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A traffic equilibrium model with paid-parking search

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Abstract

This paper describes a Wardropian traffic flow model integrated with a search model for paid parking. The occupancy rate influences the probability of finding on-street (curbside) or off-street (garage) parking spaces. We formulate the model as a mixed complementarity problem, which has the advantage that there is no need for a complete enumeration of all possible paths, and the model can be solved using readily available software. We analyze different parking policies in Zurich and find that changing the parking fee structure will lead to high efficiency gains. Incorporating household heterogeneity is not critical for the overall efficiency effects but shows important regressive, distributional effects.

JEL Classification: R41; R48.

Keywords: parking modeling; traffic congestion, cruising for parking; on-street parking, mixed complementarity format.

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1 Introduction

Downtown traffic congestion is a problem in most large cities around the world. Congestion is often aggravated by scarce parking facilities, causing drivers to cruise and spend time searching for parking spaces and thereby congesting the roads even more. In his seminal monograph on parking, Shoup (2011, Chapter 11) summarizes the studies for cruising in 13 cities around the world and finds that between 8 and 74 percent of cars are cruising and that the average searching time to find a parking space is in the range 3.5 to 19 minutes. Apart from congestion, parking behavior can also cause an externality on others by reducing the probability of finding a parking space and thereby increasing the time of cruising. Small and Verhoef (2007) estimate, based on several studies, that the average social costs of office parking can be as high as US$0.45/km. Notably, these costs are much higher than the social road costs of congestion (US$0.09/km). As any vehicle-based trip ends with parking and as parking causes externalities, the inclusion of parking behavior in transport models is essential for understanding the impacts of parking policies.

The objective of this paper is twofold. First it develops a concise, tractable and easily extensible, link-based formulation of an integrated model consisting of a traffic assignment model in the tradition of Wardrop (1952) and a parking search model.[1] The model is formulated as a mixed complementarity problem (MCP),[2] Drivers from different user classes simultaneously decide on the choice of route and parking allocation in a road network with multiple parking facilities. The externality on the probability of finding a parking space is explicitly part of the model. We make a distinction between on-street (curbside) and off-street parking (garage). We differentiate the user classes with respect to their origin and valuation of time.

A second objective is using the model for the analysis of efficiency and distributional effects of different parking fee policies in a realistic setting. From an economic point of view, the most efficient policy to reduce the congestion and other external effects of cruising is to impose a spatially differentiated parking fee that reflects the external costs (see for example

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[1]An earlier version of the model was used in Bodenbender (2013).
[2]For an introduction to the formulation of MCP models, see van Nieuwkoop (2014).

Arnott, de Palma, and Lindsey, 1991; Verhoef, Nijkamp, and Rietveld, 1995). A parking fee can reduce cruising and will also have a positive impact on congestion although driving through traffic is ignored and the parking fee does not take directly into account how the cruising driver affects congestion. We will use the model for the center of Zurich, the biggest city in Switzerland, and analyze several policies in which the fees for on-street and/or off-street parking are endogenous. These policies are compared to the existing policy and a social optimum, in which the overall time costs are minimized.

Traditional approaches have concentrated on formulating traffic assignment models as path-based models with solution procedures that rely on heuristic algorithms, or the models are formulated as variational inequality (VI) problems (see for example Patriksson, 1994; Nagurney, 2009). The MCP formulation is a special case of the VI formulation. Dafermos and Sparrow (1969) were the first to formulate the assignment problem as a VI-problem, and a few years later Aashtiani (1977) formulated it as a MCP. With the exception of a paper by Ferris, Meeraus, and Rutherford (1999), the MCP formulation, despite its advantages, is seldom used.[3] Disadvantages of the heuristic and VI procedures are first the necessity of finding the shortest paths and second, the necessity of either using specialized and costly transport model software or tedious coding of the algorithms used for solving the model. Although standard transport modeling software also has many advantages for practitioners (e.g. easy to understand, easy to apply for the existing network, no programming necessary), the software reduces the flexibility of the model (e.g. adding new features, implementing new formulations). Integrating assignment models with a parking search submodel can be difficult or even impossible if specialized software has to be used, or can make the coding even more tedious. The advantages of formulating the model as a link-based MCP are manifold as the integrated model can be solved by standard modeling software like MATLAB (The MathWorks, 2012) or GAMS (GAMS Development Corporation, 2014). This allows the researcher to concentrate completely on the model formulation, and the model can be easily extended and shared with other researchers. Furthermore, the link-based formulation has the advantage of eliminating the necessity of finding the shortest paths.

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[3]More on the differences in the several approaches can be found in the van Nieuwkoop (2014).

Our simulations show that the parking fee structure in Zurich is highly inefficient and that changing this structure can lead to high efficiency gains. The simulations also show that the existing on-street parking fees relative to the off-parking fees are substantially too low, and the implementation of endogenous parking fees would reduce the congestion and generalized costs of the agents. All scenarios, with the exception of the scenario with the endogenous on-street parking fees, would reduce the tax revenue for the city by more than 60%. Another potential problem of the investigated policies is their regressive character: The generalized costs of fees paid and time used for driving and searching for poor households increase while richer households might even gain. To investigate the distributional effects, we incorporate heterogeneous households in the model by distinguishing households by their valuation of time. It must be emphasized that although the simulations rely on actual data, the model results should be interpreted with caution. Agents can only choose between on-street or off-street parking and the routes they take. What is lacking is the choice between different modes (car, public transport and walking) and the possibility to either choose to refrain from a trip, choose another departure time or choose a different parking duration. There are several reasons why we concentrated on a static, stationary state formulation. Lam et al. (2006) show for a hypothetical example in a time-dependent network equilibrium model that the parking durations differ if people have more choice options, but the differences are small. Secondly, adding these details to the model would have meant a substantial increase in complexity and loss in the transparency in the formulation of the model and the discussion of the results. However, one of the advantages of the chosen MCP format is that these kinds of extensions can be easily implemented, and future research in this direction is planned.

This paper is organized as follows: Section 2 briefly reviews the existing literature on studies undertaken with parking models. Section 3 proceeds with a short introduction to mixed complementarity problems and describes the model features. Section 4 assesses the efficiency and distributional effects of different parking policies in a model for the center of Zurich. Finally, Section 6 offers some insights into the modeling exercise as well as directions for further research.

2 Literature overview

Existing models differ highly in the way they treat traffic and parking.[4] Few of these models consider parking and congestion in a traffic network equilibrium model setting or study the potential efficiency gains from parking policies. In the following, we discuss some of the most interesting models that are relevant for the present analysis and applications.

In a series of papers, Arnott and several co-authors were among the first to study parking in network equilibrium models.[5] Arnott, de Palma, and Lindsey (1991) use the rush-hour congestion model developed among others by Vickrey (1969) in which identical commuters travel to work at the center of a long, narrow city with zero lateral travel costs. There is only one bottleneck with congestion, where queuing can occur. People can use on-street or off-street parking. The authors find that parking fees can be at least as efficient as an optimal time-varying road toll and that competitively set parking fees are relatively inefficient. They state further that an efficient parking fee policy may be easier to implement than an efficient tolling policy for several reasons. One of these reasons is that a road toll may be regressive. With a parking fee, most low-income workers would try to avoid paying higher fees by parking further away. The authors note that a parking fee might make parking fees more progressive than road tolls when the assumption of identical commuters would be dropped. In this paper, however, we see that changing the existing parking fee structure in Zurich is highly regressive.

Arnott and Rowse (1999) develop a general equilibrium model focusing on stochastic aspects of parking, especially cruising for parking. Although the authors consider this analytical model as simplistic, they find that sound analytical work on parking policies is discouragingly difficult and thereby suggest the need for numerical simulations in the context of practical parking simulation models. Using simulation-based analysis, we find that analytical results on distributional effects for simple networks do not necessarily hold for more realistic networks.

Arnott and Inci (2006) explore the properties of a steady state model of interaction between downtown free parking and traffic congestion. They find three interesting results: First, cruising for parking results is pure dead weight loss. Secondly, the parking fee should be raised

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[4]For a classification of different parking models see Young (2008).

[5]Arnott, de Palma, and Lindsey (1991), Arnott and Rowse (1999), Arnott and Inci (2006), Arnott (2006), Arnott and Rowse (2009), and Arnott and Inci (2010).

to the point where cruising for parking is eliminated, but parking remains saturated. Thirdly, when the level of the parking fee is fixed, the amount of curbside parking should be increased to the point where cruising is eliminated, and parking is saturated. They note, however, that if all agents travel the same distance and park for the same length of time, the parking fee serves as a first-best congestion toll. If these assumptions are relaxed, decentralization of the social optimum requires both a parking fee and road pricing. For the case of Zurich described in this paper, endogenous pricing of parking indeed produces efficiency effects that are very close to the socially optimal solution.

Arnott (2006) develops an interesting model where the parking garage operator's optimal policy for investment and pricing is treated. He shows that since parking garage operators have market power, the spatial equilibrium is inefficient, and parking policy can be used to reduce the distortions. Arnott additionally adds underpriced on-street parking and mass transit and shows that a triple dividend may arise if the meter price is raised. The first dividend relates to the reduction in cruising; the second dividend reduces the level of distortionary taxation, and the third dividend refers to a reduction in overall congestion. In the example for Zurich, we see that implementing better parking policies is likely to lead to high gains in efficiency and reductions in congestion.

Arnott and Rowse (2009) combine the models developed in Arnott and Inci (2006) and Arnott (2006) and simulate a curbside parking policy for a representative medium-sized city with identical agents. They find that garage parking fees are overpriced and increase the existing distortions of underpricing of on-street parking and inefficient spacing of garage parking. One interesting question they raise is why cities do not set parking fees for on-street parking higher as this would generate welfare gains. In our simulations, we see that the actual parking fee policy is highly inefficient and that the off-street parking fees are too high.

Anderson and de Palma (2004) analyze the pricing of parking economics more formally. They show that, under certain assumptions, the social optimum can be achieved if parking lots are privately owned. Their model explicitly treats the link between occupancy rate and search costs by a stochastic process. In our model the probability of finding a spot is endogenous and linked to the number of persons searching and the number of parking spots available.

Calthrop, Proost, and Delder (2000) use the Trenen model, an urban general equilibrium model with 20 alternative transport markets, and an aggregated speed-flow function without an explicit network. Parking costs are exogenous costs added to the generalized cost functions of the cars. The model is calibrated to Brussels and is used to examine the efficiency gains from various parking policies with or without a cordon toll. They find that a combination of parking fees and cordon toll results in higher welfare gains than parking pricing or cordon tolling alone. Parking pricing alone produces higher welfare gains than cordon tolling alone.

Two other more recent models are those from Lam et al. (2006) and Balijepalli, Shepherd, and May (2008). Lam et al. (2006) developed a model in which travelers simultaneously decide on the choice of departure time, route, parking allocation (on- or off-street) and parking duration in a road network with multiple user classes and multiple parking facilities. Searching time delay is modeled using a Bureau of Public Roads function. They use a hypothetical example to show that the model generates a time-dependent network equilibrium solution as well as how the results differ from a static version of their model. Their model is formulated as a variational inequality problem that is solved by a heuristic algorithm. They stress in their conclusions that case studies on real networks are necessary.

Balijepalli, Shepherd, and May (2008) also use an integrated model with multiple user classes and different arrival and departure times. They use the model to study the choice of car parks in a small, hypothetical network as well as in managing the demand between two car parks in Leeds, UK. Their model can be solved with any standard transport modeling software such as TRIPS, EMME/3 or SATURN.

The studies mentioned show important insights and ideas for modeling parking search and analyzing parking policies in network equilibrium models, but most of the studies lack realistic examples.

3 Methodology

3.1 Mixed complementarity problems

Complementary problems can be described as systems of (non)linear constraints where the system variables are linked to the constraints with complementarity conditions (Ferris and Munson, 2014). More formally, given a function $h : \mathbb{R}^n \to \mathbb{R}^n$, lower bounds $l \in \{\mathbb{R} \cup -\infty\}^n$ and upper bounds $u \in \{\mathbb{R} \cup \infty\}^n$, we try to find $x \in \mathbb{R}^n$ such that precisely one of the following holds for each $i \in 1, \ldots, n$:

$$x_i = l_i \quad \text{and} \quad h_i(x_i) \geq 0, \quad \text{or}$$

$$x_i = u_i \quad \text{and} \quad h_i(x_i) \leq 0, \quad \text{or}$$

$$l_i < x_i < u_i \quad \text{and} \quad h_i(x_i) = 0$$

This means that the variable x_i is either at one of its bounds, or the linked function is equal to zero.

In the mixed complementarity problem (MCP) we not only have inequalities with complementary nonnegative variables, but we also have equations where the associated variables are free. In this case, the complementarity conditions become:

$$h_i(x, x) \geq 0, \quad x_i \geq 0, \quad x_i h_i(x) = 0,$$

$$h_j(x, y) = 0, \quad x_j \text{ free},$$

where we partition the set n in the sets i and j.

Complementarity models can be used for solving linear, quadratic and nonlinear programs by writing the Karush-Kuhn-Tucker optimality conditions. Complementary models can also be used for expressing a variety of economic models for both markets and games, where the problems cannot be written as a single optimization problem, or if no equivalent optimization problem exists. Examples are the famous transport problem by Dantzig, the Walras equilibrium and the von-Thunen land model. A model formulation of these examples can be found in Ferris and Munson (2014), and more examples can be found in Rutherford (1995), and Dirkse

and Ferris (1995). A good introduction to engineering and economic applications of complementarity problems can be found in Ferris and Pang (1997). A complementarity problem can often be formulated using the optimality conditions of the original problem. However, there is not always an optimization problem that corresponds to the complementarity conditions, and this is the advantage of the MCP formulation. This means that an MCP formulation allows us to solve a wider class of problems. The development of the complementarity modeling format was motivated by theoretical and practical developments in algorithms for nonlinear complementarity problems and variational inequalities. The most recent techniques are based on ideas from interior-point algorithms for linear programming (Kojima et al., 1991). Computational evidence suggests that algorithms for solving MCPs are relatively reliable and efficient, particularly for models that are not natural optimization problems. A survey of developments in the theory and applications of these methods is provided by Harker and Pang (1990).

There are at least two major reasons why a MCP formulation may be superior to the variational inequality (VI) formulation. First, the MCP formulation of the (link-based) transport problem does not rely on knowing in advance which routes are used. Ferris and Munson (2014, p. 3) state that this is the key property of a complementarity problem over a system of equations: "*If we know what arcs to send flow down, we can just solve a simple system of linear equations. However, the key to the modeling power of complementarity is that it chooses which of the inequalities satisfy as equations.*" Second, a MCP model can be solved with readily available "all-purpose" software packages, like MATLAB (The MathWorks, 2012) or GAMS (GAMS Development Corporation, 2014). Using these packages allows the researcher to concentrate on the model formulation and removes the burden of writing the algorithms to solve the model. Although stand-alone transport equilibrium models can be solved by a variety of specialized software packages,[6] these packages are geared to predefined models and cannot be easily extended in other directions. In our case, it is relatively straightforward to add the parking search sub-model. Another example can be found in van Nieuwkoop (2014), where the Wardropian model is extended with a fully-fledged Alonso-Muth-Mills model.

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[6]Correa and Stier-Moses (2010) mention the following non-exhaustive list: AIMSUN, CUBE, CONTRAM, DYNAMIT, DYNASMART, EMME/2, PARAMICS, TRANSCAD, TRANSIMS, TSIS-CORSIM, SATURN, VISUM-VISSIM, VISTA, and UROAD-UTPS.

3.2 Traffic equilibrium model with parking search

The starting point for our model is the static traffic equilibrium problem in which drivers are fully informed and look for the fastest route from their origin to destination (Wardrop, 1952). To avoid complete enumeration of all the network paths, we use the link-flow formulation of the problem. In the link-flow formulation, each origin-destination flow is treated as a different commodity.

The parking model consists of a street network and multiple types of drivers. The network is represented as a directed graph with nodes and arcs. The drivers interact on the network by searching for parking space. Households differ with respect to their origin, destination and their valuation of time. Nodes in the network can have parking garages nearby, and arcs can have on-street parking facilities (curbside) with a fixed and exogenously given capacity. After parking their car, drivers are assumed to walk to their destinations.

The model solves for a stochastic stationary state, where the probability of finding an on-street parking place on an arc or off-street depends on the capacity and the number of drivers looking for a parking space. Decisions about where to park reflect trade-offs between time (driving, search and walking), and money (parking fees).[7]

In the following description, the sets describing the model are street junctions (nodes $i \in \mathcal{N}$) and drivers are distinguished by their origin and destination pair ($(o,d) \in \mathcal{OD}$). The set of origins is a subset of the nodes. Nodes are referenced by either i or j. Arcs in the network correspond to distinct pairs of nodes. Arcs are alternatively referenced by a or by the start-end pair $(i,j) \in \mathcal{A}$.

The network problem has two classes of complementarity equilibrium conditions. The first class of conditions governs the conservation of flow for every node j by origin-destination pair. These conditions require that the number of all inflowing drivers and those having this node as an origin should be greater than or equal to the sum of the outflowing drivers. The incoming drivers are either passing through without the intention of parking in a garage or at the curbside, or they were unsuccessful in finding a curbside parking space on the arc in the

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[7]The time for parking may be significantly higher in the non-stationary case as is shown by Levy, Martens, and Benenson (2013).

direction of the node. The outgoing drivers either drive to the next node without searching, start searching for a curbside place on their way to the next node, or try to leave their car in a nearby parking garage. This condition can be formulated as follows, where we use the perpendicular symbol ($\perp$) to indicate the complementarity slackness between the constraint and the variable (see Figure 1):

$$
\underbrace{\sum_{(i,j)\in\mathscr{A}} X_{odij}}_{\text{net inflow not searching}} + \underbrace{\sum_{(i,j)\in\mathscr{A}} \pi_{ij}^S Y_{odij}}_{\text{inflow of unsuccessful curbside searching}} + \underbrace{\bar{s}_{dj}}_{\text{starting at node}} \geq
$$

$$
\underbrace{\sum_{(j,i)\in\mathscr{A}} X_{odji}}_{\text{net outflow not searching}} + \underbrace{\sum_{(j,i)\in\mathscr{A}} Y_{odji}}_{\text{searching curbside on outgoing arcs}} + \underbrace{\sum_{(j,k)\in\mathscr{K}} (1-\pi_j^G) Z_{odjk}}_{\text{parking at parking garage}} \quad (1)
$$

$$
\perp \quad T_{odj} \geq 0, \quad \forall j \in \mathscr{N},
$$

where X are the agents driving on the arc without searching, Y are the drivers searching for a curbside place, and Z are those searching for an off-street place surrounding the node. Drivers starting their trip at node j with destination d are given by $\bar{s}_{dj}$. The probabilities for not finding an on-street or off-street space are given by π_{ij}^S and π_j^Z.
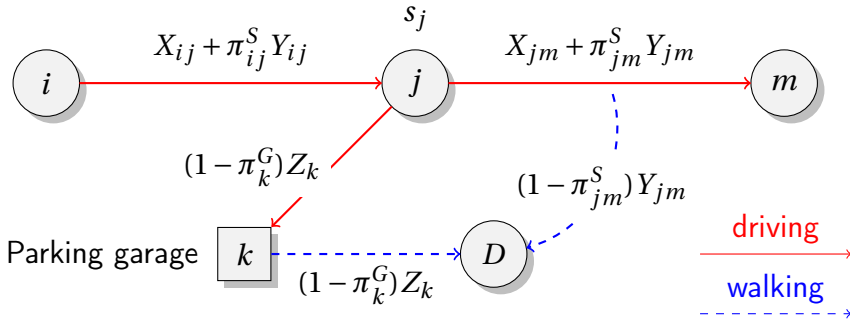
Figure 1 – Node balance for node j for all agents traveling from o to d (for convenience, all OD-indices have been dropped).

A complete trip consists of a sequence of streets where at every node the driver decides either to pass through to the next node, starts searching for an on-street parking space, or searches for an off-street parking place. Therefore, drivers can experience both types of search time. On-street parkers can even loop around if the expected trip time would be minimized. Note that in our model drivers walk from their parking space to their final destination. Therefore, there are no explicit node balance equations for the destinations.

The complementarity in this equation exists between the flow of drivers traveling from o to d through node j and the equilibrium expected time T_{odj} for these drivers to reach their destination.

The number of people finding a parking space is given by the probability of finding a space $(\hat{\pi}_a^S)$ multiplied with the number of people searching. This number is bounded by the fixed total number of parking spaces available at the arc in the observed period $(\overline{C}_a^S)$:

$$\overline{C}_a^S \geq \hat{\pi}_a^S \sum_{(o,d)\in\mathcal{OD}} Y_{oda} \quad \perp \quad \hat{\pi}_a^S \geq 0 \quad \forall a \in \mathcal{A}.$$

The probability is the complementary variable to this equation. If the number of drivers searching for an on-street parking place reaches a level that is greater than the capacity of on-street parking at that arc, the probability $\hat{\pi}^S$ will automatically decrease. The probability in this formulation should be greater or equal to zero, and is unbounded above. To impose the upper bound of 1, we redefine the probability as the probability of **not** finding a parking place (π_a^S) and reformulate the equation above accordingly:

$$\underbrace{\pi_a^S}_{\text{probability of not finding a spot}} \leq 1 - \underbrace{\frac{\overline{C}_a^S}{\sum_{(o,d)\in\mathcal{OD}} Y_{oda}}}_{\text{probability of finding a spot}} \quad \perp \quad \pi_a^S \geq 0, \quad \forall a \in \mathcal{A}. \quad (2)$$

For the probability of not finding an off-street parking space, we have an equivalent complementarity condition:

$$\pi_k^G \leq 1 - \frac{\overline{C}_k^G}{\sum_{(o,d)\in\mathcal{OD}} Z_{odk}} \quad \perp \quad \pi_k^G \geq 0, \quad \forall k \in \mathcal{K}, \quad (3)$$

where $\mathcal{K}$ is the subset of all nodes that have a parking garage.

If we assume demand-responsive off- or on-street pricing, the parking fee will be endogenous and adjust to ensure that the now exogenous probability of finding a preferred spot is equal to 1. In this case, equation (2) and (3) change to:

$$\overline{C}^S \geq \sum_{(o,d)\in\mathcal{OD}} Y_{oda} \quad \perp \quad PF_a^S \geq 0, \quad \forall a \in \mathcal{A}, \quad (4)$$

and

$$\overline{C}_k^G \geq \sum_{(o,d)\in\mathcal{OD}} Z_{odk} \quad \perp \quad PF_k^G \geq 0, \quad \forall k \in \mathcal{K}. \tag{5}$$

Depending on the scenario chosen in the simulations, either we fix the parking fee or the probability, and use the corresponding equations.

The second class of complementarity conditions ensures that, if there is a positive flow on an arc, or if drivers decide to place their car off-street in a parking garage, the corresponding time to reach the destination is minimized. There are three arbitrage conditions reflecting the three choices (passing through, on-street and off-street parking) of the drivers.

The first option for a driver with OD-pair (o, d) is to choose to drive to an adjacent node j without looking for curbside parking ("in-transit traffic"). This option has the following arbitrage condition:

$$\tau_{ij} + T_{odj} \geq T_{odi} \quad \perp \quad X_{odij} \geq 0, \quad \forall (i,j) \in \mathcal{A}, \quad \forall (o,d) \in \mathcal{OD}, \tag{6}$$

where τ_{ij} is the time for traveling from node i to j, T_j is the minimal time from node j to the final destination and T_{odi} is the minimal time from node i to the final destination. This condition is the typical arbitrage condition in the classical network problem. Note that this arbitrage condition is complementary to the number of drivers deciding to drive to the next node (X_{odij}). In equilibrium, this condition requires that the time for traveling to the final destination from node i using the arc (i, j) is either bigger or equal to the minimal time for traveling from node i to the destination. If the time in the case of using this arc is longer, the number of drivers using this link is zero. In the case of equality, there is a positive flow on this arc.

The second option is starting to search for curbside parking on the adjacent arcs around node i. The expected time to reach the destination includes the time spent searching for a parking place, the expected time spent walking to the destination plus the parking fee expressed in time units if a parking place is found and the expected time in continuing from this intersection should no parking place be available nearby. In the simulations, we introduce a parking fee for the curbside parking places, which is denoted by PF_i^S. The parking fee is

expressed in time units using the valuation of time v of the specific driver. In the simulations, we will assume that the valuation depends on the average income at the origin. Therefore, the valuation parameter v has the origin as a subscript.

The second arbitrage condition can be stated as:

$$
\underbrace{vs\tau_{ij}}_{\text{searching costs on arc}} + \underbrace{\left(1 - \pi_{ij}^S\right)\left(w_{ij}^S + \frac{PF_{ij}^S}{v_o}\right)}_{\text{walking costs and fee}} + \underbrace{\pi_{ij}^S T_{odj}}_{\text{costs after not finding}} \tag{7}
$$
$$
\geq \quad \underbrace{T_{odi}}_{\text{minimal costs of other choices}} \quad \perp \quad Y_{odij} \geq 0 \quad \forall (i,j) \in \mathcal{A},
$$

where τ_{ij} denotes the travel time on the arc, w_{ij}^S the walking time from the parking space to the destination, and vs is a multiplier for the time searching space. The multiplier is necessary to increase the driving time on the arc while searching, implicitly assuming that one drives slower. We assume a value of 1.75 for this parameter. If the left-hand side is equal to the minimum time for traveling from node i to the destination d, the number of drivers searching on this arc will be positive. If the left-hand side is greater than the right-hand side, the drivers refer from using this arc.

The third option the drivers have is to park off-street and pay the parking fee. The arbitrage condition for this choice is given by:

$$
S_k^G + (1 - \pi_k^G)\left(w_k^G + \frac{PF_k^G}{v_o}\right) + \pi_k^G T_{odk} \geq T_{odk} \tag{8}
$$
$$
\perp \quad Z_{odk} \geq 0 \quad \forall k \in \mathcal{K}, \quad (o,d) \in \mathcal{OD}.
$$

The parking fee PF^G is expressed in time units using the valuation of time of the specific driver. S_k^G is the (endogenous) searching time off-street, and w_k^G is the walking time from the node to the destination. We assume that drivers do not travel along the arc, but go straight from the node to an off-street parking possibility (see Figure 1). If the time for searching off-street and the time costs of parking are greater than the minimal time for reaching the destination from this node, drivers opt for not parking and the complementarity variable Z_{odk} is zero. If the total generalized costs in time units for parking in the garage is equal to the

minimal time, drivers decide to do so, and Z_{odk} is positive.

Note that the fee for an on-street or off-street parking place is not only influenced by the level of demand, but also by the other fees for either on- or off-street parking. If demand is higher than capacity, the probability of finding a spot is reduced and at the same time, the fee may increase. If demand is exactly equal to capacity, this can result in a zero or a positive fee depending on the level of the other fees.

The search costs for an off-street place grow non-linearly with the ratio of parked vehicles to capacity (see Axhausen et al., 1994):

$$S_k^G = \alpha_k^G \left[1 + \left(\frac{\sum_{(o,d) \in \mathcal{OD}} Z_{odk}}{\overline{C}_k^G} \right)^{\beta^G} \right] \quad \forall k \in \mathcal{K}, \tag{9}$$

where S_k^G is the search time and α_k^G and β^G are constants.

The last two equations of the model define the aggregate arc flow and the travel time on the arcs. The aggregate flow ($F_{a_{ij}}$) on a given arc is equal to the number of drivers, both those who are simply transiting the network from i to j (X_{odij}) and those who are driving from i to j in search of an available parking space (Y_{odij}):

$$F_{a_{ij}} = \sum_{od} \left(X_{odij} + Y_{odij} \right), \tag{10}$$

and the travel time on arc a_{ij} is an increasing function of the arc flow. Here we assume the Bureau of Public Roads (1964) function:

$$\tau_a = \alpha_a \left(1 + \beta_a \left[\frac{F_a}{C_a} \right]^4 \right), \tag{11}$$

where α_a is the uncongested travel time, β is the congestion coefficient and C_a is the capacity of arc a.

The equations described above form a mixed complementarity problem consisting of nine equations and can be used to solve the user equilibrium.

4 Simulations

4.1 A numerical example

We apply the model to Zurich, the biggest city in Switzerland. Zurich has a resident population of almost 400,000 people and more than 1,000,000 people live in the agglomeration. Additionally to the resident population, every day about 200,000 people come to Zurich to work, and about 100,000 people visit the city for leisure activities (Misteli, 2008). We focus on a simplified network version of the center of the city (see Figure 2) using aggregated flows and the network from MATSim, an agent-based transport simulations model (see Vrtic et al., 2005). The network consists of a directed graph with 128 nodes, 78 two-way and 61 one-way arcs.

The key figures of the network are summarized in Table 1. We assume that most of the inner part of the network is a car-free zone and only accessible by public transport or by foot. The total parking capacity of the network is 4,311 with 2,659 off-street in mostly privately owned parking garages and 1,652 on-street. We assume that 3,449 drivers search for a parking space and park their car for a duration of two hours.

Description	Value	Description	Value
Nodes	126	Off-street parking	2,659
Origins	6	On-street parking	1,652
Destinations	4	Total parking capacity	4,311
Arcs	215	Agents	3,449
Two-way arcs	78	Area size (in hectare)	257
One-way arcs	61	Length network (in km)	21

Table 1 – Information on the network.

The capacity and the fees of the 12 parking garages are summarized in Table 2. The hourly parking fees for on- and off-street parking are based on Oswald (2012). Although there is some differentiation in the on-street parking fee, we assume for simplicity an on-street parking fee of 2.5 Swiss Francs per hour (CHF/h) throughout the network. Note that most of the parking garages are located in the northwestern part of the network (small squares in Figure

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[8]The coordinate system used is based on the old Swiss reference system CH1903 introduced in 1903 (Federal Office of Topography (Swisstopo), 2008). The system comprises the definition of a reference ellipsoid (Bessel 1841) fixed in position and orientation to the old observatory in Bern (long=0m, lat=0m).
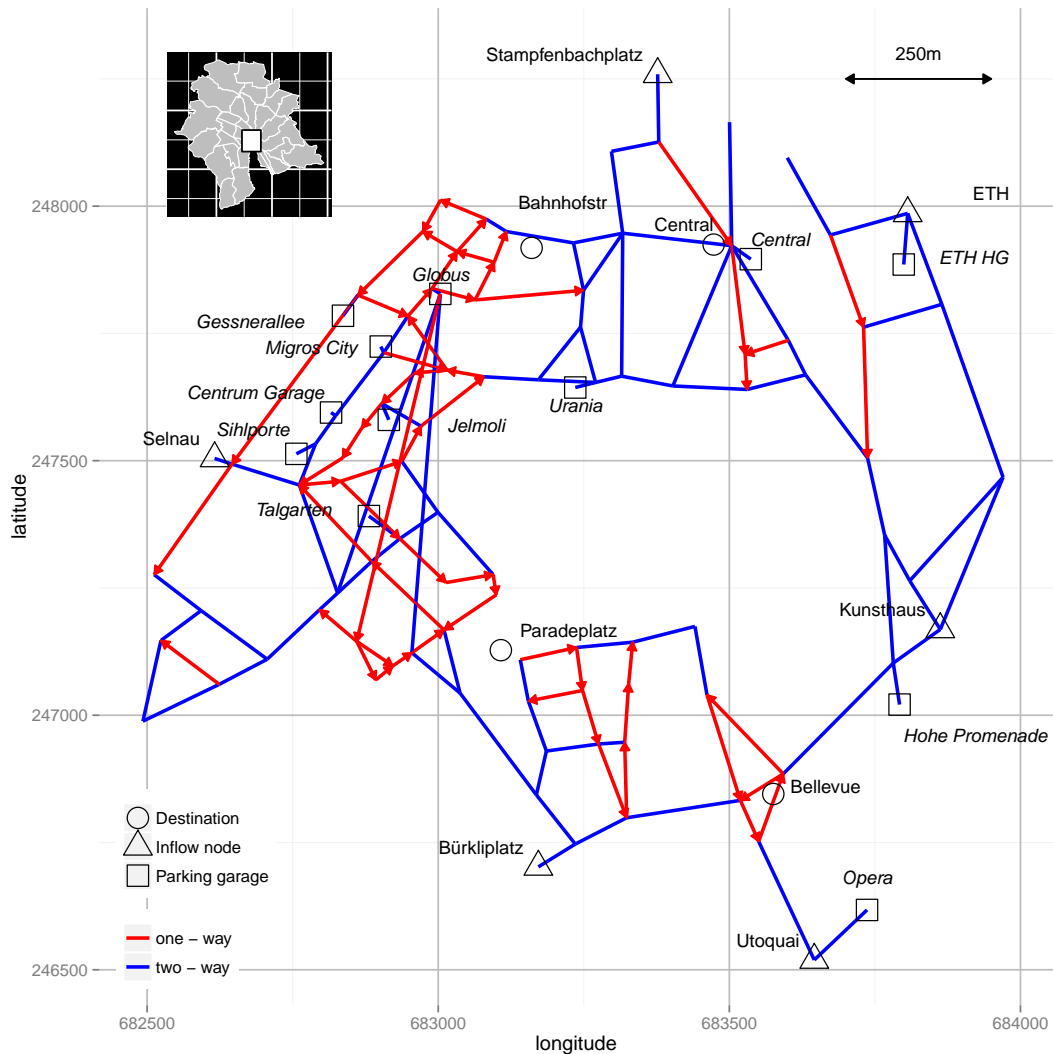
Figure 2 – Center of Zurich with origins (inflow nodes), destinations, parking garages and arcs (map of Zurich with center in white as inlay).[8]

2). Only the parking garages "Hohe Promenade" and "Opera" are located in the southeastern part of the center.

Agents enter the network at six different inflow nodes (ETH, Kunsthaus, Utoquai, Bürkliplatz, Selnau, and Stampfenbachplatz) and can choose between four different destinations in the city center (Bahnhofstrasse, Central, Bellevue and Paradeplatz). The destinations and inflow nodes are shown as circles and triangles, respectively, in Figure 2. Table 3 shows the exogenously given origin-destination matrix. We assume that 80% of all on- and off-street parking spots are used, and the visit duration is fixed to two hours.

Agents not only differ with respect to their origin but also by their valuation of time. König, Axhausen, and Abay (2001) estimated the value of travel time savings in Switzerland

Parking Garage	Capacity	CHF/h	Parking Garage	Capacity	CHF/h
Globus	170	3.50	Migros City	56	3.00
Hohe Promenade	502	4.00	ETH HG	146	2.00
Talgarten	110	4.00	Central	49	4.00
Gessnerallee	608	4.00	Jelmoli	218	3.50
Sihlporte	40	4.50	Urania	450	3.90
Centrum Garage	11	8.00	Opera	299	4.50

Table 2 – Capacities and existing parking fees for parking garages.

	Central	Bellevue	Paradeplatz	Bahnhofstr	Total
ETH	164	164	164	164	655
Kunsthaus	86	86	86	86	345
Utoquai	69	69	69	69	276
Buerkliplatz	155	155	155	155	621
Selnau	164	164	164	164	655
Stampfenbachplatz	224	224	224	224	897
Total	862	862	862	862	3,449

Table 3 – Origin-Destination Matrix.

for motorized and public travel by trip purpose using a stated choice survey of around 1,200 people. This study shows, as a review of 226 studies undertaken by Abrantes and Wardman (2011) does, that the valuation of time is rising with income or wage. Using the estimates from König, Axhausen, and Abay (2001) and the distribution of income in Zurich from Troxler (2004) we roughly estimate the valuation of time for the different agent groups (see Table 4).

Inflow node	Valuation of time (CHF/min)
ETH	0.60
Kunsthaus	0.90
Utoquai	0.80
Buerkliplatz	0.70
Selnau	0.50
Stampfenbachplatz	0.40

Table 4 – Valuation of time by agent type (origin).

4.2 Scenarios

For the policy analysis, we consider five pricing scenarios. The first scenario is the reference scenario ("Reference"), which reflects today's parking policy in Zurich: people pay a fixed

parking fee that is always lower for on-street parking (2.5 CHF/h) than for off-street parking. The off-street parking fee ranges between 2 and 8 CHF per hour (see Table 2). Using the information on the network, the fees and the origin-destination matrix we calculated the user-equilibrium. We use the solution as the reference scenario for the simulations. All other scenarios are compared with this reference scenario. The other scenarios are:

- "Pricing off-street" refers to the case of demand-responsive off-street parking pricing: In this case, the parking fee for off-street parking is endogenous, and the probability of finding an off-parking space is now exogenous and fixed to 1. This means that we use equation (5) and drop equation (3). If the demand for off-street parking on an arc exceeds the capacity of that arc, the parking fee will automatically increase until the demand is below or equal to capacity. The on-street fee is fixed at the level of the reference scenario.

- "Pricing on-street" refers to the case of demand-responsive pricing for on-street parking: Here the fees for off-street parking are fixed at the reference level, and the on-street parking fees have to adjust to ensure a probability of finding a preferred spot is equal to 1. In this case, equation (4) is used in the model, and equation (2) is dropped from the model.

- "Pricing" refers to the case of demand-responsive pricing: The parking fees for on-street and off-street parking adjust to the actual demand: The higher the demand is relative to the capacity, the higher the parking fee. In this case, equations (2) and (3) are replaced by equations (4) and (5).

- "Minimal time": In this scenario we solve the model for the minimum of the overall time costs given by

$$
\sum_{od} v_o \left[\sum_a \tau_a (X_{oda} + Y_{oda} + (1 - \pi_{oda}) w_a^C Y_{oda}) + \sum_k \left(1 - \pi_k^P\right) w_k^P Z_{odk} \right],
$$

drop the arbitrage conditions (equations (6) to (8)) and fix the fees and probabilities for not finding a parking place to zero. As in all other scenarios, time is valued by the

agents in monetary terms[9]. We minimize the weighted value of time costs and not the overall time itself. From an economic point of view, the total monetary costs and not the total time is of interest. This scenario is comparable to Wardrop's social optimum in which total time is minimized. This scenario allows us to evaluate the other scenarios relative to the social optimum and to assess the suitability of the pricing scenarios. The closer the overall time costs are to the value in this scenario, the better the policy is.

4.3   Efficiency results

Table 5 shows the generalized cost, defined as the sum of the time costs and the costs of parking fees (in 1,000 CHF), the percentage change relative to the reference scenario, the average trip time in minutes and the change in kilometers driven. We see a reduction in the generalized costs between 30 to 40 %, of the average trip time between 24 % and 40%, as well as a reduction of the total kilometers driven (31% - 40%) in all scenarios, implying that the existing fee structure in Zurich is non-optimal.[10]

	Reference	Pricing on-street	Pricing off-street	Pricing	Minimal time
Gen. costs (1,000 CHF)	57.5	57.1	35.5	28.7	21.6
Change in gen. costs	0.0%	-0.8%	-38.3%	-50.0%	-62.4%
Average trip time (in min.)	18.00	13.61	12.46	11.00	10.73
Change in km driven	0%	-35.3%	-30.9%	-37.0%	-39.9%

Table 5 – Total generalized costs in 1,000 CHF for the reference scenario, percentage change in generalized costs relative to the reference scenario, the average trip time, and the change in kilometers driven.

The big difference in generalized costs between the two scenarios in which only either the on-street parking fee or the off-street parking fee is endogenized (scenario "Pricing on-street" and "Pricing off-street", respectively) seems to be counter-intuitive: As both parking possibilities have more or less the same characteristics, changing the relative prices should lead in both scenarios to more or less the same result. This is well reflected in the small difference in the average trip time for both scenarios (13.61 versus 12.46 minutes, respectively). However,

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[9]This can easily be seen by multiplying the arbitrage equations (6) to (8) by the agent's valuation of time.
[10]Using an agent-based model for Zurich Wairach et al. (2013) find similar results.

as Table 7 shows, to set a more optimal fee structure, all fees for on-street parking have to be raised, and in the scenario with the endogenous fees for off-street parking, almost all fees have to be significantly reduced. The generalized costs are therefore in the off-street pricing scenario much higher as the fees are part of these costs. The fee costs are around 31 Million CHF in the "Pricing on-street" scenario compared to 12 Million CHF in the scenario "Pricing off-street" (see Table 10).

The greatest improvement can be reached by endogenizing parking fees for on- as well as off-street parking (scenario "Pricing"). The reductions in generalized costs, travel time and the kilometers driven are close to the optimum of scenario "Minimal time". The difference in generalized costs stems mainly from the fees levied for parking in the pricing scenario. The fee costs are 7.4 Million CHF compared to none in the minimal time scenario (Table 10).

The necessary changes in the on- and off-street parking fee structure are shown in the Tables 6 and 7.

Parking garage	Reference	Pricing on-street	Pricing off-street	Pricing
Central	4.00	4.00	5.82	3.92
Globus	3.50	3.50	2.81	1.86
Hohe Promenade	4.00	4.00	1.32	0.44
Talgarten	4.00	4.00	1.31	0.15
Gessnerallee	4.00	4.00	0.00	0.00
Sihlporte	4.50	4.50	0.00	0.00
Centrum Garage	8.00	8.00	0.00	0.00
Migros City	3.00	3.00	1.81	0.84
ETH HG	2.00	2.00	2.46	1.07
Jelmoli	3.50	3.50	0.55	0.00
Urania	3.90	3.90	2.31	1.29
Opera	4.50	4.50	0.00	0.00

Table 6 – Hourly prices for off-street parking in CHF/h for all scenarios.

	Reference	Pricing on-street	Pricing off-street	Pricing
Mean	2.50	5.29	2.50	1.26
Minimum	2.50	2.62	2.50	0.00
Maximum	2.50	7.85	2.50	3.15

Table 7 – Mean, minimal and maximal parking fees on-street parking in CHF/h.

In the scenarios with endogenous fees for off-street parking, the fees are almost all lower, and in the scenario with endogenous fees for on-street parking, the fees are all much higher

than in the reference case. These results are similar to the findings of Arnott and Rowse (2009) who showed that off-street parking fees are overpriced and increase the distortions. The minimum off-street fee can even become zero, which is in line with the early argumentation of Vickrey (1954) for an optimal parking price. A scenario with no off-street fees is politically not feasible. A feasible alternative would be to raise all on- and off-street fees by the same amount as only the relative prices matter.

The tables also show that a more optimal fee structure should be more differentiated with respect to the distance from the destinations, e.g. higher fees for on-street and off-street parking near the destinations and lower fees further away. For example, the parking garages Gessnerallee, Centrum Garage and Sihlpforte have zero parking fees as they are farther away from the destinations Paradeplatz and Bahnhofstrasse than the parking garages Globus, Jelmoli, and Talgarten. However, in some cases, the relative abundance or scarcity of on-street parking possibilities also plays a role in the level of the fees for off-street parking. For example, in the scenario with endogenous off-street parking fees, the fee for the Opera parking garage, which is located closely to the destination Bellevue is reduced to zero because there are relative more attractive on-street parking possibilities near Bellevue.

The improvements in generalized costs, average travel times and kilometers driven can be explained by looking at the probabilities of finding a parking space, the average occupancy rate, the changes in congestion, and the average in-transit, searching and walking times as shown in Table 8.

In the scenarios in which the on-street and/or off-street parking fees are endogenized, the respective probabilities of finding a parking place are set to 1. Endogenizing the parking fees will now make previously unattractive parking spaces cheaper, and the previously attractive ones with a high demand, and, therefore high searching costs, more expensive and, therefore, less attractive. If a parking fee for a parking place were too low, demand would be higher than the capacity, and the probability of finding a parking place could not be equal to 1. Based on these price signals, drivers will now be more evenly distributed over the network. This, in turn, will lead to a reduction in congestion as shown by the changes in overall flow (the number of drivers using particular streets). The congestion is reduced by more than 1% on

22

	Reference	Pricing on-street	Pricing off-street	Pricing	Minimal time
Overall probability finding					
- on-street parking	0.07	1.00	0.19	1.00	1.00
- off-street parking	0.49	0.49	1.00	1.00	1.00
Average occupancy rate off-street	0.68	0.68	0.94	0.75	0.74
Average occupancy rate on-street	1.00	1.00	0.57	0.87	0.90
Change in overall flows on arcs	0%	-38.8%	-31.8%	-40.1%	-43.2%
Changes in number of arcs:					
- with reduced congestion	0%	60.5%	63.7%	61.4%	67.4%
- with increased congestion	0%	14.4%	11.2%	11.6%	9.8%
Average times in minutes:					
- in-transit time/arc	0.46	0.43	0.45	0.43	0.42
- searching time on-street/arc	0.73	0.78	0.47	0.63	0.64
- searching time off-street/garage	2.63	2.69	1.86	1.12	1.05
- walking time	7.15	6.96	6.87	6.69	6.75
- trip time*	18.00	13.61	12.46	11.00	10.73

* Note that a trip can consist of several arcs and the sum of the average in-transit, searching and walking times on the arcs will in most cases not be equal to the average trip time.

Table 8 – Probabilities of finding parking place, occupancy rates, changes in congestion, and average in-transit, searching, walking, and trip times (in min.)

60 to 70 % of the streets, and only increases by more than 1% on 10 to 15 % of the streets. A reduction in the total flows and the congestion on the streets means that the average trip length and time is also reduced.

In the scenario with endogenized parking fees for on-street parking, the main cause for the reduction in the average trip time and kilometers driven (see Table 9) is the better distribution of drivers over the network. The changes in the average in-transit and search time on-street per arc, the searching time off-street, and the average walking are small. In the reference case, the drivers prefer the on-street parking (occupancy rate of 1) because of its relatively low fees. The better distribution of the drivers leads to trips consisting of fewer arcs used (note that a trip can consist of several arcs and the sum of the average in-transit, searching and walking times on the arcs will in most cases not be equal to the average trip time).

In the scenario with endogenized fees for off-street parking, the reduction of the parking fees for more remote off-street parking garages makes these more attractive. Therefore, fewer people will drive to the garages in the center. The reduction in off-street searching time is

caused by a more even distribution of cars over the parking garages (the occupancy rate range is 0.67 - 1.00 compared to 0.00 - 1.00 in the reference case). The average search time for on-street parking is reduced from 0.73 to 0.47 minutes because the number of drivers choosing off-street parking increases by 20.6 % (see Figure 3). This is also the reason for the slight reduction in the average off-street search times.

In the scenario with completely endogenized parking fees and the minimal time scenario, the more optimal fee structure leads to a shift from on-street to off-street parking. The average occupancy rate is higher in both scenarios.

The average walking time is only slightly reduced in all scenarios because the majority of parking garages are within a small distance from each other, and on-street parking is evenly distributed over the center of the city.

Table 9 shows the results for the number of kilometers driven and the percentage change relative to the values in scenario "Reference". In the reference scenario drivers drive almost 6000 km in the network. Of this value, 47% is due to searching on the network (the distances driven while searching off-street are not included).

	Reference	Pricing on-street	Pricing off-street	Pricing	Minimal time
Searching	2,734	-90.3%	-79.3%	-92.4%	-92.0%
In-transit	3,135	12.7%	11.4%	11.4%	5.6%
Total	5869.0	-35.3%	-30.9%	-37.0%	-39.9%

Table 9 – Total vehicle kilometers for searching and passing for the reference scenario and percentage change relative to this scenario for the other scenarios.

The overall reduction in kilometers in searching, driving and walking is in all scenarios more than 30%. Drivers now only search on the last arc of their trip and refrain from searching for off- or on-street parking on the previous arcs. Practically, this search behavior could be induced by a car guiding system with dynamic information on parking. Note that the reduction in kilometers is not proportional to the reduction in time. As shown in Table 8, the reduction in average trip time in the scenario "Pricing off-street" is greater than in the scenario "Pricing on-street", but with regard to the traveled distances the results are the other way around. This means that if Zurich aims at reducing congestion as well as car emissions with a

parking fee policy, the choice of the best scenario might not be obvious. Note, however, that gasoline consumption of cars increases with income in Zurich and, as shown in Section 4.5, the reduction in traveling in the scenario "Pricing" is higher for people with higher incomes than in the scenario "Pricing off-street". Therefore, overall reduction in emissions in this scenario might also be higher.

The changes in relative prices for off- and on-street parking are also reflected in the overall shares of the choice for the two parking possibilities (Figure 3). In all scenarios with the exception of scenario "Pricing on-street" the share of off-street parking increases. In the scenario "Pricing off-street" this share increases by 20.6% points to 72.9%. The increase in the scenario "Pricing" is less, although the fees for off-street parking are lower than in the scenario "Pricing off-street" because the fees for on-street parking in are also lower.
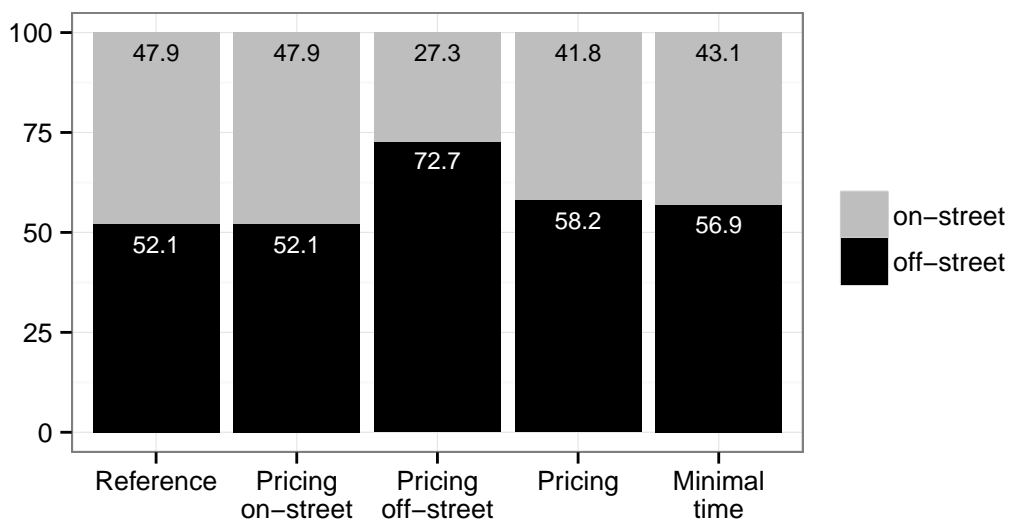
Figure 3 – Shares of total drivers choosing off- and on-street parking for all scenarios.

We have seen that the scenario with fully endogenous fees shows the highest reduction in time and general time costs. It also brings about the lowest fees for both on-street and off-street parking. If, however, the goal of the parking policy is not only to reduce congestion but also to collect money (for example to raise funds for infrastructure investments), this scenario is far from optimal. Except in the scenario "Pricing on-street", the city or parking garage owners incur high-income losses (see Table 10).

Fee revenue	Reference	Pricing on-street	Pricing off-street	Pricing	Minimal time
Off-street parking	13.3	13.3	6.4	3.1	0.0
On-street parking	8.3	17.7	4.7	4.4	0.0
Total	21.5	30.9	11.1	7.4	0.0

Table 10 – Fee revenue (in 1,000 CHF) for off-street and on-street parking across scenarios.

4.4 Heterogeneous versus homogeneous agents

We now make a short digression and compare the overall results for heterogeneous drivers with the results from the same model, but this time with a uniform valuation of time ('homogeneous drivers'). The homogeneous valuation of time (0.6 CHF/min) is calibrated such that the total generalized costs in scenario "Reference" are equal to the value in the model with heterogeneous agents. The total times for in-transit, searching and walking time slightly differ. We are, however, interested in the differences in the generalized costs. Note that in this case, minimizing the total time costs is mathematically equivalent to minimizing total time.

If we compare the results for heterogeneous agents with the results for homogeneous agents (Table 11), we see that heterogeneity in all scenarios leads to a higher reduction in generalized travel costs. This confirms, as shown by Glazer (1981), that the net gain (loss) is greater (smaller) if heterogeneity of drivers is assumed. Introducing endogeneous parking fees for on-street parking does not lead to an improvement relative to the reference case. The higher value is mainly due to the higher costs for on-street parking (total fee costs in this scenario are 31.5 Million CHF versus 21.6 Million CHF in the reference case).

	Pricing on-street	Pricing off-street	Pricing	Minimal time
Change generalized costs with heterogeneous drivers	-0.8%	-38.3%	-50.0%	-62.4%
Change generalized costs with homogeneous drivers	2.8%	-34.9%	-42.9%	-61.0%

Table 11 – Percentage changes relative to the reference scenario in generalized costs for heterogeneous and homogeneous agents.

4.5 Distributional effects

Distributional effects of parking fee policies can play an important role in the political process. The discussion on how regressive or progressive a congestion tax is critical. However, most of the discussion concentrates on comparing scenarios for drivers, who choose the same route and, therefore, travel the same distance. In reality, drivers not only choose different routes, but they also differ in where they live, and we have to take into account the number of kilometers traveled.

From a political point of view, the question that is of more interest is different: How does a policy change, reduce or increase the existing burden of the drivers? We see that in the analyzed scenarios, there is a clear tendency of increasing the burden for drivers with a lower valuation of time. As we assume that a higher valuation of time is positively correlated with higher income, the analyzed policies have a regressive effect. Although this is known for situations where we have a social optimum (see for example Layard, 1977; Santos and Rojey, 2004)[11] it is also known that this might not be the case for scenarios with non-optimal taxes. As Layard (1977) states, that "*this issue can only be settled by empirical work*". For all other scenarios, there is no clear relation between valuation and the reduction in time.
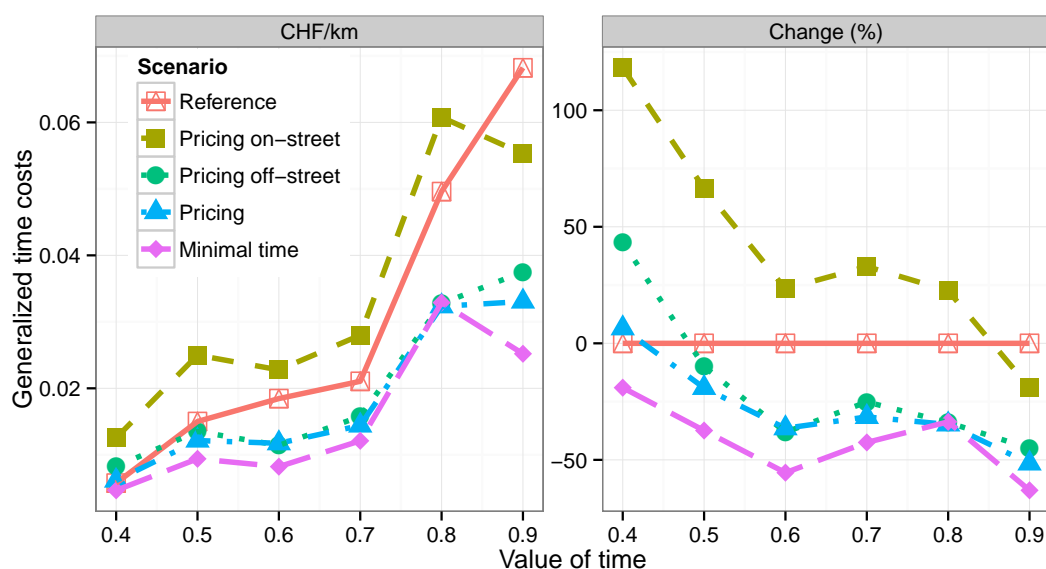
Figure 4 – Generalized costs per kilometer by the valuation of time (absolute value on the left, percentage change relative to the reference scenario on the right).

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[11]Although these papers investigate the distributional effects of congestion taxes, the results are also applicable for parking fees.

The left panel of Figure 4 shows the generalized costs per kilometer for different levels of time valuation. We have divided the overall costs by the number of kilometers driven to make the costs comparable. With some exceptions, the average cost in all scenarios increases with the valuation of time. As Figure 4 shows, there is a progressive trend. However, in some cases, we see a regressive effect, especially in the bracket between a time valuation of 0.5 and 0.6 CHF/min. The right panel of Figure 4 shows the percentage change of the general time costs. One can see that in almost all scenarios the relative change decreases or remains almost the same as the valuation of time increases. All policies are therefore clearly regressive for the low-income groups and more or less indifferent for the middle-income groups with respect to the change in generalized costs per kilometer.

Origin	Valuation	Reference	Pricing on-street	Pricing off-street	Pricing	Minimal time
Stampfenbachplatz	0.40	20.38	16.88	14.52	13.81	11.47
Selnau	0.50	17.96	14.89	14.56	12.16	10.93
ETH	0.60	17.73	15.14	11.89	10.60	10.32
Buerkliplatz	0.70	17.02	11.14	10.50	9.20	10.79
Utoquai	0.80	16.74	10.18	11.44	9.40	11.14
Kunsthaus	0.90	15.21	6.91	8.54	6.79	8.75

Table 12 – Average time of trips (in minutes) for drivers ranked according to their valuation of time.

In our numerical example demand is fixed and, therefore, the regressive effects in the minimal time scenario cannot be caused by refraining from traveling of the low-income groups with the low valuation of time. Drivers with a low valuation of time use routes that take more time (see Table 12). In the scenario "Minimal time" there is no clear relation between the valuation of time and the average travel time.

5 Conclusions

In this paper, we have developed a concise and tractable model of parking search in a traffic network with heterogeneous agents who jointly decide on on-street or off-street parking and the routes they choose. The model is formulated as a mixed-complementarity problem. This format allows the researcher to concentrate completely on the model formulation as the model

can be solved by readily available software.[12]

The numerical example for the inner city of Zurich with 128 junctions, 78 two-way and 61 one-way arcs shows that the model can give useful insights for the analysis of different parking policies. The model was used to analyze parking policies in which the fees for on-street and/or off-street parking are endogenous. These policies are compared to the existing policy and a social optimum in which the overall time costs are minimized.

The simulations show that the parking fee structure in Zurich is highly inefficient and changing this structure could lead to high efficiency gains. It also shows that the existing on-street parking fees relative to the off-street parking fees are too low. Implementation of these policies would reduce the congestion and generalized costs of the agents. It would reduce, with the exception of the scenario with endogenous on-street parking fees, the tax revenue for the city by more than 60%. Another drawback is the regressive character of all the policies: The generalized costs of poor agents will increase, and those of richer agents will decrease, as shown for a social optimum and expected for scenarios with non-optimal taxes by Layard (1977).

Comparing the results with a model with no differences in the valuation of time of the agents, reveals no significant difference in the overall welfare effects. Incorporating agent heterogeneity is nevertheless critical for being able to investigate the distributional effects of transportation policies.

Cross traffic (drivers with a destination not in the center) is not modeled. This might be a serious drawback as shown by Glazer and Niskanen (1992). In their analysis, the authors show that if "these types of drivers constitute a significant fraction of all traffic, an increase in the parking fee may have no, or even perverse, effects on congestion." However, as we analyze the policies for the center of Zurich, this is not a major drawback: In this case, travelers are better off driving around the center hence there will be almost no cross traffic.

One of the main limitations of the model used in this paper is the rather restricted choice for agents: they can only choose between on-street or off-street parking and the routes they take. What is lacking is the choice between different modes (car, public transport and walking)

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[12] The time used for solving the five scenarios on a business notebook computer is less than 5 minutes.

and the possibility to either choose to refrain from a trip, choose another departure time or choose a different parking duration. Lam et al. (2006) show for a hypothetical example in a time-dependent network equilibrium model that the parking durations differ if people have more choice options, but the differences are small. One of the advantages of the MCP format is that these kinds of extensions can be easily implemented, and future research in this direction is planned. Another possible direction for future research is the inclusion of car types differing in gasoline consumption so that the model can be used to analyze policies that aim at the reduction of emissions.

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Nomenclature

Acronyms

BPR   Bureau of Public Research

MCP   Mixed Complementarity Problem

TEP   Transport Equilibrium Problem

VI   Variational Inequality

Parameters

α_k^G Constant in BPR function for search time off-street

α_a Constant of BPR congestion function (uncongested travel time)

β^G Constant in BPR function for search time off-street

β_a Constant of BPR congestion function (uncongested travel time)

$\overline{C}_a^S$ Number of on-street parking spaces at arc a

$\overline{C}_k^G$ Number of off-street parking spaces at k

s_{dj} Agents starting their trip at node j with destination d

v_{od} Valuation of time for agent (o, d)

vs Multiplier for the searching time

w_k^G Walking time from the off-street parking to the destination

w_{ij}^S Walking time from the parking space to the destination

Sets and indices

$\mathscr{A}$ Set of arcs

$\mathscr{K}$ Set of off-street parking places (parking garages)

$\mathscr{N}$ Set of nodes

$\mathscr{OD}$ Set of pairs of destination and origin

a Arc identifier

d Destination identifier

i, j, k Node identifier

k Off-street parking possibility identifier

o Origin and households identifier

Variables

π^G	Probability of not finding a space off-street
π^S	Probability of not finding an on-street parking space
τ_{ij}	Time for traveling from node i to j
PF_g^G	Parking fee for off-street parking (garage) k
PF_a^S	Parking fee for on-street parking on arc a
S_k^P	searching time off-street
X_{odij}	Throughout flow on the arc (i, j) of agents travelling from o to d
Y_{odij}	Flow on the arc (i, j) of agents travelling from o to d searching for an on-street parking space
Z_{odj}	Agents travelling from o to d parking their car off-street at node j

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