabilities by $l_{CB}^{b_j} := -\delta^{b_j}$ for all $b_j \in B_J$. The balance sheets of banks and households are given in Table 2.A.4.

Households		Bank b_i		Bank b_j	
S_F		$d_{CB}^{b_i}$			$l_{CB}^{b_j}$
D_{H_1}	E_H	$l_M^{b_i}$	d_{H_1}	$l_M^{b_j}$	d_{H_1}

Table 2.A.4: Balance sheets at the end of Stage C, Substage 4. Source: Own illustration.

Stage C, Substage 5: Payment of Bank Equity

Now households pay the equity $E_B = \varphi D_M > 0$ pledged in $t = 1$, thereby destroying the corresponding amount of bank deposits. We use $D_H = (1 - \varphi)D_M$ to denote the remaining amount of deposits. Accordingly, $D_{H_1} = E_B + D_H$. The balance sheets of two typical banks representing a net depositor and a net borrower from the central bank are displayed in Table 2.3.

2.B Stage E—No Bank Defaults

We examine the detailed dividend payment, payback of debt, and payment process of Stage E through a series of substages. Similarly to Appendix 2.A, whenever a variable changes in some substage, we increase the index by 1, starting with the last index from Appendix 2.A.

Stage E, Substage 1: Borrowing of Banks from the Central Bank

To have enough CB deposits to guarantee payments using bank deposits, Bank b borrows from the central bank the amount of $l^{b,s}_{CB_3} = d^{b,s}_{CB_3} := D_H R^s_D + \Pi^{b,s}_B$. We use the notations

$$d^{b_i,s}_{CB_4} := d^{b_i,s}_{CB_3} + d^{b_i}_{CB} R^s_{CB}$$
$$\text{and} \quad l^{b_j,s}_{CB_4} := l^{b_j,s}_{CB_3} + l^{b_j}_{CB} R^s_{CB}.$$

The balance sheets of banks and households are given in Table 2.B.1.

Households		Bank b_i		Bank b_j	
$S_F \mathbf{R_F}$	$E_H R^s_H$	$d^{b_i,s}_{CB_4}$	$l^{b_i,s}_{CB_3}$	$d^{b_j,s}_{CB_3}$	$l^{b_j,s}_{CB_4}$
$D_H R^s_D$		$l^{b_i}_M R^s_L$	$d_H R^s_D$	$l^{b_j}_M R^s_L$	$d_H R^s_D$
$E_B R^s_E$			$\Pi^{b_i,s}_B$		$\Pi^{b_j,s}_B$
Π^s_F					

Table 2.B.1: Balance sheets at the end of Stage E, Substage 1. Source: Own illustration.

Stage E, Substage 2: Dividend Payment

Bank profits are paid as dividends to households. This creates bank deposits, and households' deposits at Bank b become $\tilde{d}^s_H := D_H R^s_D + \Pi^s_B$. The aggregate amount of households' deposits is then denoted by $\tilde{D}^s_H$. To settle these payments, each bank b transfers $\Pi_B^{b,s}$ to other banks and receives Π_B^s from other banks in the form of CB deposits. These processes impact CB deposits of Banks b_i and b_j as follows:

$$d_{CB_6}^{b_i,s} := d_{CB_4}^{b_i,s} - \Pi_B^{b_i,s} + \Pi_B^s = d_{CB}^{b_i} R_{CB}^s + D_H R_D^s + \Pi_B^s$$
$$\text{and} \quad d_{CB_5}^{b_j,s} := d_{CB_3}^{b_j,s} - \Pi_B^{b_j,s} + \Pi_B^s = D_H R_D^s + \Pi_B^s.$$

The balance sheets of banks and households are given in Table 2.B.2.

Households		Bank b_i		Bank b_j	
$S_F \mathbf{R_F}$	$E_H R_H^s$	$d_{CB_6}^{b_i,s}$	$l_{CB_3}^{b_i,s}$	$d_{CB_5}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$\tilde{D}_H^s$		$l_M^{b_i} R_L^s$	$\tilde{d}_H^s$	$l_M^{b_j} R_L^s$	$\tilde{d}_H^s$
Π_F^s					

Table 2.B.2: Balance sheets at the end of Stage E, Substage 2. Source: Own illustration.

Stage E, Substage 3: Repayment of Debt and Distribution of Profits

From the repayment of debt $S_F \mathbf{R_F}$ and the distribution of profits Π_F^s, both in terms of the consumption good, households obtain $\mathbf{f}(\mathbf{K_F})$ units of the consumption good. The balance sheets of banks and households are given in Table 2.B.3.

Stage E, Substage 4: Sale of the Consumption Good Produced by MT

Firms in MT sell the entire amount of the consumption good they have produced. Households buy it with their private deposits consisting of their wealth in terms of equity and deposits.[50] The supply of $\mathbf{K_M R_M^s}$ units of the consumption good

[50]The household receives additional deposits from the banks' dividend payments.

Households		Bank b_i		Bank b_j	
$\tilde{D}_H^s$	$E_H R_H^s$	$d_{CB_6}^{b_i,s}$	$l_{CB_3}^{b_i,s}$	$d_{CB_5}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$\mathbf{f}(\mathbf{K_F})$		$l_M^{b_i} R_L^s$	$\tilde{d}_H^s$	$l_M^{b_j} R_L^s$	$\tilde{d}_H^s$

Table 2.B.3: Balance sheets at the end of Stage E, Substage 3. Source: Own illustration.

meets the real demand $\frac{\tilde{D}_H^s}{p_C^s}$. Hence, the equilibrium price is given by

$$p_C^s = \frac{\tilde{D}_H^s}{\mathbf{K_M R_M^s}}.$$

To settle these payments, each bank b transfers $\tilde{d}_H^s$ to other banks and receives an amount $d_{M_1}^{b,s} := \alpha_M^b \tilde{d}_H^s$ from other banks in the form of CB deposits. By summing over all banks $b \in [0,1]$ in the expression of banks' profits in Equation (2.1), we obtain $L_M R_L^s = D_H R_D^s + \Pi_B^s$, which means that $d_{M_1}^{b,s} = \alpha_M^b L_M R_L^s$. This transaction impacts the CB deposits of Banks b_i and b_j as follows:

$$\begin{aligned} & d_{CB_8}^{b_i,s} := d_{CB_6}^{b_i,s} - \tilde{d}_H^s + d_{M_1}^{b_i,s} = \alpha_M^{b_i} L_M R_L^s + d_{CB}^{b_i} R_{CB}^s \\ \text{and} \quad & d_{CB_7}^{b_j,s} := d_{CB_5}^{b_j,s} - \tilde{d}_H^s + d_{M_1}^{b_j,s} = \alpha_M^{b_j} L_M R_L^s. \end{aligned}$$

The balance sheets of banks and households are given in Table 2.B.4.

Households		Bank b_i		Bank b_j	
$\mathbf{f}(\mathbf{K_F})$	$E_H R_H^s$	$d_{CB_8}^{b_i,s}$	$l_{CB_3}^{b_i,s}$	$d_{CB_7}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$\mathbf{K_M R_M^s}$		$l_M^{b_i} R_L^s$	$d_{M_1}^{b_i,s}$	$l_M^{b_j} R_L^s$	$d_{M_1}^{b_j,s}$

Table 2.B.4: Balance sheets at the end of Stage E, Substage 4. Source: Own illustration.

Stage E, Substage 5: Repayment of Loans by Firms in MT

Firms in MT pay back their loans, and bank deposits are destroyed. The balance sheets of banks and households are given in Table 2.B.5.

Households		Bank b_i		Bank b_j	
$\mathbf{f(K_F)}$	$E_H R_H^s$	$d_{CB_8}^{b_i,s}$	$l_{CB_3}^{b_i,s}$	$d_{CB_7}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$\mathbf{K_M R_M^s}$					

Table 2.B.5: Balance sheets at the end of Stage E, Substage 5. Source: Own illustration.

Stage E, Substage 6: Netting of CB Deposits and CB Liabilities

Banks net their CB deposits and CB liabilities. Using the expression of bank profits given by Equation (2.1), we obtain

$$d_{CB_8}^{b_i,s} - l_{CB_3}^{b_i,s} = \alpha_M^{b_i} L_M R_L^s + (1 - \alpha_M^{b_i}) L_M R_{CB}^s - \big((L_M - E_B) R_D^s + \Pi_B^{b_i,s}\big) = 0,$$
$$d_{CB_7}^{b_j,s} - l_{CB_4}^{b_j,s} = \alpha_M^{b_j} L_M R_L^s - (\alpha_M^{b_j} - 1) L_M R_{CB}^s - \big((L_M - E_B) R_D^s + \Pi_B^{b_j,s}\big) = 0.$$

2.C Net Positions of Banks Against the Central Bank After a Bail-out

In Table 2.6 the label $b_{i'}$ denotes banks with a non-negative net position against the central bank. For completeness, the net position is given by

$$d^{b_{i'}}_{CBT} := \begin{cases} d^{b_{i'}}_{CB} R^s_{CB} - T^s & \text{if } d^{b_{i'}}_{CB} R^s_{CB} - T^s \geq 0 \text{ and } \Pi^{b_{i'},s}_{B} \geq 0, \\ d^{b_{i'}}_{CB} R^s_{CB} - T^s + t^{b_{i'},s} & \text{if } d^{b_{i'}}_{CB} R^s_{CB} - T^s + t^{b_{i'},s} \geq 0 \text{ and } \Pi^{b_{i'},s}_{B} < 0, \text{ and} \\ t^{b_{i'},s} - T^s - l^{b_{i'}}_{CB} R^s_{CB} & \text{if } l^{b_{i'}}_{CB} R^s_{CB} + T^s - t^{b_{i'},s} \leq 0 \text{ and } \Pi^{b_{i'},s}_{B} < 0, \end{cases}$$

where T^s are the tax payments introduced in Subsection 2.2.4 representing the households' deposit withdrawals to pay taxes in State $s = l, h$ and $t^{b_{i'},s}$, the possible bail-out in State $s = l, h$ if Bank $b_{i'}$ defaults against households. Similarly, the label $b_{j'}$ denotes banks with a negative net position against the central bank:

$$l^{b_{j'}}_{CBT} := \begin{cases} l^{b_{j'}}_{CB} R^s_{CB} + T^s & \text{if } \Pi^{b_{j'},s}_{B} \geq 0, \\ T^s - d^{b_{j'}}_{CB} R^s_{CB} & \text{if } d^{b_{j'}}_{CB} R^s_{CB} - T^s < 0 \text{ and } \Pi^{b_{j'},s}_{B} \geq 0, \\ T^s - t^{b_{j'},s} - d^{b_{j'}}_{CB} R^s_{CB} & \text{if } d^{b_{j'}}_{CB} R^s_{CB} - T^s + t^{b_{j'},s} < 0 \text{ and } \Pi^{b_{j'},s}_{B} < 0, \text{ and} \\ l^{b_{j'}}_{CB} R^s_{CB} + T^s - t^{b_{j'},s} & \text{if } l^{b_{j'}}_{CB} R^s_{CB} + T^s - t^{b_{j'},s} > 0 \text{ and } \Pi^{b_{j'},s}_{B} < 0. \end{cases}$$

2.D Interbank Borrowing and Lending

In Appendix 2.D we describe how banks settle payments between agents and how banks can borrow and lend to each other, thereby creating bank assets and liabilities. Ultimately, we will be able to investigate the implications of this process for the gross rates of return on private and CB deposits in equilibrium. For ease of presentation, we omit the superscript s as the same considerations hold for both states of the world.

We use an example with two banks, b_j and b_i. Assume that Bank b_i grants a loan to Bank b_j. Then four entries in the balance sheets are created, as shown in Table 2.D.1.

Bank b_j		Bank b_i	
D_j	L_i	L_i	D_j

Table 2.D.1: Balance sheets representing interbank lending and borrowing (1/4). Source: Own illustration.

L_i represents the amount of loans granted by Bank b_i to Bank b_j, and D_j the amount of deposits held by Bank b_j at Bank b_i. We have assumed a competitive interbank market with a single gross rate of return for lending and borrowing. Since banks cannot discriminate between deposits owned by households and deposits owned by other banks, the corresponding gross rates are both equal to R_D.

We next investigate the relationship between R_{CB} and R_D. Assume first that some buyers pay with their deposits at Bank b_j and that the sellers deposit the money at Bank b_i. To settle the transfer, Bank b_j has two options. If $R_{CB} < R_D$, it will borrow from the central bank and transfer CB deposits to Bank b_i. Suppose now that $R_{CB} > R_D$. Then Bank b_j directly becomes liable to Bank b_i. The buyers' deposits at Bank b_j are replaced by a loan Bank b_i grants to Bank b_j. This loan is an asset for Bank b_i that is matched by the liability corresponding to the new sellers' deposits. As assumed in Subsection 2.2.3, Bank b_i has the right to require Bank b_j to repay its liabilities with CB deposits, which Bank b_i will do as $R_{CB} > R_D$. The balance sheets at the end of the process look exactly the same, no matter whether or not Bank b_j became liable to Bank b_i in the first place. Therefore, independently of R_D, the refinancing gross rate is equal to R_{CB}. However, assuming that no bank participating in the interbank market makes any loss by doing so requires $R_D = R_{CB}$, which we show next.

Here we prove that $R_D = R_{CB}$. By contradiction, assume first that $R_D < R_{CB}$.

Bank b_j, for example, would borrow from Bank b_i at the gross rate of return R_D and from the central bank at the gross rate of return R_{CB}, as shown in the balance sheets in Table 2.D.2.

Bank b_j	
D_j D_{CB}	L_i L_{CB}

Bank b_i	
L_i	D_j

Table 2.D.2: Balance sheets representing interbank lending and borrowing (2/4). Source: Own illustration.

Using deposits at Bank b_i, Bank b_j can now repay CB liabilities. To carry out this payment, Bank b_i has to borrow from the central bank at the gross rate of return R_{CB}. The balance sheets are given in Table 2.D.3.

Bank b_j	
D_{CB}	L_i

Bank b_i	
L_i	L_{CB}

Table 2.D.3: Balance sheets representing interbank lending and borrowing (3/4). Source: Own illustration.

Bank b_j would make positive profits from this operation, while Bank b_i would make losses. As we assumed that no bank participating in the interbank market makes any loss by doing so, $R_D < R_{CB}$ cannot occur in any equilibrium with banks.

Now assume that $R_{CB} < R_D$. Then Bank b_j would like to repay its liabilities against Bank b_i using CB deposits. This would result in the balance sheets given in Table 2.D.4.

Bank b_j would make positive profits from this operation, while Bank b_i would make losses. As we assumed that no bank participating in the interbank market makes any loss by doing so, $R_D > R_{CB}$ cannot occur in any equilibrium with banks.[51]

[51]Otherwise, we could have constellations with $R_{CB} > R_D$ and an inactive interbank market,

Bank b_j		Bank b_i	
D_j	L_{CB}	D_{CB}	D_j

Table 2.D.4: Balance sheets representing interbank lending and borrowing (4/4). Source: Own illustration.

2.E Households' Optimal Investment Choices

Lemma 6

The representative household's optimal portfolio choices are represented by three correspondences denoted by

$$\begin{aligned}
\hat{E}_B &: \mathbb{R}^7_{++} \times [0, \mathbf{W}] \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}),\\
\hat{D}_H &: \mathbb{R}^7_{++} \times \mathbb{R}_+ \times [0, \mathbf{W}] \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}),\\
\hat{S}_F &: \mathbb{R}^7_{++} \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}),
\end{aligned}$$

and given by

$$\begin{aligned}
&\Big(\hat{E}_B\big((R_E^s)_s, (R_D^s)_s, p_I, (p_C^s)_s, S_F\big),\\
&\hat{D}_H\big((R_E^s)_s, (R_D^s)_s, p_I, (p_C^s)_s, E_B, S_F\big),\\
&\hat{S}_F\big((R_E^s)_s, (R_D^s)_s, p_I, (p_C^s)_s\big)\Big) =\\
&\begin{cases}
\big(\{0\}, \{0\}, \{\mathbf{W}\}\big) \\
\qquad \textit{if } \max\left(\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right], \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right) \leq \frac{\mathbf{f}'(\mathbf{W})}{p_I},\\
\big(\{0\}, \{p_I\mathbf{W}\}, \{0\}\big) \\
\qquad \textit{if } \max\left(\frac{\mathbf{f}'(\mathbf{0})}{p_I}, \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right) < \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right],\\
\big(\{p_I\mathbf{W}\}, \{0\}, \{0\}\big) \\
\qquad \textit{if } \max\left(\frac{\mathbf{f}'(\mathbf{0})}{p_I}, \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right) < \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right],
\end{cases}
\end{aligned} \tag{2.12}$$

---

as no bank would lend in such a market. An active interbank market requires $R_D = R_{CB}$.

$$\begin{cases} \left([0, p_I \mathbf{W}], \{p_I \mathbf{W} - E_B\}, \{0\}\right) \\ \qquad \text{if } \frac{\mathbf{f'(0)}}{p_I} < \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] = \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right], \\ \left(\{0\}, \{p_I(\mathbf{W} - S_F)\}, \{\mathbf{f'}^{-1}\left(p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\}\right) \\ \qquad \text{if } \max\left(\frac{\mathbf{f'(W)}}{p_I}, \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right) < \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq \frac{\mathbf{f'(0)}}{p_I}, \\ \left(\{p_I(\mathbf{W} - S_F)\}, \{0\}, \{\mathbf{f'}^{-1}\left(p_I \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right)\}\right) \\ \qquad \text{if } \max\left(\frac{\mathbf{f'(W)}}{p_I}, \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right) < \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] \leq \frac{\mathbf{f'(0)}}{p_I}, \\ \left([0, p_I(\mathbf{W} - S_F)], \{p_I(\mathbf{W} - S_F) - E_B\}, \{\mathbf{f'}^{-1}\left(p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\}\right) \\ \qquad \text{if } \frac{\mathbf{f'(W)}}{p_I} < \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] = \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq \frac{\mathbf{f'(0)}}{p_I}. \end{cases}$$

The proof of Lemma 6 is given in Appendix 2.G.

2.F Optimal Choice of Money Creation by Banks with Capital Regulation

Lemma 7

Suppose that banks have to comply with a minimum equity ratio φ^{reg} at the end of Period $t = 0$. If $R_D^s = R_{CB}^s$ in all states $s = l, h$, the privately optimal amounts of money creation and lending by an individual bank are represented by a correspondence denoted by $\hat{\alpha}_M^{reg} : \mathbb{R}_{++}^4 \times [\varphi^{reg}, 1) \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\})$ and given by

$$\hat{\alpha}_M^{reg}\left((R_L^s)_s, (R_{CB}^s)_s, \varphi\right) =$$

$$\begin{cases} \{\frac{\varphi}{\varphi^{reg}}\} & \text{if } (R_L^s \geq R_{CB}^s \text{ for all states } s = l, h \\ & \text{with at least one strict inequality) or} \\ & \text{if } (\overline{R}_L > \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h, \text{ and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DCB}^l) \text{ or} \\ & \text{if } (\overline{R}_L > \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l, \text{ and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DCB}^h) \text{ or} \\ & \text{if } (\overline{R}_L = \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h, \text{ and } \alpha_{DH}^l < \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DCB}^l) \text{ or} \\ & \text{if } (\overline{R}_L = \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l, \text{ and } \alpha_{DH}^h < \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DCB}^h) \text{ or} \\ & \text{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h,\ \alpha_{DH}^l < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^l, \\ & \text{and } \varphi^{reg} < \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l}) \text{ or} \end{cases}$$

$$
\left\{
\begin{array}{ll}
 & \textit{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l,\ \alpha_{DH}^h < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^h, \\
 & \textit{and } \varphi^{reg} < \frac{1-\sigma}{\sigma}\frac{R_L^l - R_{CB}^l}{R_{CB}^h}), \\
\{\alpha_{DCB}^l\} & \textit{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h,\ \alpha_{DCB}^l \leq \frac{\varphi}{\varphi^{reg}}, \\
 & \textit{and } \varphi < \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}), \textit{ or} \\
 & \textit{if } (\overline{R}_L \geq \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h,\ \textit{and } \alpha_{DCB}^l \leq \frac{\varphi}{\varphi^{reg}}), \\
\{\alpha_{DCB}^h\} & \textit{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l,\ \alpha_{DCB}^h \leq \frac{\varphi}{\varphi^{reg}}, \\
 & \textit{and } \varphi < \frac{1-\sigma}{\sigma}\frac{R_L^l - R_{CB}^l}{R_{CB}^h - R_L^h}), \textit{ or} \\
 & \textit{if } (\overline{R}_L \geq \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l,\ \textit{and } \alpha_{DCB}^h \leq \frac{\varphi}{\varphi^{reg}}), \\
\left[0, \frac{\varphi}{\varphi^{reg}}\right] & \textit{if } (R_L^s = R_{CB}^s \textit{ for all states } s = l, h) \textit{ or} \\
 & \textit{if } (\overline{R}_L = \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h,\ \textit{and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DH}^l) \textit{ or} \\
 & \textit{if } (\overline{R}_L = \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l,\ \textit{and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DH}^h), \\
\{0, \frac{\varphi}{\varphi^{reg}}\} & \textit{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h,\ \alpha_{DH}^l < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^l, \\
 & \textit{and } \varphi^{reg} = \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l}) \textit{ or} \\
 & \textit{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l,\ \alpha_{DH}^h < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^h, \\
 & \textit{and } \varphi^{reg} = \frac{1-\sigma}{\sigma}\frac{R_L^l - R_{CB}^l}{R_{CB}^h}), \\
\{0, \alpha_{DCB}^l\} & \textit{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h,\ \alpha_{DCB}^l \leq \frac{\varphi}{\varphi^{reg}}, \\
 & \textit{and } \varphi = \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}), \\
\{0, \alpha_{DCB}^h\} & \textit{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l,\ \alpha_{DCB}^h \leq \frac{\varphi}{\varphi^{reg}}, \\
 & \textit{and } \varphi = \frac{1-\sigma}{\sigma}\frac{R_L^l - R_{CB}^l}{R_{CB}^h - R_L^h}), \\
\{0\} & \textit{if } (R_L^s \leq R_{CB}^s \textit{ for all states } s = l, h \\
 & \textit{with at least one strict inequality) or} \\
 & \textit{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h,\ \alpha_{DCB}^l \leq \frac{\varphi}{\varphi^{reg}}, \\
 & \textit{and } \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l} < \varphi) \textit{ or} \\
 & \textit{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l,\ \alpha_{DCB}^h \leq \frac{\varphi}{\varphi^{reg}}, \\
 & \textit{and } \frac{1-\sigma}{\sigma}\frac{R_L^l - R_{CB}^l}{R_{CB}^h - R_L^h} < \varphi) \textit{ or} \\
 & \textit{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h,\ \alpha_{DH}^l < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^l, \\
 & \textit{and } \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l} < \varphi^{reg}) \textit{ or} \\
 & \textit{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l,\ \alpha_{DH}^h < \frac{\varphi}{\varphi^{reg}} < \alpha_{DCB}^h, \\
 & \textit{and } \frac{1-\sigma}{\sigma}\frac{R_L^l - R_{CB}^l}{R_{CB}^h} < \varphi^{reg}) \textit{ or} \\
 & \textit{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^l < R_{CB}^l,\ R_{CB}^h < R_L^h,\ \textit{and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DH}^l) \textit{ or} \\
 & \textit{if } (\overline{R}_L < \overline{R}_{CB},\ R_L^h < R_{CB}^h,\ R_{CB}^l < R_L^l,\ \textit{and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DH}^h).
\end{array}
\right.
$$

The proof of Lemma 7 is given in Appendix 2.G.

2.G Proofs

Proof of Lemma 2.

As set out in Subsection 2.2.3, banks can lend to, and borrow from, each other at the gross rates $(R_D^{s*})_s$ contingently on State s. Similarly, as explained in Subsection 2.2.3, they can also borrow from, or deposit at, the central bank at the policy gross rates $(R_{CB}^s)_s$ contingently on State s. Suppose now, by contradiction, that $R_D^{s*} \neq R_{CB}^s$ for some state s. If $R_D^{s*} < R_{CB}^s$, all banks would like to become liable to other banks and use the money obtained to hold assets against the central bank, contingently on State s. Similarly, if $R_D^{s*} > R_{CB}^s$, all banks would like to become liable to the central bank and use the money obtained to hold assets against other banks, contingently on State s. As we assumed that no bank participating in the interbank market makes any loss by doing so, both cases cannot hold in an equilibrium with banks.[52] □

Proof of Proposition 1.

Let $b \in [0,1]$ denote a bank. As $R_D^s = R_{CB}^s$ in all states $s = l, h$ by Lemma 2, the expected shareholders' value of Bank b is given by

$$\mathbb{E}[\max(\alpha_M^b L_M(R_L^s - R_{CB}^s) + E_B R_D^s, 0)].$$

Suppose that $\overline{R}_L < \overline{R}_{CB}$.

- Suppose first that $R_L^s \leq R_{CB}^s$ for all states $s = l, h$ with at least one strict inequality. In this case, Bank b's expected shareholders' value is decreasing in the volume of loans. Therefore, its choice is $\alpha_M^b = 0$.
- Suppose now that $R_L^l < R_{CB}^l$ and $R_{CB}^h < R_L^h$. For these constellations Figure 2.G.1 depicts three typical cases representing the expected gross rate

[52] The mechanisms by which banks become liable to other banks or hold assets against them are explained in detail in Appendix 2.D.

of return on equity as a function of α_M^b. The three different cases are given by the comparison between the capital ratio φ and $\frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}$.

For $\alpha_M^b \leq \alpha_{DH}^l$, Bank b does not default on depositors, and its expected shareholders' value is decreasing with α_M^b, as illustrated in Figure 2.G.1. However, for $\alpha_{DH}^l < \alpha_M^b$, Bank b defaults on depositors in the bad state. Then Bank b can further increase the expected shareholders' value by granting more loans, as illustrated in Figure 2.G.1. The reason is that shareholders are protected by limited liability and due to depositors' bail-out by the government, the deposit gross rate of return of Bank b received by households in the bad state is R_D^l.

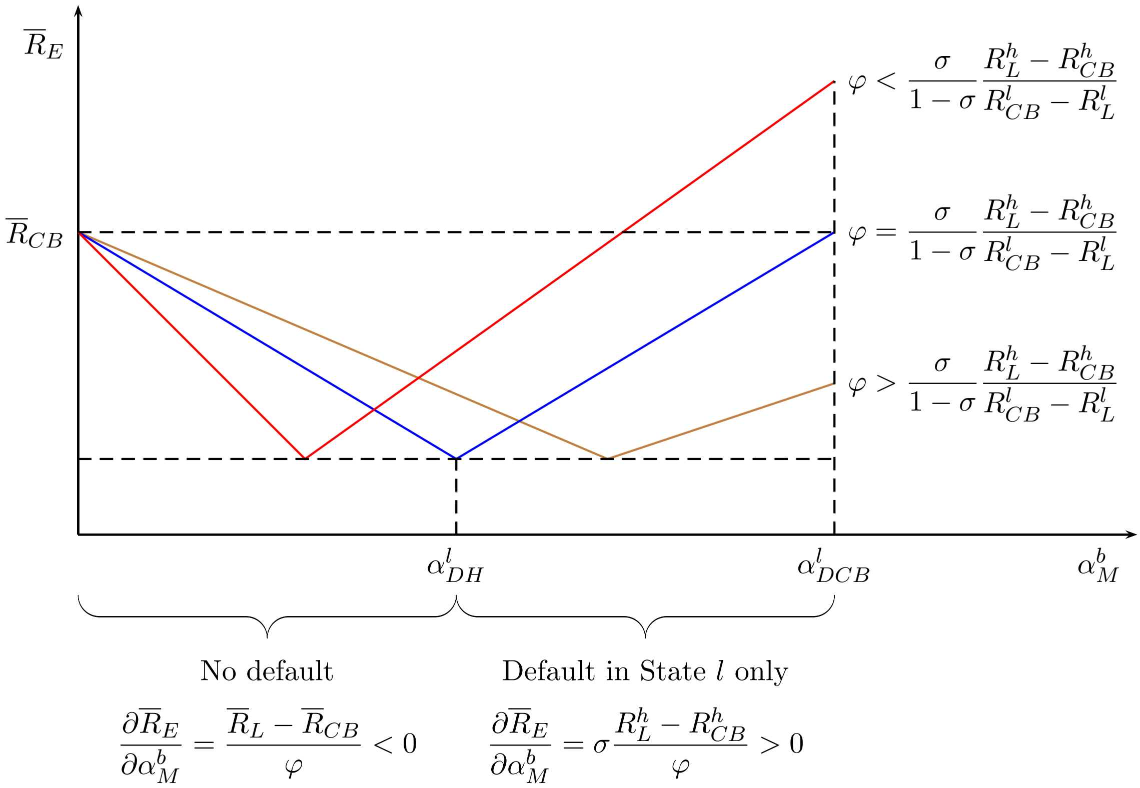

Figure 2.G.1: Expected gross rate of return on equity of Bank b as a function of α_M^b when $\overline{R}_L < \overline{R}_{CB}$ and $R_{CB}^h < R_L^h$ for three typical cases given by the comparison between the capital ratio φ and $\frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}$. α_{DH}^l and the corresponding areas of default and no default are depicted for $\varphi = \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}$. Source: Own illustration.

However, money creation levels $\alpha_M^b > \alpha_{DCB}^l$ cannot be optimal for Bank b, as it would default on the central bank and its banker would be subject to heavy penalties. Therefore, Bank b compares the expected shareholders' value with $\alpha_M^b = 0$ given by

$$E_B \overline{R}_{CB}$$

and the expected shareholders' value with $\alpha_M^b = \alpha_{DCB}^l$ given by

$$\sigma \left(\alpha_{DCB}^l L_M (R_L^h - R_{CB}^h) + E_B R_{CB}^h \right).$$

This comparison leads to the threshold equity ratio φ

$$\frac{\sigma}{1-\sigma} \frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l},$$

below which Bank b chooses $\alpha_M^b = \alpha_{DCB}^l$ and above which it chooses $\alpha_M^b = 0$.

– Suppose now that $R_L^h < R_{CB}^h$ and $R_{CB}^l < R_L^l$. Analogously to the previous case,

$$\frac{1-\sigma}{\sigma} \frac{R_L^l - R_{CB}^l}{R_{CB}^h - R_L^h}$$

is the equity ratio below which Bank b chooses $\alpha_M^b = \alpha_{DCB}^h$ and above which it chooses $\alpha_M^b = 0$.

Suppose now that $\overline{R}_L = \overline{R}_{CB}$.

– Suppose first that $R_L^s = R_{CB}^s$ for all states $s = l, h$. In this case, Bank b cannot influence its expected shareholders' value by varying its amount of loans. Therefore, $[0, +\infty)$ constitutes the set of Bank b's optimal choices.

– Suppose now that $R_L^l < R_{CB}^l$ and $R_{CB}^h < R_L^h$. In this case, for $\alpha_M^b \leq \alpha_{DH}^l$, Bank b does not default on depositors, and its expected shareholders' value is constant and equal to $E_B \overline{R}_D$. However, for $\alpha_{DH}^l < \alpha_M^b$, Bank b defaults on depositors in the bad state. Then Bank b can further increase the expected shareholders' value by granting more loans. The reason is that shareholders are protected by limited liability and due to depositors' bail-out by the government, the deposit gross rate of return of Bank b received by households in the bad state is R_D^l. However, levels of money creation $\alpha_M^b > \alpha_{DCB}^l$ cannot be optimal for Bank b, as it would default on the central bank and its banker would be subject to heavy penalties. Therefore, Bank b chooses the highest level of lending for which it does not default on the central bank. This means that Bank b chooses $\alpha_M^b = \alpha_{DCB}^l$.

– Suppose finally that $R_L^h < R_{CB}^h$ and $R_{CB}^l < R_L^l$. Analogously to the previous case, Bank b chooses $\alpha_M^b = \alpha_{DCB}^h$.

Suppose finally that $\overline{R}_L > \overline{R}_{CB}$.

– Suppose first that $R_{CB}^s \leq R_L^s$ for all states $s = l, h$ with at least one strict inequality. In this case, Bank b can increase the expected shareholders' value by granting more loans. Accordingly, its choice is $\alpha_M^b = +\infty$.

– Suppose now that $R_L^l < R_{CB}^l$ and $R_{CB}^h < R_L^h$. In this case, for $\alpha_M^b \leq \alpha_{DH}^l$, Bank b does not default on depositors, and it can increase the expected shareholders' value by increasing its lending level. However, for $\alpha_{DH}^l < \alpha_M^b$, Bank b defaults on depositors in the bad state. Then Bank b can further increase the expected shareholders' value by granting more loans. The reason is that shareholders are protected by limited liability and due to depositors' bail-out by the government, the deposit gross rate of return of Bank b received by households in the bad state is R_D^l. However, levels of money creation $\alpha_M^b > \alpha_{DCB}^l$ cannot be optimal for Bank b, as it would default on the central bank and its banker would be subject to heavy penalties. Therefore, Bank b chooses the highest level of lending for which it does not default on the central bank. This means that Bank b chooses $\alpha_M^b = \alpha_{DCB}^l$.

– Suppose finally that $R_L^h < R_{CB}^h$ and $R_{CB}^l < R_L^l$. Analogously to the previous case, Bank b chooses $\alpha_M^b = \alpha_{DCB}^h$.

We can summarize the choices of lending levels by banks, given gross rates $(R_L^s)_s$, policy choices $(R_{CB}^s)_s$, and their equity ratio φ, with the correspondence $\hat{\alpha}_M\big((R_L^s)_s, (R_{CB}^s)_s, \varphi\big)$ given in the proposition. □

Proof of Lemma 6.
Suppose first that $\max\left(\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right], \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right) \leq \frac{\mathbf{f}'(\mathbf{W})}{p_I}$. Now we define the auxiliary function

$$g_1(S_F) := \mathbf{f}(\mathbf{W}) - \left(\mathbf{f}(S_F) + p_I(\mathbf{W} - S_F)\max\left(\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right], \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right)\right).$$

It is easy to verify that, for all $S_F \in [0, \mathbf{W})$, $g_1'(S_F) < 0$. Moreover, $g_1(\mathbf{W}) = 0$. Therefore, $g_1(S_F) > 0$ for all $S_F \in [0, \mathbf{W})$, which establishes the first case in Equation (2.12).

Suppose now that $\max\left(\frac{\mathbf{f}'(\mathbf{0})}{p_I}, \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right) < \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]$. Next we consider the function

$$g_2(S_F) := p_I \mathbf{W} \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] - \left(\mathbf{f}(S_F) + p_I(\mathbf{W} - S_F)\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right),$$

which shares similar properties to g_1: for all $S_F \in [0, \mathbf{W}]$, $g_2'(S_F) > 0$, $g_2(0) = 0$, and thus $g_2(S_F) > 0$ for all $S_F \in (0, \mathbf{W}]$. Accordingly, we can apply the same argument to g_2 as previously for g_1 and obtain the second case in Equation (2.12). With similar arguments we also obtain the third and fourth cases.

Suppose finally that $\max\left(\frac{\mathbf{f}'(\mathbf{W})}{p_I}, \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right) < \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq \frac{\mathbf{f}'(\mathbf{0})}{p_I}$. Now we consider

$$\begin{aligned} g_3(S_F) := &\,\mathbf{f}\left(\mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\right) + p_I\left(\mathbf{W} - \mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\right)\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \\ &- \left(\mathbf{f}(S_F) + p_I(\mathbf{W} - S_F)\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right). \end{aligned}$$

We observe that g_3 is strictly convex in S_F, $g_3'(0) = -\mathbf{f}'(\mathbf{0}) + p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq 0$, and $g_3'(\mathbf{W}) = -\mathbf{f}'(\mathbf{W}) + p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] > 0$. Hence, on $[\mathbf{0}, \mathbf{W}]$, g_3 takes the minimum at $S_F = \mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)$, and it holds that $g_3\left(\mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\right) = 0$. Therefore, $g_3(S_F) > 0$ for all $S_F \neq \mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)$, which proves the fifth case in Equation (2.12). With similar arguments we also obtain the last two cases. □

Proof of Lemma 4.
Demands for the investment good by firms in MT and FT are directly derived

from the following shareholders' value-maximization problems:

$$\max_{\mathbf{K_M}\in[\mathbf{0},\mathbf{W}]}\{\mathbb{E}[\max(\mathbf{K_M}(\mathbf{R^s_M}p^s_C - R^s_L p_I),0)]\}$$
$$\text{s.t. } \mathbf{R^s_M}p^s_C = R^s_L p_I \text{ for all states } s = l,h$$
$$\text{and} \quad \max_{\mathbf{K_F}\in[\mathbf{0},\mathbf{W}]}\{\mathbb{E}[\max((\mathbf{f}(\mathbf{K_F}) - \mathbf{K_F}\mathbf{R_F})p^s_C,0)]\}.$$

□

Proof of Theorem 1.

Let $\mathcal{E}^*$ be an equilibrium with banks.

Then all banks choose the same level of money creation and lending, denoted by α^*_M. At the aggregate level, however, the amount borrowed by banks from the central bank has to equal the amount deposited by banks at the central bank, meaning that $\int_0^1 \alpha^b_M \mathrm{d}b = 1$, which translates into $\alpha^*_M = 1$. The result of Lemma 2 implies that we can apply Proposition 1. Thus, given gross rates of return $(R^{s*}_L)_s$, policy choices $(R^s_{CB})_s$, and the equity ratio φ^*, all banks $b \in [0,1]$ choose a lending level $\alpha^b_M \in \hat{\alpha}_M\big((R^{s*}_L)_s,(R^s_{CB})_s,\varphi^*\big)$, as given in Proposition 1. The only gross rates of return in Proposition 1 rationalizing $\alpha^*_M = 1$ are

$$R^{s*}_L = R^s_{CB}$$

for all states $s = l,h$. A direct consequence of this relation, Lemma 2, and the expression of profits in Equation (2.1) is that

$$R^{s*}_E = R^{s*}_D = R^{s*}_L = R^s_{CB} \tag{2.13}$$

for all states $s = l,h$. Moreover, due to Lemma 2 and the tie-breaking rule introduced in Subsection 2.2.3, the interbank lending market is not used in an equilibrium with banks. Finally, $\Pi^{s*}_M = 0$ for all states $s = l,h$ (see Subsection 2.2.4), which translates into

$$\mathbf{R^s_M}p^{s*}_C = R^{s*}_L p^*_I$$

for all states $s = l, h$. Given gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices p_I^* and $(p_C^{s*})_s$, households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, S_F^*\big)$ given S_F^*, $D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, E_B^*, S_F^*\big)$ given E_B^* and S_F^*, and $S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s\big)$. These correspondences are given in Lemma 6 in Appendix 2.E. Only the first, the fourth, and the seventh cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ correspond to equal nominal gross rates of return R_E^{s*} and R_D^{s*} and are hence consistent with the equality of nominal gross rates of return in Equation (2.13). However, the assumption $\mathbf{f}'(\mathbf{W}) < \overline{\mathbf{R}}_\mathbf{M} < \mathbf{f}'(\mathbf{0})$ plus $\mathbf{R}_\mathbf{M}^\mathbf{s} p_C^{s*} = R_L^{s*} p_I^*$ rule out the first and fourth cases. As in an equilibrium with banks $E_B^*, D_H^* > 0$, we thus obtain

$$\begin{aligned}
&E_B^* \in \big(0, p_I^* \left(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\right)\big),\\
&D_H^* = p_I^* \left(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\right) - E_B^*, \text{ and}\\
&S_F^* = \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M}).
\end{aligned}$$

Finally, $\mathbf{R}_\mathbf{F}^*$ can be determined by using Lemma 4 and equating the demand for the investment good $\mathbf{K}_\mathbf{F}^*$ to its supply S_F^*. With the help of the equity ratio φ^*, we can then re-write all equilibrium variables as given in Theorem 1.

In turn, it is straightforward to verify that the tuples given in Theorem 1 constitute equilibria with banks as defined in Subsection 2.2.5. □

Proof of Lemma 5.

Let $\mathcal{E}^*$ be an equilibrium with banks for which a minimum equity ratio φ^{reg} is imposed on banks at the end of Period $t = 0$. If $\alpha_M^b \geq 1$ for some bank $b \in [0, 1]$, the minimum equity ratio imposes the following constraint on money creation α_M^b:

$$\begin{aligned}
&\frac{E_B^*}{\alpha_M^b L_M^*} \geq \varphi^{reg}, \quad \text{or equivalently}\\
&\alpha_M^b \leq \frac{\varphi^*}{\varphi^{reg}}.
\end{aligned}$$

If $\alpha_M^b \leq 1$, the previous constraint becomes

$$\frac{E_B^*}{L_M^*} \geq \varphi^{reg}, \quad \text{or equivalently}$$
$$\varphi^* \geq \varphi^{reg}.$$

□

Proof of Lemma 7.
Let $b \in [0,1]$ denote a bank and assume that a minimum equity ratio $\varphi^{reg} \leq \varphi$ is imposed on banks at the end of Period $t = 0$. Using Lemma 5 and the property $R_D^s = R_{CB}^s$ for all states $s = l, h$, Bank b's maximization problem simplifies to

$$\max_{\alpha_M^b \in \left[0, \frac{\varphi}{\varphi^{reg}}\right]} \left\{ \mathbb{E}[\max(\alpha_M^b L_M(R_L^s - R_{CB}^s) + E_B R_{CB}^s, 0)] \right\}.$$

As the arguments used in this proof to investigate the impact of lending on the shareholders' value are similar to the ones given in the proof of Proposition 1, we refer readers to the proof of Proposition 1 for further details.

Suppose that $\overline{R}_L < \overline{R}_{CB}$.

- Suppose first that $R_L^s \leq R_{CB}^s$ for all states $s = l, h$ with at least one strict inequality. In this case, the expected shareholders' value of Bank b is decreasing in the volume of loans. Therefore, its choice is $\alpha_M^b = 0$.
- Suppose now that $R_L^l < R_{CB}^l$ and $R_{CB}^h < R_L^h$.
  - Suppose first that $\alpha_{DCB}^l \leq \frac{\varphi}{\varphi^{reg}}$. Then the equity ratio requirement does not impose an additional constraint on Bank b, and its optimal choice of money creation is

$$\begin{aligned} &\alpha_M^b = 0 && \text{if } \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l} < \varphi, \\ &\alpha_M^b \in \{0, \alpha_{DCB}^l\} && \text{if } \varphi = \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}, \\ \text{and} \quad &\alpha_M^b = \alpha_{DCB}^l && \text{if } \varphi < \frac{\sigma}{1-\sigma}\frac{R_L^h - R_{CB}^h}{R_{CB}^l - R_L^l}. \end{aligned}$$

- Suppose now that $\alpha^l_{DCB} > \frac{\varphi}{\varphi^{reg}}$. Then either $\alpha^l_{DH} < \frac{\varphi}{\varphi^{reg}}$ and the expected shareholders' value of Bank b is decreasing for $\alpha^b_M \in [0, \alpha^l_{DH}]$ and increasing for $\alpha^b_M \in [\alpha^l_{DH}, \frac{\varphi}{\varphi^{reg}}]$, or $\alpha^l_{DH} \geq \frac{\varphi}{\varphi^{reg}}$ and the expected shareholders' value is decreasing for $\alpha^b_M \in [0, \frac{\varphi}{\varphi^{reg}}]$. Therefore, if $\alpha^l_{DH} \geq \frac{\varphi}{\varphi^{reg}}$, the choice of Bank b is $\alpha^b_M = 0$. Suppose that $\alpha^l_{DH} < \frac{\varphi}{\varphi^{reg}}$. Then the choice of Bank b can be derived by comparison between the expected shareholders' value for $\alpha^b_M = 0$ and for $\alpha^b_M = \frac{\varphi}{\varphi^{reg}}$. Using the expression for profits in Equation (2.1) and rearranging terms establishes that the choice for Bank b is

$$
\begin{aligned}
&\alpha^b_M = 0 && \text{if } \frac{\sigma}{1-\sigma}\frac{R^h_L - R^h_{CB}}{R^l_{CB}} < \varphi^{reg},\\
&\alpha^b_M \in \{0, \tfrac{\varphi}{\varphi^{reg}}\} && \text{if } \varphi^{reg} = \frac{\sigma}{1-\sigma}\frac{R^h_L - R^h_{CB}}{R^l_{CB}},\\
\text{and } &\alpha^b_M = \tfrac{\varphi}{\varphi^{reg}} && \text{if } \varphi^{reg} < \frac{\sigma}{1-\sigma}\frac{R^h_L - R^h_{CB}}{R^l_{CB}}.
\end{aligned}
$$

- The analysis for $R^h_L < R^h_{CB}$ and $R^l_{CB} < R^l_L$ is similar to the previous one.

Suppose now that $\overline{R}_L = \overline{R}_{CB}$.

- Suppose first that $R^s_L = R^s_{CB}$ for all states $s = l, h$. Then $[0, \frac{\varphi}{\varphi^{reg}}]$ constitutes the set of Bank b's optimal choices.
- Suppose now that $R^l_L < R^l_{CB}$ and $R^h_{CB} < R^h_L$.
  - Suppose now that $\alpha^l_{DH} < \frac{\varphi}{\varphi^{reg}}$. Then the expected shareholders' value of Bank b is constant for all $\alpha^b_M \in [0, \alpha^l_{DH}]$ and increases with α^b_M in the interval $[\alpha^l_{DH}, \frac{\varphi}{\varphi^{reg}}]$. Therefore, Bank b chooses $\alpha^b_M = \min(\alpha^l_{DCB}, \frac{\varphi}{\varphi^{reg}})$.
  - Suppose now that $\alpha^l_{DH} \geq \frac{\varphi}{\varphi^{reg}}$. Then Bank b's expected shareholders' value is constant for all $\alpha^b_M \in [0, \frac{\varphi}{\varphi^{reg}}]$. Therefore, $[0, \frac{\varphi}{\varphi^{reg}}]$ constitutes the set of Bank b's optimal choices.
- The analysis for $R^h_L < R^h_{CB}$ and $R^l_{CB} < R^l_L$ is similar to the previous case.

Suppose finally that $\overline{R}_L > \overline{R}_{CB}$.

- Suppose first that $R^s_L \geq R^s_{CB}$ for all states $s = l, h$ with at least one strict inequality. In this case, Bank b can increase the expected shareholders' value by granting more loans. Therefore, its choice is $\alpha^b_M = \frac{\varphi}{\varphi^{reg}}$.

- Suppose now that $R_L^l < R_{CB}^l$ and $R_{CB}^h < R_L^h$. In this case, Bank b can increase the expected shareholders' value by granting more loans. Therefore, its choice is $\alpha_M^b = \min(\alpha_{DCB}^l, \frac{\varphi}{\varphi^{reg}})$.
- The analysis for $R_L^h < R_{CB}^h$ and $R_{CB}^l < R_L^l$ is similar to the previous case.

We can summarize our findings with the correspondence $\hat{\alpha}_M^{reg}$ given in the lemma.

□

Proof of Proposition 3.
Let $\mathcal{E}^*$ be an equilibrium with banks for which a minimum equity ratio φ^{reg} is required to be held by banks at the end of Period $t = 0$. We first note that a direct consequence is that $\varphi^* \in [\varphi^{reg}, 1)$.

Then all banks choose the same level of money creation and lending, denoted by α_M^*. At the aggregate level, however, the amount borrowed by banks from the central bank has to equal the amount deposited by banks at the central bank, meaning that $\int_0^1 \alpha_M^b \mathrm{d}b = 1$, which translates into $\alpha_M^* = 1$. The result of Lemma 2 implies that we can apply Lemma 7. Thus, given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, and the equity ratio φ^*, all banks $b \in [0,1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M^{reg}\big((R_L^{s*})_s, (R_{CB}^s)_s, \varphi^*\big)$ as given in Lemma 7. Therefore, the only gross rates of return and capital structure φ^* in Lemma 7 in Appendix 2.F rationalizing $\alpha_M^* = 1$ are such that

either Case a) ($R_L^{s*} = R_{CB}^s$ for all states $s = l, h$),
or Case b) ($\overline{R}_L^* = \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, and $\alpha_{DH}^l \geq \frac{\varphi^*}{\varphi^{reg}}$),
or Case c) ($\overline{R}_L^* = \overline{R}_{CB}$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, and $\alpha_{DH}^h \geq \frac{\varphi^*}{\varphi^{reg}}$),
or Case d) ($\overline{R}_L^* < \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, $\alpha_{DH}^l < 1$,
and $\varphi^* = \varphi^{reg} = \frac{\sigma}{1-\sigma}\frac{R_L^{h*} - R_{CB}^h}{R_{CB}^l}$),
or Case e) ($\overline{R}_L^* < \overline{R}_{CB}$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, $\alpha_{DH}^h < 1$,
and $\varphi^* = \varphi^{reg} = \frac{1-\sigma}{\sigma}\frac{R_L^{l*} - R_{CB}^l}{R_{CB}^h}$),
or Case f) ($R_L^{s*} \geq R_{CB}^s$ for all states $s = l, h$ with at least one strict inequality, and $\varphi^* = \varphi^{reg}$),
or Case g) ($\overline{R}_L^* = \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, $\alpha_{DH}^l < 1$, and $\varphi^* = \varphi^{reg}$),
or Case h) ($\overline{R}_L^* = \overline{R}_{CB}$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, $\alpha_{DH}^h < 1$, and $\varphi^* = \varphi^{reg}$),

or Case i) $(\overline{R}_L^* < \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, $\alpha_{DH}^l < 1$,
and $\varphi^* = \varphi^{reg} < \frac{\sigma}{1-\sigma}\frac{R_L^{h*}-R_{CB}^h}{R_{CB}^l})$,
or Case j) $(\overline{R}_L^* < \overline{R}_{CB}$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, $\alpha_{DH}^h < 1$,
and $\varphi^* = \varphi^{reg} < \frac{1-\sigma}{\sigma}\frac{R_L^{l*}-R_{CB}^l}{R_{CB}^h})$,
or Case k) $(\overline{R}_L^* > \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, and $\varphi^* = \varphi^{reg})$,
or Case l) $(\overline{R}_L^* > \overline{R}_{CB}$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, and $\varphi^* = \varphi^{reg})$.

Note first that for Cases f) to l), the expected gross rate of return on equity achieved by any Bank b when choosing $\alpha_M^b = 1$ is higher than the expected gross rate of return on equity when choosing $\alpha_M^b = 0$. Since the latter is equal to the expected deposit gross rate, we can conclude that in all cases f) to l) the expected gross rate of return on equity is larger than the expected deposit gross rate. Moreover, for Cases a) to e), the expected gross rate of return on equity is equal to the expected deposit gross rate.

Given gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices $p_I^* = p_C^* = 1$, households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^* = 1, p_C^* = 1, S_F^*\big)$ given S_F^*,
$D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^* = 1, p_C^* = 1, E_B^*, S_F^*\big)$ given E_B^* and S_F^*, and
$S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^* = 1, p_C^* = 1\big)$. These correspondences are given in Lemma 6 in Appendix 2.E.

For Cases f) to l), Lemma 6 implies that $D_H^* = 0$, which is excluded from the definition of an equilibrium with banks. Therefore, Cases f) to l) do not correspond to possible equilibria with banks.

For Cases a) to e), expected gross rates of return $\overline{R}_E^*$ and $\overline{R}_D^*$ are equal, and only the first, the fourth, and the seventh cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ in Appendix 2.E are consistent with $\overline{R}_E^* = \overline{R}_D^*$.

For Cases a) to c), the assumption $\mathbf{f'(W)} < \overline{\mathbf{R}}_{\mathbf{M}} < \mathbf{f'(0)}$ together with $\overline{\mathbf{R}}_{\mathbf{M}} = \overline{R}_E^* = \overline{R}_D^*$ rule out the first and fourth cases. As in an equilibrium with banks

$E_B^*, D_H^* > 0$, we obtain

$$E_B^* \in (0, \big(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\big)),$$
$$D_H^* = \big(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\big) - E_B^*, \text{ and}$$
$$S_F^* = \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M}).$$

For Cases d) and e), the assumption $\mathbf{f}'(\mathbf{W}) < \overline{\mathbf{R}}_\mathbf{M}$ together with $\overline{\mathbf{R}}_\mathbf{M} < \overline{R}_E^* = \overline{R}_D^*$ rule out the first case. As in an equilibrium with banks $E_B^*, D_H^* > 0$, we obtain

$$E_B^* \in (0, (\mathbf{W} - S_F^*)),$$
$$D_H^* = (\mathbf{W} - S_F^*) - E_B^*,$$
$$S_F^* = \begin{cases} \mathbf{f}'^{-1}\big(\overline{R}_{CB}^*\big) & \text{if } \mathbf{f}'(\mathbf{0}) \geq \overline{R}_{CB}^*, \\ 0 & \text{otherwise.} \end{cases}$$

In turn, it is straightforward to verify that the tuples found in this proof constitute equilibria with banks as defined in Subsection 2.2.5. □

Proof of Corollary 6.
Corollary 6 immediately results from Corollary 5 and from the observation that $\overline{R}_{CB} = \overline{\mathbf{R}}_\mathbf{M}$, $\mathbf{R}_\mathbf{M}^\mathbf{l} < 1 \leq \overline{\mathbf{R}}_\mathbf{M}$, and $R_{CB}^s \geq 1$ for all $s = l, h$ together imply that $R_{CB}^h > \mathbf{R}_\mathbf{M}^\mathbf{h}$.

□

2.H Example

We illustrate our results with an example. In this example we use the normalization $p_I^* = 1$, and we set households' portfolio choice to $\varphi^* = 0.4$. We use the parameter values given in Table 2.H.1.

We note that all assumptions on parameters and the function $\mathbf{f}$ are fulfilled, including Assumption 1. We now distinguish two cases:

– Either the central bank sets $(R_{CB}^l, R_{CB}^h) = (1.02, 1.02)$. Then we obtain the

$\mathbf{W}$	1
$(\mathbf{R^l_M}, \mathbf{R^h_M})$	$(0.98, 1.06)$
σ	0.5
$\mathbf{f}(\mathbf{K_F})$	$2(\mathbf{K_F} - \frac{\mathbf{K_F}^2}{2})$

Table 2.H.1: Parameter values. Source: Own illustration.

variable values given on the left side of Table 2.H.2.

– Or the central bank sets $(R^l_{CB}, R^h_{CB}) = (\mathbf{R^l_M}, \mathbf{R^h_M})$. Then we obtain the variable values given on the right side of Table 2.H.2.

In the case of price rigidities characterized by $p^{s*}_C = 1$ for $s = l, h$, the policy presented in Corollary 6 yields the following values:

$$R^l_{CB} = 1, \quad R^h_{CB} = 1.04, \quad \text{and} \quad \varphi^{reg} = 0.02.$$

(R_D^l, R_D^h) $= (R_L^l, R_L^h)$ $= (R_E^l, R_E^h)$	$(1.02, 1.02)$
$\mathbf{R_F}$	1.02
(p_C^l, p_C^h)	$(1.04, 0.96)$
$L_M = \mathbf{K_M}$	0.51
$S_F = \mathbf{K_F}$	0.49
D_H	0.31
E_B	0.20
$(\widetilde{D}_H^l, \widetilde{D}_H^h)$	$(0.52, 0.52)$
Π_M^s	0
(Π_F^l, Π_F^h)	$(0.25, 0.23)$
(Π_B^l, Π_B^h)	$(0.21, 0.21)$

(R_D^l, R_D^h) $= (R_L^l, R_L^h)$ $= (R_E^l, R_E^h)$	$(0.98, 1.06)$
$\mathbf{R_F}$	1.02
(p_C^l, p_C^h)	$(1.00, 1.00)$
$L_M = \mathbf{K_M}$	0.51
$S_F = \mathbf{K_F}$	0.49
D_H	0.31
E_B	0.20
$(\widetilde{D}_H^l, \widetilde{D}_H^h)$	$(0.50, 0.54)$
Π_M^s	0
(Π_F^l, Π_F^h)	$(0.24, 0.24)$
(Π_B^l, Π_B^h)	$(0.20, 0.22)$

Table 2.H.2: Variable values with policy gross rates $(R_{CB}^l, R_{CB}^h) = (1.02, 1.02)$ on the left side and $(R_{CB}^l, R_{CB}^h) = (0.98, 1.06)$ on the right side. Source: Own illustration.

Chapter 3

Generalizations and Variations of the Model with Money Creation

Abstract

We demonstrate that our main findings from Chapter 2 can be extended to various changes to our assumptions, such as the denomination of bonds and of profit maximization, the extent of the macroeconomic shock, the form of the production functions in the two sectors we defined, and additional states of the world. Moreover, if the real deposit rates do not adjust to the macroeconomic shock, we prove that only capital requirements that are sufficiently high can establish the existence and uniqueness of efficient equilibria with banks. If banks incur costs in real terms for the issuance of their equity, we show that governmental authorities can use monetary policy and capital requirements to implement any welfare that is arbitrarily close to the first-best welfare. Finally, we demonstrate that reserve requirements coupled with haircuts are equivalent to capital requirements as to their effect on banks' money creation.

Preliminary remark: In the generalizations and variations of Chapter 3, we use the model described in Chapter 2 and its variables, and we only re-define the variables that change with the new features examined.

3.1 Equivalence of Profit Maximization in Real Terms

In Chapter 2, banks maximize the expected shareholders' value in nominal terms. However, as only consumption matters for shareholders, banks should maximize the expected shareholders' value in real terms. In Section 3.1, we show that these two maximization problems are equivalent in the sense that they implement exactly the same equilibria with banks.

We begin by re-stating the notion of an equilibrium with banks, using the new banks' optimization problem:

Definition 3
A symmetric equilibrium with banks and shareholders' value maximization in real terms is an equilibrium with banks as defined by Definition 1, in which the banks' optimization problem is replaced by the following maximization problem:

$$\max_{\alpha_M^b \geq 0} \left\{ \mathbb{E} \left[\max \left(\alpha_M^b L_M \frac{R_L^s - R_{CB}^s}{p_C^s} + L_M \frac{R_{CB}^s - R_D^s}{p_C^s} + E_B \frac{R_D^s}{p_C^s}, 0 \right) \right] \right\}, \tag{3.1}$$

where gross rates of return $(R_D^s)_s$, $(R_L^s)_s$, and $(R_{CB}^s)_s$ as well as prices $(p_C^s)_s$ are taken as given.

Using this new definition, Proposition 1 re-writes as follows:

Proposition 5
If $R_D^s = R_{CB}^s$ in all states $s = l, h$, the privately optimal amounts of money creation and lending by an individual bank are represented by a correspondence denoted by $\hat{\alpha}_M : \mathbb{R}_{++}^6 \times (0,1) \to \mathcal{P}(\mathbb{R} \cup \{+\infty\})$ and given by

$$\hat{\alpha}_M\big((R_L^s)_s, (R_{CB}^s)_s, (p_C^s)_s, \varphi\big) =$$
$$\begin{cases} \{+\infty\} & \text{if } R_L^s \geq R_{CB}^s \text{ for all states } s = l, h \\ & \quad \text{with at least one strict inequality,} \end{cases}$$

$$
\begin{cases}
\{\alpha^l_{DCB}\} & \textit{if } (\mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] \geq \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right],\ R^l_L < R^l_{CB},\ \textit{and } R^h_{CB} < R^h_L)\ \textit{or} \\
 & \textit{if } (\mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] < \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right],\ R^h_{CB} < R^h_L,\ \textit{and } \varphi < \frac{\sigma}{1-\sigma}\frac{\mathbf{R^h_M}}{\mathbf{R^l_M}}\frac{R^l_L}{R^h_L}\frac{R^h_L - R^h_{CB}}{R^l_{CB} - R^l_L}), \\
\{\alpha^h_{DCB}\} & \textit{if } (\mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] \geq \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right],\ R^h_L < R^h_{CB},\ \textit{and } R^l_{CB} < R^l_L)\ \textit{or} \\
 & \textit{if } (\ \mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] < \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right],\ R^l_{CB} < R^l_L,\ \textit{and } \varphi < \frac{1-\sigma}{\sigma}\frac{\mathbf{R^l_M}}{\mathbf{R^h_M}}\frac{R^h_L}{R^l_L}\frac{R^l_L - R^l_{CB}}{R^h_{CB} - R^h_L}), \\
[0, +\infty) & \textit{if } R^s_L = R^s_{CB} \textit{ for all states } s = l, h, \\
\{0, \alpha^l_{DCB}\} & \textit{if } \mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] < \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right],\ R^h_{CB} < R^h_L,\ \textit{and } \varphi = \frac{\sigma}{1-\sigma}\frac{\mathbf{R^h_M}}{\mathbf{R^l_M}}\frac{R^l_L}{R^h_L}\frac{R^h_L - R^h_{CB}}{R^l_{CB} - R^l_L}, \\
\{0, \alpha^h_{DCB}\} & \textit{if } \mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] < \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right],\ R^l_{CB} < R^l_L,\ \textit{and } \varphi = \frac{1-\sigma}{\sigma}\frac{\mathbf{R^l_M}}{\mathbf{R^h_M}}\frac{R^h_L}{R^l_L}\frac{R^l_L - R^l_{CB}}{R^h_{CB} - R^h_L}, \\
\{0\} & \textit{if } (R^s_L \leq R^s_{CB} \textit{ for all states } s = l, h \\
 & \textit{with at least one strict inequality) or} \\
 & \textit{if } (\mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] < \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right],\ R^h_{CB} < R^h_L,\ \textit{and } \frac{\sigma}{1-\sigma}\frac{\mathbf{R^h_M}}{\mathbf{R^l_M}}\frac{R^l_L}{R^h_L}\frac{R^h_L - R^h_{CB}}{R^l_{CB} - R^l_L} < \varphi)\ \textit{or} \\
 & \textit{if } (\mathbb{E}\left[\frac{R^s_L}{p^s_C}\right] < \mathbb{E}\left[\frac{R^s_{CB}}{p^s_C}\right],\ R^l_{CB} < R^l_L,\ \textit{and } \frac{1-\sigma}{\sigma}\frac{\mathbf{R^l_M}}{\mathbf{R^h_M}}\frac{R^h_L}{R^l_L}\frac{R^l_L - R^l_{CB}}{R^h_{CB} - R^h_L} < \varphi).
\end{cases}
$$

The proof of Proposition 5 is straightforward and follows the proof of Proposition 1, with the difference that banks maximize their shareholders' value in real terms. This implies that the threshold equity ratio that makes both lending and defaulting against households profitable slightly differs from the one given in Proposition 1. However, our results remain qualitatively the same. We are now able to prove that equilibria are identical to the ones found in Theorem 1:

Corollary 7

The equilibria with banks where the shareholders' value is maximized by bankers in real terms are identical to the equilibria with banks where the shareholders' value is maximized by bankers in nominal terms.

The proof of Corollary 7 is given in Appendix 3.A. Finally, we simply note that in the case of price rigidities, the two maximization problems are directly identical and thus that the two models are completely identical.

3.2 Bonds Denominated in Nominal Terms

In Chapter 2, we assumed that bonds were denominated in real terms to simplify the exposition. In this section, we show how the model could be written with bonds denominated in nominal terms and we prove that the two models are equivalent in the sense that the denomination of bonds in nominal terms yields a similar definition of equilibria with banks, where the only difference is the presence of no-default conditions when bonds are denominated in nominal terms.

The only parts from Chapter 2 that change are Appendices 2.A and 2.B, where the exact payment processes are described. In Subsections 3.2.1 and 3.2.2, we re-write these appendices completely, using bonds denominated in nominal terms. In Subsection 3.2.3, we give the no-default conditions that have to be added to the definition of an equilibrium with banks.

3.2.1 Stage C

We examine the detailed payment process, investment in FT and the payment of bank equity in Stage C through a series of substages. For this purpose, we index all variables that change at some substage by an integer starting from 1.

Stage C, Substage 1: Borrowing of Banks from the Central Bank

To have enough CB deposits to guarantee payments, Bank b borrows from the central bank the amount of

$$d^b_{CB_1} := l^b_M = \alpha^b_M D_M.$$

As a result, an aggregate amount of CB deposits amounting to $D_{CB_1} = D_M > 0$ is created. The balance sheets of banks and households are given in Table 3.1.

Households	
W	E_H

Bank b	
$d^b_{CB_1}$	$l^b_{CB_1}$
l^b_M	d^b_M

Table 3.1: Bonds denominated in nominal terms—Balance sheets at the end of Stage C, Substage 1. Source: Own illustration.

Stage C, Substage 2: Sale of an Amount of Investment Good to MT

We assume that firms in FT and MT buy the highest amount of investment good they can afford and do not hold deposits in the production stage D:

$$\mathbf{W} := \frac{S_F + L_M}{p_I}.$$

Firms in MT buy $\mathbf{K_M} = \frac{L_M}{p_I}$ from households. To settle these payments, each bank b transfers $d^b_M = \alpha^b_M D_M$ to other banks and receives $d_{H_1} := D_M$ from other banks in the form of CB deposits. We note that D_{H_1} does not depend on the individual bank b, due to our assumption that households keep deposits evenly distributed across all banks at all times. The corresponding aggregate amount is denoted by D_{H_1}. This transaction impacts the CB deposits of bank b as follows:

$$d^b_{CB_2} := d^b_{CB_1} - \alpha^b_M D_M + D_M = D_M.$$

The balance sheets of banks and households are given in Table 3.2.

Stage C, Substage 3: Sequential Investment in FT and Sale of an Amount of Investment Good to FT

As the distribution of private deposits of entrepreneurs in FT across banks clearly does not matter in equilibrium, we first assume for the sake of simplicity that these private deposits are distributed uniformly across banks at all times.

Households		Bank b	
$\mathbf{K_F}$	E_H	$d^b_{CB_2}$	$l^b_{CB_1}$
D_{H_1}		l^b_M	d_{H_1}

Table 3.2: Bonds denominated in nominal terms—Balance sheets at the end of Stage C, Substage 2. Source: Own illustration.

The investment of S_F by households at the nominal return[1] R_F and the sale of an amount of investment good $\mathbf{K_F} := \frac{S_F}{p_I}$ take place sequentially in $n_I + 1$ rounds, where $n_I := \left[\frac{S_F}{D_M}\right]$.[2] A round $k \in \{1, ..., n_I + 1\}$ consists of two steps.

In the first step, households buy the highest quantity of bonds issued by FT that they can afford. To settle these payments, in round $k \in \{1, ..., n_I\}$ (resp. round $k = n_I+1$), each bank b transfers some CB deposits amounting to $d_{H_1} = D_M$ (resp. $d_{F_2} := S_F - n_I D_M$)[3] to other banks, and receives some CB deposits amounting to $d_{F_1} := D_M$ (resp. $d_{H_1} - d_{H_2} = S_F - n_I D_M$) from other banks. Clearly, these transactions do not impact the CB deposits of bank b.

In the second step, firms in FT use the proceeds of bond issuance to buy the highest affordable amount of investment good from households. To settle these payments, in round $k \in \{1, ..., n_I\}$ (resp. round $k = n_I + 1$), each bank b transfers some CB deposits amounting to $d_{F_1} = D_M$ (resp. $d_{H_1} - d_{H_2} = S_F - n_I D_M$) to other banks, and receives some CB deposits amounting to $d_{H_1} = D_M$ (resp. d_{F_2}) from other banks. Clearly, these transactions do not impact the CB deposits of bank b.

For $k \in \{1, ..., n_I\}$, the balance sheets of households, firms in FT, and banks are given in Table 3.3 for the first step and in Table 3.4 for the second step. For $k = n_I + 1$, the balance sheets of households, firms in FT, and banks are given in

[1] We note that the gross rate of return on nominal bonds is also denominated in nominal terms.

[2] We denote by $[x]$ the highest integer such that $[x] \leq x$.

[3] We define $d_{H_2} := d_{H_1} - (S_F - n_I D_M)$.

Table 3.5 for the first step and in Table 3.6 for the second step.

Households	
$\mathbf{K_F} - \frac{(k-1)D_M}{p_I}$	E_H
$S_{F_k} := kD_M$	

FT	
$\frac{(k-1)D_M}{p_I}$	$S_{F_k} = kD_M$
D_{F_1}	

Bank b	
$d^b_{CB_2}$	$l^b_{CB_1}$
l^b_M	d_{F_1}

Table 3.3: Bonds denominated in nominal terms—Balance sheets in Stage C, Substage 3, at round $k \in \{1, ..., n_I\}$, first step. Source: Own illustration.

Households	
$\mathbf{K_F} - \frac{kD_M}{p_I}$	
$S_{F_k} := kD_M$	E_H
D_{H_1}	

FT	
$\frac{kD_M}{p_I}$	$S_{F_k} = kD_M$

Bank b	
$d^b_{CB_2}$	$l^b_{CB_1}$
l^b_M	d_{H_1}

Table 3.4: Bonds denominated in nominal terms—Balance sheets in Stage C, Substage 3, at round $k \in \{1, ..., n_I\}$, second step. Source: Own illustration.

Stage C, Substage 4: Netting of CB Deposits and CB Liabilities

Now banks can net their CB deposits and CB liabilities, as no further payment has to be made before production. We use

$$\delta^b := d^b_{CB_2} - l^b_{CB_1} = (1 - \alpha^b_M)L_M$$

Households	
$\mathbf{K_F} - \frac{n_I D_M}{p_I}$	
S_F	E_H
D_{H_2}	

FT	
$\frac{n_I D_M}{p_I}$	S_F
D_{F_2}	

Bank b	
$d^b_{CB_2}$	$l^b_{CB_1}$
	d_{H_2}
l^b_M	d_{F_2}

Table 3.5: Bonds denominated in nominal terms—Balance sheets in Stage C, Substage 3, at round $k = n_I + 1$, first step. Source: Own illustration.

Households	
S_F	
D_{H_1}	E_H

FT	
$\mathbf{K_F}$	S_F

Bank b	
$d^b_{CB_2}$	$l^b_{CB_1}$
l^b_M	d_{H_1}

Table 3.6: Bonds denominated in nominal terms—Balance sheets in Stage C, Substage 3, at round $k = n_I + 1$, second step. Source: Own illustration.

to denote the net position of bank b against the central bank. We distinguish banks with claims at the central bank and banks that are debtors of the central bank:

$$\begin{aligned} & B_I := \{b_i \in [0,1] \quad \text{s.t.} \quad \delta^{b_i} \geq 0\} \\ \text{and} \quad & B_J := \{b_j \in [0,1] \quad \text{s.t.} \quad \delta^{b_j} < 0\}. \end{aligned}$$

Net claims against the central bank are denoted by $d^{b_i}_{CB} := \delta^{b_i}$ for all $b_i \in B_I$, and net liabilities are denoted by $l^{b_j}_{CB} := -\delta^{b_j}$ for all $b_j \in B_J$. The balance sheets of banks and households are given in Table 3.7.

Households		Bank b_i		Bank b_j	
S_F		$d^{b_i}_{CB}$			$l^{b_j}_{CB}$
D_{H_1}	E_H	$l^{b_i}_M$	d_{H_1}	$l^{b_j}_M$	d_{H_1}

Table 3.7: Bonds denominated in nominal terms—Balance sheets at the end of Stage C, Substage 4. Source: Own illustration.

Stage C, Substage 5: Payment of Bank Equity

Now households pay the equity $E_B := \varphi D_M > 0$ pledged in $t = 1$, thereby destroying the corresponding amount of bank deposits. We use $D_H := (1-\varphi)D_M$ to denote the remaining amount of deposits. Accordingly, $D_{H_1} = E_B + D_H$. The balance sheets of two typical banks representing a net depositor at and a net borrower from the central bank are displayed in Table 2.3 in Chapter 2.

3.2.2 Stage E—No Bank Defaults

We examine the detailed dividend payment, payback of debt, and payment process of Stage E through a series of substages. Similarly to Subsection 3.2.1, whenever a variable changes in some substage, we increase the index by 1, starting with the last index from Subsection 3.2.1.

Stage E, Substage 1: Borrowing of Banks from the Central Bank

To have enough CB deposits to guarantee payments, Bank b borrows from the central bank the amount of $l^{b,s}_{CB_3} = d^{b,s}_{CB_3} := D_H R^s_D + \Pi^{b,s}_B$. We use the notation

$$d^{b_i}_{CB_4} = d^{b_i,s}_{CB_3} + d^{b_i}_{CB} R^s_{CB},$$
$$\text{and} \quad l^{b_j,s}_{CB_4} = l^{b_j}_{CB_3} + l^{b_j}_{CB} R^s_{CB}.$$

The balance sheets of banks and households are given in Table 3.8.

Households	
$S_F R_F$	
$D_H R_D^s$	$E_H R_H^s$
$E_B R_E^s$	
Π_F^s	

Bank b_i	
$d_{CB_4}^{b_i,s}$	$l_{CB_3}^{b_i,s}$
$l_M^{b_i} R_L^s$	$d_H R_D^s$
	$\Pi_B^{b_i,s}$

Bank b_j	
$d_{CB_3}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$l_M^{b_j} R_L^s$	$d_H R_D^s$
	$\Pi_B^{b_j,s}$

Table 3.8: Bonds denominated in nominal terms—Balance sheets at the end of Stage E, Substage 1. Source: Own illustration.

Stage E, Substage 2: Dividend Payment

Bank profits are paid to households as dividends. This creates bank deposits, and the households' deposits at bank b become $\tilde{d}_H^s := D_H R_D^s + \Pi_B^s$. The aggregate amount of households' deposits is then denoted by $\tilde{D}_H^s$. To settle these payments, each bank b transfers $\Pi_B^{b,s}$ to other banks and receives Π_B^s from other banks in the form of CB deposits. These processes impact CB deposits of Banks b_i and b_j as follows:

$$\begin{aligned} & d_{CB_5}^{b_j,s} := d_{CB_3}^{b_j,s} - \Pi_B^{b_j,s} + \Pi_B^s = D_H R_D^s + \Pi_B^s, \\ \text{and} \quad & d_{CB_6}^{b_i,s} := d_{CB_4}^{b_i,s} - \Pi_B^{b_i,s} + \Pi_B^s = d_{CB}^{b_i} R_{CB}^s + D_H R_D^s + \Pi_B^s. \end{aligned}$$

The balance sheets of banks and households are given in Table 3.9.

Stage E, Substage 3: Repayment of Debt, Distribution of Profits, and Sale of an Amount of the Consumption Good to Households by FT

We assume that firms in FT and MT sell their entire stock of consumption good and do not hold deposits at the end of Period $t = 1$. The price of the consumption

Households	
$\tilde{D}_H^s$	
$S_F R_F$	$E_H R_H^s$
Π_F^s	

Bank b_i	
$d_{CB_6}^{b_i,s}$	$l_{CB_3}^{b_i,s}$
$l_M^{b_i} R_L^s$	$\tilde{d}_H^s$

Bank b_j	
$d_{CB_5}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$l_M^{b_j} R_L^s$	$\tilde{d}_H^s$

Table 3.9: Bonds denominated in nominal terms—Balance sheets at the end of Stage E, Substage 2. Source: Own illustration.

good is determined by equating demand and supply, i.e.

$$p_C^s := \frac{\tilde{D}_H^s + S_F R_F + \Pi_F^s}{\mathbf{f}(\mathbf{K_F}) + \mathbf{K_M R_M^s}}.$$

The repayment of the debt $S_F R_F$, the distribution of profits Π_F^s to households,[4] and the sale of the amount of the consumption good $\mathbf{f}(\mathbf{K_F}) = \frac{S_F R_F + \Pi_F^s}{p_C^s}$ take place sequentially in $n_C^s + 1$ rounds where $n_C^s := \left[\frac{S_F R_F + \Pi_F^s}{\tilde{D}_H^s}\right]$. A round $k \in \{1, ..., n_C^s + 1\}$ consists of two steps.

In the first step, households buy the highest amount of consumption good they can afford. To settle these payments, in round $k \in \{1, ..., n_C^s\}$ (resp. round $k = n_C^s + 1$), each bank b transfers some amount of CB deposits $\tilde{d}_H^s$ (resp. $d_{F_4,s} := S_F R_F + \Pi_F^s - n_C^s \tilde{D}_H^s$) to other banks and receives $d_{F_3,s} := \tilde{d}_H^s$ (resp. $d_{F_4,s}$) from other banks. Clearly, these transactions do not impact the CB deposits of Bank b.

In the second step, firms use the proceeds of the sale to repay their debt and distribute profits to households. To settle these payments, in round $k \in \{1, ..., n_C^s\}$ (resp. round $k = n_C^s + 1$), each bank b transfers some amount of CB deposits $d_{F_3,s}$ (resp. $d_{F_4,s}$) to other banks and receives some amount of CB deposits $\tilde{d}_H^s$ (resp. $d_{F_4,s}$) from other banks. Clearly, these transactions do not impact the CB deposits of Bank b.

[4] As bonds and profits will be repaid in the same manner, we regroup them in one position in the ensuing balance sheets.

When $k \in \{1, ..., n_C^s\}$, the balance sheets of households, firms in FT, and banks are given in Table 3.10 for the first step and in Table 3.11 for the second step. When $k = n_C^s + 1$, the balance sheets of households, firms in FT, and banks are given in Table 3.12 for the first step and in Table 3.13 for the second step.

Households	
$\frac{k\tilde{D}_H^s}{p_C^s}$	$E_H R_H^s$
$S_F R_F + \Pi_F^s - (k-1)\tilde{D}_H^s$	

FT	
$\mathbf{f(K_F)} - \frac{k\tilde{D}_H^s}{p_C^s}$	$S_F R_F + \Pi_F^s - (k-1)\tilde{D}_H^s$
$D_{F_3,s}$	

Bank b_i	
$d_{CB_6}^{b_i,s}$	$l_{CB_3}^{b_i,s}$
$l_M^{b_i} R_L^s$	$d_{F_3,s}$

Bank b_j	
$d_{CB_5}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$l_M^{b_j} R_L^s$	$d_{F_3,s}$

Table 3.10: Bonds denominated in nominal terms—Balance sheets at the end of Stage E, Substage 3, at round $k \in \{1, ..., n_C^s\}$, first step. Source: Own illustration.

Stage E, Substage 4: Sale of the Consumption Good by MT

Firms in MT sell the entire amount of the consumption good they have produced. Households buy it with their private deposits consisting of their wealth in terms of equity and deposits. The supply of $\mathbf{K_M R_M^s}$ units of the consumption good meets the real demand $\frac{\tilde{d}_H^s}{p_C^s}$. Hence, the equilibrium price is given by

$$p_C^s = \frac{\tilde{D}_H^s}{\mathbf{K_M R_M^s}}.$$

To settle these payments, each bank b transfers some amount of CB deposits $\tilde{d}_H^s$ to other banks and receives some amount of CB deposits $d_{M_1}^{b,s}$ from them. By summing over all banks $b \in [0, 1]$ in the expression of banks' profits in Equation

Households	
$\frac{k\tilde{D}^s_H}{p^s_C}$	
$S_F R_F + \Pi^s_F - k\tilde{D}^s_H$	$E_H R^s_H$
$\hat{D}^s_H$	

FT	
$\mathbf{f}(\mathbf{K_F}) - \frac{k\tilde{D}^s_H}{p^s_C}$	$S_F R_F + \Pi^s_F - k\tilde{D}^s_H$

Bank b_i	
$d^{b_i,s}_{CB_6}$	$l^{b_i,s}_{CB_3}$
$l^{b_i,s}_M R^s_L$	$\tilde{d}^s_H$

Bank b_j	
$d^{b_j,s}_{CB_5}$	$l^{b_j,s}_{CB_4}$
$l^{b_j,s}_M R^s_L$	$\tilde{d}^s_H$

Table 3.11: Bonds denominated in nominal terms—Balance sheets the end of Stage E, Substage 3, at round $k \in \{1, ..., n^s_C\}$, second step. Source: Own illustration.

(2.1), we obtain $L_M R^s_L = D_H R^s_D + \Pi^s_B$, which means that $d^{b,s}_{M_1} = \alpha^b_M L_M R^s_L$. This transaction impacts the CB deposits of Banks b_i and b_j as follows:

$$d^{b_j,s}_{CB_7} L = d^{b_j,s}_{CB_5} - \tilde{d}^s_H + d^{b_j,s}_{M_1} = \alpha^{b_j}_M L_M R^s_L$$
$$\text{and} \quad d^{b_i,s}_{CB_8} := d^{b_i,s}_{CB_6} - \tilde{d}^s_H + d^{b_i,s}_{M_1} = \alpha^{b_i}_M L_M R^s_L + d^{b_i}_{CB} R^s_{CB}.$$

The balance sheets of banks and households are given in Table 3.14.

Stage E, Substage 5: Repayment of Loans by Firms in MT

Firms in MT pay back their loans and bank deposits are destroyed. The balance sheets of banks and households are given in Table 3.15.

Households	
$\mathbf{f}(\mathbf{K_F})$	$E_H R_H^s$
$D_{H_3,s} := \tilde{D}_H^s - (S_F R_F + \Pi_F^s - n_C^s \tilde{D}_H^s)$	

FT	
$D_{F_4} := (S_F R_F + \Pi_F^s - n_C^s \tilde{D}_H^s)$	$S_F R_F + \Pi_F^s - n_C^s \tilde{D}_H^s$

Bank b_i	
$d_{CB_6}^{b_i,s}$	$l_{CB_3}^{b_i,s}$
$l_M^{b_i} R_L^s$	$d_{H_3,s}$
	$d_{F_4,s}$

Bank b_j	
$d_{CB_5}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$l_M^{b_j} R_L^s$	$d_{H_3,s}$
	$d_{F_4,s}$

Table 3.12: Bonds denominated in nominal terms—Balance sheets at the end of Stage E, Substage 3, at round $k = n_C^s + 1$, first step. Source: Own illustration.

Stage E, Substage 6: Netting of CB Deposits and CB Liabilities

Banks can net their CB deposits and CB liabilities. Using the expression of bank profits given by Equation (2.1), we obtain

$$d_{CB_7}^{b_j,s} - l_{CB_4}^{b_j,s} = \alpha_M^{b_j} L_M R_L^s - (\alpha_M^{b_j} - 1) L_M R_{CB}^s - \big((L_M - E_B) R_D^s + \Pi_B^{b_j,s}\big) = 0,$$
$$d_{CB_8}^{b_i,s} - l_{CB_3}^{b_i,s} = \alpha_M^{b_i} L_M R_L^s + (1 - \alpha_M^{b_i}) L_M R_{CB}^s - \big((L_M - E_B) R_D^s + \Pi_B^{b_i,s}\big) = 0.$$

3.2.3 No-default Conditions

As real profits in Sector FT are given by

$$\max\left(\mathbf{f}(\mathbf{K_F}) - \frac{\mathbf{K_F} R_F p_I}{p_C^s}, 0\right),$$

Households		FT	
$\mathbf{f(K_F)}$	$E_H R_H^s$	0	0
$\tilde{D}_H^s$			

Bank b_i		Bank b_j	
$d_{CB_6}^{b_i,s}$	$l_{CB_3}^{b_i,s}$	$d_{CB_5}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$l_M^{b_i} R_L^s$	$\tilde{d}_H^s$	$l_M^{b_j} R_L^s$	$\tilde{d}_H^s$

Table 3.13: Bonds denominated in nominal terms—Balance sheets at the end of Stage E, Substage 3, at round $k = n_C^s + 1$, second step. Source: Own illustration.

Households		Bank b_i		Bank b_j	
$\mathbf{f(K_F)}$	$E_H R_H^s$	$d_{CB_8}^{b_i,s}$	$l_{CB_3}^{b_i,s}$	$d_{CB_7}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$\mathbf{K_M R_M^s}$		$l_M^{b_i} R_L$	$d_{M_1}^{b_i,s}$	$l_M^{b_j} R_L^s$	$d_{M_1}^{b_j,s}$

Table 3.14: Bonds denominated in nominal terms—Balance sheets at the end of Stage E, Substage 4. Source: Own illustration.

Households		Bank b_i		Bank b_j	
$\mathbf{f(K_F)}$	$E_H R_H^s$	$d_{CB_8}^{b_i,s}$	$l_{CB_3}^{b_i,s}$	$d_{CB_7}^{b_j,s}$	$l_{CB_4}^{b_j,s}$
$\mathbf{K_M R_M^s}$					

Table 3.15: Bonds denominated in nominal terms—Balance sheets at the end of Stage E, Substage 5. Source: Own illustration.

where R_F is the nominal rate on bonds, real profits in Sector FT depend on the state of the world s, and thus production in Sector FT is now risky. As banks can always expect a certain repayment of firms in a general equilibrium, this requires a condition on R_F, i.e. that profits of firms in FT are positive in equilibrium. We impose the stronger condition that profits of firms in FT are positive for all levels of $\mathbf{K_F}$, which we can write as follows:

$$\mathbf{f}(\mathbf{W})p_C^s \geq \mathbf{W}R_F p_I \quad \text{for all states } s = l, h.$$

3.3 An Economy-wide Macroeconomic Shock

In Section 3.3, we assume that Sector FT is also affected by the macroeconomic shock. In particular, we denote the production technology in Sector FT by $\mathbf{f^s}(\cdot)$. We reformulate Assumption 1 with the following conditions:

$$\mathbb{E}[(\mathbf{f^s})'(\mathbf{W})] < \overline{\mathbf{R}}_\mathbf{M} < \mathbb{E}[(\mathbf{f^s})'(\mathbf{0})].$$

We first investigate the social planner's problem:

$$\max_{\mathbf{K_F} \in [\mathbf{0}, \mathbf{W}]} \mathbb{E}[\mathbf{R^s_M}(\mathbf{W} - \mathbf{K_F})] + \mathbb{E}[\mathbf{f^s}(\mathbf{K_F})].$$

We obtain the following proposition:

Proposition 6
There is a unique allocation $(\mathbf{K_M}, \mathbf{K_F})$ *such that*

$$\mathbf{W} = \mathbf{K_M} + \mathbf{K_F},$$
$$\textit{and} \quad \mathbb{E}[(\mathbf{f^s})'(\mathbf{K_F})] = \overline{\mathbf{R}}_\mathbf{M}.$$

This allocation is the first-best investment allocation and we denote it by $(\mathbf{K_M^{FB}}, \mathbf{K_F^{FB}})$.

The proof of Proposition 6 is given in Appendix 3.A. As production in the FT sector is now risky and as banks can always expect a certain repayment of firms in a general equilibrium, we impose the condition that profits of firms in FT are positive for all levels of $\mathbf{K_F}$ in equilibrium. The demand for the investment good of firms in FT is then directly[5] given by the following lemma:

Lemma 8
The demand for the investment good by firms in FT is represented by the correspondence denoted by $\hat{\mathbf{K}}_\mathbf{F} : \mathcal{ND} \to \mathcal{P}(\mathbb{R} \cup \{+\infty\})$ *and given by*

$$\hat{\mathbf{K}}_\mathbf{F}\big((\mathbf{R^s_F})_s\big) = \begin{cases} \{\mathbf{0}\} & \text{if } \overline{\mathbf{R}}_\mathbf{F} > \mathbb{E}[(\mathbf{f^s})'(\mathbf{0})], \\ \{\mathbf{W}\} & \text{if } \overline{\mathbf{R}}_\mathbf{F} < \mathbb{E}[(\mathbf{f^s})'(\mathbf{W})], \\ \{\tilde{\mathbf{K}}_\mathbf{F}\} & \text{otherwise,} \end{cases}$$

where

$$\mathcal{ND} = \{(\mathbf{R^s_F})_s \in \mathbb{R}^2_+ \ s.t.\ \mathbf{f^s}(\mathbf{W}) \geq \mathbf{W}\mathbf{R^s_F} \text{ for all states } s = l, h\}$$

and $\tilde{\mathbf{K}}_\mathbf{F}$ *is the unique solution of*

$$\mathbb{E}[(\mathbf{f^s})'(\mathbf{K_F})] = \overline{\mathbf{R}}_\mathbf{F}.$$

We also characterize the households' optimal portfolio choices in the following lemma:

Lemma 9
The representative household's optimal portfolio choices are represented by three correspondences denoted by

$$\begin{aligned} &\hat{E}_B : \mathbb{R}^7_{++} \times [0, \mathbf{W}] \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}), \\ &\hat{D}_H : \mathbb{R}^7_{++} \times \mathbb{R}_+ \times [0, \mathbf{W}] \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}), \\ &\hat{S}_F : \mathbb{R}^7_{++} \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}), \end{aligned}$$

[5] The proof of Lemma 8 is almost identical to the one of Lemma 10, to which we refer for further details.

and given by

$$\begin{aligned}
&\Big(\hat{E}_B\big((R_E^s)_s,(R_D^s)_s,p_I,(p_C^s)_s,S_F\big),\\
&\hat{D}_H\big((R_E^s)_s,(R_D^s)_s,p_I,(p_C^s)_s,E_B,S_F\big),\\
&\hat{S}_F\big((R_E^s)_s,(R_D^s)_s,p_I,(p_C^s)_s\big)\Big)=\\
&\begin{cases}
\big(\{0\},\{0\},\{\mathbf{W}\}\big)\\
\qquad\qquad \textit{if } \max\left(\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right],\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right)\leq\frac{\mathbb{E}[(\mathbf{f^s})'(\mathbf{W})]}{p_I},\\
\big(\{0\},\{p_I\mathbf{W}\},\{0\}\big)\\
\qquad\qquad \textit{if } \max\left(\frac{\mathbb{E}[(\mathbf{f^s})'(\mathbf{0})]}{p_I},\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right)<\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right],\\
\big(\{p_I\mathbf{W}\},\{0\},\{0\}\big)\\
\qquad\qquad \textit{if } \max\left(\frac{\mathbb{E}[(\mathbf{f^s})'(\mathbf{0})]}{p_I},\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)<\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right],\\
\big([0,p_I\mathbf{W}],\{p_I\mathbf{W}-E_B\},\{0\}\big)\\
\qquad\qquad \textit{if } \frac{\mathbb{E}[(\mathbf{f^s})'(\mathbf{0})]}{p_I}<\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]=\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right],\\
\big(\{0\},\{p_I\left(\mathbf{W}-S_F\right)\},\{\mathbf{K_F}\}\big)\\
\qquad \textit{where } \mathbf{K_F} \textit{ is the unique solution of the equation } \mathbb{E}[(\mathbf{f^s})'(\mathbf{K_F})]=p_I\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\\
\qquad\qquad \textit{if } \max\left(\frac{\mathbb{E}[(\mathbf{f^s})'(\mathbf{W})]}{p_I},\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right)<\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\leq\frac{\mathbb{E}[(\mathbf{f^s})'(\mathbf{0})]}{p_I},\\
\big(\{p_I\left(\mathbf{W}-S_F\right)\},\{0\},\{\mathbf{K_F}\}\big)\\
\qquad \textit{where } \mathbf{K_F} \textit{ is the unique solution of the equation } \mathbb{E}[(\mathbf{f^s})'(\mathbf{K_F})]=p_I\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\\
\qquad\qquad \textit{if } \max\left(\frac{\mathbb{E}[(\mathbf{f^s})'(\mathbf{W})]}{p_I},\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)<\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\leq\frac{\mathbb{E}[(\mathbf{f^s})'(\mathbf{0})]}{p_I},\\
\big([0,p_I\left(\mathbf{W}-S_F\right)],\{p_I\left(\mathbf{W}-S_F\right)-E_B\},\{\mathbf{K_F}\}\big)\\
\qquad \textit{where } \mathbf{K_F} \textit{ is the unique solution of the equation } \mathbb{E}[(\mathbf{f^s})'(\mathbf{K_F})]=p_I\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\\
\qquad\qquad \textit{if } \frac{\mathbb{E}[(\mathbf{f^s})'(\mathbf{W})]}{p_I}<\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]=\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\leq\frac{\mathbb{E}[(\mathbf{f^s})'(\mathbf{0})]}{p_I}.
\end{cases}
\end{aligned}\tag{3.2}$$

The proof of Lemma 9 is similar to the proof of Lemma 6, where $\mathbf{f}'(\mathbf{W})$ and $\mathbf{f}'(\mathbf{0})$ have to replaced by $\mathbb{E}[(\mathbf{f^s})'(\mathbf{W})]$ and $\mathbb{E}[(\mathbf{f^s})'(\mathbf{0})]$, respectively, and where the expression of $\hat{S}_F$ cannot be written as an explicit formula anymore: The intermediate value theorem ensures the existence and uniqueness of the amount of investment good $\mathbf{K_F}$ defined in the three last cases. We now obtain the following theorem:

Theorem 2
Given the policy gross rates $(R^s_{CB})_{s=l,h}$, all equilibria with banks have the following form:

$$R^{s*}_E = R^{s*}_D = R^{s*}_L = R^s_{CB}, \tag{3.3}$$

$$p^*_I = p, \quad p^{s*}_C = p\frac{R^s_{CB}}{\mathbf{R^s_M}}, \tag{3.4}$$

$$E^*_B = \varphi^* p\Big(\mathbf{W} - \mathbf{K^{FB}_F}\Big), \quad D^*_H = (1-\varphi^*)p\Big(\mathbf{W} - \mathbf{K^{FB}_F}\Big), \tag{3.5}$$

$$\tilde{D}^{s*}_H = p\Big(\mathbf{W} - \mathbf{K^{FB}_F}\Big)R^s_{CB}, \tag{3.6}$$

$$L^*_M = p\Big(\mathbf{W} - \mathbf{K^{FB}_F}\Big), \quad S^*_F = \mathbf{K^{FB}_F}, \tag{3.7}$$

$$\mathbf{K^*_M} = \mathbf{W} - \mathbf{K^{FB}_F}, \quad \mathbf{K^*_F} = \mathbf{K^{FB}_F}, \tag{3.8}$$

where the price of the investment good denoted by $p \in (0, +\infty)$, the aggregate equity ratio $\varphi^* \in (0,1)$, and $(\mathbf{R^{s*}_F})_s$ are arbitrary such that

$$\overline{\mathbf{R}}^*_\mathbf{F} = \overline{\mathbf{R}}_\mathbf{M} \quad \textit{and} \quad \mathbf{R^{s*}_F} < \frac{\mathbf{f^s(W)}}{\mathbf{W}}.$$

The equilibrium profits of firms and banks are given by

$$\Pi^{s*}_M = 0, \quad \Pi^{s*}_F = p\frac{R^s_{CB}}{\mathbf{R^s_M}}\left(\mathbf{f}\Big(\mathbf{K^{FB}_F}\Big) - \mathbf{K^{FB}_F}\mathbf{R^{s*}_F}\right), \tag{3.9}$$

$$\Pi^{s*}_B = \varphi^* p\Big(\mathbf{W} - \mathbf{K^{FB}_F}\Big)R^s_{CB}. \tag{3.10}$$

The proof of Theorem 2 is given in Appendix 3.A. Moreover, it is now straightforward to see that a shock in the FT sector neither affects qualitatively the results obtained in the case of rigid prices, nor in the case of a zero lower bound.

3.4 Two Concave Production Technologies

In Section 3.4, we assume that Sector MT is also characterized by a concave production function, which we denote by $\mathbf{f^s_M}(\cdot)$. We use $\mathbf{f_F}(\cdot)$ to denote the production function of the FT sector.

We replace Assumption 1 by the following assumptions:

Assumption 2

$$\mathbb{E}\big[(\mathbf{f_M^s})'(\mathbf{0})\big] < (\mathbf{f_F})'(\mathbf{0}) \quad \textit{and} \quad \mathbb{E}\big[(\mathbf{f_M^s})'(\mathbf{W})\big] > (\mathbf{f_F})'(\mathbf{W}),$$

and the assumption $0 < \mathbf{R_M^l} < \mathbf{R_M^h}$ by

Assumption 3

$$0 < (\mathbf{f_M^l})' < (\mathbf{f_M^h})'.$$

We first derive the first-best allocation in the following proposition:

Proposition 7
The first-best allocation $(\mathbf{K_F^{FB}}, \mathbf{K_M^{FB}})$ *chosen by a social planner maximizing household consumption is the unique solution of*

$$\begin{aligned} &\mathbf{K_M} = \mathbf{W} - \mathbf{K_F}, \\ \textit{and} \quad &\mathbb{E}[(\mathbf{f_M^s})'(\mathbf{W} - \mathbf{K_F})] = (\mathbf{f_F})'(\mathbf{K_F}). \end{aligned}$$

The proof of Proposition 7 is given in Appendix 3.A. As agents maximize their expected consumption, firms and banks maximize their profits in real terms, which we assume in the following.[6] We next turn to the firms' behavior in the MT sector.

Lemma 10
The demand for the investment good by firms in MT is represented by the corre-

[6]Recall that in Chapter 2, profit maximization in real and in nominal terms is equivalent. This was due to the special features assumed in this Chapter, but it is not a general property. The equivalence between profit maximization in real and nominal terms is given in Section 3.1.

spondence denoted by $\hat{\mathbf{K}}_{\mathbf{M}} : \mathcal{ND} \to \mathcal{P}(\mathbb{R} \cup \{+\infty\})$ *and given by*

$$\hat{\mathbf{K}}_{\mathbf{M}}\big((R_L^s)_s, p_I, (p_C^s)_s\big) = \begin{cases} \{\mathbf{0}\} & \text{if } \mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] p_I > \mathbb{E}[(\mathbf{f}_{\mathbf{M}}^{\mathbf{s}})'(\mathbf{0})], \\ \{\mathbf{W}\} & \text{if } \mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] p_I < \mathbb{E}[(\mathbf{f}_{\mathbf{M}}^{\mathbf{s}})'(\mathbf{W})], \\ \{\tilde{\mathbf{K}}_{\mathbf{M}}\} & \text{otherwise,} \end{cases}$$

where

$$\mathcal{ND} = \Big\{\big((R_L^s)_s, p_I, (p_C^s)_s\big) \in \mathbb{R}_+^5 \text{ s.t. } p_C^s \mathbf{f}_{\mathbf{M}}^{\mathbf{s}}(\mathbf{W}) \geq \mathbf{W} R_L^s p_I \text{ for all states } s = l, h\Big\},$$

and $\tilde{\mathbf{K}}_{\mathbf{M}}$ *is the unique solution of*

$$\mathbb{E}[(\mathbf{f}_{\mathbf{M}}^{\mathbf{s}})'(\mathbf{K}_{\mathbf{M}})] = \mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] p_I.$$

The proof of Lemma 10 is given in Appendix 3.A. On purpose, we exclude any default by firms in MT in any state of the world and any parameter constellation in and out of equilibrium, so that we do not have to consider situations where firms in MT default, as in a general equilibrium, banks expect a lower repayment from lending to defaulting firms in MT. Out of equilibrium, banks would lower the expected repayment until firms do not default anymore.

We now also have to account for the profits and thus for the dividends paid by firms in MT to households after production, as they may not be equal to zero anymore. However, households have no direct impact on the profits of firms in MT. Therefore, we can still use Lemma 6 with $\mathbf{f} = \mathbf{f}_{\mathbf{F}}$.

The preceding lemmata allow to characterize all equilibria with banks.

Theorem 3
Given the policy gross rates $(R_{CB}^s)_{s=l,h}$*, all equilibria with banks have the following*

form:

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^s, \quad \mathbf{R}_\mathbf{F}^* = \mathbf{f}'(\mathbf{K}_\mathbf{F}^\mathbf{FB}), \tag{3.11}$$

$$p_I^* = p, \tag{3.12}$$

$$E_B^* = \varphi^* p\Big(\mathbf{W} - \mathbf{K}_\mathbf{F}^\mathbf{FB}\Big), \quad D_H^* = (1-\varphi^*)p\Big(\mathbf{W} - \mathbf{K}_\mathbf{F}^\mathbf{FB}\Big), \tag{3.13}$$

$$\tilde{D}_H^{s*} = p\Big(\mathbf{W} - \mathbf{K}_\mathbf{F}^\mathbf{FB}\Big)R_{CB}^s, \tag{3.14}$$

$$L_M^* = p\Big(\mathbf{W} - \mathbf{K}_\mathbf{F}^\mathbf{FB}\Big), \quad S_F^* = \mathbf{K}_\mathbf{F}^\mathbf{FB}, \tag{3.15}$$

$$\mathbf{K}_\mathbf{M}^* = \mathbf{W} - \mathbf{K}_\mathbf{F}^\mathbf{FB}, \quad \mathbf{K}_\mathbf{F}^* = \mathbf{K}_\mathbf{F}^\mathbf{FB}, \tag{3.16}$$

where the aggregate equity ratio $\varphi^* \in (0,1)$, the price of the investment good denoted by $p \in (0,+\infty)$, and the price of the consumption good $(p_C^s)_{s=l,h}$ can take arbitrary values as long as

$$\max_{s\in\{l,h\}} \left(\frac{R_{CB}^s}{p_C^s} p_I\right) \leq \frac{\mathbf{f}(\mathbf{W})}{\mathbf{W}} \quad \textit{and} \quad (\mathbf{f}_\mathbf{F})'(\mathbf{K}_\mathbf{F}^\mathbf{FB}) = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right] p_I. \tag{3.17}$$

The equilibrium profits of firms and banks are given by[7]

$$\Pi_M^{s*} = \mathbf{f}_\mathbf{M}^\mathbf{s}\Big(\mathbf{W} - \mathbf{K}_\mathbf{F}^\mathbf{FB}\Big) - \Big(\mathbf{W} - \mathbf{K}_\mathbf{F}^\mathbf{FB}\Big)\frac{R_{CB}^s}{p_C^s}p, \quad \Pi_F^{s*} = \mathbf{f}_\mathbf{F}(\mathbf{K}_\mathbf{F}^\mathbf{FB}) - \mathbf{f}_\mathbf{F}'(\mathbf{K}_\mathbf{F}^\mathbf{FB})\mathbf{K}_\mathbf{F}^\mathbf{FB}, \tag{3.18}$$

$$\Pi_B^{s*} = \varphi^*\Big(\mathbf{W} - \mathbf{K}_\mathbf{F}^\mathbf{FB}\Big)\frac{R_{CB}^s}{p_C^s}p. \tag{3.19}$$

The proof of Theorem 3 is given in Appendix 3.A.

Corollary 1 continues to hold in this set-up. To demonstrate this, it suffices to show that for all $(R_{CB}^s)_s$, there exist p_I and $(p_C^s)_s$ such that both conditions in (3.17) hold, which is obviously the case. Similarly, Corollary 2 continues to hold. However, as the price of the consumption good is not anchored as in the benchmark set-up of Chapter 2, Corollary 3 has to be adapted as follows:

[7] Note that we indicate the profits in real terms, as we have used the firms' and banks' profit maximization in real terms.

Corollary 8

Given $p_I = 1$, some values of $p_C^s > 0$ for all states $s = l, h$, some value of $\varphi^* \in (0, 1)$, and the central bank policy gross rates $(R_{CB}^s)_s$ such that both conditions in (3.17) hold, all equilibrium values are uniquely determined.

With everything else equal, the levels of the ratio $\frac{R_{CB}^s}{p_C^s} p_I$ in both states $s = l, h$ essentially determine the amounts of consumption good the households earn through banks and through firms' profits from MT in each state. The expected value $\mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right] p_I$ is determined by the equation in (3.17) and thus given by fundamentals. However, the values can fluctuate across different states. Finally, we note that a similar version of Corollary 4 can be established in this set-up, where constraining the central bank policy rates, or alternatively, the prices of the consumption good so that they do not depend on the state of the world, would narrow the possible range of prices of the consumption good or the possible range of central bank policy rates, respectively.

We now explore price rigidities and capital requirements. When we assume $p_I = p_C^s = 1$ in all states $s = l, h$, the range of central bank policies $(R_{CB}^s)_s$ for which there is an equilibrium with banks is smaller compared to the possible range of central bank policies when prices are flexible. However, we do not obtain a result as strong as in Proposition 2. With price rigidities, the central bank policy rates have to fulfill

$$\overline{R}_{CB} = (\mathbf{f_F})'(\mathbf{K_F^{FB}}).$$

The equivalent version of Proposition 3 writes as follows:

Proposition 8

Suppose that prices are rigid. Then given the central bank policy rates $(R_{CB}^s)_s$ and the capital requirement φ^{reg}, in addition to the equilibria with banks given in Theorem 3 with $p_I = p_C^s = 1$ for all states $s = l, h$, there exist other equilibria with *banks and without banks' default such that for some state s,*

$$\overline{R}_L^* = \overline{R}_{CB}, \quad R_L^{s*} < R_{CB}^s, \quad \varphi^{reg} \geq 1 - \frac{R_L^{*s}}{R_{CB}^s},$$
$$\max_{s\in\{l,h\}} R_L^s \leq \frac{\mathbf{f_F}(\mathbf{W})}{\mathbf{W}}, \quad (\mathbf{f_F})'(\mathbf{K_F^{FB}}) = \overline{R}_{CB},$$

and the allocation is first-best. Moreover, there exist equilibria with banks and banks' default such that either for some state s,

$$\overline{R}_L^* < \overline{R}_{CB}, \quad R_L^{s*} < R_{CB}^s, \quad \varphi^{reg} = \frac{\sigma^{s'}}{\sigma^s}\frac{R_L^{s'*} - R_{CB}^{s'}}{R_{CB}^s},$$
$$\max_{s\in\{l,h\}} R_L^s \leq \frac{\mathbf{f_F}(\mathbf{W})}{\mathbf{W}}, \quad (\mathbf{f_F})'(\mathbf{0}) < \overline{R}_{CB}, \quad \overline{R}_L^* < \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{W})],$$
$$E_B^* \in (0, p_I^*\mathbf{W}), \quad D_H^* = p_I^*\mathbf{W} - E_B^*, \quad \textit{and} \quad S_F^* = 0,$$

or such that for some state s,

$$\overline{R}_L^* < \overline{R}_{CB}, \quad R_L^{s*} < R_{CB}^s, \quad \varphi^{reg} = \frac{\sigma^{s'}}{\sigma^s}\frac{R_L^{s'*} - R_{CB}^{s'}}{R_{CB}^s},$$
$$\max_{s\in\{l,h\}} R_L^s \leq \frac{\mathbf{f_F}(\mathbf{W})}{\mathbf{W}}, \quad (\mathbf{f_F})'(\mathbf{0}) \geq \overline{R}_{CB}, \quad \mathbb{E}\Big[(\mathbf{f_M^s})'\big(\mathbf{W} - (\mathbf{f_F})'^{-1}(\overline{R}_{CB})\big)\Big] = \overline{R}_L^*,$$
$$E_B^* \in (0, p_I^*\mathbf{W}), \quad D_H^* = p_I^*\mathbf{W} - S_F^* - E_B^*, \quad \textit{and} \quad S_F^* = (\mathbf{f_F})'^{-1}(\overline{R}_{CB}).$$

The proof of Proposition 8 is given in Appendix 3.A. The zero lower bound problem then arises as soon as

$$(\mathbf{f_F})'(\mathbf{K_F^{FB}}) < 1.$$

Then from Proposition 8, we directly obtain

Proposition 9

In the case where $(\mathbf{f_F})'(\mathbf{K_F^{FB}}) < 1$ *and the central bank is constrained by the zero lower bound in its policy choice, the equilibria with banks with the highest welfare arise when the central bank chooses* $R_{CB}^s = 1$ *for all states* $s = l, h$ *and the bank regulator imposes the capital requirement* $\varphi^{reg} = \frac{\sigma^{s'}}{\sigma^s}(R_L^{s'*} - 1)$*, if the following* *conditions hold:*

$$\max_{s\in\{l,h\}} R_L^s \le \frac{\mathbf{f_F}(\mathbf{W})}{\mathbf{W}}, \quad (\mathbf{f_F})'(\mathbf{0}) \ge 1, \quad \textit{and}$$
$$\mathbb{E}\Big[(\mathbf{f_M^s})'\big(\mathbf{W} - (\mathbf{f_F})'^{-1}(1)\big)\Big] = \overline{R}_L^*.$$

3.5 Multiple States of the World

In Section 3.5, we explore how our main results translate when there are more than two states of the world.

Suppose that there are N states of the world $s = 1, 2, \ldots, N$, where $N > 2$. We use the notation $\mathcal{N} = \{1, 2, \ldots, N\}$. Without loss of generality, we can order these states such that $(\mathbf{R_M^s})_s$ is strictly increasing in s and assume that $\sigma^s \in (0,1)$ for all $s \in \mathcal{N}$. The exposition of our model and the definition of an equilibrium are general and translate without further changes to $N > 2$ states of the world. Therefore, we focus exclusively on how our results change in this new setting. To reduce the number of notations, we will assume that banks maximize the shareholders' value in nominal terms.[8]

We first observe that Lemma 2 applies to N states of the world, as the argumentation of its proof is independent of the number of states. To ease the exposition of the results, we first introduce some notations. For given $\big((R_L^s)_s, (R_{CB}^s)_s\big) \in \mathbb{R}_{++}^{2N}$, we define

$$\mathcal{N}_{\backslash 0} = \{s \in \mathcal{N} \quad \text{s.t.} \quad R_L^s \neq R_{CB}^s\}.$$

For given $\big((R_L^s)_s, (R_{CB}^s)_s\big) \in \mathbb{R}_{++}^{2N}$ and using the previous notation, we define

$$S_{D+} = \{s \in \mathcal{N}_{\backslash 0} \quad \text{s.t.} \quad \alpha_{DCB}^s > 0\},$$

where $\alpha_{DCB}^s = \frac{R_{CB}^s}{R_{CB}^s - R_L^s}$. S_{D+} represents the set of states s where $R_{CB}^s > R_L^s$, i.e.

[8] In this setting, the result of Section 3.1 that there is complete equivalence between the maximization of the shareholders' value in nominal and in real terms continues to hold.

the set of states in which banks make losses. Using this notation, we also define

$$s_D: \quad \mathcal{N} \to \{0,1,2,\ldots,N\}$$
$$s \mapsto \begin{cases} 0 \quad \text{if } S_{D+} \setminus \{s_D(1), s_D(2), \ldots, s_D(s-1)\} = \{\}, \\ \\ \underset{s' \in S_{D+} \setminus \{s_D(1), s_D(2), \ldots, s_D(s-1)\}}{\arg\min} \left(\alpha_{DCB}^{s'}\right) \quad \text{otherwise,} \end{cases}$$

where we use the notation that $\{s_D(1), s_D(2), \ldots, s_D(s-1)\} = \{\}$ for $s = 1$. In words, s_D sorts the states in which banks make negative profits out of lending in ascending order with regard to $(\alpha_{DCB}^s)_{s \in S_{D+}}$ and attributes the value 0 to any number in $\mathcal{N}$ that is larger than the number of states in which banks make negative profits out of lending:

$$\left(\alpha_{DCB}^{s_D(s)}\right)_{s \in \{1,2,\ldots,|S_{D+}|\}} \quad \text{is weakly increasing in } s \text{ and}$$
$$s_D(s) = 0 \quad \text{for all } s \in \{|S_{D+}| + 1, \ldots, N\}.$$

Finally, given $\left((R_L^s)_s, (R_{CB}^s)_s\right) \in \mathbb{R}_{++}^{2N}$ and $\varphi \in (0,1)$, for any lending level α_M^b, we define the following two sets:

$$S_+(\alpha_M^b) = \{s \in \mathcal{N}_{\setminus 0} \quad \text{s.t.} \quad s_D(s) = 0 \quad \text{or} \quad \alpha_{DH}^s \geq \alpha_M^b\}, \quad \text{and}$$
$$S_-(\alpha_M^b) = \{s \in \mathcal{N}_{\setminus 0} \quad \text{s.t.} \quad s_D(s) > 0 \quad \text{and} \quad \alpha_{DH}^s < \alpha_M^b\}.$$

In words, $S_+(\alpha_M^b)$ denotes the set of states in which banks make positive profits after lending the amount α_M^b to firms in MT. Similarly, $S_-(\alpha_M^b)$ denotes the set of states in which banks default against households after lending the amount α_M^b to firms in MT. We note that for all lending levels α_M^b, $\mathcal{N}_{\setminus 0} = S_+(\alpha_M^b) \cup S_-(\alpha_M^b)$. Using these notations, we obtain

Proposition 10

If $R_D^s = R_{CB}^s$ in all states $s \in \mathcal{N}$, the privately optimal amounts of money creation and lending by an individual bank are represented by a correspondence denoted by

$\hat{\alpha}_M : \mathbb{R}^{2N}_{++} \times (0,1) \to \mathcal{P}(\mathbb{R} \cup \{+\infty\})$ *and given by*

$$\hat{\alpha}_M\big((R_L^s)_s, (R_{CB}^s)_s, \varphi\big) = \begin{cases} \{+\infty\} & \text{if } R_L^s \geq R_{CB}^s \text{ for all states } s \in \mathcal{N} \\ & \text{with at least one strict inequality,} \\ \left\{\alpha_{DCB}^{s_D(1)}\right\} & \text{if } \varphi < \alpha_{DCB}^{s_D(1)} \dfrac{\sum\limits_{s \in S_+\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s (R_L^s - R_{CB}^s)}{\sum\limits_{s \in S_-\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s R_{CB}^s}, \\ [0, +\infty) & \text{if } R_L^s = R_{CB}^s \text{ for all states } s \in \mathcal{N}, \\ \left\{0, \alpha_{DCB}^{s_D(1)}\right\} & \text{if } \varphi = \alpha_{DCB}^{s_D(1)} \dfrac{\sum\limits_{s \in S_+\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s (R_L^s - R_{CB}^s)}{\sum\limits_{s \in S_-\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s R_{CB}^s}, \\ \{0\} & \text{if } \alpha_{DCB}^{s_D(1)} \dfrac{\sum\limits_{s \in S_+\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s (R_L^s - R_{CB}^s)}{\sum\limits_{s \in S_-\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s R_{CB}^s} < \varphi. \end{cases}$$

The proof of Proposition 10 is given in Appendix 3.A. Proposition 10 states that whenever the intermediation margin $R_L^s - R_{CB}^s$ is negative in at least one state s of the world, the banks' behavior generally depends on the banks' equity ratio φ. Typically, a bank has an incentive to leverage as much as possible on the shareholders' limited liability when its equity ratio is low, and it has no incentive to grant any loan to firms if its equity ratio is high.

Furthermore, we note that the results of Lemma 4, the logic of the proof and the results of Theorem 1, as well as Corollaries 1 to 3 directly translate to multiple states of the world without further change. Similarly, Corollary 4 can be directly re-written with multiple states, as follows:

Corollary 9

(i) If R_{CB}^s does not depend on the state s of the economy,

$$\text{then} \quad p_C^1 > p_C^2 > \ldots > p_C^N \quad \text{and}$$
$$\frac{p_C^s}{p_C^{s'}} = \frac{\mathbf{R_M^{s'}}}{\mathbf{R_M^s}} \quad \text{for all states } s, s' \in \mathcal{N}.$$

(ii) For the central bank policy gross rates $(R_{CB}^s)_s$ characterized by

$$\frac{R_{CB}^s}{R_{CB}^{s'}} = \frac{\mathbf{R_M^s}}{\mathbf{R_M^{s'}}} \quad \text{for all states } s, s' \in \mathcal{N},$$

or equivalently by

$$\exists\, \lambda \in \mathbb{R}_{++} \quad s.t. \quad \forall s, s' \in \mathcal{N}, \quad R_{CB}^s = \lambda \mathbf{R_M^s},$$

the price of the consumption good is independent of the state of the world, i.e. $p_C^s = p_C^{s'}$ for all states $s, s' \in \mathcal{N}$.

When there are price rigidities, the results of Proposition 2 and Lemma 5 continue to hold. However, Lemma 7 has to be replaced by the following lemma:

Lemma 11

Suppose that banks have to comply with a minimum equity ratio φ^{reg} at the end of Period $t = 0$. If $R_D^s = R_{CB}^s$ in all states $s \in \mathcal{N}$, the privately optimal amounts of money creation and lending by an individual bank are represented by a correspon-

dence denoted by $\hat{\alpha}_M^{reg} : \mathbb{R}_{++}^{2N} \times [\varphi^{reg}, 1) \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\})$ *and given by*

$$\hat{\alpha}_M^{reg}\big((R_L^s)_s, (R_{CB}^s)_s, \varphi\big) =$$

$$\begin{cases}
\{\frac{\varphi}{\varphi^{reg}}\} & \text{if } R_L^s \geq R_{CB}^s \text{ for all states } s \in \mathcal{N} \\
& \text{with at least one strict inequality, or} \\
& \text{if } (\frac{\varphi}{\varphi^{reg}} \leq \alpha_{DCB}^{s_D(1)} \text{ and } (\overline{R}_L > \overline{R}_{CB} \\
& \text{or } (\alpha_{DH}^{s_D(1)} < \frac{\varphi}{\varphi^{reg}} \text{ and } \varphi^{reg} < \frac{\sum_{s \in S_+\left(\frac{\varphi}{\varphi^{reg}}\right)} \sigma^s (R_L^s - R_{CB}^s)}{\sum_{s \in S_-\left(\frac{\varphi}{\varphi^{reg}}\right)} \sigma^s R_{CB}^s}))), \\
\left\{\alpha_{DCB}^{s_D(1)}\right\} & \text{if } (\frac{\varphi}{\varphi^{reg}} > \alpha_{DCB}^{s_D(1)} \text{ and } \varphi < \alpha_{DCB}^{s_D(1)} \frac{\sum_{s \in S_+\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s (R_L^s - R_{CB}^s)}{\sum_{s \in S_-\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s R_{CB}^s}), \\
\left[0, \frac{\varphi}{\varphi^{reg}}\right] & \text{if } R_L^s = R_{CB}^s \text{ for all states } s \in \mathcal{N}, \text{ or} \\
& \text{if } \overline{R}_L = \overline{R}_{CB} \text{ and } \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DH}^{s_D(1)}, \\
\{0, \frac{\varphi}{\varphi^{reg}}\} & \text{if } (\alpha_{DH}^{s_D(1)} < \frac{\varphi}{\varphi^{reg}} \leq \alpha_{DCB}^{s_D(1)} \\
& \text{and } \varphi^{reg} = \frac{\sum_{s \in S_+\left(\frac{\varphi}{\varphi^{reg}}\right)} \sigma^s (R_L^s - R_{CB}^s)}{\sum_{s \in S_-\left(\frac{\varphi}{\varphi^{reg}}\right)} \sigma^s R_{CB}^s}), \\
\left\{0, \alpha_{DCB}^{s_D(1)}\right\} & \text{if } (\frac{\varphi}{\varphi^{reg}} > \alpha_{DCB}^{s_D(1)} \text{ and } \varphi = \alpha_{DCB}^{s_D(1)} \frac{\sum_{s \in S_+\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s (R_L^s - R_{CB}^s)}{\sum_{s \in S_-\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s R_{CB}^s}),
\end{cases}$$

$$
\begin{cases}
\{0\} & \textit{if } (\dfrac{\varphi}{\varphi^{reg}} > \alpha_{DCB}^{s_D(1)} \textit{ and } \varphi < \alpha_{DCB}^{s_D(1)} \dfrac{\sum\limits_{s \in S_+\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s (R_L^s - R_{CB}^s)}{\sum\limits_{s \in S_-\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s R_{CB}^s}), \\
& \textit{if } (\dfrac{\varphi}{\varphi^{reg}} \leq \alpha_{DCB}^{s_D(1)} \textit{ and } (\overline{R}_L < \overline{R}_{CB} \textit{ and } \alpha_{DH}^{s_D(1)} \geq \frac{\varphi}{\varphi^{reg}} \\
& \textit{or } (\alpha_{DH}^{s_D(1)} < \dfrac{\varphi}{\varphi^{reg}} \textit{ and } \dfrac{\sum\limits_{s \in S_+\left(\frac{\varphi}{\varphi^{reg}}\right)} \sigma^s (R_L^s - R_{CB}^s)}{\sum\limits_{s \in S_-\left(\frac{\varphi}{\varphi^{reg}}\right)} \sigma^s R_{CB}^s} < \varphi^{reg}))),
\end{cases}
$$

The proof of Lemma 11 is given in Appendix 3.A. Lemma 11 states that money creation is either limited by capital regulation or by the threat of heavy penalties for defaulting against the central bank. Whenever the intermediation margin $R_L^s - R_{CB}^s$ is negative in at least one state s of the world, the banks' behavior generally depends on the banks' equity ratio φ and on the minimum equity ratio φ^{reg} imposed by the bank regulator. Typically, a bank has an incentive to leverage as much as possible on the shareholders' limited liability when both its equity ratio and the capital requirement level are sufficiently low, and it has no incentive to grant any loan to firms if its equity ratio is high or if the capital requirement is too tight. We now use Lemma 11 to derive general conditions under which equilibria with banks exist when $R_{CB}^s \neq \mathbf{R_M^s}$ for some state s of the world.

Proposition 11

Suppose that prices are rigid and $R_{CB}^s \neq \mathbf{R_M^s}$ for some state s. Then there exists an equilibrium with banks if the central bank policy gross rates $(R_{CB}^s)_s$ and the capital requirement level φ^{reg} are set as follows:

(i) $\overline{R}_{CB} = \overline{\mathbf{R}}_{\mathbf{M}}$ *and* $\dfrac{1}{\alpha_{DCB}^{s_D(1)}} \leq \varphi^{reg}$, *or*

(ii) $\overline{R}_{CB} > \overline{\mathbf{R}}_{\mathbf{M}}$, $\alpha_{DH}^{s_D(1)} < 1$, *and* $\varphi^{reg} = \dfrac{\sum\limits_{s \in S_+(1)} \sigma^s (\mathbf{R_M^s} - R_{CB}^s)}{\sum\limits_{s \in S_-(1)} \sigma^s R_{CB}^s}$.

The proof of Proposition 11 is given in Appendix 3.A. Similarly, Corollary 5 translates into the following corollary.

Corollary 10
Suppose that prices are rigid and $R^s_{CB} \neq \mathbf{R^s_M}$ for some state s. Then the central bank policy gross rates $(R^s_{CB})_s$ and the capital requirement level φ^{reg} implement a socially efficient equilibrium with banks if and only if

$$\overline{R}_{CB} = \overline{\mathbf{R}}_{\mathbf{M}} \quad \textit{and} \quad \frac{1}{\alpha^{s_D(1)}_{DCB}} \leq \varphi^{reg}.$$

The intuition for Proposition 11 and Corollary 10 is identical to the one for Proposition 3 and Corollary 5. We next explore the case in which the central bank is constrained by the zero lower bound and prices are assumed to be rigid, i.e. $p^{s*}_C = p^*_I = 1$ for all states $s \in \mathcal{N}$. From Corollary 10, we obtain

Corollary 11
Suppose that prices are rigid, $1 \leq \overline{\mathbf{R}}_{\mathbf{M}}$, there exists $s_z \in \mathcal{N}$ such that $\mathbf{R^{s_z}_M} < 1 < \mathbf{R^{s_z+1}_M}$, and that the central bank is constrained by the zero lower bound ($R^s_{CB} \geq 1$ for all states $s \in \mathcal{N}$). Then there exist some central bank policy gross rates $(R^s_{CB})_s$ and capital requirement levels φ^{reg} such that the allocation of the resulting equilibrium with banks is socially efficient.

(i) The central bank policy gross rates have to satisfy $\overline{R}_{CB} = \overline{\mathbf{R}}_{\mathbf{M}}$. An example is

$$R^s_{CB} = 1 \quad \textit{for all states } s \in \{1, 2, \ldots, s_z\}, \quad \textit{and}$$
$$R^s_{CB} = \mathbf{R^s_M} - \frac{\Delta_z}{\sigma^s} \quad \textit{for all states } s \in \{s_z + 1, s_z + 2, \ldots, N\},$$

where we define

$$\Delta_z = \frac{\sum_{s=1}^{s_z} \sigma^s (1 - \mathbf{R^s_M})}{N - s_z}.$$

This example is only valid under the condition that

$$\mathbf{R_M^s} - \frac{\Delta_z}{\sigma^s} \geq 1 \quad \textit{for all states } s \in \{s_z+1, ..., N\}.$$

(ii) The regulatory capital requirement levels φ^{reg} have to satisfy

$$\varphi^{reg} \geq \frac{1}{\alpha_{DCB}^{s_D(1)}}.$$

In our example in (i), the regulatory capital requirement constraint becomes

$$\varphi^{reg} \geq 1 - \mathbf{R_M^1}.$$

The proof of Corollary 11 is given in Appendix 3.A. Corollary 11 shows that price rigidities and the zero lower bound may be countered by a suitable combination of monetary policy and capital regulation. The central bank policy gross rates given as an example ensure that in the good state, gains from money creation are sufficiently high to offset losses in the bad state. In other words, setting $R_{CB}^s < \mathbf{R_M^s}$ for all states $s > s_Z$ generates sufficient incentives for banks to lend and to create money. The capital requirement, in turn, ensures that money creation does not become excessive.

From Corollary 10 and the proof of Proposition 11, we also immediately obtain

Proposition 12
Suppose that prices are rigid, $\overline{\mathbf{R}}_\mathbf{M} < 1$, and the central bank is constrained by the zero lower bound ($R_{CB}^s \geq 1$ for all states $s \in \mathcal{N}$). Then there exists no combination of the central bank policy gross rates $(R_{CB}^s)_s$ and the capital requirement level φ^{reg} such that the allocation of the resulting equilibrium with banks is socially efficient. We obtain the following two cases:

- *If there exists a state $s_z \in \mathcal{N}$ such that $1 < \mathbf{R_M^{s_z}}$, there exist central bank policy gross rates $(R_{CB}^s)_s$ and a capital requirement level φ^{reg} implementing equilibria with banks.*
  (a) The central bank policy gross rates have to satisfy $\alpha_{DH}^{s_D(1)} < 1$. An ex-

ample is

$$R^s_{CB} = 1 \quad \textit{for all states } s \in \mathcal{N} \quad \textit{and} \quad \varphi^{reg} < 1 - \mathbf{R^1_M}.$$

(b) The regulatory capital requirement level φ^{reg} has to satisfy

$$\varphi^{reg} = \frac{\sum\limits_{s \in S_+(1)} \sigma^s(\mathbf{R^s_M} - R^s_{CB})}{\sum\limits_{s \in S_-(1)} \sigma^s R^s_{CB}}.$$

In our example in (a), the regulatory capital requirement constraint becomes

$$\varphi^{reg} = \frac{\sum\limits_{s \in S_+(1)} \sigma^s(\mathbf{R^s_M} - 1)}{\sum\limits_{s \in S_-(1)} \sigma^s} \quad \textit{and} \quad \varphi^{reg} < 1 - \mathbf{R^1_M}.$$

– *If $\mathbf{R^N_M} \leq 1$, there are no central bank policy gross rates $(R^s_{CB})_s$ and capital requirement level φ^{reg} implementing an equilibrium with banks.*

The interpretation of the results in Proposition 12 is identical to the one written in Chapter 2 after Proposition 4. In a nutshell, all our results continue to hold qualitatively with multiple states of the world.

3.6 The Role of Capital Requirements in the Case of Non-contingent Real Deposit Rates

In Section 3.6, we examine an economy in which the real deposit gross rate offered by banks cannot be written contingently on the state of the world. We obtain

Corollary 12

If the deposit gross rate of return in real terms is independent of the state of the world, no equilibrium with banks exists.

The proof of Corollary 12 is given in Appendix 3.A. Corollary 12 follows from the following considerations. With a non-contingent real gross rate of return on deposits, either banks have an incentive to increase money creation beyond the average level or they have no incentive to lend at all. This can derived from the following equations, which hold in any potential equilibrium with banks:

$$\frac{R_L^s}{p_C^s} = \frac{\mathbf{R_M^s}}{p_I},$$
$$\frac{R_{CB}^s}{p_C^s} = \frac{R_D^s}{p_C^s} = \mathbf{R_D},$$

where $\mathbf{R_D}$ denotes the deposit gross rate of return in real terms, which is independent of the state of the world. Following Proposition 1, three cases can occur, depending on the value of $\mathbf{R_D}$ compared to $\frac{\mathbf{R_M^s}}{p_I}$: Either there is no privately optimal finite amount of money creation or the privately optimal individual amount of money creation is the level at which a bank is just able to reimburse the central bank in the state when the bank makes losses, or no bank grants any loan.

In the first two cases, an individual bank would grant a larger amount of loans than the average lending level in the economy, and would borrow from the central bank the amount it does not receive in the form of households' deposits. However, in a symmetric equilibrium with banks, an individual bank cannot grant more loans and generate more money creation than the average, which means that money creation in these two cases is explosive and that the monetary system breaks down. In the case where no loan is granted, no money is created and investment is only possible in Sector FT.

We now investigate the role of capital requirements in this setting. We obtain

Proposition 13
There exists an equilibrium with banks and non-contingent real deposit gross rates of return if the central bank policy gross rates $(R_{CB}^s)_s$ and the capital requirement level φ^{reg} are set as follows:

$$\varphi^{reg} > 0 \quad and \quad R_{CB}^l \leq R_{CB}^h.$$

The equilibria can be partitioned into two sets, depending on the value of φ^{reg}:

– *The allocations of equilibria associated with $\varphi^{reg} \in \left(0, \frac{\overline{\mathbf{R}}_\mathbf{M} - \mathbf{R_M^l}}{\overline{\mathbf{R}}_\mathbf{M}}\right)$ are in-*

efficient and welfare is decreasing in φ^{reg}.

– *The allocations of equilibria associated with* $\varphi^{reg} \in \left[\frac{\overline{\mathbf{R}}_{\mathbf{M}} - \mathbf{R}^{\mathbf{l}}_{\mathbf{M}}}{\overline{\mathbf{R}}_{\mathbf{M}}}, 1\right)$ *are first-best.*

The proof of Proposition 13 is given in Appendix 3.A. We obtain the interesting result that inefficient equilibria arise for low capital requirements, while high capital requirement implement the socially efficient allocation. Higher capital requirements thus improve welfare. If we now consider price rigidities, from the proof of Proposition 13, we directly obtain

Proposition 14

There exists an equilibrium with banks, non-contingent real deposit gross rates of return, and rigid prices if the central bank policy gross rates $(R^s_{CB})_s$ *and the capital requirement level* φ^{reg} *are set as follows:*

– $R^l_{CB} = R^h_{CB} = \overline{\mathbf{R}}_{\mathbf{M}}$ *and* $\varphi^{reg} \geq \frac{\overline{\mathbf{R}}_{\mathbf{M}} - \mathbf{R}^{\mathbf{l}}_{\mathbf{M}}}{\overline{\mathbf{R}}_{\mathbf{M}}}$.

– $R^l_{CB} = R^h_{CB} > \overline{\mathbf{R}}_{\mathbf{M}}$ *and* $\varphi^{reg} = \frac{\sigma}{1-\sigma}\frac{\mathbf{R}^{\mathbf{h}}_{\mathbf{M}} - \overline{R}_{CB}}{\overline{R}_{CB}}$.

We now consider the zero lower bound problem. From Propositions 13 and 14, we obtain

Proposition 15

If $\overline{\mathbf{R}}_{\mathbf{M}} < 1$, $\mathbf{R}^{\mathbf{h}}_{\mathbf{M}} > 1$, *and the central bank is constrained by the zero lower bound, there exists an equilibrium with banks, non-contingent real deposit gross rates of return, and rigid prices if the central bank policy gross rates* $(R^s_{CB})_s$ *and the capital requirement level* φ^{reg} *are set as follows:*

$$R^l_{CB} = R^h_{CB} > \overline{\mathbf{R}}_{\mathbf{M}} \quad \textit{and} \quad \varphi^{reg} = \frac{\sigma}{1-\sigma}\frac{\mathbf{R}^{\mathbf{h}}_{\mathbf{M}} - \overline{R}_{CB}}{\overline{R}_{CB}}.$$

The welfare-maximizing policy is given by

$$1 = R^l_{CB} = R^h_{CB} > \overline{\mathbf{R}}_{\mathbf{M}} \quad \textit{and} \quad \varphi^{reg} = \frac{\sigma}{1-\sigma}(\mathbf{R}^{\mathbf{h}}_{\mathbf{M}} - 1),$$

when $\sigma\mathbf{R}^{\mathbf{h}}_{\mathbf{M}} < 1$.

If $\mathbf{R}^{\mathbf{h}}_{\mathbf{M}} < 1$, *there is no equilibrium with banks.*

We can conclude that the main results found in Chapter 2 continue to hold in this setting.

3.7 Costs of Equity Issuance

In Section 3.7, we look for the optimal policy mix when banks incur costs in real terms that are proportional to the real equity they issue.[9]

We assume in Section 3.7 that issuing an amount $\frac{E_B}{p_I}$ of real equity is costly for banks, that the costs incurred by banks are proportional to the amount of real equity, and that they are given by

$$\mathbf{c}\frac{E_B}{p_I},$$

where $\mathbf{c}$ is the cost per unit of consumption good in terms of real equity issued. Following this description, we can restate the banks' optimization problem as follows:

$$\max_{\alpha_M^b \geq 0} \left(\alpha_M^b L_M \frac{R_L^s - R_{CB}^s}{p_C^s} + E_B \frac{R_{CB}^s}{p_C^s} - \mathbf{c}\frac{E_B}{p_I}, 0 \right).$$

We first investigate the second-best welfare allocation in the presence of costs of equity issuance in real terms. The second-best allocation is given by the following social planner's maximization problem:

$$\begin{aligned} &\max_{\mathbf{K_F} \in [\mathbf{0}, \mathbf{W}]} \mathbb{E} \left[\mathbf{f}(\mathbf{K_F}) + (\mathbf{W} - \mathbf{K_F})\overline{\mathbf{R}}_\mathbf{M} - \mathbf{c}\frac{E_B}{p_I} \right] \\ \text{s.t.} \quad &\frac{E_B}{p_I} = \varphi \frac{L_M}{p_I} = \varphi(\mathbf{W} - \mathbf{K_F}). \end{aligned}$$

We replace Assumption 1 by the following assumption, which ensures that the social planner's maximization problem has an interior solution:

[9]Note that we could also assume that households incur the costs in real terms as an alternative model. In this alternative model, the costs of equity issuance would change the representative household's optimization problem, and the equilibrium properties that we would obtain would very likely differ from our findings in Section 3.7.

Assumption 4

$$\mathbf{f}'(\mathbf{W}) < \overline{\mathbf{R}}_{\mathbf{M}} - c \quad \textit{and} \quad \overline{\mathbf{R}}_{\mathbf{M}} < \mathbf{f}'(\mathbf{0}).$$

The interior solution is given by

$$\mathbf{f}'(\mathbf{K}_{\mathbf{F}}^{\mathbf{SB}}) = \overline{\mathbf{R}}_{\mathbf{M}} - c\varphi.$$

It is straightforward to see that the welfare of the second-best allocation is strictly decreasing in $\varphi \in (0, 1)$.

We now investigate how the banks' behavior is affected by the costs of equity issuance. We first calculate the value of lending above which a Bank b would default on the central bank in State s and under which it would potentially only default against households in State s. This value which we denote by α_{DCB}^s in Section 3.7 is given by the equation

$$\Pi_B^{b,s} = \alpha_{DCB}^s L_M(R_L^s - R_{CB}^s) + E_B R_{CB}^s - \mathbf{c}E_B \frac{p_C^s}{p_I} = -D_H R_{CB}^s = -(L_M - E_B)R_{CB}^s,$$

which implies that

$$\alpha_{DCB}^s = \frac{R_{CB}^s - \varphi\mathbf{c}\frac{p_C^s}{p_I}}{R_{CB}^s - R_L^s}.$$

We can now derive the expression of banks' behavior in the following proposition:

Proposition 16
The privately optimal amounts of money creation and lending by an individual bank are represented by a correspondence denoted by $\hat{\alpha}_M : \mathbb{R}_{++}^6 \times (0, 1) \to \mathcal{P}(\mathbb{R} \cup \{+\infty\})$ and given by:

$$\hat{\alpha}_M\big((R_L^s)_s, (R_{CB}^s)_s, (p_C^s)_s, \varphi\big) =$$
$$\begin{cases} \{+\infty\} & \textit{if } R_L^s \geq R_{CB}^s \textit{ for all states } s = l, h \\ & \textit{with at least one strict inequality,} \end{cases}$$

$$\begin{cases} \{\alpha_{DCB}^{s'}\} & \text{if } (\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] \geq \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right] \text{ and } R_L^{s'} < R_{CB}^{s'} \text{ for some } s \neq s') \text{ or} \\ & \text{if } (\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right], R_{CB}^s < R_L^s, \text{ and } \varphi < \varphi_1^{T,s} \text{ for some } s \neq s'), \\ [0, +\infty) & \text{if } R_L^s = R_{CB}^s \text{ for all states } s = l, h, \\ \{0, \alpha_{DCB}^{s'}\} & \text{if } \mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right], R_{CB}^s < R_L^s, \text{ and } \varphi = \varphi_1^{T,s} \text{ for some } s \neq s', \\ \{0\} & \text{in all other cases,} \end{cases}$$

where we define

$$\varphi_1^{T,s} = \frac{\frac{R_{CB}^{s'}}{p_C^{s'}}}{\frac{\sigma^{s'}}{\sigma^s}\left(\frac{R_{CB}^{s'}}{p_C^{s'}} - \frac{\mathbf{c}}{p_I}\right)\frac{p_C^s}{p_C^{s'}}\left(\frac{R_{CB}^{s'} - R_L^{s'}}{R_L^s - R_{CB}^s}\right) + \frac{\mathbf{c}}{p_I}}.$$

The proof of Proposition 16 is given in Appendix 3.A. We now investigate the equilibria with banks resulting in this setting. We obtain the following proposition:

Proposition 17
If $\mathbf{R_M^l} \geq \mathbf{c}$*, there is no equilibrium with banks. We suppose in the following that* $\mathbf{R_M^l} < \mathbf{c}$*. Given the policy gross rates* $(R_{CB}^s)_{s=l,h}$*, all equilibria with banks have the following form:*

$$R_E^{s*} = \frac{R_L^{s*} - R_{CB}^s}{\varphi^*} + R_{CB}^s, \quad R_E^{s'*} = 0, \tag{3.20}$$

$$\mathbf{R_F^*} = \mathbf{f}'(\mathbf{0}) - \max\left(0, \mathbf{f}'(\mathbf{0}) - p\mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]\right), \tag{3.21}$$

$$p_I^* = p, \quad p_C^{s*} = p\frac{R_L^{s*}}{\mathbf{R_M^s}}, \tag{3.22}$$

$$E_B^* = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F^*})\Big), \quad D_H^* = (1 - \varphi^*)p\Big(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F^*})\Big), \tag{3.23}$$

$$\tilde{D}_H^{s*} = p\Big(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F^*})\Big)R_{CB}^s, \tag{3.24}$$

$$L_M^* = p\Big(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F^*})\Big), \quad S_F^* = \mathbf{f}'^{-1}(\mathbf{R_F^*}), \tag{3.25}$$

$$\mathbf{K_M^*} = \mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F^*}), \quad \mathbf{K_F^*} = \mathbf{f}'^{-1}(\mathbf{R_F^*}), \tag{3.26}$$

where the price of the investment good denoted by $p \in (0, +\infty)$ *is arbitrary, the equity ratio* $\varphi^* \in (0, 1)$ *is given by* $\varphi^* = \frac{\mathbf{R_M^{s'}}}{\mathbf{c}}$*, and the lending gross rates of return*

$(R_L^{s*})_s$ *are arbitrary such that*

$$\begin{aligned}&R_{CB}^{s'} > R_L^{s'*}, \quad R_{CB}^{s} < R_L^{s*}, \quad \textit{and}\\ &\mathbb{E}\left[\mathbf{R}_\mathbf{M}^\mathbf{s}\frac{R_{CB}^s}{R_L^{s*}}\right] + \sigma^s(\mathbf{R}_\mathbf{M}^{\mathbf{s}'} - \mathbf{R}_\mathbf{M}^\mathbf{s}) = \sigma^{s'}\mathbf{R}_\mathbf{M}^{\mathbf{s}'}\frac{R_{CB}^{s'}}{R_L^{s'*}}\left(1 - \frac{\mathbf{R}_\mathbf{M}^{\mathbf{s}'}}{\mathbf{c}}\right).\end{aligned}$$

The equilibrium profits of firms and banks are given by

$$\Pi_M^{s*} = 0, \quad \Pi_F^{s*} = p\frac{R_L^{s*}}{\mathbf{R}_\mathbf{M}^\mathbf{s}}\left(\mathbf{f}\left(\mathbf{f}'^{-1}(\mathbf{R}_\mathbf{F}^*)\right) - \mathbf{f}'^{-1}(\mathbf{R}_\mathbf{F}^*)\mathbf{R}_\mathbf{F}^*\right), \tag{3.27}$$

$$\Pi_B^{s*} = p\Big(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R}_\mathbf{F}^*)\Big)\left(R_L^{s*} - R_{CB}^s + \varphi^* R_{CB}^s\right), \quad \Pi_B^{s'*} < 0. \tag{3.28}$$

The proof of Proposition 17 is given in Appendix 3.A. From Proposition 17, we can derive the following corollary about the welfare that can be achieved by the central bank policy gross rates:

Corollary 13
Suppose that $\mathbf{R}_\mathbf{M}^\mathbf{l} < \mathbf{c}$. There is a set of investment allocations, each one of which can be supported by any given arbitrary central bank policy gross rates $(R_{CB}^s)_s$, and these investment allocations are all dominated by the second-best investment allocation.

The proof of Corollary 13 is given in Appendix 3.A. Corollary 13 means that the central bank cannot influence the investment allocation and that the best investment allocation that can occur is dominated by the second-best allocation. The intuition runs as follows. Prices and interest rates can adjust, so that it is not possible for the central bank to implement a pre-determined allocation. Moreover, in any equilibrium with banks, banks have to default in one state to make bank equity as attractive to households as deposits. However, banks' default entails a mispricing by households and thus a misallocation, compared to the second-best allocation. In such equilibria with banks, the threat of default against the central bank combined with the costs of equity issuance limits money creation by banks.

If we now suppose that prices are rigid, a higher welfare is achieved by central bank policy gross rates that are closer to the real rates. The equity ratio is constant to prevent money creation from exploding: It is such that by lending the average amount of loans to firms, banks completely default on households and are on the brink of defaulting against the central bank. This is made possible by the costs of equity issuance.

We now investigate the impact of capital requirements on welfare when prices are rigid and examine how the regulatory authorities can improve welfare with a combination of a minimum equity ratio requirement together with the central bank policy gross rates. We first obtain a characterization of the equilibria with banks that can be implemented in this case:

Proposition 18
Suppose that prices are rigid, i.e. that $p_I^* = p_C^{s*} = 1$ for all states $s = l, h$. Depending on the policy gross rates $(R_{CB}^s)_{s=l,h}$ and the minimum equity ratio requirement $\varphi^{reg} \in (0,1)$, all equilibria with banks are of one of the following four types:

- **Type 1**: *If*

  $\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} \geq R_{CB}^s$ *for all states $s = l, h$ with at least one strict inequality and*
  $\overline{R}_{CB} = \overline{\mathbf{R}}_{\mathbf{M}} - \mathbf{c}\varphi^{reg}$,

  there is an equilibrium with banks where no bank defaults in any state $s = l, h$ and

$$\varphi^* = \varphi^{reg}, \tag{3.29}$$

$$R_D^{s*} = R_{CB}^s, \quad R_L^{s*} = \mathbf{R}_{\mathbf{M}}^{\mathbf{s}}, \quad R_E^{s*} = \frac{\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} - R_{CB}^s}{\varphi^*} + R_{CB}^s, \quad \mathbf{R}_{\mathbf{F}}^* = \overline{R}_{CB}, \tag{3.30}$$

$$E_B^* = \varphi^*\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big)\Big), \quad D_H^* = (1-\varphi^*)\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big)\Big), \tag{3.31}$$

$$\tilde{D}_H^{s*} = \Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big)\Big)R_{CB}^s, \tag{3.32}$$

$$L_M^* = \Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big)\Big), \quad S_F^* = \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big), \tag{3.33}$$

$$\mathbf{K}_{\mathbf{M}}^* = \mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big), \quad \mathbf{K}_{\mathbf{F}}^* = \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big). \tag{3.34}$$

The equilibrium profits of firms and banks are given by

$$\Pi_M^{s*} = 0, \quad \Pi_F^{s*} = \Big(\mathbf{f}\Big(\mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big) - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\mathbf{R}_\mathbf{F}^*\Big), \tag{3.35}$$

$$\Pi_B^{s*} = \Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big)\Bigg(\mathbf{R}_\mathbf{M}^\mathbf{s} - R_{CB}^s + \varphi^* R_{CB}^s\Bigg). \tag{3.36}$$

– **Type 2**: *If for some states $s \neq s'$ we have $\mathbf{R}_\mathbf{M}^{\mathbf{s}'} > \mathbf{c}$,*

$$\overline{R}_{CB} = \overline{\mathbf{R}}_\mathbf{M} - \mathbf{c}\varphi^{reg}, \quad R_{CB}^{s'} > \mathbf{R}_\mathbf{M}^{\mathbf{s}'}, \quad \textit{and}$$
$$\varphi^{reg} \geq \frac{R_{CB}^{s'} - \mathbf{R}_\mathbf{M}^{\mathbf{s}'}}{R_{CB}^{s'} - \mathbf{c}} > 0,$$

there is an equilibrium with banks where no bank defaults in any state $s = l, h$ and

$$\varphi^* = \varphi^{reg}, \tag{3.37}$$

$$R_D^{s*} = R_{CB}^s, \quad R_L^{s*} = \mathbf{R}_\mathbf{M}^\mathbf{s}, \quad R_E^{s*} = \frac{\mathbf{R}_\mathbf{M}^\mathbf{s} - R_{CB}^s}{\varphi^*} + R_{CB}^s, \quad \mathbf{R}_\mathbf{F}^* = \overline{R}_{CB}, \tag{3.38}$$

$$E_B^* = \varphi^*\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big), \quad D_H^* = (1-\varphi^*)\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big), \tag{3.39}$$

$$\tilde{D}_H^{s*} = \Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big)R_{CB}^s, \tag{3.40}$$

$$L_M^* = \Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big), \quad S_F^* = \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big), \tag{3.41}$$

$$\mathbf{K}_\mathbf{M}^* = \mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big), \quad \mathbf{K}_\mathbf{F}^* = \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big). \tag{3.42}$$

The equilibrium profits of firms and banks are given by

$$\Pi_M^{s*} = 0, \quad \Pi_F^{s*} = \Big(\mathbf{f}\Big(\mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big) - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\mathbf{R}_\mathbf{F}^*\Big), \tag{3.43}$$

$$\Pi_B^{s*} = \Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big)\Bigg(\mathbf{R}_\mathbf{M}^\mathbf{s} - R_{CB}^s + \varphi^* R_{CB}^s\Bigg). \tag{3.44}$$

– **Type 3**: *If for some states $s \neq s'$*

$$R_{CB}^{s'} > \mathbf{R_M^{s'}}, \quad \textit{and}$$
$$\varphi^{reg} = \sigma^s \frac{\mathbf{R_M^s} - R_{CB}^s}{\sigma^{s'} R_{CB}^{s'} + \sigma^s \mathbf{c}} \leq \min\left(\frac{R_{CB}^{s'} - \mathbf{R_M^{s'}}}{R_{CB}^{s'} - \mathbf{c}}, \frac{\mathbf{R_M^{s'}}}{\mathbf{c}}\right),$$

there is an equilibrium with banks where banks default in some State s' and

$$\varphi^* = \varphi^{reg}, \tag{3.45}$$
$$R_D^{s*} = R_{CB}^s, \quad R_L^{s*} = \mathbf{R_M^s}, \tag{3.46}$$
$$\mathbf{R_F^*} = \mathbf{f'(0)} - \max\left(0, \mathbf{f'(0)} - \overline{R}_{CB}\right), \tag{3.47}$$
$$R_E^{s*} = \frac{\mathbf{R_M^s} - R_{CB}^s}{\varphi^*} + R_{CB}^s, \quad R_E^{s'*} = 0, \tag{3.48}$$
$$E_B^* = \varphi^*\Big(\mathbf{W} - \mathbf{f'}^{-1}\big(\mathbf{R_F^*}\big)\Big), \quad D_H^* = (1-\varphi^*)\Big(\mathbf{W} - \mathbf{f'}^{-1}\big(\mathbf{R_F^*}\big)\Big), \tag{3.49}$$
$$\tilde{D}_H^{s*} = \Big(\mathbf{W} - \mathbf{f'}^{-1}\big(\mathbf{R_F^*}\big)\Big) R_{CB}^s, \tag{3.50}$$
$$L_M^* = \Big(\mathbf{W} - \mathbf{f'}^{-1}\big(\mathbf{R_F^*}\big)\Big), \quad S_F^* = \mathbf{f'}^{-1}\big(\mathbf{R_F^*}\big), \tag{3.51}$$
$$\mathbf{K_M^*} = \mathbf{W} - \mathbf{f'}^{-1}\big(\mathbf{R_F^*}\big), \quad \mathbf{K_F^*} = \mathbf{f'}^{-1}\big(\mathbf{R_F^*}\big). \tag{3.52}$$

The equilibrium profits of firms and banks are given by

$$\Pi_M^{s*} = 0, \quad \Pi_F^{s*} = \left(\mathbf{f}\Big(\mathbf{f'}^{-1}\big(\mathbf{R_F^*}\big)\Big) - \mathbf{f'}^{-1}\big(\mathbf{R_F^*}\big)\mathbf{R_F^*}\right), \tag{3.53}$$
$$\Pi_B^{s*} = \Big(\mathbf{W} - \mathbf{f'}^{-1}\big(\mathbf{R_F^*}\big)\Big)\left(\mathbf{R_M^s} - R_{CB}^s + \varphi^* R_{CB}^s\right), \quad \Pi_B^{s'*} < 0. \tag{3.54}$$

– **Type 4**: *If for some states $s \neq s'$ we have $\mathbf{R_M^{s'}} < \mathbf{c}$,*

$$R_{CB}^{s'} > \mathbf{R_M^{s'}}, \quad \mathbf{R_M^s} > R_{CB}^s, \quad \varphi^{reg} \leq \frac{\mathbf{R_M^{s'}}}{\mathbf{c}}, \quad \textit{and}$$
$$\mathbb{E}\left[\mathbf{R_M^s} \frac{R_{CB}^s}{R_L^{s*}}\right] + \sigma^s(\mathbf{R_M^{s'}} - \mathbf{R_M^s}) = \sigma^{s'} \mathbf{R_M^{s'}} \frac{R_{CB}^{s'}}{R_L^{s'*}}\left(1 - \frac{\mathbf{R_M^{s'}}}{\mathbf{c}}\right),$$

there is an equilibrium with banks where banks default in some State s' and

$$\varphi^* = \frac{\mathbf{R}_{\mathbf{M}}^{\mathbf{s}'}}{\mathbf{c}}, \tag{3.55}$$

$$R_D^{s*} = R_{CB}^s, \quad R_L^{s*} = \mathbf{R}_{\mathbf{M}}^{\mathbf{s}}, \tag{3.56}$$

$$\mathbf{R}_{\mathbf{F}}^* = \mathbf{f}'(\mathbf{0}) - \max\left(0, \mathbf{f}'(\mathbf{0}) - \overline{R}_{CB}\right), \tag{3.57}$$

$$R_E^{s*} = \frac{\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} - R_{CB}^s}{\varphi^*} + R_{CB}^s, \quad R_E^{s'*} = 0, \tag{3.58}$$

$$E_B^* = \varphi^*\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big)\Big), \quad D_H^* = (1-\varphi^*)\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big)\Big), \tag{3.59}$$

$$\tilde{D}_H^{s*} = \Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big)\Big)R_{CB}^s, \tag{3.60}$$

$$L_M^* = \Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big)\Big), \quad S_F^* = \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big), \tag{3.61}$$

$$\mathbf{K}_{\mathbf{M}}^* = \mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big), \quad \mathbf{K}_{\mathbf{F}}^* = \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big). \tag{3.62}$$

The equilibrium profits of firms and banks are given by

$$\Pi_M^{s*} = 0, \quad \Pi_F^{s*} = \left(\mathbf{f}\Big(\mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big)\Big) - \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big)\mathbf{R}_{\mathbf{F}}^*\right), \tag{3.63}$$

$$\Pi_B^{s*} = \Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_{\mathbf{F}}^*\big)\Big)\left(\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} - R_{CB}^s + \varphi^* R_{CB}^s\right), \quad \Pi_B^{s'*} < 0. \tag{3.64}$$

The proof of Proposition 17 is given in Appendix 3.A. This proposition has welfare implications in terms of the best regulatory policy, i.e. of the best combination of central bank gross rates $(R_{CB}^s)_s$ with a minimum equity ratio requirement $\varphi^{reg} \in (0,1)$. In this respect, we obtain the following corollary:

Corollary 14
Suppose that prices are rigid. By setting $(R_{CB}^s)_s$ appropriately in adequate combination with the minimum equity ratio requirement $\varphi^{reg} \in (0,1)$, the regulatory authorities can implement any allocation arbitrarily close to the first-best allocation.

The proof of Corollary 14 is given in Appendix 3.A. The regulatory authorities can achieve an allocation that is arbitrarily close to the first-best allocation only with policies that prevent a default against households, that involve minimum equity ratio requirements that are low enough, and that create an incentive strong enough for households to invest in bank equity.

3.8 Reserve Requirements and Haircuts

In Section 3.8, we study the impact of reserve requirements coupled with haircuts on money creation. We introduce reserve requirements as follows:

Definition 4
A minimum reserve requirement r^{reg} ($r^{reg} \in (0,1)$) requires each bank to hold more central bank reserves at the end of Period $t = 0$ than the fraction r^{reg} of its deposits. If we denote the reserve ratio of Bank b by $r^b = \frac{d^b_{CB}}{d_H}$, a reserve requirement imposes the following relationship on central bank reserves d^b_{CB}:

$$r^{reg} \leq r^b = \frac{d^b_{CB}}{d_H}.$$

We define a haircut regulation h as follows:

Definition 5
A haircut regulation h ($h \in (0,1)$) requires each bank to hold more loans to Sector MT at the end of Period $t = 0$ than a multiple $\frac{1}{1-h}$ of its CB liabilities.

The balance sheets of banks b_i and b_j which are complying with some reserve requirement r^{reg} and some haircut regulation h are given in Table 3.1. In these balance sheets, we use the following notations:

$$\begin{aligned} d^{\Delta_i}_{CB} &= l^{\Delta_i}_{CB} = \max(0, r^{reg} d_H - d^{b_i}_{CB}), \\ \text{and} \quad d^{\Delta_j}_{CB} &= l^{\Delta_j}_{CB} = r^{reg} d_H. \end{aligned}$$

In the following proposition, we investigate the impact of a minimum reserve requirement r^{reg} coupled with a haircut regulation h on money creation α^b_M by a Bank b. We obtain

Bank b_i		Bank b_j	
$d_{CB}^{\Delta_i}$	$l_{CB}^{\Delta_i}$	$d_{CB}^{\Delta_j}$	$l_{CB}^{\Delta_j}$
$d_{CB}^{b_i}$			$l_{CB}^{b_j}$
$l_M^{b_i}$	d_H	$l_M^{b_j}$	d_H
	e_B		e_B

Table 3.1: Balance sheets at the end of $t = 0$, with a combination of a minimum reserve requirement r^{reg} and a haircut regulation h. Source: Own illustration.

Proposition 19

A combination of a minimum reserve requirement r^{reg} and a haircut regulation h imposes the following constraint on money creation by Bank b:

$$\alpha_M^b \leq \frac{1 - r^{reg}(1 - \varphi)}{h}.$$

Equilibria with banks exist if and only if the equity ratio φ fulfills

$$1 - \frac{1 - h}{r^{reg}} \leq \varphi.$$

The proof of Proposition 19 is given in Appendix 3.A. From Lemma 5 in Chapter 2 and Proposition 19, we directly deduce that the impact of the reserve requirement coupled with the haircut regulation on money creation by commercial banks given in Proposition 19 is identical to the one of the minimum equity ratio requirement given in Lemma 5. In particular, we directly obtain

Proposition 20

A combination of a reserve requirement r^{reg} and a haircut regulation h imposes the same constraint on the banks' behavior as a minimum equity ratio requirement

φ^{reg} *if and only if*

$$\varphi^{reg} = \frac{\varphi h}{1 - r^{reg}(1 - \varphi)}.$$

The condition on the bank capital structure for which an equilibrium with banks exists then writes

$$\varphi^{reg} \leq \varphi,$$

or alternatively

$$1 - \frac{1 - h}{r^{reg}} \leq \varphi.$$

As a consequence, we can focus on the impact of a minimum equity ratio requirement on the banks' incentives to create money—which was performed in Chapter 2—and we deduce that similar results to the ones obtained in Chapter 2 hold when imposing a reserve requirement coupled with a haircut regulation.

Appendix

3.A Proofs

Proof of Corollary 7.
Let $\mathcal{E}^*$ be an equilibrium with banks.

Then all banks choose the same level of money creation and lending, denoted by α_M^*. At the aggregate level, however, the amount borrowed by banks from the central bank has to equal the amount deposited by banks at the central bank, meaning that $\int_0^1 \alpha_M^b \mathrm{d}b = 1$, which translates into $\alpha_M^* = 1$. The result of Lemma 2 implies that we can apply Proposition 5. Thus, given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, prices $(p_C^s)_s$, and the equity ratio φ^*, all banks $b \in [0,1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M\big((R_L^{s*})_s, (R_{CB}^s)_s, (p_C^s)_s, \varphi^*\big)$, as given in Proposition 5. The only gross rates of return in Proposition 5 rationalizing $\alpha_M^* = 1$ are

$$R_L^{s*} = R_{CB}^s$$

for all states $s = l, h$. A direct consequence of this relation, Lemma 2, and the expression of profits in Equation (3.1) is that

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^s \tag{3.65}$$

for all states $s = l, h$. Moreover, due to Lemma 2 and the tie-breaking rule introduced in Subsection 2.2.3, the interbank lending market is not used in an equilibrium with banks. Finally, $\Pi_M^{s*} = 0$ for all states $s = l, h$ (see Subsection

2.2.4), which translates into

$$\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} p_C^{s*} = R_L^{s*} p_I^*$$

for all states $s = l, h$. Given gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices p_I^* and $(p_C^{s*})_s$, households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, S_F^*\big)$ given S_F^*, $D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, E_B^*, S_F^*\big)$ given E_B^* and S_F^*, and $S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s\big)$. These correspondences are given in Lemma 6 in Appendix 2.E. Only the first, the fourth, and the seventh cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ correspond to equal nominal gross rates of return R_E^{s*} and R_D^{s*} and are hence consistent with the equality of nominal gross rates of return in Equation (3.65). However, the assumption $\mathbf{f}'(\mathbf{W}) < \overline{\mathbf{R}}_{\mathbf{M}} < \mathbf{f}'(\mathbf{0})$ plus $\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} p_C^{s*} = R_L^{s*} p_I^*$ rule out the first and fourth cases. As in an equilibrium with banks $E_B^*, D_H^* > 0$, we thus obtain

$$\begin{aligned}
&E_B^* \in \left(0, p_I^* \left(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_{\mathbf{M}})\right)\right),\\
&D_H^* = p_I^* \left(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_{\mathbf{M}})\right) - E_B^*, \text{ and}\\
&S_F^* = \mathbf{f}'^{-1}(\overline{\mathbf{R}}_{\mathbf{M}}).
\end{aligned}$$

Finally, $\mathbf{R}_{\mathbf{F}}^*$ can be determined by using Lemma 4 and equating the demand for the investment good $\mathbf{K}_{\mathbf{F}}^*$ to its supply S_F^*. With the help of the equity ratio φ^*, we can then re-write all equilibrium variables as given in Theorem 1.

In turn, it is straightforward to verify that the tuples given in Theorem 1 constitute equilibria with banks as in Section 3.1. □

Proof of Proposition 6.

We use the following auxiliary function:

$$g(\mathbf{K}_{\mathbf{F}}) = -\overline{\mathbf{R}}_{\mathbf{M}} + \mathbb{E}[(\mathbf{f}^{\mathbf{s}})'(\mathbf{K}_{\mathbf{F}})].$$

This function is strictly decreasing:

$$g'(\mathbf{K}_{\mathbf{F}}) = \mathbb{E}[(\mathbf{f}^{\mathbf{s}})''(\mathbf{K}_{\mathbf{F}})] < 0,$$

and

$$g(\mathbf{0}) = -\overline{\mathbf{R}}_{\mathbf{M}} + \mathbb{E}[(\mathbf{f^s})'(\mathbf{0})] > 0$$
$$g(\mathbf{W}) = -\overline{\mathbf{R}}_{\mathbf{M}} + \mathbb{E}[(\mathbf{f^s})'(\mathbf{W})] < 0.$$

As g is a continuous and strictly decreasing function with $g(\mathbf{0}) > 0$ and $g(\mathbf{W}) < 0$, we can apply the intermediate value theorem. Therefore, there exists a unique value $\mathbf{K_F}$ such that $g(0) = 0$, i.e.

$$\mathbb{E}[(\mathbf{f^s})'(\mathbf{K_F})] = \overline{\mathbf{R}}_{\mathbf{M}},$$

which solves the social planner's problem. □

Proof of Theorem 2.

Let $\mathcal{E}^*$ be an equilibrium with banks.

Then all banks choose the same level of money creation and lending, denoted by α_M^*. At the aggregate level, however, the amount borrowed by banks from the central bank has to equal the amount deposited by banks at the central bank, meaning that $\int_0^1 \alpha_M^b \mathrm{d}b = 1$, which translates into $\alpha_M^* = 1$. The result of Lemma 2 implies that we can apply Proposition 1. Thus, given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, and the equity ratio φ^*, all banks $b \in [0,1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M\big((R_L^{s*})_s, (R_{CB}^s)_s, \varphi^*\big)$ as given in Proposition 1. The only gross rates of return in Proposition 1 rationalizing $\alpha_M^* = 1$ are

$$R_L^{s*} = R_{CB}^s$$

for all states $s = l, h$. A direct consequence of this relation and of Lemma 2 is that

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^s \tag{3.66}$$

for all states $s = l, h$. Moreover, due to Lemma 2 and the tie-breaking rule introduced in Subsection 2.2.3, the interbank lending market is not used in an equilibrium with banks. Finally, $\Pi_M^{s*} = 0$ for all states $s = l, h$ (see Subsection

2.2.4), which translates into

$$\mathbf{R}^{\mathbf{s}}_{\mathbf{M}} p^{s*}_C = R^{s*}_L p^*_I$$

for all states $s = l, h$. Given gross rates of return $(R^{s*}_E)_s$ and $(R^{s*}_D)_s$ as well as prices p^*_I and $(p^{s*}_C)_s$, households choose $E^*_B \in \hat{E}_B\big((R^{s*}_E)_s, (R^{s*}_D)_s, p^*_I, (p^{s*}_C)_s, S^*_F\big)$ given S^*_F, $D^*_H \in \hat{D}_H\big((R^{s*}_E)_s, (R^{s*}_D)_s, p^*_I, (p^{s*}_C)_s, E^*_B, S^*_F\big)$ given E^*_B and S^*_F, and $S^*_F \in \hat{S}_F\big((R^{s*}_E)_s, (R^{s*}_D)_s, p^*_I, (p^{s*}_C)_s\big)$, where these correspondences are given in Lemma 9. Only the first, the fourth, and the seventh cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ correspond to equal nominal gross rates of return R^{s*}_E and R^{s*}_D, and thus, are consistent with the equality of nominal gross rates of return in Equation (3.66). However, the assumption $\mathbb{E}[(\mathbf{f}^{\mathbf{s}})'(\mathbf{W})] < \overline{\mathbf{R}}_{\mathbf{M}} < \mathbb{E}[(\mathbf{f}^{\mathbf{s}})'(\mathbf{0})]$ together with $\mathbf{R}^{\mathbf{s}}_{\mathbf{M}} p^{s*}_C = R^{s*}_L p^*_I$ rule out the first and fourth cases. As in an equilibrium with banks $E^*_B, D^*_H > 0$, we thus obtain

$$\begin{aligned}
&E^*_B \in (0, p^*_I\,(\mathbf{W} - \mathbf{K}^*_{\mathbf{F}})),\\
&D^*_H = p^*_I\,(\mathbf{W} - \mathbf{K}^*_{\mathbf{F}}) - E^*_B, \text{ and}\\
&S^*_F = \mathbf{K}^*_{\mathbf{F}},
\end{aligned}$$

where $\mathbf{K}^*_{\mathbf{F}}$ is the unique solution to the following equation:

$$\mathbb{E}[(\mathbf{f}^{\mathbf{s}})'(\mathbf{K}_{\mathbf{F}})] = \overline{\mathbf{R}}_{\mathbf{M}}.$$

Finally, $\mathbf{R}^*_{\mathbf{F}}$ can be determined by using Lemma 8 and equating the demand for the investment good $\mathbf{K}^*_{\mathbf{F}}$ to its supply S^*_F. Thus, with the help of the equity ratio φ^*, we can re-write all equilibrium variables as given in Theorem 2.

In turn, it is straightforward to verify that the tuples given in Theorem 2 constitute equilibria with banks as in Section 3.3. □

Proof of Proposition 7.

The social planner's maximization problem is given by

$$\max_{\mathbf{K}_{\mathbf{F}} \in [\mathbf{0}, \mathbf{W}]} \mathbb{E}[\mathbf{f}^{\mathbf{s}}_{\mathbf{M}}(\mathbf{W} - \mathbf{K}_{\mathbf{F}})] + \mathbf{f}_{\mathbf{F}}(\mathbf{K}_{\mathbf{F}}).$$

We now show that the first-order condition has a unique solution. For this, we use the notation

$$g(\mathbf{K_F}) = \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{W} - \mathbf{K_F})] - (\mathbf{f_F})'(\mathbf{K_F}).$$

We obtain

$$g'(\mathbf{K_F}) = -\mathbb{E}[(\mathbf{f_M^s})''(\mathbf{W} - \mathbf{K_F})] - (\mathbf{f_F})''(\mathbf{K_F}) > 0,$$

which means that g is a strictly increasing function over $[\mathbf{0}, \mathbf{W}]$. Moreover, Assumptions 2 and 3 imply that

$$g(\mathbf{0}) < 0 \quad \text{and} \quad g(\mathbf{W}) > 0.$$

As g is continuous over $[\mathbf{0}, \mathbf{W}]$, by the intermediate value theorem, we conclude that there exists a unique value $\mathbf{K_F} \in (\mathbf{0}, \mathbf{W})$ such that $g(\mathbf{K_F}) = 0$, which is solution of the social planner's maximization problem. □

Proof of Lemma 10.

Profits of firms in the MT sector are given by

$$\Pi_M^s = \mathbf{f_M^s}(\mathbf{K_M}) - \mathbf{K_M}\frac{R_L^s}{p_C^s}p_I$$

for each state $s = l, h$. As firms in MT do not default in any of the parameter constellations considered, their expected profits are equal to their expected shareholders' value, and the first-order condition with respect to the maximization of expected profits is given by

$$\mathbb{E}[(\mathbf{f_M^s})'(\mathbf{K_M})] = \mathbb{E}\left[\frac{R_L^s}{p_C^s}\right]p_I. \tag{3.67}$$

In the following, we use the notation

$$g(\mathbf{K_M}) = \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{K_M})] - \mathbb{E}\left[\frac{R_L^s}{p_C^s}\right]p_I.$$

We note that g is a strictly decreasing function that is continuous over $[\mathbf{0}, \mathbf{W}]$.

Suppose first that $\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] p_I > \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{0})]$. Then $g(\mathbf{K_M}) < 0$ for all $\mathbf{K_M} \in [\mathbf{0}, \mathbf{W}]$. Expected profits are thus decreasing in $\mathbf{K_M}$. Thus, $\mathbf{K_M} = 0$ maximizes expected profits.

Suppose now that $\mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] p_I < \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{W})]$. Then $g(\mathbf{K_M}) > 0$ for all $\mathbf{K_M} \in [\mathbf{0}, \mathbf{W}]$. Expected profits are thus increasing in $\mathbf{K_M}$. Thus, $\mathbf{K_M} = \mathbf{W}$ maximizes expected profits.

Suppose finally that $\mathbb{E}[(\mathbf{f_M^s})'(\mathbf{W})] \leq \mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] p_I \leq \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{0})]$. We now show that Equation (3.67) has a unique solution $\mathbf{K_M} \in [\mathbf{0}, \mathbf{W}]$. We observe that

$$g(\mathbf{0}) = \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{0})] - \mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] p_I \geq 0$$
$$\text{and} \quad g(\mathbf{W}) = \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{W})] - \mathbb{E}\left[\frac{R_L^s}{p_C^s}\right] p_I \leq 0.$$

Therefore, the intermediate value theorem applies and there exists a unique solution to Equation (3.67), which we denote by $\tilde{\mathbf{K}}_{\mathbf{M}}$. □

Proof of Theorem 3.

Let $\mathcal{E}^*$ be an equilibrium with banks.

Then all banks choose the same level of money creation and lending, denoted by α_M^*. At the aggregate level, however, the amount borrowed by banks from the central bank has to equal the amount deposited by banks at the central bank, meaning that $\int_0^1 \alpha_M^b \mathrm{d}b = 1$, which translates into $\alpha_M^* = 1$. The result of Lemma 2 implies that we can apply Proposition 1. Thus, given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, and the equity ratio φ^*, all banks $b \in [0, 1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M\big((R_L^{s*})_s, (R_{CB}^s)_s, \varphi^*\big)$ as given in Proposition 1. The only gross rates of return in Proposition 1 rationalizing $\alpha_M^* = 1$ are

$$R_L^{s*} = R_{CB}^s$$

for all states $s = l, h$. A direct consequence of this relation and of Lemma 2 is that

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^s \tag{3.68}$$

for all states $s = l, h$. Moreover, due to Lemma 2 and the tie-breaking rule introduced in Subsection 2.2.3, the interbank lending market is not used in an equilibrium with banks.

Given gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices p_I^* and $(p_C^{s*})_s$, households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, S_F^*\big)$ given S_F^*,
$D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, E_B^*, S_F^*\big)$ given E_B^* and S_F^*,
and $S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s\big)$, where these correspondences are given in Lemma 6 in Appendix 2.E. Only the first, the fourth, and the seventh cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ correspond to equal nominal gross rates of return R_E^{s*} and R_D^{s*}, and thus, are consistent with the equality of nominal gross rates of return in Equation (3.68).

Suppose first that

$$\mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right] p_I > \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{0})], \tag{3.69}$$

which is the first case in Lemma 10. Then by Lemma 10, $\mathbf{K_M^*} = 0$ and thus $\mathbf{K_F^*} = \mathbf{W}$. This corresponds only to the first case in Lemma 6. By Lemma 6, $\mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right] \leq \frac{(\mathbf{f_F})'(\mathbf{W})}{p_I}$. Together with Inequality (3.69), this inequality implies

$$\mathbb{E}[(\mathbf{f_M^s})'(\mathbf{0})] < (\mathbf{f_F})'(\mathbf{W}). \tag{3.70}$$

By our assumption that $\mathbb{E}\big[(\mathbf{f_M^s})'(\mathbf{W})\big] > (\mathbf{f_F})'(\mathbf{W})$ and by the concavity of $\mathbf{f_M^s}$ for all states $s = l, h$, we obtain $\mathbb{E}\big[(\mathbf{f_M^s})'(\mathbf{0})\big] > \mathbb{E}\big[(\mathbf{f_M^s})'(\mathbf{W})\big] > (\mathbf{f_F})'(\mathbf{W})$, which contradicts Inequality (3.70).

Suppose now that

$$\mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right] p_I < \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{W})] \tag{3.71}$$

which is the second case in Lemma 10. Then by Lemma 10, $\mathbf{K}_\mathbf{M}^* = \mathbf{W}$ and thus $\mathbf{K}_\mathbf{F}^* = \mathbf{0}$. This corresponds only to the fourth case in Lemma 6. By Lemma 6, $\frac{\mathbf{f}'(\mathbf{0})}{p_I} < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right]$. We thus obtain

$$\mathbf{f}'(\mathbf{0}) < \mathbb{E}[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{W})]. \tag{3.72}$$

By our assumption that $\mathbb{E}\big[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{0})\big] < (\mathbf{f}_\mathbf{F})'(\mathbf{0})$ and by the concavity of $\mathbf{f}_\mathbf{M}^\mathbf{s}$ for all states $s = l, h$, we obtain $\mathbb{E}\big[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{W})\big] < \mathbb{E}\big[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{0})\big] < (\mathbf{f}_\mathbf{F})'(\mathbf{0})$, which contradicts Inequality (3.72).

Thus, in any equilibrium with banks, it must be that

$$\mathbb{E}[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{K}_\mathbf{M})] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right] p_I \tag{3.73}$$

and

$$\mathbb{E}[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{W})] \leq p_I \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right] \leq \mathbb{E}[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{0})].$$

Together with the assumptions $\mathbb{E}\big[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{0})\big] < (\mathbf{f}_\mathbf{F})'(\mathbf{0})$ and $\mathbb{E}\big[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{W})\big] > (\mathbf{f}_\mathbf{F})'(\mathbf{W})$, the previous inequalities imply that

$$(\mathbf{f}_\mathbf{F})'(\mathbf{W}) < \mathbb{E}[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{W})] \leq p_I \mathbb{E}\left[\frac{R_{CB}^s}{p_C^s}\right] \leq \mathbb{E}[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{0})] < (\mathbf{f}_\mathbf{F})'(\mathbf{0}),$$

which rules out the first and fourth cases in Lemma 6. From the remaining seventh case in Lemma 6 and Equation (3.73), we obtain

$$(\mathbf{f}_\mathbf{F})'(\mathbf{K}_\mathbf{F}^*) = \mathbb{E}[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{W} - \mathbf{K}_\mathbf{F}^*)].$$

We note that Proposition 7 shows that this equation has a unique solution, which corresponds to the first-best allocation. As in an equilibrium with banks $E_B^*, D_H^* >$

0, we thus obtain

$$\begin{aligned}
&E_B^* \in (0, p_I^* \left(\mathbf{W} - \mathbf{K_F^{FB}}\right)),\\
&D_H^* = p_I^* \left(\mathbf{W} - \mathbf{K_F^{FB}}\right) - E_B^*, \text{ and}\\
&S_F^* = \mathbf{K_F^{FB}}.
\end{aligned}$$

Finally, $\mathbf{R_F^*}$ can be determined by using Lemma 4 and equating the demand for the investment good $\mathbf{K_F^*}$ to its supply S_F^*. Therefore, with the help of the equity ratio φ^*, we can re-write all equilibrium variables as given in Theorem 3.

In turn, it is straightforward to verify that the tuples given in Theorem 3 constitute equilibria with banks as in Section 3.3. □

Proof of Proposition 8.

Let $\mathcal{E}^*$ be an equilibrium with banks for which a minimum equity ratio $\varphi^{reg} \in (0, \varphi^*]$ is required to be held by banks at the end of Period $t = 0$.

Then all banks choose the same level of money creation and lending, denoted by α_M^*. At the aggregate level, however, the amount borrowed by banks from the central bank has to equal the amount deposited by banks at the central bank, meaning that $\int_0^1 \alpha_M^b \mathrm{d}b = 1$, which translates into $\alpha_M^* = 1$. The result of Lemma 2 implies that we can apply Lemma 7. Thus, given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, and the equity ratio φ^*, all banks $b \in [0,1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M^{reg}\left((R_L^{s*})_s, (R_{CB}^s)_s, \varphi^*\right)$ as given in Lemma 7. Therefore, the only gross rates of return and capital structure φ^* in Lemma 7 in Appendix 2.F rationalizing $\alpha_M^* = 1$ are such that

$$\begin{aligned}
\text{either Case a)} \quad & (R_L^{s*} = R_{CB}^s \text{ for all states } s = l, h),\\
\text{or Case b)} \quad & (\overline{R}_L^* = \overline{R}_{CB},\ R_L^{l*} < R_{CB}^l,\ R_{CB}^h < R_L^{h*}, \text{ and } \alpha_{DH}^l \geq \tfrac{\varphi^*}{\varphi^{reg}}),\\
\text{or Case c)} \quad & (\overline{R}_L^* = \overline{R}_{CB},\ R_L^{h*} < R_{CB}^h,\ R_{CB}^l < R_L^{l*}, \text{ and } \alpha_{DH}^h \geq \tfrac{\varphi^*}{\varphi^{reg}}),\\
\text{or Case d)} \quad & (\overline{R}_L^* < \overline{R}_{CB},\ R_L^{l*} < R_{CB}^l,\ R_{CB}^h < R_L^{h*},\ \alpha_{DH}^l < 1,\\
& \text{and } \varphi^* = \varphi^{reg} = \tfrac{\sigma}{1-\sigma} \tfrac{R_L^{h*} - R_{CB}^h}{R_{CB}^l}),\\
\text{or Case e)} \quad & (\overline{R}_L^* < \overline{R}_{CB},\ R_L^{h*} < R_{CB}^h,\ R_{CB}^l < R_L^{l*},\ \alpha_{DH}^h < 1,\\
& \text{and } \varphi^* = \varphi^{reg} = \tfrac{1-\sigma}{\sigma} \tfrac{R_L^{l*} - R_{CB}^l}{R_{CB}^h}),
\end{aligned}$$

or Case f) ($R_L^{s*} \geq R_{CB}^s$ for all states $s = l, h$ with at least one strict inequality, and $\varphi^* = \varphi^{reg}$),

or Case g) ($\overline{R}_L^* = \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, $\alpha_{DH}^l < 1$, and $\varphi^* = \varphi^{reg}$),

or Case h) ($\overline{R}_L^* = \overline{R}_{CB}$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, $\alpha_{DH}^h < 1$, and $\varphi^* = \varphi^{reg}$),

or Case i) ($\overline{R}_L^* < \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, $\alpha_{DH}^l < 1$, and $\varphi^* = \varphi^{reg} < \frac{\sigma}{1-\sigma}\frac{R_L^{h*}-R_{CB}^h}{R_{CB}^l}$),

or Case j) ($\overline{R}_L^* < \overline{R}_{CB}$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, $\alpha_{DH}^h < 1$, and $\varphi^* = \varphi^{reg} < \frac{1-\sigma}{\sigma}\frac{R_L^{l*}-R_{CB}^l}{R_{CB}^h}$),

or Case k) ($\overline{R}_L^* > \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, and $\varphi^* = \varphi^{reg}$),

or Case l) ($\overline{R}_L^* > \overline{R}_{CB}$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, and $\varphi^* = \varphi^{reg}$).

Note first that for Cases f) to l), the expected gross rate of return on equity achieved by any Bank b when choosing $\alpha_M^b = 1$ is higher than the expected gross rate of return on equity when choosing $\alpha_M^b = 0$. Since the latter is equal to the expected deposit gross rate, we can conclude that the expected gross rate of return on equity is larger than the expected deposit gross rate in all cases f) to l). Moreover, for Cases a) to e), the expected gross rate of return on equity is equal to the expected deposit gross rate.

Given gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices $p_I^* = p_C^* = 1$, households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^* = 1, p_C^* = 1, S_F^*\big)$ given S_F^*,
$D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^* = 1, p_C^* = 1, E_B^*, S_F^*\big)$ given E_B^* and S_F^*, and
$S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^* = 1, p_C^* = 1\big)$, where these correspondences are given in Lemma 6 in Appendix 2.E.

For Cases f) to l), Lemma 6 implies that $D_H^* = 0$, which is excluded in the definition of an equilibrium with banks. Therefore, Cases f) to l) do not correspond to possible equilibria with banks.

For Cases a) to e), expected gross rates of return $\overline{R}_E^*$ and $\overline{R}_D^*$ are equal, and only the first, the fourth, and the seventh cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ in Appendix 2.E are consistent with $\overline{R}_E^* = \overline{R}_D^*$.

Suppose in the following that we are in Case a), b), or c).

– Suppose first that

$$\overline{R}_L^* > \mathbb{E}[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{0})], \tag{3.74}$$

which is the first case in Lemma 10. Then by Lemma 10, $\mathbf{K}_\mathbf{M}^* = 0$ and thus $\mathbf{K}_\mathbf{F}^* = \mathbf{W}$. This corresponds only to the first case in Lemma 6. By Lemma 6, $\overline{R}_{CB} \leq (\mathbf{f}_\mathbf{F})'(\mathbf{W})$. Together with Inequality (3.74), this inequality implies

$$\mathbb{E}[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{0})] < (\mathbf{f}_\mathbf{F})'(\mathbf{W}). \tag{3.75}$$

By our assumption that $\mathbb{E}\big[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{W})\big] > (\mathbf{f}_\mathbf{F})'(\mathbf{W})$ and by the concavity of $\mathbf{f}_\mathbf{M}^\mathbf{s}$ for all states $s = l, h$, we obtain $\mathbb{E}\big[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{0})\big] > \mathbb{E}\big[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{W})\big] > (\mathbf{f}_\mathbf{F})'(\mathbf{W})$, which contradicts Inequality (3.75).

– Suppose now that

$$\overline{R}_L < \mathbb{E}[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{W})], \tag{3.76}$$

which is the second case in Lemma 10. Then by Lemma 10, $\mathbf{K}_\mathbf{M}^* = \mathbf{W}$ and thus $\mathbf{K}_\mathbf{F}^* = \mathbf{0}$. This corresponds only to the fourth case in Lemma 6. By Lemma 6, $\mathbf{f}'(\mathbf{0}) < \overline{R}_{CB}$. We thus obtain

$$\mathbf{f}'(\mathbf{0}) < \mathbb{E}[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{W})]. \tag{3.77}$$

By our assumption that $\mathbb{E}\big[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{0})\big] < (\mathbf{f}_\mathbf{F})'(\mathbf{0})$ and by the concavity of $\mathbf{f}_\mathbf{M}^\mathbf{s}$ for all states $s = l, h$, we obtain $\mathbb{E}\big[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{W})\big] < \mathbb{E}\big[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{0})\big] < (\mathbf{f}_\mathbf{F})'(\mathbf{0})$, which contradicts Inequality (3.77).

– Therefore, in any equilibrium with banks, it must be that

$$\mathbb{E}[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{K}_\mathbf{M})] = \overline{R}_L^* \tag{3.78}$$

and

$$\mathbb{E}[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{W})] \leq \overline{R}_{CB} \leq \mathbb{E}[(\mathbf{f}_\mathbf{M}^\mathbf{s})'(\mathbf{0})].$$

Together with the assumptions $\mathbb{E}\big[(\mathbf{f_M^s})'(\mathbf{0})\big] < (\mathbf{f_F})'(\mathbf{0})$ and $\mathbb{E}\big[(\mathbf{f_M^s})'(\mathbf{W})\big] > (\mathbf{f_F})'(\mathbf{W})$, the previous inequalities imply that

$$(\mathbf{f_F})'(\mathbf{W}) < \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{W})] \leq \overline{R}_{CB} \leq \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{0})] < (\mathbf{f_F})'(\mathbf{0}),$$

which rules out the first and fourth cases in Lemma 6. From the remaining seventh case in Lemma 6 and Equation (3.78), we obtain

$$(\mathbf{f_F})'(\mathbf{K_F^*}) = \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{W} - \mathbf{K_F^*})].$$

We note that Proposition 7 shows that this equation has a unique solution, which corresponds to the first-best allocation. As in an equilibrium with banks $E_B^*, D_H^* > 0$, we thus obtain

$$E_B^* \in (0, p_I^* \left(\mathbf{W} - \mathbf{K_F^{FB}}\right)),$$
$$D_H^* = p_I^* \left(\mathbf{W} - \mathbf{K_F^{FB}}\right) - E_B^*, \text{ and}$$
$$S_F^* = \mathbf{K_F^{FB}}.$$

Suppose finally that we are in Case d) or e).

– Suppose first that

$$\overline{R}_L^* > \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{0})], \tag{3.79}$$

which is the first case in Lemma 10. Then by Lemma 10, $\mathbf{K_M^*} = 0$ and thus $\mathbf{K_F^*} = \mathbf{W}$. This corresponds only to the first case in Lemma 6. By Lemma 6, $\overline{R}_{CB} \leq (\mathbf{f_F})'(\mathbf{W})$. Together with Inequality (3.79) and $\overline{R}_{CB} > \overline{R}_L$, this inequality implies

$$\mathbb{E}[(\mathbf{f_M^s})'(\mathbf{0})] < (\mathbf{f_F})'(\mathbf{W}). \tag{3.80}$$

By our assumption that $\mathbb{E}\big[(\mathbf{f_M^s})'(\mathbf{W})\big] > (\mathbf{f_F})'(\mathbf{W})$ and by the concavity of $\mathbf{f_M^s}$ for all states $s = l, h$, we obtain $\mathbb{E}\big[(\mathbf{f_M^s})'(\mathbf{0})\big] > \mathbb{E}\big[(\mathbf{f_M^s})'(\mathbf{W})\big] > (\mathbf{f_F})'(\mathbf{W})$, which contradicts Inequality (3.80).

– Suppose now that

$$\overline{R}_L^* < \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{W})], \tag{3.81}$$

which is the second case in Lemma 10. Then by Lemma 10, $\mathbf{K_M^*} = \mathbf{W}$ and thus $\mathbf{K_F^*} = \mathbf{0}$. This only corresponds to the fourth case in Lemma 6. In this case, we obtain

$$(\mathbf{f_F})'(\mathbf{0}) < \overline{R}_{CB}.$$

As in an equilibrium with banks $E_B^*, D_H^* > 0$, we thus obtain

$$\begin{aligned} &E_B^* \in (0, p_I^* \mathbf{W}), \\ &D_H^* = p_I^* \mathbf{W} - E_B^*, \text{ and} \\ &S_F^* = 0. \end{aligned}$$

– Suppose now that

$$\mathbb{E}[(\mathbf{f_M^s})'(\mathbf{K_M^*})] = \overline{R}_L^*. \tag{3.82}$$

Then $\mathbf{0} < \mathbf{K_M^*} < \mathbf{W}$ and $(\mathbf{f_F})'(\mathbf{0}) \geq \overline{R}_{CB}^*$. Only the seventh case in Lemma 6 corresponds to such a constellation and in this case, we obtain

$$(\mathbf{f_F})'(\mathbf{K_F^*}) = \overline{R}_{CB} \quad \text{and} \quad \mathbb{E}[(\mathbf{f_M^s})'(\mathbf{W} - \mathbf{K_F^*})] = \overline{R}_L^*$$

and thus

$$\mathbb{E}\Big[(\mathbf{f_M^s})'\big(\mathbf{W} - (\mathbf{f_F})'^{-1}(\overline{R}_{CB})\big)\Big] = \overline{R}_L^*. \tag{3.83}$$

As long as $\overline{R}_{CB}$ and $\overline{R}_L^*$ fulfill (3.83), we obtain

$$\begin{aligned} &E_B^* \in (0, (\mathbf{W} - S_F^*)), \\ &D_H^* = (\mathbf{W} - S_F^*) - E_B^*, \\ &S_F^* = (\mathbf{f_F})'^{-1}\big(\overline{R}_{CB}\big). \end{aligned}$$

In turn, it is straightforward to verify that the tuples found in this proof constitute equilibria with banks as in Section 3.4. □

Proof of Proposition 10.
Let $b \in [0,1]$ denote a bank. As $R_D^s = R_{CB}^s$ in all states $s \in \mathcal{N}$, the expected shareholders' value of Bank b writes

$$\mathbb{E}\Big[\max\big(\alpha_M^b L_M (R_L^s - R_{CB}^s) + E_B R_D^s, 0\big)\Big].$$

We obtain the following cases:

– Suppose first that $\mathcal{N}_{\setminus 0} = \{\}$. In this case, $R_L^s = R_{CB}^s$ for all states $s \in \mathcal{N}$, and Bank b cannot influence its expected shareholders' value by varying its amount of loans. Therefore, $[0, +\infty)$ constitutes the set of Bank b's optimal choices.

– Suppose now that $\mathcal{N}_{\setminus 0} \neq \{\}$. More particularly, suppose that $S_{D+} = \{\}$. In this case, $R_L^s \geq R_{CB}^s$ for all states $s \in \mathcal{N}$ with at least one strict inequality, as $\mathcal{N}_{\setminus 0} \neq \{\}$, and Bank b can increase the expected shareholders' value by granting more loans. Thus, its choice is denoted by $\alpha_M^b = +\infty$.

– Suppose now that $S_{D+} \neq \{\}$. In this case, $S_-\left(\alpha_{DCB}^{s_D(1)}\right) \neq \{\}$, as $s_D(1) \in S_-\left(\alpha_{DCB}^{s_D(1)}\right)$, and the maximum possible amount of lending by banks under the constraint that they do not default against the central bank in any state of the world is given by $\alpha_{DCB}^{s_D(1)}$. The expected shareholders' value of Bank b is clearly convex and not constant in α_M^b. Therefore, its maximum is attained at either $\alpha_M^b = 0$ or $\alpha_M^b = \alpha_{DCB}^{s_D(1)}$. The expected shareholders' values of Bank b at $\alpha_M^b = 0$ and at $\alpha_M^b = \alpha_{DCB}^{s_D(1)}$ are given respectively by

$$E_B \overline{R}_{CB} \quad \text{and} \quad \alpha_{DCB}^{s_D(1)} L_M \sum_{s \in S_+\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s (R_L^s - R_{CB}^s) + E_B \sum_{s \in S_+\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s R_{CB}^s,$$

which leads to the threshold equity ratio φ

$$\alpha_{DCB}^{s_D(1)} \frac{\sum\limits_{s \in S_+\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s (R_L^s - R_{CB}^s)}{\sum\limits_{s \in S_-\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s R_{CB}^s},$$

below which Bank b chooses $\alpha_M^b = \alpha_{DCB}^{s_D(1)}$ and above which Bank b chooses $\alpha_M^b = 0$.

We can summarize the lending level choices of lending levels by banks, given gross rates $(R_L^s)_s$, policy choices $(R_{CB}^s)_s$, and their equity ratio φ, with the correspondence
$\hat{\alpha}_M\big((R_L^s)_s, (R_{CB}^s)_s, \varphi\big)$ given in the proposition. □

Proof of Lemma 11.
Let $b \in [0,1]$ denote a bank and assume that a minimum equity ratio $\varphi^{reg} \leq \varphi$ is imposed on banks at the end of Period $t = 0$. Using Lemma 5 and the property $R_D^s = R_{CB}^s$ for all states $s \in \mathcal{N}$, Bank b's maximization problem simplifies to

$$\max_{\alpha_M^b \in \left[0, \frac{\varphi}{\varphi^{reg}}\right]} \mathbb{E}\Big[\max\big(\alpha_M^b L_M (R_L^s - R_{CB}^s) + E_B R_{CB}^s, 0\big)\Big].$$

We obtain the following cases:

- Suppose first that $\mathcal{N}_{\setminus 0} = \{\}$. In this case, $R_L^s = R_{CB}^s$ for all states $s \in \mathcal{N}$ and Bank b cannot influence its expected shareholders' value by varying its amount of loans. Thus, $[0, \frac{\varphi}{\varphi^{reg}}]$ constitutes the set of Bank b's optimal choices.
- Suppose now that $\mathcal{N}_{\setminus 0} \neq \{\}$. More particularly, suppose that $S_{D+} = \{\}$. In this case, $R_L^s \geq R_{CB}^s$ for all states $s \in \mathcal{N}$ with at least one strict inequality, as $\mathcal{N}_{\setminus 0} \neq \{\}$, and Bank b can increase the expected shareholders' value by granting more loans. Therefore, its choice is denoted by $\alpha_M^b = \frac{\varphi}{\varphi^{reg}}$.
- Suppose now that $S_{D+} \neq \{\}$. In this case, $S_-\left(\alpha_{DCB}^{s_D(1)}\right) \neq \{\}$ as $s_D(1) \in S_-\left(\alpha_{DCB}^{s_D(1)}\right)$, and the maximum possible amount of lending by banks under

the constraint that they do not default against the central bank in any state of the world is given by $\max\left(\alpha_{DCB}^{s_D(1)}, \frac{\varphi}{\varphi^{reg}}\right)$. The expected shareholders' value of Bank b is clearly convex in α_M^b. Thus, if the expected shareholders' value of Bank b is not constant between $\alpha_M^b = 0$ and $\alpha_M^b = \max\left(\alpha_{DCB}^{s_D(1)}, \frac{\varphi}{\varphi^{reg}}\right)$, its maximum is attained at either $\alpha_M^b = 0$ or $\alpha_M^b = \max\left(\alpha_{DCB}^{s_D(1)}, \frac{\varphi}{\varphi^{reg}}\right)$. The expected shareholders' values of Bank b at $\alpha_M^b = 0$, at $\alpha_M^b = \alpha_{DCB}^{s_D(1)}$, and at $\alpha_M^b = \frac{\varphi}{\varphi^{reg}}$ are given respectively by

$$E_B\overline{R}_{CB}, \tag{3.84}$$

$$\alpha_{DCB}^{s_D(1)} L_M \sum_{s\in S_+\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s(R_L^s - R_{CB}^s) + E_B \sum_{s\in S_+\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s R_{CB}^s, \quad \text{and} \tag{3.85}$$

$$\frac{\varphi}{\varphi^{reg}} L_M \sum_{s\in S_+\left(\frac{\varphi}{\varphi^{reg}}\right)} \sigma^s(R_L^s - R_{CB}^s) + E_B \sum_{s\in S_+\left(\frac{\varphi}{\varphi^{reg}}\right)} \sigma^s R_{CB}^s. \tag{3.86}$$

– For now, we suppose that the expected shareholders' value of Bank b is not constant between $\alpha_M^b = 0$ and $\alpha_M^b = \max\left(\alpha_{DCB}^{s_D(1)}, \frac{\varphi}{\varphi^{reg}}\right)$. Suppose that $\frac{\varphi}{\varphi^{reg}} \leq \alpha_{DCB}^{s_D(1)}$. If $S_-\left(\frac{\varphi}{\varphi^{reg}}\right) \neq \{\}$, the comparison between the expected shareholders' value of Bank b expressed in (3.84) and the one in (3.86) leads to the threshold equity ratio φ^{reg}

$$\frac{\sum_{s\in S_+\left(\frac{\varphi}{\varphi^{reg}}\right)} \sigma^s(R_L^s - R_{CB}^s)}{\sum_{s\in S_-\left(\frac{\varphi}{\varphi^{reg}}\right)} \sigma^s R_{CB}^s}$$

below which Bank b chooses $\alpha_M^b = \frac{\varphi}{\varphi^{reg}}$ and above which Bank b chooses $\alpha_M^b = 0$. If $S_-\left(\frac{\varphi}{\varphi^{reg}}\right) = \{\}$, the value of the expression in (3.84) is strictly larger than the one in (3.86) if and only if $\overline{R}_L < \overline{R}_{CB}$. Suppose now that $\frac{\varphi}{\varphi^{reg}} > \alpha_{DCB}^{s_D(1)}$. If $S_-\left(\alpha_{DCB}^{s_D(1)}\right) \neq \{\}$, the comparison between the expected shareholders' value of Bank b expressed in (3.84) and the one in (3.85) leads to the threshold equity ratio φ

$$\alpha_{DCB}^{s_D(1)} \frac{\sum_{s \in S_+\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s (R_L^s - R_{CB}^s)}{\sum_{s \in S_-\left(\alpha_{DCB}^{s_D(1)}\right)} \sigma^s R_{CB}^s},$$

below which Bank b chooses $\alpha_M^b = \alpha_{DCB}^{s_D(1)}$ and above which Bank b chooses $\alpha_M^b = 0$. If $S_-\left(\alpha_{DCB}^{s_D(1)}\right) = \{\}$, the value of the expression in (3.84) is strictly larger than the one in (3.86) if and only if $\overline{R}_L < \overline{R}_{CB}$.

– Finally, we suppose that the expected shareholders' value of Bank b is constant between $\alpha_M^b = 0$ and $\alpha_M^b = \max\left(\alpha_{DCB}^{s_D(1)}, \frac{\varphi}{\varphi^{reg}}\right)$. We observe that this occurs if and only if $\frac{\varphi}{\varphi^{reg}} \leq \alpha_{DH}^{s_D(1)}$ and $\overline{R}_L = \overline{R}_{CB}$.

We can summarize the choices of lending levels by banks, given gross rates $(R_L^s)_s$, policy choices $(R_{CB}^s)_s$, and their equity ratio φ with the correspondence $\hat{\alpha}_M^{reg}$ given in the lemma. □

Proof of Proposition 11.

Let $\mathcal{E}^*$ be an equilibrium with banks for which a minimum equity ratio $\varphi^{reg} \in (0, \varphi^*]$ is required to be held by banks at the end of Period $t = 0$.

Then all banks choose the same level of money creation and lending, denoted by α_M^*. At the aggregate level, however, the amount borrowed by banks from the central bank has to equal the amount deposited by banks at the central bank, meaning that $\int_0^1 \alpha_M^b \mathrm{d}b = 1$, which translates into $\alpha_M^* = 1$. The result of Lemma 2 implies that we can apply Lemma 11. Thus, given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, and the equity ratio φ^*, all banks $b \in [0,1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M^{reg}\big((R_L^{s*})_s, (R_{CB}^s)_s, \varphi^*\big)$ as given in Lemma 11. Therefore, the only gross rates of return and capital structure φ^* in Lemma 11 rationalizing $\alpha_M^* = 1$

are such that

$$
\begin{array}{rl}
\text{either Case a)} & (R_L^{s*} \geq R_{CB}^s \text{ for all states } s \in \mathcal{N} \text{ with at least one strict} \\
& \text{inequality and } \varphi^* = \varphi^{reg}), \\
\text{or Case b)} & (\overline{R}_L^* > \overline{R}_{CB} \\
& \text{or } (\alpha_{DH}^{s_D(1)} < 1 \text{ and } \varphi^* = \varphi^{reg} < \dfrac{\sum\limits_{s \in S_+(1)} \sigma^s (R_L^{s*} - R_{CB}^s)}{\sum\limits_{s \in S_-(1)} \sigma^s R_{CB}^s})), \\
\text{or Case c)} & (R_L^{s*} = R_{CB}^s \text{ for all states } s \in \mathcal{N}), \\
\text{or Case d)} & (\overline{R}_L^* = \overline{R}_{CB} \text{ and } \alpha_{DH}^{s_D(1)} \geq 1), \\
\text{or Case e)} & (\alpha_{DH}^{s_D(1)} < 1 \\
& \text{and } \varphi^* = \varphi^{reg} = \dfrac{\sum\limits_{s \in S_+(1)} \sigma^s (R_L^{s*} - R_{CB}^s)}{\sum\limits_{s \in S_-(1)} \sigma^s R_{CB}^s}).
\end{array}
$$

Note first that for Cases a) and b), the expected gross rate of return on equity achieved by any Bank b when choosing $\alpha_M^b = 1$ is higher than the expected gross rate of return on equity when choosing $\alpha_M^b = 0$. Since the latter is equal to the expected deposit gross rate, we can conclude that the expected gross rate of return on equity is larger than the expected deposit gross rate for Cases a) and b). Moreover, for Cases c) to e), the expected gross rate of return on equity is equal to the expected deposit gross rate.

Given gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices $p_I^* = p_C^* = 1$, households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^* = 1, p_C^* = 1, S_F^*\big)$ given S_F^*, $D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^* = 1, p_C^* = 1, E_B^*, S_F^*\big)$ given E_B^* and S_F^*, and $S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^* = 1, p_C^* = 1\big)$, where these correspondences are given in Lemma 6 in Appendix 2.E.

For Cases a) and b), Lemma 6 implies that $D_H^* = 0$, which is excluded in the definition of an equilibrium with banks. Therefore, Cases a) and b) do not correspond to possible equilibria with banks.

For Cases c) to e), expected gross rates of return $\overline{R}_E^*$ and $\overline{R}_D^*$ are equal, and only the first, the fourth, and the seventh cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ in Appendix 2.E are consistent with $\overline{R}_E^* = \overline{R}_D^*$.

For Cases c) and d), the assumption $\mathbf{f}'(\mathbf{W}) < \overline{\mathbf{R}}_{\mathbf{M}} < \mathbf{f}'(\mathbf{0})$ together with $\overline{\mathbf{R}}_{\mathbf{M}} = \overline{R}_E^* = \overline{R}_D^*$ rule out the first and fourth cases. As in an equilibrium with banks $E_B^*, D_H^* > 0$, we obtain

$$\begin{aligned}
&E_B^* \in (0, \big(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_{\mathbf{M}})\big)),\\
&D_H^* = \big(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_{\mathbf{M}})\big) - E_B^*, \text{ and}\\
&S_F^* = \mathbf{f}'^{-1}(\overline{\mathbf{R}}_{\mathbf{M}}).
\end{aligned}$$

For Case e), the assumption $\mathbf{f}'(\mathbf{W}) < \overline{\mathbf{R}}_{\mathbf{M}}$ together with $\overline{\mathbf{R}}_{\mathbf{M}} < \overline{R}_E^* = \overline{R}_D^*$ rule out the first case. As in an equilibrium with banks, $E_B^*, D_H^* > 0$, we obtain

$$\begin{aligned}
&E_B^* \in (0, (\mathbf{W} - S_F^*)),\\
&D_H^* = (\mathbf{W} - S_F^*) - E_B^*,\\
&S_F^* = \begin{cases} \mathbf{f}'^{-1}\big(\overline{R}_{CB}^*\big) & \text{if } \mathbf{f}'(\mathbf{0}) \geq \overline{R}_{CB}^*,\\ 0 & \text{otherwise.} \end{cases}
\end{aligned}$$

In turn, it is straightforward to verify that the tuples found in this proof constitute equilibria with banks as in Section 3.5.

Now we assume that there exists some state of the world $s \in \mathcal{N}$ such that $\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} \neq R_{CB}^s$. Therefore, Case c) cannot hold. In the following, we prove that the remaining possible cases d) and e) are equivalent to (i) and (ii) in Proposition 11, respectively.

For Case d), we observe that $\alpha_{DH}^{s_D(1)} \geq \frac{\varphi^*}{\varphi^{reg}}$ is equivalent to $\varphi^* \geq \frac{1}{\alpha_{DCB}^{s_D(1)}}$.

For Case e), we first observe that

$$\varphi^* = \varphi^{reg} = \frac{\sum\limits_{s \in S_+(1)} \sigma^s(\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} - R_{CB}^s)}{\sum\limits_{s \in S_-(1)} \sigma^s R_{CB}^s} \tag{3.87}$$

is equivalent to

$$\sum_{s \in S_-(1)} \sigma^s\big((\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} - R_{CB}^s) + \varphi R_{CB}^s\big) = \overline{\mathbf{R}}_{\mathbf{M}} - \overline{R}_{CB}.$$

Now we note that by definition of $S_{-}(1)$,

$$s \in S_{-}(1) \iff (\mathbf{R}^{\mathbf{s}}_{\mathbf{M}} - R^{s}_{CB}) + \varphi R^{s}_{CB} < 0.$$

Therefore, Equation (3.87) implies $\overline{\mathbf{R}}_{\mathbf{M}} - \overline{R}_{CB} < 0$. $\square$

Proof of Corollary 11.

Corollary 11 is an immediate consequence of Corollary 10. We will just prove that the example given by

$$\begin{aligned} &R^{s}_{CB} = 1 \quad \text{for all states } s \in \{1, 2, \ldots, s_z\}, \quad \text{and} \\ &R^{s}_{CB} = \mathbf{R}^{\mathbf{s}}_{\mathbf{M}} - \frac{\Delta_z}{\sigma^s} \quad \text{for all states } s \in \{s_z + 1, s_z + 2, \ldots, N\}, \end{aligned}$$

where

$$\Delta_z = \frac{\sum_{s=1}^{s_z} \sigma^s (1 - \mathbf{R}^{\mathbf{s}}_{\mathbf{M}})}{N - s_z},$$

fulfills the conditions for a first-best allocation. First, we can calculate

$$\begin{aligned} \overline{R}_{CB} &= \sum_{s=1}^{s_z} \sigma^s + \sum_{s=s_z+1}^{N} \sigma^s \mathbf{R}^{\mathbf{s}}_{\mathbf{M}} - \Delta_z (N - s_z) \\ &= \sum_{s=1}^{N} \sigma^s \mathbf{R}^{\mathbf{s}}_{\mathbf{M}} \\ &= \mathbb{E}[\mathbf{R}^{\mathbf{s}}_{\mathbf{M}}]. \end{aligned}$$

Finally, we note that for $s < s_z$,

$$\alpha^{s}_{DCB} = \frac{1}{1 - \mathbf{R}^{\mathbf{s}}_{\mathbf{M}}},$$

which is increasing in s. From this, we can derive the regulatory capital requirement constraint given in (ii). $\square$

Proof of Corollary 12.

Suppose that there is an equilibrium with banks denoted by $\mathcal{E}^*$ for which the deposit gross rate of return in real terms is independent of the state of the world. We denote by $\mathbf{R_D^*}$ the deposit gross rate of return in terms of the consumption good:

$$\mathbf{R_D^*} = \frac{R_D^{l*}}{p_C^{l*}} = \frac{R_D^{h*}}{p_C^{h*}}.$$

Theorem 1 implies that

$$\mathbf{R_D^*} = \frac{\mathbf{R_M^s}}{p_I^*},$$

in all states $s = l, h$, which is a contradiction to $\mathbf{R_M^l} < \mathbf{R_M^h}$. □

Proof of Proposition 13.

Let $\mathcal{E}^*$ be an equilibrium with banks for which a minimum equity ratio $\varphi^{reg} \in (0, \varphi^*]$ is required to be held by banks at the end of Period $t = 0$.

Then all banks choose the same level of money creation and lending, denoted by α_M^*. At the aggregate level, however, the amount borrowed by banks from the central bank has to equal the amount deposited by banks at the central bank, meaning that $\int_0^1 \alpha_M^b \mathrm{d}b = 1$, which translates into $\alpha_M^* = 1$. The result of Lemma 2 implies that we can apply Lemma 7. Thus, given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, and the equity ratio φ^*, all banks $b \in [0,1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M^{reg}\big((R_L^{s*})_s, (R_{CB}^s)_s, \varphi^*\big)$ as given in Lemma 7. Therefore, the only gross rates of return and capital structure φ^* in Lemma 7 in Appendix 2.F rationalizing $\alpha_M^* = 1$ are such that

either Case a) $(R_L^{s*} = R_{CB}^s$ for all states $s = l, h)$,
or Case b) $(\overline{R}_L^* = \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, and $\alpha_{DH}^l \geq \frac{\varphi^*}{\varphi^{reg}})$,
or Case c) $(\overline{R}_L^* = \overline{R}_{CB}$, $R_L^{h*} < R_{CB}^h$, $R_{CB}^l < R_L^{l*}$, and $\alpha_{DH}^h \geq \frac{\varphi^*}{\varphi^{reg}})$,
or Case d) $(\overline{R}_L^* < \overline{R}_{CB}$, $R_L^{l*} < R_{CB}^l$, $R_{CB}^h < R_L^{h*}$, $\alpha_{DH}^l < 1$,
and $\varphi^* = \varphi^{reg} = \frac{\sigma}{1-\sigma} \frac{R_L^{h*} - R_{CB}^h}{R_{CB}^l})$,

or Case e) $(\overline{R}_L^* < \overline{R}_{CB},\ R_L^{h*} < R_{CB}^h,\ R_{CB}^l < R_L^{l*},\ \alpha_{DH}^h < 1,$
and $\varphi^* = \varphi^{reg} = \frac{1-\sigma}{\sigma}\frac{R_L^{l*}-R_{CB}^l}{R_{CB}^h}),$

or Case f) $(R_L^{s*} \geq R_{CB}^s$ for all states $s = l, h$ with at least one strict inequality, and $\varphi^* = \varphi^{reg}),$

or Case g) $(\overline{R}_L^* = \overline{R}_{CB},\ R_L^{l*} < R_{CB}^l,\ R_{CB}^h < R_L^{h*},\ \alpha_{DH}^l < 1,$ and $\varphi^* = \varphi^{reg}),$

or Case h) $(\overline{R}_L^* = \overline{R}_{CB},\ R_L^{h*} < R_{CB}^h,\ R_{CB}^l < R_L^{l*},\ \alpha_{DH}^h < 1,$ and $\varphi^* = \varphi^{reg}),$

or Case i) $(\overline{R}_L^* < \overline{R}_{CB},\ R_L^{l*} < R_{CB}^l,\ R_{CB}^h < R_L^{h*},\ \alpha_{DH}^l < 1,$
and $\varphi^* = \varphi^{reg} < \frac{\sigma}{1-\sigma}\frac{R_L^{h*}-R_{CB}^h}{R_{CB}^l}),$

or Case j) $(\overline{R}_L^* < \overline{R}_{CB},\ R_L^{h*} < R_{CB}^h,\ R_{CB}^l < R_L^{l*},\ \alpha_{DH}^h < 1,$
and $\varphi^* = \varphi^{reg} < \frac{1-\sigma}{\sigma}\frac{R_L^{l*}-R_{CB}^l}{R_{CB}^h}),$

or Case k) $(\overline{R}_L^* > \overline{R}_{CB},\ R_L^{l*} < R_{CB}^l,\ R_{CB}^h < R_L^{h*},$ and $\varphi^* = \varphi^{reg}),$

or Case l) $(\overline{R}_L^* > \overline{R}_{CB},\ R_L^{h*} < R_{CB}^h,\ R_{CB}^l < R_L^{l*},$ and $\varphi^* = \varphi^{reg}).$

Note first that for Cases f) to l), the expected gross rate of return on equity achieved by any Bank b when choosing $\alpha_M^b = 1$ is higher than the expected gross rate of return on equity when choosing $\alpha_M^b = 0$. Since the latter is equal to the expected deposit gross rate, we can conclude that the expected gross rate of return on equity is larger than the expected deposit gross rate for all cases f) to l). Moreover, for Cases a) to e), the expected gross rate of return on equity is equal to the expected deposit gross rate.

Given gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices p_I^* and $(p_C^{s*})_s$, households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, S_F^*\big)$ given S_F^*,
$D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, E_B^*, S_F^*\big)$ given E_B^* and S_F^*,
and $S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s\big)$, where these correspondences are given in Lemma 6 in Appendix 2.E. Only the fourth and the seventh cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ in Appendix 2.E are consistent with D_H^*, $E_B^* > 0$. Therefore,

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \mathbf{R}_\mathbf{D}^*. \tag{3.88}$$

For Case a), $R_L^{s*} = R_D^{s*}$ for all states $s = l, h$. Thus,

$$\frac{R_L^{s*}}{p_C^{s*}} = \frac{\mathbf{R_M^s}}{p_I^*} = \frac{R_D^{s*}}{p_C^{s*}} = \mathbf{R_D^*},$$

which cannot hold as $\mathbf{R_M^l} < \mathbf{R_M^h}$.

For Cases b) and c), the gross rate of return on equity is given by

$$R_E^{s*} = \frac{R_L^{s*} - R_{CB}^s}{\varphi^*} + R_{CB}^s.$$

Using $\frac{R_{CB}^s}{p_C^{s*}} = \mathbf{R_D^*}$ and $p_I^* R_L^{s*} = p_C^{s*}\mathbf{R_M^s}$, we obtain

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \mathbf{R_D^*} + \frac{\overline{\mathbf{R}}_\mathbf{M}/p_I^* - \mathbf{R_D^*}}{\varphi^*}.$$

From Equation (3.88), we obtain $\mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right] = \mathbf{R_D^*} = \frac{\overline{\mathbf{R}}_\mathbf{M}}{p_I^*}$. We note that this allocation is efficient. Moreover, as $\mathbf{R_D^*} = \frac{\overline{\mathbf{R}}_\mathbf{M}}{p_I^*}$, then $\frac{\mathbf{R_M^l}}{p_I^*} < \mathbf{R_D^*} < \frac{\mathbf{R_M^h}}{p_I^*}$ and

$$\frac{R_L^{l*}}{p_C^{l*}} = \frac{\mathbf{R_M^l}}{p_I^*} < \frac{R_{CB}^l}{p_C^{l*}} = \mathbf{R_D^*} = \frac{R_{CB}^h}{p_C^{h*}} < \frac{\mathbf{R_M^h}}{p_I^*} = \frac{R_L^{h*}}{p_C^{h*}}.$$

From these inequalities, we obtain

$$R_L^{l*} < R_{CB}^l \quad \text{and} \quad R_{CB}^h < R_L^{h*},$$

which excludes Case c). Finally, the condition $\alpha_{DH}^l \geq \frac{\varphi^*}{\varphi^{reg}}$ is equivalent to

$$\varphi^{reg} \geq \frac{\mathbf{R_D^*} - \mathbf{R_M^l}/p_I^*}{\mathbf{R_D^*}}.$$

For Case d), the gross rate of return on equity is given by

$$\frac{R_E^{l*}}{p_C^{l*}} = 0 \quad \text{and} \quad \frac{R_E^{h*}}{p_C^{h*}} = \frac{\mathbf{R_M^h}/p_I^* - \mathbf{R_D^*}}{\varphi^*} + \mathbf{R_D^*}.$$

We can thus write the expected gross rate of return on equity as

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \sigma\left(\frac{\mathbf{R_M^h}/p_I^* - \mathbf{R_D^*}}{\varphi^*} + \mathbf{R_D^*}\right),$$

which is equal to $\mathbf{R_D^*}$ according to Equation (3.88). This equality implies that

$$p_C^{h*} = p_C^{l*},$$

where we used that $(1-\sigma)\frac{R_{CB}^l}{p_C^{h*}} = \frac{\sigma}{\varphi^*}\left(\frac{\mathbf{R_M^h}}{p_I^*} - \mathbf{R_D^*}\right)$. We denote the price of the consumption good by p_C^* in both states. Thus,

$$\mathbf{R_D^*} p_C^* = R_{CB}^h = R_{CB}^l.$$

Moreover, dividing both sides of the inequality $\overline{R}_L^* < \overline{R}_{CB}$ by p_C^* yields

$$\frac{\overline{\mathbf{R}}_\mathbf{M}}{p_I^*} < \mathbf{R_D^*} = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right].$$

We note that this allocation is inefficient. We can easily verify that $\alpha_{DH}^l < 1$.

For Case e), similar derivations as in Case d) yield

$$p_C^{h*} = p_C^{l*} = p_C^*.$$

Moreover, $\frac{\overline{\mathbf{R}}_\mathbf{M}}{p_I^*} < \mathbf{R_D^*}$ implies $\frac{\mathbf{R_M^l}}{p_I^*} < \mathbf{R_D^*}$ and

$$\frac{R_L^{l*}}{p_C^*} = \frac{\mathbf{R_M^l}}{p_I^*} < \frac{R_{CB}^l}{p_C^*} = \mathbf{R_D^*}.$$

From these inequalities, we obtain

$$R_L^{l*} < R_{CB}^l,$$

which excludes Case e).

For Case f), the inequalities $R_L^{s*} \geq R_{CB}^s$ for all states $s = l, h$ imply that

$$\frac{\overline{\mathbf{R}}_{\mathbf{M}}}{p_I^*} > \mathbf{R}_{\mathbf{D}}^*. \tag{3.89}$$

Moreover, combining to Equation (3.88) with

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \mathbf{R}_{\mathbf{D}}^* + \frac{\overline{\mathbf{R}}_{\mathbf{M}}/p_I^* - \mathbf{R}_{\mathbf{D}}^*}{\varphi^*},$$

we obtain $\frac{\overline{\mathbf{R}}_{\mathbf{M}}}{p_I^*} = \mathbf{R}_{\mathbf{D}}^*$, which is in contradiction with Equation (3.89).

For Case g), the gross rates of return on equity are given by

$$\frac{R_E^{l*}}{p_C^{l*}} = 0 \quad \text{and} \quad \frac{R_E^{h*}}{p_C^{h*}} = \frac{\mathbf{R}_{\mathbf{M}}^{\mathbf{h}}/p_I^* - \mathbf{R}_{\mathbf{D}}^*}{\varphi^*} + \mathbf{R}_{\mathbf{D}}^*.$$

We can thus write the expected gross rate of return on equity as

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \sigma\left(\frac{\mathbf{R}_{\mathbf{M}}^{\mathbf{h}}/p_I^* - \mathbf{R}_{\mathbf{D}}^*}{\varphi^*} + \mathbf{R}_{\mathbf{D}}^*\right),$$

which is equal to $\mathbf{R}_{\mathbf{D}}^*$ according to Equation (3.88). This implies that

$$\varphi^* = \frac{\sigma}{1-\sigma}\frac{\mathbf{R}_{\mathbf{M}}^{\mathbf{h}}/p_I^* - \mathbf{R}_{\mathbf{D}}^*}{\mathbf{R}_{\mathbf{D}}^*}. \tag{3.90}$$

By inserting Equation (3.90) into the inequality $\varphi^* < 1$, we obtain

$$\frac{\mathbf{R}_{\mathbf{M}}^{\mathbf{h}}}{p_I^*} < \frac{\mathbf{R}_{\mathbf{D}}^*}{\sigma}.$$

Inserting the expression of φ^* in Equation (3.90) into the equality $\alpha_{DH}^l = \varphi^* \frac{R_{CB}^l}{R_{CB}^l - R_L^{l*}}$ and using $\overline{R}_L^* = \overline{R}_{CB}$ yields

$$\alpha_{DH}^l = \frac{\sigma}{1-\sigma}\frac{\mathbf{R}_{\mathbf{M}}^{\mathbf{h}}/p_I^* - \mathbf{R}_{\mathbf{D}}^*}{\mathbf{R}_{\mathbf{D}}^* - \mathbf{R}_{\mathbf{M}}^{\mathbf{l}}/p_I^*} = \frac{p_C^{l*}}{p_C^{h*}} = \frac{R_{CB}^l}{R_{CB}^h}.$$

Therefore, the inequality $\alpha^l_{DH} < 1$ implies that

$$\mathbf{R_D^*} > \overline{\mathbf{R}}_\mathbf{M}/p_I^*, \quad p_C^{l*} < p_C^{h*}, \quad \text{and} \quad R^l_{CB} < R^h_{CB}.$$

We note that this allocation is inefficient, as $\mathbf{R_D^*} > \overline{\mathbf{R}}_\mathbf{M}/p_I^*$.

For Case h), similar derivations as in Case g) give $R^h_{CB} < R^l_{CB}$ and thus

$$\frac{R_L^{h*}}{p_C^{h*}} = \frac{\mathbf{R_M^h}}{p_I^*} < \frac{R^h_{CB}}{p_C^{h*}} = \mathbf{R_D^*} = \frac{R^l_{CB}}{p_C^{l*}} < \frac{\mathbf{R_M^l}}{p_I^*} = \frac{R_L^{l*}}{p_C^{l*}},$$

which contradicts $\mathbf{R_M^l} < \mathbf{R_M^h}$.

For Case i), the gross rates of return on equity are given by

$$\frac{R_E^{l*}}{p_C^{l*}} = 0 \quad \text{and} \quad \frac{R_E^{h*}}{p_C^{h*}} = \frac{\mathbf{R_M^h}/p_I^* - \mathbf{R_D^*}}{\varphi^*} + \mathbf{R_D^*}.$$

We can thus write the expected gross rate of return on equity as

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \sigma\left(\frac{\mathbf{R_M^h}/p_I^* - \mathbf{R_D^*}}{\varphi^*} + \mathbf{R_D^*}\right),$$

which is equal to $\mathbf{R_D^*}$ according to Equation (3.88). This implies that

$$\varphi^* = \frac{\sigma}{1-\sigma}\frac{\mathbf{R_M^h}/p_I^* - \mathbf{R_D^*}}{\mathbf{R_D^*}}. \tag{3.91}$$

By inserting Equation (3.91) into the inequality $\varphi^* < 1$, we obtain

$$\frac{\mathbf{R_M^h}}{p_I^*} < \frac{\mathbf{R_D^*}}{\sigma}.$$

Inserting the expression of φ^* in Equation (3.91) into the equality $\alpha^l_{DH} = \varphi^* \frac{R^l_{CB}}{R^l_{CB} - R_L^{l*}}$ and using $\overline{R}_L^* < \overline{R}_{CB}$ yields

$$\alpha^l_{DH} = \frac{\sigma}{1-\sigma}\frac{\mathbf{R_M^h}/p_I^* - \mathbf{R_D^*}}{\mathbf{R_D^*} - \mathbf{R_M^l}/p_I^*} < \frac{p_C^{l*}}{p_C^{h*}} = \frac{R^l_{CB}}{R^h_{CB}}.$$

Inserting the expression of φ^* in Equation (3.91) into the inequality

$\varphi^* < \frac{\sigma}{1-\sigma}\frac{R_L^{h*}-R_{CB}^h}{R_{CB}^l}$ yields

$$p_C^{l*} < p_C^{h*} \quad \text{and} \quad R_{CB}^l < R_{CB}^h.$$

Moreover, the inequality $\alpha_{DH}^l < 1$ implies that

$$\mathbf{R}_\mathbf{D}^* > \overline{\mathbf{R}}_\mathbf{M}/p_I^*.$$

We note that this allocation is inefficient.

For Case j), similar derivations as in Case i) yield

$$\frac{R_L^{h*}}{p_C^{h*}} = \frac{\mathbf{R}_\mathbf{M}^\mathbf{h}}{p_I^*} < \frac{R_{CB}^h}{p_C^{h*}} = \mathbf{R}_\mathbf{D}^* = \frac{R_{CB}^l}{p_C^{l*}} < \frac{\mathbf{R}_\mathbf{M}^\mathbf{l}}{p_I^*} = \frac{R_L^{l*}}{p_C^{l*}},$$

which contradicts $\mathbf{R}_\mathbf{M}^\mathbf{l} < \mathbf{R}_\mathbf{M}^\mathbf{h}$.

For Cases k), suppose first that $\alpha_{DH}^l \geq 1$, i.e. that banks do not default. Then the gross rate of return on equity is given by

$$R_E^{s*} = \frac{R_L^{s*} - R_{CB}^s}{\varphi^*} + R_{CB}^s.$$

Using $\frac{R_{CB}^s}{p_C^{s*}} = \mathbf{R}_\mathbf{D}^*$ and $p_I^* R_L^{s*} = p_C^{s*}\mathbf{R}_\mathbf{M}^\mathbf{s}$, we obtain

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \mathbf{R}_\mathbf{D}^* + \frac{\overline{\mathbf{R}}_\mathbf{M}/p_I^* - \mathbf{R}_\mathbf{D}^*}{\varphi^*}.$$

From Equation (3.88), we obtain $\mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right] = \mathbf{R}_\mathbf{D}^* = \frac{\overline{\mathbf{R}}_\mathbf{M}}{p_I^*}$. We note that this allocation is efficient. Moreover, as $\mathbf{R}_\mathbf{D}^* = \frac{\overline{\mathbf{R}}_\mathbf{M}}{p_I^*}$, then $\frac{\mathbf{R}_\mathbf{M}^\mathbf{l}}{p_I^*} < \mathbf{R}_\mathbf{D}^* < \frac{\mathbf{R}_\mathbf{M}^\mathbf{h}}{p_I^*}$ and

$$\frac{R_L^{l*}}{p_C^{l*}} = \frac{\mathbf{R}_\mathbf{M}^\mathbf{l}}{p_I^*} < \frac{R_{CB}^l}{p_C^{l*}} = \mathbf{R}_\mathbf{D}^* = \frac{R_{CB}^h}{p_C^{h*}} < \frac{\mathbf{R}_\mathbf{M}^\mathbf{h}}{p_I^*} = \frac{R_L^{h*}}{p_C^{h*}}.$$

From these inequalities, we obtain

$$R_L^{l*} < R_{CB}^l \quad \text{and} \quad R_{CB}^h < R_L^{h*}.$$

Case l) is excluded when $\alpha_{DH}^{h} \geq 1$, as the same previous inequalities can then also be derived and contradicts those of Case l). The condition $\alpha_{DH}^{l} \geq 1$ is equivalent to

$$\varphi^{reg} \geq \frac{\mathbf{R_D^*} - \mathbf{R_M^l}/p_I^*}{\mathbf{R_D^*}}.$$

Finally, the condition $\overline{R}_L^* > \overline{R}_{CB}$ translates into

$$\frac{R_{CB}^l}{R_{CB}^h} = \frac{p_C^{l*}}{p_C^{h*}} < \frac{\sigma}{1-\sigma}\frac{\mathbf{R_M^h}/p_I^* - \mathbf{R_D^*}}{\mathbf{R_D^*} - \mathbf{R_M^l}/p_I^*}.$$

Suppose now that $\alpha_{DH}^{l} < 1$, i.e. that banks default in the bad state. Then the gross rates of return on equity are given by

$$\frac{R_E^{l*}}{p_C^{l*}} = 0 \quad \text{and} \quad \frac{R_E^{h*}}{p_C^{h*}} = \frac{\mathbf{R_M^h}/p_I^* - \mathbf{R_D^*}}{\varphi^*} + \mathbf{R_D^*}.$$

We can thus write the expected gross rate of return on equity as

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^s}\right] = \sigma\left(\frac{\mathbf{R_M^h}/p_I^* - \mathbf{R_D^*}}{\varphi^*} + \mathbf{R_D^*}\right),$$

which is equal to $\mathbf{R_D^*}$ according to Equation (3.88). This implies that

$$\varphi^* = \frac{\sigma}{1-\sigma}\frac{\mathbf{R_M^h}/p_I^* - \mathbf{R_D^*}}{\mathbf{R_D^*}}. \tag{3.92}$$

By inserting Equation (3.92) into the inequality $\varphi^* < 1$, we obtain

$$\frac{\mathbf{R_M^h}}{p_I^*} < \frac{\mathbf{R_D^*}}{\sigma}.$$

Inserting the expression of φ^* in Equation (3.92) into the equality $\alpha_{DH}^l = \varphi^* \frac{R_{CB}^l}{R_{CB}^l - R_L^{l*}}$ and using $\overline{R}_L^* > \overline{R}_{CB}$ yields

$$\alpha_{DH}^l = \frac{\sigma}{1-\sigma}\frac{\mathbf{R_M^h}/p_I^* - \mathbf{R_D^*}}{\mathbf{R_D^*} - \mathbf{R_M^l}/p_I^*} > \frac{p_C^{l*}}{p_C^{h*}} = \frac{R_{CB}^l}{R_{CB}^h}.$$

Therefore, the inequality $\alpha^l_{DH} < 1$ implies that

$$p^{l*}_C < p^{h*}_C, \quad R^l_{CB} < R^h_{CB}, \quad \text{and} \quad \mathbf{R}^*_{\mathbf{D}} > \overline{\mathbf{R}}_{\mathbf{M}}/p^*_I.$$

We note that Case l) is again excluded by $p^{l*}_C < p^{h*}_C$ and that the allocation is inefficient.

We observe that all efficient equilibria fulfills either

$$(P^1_{FB}) : \quad \begin{cases} \dfrac{\overline{\mathbf{R}}_{\mathbf{M}}}{p^*_I} &= \mathbf{R}^*_{\mathbf{D}}, \\ \varphi^* \geq \varphi^{reg} &\geq \dfrac{\mathbf{R}^*_{\mathbf{D}} - \mathbf{R}^{\mathbf{l}}_{\mathbf{M}}/p^*_I}{\mathbf{R}^*_{\mathbf{D}}}, \\ R^l_{CB} &= R^h_{CB}, \\ \mathbf{R}^*_{\mathbf{D}} &= \dfrac{R^l_{CB}}{p^{l*}_C} = \dfrac{R^h_{CB}}{p^{h*}_C}, \end{cases}$$

or

$$(P^2_{FB}) : \quad \begin{cases} \dfrac{\overline{\mathbf{R}}_{\mathbf{M}}}{p^*_I} &= \mathbf{R}^*_{\mathbf{D}}, \\ \varphi^{reg} = \varphi^* &\geq \dfrac{\mathbf{R}^*_{\mathbf{D}} - \mathbf{R}^{\mathbf{l}}_{\mathbf{M}}/p^*_I}{\mathbf{R}^*_{\mathbf{D}}}, \\ R^l_{CB} &< R^h_{CB}, \\ \mathbf{R}^*_{\mathbf{D}} &= \dfrac{R^l_{CB}}{p^{l*}_C} = \dfrac{R^h_{CB}}{p^{h*}_C}. \end{cases}$$

Moreover, all inefficient equilibria fulfills

$$(P_I) : \quad \begin{cases} \dfrac{\overline{\mathbf{R}}_{\mathbf{M}}}{p^*_I} &< \mathbf{R}^*_{\mathbf{D}}, \\ \varphi^{reg} = \varphi^* &= \dfrac{\sigma}{1-\sigma}\dfrac{\mathbf{R}^{\mathbf{h}}_{\mathbf{M}}/p^*_I - \mathbf{R}^*_{\mathbf{D}}}{\mathbf{R}^*_{\mathbf{D}}} > 0, \\ R^l_{CB} &\leq R^h_{CB}, \\ \mathbf{R}^*_{\mathbf{D}} &= \dfrac{R^l_{CB}}{p^{l*}_C} = \dfrac{R^h_{CB}}{p^{h*}_C}. \end{cases}$$

Reciprocally, we first prove that the tuples described in (P^1_{FB}) and (P^2_{FB}) constitute efficient equilibria with a non-contingent real gross rate of deposit as in Section 3.6.

– Suppose that a tuple $\mathcal{E}$ verifies (P^1_{FB}). We deduce from the first equation

$$\frac{R^{l*}_L}{p^{l*}_C} = \frac{\mathbf{R^l_M}}{p^*_I} < \frac{R^l_{CB}}{p^{l*}_C} = \mathbf{R^*_D} = \frac{R^h_{CB}}{p^{h*}_C} < \frac{\mathbf{R^h_M}}{p^*_I} = \frac{R^{h*}_L}{p^{h*}_C}$$

and thus $R^{l*}_L < R^l_{CB} = R^h_{CB} < R^{h*}_L$. Using the fourth equation of (P^1_{FB}), we obtain $p^{l*}_C = p^{h*}_C$. The first equation of (P^1_{FB}) then implies that $\overline{R}^*_L = \overline{R}_{CB}$ and the second equation that $\alpha^l_{DH} \geq \frac{\varphi^*}{\varphi^{reg}}$, which means that banks do not default. Using the fact that banks do not default as well as the first equation, we obtain

$$\mathbb{E}\left[\frac{R^{s*}_E}{p^{s*}_C}\right] = \mathbf{R^*_D}.$$

This tuple is the same as the tuple in Case b), and it constitutes an equilibrium with banks, with a non-contingent real gross deposit rate of return.

– Suppose now that the tuple $\mathcal{E}$ verifies (P^2_{FB}). We deduce from the first equation that

$$\frac{R^{l*}_L}{p^{l*}_C} = \frac{\mathbf{R^l_M}}{p^*_I} < \frac{R^l_{CB}}{p^{l*}_C} = \mathbf{R^*_D} = \frac{R^h_{CB}}{p^{h*}_C} < \frac{\mathbf{R^h_M}}{p^*_I} = \frac{R^{h*}_L}{p^{h*}_C}$$

and thus $R^{l*}_L < R^l_{CB} < R^h_{CB} < R^{h*}_L$. Using the fourth equation of (P^2_{FB}), we obtain $p^{l*}_C < p^{h*}_C$. The first equation of (P^2_{FB}) then implies that

$$\frac{p^{l*}_C}{p^{h*}_C} = \frac{1-\sigma}{\sigma}\frac{R^l_{CB} - R^{l*}_L}{R^{h*}_L - R^h_{CB}}.$$

From $p^{l*}_C < p^{h*}_C$ and the previous equation, we derive $\overline{R}^*_L > \overline{R}_{CB}$. Moreover, the second equation implies that $\alpha^l_{DH} \geq \frac{\varphi^*}{\varphi^{reg}}$, which means that banks do not default. Using the fact that banks do not default as well as the first equation, we obtain

$$\mathbb{E}\left[\frac{R^{s*}_E}{p^{s*}_C}\right] = \mathbf{R^*_D}.$$

This tuple is the same as the tuple in Case k), and it constitutes an equilibrium with banks, with a non-contingent real gross deposit rate of return.

– Suppose now that a tuple $\mathcal{E}$ verifies (P_I). We deduce from the first two inequalities

$$\frac{R_L^{l*}}{p_C^{l*}} = \frac{\mathbf{R_M^l}}{p_I^*} < \frac{R_{CB}^l}{p_C^{l*}} = \mathbf{R_D^*} = \frac{R_{CB}^h}{p_C^{h*}} < \frac{\mathbf{R_M^h}}{p_I^*} = \frac{R_L^{h*}}{p_C^{h*}},$$

and thus $R_L^{l*} < R_{CB}^l \leq R_{CB}^h < R_L^{h*}$. Using the fourth equation of (P_I), we obtain $p_C^{l*} \leq p_C^{h*}$. The second equation of (P_I) implies that

$$\alpha_{DH}^l = \frac{\sigma}{1-\sigma}\frac{\mathbf{R_M^h}/p_I^* - \mathbf{R_D^*}}{\mathbf{R_D^*} - \mathbf{R_M^l}/p_I^*} < \frac{\varphi^*}{\varphi^{reg}} = 1,$$

which means that banks default in the bad state. Using this fact as well as the second equation, we obtain

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \mathbf{R_D^*}.$$

We now differentiate the two cases where $R_{CB}^l = R_{CB}^h$ and $R_{CB}^l < R_{CB}^h$:

– Suppose first that $R_{CB}^l = R_{CB}^h$. The fourth equation of (P_I) implies that $p_C^{l*} = p_C^{h*}$. As a consequence, $\mathbf{R_M^h}/p_I^* < \mathbf{R_D^*}$ implies that $\overline{R}_L^* < \overline{R}_{CB}$. Finally, the second equation of (P_I) implies that $\varphi^* = \varphi^{reg} = \frac{\sigma}{1-\sigma}\frac{R_L^{h*}-R_{CB}^h}{R_{CB}^l}$. This tuple is the same as the tuple in Case d), and it constitutes an equilibrium with banks, with a non-contingent real gross deposit rate of return.

– Suppose now that $R_{CB}^l < R_{CB}^h$. The second equation of (P_I) then implies that

$$\begin{aligned}\varphi^* = \varphi^{reg} &= \frac{\sigma}{1-\sigma}\frac{\mathbf{R_M^h}/p_I^* - \mathbf{R_D^*}}{\mathbf{R_D^*}} \\ &= \frac{\sigma}{1-\sigma}\frac{R_L^{h*} - R_{CB}^h}{R_L^{l*}}\frac{p_C^{l*}}{p_C^{h*}} < \frac{\sigma}{1-\sigma}\frac{R_L^{h*} - R_{CB}^h}{R_L^{l*}}.\end{aligned}$$

This tuple covers Cases g), i), and k), and it constitutes an equilibrium with banks, with a non-contingent real deposit rate of return.

We finally observe that

$$\frac{\sigma}{1-\sigma}\frac{\mathbf{R_M^h}/p_I^* - \mathbf{R_D^*}}{\mathbf{R_D^*}} < \frac{\overline{\mathbf{R}}_\mathbf{M} - \mathbf{R_M^l}}{\overline{\mathbf{R}}_\mathbf{M}}$$

for all $\mathbf{R_D^*} > \mathbf{R_M^h}/p_I^*$ and that in the constellation given by (P_I), $\mathbf{R_D^*} - \overline{\mathbf{R}}_\mathbf{M}/p_I^*$. Welfare is thus decreasing in φ^{reg}. □

Proof of Proposition 16.

The proof is similar to the one of Proposition 1. The only difference is the calculation of the equity threshold, which we denote by $\varphi_1^{T,s}$: It equalizes the shareholders' value when banks lend $\alpha_M^b = \alpha_{DCB}^{s'}$ and $\alpha_M^b = 0$. This equity threshold is given by the following equality:

$$\sigma^s\left(\frac{R_{CB}^{s'} - \varphi_1^{T,s}\mathbf{c}\frac{p_C^{s'}}{p_I}}{R_{CB}^{s'} - R_L^{s'}}\frac{L_M}{p_C^s}(R_L^s - R_{CB}^s) + E_B\frac{R_{CB}^s}{p_C^s} - \mathbf{c}\frac{E_B}{p_I}\right) = \mathbb{E}\left[E_B\frac{R_{CB}^s}{p_C^s}\right] - \mathbf{c}\frac{E_B}{p_I},$$

which yields the value $\varphi_1^{T,s}$ given in Proposition 16. □

Proof of Proposition 17.

Let $\mathcal{E}^*$ be an equilibrium with banks.

Then all banks choose the same level of money creation and lending, denoted by α_M^*. At the aggregate level, however, the amount borrowed by banks from the central bank has to equal the amount deposited by banks at the central bank, meaning that $\int_0^1 \alpha_M^b \mathrm{d}b = 1$, which translates into $\alpha_M^* = 1$. From Proposition 16, we obtain that given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, consumption prices $(p_C^{s*})_s$, and the equity ratio φ^*, all banks $b \in [0,1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M\big((R_L^{s*})_s, (R_{CB}^s)_s, (p_C^{s*})_s, \varphi^*\big)$ as given in Proposition 16.

Now suppose first that $\mathbf{R_M^l} \geq \mathbf{c}$. Then as $\varphi^*\mathbf{c}\frac{p_C^{s*}}{p_I^*} < \mathbf{R_M^s}\frac{p_C^{s*}}{p_I^*} = R_L^{s*}$ for all states $s = l, h$ and for all values $\varphi^* \in (0,1)$, $\alpha_{DCB}^s > 1$ for all states $s = l, h$ and all values $\varphi^* \in (0,1)$. The only gross rates of return in Proposition 16 rationalizing $\alpha_M^* = 1$ would thus be

$$R_L^{s*} = R_{CB}^s$$

for all states $s = l, h$. A direct consequence of this relation and of Lemma 2 would be that

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^s \tag{3.93}$$

for all states $s = l, h$. We would then obtain the following relationships between real gross rates of return:

$$\mathbb{E}\left[\frac{R_D^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] = \frac{\overline{\mathbf{R}}_{\mathbf{M}}}{p_I^*} \quad \text{and}$$
$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \frac{\overline{\mathbf{R}}_{\mathbf{M}} - \mathbf{c}}{p_I^*} < \mathbb{E}\left[\frac{R_D^{s*}}{p_C^{s*}}\right],$$

as $\Pi_M^* = \mathbf{R}_{\mathbf{M}}^{\mathbf{s}} p_C^{s*} - R_L^{s*} p_I^* = 0$. Given gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices p_I^* and $(p_C^{s*})_s$, households would choose
$E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, S_F^*\big)$ given S_F^*,
$D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, E_B^*, S_F^*\big)$ given E_B^* and S_F^*, and
$S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s\big)$, where these correspondences are given in Lemma 6 in Appendix 2.E in Chapter 2. None of the seven cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ would correspond to both a real gross rate of return on equity smaller than the real gross rate of return on deposits and to the condition $E_B^* > 0$. We conclude that there is no equilibrium with banks whenever $\mathbf{R}_{\mathbf{M}}^{\mathbf{l}} \geq \mathbf{c}$.

Suppose in the remainder of the proof that $\mathbf{R}_{\mathbf{M}}^{\mathbf{l}} < \mathbf{c}$. Then as $\varphi^* \mathbf{c} \frac{p_C^{s*}}{p_I^*} \geq \mathbf{R}_{\mathbf{M}}^{\mathbf{l}} \frac{p_C^{s*}}{p_I^*} = R_L^{s*}$ for high enough values $\varphi^* \in (0,1)$, there exists a State $s' = l, h$ such that $\alpha_{DCB}^{s'} \leq 1$ for high enough values $\varphi^* \in (0,1)$. Similarly to the previous case, there is no equilibrium with banks if $\varphi^* \neq \frac{\mathbf{R}_{\mathbf{M}}^{\mathbf{s}'}}{\mathbf{c}}$. Therefore, in any equilibrium with banks $\varphi^* = \frac{\mathbf{R}_{\mathbf{M}}^{\mathbf{s}'}}{\mathbf{c}}$.

From the previous analysis, we conclude that the only gross rates of return rationalizing households' investment $D_H^*, E_B^* > 0$ given in Lemma 6 in Appendix 2.G of Chapter 2 would be such that

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_D^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right].$$

However, from Proposition 16, we conclude that Bank b is indifferent between $\alpha_M^b = 0$ and $\alpha_M^b = \alpha_{DCB}^{s'}$ if and only if

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right] - \frac{\mathbf{c}}{p_I^*}.$$

We can therefore exclude the fourth and similarly the fifth cases in the definition of the correspondence in Proposition 16. We note that when $R_{CB}^{s'} > R_L^{s'*}$ and $R_{CB}^{s} < R_L^{s*}$,

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right],$$

where R_E^{s*} is calculated for $\alpha_M^b = \alpha_{DCB}^{s'}$, is a sufficient condition for banks to choose $\alpha_M^b = \alpha_{DCB}^{s'}$ and is moreover a necessary condition from households' investment perspective. Thus, the remaining cases in the definition of the correspondence in Proposition 16 are summarized by

$$R_{CB}^{s'} > R_L^{s'*}, \quad R_{CB}^{s} < R_L^{s*}, \quad \text{and} \quad \mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right].$$

Banks' default in State s' and the latter equality can be written as follows:

$$\sigma^s\left(\frac{R_{CB}^{s'} - \varphi^*\mathbf{c}\frac{p_C^{s'*}}{p_I^*}}{R_{CB}^{s'} - R_L^{s'*}} L_M^* \frac{R_L^{s*} - R_{CB}^{s}}{p_C^{s*}} + E_B^*\frac{R_{CB}^{s}}{p_C^{s*}} - \mathbf{c}\frac{E_B^*}{p_I^*}\right) = \mathbb{E}\left[E_B^*\frac{R_{CB}^{s}}{p_C^{s*}}\right],$$

which also writes

$$\sigma^s\left(\frac{R_{CB}^{s'} - \varphi^*\mathbf{c}\frac{p_C^{s'*}}{p_I^*}}{R_{CB}^{s'} - R_L^{s'*}} \frac{R_L^{s*} - R_{CB}^{s}}{p_C^{s*}} + \varphi^*\left(\frac{R_{CB}^{s}}{p_C^{s*}} - \frac{\mathbf{c}}{p_I^*}\right)\right) = \varphi^*\mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right].$$

The latter equation is equivalent to

$$\sigma^s\Big(R_{CB}^{s'} - \varphi^*\mathbf{c}\frac{p_C^{s'*}}{p_I^*}\Big) - \sigma^s\varphi^*\frac{\mathbf{c}}{p_I^*}p_C^{s*}\frac{R_{CB}^{s'} - R_L^{s'*}}{R_L^{s*} - R_{CB}^{s}} = \varphi^*\sigma^{s'}\frac{R_{CB}^{s'}}{p_C^{s'*}}p_C^{s*}\frac{R_{CB}^{s'} - R_L^{s'*}}{R_L^{s*} - R_{CB}^{s}},$$

from which we can derive an expression for the equity ratio φ^*:

$$\varphi^* = \frac{\frac{R_{CB}^{s'}}{p_C^{s'*}}}{\left(\frac{\mathbf{c}}{p_I^*} + \frac{\sigma^{s'}}{\sigma^s}\frac{R_{CB}^{s'}}{p_C^{s'*}}\right)\frac{p_C^{s*}}{p_C^{s'*}}\frac{R_{CB}^{s'} - R_L^{s'*}}{R_L^{s*} - R_{CB}^{s}} + \frac{\mathbf{c}}{p_I^*}}.$$

Using $\varphi^* = \frac{\mathbf{R}_\mathbf{M}^{\mathbf{s}'}}{\mathbf{c}}$, we can re-write this equation as follows:

$$\frac{\mathbf{R}_\mathbf{M}^{\mathbf{s}'}}{\mathbf{c}}\left(\left(\mathbf{c} + \frac{\sigma^{s'}}{\sigma^s}p_I^*\frac{R_{CB}^{s'}}{p_C^{s'*}}\right)\frac{(R_{CB}^{s'} - R_L^{s'*})/p_C^{s'*}}{(R_L^{s*} - R_{CB}^{s})/p_C^{s*}} + \mathbf{c}\right) = p_I^*\frac{R_{CB}^{s'}}{p_C^{s'*}},$$

which is equivalent to

$$\mathbf{R}_\mathbf{M}^{\mathbf{s}'}\left(\sigma^s\frac{R_L^{s*} - R_{CB}^{s}}{p_C^{s*}} + \sigma^s\frac{R_{CB}^{s'} - R_L^{s'}}{p_C^{s'}} + \frac{\sigma^{s'}}{\mathbf{c}}p_I^*\frac{R_{CB}^{s'}}{p_C^{s'*}}\frac{R_{CB}^{s'} - R_L^{s'*}}{p_C^{s'*}}\right)$$
$$= \sigma^s p_I^*\frac{R_{CB}^{s'}}{p_C^{s'*}}\frac{R_L^{s*} - R_{CB}^{s}}{p_C^{s*}}.$$

We re-write this equation using the notation[10]

$$\begin{aligned} I^* :=& p_I^*\mathbb{E}\left[\left(\frac{R_{CB}^{s}}{p_C^{s*}} - \frac{R_L^{s*}}{p_C^{s*}}\right) + \varphi^*\mathbf{c}\right] \\ =& p_I^*\mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right] + \sigma^s(\mathbf{R}_\mathbf{M}^{\mathbf{s}'} - \mathbf{R}_\mathbf{M}^{\mathbf{s}}), \end{aligned}$$

which is equivalent to

$$\frac{R_L^{s*} - R_{CB}^{s}}{p_C^{s*}} = \frac{\sigma^{s'}}{\sigma^s}\frac{R_{CB}^{s'} - R_L^{s'*}}{p_C^{s'*}} + \left(\frac{\mathbf{R}_\mathbf{M}^{\mathbf{s}'}}{p_I^*} - \frac{I^*}{p_I^*}\right)\frac{1}{\sigma^s}$$

as follows:

$$\mathbf{R}_\mathbf{M}^{\mathbf{s}'}\left(\frac{\mathbf{R}_\mathbf{M}^{\mathbf{s}'} - I^*}{p_I^*} + \frac{R_{CB}^{s'} - R_L^{s'*}}{p_C^{s'*}} + \frac{\sigma^{s'}}{\mathbf{c}}p_I^*\frac{R_{CB}^{s'}}{p_C^{s'*}}\frac{R_{CB}^{s'} - R_L^{s'*}}{p_C^{s'*}}\right)$$
$$= p_I^*\frac{R_{CB}^{s'}}{p_C^{s'*}}\left(\sigma^{s'}\frac{R_{CB}^{s'} - R_L^{s'*}}{p_C^{s'*}} + \frac{\mathbf{R}_\mathbf{M}^{\mathbf{s}'} - I^*}{p_I^*}\right).$$

[10] We note that $I = 0$ if the investment allocation is second-best and I is a measure of the intensity of the inefficiency compared to the second-best allocation.

Finally, this equation can be re-written as

$$\begin{aligned} I^* &= \sigma^{s'} p_I^* \frac{R_{CB}^{s'}}{p_C^{s'*}} \left(1 - \frac{\mathbf{R}_\mathbf{M}^{\mathbf{s}'}}{\mathbf{c}}\right) \\ &= \sigma^{s'} \mathbf{R}_\mathbf{M}^{\mathbf{s}'} \frac{R_{CB}^{s'}}{R_L^{s'*}} \left(1 - \frac{\mathbf{R}_\mathbf{M}^{\mathbf{s}'}}{\mathbf{c}}\right). \end{aligned}$$

We conclude that there are equilibria with banks given by

$$\begin{aligned} &E_B^* \in \Big(0, p_I^* \left(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R}_\mathbf{F}^*)\right)\Big), \\ &D_H^* = p_I^* \left(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R}_\mathbf{F}^*)\right) - E_B^*, \text{ and} \\ &S_F^* = \mathbf{f}'^{-1}(\mathbf{R}_\mathbf{F}^*), \end{aligned}$$

where $\mathbf{R}_\mathbf{F}^*$ can be determined by using Lemma 4 and equating the demand for the investment good $\mathbf{K}_\mathbf{F}^*$ to its supply S_F^*:

$$\mathbf{R}_\mathbf{F}^* = \mathbf{f}'(\mathbf{0}) - \max\left(0, \mathbf{f}'(\mathbf{0}) - p_I^* \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]\right).$$

Therefore, with the help of the equity ratio φ^*, we can re-write all equilibrium variables as given in Proposition 16.

In turn, it is straightforward to verify that the tuples given in Proposition 17 constitute equilibria with banks as in Section 3.7. □

Proof of Corollary 13.
For any central bank policy gross rates $(R_{CB}^s)_s$, equilibria with banks are given by Proposition 16. These equilibria with banks are essentially characterized by two parameters $(R_{CB}^s)_s$ and three variables, $(R_L^{s*})_s$ as well as p_I^*. Clearly, the price of the investment good has no impact on the investment allocation. Once the central bank policy gross rates $(R_{CB}^s)_s$ are set, there is a continuum of investment allocation supported by equilibria with banks that are characterized by

$$R_{CB}^{s'} > R_L^{s'*}, \quad R_{CB}^s < R_L^{s*}, \quad \text{and} \quad I^* = \sigma^{s'} \mathbf{R}_\mathbf{M}^{\mathbf{s}'} \frac{R_{CB}^{s'}}{R_L^{s'*}} \left(1 - \frac{\mathbf{R}_\mathbf{M}^{\mathbf{s}'}}{\mathbf{c}}\right).$$

For any given central bank policy gross rates $(R^s_{CB})_s$, if $\mathbf{R^h_M} \geq \mathbf{c}$, the set of possible investment allocations is given by

$$I^* = \sigma^l \mathbf{R^l_M} A \left(1 - \frac{\mathbf{R^l_M}}{\mathbf{c}}\right),$$

where A can take any value in $(1, +\infty)$. For any given central bank policy gross rates $(R^s_{CB})_s$, if $\mathbf{R^h_M} < \mathbf{c}$, the set of possible investment allocations is given by

$$I^*_1 := \sigma^l \mathbf{R^l_M} A \left(1 - \frac{\mathbf{R^l_M}}{\mathbf{c}}\right), \quad \text{and by}$$
$$I^*_2 := \sigma^h \mathbf{R^h_M} B \left(1 - \frac{\mathbf{R^h_M}}{\mathbf{c}}\right),$$

where A, B can take any value in $(1, +\infty)$. We first note that any possible investment allocation is dominated by the second-best allocation. Furthermore, we note that depending on the choice of parameters, the set of possible investment allocations may even by reduced to $(\mathbf{K^*_F}, \mathbf{K^*_M}) = (\mathbf{0}, \mathbf{W})$. □

Proof of Proposition 18.
Let $\mathcal{E}^*$ be an equilibrium with banks with rigid prices.

Then all banks choose the same level of money creation and lending, denoted by α^*_M. At the aggregate level, however, the amount borrowed by banks from the central bank has to equal the amount deposited by banks at the central bank, meaning that $\int_0^1 \alpha^b_M \mathrm{d}b = 1$, which translates into $\alpha^*_M = 1$. From Proposition 16, we obtain that given gross rates of return $(R^{s*}_L)_s$, policy choices $(R^s_{CB})_s$, consumption prices $(p^{s*}_C)_s$, and the equity ratio φ^*, all banks $b \in [0,1]$ choose a lending level α^b_M maximizing their expected gross rate of return on equity in real terms. Analogously to the derivation of Proposition 16 we can easily derive a proposition similar to Lemma 7 in Appendix 2.F in Chapter 2 that gives the expression for the privately optimal amounts of money creation and lending by an individual bank when there are costs of equity issuance.

We note that banks will create the maximum amount of money that will fulfill the capital requirement φ^{reg} and which—at the same time—will preserve them from defaulting against the central bank. The limit case where Bank b is indifferent between this amount of money creation and $\alpha_M^b = 0$ cannot hold in an equilibrium with banks, as the return on equity in such cases would be

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right] - \frac{\mathbf{c}}{p_I^*},$$

and a necessary condition from households' investment perspective is

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right],$$

due to the conditions that $D_H^*, E_B^* > 0$.

From this analysis, in any equilibrium with banks, we can rule out all cases where banks are indifferent between a maximum amount of money creation and no money creation, as well as all cases where banks do not wish to create any money. In the remaining cases, either the banks' demand for money is limited by the capital requirement—and in these cases $\alpha_M^b = \frac{\varphi^*}{\varphi^{reg}} = 1$—, or the demand is limited by the heavy penalties bankers have to pay when their bank defaults against the central bank. In the latter cases, $\alpha_M^b = \alpha_{DCB}^{s'} = 1$ for some State s'. The former cases can be summarized as follows in Cases a) to c) and the latter cases can be summarized in Case d):

Case a) ($\mathbf{R_M^s} \geq R_{CB}^s$ for all states $s = l, h$
with at least one strict inequality),
Case b) ($\overline{\mathbf{R}}_\mathbf{M} > \overline{R}_{CB}$, $\mathbf{R_M^{s'}} < R_{CB}^{s'}$, and $\frac{\varphi^*}{\varphi^{reg}} \leq \alpha_{DH}^{s'}$),
Case c) ($\mathbf{R_M^{s'}} < R_{CB}^{s'}$, $R_{CB}^s < \mathbf{R_M^s}$, and $\alpha_{DH}^{s'} < \frac{\varphi^*}{\varphi^{reg}} \leq \alpha_{DCB}^{s'}$),
Case d) ($\mathbf{R_M^{s'}} < R_{CB}^{s'}$, $R_{CB}^s < \mathbf{R_M^s}$, and $\alpha_{DCB}^{s'} \leq \frac{\varphi^*}{\varphi^{reg}}$),

with the condition that the real expected gross rates of return on equity are equal to the real expected gross rates on deposits in all cases, and where

$$\alpha_{DH}^s = \varphi^* \frac{R_{CB}^s - \mathbf{c}}{R_{CB}^s - \mathbf{R_M^s}}.$$

Suppose first that the central bank policy rates $(R_{CB}^s)_s$ are as given in Case a).

Then banks do not default in any state and the condition stemming from the households' optimization problem that the real gross rates of return on equity are equal to the real gross rates on deposits implies that

$$\mathbb{E}\left[\frac{\varphi^*}{\varphi^{reg}}\frac{\mathbf{R}^{\mathbf{s}}_{\mathbf{M}} - R^s_{CB}}{\varphi^*} + R^s_{CB} - \mathbf{c}\right] = \overline{R}_{CB},$$

which is equivalent to

$$\overline{R}_{CB} = \overline{\mathbf{R}}_{\mathbf{M}} - \mathbf{c}\varphi^{reg},$$

as $\varphi^* = \varphi^{reg}$.

Suppose now that the central bank policy rates $(R^s_{CB})_s$ and the equity ratio requirement φ^{reg} are as given in Case b). Then banks do not default in any state and the condition from the households' optimization problem that the real gross rates of return on equity are equal to the real gross rates on deposits implies, similarly to Case a), that

$$\overline{R}_{CB} = \overline{\mathbf{R}}_{\mathbf{M}} - \mathbf{c}\varphi^{reg},$$

as $\varphi^* = \varphi^{reg}$. Moreover, the condition $\frac{\varphi^*}{\varphi^{reg}} \leq \alpha^{s'}_{DH}$ is equivalent to

$$\varphi^{reg} \geq \frac{R^{s'}_{CB} - \mathbf{R}^{\mathbf{s'}}_{\mathbf{M}}}{R^{s'}_{CB} - \mathbf{c}} > 0,$$

which can only hold if $\mathbf{R}^{\mathbf{s'}}_{\mathbf{M}} > \mathbf{c}$.

Suppose now that the central bank policy rates $(R^s_{CB})_s$ and the equity ratio requirement φ^{reg} are as given in Case c). Then banks default in State s' and the condition from the households' optimization problem that the real gross rates of return on equity are equal to the real gross rates on deposits implies that

$$\sigma^s\left(\frac{\varphi^*}{\varphi^{reg}}\frac{\mathbf{R}^{\mathbf{s}}_{\mathbf{M}} - R^s_{CB}}{\varphi^*} + R^s_{CB} - \mathbf{c}\right) = \overline{R}_{CB},$$

which is equivalent to

$$\varphi^{reg} = \sigma^s \frac{\mathbf{R^s_M} - R^s_{CB}}{\sigma^{s'} R^{s'}_{CB} + \sigma^s \mathbf{c}}, \tag{3.94}$$

as $\varphi^* = \varphi^{reg}$. Moreover, the condition $\alpha^{s'}_{DH} < \frac{\varphi^*}{\varphi^{reg}}$ is equivalent to

$$\varphi^{reg} < \frac{R^{s'}_{CB} - \mathbf{R^{s'}_M}}{R^{s'}_{CB} - \mathbf{c}}$$

and the condition that $\frac{\varphi^*}{\varphi^{reg}} \leq \alpha^{s'}_{DCB}$ to

$$\varphi^{reg} \leq \frac{\mathbf{R^{s'}_M}}{\mathbf{c}}.$$

We can also calculate in this case the value of $I^* := \overline{R}_{CB} - \overline{\mathbf{R}}_{\mathbf{M}} + \mathbf{c}\varphi^{reg}$ using Equation (3.94). We obtain

$$I^* = (R^{s'}_{CB} - \mathbf{c})\sigma^{s'} \left(\frac{R^{s'}_{CB} - \mathbf{R^{s'}_M}}{R^{s'}_{CB} - \mathbf{c}} - \varphi^{reg} \right) > 0.$$

Suppose finally that the central bank policy rates $(R^s_{CB})_s$ and the equity ratio requirement φ^{reg} are as given in Case d). Then banks default in some State s' and the possible equilibria with banks are described in Proposition 17. In addition, the equity ratio requirement φ^{reg} has to fulfill the following inequality:

$$\alpha^{s'}_{DCB} \leq \frac{\varphi^*}{\varphi^{reg}},$$

which is equivalent to

$$\frac{\mathbf{R^{s'}_M}}{\mathbf{c}} = \varphi^* \geq \frac{R^{s'}_{CB}\varphi^{reg}}{R^{s'}_{CB} - \mathbf{R^{s'}_M} + \varphi^{reg}\mathbf{c}}.$$

This inequality re-writes

$$\varphi^{reg} \leq \frac{\mathbf{R^{s'}_M}}{\mathbf{c}}.$$

In turn, it is straightforward to verify that the tuples given in Proposition 18 constitute equilibria with banks. □

Proof of Corollary 14.
If the central bank policy rates (R^s_{CB}) and the equity ratio requirement φ^{reg} are chosen such that

$$R^s_{CB} = \mathbf{R^s_M} - \sigma^s \mathbf{c}\varphi^{reg},$$

we obtain

$$\mathbf{R^s_M} > R^s_{CB} \quad \text{for all states } s = l, h \text{ and } \quad \overline{R}_{CB} = \overline{\mathbf{R}}_{\mathbf{M}} - \mathbf{c}\varphi^{reg},$$

and the welfare in these equilibria with banks converges to the one implied by the first-best allocation when φ^{reg} converges to 0. □

Proof of Proposition 19.
Suppose that a minimum reserve requirement $r^{reg} \in (0, 1)$ and a haircut regulation $h \in (0, 1)$ are imposed on each Bank b at the end of Period $t = 0$.

Then a Bank b_i has to borrow the amount $\max(0, r^{reg} d_H - d^{b_i}_{CB})$ of central bank money at the end of Period $t = 0$ to fulfill the reserve requirement r^{reg}. The maximum amount of reserves which Bank b_i can borrow from the central bank is given by $(1-h)l^{b_i}_M$.[11] Therefore, the following constraint should hold in equilibrium at the end of Period $t = 0$:

$$\max(0, r^{reg} d_H - d^{b_i}_{CB}) \leq (1 - h)l^{b_i}_M,$$

which is equivalent to

$$0 \leq \alpha^{b_i}_M \leq \frac{1 - r^{reg}(1 - \varphi)}{h},$$

where $\alpha^{b_i}_M \leq 1$.

[11]Note that banks are indifferent between borrowing any lower reserve level as soon as it fulfills the reserve requirement, as the gross rate of return charged for CB liabilities is equal to the gross rate of return for holding CB deposits.

Similarly, a Bank b_j has to borrow the amount $r^{reg}d_H$ of central bank money at the end of Period $t = 0$ to fulfill the reserve requirement r^{reg}. The maximum amount of reserves which Bank b_j can borrow from the central bank is given by $(1-h)l_M^{b_j}$. Therefore, the following constraint should hold in equilibrium at the end of Period $t = 0$:

$$r^{reg}d_H + l_{CB}^{b_j} \leq (1-h)l_M^{b_j},$$

which is equivalent to

$$\alpha_M^{b_j} \leq \frac{1 - r^{reg}(1-\varphi)}{h},$$

where $\alpha_M^{b_j} \geq 1$. We note that for any Bank b, the constraint is given by

$$\alpha_M^{b} \leq \frac{1 - r^{reg}(1-\varphi)}{h}.$$

□

Chapter 4

More Sophisticated Households' Problems in the Model with Money Creation

Abstract[1]

We demonstrate that our main findings from Chapter 2 can be extended to a sophistication of the households' problem, i.e. to the introduction of households' risk aversion and of households' savings decisions.

Preliminary remark: In the models with more sophisticated households' problems of Chapter 4, we use the model described in Chapter 2 and its variables, and we only re-define the variables that change with the new features examined.

[1]The research on which this chapter is based was supported by the SNF project no. 100018_165491/1 "Money Creation by Banks, Monetary Policy, and Regulation".

4.1 Households' Risk Aversion

4.1.1 General Concave Utility Function

In Section 4.1, we assume that households are risk-averse and we investigate how this impacts our results. In Subsection 4.1.1, we use a general concave utility function. First, to simplify the analysis, we use more restrictive Inada Conditions on the production function, as follows:

Assumption 5

$$\lim_{\mathbf{K_F}\to\mathbf{0}} \mathbf{f}'(\mathbf{K_F}) = \infty \quad \textit{and} \quad \lim_{\mathbf{K_F}\to\mathbf{W}} \mathbf{f}'(\mathbf{K_F}) = 0.$$

Moreover, we assume that households maximize their expected utility $\mathbb{E}[U(\mathbf{C^s})]$, which they will obtain from the consumption of the physical good in Period $t = 1$. In particular, we assume that U is increasing, strictly concave, and twice differentiable. The definition of an equilibrium with banks becomes

Definition 6
Given the central bank policy gross rates $(R^s_{CB})_s$, a symmetric equilibrium with banks of the sequential market process described in Subsection 2.2.4 is defined as a tuple

$$\begin{aligned}\mathcal{E} := \Big(&(R^s_E)_s, (R^s_D)_s, (R^s_L)_s, \mathbf{R_F},\\ &p_I, (p^s_C)_s,\\ &E_B, D_H, (\tilde{D}^s_H)_s, L_M, S_F,\\ &\mathbf{K_M}, \mathbf{K_F}\Big)\end{aligned}$$

consisting of positive and finite gross rates of return, prices, savings, bank deposits D_H at the end of Stage C of Period $t = 0$, bank deposits $(\widetilde{D}^s_H)_s$ at Stage E of Period $t = 1$, and the corresponding physical investment allocation, such that

– *households hold some private deposits $D_H > 0$ at the end of Stage C,*

– *households maximize their expected utility*

$$\max_{\{D_H,E_B,S_F\}} \mathbb{E}\big[U(\mathbf{C^s})\big]$$
$$s.t. \quad E_B + D_H + p_I S_F = p_I \mathbf{W},$$
$$and \quad \mathbf{C^s} = E_B \frac{R_E^s}{p_C^s} + D_H \frac{R_D^s}{p_C^s} + \mathbf{f}(S_F),$$

taking gross rates of return $(R_E^s)_s$ *and* $(R_D^s)_s$ *as well as prices* p_I *and* $(p_C^s)_s$ *as given,*

– *firms in MT and FT as well as each bank* $b \in [0,1]$ *maximize their expected shareholders' value given respectively by*

$$\max_{\mathbf{K_M}\in[\mathbf{0},\mathbf{W}]} \{\mathbb{E}[\max(\mathbf{K_M}(\mathbf{R_M^s} p_C^s - R_L^s p_I), 0)]\},$$
$$s.t.\ \mathbf{R_M^s} p_C^s = R_L^s p_I \ for\ s = l, h,$$
$$\max_{\mathbf{K_F}\in[\mathbf{0},\mathbf{W}]} \{\mathbb{E}[\max((\mathbf{f}(\mathbf{K_F}) - \mathbf{K_F}\mathbf{R_F})p_C^s, 0)]\},$$
$$and \quad \max_{\alpha_M^b \geq 0} \{\mathbb{E}[\max(\alpha_M^b L_M \frac{R_L^s - R_{CB}^s}{p_C^s} + L_M \frac{R_{CB}^s - R_D^s}{p_C^s} + E_B \frac{R_D^s}{p_C^s}, 0)]\},$$

taking gross rates of return $(R_D^s)_s$*,* $(R_L^s)_s$*, and* $\mathbf{R_F}$ *as well as prices* p_I *and* $(p_C^s)_s$ *as given,*

– *all banks choose the same level of money creation, and*

– *markets for investment and consumption goods clear in each state.*

All lemmata and propositions that do not concern households remain unchanged. We now examine how Lemma 6 is affected by our new assumption of risk-averse households. For this, as we exclude corner solutions with $D_H = 0$ or $E_B = 0$ in Definition 6, only interior solutions $D_H, E_B, S_F > 0$ to the households' maximization problem are relevant for the ensuing analysis. To state the households' portfolio choice, we need some mathematical definitions given as follows:

Lemma 12

We first define the following functions:

$$g_E(E_B, S_F) = \mathbb{E}\left[U'\left(E_B \frac{R_E^s}{p_C^s} + \big(p_I(\mathbf{W} - S_F) - E_B\big)\frac{R_D^s}{p_C^s} + \mathbf{f}(S_F)\right)\frac{R_D^s - R_E^s}{p_C^s}\right],$$
$$h_E(E_B, S_F) = \mathbb{E}\left[U'\left(E_B \frac{R_E^s}{p_C^s} + \big(p_I(\mathbf{W} - S_F) - E_B\big)\frac{R_D^s}{p_C^s} + \mathbf{f}(S_F)\right)\left(\frac{\mathbf{f}'(S_F)}{p_I} - \frac{R_D^s}{p_C^s}\right)\right].$$

Then for all $E_B \in (0, p_I\mathbf{W})$, there is a unique solution to

$$h_E(E_B, S_F) = 0,$$

which we denote by $S_F^{h_E}(E_B)$. Finally, we define the function $X_E(E_B)$ for all $E_B \in (0, p_I\mathbf{W})$ by

$$X_E(E_B) = g_E\big(E_B, S_F^{h_E}(E_B)\big).$$

The function $X_E(\cdot)$ is strictly increasing.
If $\frac{R_E^s}{p_C^s} < \frac{R_D^s}{p_C^s} < \frac{R_D^{s'}}{p_C^{s'}} < \frac{R_E^{s'}}{p_C^{s'}}$ for states $s \neq s'$, there is a unique solution to

$$h_E\left(E_B, \mathbf{W} - \frac{E_B}{p_I}\right) = 0, \tag{4.1}$$

which we denote by E_B^{MAX}.

Similarly, we define the following functions:

$$g_D(D_H, S_F) = \mathbb{E}\left[U'\left(D_H \frac{R_D^s}{p_C^s} + \big(p_I(\mathbf{W} - S_F) - D_H\big)\frac{R_E^s}{p_C^s} + \mathbf{f}(S_F)\right)\frac{R_E^s - R_D^s}{p_C^s}\right],$$
$$h_D(D_H, S_F) = \mathbb{E}\left[U'\left(D_H \frac{R_D^s}{p_C^s} + \big(p_I(\mathbf{W} - S_F) - D_H\big)\frac{R_E^s}{p_C^s} + \mathbf{f}(S_F)\right)\left(\frac{\mathbf{f}'(S_F)}{p_I} - \frac{R_E^s}{p_C^s}\right)\right].$$

Then for all $D_H \in (0, p_I\mathbf{W})$, there is a unique solution to

$$h_D(D_H, S_F) = 0,$$

which we denote by $S_F^{h_D}(D_H)$. Finally, we define the function $X_D(D_H)$ for all

$D_H \in (0, p_I \mathbf{W})$ by

$$X_D(D_H) = g_D\big(D_H, S_F^{h_D}(D_H)\big).$$

The function $X_D(\cdot)$ is strictly increasing.
If $\frac{R_D^s}{p_C^s} < \frac{R_E^s}{p_C^s} < \frac{R_E^{s'}}{p_C^{s'}} < \frac{R_D^{s'}}{p_C^{s'}}$ for states $s \neq s'$, there is a unique solution to

$$h_D\left(D_H, \mathbf{W} - \frac{D_H}{p_I}\right) = 0, \tag{4.2}$$

which we denote by D_H^{MAX}.

The proof of the claims given in Lemma 12 is given in Appendix 4.A. We now obtain

Proposition 21
The existence of an interior solution with $D_H, E_B, S_F > 0$ to the representative household's expected utility maximization problem requires either

(i) $R_E^s = R_D^s$ *for both states* $s = l, h$, *or*

(ii) $\frac{R_E^s}{p_C^s} < \frac{R_D^s}{p_C^s} < \frac{R_D^{s'}}{p_C^{s'}} < \frac{R_E^{s'}}{p_C^{s'}}$ *for states* $s \neq s'$ *and* $X_E(0) < 0 < X_E(E_B^{MAX})$, *or*

(iii) $\frac{R_D^s}{p_C^s} < \frac{R_E^s}{p_C^s} < \frac{R_E^{s'}}{p_C^{s'}} < \frac{R_D^{s'}}{p_C^{s'}}$ *for states* $s \neq s'$ *and* $X_D(0) < 0 < X_D(D_H^{MAX})$.

We distinguish the following cases, depending on the parameter values $(R_E^s)_s$, $(R_D^s)_s$, p_I, and $(p_C^s)_s$:

– *In Case (i), households are indifferent between different capital structures, and their portfolio choice is given by*

$$\Big(\hat{E}_B\big((R_E^s)_s, (R_D^s)_s, p_I, (p_C^s)_s\big),$$
$$\hat{D}_H\big((R_E^s)_s, (R_D^s)_s, p_I, (p_C^s)_s, E_B, S_F\big),$$
$$\hat{S}_F\big((R_E^s)_s, (R_D^s)_s, p_I, (p_C^s)_s, E_B\big)\Big) =$$

$$\Big((0, E_B^{MAX}), \{p_I\left(\mathbf{W} - S_F\right) - E_B\}, \{S_F^{h_E}(E_B)\}\Big),$$

where E_B^{MAX} *denotes the unique solution of* $h_E\left(E_B, \mathbf{W} - \frac{E_B}{p_I}\right) = 0$.

– *In Case (ii), the interior solution is unique and given by*

$$\begin{aligned}\Big(&\hat{E}_B\big((R_E^s)_s, (R_D^s)_s, p_I, (p_C^s)_s\big),\\ &\hat{D}_H\big((R_E^s)_s, (R_D^s)_s, p_I, (p_C^s)_s, S_F, E_B\big),\\ &\hat{S}_F\big((R_E^s)_s, (R_D^s)_s, p_I, (p_C^s)_s, E_B\big)\Big) =\end{aligned}$$

$$\Big(\{E_B^{g_E}\}, \{p_I(\mathbf{W} - S_F) - E_B\}, \{S_F^{h_E}(E_B)\}\Big),$$

where $E_B^{g_E}$ *is the unique solution of* $X_E(E_B) = 0$.

– *In Case (iii), the interior solution is unique and given by*

$$\begin{aligned}\Big(&\hat{E}_B\big((R_E^s)_s, (R_D^s)_s, p_I, (p_C^s)_s, S_F, D_H\big),\\ &\hat{D}_H\big((R_E^s)_s, (R_D^s)_s, p_I, (p_C^s)_s\big),\\ &\hat{S}_F\big((R_E^s)_s, (R_D^s)_s, p_I, (p_C^s)_s, D_H\big)\Big) =\end{aligned}$$

$$\Big(\{p_I(\mathbf{W} - S_F) - D_H\}, \{D_H^{g_D}\}, \{S_F^{h_D}(D_H)\}\Big),$$

where $D_H^{g_D}$ *denotes the unique solution of* $X_D(D_H) = 0$.

– *For all other parameter values of* $(R_E^s)_s$, $(R_D^s)_s$, p_I, *and* $(p_C^s)_s$, *there is no interior solution to the representative household's maximization problem.*

The proof of Proposition 21 is given in Appendix 4.A. We now define and then give a characterization of a first-best allocation with risk-averse households. We define the first-best allocation as follows:

Definition 7

A first-best allocation is a feasible allocation denoted by $(\mathbf{K_F^{FB}}, \mathbf{K_M^{FB}})$ *which maximizes the representative household's utility given by*

$$\mathbb{E}\Big[U\big(\mathbf{K_M}\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F})\big)\Big].$$

We can show that there is a unique first-best allocation, which we characterize in the following proposition.

Proposition 22
There is a unique first-best allocation $(\mathbf{K_F^{FB}}, \mathbf{K_M^{FB}})$. *This allocation is the unique solution of the following system of equations:*

$$\mathbb{E}\Big[U'\big((\mathbf{W}-\mathbf{K_F})\mathbf{R_M^s}+\mathbf{f}(\mathbf{K_F})\big)\mathbf{f}'(\mathbf{K_F})\Big] = \mathbb{E}\Big[U'\big((\mathbf{W}-\mathbf{K_F})\mathbf{R_M^s}+\mathbf{f}(\mathbf{K_F})\big)\mathbf{R_M^s}\Big],$$
$$\mathbf{K_F}+\mathbf{K_M}=\mathbf{W}.$$

The proof of Proposition 22 is given in Appendix 4.A. We can now characterize the symmetric equilibria with banks as defined in Definition 6, and compare them in terms of welfare to the first-best allocation. We obtain

Theorem 4
Given the policy gross rates $(R^s_{CB})_{s=l,h}$, *all equilibria with banks have the following form:*

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^s, \quad \mathbf{R_F^*} = \mathbf{f}'\big(\mathbf{K_F^{FB}}\big), \tag{4.3}$$
$$p_I^* = p, \quad p_C^{s*} = p\frac{R_{CB}^s}{\mathbf{R_M^s}}, \tag{4.4}$$
$$E_B^* = \varphi^* p\Big(\mathbf{W}-\mathbf{K_F^{FB}}\Big), \quad D_H^* = (1-\varphi^*)p\Big(\mathbf{W}-\mathbf{K_F^{FB}}\Big), \tag{4.5}$$
$$\tilde{D}_H^{s*} = p\Big(\mathbf{W}-\mathbf{K_F^{FB}}\Big)R_{CB}^s, \tag{4.6}$$
$$L_M^* = p\Big(\mathbf{W}-\mathbf{K_F^{FB}}\Big), \quad S_F^* = \mathbf{K_F^{FB}}, \tag{4.7}$$
$$\mathbf{K_M^*} = \mathbf{W}-\mathbf{K_F^{FB}}, \quad \mathbf{K_F^*} = \mathbf{K_F^{FB}}, \tag{4.8}$$

where the price of the investment good denoted by $p \in (0,+\infty)$ *and the aggregate equity ratio* $\varphi^* \in (0,1)$ *are arbitrary. The equilibrium profits of firms and banks are given by*

$$\Pi_M^{s*} = 0, \quad \Pi_F^{s*} = p\frac{R_{CB}^s}{\mathbf{R_M^s}}\Big(\mathbf{f}\big(\mathbf{K_F^{FB}}\big)-\mathbf{K_F^{FB}}\mathbf{f}'\big(\mathbf{K_F^{FB}}\big)\Big), \tag{4.9}$$
$$\Pi_B^{s*} = \varphi^* p\Big(\mathbf{W}-\mathbf{K_F^{FB}}\Big)R_{CB}^s. \tag{4.10}$$

The proof of Theorem 4 is given in Appendix 4.A. We now illustrate our results with a specific utility function.

4.1.2 Quadratic Utility Function

In Subsection 4.1.2, we use a quadratic utility function given by

$$U(\mathbf{C^s}) = \mathbf{C^s} - \frac{b}{2}\big(\mathbf{C^s}\big)^2,$$

where

$$b \leq \frac{1}{\mathbf{f}'(\mathbf{0})\mathbf{W} + \mathbf{W}\mathbf{R_M^h}},$$

so that $U'(\mathbf{C^s}) = 1 - b\mathbf{C^s} > 0$ for all possible consumption levels $\mathbf{C^s}$. The first-best allocation is given by the following optimization problem:

$$\max_{\mathbf{K_F},\mathbf{K_M}\in[\mathbf{0},\mathbf{W}]} \mathbb{E}\Big[U\big(\mathbf{K_M}\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F})\big)\Big]$$
$$\text{s.t.} \quad \mathbf{K_M} + \mathbf{K_F} = \mathbf{W}.$$

We use a linear production function for $\mathbf{f}$ and we denote $\mathbf{f}'(\mathbf{K_F}) = \mathbf{f}'$ for all $\mathbf{K_F} \in [\mathbf{0}, \mathbf{W}]$. We assume that $\mathbf{R_M^l} < \mathbf{f}' < \overline{\mathbf{R}}_\mathbf{M}$ and similarly to the proof of Proposition 22, we can show that the latter optimization problem has a unique solution given by

$$\mathbb{E}\Big[U'\big((\mathbf{W} - \mathbf{K_F})\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F})\big)\mathbf{f}'\Big] = \mathbb{E}\Big[U'\big((\mathbf{W} - \mathbf{K_F})\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F})\big)\mathbf{R_M^s}\Big]. \quad (4.11)$$

Solving Equation (4.11) gives the following investment allocation:

$$\mathbf{K_M^{FB}} = \frac{(\overline{\mathbf{R}}_\mathbf{M} - \mathbf{f}')(\mathbf{W}\mathbf{f}' + \frac{1}{b})}{\mathbb{E}[(\mathbf{R_M^s})^2] + (\mathbf{f}')^2},$$
$$\mathbf{K_F^{FB}} = \mathbf{W} - \mathbf{K_M^{FB}}.$$

We note that $\mathbf{K_M^{FB}} > 0$ and for $\mathbf{f}'$ sufficiently close to $\overline{\mathbf{R}}_\mathbf{M}$, we obtain $\mathbf{K_M^{FB}} < \mathbf{W}$, which we assume in the following. We note that $\mathbf{K_M^{FB}}$ is decreasing with respect to

b: As b is a measure of the intensity of households' risk aversion, more risk-averse households that can choose to directly invest in the two production technologies will shift investment to the less risky one, which means that $\mathbf{K_M^{FB}}$ decreases when households become more risk-averse. Moreover, we can show that the first-best level of investment in the FT sector is larger with risk-averse than with risk-neutral households. As the production function is linear, the households' maximization problem does not have an interior solution when households are risk-neutral and $\mathbf{f'} \neq \overline{\mathbf{R}}_\mathbf{M}$. For $\mathbf{f'} < \overline{\mathbf{R}}_\mathbf{M}$, risk-neutral households invest all their endowment in the risk technology, whereas the maximization problem of risk-averse households has an interior solution. It is thus straightforward to see that risk-averse households invest less in the risky technology than risk-neutral households, everything else being equal.

We now investigate whether the first-best allocation is indeed implemented in any equilibrium with banks, as suggested by Theorem 4.

Analogously to the derivations in the proof of Theorem 4, we obtain the following equations in any equilibrium with banks:

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^{s} = \frac{\mathbf{R_M^s} p_C^{s*}}{p_I^*}.$$

We note that the solution of the households' maximization problem is clearly interior, as $\mathbf{R_M^l} < \mathbf{f'} < \overline{\mathbf{R}}_\mathbf{M}$ and $\mathbf{f'}$ is sufficiently close to $\overline{\mathbf{R}}_\mathbf{M}$. The first-order conditions of the households' maximization problem write

$$\mathbb{E}\left[(1 - b\mathbf{C^{s*}})\frac{R_D^{s*} - R_E^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[(1 - b\mathbf{C^{s*}})\left(\frac{\mathbf{f'}}{p_I^*} - \frac{R_D^{s*}}{p_C^{s*}}\right)\right] = 0.$$

Using $\frac{R_D^{s*}}{p_C^{s*}} = \frac{\mathbf{R_M^s}}{p_I^*}$, we can re-write the second equality as follows:

$$\mathbb{E}\Big[U'\big((\mathbf{W} - \mathbf{K_F})\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F})\big)\mathbf{f'}\Big] = \mathbb{E}\Big[U'\big((\mathbf{W} - \mathbf{K_F})\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F})\big)\mathbf{R_M^s}\Big], \quad (4.12)$$

which is the equality characterizing the first-best allocation. We conclude that the first-best allocation is indeed implemented in any equilibrium with banks.

4.2 The Households' Savings Decisions

In Section 4.2, the physical good at $t = 0$ cannot only be invested to produce some amount of consumption good in Period $t = 1$, it can also be consumed directly in Period $t = 0$—the remaining amount being invested. The representative household is risk-neutral and discounts future consumption in Period $t = 1$ at rate β. We denote consumption in Period $t = 0$ by $\mathbf{C_0}$ and the one occurring in State s in Period $t = 1$ by $\mathbf{C_1^s}$. Moreover, we use $\mathbf{I}$ to denote the amount that the representative household saves in Period $t = 0$ for future investment in Period $t = 1$. The definition of an equilibrium with banks is then as follows:

Definition 8
Given the central bank policy gross rates $(R^s_{CB})_s$, a symmetric equilibrium with banks is defined as a tuple

$$\begin{aligned}\mathcal{E} := \Big(& (R^s_E)_s, (R^s_D)_s, (R^s_L)_s, \mathbf{R_F}, \\ & p_I, (p^s_C)_s, \\ & E_B, D_H, (\tilde{D}^s_H)_s, L_M, S_F, \\ & \mathbf{I}, \mathbf{K_M}, \mathbf{K_F} \Big)\end{aligned}$$

consisting of positive and finite gross rates of return, prices, savings, bank deposits D_H at the end of Stage C of Period $t = 0$, bank deposits $(\widetilde{D}^s_H)_s$ at Stage E of Period $t = 1$, and the corresponding physical investment allocation such that

- *households hold some private deposits $D_H > 0$ at the end of Stage C,*
- *households maximize their expected utility*

$$\max_{\{D_H, E_B, S_F, \mathbf{I} \in [\mathbf{0}, \mathbf{W}]\}} \Big\{ U := \mathbf{C_0} + \beta \mathbb{E}\big[\mathbf{C_1^s}\big] \Big\}$$

$$s.t. \quad \begin{cases} \mathbf{C_0} &= \mathbf{W} - \mathbf{I}, \\ \mathbf{C_1^s} &= E_B \frac{R^s_E}{p^s_C} + D_H \frac{R^s_D}{p^s_C} + \mathbf{f}(S_F), \end{cases}$$

$$and \quad E_B + D_H + p_I S_F = p_I \mathbf{I},$$

taking gross rates of return $(R_E^s)_s$ *and* $(R_D^s)_s$ *as well as prices* p_I *and* $(p_C^s)_s$ *as given,*

– *firms in MT and FT as well as each bank* $b \in [0,1]$ *maximize their expected shareholders' value given respectively by*

$$\begin{aligned}
&\max_{\mathbf{K_M}\in[\mathbf{0},\mathbf{W}]}\{\mathbb{E}[\max(\mathbf{K_M}(\mathbf{R^s_M}p_C^s - R_L^s p_I),0)]\},\\
&s.t.\ \mathbf{R^s_M}p_C^s = R_L^s p_I \ \textit{for}\ s = l,h,\\
&\max_{\mathbf{K_F}\in[\mathbf{0},\mathbf{W}]}\{\mathbb{E}[\max((\mathbf{f}(\mathbf{K_F}) - \mathbf{K_F}\mathbf{R_F})p_C^s,0)]\},\\
\textit{and}\quad &\max_{\alpha_M^b\geq 0}\{\mathbb{E}[\max(\alpha_M^b L_M\frac{R_L^s - R_{CB}^s}{p_C^s} + L_M\frac{R_{CB}^s - R_D^s}{p_C^s} + E_B\frac{R_D^s}{p_C^s},0)]\},
\end{aligned}$$

taking gross rates of return $(R_D^s)_s$*,* $(R_L^s)_s$*, and* $\mathbf{R_F}$ *as well as prices* p_I *and* $(p_C^s)_s$ *as given,*

– *all banks choose the same level of money creation, and*

– *markets for investment and consumption goods clear in each state.*

All lemmata and propositions not concerning households clearly stay unchanged. We now examine how Lemma 6 is affected by our new assumption that households make a consumption-savings decision in Period $t = 0$. We obtain the following lemma:

Lemma 13

The representative household's optimal portfolio and consumption-savings choices are represented by four correspondences denoted by

$$\begin{aligned}
\hat{E}_B &: \mathbb{R}^7_{++} \times [0,\mathbf{W}]^2 \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}),\\
\hat{D}_H &: \mathbb{R}^7_{++} \times \mathbb{R}_+ \times [0,\mathbf{W}]^2 \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}),\\
\hat{S}_F &: \mathbb{R}^7_{++} \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}),\\
\hat{\mathbf{I}} &: \mathbb{R}^7_{++} \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\}),
\end{aligned}$$

and given by

$$
\begin{aligned}
&\Big(\hat{E}_B\big((R_E^s)_s,(R_D^s)_s,p_I,(p_C^s)_s,S_F,\mathbf{I}\big),\\
&\hat{D}_H\big((R_E^s)_s,(R_D^s)_s,p_I,(p_C^s)_s,E_B,S_F,\mathbf{I}\big),\\
&\hat{S}_F\big((R_E^s)_s,(R_D^s)_s,p_I,(p_C^s)_s\big),\\
&\hat{\mathbf{I}}\big((R_E^s)_s,(R_D^s)_s,p_I,(p_C^s)_s\big)\Big)=\\
&\begin{cases}
\big(\{0\},\{0\},\{\mathbf{W}\},\{\mathbf{W}\}\big)\\
\qquad \textit{if } \max\left(\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right],\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right],\frac{1}{\beta p_I}\right)\le\frac{\mathbf{f'(W)}}{p_I},\\
\big(\{0\},\{p_I\mathbf{I}\},\{0\},[\mathbf{0},\mathbf{W}]\big)\\
\qquad \textit{if } \max\left(\frac{\mathbf{f'(0)}}{p_I},\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right)<\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]=\frac{1}{\beta p_I},\\
\big(\{0\},\{p_I\mathbf{W}\},\{0\},\{\mathbf{W}\}\big)\\
\qquad \textit{if } \max\left(\frac{\mathbf{f'(0)}}{p_I},\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right],\frac{1}{\beta p_I}\right)<\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right],\\
\big(\{0\},\{0\},\{0\},\{\mathbf{0}\}\big)\\
\qquad \textit{if } \max\left(\frac{\mathbf{f'(0)}}{p_I},\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right],\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)<\frac{1}{\beta p_I},\\
\big(\{p_I\mathbf{I}\},\{0\},\{0\},[\mathbf{0},\mathbf{W}]\big)\\
\qquad \textit{if } \max\left(\frac{\mathbf{f'(0)}}{p_I},\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)<\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]=\frac{1}{\beta p_I},\\
\big(\{p_I\mathbf{W}\},\{0\},\{0\},\{\mathbf{W}\}\big)\\
\qquad \textit{if } \max\left(\frac{\mathbf{f'(0)}}{p_I},\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right],\frac{1}{\beta p_I}\right)<\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right],\\
\big([0,p_I\mathbf{I}],\{p_I\mathbf{I}-E_B\},\{0\},[\mathbf{0},\mathbf{W}]\big)\\
\qquad \textit{if } \frac{\mathbf{f'(0)}}{p_I}<\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]=\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]=\frac{1}{\beta p_I},\\
\big([0,p_I\mathbf{W}],\{p_I\mathbf{W}-E_B\},\{0\},\{\mathbf{W}\}\big)\\
\qquad \textit{if } \max\left(\frac{\mathbf{f'(0)}}{p_I},\frac{1}{\beta p_I}\right)<\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]=\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right],\\
\big(\{0\},\{0\},\{\tilde{\mathbf{I}}\},\{\tilde{\mathbf{I}}\}\big)\\
\qquad \textit{if } \max\left(\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right],\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right],\frac{\mathbf{f'(W)}}{p_I}\right)<\frac{1}{\beta p_I}\le\frac{\mathbf{f'(0)}}{p_I},\\
\big(\{0\},\{p_I(\mathbf{I}-S_F)\},\{\tilde{\mathbf{I}}\},[\tilde{\mathbf{I}},\mathbf{W}]\big)\\
\qquad \textit{if } \max\left(\frac{\mathbf{f'(W)}}{p_I},\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right)<\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]=\frac{1}{\beta p_I}\le\frac{\mathbf{f'(0)}}{p_I},\\
\big(\{0\},\{p_I(\mathbf{W}-S_F)\},\{\mathbf{f'}^{-1}\left(p_I\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\},\{\mathbf{W}\}\big)\\
\qquad \textit{if } \max\left(\frac{\mathbf{f'(W)}}{p_I},\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right],\frac{1}{\beta p_I}\right)<\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\le\frac{\mathbf{f'(0)}}{p_I},\\
\big(\{p_I(\mathbf{I}-S_F)\},\{0\},\{\tilde{\mathbf{I}}\},[\tilde{\mathbf{I}},\mathbf{W}]\big)\\
\qquad \textit{if } \max\left(\frac{\mathbf{f'(W)}}{p_I},\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)<\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]=\frac{1}{\beta p_I}\le\frac{\mathbf{f'(0)}}{p_I},\\
\big(\{p_I(\mathbf{W}-S_F)\},\{0\},\{\mathbf{f'}^{-1}\left(p_I\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\right)\},\{\mathbf{W}\}\big)\\
\qquad \textit{if } \max\left(\frac{\mathbf{f'(W)}}{p_I},\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right],\frac{1}{\beta p_I}\right)<\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]\le\frac{\mathbf{f'(0)}}{p_I},
\end{cases}
\end{aligned}
$$

$$\begin{cases} \big([0, p_I(\mathbf{I} - S_F)], \{p_I(\mathbf{I} - S_F - E_B)\}, \{\mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\}, [\tilde{\mathbf{I}}, \mathbf{W}]\big) \\ \qquad \text{if } \frac{\mathbf{f}'(\mathbf{W})}{p_I} < \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] = \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] = \frac{1}{\beta p_I} \leq \frac{\mathbf{f}'(\mathbf{0})}{p_I}, \\ \big([0, p_I(\mathbf{W} - S_F)], \{p_I(\mathbf{W} - S_F - E_B)\}, \{\mathbf{f}'^{-1}\left(p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]\right)\}, \{\mathbf{W}\}\big) \\ \qquad \text{if } \max\left(\frac{\mathbf{f}'(\mathbf{W})}{p_I}, \frac{1}{\beta p_I}\right) < \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] = \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] \leq \frac{\mathbf{f}'(\mathbf{0})}{p_I}, \end{cases} \tag{4.13}$$

where we define $\tilde{\mathbf{I}} := \mathbf{f}'^{-1}(1/\beta)$.

The proof of Lemma 13 is given in Appendix 4.A. We can now investigate the equilibria arising in this set-up. We obtain

Theorem 5

The existence and the form of potential equilibria with banks depend on the model parameters. We distinguish four cases:

(i) If $\dfrac{1}{\beta} < \overline{\mathbf{R}}_{\mathbf{M}}$*, given the policy gross rates* $(R_{CB}^s)_{s=l,h}$*, all equilibria with banks take the following form:*

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^s, \quad \mathbf{R}_{\mathbf{F}}^* = \overline{\mathbf{R}}_{\mathbf{M}}, \tag{4.14}$$

$$p_I^* = p, \quad p_C^{s*} = p\frac{R_{CB}^s}{\mathbf{R}_{\mathbf{M}}^{\mathbf{s}}}, \tag{4.15}$$

$$E_B^* = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\Big), \quad D_H^* = (1 - \varphi^*)p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\Big), \tag{4.16}$$

$$\tilde{D}_H^{s*} = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\Big)R_{CB}^s, \tag{4.17}$$

$$L_M^* = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\Big), \quad S_F^* = \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big), \tag{4.18}$$

$$\mathbf{I}^* = \mathbf{W}, \quad \mathbf{K}_{\mathbf{M}}^* = \mathbf{I} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big), \quad \mathbf{K}_{\mathbf{F}}^* = \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big), \tag{4.19}$$

where the price of the investment good denoted by $p \in (0, +\infty)$ *and the aggregate equity ratio* $\varphi^* \in (0, 1)$ *are arbitrary. The equilibrium profits of firms and banks are given by*

$$\Pi_M^{s*} = 0, \quad \Pi_F^{s*} = p\frac{R_{CB}^s}{\mathbf{R}_{\mathbf{M}}^{\mathbf{s}}}\left(\mathbf{f}\Big(\mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\Big) - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\overline{\mathbf{R}}_{\mathbf{M}}\right), \tag{4.20}$$

$$\Pi_B^{s*} = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\Big)R_{CB}^s. \tag{4.21}$$

(ii) If $\frac{1}{\beta} = \overline{\mathbf{R}}_{\mathbf{M}}$*, given the policy gross rates* $(R^s_{CB})_{s=l,h}$*, all equilibria with banks take the following form:*

$$R^{s*}_E = R^{s*}_D = R^{s*}_L = R^s_{CB}, \quad \mathbf{R}^*_{\mathbf{F}} = \overline{\mathbf{R}}_{\mathbf{M}}, \tag{4.22}$$

$$p^*_I = p, \quad p^{s*}_C = p\frac{R^s_{CB}}{\mathbf{R}^{\mathbf{s}}_{\mathbf{M}}}, \tag{4.23}$$

$$E^*_B = \varphi^* p\Big(\mathbf{S} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\Big), \quad D^*_H = (1-\varphi^*)p\Big(\mathbf{S} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\Big), \tag{4.24}$$

$$\tilde{D}^{s*}_H = p\Big(\mathbf{S} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\Big)R^s_{CB}, \tag{4.25}$$

$$L^*_M = p\Big(\mathbf{S} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\Big), \quad S^*_F = \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big), \tag{4.26}$$

$$\mathbf{I}^* = \mathbf{S}, \quad \mathbf{K}^*_{\mathbf{M}} = \mathbf{S} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big), \quad \mathbf{K}^*_{\mathbf{F}} = \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big), \tag{4.27}$$

where the price of the investment good denoted by $p \in (0, +\infty)$*, the aggregate equity ratio* $\varphi^* \in (0,1)$*, and the savings* $\mathbf{S} \in [\tilde{\mathbf{I}}, \mathbf{W}]$ *are arbitrary. The equilibrium profits of firms and banks are given by*

$$\Pi^{s*}_M = 0, \quad \Pi^{s*}_F = p\frac{R^s_{CB}}{\mathbf{R}^{\mathbf{s}}_{\mathbf{M}}}\left(\mathbf{f}\Big(\mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\Big) - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\overline{\mathbf{R}}_{\mathbf{M}}\right), \tag{4.28}$$

$$\Pi^{s*}_B = \varphi^* p\Big(\mathbf{S} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_{\mathbf{M}}\big)\Big)R^s_{CB}. \tag{4.29}$$

(iii) If $\overline{\mathbf{R}}_{\mathbf{M}} < \frac{1}{\beta} \leq \mathbf{f}'(\mathbf{0})$*, given any policy gross rates* $(R^s_{CB})_{s=l,h}$*, there is no equilibrium with banks. The allocation is as follows:*

$$\mathbf{I}^* = \tilde{\mathbf{I}}, \quad \mathbf{K}^*_{\mathbf{M}} = \mathbf{0}, \quad \mathbf{K}^*_{\mathbf{F}} = \tilde{\mathbf{I}}. \tag{4.30}$$

(iv) If $\mathbf{f}'(\mathbf{0}) < \frac{1}{\beta}$*, given any policy gross rates* $(R^s_{CB})_{s=l,h}$*, there is no equilibrium with banks. The allocation is as follows:*

$$\mathbf{I}^* = \mathbf{0}, \quad \mathbf{K}^*_{\mathbf{M}} = \mathbf{0}, \quad \mathbf{K}^*_{\mathbf{F}} = \mathbf{0}. \tag{4.31}$$

The proof of Theorem 5 is given in Appendix 4.A. This theorem describes the classical consumption-savings trade-off: When the real interest rate is high, households are more willing to save and when it is low, households increase their immediate consumption. In terms of welfare, we now show that the equilibria found in Theorem 5 all implement the first-best allocation. We obtain

Proposition 23
The equilibria with and without banks found in Theorem 5 all implement the first-best allocation.

The proof of Proposition 23 is given in Appendix 4.A. Note that if $\overline{\mathbf{R}}_{\mathbf{M}} < \frac{1}{\beta}$, no equilibrium with banks exist. In particular, if $\overline{\mathbf{R}}_{\mathbf{M}} < 1$, there is no equilibrium with banks for all $\beta < 1$. This differs from the zero-lower bound problem in Chapter 2, as the allocation implemented when $\overline{\mathbf{R}}_{\mathbf{M}} < \frac{1}{\beta}$ is first-best and as there is thus no welfare improvement possible by regulation. When $\overline{\mathbf{R}}_{\mathbf{M}} \geq \frac{1}{\beta}$, equilibria with banks exist and the results concerning capital requirements and their welfare benefits from Chapter 2 in the case where prices are rigid and $R_{CB}^s \neq \mathbf{R}_{\mathbf{M}}^{\mathbf{s}}$ for some state s continue to hold in this set-up.

Appendix

4.A Proofs

Proof of Lemma 12.
Let $E_B \in (0, p_I \mathbf{W})$. From the definition of h_E and the Inada Conditions, we obtain

$$\lim_{S_F \to 0} h_E(E_B, S_F) > 0 \quad \text{and} \quad \lim_{S_F \to \mathbf{W}} h_E(E_B, S_F) < 0.$$

Moreover, to examine the monotonicity of h_E with respect to S_F, we can calculate its partial derivate with respect to S_F as follows:

$$\begin{aligned}
&\frac{\partial h_E}{\partial S_F}(E_B, S_F) = \\
&\mathbb{E}\left[\frac{\mathbf{f}''(S_F)}{p_I} U'\left(E_B \frac{R_E^s}{p_C^s} + \big(p_I(\mathbf{W} - S_F) - E_B\big)\frac{R_D^s}{p_C^s} + \mathbf{f}(S_F)\right)\right] + \\
&p_I \mathbb{E}\left[\left(\frac{\mathbf{f}'(S_F)}{p_I} - \frac{R_D^s}{p_C^s}\right)^2 U''\left(E_B \frac{R_E^s}{p_C^s} + \big(p_I(\mathbf{W} - S_F) - E_B\big)\frac{R_D^s}{p_C^s} + \mathbf{f}(S_F)\right)\right] \\
&< 0,
\end{aligned}$$

as U and f are concave. As $S_F \mapsto h_E(E_B, S_F)$ is continuous, by the intermediate value theorem, for all $E_B \in (0, p_I \mathbf{W})$, there exists a unique value, which we denote by $S_F^{h_E}(E_B)$, such that

$$h_E(E_B, S_F) = 0.$$

We note that $E_B \mapsto S_F^{h_E}(E_B)$ is continuously differentiable. We can now use the stated definition of X_E and we note that X_E is also continuously differentiable in E_B. We first investigate the monotonicity of X_E by calculating its derivative as follows:

$$\frac{dX_E}{dE_B} = \frac{\partial g_E}{\partial E_B} + \frac{\partial g_E}{\partial S_F}\frac{dS_F^{h_E}}{dE_B}. \tag{4.32}$$

We now calculate the first term in the addition as follows:

$$\begin{aligned}\frac{\partial g_E}{\partial E_B} =& -\mathbb{E}\left[U''\left(E_B\frac{R_E^s}{p_C^s} + \big(p_I(\mathbf{W} - S_F) - E_B\big)\frac{R_D^s}{p_C^s} + \mathbf{f}(S_F)\right)\left(\frac{R_D^s - R_E^s}{p_C^s}\right)^2\right] \\ &> 0.\end{aligned}$$

For the second term in Equation (4.32), we note that

$$\begin{aligned}&\frac{\partial g_E}{\partial S_F} = \\ &\mathbb{E}\left[\left(\mathbf{f}'(S_F) - p_I\frac{R_D^s}{p_C^s}\right)\frac{R_D^s - R_E^s}{p_C^s}U''\left(E_B\frac{R_E^s}{p_C^s} + \big(p_I(\mathbf{W} - S_F) - E_B\big)\frac{R_D^s}{p_C^s} + \mathbf{f}(S_F)\right)\right] \\ &= -p_I\frac{\partial h_E}{\partial E_B}.\end{aligned}$$

Moreover, we note that

$$\frac{dS_F^{h_E}}{dE_B} = -\frac{\partial h_E/\partial E_B}{\partial h_E/\partial S_F}.$$

Thus, Equation (4.32) re-writes as follows:

$$\begin{aligned}\frac{dX_E}{dE_B} =& \frac{\frac{\partial g_E}{\partial E_B}\frac{\partial h_E}{\partial S_F} - \frac{\partial g_E}{\partial S_F}\frac{\partial h_E}{\partial E_B}}{\frac{\partial h_E}{\partial S_F}} \\ =& - \frac{1}{p_I \frac{\partial h_E}{\partial S_F}} \left(\mathbb{E}\left[U''(\mathbf{C^s}) \left(\frac{R_D^s - R_E^s}{p_C^s} \right)^2 \right] \mathbb{E}\left[U''(\mathbf{C^s}) \left(\mathbf{f}'(S_F) - \frac{R_D^s}{p_C^s} p_I \right)^2 \right] \right. \\ & \left. - \mathbb{E}\left[\left(\mathbf{f}'(S_F) - \frac{R_D^s}{p_C^s} p_I \right) \left(\frac{R_D^s - R_E^s}{p_C^s} \right) U''(\mathbf{C^s}) \right]^2 \right) \\ & + \frac{\mathbb{E}[\mathbf{f}''(S_F) U'(\mathbf{C^s})] \mathbb{E}\left[U''(\mathbf{C^s}) \left(\frac{R_D^s - R_E^s}{p_C^s} \right)^2 \right]}{-p_I \frac{\partial h_E}{\partial S_F}}.\end{aligned}$$

Using the Theorem of Cauchy-Schwarz, which is given by

$$\mathbb{E}\Big[(Y^s)^2\Big] \mathbb{E}\Big[(Z^s)^2\Big] \geq \mathbb{E}[Y^s Z^s]^2,$$

with

$$\begin{aligned} Y^s &= \sqrt{-U''(\mathbf{C^s})} \frac{R_D^s - R_E^s}{p_C^s} \\ \text{and} \quad Z^s &= \sqrt{-U''(\mathbf{C^s})} \left(\mathbf{f}'(S_F) - \frac{R_D^s}{p_C^s} p_I \right), \end{aligned}$$

we obtain

$$\begin{aligned} &\left(\mathbb{E}\left[U''(\mathbf{C^s}) \left(\frac{R_D^s - R_E^s}{p_C^s} \right)^2 \right] \mathbb{E}\left[U''(\mathbf{C^s}) \left(\mathbf{f}'(S_F) - \frac{R_D^s}{p_C^s} p_I \right)^2 \right] \right. \\ & \qquad \left. - \mathbb{E}\left[\left(\mathbf{f}'(S_F) - \frac{R_D^s}{p_C^s} p_I \right) \left(\frac{R_D^s - R_E^s}{p_C^s} \right) U''(\mathbf{C^s}) \right]^2 \right) > 0 \end{aligned}$$

and as $\partial h_E / \partial S_F < 0$, we obtain

$$\frac{dX_E}{dE_B} > 0.$$

From the definition of h_E, we obtain

$$h_E\left(E_B, \mathbf{W} - \frac{E_B}{p_I}\right) = \mathbb{E}\left[\left(\frac{\mathbf{f}'\left(\mathbf{W} - \frac{E_B}{p_I}\right)}{p_I} - \frac{R_D^s}{p_C^s}\right) U'\left(E_B \frac{R_E^s}{p_C^s} + \mathbf{f}\left(\mathbf{W} - \frac{E_B}{p_I}\right)\right)\right].$$

We first note that

$$\lim_{E_B \to 0} h_E\left(E_B, \mathbf{W} - \frac{E_B}{p_I}\right) < 0 \quad \text{and that} \quad \lim_{E_B \to p_I \mathbf{W}} h_E\left(E_B, \mathbf{W} - \frac{E_B}{p_I}\right) > 0.$$

As $E_B \mapsto h_E(E_B, \mathbf{W} - E_B/p_I)$ is a continuous function, by the intermediate value theorem there exists a solution E_B^{MAX} to the equation

$$h_E\left(E_B, \mathbf{W} - \frac{E_B}{p_I}\right) = 0.$$

We now show that when $\frac{R_E^s}{p_C^s} < \frac{R_D^s}{p_C^s} < \frac{R_D^{s'}}{p_C^{s'}} < \frac{R_E^{s'}}{p_C^{s'}}$ for some states $s \neq s'$, this solution is unique.

Suppose that $\frac{R_E^s}{p_C^s} < \frac{R_D^s}{p_C^s} < \frac{R_D^{s'}}{p_C^{s'}} < \frac{R_E^{s'}}{p_C^{s'}}$ for some states $s \neq s'$. Then the equation $h_E(E_B, \mathbf{W} - \frac{E_B}{p_I}) = 0$ implies that $\frac{\mathbf{f}'\left(\mathbf{W} - \frac{E_B^{MAX}}{p_I}\right)}{p_I} \in \left(\frac{R_D^s}{p_C^s}, \frac{R_D^{s'}}{p_C^{s'}}\right)$. We can calculate

$$\frac{dh_E\left(E_B, \mathbf{W} - \frac{E_B}{p_I}\right)}{dE_B} = \mathbb{E}\left[-\frac{\mathbf{f}''\left(\mathbf{W} - \frac{E_B}{p_I}\right)}{p_I^2} U'\left(E_B \frac{R_E^s}{p_C^s} + \mathbf{f}\left(\mathbf{W} - \frac{E_B}{p_I}\right)\right)\right.$$
$$\left. -U''\left(E_B \frac{R_E^s}{p_C^s} + \mathbf{f}\left(\mathbf{W} - \frac{E_B}{p_I}\right)\right)\left(\frac{\mathbf{f}'\left(\mathbf{W} - \frac{E_B}{p_I}\right)}{p_I} - \frac{R_D^s}{p_C^s}\right)\left(\frac{\mathbf{f}'\left(\mathbf{W} - \frac{E_B}{p_I}\right)}{p_I} - \frac{R_E^s}{p_C^s}\right)\right].$$

For all E_B such that $\frac{\mathbf{f}'\left(\mathbf{W} - \frac{E_B}{p_I}\right)}{p_I} \in \left(\frac{R_D^s}{p_C^s}, \frac{R_D^{s'}}{p_C^{s'}}\right)$, $\frac{dh_E\left(E_B, \mathbf{W} - \frac{E_B}{p_I}\right)}{dE_B} > 0$. Therefore, there is a unique solution to the equation $h_E(E_B, \mathbf{W} - \frac{E_B}{p_I}) = 0$.

By symmetry when we replace E_B by D_H and R_E^s by R_D^s, we obtain similar properties for h_D and X_D. $\square$

Proof of Proposition 21.
An interior solution D_H, E_B, and S_F to the representative household's expected utility maximization problem fulfills the following system of equations:

$$\begin{aligned} & g_E(E_B, S_F) = 0, \\ & h_E(E_B, S_F) = 0, \\ \text{and} \quad & E_B + D_H + p_I S_F = p_I \mathbf{W}. \end{aligned}$$

This system is clearly equivalent to

$$\begin{aligned} & g_D(D_H, S_F) = 0, \\ & h_D(D_H, S_F) = 0, \\ \text{and} \quad & E_B + D_H + p_I S_F = p_I \mathbf{W}. \end{aligned}$$

We distinguish the following cases:

- Note first that there is no interior solution with $R_D^s \leq R_E^s$ for all states $s = l, h$ with at least one strict inequality or $R_D^s \geq R_E^s$ for all states $s = l, h$ with at least one strict inequality, as in the former case, $g_E(E_B, S_F) < 0$ and in the latter case, $g_E(E_B, S_F) > 0$ for all $E_B, S_F > 0$.
- Assume now that $R_E^s = R_D^s$ for all states $s = l, h$. Then $g_E(E_B, S_F) = 0$ for all $E_B, S_F > 0$. Moreover, as $\frac{dh_E\left(E_B, \mathbf{W} - \frac{E_B}{p_I}\right)}{dE_B} > 0$ and $h_E(E_B, \mathbf{W} - \frac{E_B}{p_I}) < 0$ for E_B small enough and $h_E(E_B, \mathbf{W} - \frac{E_B}{p_I}) > 0$ for E_B large enough, the equation $h_E(E_B, \mathbf{W} - \frac{E_B}{p_I}) = 0$ has a unique solution denoted by E_B^{MAX}. We now focus on the equation $h_E(E_B, S_F) = 0$. As $\frac{\partial h_E}{\partial S_F} < 0$ and $h_E(E_B, 0) > 0$, the previous equation has a solution $S_F \in (0, \mathbf{W} - \frac{E_B}{p_I})$ if and only if $h_E(E_B, \mathbf{W} - \frac{E_B}{p_I}) < 0$, i.e. if and only if $E_B < E_B^{MAX}$. This solution is then unique and we denote it by $S_F^{h_E}(E_B)$. As $g_E(E_B, S_F) = 0$ for all $E_B, S_F > 0$, $X_E(E_B) = 0$ if and only if $E_B \in (0, E_B^{MAX})$.
- Assume now that $\frac{R_E^s}{p_C^s} < \frac{R_D^s}{p_C^s} < \frac{R_D^{s'}}{p_C^{s'}} < \frac{R_E^{s'}}{p_C^{s'}}$ for some states $s \neq s'$. We know from Lemma 12 that there is a unique solution to the equation $h_E(E_B, \mathbf{W} - \frac{E_B}{p_I}) = 0$, which is denoted by E_B^{MAX}. Moreover, $h_E(E_B, \mathbf{W} - \frac{E_B}{p_I}) < 0$ when E_B is small enough and $h_E(E_B, \mathbf{W} - \frac{E_B}{p_I}) > 0$ when E_B is large enough. Thus,

$h_E(E_B, \mathbf{W} - \frac{E_B}{p_I}) < 0$ for all $E_B < E_B^{MAX}$ and $h_E(E_B, \mathbf{W} - \frac{E_B}{p_I}) > 0$ for all $E_B > E_B^{MAX}$. We now focus on the equation $h_E(E_B, S_F) = 0$. As $\frac{\partial h_E}{\partial S_F} < 0$ and $h_E(E_B, 0) > 0$, the previous equation has a solution $S_F \in (0, \mathbf{W} - \frac{E_B}{p_I})$ if and only if $h_E(E_B, \mathbf{W} - \frac{E_B}{p_I}) < 0$, i.e. if and only if $E_B < E_B^{MAX}$. We denote this solution by $S_F^{h_E}(E_B)$. As $\frac{dX_E}{dE_B} > 0$, $X_E(E_B) = 0$ has a solution $E_B \in (0, E_B^{MAX})$ if and only if $X_E(0) < 0 < X_E(E_B^{MAX})$. This solution is then unique.

– The same analysis can be done by symmetry with g_D, h_D, and X_D when R_E^s is replaced by R_D^s and E_B by D_H.

□

Proof of Proposition 22.

By Definition 7, a first-best allocation $(\mathbf{K_F^{FB}}, \mathbf{K_M^{FB}})$ maximizes the welfare function

$$\mathbb{E}\Big[U\big(\mathbf{K_M}\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F})\big)\Big]$$

under the constraint $\mathbf{K_F} + \mathbf{K_M} = \mathbf{W}$. This welfare function can thus be expressed in terms of $\mathbf{K_F}$ only and we denote it by $g(\mathbf{K_F})$. First, we note that g is strictly concave:

$$\begin{aligned} g''(\mathbf{K_F}) =& \frac{d}{d\mathbf{K_F}}\Big(\mathbb{E}\Big[U'\big((\mathbf{W} - \mathbf{K_F})\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F})\big)\big(\mathbf{f}'(\mathbf{K_F}) - \mathbf{R_M^s}\big)\Big]\Big) \\ =& \Big(\mathbb{E}\Big[U''\big((\mathbf{W} - \mathbf{K_F})\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F})\big)\big(\mathbf{f}'(\mathbf{K_F}) - \mathbf{R_M^s}\big)^2\Big]\Big) \\ &+ \Big(\mathbb{E}\Big[U'\big((\mathbf{W} - \mathbf{K_F})\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F})\big)\mathbf{f}''(\mathbf{K_F})\Big]\Big) < 0, \end{aligned}$$

as U and $\mathbf{f}$ are strictly concave. Thus, g has a unique maximum. Because of the Inada Conditions, this maximum has to be attained for some $\mathbf{K_F} \in (\mathbf{0}, \mathbf{W})$. As consequence, $(\mathbf{K_F^{FB}}, \mathbf{K_M^{FB}})$ is unique and is characterized by the first-order condition given by

$$\mathbb{E}\Big[U'\big((\mathbf{W} - \mathbf{K_F})\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F})\big)\mathbf{f}'(\mathbf{K_F})\Big] = \mathbb{E}\Big[U'\big((\mathbf{W} - \mathbf{K_F})\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F})\big)\mathbf{R_M^s}\Big].$$

□

Proof of Theorem 4.

Let $\mathcal{E}^*$ be an equilibrium with banks.

Then all banks choose the same level of money creation and lending, denoted by α_M^*. At the aggregate level, however, the amount borrowed by banks from the central bank has to equal the amount deposited by banks at the central bank, meaning that $\int_0^1 \alpha_M^b \mathrm{d}b = 1$, which translates into $\alpha_M^* = 1$. The result of Lemma 2 implies that we can apply Proposition 1. Thus, given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, and the equity ratio φ^*, all banks $b \in [0,1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M\big((R_L^{s*})_s, (R_{CB}^s)_s, \varphi^*\big)$ as given in Proposition 1. The only gross rates of return in Proposition 1 rationalizing $\alpha_M^* = 1$ are

$$R_L^{s*} = R_{CB}^s$$

for all states $s = l, h$. A direct consequence of this relation and of Lemma 2 is that

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^s \tag{4.33}$$

for all states $s = l, h$. Moreover, due to Lemma 2 and the tie-breaking rule introduced in Subsection 2.2.3, the interbank lending market is not used in an equilibrium with banks. Finally, $\Pi_M^{s*} = 0$ for all states $s = l, h$ (see Subsection 2.2.4), which translates into

$$\mathbf{R_M^s} p_C^{s*} = R_L^{s*} p_I^*$$

for all states $s = l, h$. Given gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices p_I^* and $(p_C^{s*})_s$, households choose E_B^*, D_H^*, and S_F^* according to Proposition 21. Only the case (i) in Proposition 21 corresponds to equal nominal gross rates of return R_E^{s*} and R_D^{s*}, and thus, are consistent with the equality of nominal gross rates of return in Equation (4.33). We thus obtain

$$\begin{aligned}
&E_B^* \in (0, E_B^{MAX}), \\
&D_H^* = p_I^* \left(\mathbf{W} - S_F^*\right) - E_B^*, \text{ and} \\
&S_F^* = S_F^{h_E}(E_B^*).
\end{aligned}$$

As $R_D^{s*} = R_E^{s*} = R_{CB}^s = p_C^{s*}\frac{\mathbf{R_M^s}}{p_I^*}$, $\mathbf{K_F^*} = S_F^{h_E}(E_B^*)$ solves the following equation:

$$\mathbb{E}\Big[U'\big((\mathbf{W}-\mathbf{K_F})\mathbf{R_M^s}+\mathbf{f}(\mathbf{K_F})\big)\mathbf{f}'(\mathbf{K_F})\Big] = \mathbb{E}\Big[U'\big((\mathbf{W}-\mathbf{K_F})\mathbf{R_M^s}+\mathbf{f}(\mathbf{K_F})\big)\mathbf{R_M^s}\Big].$$

From the proof of Proposition 22, we note that $\mathbf{K_F^{FB}}$ solves the unique solution of this equation. Thus, $\mathbf{K_F^*} = \mathbf{K_F^{FB}}$. Finally, $\mathbf{R_F^*}$ can be determined by using Lemma 4 and equating the demand for the investment good $\mathbf{K_F^*}$ to its supply S_F^*. Thus, with the help of the equity ratio φ^*, we can re-write all equilibrium variables as given in Theorem 4.

In turn, it is straightforward to verify that the tuples given in Theorem 4 constitute equilibria with banks as defined in Definition 6. □

Proof of Lemma 13.
The Lagrangean for this maximization problem writes

$$\begin{aligned} L_H = &(\mathbf{W}-\mathbf{I}) + \beta\mathbb{E}\left[E_B\frac{R_E^s}{p_C^s} + D_H\frac{R_D^s}{p_C^s} + \mathbf{f}(S_F)\right] \\ &- \lambda_I(E_B + D_H + p_I S_F - p_I\mathbf{I}) - \gamma_I(\mathbf{I}-\mathbf{W}), \end{aligned}$$

where λ_I and γ_I denote the Lagrange parameters associated with the constraints $p_I\mathbf{I} = E_B + D_H + p_I S_F$ and $\mathbf{W} \geq \mathbf{I}$, respectively. As the objective function of the households' maximization problem is concave and the constraints are linear, the Kuhn-Tucker Conditions for an optimum are necessary and sufficient. By writing these conditions and solving for the system, we will thus find all possible solutions. The system of equations writes

$$\begin{cases} \beta\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] \leq \lambda_I, & E_B \geq 0, \quad 0 = E_B\left(\beta\mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] - \lambda_I\right), \\ \beta\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq \lambda_I, & D_H \geq 0, \quad 0 = D_H\left(\beta\mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] - \lambda_I\right), \\ \beta\mathbf{f}'(S_F) - p_I\lambda_I \leq 0, & S_F \geq 0, \quad 0 = S_F\big(\beta\mathbf{f}'(S_F) - p_I\lambda_I\big), \\ -1 + \lambda_I p_I - \gamma_I \leq 0, & \mathbf{I} \geq 0, \quad 0 = \mathbf{I}\big(-1 + \lambda_I p_I - \gamma_I\big), \\ \gamma_I \geq 0, & \mathbf{I} \leq \mathbf{W}, \quad 0 = \gamma_I(\mathbf{I}-\mathbf{W}), \\ & \lambda_I \geq 0, \quad 0 = S_F p_I + E_B + D_H - p_I\mathbf{I}. \end{cases}$$

We first solve for possible constellations with $\gamma_I = 0$. We first treat the case with

$\mathbf{I} = \mathbf{0}$. In this case, $E_B = D_H = S_F = 0$ and $\lambda_I > 0$. We thus obtain

$$\begin{cases} \beta \mathbf{f}'(\mathbf{0}) - 1 \leq 0, \\ \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] \leq \frac{1}{\beta p_I}, \\ \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq \frac{1}{\beta p_I}. \end{cases}$$

We note that this case corresponds to the fourth case in the definition of the correspondence in Lemma 13.

Assume now that $I > 0$. Then either $E_B \neq 0$ or $D_H \neq 0$ or $S_F \neq 0$. We thus investigate the following cases:

- $E_B, D_H = 0 < S_F$ and in this case, $\lambda_I > 0$. We thus obtain

$$\begin{cases} \beta \mathbf{f}'(S_F) = 1, \\ \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] \leq \frac{1}{\beta p_I}, \\ \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq \frac{1}{\beta p_I}, \\ S_F = \mathbf{I}. \end{cases}$$

  This implies that $\mathbf{f}'(\mathbf{W}) \leq \frac{1}{p_I} \leq \mathbf{f}'(\mathbf{0})$. This case corresponds to the ninth case[2] in the definition of the correspondence in Lemma 13.

- $E_B = 0 < D_H, S_F$ and in this case, $\lambda_I > 0$. We thus obtain

$$\begin{cases} \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] = \frac{1}{\beta p_I} = \frac{\mathbf{f}'(S_F)}{p_I}, \\ \mathbf{f}'(\mathbf{W}) \leq \frac{1}{\beta} \leq \mathbf{f}'(\mathbf{0}). \end{cases}$$

  This case corresponds to the tenth case in the definition of the correspondence in Lemma 13.

- $D_H = 0 < E_B, S_F$ and in this case, $\lambda_I > 0$. We thus obtain

$$\begin{cases} \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] = \frac{1}{\beta p_I} = \frac{\mathbf{f}'(S_F)}{p_I}, \\ \mathbf{f}'(\mathbf{W}) \leq \frac{1}{\beta} \leq \mathbf{f}'(\mathbf{0}). \end{cases}$$

[2] By this statement and following similar ones, we generally exclude the boundary cases, which will not be addressed specifically.

This case corresponds to the twelfth case in the definition of the correspondence in Lemma 13.

– $S_F = 0 < E_B, D_H$ and we thus obtain

$$\begin{cases} \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] = \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] = \frac{1}{\beta p_I}, \\ \mathbf{f}'(\mathbf{0}) \leq \frac{1}{\beta}. \end{cases}$$

This case corresponds to the seventh case in the definition of the correspondence in Lemma 13.

– $S_F = E_B = 0 < D_H$ and in this case, we thus obtain

$$\begin{cases} \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] = \frac{1}{\beta p_I}, \\ \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] \leq \frac{1}{\beta p_I}, \\ \mathbf{f}'(\mathbf{0}) \leq \frac{1}{\beta}. \end{cases}$$

This case corresponds to the second case in the definition of the correspondence in Lemma 13.

– $S_F = D_H = 0 < E_B$ and in this case, we thus obtain

$$\begin{cases} \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] = \frac{1}{\beta p_I}, \\ \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq \frac{1}{\beta p_I}, \\ \mathbf{f}'(\mathbf{0}) \leq \frac{1}{\beta}. \end{cases}$$

This case corresponds to the fifth case in the definition of the correspondence in Lemma 13.

– $E_B, D_H, S_F > 0$ and in this case, we thus obtain

$$\begin{cases} \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] = \frac{1}{\beta p_I}, \\ \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] = \frac{1}{\beta p_I}, \\ \mathbf{f}'(S_F) = \frac{1}{\beta}, \\ \mathbf{f}'(\mathbf{W}) \leq \frac{1}{\beta} \leq \mathbf{f}'(\mathbf{0}). \end{cases}$$

This case corresponds to the fourteenth case in the definition of the correspondence in Lemma 13.

We now tackle the case with $\gamma_I > 0$. Then $\mathbf{I} = \mathbf{W}$ and we thus obtain the following cases:

- $E_B, D_H = 0 < S_F$ and in this case, $\lambda_I > 0$. We thus obtain

$$\begin{cases} \lambda_I = \frac{\beta \mathbf{f}'(\mathbf{W})}{p_I}, \\ \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] \leq \frac{\mathbf{f}'(\mathbf{W})}{p_I}, \\ \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq \frac{\mathbf{f}'(\mathbf{W})}{p_I}, \\ \mathbf{f}'(\mathbf{W}) \geq \frac{1}{\beta}. \end{cases}$$

  This case corresponds to the first case in the definition of the correspondence in Lemma 13.

- $E_B = 0 < D_H, S_F$ and in this case, $\lambda_I > 0$. We thus obtain

$$\begin{cases} \mathbf{f}'(S_F) \geq \frac{1}{\beta}, \\ \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] \leq \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] = \frac{\mathbf{f}'(S_F)}{p_I}, \\ \mathbf{f}'(\mathbf{W}) \leq p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq \mathbf{f}'(\mathbf{0}). \end{cases}$$

  This case corresponds to the eleventh case in the definition of the correspondence in Lemma 13.

- $D_H = 0 < E_B, S_F$ and in this case, $\lambda_I > 0$. We thus obtain

$$\begin{cases} \mathbf{f}'(S_F) \geq \frac{1}{\beta}, \\ \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] = \frac{\mathbf{f}'(S_F)}{p_I}, \\ \mathbf{f}'(\mathbf{W}) \leq p_I \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] \leq \mathbf{f}'(\mathbf{0}). \end{cases}$$

  This case corresponds to the thirteenth case in the definition of the correspondence in Lemma 13.

- $S_F = 0 < E_B, D_H$ and we thus obtain

$$\begin{cases} \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] = \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \geq \frac{1}{\beta p_I}, \\ \mathbf{f}'(\mathbf{0}) \leq p_I \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] = p_I \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]. \end{cases}$$

This case corresponds to the eighth case in the definition of the correspondence in Lemma 13.

– $S_F = E_B = 0 < D_H$ and in this case, we thus obtain

$$\begin{cases} \beta \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] = \lambda_I, \\ \frac{\mathbf{f}'(\mathbf{0})}{p_I} \leq \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right], \\ \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] \leq \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right], \\ \frac{1}{\beta p_I} \leq \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right]. \end{cases}$$

This case corresponds to the third case in the definition of the correspondence in Lemma 13.

– $S_F = D_H = 0 < E_B$ and in this case, we thus obtain

$$\begin{cases} \beta \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] = \lambda_I, \\ \frac{\mathbf{f}'(\mathbf{0})}{p_I} \leq \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right], \\ \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] \leq \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right], \\ \frac{1}{\beta p_I} \leq \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right]. \end{cases}$$

This case corresponds to the sixth case in the definition of the correspondence in Lemma 13.

– $E_B, D_H, S_F > 0$ and in this case, we thus obtain

$$\begin{cases} \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] = \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] = \frac{\lambda_I}{\beta} = \frac{\mathbf{f}'(S_F)}{p_I} \geq \frac{1}{\beta p_I}, \\ \mathbf{f}'(\mathbf{W}) \leq \mathbb{E}\left[\frac{R_E^s}{p_C^s}\right] = \mathbb{E}\left[\frac{R_D^s}{p_C^s}\right] = \frac{\lambda_I}{\beta} = \frac{\mathbf{f}'(S_F)}{p_I} \leq \mathbf{f}'(\mathbf{0}). \end{cases}$$

This case corresponds to the fifteenth case in the definition of the correspondence in Lemma 13.

□

Proof of Theorem 5.

All arguments not related to households' portfolio and consumption-savings choice carry on from Theorem 1. These arguments imply that in any equilibrium with banks,

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^s = \frac{\mathbf{R}_\mathbf{M}^\mathbf{s} p_C^{s*}}{p_I^*}.$$

Households make their portfolio and consumption-savings decision according to Lemma 13. The only choices that are consistent with the above equality between gross rates of return are given in Cases 4, 9, 14, and 15 of the definition of the correspondence in Lemma 13. Banks exist only in Cases 14 and 15. We thus obtain Theorem 5. □

Proof of Proposition 23.
The first-best allocation is obtained by maximizing the households' utility subject to their budget constraint, as follows:

$$\max_{\{\mathbf{K_F},\mathbf{K_M},\mathbf{I}\}} \Big\{ U := \mathbf{C_0} + \beta \mathbb{E}\big[\mathbf{C_1^s}\big] \Big\}$$

$$\text{s.t.} \quad \begin{cases} \mathbf{C_0} = \mathbf{W} - \mathbf{I}, \\ \mathbf{C_1^s} = \mathbf{K_M}\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F}), \end{cases}$$

$$\text{and} \quad \mathbf{K_F} + \mathbf{K_M} = \mathbf{I}.$$

We can re-write this maximization problem as follows:

$$\max_{\{\mathbf{K_F},\mathbf{I}\}} \Big\{ U := \mathbf{W} - \mathbf{I} + \beta(\mathbf{I} - \mathbf{K_F})\overline{\mathbf{R}}_\mathbf{M} + \mathbf{f}(\mathbf{K_F}) \Big\}$$

$$\text{s.t.} \quad \mathbf{K_F}, \mathbf{I} \geq 0,$$

$$\mathbf{I} \geq \mathbf{K_F},$$

$$\text{and} \quad \mathbf{W} \geq \mathbf{I}.$$

We denote by

$$L = (\mathbf{W} - \mathbf{I}) + \beta\big((\mathbf{I} - \mathbf{K_F})\overline{\mathbf{R}}_\mathbf{M} + \mathbf{f}(\mathbf{K_F})\big) - \lambda(\mathbf{K_F} - \mathbf{I}) - \gamma(\mathbf{I} - \mathbf{W})$$

the Lagrangean associated with this maximization problem, where λ and γ denote the Lagrange parameters corresponding to the constraints $\mathbf{I} \geq \mathbf{K_F}$ and $\mathbf{W} \geq \mathbf{I}$, respectively. As the objective function of the households' utility maximization problem is concave and the constraints are linear, the Kuhn-Tucker Conditions for an optimum are necessary and sufficient. By writing theses conditions and solving for the system, we will thus find all possible solutions. The system of equations writes

$$\frac{\partial L}{\partial \mathbf{I}} = -1 + \beta \overline{\mathbf{R}}_{\mathbf{M}} + \lambda - \gamma \leq 0, \quad \mathbf{I} \geq 0, \quad \mathbf{I}(-1 + \beta \overline{\mathbf{R}}_{\mathbf{M}} + \lambda - \gamma) = 0,$$

$$\frac{\partial L}{\partial \mathbf{K_F}} = -\beta \overline{\mathbf{R}}_{\mathbf{M}} + \beta \mathbf{f}'(\mathbf{K_F}) - \lambda \leq 0, \quad \mathbf{K_F} \geq 0, \quad \mathbf{K_F}(-\beta \overline{\mathbf{R}}_{\mathbf{M}} + \beta \mathbf{f}'(\mathbf{K_F}) - \lambda) = 0,$$

$$\mathbf{I} \geq \mathbf{K_F}, \quad \lambda \geq 0, \quad \lambda(\mathbf{I} - \mathbf{K_F}) = 0,$$

$$\mathbf{W} \geq \mathbf{I}, \quad \gamma \geq 0, \quad \gamma(\mathbf{W} - \mathbf{I}) = 0.$$

Suppose first that $\mathbf{K_F} = \mathbf{0}$. Then $\mathbf{I} = \mathbf{0} < \mathbf{W}$ and thus $\gamma = 0$. Then

$$\frac{\partial L}{\partial \mathbf{I}} + \frac{\partial L}{\partial \mathbf{K_F}} \leq 0,$$

is equivalent to $\mathbf{f}'(\mathbf{0}) \leq \frac{1}{\beta}$. In this case, any $\lambda \in [\beta\big(\mathbf{f}'(\mathbf{0}) - \overline{\mathbf{R}}_{\mathbf{M}}\big), 1 - \beta \overline{\mathbf{R}}_{\mathbf{M}}]$ is a parameter fulfilling $\frac{\partial L}{\partial \mathbf{K_F}}$ and $\frac{\partial L}{\partial \mathbf{I}} \leq 0$, and defines a particular optimum. This allocation corresponds to the one given in Theorem 5 in Case (*iv*).

Suppose now that $\mathbf{K_F} > 0$. Then $\mathbf{I} > 0$ and we obtain the following four cases:

- $\lambda = \gamma = 0$ and in this case,

$$\begin{cases} -1 + \beta \overline{\mathbf{R}}_{\mathbf{M}} = 0, \\ \mathbf{f}'(\mathbf{K_F}) = \overline{\mathbf{R}}_{\mathbf{M}}. \end{cases}$$

  Thus, any investment level $\mathbf{I} \in [\mathbf{f}'(\overline{\mathbf{R}}_{\mathbf{M}}), \mathbf{W}]$ defines a particular optimum. These allocations correspond to the ones given in Theorem 5 in Case (*ii*).

– $\lambda = 0 < \gamma$ and in this case, $\mathbf{I} = \mathbf{W}$ and

$$\begin{cases} -1 + \beta\overline{\mathbf{R}}_{\mathbf{M}} - \gamma = 0, \\ \mathbf{f}'(\mathbf{K_F}) = \overline{\mathbf{R}}_{\mathbf{M}}. \end{cases}$$

This implies that $\overline{\mathbf{R}}_{\mathbf{M}} > \frac{1}{\beta}$. This allocation correspond to the one given in Theorem 5 in Case (i).

– $\gamma = 0 < \lambda$ and in this case, $\mathbf{K_F} = \mathbf{I}$ and

$$\begin{cases} -1 + \beta\overline{\mathbf{R}}_{\mathbf{M}} - \lambda = 0, \\ -\beta\overline{\mathbf{R}}_{\mathbf{M}} + \beta\mathbf{f}'(\mathbf{K_F}) - \lambda = 0. \end{cases}$$

This implies that $\overline{\mathbf{R}}_{\mathbf{M}} < \frac{1}{\beta}$ and $\mathbf{f}'(\mathbf{K_F}) = \frac{1}{\beta}$. This equation has a solution $\mathbf{K_F} \in [\mathbf{0}, \mathbf{W}]$ if and only if $\mathbf{f}'(\mathbf{0}) \geq \frac{1}{\beta}$. This allocation corresponds to the one given in Theorem 5 in Case (iii).

– $\lambda, \gamma > 0$ and in this case, $\mathbf{K_F} = \mathbf{I} = \mathbf{W}$ and

$$\begin{cases} -1 + \beta\overline{\mathbf{R}}_{\mathbf{M}} + \lambda - \gamma = 0, \\ -\beta\overline{\mathbf{R}}_{\mathbf{M}} + \beta\mathbf{f}'(\mathbf{K_F}) - \lambda = 0. \end{cases}$$

This implies that $\mathbf{f}'(\mathbf{W}) > \overline{\mathbf{R}}_{\mathbf{M}}$, which is excluded by assumption. Thus, there is no optimum with $\gamma, \lambda > 0$.

□

Chapter 5

Changes in Critical Features of the Model with Money Creation

Abstract[1]

We investigate whether the model of Chapter 2 is robust to changes in critical features. We demonstrate that inefficient asymmetric equilibria may arise when prices are flexible, but that these inefficient equilibria are eliminated if capital requirements are sufficiently high, so that only the efficient equilibria with banks persist. We find that deposit insurance increases welfare in our setting. If there are financial frictions at bank level and if these frictions are not too intense, we prove that the second-best allocation can only be implemented by a combination of monetary policy and capital regulation. Finally, we outline alternative monetary architectures in which money is solely created by the central bank, and we discuss some of their properties.

Preliminary remark: In the models of Chapter 5, we use the model described in Chapter 2 and its variables, and we only re-define the variables that change with the new features examined.

[1]Section The research on which this chapter is based was supported by the SNF project no. 100018_165491/1 "Money Creation by Banks, Monetary Policy, and Regulation".

5.1 Asymmetric Equilibria

In Section 5.1, we extend our results to asymmetric equilibria with banks by allowing banks to display different behaviors in an equilibrium with banks. We first focus on the case with flexible prices and without capital requirement. We then describe the equilibria that arise when a capital requirement—in the form of a minimum required equity ratio—has to be held by banks. We show that the there are inefficient equilibria and efficient equilibria with banks when capital requirements are sufficiently low, but we also prove that inefficient equilibria with banks are eliminated by capital requirements that are sufficiently high. Finally, we investigate how our results change under rigid prices.

To take the fact that banks' behavior can now differ across banks into account, we have to re-define the notion of an equilibrium with banks: An equilibrium with banks is now defined as an equilibrium with banks as in Definition 1, yet where the constraint that all banks choose the same level of money creation is relaxed. The variable α_M^b can vary across banks b. Without loss of generality, we can assume that α_M^b is increasing in b. Finally, the gross rate of return on equity and banks' profits can vary from banks to banks, depending on whether they default. We use $R_E^{b,s}$ to denote the gross rate of return on equity of Bank b in State s.

5.1.1 Flexible Prices

From Proposition 1, we obtain the following theorem:

Proposition 24
Given the policy gross rates $(R_{CB}^s)_{s=l,h}$, equilibria with banks take one of the following two forms:

– **Type 1**: *No bank defaults and in such equilibria with banks,*

$$R_E^{s*} = R_D^{s*} = R_L^{s*} = R_{CB}^s, \quad \mathbf{R_F^*} = \overline{\mathbf{R}}_\mathbf{M}, \tag{5.1}$$

$$p_I^* = p, \quad p_C^{s*} = p\frac{R_{CB}^s}{\mathbf{R_M^s}}, \tag{5.2}$$

$$E_B^* = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big), \quad D_H^* = (1-\varphi^*)p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big), \tag{5.3}$$

$$\tilde{D}_H^{s*} = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big)R_{CB}^s, \tag{5.4}$$

$$L_M^* = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big), \quad S_F^* = \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big), \tag{5.5}$$

$$\mathbf{K_M^*} = \mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big), \quad \mathbf{K_F^*} = \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big), \tag{5.6}$$

where the price of the investment good denoted by $p \in (0, +\infty)$ *and the aggregate equity ratio* $\varphi^* \in (0,1)$ *are arbitrary. The equilibrium profits of firms and banks are given by*

$$\Pi_M^{s*} = 0, \quad \Pi_F^{s*} = p\frac{R_{CB}^s}{\mathbf{R_M^s}}\left(\mathbf{f}\Big(\mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big) - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\overline{\mathbf{R}}_\mathbf{M}\right), \tag{5.7}$$

$$\Pi_B^{s*} = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big)R_{CB}^s. \tag{5.8}$$

In such equilibria with banks, the level of money creation α_M^b *of an individual Bank* b *is indeterminate. The only constraint that has to hold is given by*

$$\int_0^1 \alpha_M^b \,\mathrm{d}b = 1.$$

– **Type 2**: *Banks* $b_d \in \left[\frac{R_L^{s'*}}{R_{CB}^{s'}}, 1\right]$ *default in State* s' *and*

Banks $b_n \in \left[0, \frac{R_L^{s'*}}{R_{CB}^{s'}}\right]$ *do not default in any state* $s = l, h$ *and* *in such equilibria with banks,*

$$R_E^{b_d,s*} = \frac{R_{CB}^{s'}}{\varphi^*}\frac{R_L^{s*} - R_{CB}^s}{R_{CB}^{s'} - R_L^{s'*}} + R_{CB}^s, \quad R_E^{b_d,s'*} = 0, \tag{5.9}$$

$$R_E^{b_n,s*} = R_D^{s*} = R_{CB}^s, \tag{5.10}$$

$$\mathbf{R}_\mathbf{F}^* = \mathbf{f}'(\mathbf{0}) - \max\left(0, \mathbf{f}'(\mathbf{0}) - p_I^*\mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]\right), \tag{5.11}$$

$$p_I^* = p, \quad p_C^{s*} = p\frac{R_L^{s*}}{\mathbf{R}_\mathbf{M}^\mathbf{s}}, \tag{5.12}$$

$$E_B^* = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big), \quad D_H^* = (1-\varphi^*)p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big), \tag{5.13}$$

$$\tilde{D}_H^{s*} = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big)R_{CB}^s, \tag{5.14}$$

$$L_M^* = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big), \quad S_F^* = \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big), \tag{5.15}$$

$$\mathbf{K}_\mathbf{M}^* = \mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big), \quad \mathbf{K}_\mathbf{F}^* = \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big), \tag{5.16}$$

where the price of the investment good denoted by $p \in (0, +\infty)$*, the aggregate equity ratios* $\varphi^* \in (0,1)$*, and the lending gross rates* $(R_L^{s*})_s$ *are arbitrary such that*

$$R_{CB}^s < R_L^{s*}, \quad \mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right], \quad \textit{and} \tag{5.17}$$

$$\varphi^* = \frac{\sigma^s}{\sigma^{s'}}\frac{p_C^{s'*}}{p_C^{s*}}\frac{R_L^{s*} - R_{CB}^s}{R_{CB}^{s'} - R_L^{s'*}}. \tag{5.18}$$

The equilibrium profits of firms and banks are given by

$$\Pi_M^{s*} = 0, \quad \Pi_F^{s*} = p\frac{R_{CB}^s}{\mathbf{R}_\mathbf{M}^\mathbf{s}}\left(\mathbf{f}\Big(\mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big) - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\mathbf{R}_\mathbf{F}^*\right), \tag{5.19}$$

$$\Pi_B^{b_n,s*} = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big)R_{CB}^s, \quad \textit{and} \tag{5.20}$$

$$\Pi_B^{b_d,s*} = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big)\left(R_{CB}^{s'}\frac{R_L^{s*} - R_{CB}^s}{R_{CB}^{s'} - R_L^{s'*}} + \varphi^* R_{CB}^s\right), \quad \Pi_B^{b_d,s'*} < 0. \tag{5.21}$$

In such equilibria with banks, the level of money creation α_M^b *of an individual*

Bank b is given by

$$\alpha_M^b = \begin{cases} 0 & \text{if } b \in \left[0, \frac{R_L^{s'*}}{R_{CB}^{s'}}\right], \\ \frac{R_{CB}^{s'}}{R_{CB}^{s'} - R_L^{s'*}} & \text{if } b \in \left[\frac{R_L^{s'*}}{R_{CB}^{s'}}, 1\right]. \end{cases}$$

The proof of Proposition 24 is given in Appendix 5.A. We now examine the equilibria with banks in the case where capital requirements are imposed. We obtain the following proposition:

Proposition 25
Given the policy gross rates $(R_{CB}^s)_{s=l,h}$, equilibria with banks take one of the following three forms:

– **Type 1**: *No bank defaults and in such equilibria with banks,*

$$R_E^{s*} = \frac{R_L^{s*} - R_{CB}^s}{\varphi^*} + R_{CB}^s, \tag{5.22}$$

$$R_D^{s*} = R_{CB}^s, \quad \mathbf{R_F^*} = \overline{\mathbf{R}}_\mathbf{M}, \tag{5.23}$$

$$p_I^* = p, \quad p_C^{s*} = p\frac{R_{CB}^s}{\mathbf{R_M^s}}, \tag{5.24}$$

$$E_B^* = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big), \quad D_H^* = (1-\varphi^*)p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big), \tag{5.25}$$

$$\tilde{D}_H^{s*} = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big)R_{CB}^s, \tag{5.26}$$

$$L_M^* = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big)\Big), \quad S_F^* = \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big), \tag{5.27}$$

$$\mathbf{K_M^*} = \mathbf{W} - \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big), \quad \mathbf{K_F^*} = \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}\big), \tag{5.28}$$

where the price of the investment good denoted by $p \in (0, +\infty)$ and the aggregate equity ratio φ^*, and the lending gross rates $(R_L^{s*})_s$ are arbitrary such that

$$\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right] \quad \text{and} \quad \max_{s\in\{l,h\}}\left(1 - \frac{R_L^{s*}}{R_{CB}^s}\right) \leq \varphi^{reg} \leq \varphi^*.$$

The equilibrium profits of firms and banks are given by

$$\Pi_M^{s*} = 0, \quad \Pi_F^{s*} = p\frac{R_{CB}^s}{\mathbf{R_M^s}}\left(\mathbf{f}\Big(\mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\Big) - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\overline{\mathbf{R}}_\mathbf{M}\right), \tag{5.29}$$

$$\Pi_B^{s*} = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\Big)R_{CB}^s. \tag{5.30}$$

In such equilibria with banks, the level of money creation α_M^b of an individual Bank b is indeterminate. The two constraints that have to hold are given by

$$\int_0^1 \alpha_M^b \, \mathrm{d}b = 1 \quad \textit{and} \quad \alpha_M^b \in \left[0, \frac{\varphi^*}{\varphi^{reg}}\right].$$

– **Type 2**: *Banks $b_d \in \left[\frac{R_L^{s'*}}{R_{CB}^{s'}}, 1\right]$ default in State s' and*
Banks $b_n \in \left[0, \frac{R_L^{s'*}}{R_{CB}^{s'}}\right]$ do not default in any state $s = l, h$ and
in such equilibria with banks,

$$R_E^{b_d,s*} = \frac{R_{CB}^{s'}}{\varphi^*}\frac{R_L^{s*} - R_{CB}^s}{R_{CB}^{s'} - R_L^{s'*}} + R_{CB}^s, \quad R_E^{b_d,s'*} = 0, \tag{5.31}$$

$$R_E^{b_n,s*} = R_D^{s*} = R_{CB}^s, \tag{5.32}$$

$$\mathbf{R_F^*} = \mathbf{f}'(\mathbf{0}) - \max\left(0, \mathbf{f}'(\mathbf{0}) - p_I^*\mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]\right), \tag{5.33}$$

$$p_I^* = p, \quad p_C^{s*} = p\frac{R_L^{s*}}{\mathbf{R_M^s}}, \tag{5.34}$$

$$E_B^* = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F^*})\Big), \quad D_H^* = (1-\varphi^*)p\Big(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F^*})\Big), \tag{5.35}$$

$$\tilde{D}_H^{s*} = p\Big(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F^*})\Big)R_{CB}^s, \tag{5.36}$$

$$L_M^* = p\Big(\mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F^*})\Big), \quad S_F^* = \mathbf{f}'^{-1}(\mathbf{R_F^*}), \tag{5.37}$$

$$\mathbf{K_M^*} = \mathbf{W} - \mathbf{f}'^{-1}(\mathbf{R_F^*}), \quad \mathbf{K_F^*} = \mathbf{f}'^{-1}(\mathbf{R_F^*}), \tag{5.38}$$

where the price of the investment good denoted by $p \in (0, +\infty)$, the aggregate equity ratios $\varphi^* \in [\varphi^{reg}, 1)$, and the lending gross rates $(R_L^{s*})_s$ are arbitrary *such that*

$$\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right], \quad \textit{and} \tag{5.39}$$

$$0 < \varphi^{reg}\frac{R_{CB}^{s'}}{R_{CB}^{s'} - R_L^{s'*}} \leq \varphi^* = \frac{\sigma^s}{\sigma^{s'}}\frac{p_C^{s'*}}{p_C^{s*}}\frac{R_L^{s*} - R_{CB}^{s}}{R_{CB}^{s'} - R_L^{s'*}}. \tag{5.40}$$

The equilibrium profits of firms and banks are given by

$$\Pi_M^{s*} = 0, \quad \Pi_F^{s*} = p\frac{R_{CB}^{s}}{\mathbf{R_M^s}}\left(\mathbf{f}\Big(\mathbf{f}'^{-1}\big(\mathbf{R_F^*}\big)\Big) - \mathbf{f}'^{-1}\big(\mathbf{R_F^*}\big)\mathbf{R_F^*}\right), \tag{5.41}$$

$$\Pi_B^{b_n,s*} = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R_F^*}\big)\Big)R_{CB}^{s}, \quad \textit{and} \tag{5.42}$$

$$\Pi_B^{b_d,s*} = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R_F^*}\big)\Big)\left(R_{CB}^{s'}\frac{R_L^{s*} - R_{CB}^{s}}{R_{CB}^{s'} - R_L^{s'*}} + \varphi^* R_{CB}^{s}\right), \quad \Pi_B^{b_d,s'*} < 0. \tag{5.43}$$

In such equilibria with banks, the level of money creation α_M^b *of an individual bank* b *is given by*

$$\alpha_M^b = \begin{cases} 0 & \textit{if } b \in \left[0, \frac{R_L^{s'*}}{R_{CB}^{s'}}\right], \\ \frac{R_{CB}^{s'}}{R_{CB}^{s'} - R_L^{s'*}} & \textit{if } b \in \left[\frac{R_L^{s'*}}{R_{CB}^{s'}}, 1\right]. \end{cases}$$

– **Type 3**: *Banks* $b_d \in \left[1 - \frac{\sigma^s}{\sigma^{s'}}\frac{p_C^{s'*}}{p_C^{s*}}\frac{R_L^{s*} - R_{CB}^{s}}{\varphi^* R_{CB}^{s'}}, 1\right]$ *default in State* s' *and* *Banks* $b_n \in \left[0, 1 - \frac{\sigma^s}{\sigma^{s'}}\frac{p_C^{s'*}}{p_C^{s*}}\frac{R_L^{s*} - R_{CB}^{s}}{\varphi^* R_{CB}^{s'}}\right]$ *do not default in any state* $s = l, h$ *and* *in such equilibria with banks,*

$$R_E^{b_d,s*} = \frac{R_L^{s*} - R_{CB}^s}{\varphi^{reg}} + R_{CB}^s, \quad R_E^{b_d,s'*} = 0, \tag{5.44}$$

$$R_E^{b_n,s*} = R_D^{s*} = R_{CB}^s, \tag{5.45}$$

$$\mathbf{R}_\mathbf{F}^* = \mathbf{f}'(\mathbf{0}) - \max\left(0, \mathbf{f}'(\mathbf{0}) - p_I^* \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]\right), \tag{5.46}$$

$$p_I^* = p, \quad p_C^{s*} = p\frac{R_L^{s*}}{\mathbf{R}_\mathbf{M}^\mathbf{s}}, \tag{5.47}$$

$$E_B^* = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big), \quad D_H^* = (1-\varphi^*)p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big), \tag{5.48}$$

$$\tilde{D}_H^{s*} = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big)R_{CB}^s, \tag{5.49}$$

$$L_M^* = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big), \quad S_F^* = \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big), \tag{5.50}$$

$$\mathbf{K}_\mathbf{M}^* = \mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big), \quad \mathbf{K}_\mathbf{F}^* = \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big), \tag{5.51}$$

where the price of the investment good denoted by $p \in (0, +\infty)$*, the aggregate equity ratios* $\varphi^* \in [\varphi^{reg}, 1)$*, and the lending gross rates* $(R_L^{s*})_s$ *are arbitrary such that*

$$\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right] \quad \textit{and} \quad \varphi^* \frac{R_{CB}^{s'} - R_L^{s'*}}{R_{CB}^{s'}} < \varphi^{reg} = \frac{\sigma^s}{\sigma^{s'}} \frac{p_C^{s'*}}{p_C^{s*}} \frac{R_L^{s*} - R_{CB}^s}{R_{CB}^{s'}}. \tag{5.52}$$

The equilibrium profits of firms and banks are given by

$$\Pi_M^{s*} = 0, \quad \Pi_F^{s*} = p\frac{R_{CB}^s}{\mathbf{R}_\mathbf{M}^\mathbf{s}}\left(\mathbf{f}\Big(\mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big) - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\mathbf{R}_\mathbf{F}^*\right), \tag{5.53}$$

$$\Pi_B^{b_n,s*} = \varphi^* p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big)R_{CB}^s, \quad \textit{and} \tag{5.54}$$

$$\Pi_B^{b_d,s*} = p\Big(\mathbf{W} - \mathbf{f}'^{-1}\big(\mathbf{R}_\mathbf{F}^*\big)\Big)\left(R_{CB}^{s'}\frac{R_L^{s*} - R_{CB}^s}{R_{CB}^{s'} - R_L^{s'*}} + \varphi^* R_{CB}^s\right), \quad \Pi_B^{b_d,s'*} < 0. \tag{5.55}$$

In such equilibria with banks, the level of money creation α_M^b *of an individual*

Bank b is given by

$$\alpha_M^b = \begin{cases} 0 & \text{if } b \in \left[0, 1 - \frac{\sigma^s}{\sigma^{s'}} \frac{p_C^{s'*}}{p_C^{s*}} \frac{R_L^{s*} - R_{CB}^s}{\varphi^* R_{CB}^{s'}}\right], \\ \frac{\varphi^*}{\varphi^{reg}} & \text{if } b \in \left[1 - \frac{\sigma^s}{\sigma^{s'}} \frac{p_C^{s'*}}{p_C^{s*}} \frac{R_L^{s*} - R_{CB}^s}{\varphi^* R_{CB}^{s'}}, 1\right]. \end{cases}$$

The proof of Proposition 25 is given in Appendix 5.A. We observe that there are three different types of equilibria with banks in Proposition 25 and that both inefficient and efficient equilibria with banks arise. We can now investigate how monetary policy and capital requirements can remove the inefficient equilibria with banks. We obtain

Proposition 26

- *For any given central bank policy gross rates* $(R_{CB}^s)_s$*, there exists an efficient equilibrium with banks for any capital requirement* $\varphi^{reg} \in [0, 1)$.
- *The minimum equity ratio requirement* $\varphi^{reg} \in [0, 1)$ *can remove inefficient equilibria with banks if and only if*

$$\varphi^{reg} \geq \frac{\max_{s=l,h}\left(\sigma^s \mathbf{R_M^s}\right)}{\overline{\mathbf{R}}_\mathbf{M}}.$$

The proof of Proposition 26 is given in Appendix 5.A. We now investigate the equilibria with banks, when prices are rigid.

5.1.2 Rigid Prices

In contrast to the results in Chapter 2, there are equilibria with banks even without any capital requirement when $R_{CB}^s \neq \mathbf{R_M^s}$. From Proposition 24, we directly obtain

Proposition 27

Suppose that prices are rigid and $R_{CB}^s \neq \mathbf{R_M^s}$ *for some state s. Then there exists an equilibrium with banks if and only if the central bank policy gross rates* $(R_{CB}^s)_s$ *are set as follows:* $\overline{R}_{CB} > \overline{\mathbf{R}}_\mathbf{M}$.

Similarly to Proposition 3 in Chapter 2, we directly obtain from Proposition 25

Proposition 28
Suppose that prices are rigid and $R^s_{CB} \neq \mathbf{R^s_M}$ *for some state* s*. Then there exists an equilibrium with banks if and only if the central bank policy gross rates* $(R^s_{CB})_s$ *and the capital requirement level* φ^{reg} *are set as follows:*

(i) $\overline{R}_{CB} = \overline{\mathbf{R}}_{\mathbf{M}}$ *and* $\max_{s=l,h}\left(\frac{R^s_{CB}-\mathbf{R^s_M}}{R^s_{CB}}\right) \leq \varphi^{reg}$.

(ii) $\overline{R}_{CB} > \overline{\mathbf{R}}_{\mathbf{M}}$ *and* $0 \leq \varphi^{reg} \leq \max_{s\neq s'}\left(\frac{\sigma^s}{\sigma^{s'}}\frac{\mathbf{R^s_M}-R^s_{CB}}{R^{s'}_{CB}}\right) < 1$.

We observe that this proposition is similar to Proposition 3 in Chapter 2, with the difference that there are equilibria even without capital requirements. Moreover, we can also note that the efficiency of equilibria with banks can only be restored with capital requirements when $R^s_{CB} \neq \mathbf{R^s_M}$ for some state s. We indeed obtain a corollary with regard to welfare that is essentially identical to Corollary 5 in Chapter 2. In the presence of the zero lower bound problem, it is straightforward that the results that we obtain are similar to the ones given in Corollary 6 and Proposition 4 in Chapter 2, the only difference being that no capital requirement is needed in Proposition 4 to guarantee the existence of an equilibrium with banks.

5.2 Money Creation in the Absence of Deposit Insurance

In Section 5.2, we suppose that banks defaulting on households are not bailed out by the government. In this setting, we obtain the following proposition:

Proposition 29
When defaulting banks are not bailed out by the government, there never exists an equilibrium with banks and default.

The proof of Proposition 29 is given in Appendix 5.A. This proposition enables us to use all our previous results of Chapter 2, in each of which the possibility of banks' default is removed. Thus, there is no change when prices are flexible.

However, when prices are rigid, equilibria with banks and default disappear and in these cases, only the equilibria without banks remain, whose welfare is lower than the welfare of equilibria with banks and default. We can thus conclude that deposit insurance increases welfare in our model.

5.3 Financial Frictions

In Section 5.3, we explore situations in which the economy described in Chapter 2 is affected by financial frictions.[2] We introduce a well-known financial friction into our model: Bankers cannot pledge the entire return from their investment to depositors or shareholders and hence, they receive a non-pledgeable income for carrying out their monitoring activities, which is proportional to the repayments $\theta\alpha_M^b L_M R_L^s$ at $t = 1$, where $\theta \in (0, 1)$. This financial friction arises from several theories on the micro-foundation of such frictions such as moral hazard in the sense of Holmström and Tirole (1997), asset diversion (Gertler and Karadi, 2011; Gertler and Kiyotaki, 2011), and inalienability of human capital (Diamond and Rajan, 2001; Hart and Moore, 1994).

Following these approaches, the financial frictions are integrated into our model in the following form: Bankers need to be paid the amount $\theta\alpha_M^b L_M R_L^s$ in the form of deposits in Period $t = 1$ to ensure that they behave—do monitor entrepreneurs and do not divert assets, for instance. Bankers will use these deposits to buy an amount of consumption good, like households.

We assume that bankers are risk-neutral. Hence, they aim at maximizing their own expected consumption instead of their expected shareholders' value:[3]

$$\mathbb{E}\left[\frac{\theta\alpha_M^b L_M R_L^s}{p_C^s}\right]. \tag{5.56}$$

[2]Some form of financial frictions is already present in the economy in Chapter 2, but these frictions are eliminated by banks when they monitor entrepreneurs running firms in Sector MT.

[3]The central bank's and the bank regulators' objective function is more subtle in this case. We will focus on policies that maximize the households' welfare. However, policies could also be derived by maximizing a utilitarian welfare function that bankers' and households' utilities would enter with some weights. This is left to future research.

Since the price of physical goods and aggregate lending cannot be influenced by an individual banker, bankers aim at maximizing their expected consumption by choosing α_M^b under the constraint that their bank does not default against the central bank.[4] We obtain

Lemma 14

In the presence of financial frictions, the privately optimal amounts of money creation and lending by an individual bank are represented by a correspondence denoted by $\hat{\alpha}_M : \mathbb{R}^4_{++} \to \mathcal{P}(\mathbb{R}_+ \cup \{+\infty\})$ *and given by*

$$\hat{\alpha}_M\big((R_L^s)_s, (R_{CB}^s)_s\big) = \begin{cases} \{+\infty\} \\ \quad \text{if } R_L^s \geq R_{CB}^s \text{ in all states } s = l, h, \\ \{\alpha_{DCB}^s\} \\ \quad \text{if } R_L^s < R_{CB}^s \text{ in only one state } s = l \text{ or } h, \\ \{\min\big(\alpha_{DCB}^l, \alpha_{DCB}^h\big)\} \\ \quad \text{if } R_L^s < R_{CB}^s \text{ in all states } s = l, h. \end{cases}$$

Lemma 14 follows directly from the observation that bankers want to increase money creation as much as possible, but must avoid a default against the central bank. The formal proof of Lemma 14 is given in Appendix 5.A. An important implication of Lemma 14 is that no constellation of gross rates $(R_L^s)_s$ and $(R_{CB}^s)_s$ is compatible with $\alpha_M^b = 1$ for any Bank b in an equilibrium with banks. The reason is that bankers aim at increasing money creation and lending to raise their own income. The only constraint is the threat of default against the central bank. If $R_L^s \geq R_{CB}^s$ for $s = l, h$, there is no limit to money creation, as an individual bank can always pay back obligations against the central bank that occur in the payment process. If $R_L^s < R_{CB}^s$ for one or both states of the world, there is a limit to money creation and lending by an individual bank, given some average money creation and lending by banks. However, an individual bank would like to increase lending beyond average, as it will only default against the central bank if money creation reaches α_{DCB}^l or α_{DCB}^h. However, as all banks face the same incentives, no finite money creation or lending is possible in an equilibrium with

[4] We continue to assume that bankers face severe penalties if they default against the central bank. These penalties are assumed to be higher than expected consumption.

banks. The breakdown of money creation thus only allows equilibria without banks. We summarize these observations in the following proposition:

Proposition 30
In the presence of financial frictions,

- *no equilibrium with banks exists, and*
- *the no-bank equilibrium exists and is unique.*

Since the equilibrium without banks is inefficient, we next explore whether a capital requirement as defined in Definition 2 in Chapter 2 can reduce or even eliminate this inefficiency. We first characterize money creation by an individual bank when this bank faces a capital requirement.

Lemma 15
Suppose that financial frictions are present and banks have to comply with a minimum equity ratio φ^{reg} *at the end of Period* $t = 0$. *The privately optimal amounts of money creation and lending by an individual bank are represented by a correspondence denoted by* $\hat{\alpha}_M^{reg} : \mathbb{R}^4_{++} \times [\varphi^{reg}, 1) \to \mathcal{P}(\mathbb{R} \cup \{+\infty\})$ *and given by*

$$\hat{\alpha}_M^{reg}\big((R_L^s)_s, (R_{CB}^s)_s, \varphi\big) = \begin{cases} \{\frac{\varphi}{\varphi^{reg}}\} \\ \quad \textit{if } R_L^s \geq R_{CB}^s \textit{ in all states } s = l, h, \\ \{\min(\alpha_{DCB}^s, \frac{\varphi}{\varphi^{reg}})\} \\ \quad \textit{if } R_L^s < R_{CB}^s \textit{ in just one state } s = l \textit{ or } h, \\ \{\min\big(\alpha_{DCB}^l, \alpha_{DCB}^h, \frac{\varphi}{\varphi^{reg}}\big)\} \\ \quad \textit{if } R_L^s < R_{CB}^s \textit{ in all states } s = l, h. \end{cases}$$

The proof of Lemma 15 is straightforward and follows the proof of Lemma 14. For the sake of completeness, the proof of Lemma 15 is given in Appendix 5.A. A capital requirement limits money creation and is thus effective when bankers aim at increasing money creation to generate more rents.

Similarly to Lemma 2, we next establish how the deposit gross rates are related to the policy gross rates. Since banks can grant loans to or can borrow from other banks, we obtain

Lemma 16

In any equilibrium with banks, the nominal lending gross rates on the interbank market satisfy

$$R_D^{s*} = R_{CB}^s \quad \textit{for all states } s = l, h.$$

The proof of Lemma 16 can be found in Appendix 5.A. It is also based on a simple arbitrage argument: Any differential in the gross rates could be used in the interbank market by borrowing or lending to increase bankers' expected consumption.

With the help of Lemmata 15 and 16, we can establish conditions that the policy rates and capital regulation have to fulfill to restore the existence of equilibria with banks:

Proposition 31

In the presence of financial frictions, there exist central bank policy gross rates $(R_{CB}^s)_s$ *and capital requirement levels* φ^{reg} *such that an equilibrium with banks exists. The policy gross rates* $(R_{CB}^s)_s$ *and the capital requirement level* φ^{reg} *have to fulfill*

- $\varphi^{reg} \geq \max_{s \in \{l,h\}} \left\{ \frac{R_{CB}^s - R_L^{s*}(1-\theta)}{R_{CB}^s} \right\}$, *or*
- $\varphi^{reg} = \frac{\sigma}{1-\sigma} \frac{R_L^{l*}}{R_L^{h*}} \frac{\mathbf{R_M^h}}{\mathbf{R_M^l}} \frac{R_L^{h*}(1-\theta) - R_{CB}^h}{R_{CB}^l} < \frac{R_{CB}^l - R_L^{l*}(1-\theta)}{R_{CB}^l}$, *or*
- $\varphi^{reg} = \frac{1-\sigma}{\sigma} \frac{R_L^{h*}}{R_L^{l*}} \frac{\mathbf{R_M^l}}{\mathbf{R_M^h}} \frac{R_L^{l*}(1-\theta) - R_{CB}^l}{R_{CB}^h} < \frac{R_{CB}^h - R_L^{h*}(1-\theta)}{R_{CB}^h}$.

The proof of Proposition 31 is given in Appendix 5.A. The intuition for Proposition 31 runs as follows: First, note that banks cannot inflate their balance sheets by lending more than the average bank if and only if $\varphi^* = \varphi^{reg}$. Moreover, if the banks' equity ratio is sufficiently high, as in the first case of Proposition 31, no bank defaults, and investment in MT is decreasing in the intensity of financial frictions, which is measured by θ. In the last two cases, all banks default in one state of the world. As depositors do not take into account the impact of their investment on the lump-sum taxes that are sued to bail out their deposits, investment in MT becomes relatively more attractive and hence, its level is higher than in the no-default case. Financial frictions and bank defaults are thus two countervailing effects, whose impact on the households' and the bankers' consumption is a priori not clear. From the proof of Proposition 31, we obtain

Proposition 32

When prices are rigid and there are financial frictions, there are central bank policy gross rates $(R_{CB}^s)_s$ and capital requirement levels φ^{reg} implementing the second-best allocation if and only if

$$\theta < \frac{\overline{\mathbf{R}}_{\mathbf{M}} - \mathbf{f}'(\mathbf{W})}{\overline{\mathbf{R}}_{\mathbf{M}}}.$$

In this case, the second-best allocation is implemented if and only if the central bank policy gross rates $(R_{CB}^s)_s$ and the capital requirement level φ^{reg} fulfill

$$\varphi^{reg} \geq \max_{s \in \{l,h\}} \left\{ \frac{R_{CB}^s - \mathbf{R}_{\mathbf{M}}^{\mathbf{s}}(1-\theta)}{R_{CB}^s} \right\}.$$

An example of such a policy combination is given by

$$R_{CB}^s = \mathbf{R}_{\mathbf{M}}^{\mathbf{s}}(1-\theta) \quad \textit{and} \quad \varphi^{reg} \in (0,1).$$

The proof of Proposition 32 is given in Appendix 5.A. The allocation associated with equilibria without banks' default maximizes the households' utility subject to the bankers' incentive constraint. Thus, the policy combination of central bank gross rates and capital regulation as defined in Proposition 32 implements the second-best allocation for households. Moreover, if financial frictions are too intense, investment in MT can only appear to be profitable for households in the case when banks default.

5.4 Money Creation by the Central Bank

5.4.1 Overview and Money Supply Schemes

The two models we will elaborate on in Section 5.4 are variations of the model described in Chapter 2, for which the money creation process is entirely administrated by the central bank and for which deposits held at banks have to be covered by an equal amount of central bank reserves.[5]

In the two models of Section 5.4, we assume that money is created at the beginning of time by the central bank, when it grants loans to banks. Banks can also borrow from the central bank to be able to make dividend payments to households. As banks will make profits from lending the money to firms in MT, they have a demand function for money from the central bank. The amount of money supplied to an individual bank is defined by one of the two following supply schemes:

- Either the central bank fulfills any banks' demand for money, which we will call the "no-rationing scheme",
- or the central bank rations banks and grants the same finite amount of loans to every bank ($\alpha_M^b = 1$ for all banks b), if all banks demand an amount of loans above the average. We call this set-up the "rationing scheme".

We still assume that bankers have to pay heavy penalties for defaulting against the central bank. Finally, in both models, we assume that $R_D^s = R_{CB}^s$ for all states $s = l, h$, which means that there is a perfect pass-through between the deposit gross rate and the central bank policy gross rate.[6]

In our first model in Subsection 5.4.2, the deposits that households or firms use to make payments are either held directly at the central bank or are off the banks' balance sheets. This essentially means that banks cannot default on these deposits. In our second model in Section 5.4.3, these deposits are held on the banks' balance sheets and banks can default on them, although these deposits are covered at 100% by central bank reserves.

[5]Section 5.4 is based on joint work with Hans Gersbach (see Faure and Gersbach (2017)).

[6]This assumption simplifies considerably the analysis. We note that in the rationing scheme, there may be other more meaningful assumptions, but they would entail much more involved analyses.

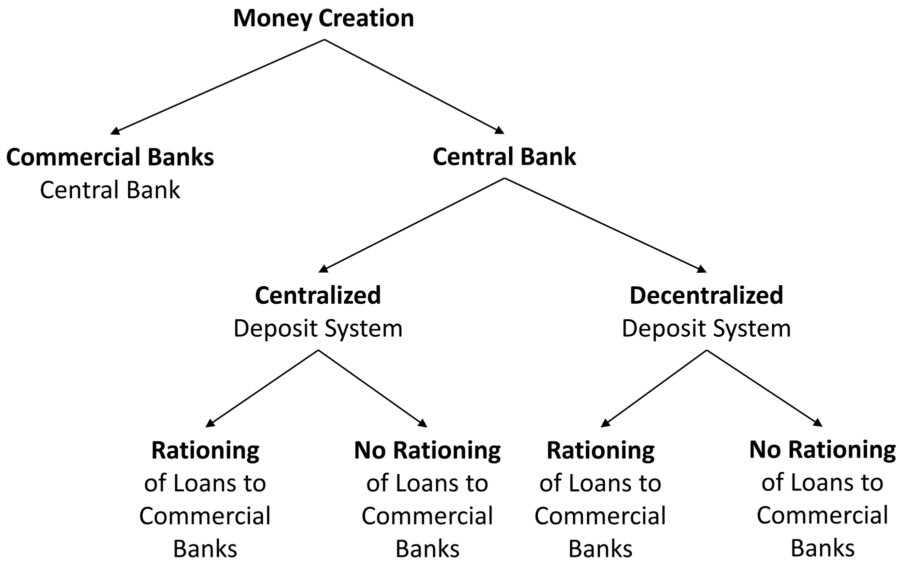

Figure 5.1: The various monetary architectures discussed in Section 5.4. Source: Own illustration.

5.4.2 Centralized Deposit System

We call a monetary system in which only the central bank can create money, all deposits are fully covered by central bank reserves, and all agents have an account at the central bank only "a Centralized Deposit System". In this monetary system, banks cannot default against households, as deposits are not held at commercial banks. Depositing and lending are thus completely decoupled. As an alternative and equivalent formulation, we could assume that deposits and the corresponding reserves are held off the banks' balance sheets. Banks are thus solely financed by equity and by loans from the central bank and they have a demand function for money from the central bank.

Period t = 0

The central bank grants loans to banks. Each bank b has a demand function for loans l^b_{CB}. Depending on whether banks are rationed, the amount of loans that the central bank supplies is 1 or l^b_{CB}. Then banks grant loans to firms, and at the same time, transfer the deposits they have obtained from the central bank to them. Firms in MT now hold deposits at the central bank and use them to buy some amount of investment good from households. Households use some deposits $d_B = e_B$ to invest in bank equity. The balance sheets before the macroeconomic shock takes place are given in Table 5.1.

Households		Firms in MT		Bank b		Central Bank	
S_F	E_H	$\mathbf{K_M}$	L_M	l^b_M	l^b_{CB}	L_{CB}	D_H
D_H				d_B	e_B		D_B
E_B							

Table 5.1: Centralized Deposit System: The balance sheets of agents before the macroeconomic shock. Source: Own illustration.

Period t = 1

After production and the macroeconomic shock, banks demand an amount of loans $\max(e_B R_E^{b,s} - d_B R_{CB}^s, 0)$ from the central bank to be able to pay the dividends. Then banks pay dividends to households and households buy the investment good. Firms repay their loans and bonds, and banks reimburse their loans to the central bank. Money is destroyed only at the end of Period $t = 1$, when banks repay their loans to the central bank.

Informal Results

In a Centralized Deposit System, the default of banks against households is not possible, and this excludes the existence of inefficient equilibria with banks. Moreover, the rationing and the no-rationing schemes are equivalent, as banks demanding more money than the average always have a return on equity exceeding the cost of capital, even when they only grant the average level of loans to firms in MT. As a consequence, equilibria with banks are characterized by intermediation margins that prevent banks from defaulting and make banking an attractive investment in the form of both equity and deposits in any equilibrium with banks. In any equilibrium with banks, the banks' equity ratio is large enough compared to the intermediation margins, so that banks do not default in any state.

5.4.3 Decentralized Deposit System

We call a monetary system in which only the central bank can create money, all deposits are fully covered by central bank reserves, and only commercial banks have an account at the central bank a "Decentralized Deposit System". In such a monetary system, banks can default against households, and depositing and lending are thus not completely decoupled. Banks are thus financed by equity, by a loan from the central bank, and by households' deposits, and they have a demand function for money from the central bank.

Period t = 0

The central bank grants loans to banks. Each bank b has a demand function for loans l^b_{CB}. Depending on whether banks are rationed, the amount of loans that the central bank supplies is 1 or l^b_{CB}. Then banks grant loans l^b_M to firms, which use the deposits to buy the investment good from households. Households invest in bonds and use some of these deposits to invest in bank equity. The total amount of reserves held by any bank is denoted by d_{CB} and fulfills $d_{CB} = d_H + e_B$. At all times, banks hold reserves that are equal to the deposits agents hold at them. The balance sheets before the macroeconomic shock takes place are given in Table 5.2.

Period t = 1

After production and the macroeconomic shock, either some banks default or no bank defaults. Banks demand an amount of loans $\max(e_B R^{b,s}_E + d_H R^s_D - d_{CB} R^s_{CB}, 0)$ from the central bank to be able to pay the dividends. Then banks pay dividends to households and households buy the investment good from MT. Firms repay

Households		Firms in MT		Bank b		Central Bank	
S_F	E_H	$\mathbf{K_M}$	L_M	l^b_M	l^b_{CB}	L_{CB}	D_{CB}
D_H				d_{CB}	d_H		
E_B					e_B		

Table 5.2: Decentralized Deposit System: The balance sheets of agents before the macroeconomic shock. Source: Own illustration.

their loans and bonds and banks reimburse their loans to the central bank. Money is destroyed only at the end of Period $t = 1$, when banks repay their loans to the central bank.

Informal Results

In a Decentralized Deposit System, the default of banks against households is possible. Such a monetary system with a no-rationing scheme is equivalent to the monetary system described in Chapter 2, as banks demand and obtain an infinite amount of money as soon as there is a small intermediation margin. In the rationing scheme, banks are rationed if they all demand more money than the average, which can only occur in equilibrium when banks default for a lending level that is equal to the average level. There are such inefficient equilibria with banks with distorted investment across sectors of production. However, for sufficiently high capital requirements coupled with an adequate monetary policy, these equilibria with banks can be eliminated, leaving a larger set of efficient equilibria with banks than the one prevailing in Chapter 2.

Appendix

5.A Proofs

Proof of Proposition 24.
Let $\mathcal{E}^*$ be an equilibrium with banks.

The result of Lemma 2 implies that we can apply Proposition 1. Thus, given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, and the equity ratio φ^*, all banks $b \in [0,1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M\big((R_L^{s*})_s, (R_{CB}^s)_s, \varphi^*\big)$ as given in Proposition 1. The only gross rates of return in Proposition 1 rationalizing $\int_0^1 \alpha_M^b \mathrm{d}b = 1$ are

$$\begin{aligned} &(R_L^{s*} = R_{CB}^s \quad \text{for all states } s = l, h) \\ \text{and} \quad &(\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right], \quad R_{CB}^s < R_L^{s*}, \quad \text{and} \quad \varphi^* = \frac{\sigma^s}{\sigma^{s'}} \frac{p_C^{s'*}}{p_C^{s*}} \frac{R_L^{s*} - R_{CB}^s}{R_{CB}^{s'} - R_L^{s'*}}). \end{aligned} \tag{5.57}$$

In the case of equal nominal gross rates of return $R_L^{s*} = R_{CB}^s$, we obtain the same equilibria as in Theorem 1. We assume now that Equations in (5.57) hold. We denote banks choosing $\alpha_M^b = 0$ by b_n and banks choosing $\alpha_M^b = \alpha_{DCB}^{s'}$ by b_d. Moreover, we use m_n to denote the measure of banks choosing $\alpha_M^b = 0$. This case constitutes an equilibrium with banks, if the equation $\int_0^1 \alpha_M^b \,\mathrm{d}b = 1$ holds, which is equivalent to

$$\int_{b\in[0,m_n]} 0 \,\mathrm{d}b + \int_{b\in[m_n,1]} \alpha_{DCB}^{s'} \,\mathrm{d}b = (1-m_n)\frac{R_{CB}^{s'}}{R_{CB}^{s'} - R_L^{s'}} = 1.$$

The previous equation enables to derive the expression of m_n as follows:

$$m_n = \frac{R_L^{s'*}}{R_{CB}^{s'}}.$$

A direct consequence of Lemma 2 is that

$$R_E^{b_n,s*} = R_D^{s*} = R_L^{s*} = R_{CB}^{s} \tag{5.58}$$

for all states $s = l, h$. Moreover, due to Lemma 2 and the tie-breaking rule introduced in Subsection 2.2.3, the interbank lending market is not used in an equilibrium with banks. Finally, $\Pi_M^{s*} = 0$ for all states $s = l, h$ (see Subsection 2.2.4), which translates into

$$\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} p_C^{s*} = R_L^{s*} p_I^*$$

for all states $s = l, h$. From the proof of Proposition 1, we can also give the gross rates of return on equity of banks b_d which default in State s', as follows:

$$R_E^{b_d,s'*} = 0 \quad \text{and} \quad R_E^{b_d,s*} = \alpha_{DCB}^{s'*} \frac{R_L^{s*} - R_{CB}^{s}}{\varphi^*} + R_{CB}^{s},$$

which directly implies the expressions of the gross rates of return on equity and the profits of banks b_d given in Proposition 24. As banks are indifferent between $\alpha_M^b = 0$ and $\alpha_M^b = \alpha_{DCB}^b$, we note that in these equilibria with banks,

$$\mathbb{E}\left[\frac{R_E^{b_n,s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_E^{b_d,s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_D^{s*}}{p_C^{s*}}\right]. \tag{5.59}$$

Given gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices p_I^* and $(p_C^{s*})_s$, households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, S_F^*\big)$ given S_F^*, $D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, E_B^*, S_F^*\big)$ given E_B^* and S_F^*, and $S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s\big)$, where these correspondences are given in Lemma 6 in Appendix 2.E. Only the first, the fourth, and the seventh cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ are consistent with the equality of real gross rates of return in Equation (5.59). However, the assumption

$\mathbf{f}'(\mathbf{W}) < \overline{\mathbf{R}}_{\mathbf{M}} = p_I^* \mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right]$ together with $\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right]$ rule out the first case. As in an equilibrium with banks $E_B^*, D_H^* > 0$, we obtain in the case where $\mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right] \leq \frac{\mathbf{f}'(\mathbf{0})}{p_I^*}$ the following equilibrium variables:

$$\begin{aligned}
E_B^* &\in \left(0, p_I^* \left(\mathbf{W} - \mathbf{f}'^{-1}\left(p_I^* \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]\right)\right)\right), \\
D_H^* &= p_I^* \left(\mathbf{W} - \mathbf{f}'^{-1}\left(p_I^* \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]\right)\right) - E_B^*, \text{ and} \\
S_F^* &= \mathbf{f}'^{-1}\left(p_I^* \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]\right)
\end{aligned}$$

and in the case where $\mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right] > \frac{\mathbf{f}'(\mathbf{0})}{p_I^*}$,

$$\begin{aligned}
E_B^* &\in (0, p_I^* \mathbf{W}), \\
D_H^* &= p_I^*(\mathbf{W} - E_B^*), \text{ and} \\
S_F^* &= 0.
\end{aligned}$$

Finally, $\mathbf{R}_{\mathbf{F}}^*$ can be determined by using Lemma 4 and equating the demand for the investment good $\mathbf{K}_{\mathbf{F}}^*$ to its supply S_F^*. Thus, with the help of the equity ratio φ^*, we can re-write all equilibrium variables as given in Proposition 24.

In turn, it is straightforward to verify that the tuples given in Proposition 24 constitute equilibria with banks as defined in Subsection 2.2.5, where the constraint that all banks choose the same money creation level is relaxed. □

Proof of Proposition 25.

Let $\mathcal{E}^*$ be an equilibrium with banks for which a minimum equity ratio $\varphi^{reg} \in (0, \varphi^*]$ is required to be held by banks at the end of Period $t = 0$.

The result of Lemma 2 implies that we can apply Lemma 7. Thus, given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, and the equity ratio φ^*, all banks $b \in [0, 1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M^{reg}\left((R_L^{s*})_s, (R_{CB}^s)_s, \varphi^*\right)$ as given in Lemma 7.

We first examine the symmetric equilibria with banks, in which all banks choose the same level of money creation. These equilibria with banks require constellations similar to the ones given in Proposition 3. We draw on these results and on Lemma 7 to directly generalize Proposition 3 for the case of flexible prices. The two resulting types of symmetric equilibria with banks are described in the following:

– Banks do not default and either

$$R_L^{s*} = R_{CB}^s \quad \text{for all states } s = l, h$$

or

$$\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right] \quad \text{and} \quad 0 < 1 - \frac{R_L^{s'*}}{R_{CB}^{s'}} \leq \varphi^{reg}.$$

In this case,

$$\begin{aligned} &E_B^* \in (0, \big(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\big)), \\ &D_H^* = \big(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\big) - E_B^*, \text{ and} \\ &S_F^* = \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M}). \end{aligned}$$

– Some banks default in one state, say s', and

$$\begin{aligned} &\varphi^* = \varphi^{reg} = \frac{\sigma^s}{\sigma^{s'}} \frac{p_C^{s'*}}{p_C^{s*}} \frac{R_L^{s*} - R_{CB}^s}{R_{CB}^{s'}}, \\ &\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right], \quad R_L^{s'*} < R_{CB}^{s'}, \quad \text{and} \quad \alpha_{DH}^{s'} < 1. \end{aligned}$$

In this case,

$$\begin{aligned} &E_B^* \in (0, (\mathbf{W} - S_F^*)), \\ &D_H^* = (\mathbf{W} - S_F^*) - E_B^*, \\ &S_F^* = \begin{cases} \mathbf{f}'^{-1}\big(p_I^* \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]\big) & \text{if } \mathbf{f}'(\mathbf{0}) \geq p_I^* \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right], \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$

We now turn to potential asymmetric equilibria with banks. According to Lemma

7 in Appendix 2.F, the gross rates of return rationalizing more than one choice of level money creation are such that

either Case a) $(R_L^{s*} = R_{CB}^s$ for all states $s = l, h)$,
or Case b) $(\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]$, $R_L^{s'*} < R_{CB}^{s'}$, and $\alpha_{DH}^{s'} \geq \frac{\varphi^*}{\varphi^{reg}}$ for $s' = l$ or $h)$,
or Case c) $(\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]$, $R_{CB}^s < R_L^{s*}$, $\alpha_{DH}^{s'} < \frac{\varphi^*}{\varphi^{reg}} < \alpha_{DCB}^{s'}$,
and $\varphi^{reg} = \frac{\sigma^s}{\sigma^{s'}} \frac{p_C^{s'*}}{p_C^{s*}} \frac{R_L^{s*} - R_{CB}^s}{R_{CB}^{s'}} \leq \varphi^*)$,
or Case d) $(\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]$, $R_{CB}^s < R_L^{s*}$, $\alpha_{DCB}^{s'} \leq \frac{\varphi^*}{\varphi^{reg}}$,
and $\varphi^{reg} \leq \varphi^* = \frac{\sigma^s}{\sigma^{s'}} \frac{p_C^{s'*}}{p_C^{s*}} \frac{R_L^{s*} - R_{CB}^s}{R_{CB}^{s'} - R_L^{s'*}})$.

Similarly to the analysis in the proof of Proposition 24, Case a) gives rise to the same equilibria with banks and without default with the exception that the level of money created by an individual bank is limited as follows:

$$\alpha_M^b \in \left[0, \frac{\varphi^*}{\varphi^{reg}}\right]. \tag{5.60}$$

Suppose now that the gross rates of return fulfill the conditions given in Case b). In this case, no bank defaults and

$$\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right] \quad \text{and} \quad 0 < 1 - \frac{R_L^{s'*}}{R_{CB}^{s'}} \leq \varphi^{reg}.$$

Then households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, p_C^*, S_F^*\big)$ given S_F^*, $D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, p_C^*, E_B^*, S_F^*\big)$ given E_B^* and S_F^*, and
$S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, p_C^*\big)$, where these correspondences are given in Lemma 6 in Appendix 2.E. Only the seventh case in the definition of the correspondences given in Lemma 6 is consistent with equal real gross rates of return as well as positive equity and deposits. Thus, we can conclude that

$$\begin{aligned}
&E_B^* \in (0, \big(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\big)),\\
&D_H^* = \big(\mathbf{W} - \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M})\big) - E_B^*, \text{ and}\\
&S_F^* = \mathbf{f}'^{-1}(\overline{\mathbf{R}}_\mathbf{M}).
\end{aligned}$$

In this case, the level of money created by an individual bank is limited by the same constraint as in (5.60).

Suppose now that the gross rates of return fulfill the conditions given in Case c). In this case, some banks default and

$$\begin{aligned}
&\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right], \quad \text{and} \\
&\varphi^* \frac{R_{CB}^{s'} - R_L^{s'*}}{R_{CB}^{s'}} < \varphi^{reg} = \frac{\sigma^s}{\sigma^{s'}} \frac{p_C^{s'*}}{p_C^{s*}} \frac{R_L^{s*} - R_{CB}^{s}}{R_{CB}^{s'}} \leq \varphi^*.
\end{aligned}$$

Then households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, p_C^*, S_F^*\big)$ given S_F^*, $D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, p_C^*, E_B^*, S_F^*\big)$ given E_B^* and S_F^*, and $S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, p_C^*\big)$, where these correspondences are given in Lemma 6 in Appendix 2.E. Only the fourth and seventh cases in the definition of the correspondences given in Lemma 6 are consistent with equal real gross rates of return on equity and on deposits as well as positive equity and deposits. Thus, we can conclude that in the case where $\mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right] \leq \frac{\mathbf{f}'(\mathbf{0})}{p_I^*}$, we obtain

$$\begin{aligned}
&E_B^* \in \left(0, p_I^* \left(\mathbf{W} - \mathbf{f}'^{-1}\left(p_I^* \mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right]\right)\right)\right), \\
&D_H^* = p_I^* \left(\mathbf{W} - \mathbf{f}'^{-1}\left(p_I^* \mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right]\right)\right) - E_B^*, \text{ and} \\
&S_F^* = \mathbf{f}'^{-1}\left(p_I^* \mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right]\right),
\end{aligned}$$

and in the case where $\mathbb{E}\left[\frac{R_{CB}^{s}}{p_C^{s*}}\right] > \frac{\mathbf{f}'(\mathbf{0})}{p_I^*}$,

$$\begin{aligned}
&E_B^* \in (0, p_I^* \mathbf{W}), \\
&D_H^* = p_I^* (\mathbf{W} - E_B^*), \text{ and} \\
&S_F^* = 0.
\end{aligned}$$

In this case, the measure of banks defaulting against households that we denote by m_d is given by the following equality:

$$\int_0^1 \alpha_M^b \,\mathrm{d}b = m_d \frac{\varphi^*}{\varphi^{reg}} = 1,$$

which is equivalent to

$$m_d = \frac{\sigma^s}{\sigma^{s'}} \frac{p_C^{s'*}}{p_C^{s*}} \frac{R_L^{s*} - R_{CB}^s}{\varphi^* R_{CB}^{s'}}.$$

Suppose finally that the gross rates of return fulfill the conditions given in Case d). In this case, some banks default and

$$\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right] \quad \text{and} \quad 0 < \varphi^{reg} \frac{R_{CB}^{s'}}{R_{CB}^{s'} - R_L^{s'*}} \leq \varphi^* = \frac{\sigma^s}{\sigma^{s'}} \frac{p_C^{s'*}}{p_C^{s*}} \frac{R_L^{s*} - R_{CB}^s}{R_{CB}^{s'} - R_L^{s'*}}.$$

In this case, households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, p_C^*, S_F^*\big)$ given S_F^*, $D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, p_C^*, E_B^*, S_F^*\big)$ given E_B^* and S_F^*, and $S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, p_C^*\big)$, where these correspondences are given in Lemma 6 in Appendix 2.E. Only the fourth and seventh cases in the definition of the correspondences given in Lemma 6 are consistent with equal real gross rates of return on equity and on deposits as well as positive equity and deposits. Thus, we can conclude that in the case where $\mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right] \leq \frac{\mathbf{f}'(\mathbf{0})}{p_I^*}$, we obtain

$$\begin{aligned}
E_B^* &\in \left(0, p_I^* \left(\mathbf{W} - \mathbf{f}'^{-1}\left(p_I^* \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]\right)\right)\right), \\
D_H^* &= p_I^* \left(\mathbf{W} - \mathbf{f}'^{-1}\left(p_I^* \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]\right)\right) - E_B^*, \text{ and} \\
S_F^* &= \mathbf{f}'^{-1}\left(p_I^* \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]\right),
\end{aligned}$$

and in the case where $\mathbb{E}\left[\frac{R^s_{CB}}{p^{s*}_C}\right] > \frac{\mathbf{f'(0)}}{p^*_I}$,

$$\begin{aligned} &E^*_B \in (0, p^*_I \mathbf{W}),\\ &D^*_H = p^*_I(\mathbf{W} - E^*_B), \text{ and}\\ &S^*_F = 0. \end{aligned}$$

In this case, the measure of banks defaulting against households that we denote by m_d is given by the following equality:

$$\int_0^1 \alpha^b_M \, \mathrm{d}b = m_d \alpha^{s'}_{DCB} = 1,$$

which is equivalent to

$$m_d = 1 - \frac{R^{s'*}_L}{R^{s'}_{CB}}.$$

Finally, for Cases a) to d), $\mathbf{R}^*_\mathbf{F}$ can be determined by using Lemma 4 and equating the demand for the investment good $\mathbf{K}^*_\mathbf{F}$ to its supply S^*_F. Thus, with the help of the equity ratio φ^*, we can re-write all equilibrium variables as given in Proposition 24.

In turn, it is straightforward to verify that the tuples found in this proof constitute equilibria with banks as defined in Subsection 2.2.5, where the constraint that all banks choose the same money creation level is relaxed. □

Proof of Proposition 26.

Let $\varphi^{reg} \in [0, 1)$ be a minimum equity ratio requirement and $(R^s_{CB})_s$ the central bank policy gross rates. From Propositions 25 and 24, we conclude that the equilibrium with banks such that $R^{s*}_L = R^s_{CB}$ for all states $s = l, h$ exists and is independent of the capital requirement $\varphi^{reg} \in [0, 1)$ as

$$1 - \frac{R^{s*}_L}{R^s_{CB}} = 0$$

for all states $s = l, h$.

We now prove the second claim in Proposition 26. Let $\varphi^{reg} \in [0,1)$ be a minimum equity ratio requirement and $(R^s_{CB})_s$ be the central bank policy gross rates. Let $\varphi^* \in [\varphi^{reg}, 1)$ be the prevailing equity ratio of banks. We look for necessary and sufficient conditions for an equilibrium with banks of the second type in Proposition 25 and we assume without loss of generality that banks default in State s'. Let $\epsilon > 0$. We define

$$R_L^{s*} = R_{CB}^s(1+\epsilon).$$

We also define

$$R_L^{s'*} = \frac{\varphi^*(1+\epsilon)\sigma^{s'}\mathbf{R}_{\mathbf{M}}^{\mathbf{s}'}R_{CB}^{s'}}{\epsilon\sigma^s\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} + (1+\epsilon)\sigma^{s'}\mathbf{R}_{\mathbf{M}}^{\mathbf{s}'}\varphi^*}. \tag{5.61}$$

Equation (5.61) implies that

$$\varphi^* = \frac{\sigma^s}{\sigma^{s'}}\frac{R_L^{s'*}}{R_L^{s*}}\frac{\mathbf{R}_{\mathbf{M}}^{\mathbf{s}}}{\mathbf{R}_{\mathbf{M}}^{\mathbf{s}'}}\frac{R_L^{s*} - R_{CB}^s}{R_{CB}^{s'} - R_L^{s'*}}.$$

The existence of an equilibrium with banks requires $R_L^{s'*} < R_{CB}^{s'}$, which, in turn, requires ϵ to be large enough. To complete the definition of the equilibrium with banks, the following inequalities have to be shown:

$$\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right] \quad \text{and} \quad \varphi^{reg}\frac{R_{CB}^{s'}}{R_{CB}^{s'} - R_L^{s'*}} \le \varphi^*.$$

The first inequality also writes

$$\begin{aligned}\sigma^s\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} + \sigma^{s'}\mathbf{R}_{\mathbf{M}}^{\mathbf{s}'} <& \sigma^s\frac{\mathbf{R}_{\mathbf{M}}^{\mathbf{s}}}{1+\epsilon} + \sigma^{s'}\mathbf{R}_{\mathbf{M}}^{\mathbf{s}'}\frac{R_{CB}^{s'}}{R_L^{s'}}\\ =& \sigma^s\frac{\mathbf{R}_{\mathbf{M}}^{\mathbf{s}}}{1+\epsilon} + \frac{\epsilon\sigma^s\mathbf{R}_{\mathbf{M}}^{\mathbf{s}} + (1+\epsilon)\varphi^*\sigma^{s'}\mathbf{R}_{\mathbf{M}}^{\mathbf{s}'}}{(1+\epsilon)\varphi^*},\end{aligned}$$

which is equivalent to $\varphi^* < 1$. This inequality holds in any equilibrium with banks. The following inequality is thus a sufficient and necessary condition for the existence of an inefficient equilibrium with banks of the second type in Proposition 25, given central bank policy gross rates $(R^s_{CB})_s$ and the minimum equity ratio requirement φ^{reg}:

$$\varphi^{reg} \leq \varphi^* \left(1 - \frac{R_L^{s'*}}{R_{CB}^{s'}}\right),$$

which is equivalent to

$$\varphi^{reg} \leq \frac{\varphi^* \epsilon \sigma^s \mathbf{R_M^s}}{\epsilon \sigma^s \mathbf{R_M^s} + \varphi^*(1+\epsilon)\sigma^{s'} \mathbf{R_M^{s'}}}.$$

The right-hand side of this inequality is increasing in ϵ and in φ^*. As any ϵ that is sufficiently large qualifies for the existence of an equilibrium with banks as described above given that the previous inequality holds, there is such an equilibrium with banks if and only if

$$\varphi^{reg} < \lim_{\varphi \to 1} \lim_{\epsilon \to \infty} \frac{\varphi^* \epsilon \sigma^s \mathbf{R_M^s}}{\epsilon \sigma^s \mathbf{R_M^s} + \varphi^*(1+\epsilon)\sigma^{s'} \mathbf{R_M^{s'}}} = \frac{\sigma^s \mathbf{R_M^s}}{\sigma^s \mathbf{R_M^s} + \sigma^{s'} \mathbf{R_M^{s'}}}.$$

For any such φ^{reg} there are indeed an $\epsilon > 0$ and an equity ratio $\varphi^* < 1$ that are both sufficiently high such that

$$\varphi^{reg} \leq \frac{\varphi^* \epsilon \sigma^s \mathbf{R_M^s}}{\epsilon \sigma^s \mathbf{R_M^s} + \varphi^*(1+\epsilon)\sigma^{s'} \mathbf{R_M^{s'}}}.$$

As another direct consequence, we obtain that if

$$\varphi^{reg} \geq \frac{\sigma^s \mathbf{R_M^s}}{\sigma^s \mathbf{R_M^s} + \sigma^{s'} \mathbf{R_M^{s'}}},$$

there is no equilibrium with banks as described above.

Let $\varphi^{reg} \in (0,1)$ be a minimum equity ratio requirement and $(R_{CB}^s)_s$ be the central bank policy gross rates. Let $\varphi^* \in [\varphi^{reg}, 1)$ be the prevailing equity ratio of banks. We look for necessary and sufficient conditions for an equilibrium with banks of the third type in Proposition 25 and we assume without loss of generality that banks default in State s'. Let $\epsilon > 0$. We define

$$R_L^{s*} = R_{CB}^s(1+\epsilon).$$

We also define

$$R_L^{s'*} = \varphi^{reg} R_{CB}^{s'} \frac{\sigma^{s'}}{\sigma^s} \frac{1+\epsilon}{\epsilon} \frac{\mathbf{R}_\mathbf{M}^{\mathbf{s}'}}{\mathbf{R}_\mathbf{M}^\mathbf{s}}. \tag{5.62}$$

Equation (5.62) implies that

$$\varphi^{reg} = \frac{\sigma^s}{\sigma^{s'}} \frac{R_L^{s'*}}{R_L^{s*}} \frac{\mathbf{R}_\mathbf{M}^\mathbf{s}}{\mathbf{R}_\mathbf{M}^{\mathbf{s}'}} \frac{R_L^{s*} - R_{CB}^s}{R_{CB}^{s'}}.$$

The existence of an equilibrium with banks requires $R_L^{s'} < R_{CB}^{s'}$, which, in turn, requires ϵ to be large enough. To complete the definition of the equilibrium with banks, the following inequalities have to be shown:

$$\mathbb{E}\left[\frac{R_L^{s*}}{p_C^{s*}}\right] < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right] \quad \text{and} \quad \varphi^* \frac{R_{CB}^{s'} - R_L^{s'*}}{R_{CB}^{s'}} < \varphi^{reg}.$$

The first inequality also writes

$$\begin{aligned} \sigma^s \mathbf{R}_\mathbf{M}^\mathbf{s} + \sigma^{s'} \mathbf{R}_\mathbf{M}^{\mathbf{s}'} <& \sigma^s \frac{\mathbf{R}_\mathbf{M}^\mathbf{s}}{1+\epsilon} + \sigma^{s'} \mathbf{R}_\mathbf{M}^{\mathbf{s}'} \frac{R_{CB}^{s'}}{R_L^{s'*}} \\ =& \sigma^s \frac{\mathbf{R}_\mathbf{M}^\mathbf{s}}{1+\epsilon} + \frac{\epsilon \sigma^s \mathbf{R}_\mathbf{M}^\mathbf{s}}{(1+\epsilon)\varphi^{reg}}, \end{aligned}$$

which is equivalent to

$$\epsilon \left(\frac{1}{\varphi^{reg}} - 1 - \frac{\sigma^{s'} \mathbf{R}_\mathbf{M}^{\mathbf{s}'}}{\sigma^s \mathbf{R}_\mathbf{M}^{\mathbf{s}'}} \right) > \frac{\sigma^{s'} \mathbf{R}_\mathbf{M}^{\mathbf{s}'}}{\sigma^s \mathbf{R}_\mathbf{M}^\mathbf{s}}. \tag{5.63}$$

Finally, the inequality $\varphi^* \frac{R_{CB}^{s'} - R_L^{s'*}}{R_{CB}^{s'}} < \varphi^{reg}$ writes

$$\varphi^{reg} > \frac{\varphi^* \epsilon \sigma^s \mathbf{R}_\mathbf{M}^\mathbf{s}}{\epsilon \sigma^s \mathbf{R}_\mathbf{M}^\mathbf{s} + \varphi^*(1+\epsilon)\sigma^{s'} \mathbf{R}_\mathbf{M}^{\mathbf{s}'}}. \tag{5.64}$$

Equation (5.64) holds for any φ^* sufficiently close to φ^{reg}. Equation (5.63) holds if and only if ϵ is sufficiently large and

$$\varphi^{reg} < \frac{\sigma^s \mathbf{R}_\mathbf{M}^\mathbf{s}}{\sigma^s \mathbf{R}_\mathbf{M}^\mathbf{s} + \sigma^{s'} \mathbf{R}_\mathbf{M}^{\mathbf{s}'}}. \tag{5.65}$$

As a consequence, similarly to the reasoning for the case of equilibria with banks for the second type, Equation (5.65) is a necessary and sufficient condition for the existence of an equilibrium with banks of the third type in Proposition 25. □

Proof of Proposition 29.
Suppose that there is an equilibrium with banks and that in this equilibrium, banks default in some state s'. Then households anticipate the banks' default and expect a lower return on their deposits in State s'. Thus, this cannot be an equilibrium with banks. □

Proof of Lemma 14.
The banker's objective function of Bank $b \in [0, 1]$ is given by

$$\mathbb{E}\left[\frac{\theta \alpha_M^b L_M R_L^s}{p_C^s}\right].$$

Hence, this banker chooses the highest level α_M^b such that Bank b does not default against the central bank. We distinguish the following cases:

- Suppose that $R_L^s \geq R_{CB}^s$ in all states $s = l, h$. Then for any level $\alpha_M^b \in \mathbb{R}_+$, Bank b does not default against the central bank. Therefore, its optimal choice is denoted by $\alpha_M^b = +\infty$.
- Suppose that $R_L^s < R_{CB}^s$ in all states $s = l, h$. Then the highest level α_M^b for which Bank b does not default against the central bank is

$$\alpha_M^b = \min\left\{\alpha_{DCB}^l, \, \alpha_{DCB}^h\right\}.$$

- Suppose that $R_L^s < R_{CB}^s$ in just one state s. Then the highest level α_M^b for which Bank b does not default against the central bank is $\alpha_M^b = \alpha_{DCB}^s$.

We can summarize the choices of lending levels by bankers, given gross rates $(R_L^s)_s$ and policy choices $(R_{CB}^s)_s$ with the correspondence $\hat{\alpha}_M\big((R_L^s)_s, (R_{CB}^s)_s\big)$ given in the lemma. □

Proof of Lemma 15.

Let $b \in [0,1]$ denote a bank and assume that a minimum equity ratio $\varphi^{reg} \in (0, \varphi]$ is required to be held by banks at the end of Period $t = 0$. Using Lemma 5, the banker's maximization problem can be written as

$$\max_{\alpha_M^b \in \left[0, \frac{\varphi}{\varphi^{reg}}\right]} \mathbb{E}\left[\frac{\theta \alpha_M^b L_M R_L^s}{p_C^s}\right]$$

subject to the constraint that it does not default against the central bank. We distinguish the following cases:

- Suppose that $R_L^s \geq R_{CB}^s$ for all states $s = l, h$. Then as Banker b's objective function is increasing in α_M^b on $[0, \frac{\varphi}{\varphi^{reg}}]$ and as its bank does not default against the central bank for any $\alpha_M^b \in [0, \frac{\varphi}{\varphi^{reg}}]$, the banker chooses $\alpha_M^b = \frac{\varphi}{\varphi^{reg}}$.
- Suppose now that $R_L^s < R_{CB}^s$ for all states $s = l, h$. Then the banker chooses the highest lending level that is compatible with the minimum equity ratio on the one hand and the no-default condition against the central bank on the other, i.e.

$$\alpha_M^b = \min\left(\alpha_{DCB}^l, \alpha_{DCB}^h, \frac{\varphi}{\varphi^{reg}}\right).$$

- Suppose finally that $R_L^s < R_{CB}^s$ in just one state $s \in \{l, h\}$. Then the banker chooses the highest lending level that is compatible with the minimum equity ratio and the no-default condition against the central bank, i.e.

$$\alpha_M^b = \min\left(\alpha_{DCB}^s, \frac{\varphi}{\varphi^{reg}}\right).$$

We can summarize our findings with the correspondence $\hat{\alpha}_M^{reg}$ given in the lemma.

□

Proof of Lemma 16.

Due to the Inada Conditions,[7] in any equilibrium with banks, a positive amount of investment good is invested in FT. Lemma 2 shows that any differential between the deposit gross rates and the central bank policy gross rates can be used by bankers to infinitely increase their expected shareholders' value. In this case, as the expected return on equity becomes arbitrarily large, bank equity becomes the most profitable investment for households according to Lemma 6, and the households shift their investment from FT to MT. As bankers can increase their expected consumption, this cannot be an equilibrium with banks. Thus, an equilibrium with banks requires $R_D^{s*} = R_{CB}^s$ for all states $s = l, h$. □

Proof of Proposition 31.

Let $\mathcal{E}^*$ be an equilibrium with banks for which a minimum equity ratio $\varphi^{reg} \in (0, \varphi^*]$ is required to be held by banks at the end of Period $t = 0$.

Then all banks choose the same level of money creation and lending, denoted by α_M^*. At the aggregate level, however, the amount borrowed by banks from the central bank has to equal the amount deposited by banks at the central bank, meaning that $\int_0^1 \alpha_M^b \mathrm{d}b = 1$, which translates into $\alpha_M^* = 1$. As Lemma 15 applies, given gross rates of return $(R_L^{s*})_s$, policy choices $(R_{CB}^s)_s$, and the equity ratio φ^*, all banks $b \in [0, 1]$ choose a lending level $\alpha_M^b \in \hat{\alpha}_M^{reg}\big((R_L^{s*})_s, (R_{CB}^s)_s, \varphi^*\big)$. Thus, the only capital structure φ^* in Lemma 15 rationalizing $\alpha_M^* = 1$ is such that $\varphi^* = \varphi^{reg}$. Reciprocally, if $\varphi^* = \varphi^{reg}$, each bank b will choose $\alpha_M^b = 1$. Finally, Lemma 16 implies the relationship $R_D^{s*} = R_{CB}^s$ for all states $s = l, h$.

As bankers receive an amount $\theta L_M^* R_L^{s*}$, the shareholders' value of Bank b in State s amounts to

$$\max\Big(L_M^*\big(R_L^{s*}(1-\theta) - R_{CB}^s\big) + E_B^* R_{CB}^s, 0\Big).$$

We can distinguish three cases:

[7] We now need a stronger assumption than the Inada Conditions of Chapter 2. This proof requires that $\lim_{\mathbf{K_F}\to\mathbf{0}} \mathbf{f}'(\mathbf{K_F}) = +\infty$, so that we make this assumption.

– Suppose first that

$$\varphi^* \geq \max_{s\in\{l,h\}} \left\{ \frac{R^s_{CB} - R^{s*}_L(1-\theta)}{R^s_{CB}} \right\}. \tag{5.66}$$

Condition (5.66) means that no bank defaults in any state of the world. The expected gross rate of return on equity in real terms is thus given by

$$\mathbb{E}\left[\frac{R^{s*}_E}{p^{s*}_C}\right] = \mathbb{E}\left[\frac{R^{s*}_L(1-\theta) - R^s_{CB}}{\varphi^* p^{s*}_C}\right] + \mathbb{E}\left[\frac{R^s_{CB}}{p^{s*}_C}\right]. \tag{5.67}$$

Given gross rates of return $(R^{s*}_E)_s$ and $(R^{s*}_D)_s$ as well as prices p^*_I and $(p^{s*}_C)_s$, households choose $E^*_B \in \hat{E}_B\big((R^{s*}_E)_s, (R^{s*}_D)_s, p^*_I, (p^{s*}_C)_s, S^*_F\big)$ given S^*_F, $D^*_H \in \hat{D}_H\big((R^{s*}_E)_s, (R^{s*}_D)_s, p^*_I, (p^{s*}_C)_s, E^*_B, S^*_F\big)$ given E^*_B and S^*_F, and $S^*_F \in \hat{S}_F\big((R^{s*}_E)_s, (R^{s*}_D)_s, p^*_I, (p^{s*}_C)_s\big)$, where these correspondences are given in Lemma 6 in Appendix 2.E. Only the fourth and the seventh cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ in Appendix 2.E are consistent with the equilibrium conditions $E^*_B > 0$ and $D^*_H > 0$. Both cases require the equality of expected gross rates of return on equity and deposits in real terms:

$$\mathbb{E}\left[\frac{R^{s*}_E}{p^{s*}_C}\right] = \mathbb{E}\left[\frac{R^s_{CB}}{p^{s*}_C}\right]. \tag{5.68}$$

Together with Equalities (5.67) and $R^{s*}_L p^*_I = \mathbf{R^s_M} p^{s*}_C$, Equality (5.68) implies that

$$\frac{\overline{\mathbf{R}}_\mathbf{M}}{p^*_I}(1-\theta) = \mathbb{E}\left[\frac{R^s_{CB}}{p^{s*}_C}\right].$$

Thus, $\frac{\overline{\mathbf{R}}_\mathbf{M}}{p^*_I} > \mathbb{E}\left[\frac{R^s_{CB}}{p^{s*}_C}\right]$, which together with condition $\mathbf{f}'(\mathbf{0}) > \overline{\mathbf{R}}_\mathbf{M}$ rules out the fourth case. Thus, we obtain

$$\begin{aligned} &E^*_B \in (0, p^*_I(\mathbf{W} - S^*_F)), \\ &D^*_H = p^*_I(\mathbf{W} - S^*_F) - E^*_B, \\ &S^*_F = \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}(1-\theta)\big). \end{aligned}$$

However, this can only correspond to an equilibrium with banks if

$$\frac{\mathbf{f}'(\mathbf{W})}{p_I^*} < \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right] = \frac{\overline{\mathbf{R}}_{\mathbf{M}}}{p_I^*}(1-\theta),$$

implying that

$$\theta < \frac{\overline{\mathbf{R}}_{\mathbf{M}} - \mathbf{f}'(\mathbf{W})}{\overline{\mathbf{R}}_{\mathbf{M}}}.$$

In turn, one can directly verify that the tuples found in this case constitute equilibria with banks and financial frictions.

– Suppose now that

$$\varphi^* \leq \min_{s\in\{l,h\}} \left\{ \frac{R_{CB}^s - R_L^{s*}(1-\theta)}{R_{CB}^s} \right\}.$$

This condition implies that the shareholders' value of banks is zero in both states. Thus, $R_E^{s*} = 0$ in both states $s = l, h$. Given gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices p_I^* and $(p_C^{s*})_s$, households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, S_F^*\big)$ given S_F^*,
$D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, E_B^*, S_F^*\big)$ given E_B^* and S_F^*, and
$S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s\big)$, where these correspondences are given in Lemma 6 in Appendix 2.E. Only the fourth and the seventh cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ in Appendix 2.E are consistent with the equilibrium conditions $E_B^* > 0$ and $D_H^* > 0$. As they are not compatible with $R_E^{s*} = 0$ in all states $s = l, h$, we can conclude that there is no equilibrium with banks.

– Suppose finally that

$$\min_{s\in\{l,h\}} \left\{ 1 - \frac{R_L^{s*}}{R_{CB}^s}(1-\theta) \right\} < \varphi^* < \max_{s\in\{l,h\}} \left\{ 1 - \frac{R_L^{s*}}{R_{CB}^s}(1-\theta) \right\}. \tag{5.69}$$

This condition implies that the shareholders' value of banks is zero in one state, say w.l.o.g. State l, and that this value is positive in the other state, h. As a consequence, the expected gross rate of return on equity in real terms is given by

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \sigma\left(\frac{R_L^{h*}(1-\theta) - R_{CB}^h}{\varphi^* p_C^{h*}} + \frac{R_{CB}^h}{p_C^{h*}}\right). \tag{5.70}$$

Given gross rates of return $(R_E^{s*})_s$ and $(R_D^{s*})_s$ as well as prices p_I^* and $(p_C^{s*})_s$, households choose $E_B^* \in \hat{E}_B\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, S_F^*\big)$ given S_F^*, $D_H^* \in \hat{D}_H\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s, E_B^*, S_F^*\big)$ given E_B^* and S_F^*, and $S_F^* \in \hat{S}_F\big((R_E^{s*})_s, (R_D^{s*})_s, p_I^*, (p_C^{s*})_s\big)$, where these correspondences are given in Lemma 6 in Appendix 2.E. Only the fourth and the seventh cases of the definition of the correspondences $\hat{E}_B$, $\hat{D}_H$, and $\hat{S}_F$ in Appendix 2.E are consistent with the equilibrium conditions $E_B^* > 0$ and $D_H^* > 0$. Both cases require the equality of expected gross rates of return on equity and deposits in real terms:

$$\mathbb{E}\left[\frac{R_E^{s*}}{p_C^{s*}}\right] = \mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right].$$

By combining this equality with Equality (5.70), we find the following relationship:

$$\varphi^* = \frac{\sigma}{1-\sigma}\frac{p_C^{l*}}{p_C^{h*}}\frac{R_L^{h*}(1-\theta) - R_{CB}^h}{R_{CB}^l}.$$

Thus, $p_I^*\mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right] > \mathbf{f}'(\mathbf{W})$ has to hold and

$$\begin{aligned} &E_B^* \in (0, p_I^*(\mathbf{W} - S_F^*)), \\ &D_H^* = p_I^*(\mathbf{W} - S_F^*) - E_B^*, \\ &S_F^* = \begin{cases} \mathbf{f}'^{-1}\left(p_I^*\mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right]\right) & \text{if } \mathbf{f}'(\mathbf{0}) \geq p_I^*\mathbb{E}\left[\frac{R_{CB}^s}{p_C^{s*}}\right], \\ \mathbf{0} & \text{otherwise.} \end{cases} \end{aligned}$$

Finally, this can only correspond to an equilibrium with banks if $\varphi^* \in (0,1)$, which implies

$$0 < \frac{\sigma}{1-\sigma}\frac{p_C^{l*}}{p_C^{h*}}\frac{R_L^{h*}(1-\theta) - R_{CB}^h}{R_{CB}^l} < 1,$$

and if Conditions in (5.69) hold. In turn, it is straightforward to verify that the tuples found in this case constitute equilibria with banks and financial frictions.

The case when banks default in the good state is similar.

□

Proof of Proposition 32.
As we investigate how the households' consumption can be maximized without taking into account the impact on the bankers' consumption, the second-best allocation is given by the following maximization problem:

$$\begin{aligned} &\max_{(\mathbf{K_M},\mathbf{K_F},\mathbf{C_B^s})} \mathbb{E}[\mathbf{K_M}\mathbf{R_M^s} + \mathbf{f}(\mathbf{K_F}) - \mathbf{C_B^s}] \\ \text{s.t.} \quad & \mathbf{K_M} + \mathbf{K_F} = \mathbf{W} \\ \text{s.t.} \quad & \mathbf{C_B^s} \geq \theta \mathbf{K_M}\overline{\mathbf{R}}_\mathbf{M}, \end{aligned}$$

where $\mathbf{C_B^s}$ denotes the bankers' consumption in State $s = l, h$ and the second condition represents the bankers' incentive-compatibility constraint. The above optimization problem has clearly a unique solution given by

$$\begin{aligned} \mathbf{K_F^*} &= \mathbf{f}'^{-1}\big(\overline{\mathbf{R}}_\mathbf{M}(1-\theta)\big), \\ \mathbf{K_M^*} &= \mathbf{W} - \mathbf{K_F^*}, \quad \text{and} \\ \mathbf{C_B^{s*}} &= \theta \mathbf{K_M^*}\mathbf{R_M^s}. \end{aligned}$$

From the proof of Proposition 31, we show in the following that in any equilibrium with banks' default,

$$\overline{R}_{CB} > \overline{\mathbf{R}}_\mathbf{M}(1-\theta), \tag{5.71}$$

which directly implies that investment in MT is always higher than the second-best level, when banks default. The second-best allocation is thus only attained without banks' default.

We now show Inequality (5.71). Consider first the case when banks default in the bad state of the world. Then from Proposition 31, we obtain

$$\varphi^{reg} = \frac{\sigma}{1-\sigma}\frac{\mathbf{R}_{\mathbf{M}}^{\mathbf{h}}(1-\theta) - R_{CB}^{h}}{R_{CB}^{l}} < \frac{R_{CB}^{l} - \mathbf{R}_{\mathbf{M}}^{\mathbf{l}}(1-\theta)}{R_{CB}^{l}}.$$

This inequality can also be re-written

$$\overline{R}_{CB} > \overline{\mathbf{R}}_{\mathbf{M}}(1-\theta),$$

which is Inequality (5.71). The case when banks default in the good state of the world is similar. □

Chapter 6

Further Extensions and Outlook

Abstract

We outline further possible extensions to the model of Chapter 2 and we provide an outlook for future research based on inside money creation out of thin air.

6.1 Further Extensions

6.1.1 Financial Frictions Without Deposit Insurance

This extension combines two features: financial frictions and the absence of deposit insurance. They have been dealt with separately in Subsections 5.3 and 5.2, respectively. In the following, we only outline the different steps and problems arising when solving for the equilibria with and without capital requirements.

First, the level of money creation for different capital requirements and central bank policy rates should be derived from the bankers' incentive problem. To do that, the influence of the absence of deposit insurance on bankers' incentive would have to be clarified. Then all other lemmata and results in Chapter 2, together with the fact that households' deposits are not bailed out if some banks default against households, could be used to find the equilibria with banks.

6.1.2 Lending to Bank-specific Production Technologies

This setting is characterized by the banks' heterogeneity regarding the production function of the firms they can lend to. The production function of firms which can only borrow from bank b would be a concave production function that could be denoted by $\mathbf{f_M^{s,b}}(\mathbf{K_M^b})$, where $\mathbf{K_M^b}$ would denote the amount of investment good bought by the firms financed by bank b. As banks would now be heterogeneous, we would have to depart from our investigation limited to symmetric equilibria with banks and also consider asymmetric equilibria with banks.

In this set-up, some banks may want to grant a larger amount of loans to firms than other banks. The banks' incentive to create money would have to be investigated. All other agents' optimization problem would remain unaffected. However, the feature that firms in MT would make profits in some state of the world would have an impact on the formulation of the households' maximization problem. Inefficient equilibria with banks and default could not be excluded a priori. In the case of rigid prices, however, this feature may allow smoother distortions than the ones caused by a complete breakdown of equilibria with banks.

6.1.3 Two Central Bank Policy Rates

In practice, the interest rate charged for borrowing central bank money is higher than the interest rate paid on central bank deposits. We could thus differentiate these two rates in our model to investigate whether this feature has a significant qualitative effect on our results. We would denote the gross rate of return charged for borrowing central bank money by $R^s_{CB,l}$ and the gross rate of return paid on central bank deposits by $R^s_{CB,d}$.

We would then first note that R^s_D would be equal to $R^s_{CB,d}$ from our previous assumption that banks cannot differentiate between households' and other banks' deposits. Moreover, we would also note that $R^s_{CB,l}$ would have to be larger than $R^s_{CB,d}$, as otherwise banks would borrow an infinite amount of money from the central bank. It would then be straightforward that the central bank would make profits. We would assume some kind of redistribution of these profits: For example, these profits could be redistributed as a lump sum to households.

The main challenge would be the analysis of the bankers' maximization problem. Knowing the level of money creation for each possible combination of gross rates of return and capital structures would, however, allow to find all possible equilibria and to examine whether the additional central bank's policy degree of freedom could improve welfare, compared to the model of Chapter 2.

6.2 Outlook and Conclusion

Several research directions may be worth investigating in future research.

First, integrating inside money creation into a dynamic setting, either with infinitely-lived agents or in an overlapping-generation model would probably reveal new insights regarding price stability and financial stability. The model developed in Chapter 2 could then provide a framework that simplifies the exposition of a new, dynamic model with inside money creation.

Second, as most banking models are expressed in real terms and thus use the loanable-funds approach, a precise investigation whether their conclusions continue to hold in a monetary architecture with inside money creation would be very interesting. For example, banking was integrated into a dynamic growth model by Gersbach et al. (2015c), using the loanable-funds model of banking. It could be worthwhile to examine whether their conclusions still hold with the more realistic financing-through-money-creation approach. In general, the issue whether the creation of inside money by commercial banks allows the existence of equilibria with asset price bubbles, excessive lending, or boom-bust cycles should be examined. First analyses include Jakab and Kumhof (2015), who show that the bank-lending volatility is larger in the financing-through-money-creation approach than in the loanable-funds approach.

Third, the financial crisis of 2007/2008 has revived claims that the creation of money by commercial banks when they grant loans has played a role in this boom-bust cycle and that alternative monetary architectures may be more efficient in terms of financial stability, in terms of the appropriate amount of money created, and even with regard to the appropriate choice of borrowers. These claims that asset price bubbles or boom-bust lending cycles may be avoided or mitigated in other monetary architectures and that these monetary architectures would then react differently to shocks should be dealt with thoroughly. Benes and Kumhof (2012), for instance, provide arguments that advocate the Chicago Plan and are based on a DSGE model with inside money creation. However, their analysis should be completed by other micro-founded theoretical models.

Fourth, pioneering analyses of DSGE models characterized by inside money creation were performed by Benes and Kumhof (2012) and Jakab and Kumhof (2015). It would certainly be fruitful to further investigate their approach, which might yield more accurate predictions regarding inflation and thus might be a better tool for the assessment and the design of monetary policy.

Fifth, the modeling of inside money creation might help understand the problem caused by so-called "zombie banks"[1] or more generally, to understand and find potential solutions to the debt-overhang problem.

Sixth, the rise of cryptocurrencies such as Bitcoin or Ethereum has triggered a strong interest among central banks. Major central banks are investigating whether these alternative currencies pose a threat to monetary or financial stability and whether they undermine the efficiency of the central bank policies.[2] Moreover, major central banks are examining the case for national cryptocurrencies that would be based on the blockchain technology.[3] Since 2014, the People's Bank of China is actively investigating such a digital national currency,[4] which would be issued within the next fews years, with the ultimate goal to replace coins and banknotes. The introduction of such a digital currency would entail a comprehensive overhaul of the monetary system. Ultimately, banks will have to attract depositors with higher deposit rates, as the depositors' alternative would be to hold the digital currency that would not involve any credit risk. If banks

[1] According to the definition in Financial Times Lexicon (no date b), a zombie bank is a bank *"that is insolvent but continues to operate until its fate is resolved by closure or merger"*.

[2] Ali et al. (2014) from the Bank of England assesses whether private digital currencies pose a threat to monetary or financial stability, for instance.

[3] Barrdear and Kumhof (2016) from the Bank of England use a DSGE model to assess the impact of the introduction of such a digital national currency on GDP.

[4] See Zhao (2017).

do not offer higher rates, households may stop holding deposits at banks, and the monetary architecture will converge to the one described in Subsection 5.4.2.

The framework developed in this thesis will allow to analyze some aspects of monetary policy, capital regulation, and financial stability in such new monetary architectures. Such analysis is necessary before making changes to the current architecture. It should also determine which monetary architecture is most beneficial for monetary and financial stability, as well as for an efficient intermediation of funds between borrowers and lenders.

Seventh, it will be essential to introduce a government and its role in public good provision into future economic models with inside money creation because money creation is highly intertwined with the money market via repurchase agreements, which are essentially based on government bonds. Such a model would allow to answer key questions such as to whether money creation by banks is a subsidy to banks. This claim is made by some organizations that promote a ban on private money creation.[5] Such more comprehensive models would also yield insights on the impact of quantitative easing on the real economy. Finally, the government and its role in public good provision could be introduced into economic models in which only the central bank can create money. This would allow to compare a monetary architecture in which government spending is the only way to create money and a monetary architecture in which money is only created when the central bank grants loans to banks.

Finally, our model could also provide the basis for an extended macroeconomic model of an open economy, with tradable and non-tradable goods. This model could be used to examine the impact of the purchase of foreign or domestic assets by the central bank on the exchange rate, on the households' consumption and investment decisions, as well as on output.

[5]The non-profit organization called "Positive Money" in the United Kingdom is an example of such an organization: See Ryan-Collins (2017).

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