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# Time-varying mixed frequency forecasting: A real-time experiment

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**Abstract**

This paper tests the usefulness of time-varying parameters when forecasting with mixed-frequency data. For this we compare the forecast performance of bridge equations and unrestriced MIDAS models with constant and time-varying parameters. An out-of-sample forecasting exercise with US real-time data shows that the use of time-varying parameters does not improve forecasts significantly over all vintages. However, since the Great Recession, forecast errors are smaller when forecasting with bridge equations due to the ability of time-varying parameters to incorporate gradual structural changes faster.

*JEL classifications:*
*Keywords:* Forecasting, Bayesian, mixed frequency data, time-varying parameters

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# 1 Introduction

Policy makers need up-to-date information on the state of the economy in order to efficiently implement policy actions. This need has lead to the emergence of models that can incorporate readily available up-to-date high frequency data into econometric models. Two econometric approaches which allow the combination of different frequencies are bridge models and (restricted and unrestricted) Mixed Data Sampling (MIDAS) models.

Another problem when using up-to-date data are temporal instabilities of the parameters which are difficult to detect at the current edge of the data. This difficulty can be addressed to some degree by using time-varying parameters where the parameters follow a random-walk process. The aim of this paper is to analyse whether the use of time-varying parameters gives an advantage when forecasting with mixed frequency data compared to established methods, in this case ordinary least squares (OLS).

A few approaches have been tested in the literature so far. Carriero et al. (2012) use a Bayesian mixed-frequency regression model with stochastic volatility and find some usefulness of using stochastic volatility for forecasting US GDP but not when using time-varying parameters. Galvão (2013) uses a transition function that governs for some parameters the change in parameters in MIDAS regressions. Guerin & Marcellino (2013) propose a Markov-Switching MIDAS approach which allows for switches between a small number of regimes. Schumacher (2014) analyses MIDAS regressions with time-varying parameters for Euro area GDP and corporate bonds spreads by using a particle filter to deal with non-linearities in the MIDAS equation.

This paper extends the literature by using time-varying parameters in both bridge equations and unrestricted MIDAS models. Additionally, we compare forecast performance of the different models and methods for a long forecast horizon. For this analysis we employ a large real-time data set for the US.

The real-time data set of US data contains both quarterly data (GDP) as well as monthly data. We use 11 monthly standard business cycle indicators and their growth rates (month-on-month, 3-month change, year-on-year) to predict quarter on quarter GDP growth. The real-time data set ranges from 1970 until mid-2013.

Due to technical restrictions we can only incorporate few lags, as unrestricted

MIDAS models tend to get over-parametrized fast. Still, even a minimum specification includes enough information for now- and short-term forecasting. Albeit forecasts are made with the use of a single variable, we also look at forecast combinations of the individual models, both as an unweighted average and as weighted average based on the past forecast performance. We find that the use of time-varying parameters does not significantly improve forecast performance of bridge equations over all vintages. But the possibility to incorporate gradual structural changes can help when forecasting recessions and especially the phase since the Great Recession. Economic relationships between variables have changed since the Great Recession. This is the reason why forecasting with bridge models using time-varying parameters is superior to forecasting with OLS. The results are also robust when estimating with a rolling window instead of an expanding window.

The paper is structured as follows: Section 2 explains the method and the estimation strategy. Section 3 discusses the used data set and Section 4 presents an analysis of the real-time parameter estimates of the used models and methods over all vintages. The results of the real-time experiment are shown in Section 5. Section 6 concludes.

# 2 Mixed-frequency models with time-varying parameters

## 2.1 Model setup

We employ two standard single equation mixed frequency models for forecasting, namely bridge equations and unrestricted MIDAS models. Bridge equations are common in policy organizations due to their simplicity and transparency. The general idea behind bridge equations is to explain a low-frequency variable by a time-aggregated contemporary high frequency variable. First, forecasts of the high frequency variable are generated by using an additional model, normally an autoregressive process. These forecasts are then time-aggregated to the lower frequency. Both equations can be easily estimated by OLS. Forecasts are then done iteratively by using the previously obtained parameters as well as the high- and low-frequency forecasts. Early applications of bridge equations in the literature can be found for example in Ingenito & Trehan (1996) or Baffigi et al. (2004) as well at central banks like ECB (2008) or Bundesbank (2013).

Mixed data sampling (MIDAS) proposed by Ghysels et al. (2004), building on distributed lag models like Almon (1965), is another single equation approach which is able to handle time series with different frequencies even in the presence of a long lag structure. The high-frequency variable is not time-aggregated but directly related to the low-frequency variable. As this approach can lead to a high number of parameters to be estimated, lag polynomials are used to decrease the necessary number of parameters. The estimation can be done by non-linear least squares (Ghysels et al., 2007). Early applications of this method were mostly with financial data. More recently MIDAS has also been used for macroeconomic data for example in Clements & Galvão (2008) and Clements & Galvão (2009) or Armesto et al. (2010) and Andreou et al. (2011). Foroni et al. (2015) have shown, that if differences in frequencies are small, for instance for a mixture of quarterly and monthly data, an unrestricted MIDAS setup (U-MIDAS) is equivalent or even superior compared with standard MIDAS setups. An unrestricted MIDAS setup requires less computational and modelling efforts compared with standard MIDAS setups. As the U-MIDAS approach represents a compromise between parsimony, simplicity and accuracy it is often used for nowcasting (Aprigliano et al., 2016).

In order to test the usefulness of time-varying parameters for forecasting both models are estimated in their standard form by using ordinary least squares as well as in a Bayesian state-space framework with time-varying coefficients. The time-varying approaches are presented in the next sub-chapters.

**Time-varying bridge equations**

We specify the bridge equation model using lags of a quarterly variable $y_t$ as well as time aggregated quarterly values of a monthly variable $x_t^q$. This is defined as follows:

$$y_t = c_t + \sum_{i=1}^{p} \beta_{i,t} L^i y_t + \sum_{j=0}^{n} \gamma_{j,t} L^j x_t^q + \epsilon_t \tag{1}$$

$L^i$ and $L^j$ indicate the lag operator for lag lengths $p$ and $n$ respectively. The parameter $\beta_{i,t}$ for each $i$th lag of variable $y_t$ and the parameter $\gamma_{j,t}$ for the $j$th lag of the time-aggregated high-frequency variable $x_t^q$ as well as the constant $c_t$ are time-varying and follow a random walk. With the bridge equations parameters as $a_t = [c_t \ \beta_{1,t} \ ... \ \beta_{p,t} \ \gamma_{1,t} \ ... \ \gamma_{n,t}]$ the measurement equation can be written as:

$$y_t = a_t \begin{bmatrix} 1 \\ y_{t-1} \\ \vdots \\ y_{t-p} \\ x_t^q \\ \vdots \\ x_{t-n}^q \end{bmatrix} + \epsilon_t \tag{2}$$

The state equation models the random walk behaviour of the time-varying parameters:

$$a_t = a_{t-1} + \nu_t \tag{3}$$

The variance-covariance matrix of the innovations is block-diagonal:

$$\begin{pmatrix} \epsilon_t \\ \nu_t \end{pmatrix} \sim N(0, V), V = \begin{pmatrix} \sigma^2 & 0 \\ 0 & Q \end{pmatrix} \tag{4}$$

The forecast procedure for bridge equations consists of three steps. Firstly, the forecasts for the high-frequency variable are generated. This is normally done by using an autoregressive model. The model is estimated by using OLS and the lag length is optimized according to the Bayesian Information Criteria (BIC). In a second step, the high-frequency forecasts are time-aggregated to the lower frequency. Thirdly, the forecasts are plugged into the above specification in order to compute forecasts of the quarterly variable. The forecasts are computed iteratively. In this application, we analyse both bridge models using OLS and time-varying parameters. In both cases the high-frequency forecasts are done by OLS as described in the first step. When forecasting using OLS, the estimated parameters are used for forecasting. In the case of time-varying parameters only the parameters of the last quarter in each vintage are used. The real-time estimates are described and analysed in section 4.

**Time-varying U-MIDAS model**

The unrestricted MIDAS approach was promoted by Foroni et al. (2015). Instead of approximating the parameters of each high-frequency observation of the high frequency variable $x_t$ with the help of polynomials, this approach estimates the weights as unrestricted parameters. The number of parameters depends both on the number of high frequency periods $m$ inside the low frequency period as well as the number of low frequency period lags $n$. Depending on the frequency of the high-frequency variable and the number of desired lags, this approach can easily be over-parametrized. But when frequency differences are small as with macroeconomic data, the results of the U-MIDAS are similar or even slightly superior to the MIDAS approach. For instance, when $x_t$ is a monthly variable and $y_t$ a quarterly variable, then $m = 3$. Forecasts in U-MIDAS models are done directly, thus the model is specified with respect to the desired forecast horizon $h$. The aim is to find the specification that would have predicted $y_t$ $h$ quarters ahead. The functional form of the U-MIDAS can be written as:

$$y_t = c_t + \sum_{i=h}^{p+h} \beta_{i,t} L^i y_t + \sum_{k=mh}^{K} \gamma_{k,t} L^{k/m} x_{t+l} + \epsilon_t \tag{5}$$

$L^{k/3}$ is a lag operator for monthly variables which is defined as $x_{t-1/3} = L^{1/3} x_t$ and $l$ is defined as the lead of the high-frequency variable on the low-frequency variable. As the models is specified for each forecast horizon $h$, $K = mnh$. The forecasts are done directly by plugging the most recent data into the formula using the estimated parameters for each forecast horizon $h$. Thus, with the U-MIDAS parameters as $a_t = [c_t \; \beta_{h,t} \; \dots \; \beta_{p+h,t} \; \gamma_{nh/3,t} \; \dots \; \gamma_{K/3,t}]$ the measurement equation can be written as

$$y_t = a_t \begin{pmatrix} 1 \\ y_{t-h} \\ \vdots \\ y_{t-(p+h)} \\ x_{t+l-nh/3} \\ \vdots \\ x_{t+l-K/3} \end{pmatrix} + \epsilon_t \tag{6}$$

and the state equation as

$$a_t = a_{t-1} + \nu_t \tag{7}$$

with the variance-covariance matrix:

$$\begin{pmatrix} \epsilon_t \\ \nu_t \end{pmatrix} \sim N(0, V), V = \begin{pmatrix} 1 & 0 \\ 0 & Q \end{pmatrix} \tag{8}$$

## 2.2 Estimation

Due to the complexity of the estimation of time-varying parameters, classical methods lead to problems when encountering peaks in regions of low probability and thus can lead to unreasonable results. A Bayesian framework offers the tools to circumvent these problems. We adopt a Bayesian approach and use the Markov Chain Monte Carlo (MCMC) method for the estimation of the time-varying model following Cogley & Sargent (2001), Cogley & Sargent (2005), Primiceri (2005). More precisely we employ a Gibbs sampling algorithm which involves the following steps. Firstly, the matrix $V$ is initialized. The initial values in this step are set to $\sigma_0^2$ being 0.01 and $Q_0 \sim IW(k_Q^2 I_s, T_Q)$ with $s$ being the number of states, the scaling factor $k_Q$ set to 0.05 and the shape parameter $T_Q$ set to $dim(\mathrm{Q})$ + 2. As suggested by DeJong (1991), for the initial conditions $a_0$ we chose an uninformative prior centered at zero with a high variance $p_0$ of 1000. In a second step $a^T$ is sampled from the conditional probability $p(a^T|y^T, x^T, V)$, given the previous results for the variance-covariance matrix $V$ as well as the actual data $y^T$ and $x^T$. In a third step, conditional on the data $y^T$ and $x^T$ as well as the previous draws for the parameter vector $a^T$ the innovations of $\nu_t$ are treated as observable. Thus, $V$ can be sampled by sampling $Q$ from $p(Q|y^T, x^T, a^T)$. The second and third step are then repeated for a number of iterations. In this case we set the number of iterations to 50 000 while discarding the first 80% as burn-in.

# 3 Data

The real-time data used in our analysis comprise quarterly real seasonally adjusted GDP as well as the following monthly data: the consumer price index, industrial production, housing starts and the unemployment rate. Data sources are the ArchivaL Federal Reserve Economic Data (ALFRED) published by the Federal Reserve Bank of St. Louis and the Federal Reserve Bank of Philadelphia Real-Time Data Set for Macroeconomists (RTDSM). In addition, we employ a set of time series that are not subject to data revisions: the ISM indices for manufacturing, supplier delivery times and orders, the S&P 500 stock market index, the 3-month treasury bill yield, the 10-year treasury bond yield and average weekly hours worked by production and supervisory workers. Following common practice we time-aggregate all variables with a higher than monthly frequency by using their end-of-month values (Carriero et al., 2012 and Schorfheide &

Song, 2013, e.g.) Table 1 provides precise data definitions. We use different data transformations for our forecast evaluation, namely year-on-year growth rates, 3-month growth rates as well as month-on-month growth rates.

The real-time dataset comprises 344 vintages covering the time frame January 1970 to August 2013. The first vintage starts in January 1970 and includes all data available until the end of January 1985. In order to use an expanding window setup each following vintage one month is added, i.e. the second vintage starts in January 1970 and includes all data available until the end of February 1985, and so on.

The choice of actual realizations is a delicate issue in a real time data context (cf. the discussions in Croushore, 2006, Romer & Romer, 2000, and Sims, 2002). There have been several benchmark revisions in the time series that we use; the latest revision for GDP occurred in mid-2014 and included a substantial redefinition of gross fixed capital formation which accounts for 20 percent of GDP. A forecaster in, e.g., 1985 could not have predicted such a definition change. Thus we follow Romer & Romer (2000), Faust & Wright (2009) and Carriero et al. (2012) who propose to use the second estimate of quarterly GDP as the actual realization with which we compare our forecasts.

In order to reproduce the available information a forecaster would have had at each forecast date, i.e. at the end of each month $m$, we need to take differing publication lags – so called ragged edges – into account. For most monthly variables the (first) release for each month is not available directly at the end of the month, but is published with a time lag of up to one month. Only the S&P 500 index, the 10-year treasury bond yield and the 3-month treasury bill yield are available directly at the end of each month. The latter three variables are released daily and are never revised. Equally, for the quarterly variable, namely GDP, the (first) release for each quarter $q$ is published with a time lag of up to one month. Consequently, when forecasting at the end of a quarter $q$ the releases for quarter $q$ cannot be used for forecasting but must be backcasted using all information available until the end of $q$.

# 4 Real-time parameter estimates

To start our analysis we look at the real-time estimates of the parameters that are used for the forecasts, starting with the estimates of the bridge equations. In total, over all vintages we have time series with over 300 observations. They contain the parameters of the bridge equations for each variable both for the estimates using OLS and the TVP parameters. In the TVP case we only show the parameter values of the last quarter of each vintage (which are the ones used for the calculation of the forecast). Figure 1 shows the OLS and TVP parameters of the bridge equations using the month-on-month growth rate of industrial production and the 3-month growth rate of housing starts.[1] As can be seen from the figure, the OLS parameters that are used to produce the forecasts are relatively stable over the vintages and exhibit a much lower variance than the time-varying parameters. Nonetheless the estimated time-varying parameters do not show a lot of sudden jumps and do not, at least at first glance seem to introduce much noise.

The same analysis is done for the parameters of the U-MIDAS equations. As forecasts with the U-MIDAS method are done directly in contrast to the iterative approach of bridge equations, for each forecast horizon $h$ and each variable we have a set of $1, ..., H$ estimated parameters both for OLS and TVP. Additionally, parameters change with each month of each quarter. Thus, for sake of simplicity, in figure 2 we show only the estimates of the first month $m1$ of each quarter for horizons $h = 1, ..., 5$. The figure shows the constant of the U-MIDAS specification when using the 3-month growth rate of housing starts as high-frequency variable. In case of U-MIDAS, the estimated parameters are a bit more volatile compared to the parameters for the bridge equations in figure 1 which could introduce more noise into the forecasts. In the next section we will analyse whether this parameter volatility is actually a problem for the forecast performance of the different models.

# 5 Real-time out-of-sample results

## 5.1 Forecast comparison

In this section the forecast results of the different models and methods are presented. Namely, these are bridge equations and U-MIDAS models estimated by either OLS or time-varying parameters in a Bayesian state space setup (TVP). In order to coherently compare the forecasts of the models and methods we have to make sure that always the same information is used, so that differences in forecast performance result only from the different models / methods. As U-MIDAS models tend to get overparametrized quickly

[1] An overview over all used parameters can be found in Appendix A.1.

we restrict the data used in this forecasting exercise to one lagged quarter of low-frequency data and three months of high-frequency data. For the comparison of the forecast performance of the different models we calculate the relative root mean squared forecast error (RMSFE) for each model and forecast horizon $h$ which is defined as:

$$\Delta RMSFE_h \quad = \quad 100 * \left( \frac{RMSFE_h^{Model\ 1} - RMSFE_h^{Model\ 2}}{RMSFE_h^{Model\ 2}} \right).$$

We refer to $\Delta RMSFE_h$ as the relative change in the RMSFE. The *more negative* $\Delta RMSFE_h$ is, the *better* performs model 1 compared to the respective benchmark model in terms of predictive power.

In order to test if the two forecasts are actually different from each other we employ the Diebold-Mariano test for equality of forecast accuracy (Diebold & Mariano, 1995). For this we use a quadratic loss function differential $d$ between the forecast errors $\epsilon$ of the models 1 and 2 at time $t$ for the period $t + h$:

$$d_t = (\epsilon^1_{(t+h|t)})^2 - (\epsilon^2_{(t+h|t)})^2$$

The Diebold-Mariano test statistic $DM$ is defined as

$$DM = \frac{\bar{d}}{\sqrt{(1/T) * LRV(d)}}$$

with $\bar{d}$ being the sample mean of $d_t$, $T$ being the number of forecasts and the estimated long run variance (LRV) which corrects for possible serial correlation of the loss differentials $d$ for forecast horizons $h > 1$:

$$LRV(d) = \mathrm{Var}(d) + 2 \sum_{k=1}^{h-1} \mathrm{Cov}(d_t, d_{t-k})$$

Under the null hypothesis of equal forecast accuracy the test statistic is asymptotically N(0,1) distributed, thus the null hypothesis will be rejected it the test statistic falls outside the range of -1.96 and 1.96 at the 5 percent significance level. The significance is incorporated in the following subsections in the graphs for the relative RMSFE. In case the null hypothesis is not rejected the relative RMSFE will be in a dashed line. If the Diebold-Mariano test indicates a significant difference in forecast accuracy the lines will be solid. In the following sections the different combinations of models and methods will be tested separately.

## 5.2 Performance over all vintages

When estimating with OLS, the forecast performance of bridge equations and U-MIDAS models over all vintages turns out to be basically equal. This is in line with previous work by Marcellino et al. (2006) who find that direct forecasting can be at least as accurate as indirect forecasting. Bridge models have a slightly smaller RMSFE for some of the used variables like the ISM total or hours worked, whereas for some other variables the U-MIDAS models have smaller forecast errors. Figure 3 shows the relative RMSFE on average over all variables as well as the results for a forecast combination using forecast errors of the previous 4 quarters of each vintage to weight forecasts.[2] The figure shows a slightly lower RMSFE for bridge equations up to a maximum of 9 percent. But the results are hardly significant. Only for forecast horizons from 7 to 9 months ahead the Diebold-Mariano test indicates actual differences in forecast accuracy (pointed out by the solid line for those months). But even in those cases the difference in forecast accuracy is very small.

The picture changes somewhat when using time-varying parameters to compare those two models: In most cases the forecast errors of bridge models are smaller compared to the U-MIDAS specification. Still, when using short term interest rates or industrial production the RMSFE is somewhat smaller for the U-MIDAS setup compared to the bridge equations. Over all variables, as can be seen in figure 4 the RMSFE is clearly – and for shorter forecast horizons significantly – smaller for bridge equations than for U-MIDAS models. This result could be due to an introduction of noise when using time-varying parameters with U-MIDAS as shown in chapter 4.

This result can also be seen in the direct comparison of OLS and TVP in the case of U-MIDAS models: For almost all variables the forecast errors are smaller when using OLS and also the average and combination forecasts show a clear outperformance by OLS over TVP (figure 5). The differences in forecast accuracy are also significant for shorter forecast horizons. In the case of bridge equations, the results are not that distinct.

[2] The graphs for all variables and all forecast comparisons can be seen in Appendix A.2.

The forecasts errors for almost all variables are smaller when using OLS instead of TVP. But, as figure 6 shows, the differences in forecast accuracy are not statistically different, except for a forecast horizon of 7 to 9 months when using a combined forecast instead of an average over all variables. But even in this case the differences in forecast performance are rather small.

In summary, over all vintages, the use of time-varying parameters does not give an advantage when forecasting with mixed-frequency models. The estimation of U-MIDAS models with time-varying parameters mostly introduces noise due to the more volatile monthly data. Forecast errors of U-MIDAS models when using TVP are clearly worse. So when deciding between bridge equations and U-MIDAS models, it might be slightly beneficial using bridge equations when using TVP. But bridge equations when using TVP are, over all vintages, also not clearly better compared to bridge equations estimated with OLS.

## 5.3 Performance in times of structural changes

The higher flexibility of time-varying parameters and thus the possibility to incorporate gradual structural changes could lead to a better performance of models using this technique in some special phases. In order to test this conjecture a sub-sample of our data is created. Specifically, we look at the Great Recession and the subsequent upswing. It can be argued that with the Great Recession economic relationships between GDP and different indicators changed. In this case, a model using time-varying parameters should be able to react faster to such changes than a simple OLS model with an expanding window.

In the following, for the sake of presentation, only the average forecast results of the previous analysis are presented.[3] Figure 7 shows that since the Great Recession bridge equations are slightly superior than U-MIDAS models both when using OLS or TVP. When forecasting with bridge equations it was advantageous to use TVP instead of OLS. The relative RMSFE lies now mostly below zero and is, at least for longer forecast horizons, significant. This result can be explained by the change in the time-varying parameters in 2009. Figure 8 shows as example the time-varying parameter of the autoregressive term in bridge models when using the short-term interest rate as high-frequency variable over several vintages. Beginning in 2009

[3]All results can be found in Appendix A.3

the parameters shift upwards and show a very different dynamic also in 2010 and 2011. Similar reactions can be found using other high-frequency variables. This higher flexibility allows for better forecasts after the Great Recession.

In order to check the robustness of the results for the full sample we split the sample at the beginning of the Great Recession. As can be seen in figure 9 most results do not differ too much between the sample before the Great Recession and afterwards. Bridge models still are preferable compared to U-MIDAS when using OLS (albeit not significantly anymore) and when using TVP. Also when forecasting with U-MIDAS models OLS should be the preferred estimation method. The biggest differences between the samples occur when using bridge models for forecasting. Before the Great Recession the estimation with OLS was slightly superior. During and after the Great Recession the use of TVP is slightly superior, increasingly so with longer forecast horizons. In order to check if this result stays robust we estimate the OLS models using a rolling window instead of an expanding window. This allows the parameters to be more flexible and to adapt faster to such structural changes. While the OLS parameters are more flexible they do not adapt as fast time-varying parameters and even with rolling windows of 36, 48 and 60 quarters the results are robust. In summary, the use of time-varying parameters can be beneficial when using bridge models for forecasting especially since the Great Recession.

# 6 Conclusion

To study the usefulness of time-varying parameters for forecasting with mixed-frequency data we compared different forecasting models and estimation methods. We used two standard mixed-frequency forecasting models, namely bridge equation and unrestricted MIDAS (U-MIDAS) models. These models were estimated with standard ordinary least squares (OLS) and in a Bayesian state space framework that allows for the estimation of time-varying parameters.

Time-varying parameters offer the possibility to address potential temporal instabilities which are hard to detect, especially when working at the current edge of the data. By using the estimated parameters for the latest quarter, forecasts could be improved compared to using OLS parameters that do not include potential shifts in the correlation of different variables.

To test the forecast performance of bridge equations and U-MIDAS models estimated with OLS and time-varying parameters (TVP) we conduct a real-time experiment using US data with vintages from January 1985 until August 2013. We compare the out-of-sample forecasts of all models and methods separately. We find that when using OLS, bridge equations and U-MIDAS models perform almost equally over all vintages. When using TVP, bridge equations perform significantly better. Due to the more volatile high-frequency data used in U-MIDAS models in contrast to the time-aggregated data in bridge equations the time-varying parameters seem to introduce noise compared to the estimation with OLS. When using bridge equations the estimation method does not seem to matter much: the forecast performance is roughly the same when using OLS or TVP and the differences are hardly significant for most forecast horizons. For U-MIDAS models the classic estimation method is clearly and significantly better for forecasting.

We also analyse if the higher flexibility of time-varying parameters and thus the possibility to incorporate gradual structural changes could lead to a better performance of models using this technique in times of structural changes. We check a sub-sample of our data, namely the period since the start of the Great Recession. While most results change only slightly in the sub-sample, the results for bridge models stand out. Forecast errors are smaller when using time-varying parameters compared to OLS and get smaller for longer forecast horizons. The results are robust even when estimating the bridge models with a rolling window. Even though models with a rolling window can adapt faster to structural changes compared to the use of an expanding window, the time-varying parameters are able to react faster to the changes after the Great Recession. In summary, over all vintages the use of time-varying parameters did not improve forecast results significantly. However, since the Great Recession it was advantageous to use time-varying parameters when forecasting with bridge models due to the higher flexibility of those models.

# References

Almon, S. (1965). The distributed lag between capital appropriations and expenditures. *Econometrica*, (pp. 178–196).

Andreou, E., Ghysels, E., & Kourtellos, A. (2011). Forecasting with mixed-frequency data. In *In Oxford Handbook of Economic Forecasting*, (pp. 225–245).

Aprigliano, V., Foroni, C., Marcellino, M., Mazzi, G., & Venditti, F. (2016). A daily indicator of economic growth for the euro area. Tech. rep.

Armesto, M. T., Engemann, K. M., & Owyang, M. T. (2010). Forecasting with Mixed Frequencies. *Federal Reserve Bank of St. Louis Review*, *92*(6), 521–536.

Baffigi, A., Golinelli, R., & Parigi, G. (2004). Bridge models to forecast the euro area GDP. *International Journal of Forecasting*, *20*(3), 447–460.

Bundesbank (2013). Forecasting models in short-term business cycle analysis - a workshop report. *Deutsche Bundesbank Monthly Report September 2013*, (pp. 69–83).

Carriero, A., Clark, T. E., & Marcellino, M. (2012). Real-time nowcasting with a Bayesian mixed frequency model with stochastic volatility. *Federal Reserve Bank of Cleveland Working Papers*, *12-27*. Federal Reserve Bank of Cleveland.

Clements, M. P., & Galvão, A. B. (2008). Macroeconomic Forecasting With Mixed-Frequency Data. *Journal of Business & Economic Statistics*, *26*, 546–554.

Clements, M. P., & Galvão, A. B. (2009). Forecasting US output growth using leading indicators: an appraisal using MIDAS models. *Journal of Applied Econometrics*, *24*(7), 1187–1206.

Cogley, T., & Sargent, T. J. (2001). Evolving Post-World War II U.S. Inflation Dynamics. In *NBER Macroeconomics Annual 2001, Volume 16*, NBER Chapters, (pp. 331–388). National Bureau of Economic Research, Inc.

Cogley, T., & Sargent, T. J. (2005). Drift and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S. *Review of Economic Dynamics*, *8*(2), 262–302.

Croushore, D. (2006). *Forecasting with Real-Time Macroeconomic Data*, vol. 1 of *Handbook of Economic Forecasting*, chap. 17, (pp. 961–982). Elsevier.

DeJong, P. (1991). The diffuse kalman filter. *The Annals of Statistics*, *19*(2), 1073–1083.

Diebold, F. X., & Mariano, R. S. (1995). Comparing predictive accuracy. *Journal of Business & Economic Statistics*, *13(3)*, 253–263.

ECB (2008). Short-term forecasts of economic activity in the euro area. *ECB Monthly Bulletin April 2008*, (pp. 69–74).

Faust, J., & Wright, J. H. (2009). Comparing greenbook and reduced form forecasts using a large realtime dataset. *Journal of Business & Economic Statistics*, *27*(4), 468–479.

Foroni, C., Marcellino, M., & Schumacher, C. (2015). Unrestricted mixed data sampling (MIDAS): MIDAS regressions with unrestricted lag polynomials. *Journal of the Royal Statistical Society Series A*, *178*(1), 57–82.

Galvão, A. B. (2013). Changes in predictive ability with mixed frequency data. *International Journal of Forecasting*, *29*(3), 395–410.

Ghysels, E., Santa-Clara, P., & Valkanov, R. (2004). The MIDAS Touch: Mixed Data Sampling Regression Models. *CIRANO Working Papers*, *20*, 1–33.

Ghysels, E., Sinko, A., & Valkanov, R. (2007). MIDAS Regressions: Further Results and New Directions. *Econometric Reviews*, *26*(1), 1–48.

Guerin, P., & Marcellino, M. (2013). Markov-Switching MIDAS Models. *Journal of Business & Economic Statistics*, *31*(1), 45–56.

Ingenito, R., & Trehan, B. (1996). Using monthly data to predict quarterly output. *Economic Review*, (pp. 3–11).

Marcellino, M., Stock, J. H., & Watson, M. W. (2006). A comparison of direct and iterated multistep AR methods for forecasting macroeconomic time series. *Journal of Econometrics*, *135*(1-2), 499–526.

Primiceri, G. E. (2005). Time Varying Structural Vector Autoregressions and Monetary Policy. *Review of Economic Studies*, *72*(3), 821–852.

Romer, D. H., & Romer, C. D. (2000). Federal reserve information and the behavior of interest rates. *American Economic Review*, *90*(3), 429–457.

Schorfheide, F., & Song, D. (2013). Real-time forecasting with a mixed-frequency VAR. *NBER Working Papers*, *19712*. National Bureau of Economic Research.

Schumacher, C. (2014). MIDAS regressions with time-varying parameters: An application to corporate bond spreads and GDP in the Euro area. Tech. rep.

Sims, C. A. (2002). The role of models and probabilities in the monetary policy process. *Brookings Papers on Economic Activity*, *33*(2), 1–62.

# 7 Figures and Tables

Table 1: **Data overview**

| VARIABLE NAME | FREQUENCY | REAL TIME | SOURCE | NOTES |
|---|---|---|---|---|
| *Real seasonal adjusted GDP* | quarterly | yes | RTDSM | |
| *Industrial production index: manufacturing* | monthly | yes | RTDSM | From 1997M12 to 1998M10 historical data from 1969 to 1970 were not available in the data set. As there do not seem to be any revisions in between, we make the assumption that there were no revisions. |
| *Consumer price index* | monthly | yes | ALFRED | For some months two vintages were available. In those cases we used the later publications. This was the case for 2000M9, 2005M2, 2006M2, 2007M2, 2008M2, 2009M2, 2010M2, 2011M2 and 2012M2. |
| *Housing starts* | monthly | yes | RTDSM | There were no observations available for the first publication of 1995M11 and 1995M12. We make the assumption that in those cases no revision took place and use the same values as in the second publication. |
| *Umemployment rate* | monthly | yes | ALFRED | We used the real-time data from ALFRED data base because the RTDSM only offers quarterly vintages. |
| *ISM index for manufacturing (= ISM total)* | monthly | no | Institute for Supply Management | |
| *ISM index for supplier delivery times* | monthly | no | Institute for Supply Management | |
| *ISM index for orders* | monthly | no | Institute for Supply Management | |
| *Average weekly hours of production and supervisory workers* | monthly | no | US Bureau of Labor Statistics | |
| *S&P 500 stock market index* | monthly | no | Thomson Reuters Datastream | |
| *3-month treasury bill yield* | monthly | no | Thomson Reuters | |
| *10-year treasury bond yield* | monthly | no | Thomson Reuters | |

Figure 1: **Parameters for bridge equation**

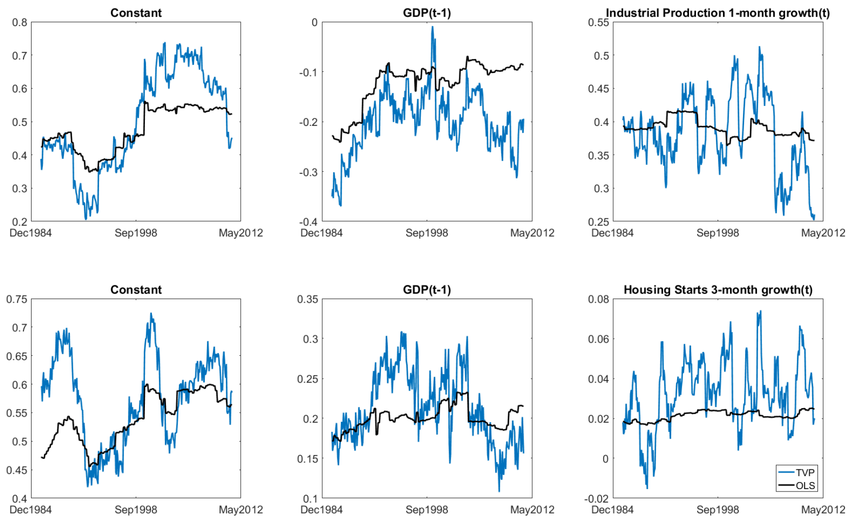



Figure 2: **Parameters for U-MIDAS model**

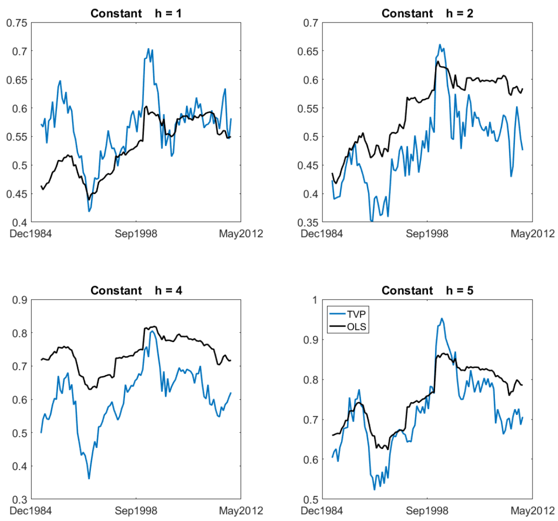
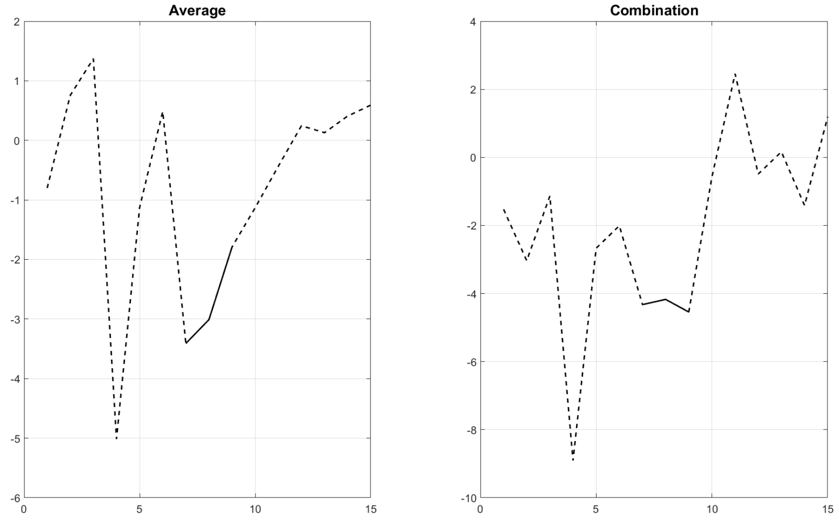
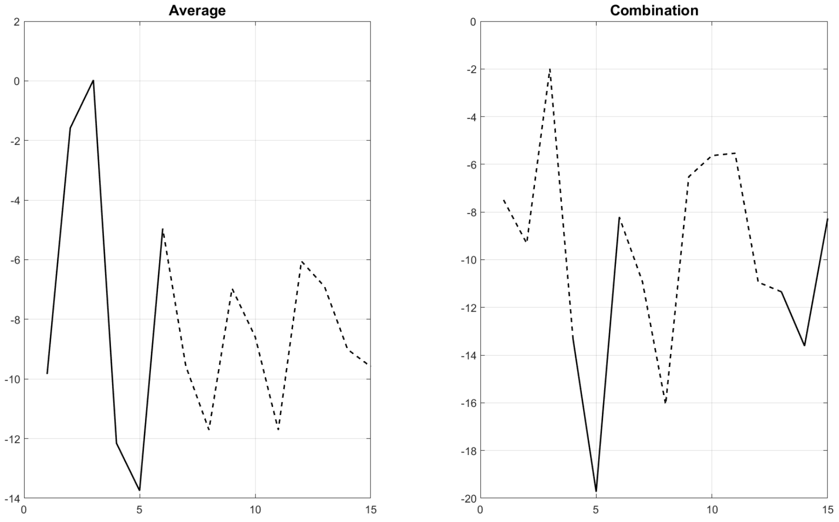


Figure 3: **Rel. RMSFE: Bridge vs U-MIDAS using OLS**



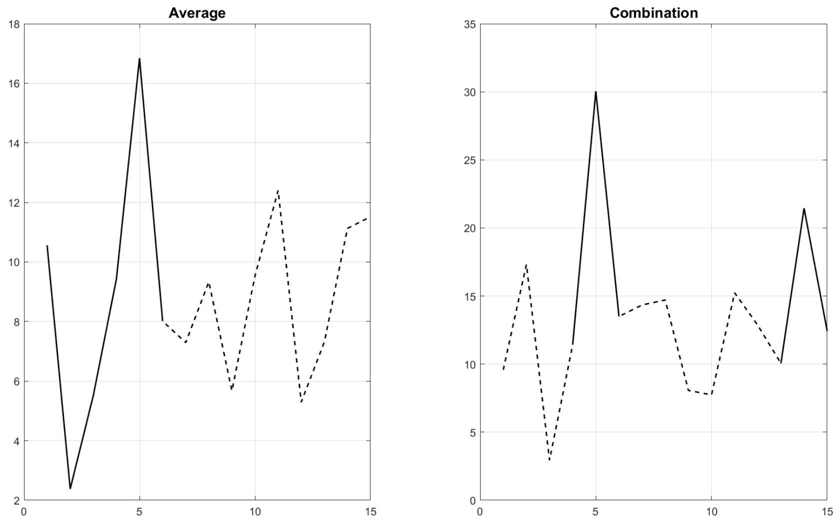

Note: The dashed line shows the relative RMSFE of bridge equations compared to the benchmark model U-MIDAS. Values below zero indicate the smaller percentage forecast error of bridge equations compared to U-MIDAS. The solid line indicates significant results according to the Diebold-Mariano test.

Figure 4: **Rel. RMSFE: Bridge vs U-MIDAS using TVP**



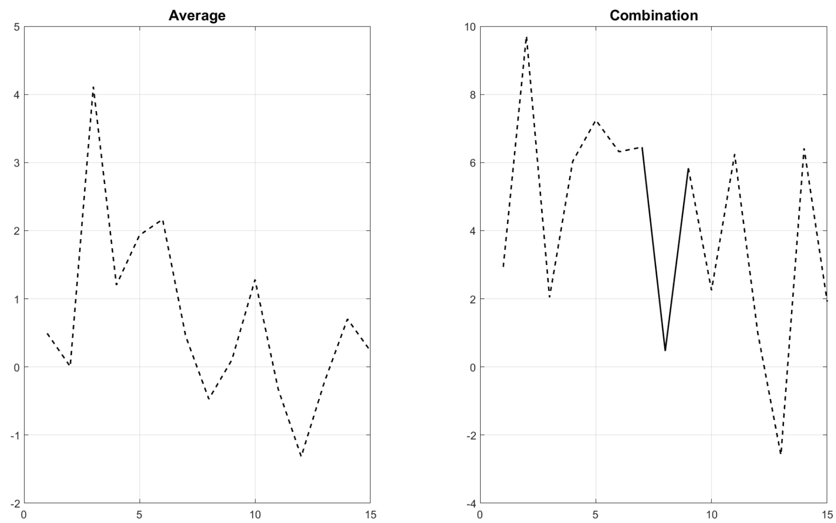

Note: The dashed line shows the relative RMSFE of bridge equations compared to the benchmark model U-MIDAS. Values below zero indicate the smaller percentage forecast error of bridge equations compared to U-MIDAS. The solid line indicates significant results according to the Diebold-Mariano test.

Figure 5: **Rel. RMSFE: TVP vs OLS using U-MIDAS**



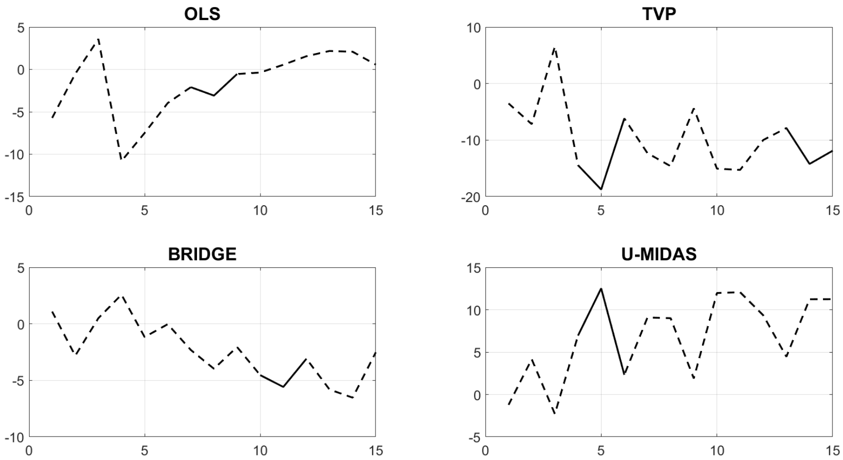

Note: The dashed line shows the relative RMSFE of TVP compared to the benchmark method OLS. Values below zero indicate the smaller percentage forecast error of TVP compared to OLS. The solid line indicates significant results according to the Diebold-Mariano test.

Figure 6: **Rel. RMSFE: TVP vs OLS using Bridge**




Note: The dashed line shows the relative RMSFE of TVP compared to the benchmark method OLS. Values below zero indicate the smaller percentage forecast error of TVP compared to OLS. The solid line indicates significant results according to the Diebold-Mariano test.

Figure 7: **Rel. RMSFEs for Great Recession**




Note: The four graphs show the relative RMSFE of the four previously used cases for the average forecast over all variables. The graph OLS shows the case of Bridge vs U-MIDAS using OLS. Graph TVP shows the case Bridge vs U-MIDAS using TVP. The graph BRIDGE depicts the comparison TVP vs OLS using Bridge and respectively in graph U-MIDAS the case TVP vs OLS using U-MIDAS. Values below zero indicate the smaller percentage forecast error of the benchmark model. The solid line indicates significant results according to the Diebold-Mariano test.

Figure 8: **Time-varying parameters over different vintages**

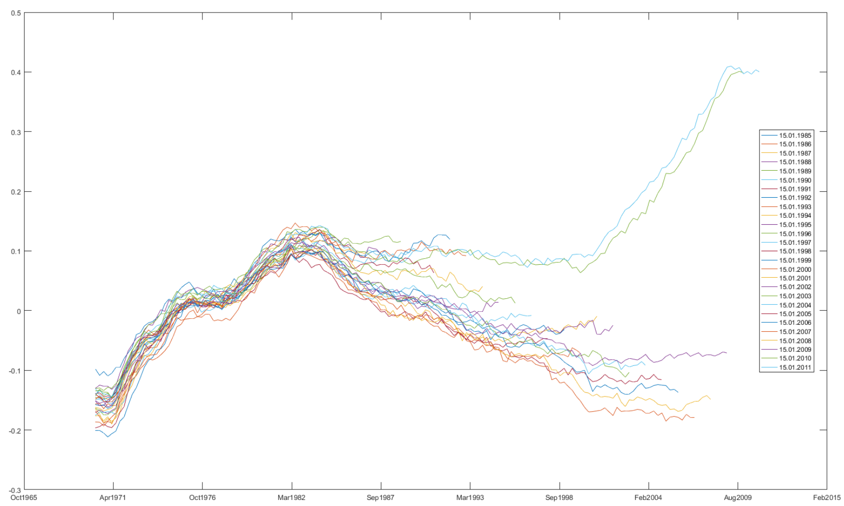
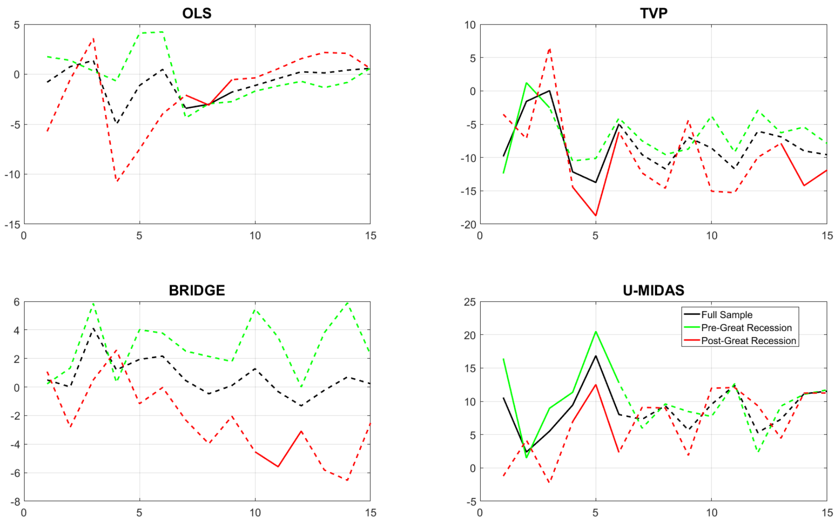


Figure 9: **Rel. RMSFEs for split sample**




Note: The four graphs show the relative RMSFE of the four previously used cases for the average forecast over all variables for 3 different samples. The graph OLS shows the case of Bridge vs U-MIDAS using OLS. Graph TVP shows the case Bridge vs U-MIDAS using TVP. The graph BRIDGE depicts the comparison TVP vs OLS using Bridge and respectively in graph U-MIDAS the case TVP vs OLS using U-MIDAS. Values below zero indicate the smaller percentage forecast error of the benchmark model. The solid line indicates significant results according to the Diebold-Mariano test.

# A Appendix

## A.1 Parameter estimates for all models and forecast horizons

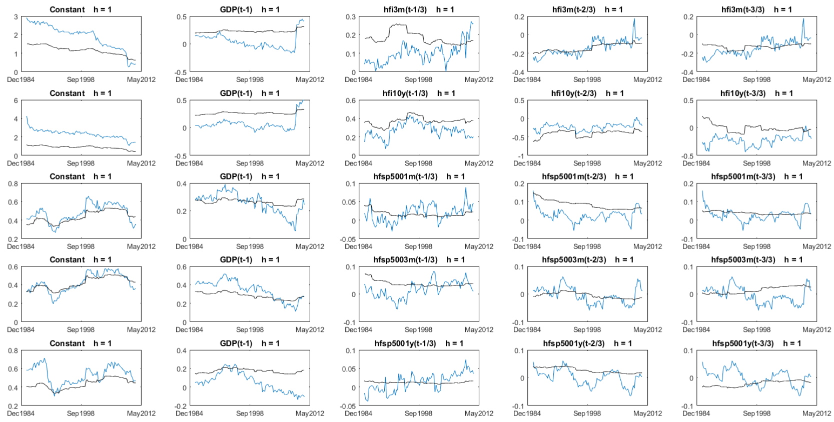
In this section the parameter estimates for all models and forecast horizons are presented. In order to safe space in the graphs the used variables are abbreviated. An overview can be found in Table 2. If the variable name has no ending, it is used in its original form. Three different transformations are also used, when appropriate, namely year-on-year growth rates, 3-month growth rates as well as month-on-month growth rates. These are labelled by the addition of $1y$, $3m$ or $1m$ respectively as ending of the variable name.

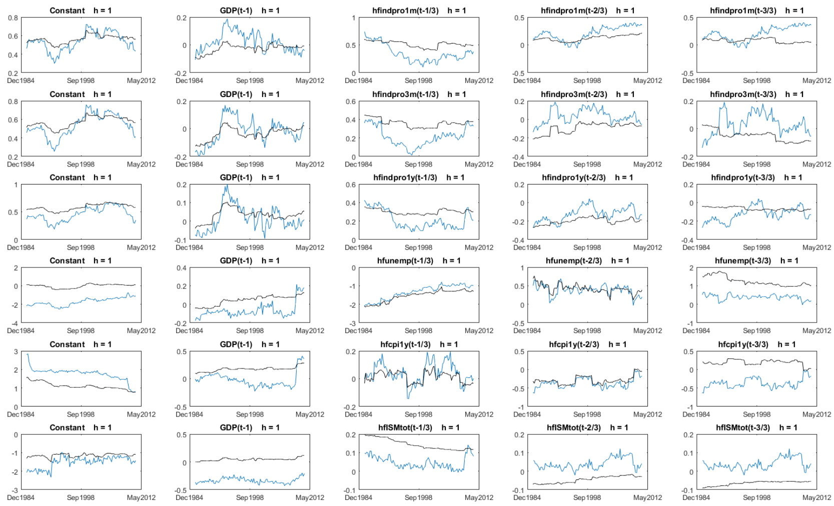
Table 2: **Abbreviations of variables**

| VARIABLE NAME | ABBREVIATION |
|---|---|
| *Industrial production index: manufacturing* | hfindpro |
| *Consumer price index* | hfcpi |
| *Housing starts* | hfhstarts |
| *Umemployment rate* | hfunemp |
| *ISM index for manufacturing* | hfISMtot |
| *ISM index for supplier delivery times* | hfISMsupply |
| *ISM index for orders* | hfISMorder |
| *Average weekly hours of production and supervisory workers* | hfhworked |
| *S&P 500 stock market index* | hfsp500 |
| *3-month treasury bill yield* | hfi3m |
| *10-year treasury bond yield* | hfi10y |

Figure 10: **Parameters for bridge equation**

Figure 11: **Parameters for U-MIDAS model with h = 1**





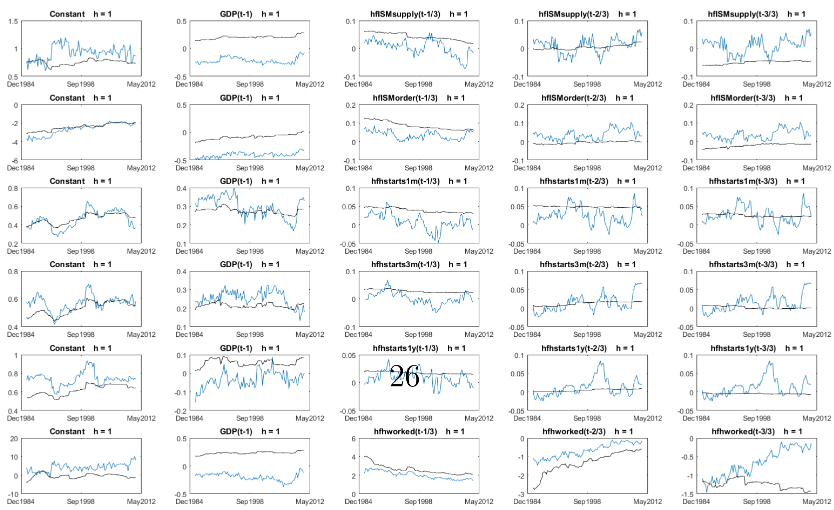
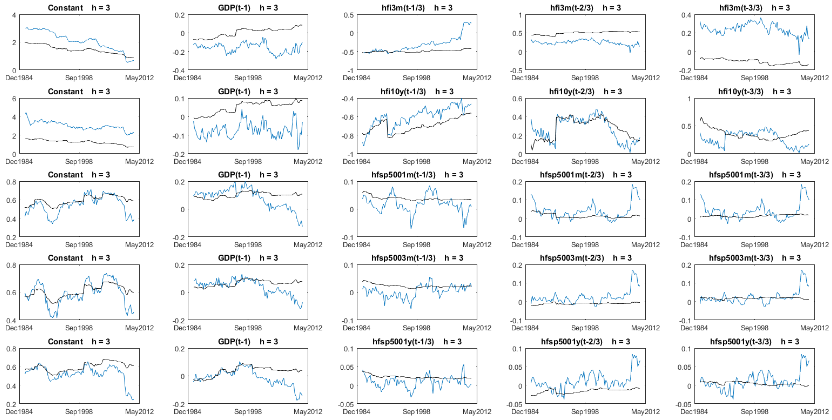
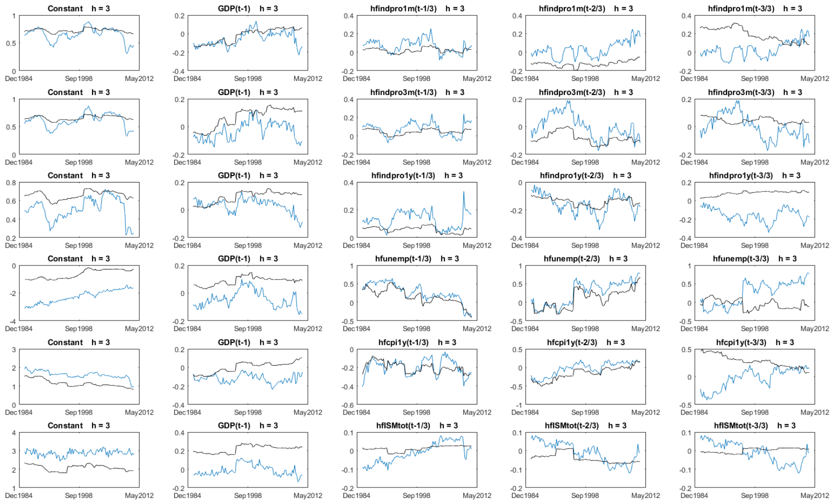


Figure 12: **Parameters for U-MIDAS model with h = 2**

Figure 13: **Parameters for U-MIDAS model with h = 3**





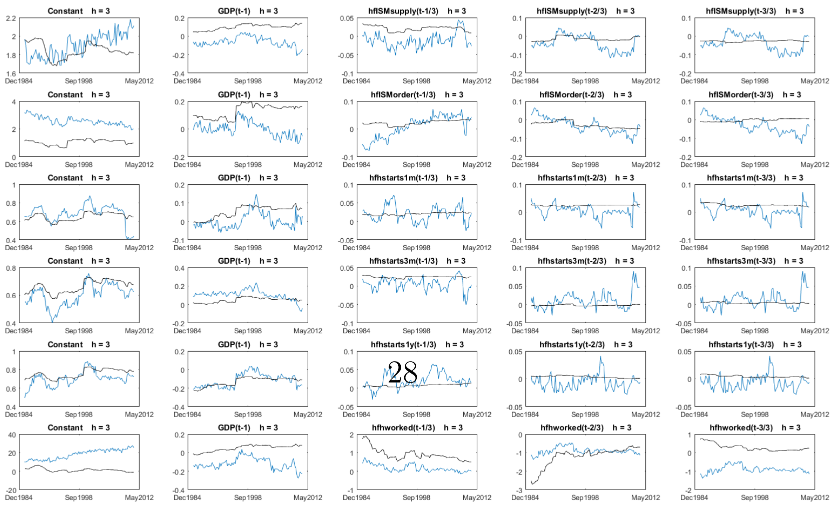
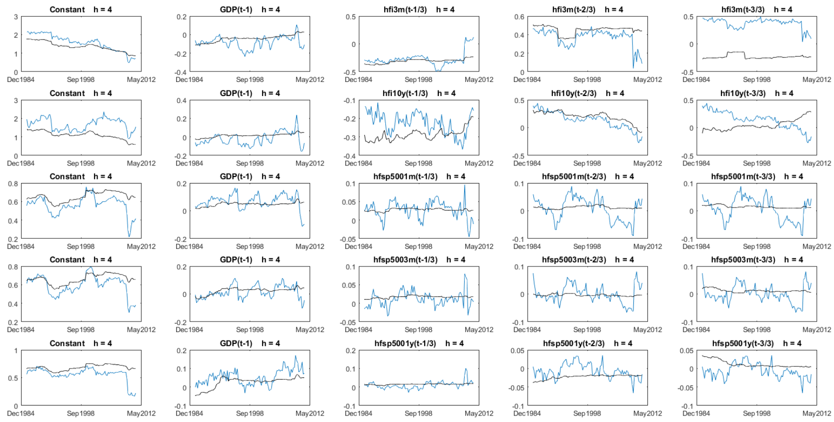
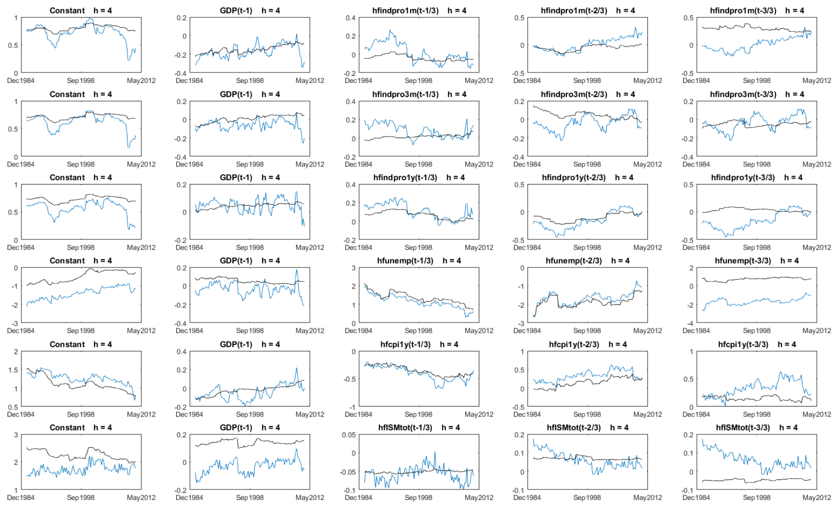


Figure 14: **Parameters for U-MIDAS model with h = 4**



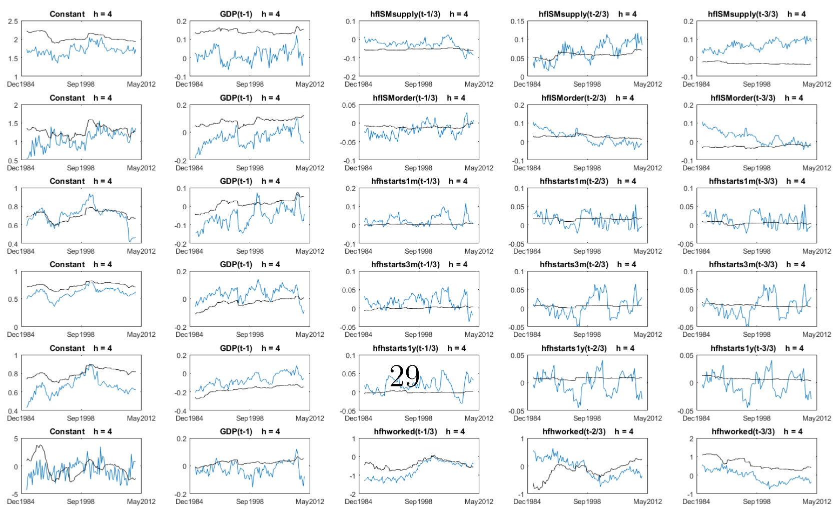
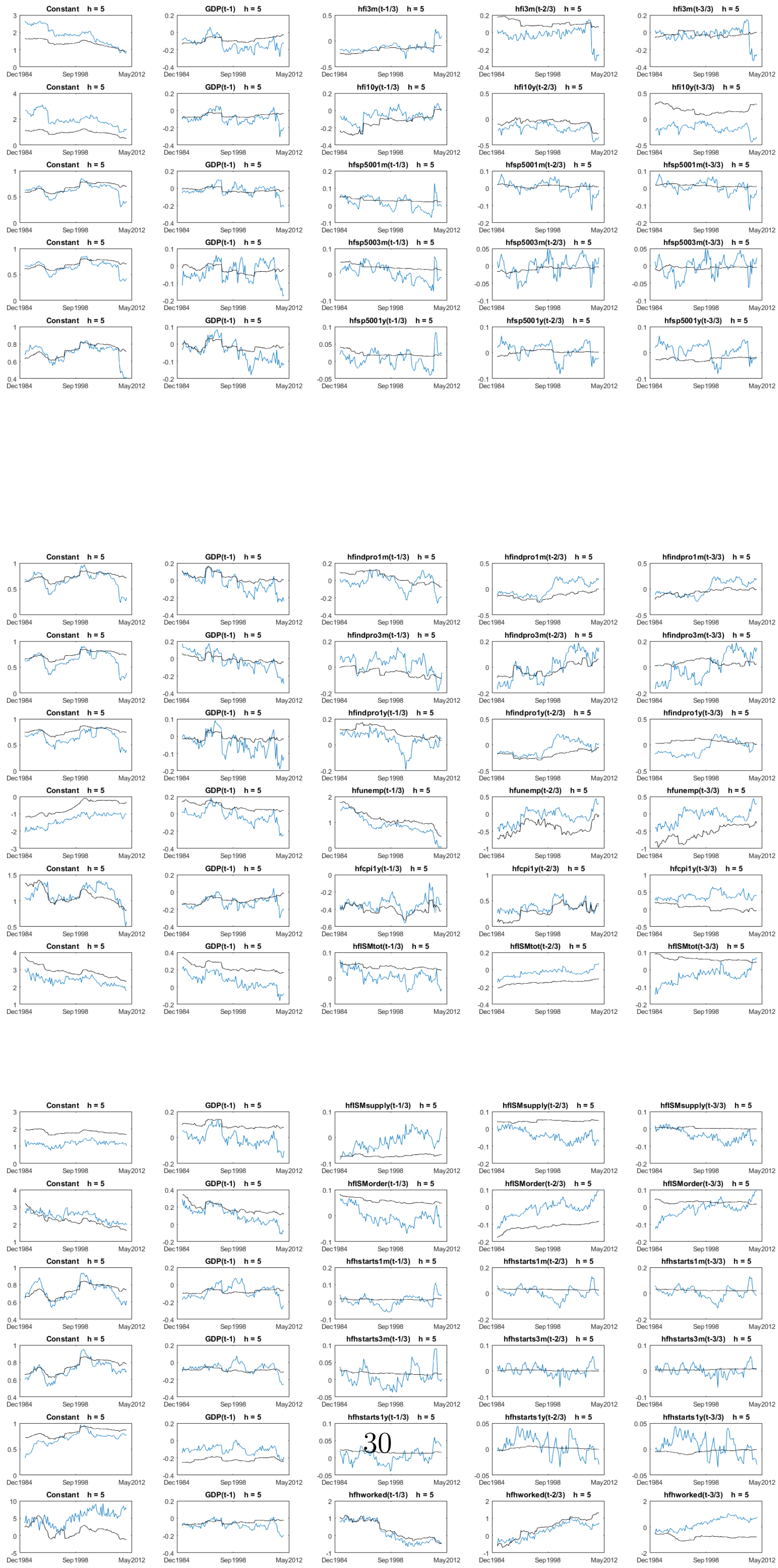
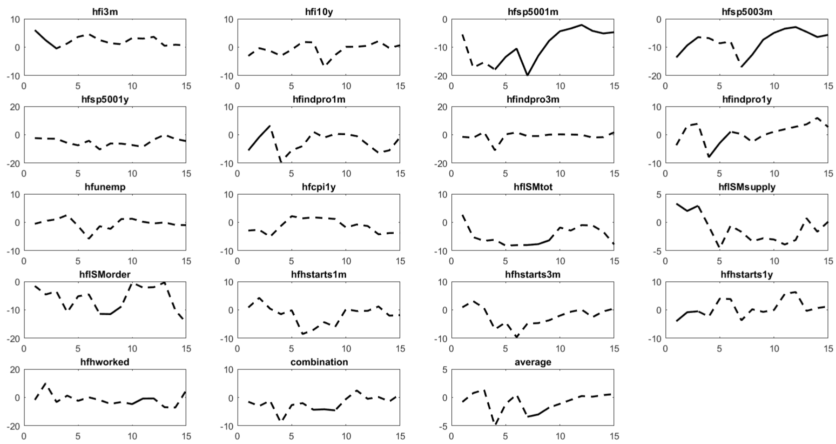



Figure 15: **Parameters for U-MIDAS model with h = 5**



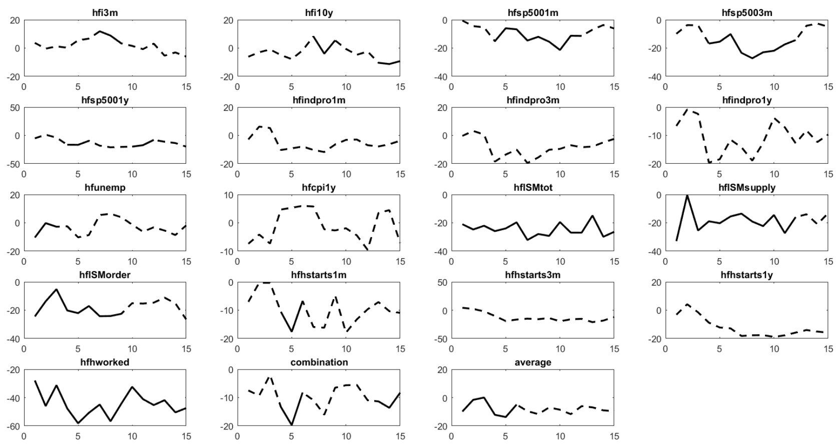
## A.2 Results for full sample for all models

Figure 16: **Rel. RMSFE: U-MIDAS vs Bridge using OLS**



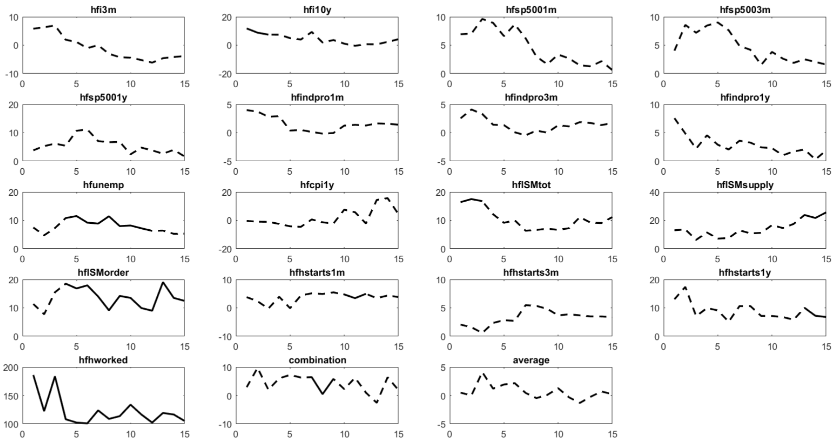

Note: The dashed line shows the relative RMSFE of bridge equations compared to the benchmark model U-MIDAS. Values below zero indicate the smaller percentage forecast error of bridge equations compared to U-MIDAS. The solid line indicates significant results according to the Diebold-Mariano test.

Figure 17: **Rel. RMSFE: U-MIDAS vs Bridge using TVP**



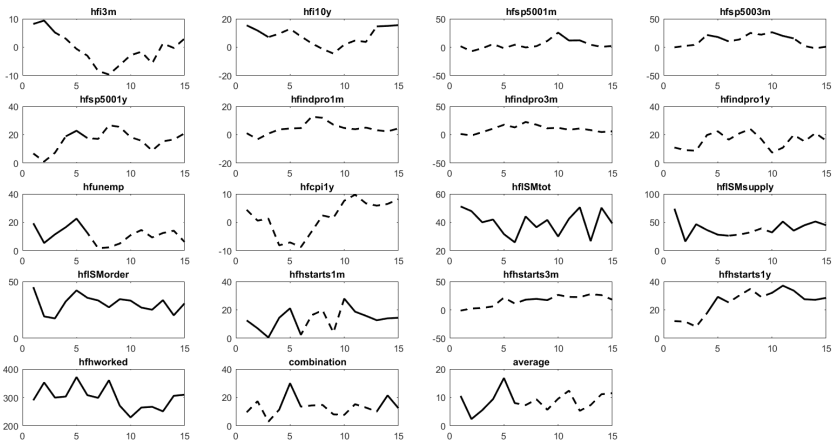

Note: The dashed line shows the relative RMSFE of bridge equations compared to the benchmark model U-MIDAS. Values below zero indicate the smaller percentage forecast error of bridge equations compared to U-MIDAS. The solid line indicates significant results according to the Diebold-Mariano test.

Figure 18: **Rel. RMSFE: OLS vs TVP using Bridge**




Note: The dashed line shows the relative RMSFE of TVP compared to the benchmark method OLS. Values below zero indicate the smaller percentage forecast error of TVP compared to OLS. The solid line indicates significant results according to the Diebold-Mariano test.

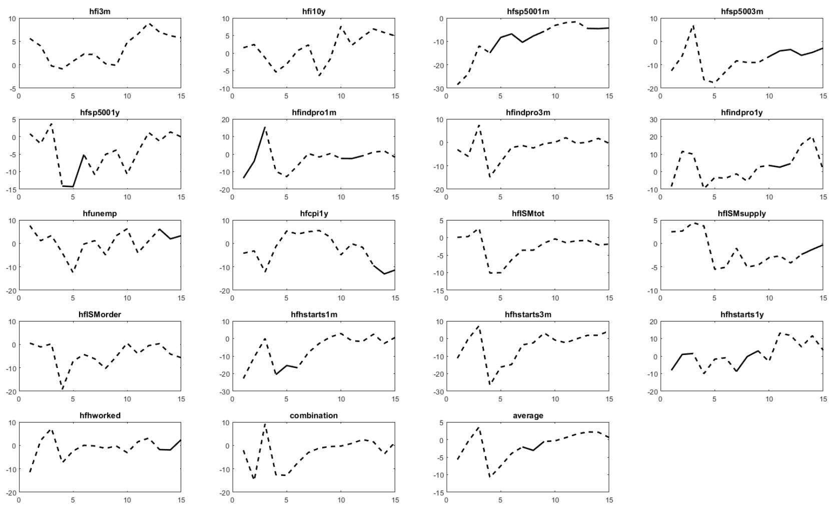
Figure 19: **Rel. RMSFE: OLS vs TVP using U-MIDAS**




Note: The dashed line shows the relative RMSFE of TVP compared to the benchmark method OLS. Values below zero indicate the smaller percentage forecast error of TVP compared to OLS. The solid line indicates significant results according to the Diebold-Mariano test.

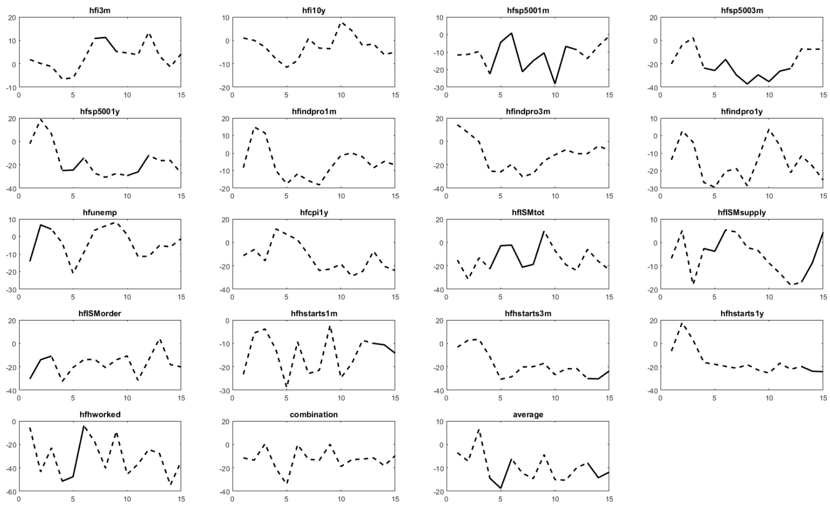
## A.3 Results for Great Recession

Figure 20: **Rel. RMSFE: U-MIDAS vs Bridge using OLS**



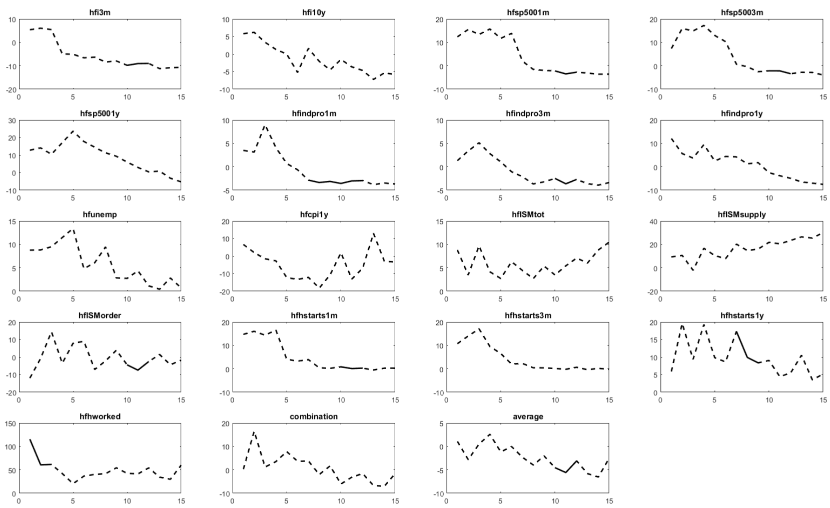

Note: The dashed line shows the relative RMSFE of bridge equations compared to the benchmark model U-MIDAS. Values below zero indicate the smaller percentage forecast error of bridge equations compared to U-MIDAS. The solid line indicates significant results according to the Diebold-Mariano test.

Figure 21: **Rel. RMSFE: U-MIDAS vs Bridge using TVP**



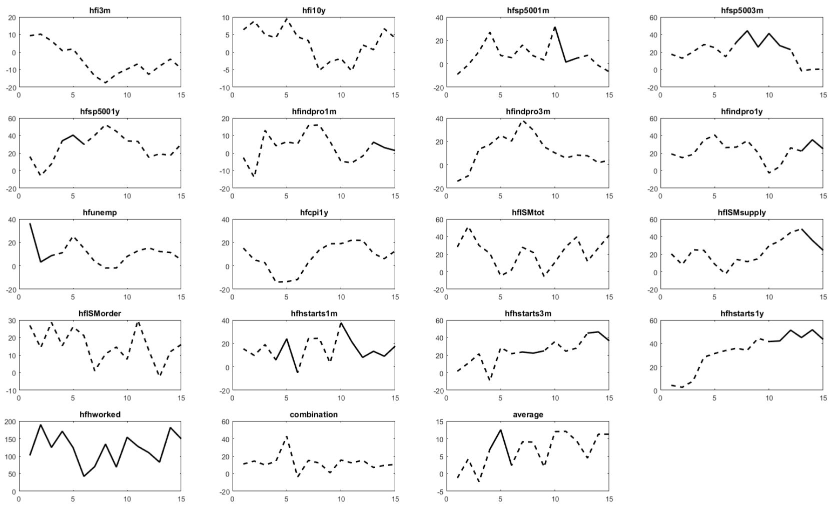

Note: The dashed line shows the relative RMSFE of bridge equations compared to the benchmark model U-MIDAS. Values below zero indicate the smaller percentage forecast error of bridge equations compared to U-MIDAS. The solid line indicates significant results according to the Diebold-Mariano test.

Figure 22: **Rel. RMSFE: OLS vs TVP using Bridge**




Note: The dashed line shows the relative RMSFE of TVP compared to the benchmark method OLS. Values below zero indicate the smaller percentage forecast error of TVP compared to OLS. The solid line indicates significant results according to the Diebold-Mariano test.

Figure 23: **Rel. RMSFE: OLS vs TVP using U-MIDAS**




Note: The dashed line shows the relative RMSFE of TVP compared to the benchmark method OLS. Values below zero indicate the smaller percentage forecast error of TVP compared to OLS. The solid line indicates significant results according to the Diebold-Mariano test.