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Optimal Interference Management In Multi-Antenna, Multi-Cell Systems

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Abstract— Intercell interference is a major limiting factor in wireless multi-cell networks. Recently, it has been shown that significant performance gains can be achieved by cooperation between base stations. Different degrees of cooperation are possible: From full cooperation, where multiple base stations form a virtual antenna array, to weak cooperation, where base stations take into consideration the interference caused to users in neighboring cells. In this work, weak cooperation in the form of interference management is investigated. The base stations are equipped with multiple antennas, while the mobile terminals only have a single antenna. Due to the spatial degrees of freedom, a base station can serve multiple users in the same slot. Each base station performs beamforming, user group selection, and scheduling, while the terminals treat interference as noise. The corresponding resource allocation problem is cast as a utility maximization problem, which includes common performance objectives such as sum-throughput, max-min fairness, and proportional fairness. Due to interference, the resulting utility maximization problem is a nonconvex optimization problem. Still, after a suitable reformulation, the problem can be solved to global optimality using the framework of monotonic optimization. In other words, we provide a framework for computing the jointly optimal beamforming, user selection, and scheduling strategy for each base station, under an arbitrary utility objective.

I. INTRODUCTION

The interference management problem in the downlink of a cellular system is considered. The cellular system consists of multiple base stations (BS) and a set of mobile stations (MS). Moreover, the system is partitioned into cells, where each cell consists of a base station and a subset of the mobile stations. The base stations are assumed to have multiple antennas. In the downlink, the base stations transmit independent information to the mobile stations. In a conventional cellular system, each base station transmits to the mobile stations within its cell, without taking into consideration the interference caused in neighboring cells. As a result, the performance of the cellular downlink is limited by intercell interference.

Cooperation between base stations can help mitigate intercell interference and thereby improve system performance. Different degrees of cooperation are possible. Maximal performance is achieved by coordinated transmission [1]. In coordinated transmission, the base stations are connected by a high-speed backbone link, enabling them to act as a single transmitter, meaning that the antennas of all base stations form a single antenna array, and the signals of all users are jointly encoded across all base stations [1]. The coordinated transmission scheme requires that the data signals and channel

state information for all users are available at each base station. Moreover, in order to enable coherent reception, each mobile station needs to be synchronized with all base stations.

In this work, a weaker form of cooperation between base stations is considered. As in a conventional system, base stations act as separate transmitters, meaning that the data signals of one user are only available at one of the base stations. Moreover, each mobile station is only synchronized with one base station. Interference from signals intended for other users is treated as noise. Due to the availability of multiple transmit antennas, base stations can choose transmit covariance matrices for transmission to their associated mobile stations. In the following, a choice of transmit covariance matrices for all base stations is denoted as a transmit strategy. Evidently, system performance can be improved if the choice of a transmit strategy is coordinated among base stations, taking into account intercell interference.

Each choice of a transmit strategy yields a certain system performance. Different models for the map from transmit strategy to system performance are possible. In this work, a generic utility model is used. Utility-based models have seen wide application in resource allocation for wireless networks, see, e.g., [2]. By allowing the base stations to switch between transmit strategies during one transmit interval, a further improvement of system performance is possible. Such a switching between strategies can be interpreted as *scheduling*.

Finding the optimal transmit strategies (with or without scheduling) in a coordinated manner represents a utility maximization problem. The presence of interference generally results in a nonconvex optimization problem. There exist resource allocation problems in the multi-cell downlink that can be reformulated as convex problems, such as the minimization of total transmit power under target rate constraints [3]. For the utility maximization problem considered in this work, however, it is generally not possible to find a convex reformulation. As a result, standard tools from convex optimization cannot be applied to find the optimal transmit strategies. Based on a framework proposed in [4], this work uses methods from deterministic global optimization to compute the optimal transmit strategies.

In the case that each base station serves only one mobile station, our system setup corresponds to a multiple-input, single-output interference channel (MISO IFC) with single-user decoding. Recently, a number of works have explored

the properties of the MISO IFC under single-user decoding [5], [6], [7]. For the two-user MISO IFC without scheduling, a method to find the optimal transmit strategies for a given utility model is proposed in [8].

Notation: Lowercase bold letters and uppercase bold letters denote vectors and matrices, respectively. The trace of a square matrix \mathbf{Q} is $\text{tr}(\mathbf{Q})$. We write $\mathbf{Q} \succeq \mathbf{0}$ to say that a Hermitian matrix \mathbf{Q} is positive semidefinite. The symbol \mathbb{R}_+ denotes the set of nonnegative real numbers. Order relations \geq and \leq are defined component-wise. A subset \mathcal{R} of \mathbb{R}_+^K is comprehensive if $\mathbf{s} \in \mathcal{R}$ and $\mathbf{0} \leq \mathbf{s}' \leq \mathbf{s}$ implies $\mathbf{s}' \in \mathcal{R}$. A function u is increasing if $\mathbf{s}' \leq \mathbf{s}$ implies $u(\mathbf{s}') \leq u(\mathbf{s})$, provided both \mathbf{s} and \mathbf{s}' are in the domain of u .

II. SYSTEM MODEL

Downlink transmission in a cellular network is considered. The network consists of B multi-antenna base stations and K single-antenna mobile stations. Base station b is equipped with M transmit antennas and sends independent information to each of its associated MS, where the set of associated MS is denoted by $\mathcal{K}_b \subset \{1, \dots, K\}$. Each MS is associated with one BS, i.e., $\mathcal{K}_b \cap \mathcal{K}_c = \emptyset$ if $b \neq c$ and

$$\bigcup_{b=1}^B \mathcal{K}_b = \{1, \dots, K\}.$$

Let \mathbf{x}_k denote the signal transmitted to MS k by the associated BS. The signal transmitted by base station b is the superposition of the signals transmitted to each of its associated MS. Accordingly, the received signal at MS k is given by

$$y_k = \sum_{q=1}^K \mathbf{h}_{q,k}^H \mathbf{x}_q + \eta_k,$$

where $\mathbf{h}_{q,k}^H \in \mathbb{C}^{1 \times N}$ is the channel from the base station associated with MS q to MS k , and η_k is circularly symmetric AWGN with zero mean and variance σ^2 .

Each BS encodes information separately for each of its associated MS using Gaussian codebooks. Each MS receives independent information. Accordingly, the signal \mathbf{x}_k sent to MS k is independent of the signals to all other MS. Furthermore, it is assumed that each transmit signal \mathbf{x}_k is a circularly symmetric Gaussian random variable with zero mean and covariance matrix $\mathbf{Q}_k \in \mathbb{C}^{M \times M}$. Finally, all MS treat interference as noise. A transmit strategy \mathbf{Q} is a K -tuple of transmit covariance matrices, one for each MS:

$$\mathbf{Q} = (\mathbf{Q}_1, \dots, \mathbf{Q}_K).$$

For each transmit strategy \mathbf{Q} , an achievable rate of MS k is given by

$$r_k(\mathbf{Q}) = \log_2 \left(1 + \frac{\mathbf{h}_{k,k}^H \mathbf{Q}_k \mathbf{h}_{k,k}}{\sigma^2 + \sum_{q \neq k} \mathbf{h}_{q,k}^H \mathbf{Q}_q \mathbf{h}_{q,k}} \right)$$

The transmitted signal from each BS is subject to a transmit power constraint,

$$\sum_{k \in \mathcal{K}_b} \text{tr}(\mathbf{Q}_k) \leq P_b, \quad b = 1, \dots, B.$$

Accordingly, the set of feasible transmit strategies is given by

$$\mathcal{Q} = \left\{ \mathbf{Q} : \mathbf{Q}_k \succeq \mathbf{0}, \forall k, \sum_{k \in \mathcal{K}_b} \text{tr}(\mathbf{Q}_k) \leq P_b, \forall b \right\}.$$

A rate region \mathcal{R} is defined as the set of rate tuples achievable by a feasible choice of \mathbf{Q} ,

$$\mathcal{R} = \{\mathbf{r}(\mathbf{Q}) : \mathbf{Q} \in \mathcal{Q}\}.$$

The rate region \mathcal{R} is compact and comprehensive. In general, however, the rate region \mathcal{R} is not convex. A convex rate region \mathcal{C} is obtained by taking the convex hull of \mathcal{R} . Due to the fact that \mathcal{R} is a comprehensive set, each point in \mathcal{C} can be written as the convex combination of at most K points in \mathcal{R} : For each $\mathbf{s} \in \mathcal{C}$, there exist K transmit strategies $\mathbf{Q}^1, \dots, \mathbf{Q}^K$ and coefficients b_1, \dots, b_K such that $\mathbf{Q}^k \in \mathcal{Q}$, $b_k \geq 0$, $\sum b_k = 1$, and

$$\mathbf{s} = \sum_{k=1}^K b_k \mathbf{r}(\mathbf{Q}^k).$$

Accordingly, the convex hull operation can be interpreted as *scheduling* between K transmit strategies, with scheduling coefficients b_1, \dots, b_K . Moreover, the convex hull of a comprehensive set is comprehensive, hence \mathcal{C} is comprehensive. In the following, let \mathbf{Q}' denote a vector of transmit strategies, $\mathbf{Q}' = (\mathbf{Q}^1, \dots, \mathbf{Q}^K)$, and let $\mathbf{b} = (b_1, \dots, b_K)$.¹

III. INTERFERENCE MANAGEMENT

In general, transmission to MS k causes interference at all MS q with $q \neq k$. On the other hand, reducing the interference caused at MS q reduces the achievable rate for MS k . The goal of interference management is to adapt the system parameters in such a way that overall system performance is maximized. In this work, it is assumed that overall system performance is measured by a utility function u that maps a rate vector $\mathbf{s} \in \mathbb{R}_+^K$ into a scalar utility value $u(\mathbf{s})$. The utility function u is assumed to be continuous and increasing. Commonly used utility models are

$$\begin{aligned} u_{\text{WSR}}(\mathbf{s}) &= \boldsymbol{\lambda}^T \mathbf{s} && \text{(weighted sum-rate),} \\ u_{\text{MM}}(\mathbf{s}) &= \min_k s_k && \text{(max-min fairness),} \\ u_{\text{PF}}(\mathbf{s}) &= \sum_k \ln(s_k) && \text{(proportional fairness).} \end{aligned}$$

Without scheduling, interference management corresponds to determining a feasible transmit strategy \mathbf{Q} such that $u(\mathbf{r}(\mathbf{Q}))$ is maximized:

$$\max_{\mathbf{Q}} u(\mathbf{r}(\mathbf{Q})) \quad \text{s.t.} \quad \mathbf{Q} \in \mathcal{Q}. \quad (1)$$

Due to the nonconcavity of the rate map \mathbf{r} , problem (1) is generally a nonconvex optimization problem, regardless of the properties of u . Moreover, problem (1) offers no

¹By adapting the results from [6] and [7], it can be shown that beamforming is optimal, i.e., it is sufficient to consider covariance matrices of rank 1. Based on this result, the problem can also be formulated using beamforming vectors instead of covariance matrices, cf. [7].

further structure with respect to the parameters \mathbf{Q} . Including scheduling makes the interference management problem even harder: Instead of finding a single transmit strategy \mathbf{Q} , it is now necessary to find a vector \mathbf{Q}' of K feasible transmit strategies and a scheduling vector \mathbf{b} such that the resulting rate vector maximizes utility.

Interference management is optimal if the globally optimal solution is found. However, finding a globally optimal solution of problem (1) directly by operating in the space of transmit strategies is practically impossible, due to the fact that problem (1) is nonconvex and the dimension of the search space is prohibitively high for global methods.² With scheduling, the dimension of the search space is further increased.

The key to finding globally optimal solutions is a rate space approach [4], which basically corresponds to a change of the optimization domain. Without scheduling, the rate space problem is given by

$$\max_{\mathbf{s}} u(\mathbf{s}) \quad \text{s.t.} \quad \mathbf{s} \in \mathcal{R}. \quad (2)$$

Clearly, if \mathbf{s}^* is a global maximizer of (2), then there exists \mathbf{Q}^* such that $\mathbf{s}^* = \mathbf{r}(\mathbf{Q}^*)$ and \mathbf{Q}^* is a global maximizer of (1). The rate space approach provides two major advantages: First, the rate region \mathcal{R} is comprehensive, while the utility function u is increasing. Hence, the rate space problem is a monotonic optimization problem [9], and can be solved by using a generic algorithm for monotonic optimization. Second, the dimension of the search space is reduced to K , the number of MS, and is independent of M .

The rate space problem for the case that scheduling is included is obtained by replacing \mathcal{R} by \mathcal{C} in (2):

$$\max_{\mathbf{s}} u(\mathbf{s}) \quad \text{s.t.} \quad \mathbf{s} \in \mathcal{C}. \quad (3)$$

Due to the fact that \mathcal{C} is also comprehensive, the resulting rate space problem is again a monotonic optimization problem. If the utility function u is concave, the rate space problem with rate region \mathcal{C} is a convex problem.

IV. SOLVING THE RATE SPACE PROBLEM

A general framework for solving rate space problems in the form of (2) and (3) is provided in [4]. The framework is based on the polyblock algorithm [9], a deterministic global optimization algorithm for solving monotonic optimization problems. As a global method that uses a black-box model of objective function and feasible set, the worst case computational complexity of the polyblock algorithm increases at least exponentially in K [10]. In practice, it can be observed that computing the globally optimal solutions is practically feasible for a small to moderate number of users only ($K \leq 10$). Moreover, the computational complexity of the polyblock algorithm limits the applicability of the framework to off-line computation. Nevertheless, by using global methods it is possible to compute the ultimate performance bounds for a given system configuration and a corresponding interference

²Clearly, there exist special cases that result in a sufficiently low problem dimension, such as $M = 1$ and K small.

management strategy which is guaranteed to be globally optimal. The only prerequisite for applying the framework from [4] is the availability of a membership test for the rate region \mathcal{R} . In [4], the single-cell case is considered. For the multi-cell case, a membership test can be formulated as follows: A rate vector \mathbf{s} is element of \mathcal{R} if and only if there exists \mathbf{Q} in \mathcal{Q} such that

$$\mathbf{s}_k = r_k(\mathbf{Q}), \quad \forall k. \quad (4)$$

Re-arranging (4) yields the condition

$$\mathbf{h}_{k,k}^H \mathbf{Q}_k \mathbf{h}_{k,k} - \beta_k \sum_{q \neq k} \mathbf{h}_{q,k}^H \mathbf{Q}_q \mathbf{h}_{q,k} = \beta_k \sigma^2, \quad \forall k,$$

with $\beta_k = 2^{s_k} - 1$. The following feasibility test is obtained:

$$\begin{aligned} & \text{find} \quad (\mathbf{Q}_1, \dots, \mathbf{Q}_K) \\ & \text{s.t.} \quad \mathbf{Q}_k \succeq \mathbf{0}, \quad \forall k, \\ & \quad \sum_{k \in \mathcal{K}_b} \text{tr}(\mathbf{Q}_k) \leq P, \quad \forall b, \\ & \quad \mathbf{h}_{k,k}^H \mathbf{Q}_k \mathbf{h}_{k,k} - \beta_k \sum_{q \neq k} \mathbf{h}_{q,k}^H \mathbf{Q}_q \mathbf{h}_{q,k} = \beta_k \sigma^2, \quad \forall k. \end{aligned} \quad (5)$$

Problem (5) is a semidefinite program (SDP), i.e., a convex problem and efficiently solvable.³

V. NUMERICAL RESULTS

In order to illustrate the impact of optimal interference management, the optimal transmit strategies are computed for an exemplary channel realization. A system with $B = 2$ base station and $K = 4$ mobile stations is considered, with $\mathcal{K}_1 = \{1, 2\}$ and $\mathcal{K}_2 = \{3, 4\}$. Each base station has $M = 2$ transmit antennas and a transmit power budget of $P = 10^{1.5}$. The noise variance at each receiver is $\sigma^2 = 1$. As a reference strategy, we consider the case where \mathbf{Q}_k is chosen such that it perfectly matches its channel and transmit power is divided equally among all associated MS:

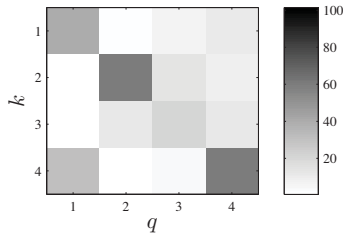
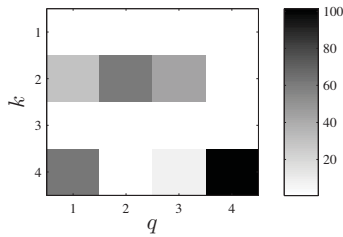
$$\mathbf{Q}_k = 0.5P \mathbf{h}_{k,k} \mathbf{h}_{k,k}^H / \text{tr}(\mathbf{h}_{k,k} \mathbf{h}_{k,k}^H), \quad \forall k.$$

This case is denoted as *no coordination*, as it considers neither intra- nor inter-cell interference. Figure 1 shows the path gains $\mathbf{h}_q^H \mathbf{Q}_k \mathbf{h}_q$ in case of no coordination. The diagonal entries in Figure 1 correspond to the signal paths to the four MS. It can be observed that the channels to MS 2 and 4 are best, while MS 3 has the weakest channel. The off-diagonal entries in Figure 1 correspond to interference. As an example, the signal to MS 4 causes significant interference at MS 1.

Figure 2 shows the path gains resulting from a choice of covariance matrices that maximizes the sum of rates. MS 1 and MS 3 are allocated zero transmit power – it is optimal to switch them off. Moreover, it can be observed that the signals to the active MS only cause interference at the inactive MS.

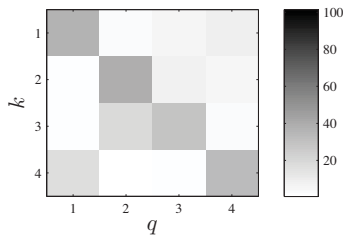
In Figure 3, the transmit strategy is chosen such that the resulting rate vector is max-min fair in \mathcal{R} (i.e., no scheduling). For max-min fairness, no MS can be switched-off. The result is

³Based on the optimality of beamforming, the feasibility test can also be formulated as a second order cone program (SOCP), cf. [7].


 Fig. 1. Path gains $h_q^H Q_k h_q$, no coordination.

 Fig. 2. Path gains $h_q^H Q_k h_q$, sum-rate maximization.

a significant amount of interference, due to insufficient degrees of freedom. Moreover, comparing Figure 3 to Figure 1 shows that it is optimal for BS 1 to transmit with a total power less than P .

Table I shows the optimal rate vectors and corresponding utility values for different performance objectives. The first row corresponds to the rate vector resulting from no cooperation. In the second row, the transmit strategy maximizes the sum of the users' rates. For sum-rate maximization, scheduling is not needed. Rows 3 and 4 correspond to a transmit strategy that is optimal under the max-min and proportional fairness objective, respectively. For the results in rows 3 and 4, optimization is over \mathcal{R} (no scheduling). Whereas sum-rate maximization can achieve a significant gain over the no cooperation case, the benefit of cooperation is significantly lower in case of max-min and proportional fairness. This result is due to the fact that max-min and proportional fairness enforce non-zero rate for all users. Without scheduling, this implies that all users have to be active at the same time. However, there are


 Fig. 3. Path gains $h_q^H Q_k h_q$, max-min fairness.

	r_1	r_2	r_3	r_4	u_{SR}	u_{MM}	u_{PF}
NoCo	1.14	2.21	0.85	1.47	5.68	0.85	1.15
SR	0.00	5.96	0.00	6.81	12.77	0.00	-Inf
MM	1.40	1.40	1.40	1.40	5.61	1.40	1.35
PF	1.63	1.81	0.96	1.43	5.83	0.96	1.39
MM-S	2.96	2.96	2.96	2.96	11.84	2.96	4.34
PF-S	2.97	2.98	2.72	3.41	12.08	2.72	4.41

 TABLE I
 RATES AND UTILITY VALUES FOR DIFFERENT STRATEGIES

k	r_1	r_2	r_3	r_4	b_k
1	0.00	5.96	0.00	6.81	0.43
2	5.95	0.00	5.44	0.00	0.50
3	0.00	5.42	3.75	0.00	0.05
4	0.00	5.58	3.65	0.00	0.02

 TABLE II
 OPTIMAL SCHEDULE FOR MAX-MIN FAIRNESS

only two spatial degrees of freedom available, hence it is not possible to properly separate users. Rows 5 and 6 show the optimal rates for the case that jointly optimal scheduling and beamforming is performed. The gains of optimal scheduling are significant – in case of max-min, the minimal rate more than doubles by including scheduling.

Table II shows the optimal rate vectors and scheduling coefficients for max-min fairness. It can be observed that it is optimal to have only two users active at a given time. While this result can be expected (as $M = 2$), it is not a priori clear which two users are grouped together.

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