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Applied Decision-Making in Civil Engineering

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Oliver Kübler
Dipl.-Ing., University of Karlsruhe
born 18 May 1974
citizen of Germany

accepted on the recommendation of

Prof. Dr. Michael Havbro Faber, examiner
Prof. Dr.-Ing. Rüdiger Rackwitz, co-examiner

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Preface

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Abstract

Civil engineering facilities constitute a crucial backbone of any society and represent a considerable part of its asset. These facilities allow us to undertake activities, such as the production and transport of energy, they provide space for living or business activities or they allow for the transportation of persons and goods by private and public transportation.

Civil engineering is concerned with the optimal management of civil engineering facilities. This not only comprises the design and construction of such facilities. They also have to be operated, maintained, inspected and, if necessary, repaired and/ or decommissioned. Given a specific budget, civil engineers aim to maximize the utility of these facilities. Finally, this involves engineering decision-making throughout the life cycle of such facilities.

To start with, decision theory is reviewed regarding its applicability in civil engineering. For engineering decision-making, Bayesian decision theory combined with methods of structural reliability provides a consistent and applicable basis for the optimal management of civil engineering facilities. Besides probabilities, consequences need to be assessed for decision-making. A framework for their consistent consideration is introduced. It also accounts for socioeconomic consequences which often are referred to as indirect or follow-up consequences. As an example, consequences due to business interruption are reviewed. It is found that the consideration of follow-up consequences can be crucial for the identification of the optimal decision.

Several approaches aim to optimize the utility of civil engineering facilities. The most general approach maximizes the expected life cycle benefit. It is shown that the minimization of the expected life cycle costs is equivalent to this approach, if the expected revenue is independent of the decision/ design variable. Moreover, it is shown that within the life cycle modeling it is possible to consider whether failed structures are reconstructed or not. Also the effect of deterioration processes can be taken into account. This includes both, the effect of the deterioration process on inspection results and secondly on the residual structural resistance.

Acceptability of decision alternatives can be assessed on the basis of the life quality index (LQI). The LQI is a compound social indicator from which acceptance criteria in terms of life saving costs can be derived. The latter can be introduced into the above mentioned optimization problem. In the present work the LQI is reviewed on the basis of microeconomics consumption theory. On that basis, a simple framework is introduced, which allows to interpret a correlation that is observed between the life expectancy and the gross domestic product per capita, as the result of rational decision-making with regard to risk to life.

Finally, it is shown that the described decision framework provides a basis for the calibration of modern structural design codes. Moreover, principal studies show the applicability of decision theory in civil engineering, e.g. the optimal design of different types of structures.

Kurzfassung

Wir wohnen und arbeiten in Bauwerken, wir brauchen sie für den Transport von Gütern und Personen und wir produzieren mit ihnen Energie. Dies sind nur einige Beispiele die zeigen, dass Bauwerke das Fundament gesellschaftlicher Aktivitäten sind. Zugleich stellen sie auch Vermögensgüter der Gesellschaft dar.

Ziel des Bauingenieurwesens ist das optimale Managen von Bauwerken. Dies beinhaltet nicht nur ihre Bemessung und Konstruktion. Bauwerke müssen auch bewirtschaftet, unterhalten, inspiziert und gegebenenfalls repariert oder rückgebaut werden. Bei vorgegebenen Mitteln zielen Bauingenieure auf die Nutzenmaximierung von Bauwerken und dies erfordert das Fällen von Entscheidungen während des gesamten Lebenszyklus der Bauwerke.

Zunächst wird die Anwendbarkeit der Entscheidungstheorie auf das Bauwesen überprüft. Dabei stellt sich die Kombination der Bayes'schen Entscheidungstheorie mit der Zuverlässigkeitstheorie als eine konsistente und anwendbare Basis heraus. Um Entscheidungen rational zu fällen müssen neben Wahrscheinlichkeiten auch Konsequenzen quantifiziert werden. Ein Rahmen für die konsistente Ermittlung von Konsequenzen wird eingeführt, mit besonderem Augenmerk bezüglich sozioökonomischer Konsequenzen. Oft werden solche Konsequenzen auch als indirekte Konsequenzen oder Folgekonsequenzen bezeichnet. Es stellt sich heraus, dass die Berücksichtigung von Folgekonsequenzen die Entscheidungsfindung entscheidend beeinflussen kann. Als konkretes Beispiel werden die Konsequenzen bezüglich Betriebsausfall näher untersucht.

Mehrere Prinzipien zielen auf die Nutzenmaximierung von Bauwerken. Die generellste Herangehensweise maximiert den erwarteten Lebenszyklusgewinn. Dazu ist die Minimierung der Lebenszykluskosten äquivalent, wenn die zu erwartenden Einnahmen von den Entscheidungsvariablen unabhängig sind. Des Weiteren wird gezeigt wie in der Lebenszyklusmodellierung die Entscheidung berücksichtigt wird, ob eingestürzte Bauwerke wieder aufgebaut werden, oder nicht. Auch Schädigungsprozesse können und müssen berücksichtigt werden. Sie haben Einfluss auf Inspektionsergebnisse wie auch auf einen reduzierten Widerstand.

Ob Entscheidungsalternativen akzeptierbar sind kann mittels des Lebensqualitätsindex (LQI) bestimmt werden. Der LQI ist ein zusammengesetzter Sozialindikator von dem Akzeptanzkriterien in Form von Rettungskosten abgeleitet werden können. Letztere können im oben beschriebenen Optimierungsproblem berücksichtigt werden. Die vorliegende Arbeit betrachtet den LQI im Kontext der Konsumtheorie der Mikroökonomie. Darauf basierend wird ein simpler Rahmen geschaffen, der die beobachtete Korrelation zwischen Lebenserwartung und Bruttoinlandsprodukt pro Kopf als Ergebnis rationalen Handelns interpretiert.

Die Arbeit schliesst mit Studien, die die Anwendbarkeit der Entscheidungstheorie im Bauwesen zeigen, stellt sie doch heute die Basis der Normenkalibrierung dar.

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1 Introduction

1.1 Context

Civil engineering facilities allow us to undertake activities, such as the production and transport of energy, they provide space for living or business activities or they allow for the transportation of persons and goods by private and public transport using roads, tunnels, bridges, railways, airports, harbors etc. Hence, civil engineering facilities constitute a crucial backbone of any society and represent a considerable part of its asset.

Therefore, civil engineering is concerned with the optimal management of civil engineering facilities. This not only comprises the design and construction of such facilities. They also have to be operated, maintained, inspected and, if necessary, repaired or decommissioned. In case there are doubts with regard to the implied safety of such a facility, the structure has to be reassessed and, if necessary, rehabilitated. This means, the whole life cycle of such structures and the associated consequences need to be considered within the process of the management of facilities which involve decision-making throughout the life cycle of a structure.

However, the consequences that may occur are subject to uncertainty in their likelihood and also in their extent. During the long history of civil engineering, civil engineers learned to deal with such uncertainties in decision-making. Their major tool therefore was and still is a simple but conservative modeling with regard to the uncertain measures of consideration. Then a sensitivity analysis reveals its importance for the identified optimal decision. If it is found to be relevant, more emphasis has to be given to modeling and/ or controlling the uncertain variable.

Another way to deal with uncertainty in civil engineering was mentioned already a long time ago, see e.g. Mayer (1926). The possibility was suggested to quantify uncertainty in terms of probability. But it needed first the development of methods for structural reliability analysis to become applicable in civil engineering. This is because structural reliability analysis makes it possible to quantify probabilities for events, which are rarely observable, e.g. a structural failure.

Having expressed uncertainties in probabilistic terms, it is straightforward to incorporate it into risk-based decision-making as proposed in JCSS (2001b). On that basis, it is possible to calculate the expected utility, which is associated with different possible alternatives so that the optimal alternative, e.g. the optimal structural design, the optimal service life of the structure or the optimal inspection intervals and methods can be identified. This means, the decision alternative is selected which maximizes the expected utility of the facility.

Combining decision theory with Bayes' theorem, the Bayesian decision theory is obtained. This allows for the integration of new information into the decision process and to evaluate the value of such information. Today, it is a crucial tool for the assessment of existing structures and for the identification of the best inspection and maintenance plan.

In short, the aim of civil engineering can be described as providing civil engineering facilities that serve their purposes best, at low costs and with a high level of safety. However, costs and safety are competing with each other, and the engineer has to find a balance between these two factors. In the past, engineering understanding was used in connection with the trial and error

method to find an optimal balance. However, today this optimization is augmented by a new approach to model public preferences using a compound societal index such as the life quality index. On that basis, acceptance criteria can be derived that can be incorporated in the evaluation of the performance of civil engineering facilities.

1.2 Scope

The work reviews the main aspects of engineering decision-making and puts them together to obtain a consistent basis for decision-making. Hereby, focus is directed towards:

- A more realistic life cycle modeling of civil engineering facilities that besides all negative consequences also takes into account the positive contributions of revenues. Hereby, the consequences are considered to be time-variant, as well as the structural reliability. The latter may result from deterioration processes acting on the structure or from an in-stationary loading. The resulting time-variant reliability can then be modeled using a non-homogenous stochastic process, such as the non-homogenous Poisson process. The effect of possible reconstruction of failed structures is also investigated.
- The modeling of consequences including the consideration of direct, as well as indirect consequences in engineering decision-making. This also comprises the modeling of preferences by basing on the life quality index. From this index immaterial consequences and acceptance criteria can be derived. Empirical data for socioeconomic indicators that show a correlation are investigated and their influence on the life quality index is discussed. In particular, the correlation between the gross domestic product per capita and the life expectancy is investigated, as well as the development of the work time fraction with the gross domestic product per capita

1.3 Overview

The work is structured into three parts, see Figure 1.1. Part I provides an overview of the fundamentals required for engineering decision-making. This comprises Chapter 2 which gives an overview of decision theory and Chapter 3 which deals with the quantification of uncertainty in terms of probabilities.

Part II deals with the assessment and modeling of consequences and preferences. Chapter 4 formulates a framework for the assessment of consequences, whereas Chapter 5 deals with the modeling of preferences using a compound social indicator, namely the life quality index. It is shown in Chapter 6 that from this index risk acceptance criteria can be derived. These criteria can then be considered in engineering decision-making. Chapter 7 discusses briefly risk aversion which is implied with the life quality index. Finally, Appendix A and Appendix B provide an introduction to the basic macro- and microeconomic concepts that are utilized in the previously mentioned chapters.

In Part III, Applied decision theory in civil engineering, Chapter 8 puts the relevant components of engineering decision-making into a consistent framework. Then, the applicability is demonstrated by means of principal studies in Chapter 9. Moreover, it is shown in Chapter 10

1 Introduction

**Part I:
Fundamentals**

- 2 Decision Theory
- 3 Uncertainty and Probability

**Part II:
Consequences and Preferences**

- 4 Consequence Modelling
- 5 The Life Quality Index
- 6 Acceptance Criteria
- 7 LQI and Risk Aversion
- A Basic Macroeconomic Concepts
- B Basic Microeconomic Concepts

**Part III:
Applied Decision Theory in Civil Engineering**

- 8 Decision and Optimal Design
- 9 Principal Studies on Optimal Design
- 10 Code Calibration

11 Summary and Conclusions

Figure 1.1: Overview of the thesis.

how the described decision framework provides a basis for the calibration of modern structural design codes.

Part I

Fundamentals

2 Decision Theory

Today, decision theory provides the framework for engineering decision-making and for risk assessment and management, see e.g. JCSS (2001a). In civil engineering, decision theory is the rational basis for solving practical engineering decision problems, such as the optimal design of structures, assessment of existing structures, optimization of inspection and maintenance plans or the calibration of modern structural design codes. Activities involving civil engineering facilities, e.g. the design and operation of a power plant, imply consequences for the owner, operator and not least society. These consequences can be positive, e.g. the owner's revenue or the power plant's utility to the economy and households, but negative consequences can also be associated, e.g. the power plant's construction costs. Decision theory weights the consequences according to the preferences of the decision maker in a consistent and transparent manner to provide a rational decision support.

2.1 Classification of Decision Theory

Decision theory is classified into descriptive and prescriptive decision theory. Descriptive decision theory aims to describe and understand decisions made in reality by real persons which also sometimes behave irrationally. The prescriptive decision theory – also called normative decision theory – provides a basis for rational decision-making, choosing the optimal alternative and supporting decision makers. In the following, decision theory is always referred to as normative decision theory, because engineering decisions should be rational.

Moreover, decision theory can distinguish between decision-making by an individual or a group. Regarding decision-making by a group, the whole group can be treated in decision theory as an individual, provided that the group members have unitary preferences. Decision-making can also be studied in the case when the decision process involves a rationally acting opponent. This part of decision theory is studied using game theory.

Following Luce and Raiffa (1957), decision theory can also be divided into decision-making under:

- *Certainty*,
- *Risk* or
- *Uncertainty*.

Decision-making under *certainty* studies the case where actions without exceptions lead to a specific known outcome. In the case where an action may lead to several outcomes with known probabilities, we speak of decision under *risk*. Decision under *uncertainty* is referred to the situation where actions may lead to consequences with unknown probabilities. In Figure 2.1, it is seen that rational decision-making of individuals under risk is only one of several possibilities of studying decision-making. But for engineering decision-making it is definitely the most important one because:

| Decision Theory | | | | |
|-----------------|-------------------------|------|-------------|-------------|
| Descriptive | Prescriptive/ Normative | | | |
| | Individual | | Group | Game Theory |
| | Certainty | Risk | Uncertainty | |
| | | | | |

Figure 2.1: Decision theory classification.

1. Engineering decisions should be rational decisions and therefore identified using normative decision theory.
2. If a group has unitary preferences then decision-making by a group can be modeled as decision-making by an individual.
3. Game theory involves a rationally acting opponent. However, engineering decision problems involve more often random natural phenomena than rationally acting opponents.
4. Decision-making under certainty is a special case of decision-making under risk.
5. Decision-making under uncertainty can be studied using a priori probabilities which can also be subjective probabilities.

In the following an introduction to normative decision-making of individuals under risk is given starting with decision-making under uncertainty.

2.2 Decision under Uncertainty

Consider the following situation in which a decision has to be made. Three possible actions/alternatives may be chosen, namely a_1 , a_2 and a_3 . Each action may lead to four different events/states θ_1 , θ_2 , θ_3 and θ_4 that are also called outcomes. But the probabilities with which these events occur are unknown. The consequences associated with action i and event j are denoted by c_{ij} and it is assumed that they may be expressed monetarily by $\chi(c_{ij})$ (e.g. revenues or losses) and numerically by the utility $u(c_{ij})$. For instance, c_{ij} may be the repair of a structure, $\chi(c_{ij})$ are the associated costs and $u(c_{ij})$ is the associated utility, which generally is expressed as a function of χ , $u(\chi(c_{ij}))$. As the probabilities are not known, a decision has to be made under uncertainty. The decision problem is illustrated in Table 2.1. The first column contains the different possible actions and the first row summarizes the possible states. They form a matrix containing the

consequences. Table 2.2 is constructed in the same way, however this table lists the associated utility instead.

| | θ_1 | θ_2 | θ_3 | θ_4 |
|-------|------------|------------|------------|------------|
| a_1 | c_{11} | c_{12} | c_{13} | c_{14} |
| a_2 | c_{21} | c_{22} | c_{23} | c_{24} |
| a_3 | c_{31} | c_{32} | c_{33} | c_{34} |

Table 2.1: Decision under uncertainty with associated consequences.

| | θ_1 | θ_2 | θ_3 | θ_4 |
|-------|-------------|-------------|-------------|-------------|
| a_1 | $u(c_{11})$ | $u(c_{12})$ | $u(c_{13})$ | $u(c_{14})$ |
| a_2 | $u(c_{21})$ | $u(c_{22})$ | $u(c_{23})$ | $u(c_{24})$ |
| a_3 | $u(c_{31})$ | $u(c_{32})$ | $u(c_{33})$ | $u(c_{34})$ |

Table 2.2: Decision under uncertainty, in which consequences are expressed by utilities.

Before some decision criteria for decision-making under uncertainty are introduced, Table 2.3 is introduced as an example where the utilities associated with the actions and states are expressed in numbers. To provide decision support, several decision criteria exist, namely the

| | θ_1 | θ_2 | θ_3 | θ_4 |
|-------|------------|------------|------------|------------|
| a_1 | 6 | 4 | 3 | 7 |
| a_2 | 1 | 1 | 1 | 20 |
| a_3 | -4 | 10 | 12 | 15 |

Table 2.3: Simple example illustrating a decision problem under uncertainty.

maximin, *maximax*, *Hurwicz*, *Laplace* and the *Niehans-Savage* criterion, see e.g. Laux (2003) or Luce and Raiffa (1957). The Niehans-Savage criteria is also called the minimax regret rule which is not to be mistaken for the minimax theorem of game theory. In the latter a rationally acting opponent is involved, see Luce and Raiffa (1957).

Each decision criterion defines a preference function $\Pi(a_i)$ for each alternative a_i and selects the optimum one with $\Pi(a_i) = \Pi^*$ according to a prescribed rule. For instance, the *maximin* criterion aims to maximize the minimum outcome.

$$\Pi^* = \max_i (\Pi(a_i)) = \max_i \left(\min_j u(c_{ij}) \right) \quad (2.1)$$

This criterion is used by pessimistic decision makers who expects that always the least preferable event will occur. Considering Table 2.3, the decision maker therefore chooses a_1 . An optimistic decision maker takes the alternative a_2 because this alternative yields the highest outcome and he or she is confident that the most preferable outcome occurs. This strategy is called the *maximax* decision rule. In this case Π^* is given as follows.

$$\Pi^* = \max_i (\Pi(a_i)) = \max_i \left(\max_j u(c_{ij}) \right) \quad (2.2)$$

2 Decision Theory

The *Hurwicz* rule is simply a linear combination of these both criteria (maximin and maximax). Therefore, it uses a factor $\alpha \in [0, 1]$ which weights the criteria.

$$\Pi^* = \max_i \left(\alpha \max_j u(c_{ij}) + (1 - \alpha) \min_j u(c_{ij}) \right). \quad (2.3)$$

It is easily seen that for $\alpha = 1$ the maximax and for $\alpha = 0$ the maximin rule is obtained.

The *Niehans-Savage* decision criterion calculates a decision maker's regret for each alternative and chooses the action which yields the minimal regret. The regret is simply calculated as the difference between the utility of the realized event for any action and the utility of the realized event for the selected action. For instance, considering Table 2.3 and if θ_1 occurs and we selected a_1 then our regret would be $u(c_{11}) - u(c_{11}) = 6 - 6 = 0$. However, if we had selected a_2 , then our regret would have been $u(c_{11}) - u(c_{21}) = 6 - 1 = 5$.

$$\Pi^* = \min_i \left(\max_k \left(\max_j u(c_{ij}) - u(c_{ik}) \right) \right) \quad (2.4)$$

The *Laplace* decision rule makes use of Johann Bernoulli's principle of insufficient reason. This principle states that if we are ignorant of the way an event can occur and if we have no reason to believe that one event will be more likely, then we should consider the events as equally likely. This implies that the expected value of the consequences is calculated with an a priori uniform probability distribution. For n events we obtain for Π^* :

$$\Pi^* = \max_i \left(1/n \sum_{j=1}^n u(c_{ij}) \right). \quad (2.5)$$

Table 2.4 summarizes the optimal decisions according to the different decision criteria. It is seen that the maximin criterion shows a minimum of 3 for a_1 . 1 and -4 are obtained for a_2 and a_3 , respectively. Hence, the alternative a_1 is the optimal decision according to the maximin criteria. It is seen that, depending on the selected criteria, different alternatives are selected. Laux (2003) and Dörsam (2003) present decision criteria for the illustrated decision problem in more detail and also discuss their shortcomings. But which decision rule should be used? Intuitively,

| | Maximin | Maximax | Hurwicz ($\alpha = 0.5$) | Niehans-Savage | Laplace |
|------------------|-------------|--------------|-------------------------------|----------------|-------------|
| a_1 | 3.00 | 7.00 | 5.00 | 13.00 | 5.00 |
| a_2 | 1.00 | 20.00 | 10.50 | 11.00 | 5.75 |
| a_3 | -4.00 | 15.00 | 5.50 | 10.00 | 8.25 |
| Optimal decision | a_1 | a_2 | a_2 | a_3 | a_3 |

Table 2.4: Optimal identified decisions for different decision criteria.

when faced with decisions under uncertainty as given by Table 2.3, many individuals explicitly or implicitly make use of the Laplace criterion. They also use subjective probabilities i.e. they estimate the likelihood of the consequences and weight the consequences accordingly. However, the Laplace criterion utilizes a uniform distribution of probabilities and also if subjective probabilities are used, the probability concept is utilized. Therefore, strictly speaking, the decision is made *under risk*.

2.3 Decision under Risk

Decision-making under risk aims to select among several actions a_i from the *action space* A the single action $a_i = a^*$ which is most preferable. Within the decision theoretical framework only actions which are elements of $A = \{a_1, \dots, a_n\}$ can be considered. Hence, if the optimal action is not an element of A , then the action identified by the decision framework is suboptimal. This highlights the importance of the consideration of all relevant actions a_i .

Making decisions depends on different *states of the world*. The possible states θ_i considered in the decision process are elements of the state space $\Theta = \{\theta_1, \dots, \theta_n\}$. Θ as well as A may be a finite set of discrete variables, or they may be countable or uncountably infinite. For instance, when the partial safety factor γ of a structural code is to be optimized, then the action space $A = \{\gamma\}$ can be defined to be uncountably infinite, e.g. $\gamma \in \mathbb{R}$ or countably infinite, e.g. $\gamma \in [0.00, 0.05, 0.10, \dots]$. For illustrative purposes, in the following the action space as well as the state space are considered to be finite, but the application of decision theory to infinite action and/ or state spaces is straightforward.

Each action and state may yield consequences $c(a_i, \theta_j)$ which depend on the action a_i and the state θ_j , whereas $c(a_i, \theta_j)$ is an element of the consequence space C . Moreover, it is assumed that the consequences may be evaluated numerically as a utility $u(a, \theta)$ and monetarily as a value $\chi(a_i, \theta_j)$. And to each state of the state space, probabilities of occurrence $P(\theta_j|a_i)$ can be allocated which may be conditioned by the action a_i . For instance, reducing traffic over a bridge reduces the likelihood of an extreme load combination and in turn the probability of failure.

Finally, the decision is made which yields the highest *expected utility* $E_{\Theta}[u|a_i]$. Later it is shown that decision-making according to the *expected value* $E_{\Theta}[\chi|a_i]$ is a specific case of decision-making according to expected utilities.

2.3.1 Graphical Representation of Decision Problems

For a simple example, Table 2.5 summarizes the relevant parameters used for risk-based decision-making. In addition to the already introduced tables such as Table 2.1, Table 2.5 also summarizes the likelihood of occurrence of the different states θ_j by expressing it in terms of probabilities $P(\theta_j|a_i)$. If these probabilities are given, then the expected utility associated with an action $E_{\Theta}[u|a_i]$ can be evaluated. For the simple example considered here, this form of representation is very useful; however, for practical purposes decision trees and influence diagrams, an extension of Bayesian nets, are far more valuable.

| | | θ_1 | θ_2 | θ_3 | θ_4 |
|-------|---------------------|--------------------|--------------------|--------------------|--------------------|
| a_1 | $E_{\Theta}[u a_1]$ | $u(a_1, \theta_1)$ | $u(a_1, \theta_2)$ | $u(a_1, \theta_3)$ | $u(a_1, \theta_4)$ |
| | | $P(\theta_1 a_1)$ | $P(\theta_2 a_1)$ | $P(\theta_3 a_1)$ | $P(\theta_4 a_1)$ |
| a_2 | $E_{\Theta}[u a_2]$ | $u(a_2, \theta_1)$ | $u(a_2, \theta_2)$ | $u(a_2, \theta_3)$ | $u(a_2, \theta_4)$ |
| | | $P(\theta_1 a_2)$ | $P(\theta_2 a_2)$ | $P(\theta_3 a_2)$ | $P(\theta_4 a_2)$ |
| a_3 | $E_{\Theta}[u a_3]$ | $u(a_3, \theta_1)$ | $u(a_3, \theta_2)$ | $u(a_3, \theta_3)$ | $u(a_3, \theta_4)$ |
| | | $P(\theta_1 a_3)$ | $P(\theta_2 a_3)$ | $P(\theta_3 a_3)$ | $P(\theta_4 a_3)$ |

Table 2.5: Utility and consequences for decision-making under risk.

2.3.1.1 Decision/ Event Tree

A decision/ event tree is an excellent tool for making selections among several possible actions. A decision tree is a logical tree which structures the components for decision-making systematically. All actions and states are ordered logically and identified by a rectangular decision node or a circular chance node. Finally probabilities associated with the different states and expected values can be indicated. An example of a decision/ event tree is shown in Figure 2.2, see also Benjamin and Cornell (1970) and Faber (2004). However, in practical decision problems the

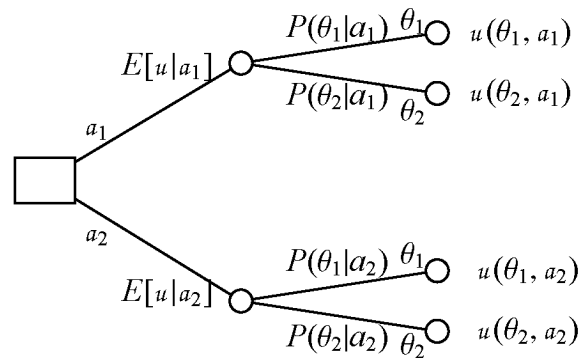


Figure 2.2: A simple decision tree.

number of alternatives as well as the number of outcomes can be large; therefore, systematic analysis with decision trees can easily become complex and the overview is lost. Bayesian nets and influence diagrams are more efficient in this regard.

2.3.1.2 Bayesian Probabilistic Nets and Influence Diagrams

A Bayesian probabilistic net is a directed acyclic graph with edges and variables. The latter have mutually exclusive states. In addition, to each variable A with parents B_1, \dots, B_n there exists a probability table $P(A|B_1, \dots, B_n)$.

In order to describe decision problems, Bayesian probabilistic nets are augmented by decision and utility nodes, and if a directed graph comprises all decision nodes, an influence diagram is obtained. Figure 2.3 shows a simple influence diagram equivalent to the decision tree in Figure 2.2. A short introduction to Bayesian nets is given in Faber (2004) and more details are found in Jensen (2001) and Pearl (1988).

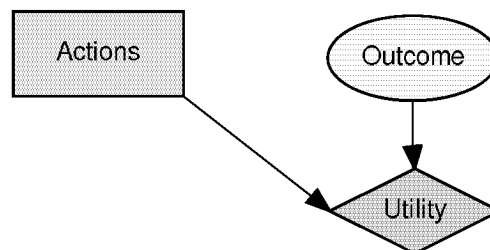


Figure 2.3: A simple influence diagram.

2.3.2 Bernoulli

Today's decision theory is based on Daniel Bernoulli's proposition of 1738 to use the following fundamental rule: "If the utility of each possible profit expectation is multiplied by the number of ways in which it can occur, and we then divide the sum of these products by the total number of possible cases, a mean utility will be obtained, and the profit which corresponds to this utility will equal the value of the risk in question", see the English translation of Bernoulli's text, D. Bernoulli (1954). Due to Bernoulli's influence, in the German literature decision-making under risk is also referred as the Bernoulli Principle, whereas the English literature also refers to it as the decision theory according to von Neumann and Morgenstern because they proved that Bernoulli's principle follows from a set of axioms of rational behavior, see Section 2.3.3. Finally, utilitarian decision theory is also an equivalent term used in the literature for decision-making under risk.

Considering Table 2.5 the simple example, the expected utility is obtained by

$$E_{\Theta}[u|a_i] = \sum_{j=1}^n P(\theta_j|a_i)u(a_i, \theta_j) \quad (2.6)$$

and the maximum utility u^* by

$$u^* = \max_i (E_{\Theta}[u|a_i]). \quad (2.7)$$

Based on the works of Pascal and Fermat, Bernoulli's contemporaries already calculated expected values of actions. However, they calculated the expectation of monetary values $\chi(a_i, \theta_j)$ such as revenues or losses which are associated with the consequences $c(a_i, \theta_j)$. Bernoulli's contribution was the introduction of utility. Thereby, he was inspired by his cousin Nicolaus Bernoulli who formulated the following problem. It became known as the *St. Petersburg problem*.

2.3.2.1 St. Petersburg Problem

Nicolaus Bernoulli formulated the St. Petersburg problem and Daniel Bernoulli solved it. The St. Petersburg problem considers Peter who tosses a coin. Peter will continue tossing until the coin should land 'heads'. If the coin shows 'heads' on the very first throw Peter will give Paul one ducat, two ducats if he gets it on the second, four if on the third, and so on. This means that with each additional throw the number of ducats to be paid is doubled, see D. Bernoulli (1954). Figure 2.4 shows that for the n^{th} throw the probability to get "heads" is $1/2^n$ and the gain is 2^{n-1} , i.e. the expected value for each throw is one half. As an infinite amount of tosses are possible, the expected value of the game is infinite. Hence, a person deciding according to the expected value would be willing to invest any amount in order to take part in this game. But "any fairly reasonable man would sell his chance with great pleasure for twenty ducats", D. Bernoulli (1954).

$$E[X] = \frac{1}{2} 2^0 + \frac{1}{2^2} 2^1 + \dots + \frac{1}{2^n} 2^{n-1} + \dots = \sum_{i=1}^{\infty} \frac{1}{2} = \infty \quad (2.8)$$

Daniel Bernoulli stated that it is not the wealth χ which influences people's choice. The preferences of the people are rather reflected by the utility $u(\chi)$ which people get of the wealth. In

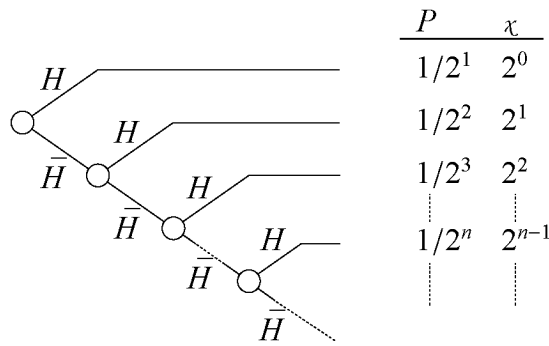


Figure 2.4: The St. Petersburg problem.

order to derive such a utility function, Bernoulli assumed that "any increase in wealth, no matter how insignificant, will always result in an increase in utility which is inversely proportionate to the quantity of goods already possessed". This is expressed in Equation 2.9 which leads to the utility function $u(\chi)$, given by Equation 2.10.

$$du = b \frac{d\chi}{\chi} \tag{2.9}$$

$$u = b \log \frac{\chi}{a} \tag{2.10}$$

Utilizing this utility function, it is easily seen that the infinite series and therefore $E[u]$ is converging to a finite value. With the introduced relation between utility and wealth, Bernoulli also showed that it is possible to model a person's preference to buy an insurance. In economics the concave relation between utility and wealth is explained by the law of diminishing marginal utilities, see Annex B.3.3.1.

2.3.3 Axioms of Decision Theory

To some people, Bernoulli's proposition to use expected utility may still seem to be arbitrarily chosen but it is proven that decisions on this basis are in accordance with rational behavior. More than 200 years after Daniel Bernoulli stated his principle, von Neumann and Morgenstern (1943) introduced a set of axioms of rational behavior. Based on these simple axioms, the researchers show in a lengthy mathematical proof that from these axioms it follows that decisions have to be made based on expected utility.

For this reason, if a decision maker agrees on the stated axioms, it follows that decisions should be made according to expected utilities. In the literature several axiom systems can be found. In the following the system proposed by Luce and Raiffa (1957) is introduced and its meaning is illustrated, followed by a critical review of the axioms.

2.3.3.1 Axiom System of Luce and Raiffa

Luce and Raiffa's system of axioms comprise six axioms, namely the *ordering*, *reduction*, *continuity*, *substitutability*, *transitivity* and the *monotonicity* axiom. These axioms will be introduced

using the term lottery. A lottery \mathcal{L} is simply a set of possible outcomes or consequences c_i with associated probabilities p_i . A lottery may therefore be written as $\mathcal{L} = (p_1 c_1, p_2 c_2)$ with $p_2 = 1 - p_1$.

Axiom 1: Ordering and transitivity

Ordering: Let \mathcal{C} be the set of consequences, and c_1, c_2 and c_3 are elements of \mathcal{C} . The ordering axiom states that decision makers can express their preferences between two consequences c_1 and c_2 so that:

$$\begin{aligned} c_1 > c_2 &: c_1 \text{ is preferable to } c_2, \\ c_1 < c_2 &: c_2 \text{ is preferable to } c_1 \text{ or} \\ c_1 \sim c_2 &: c_1 \text{ is equivalent to } c_2. \end{aligned}$$

Transitivity: The ordering of preferences is transitive if from $c_1 > c_2$ and $c_2 > c_3$ it follows that $c_1 > c_3$.

Axiom 2: Reduction of compound lotteries

Based on the consequences c_1, c_2, \dots, c_n , the simple lotteries S and $\mathcal{L}^{(i)}$ with $i = 1, \dots, m$ can be constructed. In addition, a compound lottery \mathcal{L}^C can be constructed out of the m simple lotteries $\mathcal{L}^{(i)}$. Hereby, the associated probabilities are denoted with $p_i, p_i^{(j)}$ and q_i .

$$\begin{aligned} S &= (p_1 c_1, p_2 c_2, \dots, p_n c_n) \\ \mathcal{L}^{(i)} &= (p_1^{(i)} c_1, p_2^{(i)} c_2, \dots, p_n^{(i)} c_n). \\ \mathcal{L}^C &= (q_1 \mathcal{L}^{(1)}, q_2 \mathcal{L}^{(2)}, \dots, q_m \mathcal{L}^{(m)}) \end{aligned}$$

\mathcal{L}^C is equivalent to S , i.e. $\mathcal{L}^C \sim S$, if the probabilities of S can be computed according to probability theory, i.e.

$$p_j = q_1 p_j^{(1)} + q_2 p_j^{(2)} + \dots + q_m p_j^{(m)}, \quad j = 1, \dots, n.$$

Axiom 3: Continuity

Given the preference ordering $c_1 > c_i > c_n$, there exists a value $p_i \in (0, 1)$ such that the lottery c_i is equivalent to $\tilde{c}_i = (p_i c_1, (1 - p_i) c_n)$.

Axiom 4: Substitutability

In any lottery $\mathcal{L} = (p_1 c_1, \dots, p_i c_i, \dots, p_n c_n)$, \tilde{c}_i is substitutable for c_i that is:

$$(p_1 c_1, \dots, p_i c_i, \dots, p_n c_n) \sim (p_1 c_1, \dots, p_i \tilde{c}_i, \dots, p_n c_n).$$

Axiom 5: Transitivity among lotteries

Preference and indifference ordering among lotteries are transitive relations.

Axiom 6: Monotonicity

If $c_1 > c_n$ then the lottery $\mathcal{L}^{(1)} = (w_1 c_1, (1 - w_1) c_n)$ is preferred to the lottery $\mathcal{L}^{(2)} = (w_2 c_1, (1 - w_2) c_n)$ if and only if $w_1 > w_2$.

These axiomatic system may be found to be somewhat abstract. Therefore, their meaning is illustrated by means of a simple decision problem in Figure 2.5. Consider the case, where a decision among two actions (choose lottery $\mathcal{L}^{(1)}$ or $\mathcal{L}^{(2)}$) has to be made. Each of the lotteries has three possible outcomes θ_1, θ_2 , and θ_3 with probabilities $P(\theta_1) = q_1, P(\theta_2) = q_2$ and $P(\theta_3) = q_3$. The consequences associated with θ_1, θ_2 , and θ_3 , are c_2, c_3 and c_6 for lottery $\mathcal{L}^{(1)}$ and c_1, c_4 and c_5 for lottery $\mathcal{L}^{(2)}$, respectively.

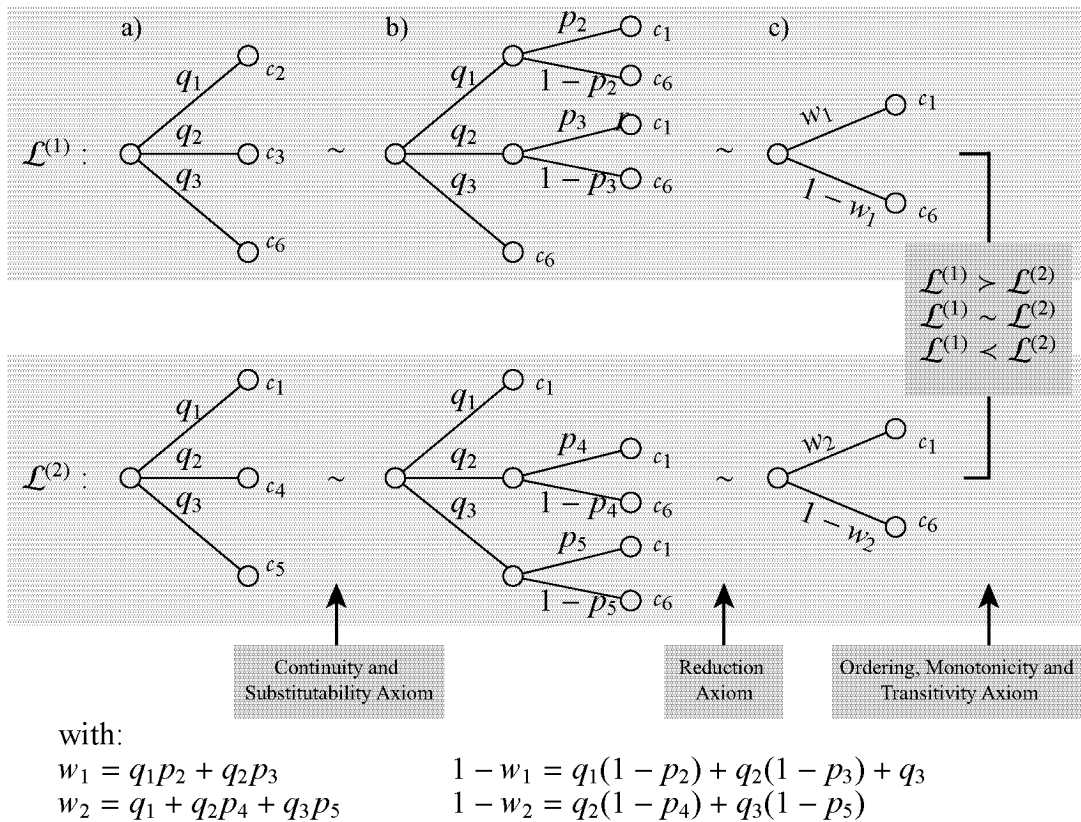


Figure 2.5: Illustration of the axioms of decision theory by means of lotteries.

According to the ordering axiom, the decision maker is able to express his or her preferences such that a transitive ordering $c_1 > c_2 > \dots > c_6$ is obtained. The most preferable consequence is c_1 and the least preferable one is c_6 . Based on these consequences a simple lottery $\tilde{c}_2 = (p c_1, (1 - p)c_6)$ is constructed, where p is the probability that the lottery's outcome is c_1 . If $p = 1$ then c_1 is the certain outcome of \tilde{c}_2 and from the preference ordering it follows that \tilde{c}_2 is preferable to c_2 . Whereas if $p = 0$, then c_2 is the preferable choice. The continuity axiom states that starting from $p = 0$, where c_2 is preferable to \tilde{c}_2 , p can be increased until for $p = p_2$ the decision maker is indifferent between the simple lottery $\tilde{c}_2 = (p_2 c_1, (1 - p_2)c_6)$ and c_2 , this principle is illustrated in Figure 2.6.

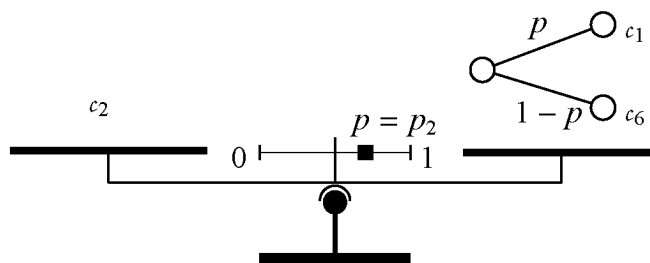


Figure 2.6: Illustration of the continuity axiom.

According to the substitutability axiom, the consequence c_2 in the lottery $\mathcal{L}^{(1)}$ can be replaced by the simple lottery \tilde{c}_2 . This is illustrated in Figure 2.5. It is seen that the same principle is applied to substitute c_3 . Moreover, it is seen that by means of the continuity and substitutability axiom the decision tree may be reduced such that it comprises only two consequences. In Figure 2.5 it is seen that using the reduction axiom the decision tree can be simplified even more. Each lottery consists of the two possible consequences c_1 and c_6 . The probability to obtain c_1 is $w_1 = q_1p_2 + q_2p_3$ for lottery $\mathcal{L}^{(1)}$ and $w_2 = q_1 + q_2p_4 + p_3p_5$ for lottery $\mathcal{L}^{(2)}$, respectively. Using the monotonicity axiom the preferences among the lotteries can be ordered and the most preferable identified. For instance, if $w_1 > w_2$ then lottery $\mathcal{L}^{(1)}$ is preferable.

In Figure 2.5 it is seen that with the help of the axioms of decision theory any decision problem may be reduced to a choice among simple lotteries with two consequences. It is emphasized that in Figure 2.5 no utilities have been assigned. Consider now that utilities can be assigned to the consequences and that the utility function has the following characteristics:

$$c_1 > c_2 \Leftrightarrow u(c_1) > u(c_2) \quad (2.11)$$

$$u(p c_1 + (1 - p)c_2) = pu(c_1) + (1 - p)u(c_2). \quad (2.12)$$

As c_2 is equivalent to $(p_2c_1, (1 - p_2)c_6)$, it follows that $u(c_2) = p_2u(c_1) + (1 - p_2)u(c_6)$. If $u(c_1)$ is chosen to be one and $u(c_6)$ zero, then $u(c_2)$ is equal to p_2 . Substituting p_2 and p_3 by $u(c_2)$ and $u(c_3)$ in w_1 it is seen that w_1 is simply the expected utility of $\mathcal{L}^{(1)}$. Finally, the monotonicity axiom implies that the action is chosen which yields the highest expected utility.

Moreover, it is mathematically proven that from the postulated axioms it follows that rational decisions have to be made according to expected utilities. This proof is unambiguous. Nonetheless, the axioms may be criticized. The most criticized axioms are reviewed below.

2.3.3.2 Criticizing the Axioms

Ordering

Doubts may be raised whether a decision maker can always order two consequences, e.g. $c_1 > c_2$. But how can a decision maker be supported without knowing his or her preferences? Competing approaches to decision-making also require the decision maker's preferences to be expressed.

Continuity

The continuity axiom is questioned because it is doubted that a decision maker is able to determine a probability p such that he or she is indifferent between two lotteries. Critics of this axiom formulated the following example. The first lottery offers the decision maker with certainty 1 USD whereas the second lottery yields 2 USD with probability $(1 - p)$ but with probability p the decision maker risks his or her life. In other words, the critics question whether a decision maker would risk his or her life for such a small benefit, no matter how small p is.

But this criticism can easily be removed by simply observing people in their everyday life. For instance, people risk their lives by crossing a street to get the latest newspaper.

Reduction of compound lotteries

The reduction axiom states that the decision maker is indifferent in regard to a simple lottery and a compound lottery if the probabilities can be computed by means of probability

calculus. This means that no utility of gambling is considered. Certainly, without the pleasure of gambling a rational person would not be interested in playing e.g. roulette. Nonetheless, if the utility can be assessed numerically, it can be included in the decision process.

But in the light of decision-making which e.g. considers the resource allocation of society, the utility or the averseness of an individual with regard to gambling should not be considered.

Transitivity

Transitivity among the stated preferences is important. To illustrate the effect of intransitivity, consider Peter who prefers to have a flat in the mountains c_1 compared to a flat in the city c_2 . Moreover, he prefers a flat in the city c_2 compared to a flat at the sea c_3 but at the same time he finds a flat at the sea c_3 more preferable to a flat in the mountains c_1 , i.e.: $c_1 \succ c_2$, $c_2 \succ c_3$ and $c_1 \prec c_3$

Consider now that Peter owns a flat in the city c_2 . According to the ordering he is willing to pay a certain sum x_1 to exchange c_2 to get c_1 . But he would exchange also c_1 with c_3 and after that c_3 with c_2 and each time he invests a certain sum, say x_2 and x_3 , to get the more preferable flat. But at the end, Peter owns his flat in the city again and he has paid out the amount $x_1 + x_2 + x_3$.

This example clearly shows that intransitivity is irrational. Nonetheless, sometimes real people behave intransitively and irrationally. There are examples, which trap even the strongest supporters of utility theory, see Allais (1953). The inconsistent behavior arises from the fact that when the axioms of decision theory are accepted, each decision problem can be reduced step by step using the stated axioms. Real persons, however, easily lose the overview of complex decision problems leading to inconsistencies. However, the consideration of this is a topic of descriptive decision theory.

2.3.4 Utility Function and Risk Averseness

Generally, the utility function $u(c)$ is a relation $u : C \rightarrow \mathbb{R}$ which assigns to each element of the consequence space C a real value. Equation 2.11 requires that during the transformation from C to \mathbb{R} , an assigned ordering of consequences is not lost and is reflected by $u(c)$.

Often consequences are expressed on a numerical scale, e.g. revenues, average daily traffic, number of destroyed buildings after an earthquake etc. Let us denote the monetary consequences by x , and $u(x)$ is the assigned utility. In this case, $u(x)$ is a mapping $u : \mathbb{R} \rightarrow \mathbb{R}$. Moreover, if a higher value of x is more preferable to a smaller one then Equation 2.11 requires that the utility function $u(x)$ is a strictly monotonic increasing function and strictly monotonic decreasing, if a smaller value of x is always preferable.

The continuity axiom should not be wrongly interpreted; it does not require that the utility function is continuous. The utility function may have discontinuities. However, in practice, discontinuous utility functions are not important.

Besides a specific utility function $u(x)$, there exists an infinite amount of utility functions which represent the same preferences and yield the same optimal decision. Any utility function is determined up to a positive linear transformation.

$$\tilde{u}(x) = au(x) + b, \quad a, b \in \mathbb{R}, \quad a > 0 \quad (2.13)$$

Using the properties of the expectation operation, the expectation of $\tilde{u}(\chi)$ is $aE[u(\chi)] + b$. As neither a multiplication with a positive value a nor an addition with b changes the ordering of alternatives, both utility functions $u(\chi)$ and $\tilde{u}(\chi)$ identify the same optimal decision alternative. For instance, in a decision problem the costs for maintenance can be considered by means of negative numbers (maintenance costs) if they occur. But it is also valid to consider beforehand that these expenses for maintenance are already considered to be spent with certainty. In this case, any maintenance action which is not required is then equivalent to a positive gain. Independent of how maintenance expenses are considered, the same optimal alternative is identified.

2.3.4.1 Risk Aversion

The concave utility function introduced by Bernoulli (Equation 2.10) weights benefits less and at the same time emphasizes losses. A decision maker is called to be risk averse, if his or her preferences are expressed by a concave utility function. If the utility function is convex, the decision maker is said to be risk prone. A neutral decision maker is characterized by a linear utility function. In other words, a risk averse decision maker is not willing to pay the expected value for a fair bet/ game, whereas a risk prone decision maker would pay even more. This is illustrated by means of the certain equivalent.

2.3.4.1.1 Certain Equivalent

The certain equivalent ε is the value on the χ axes which corresponds to the expected utility

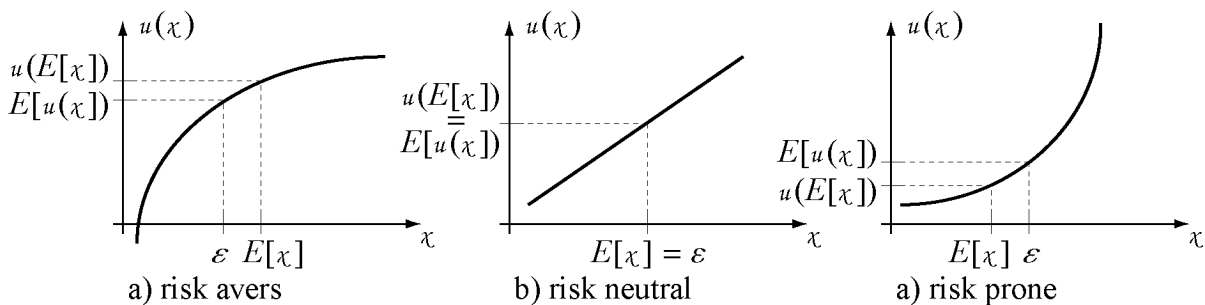


Figure 2.7: Certain equivalent ε and risk attitudes.

$E[u(\chi)]$. It is determined by

$$\varepsilon = u^{-1}(E[u(\chi)]), \tag{2.14}$$

where, $u^{-1}(y)$ is the inverse utility function. For risk averse decision makers, $u(\chi)$ is a concave function. From Jensen's inequality it follows that the expected utility of a risk averse decision maker, is always smaller than the utility of the expected value $E[u(\chi)] < u(E[\chi])$, and from Equation 2.11 it follows that

$$\varepsilon \leq E[\chi]. \tag{2.15}$$

ε is the equivalent which a decision maker would invest in a game with expected value $E[\chi]$. The difference $E[\chi] - \varepsilon$ is called the risk premium and is the amount which a decision maker would be willing to buy an insurance. From Figure 2.7 it is seen that a risk averse decision maker

would take the opportunity and invest in a game if it costs less than ε with $\varepsilon < E[\chi]$, whereas the risk prone decision maker would be willing to pay more than the game is expected to return. In the long run both strategies lead to smaller gains. This highlights the importance of preference modeling of decision makers allocating resources, the topic of the Chapters 4–7.

2.4 Bayesian Decision Theory

Bayesian decision theory is the straightforward combination of two strong concepts, namely decision theory and the theorem of Reverend Thomas Bayes. Bayes' theorem permits not only the inclusion of new information in the decision process as it becomes available, but it is also able to evaluate the value of additional information before it is obtained. This enables engineers to evaluate the economic efficiency of experiments/ inspections before they are actually carried out. Then the most efficient experiment can be selected from which information is gained and incorporated in the decision process. Finally, the new information is incorporated consistently in the decision framework, see also Raiffa and Schlaifer (1961), Benjamin and Cornell (1970) and Faber (2004).

Bayesian decision analysis can be performed in three different situations depending on the state of information. The most simple is the *prior decision analysis*. When new information is available, it can be considered in a *posterior decision analysis*. Finally, *pre-posterior decision analysis* considers unknown results of possible experiments and the most efficient experiment is identified.

2.4.1 Prior Decision Analysis

Prior decision analysis evaluates the expected utility of each alternative and selects the most preferable one with the highest expected utility. Therefore, utilities and probabilities have to be assigned to each possible outcome. Prior decision analysis considers probabilities $P(\theta_j|a_i)$ which are *a priori* available without additional experiments. They are based on available information and experience. Prior probabilities are denoted $P'(\theta_j|a_i)$ and the expected utility of a prior decision analysis is obtained to:

$$E[u] = \max_i E'_{\Theta}[u(a_i, \theta_j)]. \quad (2.16)$$

In Equation 2.16, $E'[\]$ indicates that the expectation operation is carried out using *a priori* information.

2.4.2 Posterior Decision Analysis

In addition to the a priori information $P'(\theta_j|a_i)$, new information, e.g. in terms of observations $\mathbf{o} = (o_1, \dots, o_k)^T$, may become available, then the posterior probabilities $P''(\theta_j|a_i)$ can be calculated using Bayes' theorem, see also Section 3.3.

$$P''(\theta_j|a_i) = \frac{P(\mathbf{o}|\theta_j, a_i)P'(\theta_j|a_i)}{\sum_{k=1}^n P(\mathbf{o}|\theta_k, a_i)P'(\theta_k|a_i)} \quad (2.17)$$

$P(\mathbf{o}|\theta_j, a_i)$ is also denoted as likelihood, $P'(\theta_j|a_i)$ is the prior probability and $\sum_{k=1}^n P(\mathbf{o}|\theta_k, a_i)P'(\theta_k|a_i)$ is a normalizing constant.

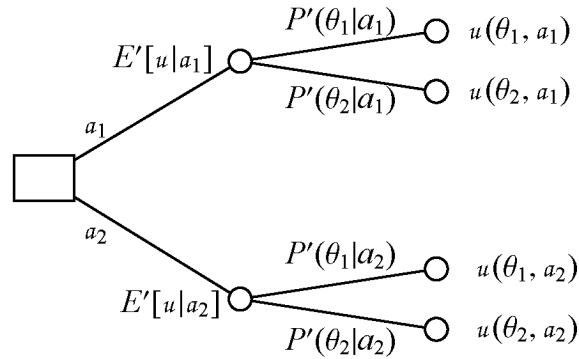


Figure 2.8: Prior decision analysis with two alternatives and two states.

After having updated the probabilities, the structure of the decision analysis is the same as for the prior decision analysis and the maximum expected utility is:

$$E[u] = \max_i E''_{\Theta}[u(a_i, \theta_j)], \quad (2.18)$$

where $E''[\cdot]$ is the expectation operation using posterior probabilities. Figure 2.8 and Figure 2.9 illustrate a simple decision problem with two alternatives and two possible states. From these figures it is seen that they only differ in the way probabilities are incorporated.

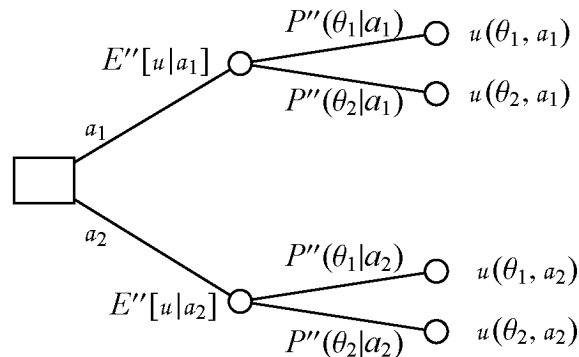


Figure 2.9: Posterior decision analysis with two alternatives and two states.

2.4.3 Pre-posterior Decision Analysis

Pre-posterior decision analysis is a posterior decision analysis which considers the outcome of experiments before they have actually been carried out. Figure 2.10 shows the decision tree illustrating the pre-posterior decision process. The decision tree shows that first an experiment $e \in \mathcal{E}$ can be selected, where \mathcal{E} is the set of possible experiments, e.g. a set of possible inspection methods. The selected experiment will lead to a specific outcome z with z as an element of the sample space \mathcal{Z} . Given the experiment's outcome, an action a of the space of terminal acts \mathcal{A} can be implemented which will lead to the outcome θ element of the state space Θ . Finally, to each combination (e, z, a, θ) , a utility $u(e, z, a, \theta)$ is assigned. Considering a specific

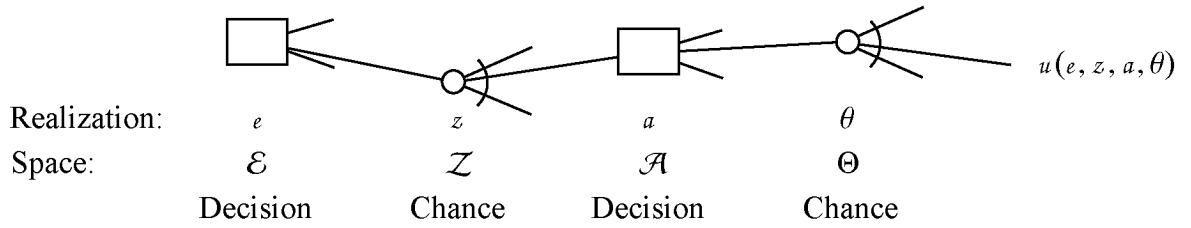


Figure 2.10: Pre-posterior decision analysis.

(e, z) , the posterior probability $P''(\theta|a)$ can be evaluated. Then the utility for a specific a is $E''_{\Theta|z, e, a}[u(e, z, a, \theta)]$. For a given outcome z after having selected e , the decision alternative with the maximal utility is selected. When the prior probability $P'(z)$ is considered the expected utility for each experiment is assessed. Finally, the optimal strategy (e, a) is identified by:

$$E[u] = \max_e E'_{z|e} [\max_a E''_{\Theta|z} [u(e, z, a, \theta)]] \quad (2.19)$$

By means of Equation 2.19, a decision analysis is carried out with *unknown* outcomes. This means that the outcome of experiments is considered without performing them beforehand. Thereby, the cost of experiments can be compared with the information they provide, and the value of the additional information can be determined. If several different experiments are possible, the decision maker can also identify the best experiment; and given its outcome, he or she immediately identifies the best action.

Today, pre-posterior decision analysis is a crucial tool for the assessment of existing structures or to establish risk-based inspection and maintenance plans. In both engineering problems prior information is available and additional information can be *bought* using different methods.

2.4.3.1 Extensive and Normal Form

Pre-posterior decision analysis can be performed using either the extensive or the normal form. The decision analysis as introduced and represented by Equation 2.19 implicitly uses the extensive form. It was shown that the extensive form aims to identify the optimal strategy (e, a) . Also the normal form aims to find an optimal strategy; however, the strategy is formulated differently. The strategy consists of both the selected experiment e and a decision rule $d(z)$ which defines the action to be taken if z occurs. The expected utility then becomes

$$E[u] = \max_e \max_d E'_{\Theta, \mathcal{Z}|e} [u(e, z, d(z), \theta)]. \quad (2.20)$$

In Raiffa and Schlaifer (1961) the mathematical equivalence of Equation 2.19 and Equation 2.20 is proven.

Generally, the decision rule $d(z)$ is simply a mapping from z in \mathcal{Z} to $d(z)$ in \mathcal{A} and is conditioned by the experiment e . If the decision rule is determined and the probability structure is identified, the probability $P(a|e, d, \theta)$ can be assessed such that the decision rule will lead to alternative a . $P(a|e, d, \theta)$ is also called performance characteristics of d for e .

2.5 Formulation of the Objective and Asset Management

The theoretical and practical framework of decision-making was introduced in the foregoing sections and it has been shown that decision theory is a rational tool to support decision makers. Nonetheless, the objective of decision-making has not been clearly formulated. This is done in the present section, where the objective of decision-making in economics is studied.

In economics, the sectors of finance and investment help individuals, business enterprises and organizations to invest their assets beneficially. Not least today, the institutions managing public assets which are required to allocate the public resources follow the same principle: *maximize the asset by investing in beneficial projects*. But how can beneficial projects be identified?

2.5.1 Investment Criteria

Investment criteria are needed to identify beneficial investments. Cost comparison is the simplest investment criterion. For instance, two different concrete mixers are compared in terms of their costs and the cheaper mixer is identified as the more beneficial investment. But this simple criterion may be too simple. It is better to also include the ability of the concrete mixers to generate revenue in the investment consideration. For instance, this is accounted for in the following investment criteria: net revenue, simple rate of return or amortization calculation. These simple criteria are straightforward and are often applied in the everyday life of individuals, enterprises and organizations. However, they fail to address long term investments appropriately because they neglect the time-variant structure of costs and revenues.

Long term investments are characterized by a long time period between the beginning of the investment and its end. Here, *long* has to be understood so that future consequences need to be discounted.

2.5.1.1 Discounting in Economics

In economics, discounting is the main method of evaluating future cash flows, i.e. consequences of a project such as costs and revenues. The method was developed in the years after the 1929 market crash. Generally, Williams (1938) is regarded as having first formally expressed discounting, see also Preinreich (1935) and Guild (1931).

Discounting allows for comparison of cash flows $x_n(t)$ occurring at different points in time on a common basis. The index n indicates that the cash flows (costs and revenues) are nominal values corresponding to the time t . Generally speaking, discounting reduces nominal costs or revenues by a discount factor $r(t)$ to receive the corresponding present value x_r associated with time t_0 . The index r reflects that x_r represents a real value with regard to t_0 .

$$x_r = r(t) x_n(t) \quad (2.21)$$

Using the annual rate or interest i_r , the discount factor $r(t)$ is given by Equation 2.22. It can also be formulated as given in Equation 2.23. Both equations are related by $i'_r = \ln(1 + i_r)$.

$$r(t) = (1 + i_r)^{-(t-t_0)} \quad (2.22)$$

$$= e^{-(t-t_0)i'_r} \quad (2.23)$$

Figure 2.11 illustrates why cash flows/ consequences must be discounted. It illustrates the effect of compound interest (interest on interest). If the sum x_r is invested at time t_0 on a bank account,

then the investor can receive at $t = t_i$ the amount $x_r(1 + i_r)^{(t_i-t_0)}$. In other words, $x_r(1 + i_r)^{(t_i-t_0)}$ at time t_i is equivalent to x_r at t_0 . Respectively, $x_n(t_i)$ at t_i is equivalent to $x_n(t_i)r(t_i)$ at time t_0 , with $r(t)$ as defined above. Investment criteria which account for the effect of discounting are e.g.

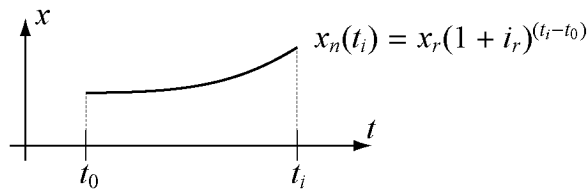


Figure 2.11: Effect of compound interest.

the net present value and the internal rate of return, see also Schierenbeck (2003) or Burdelski (n.a.). The internal rate of return is defined as the interest rate that gives a net present value of zero. Therefore, both criteria yield the same decision.

2.5.2 Net Present Value

The net present value (NPV) is simply the sum of all present valued cash flows (costs and revenues) which are associated with a specific project such as a civil engineering structure. Hereby, the *NPV* takes into account all consequences throughout the project's lifetime, e.g. life cycle of a structure. If the *NPV* of a project is positive then it is economically feasible to invest. However, another project might be more beneficial. The project which yields the highest *NPV* is generally selected. In certain situations, the revenues may be neglected. In this case, Equation 2.24 formulates the life cycle costs, see also Chapter 8.

$$NPV = \sum_i r(t_i)x_{n,i}(t_i) \quad (2.24)$$

In the consideration of *NPV* the cash flows occur with certainty – an assumption which in practical engineering cannot be presumed; but the consequences may be weighted with their probability of occurrence such that the investment criterion is the expected net present value $E[NPV]$.

In economics the *NPV* is undoubtedly the predominating investment criterion. In engineering it is also well accepted that cash flows need to be discounted because investing the assets in bank accounts or in bond funds is always a valid and reliable decision alternative. The topic of discounting is also addressed in Section 8.4. In Section 6.1.1.5 discounting of life saving costs is discussed.

It is noted that t_0 can be chosen arbitrarily. Often the point in time is chosen at which the decision is to be made. By means of Equation 2.21, the *NPV* may be calculated for any point in time without changing the identified optimal decision. It is seen that Equation 2.21 is a special positive linear transformation of a utility function which does not influence the ordering of the decision outcome.

Hence, maximizing the expected net present value is the formulated objective leading to a maximization of the asset. Straightforwardly, this formulated objective can be implemented in engineering if this is the only objective. In cases, where more than one objective is identified,

e.g. when consequences are not expressed in monetary terms such as consequences to the environment, then a utility function $u(c_1, c_2)$ depending on two or more variables c_i can be established, see Chapter 5 and Annex B.

2.5.3 Portfolio Selection

A set of civil engineering facilities can be regarded as a portfolio such as a portfolio of shares. Both need to be managed professionally. Today's portfolio theory was introduced by Markowitz (1952). It determines all combinations of a portfolio which are elements of the *efficient set*. This set characterizes all efficient portfolios which for a given mean value gives the least variance. In the space of all possible portfolios the set of efficient portfolios consists of a series of connected line segments. Transferred into the mean value – variance space (μ - σ space), the efficient set forms a series of connected parabolic segments. At one end of the series there is the portfolio with the maximum expected value and at the other end is the portfolio with the least variance.

Only in the case when the set of possible portfolios contains a share which at the same time has the maximum expected return and also the least variance, then the maximization of the expected value yields also the minimum variance; the portfolio contains only a single security with the maximum return. In all other cases no single portfolio is identified to be optimal. There is an infinite amount of portfolios which are elements of the efficient set and only with a predefined variance or mean value, a single portfolio is identified to be optimal. An adequate solution in finding the optimal portfolio would comprise a formulation of the preferences in the μ - σ space as described in Laux (2003).

2.5.4 Value at Risk

Another measure to quantify the risk of portfolios is the value at risk (VaR). It is widely used in economics and finance but also in engineering decision-making.

In the early 1990s, three incidents showed the need to quantify extreme losses of portfolios, namely the near bankruptcy of the Metallgesellschaft in 1993, the speculation of the treasurer of Orange County in 1994 and the bankruptcy of the renowned Barings Bank in 1995. In that period, VaR was already introduced and implemented by institutional investors and finally it entered into the Basel Capital Accords BCBS (2004).

To quantify risks using VaR, three measures need to be predefined:

1. the reference period (e.g. one day),
2. the reference quantile (e.g. 99%) and
3. the unit used to measure the portfolio (e.g. USD).

For instance, VaR can be defined as the $\alpha = 99\%$ quantile of a loss distribution with a one day reference period that is measured in USD. If e.g. a portfolio has a one day 99% VaR of 1 million USD, it is expected that the probability of an eventual loss that exceeds 1 million USD next day is less than 1%. Denoting the loss with L , VaR can be written as

$$VaR_{\alpha}(L) = F_L^{-1}(\alpha). \quad (2.25)$$

A measure often encountered together with VaR is the expected shortfall. The expected shortfall is simply defined as the expected loss given the loss exceeds the VaR.

$$ES_\alpha(L) = E[L|L \geq VaR_\alpha(L)] \quad (2.26)$$

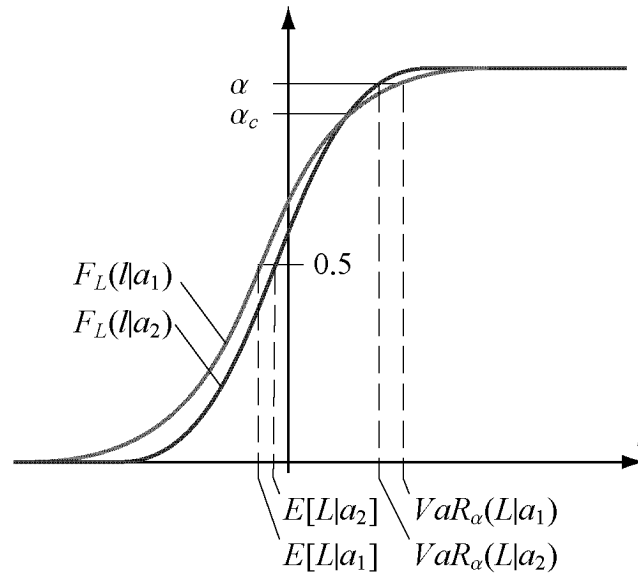


Figure 2.12: Inconsistency between value at risk and utilitarian decision theory.

It is noted that the VaR concept is clearly able to identify within portfolios risk contributors having distribution functions with long tails. Subsequently risk reducing measures can be implemented. But it can be shown that decision-making only on the basis of VaR can lead to decisions that are not consistent with utility-based decision theory. Figure 2.12 illustrates two symmetric loss distribution functions corresponding to two actions, namely a_1 and a_2 . Both have negative expected values $E[L|a_1]$ and $E[L|a_2]$, i.e. a gain is expected. $E[L|a_1]$ is smaller than $E[L|a_2]$ and therefore a_1 is preferred if decisions are made according to expected utilities¹. However, as seen in the diagram, the VaR associated with a_1 is larger than the VaR for a_2 . Therefore, this simple example shows that decisions according to VaR are not always consistent with utilitarian decision theory. Furthermore, it is seen that VaR based decisions depend on whether α is chosen to be smaller or larger than α_c .

2.6 Risk Assessment and Management

Hazard is an adverse event and risk is a measure of it. In engineering decision-making it is quantified as the expected consequences which are associated with an activity, JCSS (2004) and Schneider (1996). Mathematically this is represented by Equation 2.6.

Today risk assessment and management forms the basis for the structural design, reassessment, operation as well as inspection and maintenance planning of civil engineering facilities.

¹Using a risk neutral utility function.

From Equation 2.6 it is seen that risk assessments are embedded in the framework of normative decision theory. A complete risk assessment needs to address all relevant adverse events. But it also considers the activity's benefits which are positive such as revenues or the ability to cross a valley using a bridge.

2.6.1 Risk Assessment Principles

Public management of societal risks aims to serve the common good in a consistent, transparent and defensible manner. To achieve this, Nathwani, Lind, and Pandey (1997) formulated four principles for managing risk to the public, see also Pandey and Nathwani (2003), Pandey and Nathwani (2004). They state the principle of:

- Accountability
- Maximum net benefit
- Compensation
- Life measure

2.6.1.1 Accountability

The accountability principle states that decision for the public must be:

- Open,
- Quantified,
- Defensible,
- Consistent and
- Applied across the complete range of hazards.

Only open and transparent risk assessments assure that the preferences of the public are well addressed. If it is transparent then it is defensible at the same time and supports decision makers when an unpopular decision has to be justified. Risks from various hazards are only comparable if they have been analyzed consistently. And the quantified risks of all hazards permit the allocation of societal resources such that the preferences of society are best met, i.e. maximizing the net benefit.

2.6.1.2 Maximum Net Benefit

Decision-making according to the maximum net benefit is strictly in line with decision theory. The latter was already discussed at length. Here, the difficulty lies in the assessment of benefits to society with regard to *immaterial consequences* measures such as the quality of the environment and a healthy life. Today, the latter can be assessed using the life quality index, see Chapter 5 and 6.

2.6.1.3 Compensation

Not all parties of society benefit from hazardous activities. Some persons take higher risks and can or have to be compensated. The chosen activity is then beneficial if these persons are fully compensated and if there is still a net benefit. However, the principle does not state how compensation may be quantified.

2.6.1.4 Life Measure

The principle of maximum net benefit to society should address the length of life. A reliable and broadly accepted measure to assess the length of life is the life expectancy. An activity's effect on the life expectancy can and should be taken into account in the net benefit of society.

2.6.2 Risk Management

Risk management is the process which focuses on efficient and effective management of potential opportunities and adverse hazards. Risk management involves all aspects of risk assessment and it can be structured into a generic format which is illustrated in Figure 2.13. It is emphasized that this format is independent of a considered industry or branch thereof. Therefore, it can be seen as an overall decision and management framework.

Figure 2.13, which is taken from AS/NZS 4360:1999, shows the required steps for risk management. Definition of the context is the most important step. The strategic and organizational context needs to be identified or defined. For instance, answers need to be found to questions such as: Who are the decision makers? And which other parties are affected by the implementation of possible actions? In addition, the system has to be identified. A further crucial part of the first step is the choice of the acceptance criteria to be used. A risk screening may help to identify all relevant hazards and opportunities which after that are thoroughly analyzed in terms of their potential consequences and the likelihood of occurrence. The analyzed risk can then be assessed/ evaluated, whether it is acceptable or not. In case it is not acceptable, appropriate risk treatments can be incorporated such as risk mitigation, reduction or transfer. The review and monitoring of the individual steps accompany the whole risk management process as well as communication with the decision makers and stakeholders. More details are found in AS/NZS 4360:1999 or Faber (2004).

2.7 Summary

Many people making decisions under uncertainty, i.e. making a decision without knowledge about the probability of events, utilize the Laplace criterion. This criterion is based on Johann Bernoulli's principle of insufficient reason and considers each event to occur equal likely, i.e. a uniform probability distribution is intuitively used. However, strictly speaking this is referred to decision-making under risk, decision-making when probabilities can be assigned to the occurrence of events.

Today's decision-making under risk is based on Daniel Bernoulli's proposition of 1738 to use expected utilities. More than 200 years later, von Neumann and Morgenstern proved that if their Axiom system is accepted, rational decisions have to be made according to expected utilities. The considered utility function expresses the preferences of the decision maker and

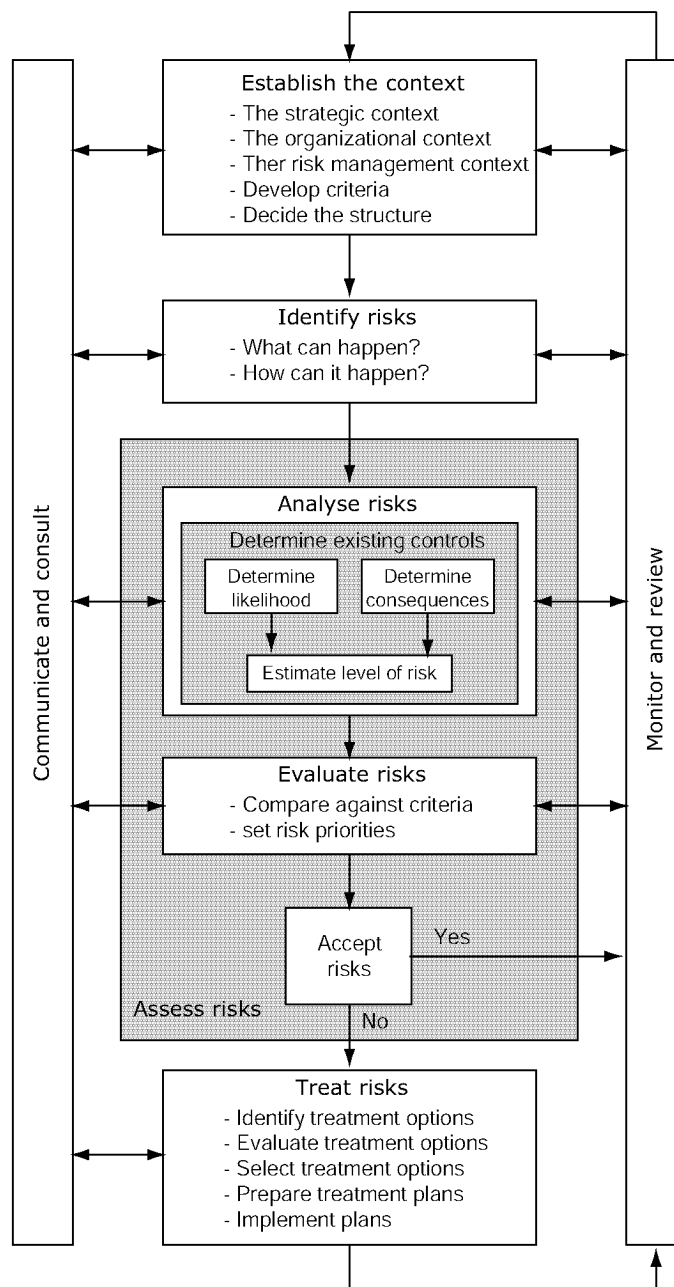


Figure 2.13: Risk management process according to AS/NZS 4360:1999.

risk prone, neutral as well as risk averse behavior can be described. But by applying a risk prone or risk averse utility function to decision-making, it can be shown that the gain in the long run is smaller than the expected value. This highlights the importance of an appropriate modeling of preferences.

It is possible to combine the two strong concepts, rational decision theory and Bayes' theorem. Three possible decision analyses can be distinguished, namely *prior*, *posterior* and *pre-posterior* decision analysis. Whereas prior decision analysis considers only information which is already available, the posterior decision analysis is able to account for new information; and using the

2 Decision Theory

pre-posterior decision analysis it is possible to identify an optimal strategy to perform experiments, e.g. inspections and subsequent actions before the experiments are actually carried out. Today, pre-posterior decision analysis is a crucial tool for the assessment of existing structures and risk-based inspection and maintenance planning.

Only if the objective of decision-making is clearly identified, is it able to serve the decision maker to allocate his or her resources beneficially. Using investment criteria beneficial projects or actions are identified. For instance, cost comparison is a simple and often utilized criterion. However, it fails take into account the revenues as well as the time-variant structure of costs and revenues. Comparing costs and revenues occurring at different points in time is only possible by using the concept of discounting. The predominant investment criterion utilized in economics is the net present value that discounts costs and revenues to obtain the present value. The net present value is simply the sum of all present valued costs and revenues which are associated with the investment, such as a civil engineering facility. If costs and revenues are uncertain, the project with the maximum expected net present value is preferred.

Risk is a measure of a hazard. Mathematically formulated, it is the expected value of all consequences and this highlights its closeness to decision theory. Risk assessment and management is today's framework for decision-making in civil engineering, such as the design, assessment and inspection of civil engineering structures. In order to make decisions for the public, the risk assessment principle of accountability, maximum net benefit, compensation and life measure are presented and a general risk management procedure is outlined.

3 Uncertainty and Probability

The foregoing chapter clearly shows that for rational decision-making two ingredients need to be quantified, namely consequences and uncertainty. Each decision problem involves uncertainty and there may be uncertainty in the magnitude of consequences and in the likelihood of occurrence. To be able to perform a quantitative risk assessment and risk-based decision-making, uncertainty needs to be expressed quantitatively in terms of probabilities. The appropriate modeling of these uncertainties is crucial for the identification of the optimal decision.

No introduction into statistics and basic probability theory is given here. These topics are covered by many standard books, e.g. Benjamin and Cornell (1970) or Faber (2004). The present chapter first categorizes different types of uncertainty. Then it addresses the different possible interpretations of the probability concept. This is followed by Bayesian updating and an introduction to Structural Reliability Analysis (SRA), a branch of probability theory developed by engineers in the last few decades. First, the important time-invariant case is presented followed by the time-variant SRA. The latter requires knowledge of stochastic processes; hence, an introduction to the important Poisson process is provided. The chapter closes with a brief discussion of outcrossing rates.

3.1 Types of Uncertainty

One can classify uncertainty into two types, namely:

- *aleatoric uncertainty* influenced by natural fluctuation/ variability, and
- *epistemic uncertainty* due to insufficient knowledge.

3.1.1 Aleatoric Uncertainty

Aleatoric uncertainty — from *alea*, Greek for dice — is the natural fluctuation or variation of the considered quantity. This variation is inherent to the considered quantity, which is why this type of uncertainty is also called inherent or physical uncertainty. But not only physical quantities such as yield strength or wind pressure are random. For instance, the value of government bonds or human errors are random as well.

3.1.2 Epistemic Uncertainty

Epistemic stems from *episteme*, Greek for science which is also translated as knowledge. Epistemic uncertainty results from insufficient knowledge of the considered phenomena. There are two sources for epistemic uncertainty: *model uncertainty* and *statistical uncertainty*.

Model Uncertainty

Models that are used to describe real phenomena are only approximations and there are

different models of various levels of detail. They result from different simplifications they imply such as disregard of influencing variables or interrelation of variables. If model prediction and real observations are available, then the model uncertainty can be quantified as a random variable and the performance of the different models may be compared. In addition, the sensitivity of the identified decision with regard to the selected model may be assessed when it is decided which model to select.

Statistical Uncertainty

Data of observations are often scarce and limited. Therefore, if a phenomenon that is subject to fluctuations is modelled by a random variable, the parameters associated to this variable, e.g. the mean and the standard deviation, cannot be assessed exactly due to the lack of data. Therefore, these parameters are uncertain themselves and may be modeled as random variables as well. However, if additional observations are provided then the statistical uncertainty can be reduced.

Whereas epistemic uncertainty can be reduced by improved models and/ or additional statistical observations, the aleatoric uncertainty remains unchanged. It only changes if the quantity of interest is modified itself.

In many engineering decision problems, natural fluctuation and insufficient information are the most important source of uncertainty. However, Nathwani et al. (1997) also list *conflict* and *vagueness* as additional types of uncertainty which may become relevant.

3.1.3 Conflict and Vagueness

Conflict

All activities involving large hazard potentials carry the possibility of conflicts. Unless all stakeholders benefit from the activity in the same way, their preference ordering may differ and the behavior of the stakeholders is not predictable, i.e. uncertain. Game theory is able to predict such behavior and with compensations, the preference ordering may be brought in line to solve conflicts.

Vagueness

Vagueness is uncertainty in definitions. For instance, the deflection of a beam may be characterized as medium, large or very large. Fuzzy theory deals with vagueness and treats this as a fuzzy quantity. However, for quantitative risk assessments vagueness should be reduced to a minimum. Nathwani et al. (1997) state that "Quantitative risk analysis is not possible if you are not clear what you are talking about" (153). There is also no generally accepted way to map fuzzy elements into crisp quantities. Quantities must be made *crisp*, i.e. not fuzzy. By definitions and standardizations vagueness may be reduced to a negligible level. For instance, the deflection of a beam can be defined to be unacceptable if it is larger than 2% of the beam's span.

3.2 Interpretation of Probability

With the evolution of probability theory and statistics, the interpretation of the probability concept evolved as well. Today, the *classical*, *frequentistic* or the *subjective* interpretation of probabilities are considered.

Classical

The classical interpretation considers the probability of an event as the ratio of the number of cases favorable to the event to the number of all possible cases. This requires two things. On the one hand, full knowledge is needed of the considered phenomenon to identify all possible basic cases. On the other hand, for each basic case the same likelihood is assigned. Then the number of favorable events is calculated by means of combinatorics. These criteria are fulfilled by many games, e.g. throwing fair dices or flipping coins. For instance, considering the throw of a dice it can exhibit the numbers 1 to 6, which represents the basic events, i.e. the number of the basic events is 6. If the probability is of interest to obtain an odd number (1, 3 or 5), i.e. 3 favorable events then the probability is evaluate to $3/6 = 0.5$.

Frequentistic

The probability is the limit of the relative frequency of observable events, when the number of observations approaches infinity, von Mises (1928). In contrast to the classical interpretation, the frequentistic interpretation does not require complete knowledge of the considered phenomenon. It allows for derivation of knowledge based on experimental observations, see also J. Bernoulli (1713).

Subjective

Probability is interpreted as a degree of belief that a considered event will occur. It goes without saying that, if possible, experts should express their belief to obtain best engineering judgment.

Engineering decision problems are characterized by the fact that generally partial solutions can be found by using one of the three interpretations. But finding the most appropriate global solution involves all three. Their consistent combination is also referred to as Bayesian modeling, see JCSS (2001b).

3.3 Bayes' Theorem and Updating

Inspections and reassessments of existing structures yield additional information, e.g. by non-destructive tests. In these cases Bayes' theorem is particularly useful because reliability can be assessed conditioned by the made observations. This process is also referred to as *updating*. In probability theoretical terms, the Bayesian theorem is nothing other than a conditional probability.

Consider the sample space Ω with event A and mutually exclusive events E_i (with $i \in \{1, \dots, n\}$). Figure 3.1 illustrates this sample space for the case $n = 5$. From this figure it is seen that $P(A)$ can be written as

$$P(A) = \sum_{j=1}^n P(E_j \cap A). \quad (3.1)$$

Using the definition of conditional probabilities $P(A|E_j) = P(E_j \cap A)/P(E_j)$ – we further obtain

$$P(A) = \sum_{j=1}^n P(A|E_j)P(E_j). \quad (3.2)$$

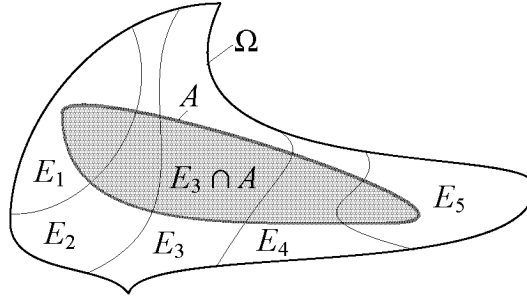


Figure 3.1: Sample space Ω with mutually exclusive events E_1 and event A .

If the event A is observed and one is interested in the event E_i , its probability is given by

$$P(E_i|A) = \frac{P(E_i \cap A)}{P(A)}. \quad (3.3)$$

Replacing $P(E_i \cap A)$ by $P(A|E_i)P(E_i)$ leads to

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{P(A)}, \quad (3.4)$$

and substituting $P(A)$ in the denominator by Equation 3.2 yields

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{\sum_{j=1}^n P(A|E_j)P(E_j)}. \quad (3.5)$$

This equation is also referred to as *Bayes' theorem*. In Equation 3.5, the individual terms are given names:

- $P(E_i)$ is called *prior* and represents the information which was available before the event A was observed. Often it is denoted by a prime $P'(E_i)$.
- $P(A|E_i)$ is also referred to as *likelihood*.
- $P(E_i|A)$ is called *posterior* and is the updated probability. Generally, two primes $P''(E_i|A)$ are used to distinguish it from the prior probability.
- $\sum_{j=1}^n P(A|E_j)P(E_j)$ is a normalizing constant.

Therefore, Bayes' theorem may be written as:

$$\text{Posterior} = \frac{\text{Likelihood} \cdot \text{Prior}}{\text{Normalizing Constant}}. \quad (3.6)$$

Bayes' theorem is universally applicable in engineering. For instance, it is successfully applied in structural engineering to update resistances after proof loading, it is indispensable for re-assessments of existing structures, e.g. risk-based inspection planning, and it may also be used for statistical inference. In Section 10.6.1, it is shown that Bayes' theorem can also be applied for code calibration to update characteristic values together with partial safety factors.

3.4 Structural Reliability Analysis

Not all uncertain quantities are observable, particularly in civil engineering, e.g. the important case of structural failure. Data is scarce and often inconsistent. The latter results from the fact that civil engineering structures are mostly unique and are not manufactured in mass production.

However, if the event of interest, e.g. structural failure, can be described by a functional relation $g(\mathbf{x})$, methods of structural reliability analysis (SRA) can be utilized to determine the associated probability. $g(\mathbf{x})$ is a limit state function or performance function and $\mathbf{x} = (x_1, \dots, x_n)^T$ is the vector of variables required to describe the considered event.

It goes without saying that for the probabilistic evaluation of $g(\mathbf{x})$, the probability structure of the basic random variables $\mathbf{X} = (X_1, \dots, X_n)^T$ is needed. It is given by the joint cumulative distribution function $F_{\mathbf{X}}(\mathbf{x})$. Here, the notation of probability theory distinguishes between a random variable, e.g. X_1 and a possible realization of it x_1 .

Generally, methods of structural safety are not limited to calculate failure probabilities. In principle, any event which can be described by a limit state function $g(\mathbf{x})$ can be analyzed, such as the effects of deterioration on inspection results. SRA is applicable to any field of research or industry, e.g. in finance to determine the benefits or losses of a specific portfolio.

Already Mayer (1926) formulated the possibility to base structural safety on probability theory to avoid inconsistent and irrational structural design. Wierzbicki (1936) made an attempt to calculate failure probabilities. But it was Freudenthal (1947), (1956) who introduced the fundamental case of structural reliability.

3.4.1 Fundamental Case of Structural Reliability

The fundamental case considers a simple structural system consisting of the load S and the resistance R . The failure \mathcal{F} is defined as the event when the load exceeds the resistance. That is:

$$\mathcal{F} = \{\mathbf{x} | g(\mathbf{x}) \leq 0\} \quad (3.7)$$

with

$$g(\mathbf{x}) = r - s. \quad (3.8)$$

The corresponding failure probability can be evaluated as follows. The conditional failure probability given that the load takes the value $S = x$ is given by $F_R(x)$. The unconditional failure probability is obtained, if the conditional failure probability is integrated over the possible domain of R and S and weighted by the probability density function $f_S(x)$ of the loading. This is represented by Equation 3.9 and illustrated in Figure 3.2.

$$P(\mathcal{F}) = \int_{-\infty}^{\infty} F_R(x) f_S(x) dx \quad (3.9)$$

3.4.2 The Reliability Index

Another measure of structural safety was introduced by Cornell (1969a). In this context the contribution of Basler (1960) should be mentioned as well. Today, this measure is known as the reliability index β . The definition starts with the already introduced limit state function

3 Uncertainty and Probability

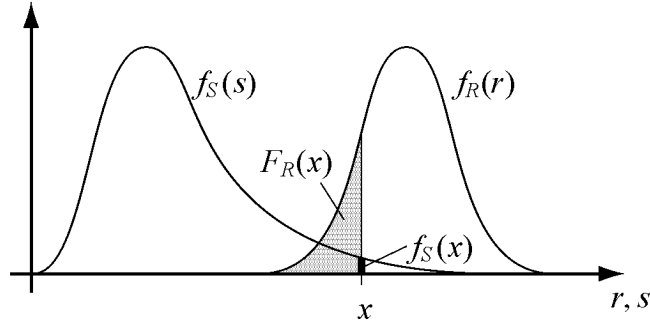


Figure 3.2: The fundamental case of structural reliability analysis.

given by Equation 3.8 and substitutes the realizations r and s by the random variables R and S . Consequently, the limit state becomes a function of random variables and therefore a random quantity itself. It is also called safety margin M and it is denoted by

$$M = g(\mathbf{X}) = R - S. \quad (3.10)$$

Probability theory allows for the computation of the mean value of the safety margin μ_M and its standard deviation σ_M . They are calculated using the mean and standard deviation of the resistance μ_R, σ_R and load μ_S, σ_S . If these variables are correlated, then the correlation coefficient ρ_{RS} needs to be considered to calculate σ_M .

$$\mu_M = \mu_R - \mu_S \quad (3.11)$$

$$\sigma_M = \sqrt{\sigma_R^2 + \sigma_S^2 - 2\rho_{RS}\sigma_R\sigma_S} \quad (3.12)$$

The failure probability is identical to the probability that M is smaller or equal to zero, which for normally distributed variables is given by Equation 3.14.

$$P(\mathcal{F}) = P(M \leq 0) \quad (3.13)$$

$$= \Phi\left(-\frac{\mu_M}{\sigma_M}\right) \quad (3.14)$$

Using Cornell's definition of the reliability index

$$\beta = \frac{\mu_M}{\sigma_M}, \quad (3.15)$$

the failure probability is obtained as

$$P(\mathcal{F}) = \Phi(-\beta). \quad (3.16)$$

The reliability index is thus directly related to the failure probability via the standard normal distribution function $\Phi(x)$. This makes it a measure of structural safety.

Figure 3.3 shows that the reliability index times the standard deviation is the distance between the mean value of the safety margin and the origin (if M is normally distributed). It is seen that if β increases, then the hatched area, which represents the failure probability, decreases.

However, the failure probability given by Equation 3.15 is not invariant. For instance, consider the limit state function $\ln(r) - \ln(s) \leq 0$. It is also a valid formulation for structural failure. If

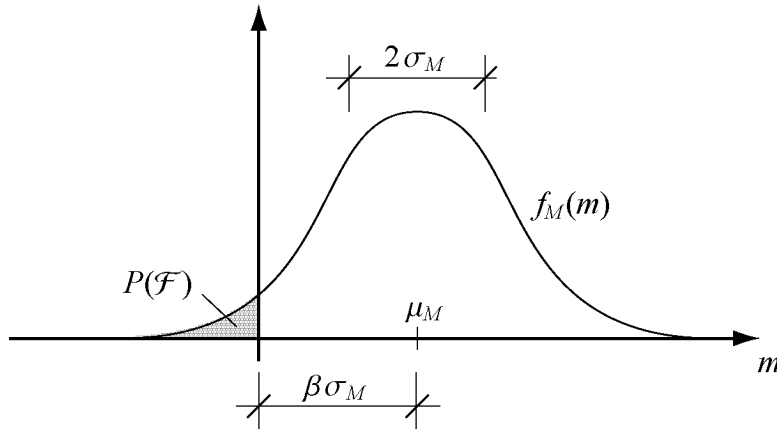


Figure 3.3: Graphical representation of the reliability index β .

R and S are lognormally distributed, then the probability of failure can be calculated according to Equation 3.17. The difference to Equation 3.14 is obvious. To circumvent the invariance problem, Hasofer and Lind (1974) proposed a modified formulation for the reliability index.

$$P(\mathcal{F}) = \Phi \left(\frac{\ln \left(\frac{\mu_S}{\mu_R} \sqrt{\frac{V_R^2 + 1}{V_S^2 + 1}} \right)}{\sqrt{\ln((V_R^2 + 1)(V_S^2 + 1))}} \right) \quad (3.17)$$

3.4.3 Reliability Index of Hasofer and Lind

Based on Equations 3.10 to 3.14 the reliability index can be extended to consider more than two variables while at the same time the limit state function may be nonlinear. First by a Taylor series expansion at a point $\tilde{\mathbf{x}}$ the nonlinear limit state function $g(\mathbf{x})$ may be linearized to $g_L(\mathbf{x})$ and for this function the mean and standard deviation are readily obtained, as well as the reliability index. As the reliability index depends on $\tilde{\mathbf{x}}$, an iteration scheme yields the final reliability index. If the normal variables \mathbf{X} are correlated, they can be transformed to obtain independent variables \mathbf{Z} , which in turn by $u_i = (z_i - \mu_{Z_i})/\sigma_{Z_i}$ transforms \mathbf{Z} into the standard normal space \mathbf{U} .

Hasofer and Lind (1974) also give the reliability index an illustrative meaning. By transforming the problem into the standard normal space, the reliability index is characterized as the shortest distance from the origin to the limit state function $g(\mathbf{u}) = 0$. Due to this, the calculation of the reliability index may also be formulated as a minimization problem with the condition that \mathbf{u} is an element of $\{\mathbf{u}|g(\mathbf{u}) = 0\}$.

$$\begin{aligned} \beta &= \min_{\mathbf{u}} |\mathbf{u}| \\ \text{s.t. } &g(\mathbf{u}) = 0 \end{aligned} \quad (3.18)$$

Here, $|\mathbf{u}| = \sqrt{\sum_i u_i^2}$ is the norm of \mathbf{u} , i.e. the length of the vector \mathbf{u} .

3.4.4 General Case

The methods introduced in the last two sections are exact, provided the basic variables are normally distributed. Unfortunately, this case is rarely met in practical applications. For a single limit state function the most general formulation of the failure probability is

$$P(\mathcal{F}) = \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}. \quad (3.19)$$

The limit state function $g(\mathbf{x})$ divides the sample space Ω into two subsets, namely the safe domain $\mathcal{S} = \{\mathbf{x} | g(\mathbf{x}) > 0\}$ and the failure domain $\mathcal{F} = \{\mathbf{x} | g(\mathbf{x}) \leq 0\}$. A subset of \mathcal{F} is the set where $g(\mathbf{x})$ is equal to zero $\mathcal{G} = \{\mathbf{x} | g(\mathbf{x}) = 0\}$. The failure probability is obtained by integration over the failure domain \mathcal{F} weighted by $f_{\mathbf{X}}(\mathbf{x})$, where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function (jpdf) of the random variables. Generally, \mathbf{X} are non-normally distributed and possibly dependent variables.

A solution to the above integral is to transform \mathbf{X} into independent standard normal distributed variables \mathbf{U} with jpdf $\varphi(\mathbf{u})$. If such a transformation $\mathbf{x} = \mathbf{T}(\mathbf{u})$ exists, the failure probability can be written as given by Equation 3.20, and the methods introduced in the last two sections may be applied to obtain $P(\mathcal{F})$. This procedure is also referred to as first order reliability method (FORM).

$$P(\mathcal{F}) = \int_{g(\mathbf{T}(\mathbf{u})) \leq 0} \varphi(\mathbf{u}) d\mathbf{u} \quad (3.20)$$

3.4.4.1 Transformation

Several transformations are able to map non-normal variables \mathbf{X} into independent standard normal variables \mathbf{U} , namely the *normal tail*, *Rosenblatt*, *Nataf* and the *Hermite polynomial* transformation.

If the variables \mathbf{X} are uncorrelated, the normal tail transformation

$$p_i = F_{X_i}(x_i) = \Phi(u_i), \quad i \in [1, \dots, n] \quad (3.21)$$

or

$$x_i = F_{X_i}^{-1}(\Phi(u_i)), \quad i \in [1, \dots, n] \quad (3.22)$$

can be used. It is seen that this transformation preserves the same probability content p_i , see Figure 3.4. However, it is only applicable if the variables are not correlated.

If they are dependent, the Rosenblatt transformation can be used which is the most general transformation. It requires knowledge of the joint cumulative distribution function $F_{\mathbf{X}}(\mathbf{x})$. Using the definition of conditional probability we obtain

$$F_{\mathbf{X}}(\mathbf{x}) = F_{X_n}(x_n | x_{n-1}, \dots, x_1) \dots F_{X_2}(x_2 | x_1) F_{X_1}(x_1), \quad (3.23)$$

and the Rosenblatt transformation is given by following set of equations.

$$\begin{aligned} x_1 &= F_{X_1}^{-1}(\Phi(u_1)) \\ x_2 &= F_{X_2}^{-1}(\Phi(u_2) | x_1) \\ &\vdots \\ x_n &= F_{X_n}^{-1}(\Phi(u_n) | x_{n-1}, \dots, x_1) \end{aligned} \quad (3.24)$$

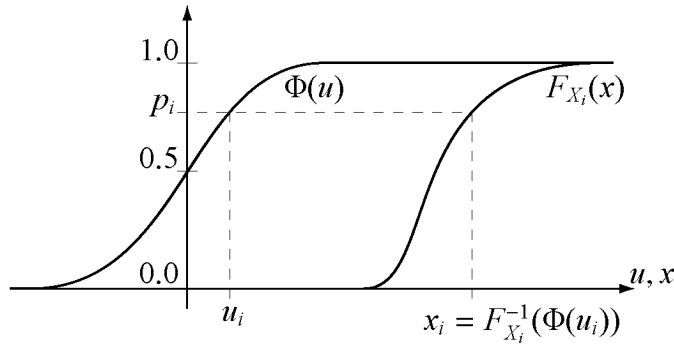


Figure 3.4: Tail transformation.

It is seen that if the variables \mathbf{X} are independent, the normal tail transformation is obtained. However, in practical engineering cases it is seldom the case that the full probability structure is available as required for the Rosenblatt transformation.

The Nataf transformation only requires the cumulative distribution functions of the marginals $F_{X_i}(x_i)$ and the correlation matrix $\boldsymbol{\rho} = \rho_{ij}$. In most cases, these quantities are available and an approximation to $F_{\mathbf{X}}(\mathbf{x})$ can be constructed. The Nataf transformation is obtained by the transformation of the marginals \mathbf{x} to \mathbf{u} .

$$u_i = \Phi^{-1}(F_{X_i}(x_i)), \quad i \in [1, \dots, n] \quad (3.25)$$

However, the variables \mathbf{U} are only independent if the variables \mathbf{X} are independent. Otherwise, its correlation structure $\boldsymbol{\rho}^o$ is a function of $\boldsymbol{\rho}$.

$$\rho_{ij}^o = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} z_i z_j \phi_2(y_i, y_j; \rho_{ij}^o) dy_i dy_j \quad (3.26)$$

Liu and Der Kiureghian (1986) showed that $\boldsymbol{\rho}^o$ can be calculated by Equation 3.26, where $z_i = (F_{X_i}^{-1}(\Phi(y_i)) - \mu_{X_i})/\sigma_{X_i}$; approximate expressions for the ratio R were also determined.

$$R = \frac{\rho_{ij}^o}{\rho_{ij}}. \quad (3.27)$$

Melchers (1999) states that for all practical cases with the exception of the shifted exponential distribution, the factor R lies in the interval $[0.9, 1.1]$. With regard to the accuracy with which the correlation structure can be obtained, it is often sufficient to assume $\boldsymbol{\rho}^o = \boldsymbol{\rho}$, see also Der Kiureghian and Liu (1986).

Another approximation for $F_{\mathbf{X}}(\mathbf{x})$ can be obtained by the use of Hermitian polynomials, see Winterstein and Bjerager (1987) or Ditlevsen and Madsen (2005).

3.4.4.2 Second Order Reliability Method

Even if the limit state is formulated as a linear function, the transformation of variables makes it nonlinear. Therefore, limit state functions are generally nonlinear in the standard normal space and a first order approximation might be inadequate.

It follows from this idea to expand the limit state function into a Taylor series such that the consideration of the second order terms yields a better approximation to Equation 3.20. Breitung

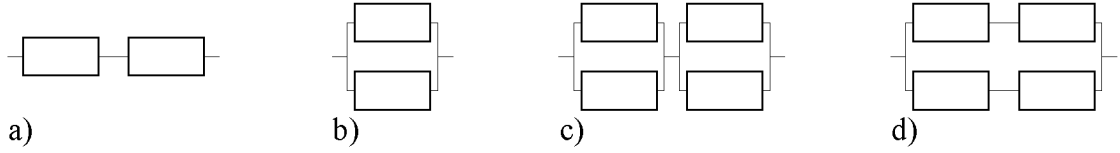


Figure 3.5: System representation of a) series, b) parallel and mixed systems c) and d).

(1984) showed that the probability content of a hyperparabolic failure surface can be approximated by the following equation.

$$P(\mathcal{F}) = \Phi(-\beta) \prod_{i=1}^{n-1} \sqrt{1 - \beta\kappa_i} \quad (3.28)$$

Here, β is the reliability index obtained by first order reliability methods and κ_i represents the principal curvatures at the design point \mathbf{u}^* where $g(\mathbf{u}^*) = 0$. From this, it follows that in order to apply second order reliability methods (SORM), the limit state function must be continuous and twice differentiable. The equation also shows that if the curvatures converge to zero, the first order solution is obtained. It is also seen that the expression is singular for $\beta = 1/\kappa_i$. Better approximations for Equation 3.28 are available and can be found e.g. in Rackwitz (2002).

3.4.5 Reliability of Systems

The performance of real structures can rarely be described by a single limit state function. They constitute rather systems at different levels of complexity. Structures may be composed of several elements or subject to different loads and deterioration processes. Besides the ultimate limit states, serviceability and durability may be of interest, as well.

For this reason, Equation 3.19 is generalized to Equation 3.29 which integrates over the failure set \mathcal{F} . This set may explicitly consider multiple limit states $g_i(\mathbf{x})$.

$$P(\mathcal{F}) = \int_{\mathcal{F}} f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x}. \quad (3.29)$$

To formulate the failure set \mathcal{F} , it is advantageous to first express it in terms of a system representation. Systems can be classified as *series*, *parallel* or *mixed* systems. Examples for mixed systems are the *series system of parallel systems* and the *parallel system of series systems*, see Figure 3.5.

A series system fails if at least one of its components fails; and if the components' failures are described by $g_i(\mathbf{x})$, the failure is defined by the union $\mathcal{F} = \{\cup_{i=1}^n g_i(\mathbf{X}) \leq 0\}$. Failure of a parallel system is observed, if all its components fail, which is given by the intersection $\mathcal{F} = \{\cap_{i=1}^n \{g_i(\mathbf{X}) \leq 0\}\}$, see Figure 3.6.

Figure 3.6 shows two linear limit state functions in the standard normal space. It is seen that the failure probability for parallel systems is given by Equation 3.30 and for series systems by Equation 3.31. $\Phi_n(\cdot; \cdot)$ is the multi normal distribution and $\boldsymbol{\beta}$ is the vector with the reliability indexes of the safety margins, and $\boldsymbol{\rho}$ is the correlation matrix of the safety margins.

$$P(\mathcal{F}) = \Phi_n(-\boldsymbol{\beta}; \boldsymbol{\rho}) \quad (3.30)$$

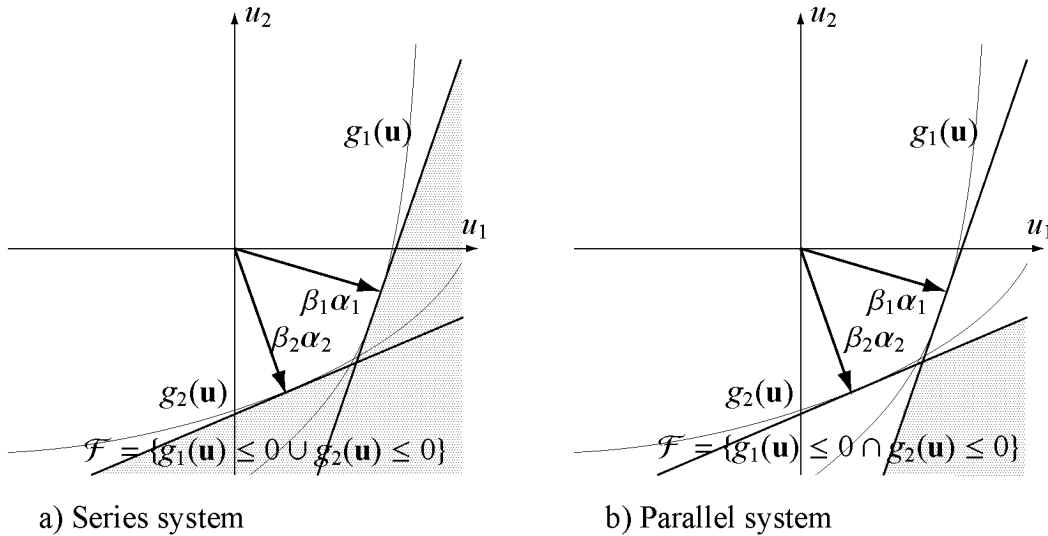


Figure 3.6: System reliability in two dimensional standard normal space with two limit states $g_1(\mathbf{u})$ and $g_2(\mathbf{u})$.

$$P(\mathcal{F}) = 1 - \Phi_n(\boldsymbol{\beta}; \boldsymbol{\rho}) \quad (3.31)$$

It is noted that the multi normal distribution $\Phi_n(\mathbf{u}; \boldsymbol{\rho})$ is in general difficult to solve; but approximations are available in the standard books on reliability theory, e.g. Madsen, Krenk, and Lind (1986), Melchers (1999), Ditlevsen and Madsen (2005), Rackwitz (2004) and Faber (2004).

3.5 Time-Variant Structural Reliability

Engineering problems are typically time-variant. Loads such as live loads, wind and snow vary with time as well as resistances which are subject to deterioration, e.g. corrosion, fatigue, etc. Therefore, the failure probability is a function of time.

$$P(\mathcal{F}(t)) = \int_{\mathcal{F}(t)} f_{\mathbf{X}(t)}(\mathbf{x}(t)) d\mathbf{x}(t) \quad (3.32)$$

Often, e.g. when the time-variant structure of the load can be expressed by an extreme value distribution, the time-variant reliability can be formulated as a time invariant reliability problem. However, not all practical cases can be covered. Generally, the engineer is interested in the probability that in a time period $[0, t]$ the structure will fail. It is denoted by

$$P(T \leq t) = F_1(t) = 1 - R(t). \quad (3.33)$$

T is the random time of a first failure, $F_1(t)$ is the associated cumulative distribution function and $R(t)$ is the reliability function. By differentiation, the probability distribution function of T is obtained.

$$f_1(t) = \frac{d}{dt} F_1(t) \quad (3.34)$$

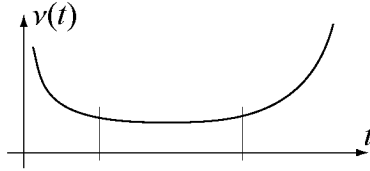


Figure 3.7: A characteristic hazard function $h(t)$.

From classical reliability theory, another reliability measure is known, the hazard function $\nu(t)$. It is defined as the conditional failure probability for the time interval $[t, t + dt]$ given that no failure occurred before t , i.e.

$$\nu(t) = P(t \leq T \leq t + dt | T > t) \quad (3.35)$$

$$= \frac{P(t \leq T \leq t + dt)}{P(T > t)}. \quad (3.36)$$

Therefore, the hazard function is given by the following equation.

$$\nu(t) = \frac{f_1(t)}{1 - F_1(t)} \quad (3.37)$$

A characteristic hazard function is shown in Figure 3.7. Due to birth defects, the hazard function is high at the beginning. Then it decreases and reaches a plateau and failures occur at a constant rate by chance. After that, when the structure reaches its design life or goes beyond, deterioration and aging increase the failure probability, as well as the hazard function. With the use of Equation 3.34, Equation 3.37 can be rewritten as

$$\nu(t) = \frac{F_1'(t)}{1 - F_1(t)} \quad (3.38)$$

$$= -\frac{R'(t)}{R(t)} \quad (3.39)$$

The integration of the last equation yields the reliability function.

$$R(t) = R(0) e^{-\int_0^t \nu(\tau) d\tau} \quad (3.40)$$

Using Equation 3.33, $F_1(t)$ is readily obtained and $f_1(t)$ follows by differentiation.

$$f_1(t) = \nu(t) e^{-\int_0^t \nu(\tau) d\tau} \quad (3.41)$$

It is seen from the above equations that if either $f_1(t)$, $F_1(t)$ or $\nu(t)$ is given, the two other terms may be derived. In the next section it is shown that Equation 3.41 is the probability distribution function of the first passage time of a non-homogeneous Poisson process.

3.5.1 The Poisson Process

Randomness of quantities varying in time and/ or location can be described by stochastic processes. A stochastic process $\{X(t, \mathbf{s}) | t \in T, \mathbf{s} \in \mathbf{S}\}$ is a parametered family of random variables, whereby the parameters t, \mathbf{s} are elements of the indexing sets T, \mathbf{S} . A process only parametered

with the location vector \mathbf{s} is generally referred to as a random field. But such fields can also depend on time. Random processes can be typically distinguished between discrete and continuous processes depending on whether it is a family of discrete or continuous random variables. However, the process may also be discrete or continuous in terms of the index parameters. If the complete probability structure of the process is independent of a shift of the parameter, the process is called stationary or homogeneous. If just the first two moments are independent, the process is called weakly stationary. In addition, it is weakly ergodic, if the mean and autocorrelation function can be assessed by one single realization, and it is strictly ergodic, if this holds for all higher moments, as well.

Many events, e.g. structural failure, take place at discrete points in time. Such events are appropriately modelled by counting processes. The most widely known counting process is the Poisson process with discrete counts and a continuous parameter t . According to Parzen (1962), a stationary Poisson process is defined by the following three characteristics.

1. The events/ arrivals are independent.
2. The probability of an event occurring in $(t, t + \Delta t]$ is asymptotically proportional to Δt .
3. Simultaneous occurrences in $(t, t + \Delta t]$ are negligible for $\Delta t \rightarrow 0$.

If just points 1 and 3 are fulfilled then the Poisson process is non-homogeneous. If, moreover, the simultaneous occurrence of events is not negligible, the counting process may be described by the generalized Poisson process, see Lin (1967).

3.5.1.1 The Differential Equation of the Poisson Process

The event $\{N(t + dt) = n\}$ may take place in two ways. Either at time t , n occurrences were already counted and no further event occurs, or $n - 1$ events occurred until t and an additional event occurs in $(t, t + dt]$. The probability for $\{N(t + dt) = n\}$ is written as

$$P(n, t + dt) = P(n, t)P(0, dt) + P(n - 1, t)P(1, dt). \quad (3.42)$$

If the failure rate is given by $\lambda(t)$, with

$$\lambda(t) = \lim_{dt \rightarrow 0} \frac{P(\text{one event in } (t, t + dt])}{dt}, \quad (3.43)$$

$P(1, dt) = \lambda(t)dt$ is obtained and its complement $P(0, dt) = 1 - \lambda(t)dt$. In this case one obtains

$$P(n, t + dt) = P(n, t)(1 - \lambda(t)dt) + P(n - 1, t)\lambda(t)dt \quad (3.44)$$

and by rearranging

$$\frac{P(n, t + dt) - P(n, t)}{dt} = \lambda(t)(P(n - 1, t) - P(n, t)). \quad (3.45)$$

Taking the limit $dt \rightarrow 0$, the following differential equation is obtained.

$$\frac{d}{dt}P(n, t) + \lambda(t)P(n, t) = \lambda(t)P(n - 1, t) \quad (3.46)$$

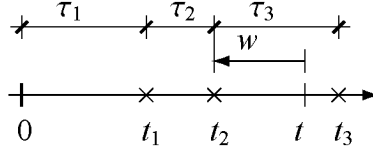


Figure 3.8: Realization of a renewal process with waiting times t_i , inter-arrival times τ_i and backward recurrence time w .

A solution to this equation is given by following integral.

$$P(n, t)e^{\int_0^t \lambda(\tau)d\tau} = \int_0^t \lambda(\tau)P(n-1, \tau)e^{\int_0^{\tau} \lambda(\tau)d\tau} d\tau + C \quad (3.47)$$

With $P(-1, t) = 0$, $P(0, 0) = 1$ and $n = 0$ one obtains

$$P(0, t) = e^{-\int_0^t \lambda(\tau)d\tau}, \quad (3.48)$$

and for the general case

$$P(n, t) = \frac{1}{n!} \left[\int_0^t \lambda(\tau)d\tau \right]^n e^{-\int_0^t \lambda(\tau)d\tau}. \quad (3.49)$$

3.5.1.2 Waiting Time, Renewal Density and Backward Recurrence Time

It is seen that $P(0, t)$ is equivalent to the reliability function $R(t)$. As this is the complement of the first occurrence, Equation 3.48 also defines for the first occurrence – also called first passage time – the cumulative distribution function $F_1(t)$ and its probability density function $f_1(t)$.

$$F_1(t) = 1 - e^{-\int_0^t \lambda(\tau)d\tau} \quad (3.50)$$

$$f_1(t) = \lambda(t)e^{-\int_0^t \lambda(\tau)d\tau} \quad (3.51)$$

The waiting time to the n^{th} arrival can be constructed by the probability that $n-1$ arrivals occurred until t multiplied by the probability that an event occurs in $(t + dt)$. Thus, it is given by

$$f_n(t) = \frac{\lambda(t)}{(n-1)!} \left[\int_0^t \lambda(\tau)d\tau \right]^{n-1} e^{-\int_0^t \lambda(\tau)d\tau}. \quad (3.52)$$

The stationary Poisson process is a special case of a *renewal process*. Generally, a renewal process is a counting process with independent identically distributed inter-arrival times τ_i . The renewal function $H(t)$ is defined as the expected number of occurrences – also called renewals – in $[0, t]$.

$$H(t) = E[N(t)] = \sum_{n=1}^{\infty} nP(n, t). \quad (3.53)$$

This is equal to

$$H(t) = \sum_{n=1}^{\infty} n(F_n(t) - F_{n-1}(t)) = \sum_{n=1}^{\infty} nF_n(t) - \sum_{n=2}^{\infty} (n-1)F_n(t) \quad (3.54)$$

$$= \sum_{n=1}^{\infty} F_n(t). \quad (3.55)$$

By differentiation we obtain the renewal density

$$h(t) = \frac{d}{dt}H(t) = \sum_{n=1}^{\infty} f_n(t) \quad (3.56)$$

which for $dt \rightarrow 0$ is the probability that within the interval $[t, t + dt]$ at least one event occurs.

$$h(t) = \lim_{dt \rightarrow 0} \frac{P(\text{one or more renewals in } [t, t + dt])}{dt} \quad (3.57)$$

With Equations 3.56 and 3.52, the renewal density of a non-homogeneous Poisson process is

$$h(t) = \sum_{n=1}^{\infty} \frac{\lambda(t)}{(n-1)!} \left[\int_0^t \lambda(\tau) d\tau \right]^{n-1} e^{-\int_0^t \lambda(\tau) d\tau}. \quad (3.58)$$

Using the exponential series $\sum_{i=0}^{\infty} \frac{x^i}{i!} = e^x$, it is seen that the renewal density of a non-homogeneous Poisson process is given by

$$h(t) = \lambda(t). \quad (3.59)$$

For another case, when the waiting times are normally distributed with mean $n\mu$ and standard deviation $\sqrt{n}\sigma$ – with $n \in \mathbb{N}^*$ – the renewal density becomes

$$h(t) = \sum_{n=1}^{\infty} \frac{1}{\sigma\sqrt{n}} \varphi\left(\frac{t - n\mu}{\sigma\sqrt{n}}\right). \quad (3.60)$$

If $f_1(t)$ is given, the probability density functions of the remaining waiting times may be determined by the convolution integral

$$f_n(t) = \int_{-\infty}^{\infty} f_{n-1}(\tau) f_1(t - \tau) d\tau. \quad (3.61)$$

An important result of renewal theory is that if $f(t) \rightarrow 0$ for $t \rightarrow \infty$, the asymptotic renewal density is given by the following term, see Cox (1962), where $E[T]$ is the expected value of the first occurrence.

$$\lim_{t \rightarrow \infty} h(t) = 1/E[T] \quad (3.62)$$

Other useful concepts derived from renewal theory are the forward and backward recurrence times. The backward recurrence time w is the time measured backwards from t to the most recent failure event at or before t , see Figure 3.8. The probability density function of the recurrence time $f_w(w)$ is given by the probability that within an infinitesimal time interval at $t - w$ a renewal occurs multiplied with the probability that no further renewal occurs until t .

$$f_w(w) = h(t - w)R(w) \quad (3.63)$$

The derivation for the forward recurrence time follows analogously, see Cox (1962).

3.5.2 First Passage Problem and Outcrossings

In the foregoing it was shown how the time-dependent reliability of structures may be assessed. However, in practical engineering problems this is not known and the probability that in $[0, t]$ a failure occurs $P_{\mathcal{F}}(t)$ can only be approximated, see Rackwitz (2004).

$$P_{\mathcal{F}}(t) = P(\{X(0) \in \mathcal{F}\} \cup \{N(t) > 0\}) \quad (3.64)$$

$$= P(X(0) \in \mathcal{F}) + P(\{X(0) \in \mathcal{S}\} \cap \{N(t) > 0\}) \quad (3.65)$$

$$= P(X(0) \in \mathcal{F}) + P(X(0) \in \mathcal{S})P(N(t) > 0|X(0) \in \mathcal{S}) \quad (3.66)$$

$$= P(X(0) \in \mathcal{F}) + P(X(0) \in \mathcal{S}) \sum_{n=1}^{\infty} P(N(t) = n|X(0) \in \mathcal{S}) \quad (3.67)$$

$$\leq P(X(0) \in \mathcal{F}) + P(X(0) \in \mathcal{S}) \sum_{n=1}^{\infty} nP(N(t) = n|X(0) \in \mathcal{S}) \quad (3.68)$$

$$= P(X(0) \in \mathcal{F}) + P(X(0) \in \mathcal{S})E[N(t)|X(0) \in \mathcal{S}] \quad (3.69)$$

$$\leq P_{\mathcal{F}}(0) + E[N(t)] \quad (3.70)$$

$P(A \cup B) = P(A) + P(\bar{A} \cap B)$ is used to obtain Equation 3.65. In Equation 3.68, the factor n is introduced, so that the sum corresponds to the conditional expected number of occurrences $E[N(t)|X(0) \in \mathcal{S}]$. The inequality sign follows directly from comparison of the sums in Equations 3.67 and 3.68. Finally, $P(A \cap B) \leq \min\{P(A), P(B)\}$ is used to obtain the final approximation for $P_{\mathcal{F}}(t)$. Using the definition of the expected number of occurrences

$$E[N(t)] = \int_0^t \nu^+(\tau) d\tau, \quad (3.71)$$

the failure probability may be written as

$$P_{\mathcal{F}}(t) = P_{\mathcal{F}}(0) + \int_0^t \nu^+(\tau) d\tau. \quad (3.72)$$

Here $\nu^+(t)$ is the mean outcrossing rate with

$$\nu^+(t) = \lim_{dt \rightarrow 0} \frac{1}{dt} P(\text{one event in } (t, t + dt]) \quad (3.73)$$

Another possibility to assess $P_{\mathcal{F}}(t)$ is by obtaining an expression for $f_1(t)$. According to Equation 3.73, $\nu^+(t)$ is the probability that in $(t, t + dt]$ with $dt \rightarrow 0$ a first event occurs, represented by $f_1(t)$, or that an n^{th} event occurs, with $n > 1$. As occurrences are mutually exclusive, the probability can be written as

$$\nu^+(t) = f_1(t) + \int_0^t \kappa(t|\tau) f_1(\tau) d\tau. \quad (3.74)$$

$\kappa(t|\tau)$ is the conditional crossing rate given a previous first excursion at time τ . From Equation 3.74 it is seen that $\nu^+(t)$ is an upper bound for $f_1(t)$ because the integral is non-negative. Generally, $f_1(t)$ may be obtained by solving the integral equation 3.74 with kernel $\kappa(t|\tau)$. A well known solution is obtained if we assume $\kappa(t|\tau) = \nu^+(t)$, namely:

$$F_1(t) = 1 - e^{\int_0^t -\nu^+(\tau) d\tau}. \quad (3.75)$$

In Equation 3.74, substituting $v^+(t)$ by $h(t)$ and $\kappa(t|\tau)$ by $h(t-\tau)$, one obtains the integral equation of the renewal theory, see Cox and Miller (1965).

Generally, the reliability of a structure is conditional on the vector of non-ergodic variables \mathbf{R} and the vector of ergodic sequences \mathbf{Q} . Using Jensen's inequality, the expectation operations $E_{\mathbf{R}}[\cdot]$ and $E_{\mathbf{Q}}[\cdot]$ with respect to \mathbf{R} and \mathbf{Q} can be performed together with the computation as described by Schall, Faber, and Rackwitz (1991).

$$F_1(t) \leq 1 - e^{-E_{\mathbf{R}}[E_{\mathbf{Q}}[\int_0^t v^+(\tau, \mathbf{R}, \mathbf{Q}) d\tau]]} \quad (3.76)$$

That is, if the outcrossing rate can be assessed, the probability of a first passage can be calculated using Equation 3.76. For instance, if the underlying process is differentiable, the outcrossing rate is determined by Rice's formula, see Rice (1944) and Rice (1945).

$$v(t) = \int_{\dot{\xi}}^{\infty} f_{X\dot{X}}(\xi, \dot{x})(\dot{x} - \xi) d\dot{x} \quad (3.77)$$

For other processes, e.g. the Poisson spike process, rectangular square wave process etc., equations for the outcrossing rates are available based on Equation 3.73, see e.g. Melchers (1999).

3.6 Summary

Uncertainty can be divided into aleatoric and epistemic uncertainties. Aleatoric uncertainties are influenced by chance whereas epistemic uncertainty is due to insufficient knowledge. The latter is due to imperfect models that are used to describe the real nature and due to limited observations.

By means of the probability concept, uncertainty may be quantified consistently. There are three possible ways to interpret probability, namely the classical, the frequentistic or the subjective interpretation. Each of them may be used for part solutions of engineering problems. However, these interpretations may be integrated, which is also referred to as Bayesian modeling or Bayesian interpretation of probabilities. A useful tool for Bayesian modeling is Bayes' theorem.

Bayes' theorem facilitates the integration of new information, e.g. of new observations in probabilistic modeling. It considers prior probability and updates it by the likelihood of the made observations in order to obtain the posterior probability. Bayes' theorem is universally applicable in structural engineering, and it was successfully applied, e.g. in proof load tests; moreover it is an indispensable tool for the assessment of existing structures.

Not all quantities are observable or may be observed in quantities permitting the formulation of reliable probabilistic models. However, if a physical relation between probabilistically modelled quantities is provided, a structural reliability analysis (SRA) may be performed to assess the probabilities of given events or the corresponding reliability index. First order and second order reliability methods (FORM/ SORM) are two efficient SRA methods, among others, e.g. simulations. They account for non-normally and dependent variables and also for system effects.

If the phenomenon of interest is of a time-variant nature, it generally must be considered by time-variant SRA; although some special cases may be formulated in terms of time-invariant problems. Time-invariant SRA requires knowledge of stochastic processes; therefore an introduction to the important Poisson process is given. For this counting process, waiting times,

3 Uncertainty and Probability

inter-arrival times as well as backward recurrence times and the renewal density are discussed, and they are formulated for the non-homogeneous Poisson process. Finally, a brief outline on outcrossing rates is provided.

Part II

Consequences and Preferences

4 Consequence Modeling

Engineering decision-making requires uncertainty analysis and the proper modeling of consequences. Besides the likelihood of an event, also its extent is subject to uncertainty so that consequence modeling involves all considerations outlined in the previous chapter.

First, the present chapter categorizes consequences into different types. Then a systematic framework to assess consequences is introduced and it is illustrated for the case of business interruption losses. Moreover, follow-up consequences are discussed and it is shown how decisive they may be in engineering decision-making. Are follow-up consequences and business interruption losses societal consequences? And should engineering decision-making lead to an optimization of the total consequence to society? These questions are then discussed. The chapter closes with an overview of the consequences regarding the failure of the twin towers of the World Trade Center (WTC).

4.1 Categorization of Consequences

Structural failures may lead to manifold consequences. To assess consequences systematically, it is helpful to categorize consequences into different types. It is helpful to select the categories so that they are mutually exclusive. This allows obtaining the total consequence by simply adding up the different types of consequences.

At first, consequences are generally differentiated whether they constitute consequences to *humans*, the *environment*, the *economy* or *cultural assets*.

In addition to this scheme they can also be categorized whether they are substitutable or not, i.e. they are either *material* or *immaterial* consequences. If for instance consequences with regard to fatalities are considered, i.e. consequences to humans, it is seen that on the one hand the lost income of dependants can be substituted. But on the other hand the compensation for pain and suffering is immaterial.

Whereas a common basis exists for the assessment of the lost income of dependants, see DOJ (2004) and Faber, Kübler, Fontana, and Knobloch (2004), the compensation for compensation payments with regard to pain and suffering are assessed differently in different countries. Generally, in Europe this amount is rather low. For instance, 7,500 pounds (approx. 14,000 USD) is paid in Great Britain and 30,000 CHF (approx. 24,000 USD) in Switzerland. In Germany, the maximum amount which can be claimed is 5,115 € (approx. 6,300 USD). Willingmann (2002) explains why these compensation payments are so low and recalls the intention of the creators of the Bürgerliches Gesetzbuch (BGB), the German civil code. Willingmann quotes that "from monetary compensation for immaterial losses, 'only the bad elements of society would benefit; selfishness and greed would increase, and numerous legal processes would result from dishonest motives.'"¹

¹Translated by the author.

In addition to the lost income of dependants and the compensation for pain and suffering, the societal willingness to invest in safety measures needs to be taken into account in engineering decision-making. The societal willingness to invest in safety is an immaterial consequence, as well. It is introduced in Section 6.1.

Furthermore, consequences can be differentiated into *direct* and *indirect* consequences. As suggested by the name, direct consequences are considered to be directly related to the considered structure and hazard, whereas indirect consequences occur as a result of direct consequences. They are also denoted as *follow-up* consequences. It is obvious that this frequently used differentiation is subject to a subjective assessment of what is interpreted as a direct or indirect consequence. Figure 4.1 illustrates the possible categorization of consequences and highlights as an example immaterial and indirect consequences to humans.

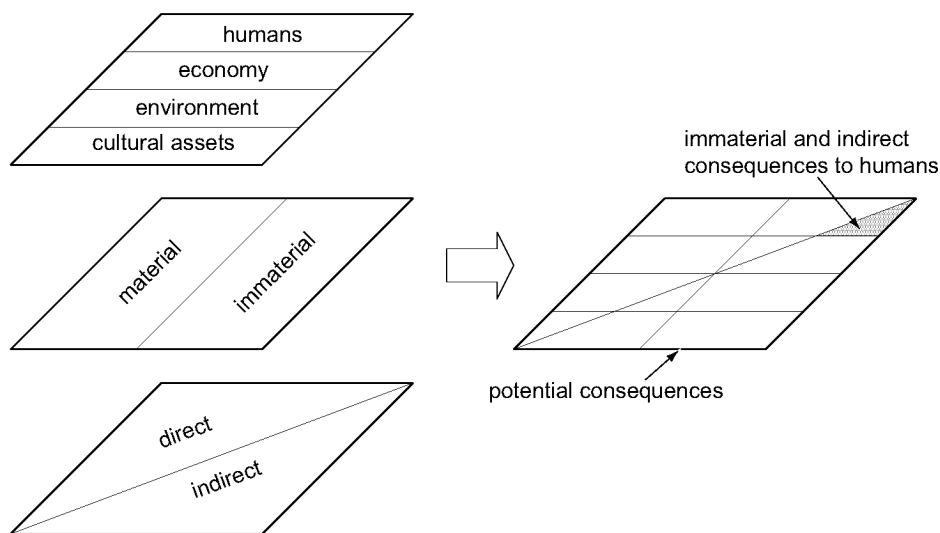


Figure 4.1: Categorization of consequences.

It can also be sought of other differentiations, e.g. whether these consequences are short, medium or long-term consequences etc. To identify such additional categories and also to identify relevant consequences, *hazard pointers* may be used, see Faber (2004).

4.2 Framework to Assess Consequences

Consequences occur as an amount of lost, damaged or destroyed assets. Examples of this are e.g. amount of destroyed or damaged office space, roads, water supply systems, sewage systems, etc. These amounts also indicate the intensity of the consequences. When assessing risks related to civil engineering structures, the amount of lost, damaged or destroyed assets due to an adverse event can be known precisely, with a negligible variability or is uncertain. For instance when a structure fails completely, the destroyed gross floor area of that building is known precisely. But if it fails only partially, it is generally subject to uncertainty. The amount of lost, damaged or destroyed assets can be summarized in the random vector $\mathbf{a} = (a_1, a_2, \dots, a_{n_a})^T$, where n_a is the number of considered assets. a_j may also be described by means of a damage factor d_j . This approach is especially useful, when an inventory list is already available. Then a damage

factor d_j may be associated with each element ε_j of the inventory list represented by the vector $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_{n_a})^T$. The damage factor is larger or equal to zero and smaller or equal to one. Then a_j is obtained by $a_j = d_j \varepsilon_j$ or when \mathbf{D} is a diagonal matrix with diagonal elements d_j , by $\mathbf{a} = \mathbf{D}\boldsymbol{\varepsilon}$. For specific hazards it is possible to interrelate the damage factor with hazard-specific measures, see e.g. Voortman (2003), HAZUS (National Institute of Building Science (1999)) or Bayraktarli, Ulfkjaer, Yazgan, and Faber (2005).

All consequences should be expressed in the same units, generally monetary units. This is easy to achieve for all material losses that are substitutable. Consider a consequence type such as clean-up costs, property losses etc. and let $c_{i,j}$ be the unit costs associated with a_j . In this case, $\mathbf{c}_i^T \mathbf{a}$ expresses the corresponding monetary value of the consequence. Uncertainty may be related to the unit costs, as well. Then, $\mathbf{c}_i = (c_{i,1}, c_{i,2}, \dots, c_{i,n_a})^T$ is a random vector. However, it may be sufficiently accurate to use expected values or necessary due to lack of information. In order to account for the variation of costs with respect to time, the function $\varphi_i(t)$ is introduced. It considers variation of costs with time. $\varphi_i(t)$ is subject to uncertainty as well and may be described – if appropriate – by a random process together with a specific correlation function. Finally, $C_i(t)$ reflects the monetary consequences of type i of an incident occurring at time t , e.g. structural failure, and can be expressed as follows.

$$C_i(t) = m_i \varphi_i(t) \mathbf{c}_i^T \mathbf{a} \quad (4.1)$$

$$= m_i \varphi_i(t) \mathbf{c}_i^T \mathbf{D} \boldsymbol{\varepsilon} \quad (4.2)$$

Here, m_i considers the multiplier effect, which models a reduced consumption of the affected parts of society (persons, enterprises and organizations). An introduction to the multiplier model is given in Annex A. Finally, the total consequence due to an incident at time t are obtained by summing up all mutually exclusive types of consequences.

$$C(t) = \sum_i C_i(t) \quad (4.3)$$

$C(t)$ is a function of random variables and needs therefore to be treated as such. Generally, in a risk analysis all consequences which are relevant and meaningful for the underlying decision-making have to be identified. If the relevance of a consequence type cannot a priori be estimated, it should be considered in a preliminary analysis. A sensitivity study will then reveal its relevance by showing its influence on the identified optimal decision.

Equations 4.1 to 4.3 provide a basis to assess any type of consequence. In the following subsection it is illustrated how business interruption may be assessed.

4.2.1 Business Interruption

Business losses may result from adverse events. Enterprises may be forced to temporarily cease their activities to provide access for rescue teams or due to the danger of building collapse. In addition, if a structure fails, enterprises may be affected so that they lose the basis for doing their business (e.g. loss of crucial data). However, the latter type of consequence is not considered in the following.

The failure of the World Trade Center showed that failure of civil engineering structures can lead to consequences that reach beyond the experience that civil engineers made in the past. The extent of the consequences associated with the WTC failure made it necessary to use concepts

4 Consequence Modeling

known from macroeconomics in order to assess business interruption losses. The economic loss due to business interruption can be assessed by calculating the lost gross domestic product (GDP) or more precisely the lost value added. In Annex A it is argued that the GDP and also the value added should be adjusted for depreciation such that instead of the GDP, the net domestic product is used. Using the adjusted value added the benefit generated by the enterprises, the income of employees, taxes, rents and capital costs are accounted for. If the data is available, the damage assessment is straightforward. Another big advantage of modeling consequences by means of the value added is that the assessed damage is independent of the tax, social and juridical system of the considered geographical region. This approach permits one to assess damages without indicating who bears to what extent the consequences. If however the damage distribution among the members of society is known, then the latter can also be calculated. But generally, for the structural design this is not required because all consequences need to be considered, see Section 4.3.1.

The economic loss due to business interruption due to an event occurring at time t is given in Equation 4.4. The assessment process is also illustrated in Figure 4.2.

$$C_B(t) = \int_t^{\infty} \iint_{\Omega} m_B(\mathbf{x}) \varphi_B(\tau) d_B(\mathbf{x}, \tau) (p_B(\mathbf{x}) + \Delta p_B(\mathbf{x})) d\mathbf{x} d\tau. \quad (4.4)$$

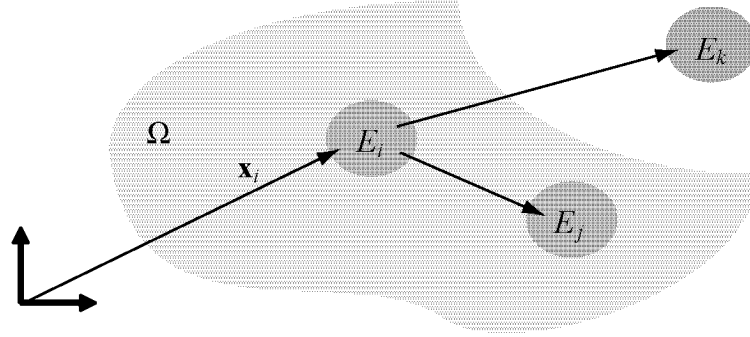


Figure 4.2: Assessment of losses due to business interruption.

To calculate the economic loss one has to consider all enterprises and facilities generating added value. To do this, Equation 4.4 integrates over the affected geographical region Ω . Business interruption losses involve temporary cessation, which may differ from enterprise to enterprise. This is accounted for in Equation 4.4 by the integration over time. The first term of the integrand $m_B(\mathbf{x})$ is the factor accounting for the multiplier effect and $\varphi_B(\tau)$ considers the time variation of the value added. The third term $d_B(\mathbf{x}, \tau)$ is the damage factor. It describes the damage according to the spatial and temporal distribution. $d_B(\mathbf{x}, \tau)$ is smaller or equal to one and larger or equal to zero. $d_B(\mathbf{x}, \tau) = 0$ represents no damage and $d_B(\mathbf{x}, \tau) = 1$ is equivalent to complete damage. For any adverse event that can be reasonably assumed, $d_B(\mathbf{x}, \tau)$ approaches zero as τ goes to infinity.

$p_B(\mathbf{x})$ is the productivity, which is the adjusted value added per area. In general, the productivity can be calculated for each production factor, namely land, labor and capital. Thereby for instance, the value added per employee or facility is obtained. Figure 4.2 illustrates that $p_B(\mathbf{x})$ accounts for the value added produced by the entity E_i per time unit at the location \mathbf{x} . The

factor $\Delta p_B(\mathbf{x})$ considers the interrelation of E_i with the surrounding economy. But $\Delta p_B(\mathbf{x})$ only accounts for the lost value added of interrelated entities outside of Ω such as E_k . The damage of the interrelated enterprises and facilities lying inside of Ω such as E_j are already considered in the integration. This means that if E_j depends on E_i and if E_i is affected for a longer period than E_j , then this interrelation is to be included in $d_B(\mathbf{x}, \tau)$. Hence, $d_B(\mathbf{x}, \tau)$ may also account for follow-up consequences in the same way as $m_B(\mathbf{x})$ and $\Delta p_B(\mathbf{x})$ do.

4.2.2 Follow-Up Consequences

Follow-up consequences often give rise to risk averse behavior of decision makers. Often decision makers do not feel comfortable with Equation 2.6 because follow-up consequences are not explicitly accounted for. The decision maker may also be uncertain about the assessment of the occurrence probabilities $P(\theta_j|a_i)$ in Equation 2.6. In principle misjudging an outcome's utility and probability of occurrence may both lead to over- and underestimation of the expected utility and give rise to risk averse behavior.

Faber and Maes (2003) reformulate Equation 2.6 to explicitly account for the utility of follow-up consequences and also epistemic uncertainty.

$$E[u(a_i)] = E_{\mathbf{e}} \left[\sum_{i=1}^{n_{\theta}} u(\theta_i, a_i) P(\theta_i|a_i, \mathbf{e}) + \sum_{j=1}^{m_{\theta}} u_{fu}(\theta_j, a_i) P(\theta_j|a_i, \mathbf{e}) \right] \quad (4.5)$$

Here, n_{θ} is the number of possible discrete outcomes associated with alternative a_i , $P(\theta_i|a_i, \mathbf{e})$ is the probability that the outcome θ_i occurs and $u(\theta_i, a_i)$ is the associated utility. Both are conditioned by the decision alternative a_i . The probability is also conditioned by the epistemic uncertainty \mathbf{e} . The additional term in Equation 4.5 considers *follow-up* consequences. In this term m_{θ} is the number of different combinations θ_j of one or more of the θ_i and $P(\theta_j|a_i, \mathbf{e})$ is the probability that this combination occurs. Finally, $u_{fu}(\theta_j, a_i, \mathbf{e})$ is the utility considering the follow-up consequences. For more on the interpretation of Equation 4.5, the reader is referred to Faber and Maes (2003).

4.3 Total Consequence to Society and Liability

The total consequence to society needs to be addressed in order to model the multiplier effect appropriately. But also with regard to the design of civil engineering structures or the assessment of risks associated with such facilities, it is often argued that the total consequence to society have to be considered. Using concepts of macroeconomics, it is seen that adverse events do not lead to costs and losses only. There are also positive effects. For instance, when a natural hazard, e.g. an avalanche or flood, affects a specific region, then clean-up and repair actions take place and destroyed buildings are reconstructed. Among others the construction sector will benefit yielding higher profits and possibly employ more workers. Considering also positive effects makes the total consequence to society smaller than the amount that only considers losses. Sometimes it is zero or in the extreme case even positive. In these cases, the positive effects are equal to or larger than the negative consequences. The following example represents such a case. In February 1993, a bomb attack was carried out in the basement of the World Trade Center complex. After the incident, the complex owner, the Port Authority of New York and New

Jersey, estimated that the incident generated jobs and economic activity, which would result in a net gain of 200 million USD, see Darton (1999). But if positive effects are taken into account, then it is often neglected to subtract the effect of planned renewals, because these planned renewals are difficult to assess. For instance, consider a structure that fails the day before it is decommissioned. In this case, the positive effect from the failure is small.

Another source of positive effects results from the fact that reconstructed facilities generally show improvements, when compared to the structures replaced. This may allow for a higher productivity due to technological improvements or a better design concept. For instance, after the second terrorist attack on the WTC in 2001, the City of New York took advantage to redesign and improve the transportation infrastructure in Lower Manhattan. However, such improvements are difficult to assess within the framework of engineering decision-making.

Regarding the preceding outline, it follows that engineering decision-making does not aim to optimize the total consequence to society because the positive effects and the effects of planned renewals are difficult to assess. Secondly, the total consequence to society can be small, zero or even positive. If it is zero, the objective function is indifferent to the structural design and any design is a valuable one. If the total consequence is positive, then the least reliable structure is identified as being optimal which contradicts engineering experience. Therefore, another principle is investigated.

4.3.1 Liability

Even if the total consequence to society resulting from an adverse event is zero, a part of society is affected negatively. In that case the negatively affected persons' right to be safeguarded was not considered. The right to be safeguarded follows from the *principle of restricted liberty*, see Zandvoort (2004). The principle says that: everyone is free to do what he or she pleases to do, unless no other person is harmed. Violating this principle the *principle of strict liability* forces actors unconditionally to compensate the part of society affected negatively by their actions. The implication of these principles on the legal systems and decision-making are further discussed in Zandvoort (2004).

Without liability the decision maker may only take into account the consequences directly related to his or her utility and neglect follow-up consequences imposed on others. Considering strict liability the decision maker must take into account the negative effects of his or her actions on others. For the design, construction, operation and decommissioning of civil engineering facilities, this means that owners and operators of such structures are responsible for their activities and have to consider in their decision-making all potential consequences imposed from their activities on others.

4.4 The Failure of the WTC Twin Towers

The Twin Towers of the WTC in Lower Manhattan were hit by airplanes on September 11, 2001. The high-rise buildings, 415 and 417 m high, were capable of bearing the loads after being damaged and showed a robust performance so that most occupants were able to leave the building, NIST (2005). However, they were not designed to resist in a damaged state such fires as were ignited by the fuel of the airplanes. FEMA (2002) and NIST (2005) provide a detailed analysis of the structures that were destroyed as a result of this terrorist attack. The

resulting consequences are studied in Faber et al. (2004) where a thorough list of consequences is summarized. Only a brief overview is given here. It is noted that consequences that only resulted because of the specific exposure, i.e. the act of terrorism, are not considered in this summary. For instance, as a result of this, the reaction of the stock exchanges are not included.

Figure 4.3 illustrates an overview of relevant consequence types for failures of high-rise buildings. Although extensive, it is not exhaustive. At first, rescue and clean-up costs are listed besides the loss of the WTC towers themselves and the damage to the surrounding buildings and facilities. In addition, the damaged or destroyed inventory which was contained in the buildings and facilities is accounted for. Furthermore, the impact on the economy is accounted for in terms of business interruption and lost rents. Moreover, lost cultural assets form a material loss, which can be evaluated, but they also constitute a cultural loss that is difficult to quantify. In addition to this, environmental consequences may have effects on plants, animals and not least humans. Finally, the consequences due to fatalities are considered.

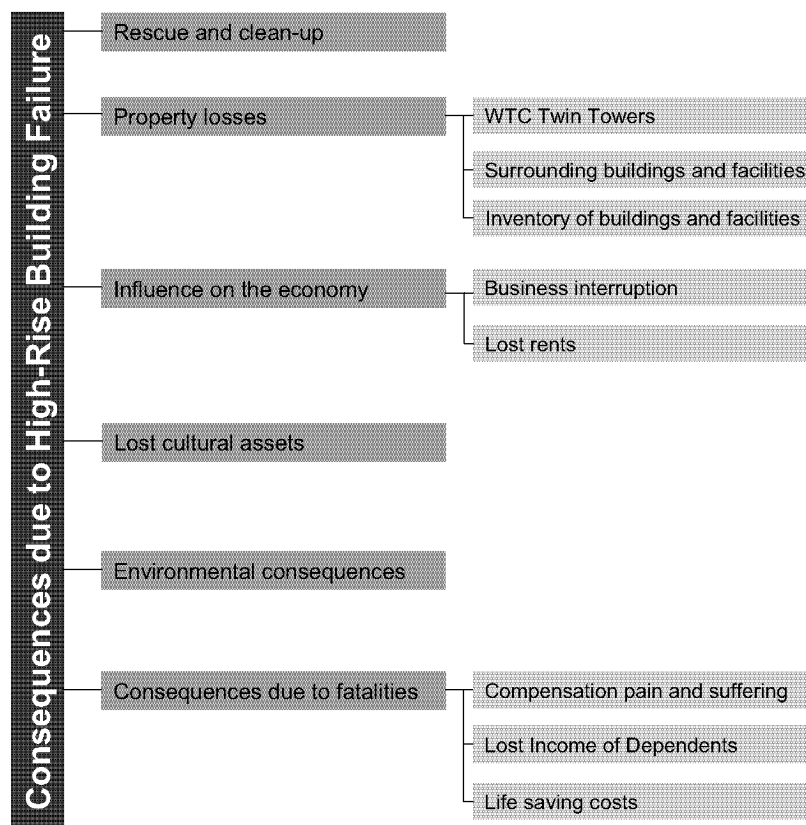


Figure 4.3: Consequences due to high-rise building failure.

The World Trade Center towers WTC1 and WTC2 were built in the late 60's and early 70's. At that time the towers cost 900 million USD. On September 11, 2001, both towers collapsed after a terrorist attack. The succeeding rescue efforts and clean-up were finished earlier and were with 1.7 billion USD² cheaper than expected. Today, a reconstruction of the towers would cost 4.7 billion USD and the replacement costs of other destroyed buildings are estimated to

²According to the 'échelle courte' convention: 1 billion = 10⁹.

| Type of Consequence | | Low | High |
|-----------------------------------|-----|-------------|-------------|
| Rescue and clean-up | | 1.7 | 1.7 |
| Property | | 19.2 | 19.2 |
| <i>WTC Twin Towers</i> | 4.7 | | |
| <i>Other destroyed buildings</i> | 2.0 | | |
| <i>Damaged buildings</i> | 4.3 | | |
| <i>Inventory</i> | 5.2 | | |
| <i>Infrastructure</i> | 3.0 | | |
| Persons | | 10.0 | 10.0 |
| <i>Fatality</i> | 8.5 | | |
| <i>Injured</i> | 1.5 | | |
| Environmental and cultural assets | | 0.1 | 0.1 |
| Impact to economy | | 9.1 | 66.2 |
| <i>Business</i> | 7.2 | 64.3 | |
| <i>Infrastructure</i> | 0.7 | 0.7 | |
| <i>Rents</i> | 1.2 | 1.2 | |
| Total | | 40.1 | 97.2 |

Table 4.1: Summary of consequences due to the WTC failure (in billion USD).

2.0 billion USD. Repair of the damaged buildings is expected to cost 4.3 billion USD while the inventory stored within these buildings is estimated to be 5.2 billion USD. The damage to the surrounding infrastructure is estimated to be 3.0 billion USD. This includes damage to the PATH system, the MTA subway as well as the damage to power, gas, steam and telecommunication facilities.

Consequences due to fatalities are often seen as the most difficult to assess. However, on the basis of the life quality index the societal willingness for safety investments can be derived as outlined in Chapter 6. Moreover, the compensation of the dependants of the fatalities may also depend on the societal and legal system of the considered country. In case of the WTC Twin Towers failures, the Victim Compensation Fund compensated the dependants of the fatalities. Based on the compensation scheme an average compensation of 2.9 million USD per fatality is obtained summarizing to a total cost of compensation of 8.5 billion USD. Additionally, injured people were compensated with 1.5 billion USD. These numbers include offsets³ and the persons involved in the Pentagon assault, see DOJ (2004).

Due to the considerable reconstruction period, the failure of the World Trade Center will imply an estimated loss in rents of 1.2 billion USD. Business interruption losses are estimated as low as 7.2 billion USD and range up to 64.3 billion USD. The difference arises from uncertainties, which are inherent to the modeling of the economic development. Also very influential is the expert judgment on how much of the economic loss may be allocated to the events of September 11 and how much to the economic recession that already started in December 2000. Finally, the analyzed reports, which assess the impact on the economy, vary in the period for which they estimate the economic impact. Economic losses associated with the infrastructural facilities

³Offsets are payments that claimants have received from other sources due to the event of September 11, 2001. For instance, offsets are payments from life insurances, pension funds, etc. In order to calculate the compensation payments, offsets are subtracted from the determined loss, see DOJ (2004).

are estimated to 0.7 billion USD. 20 million USD are committed to establish a health registry, which will follow up the health condition of up to 200,000 people due to environmental impacts. However, the 20 million USD will not cover future compensation payments to these people. For instance, more than one thousand firemen filed lawsuits against the City of New York with a total claim corresponding to 12 billion USD. Moreover, artworks and cultural assets were lost. The art works situated within the WTC were insured to a value of 100 million USD. However, this does not account for the non-reproducible loss of masterpieces of Miró, Rodin, Picasso, as well as the cultural assets, which were stored in the Greek Orthodox St. Nicholas church.

Given the listed consequences and the studies regarding the consequences to the economy the total consequence associated with the failure of the World Trade Center ranges between 40.1 to 97.2 billion USD, which corresponds to 8.5 to 20.7 times the reconstruction costs of the Twin Towers. It is seen that if only the property loss directly related to the WTC Twin Towers is considered, then a major part of the consequences is not considered. Neglecting these follow-up consequences may give rise to risk aversion as described in Faber et al. (2004).

4.5 Summary

Categorization of consequences helps to assess consequences systematically. Generally, they are classified into consequences to humans, the environment, the economy and cultural assets. Moreover, consequences are also differentiated whether they are substitutable i.e. material or immaterial consequences. Examples of immaterial losses are the compensation for pain and suffering or the willingness to pay to avoid a loss of reputation. Furthermore, consequences are classified into direct and indirect consequences. As indicated, direct consequences are considered to be directly related to the considered structure and hazard, whereas indirect consequences occur as a result of direct consequences. They are also denoted as follow-up consequences. It is obvious that this frequently used differentiation is subject to a subjective consideration of what is interpreted as being directly or indirectly related. In the process of consequence assessment, it is helpful to categorize consequences into different types that are mutually exclusive. This allows obtaining the total consequence by simply adding up the different types of consequences.

In addition, a framework is introduced to support consequence assessment. It is illustrated by means of business interruption losses that are assessed making use of the gross domestic product or the value added. Also the consequences resulting from the multiplier effect are accounted for. Such consequences are follow-up consequences and may give rise to a risk averse behavior. Faber and Maes (2003) therefore introduced a formulation of expected utility that explicitly accounts for any type of follow-up or indirect consequences.

It is also discussed whether the objective of structural design should be the optimization of the total consequence to society. It is argued that engineering decision-making does not aim to optimize the total consequence to society. This is because positive effects and the effects of planned renewals are difficult to assess and secondly the total effect to society can be small, zero or even positive. If it is zero, the objective function is indifferent to the structural design and any design is a valuable one. This contradicts engineering experience. Moreover, it is shown that liability principles form a basis for engineering decision-making. Decision makers need to take responsibility with regard to the direct and indirect consequences resulting from their actions. Therefore, actors have to take into account within their decisions the consequences of others and in case of occurrence, to compensate them.

4 Consequence Modeling

Finally, an overview of the consequences associated with the structural failure of the World Trade Center Twin Towers is given. It is clearly seen that indirect or follow-up consequences may constitute a major part of the consequences. It follows directly from the application of decision theory that their inclusion is significant to derive acceptable safety levels, see also Faber et al. (2004).

5 The Life Quality Index

Not least in civil engineering, engineering decision-making may involve consequences due to fatalities. As discussed in Chapter 4 these consequences comprise on the one hand, the lost income of dependants which is a material loss, but on the other hand it involves immaterial losses such as compensation of dependants for pain and suffering as well as the willingness to pay for safety.

In order to account for the willingness to pay for safety, Nathwani et al. (1997) developed the compound social index, the life quality index. From this measure, the willingness to invest in safety can be derived and taken into account in engineering decision-making.

Lind (1994) introduces the methodology to assess risks which also involve risks to persons. Lind defines a quantitative safety criteria derived from the compound social indicator namely the life product index (LPI). On that basis Nathwani et al. (1997) set up a new compound social indicator, the life quality index (LQI). This index is composed of three social indicators, namely the gross domestic product per capita g , the life expectancy l and the fraction of life spent to earn a living w . From the LQI, acceptable life saving costs can be derived as discussed in Chapter 6.

The chapter first introduces the classical derivation of the LQI and the LQI acceptance criteria. Thereafter, the correlation between two social indicators of the LQI is studied. Finally, the LQI is revisited in the light of microeconomic consumption theory, which is able to give the observed correlation a distinct meaning.

5.1 Its Classical Derivation

Lind (1994) introduces a compound social indicator L , which is a function of several social indicators a, b , etc.

$$L = L(a, b, \dots, g, \dots, l, \dots) \quad (5.1)$$

L may be a function of an arbitrary number of indicators, but in the following only the two important indicators are considered, namely the gross domestic product per capita g and the life expectancy l , since structural engineering may influence both. At a later point w , the proportion of time spent at work, is added.

If L is differentiable, the total differential dL is given as

$$dL = \frac{\partial L}{\partial g} dg + \frac{\partial L}{\partial l} dl. \quad (5.2)$$

It is seen that dL vanishes, i.e. $dL = 0$, if

$$\frac{dl}{dg} = -\frac{\frac{\partial L}{\partial g}}{\frac{\partial L}{\partial l}}. \quad (5.3)$$

This Equation 5.3 shows the fundamental principle behind the LQI:

Life time is exchangeable with wealth and vice versa.

To obtain the LQI, Nathwani et al. (1997) formulate L as the product of two functions $f_g(g)$ and $f_r(r)$. Whereas, $f_g(g)$ represents the intensity or quality of life and $f_r(r)$ represents the duration or quantity of life. Hereby, $r = (1 - w)l$ is the leisure time not spent at work, with w as the proportion of life time spent at work. Therefore, L is written as

$$L = f_g(g)f_r(r) \quad (5.4)$$

and the relative differential as follows

$$\frac{dL}{L} = \left(\frac{g}{f_g(g)} \frac{df_g(g)}{dg} \right) \frac{dg}{g} + \left(\frac{r}{f_r(r)} \frac{df_r(r)}{dr} \right) \frac{dr}{r} \quad (5.5)$$

$$= k_g \frac{dg}{g} + k_r \frac{dr}{r} \quad (5.6)$$

with the elasticities k_g and k_r given by

$$k_g = \frac{g}{f_g(g)} \frac{df_g(g)}{dg} \quad (5.7)$$

$$k_r = \frac{r}{f_r(r)} \frac{df_r(r)}{dr}. \quad (5.8)$$

The next step in the LQI derivation is also called the *universality requirement*, see Rackwitz (2005). It is assumed that the ratio k_g/k_r is constant. This reflects that a relative change in g is proportional to the relative change in r , independent of the actual values of g and l . This assumption is fulfilled if k_g and k_r are constants so that two simple first order differential equations are obtained. They can be readily solved and one obtains: $f_g(g) = g^{c_g}$ and $f_r(r) = r^{c_r}$ with c_g and c_r as constants. Thus L can be written as

$$L = g^{c_g} r^{c_r} \quad (5.9)$$

$$= g^{c_g} ([1 - w]l)^{c_r}. \quad (5.10)$$

Considering only a single year, g is assumed to be proportional to w , i.e. $g = pw$ with p as a constant productivity.

$$L = (pw)^{c_g} ([1 - w]l)^{c_r}. \quad (5.11)$$

Considering L as an utility function, the optimum is obtained by setting dL/dw equal to zero.

$$\frac{dL}{dw} = 0 \quad (5.12)$$

From this assumption one obtains a relation between c_g and c_r .

$$c_g = c_r \frac{w}{1 - w} \quad (5.13)$$

Originally, Nathwani et al. (1997) assumed that g is proportional to wl . Nonetheless, both approaches lead to Equation 5.13. Pandey (2005) shows that also a nonlinear relation between g and w can be considered to derive an LQI formulation, see Section 5.1.3.

It can be shown that with $c_g + c_r = k_c$, where $k_c \in \mathbb{R}_+$, a set of utility functions is determined. Hereby, any utility function of this set can be obtained from a monotonic transformation of any other utility function of the set. Due to this characteristic, the utility functions exhibit the same

indifference curves. Therefore, the LQI criterion, which will be derived later, is invariant to the actual value of k_c , see also Annex B. Using $c_g + c_r = 1$ one finally obtains

$$c_q = w \tag{5.14}$$

$$c_r = 1 - w \tag{5.15}$$

and the original life quality index results to

$$L_o = g^w l^{1-w} (1 - w)^{1-w}. \tag{5.16}$$

In Figure 5.1, the factor $(1 - w)^{1-w}$ is shown, whereby observed values for w lie between 0.05 and 0.15, see Rackwitz (2005). In developing countries w may increase to 0.20. Most values cluster around the mean of 0.105. This means, the factor $(1 - w)^{1-w}$ varies from 0.95 to 0.87 for developed countries (less than 10%). Due to this, it is generally considered as a good approximation to omit the factor $(1 - w)^{1-w}$. In doing so, the classical form of the LQI is obtained indicated by the index c .

$$L_c = g^w l^{1-w} \tag{5.17}$$

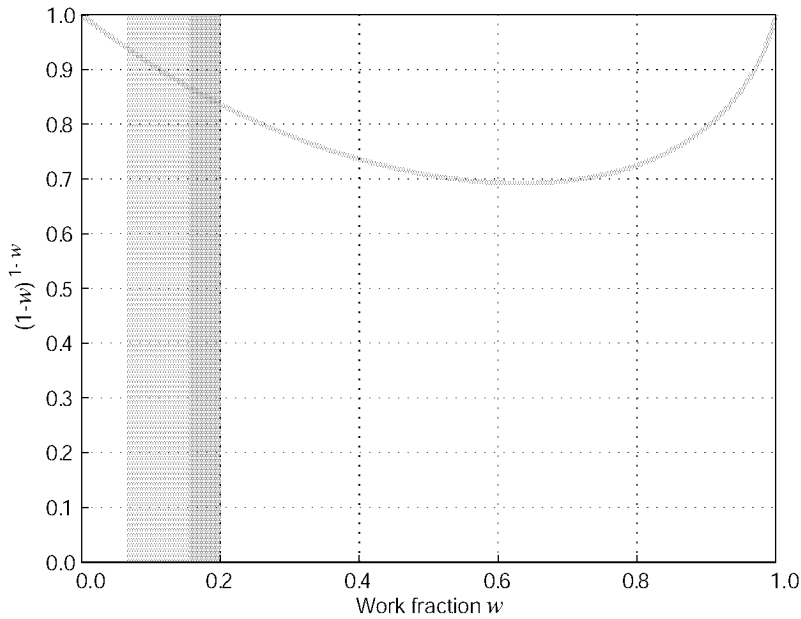


Figure 5.1: The factor $(1 - w)^{1-w}$ as a function of w .

5.1.1 The Reformulations L_q and L_R

For $g = 20,000$ and $l = 75$ Figure 5.2 plots L_c over w . It is seen that increasing w yields a higher life quality. Rackwitz (2003b) considers it as an inconsistency and proposes the following modified formulation. First, the $(1 - w)^{1-w}$ root of L_c is taken, which is a monotonically increasing transformation to obtain $L_q = g^{w/(1-w)} l$. In Figure 5.2 it is seen that the preference ordering is thereby maintained by the obtained utility function L_q . Secondly, the term is divided by

$$q = w/(1 - w), \tag{5.18}$$

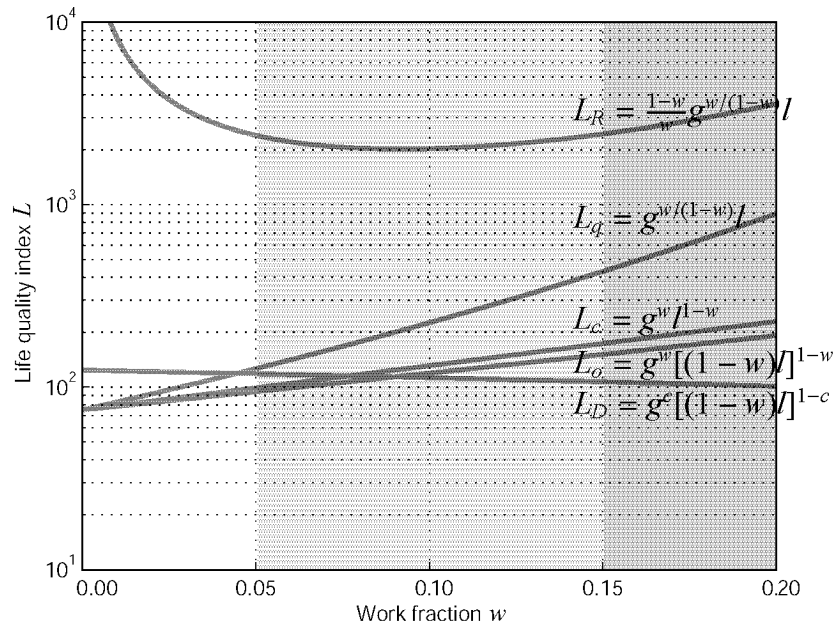


Figure 5.2: Comparison of different LQI formulation L_c , L_o , L_q , L_R and L_D .

which is a monotonically decreasing transformation. Therewith the preference ordering is modified. In doing so, values of $w < 0.10$ are preferred to $w = 0.10$, whereas at the same time the preference ordering for $w > 0.1$ is maintained. However, considering the small variation of w for a developed society the LQI values do not differ much. Therefore, the LQI is reformulated to

$$L_R = \frac{g^q}{q} l. \tag{5.19}$$

It is noted that $u(x) = x^q/q$ is a utility function with a constant relative risk aversion that is frequently used in economics, see also Chapter 7.

5.1.2 The Reformulation L_D

Ditlevsen and Friis-Hansen (2005) state that for the integration of the LQI from its relative differential formulation, the factors c_g and c_r need to be constant and cannot vary with w . With c as a constant they reformulate the LQI as

$$L_D = g^c [(1-w)l]^{1-c}. \tag{5.20}$$

Figure 5.2 also shows L_D for $c = 0.09$ ¹. It is seen that this formulation has the characteristic that with increasing w (less leisure) the life quality is decreasing for the same values of $g = 20,000$ and $l = 75$. This characteristic is thoroughly discussed in Section 5.5.

¹ $c \approx 0.09$ is obtained from data representing Denmark, see Figure 5.22

5.1.3 LQI Formulation on the Basis of the Cobb-Douglas Production Function

From empirical data Cobb and Douglas derived a production function for the GDP , which is a function of labor L , capital K and the technology A .

$$GDP = AK^{1-\beta}L^\beta \quad (5.21)$$

The factor β is a constant, which for developed countries is determined to $\beta = 2/3$, see Pandey (2005). Rackwitz (2005) finds a similar value, namely $\beta = 0.7$.

With the labor input equal to w times the population size N , the GDP is given by

$$GDP = AK^{1-\beta}(wN)^\beta \quad (5.22)$$

and the GDP per capita g by

$$g = GDP/N = A(K/N)^{1-\beta}w^\beta, \quad (5.23)$$

whereby $A(K/N)^\alpha$ is substituted by the constant k to obtain

$$L_P = (kw^\beta)^q(1-w)l. \quad (5.24)$$

Finally, the labor leisure optimization $\frac{dL_P}{dw} = 0$ leads to

$$q = \frac{1}{\beta} \frac{w}{1-w}. \quad (5.25)$$

Considering these different LQI formulations, it can be said that independent of the utilized formulation, the indifference curve in the $g-l$ plane are identical, if the exponents are chosen accordingly.

5.2 The LQI Acceptance Criterion

It was already stated that the fundamental idea behind the LQI concept is that life time and wealth are exchangeable. This idea is expressed when the relative differential of the LQI is formulated.

$$\frac{dL}{L} = \left(\frac{\partial L}{\partial g} \frac{g}{L} \right) \frac{dg}{g} + \left(\frac{\partial L}{\partial l} \frac{l}{L} \right) \frac{dl}{l} \quad (5.26)$$

With $L_o = g^w[(1-w)l]^{1-w}$, $\frac{\partial L}{\partial g} = wg^{w-1}[(1-w)l]^{1-w}$ and $\frac{\partial L}{\partial l} = (1-w)g^w(1-w)^{1-w}l^{-w}$ one finally obtains

$$\frac{dL}{L} = w \frac{dg}{g} + (1-w) \frac{dl}{l}. \quad (5.27)$$

This equation shows how a change in g and/ or l influences L . For instance, if g is changed by 1% ($dg = 0.01g$), L is changed by $w\%$. The same interpretation holds for a change in life expectancy l . A natural limit is obtained for $dL/L = 0$. On this basis, any activity yielding an

5 The Life Quality Index

increase in life quality, i.e. $\frac{dl}{l} \geq 0$, can be considered as being acceptable. Therefore, the LQI acceptance criterion is formulated as:

$$\boxed{w \frac{dg}{g} + (1 - w) \frac{dl}{l} \geq 0.} \quad (5.28)$$

Considering the inequality as an equality sign, the *marginal rate of substitution* $MRS = \frac{dg}{dl}$ can be calculated.

$$\frac{dg}{dl} = -\frac{1 - w}{w} \frac{g}{l} \quad (5.29)$$

The MRS is also known as the *marginal willingness to pay*. However, it is only meaningful if dl is small. In all other cases, the *willingness to pay* dg has to be evaluated. The interpretation of these equations leads to an LQI-based acceptance criterion as formulated in Chapter 6.

5.3 The Correlation between the GDP per Capita and the Life Expectancy

Figure 5.3 illustrates the life expectancy l and the GDP per capita g for 175 countries. The data are taken from the Human Development Report, UNDP (2004), see also Skjong (2002) and Rackwitz (2005). In both figures, the life expectancy as well as the GDP per capita refer to the year 2002. In order to be able to compare the GDP per capita, it is given in purchasing power parity expressed in US dollars (PPP USD). The left figure shows a nonlinear relation between g and l . Taking the logarithm of g , the right figure is obtained and the correlation coefficient increases from $\rho_{g,l} = 0.58$ to $\rho_{\log(g),l} = 0.76$. Considering these figures, a correlation between

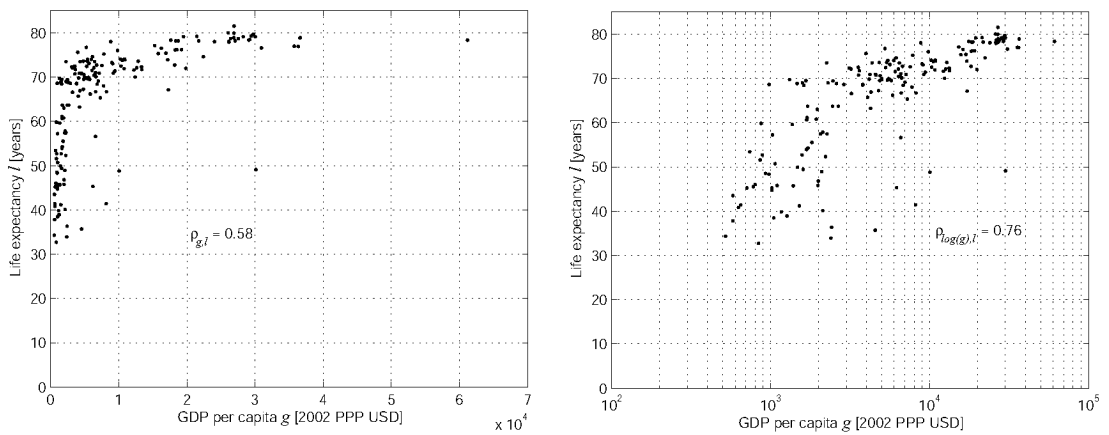


Figure 5.3: Correlation between g and l .

the two measures is observed that might indicate a dependency. Starting from the following differential equation that fulfills the conditions $\lim_{g \rightarrow \infty} \frac{dl}{l} = 0$ and $\lim_{g \rightarrow 0} \frac{dl}{l} = \infty$,

$$\frac{dl}{l} = q \frac{dg}{g} \quad (5.30)$$

a relation between l and g is obtained with $q \in [0, 1]$. Hereby g_0 and l_0 represent initial conditions. Later in Chapter 5.4.4, this relation is given an illustrative meaning.

$$l = g^q g_0^{-q} l_0 \tag{5.31}$$

The relation implied by Equation 5.31 is now investigated using empirical data.

5.3.1 Comparison with Empirical Data

Equation 5.31 is evaluated with empirical data from World Bank (2003). The World Development Indicators of World Bank provide an extensive collection of social indicators. There are time series for 208 countries, and many time series cover the period from 1960 to 2001. The life

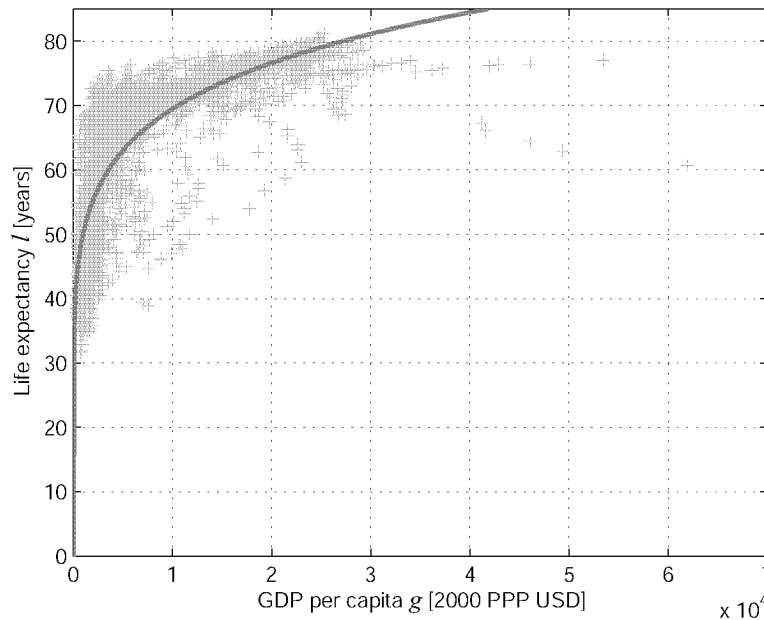


Figure 5.4: Life expectancy l and GDP per capita g of 166 countries, 1960 - 2000.

expectancy is plotted versus the GDP per capita in Figure 5.4 for 166 countries over the period from 1960-2000. Thereby, the GDP per capita has been adjusted for the different purchasing powers and for inflation. Consequently 2,611 pairs are plotted in the figure. In addition, the curve according to Equation 5.31 is plotted with values for g_0 , e_0 and q that fit the data points best using the least square method.

In Figure 5.5, the data for all countries is plotted again for the time period from 1960 to 2000. However, the data representing OECD countries are emphasized together with the fitted curve for these countries. It is seen from Figure 5.5 and 5.4 that the two data sets result in different curves. The curve fitted with data from OECD countries is steeper for small values of g and more flat for higher values of g . Nonetheless, the fitted curves are able to describe the shape given by the 657 data points.

For single countries the pairs of g and l can be plotted as well, e.g. Figure 5.6 shows the development of Denmark. It is seen that the development of Denmark during the last four decades can

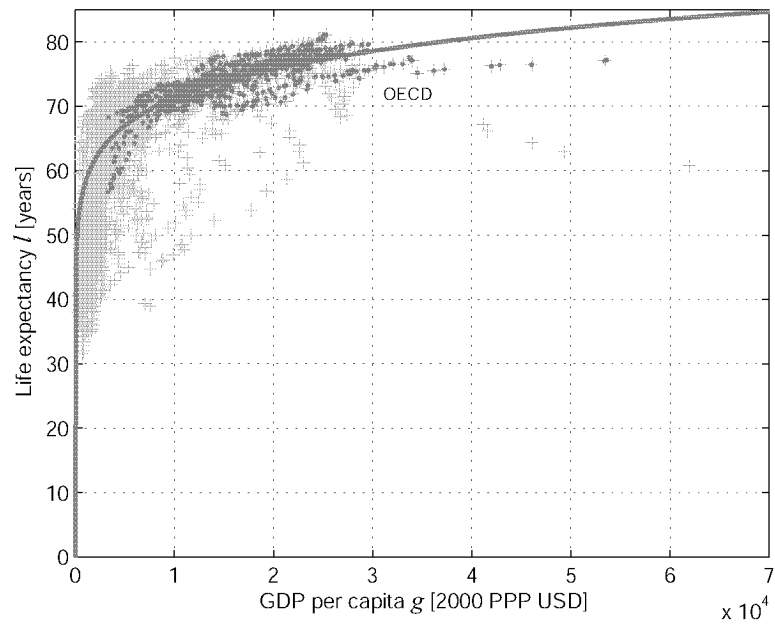


Figure 5.5: Life expectancy l and GDP per capita g of 28 OECD countries, 1960 - 2000.

also be appropriately described by a linear function, which is observed for most OECD countries, since they are highly developed. A country that exhibits a pronounced nonlinear relation is Hong Kong. Its development is seen in Figure 5.7. Adding Singapore and South Korea, Figure 5.8 is obtained, which illustrates the development of the selected Asian countries.

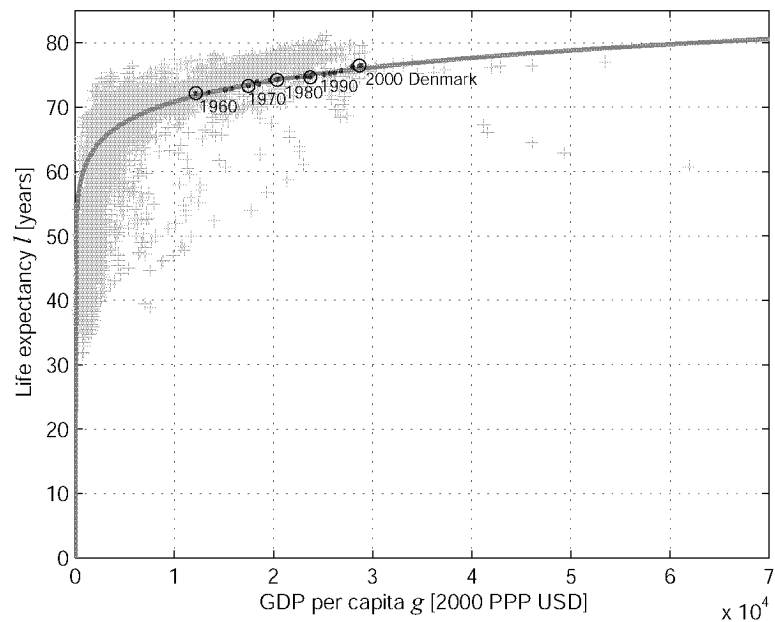


Figure 5.6: Development of Denmark, 1960 - 2000.

It is clearly seen that the parameters of Equation 5.31 can be fitted so that the curves coincide

5.3 The Correlation between the GDP per Capita and the Life Expectancy

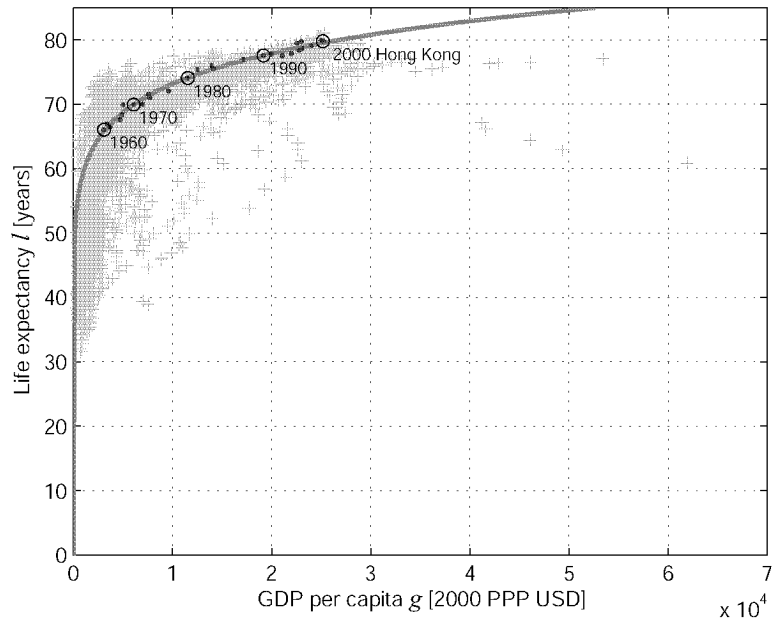


Figure 5.7: Development of Hong Kong, 1960 - 2000.

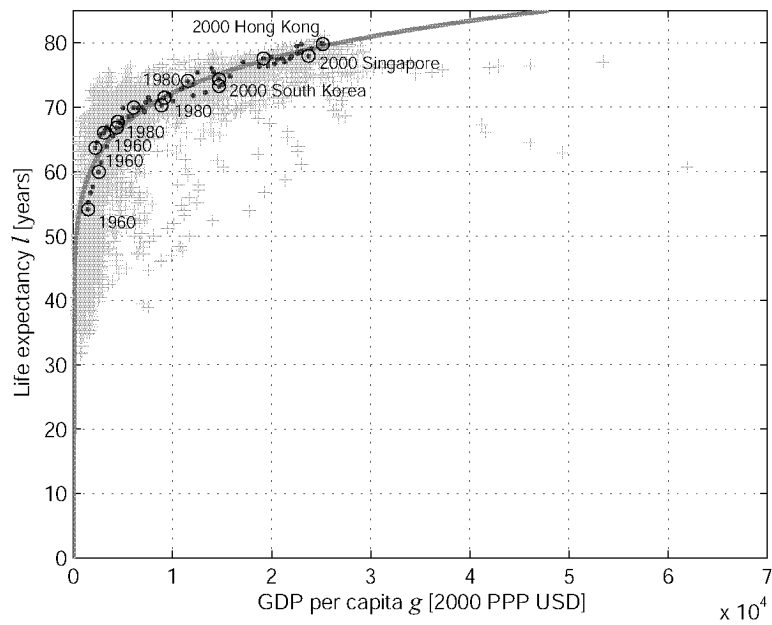


Figure 5.8: Development of Hong Kong, Singapore and South Korea, 1960 - 2000.

well with the observed data. But not all developments can be described by Equation 5.31, simply because not all effects influencing g and l are considered by this equation. Figure 5.9 shows the development of three countries namely Botswana, Saudi Arabia and Kuwait. It is seen that these developments cannot be accounted for with the derived equation. It is noted that the added trend lines in Figure 5.9 have only qualitative character. They help to distinguish the developments of the different countries.

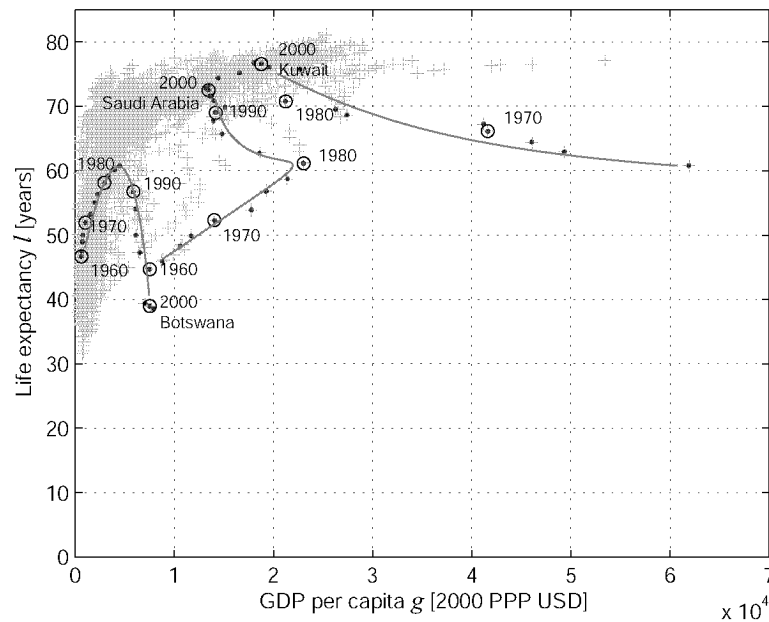


Figure 5.9: Development of Kuwait, Saudi Arabia and Botswana, 1960 - 2000.

Figure 5.9 illustrates the effect of the population increase of Kuwait. The population increased to more than six times of its initial value, whereas in the same period its real GDP doubled. Furthermore, the positive effect of the oil boom on the Saudi Arabian economy can be observed. However, after 1980 its economic growth was smaller than the population growth. The development of Botswana is characteristic for many sub-Saharan countries. Today, the population of Botswana has a life expectancy which is smaller than it had forty years ago, evidently due to AIDS, see UNAIDS (2004).

5.3.2 Ratio of Time Spent to Earn a Living

In the next section it is shown that the fitting parameter q may be related to w , an average person's ratio of time spent to work, as given by Equation 5.18. Recently the question came up: what value should be chosen for w ? See Ditlevsen (2004). Considering the figure above, it seems that the parameter values of the fitted curves might help to answer this question. It even might be possible to argue for a specific value presented in the current literature, see e.g. Nathwani et al. (1997), Ditlevsen and Friis-Hansen (2005) or Rackwitz (2005).

Nathwani et al. (1997) propose that w equals 0.125. A demographic study by Rackwitz (2005) shows a mean value of $E[w] = 0.129$, which corresponds to a mean value for q of $E[q] = 0.148$. Ditlevsen and Friis-Hansen (2005) introduce a constant c replacing w in the exponent of the classical LQI. By means of data related to Denmark, a value around 0.085 is determined. Pandey and Nathwani (2004) also support the value 0.125 but refer to literature, which indicate values of around 0.2 and even up to 0.4, see e.g. Shepard and Zeckhauser (1984).

Table 5.1 summarizes the values found in the literature together with the fitting parameters. The general form of Equation 5.31 can be described by two parameters; however, the equation has three parameters to be fitted, namely g_0 , l_0 and q . Therefore, g_0 is arbitrarily set to the value

| Country | Period | $g_0 = g_{2000}^a$ | l_0^b | l_{2000}^c | q | w |
|-----------------------------------|-----------|--------------------|---------|--------------|-------|-------|
| 166 countries | 1960–2000 | 8,871 | 68.28 | 65.35 | 0.141 | 0.123 |
| OECD | 1960–2000 | 22,734 | 76.65 | 77.00 | 0.089 | 0.082 |
| Denmark | 1960–2000 | 28,677 | 75.97 | 76.43 | 0.066 | 0.062 |
| Germany | 1960–2000 | 25,101 | 77.36 | 77.53 | 0.171 | 0.146 |
| Hong Kong | 1960–2000 | 25,181 | 79.53 | 79.82 | 0.090 | 0.083 |
| Luxembourg | 1960–2000 | 53,405 | 78.07 | 77.04 | 0.095 | 0.087 |
| Switzerland | 1960–2000 | 27,782 | 78.90 | 79.70 | 0.210 | 0.173 |
| USA | 1960–2000 | 33,960 | 77.15 | 77.07 | 0.123 | 0.109 |
| Japan | 1960–2000 | 25,281 | 80.20 | 81.07 | 0.107 | 0.097 |
| Nathwani et al. (1997) | | | | | | 0.125 |
| Rackwitz (2005) | | | | | 0.148 | 0.129 |
| Ditlevsen and Friis-Hansen (2005) | | | | | | 0.085 |
| Shepard and Zeckhauser (1984) | | | | | 0.200 | 0.167 |

^ain 2000 PPP USD

^bfitted value for 2000

^cactual life expectancy in 2000

Table 5.1: Comparison of fitting parameters and values from literature.

corresponding to the GDP per capita value of the year 2000, i.e. g_{2000} . The remaining parameters are fitted using the least square method. The fitted value l_0 is then compared with the actual life expectancy for the year 2000 l_{2000} . Except for the second row that considers 166 countries, the difference between these values is less than 1.4%. For the case when g_0 and l_{2000} reflect indicators of a group of societies, the arithmetic mean of the indicators has been taken. In Table 5.1, it is seen that the factor q of the fitted curves yields values for w covering the values found in the literature. The highest value is 0.173 corresponding to the data from Switzerland and the lowest value is 0.062 referring to the data from Denmark. Considering the 166 countries from which the data was available, a value for w is obtained close to the value proposed by Nathwani et al. (1997). For Switzerland and Germany, high values are obtained close to the suggestions of Shepard and Zeckhauser (1984). Considering the data from the OECD countries, a low value is obtained; it can also be obtained for individual societies, such as Hong Kong and Luxembourg. These values are close to the value proposed by Ditlevsen and Friis-Hansen (2005). The value of Ditlevsen and Friis-Hansen, however, was obtained with data representing the development of Denmark, and for this country Table 5.1 indicates a value of 0.062 which is smaller.

Finally, the fitted values of the parameter q and the corresponding w do not support a specific value that is published in the literature. But the values cannot be rejected, either. This is because the values of the fitted parameter lie in the same region as the values found by other studies.

However, the question remains: Is Equation 5.31 related to the life quality index and is there a causal relation? A possible explanation is given in the following section.

5.4 LQI and Economic Behavior

Correlation between the life expectancy l and the GDP per capita g may result from three cases:

- It can result from a *direct causal relation* between the observed measures,
- an *indirect causal relation*, e.g. if both are influenced by a third quantity, or
- it can simply result from *coincidence*.

Considering the underlying relation between l and g it seems that there is not a direct relation between l and g because examples can be constructed that are inconsistent with the observed relation. For instance, consider the case when a safety measure is implemented. It is possible that the measure is effective but reduces mortality only of persons older than the retirement age. The measure would therefore increase life expectancy l and at the same time increase the population size. Nonetheless, the GDP is unaffected so that g decreases. The example therefore supports the assumption that l and g are not directly related.

In the following it is studied, whether the observed correlation between l and g is indirectly related as a result from economic behavior of decision makers. It is assumed that public decision-making aims to maximize life quality with a given budget. Hereby, life quality is expressed in terms of a utility function, the life quality index.

5.4.1 Preferences and Utility

Society comprises individuals, enterprises, organizations and the state which is represented by public institutions and organizations. All these members are decision makers and they invest their available budgets in goods and services. Goods and services e.g. can be a part of the infrastructure, such as buildings and bridges but also medication, medical treatment, education, entertainment, etc. Decision makers buy these goods and services privately or on behalf of their organizations. For individuals Maslow (1970) introduced a hierarchy of needs that are visualized in Figure 5.10. The physiological needs (food, drink, air, etc.) are fundamental. Only if they are satisfied does the decision maker try to satisfy other needs, e.g. safety, social relations, esteem and self actualization. It might be questionable whether the presented scheme is suitable to describe needs of enterprises but it is appropriate for individuals and therefore the population of a society. Statistics that quantify the performance of a state are called social indicators. They

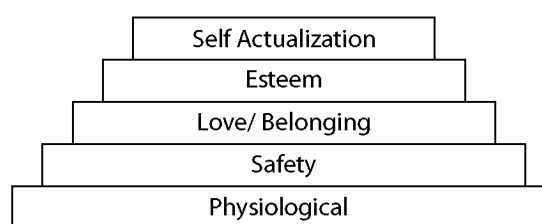


Figure 5.10: Maslow's hierarchy of needs.

aim to quantify how societal needs are met. The most important and well-known indicators are the life expectancy l and the gross domestic product per capita g . But additional meaningful indicators can always be added, e.g. unemployment or literacy rate. However, single social indicators only focus on a specific societal issue. They fail to represent the overall state. The latter is better reflected by compound social indicators, such as the human development index

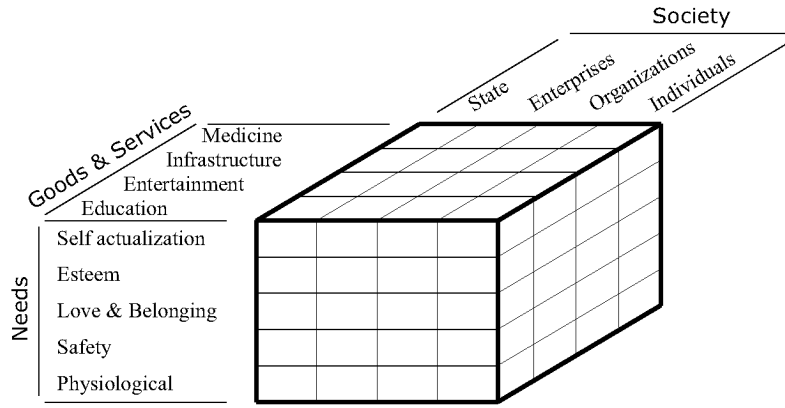


Figure 5.11: Society consumes goods to satisfy needs.

(HDI) (UNDP (2004)), the life product index (LPI) (Lind (1994)) or the life quality index (LQI) (Nathwani et al. (1997)). They are more adequate to reflect the performance of a society.

To generate such a compound social indicator, the relevant social indicators are put into a basket of societal indicators $\mathbf{x} = (l, g, \dots)^T$. Postulating that a utility can be assigned, we denote it by $L(\mathbf{x})$ and call it the life quality index. If the decision maker is indifferent between the two baskets \mathbf{x}_1 and \mathbf{x}_2 , then their utility is identical. This preference statement can be visualized by indifference curves or curves of constant L , see also Annex B.

If the considered societal indicators can be regarded as *normal goods* that are liked to be consumed at the same time, then the preferences are expressed by convex indifference curves, see Annex B. Normal goods are goods that are wanted goods for which no satiation is achieved, i.e. *more is always more*. Regarding the GDP per capita and the life expectancy, this assumption seems to be reasonable. A utility function with convex indifference curves is the life quality index as derived in Section 5.1. With $q = w/(1 - w)$ the LQI can be written as

$$L = g^q l, \quad (5.32)$$

the indifference curve crossing the point $(g_0, e_0)^T$ by

$$l = g^{-q} g_0^q l_0 \quad (5.33)$$

and the marginal rate of substitution (MRS) is given by

$$MRS = \frac{dg}{dl} = -\frac{1}{q} \frac{g}{l}. \quad (5.34)$$

The MRS is also known as the marginal willingness to pay. It reflects how much the decision maker is willing to exchange g for l , see also Annex B. Lines perpendicular to the indifference curves crossing $(g_0, e_0)^T$ are obtained by

$$l = \sqrt{g^2/q - g_0^2/q - l_0^2}. \quad (5.35)$$

L can be interpreted as the utility of a society regarding a specific year. It is also possible to formulate the expected remaining utility $U(a)$ for an individual

$$U(a) = \int_a^\infty u(\tau)r(\tau)R(\tau)d\tau, \quad (5.36)$$

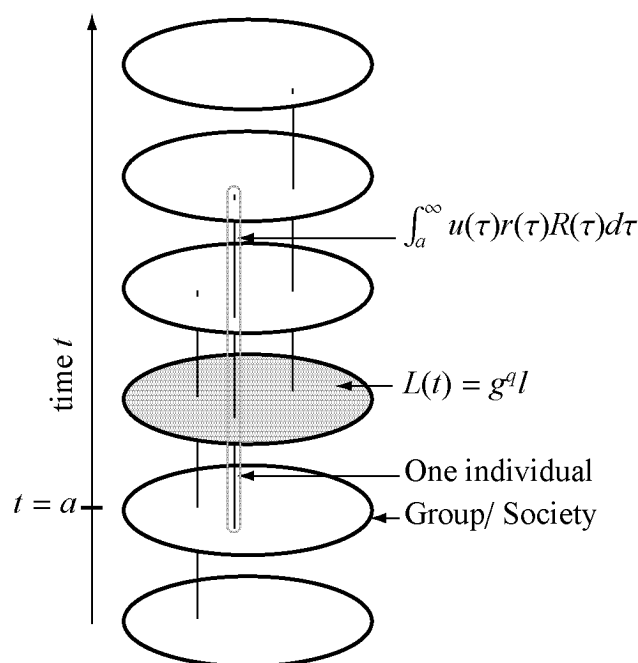


Figure 5.12: Comparison of utility formulations.

see e.g. Pandey and Nathwani (2004) or it can also be formulated for multiple individuals Rackwitz, Lentz, and Faber (2005) or Nishijima, Straub, and Faber (2005). In Equation 5.36, a is the age of the considered individual, $u(t)$ the time variant utility, $r(t)$ a discounting function and $R(t)$ the likelihood that the considered person lives to year t . The approaches are illustrated in Figure 5.12. Considering Figure 5.12, it is meaningful that the social indicators needed for the LQI calibration refer to the same year. However, regarding utility formulations of individuals, they should be evaluated on the basis of cohorts and therefore use predictive cohort life table, e.g. as described in Rackwitz (2005). In addition, the utility of individuals can be average over the society's age distribution to obtain the utility of an average individual.

5.4.2 Budget Constraint and Technology

5.4.2.1 Budget Constraint

The GDP is generally considered as the annual income or budget of a society. There might be better statistics to describe a society's budget, e.g. national income, but all are related to the GDP.

If the GDP is considered as the budget of a society, then g is the available annual amount for an individual of society. This amount can be used to satisfy the needs of an average individual. In the following we consider that a part of the budget is spent on measures increasing the life expectancy. This amount is indicated by c_l , whereas the remaining amount c_c is available for consumption to satisfy all other needs. Implicitly, the assumption is made that goods and services can be bought to satisfy needs. This implies that the needs are neglected, which cannot be satisfied by the purchase of goods and services. Considering this, the budget constraint is

written as

$$g = c_l + c_c. \quad (5.37)$$

This means, g is the available budget and society cannot spend more money on safety and other needs than is available. Hence, the possibility to borrow money is neglected.

5.4.2.2 Technological Constraint

From the amount g , the fraction c_l is used to invest into activities increasing the life expectancy, e.g. a better medical care, improved safety in traffic, safety against natural hazards, etc.

Hereby, it does not matter, whether the investment into safety is paid by individuals or by the public. Independent of it, a rational decision maker would first invest in the most efficient safety measures. Here, efficiency can be evaluated by $\frac{\Delta l}{\Delta c_l}$, the ratio of change in life expectancy and the associated costs. An example of this is given in Chapter 6, see also Table 6.1.

Arranging the safety measures according to their efficiency, a curve is obtained that characterizes the technologically achievable life expectancy l_t given an investment c_l . In the following, this curve is referred to as the technology curve. From the above described selection strategy two conditions can be formulated that the technology curve must fulfill.

1. If $c_l = 0$, any additional small investment into safety will yield a disproportionately high increase in life expectancy.
2. Increasing the investment c_l more and more, the efficiency of the safety measures reduces which is expressed by a decreasing marginal life expectancy.

Mathematically these conditions can be formulated by the following equations:

$$\lim_{c_l \rightarrow 0} \frac{dl_t(c_l)}{dc_l} = \infty \quad (5.38)$$

$$\lim_{c_l \rightarrow \infty} \frac{dl_t(c_l)}{dc_l} = 0. \quad (5.39)$$

A differential equation fulfilling the above equations is given by

$$\frac{dl_t}{dc_l} = b_t \frac{l_t}{c_l}, \quad (5.40)$$

which is integrated to

$$l_t = a_t c_l^{b_t}. \quad (5.41)$$

Here, $a_t \in \mathbb{R}_+$ and $b_t \in (0, 1)$ are constants where the index t indicates that the parameters are related to the technology curve. The technologically achievable life expectancy l_t is indicated by the index t , as well.

Substituting the budget constraint into Equation 5.41, one obtains

$$l_t = a_t (g - c_c)^{b_t}, \quad (5.42)$$

see also Figure 5.13. The derivative of c_c with respect to l_t is an important measure. It is the marginal opportunity costs. This means that with the technology curve the opportunity is provided to increase l_t infinitesimally at the unit cost of dc_c/dl_t .

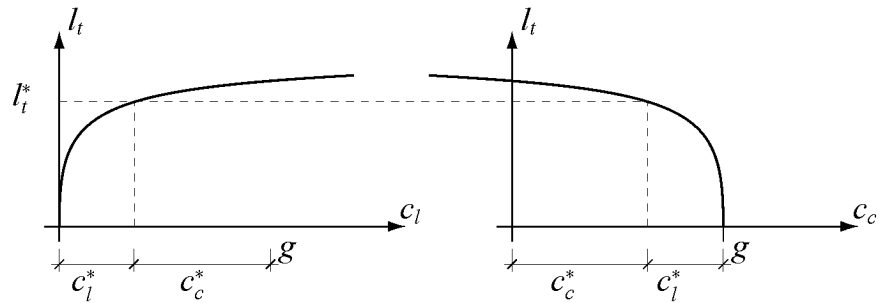


Figure 5.13: Technology curve l_t .

5.4.3 Optimal Choice

The LQI indifference curves are given by Equation 5.33. If a relation between g and c_c is given, then these indifference curves may be mapped into the $l - c_c$ space as illustrated in Figure 5.14. Figure 5.14 illustrates the technology curve l_t together with three indifference curves of the LQI, where, l_{L3} is preferable to l_{L2} and l_{L1} . As discussed in Annex B, the decision maker chooses the optimum as the point where the marginal rate of substitution is equal to the marginal opportunity costs². This means that the technology curve is tangent to the highest affordable indifference curve. It is noted that this tangent condition is a necessary but not a sufficient condition, see Annex B. Figure 5.15 identifies for different levels of g , namely g_1 to g_3 , the optimal points that

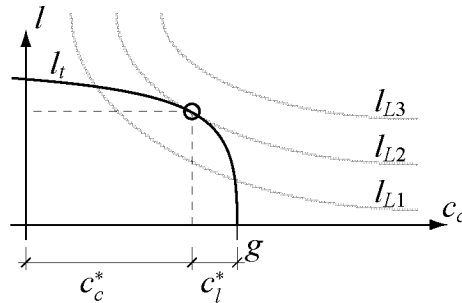


Figure 5.14: Indifference curves l_{L1} , l_{L2} and l_{L3} , the technologically achievable life expectancy l_t and the optimal choice.

are connected by the trajectory l_o . If the optimal values for c_l are plotted versus g , the *Engel curve* is obtained which is shown in Figure 5.16. The Engel curve is introduced in Annex B.

5.4.4 Verification with Empirical Data

It is straightforward to study, whether the observed correlation between l and g can be explained as the result of economic behavior.

²Optimal points at the boundaries are not considered.

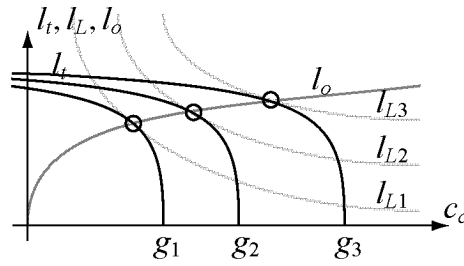


Figure 5.15: Optimal choice for different budgets g_1 , g_2 and g_3 .

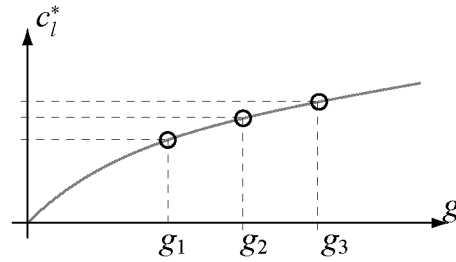


Figure 5.16: Engel curve for investments in life safety.

To start with, a special Engel curve for c_c is assumed. Independently of the actual value of g , it is assumed that a constant fraction κ_l of g is spent to increase life expectancy. This means

$$c_c^* = (1 - \kappa_l)g. \tag{5.43}$$

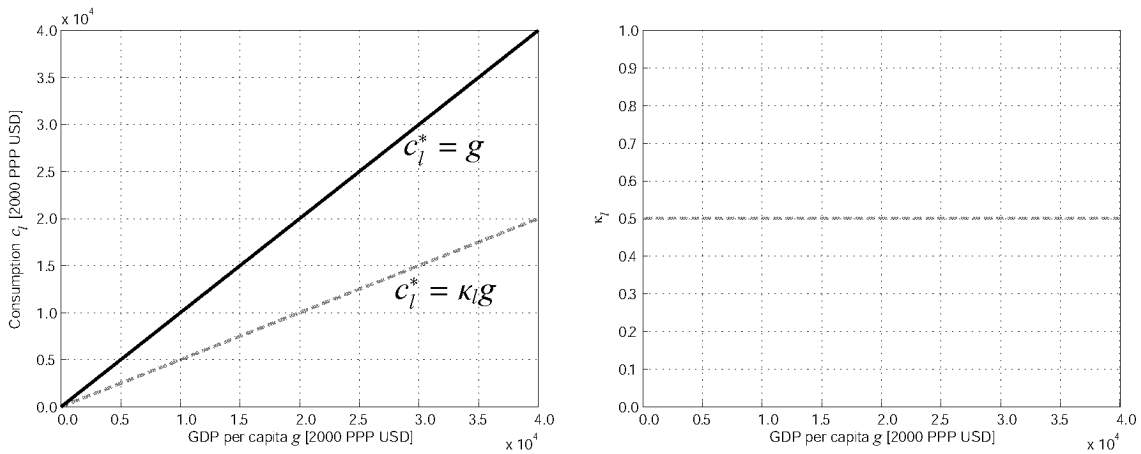


Figure 5.17: Assumed preferences for investment into safety represented by an Engel curve (left) and the corresponding fraction of g (right).

In other words, it is assumed that the decision maker expresses homothetic preferences with respect to investments in life safety, see Annex B.5.1. But this does not imply homothetic

5 The Life Quality Index

preferences with respect to life expectancy because of the nonlinear form of the technology curve.

Given Equation 5.43, the technology curve can be reformulated as

$$l_t = a_t \left(\frac{c_c^*}{1 - \kappa_l} - c_c \right)^{b_t} . \quad (5.44)$$

Hereby, it is important to note that c_c^* is a constant that refers to an optimum given a specific g and κ_l as seen in Figure 5.13.

Equation 5.43 is also used to rewrite the indifference curves, which at least are valid in the neighborhood of c_c^* .

$$l_L = a_L \left(\frac{c_c}{1 - \kappa_l} \right)^{-b_L} \quad (5.45)$$

Considering Equation 5.31 as the optimal path of a society and substituting Equation 5.43 into it one obtains

$$l_o = a_o \left(\frac{c_c}{1 - \kappa_l} \right)^{b_o} . \quad (5.46)$$

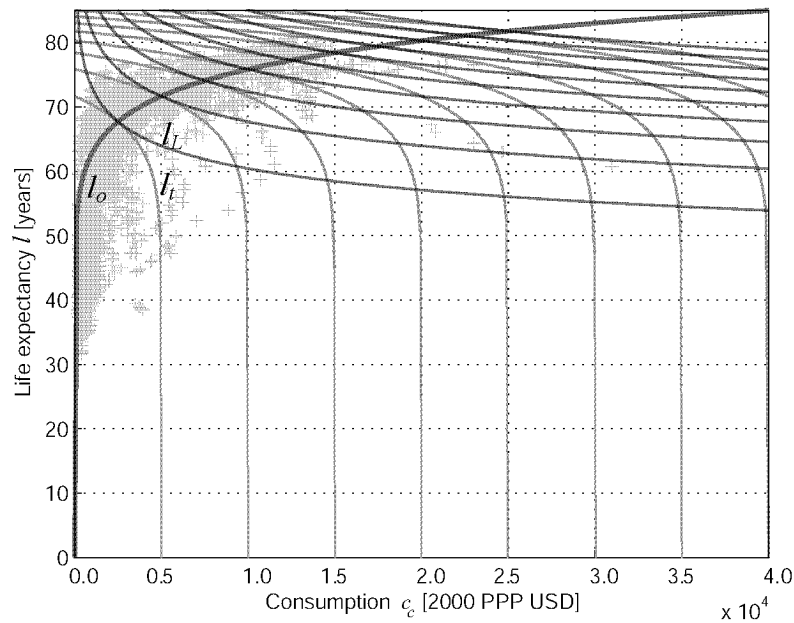


Figure 5.18: Illustration of the optimal decision process together with empirical data.

Equation 5.46 represents the optimal path of a society, if for a specific g (or c_c^*) the following conditions are fulfilled.

- $l_L(c_c^*) = l_o(c_c^*)$
- $l_t(c_c^*) = l_o(c_c^*)$

$$\bullet \frac{dl_I(c_c^*)}{dc_c} = \frac{dl_L(c_c^*)}{dc_c}$$

From these conditions the parameters defining the technology, indifference and Engel curves are determined as follows:

$$b_L = b_o \quad (5.47)$$

$$a_L = a_0 \left(\frac{c_c^*}{1 - \kappa_l} \right)^{2b_o} \quad (5.48)$$

$$b_t = b_o \quad (5.49)$$

$$a_t = a_o \kappa_l^{-b_o} \quad (5.50)$$

$$\kappa_l = 0.5. \quad (5.51)$$

Figure 5.18 summarizes the indifference curves, the technology curves for different levels of g and the optimal path. The latter is drawn with values from Table 5.1 representing the OECD countries and finally, data of l and g is plotted from World Bank (2003).

5.4.5 Discussion

50% of g is spent on actions increasing life expectancy. This is the interpretation of Equation 5.51. At first glance, this amount is generally assessed as being large. However, this amount is the sum of all costs related to measures increasing the life expectancy. This includes not only costs for structural safety or medical services, but also costs related to security, e.g. police, firemen as well as parts of costs for alimentation, accommodation etc.

On the other hand, this amount can be seen as being too low for extremely low incomes. Considering individuals with very low incomes, it is known that they tend to spend their complete budget to satisfy their fundamental needs. This means that the assumed Engel curve can only be regarded as an approximation. A more realistic curve of c_c^* might converge to g if g approaches zero, i.e. $\lim_{g \rightarrow 0} \kappa_l(g) = 1$. For larger values of g , $\kappa_l(g)$ may converge to a constant or to zero.

Another critical point of the framework outlined is that the technology curve is implicitly assumed to be constant for all considered years. As seen in Figure 5.18, this assumption still seems to be appropriate for the considered case but is it realistic? A more exhaustive and realistic picture is only obtained if the development of the technology curve is appropriately taken into account, a task involving enormous efforts because the effect on life safety of all goods and services needs to be assessed as well as their costs. An example on how costs associated with life safety can be evaluated is given in Schneider (2000a). Also the curve's development over time should be considered which depends e.g. on the development of medical science.

Equations 5.47 and 5.49 should not be interpreted in the way that the technology curve and the indifference curve are interrelated. The preferences of the decision maker has no influence on the technology. On the other hand, there is no doubt that the technological development affects the choice of the decision maker but not his preferences. These equations result from simplistic assumptions for the technology and the Engel curves as well the optimal path. These assumptions need to be refined.

Open questions remain, but the framework outlined combines economic consumption behavior together with the life quality index. On that basis it is possible to construct simple approximations so that the correlation which is found between g and l can be interpreted as an optimal

path that a rational decision maker would pursue in order to optimally allocate his available resources so that his preferences are best met. Mathematically this is formulated by

$$\begin{aligned} \max_{c_l} L(l(c_l), c_c) \\ \text{s.t. } g - c_l - c_c = 0. \end{aligned} \tag{5.52}$$

As a result of the postulated assumptions, the outlined study has an academic character. Refining these assumptions might reduce this characteristic. This is desirable because the verification of this study has the potential to:

1. explain why a correlation between g and l does not need to be considered in the derivation of the LQI because the observed correlation results from rational decision-making maximizing the life quality with a given budget constraint.
2. interpret Figure 5.18 as an empirical verification that public decisions are made on the basis that time is exchangeable with wealth – the fundamental postulate of the LQI.

5.5 LQI and Work Time Optimization

Another optimization takes place with regard to the fraction w as assumed in the LQI derivation. In the following this optimization process is reviewed, where first a plausible outline of the LQI is investigated.

5.5.1 Plausible Outline of the LQI

The LQI is a compound social indicator of three societal indicators namely g , l and w . In the following consideration, l is kept constant and the LQI is studied with regard to g and w .

To start with, Figure 5.19 shows the w - g plane, where indifference curves of the LQI can be plotted. Before analyzing specific formulations for the LQI, some plausibility considerations are studied. Figure 5.19 shows the w - g plane where two borders are highlighted, namely $g = 0$ and $w = 1$. It is generally agreed that both borders are not desirable. It therefore should yield a low LQI value \tilde{L} , e.g. $\tilde{L} = 0$. If g is considered as a normal good and w as an unwanted good³, then g should be increased and/ or w should be decreased to obtain a more preferable combination (w, g) . An indifference curve representing this preferences is indicated in Figure 5.19 together with the arrow that indicates increasing utility.

On that basis it is possible to consider whether the indifference curves of different LQI formulations reflect the discussed considerations.

For the different LQI formulations

$$L_o = g^w [(1 - w)l]^{1-w} \tag{5.53}$$

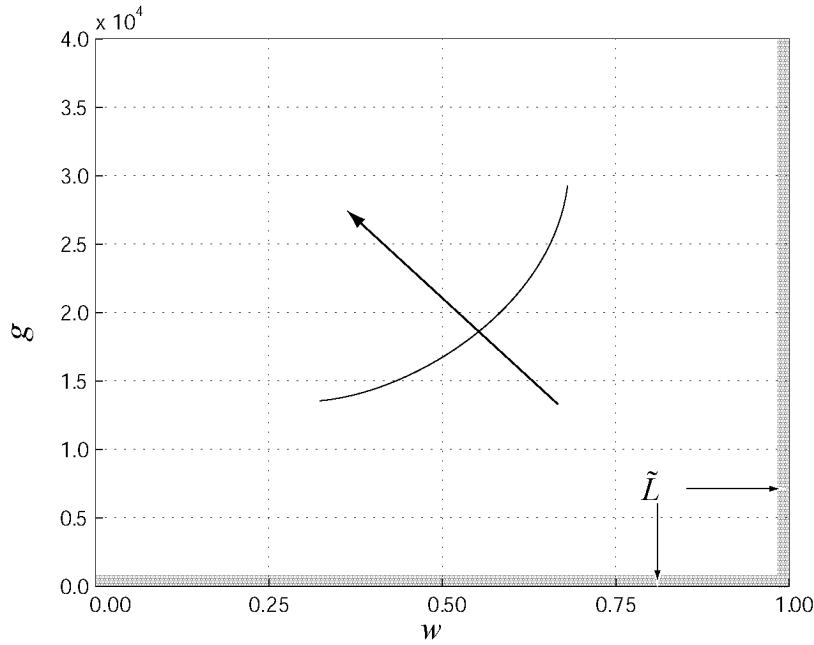
$$L_c = g^w l^{1-w} \tag{5.54}$$

$$L_q = g^{w/(1-w)} l \tag{5.55}$$

$$L_D = g^c [(1 - w)l]^{1-c} \tag{5.56}$$

$$L_R = \frac{1 - w}{w} g^{w/(1-w)} l \tag{5.57}$$

³The Classification of goods is described in Annex B.


 Figure 5.19: Reasonable outline of L .

the indifference curves are given by

$$g_o = L_o^{1/w} [(1-w)l]^{1-1/w} \quad (5.58)$$

$$g_c = L_c^{1/w} l^{1-1/w} \quad (5.59)$$

$$g_q = L_q^{(1-w)/w} l^{(w-1)/w} \quad (5.60)$$

$$g_D = L_D^{1/c} [(1-w)l]^{1-1/c} \quad (5.61)$$

$$g_R = L_R^{1/w-1} \left(\frac{w}{1-w} \right)^{1/w-1} l^{1-1/w}. \quad (5.62)$$

In Figure 5.20 and 5.21, the indifference curves g_o and g_R are shown together with the arrow indicating increasing preferences. It is seen that L_o increases, if g increases but also if w increases. The latter expresses w as a normal good which contradicts the plausibility considerations. Qualitatively, the same indifference curves as for L_o are obtained for L_c and L_q . L_R shows completely different indifference curves and it is seen in Figure 5.21 that for $w < 0.07$, w is an unwanted good and for larger values it is a normal good. The indifference curves for L_D are shown in Figure 5.22. It is seen that these indifference curves fulfill the plausibility considerations. In addition to the indifference curves, the other lines are plotted which result from the optimization consideration with regard to labor supply.

5.5.2 Labor Supply

On the basis of microeconomic consumption theory the quantity of labor that an individual is willing to supply to the labor market can be analyzed. This is studied using a utility function that is maximized. Considering the LQI as such a utility function it is studied how the optimal fraction w can be determined. It is assumed that the productivity with respect to the work fraction

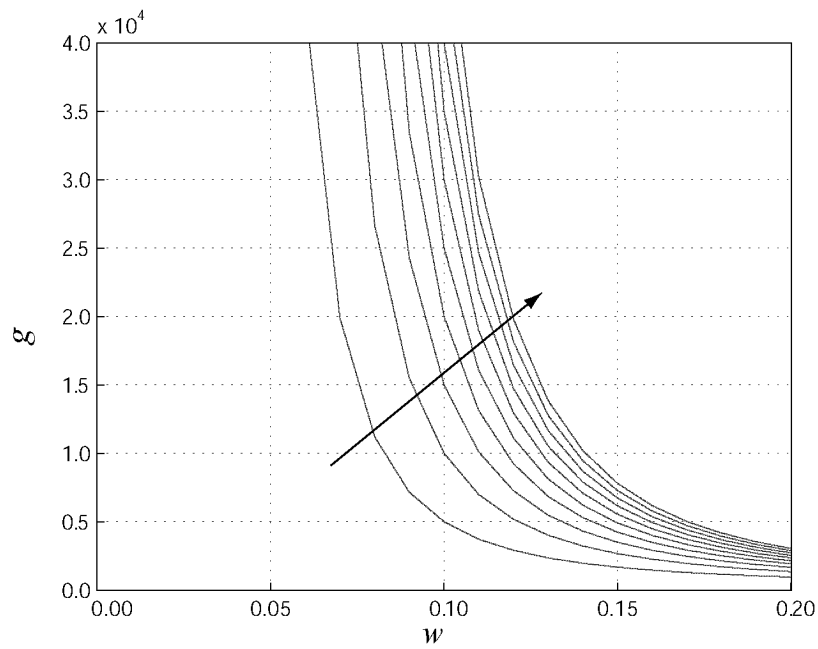


Figure 5.20: Indifference curves for $L_o = g^w[(1-w)l]^{1-w}$.

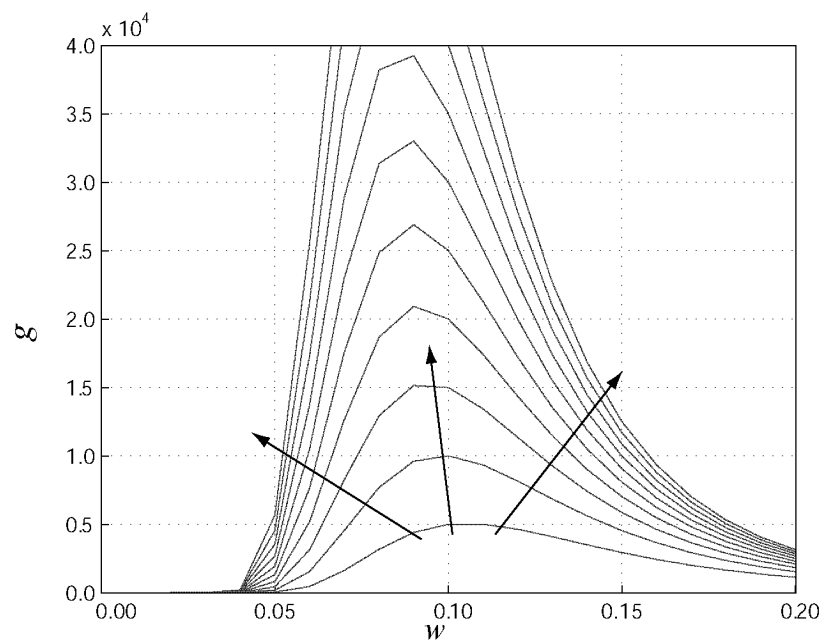


Figure 5.21: Indifference curves for $L_R = \frac{1-w}{w} g^{1-w} l$.

is p . p is a function of several factors but it is considered to be independent of w . On that basis the GDP per capita is written as

$$g = pw. \tag{5.63}$$

This means that given a specific p it is possible to increase w to obtain a higher g . The linear function $g = pw$ is plotted in Figure 5.22 for different values of p .

Given the utility function by Equation 5.56 and the constraint by Equation 5.63, one can formulate the optimization problem as follows:

$$\begin{aligned} \max_w L_D &= g^c[(1-w)l]^{1-c} \\ \text{s.t. } g &= pw. \end{aligned} \quad (5.64)$$

The solution to this optimization problem is obtained by formulating the Lagrange function

$$\mathcal{L} = g^c[(1-w)l]^{1-c} + \lambda(pw - g) \quad (5.65)$$

with the Lagrange multiplier λ . Setting the partial derivatives $\frac{\partial \mathcal{L}}{\partial g}$, $\frac{\partial \mathcal{L}}{\partial w}$ and $\frac{\partial \mathcal{L}}{\partial \lambda}$ to zero, a set of equations is obtained from which the optimal choice is derived. All optimal points lie on the line:

$$w = c = \text{constant}. \quad (5.66)$$

This line is also drawn in Figure 5.22. Finally, this figure is augmented with data representing the development of Denmark from 1948 to 2003. The data is taken from Ditlevsen and Friis-Hansen (2005).

It is seen that in the early years of 1948, the data and the derived optimal path do not agree well. However, after 1974, when w is smaller than 0.1, the data and the optimal path $w = c$ coincide well. Hereby, the value for c was chosen to be 0.09 close to 0.085, the value derived by Ditlevsen and Friis-Hansen (2005). Ditlevsen and Friis-Hansen (2005) formulate that a societal economy develops towards an equilibrium state w_e , in which the LQI is insensitive to variations of w .

Figure 5.23 shows the development of 19 OECD countries in the period from 1960 to 2000. Hereby the values for g have been taken from World Bank (2003) and the OECD data is used to calculate the factor w , Friis Hansen (2005). Friis Hansen (2005) calculates the work fraction w by multiplying the number of workers N_w by the average annual working hours per worker h_w . This amount is then divided by the population size N_p and the amount of hours per year $h_a = 360 \cdot 24$ h, i.e.

$$w = \frac{N_w h_w}{N_p h_a}. \quad (5.67)$$

From this equation it is seen that w is influenced by many factors, e.g. unemployment, economic growth, demographic structure of the population such as the workforce participation of females, immigration, etc.

Figure 5.23 shows the developments of OECD countries for the period 1960–2000. It is seen that the value w for selected countries is roughly constant, e.g. for Australia, Norway or the United Kingdom. However, there are countries that show a reduction in w while g increases, e.g. Belgium, Denmark or Japan. On the other hand, there are countries which show an increase in w if g increases, e.g. Canada and the United States of America. Considering the period 1960–2000 and all 19 OECD countries, the maximal value is obtained for w to 0.126 (Japan, 1970) and the smallest value is $w = 0.061$ (Netherlands, 1993). Generally, a trend supporting Equation 5.66 can be observed, especially if the time series for all 19 OECD countries are plotted in

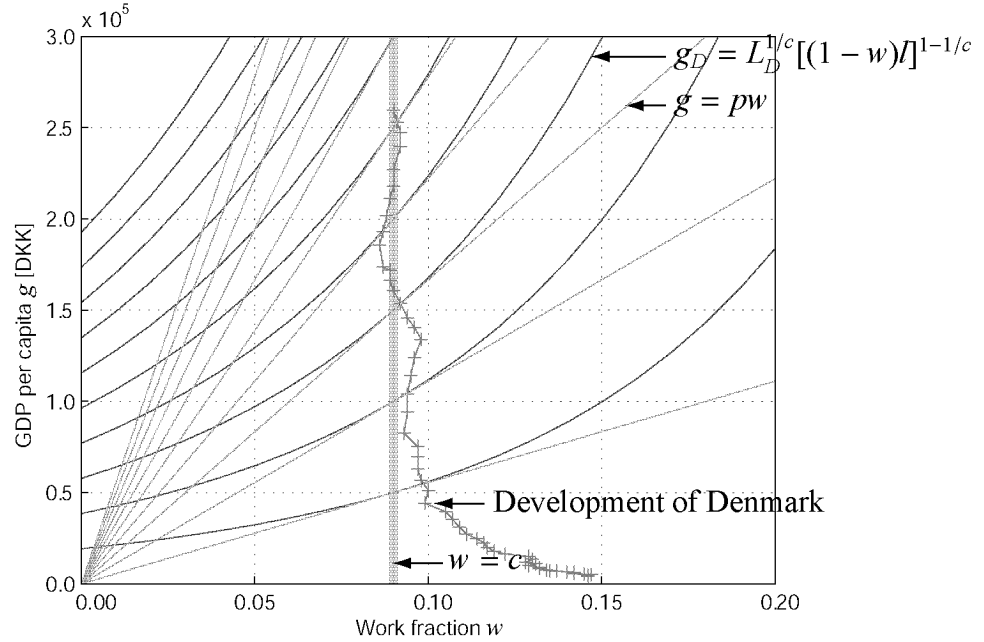


Figure 5.22: Indifference curves for $L_D = (g^c[(1-w)l]^{1-c})$, curves of possibly achievable $g = pw$, the optimal path $w = c$ and data representing Denmark for the period 1948–2003.

the diagram with w ranging from 0 to 1. However, it is noted that $w = 1$ is not achievable. Considering the development of Denmark in Figure 5.22 and 5.23 it is mentioned that the values for g in Figure 5.22 do not account for inflation and cover a longer time period.

Based on this interpretation, it is of interest to consider the relationship

$$g = p'(1-w)^{1-1/c} \quad (5.68)$$

as derived in Ditlevsen and Friis-Hansen (2005). Here, the constants p and K as given in Ditlevsen and Friis-Hansen (2005) are summarized in p' .

Considering this, one can formulate the optimization problem as

$$\begin{aligned} \max_{g,w} L_D &= g^c[(1-w)l]^{1-c} \\ \text{s.t.} \quad g &= p'(1-w)^{1-1/c} \end{aligned} \quad (5.69)$$

and solve it. It is seen that any value of w is valid, so that not a single optimal path exists. There are an infinite number of them. This is because the constraint given by Equation 5.68 is identical to the indifference curves given by Equation 5.61 and for any specific p' any value of w is optimal. This is also what Ditlevsen and Friis-Hansen (2005) aimed at when deriving Equation 5.68. The aim was that dL/L should be insensitive to variations of w .

Taking as a basis $L_D = g^c[(1-w)l]^{1-c}$ one can formulate the relative differential:

$$\frac{dL}{L} = c \frac{dg}{g} + (1-c) \frac{dl}{l} - (1-c) \frac{dw}{1-w}. \quad (5.70)$$

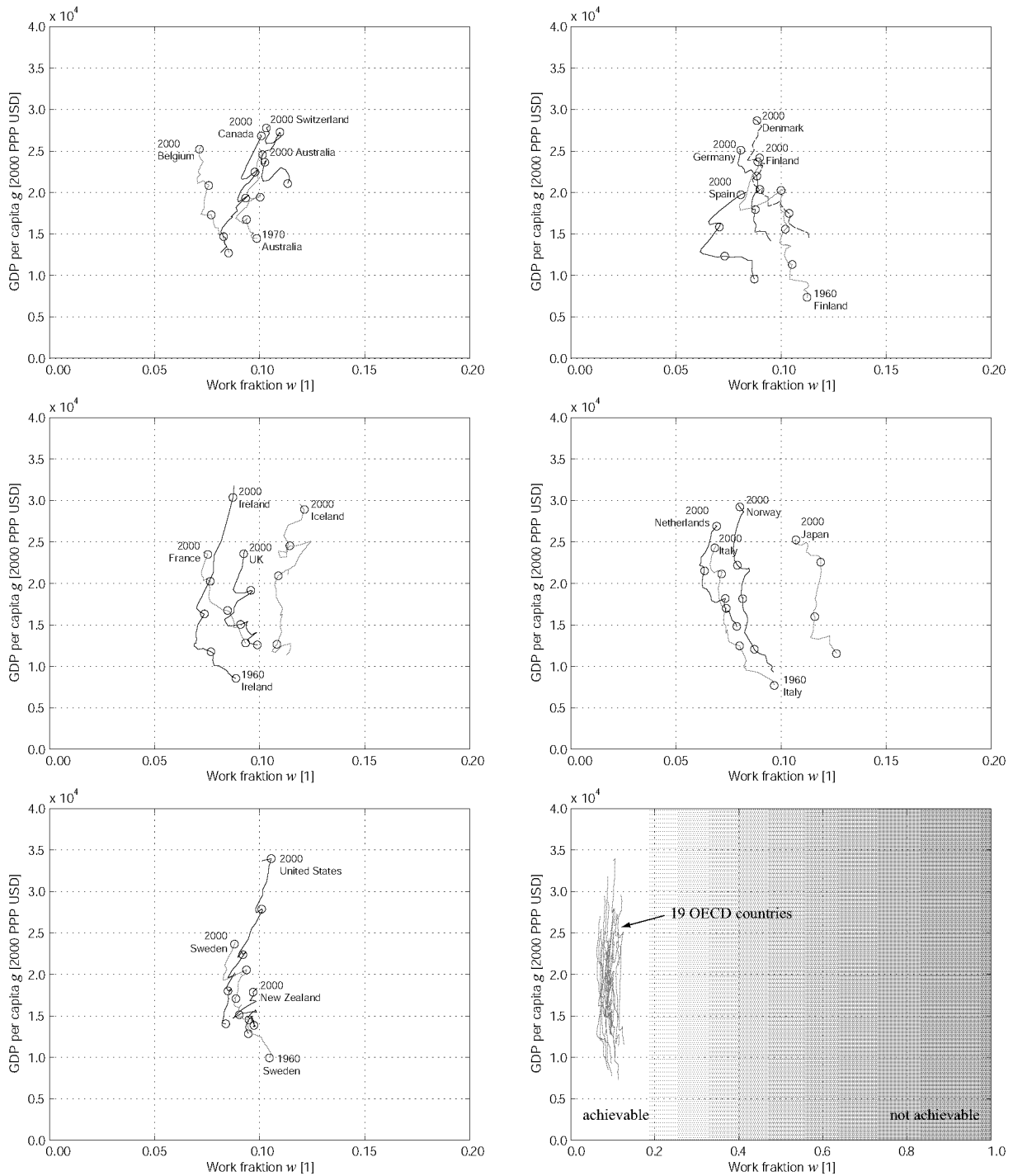


Figure 5.23: w - g development of OECD countries 1960–2000. Decades are indicated by circles.

Keeping l constant, i.e. $dl = 0$ and setting $\frac{dL}{L} = 0$ one obtains the marginal opportunity costs with respect to w .

$$\frac{dg}{dw} = \frac{1-c}{c} \frac{q}{1-w} \tag{5.71}$$

(5.72)

And further:

$$dg = \frac{1-c}{c} \frac{q}{1-w} dw. \quad (5.73)$$

Considering a developed society so that it has optimized a stable value for w , namely w_0 , one obtains with $c = w_0$ and $p = g/w_0$

$$dg = p dw. \quad (5.74)$$

Equation 5.74 is also obtained using Equation 5.63. dw can be interpreted as the change in leisure and p is the productivity so that dg can be assessed.

This equation represents the willingness to exchange dw for dg . From the equation it is seen that dg has the same sign as dw . This means that a reduction in w (more leisure) is compensated by a reduction in g (reduced wealth).

In economics the same reasoning is applied to quantify for individuals the optimal share of time to be spent at work. In economics the optimal path which individuals follow is denoted *labor supply curve*. It is not linear for individuals, see Varian (2003), but the labor supply curve shown in Figure 5.22 considers all individuals of a given society. Considering the utility function of an individual and his or her budget constraint, the marginal opportunity cost to exchange time versus money can be derived. According to Varian (2003) it is simply the wage rate. Therefore, the wage rate is also known as the *opportunity cost of leisure*. This is not to say that by reducing leisure, e.g. by one hour due to a traffic jam, an individual earns the equivalent amount of money. But the individual is willing to exchange one hour of his life for that amount. Therefore, the consequences resulting from traffic delays are quantified on that basis. Sometimes these costs are also called *user costs*.

5.6 Summary

The original derivation of the life quality index (LQI) is given and other formulations of the LQI are discussed.

Thereafter, the correlation between g and l is studied, which is observed for many countries. It is studied whether the observed correlation may result from economic behavior of rational decision-making. That is, the decision maker chooses the best combination of life expectancy and consumption that is affordable. The outlined framework is able to formally combine economics and consumption behavior together with the life quality index. Open questions remain. However, if the described framework can be verified, the observed relation may be interpreted as an empirical proof that societal decisions involving risk to life are made on a decision theoretical basis as outlined in the present thesis.

Finally, different LQI formulations are checked against plausibility considerations with regard to possible combinations of w and g . On that basis, several LQI formulations are rejected for not appropriately reflecting the preferences of the public. This means, the exponent in the LQI formulation needs to be constant. The remaining LQI formulation $L_D = g^c[(1-w)l]^{1-c}$ is thereafter investigated, whether it is possible to derive an optimal development of w that a society would follow. This path, which in economics is known as labor supply curve, is derived as a straight line $w = c$, i.e. independent of g . Comparing the analytical solution with empirical data, a tendency is observed supporting the framework.

6 Acceptance Criteria

In the foregoing chapter, the LQI was introduced and it was illustrated that this compound social indicator is able to support decision makers that need to balance between monetary consequences and safety. Nonetheless, it did not explicitly show how acceptance criteria may be derived to incorporate them into engineering decision-making.

First, the present chapter introduces LQI-based risk acceptance criteria. For the sake of completeness, commonly applied acceptance criteria are presented thereafter. These criteria comprise the Farmer diagram, the fatal accident rate and structural design codes.

6.1 LQI-Based Acceptance Criteria

In the following, LQI-based acceptance criteria are distinguished into life saving costs (LSC) and the acceptance criterion that is derived on the basis of mortality changes. As LSC reflect the preferences of society to avert fatalities, these costs can be incorporated in lieu of compensation costs into the optimization procedure as outlined in Chapter 8. In addition, acceptability can be verified based on mortality changes, as is shown in Section 6.1.2.

6.1.1 Life Saving Costs

The value of life is infinite. However, putting in utilitarian decision analysis an infinite value to his own life, a person would not be willing to cross the street in order to get the latest newspaper. This contradicts our daily made observations. For engineering decision-making it is necessary to identify an amount that is acceptable to spend for safety which reflects the preferences of the public. Because any risk reduction measure can be evaluated by comparing the associated costs and the number of lives it saves, i.e. the quotient x USD per number of lives saved can be calculated. But this is not to say that this fraction evaluates the value of life. It rather evaluates the efficiency of the considered safety measure. If a limit for acceptable LSC is determined then this amount can be interpreted as the *price that a safety measure may be more expensive, if it saves an additional person's live*. In engineering decision analysis as outlined in Chapter 8, this is considered by penalizing an activity with the *acceptable life saving costs* if it involves a fatality.

Even if a decision maker rejects life saving costs *LSC* as a basis for decision-making, it is possible to calculate the implied life saving costs for any activity or decision this decision maker accepted in the past. The following illustrative example is taken from Schneider (2000b).

Consider a road junction that is particularly dangerous. Due to traffic accidents 2 persons die on average each year. This number might be reduced to almost zero, if it is decided to redevelop the junction. It would cost 8,000,000 USD or on an annual scale – with a service life of ten years – 800,000 USD. Of that amount, 75% may be attributed to risk reduction while the remaining sum increases road comfort and reduction of noise, etc. Therefore, one can calculate

| Life saving costs CHF per life saved | |
|---|--------------------------------------|
| 100 | Multiple vaccine 3rd World |
| $1 \cdot 10^3$ | |
| $2 \cdot 10^3$ | Installation of x-ray equipment |
| $5 \cdot 10^3$ | Wearing motor cycle helmet |
| $10 \cdot 10^3$ | Providing cardio-equipped ambulance |
| $20 \cdot 10^3$ | Tuberculosis checks |
| $50 \cdot 10^3$ | Providing helicopter for emergencies |
| $100 \cdot 10^3$ | Safety belts in cars |
| to | Reconstructing road junctions |
| | Providing kidney dialysis units |
| $500 \cdot 10^3$ | Building structures |
| $1 \cdot 10^6$ | |
| $5 \cdot 10^6$ | Zurich fast rail system, AlpTransit |
| $10 \cdot 10^6$ | Swiss earthquake code |
| $20 \cdot 10^6$ | Safety measures in mines USA |
| $50 \cdot 10^6$ | |
| $100 \cdot 10^6$ | Tall Buildings regulations in GB |
| $1 \cdot 10^9$ | Removal of asbestos from schools |

Table 6.1: Life saving costs.

the implied life saving costs as follows:

$$LSC = \frac{0.75 \cdot 800,000 \text{ USD}}{2 \text{ lives saved}} = 300,000 \text{ USD/ live saved.} \quad (6.1)$$

This means, if it is decided to redevelop the junction, it is implicitly accepted to spend 300,000 USD per life saved. In Table 6.1 it is seen that on this basis the efficiency of different risk reduction measures can be compared with each other. Table 6.1 is taken from Schneider (2000b), see also Schneider (1996).

On the other hand, accepted LSC can be studied. This means that for made decisions implied LSC are calculated, see e.g. Skjong (2002) or Rackwitz (2005). For instance, regarding the tunnel project Alptransit, acceptable life saving costs have been considered to be 10 million CHF (approx. 7.6 million USD), see Schneider (2000b). In the following, different possibilities to derive LSC are outlined.

6.1.1.1 Skjong and Ronold's LSC

From the historical point of view, the first LQI-based LSC was published by Skjong and Ronold (1998). Based on the LQI criterion (Equation 5.28) and simple approximations, Skjong and Ronold (1998) derive acceptable life saving costs which thereafter are denoted LSC_{SR} . In the literature LSC_{SR} is also referred to as *ICAF* standing for *implied costs of averting a fatality*.

It represents an upper limit for life saving costs that can be accepted. Reformulating Equation 5.28 yields the annual acceptable costs per change in life expectancy.

$$dg \geq -\frac{1-w}{w} g \frac{dl}{l} \quad (6.2)$$

Considering the differentials as differences and taking the absolute value one obtains

$$\Delta g_{SR} = \frac{1-w}{w} g \frac{\Delta l}{l}. \quad (6.3)$$

This equation can be interpreted for an individual of age a exposed to a specific hazard. If the hazard occurs and kills the person, the life expectancy of the person is reduced by $\Delta l = l(a) - a$. $l(a)$ is the age dependent life expectancy, i.e. $l(a) = a + \int_a^\infty \frac{R(t)}{R(a)} dt$ with $R(t)$ as the survival probability of an individual. Skjong and Ronold (1998) state that on average one can assume Δl to be $l/2$. So one obtains the acceptable annual life saving costs as

$$\Delta g_{SR} = \frac{1-w}{w} \frac{g}{2}, \quad (6.4)$$

which on average have to be spent $\Delta l = l/2$ years. Finally, the life saving costs are obtained as

$$LSC_{SR} = \Delta g_{SR} \Delta l \quad (6.5)$$

$$= \frac{1-w}{w} \frac{gl}{4}. \quad (6.6)$$

6.1.1.2 Life Saving Costs LSC_R

Rackwitz (2003a), proposes a different interpretation of Equation 5.28. The variables are separated and the equation is integrated from g to $g + \Delta g$ on the one side and from l to $l + \Delta l$ on the other side. These operations yield the annual increment Δg to which an average person is indifferent, if the life expectancy is changed by Δl years.

$$\frac{dg}{g} = -\frac{1-w}{w} \frac{dl}{l} \quad (6.7)$$

$$\Delta g_R = \left[\left(\frac{l + \Delta l}{l} \right)^{-\frac{1-w}{w}} - 1 \right] g \quad (6.8)$$

From Equation 6.8 it is seen that for positive Δl Δg_R is larger than $-g$ and smaller or equal to 0. Since Δg_R represents annual costs Rackwitz (2003a) multiplies it with Δl to obtain the life saving costs $LSC_{R,i}$ of an individual.

$$LSC_{R,i}(\Delta l) = \left[\left(1 + \frac{\Delta l}{l} \right)^{-\frac{1-w}{w}} - 1 \right] g \Delta l \quad (6.9)$$

As people may be affected by adverse events at different ages, Rackwitz (2003a) proposes to integrate $LSC_{R,i}$ over the age distribution $f_A(a)$ of the considered society. In this case Δl is substituted by $l(a) - a$.

$$LSC_R = E_A[LSC_{R,i}] = \int_0^\infty LSC_{R,i}(l(a) - a) f_A(a) da \quad (6.10)$$

In order to avoid the expectation operation, Rackwitz (2003a) proposes an approximation of LSC_R as follows.

$$LSC_R \approx LSC_{R,i}(l/2) = \left[\left(\frac{3}{2} \right)^{-\frac{1-w}{w}} - 1 \right] \frac{gl}{2} \quad (6.11)$$

Considering the value of $w = 0.125$, it is seen that LSC_R is 3.7 times smaller than LSR_{SR} .

6.1.1.3 LSC_R reinterpreted

LSC_R can be reformulated by simply applying different integration boundaries. The LQI criterion, Equation 6.2, can be integrated from $(g - \Delta g)$ to g on the left side and from $(l - \Delta l)$ to l on the other. In this case one obtains Δg_{mod} as follows.

$$\Delta g_{mod} = g \left[1 - \left(\frac{l}{l - \Delta l} \right)^{\frac{1-w}{w}} \right] \quad (6.12)$$

and finally

$$LSC_{mod} \approx LSC_{mod,i}(l/2) = \left[1 - (2)^{\frac{1-w}{w}} \right] \frac{gl}{2}. \quad (6.13)$$

It is immediately seen that Δg_{mod} follows from Δg_R if in Equation 6.8 the algebraic sign is changed and if Δl is substituted by $-\Delta l$, i.e. $\Delta g_{mod}(\Delta l) = -\Delta g_R(-\Delta l)$. However, the further derived life saving costs are very different. For $w = 0.125$ LSC_{mod} is more than 35 times larger than LSC_{SR} .

The implication of this is illustrated in Figure 6.1. The figure shows indifference curves for $w = 0.15$ or respectively $q = 0.177$. One of these curves is emphasized. It crosses the point at which $g = 20,000$ PPP USD and $l = 70$ years. For illustrative purposes Δl is chosen equal to 10 years and the different Δg are calculated. The following absolute values are obtained.

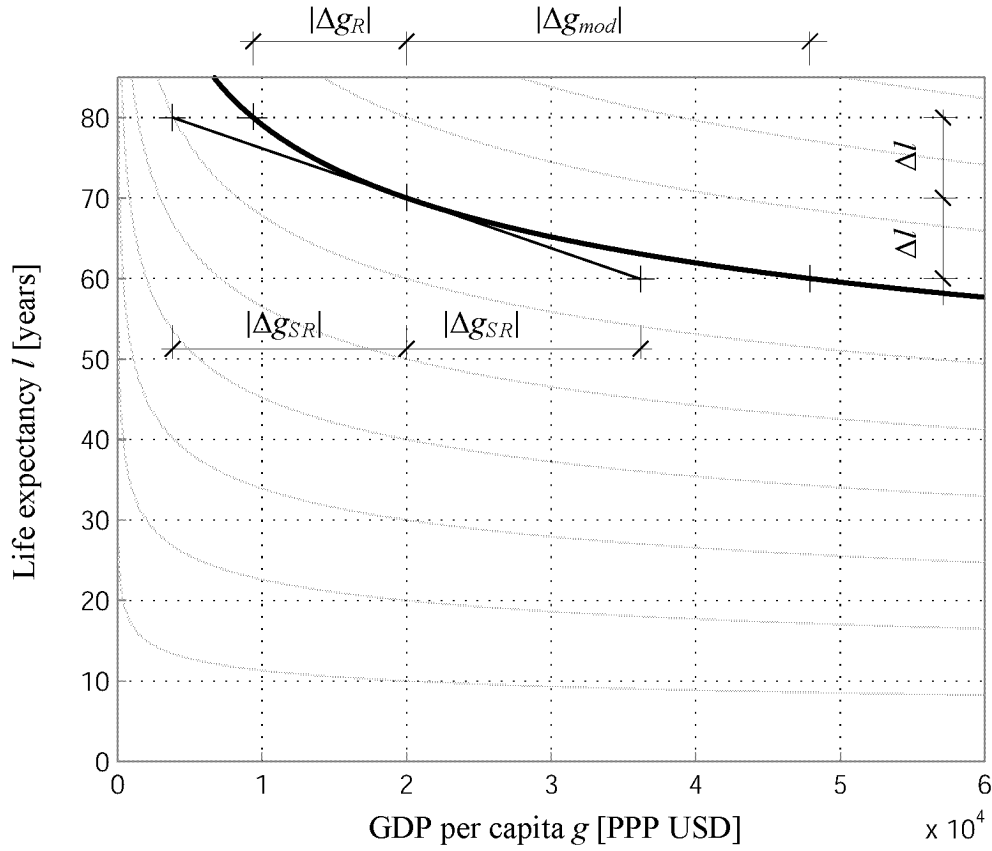
- $|\Delta g_{SR}| = 16,190$ PPP USD
- $|\Delta g_R| = 10,616$ PPP USD
- $|\Delta g_{mod}| = 27,907$ PPP USD

It is noted that the same absolute value is obtained for Δg_{SR} as for Δg_{mod} if the negative value of Δl is utilized.

Figure 6.1 illustrates the various meanings of the different Δg . It is seen that Δg_{SR} is obtained by a linear extrapolation using the slope of the indifference curve at the point of interest. The indifference curve's slope is also known as the marginal rate of substitution (MRS) which in this case can be interpreted as the marginal willingness to pay for life expectancy. This implies that it is only appropriate to use Δg_{SR} if the considered safety measures change life expectancy only marginally.

If the change in life expectancy is not marginal, then Δg_R or Δg_{mod} should be used as shown in Figure 6.1. Both take into account the indifference curve's nonlinearity. As indifference curves represent the preferences of decision makers, they indicate a combination of g and l to which the decision maker is indifferent. In order to increase the life expectancy by Δl , the decision maker is willing to reduce g by Δg_R . From the figure it is seen that with $\Delta l > 0$, the absolute value of Δg_R is larger or equal to zero but cannot be larger than g . It is simply impossible to reduce g to negative values. $|\Delta g_R| = g$ would imply that life expectancy has to become infinite in order to compensate for the reduction in g .

A decrease in life expectancy by Δl is considered to be compensated, if g is increased by Δg_{mod} . This means that Δg_{mod} represents the willingness to avoid a reduction of Δl . Δg_{mod} is larger or equal to zero and it can become infinite, if life expectancy is reduced to zero, see also *LQI and the Asteroid* in Section 7.3.


 Figure 6.1: Interpretation of the different Δg .

6.1.1.4 Adverse Events as a Stochastic Process

Another approach to assess the change in life expectancy is proposed by Ditlevsen (2003) and Ditlevsen (2004). The relative change in life expectancy is calculated due to an event causing fatalities. Hereby, it is postulated that a person's life may either end due to natural death or as the result of a fatal event. Following this the random lifetime T_L may be formulated as

$$T_L = \min[T_N, T_F], \quad (6.14)$$

where T_N is the random time of a natural death and T_F the occurrence time of a fatal event. On that basis, the probability density distribution of T_L and its expected value can be formulated.

$$f_{T_L} = \bar{F}_{T_F}(t)f_{T_N}(t) + \bar{F}_{T_N}(t)f_{T_F}(t) \quad (6.15)$$

$$E[T_L] = \int_0^{\infty} t f_{T_L}(t) dt \quad (6.16)$$

$$= \int_0^{\infty} \bar{F}_{T_L}(t) dt \quad (6.17)$$

$$= \int_0^{\infty} \bar{F}_{T_N}(t) \bar{F}_{T_F}(t) dt \quad (6.18)$$

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$\bar{F}_{T_F}(t)$ and $\bar{F}_{T_N}(t)$ are the complements of $F_{T_F}(t)$ and $F_{T_N}(t)$, respectively. $E[T_L]$ can also be formulated as

$$E[T_L] = E[T_L|T_F < T_N]P(T_F < T_N) + E[T_L|T_N < T_F]P(T_N < T_F) \quad (6.19)$$

where $E[T_L|T_F < T_N]$ is the life expectancy of an individual killed by an accident and $P(T_F < T_N)$ is the corresponding probability of occurrence. $E[T_L|T_N < T_F] = E[T_N]$ is the life expectancy, if the person dies naturally, which is likely with probability $P(T_N < T_F) = 1 - P(T_F < T_N)$.

Therefore, the conditional relative change in life expectancy can be written as

$$\frac{\Delta I_c}{I} = \frac{E[T_N] - E[T_L|T_F < T_N]}{E[T_L]}, \quad (6.20)$$

which is identical to

$$\frac{\Delta I_c}{I} = \frac{E[T_N] - E[T_L]}{E[T_L]P(T_F < T_N)}. \quad (6.21)$$

Considering now the underlying process of such fatal events as a stationary Poisson process, i.e. with independent and identically distributed inter-arrival times following an exponential distribution with $\bar{F}_{T_F} = e^{-\lambda t}$ one obtains

$$E[T_L] = \int_0^{\infty} e^{-\lambda t} \bar{F}_{T_N}(t) dt \quad (6.22)$$

$$= \frac{1}{\lambda} \int_0^{\infty} (1 - e^{-\lambda t}) f_{T_N}(t) dt. \quad (6.23)$$

The integral in the last equation is identical to $P(T_F < T_N)$ and $1/\lambda$ is $E[T_F]$. Therefore one can write

$$E[T_L] = E[T_F]P(T_F < T_N), \quad (6.24)$$

which can be substituted into Equation 6.21 to finally obtain

$$\frac{\Delta I_c}{I} = \frac{E[T_N] - E[T_L]E[T_F]}{E[T_L]E[T_F]} \quad (6.25)$$

$$= \frac{E[T_N] - \int_0^{\infty} e^{-\lambda t} \bar{F}_{T_N}(t) dt}{\lambda \left(\int_0^{\infty} e^{-\lambda t} \bar{F}_{T_N}(t) dt \right)^2}. \quad (6.26)$$

Using l'Hospital's rule one can evaluate the limit for $\lambda \rightarrow 0$. With

$$\lim_{\lambda \rightarrow 0} \frac{d}{d\lambda} E[T_L] = -\frac{1}{2} E[T_N]^2 \quad (6.27)$$

and

$$\lim_{\lambda \rightarrow 0} \frac{d}{d\lambda} (E[T_F]^{-1} E[T_L]^2) = E[T_N]^2 \quad (6.28)$$

one obtains

$$\lim_{\lambda \rightarrow 0} \frac{\Delta I_c}{I} = \frac{1}{2} \frac{E[T_N]^2}{E[T_N]^2} = \frac{1}{2} (1 + V_{T_N}^2) \quad (6.29)$$

$$\approx \frac{1}{2} (1 + V_{T_L}^2). \quad (6.30)$$

Finally, from Equation 6.25 and 6.30 we obtain the relative change of the unconditioned life expectancy $\frac{\Delta l}{l} = \frac{E[T_N] - E[T_L]}{E[T_L]}$ to

$$\frac{\Delta l}{l} = \frac{1}{2}(1 + V_{T_N}^2)p \tag{6.31}$$

$$\tag{6.32}$$

with $p = \frac{E[T_L]}{E[T_F]}$.

Ditlevsen (2004) argues that the probability of an individual to be affected by the adverse event is p per lifetime, i.e. $p/E[L]$ per year. Then based on the LQI criterion Ditlevsen derives the maximal acceptable life saving costs to

$$LSC_D = \frac{1-w}{w} \frac{1 + V_{T_L}^2}{2} l g. \tag{6.33}$$

Here, $E[T_L] = l$. It is seen that, compared to the LSC of Skjong and Ronold, Ditlevsen derives costs that are higher by a factor of $2(1 + V_{T_L}^2)$. With $V_{T_L} \approx 0.2$ this factor becomes 2.08. This difference results mostly from the fact that Ditlevsen considers costs for safety reduction to be spent during the whole lifetime of an individual, whereas Skjong and Ronold (1998) consider only the half period.

6.1.1.5 Discounting of Life Saving Costs

The preceding sections introduced several LQI-based acceptance criteria in terms of life saving costs that can be considered in engineering decision-making. As activities that involve civil engineering facilities represent long-term commitments, the question arises whether life saving costs should be discounted and if so, at what discount rate.

Paté-Cornell (1984) argues that an investor has the opportunity to invest today x USD in safety measures that on average result in n saved lives. On the other hand, the decision maker can also invest his or her money with interest in stocks, bonds, etc. with a real rate of interest i_r .

Therefore, $x(1 + i_r)^t$ USD are available in t years. Assuming no technological progress, $n \frac{x(1+i_r)^t}{x} > n$ lives can be saved in the future. On this basis one obtains the equivalent LSC to be $\frac{x}{n}$ and $\frac{x}{n(1+i_r)^t}$. It is seen that without considering discounting, it is more efficient to save lives in the future than today. Therefore, an investor might excuse himself for not investing in safety today. This example shows that life saving costs must be discounted so that the same amount of today's wealth is spent for life saving measures at different points in time, see also Lind (1994).

The present section argues for discounting of life saving costs as it is done, if these consequences are implemented into Equation 8.9. This is required for a consistent allocation of today's and future resources for safety measures. Approaches to identify appropriate rates of interest are briefly discussed in Section 8.4.

6.1.2 The LQI acceptance criterion based on mortality calculations

The LQI acceptance criterion as considered by Nathwani et al. (1997) and Rackwitz (2005) makes use of demography and calculates the effect of mortality changes on the life expectancy. Hereby, the life expectancy is calculated using methods identical to methods of time-variant

structural reliability, see Chapter 3. Considering the age of death as a first-passage problem one can write the life expectancy at birth l as the integral over the survival function $R(t)$.

$$l = \int_0^{\infty} R(t) dt \quad (6.34)$$

In terms of structural reliability, $R(t)$ can be considered as the reliability function. For non-homogeneous Poisson processes $R(t)$ can be calculated, if the hazard function $\nu(t)$ is known. In demography this function is called the age-dependent mortality rate. From life tables, $\nu(t)$ can be readily calculated to finally obtain the life expectancy.

$$l = \int_0^{\infty} e^{-\int_0^t \nu(\tau) d\tau} dt \quad (6.35)$$

In addition, for a stable population with population growth rate n the probability density function of the age distribution can be calculated

$$f_A(a) = \frac{e^{-na} R(a)}{\int_0^{\infty} e^{-na} R(a) da} \quad (6.36)$$

For $n = 0$, i.e. a stationary population, one obtains

$$f_A(a) = \frac{R(a)}{l} \quad (6.37)$$

In addition to the age-dependent mortality, the crude mortality m and its change dm are of use. Assuming that relative changes in crude mortality $\pi = dm/m$ lead to a constant proportional change of $\nu(t)$, i.e. independent of age, one can formulate the new age-dependent mortality $\nu_{\pi}(t)$ as

$$\nu_{\pi}(t) = \nu(t)(1 + \pi). \quad (6.38)$$

Substituting this term into Equation 6.35, one can expand it into a McLaurin series.

$$l(\pi) = \sum_{i=0}^{\infty} \pi^i \frac{d^i}{d\pi^i} \int_0^{\infty} e^{-\int_0^t \nu(\tau)(1+\pi) d\tau} dt \Big|_{\pi=0} \quad (6.39)$$

Taking into account only the first order term, one obtains following approximation.

$$\frac{dl}{l} = \frac{\frac{d}{d\pi} \int_0^{\infty} e^{-\int_0^t \nu(\tau)(1+\pi) d\tau} da}{\int_0^{\infty} R(a) da} \pi = \frac{\frac{d}{d\pi} \int_0^{\infty} R(a)^{1+\pi} da}{\int_0^{\infty} R(a) da} \pi \quad (6.40)$$

$$= \frac{\int_0^{\infty} \ln(R(a)) R(a)^{1+\pi} da}{\int_0^{\infty} R(a) da} \pi \quad (6.41)$$

Denoting the fraction by $-c_{\pi}$ the following equation is obtained.

$$\frac{dl}{l} = -c_{\pi} \frac{dm}{m} = -C_{\pi} dm \quad (6.42)$$

For developed countries c_{π} is obtained as $c_{\pi} \approx 0.15$. With $m \approx 0.01$ one obtains $C_{\pi} \approx 15$. For developing countries, the factor c_{π} and C_{π} may rise to more than 0.5 and 50, respectively.

From Equation 6.42 it is seen that a 1% increase in mortality reduces life expectancy by $c_\pi\%$. In general, c_π is an indicator for the shape of the reliability function. It is larger than or equal to zero and is smaller or equal to one. $c_\pi = 0$ represents the case when all people die at the same age which correspond to a rectangular reliability function.

The mortality regime that is considered by Equation 6.40 is characteristic for toxicologic exposures. In Rackwitz (2005), other mortality regimes are analyzed, as well. For the important case of structural failure, it can be considered that independent of the age, the change in mortality is Δ so that $v_\Delta(t) = v(t) + \Delta$. On that basis, the proportionality factor C_Δ is obtained to $C_\Delta = 45$. The implication of the selected mortality regimes and their effect on life expectancy, as well as the LQI-based acceptance criterion are studied and discussed in Rackwitz (2005), see also Lentz (2006).

The LQI criterion can be written as

$$dg_\Delta \geq \frac{1-w}{w} g C_\Delta dm. \quad (6.43)$$

Introducing $G_\Delta = \frac{1-w}{w} g C_\Delta$ and considering the limit of the inequality above, the equation can be reformulated to

$$dg_\Delta = G_\Delta dm, \quad (6.44)$$

where G_Δ^1 takes values around 5 million USD. In order to evaluate G_Δ , Rackwitz (2005) proposes to use just a fraction of g , namely $\tilde{g} \approx 0.7g$. The remaining part of g is bound for investments and should not be considered for safety improvements because it is the basis for a continuous development of society.

However, G_Δ neither accounts for the age distribution of the considered society, nor does it consider discounting. In Rackwitz (2005) it is shown that if consumption of an individual is assumed to be constant, discounting can be considered by discounting the life expectancy. Thereby, the discounted life expectancy l_d , a fictitious measure, is obtained. As l_d is a function of the individuals age, the age averaged measure $E_A \left[\frac{dl_d(a)}{l_d(a)} \right]$ can be computed and in turn the corresponding values $C_{\pi,d,a}$ and $C_{\Delta,d,a}$. The indexes d and a indicate that discounting and age-averaging have been accounted for. Whereas C_π and C_Δ differ significantly, the coefficients $C_{\pi,d,a}$ and $C_{\Delta,d,a}$ do not differ much, at least for OECD countries, see Table 6.2. Therefore, $G_{\pi,d,a}$ and $G_{\Delta,d,a}$ are roughly the same and lie for OECD countries around 2 million USD.

6.1.2.1 Application to Technical Facilities

In order to evaluate Equation 6.44 for acceptability assessments with regard to infrastructural facilities, the change in the mortality rate dm has to be evaluated. The mortality rate m is the expected annual number of fatalities N_f divided by the population size, i.e.

$$m = \frac{N_f}{N_{pop}}. \quad (6.45)$$

Treating N_{pop} as a constant, the differential is given by

$$dm = \frac{dN_f}{N_{pop}}. \quad (6.46)$$

¹Although G_Δ does not quantify the value of life, this quantity is also called the *societal value of a statistical life*.

| Country | C_{Δ} | C_{π} | $C_{\Delta,d,a}$ | $C_{\pi,d,a}$ | $G_{\Delta,d,a}$ [10^6 USD] |
|-------------|--------------|-----------|------------------|---------------|--------------------------------|
| Austria | 43 | 12 | 16 | 17 | 1.9 |
| Canada | 43 | 18 | 17 | 21 | 1.8 |
| Denmark | 42 | 13 | 16 | 17 | 1.7 |
| France | 43 | 13 | 16 | 18 | 1.9 |
| Germany | 44 | 12 | 16 | 16 | 1.9 |
| Italy | 42 | 12 | 15 | 17 | 1.8 |
| Switzerland | 43 | 13 | 16 | 19 | 1.8 |
| UK | 42 | 12 | 18 | 17 | 1.7 |
| USA | 44 | 16 | 18 | 18 | 2.1 |

Table 6.2: Social indicators for selected countries.

For civil engineering facilities, the change in the number of fatalities dN_f can be assessed by

$$dN_f = N_{PE}kd\lambda. \quad (6.47)$$

$d\lambda$ is the change of the occurrence rate of a structural failure, k is a person's probability of being killed given the incident occurs² and N_{PE} is the number of people exposed to the hazard. Rackwitz (2005) gives hazard-related values for k . Typically, these values range from 10^{-4} to 10^0 , e.g. for earthquakes, a reasonable value lies between 0.01 and 0.10. Finally, the LQI criterion can be written as

$$dC_a = dg_{\Delta}N_{pop} = G_{\Delta,d,a}kN_{PE}d\lambda. \quad (6.48)$$

With the acceptable annual costs per capita dg_{Δ} and the population size N_{pop} , one obtains dC_a the acceptable annual costs for a safety investments. If C_a and λ are functions of the design/decision variable z , one can also determine the acceptable design by

$$\frac{dC_a(z)}{dz} = G_{\Delta,d,a}kN_{PE} \frac{d\lambda(z)}{dz}. \quad (6.49)$$

For structures exposed to natural hazards it is straightforward and further meaningful to formulate $\lambda = P_{f,h}\lambda_h$ with λ_h as occurrence rate of the considered hazard and $P_{f,h}$ as the conditional failure probability given the hazard occurs. On this basis the influence of structural design can be analyzed on $P_{f,h}$, λ and C_a .

6.2 Prescribed Acceptance Criteria

Prescribed risk criteria are widely applied in engineering decision-making to assess whether a risk is acceptable or not. In the following, it is outlined how the Farmer diagram, the fatal accident rate and structural design codes may be used for risk assessments. As shown in Section 9.1, such prescribed risk criteria can be calibrated on the basis of the LQI.

²Here assumed to be constant, i.e. independent of the structural design. However, the consideration of a dependency is straightforward.

6.2.1 Farmer Diagram

A popular criterion for risk assessments are Farmer diagrams or so called F-N curves. The latter name results from the fact that acceptable frequencies (F) are plotted over the potential number (N) of fatalities. However, this type of curve is not at all limited to fatalities. This will be shown by means of the *Störfallverordnung*, see StfV (1991). Generally, the acceptable frequency is so small that the approximation 'Frequency = Probability' is valid. Therefore, one also finds diagrams that plot the acceptable probabilities over consequences. It is noted that with the acceptable frequency or probability a reference period is associated with it; typically, a one year period is chosen.

Generally, two lines separate the domain of the F-N diagram into three domains. The lower domain is the domain with acceptable risks and the upper domain represents unacceptable risks. The area in between marks the domain where acceptability is decided on cost effectiveness considerations. This principle is also known as ALARP standing for *as low as reasonably practicable*. In Figure 6.2 a particularity of the *Störfallverordnung* is seen. An additional domain is added that considers negligible consequences.

All F-N curves have the characteristic that with increasing consequences the acceptable probability is decreasing. In Figure 6.2 the acceptable probability is decreasing more rapidly than the consequences are increasing. For instance, whereas for 10 fatalities a probability of $10^{-7}/a$ is acceptable, only a probability of $10^{-9}/a$ is acceptable, if the number of fatalities is 100. That is, if the the number of fatalities is increased by an order of magnitude, the acceptable probability is reduced by two orders. Therewith, risk averseness is taken into account. However, there are also risk neutral F-N curves, see e.g. Jonkman, van Gelder, and Vrijling (2003).

The advantage of Farmer diagrams is that in these diagrams risk profiles of activities can be illustrated. This helps to compare various actions associated with different locations and/ or hazards. However, Farmer diagrams may not be consistent with decision theory as outlined in Chapter 2. For instance, consider the two discrete risk profiles A1 and A2 in Figure 6.2 on the right. Both profiles have the same expected consequences of $4.4 \cdot 10^{-7}$; however, A1 is considered to be acceptable, whereas the other alternative A2 requires an additional cost efficiency analysis, see also Hoej and Kroon (2001).

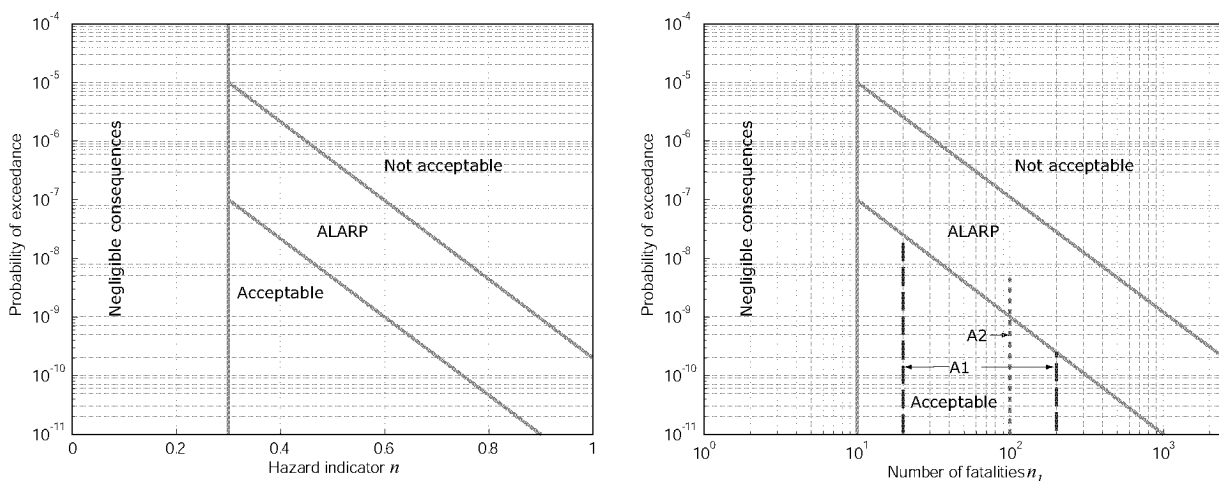


Figure 6.2: FN diagram according to the Swiss *Störfallverordnung*.

6.2.1.1 The Swiss Störfallverordnung

Two incidents lead to the Störfallverordnung, the Swiss regulation dealing with hazardous incidents. After the Seveso incident in 1976, protection against catastrophes was tied to the environmental protection law. Then after the chemical accident at Schweizerhalle (near Basel) in 1986, the need for a regulation was expressed leading to the Störfallverordnung which was put into force in 1991.

The left diagram in Figure 6.2 shows the F-N diagram according to StFV (1991). The acceptable probability is plotted over the hazard indicator n which ranges from 0 to 1. Values of n smaller than 0.3 represent accidents, and values larger than 0.5 are interpreted as catastrophes. The domain in between, i.e. $0.3 < n < 0.5$, characterizes major accidents.

Introducing the hazard indicator n has the benefit that different risks may be mapped into the same diagram. Therefore, they may be compared with each other. StFV (1991) considers 6 different consequence indicators namely:

1. n_1 number of fatalities,
2. n_2 number of injured,
3. n_3 amount of contaminated surface water in [km²],
4. n_4 amount of contaminated subsurface water, loss in [person month],
5. n_5 amount of contaminated soil in [km² a] and
6. n_6 material damage in [Million CHF (Base year 1996)].

In the case when a hazard leads to a combination of the above mentioned consequences, the dominant consequence is chosen and the hazard indicator is determined according to the scheme shown in Figure 6.3. Based on this transformation, the left diagram in Figure 6.2 can be mapped to obtain the diagram on the right.

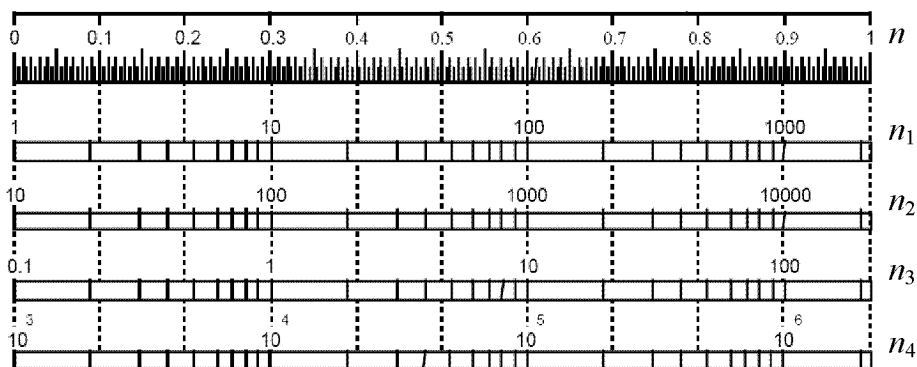


Figure 6.3: Evaluation of the hazard indicator n .

6.2.2 Fatal Accident Rate

The fatal accident rate (FAR) is a risk criterion that is established to quantify risks for specific groups of people, e.g. users or operational personnel. Besides FAR, other risk criteria to quantify risk are set up by authorities, see Paté-Cornell (1994).

The FAR is defined as the average number of fatalities within a predefined period of exposure, e.g. 10^8 hours of work. FAR is often used in quantitative risk analysis (QRA) and it is applied in the aviation and offshore industries, e.g. see Stahl, Aune, Gebara, and Cornell (1998). But risks due to building fires may be quantified as well, see Maag (2004).

For instance in the offshore industry, the FAR is defined as follows, see e.g. in Faber (2004).

$$FAR = \frac{PLL \cdot 10^8 \text{h}}{N_p H_p} \quad (6.50)$$

PLL is the potential loss of lives per annum, i.e. the annual expected number of fatalities. If the annual probability of occurrence of an event involving fatalities is given by P_f , and if N_f is the number of potential fatalities, it is calculated as

$$PLL = P_f N_f. \quad (6.51)$$

N_p is the number of persons exposed and H_p are the annual hours of exposure, e.g. if persons spend 24h a day at the facility, H_p is determined by $H_p = 360 \cdot 24 = 8,760$ h/year. Typical values for acceptable FAR range from 10 to 15.

6.2.3 Structural Design Codes

The most frequently applied risk acceptance criteria in civil engineering are structural design codes. Today, modern structural design codes provide a cost and time efficient framework for structural design that ensures an acceptable level of reliability/ safety. In order to achieve an acceptable safety level – e.g. a safety level according to the JCSS Probabilistic Model Code JCSS (2001b) – structural design codes need to be calibrated. Due to its importance, a whole chapter is devoted to this issue, see Chapter 10.

6.3 Summary

The value of life is infinite. However, by means of life saving cost the effectiveness of safety measures may be evaluated and can be compared to each other. If a safety measure is more cost efficient than an accepted value, then it can be implemented. The acceptable LSC is such a measure and it can be interpreted as the costs that an activity or safety measure can be more expensive, if it saves an anonymous person's life.

Even if this basis for decision-making is rejected, implied LSC that have been accepted can be calculated for any decision involving risk to life. This includes already made decisions, as well.

Based on the LQI criterion, it is shown how acceptance criteria may be derived. First, the life saving costs according to Skjong and Ronold are introduced. Their acceptable LSC considers the LQI criterion as the marginal rate of substitution between the life expectancy l and GDP per capita g . If however, the changes in l are not marginal, LSC according to the proposal of

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Rackwitz are more meaningful. It is also shown that the derived criteria also vary according to whether the derived annual costs need to be spent for l or $l/2$ years. This indicates that LQI research is still under development; but it converges.

Furthermore, the LQI acceptance criterion is introduced that is based on changes in mortality. The assessment of the relative change in life expectancy depending on mortality regimes is discussed and also the effect of discounting and age averaging.

The chapter closes with the introduction of other commonly applied acceptance criteria, namely the Farmer diagram, the fatal accident rate and structural design codes. It is shown that decision-making purely on the basis of F-N curves can be inconsistent with utilitarian decision theory.

7 LQI and Risk Aversion

Public decision-making involves a controversially discussed concept, namely *risk aversion*. After having defined normative decision theory under risk as the basis for engineering decision-making, the optimal decision is still sensitive to the underlying modeling of risk aversion.

The appropriate modeling of risk averseness is needed for two aims. Firstly, it is required for decision-making involving rationally acting opponents. Because only if the preferences of the opponents are modelled accurately, may their actions be predicted. Assessing their preferences also implies the assessment of their attitude towards risk. Secondly, risk aversion needs to be modelled for engineering decision-making in general. Only if the preferences of the decision maker are taken into account, may the optimal decision be identified. Considering the preferences of the decision maker also implies his or her attitude towards risk.

7.1 Reasons for Risk Aversion

There are different reasons for risk averse behavior. A list is given here that is not regarded as exhaustive, but it includes the main driving forces for risk averseness. It includes the assessment of follow-up consequences and uncertainties as well as the law of diminishing marginal utility. Moreover, multi-criteria decision-making and the possibility of not having unitary preferences are addressed as reasons for risk averse behavior.

Assessment of follow-up consequences

Indirect or also called follow-up consequences are difficult to assess. If they are not taken into account for structural design, a structure may be identified as being optimal that has a significantly lower reliability than is actually optimal, see Faber et al. (2004). In such cases decision makers who are aware of the significance of follow-up consequences compensate for not having quantified follow-up consequences with an risk averse attitude. In addition, the assessment of follow-up consequences is often subject to epistemic uncertainty.

Assessment of uncertainties

Expressing uncertainties in probabilistic terms involves epistemic uncertainties due to lack of knowledge, i.e. model uncertainty and statistical uncertainty. In some cases it is also necessary to express uncertainty using expert judgment with underlying high epistemic uncertainties. In such cases decision makers may tend to compensate lack of knowledge by risk averse behavior.

Law of diminishing marginal utility

In economics, it is well accepted that with increasing assets an additional asset increase does not yield a proportional increase in utility. The underlying principle is also known as the law of diminishing marginal utility introduced in Annex B.3.3.1. The law of diminishing marginal utility leads to a concave utility function that implies risk averseness.

Multi criteria decision analysis

Faced with the task of fulfilling multiple objectives, decision makers might not have expressed their preferences clearly among the various possible alternatives. This is more often the case than not, if socioeconomic consequences are involved such as the risk to life. For instance, LQI-based life saving costs provide a way out of this situation, see Chapter 6.

Non-unitary preferences of stakeholders

For instance, political decisions often involve risk averse behavior because the responsible decision maker may not have unitary preferences with the public. He or she might be willing to avoid negative outcomes of decisions that were made in his or her name. This kind of behavior is especially inviting as additional costs, e.g. for increased safety, are paid by the public and not by the decision maker him or herself.

These listed reasons can lead to risk averse behavior and it should be noted that an inappropriate risk averseness is paid by the expense of availing opportunities, see Section 2.3.4.1.1. Thus, risk averseness has to be modelled and quantified appropriately.

7.2 Risk Aversion Measure

A well known measure to quantify risk averseness implied by a utility function is the Arrow Pratt measure, see Pratt (1964). Although, concavity, i.e. the second derivative of the utility function, reflects risk averseness, it is not an appropriate measure because it is not invariant under a positive linear transformation, see Equation 2.13. To circumvent this, the curvature of the utility function is divided by the slope and multiplied with minus one. The latter ensures a positive value for the measure of *absolute risk aversion* $\iota(x)$, if the behavior is risk averse.

$$\iota(x) = -\frac{u''(x)}{u'(x)} \quad (7.1)$$

$$= -\frac{d}{dx} \ln(u'(x)) \quad (7.2)$$

$\iota(x)$ is a function of the local risk aversion at x . If $\iota(x) = \iota$ is constant, the following utility functions are obtained.

$$\begin{aligned} u(x) &= x, & \iota &= 0 \\ u(x) &= e^{-\iota x}, & \iota &< 0 \\ u(x) &= -e^{-\iota x}, & \iota &> 0 \end{aligned} \quad (7.3)$$

Besides the measure of absolute risk aversion $\iota(x)$, there is the measure of *relative risk aversion* $\iota^*(x)$.

$$\iota^*(x) = -\frac{du'(x)/dx}{u'(x)} \cdot \frac{1}{x} \quad (7.4)$$

$$= \iota(x)x \quad (7.5)$$

A constant relative risk aversion $\iota^*(x) = \iota^*$ is implied by the following utility functions.

$$\begin{aligned} u(x) &= x^{1-\iota^*}, & \iota^* &\neq 1 \\ u(x) &= \ln(x), & \iota^* &= 1 \end{aligned} \quad (7.6)$$

In economics, it is preferred to use the positive linear transformation $\tilde{u}(x) = \frac{x^{1-t^*}}{1-t^*}$ instead of $u(x) = x^{1-t^*}$ because for $(1 - t^*) \rightarrow 0$ the utility function $\tilde{u}(x)$ converges to $\ln(x)$. Nonetheless, both utility functions represent the same preferences. Comparing Equation 7.6 with the classical LQI, $L_c = g^w l^{1-w}$, it is seen with $(0 \leq w \leq 1)$ that the LQI implies a constant relative risk measure of $t_g^* = 1 - w$ with regard to g and $t_l^* = w$ with regard to l . A case where risk aversion implied in the LQI plays a significant role is outlined in the following.

7.3 LQI and the Asteroid

The earth is regularly hit by asteroids, mostly with insignificant consequences. In the following, decision-making involving the hazard of an asteroid impact is investigated. The consideration is limited to asteroids with the potential of causing a large number of fatalities so that the change in life expectancy is not marginal.

In order to study the decision-making process, two cases are distinguished, namely the conditional and the unconditional case. The conditional case considers that the public is aware that an asteroid would hit their territory very soon, leading to catastrophic consequences. In the unconditional case, the population is aware of the potential threat resulting from an asteroid impact but it considers the event to be unlikely to happen in the near future.

7.3.1 The Conditional Case

In the conditional case, the public is faced with the threat of a catastrophic event leading to a large number of fatalities and reducing life expectancy such that the associated change dI/I is not marginal, i.e. not small. In this case, the assessment of acceptable life saving costs assessed according to Equation 6.6 and 6.33 is inappropriate. Equation 6.8 or 6.12 have to be considered for changes in life expectancy that are not marginally. Equation 6.12 assesses the willingness of the public to avoid the hazard. In the extreme case, the whole of society is affected by the hazard with certainty so that $|\Delta I| = I$. This in turn makes Δg become infinite, see also Figure 6.1. The interpretation of this is that in this specific case, the public would be willing to spend anything to avoid the hazard it faces.

7.3.2 The Unconditional Case

In the unconditional case, the public is aware of the threat but it considers the occurrence probability to be very low. This means the mortality caused by an asteroid impact is multiplied by the occurrence probability and the change in I becomes marginal, i.e. small. In this case the hazard from asteroid impacts is regarded as a contributor, such as many other risk contributors, e.g. traffic accidents, environmental pollution or building safety. Only if safety measures against asteroid impact are assessed to be more efficient than other safety measures, will they be implemented.

This consideration is not limited to asteroid impacts only. Generally, it can apply to any hazard. Independent of the specific hazard scenario, in the extreme case, the public and also individuals are willing do anything for their self-preservation.

7.4 Small Consequences

Consequences associated with civil engineering facilities can e.g. be large, major or catastrophic. For instance, the failure of the WTC Twin Towers represents such a case where follow-up consequences were many times larger than the direct failure consequences of the two towers, see Faber et al. (2004). Considering this specific incident, it is noted that many decisions that were made following this event are attributed to the fact that the towers failed due to an act of terrorism.

However, most consequences associated with single civil engineering facilities can a priori be considered as being small, if they are regarded on a socioeconomic scale. For instance, the WTC consequences that have been assessed represent less than 1% of the US GDP. Considering small consequences Δx , the relevant domain is limited to around the current value x_0 (g or l). If the consequences are small enough, the utility function $u(x)$ can be well represented by a linear hyper plane $\tilde{u}(x)$ as seen in Figure 7.1. Therefore are in the context of societal decision making risk neutral utility functions appropriate. An introduction of a risk aversion factors as an approximation for more rigorous assessments of consequences will in general only coincidentally lead to optimal decisions and should be avoided.

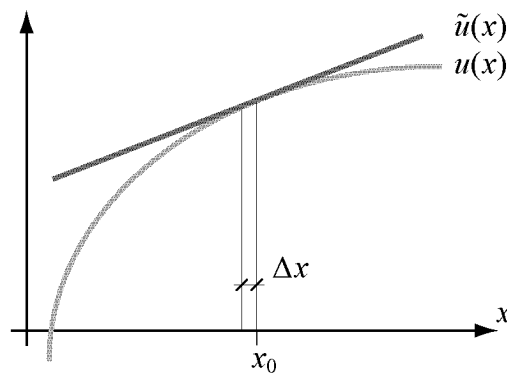


Figure 7.1: Small consequences.

7.5 Perception of Risks

In Section 7.1 it is discussed that the difficulties in the assessment of consequences and uncertainties may lead to risk averse behavior. These assessments may depend on the person that assesses the risks and therefore may differ between different persons, e.g. lay-people and experts assess/perceive risks differently, see Slovic (1987). However Nathwani et al. (1997) argue that if risks are assessed by experts, it should be given more weight compared to the perception of lay-people. As experts are humans, their judgement may be erroneous, as well. However in general, they are more familiar with the considered phenomenon and have more relevant information available in order to carry out transparent risk assessments.

7.6 Summary

Even if it is agreed on the basis of utilitarian decision-making, decision-making is still sensitive to the modeling of risk aversion.

The main reasons for risk aversion are that decision makers compensate for unquantified follow-up consequences, uncertainties or socioeconomic consequences. Furthermore, the law of diminishing marginal utility stimulates risk averse behavior, as well as non-unitary preferences of the stakeholders, e.g. between the decision makers and the group on behalf of which the decision maker acts.

Arrow and Pratt introduced two measures to quantify risk aversion. It is shown that the classical LQI formulation implies risk averse preferences with regard to wealth and life expectancy. This is illustrated by means of a simple example. It shows that the LQI formulation in the extreme is also able to describe self-preservation of individuals or societies. However, consequences associated with single civil engineering facilities are mostly small, if considered on a socioeconomic scale.

Part III

Applied Decision Theory in Civil Engineering

8 Decision and Optimal Design

The two preceding parts introduced different topics of engineering decision-making. The present chapter puts the different topics together into a consistent framework for engineering decision-making, such as the optimal design of structures.

First, the objective of structural design is discussed and approaches to it are identified such as the life cycle benefit optimization. This measure is subject to uncertainty and therefore decision-making needs to take into account the involved uncertainty by evaluating the expected value. In order to do so, all events leading to relevant consequences (costs and revenues) have to be accounted for. Among others, this includes operation and maintenance activity, potential inspections and repairs, as well as possible structural failures, reconstructions and decommissioning. These considerations depend on the modelling of the system and therefore on expert judgment. Due to this fact, the influence of system modelling on the expected life cycle benefit is investigated. Finally, after having discussed notional probabilities and the influence of gross errors, the decisive topic, discounting is briefly addressed.

8.1 Objective of Structural Design

Paragraph 2.3.1 of SIA 260:2003 defines the demand on structures and thereby the mission statement for civil engineers.

2.3.1 Apart from being appropriately integrated, configured and reliable, a structure should be economic, robust and durable.

Focusing on the aspects of structural design, this paragraph which similarly is found in modern structural design codes states that: Structures should be appropriately safe, robust, durable and economic. Safety, robustness and durability are competing with economics. Due to this, a framework or design principle to balance these characteristics has to be identified.

If a utility function U is able to consider the preferences with respect to these characteristics, e.g. the LQI, the mission statement can be formulated to maximize $U(\mathbf{z})$. U is depending of decision/ design variables \mathbf{z} . In addition, any practical decision problems requires the consideration of additional constraints $h_i(\mathbf{z}) = 0$ and $h'_j(\mathbf{z}) \leq 0$. Finally, the optimization problem can be written as

$$\begin{aligned} \max_{\mathbf{z}} U(\mathbf{z}) & & (8.1) \\ \text{s.t. } h_i(\mathbf{z}) = 0, \quad i = 1, \dots, n_i \\ h'_j(\mathbf{z}) \leq 0, \quad j = 1, \dots, n_j, \end{aligned}$$

see also Keeney and Raiffa (1976).

The development of the LQI is still in progress and the application of different LQI formulations, e.g. L_c or L_q can lead to different optimal decision variables. This is because they differ in terms of how they account for risk aversion, see Chapter 7. However, the derived LSC are

the same for both formulations L_c and L_q , and the derived LSC can be considered in other well known design principles.

Many design principles have been proposed and applied during the long history of civil engineering. It was always the aim to build the best structures with the available resources. Initially, the main resource was the material that was used for construction. Therefore, the volume of the material or respectively the material costs were minimized. Labor was and still is another important resource that is needed for construction. Given different design alternatives with varying demands for labor and material, it can be optimal to choose the alternative where more material is used but less labor. Hence, the overall construction costs are minimized. In the following, it is considered that construction costs can be distinguished by variable costs $C_{C,var}$ that are influenced by decision/ design variables and fixed costs $C_{C,fix}$ that are independent of it. But only considering the construction costs for the design of structures may not be optimal at all. This was especially experienced in recent decades, where large sums were spent to correct for the effects resulting from deterioration processes. The aim of the life cycle cost principle is to take into account all potential costs. It accounts for all events yielding negative consequences that are associated with the investigated structure. This includes, operation C_O , maintenance C_M , inspection C_I and repair costs C_{Rep} , as well as decommissioning costs C_D and the consequences of potential structural failure C_F . The latter have to account for all consequences associated directly or indirectly to the adverse event – see Chapter 4 – which also includes the willingness to pay for safety as outlined in Chapter 5-7. The different approaches to optimal design are illustrated in Figure 8.1.

| | | | |
|--------------------|--|--|---------|
| Life cycle benefit | | | |
| Life cycle costs | | | Revenue |
| Construction costs | | Inspection, maintenance, failure and decommissioning costs | |
| Material costs | Labor, project development costs, etc. | | |



 Approach to optimal design
 Considered consequences (costs and revenues)

Figure 8.1: Approaches to optimal design.

$$LCC = (C_{C,var} + C_{C,fix}) + C_O + C_I + C_M + C_{Rep} + C_F + C_D \quad (8.2)$$

$$\begin{aligned}
 LCB &= (C_{C,var} + C_{C,fix}) + C_O + C_M + C_I + C_{Rep} + C_F + C_D + C_{Rev} \\
 &= LCC + C_{Rev}
 \end{aligned} \quad (8.3)$$

In this figure, it is seen that if the life cycle costs LCC are considered together with the revenues C_{Rev} , the life cycle benefit LCB is obtained. Moreover, it is seen that the maximization of the life cycle benefit is the most general approach. In Section 9.1, a principal study illustrates that in specific cases the inclusion of revenues may lead to more reliable structures. In Figure 8.1, it is also shown that the life cycle benefit approach may yield the same optimal design decision as the minimization of the life cycle costs. This is the case, if the consideration of revenues is not decisive for the identified optimal decision, i.e. the optimal design variables. In this case, $\frac{\partial C_{Rev}}{\partial z_i} = 0$ with z_i as the decision/ design variables. In cases where it is meaningful to also neglect labor, operation, maintenance, inspection, repair, failure etc., the minimization of the material costs leads to the maximization of the life cycle benefit.

Generally, the life cycle benefit is considered for a single structure, but Rosenblueth and Mendoza (1971) show that it is possible to look beyond the service life of a single structure. It is also possible to take into account the effects of subsequent structures. In this case, one should rather speak of the life cycle benefit of an activity, an activity that requires civil engineering facilities. Activities like this are transportation of persons and goods by individual and public transport using streets, tunnels, bridges, railways, airports, harbors etc. or to provide space for business activities such as industrial production, retail or offices for consulting, finance, insurance, real estate, education, etc. Another important sector, which depends on civil engineering facilities, is energy supply. Hence, civil engineering facilities represent a crucial part of the backbone of any society.

8.1.1 Expected Life Cycle Benefit

The safe and reliable operation of civil engineering facilities is of paramount importance for the continued development of societies. However, the investment bound up in the development and maintenance of such facilities constitutes a considerable percentage of the available resources. As a consequence of this, a way to balance these characteristics is required. As described before, the life cycle benefit optimization is an appropriate approach to consider all consequences associated with an activity. However, the events leading to consequences and the resulting consequences are subject to uncertainty. According to normative decision-making under risk as introduced in Chapter 2, decisions should be made according to expected utilities. Given the life cycle benefit as the utility function, the expected life cycle benefit should be maximized. As structures represent long term commitments, the cash-flows (negative costs and positive revenues) need to be *discounted*, see Section 2.5.1.1. Considering this, the proposed design principle is equivalent to the *maximization of the expected net present value*, see Section 2.5.2. Figure 8.2 illustrates the approach. It is seen that structural design variables influences both, the structural reliability and the associated consequences. In order to assess the structural reliability, the resistance and the exposure of the structure need to be considered. With regard to the exposure, all loads e.g. wind, dead, live and fire load have to be taken into account, as well as the environmental exposure leading to deterioration of the structural resistance. Together with the associated consequences, which are at first divided into consequences to humans, the economy, cultural assets and the environment – for other categorizations of consequences see Chapter 4 – the life cycle benefit can be evaluated, and if all aspects are expressed in probabilistic and monetary/ utilitarian terms its expected value B can be determined.

$$B = E[LCB] \quad (8.4)$$

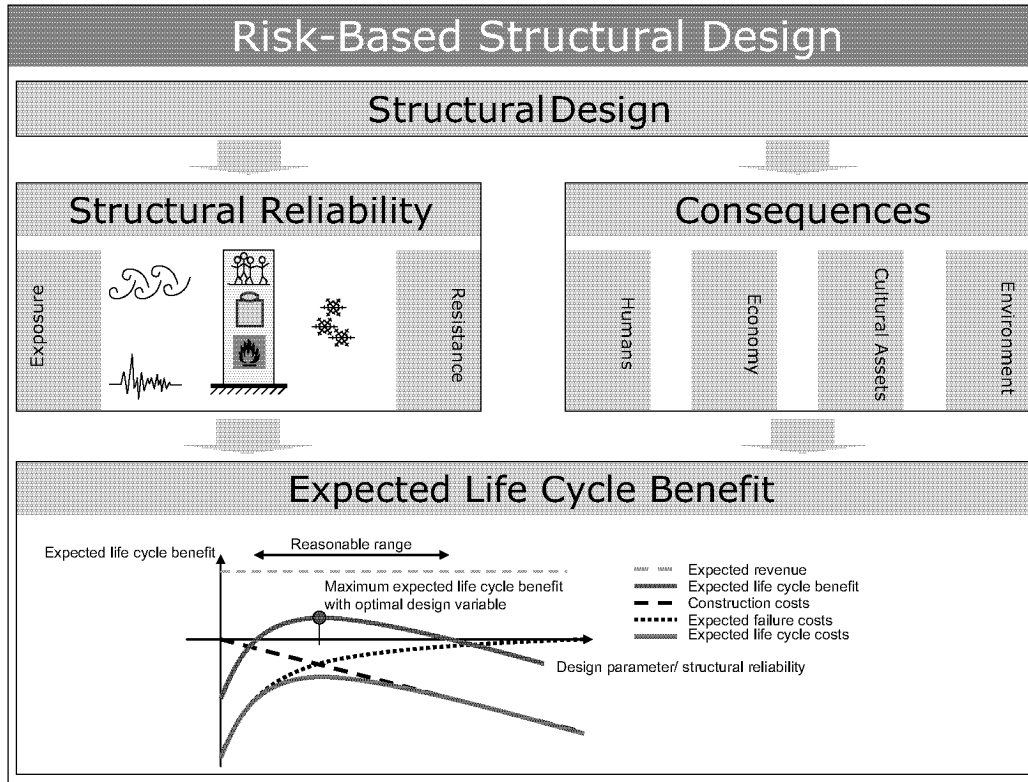


Figure 8.2: Framework for risk-based structural design.

$$B = E[C_{C,var}] + E[C_{C,fix}] + E[C_O] + E[C_M] + E[C_I] + E[C_{Rep}] + E[C_F] + E[C_D] + E[C_{Rev}] \quad (8.5)$$

If the design variables – e.g. the cross-sectional area, section modulus, material characteristics or the design lifetime – are summarized in the vector \mathbf{z} , the expected life cycle benefit can be written as

$$B = B(\mathbf{z}) \quad (8.6)$$

$$= E[C_{C,var}(\mathbf{z})] + E[C_{C,fix}] + E[C_O(\mathbf{z})] + E[C_M(\mathbf{z})] + E[C_I(\mathbf{z})] + E[C_{Rep}(\mathbf{z})] + E[C_F(\mathbf{z})] + E[C_D(\mathbf{z})] + E[C_{Rev}(\mathbf{z})] \quad (8.7)$$

and the optimum design \mathbf{z}^* is finally determined, which maximizes B , i.e.

$$B^* = B(\mathbf{z}^*) = \max_{\mathbf{z}} B(\mathbf{z}). \quad (8.8)$$

8.1.2 Life Cycle Modeling

The principle of the expected life cycle benefit maximization for structural design was already introduced by Rosenblueth and Mendoza (1971). Almost 30 years later, the idea was taken up again by Rackwitz (2000), who proposes it as the basis of code making. Besides negative costs, these authors also consider positive revenues that are generated from the activity the structure supports. Moreover, they also address the possibility of structures failing and their possible

reconstruction. Hereby, structural failures are treated as realizations of Renewal processes. For instance, it can be thought of a non-homogenous Poisson process that results from non-stationary loads or resistances, e.g. due to deterioration processes such as fatigue or corrosion.

In order to identify the optimum life cycle costs, Faber (1997) formulates a pre-posterior decision problem in accordance with Bayesian decision theory, see Section 2.4. Using decision trees permits him to account for the effects of maintenance, inspections and repairs. Furthermore, by use of Bayes' theorem, probabilities may be updated, when additional information is available, see Section 3.3.

Kübler and Faber (2003) integrate these two developments and the expected life cycle benefit is formulated, so that the optimal design together with the optimal inspection and maintenance plan can be assessed. Following this, the expected life cycle benefit can be written as

$$B(\mathbf{z}, \mathbf{i}, d) = E_{\mathbf{X}, \Theta, \Psi | \mathbf{z}, \mathbf{i}, d} [LCB(\mathbf{z}, \mathbf{i}, \Psi, d(\Psi, \mathbf{t}_f), \mathbf{x})]. \quad (8.9)$$

In Equation 8.9, $B(\mathbf{z}, \mathbf{i}, d)$ is the expected life cycle benefit and it is obtained when the expectation operation $E[\cdot]$ is performed over LCB . The expected life cycle benefit is a function of the decision variables \mathbf{z} , \mathbf{i} and d . \mathbf{z} is a vector containing the structural design variables such as section modulus or the cross-sectional area. $\mathbf{i} = (\mathbf{t}_i, \mathbf{l}_i, \mathbf{q}_i)^T$ is the vector containing the parameters of the inspection plan, namely $\mathbf{t}_i = (t_{i,1}, t_{i,2}, \dots, t_{i,n_i})^T$ the vector with n_i inspection times, \mathbf{l}_i the vector of inspection locations and \mathbf{q}_i , the vector containing the different possible inspection qualities. The latter depends on the chosen inspection method. The uncertain inspection results are represented by the random vector Ψ . Based on normal form representation (Section 2.4.3.1) $d(\Psi, \mathbf{t}_f)$ is defined as a decision rule, which describes the rehabilitation strategy following an inspection result $\Psi = \psi$ or a structural failure at the times $\mathbf{T}_f = \mathbf{t}_f$. Finally, \mathbf{X} is a vector containing the basic random variables required for the probabilistic modeling of loads, material characteristics and consequences. The optimal structural design together with the best inspection and maintenance plan $(\mathbf{z}^*, \mathbf{i}^*, d^*)$ is finally identified by the values maximizing the expected life cycle benefit B . Principal studies showing the application of Equation 8.9 are given in Chapter 9.

8.2 System Modelling

As described in the foregoing, the expected life cycle benefit can be evaluated for any activity supported by a civil engineering structure. However, it depends on the underlying system modelling S_i , which also involves engineering judgment, see Faber and Maes (2005). All uncertainties that are expressed in probabilistic terms – inherent, model and statistical uncertainty – may be taken into account so that the conditional expected life cycle benefit is obtained.

$$B_i = E [LCB | S_i] \quad (8.10)$$

Figure 8.3 illustrates the single components out of which systems can be defined. These components represent risk contributors, i.e. events, the associated uncertainties and/or consequences. The white boxes represent components that are irrelevant for the decision problem and subsequently do not change the identified optimal decision. The grey risk contributors are needed to fully represent the *true* system S^* . But not all are known, recognized, taken into account etc., see Figure 8.4 according to Schneider (1996). Given different B_i it is consistent to consider the identification of the optimal decision within the framework of decision-making

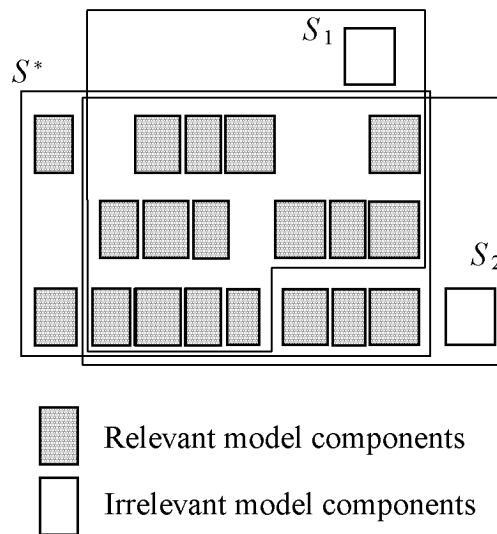


Figure 8.3: System definition.

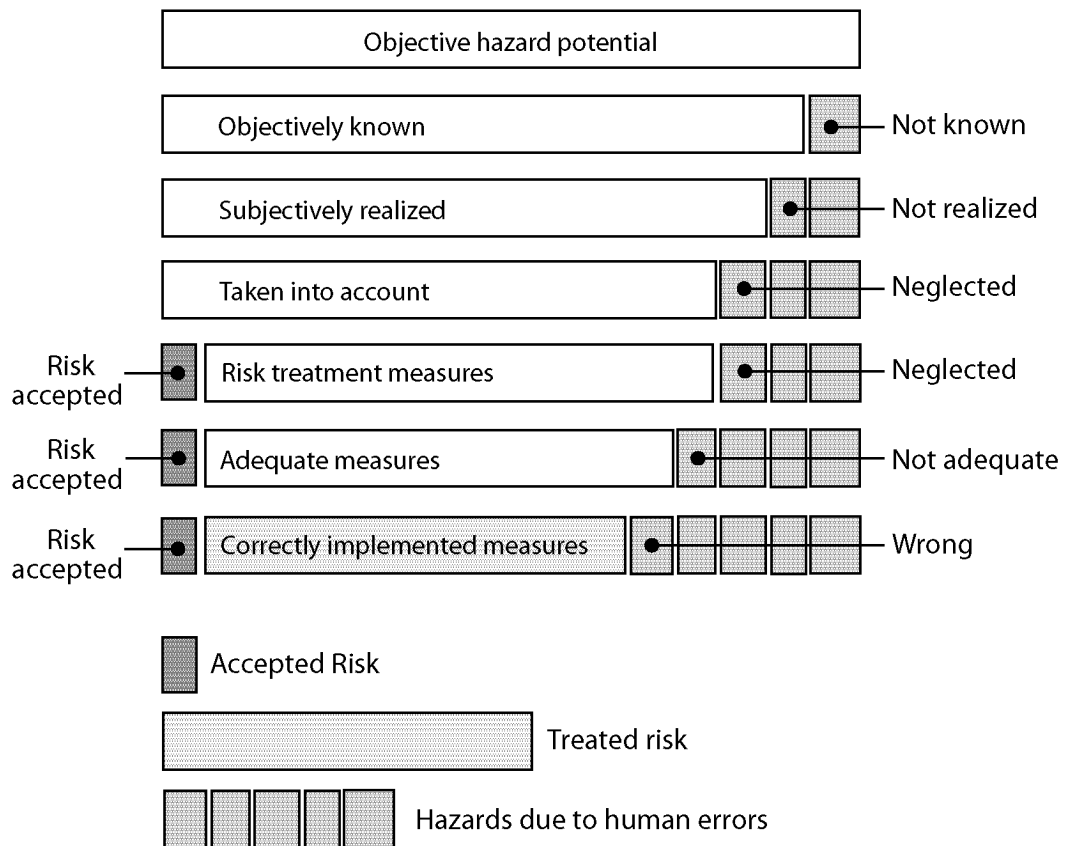


Figure 8.4: Hazard potential and residual risk.

under uncertainties. In this decision framework, several decision criteria may be applied e.g. the Hurwicz, Niehans-Savage or Laplace criteria using the corresponding preference function Π ,

see Chapter 2. The Laplace criterion makes use of Bernoulli's principle of insufficient reasoning and evaluates the different system definitions as being equally representative of the true system S^* . The ideas to use subjective instead of uniformly distributed probabilities is straightforward and allows for the combination of additional available expert judgement. Considering expert judgment using subjective probabilities consistently combines all available knowledge. However, the expected value obtained is again a conditional expected value. It is assumed that with consistently increased knowledge about the system, the system definition converges to S^* .

8.3 On Notional Probabilities and Gross Errors

In the foregoing, optimum structural design is introduced as a decision problem. The underlying procedure assumes that failure consequences and failures probabilities can be assessed so that optimal decision variables can be derived yielding the highest expected utility.

Therefore, the assessment of failure probabilities is crucial for risk-based optimum design as well as decision-making. Critiques of structural reliability methods refer to works such as Matoussek and Schneider (1976), which show that observed structural failures practically always result from human errors. Therefore, criticism is directed towards the calculated failure probabilities. It is stated that the calculated probabilities are fictitious measures of safety and the term notional probabilities is used. The calculated probabilities are also suggested to be surrogates for the observed relative frequencies because gross errors are often neglected in risk-based optimum design. Another point of criticism is that the choice of the probabilistic models can influence the analysis outcome considerably. In the following, the influence of notional probabilities on optimal design is analyzed, and then, the question of the most appropriate probabilistic model is discussed.

8.3.1 Notional Probabilities

Detailed summaries of failure causes are found in e.g. Matoussek and Schneider (1976). As mentioned, the calculated nominal failure probabilities p_{fn} using methods of structural reliability do not represent relative frequencies that can be observed. This is because gross errors from organizational and human factors are often neglected. One reason is that this quantity is hard to assess statistically, but "this is not to say, however, that it is nonprobabilistic in nature", Cornell (1969a).

The influence of gross errors on the failure probability is indicated by p_{fg} . Assuming independency, the failure probability p_f is $p_f = p_{fn} + p_{fg}$. From this, it is seen that the inclusion of human and organizational factors is important to assess realistic failure probabilities. However, are they important for optimal design?

8.3.2 Optimization Including Gross Errors

The following argumentation is taken from Melchers (1999), see also Ditlevsen (1983) and Baker and Wyatt (1979). Consider the simple case, where the expected utility may be represented with the expected total costs.

$$E[C_T] = C_C(p_{fn}) + (p_{fn} + p_{fg})C_F \quad (8.11)$$

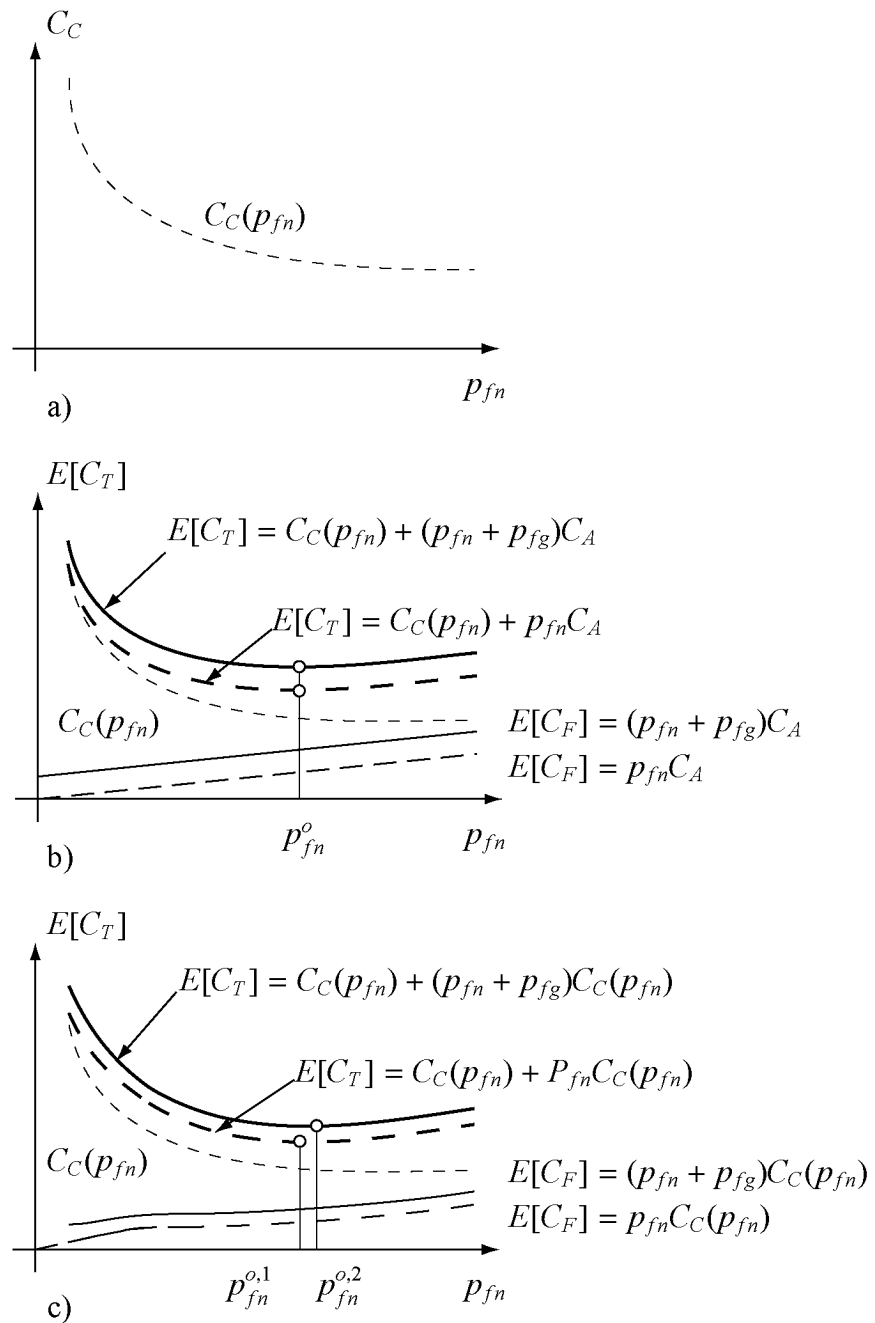


Figure 8.5: Optimization to find the optimal nominal probability p_{fn}^o .

The expected total costs $E[C_T]$ depend on the construction costs $C_C(p_{fn})$ which are a function of the nominal probability p_{fn} . In Figure 8.5a, it is seen that a low nominal failure probability is achieved with high construction costs, whereby costs decrease, if p_{fn} increases. In addition, the present valued failure costs C_F may contribute to the total cost. Its probability of occurrence is $p_f = p_{fn} + p_{fg}$, where p_{fg} accounts for gross errors (human and organizational factors). Generally, C_F comprises reconstruction costs and additional costs C_A , e.g. costs for clean-up.

For convenience it is assumed that the reconstruction costs are equal to the construction costs C_C . Then two extremes are possible:

1. : $C_F \approx C_A$, if $C_A \gg C_C$ and

2. : $C_F \approx C_C$, if $C_A \ll C_C$.

The two cases are represented in 8.5b and 8.5c. In Figure 8.5b, the case $C_F = C_A$ is illustrated. It is seen that the influence of p_{fg} has no effect on the optimal choice of p_{fn} . This is because p_{fg} only influences the expected failure costs and the inclusion of p_{fg} represents a parallel shift of $E[C_F]$ as well as $E[C_T]$. Therefore, the same optimal nominal failure probability is obtained p_{fn}^o .

The second extreme, $C_F = C_C$, is illustrated in Figure 8.5c. It is seen that despite the influence of p_{fg} the optimal choices probabilities $p_{fn}^{o,1}$ and $p_{fn}^{o,2}$ do not differ much, provided that p_{fn} is independent of p_{fg} . It is assumed that e.g. failures from bad workmanship are independent of the safety level given by the structural design. However, p_{fn} and p_{fg} may be slightly positively correlated because an increase in ductile capacity reduces p_{fn} but also makes the structure more reliable gross errors, i.e. reduces p_{fg} . On the other hand, it can be said that an increase or decrease of p_{fg} has no impact on p_{fn} .

Following this argumentation, the process of finding the optimal reliability level may be decoupled into two optimization problems.

1. Optimization of the optimal nominal reliability expressed in p_{fn}^o .

2. Optimization of the optimal reliability level regarding gross errors expressed in p_{fg}^o .

The global optimum is then obtained by solving each optimization problem. In addition, a third optimization level may be added, which considers the optimal resource allocation with regard to measures not affecting the structures reliability e.g. the choice of the optimal floor covering of a building described by the decision variable a_f . Whereas only the first optimization problem is needed to find the optimum structural design and partial safety factors, all three contributors (a_f, p_{fn}, p_{fg}) need to be considered, if the utility of the structure is compared with other possible investments.

8.3.3 The Influence of Probabilistic Models

If p_{fn} is identified, critiques may still counter that there is an infinite number of possible designs which yield the nominal failure probability p_{fn} . It is only a question of how to choose the underlying probabilistic model. Figure 8.6 shows the influence of two probabilistic models. It compares the partial safety factor for a resistance variable γ_M as described in Chapter 10. Using the classical method to derive partial safety factors, the factor is determined by Equations 10.21 and 10.25. The first equation considers the basic variable to be normally distributed, whereas the second considers a lognormally distributed variable. The sensitivity factor was chosen to $\alpha_R = 0.8$ valid for dominant resistance variables. Moreover, the coefficient of variation is selected $V_R = 0.10$ and the quantile defining the characteristic value 5%, i.e. $k_R = 1.64$. From Figure 8.6 it is seen that depending on the chosen distribution, for $\gamma_M = 1.3$ a reliability index of $\beta \approx 4.5$ or $\beta \approx 5.3$ may be obtained. This is considerably different.

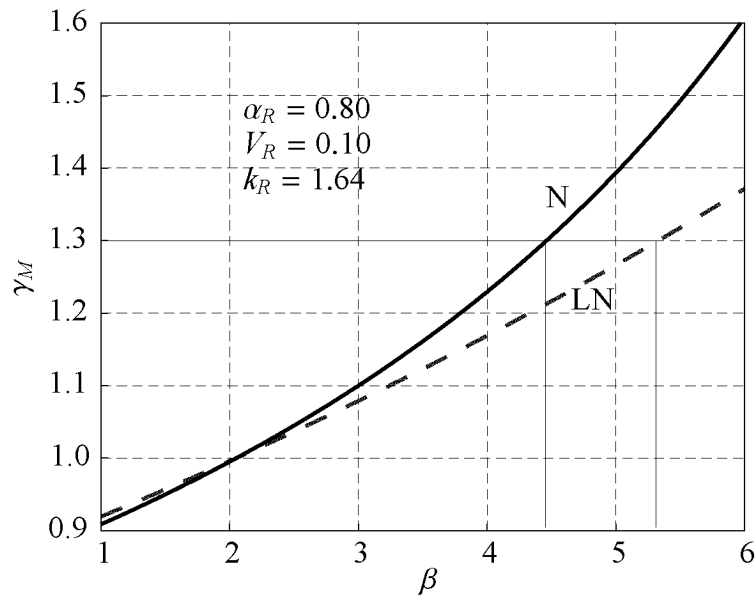


Figure 8.6: Resistance partial safety factors for normal and lognormal model.

This simple example shows that reliability analyses are only comparable, if a common probabilistic model is applied that is appropriate for the considered case. In section 8.2 it is argued that the most appropriate approach is to bring together expertise and integrate it in the decision framework. Considering models for structural reliability analysis, this task has been taken over by the Joint Committee on Structural Safety (JCSS) which publishes the Probabilistic Model Code, JCSS (2001b). A common basis for probabilistic models is available and will consequently be improved and complemented.

8.4 Discounting for Optimal Design

Investments in civil engineering structures represent long term commitments. Therefore, costs as well as revenues need to be discounted, see Section 2.5.1.1. This also includes life saving costs as argued in Section 6.1.1.5. However, neither section indicate how an appropriate rate of interest is quantified. One can consider many interest rates. On the financial market, there are short or long term, before or after tax and high or low risk interest rates etc. A criterion for the appropriate choice of interest rates is that it should reflect the preferences of the decision maker.

Corotis (2005) outlines the differences between private and public investments and argues that the rate of interest can be different for public and private investors. Rackwitz et al. (2005) and Rackwitz (2005) discuss the aspect of public discounting at length. These works suggested an intergenerational discounting model which takes into account the economic growth per capita δ . It is also called the *natural interest rate*. In addition, the elasticity of marginal consumption ϵ is considered and the rate of pure time preference ρ . The latter takes into account that individuals rather prefer to consume earlier than later. In this way, the interest rate γ is obtained.

$$\gamma = \rho + \delta\epsilon \quad (8.12)$$

The model suggested by Rackwitz et al. (2005) considers that all affected generations discount with γ . However, the consequences for the unborn generations are discounted only with $\delta\epsilon$. Using the same basis, Nishijima et al. (2005) determine public interest rates net of inflation averaging around 2% annually, i.e. significantly lower than assigned for private investments.

8.5 Summary

The task of civil engineers is to provide safe and economic structures. However, safety and economics compete with each other so that these characteristics need to be balanced to obtain an optimum. This can be achieved using different design approaches. The most important approaches are the minimization of the material, construction or the life cycle costs. A more general approach aims to maximize the life cycle benefit. It considers revenues besides construction, operation, inspection, maintenance, repair, failure and decommissioning costs. As these costs and revenues are subject to uncertainty, the expected utility, i.e. the expected life cycle benefit has to be evaluated. Decision-making on that basis is equivalent to decision-making according to the expected net present value.

This means that all uncertainties that are expressed probabilistically can be taken into account in decision-making. If uncertainties still remain, decision-making under uncertainties is the appropriate framework. Here, several decision criteria using preference functions can be applied, which can lead to different optimal decisions. Intuitively, many decision makers make use of the Laplace criterion that assumes all possible states as being equally likely. However, when expert judgment is available, then it is straightforward to express it in terms of subjective probabilities. This approach allows e.g. code committees to consistently combine all available knowledge.

A criticism often directed towards risk-based optimum design is that the identified failure probabilities represent so called notional probabilities. Indeed, for the determination of an observable failure frequency, it is necessary to include gross errors, as well. However, for the objective of optimal resource allocation with regard to structural reliability, it is shown that the inclusion of gross errors is generally not relevant. Moreover, it is shown that the overall optimization can be separated into several optimization problems.

Finally, a very decisive component of decision analysis is addressed, namely discounting. It is argued that the interest rate needs to represent the preferences of the decision maker. Corotis (2005) discusses the differences between public and private investments that argue for different discount rates. Finally, an approach is mentioned that considers the personal preferences of cohorts, Rackwitz et al. (2005).

9 Principal Studies on Optimal Design

The preceding chapter summarizes the different aspects of engineering decision-making into a decision theoretical framework. It concludes that decisions should maximize the expected life cycle benefit of the activity that is supported by a civil engineering facility. This framework was first introduced by Rosenblueth and Mendoza (1971) and taken up again by Rackwitz (2000).

The present chapter illustrates the proposed framework, by means of principal studies. Equation 8.9 can be seen as a very general representation of the expected life cycle benefit, but given the variety of activities, structures and systems, the expected life cycle benefit evaluation requires the consideration of different contributors, e.g. deterioration effects, possible reconstruction of failed structures, revenues etc.

The following principal studies illustrate how these effects can be considered within a life cycle approach. The first study shows the optimal design of an offshore platform, where it is shown that different design approaches identify different designs to be optimal. In addition, the effects of influencing variables are studied. Following this, the optimum design of a concrete bridge is analyzed that takes into account the influence of deterioration, inspections and possible repairs. Finally, aspects considering the possibility of reconstructions are discussed.

9.1 Optimal Design of Offshore Platforms

During recent years, an increased focus has been directed towards the aspects of optimality and acceptance criteria for the design and operation of offshore facilities, e.g. Stahl et al. (1998), Skjong and Ronold (1998) and Pinna, Ronalds, and Andrich (2001). Decisions on structural strength parameters, such as the reserve strength ratio RSR for fixed steel offshore structures, may be optimized on the basis of considerations of past practice and cost benefit considerations together with the legislative requirements for the safety of personnel imposed by regulatory bodies. So far, considerations regarding the optimal design and acceptance criteria for the design and operation of offshore facilities differentiate the consequences of failure between cost consequences and fatalities. Furthermore, the aspect of benefit is largely overseen. As shown in Chapter 6, the consequences of fatalities in case of failures may be included in terms of acceptable life saving costs derived from the life quality index. Furthermore, the benefit associated with a given activity, such as the installation and operation of an offshore production facility, can and should be taken into account in the design considerations.

This principal study illustrates the framework outlined in Chapter 8 for a special type of application. The general decision problem of optimal design of an offshore facility is assessed taking into account the specific production profile for the considered facility and also including the possibility that the facility fails and is reconstructed, but only if it is economically feasible. By considering the data on production profiles and design costs given in Almlund (1991), a series of parameter studies are performed.

In the present study the construction costs C_C , the revenue obtained through production C_{Rev} , the failure costs C_F and the reconstruction costs C_{Rec} are included in the life cycle benefit. The

capital letter C denotes that costs and revenues are consequences, whereas the index specifies the consequence type. Each of these consequences depends on a set of design parameters, which for the present case are represented by the reserve strength ratio RSR .

9.1.1 A Simplified Approach

Within the optimal design consideration of a structure, it should be taken into account that the structure may fail in the future and that it may be reconstructed, if feasible. This again depends on the time variation of the revenue. So far, mainly two reconstruction strategies have been considered in the literature, namely:

1. the structure is abandoned after a failure and
2. failure leads directly to a reconstruction.

As discussed in Rosenblueth and Mendoza (1971) and Rackwitz (2000), these two strategies represent extreme but convenient cases because analytical solutions may readily be derived. Thereby, Rosenblueth and Mendoza (1971) distinguishes between *failure upon construction*, i.e. immediately after the construction and the more realistic case *failure due to time-variant loads and/ or resistances*. For offshore platforms, time-variant failures need to be considered, whereby a reconstruction strategy in between the two extremes might be appropriate that depends on the specifics of the considered type of structure. Due to the typical lifetime of offshore oil and gas fields, the approach utilized within the present study considers the possibility of a single reconstruction, but only if this is economically feasible. The pursued approach is illustrated by the decision/ event tree shown in Figure 9.1. The first node represents the design and

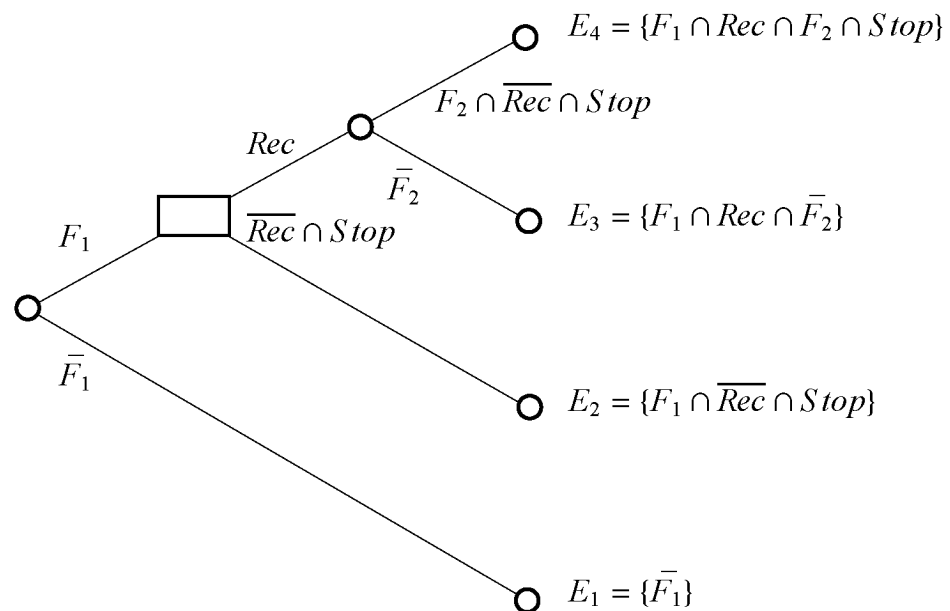


Figure 9.1: Event/ decision tree illustrating the simplified approach.

construction of the structure. At this time, the decision is made with regard to the reliability of the structure. After the structure has been realized, two events may follow. The structure may

survive (event E_1) or it may fail. If the structure fails, there are again two possibilities. Either it is economically feasible to reconstruct the structure or it is not (event E_2). If economically feasible, the structure is reconstructed and subsequently again two events may follow. Either the structure fails (event E_4) or the structure survives (event E_3). Hence, in the present study, only the expected costs and revenues up to the time of the second failure are taken into account. The expected life cycle benefit may be written as

$$E[LCB] = E[C_{Rev}] - E[C_C] - E[C_{Rec}] - E[C_F], \quad (9.1)$$

where the expected value operations are performed in regard to the uncertainties associated with the loading and the load capacity of the structure. The revenues and costs are discounted by a discounting function $r(t)$ where, γ is equal to $\ln(1 + \gamma')$ and γ' denotes the annual interest rate. t is the time at which the consequences (revenues or costs) occur.

$$r(t) = e^{-\gamma t} \quad (9.2)$$

Hence, $E[LCB]$ is the expected net present value of the activity. It can be written as

$$E[LCB] = \int_0^T i(t)r(t)R_{i,2}(t)dt - C_C - \int_0^{t_0} C_{Rec}r(t)f_1(t)dt - \sum_{n=1}^2 \int_0^T C_F r(t)f_n(t)dt. \quad (9.3)$$

T is the design lifetime, $i(t)$ the revenues as a function of time, $r(t)$ the discounting function, $R_{i,2}$ the "revenue's reliability function" considering two possible failures. The reliability function of the revenue expresses the probability that at time t the revenue is obtained. It should not be confused with the structural reliability function. C_C are the construction costs, C_{Rec} the reconstruction costs, C_F the failure costs and $f_n(t)$ is the probability density function of the time to the n^{th} failure. Finally, t_0 is the latest point in time for an economically feasible reconstruction.

9.1.2 Evaluation of the Expected Life Cycle Benefit

The evaluation of Equation 9.3 requires the analysis of $R_{i,2}$ and $f_n(t)$, for which the underlying failure process needs to be determined. Assuming that failures occur as realizations of a stationary Poisson process, it is possible to analytically evaluate Equation 9.3 as given in Kübler and Faber (2004). Starting from the distribution function of the time to a second failure

$$F_2(t) = \iint_{\tau_1 + \tau_2 \leq t \cap \tau_1 \leq t_0} f(\tau_1, \tau_2) d\tau_1 d\tau_2, \quad (9.4)$$

one can calculate the reliability function of the revenue

$$R_{i,2}(t) = \begin{cases} e^{-\lambda t} (1 + \lambda t); & t \leq t_0 \\ e^{-\lambda t} (1 + \lambda t_0); & t > t_0 \end{cases}. \quad (9.5)$$

In Equation 9.4, τ_1 and τ_2 are two failure inter-arrival times. The sum of these is the time of the second failure. Inter-arrival times of Poisson processes are mutually independent. Therefore, the

joint probability density function is simply the product of the two marginal probability density functions with occurrence rate λ . For the evaluation of Equation 9.4, a case differentiation is necessary because the first inter-arrival time τ_1 is limited by t_0 .

In order to be able to study arbitrary revenue functions, $i(t)$ is approximated as a polynomial multiplied by an exponential term.

$$i(t) = \sum_{i=0}^n a_i t^i e^{bt}, \quad b, a_i \in \mathbb{R}; n \in \mathbb{N}_0 \quad (9.6)$$

Given these assumptions, the expected revenue can be evaluated analytically.

$$\begin{aligned} E[C_{Rev}] &= \sum_{i=0}^n \sum_{j=0}^i (-1)^j \frac{i!}{(i-j)!} \frac{a_i}{A^{1+j}} \left(t_0^{i-j} e^{At_0} - 0^{i-j} \right) \\ &+ \lambda \sum_{i=0}^n \sum_{j=0}^{i+1} (-1)^j \frac{(i+1)!}{(i+1-j)!} \frac{a_i}{A^{1+j}} \left(t_0^{i+1-j} e^{At_0} - 0^{i+1-j} \right) \\ &+ (1 + \lambda t_0) \sum_{i=0}^n \sum_{j=0}^i (-1)^j \frac{i!}{(i-j)!} \frac{a_i}{A^{1+j}} \left(T^{i-j} e^{AT} - t_0^{i-j} e^{At_0} \right) \end{aligned} \quad (9.7)$$

Here, the mathematical convention $0^0 = 1$ is used. Moreover, the variable A is introduced which is equal to $A = b - \gamma - \lambda$. It is seen that Equation 9.7 is a function of t_0 which is determined by the reconstruction decision.

9.1.2.1 Modeling the Reconstruction Decision

After a first failure, the decision maker has to consider, whether or not to reconstruct the failed structure. Essentially, a decision to reconstruct should only be made, if it can be shown that the remaining expected life cycle benefit $E[LCB_{Rec}(t)]$ associated with the decision is positive.

$$E[LCB_{Rec}(t)] = E[C_I(t)] - E[C_{Rec}(t)] - E[C_F(t)] \quad (9.8)$$

$$\begin{aligned} &= \int_t^T i(\tau) \delta(\tau) R_{i,1}(\tau - t) d\tau - C_C r(t) \\ &\quad - \int_t^T C_F r(\tau) f_1(\tau - t) d\tau \end{aligned} \quad (9.9)$$

Equation 9.8 can also be solved analytically

$$\begin{aligned} E[LCB_{Rec}(t)] &= \sum_{i=0}^n \sum_{j=0}^i \frac{(-1)^j i!}{(i-j)!} \frac{a_i}{A^{1+j}} e^{\lambda t} \left(T^{i-j} e^{AT} - t^{i-j} e^{At} \right) \\ &\quad - e^{-\gamma t} C_C \\ &\quad + e^{\lambda T} C_F \frac{\lambda}{\gamma + \lambda} \left(e^{-(\gamma + \lambda)T} - e^{-(\gamma + \lambda)t} \right); \end{aligned} \quad (9.10)$$

however, generally t_0 needs to be evaluated numerically. The remaining terms of Equation 9.3 can be expressed analytically, too. The complete derivation is given in Kübler and Faber (2004).

9.1.3 Application to the Optimal Design of Monopod Offshore Structures

The introduced framework is applied for the design of an offshore monopod steel structure. The decision problem is to determine the optimal RSR for different situations in terms of construction costs, failure consequences and interest rates. Revenue and cost models have been established according to Almlund (1991) and Pinna et al. (2001). Consequently, oil and gas production facilities are regarded as whole systems composed of several subsystems like topsides, substructures, platform well systems, sub sea well systems and offloading/ pipeline systems. The costs for project teams, engineering and insurances, as well as the production profiles and processing costs are also included in the data of Almlund (1991). All revenue and cost models are normalized with respect to the fixed construction costs. Furthermore, a relation between RSR and the annual failure rate λ , as well as the elastic section modulus W_{el} , is established.

9.1.3.1 Revenue

Special focus has been on the revenue model. The revenue function of an offshore facility can be subdivided into three phases, namely the *build-up*, the *plateau* and the *decline phases*. In the build-up phase, the producers are installed and set on stream. During the plateau phase, the production is limited by the processing or transport capacity, and the maximal annual revenue is obtained. From this amount the processing and transport costs have to be subtracted. The maximal annual net revenue is set relative to the total fixed construction costs, also taken from Almlund (1991). Thus, the maximal annual net revenue approximately amounts to 84% of the total fixed construction costs. The decline phase succeeds the plateau phase and during this phase, the oil and/ or gas production decreases exponentially, which is described by the decline factor δ_{dec} . For water injected processing, this factor lies in the range 0.03 to 0.22 and depends on several reservoir and production specific parameters. For this example, the decline factor was assumed to be $\delta_{dec} = 0.22$ and the time t_{op} , i.e. the beginning of the declining phase was set to $t_{op} = 6$ years. The decline phase ends with the decommissioning of the structure, when production is no longer economically feasible. This point in time is assumed to be $T = 25$ years after the production is started. Figure 9.2 shows the net revenue function and a simple normalized approximation based on Equation 9.6.

9.1.3.2 Construction Costs

In addition, Kübler and Faber (2004) formulate the section modulus and the construction costs as a function of the design variable RSR . For representational convenience, all costs are normalized to the fixed construction costs. Based on Equation 9.16, it is shown that the sectional modulus can be formulated as

$$W_{el}(RSR) = 2.806RSR, \quad (9.11)$$

and the construction costs are given by

$$C_C(RSR) = 1 + \frac{\rho_{SS}\rho_{SSvar}}{1 - \rho_{SS}\rho_{SSvar}} \left(\frac{RSR}{1.6} \right)^{\frac{2}{3}}. \quad (9.12)$$

Almlund (1991) found that substructure costs are in the range of 7 to 32 percent of the total cost, depending on the type of production facility. For monopod structures, a proportion of $\rho_{SS} = 20\%$

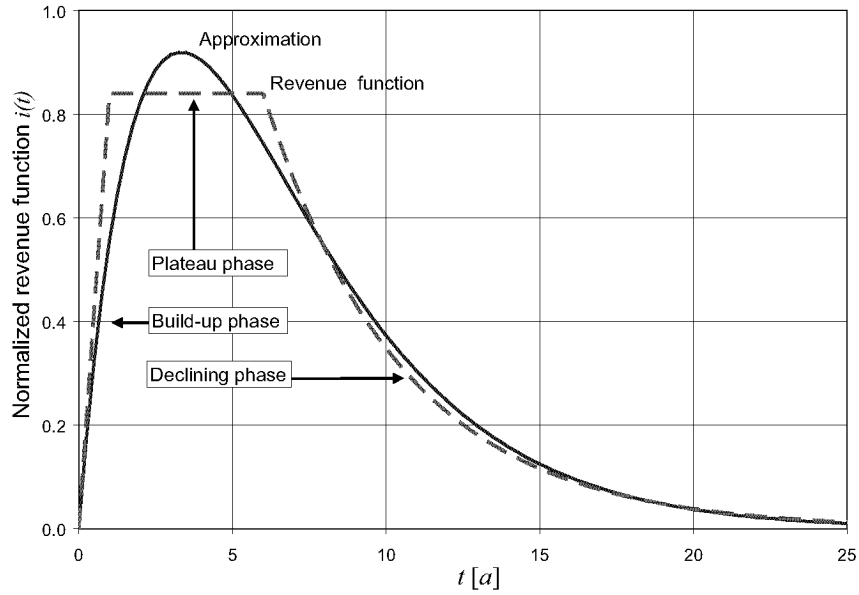


Figure 9.2: Normalized revenue function $i(t)$ and the applied approximation.

is found appropriate. The substructure costs contain a fixed part, which is independent of the design parameter RSR. As these costs amount to 50% of the substructure costs, the proportion of variable costs averages to $\rho_{var} = \rho_{SS}\rho_{SSvar} = 10\%$.

9.1.3.3 Failure Consequences

In addition, failure consequences need to be considered. Material losses C_{FM} as well as immaterial losses C_{FF} need to be considered.

$$C_F = C_{FM} + C_{FF} \quad (9.13)$$

The failure costs due to material losses are implemented as a linear function of the construction costs. Pinna et al. (2001) indicate failure costs to construction costs ratios ρ_{CF} to lie in the range of 3 to 7. With the condition of a short reconstruction period a ratio of 10 may be appropriate. In case of extraordinarily severe failure consequences, including both complete failure and clean up costs, the ratio may even be as high as 20. Since the reconstruction costs are already included, they have to be subtracted from the failure costs.

According to Chapter 6, immaterial losses due to fatalities can be taken into account in engineering decision-making using acceptable life saving costs. On that basis, the immaterial losses due to structural failure is given by the product of the expected number of fatalities NF and LSC . For the following parameter study, a value of two million USD is chosen. In order to analyze different construction costs, the factor ρ_{LSC} is introduced as the ratio of LSC to the fixed construction costs $C_{C,fix}$. Following this, the total failure costs may be assessed as

$$C_F = \rho_{CF}C_C + \rho_{LSC}NF C_{C,fix}. \quad (9.14)$$

| Variable X | Description | Dist. type | $E[X]$ | V_X | Fractile | Char. value X_k |
|--------------|----------------------|------------|----------|-------|----------|-------------------|
| f_y | Yield strength | LN | 1 | 0.15 | 5% | 0.77 |
| b | Load scale parameter | LN | 1 | 0.20 | 54% | 1.00 |
| H | Wave height | G | 1 | 0.15 | 99% | 1.47 |
| W_{el} | Section modulus | Det. | W_{el} | – | – | W_{el} |

Table 9.1: Random variables and their parameters.

9.1.3.4 Reliability Analysis

Finally, to determine the annual failure rate λ as a function of RSR , a probabilistic model according to Stahl et al. (1998) and Faber and Sørensen (1999) has been applied. The limit state function for ultimate collapse is given as

$$g(\mathbf{x}) = W_{el}f_y - bH^2. \quad (9.15)$$

The description of the random variables and their parameters is given in Table 9.1. The random variables have been normalized with mean values equal to one and representative coefficients of variation. The RSR is defined as the ratio of the characteristic resistance to the characteristic loading. Thus, a linear relation between RSR and the section modulus is obtained.

$$W_{el} = \frac{b_k H_k^2}{f_{y,k}} RSR \quad (9.16)$$

The probabilistic evaluation of Equation 9.15 yields the relation between the RSR and the annual probability of failure $P_{f,a}$ and the reliability index β_a (both with a reference period of one year), see Figure 9.3. By means of the annual probability of failure, the annual failure rate becomes

$$\lambda = \ln\left(\frac{1}{1 - P_{f,a}}\right). \quad (9.17)$$

9.1.4 Sensitivity Studies

Given the formulated framework, studies are performed to investigate the sensitivity of influencing variables.

9.1.4.1 Sensitivity to Failure Costs

In Figure 9.4, the expected life cycle benefit is illustrated for different failure costs and for an annual interest rate equal to $\gamma' = 6\%$. The expected life cycle benefit has a maximum corresponding to an optimum value of RSR , hereafter denoted as RSR_{opt} . Figure 9.4 shows that an increasing ρ_{CF} and therefore increasing failure costs, reduces the expected life cycle benefit. For high values of RSR , and therefore low failure rates, $E[LCB]$ converges to the expected life cycle benefit without considering any failure at all. Furthermore, it appears that additional safety might be bought at relatively low costs. However, the decision maker and/ or investor usually prefers to minimize up-front investments. For this reason, it is important to choose the optimal reserve strength ratio RSR_{opt} as the basis for the design. The optimal reserve strength ratios taken from Figure 9.4 are summarized in Figure 9.5. This figure illustrates the relation between

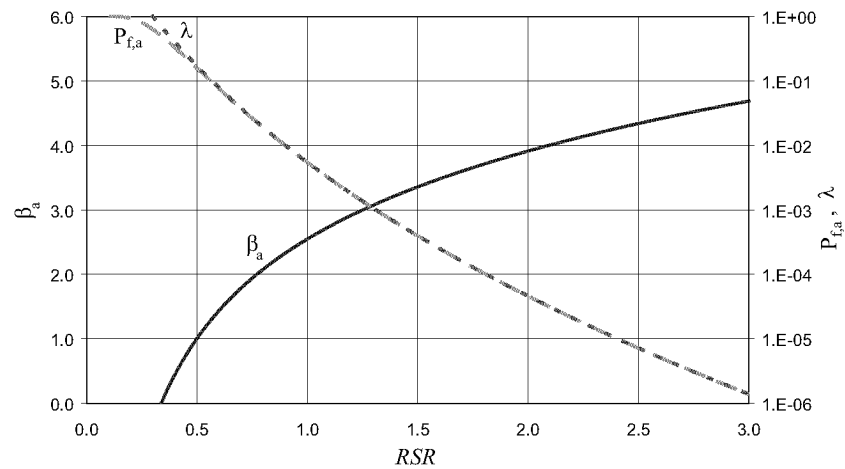


Figure 9.3: Annual probability of failure $P_{f,a}$, annual failure rate λ and the corresponding reliability index β_a as a function of RSR .

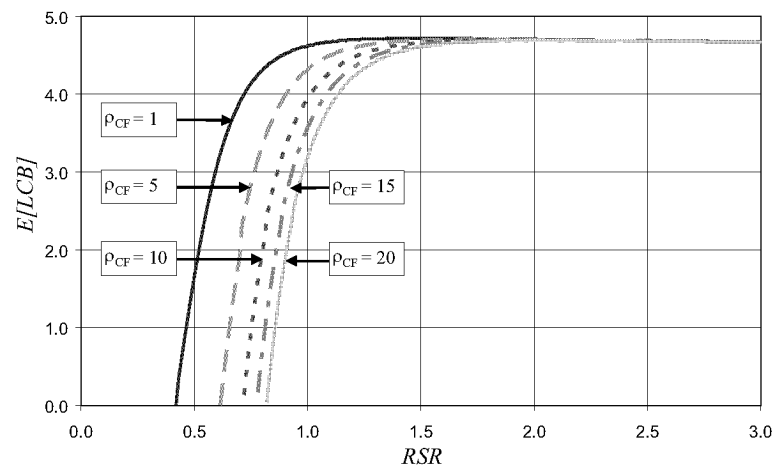


Figure 9.4: Expected life cycle benefit $E[LCB]$ for different failure costs ratios ρ_{CF} .

the failure to construction cost ratio ρ_{CF} and RSR_{opt} . It is seen that for increasing failure costs ρ_{CF} , RSR_{opt} also increases. According to Figure 9.5, RSR_{opt} for monopod structures lies in the range from 1.50 to 1.92, if the failure costs are one to ten times higher than the construction costs. These values obtained for RSR_{opt} are consistent with current practice. In case of very high failure costs, a RSR up to $RSR_{opt} = 2.1$ may be optimal.

9.1.4.2 Impact of Potential Fatalities

In order to analyze the effect of possible fatalities, Equation 9.14 is studied. For constant material losses with $\rho_{CF} = 3$, a parameter study is performed. As illustrated in Figure 9.6, a Farmer diagram can be derived using the relation between RSR_{opt} and the corresponding occurrence rate of the failure λ . It is seen that λ decreases for increasing numbers of fatalities.

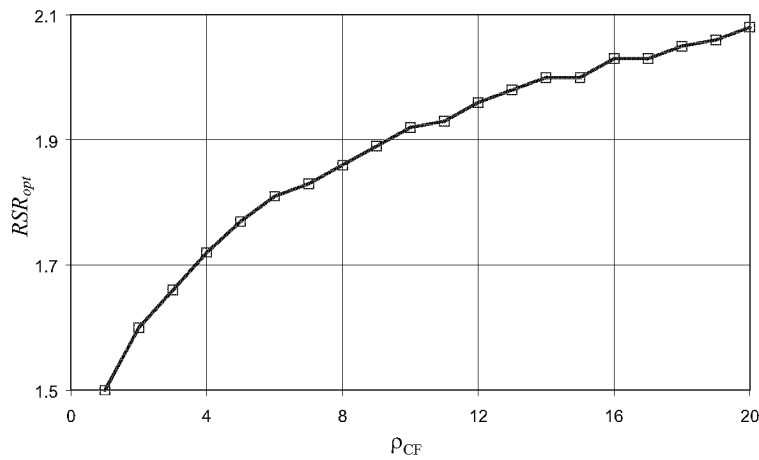


Figure 9.5: Optimal reserve strength ratio RSR_{opt} for different failure costs ratios ρ_{CF} .

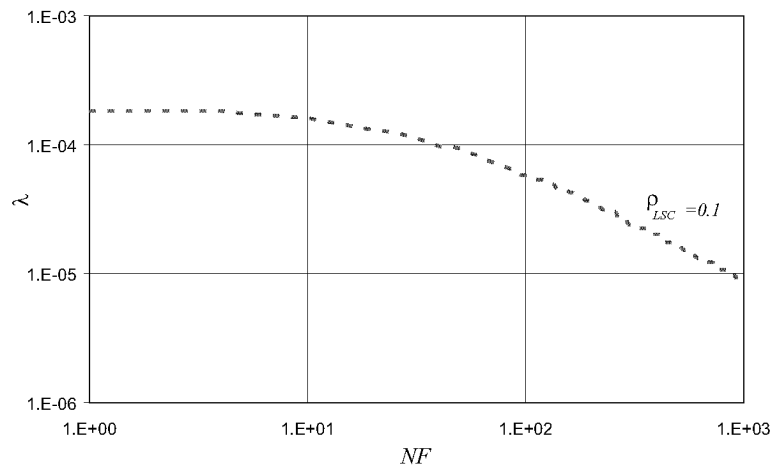


Figure 9.6: Derived F-N curves.

9.1.4.3 Significance of Interest Rates

Another factor, which significantly influences the expected life cycle benefit, is the interest rate γ' . In order to study the importance of the interest rate, $E[LCB]$ has been evaluated for different interest rates. Figure 9.7 shows that for lower interest rates, the life cycle benefit is higher than for higher interest rates. This is because the future benefits outweigh future costs (the activity is beneficial), and both are weighted more for lower interest rates. In case of lower interest rates, the ratio of the present valued failure costs to the construction costs increases. Thus, a higher RSR ensures an optimal design. This relation is illustrated in Figure 9.8.

9.1.4.4 Comparison of Different Approaches to Optimal Design

In order to study the significance of the proposed approach, it is compared with other approaches to optimal design. These approaches can be divided into two categories, namely: 1) the ap-

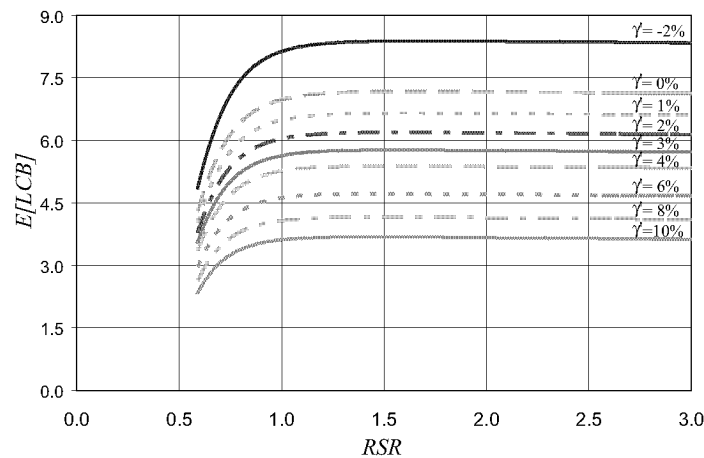


Figure 9.7: Significance of interest rate γ' .

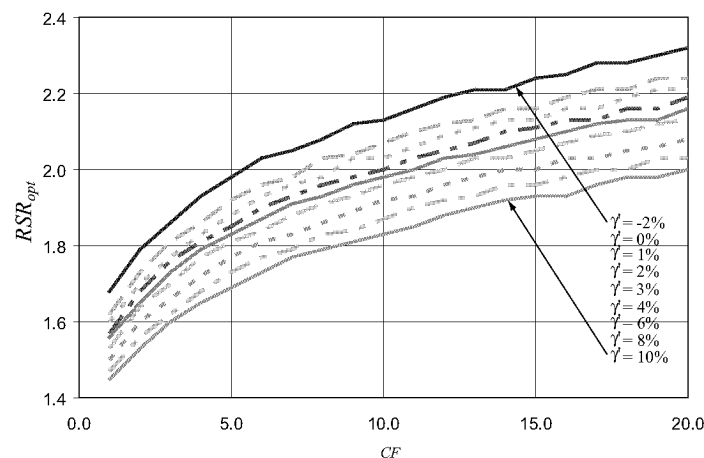


Figure 9.8: Optimal RSR_{opt} for different interest rates r and failure costs ρ_{CF} .

proach, where the design decision problem is formulated as a maximization problem of the expected life cycle benefit $E[LCB]$, and 2) where the design decision problem is formulated as a minimization problem of the expected life cycle costs $E[LCC]$. These two general approaches have been applied considering three different reconstruction strategies. Strategy a) takes into account only one possible failure of the structure, whereby strategy b) considers two possible failures. Strategy c) considers two possible failures if a reconstruction is economically feasible.

For ρ_{CF} , $\gamma' = 6\%$ and the described strategies, Figure 9.9 shows the expected life cycle benefit $E[LCB]$ and the absolute value of the expected life cycle costs $E[LCC]$. For large RSR , the $E[LCB]$ and the $E[LCC]$ only decrease and increase slightly. This is due to the slow increase in the construction costs. From Figure 9.9, it is seen that $E[LCB]$ is always largest for strategy c), followed by strategy b) and strategy a). Furthermore, Figure 9.9 shows that strategy b) leads to the highest expected life cycle costs $E[LCC]$. For strategy a), one obtains the lowest $E[LCC]$ because its costs are also included in the costs of strategy b) and c). For large values of RSR , the

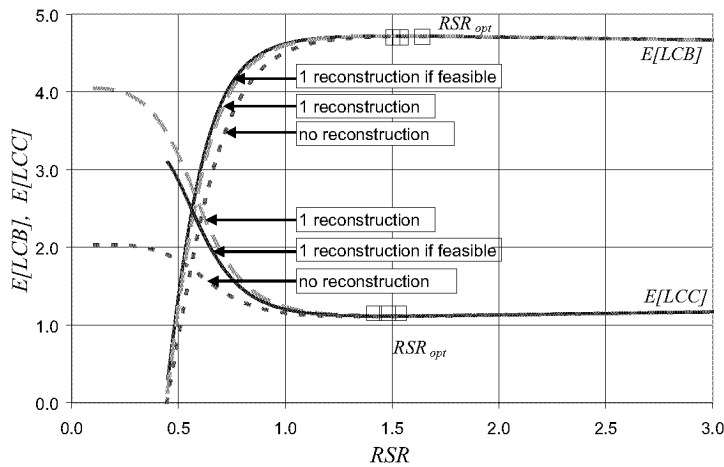


Figure 9.9: Comparison of different approaches.

| RSR_{opt} | Reconstruction strategy | | |
|-----------------|-------------------------|------------------|--------------------------|
| Approach | a) No reconstr. | b) One reconstr. | c) Reconstr. if feasible |
| 1) max $E[LCB]$ | 1.63 | 1.53 | 1.50 |
| 2) min $E[LCC]$ | 1.38 | 1.53 | 1.48 |

Table 9.2: Optimal RSR_{opt} for different approaches

expected present valued costs converge to the construction cost.

Table 9.2 summarizes the RSR_{opt} values obtained by the different approaches and reconstruction strategies. In this special case, RSR_{opt} varies from 1.38 to 1.63, i.e. (18%), depending on the chosen approach and reconstruction strategy. It is seen that generally different optimal values for RSR are identified by the different approaches and strategies. Thus, it can be stated that:

A minimization of the expected life cycle costs does not automatically yield the highest expected life cycle benefit.

9.2 Deterioration of Bridges

Another principal study was carried out by Kübler and Faber (2003). It illustrates the possibility to consider the optimum design and inspection and maintenance plan of structures subject to deterioration. This can be done by assuming a specific reconstruction strategy. For the appropriate consideration of all possible consequences, event and decision trees are very helpful.

9.2.1 Event Trees and Probabilities

For the purpose of assessing life cycle costs and revenues, decision/ event trees may be utilized, see Section 2.3.1.1. Within these logical trees, all events, which may occur during the design lifetime of the activity, can be illustrated in a logical order. If E_i is the i^{th} component of the tree

with j_i possible states, then $E_{i,k}$ is the event with the i^{th} component in state k . If n is the number of components, then the total number of paths is given by

$$n_U = \prod_{i=1}^n j_i. \quad (9.18)$$

The occurrence in time of these events and their logical order depend on the structural design \mathbf{z} and the inspection and maintenance plan (\mathbf{i}, \mathbf{d}) as outlined in the previous chapter. A path U_k is defined as the intersection of events.

$$\{U_k\} = \{E_{1,k_1} \cap \dots \cap E_{n,k_n}\} \quad (9.19)$$

The associated probability is

$$P(U_k) = P(E_{1,k_1} \cap \dots \cap E_{n,k_n}). \quad (9.20)$$

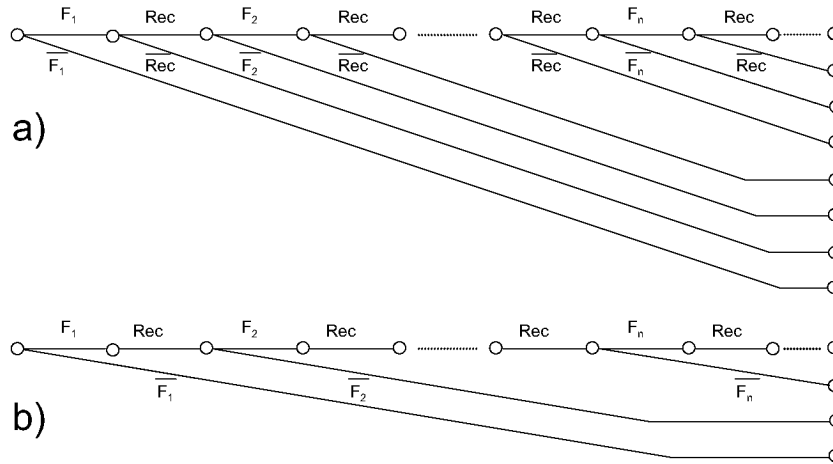


Figure 9.10: Possible failures and reconstructions within a time interval.

An event tree is shown in Figure 9.10a. It illustrates all possible failure events for a given time interval, e.g. time between inspections. It is illustrated that the initial structure might survive this time period, indicated by $\overline{F_1}$; however, it might fail, as well. The latter case is indicated by F_1 . In case the structure fails, it might be reconstructed or not, represented by the events Rec and \overline{Rec} , respectively. This decision will be made according to its economical feasibility. If the structure is not reconstructed, the activity supported by the structure ceases. If the structure is reconstructed, the activity continues. However, the new structure might fail as well, denoted by the event F_2 and a reconstruction decision has to be made again. Although unlikely, following this scheme, an infinite number of failures is possible.

For the case when it can be assumed that a failed structure will always be reconstructed, the event tree can be simplified as illustrated in Figure 9.10b. This assumption seems to be justified for all structures that support essential activities.

Figure 9.11a illustrates an event tree representing all events, which might occur between inspections. In this figure, S denotes the survival event and \overline{S} the event that at least one structural failure has occurred. Rec denotes the event of reconstruction of the last failed structure, \overline{Rec} the

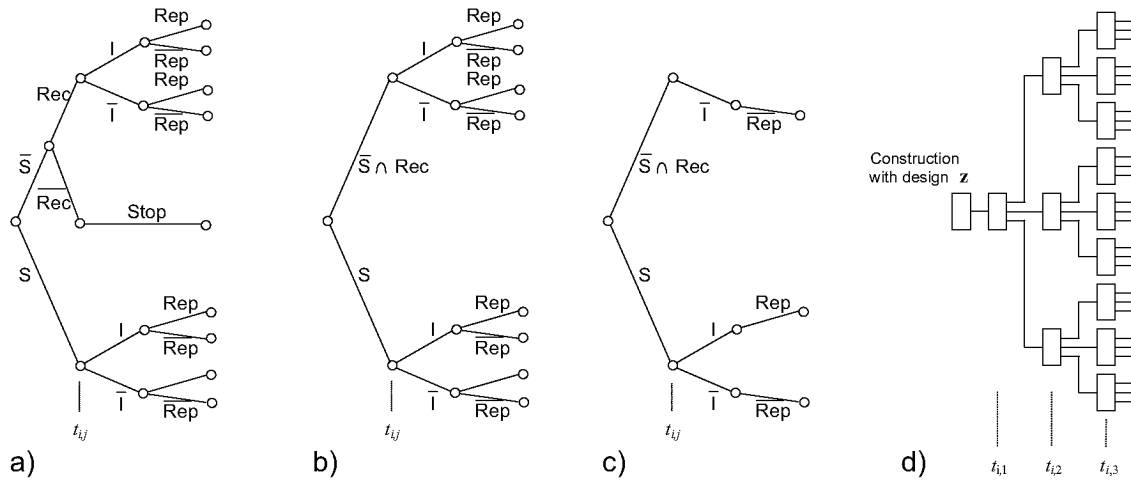


Figure 9.11: Simplification of the considered decision/ event tree.

event that the structure is not reconstructed and that the activity will thus cease. If an inspection is performed, the event I represents having found an indication of deterioration and the event Rep denotes that a repair action is implemented. The complementary events are \bar{I} and \overline{Rep} , respectively.

9.2.1.1 Reconstruction and Repair Strategy

A decision rule $d(\psi, t_f)$ is introduced to define the decisions made with regard to repairs or reconstructions depending on the specific outcome of the performance of the structure (e.g. failure at time t) or the result of the inspections $\Psi = \psi$. As already mentioned, for the case of structural failure, there are two decision alternatives. The first alternative is that a failed structure will be removed and the activity it supports will cease, whereas the second alternative is the reconstruction of the failed structure. This decision must be performed according to its economic feasibility. In the following however, for the considered types of structure it is assumed that a reconstruction is always the optimal decision and that failed structures will be systematically reconstructed without changing the original level of reliability. This reconstruction strategy is illustrated in Figure 9.10b for a considered time interval, and Figure 9.11b shows the event tree between two inspections, augmented with this reconstruction strategy.

Furthermore, decision rules for repair actions can be defined. This simplifies the event tree even more and keeps the event tree manageable and numerically tractable. For further derivations the following assumptions are made:

1. If a structure fails between inspections, it will be reconstructed. A consecutive inspection will not give an indication of deterioration, and no repair will be made.
2. If a structure survives the time between inspections, an inspection might give an indication of deterioration which automatically will lead to a repair of the structure. No repair will be performed, if the inspection result does not indicate deterioration.

Implementing these assumptions, Figure 9.11c is obtained. Finally, Figure 9.11d illustrates a full event tree considering three inspections. The top event is the construction with design z .

9.2.1.2 Evaluation of Probabilities

In probabilistic terms, a path in the event tree is the intersection of events. This intersection can be expressed as a sequence of events, where the structural reliability and the probability of indication of deterioration are conditioned by the events that occurred before. However, if a repair or a reconstruction is performed at time t_{li} , the initial resistance will be reestablished, and the events after this point in time will be independent of the events that occurred before t_{li} . Generally, information about performed inspections has to be used to express the conditional probability of getting a specific inspection result at a later point in time. Furthermore, it can be used to update the structural reliability; however, in the following this possibility is not pursued.

In the present study it is assumed, that the cumulative distribution function of a first failure can be described on the basis of a non-homogeneous Poisson process and the outcrossing rate, i.e. by means of Equation 3.75. However, this cumulative distribution function (cdf) is only an asymptotic approximation of the first passage cdf. This means that this assumption is only valid, if outcrossing rates are small so that simultaneous occurrences in an infinite time interval are negligible. Whereas for the considered study this assumption is meaningful – the expected value of T_1 is smaller than the design lifetime – it cannot be generally assumed to be fulfilled for structures that are subject to deterioration. Then the cdf could be assessed by means of Monte Carlo simulations and the renewal density by means of Equations 3.74 and 3.56.

The reliability function for a specific path in the event tree is given by Equation 9.21 and the probability that the structure does not survive the time interval $\Delta t_j = [t_{i,j-1}, t_{i,j}] = \{t \in \mathbb{R}_+ | t_{j-1} \leq t \leq t_j\}$ is given by Equation 9.22.

$$R(t|U_k) = \exp \left[- \int_0^{t-t_{li}} v^+(\tau) d\tau \right] \quad (9.21)$$

$$P(\bar{S}(t_{i,j}) | U_k) = R(t_{i,j-1} | U_k) - R(t_{i,j} | U_k) \quad (9.22)$$

In these equations, $v^+(t)$ is the outcrossing rate, and t_{li} is the last point in time the initial resistance was reestablished (either by repair or reconstruction), whereas t_{li} depends on the path U_k . Figure 9.12 illustrates the influence of a repair and reconstruction actions at $t_{i,1}$, $t_{f,1}$ and $t_{i,2}$ on the structural resistance and the outcrossing rate.

The probability of indication for deterioration at time $t_{i,j}$ has to be considered for three cases. The first case considers that the structure does not survive the time interval $\Delta t_{i,j}$. In this case, t_{li} is larger than $t_{i,j-1}$. According to assumption 1), an inspection after a reconstruction will not indicate deterioration.

$$P(I(t_{i,j}) | Rec(t_{i,j}) \cap \bar{S}(t_{i,j})) = 0 \quad (9.23)$$

The second case considers a repair made at $t_{i,j-1}$, then $t_{li} = t_{i,j-1}$ and the conditional probability of indication for deterioration is

$$P(I(t_{i,j}) | Rep(t_{i,j-1}) \cap I(t_{i,j-1})) = P(I(t_{i,j} - t_{i,j-1})) \quad (9.24)$$

Finally, the case is considered, in which the structure is neither repaired nor reconstructed. In this case, t_{li} is smaller than $t_{i,j-1}$. The probability of indication for deterioration is conditioned by the results of the most recent inspection results. If $\bar{I}(t_{i,k})$ is the first inspection result not

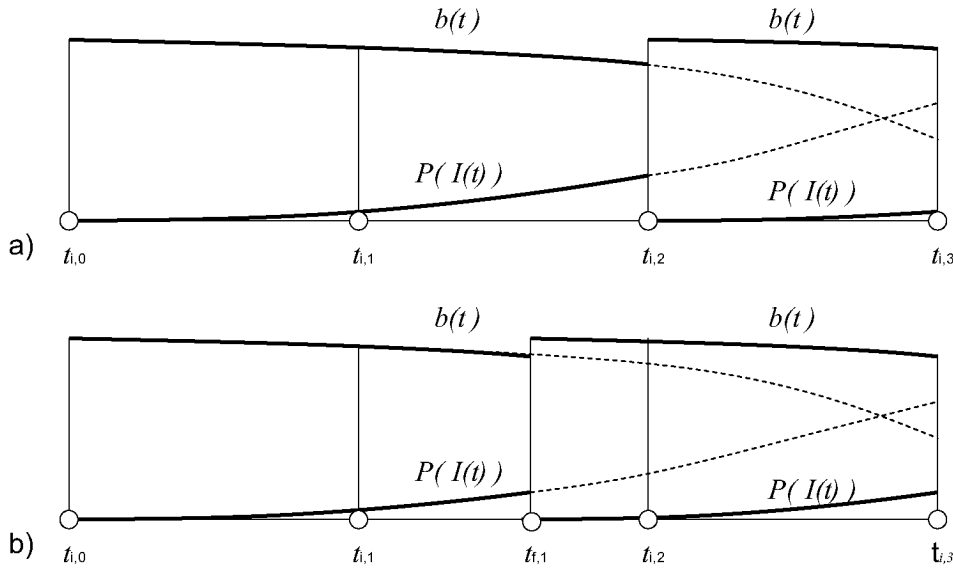


Figure 9.12: Influence of deterioration, repair and reconstruction on the structural resistance $b(t)$ and the probability of deterioration indication $P(I(t))$.

leading to a repair, and if no repair or reconstruction is performed between $t_{i,k}$ and $t_{i,j}$, then the conditional probability of obtaining an indication of deterioration at time $t_{i,j}$ is

$$P(I(t_{i,j} - t_{li}) | \bar{I}(t_{i,j-1} - t_{li}) \cap \dots \cap \bar{I}(t_{i,k} - t_{li})) = \frac{P(I(t_{i,j} - t_{li}) \cap \bar{I}(t_{i,j-1} - t_{li}) \cap \dots \cap \bar{I}(t_{i,k} - t_{li}))}{P(\bar{I}(t_{i,j-1} - t_{li}) \cap \dots \cap \bar{I}(t_{i,k} - t_{li}))} \quad (9.25)$$

The event of indication of deterioration $I(t)$ at a certain point in time t can be expressed in terms of a safety margin $g_I(\mathbf{X}, t)$. By means of structural reliability methods, such as FORM/ SORM or Monte Carlo simulations, these function can be evaluated. The probability of deterioration indication is formulated as $P(I(t)) = P(g_I(\mathbf{X}, t) \leq 0)$, and $P(\bar{I}(t)) = P(g_I(\mathbf{X}, t) > 0)$ is the probability of no indication. If the same inspection method is used at the inspection, the basic random variables \mathbf{X} entering the safety margin remain unchanged, and therefore, the events representing the indication of deterioration at two points in time tend to be highly correlated. If the safety margins are fully dependent and if the limit state function $g_I(\mathbf{x}, t)$ is a non-increasing function for constant \mathbf{x} and increasing t , Equation 9.25 becomes

$$P(I(t_{i,j} - t_{li}) | \bar{I}(t_{i,j-1} - t_{li}) \cap \dots \cap \bar{I}(t_{i,k} - t_{li})) = P(I(t_{i,j} - t_{li}) | \bar{I}(t_{i,j-1} - t_{li})) \quad (9.26)$$

$$= \frac{P(I(t_{i,j} - t_{li})) - P(I(t_{i,j-1} - t_{li}))}{1 - P(I(t_{i,j-1} - t_{li}))}, \quad (9.27)$$

as shown in Kübler and Faber (2003).

9.2.1.2.1 Construction Costs

At least in the neighborhood of the optimum, the construction costs can be modelled as a linear

function of the design variables. Uncertainties related to the coefficients $C_{C,i}$ can be accounted for by including them in the vector \mathbf{X} .

$$E[C_C(\mathbf{z}, \mathbf{i}, d)] = E_{\mathbf{X}} \left[C_{C,0} + \sum_{j=1}^p C_{C,j} z_j \right] \quad (9.28)$$

9.2.1.2.2 Assessment of Revenues

For the assessment of the expected revenue an integration over the design lifetime has to be performed, where the integrand is the product of the discounted revenue and the probability that the revenue is obtained. For the considered reconstruction strategy, the probability that a revenue is obtained – this is the probability that the activity is not stopped 1- is unity for any point in time. For this case the expected value of the revenue is given by

$$E[C_{Rev}(\mathbf{z}, \mathbf{i}, d)] = E_{\mathbf{X}} \left[\int_0^T C_{Rev}(\tau) r(\tau) d\tau \right]. \quad (9.29)$$

Here, T is the design lifetime of the activity and $C_{Rev}(t)$ is the time-variant revenue weighted by the discounting function $r(t)$. It is seen that the expected revenue is independent of the structural design variable and independent of the inspection and maintenance plan. Hence, for the optimization of these parameters, the expected revenue can be neglected.

9.2.1.2.3 Inspection Costs

The expected inspection costs are simply the sum of the costs for the single inspections weighted by the discounting function. The inspection times are summarized in the vector \mathbf{t}_i . $t_{i,j}$ is the j^{th} inspection time given by the inspection plan, and n_i is the number of inspections. $\delta(t)$ is the Dirac delta function. $C_I(t)$ are the time-variant inspection costs.

$$E[C_I(\mathbf{z}, \mathbf{i}, d)] = E_{\mathbf{X}|\mathbf{i}} \left[\sum_{j=1}^{n_i} \int_0^T \delta(\tau - t_{i,j}) C_I(\tau) r(\tau) d\tau \right] \quad (9.30)$$

9.2.1.2.4 Repair Costs

The expected value of the repair costs can be expressed as follows.

$$E[C_R(\mathbf{z}, \mathbf{i}, d)] = E_{\mathbf{X}, \Theta, \mathbf{S}|\mathbf{z}, \mathbf{i}, d} \left[\sum_{k=1}^{n_U} \sum_{j=1}^{n_{rep,Uk}} \int_0^T C_R(\tau) r(\tau) \delta(\tau - t_{rep,j}) P(U_k) d\tau \right] \quad (9.31)$$

This equation sums over all possible paths U_k of the event tree, where n_U is the number of paths. The second sum considers the repair times \mathbf{t}_{rep} for a given path U_k and associated probability $P(U_k)$. Here, $t_{rep,j}$ is the time of the j^{th} of $n_{rep,Uk}$ repairs associated with path U_k . Finally $C_R(t)$ denotes the time-variant repair costs.

9.2.1.2.5 Decommissioning Costs

For the underlying reconstruction strategy, where failed structures will be reconstructed successively, decommissioning will be made at the end of the design lifetime. Then the expected value is given as follows

$$E [C_D(\mathbf{z}, \mathbf{i}, d)] = E_X [C_D(T)r(T)]. \quad (9.32)$$

$C_D(T)$ are the time-varying decommissioning costs and T is the design lifetime of the activity. Again, uncertainties related to the consequence (cost) model can be accounted for, if they are included in \mathbf{X} .

9.2.1.2.6 Failure Costs

The expected value of the failure costs is given by the following equation.

$$E [C_F(\mathbf{z}, \mathbf{i}, d)] = E_{\mathbf{X}, \Theta, S | \mathbf{z}, \mathbf{i}, d} \left[\sum_{k=1}^{n_U} \sum_{j=0}^{n_i} \int_{t_{i,j}}^{t_{i,j+1}} C_F(\tau)r(\tau)P(U_k)h(\tau|U_k)d\tau \right] \quad (9.33)$$

Again, this equation sums over all paths U_k integrating piecewise over each interval between inspections for the entire design lifetime. $C_F(t)$ are the time-variant failure costs weighted by the discounting function. $P(U_k)$ is the probability of the path event and $h(t|U_k)$ is the renewal density conditioned by the events of the path. Equation 9.34 shows that if no failure occurs within $\Delta t_{i,j}$, the conditional renewal density is zero, otherwise it is the renewal density divided by the probability that the structure does not survive in $\Delta t_{i,j}$. Again, t_{li} is the most recent point in time at which the initial resistance was reestablished.

$$h(t|U_k) = \begin{cases} \frac{h(t-t_{li})}{P(\bar{S}(t_{i,j})|U_k)}, \bar{S} & \text{in } \Delta t_{i,j} \\ 0, S & \text{in } \Delta t_{i,j} \end{cases} \quad (9.34)$$

9.2.1.3 Renewal Density

The renewal density is introduced in Chapter 3. If a repair or a reconstruction is made at t_{li} , the initial resistance is reestablished. The renewal density function considering this is the initial renewal function, which is shifted by t_{li} to the right.

$$h(t|Rec \text{ or } Rep \text{ at } t_{li}) = h(t - t_{li}) \quad (9.35)$$

Only t_{li} is of interest because repairs or reconstructions performed before do not influence the reliability after t_{li} . For a certain point in time t and for a given path U_k , t_{li} can be determined as follows

$$t_{li} = \max(\mathbf{t}_{rep}^T, \mathbf{t}_{rec}^T, 0) \wedge t_{li} \leq t. \quad (9.36)$$

In this expression \mathbf{t}_{rep} is the vector of repair times and \mathbf{t}_{rec} is the vector of times of reconstruction. Both are uniquely determined by the path.

9.2.1.3.1 Backward Recurrence Time

In the foregoing it is mentioned that the repair times t_{rep} and the reconstruction times t_{rec} are required in order to facilitate updating of the renewal density and the probability of obtaining an indication of deterioration. The repair times are known exactly, if the time differences between inspection, indication of deterioration and the successive repair are neglected. However, the times at which failure and reconstructions occur are uncertain. For a given path, however, the intervals in which a structure does not survive are known. By means of the backward recurrence time w , see Chapter 3, the life time distribution of the current structure can be assessed.

For non-homogenous Poisson processes the mean value of the backward recurrence time conditional on failure in the interval $\Delta t_{i,j}$ is

$$E[W|\bar{S} \text{ in } \Delta t_{i,j}] = \int_{t_{i,j-1}}^{t_{i,j}} w \frac{h(t-w) e^{-\int_0^w v^+(\tau) d\tau}}{P(\bar{S}(t_{i,j})|U_k)} dw \quad (9.37)$$

with $\Delta t_{i,j} = [t_{i,j-1}, t_{i,j}]$, i.e. $t_{i,j-1} \leq w \leq t_{i,j}$. For practical cases, it is assumed that the mean value of the backward recurrence time can be used for determining the reconstruction time. Otherwise, the expectation operation has to be performed over the costs affected by the reconstruction time. This assumption is especially valid for a small time interval $\Delta t_{i,j}$ in which the resistances do not significantly change and the load process is stationary. Moreover, in most practical situations in structural engineering, failure rates are low, and the probability density distribution of the backward recurrence time is almost uniformly distributed within the considered interval. Hence, the mean value of the conditional recurrence-time is close to the mean value of the time interval between inspections, i.e. $t_{f,j} \approx E[W] \approx \frac{1}{2}(t_{i,j} + t_{i,j-1})$.

9.2.2 Optimum Design of Concrete Structures

The described approach is illustrated by means of the optimal design of the concrete cover of a concrete bridge. The life cycle benefit of a concrete bridge subject to chloride-induced corrosion is investigated. It is assumed that visual inspections are carried out at equidistant time intervals of 10 years and the design lifetime is set to $T = 100$ years. If corrosion is visible at the surface, the structure is assumed to be repaired. The structure considered is a simply supported bridge, consisting of parallel T-beams. The mean value of the concrete cover is chosen as the design variable to be optimized and the structural design is chosen so that the initial reliability is the same for different concrete covers. The reliability index of the structure before deterioration is $\beta = 4.47$ referring to a one year reference period.

For the evaluation of the expected costs, the influences of the deterioration process on the inspection result and the structural resistance are required.

9.2.2.1 Chloride-Induced Corrosion

According to Engelund, Sørensen, and Sørensen (1999), the time until initiation of chloride-induced corrosion may be given as

$$T_{CI} = \frac{z^2}{4D} \left(\text{erf}^{-1} \left(1 - \frac{C_{CR}}{C_S} \right) \right)^{-2}. \quad (9.38)$$

| Variable X | Dist. Type | $E[X]$ | V_X |
|-----------------------------|------------|--------|-------|
| z [mm] | LN | $E[z]$ | 0.20 |
| D [mm ² /a] | LN | 40.00 | 0.25 |
| C_S [wt.% of concrete] | LN | 0.40 | 0.20 |
| C_{CR} [wt.% of concrete] | LN | 0.40 | 0.33 |
| X_{CI} [1] | LN | 1.00 | 0.05 |
| T_P [a] | LN | 7.50 | 0.25 |

Table 9.3: Probabilistic model for the time to corrosion initiation and visual corrosion.

Where T_{CI} is the time to corrosion initiation, z is the concrete cover, D the diffusion coefficient, C_{CR} the critical chloride concentration and C_S is the concentration of chlorides on the concrete surface. The time between initiation of corrosion T_{CI} and the point in time at which corrosion is visual on the surface of the structure T_{VC} is called the propagation time T_P . To these variables probabilistic models may be assigned that closely follow the models proposed by Faber and Sørensen (2002), see Table 9.3.

The corresponding safety margin for the time until initiation of corrosion and the time until visual corrosion are given by the following equations, where X_{CI} is the associated model uncertainty.

$$g_{CI}(\mathbf{X}, t) = X_{CI}T_{CI} - t \quad (9.39)$$

$$g_{VC}(\mathbf{X}, t) = X_{CI}T_{CI} + T_P - t \quad (9.40)$$

Using FORM/ SORM methods, the probability of corrosion initiation and visual corrosion can be evaluated as a function of time.

9.2.2.2 Reliability Analysis of Concrete Bridges

9.2.2.2.1 Degradation of the Resistance of Concrete Bridges

The degradation of the resistance of concrete bridges is described by using the model suggested by Mori and Ellingwood (1993) and Enright and Frangopol (1998). In this model, the time-variant resistance $b(t)$ is taken as the product of the initial resistance b_0 and the resistance degradation function $b_b(t)$.

$$b(t) = b_0 b_b(t) \quad (9.41)$$

$$b_b(t) = \begin{cases} 1, & t < t_{CI} \\ 1 - k_1(t - t_{CI}) + k_2(t - t_{CI})^2, & t \geq t_{CI} \end{cases} \quad (9.42)$$

the degradation function $b_b(t)$ is a parabolic function with parameters given in Table 9.4. For the present example, a bridge with medium degradation rate is assumed. Moreover, it is assumed that the coefficient of variation for k_1 is $V_{k_1} = 0.10$ and t_{CI} , the time to corrosion initiation, is evaluated by Equation 9.38.

| Degradation case | Degradation rate | $E[k_1]$ | $E[k_2]$ |
|------------------|------------------|----------|----------|
| I | Low | 0.0005 | 0 |
| II | Medium | 0.005 | 0 |
| III | High | 0.0100 | 0.00005 |

Table 9.4: Model for resistance degradation.

| Variable | Value | Variable | Value |
|----------|------------------------|----------|--------|
| C_R | 0.25, 0.50, 0.75, 1.00 | C_I | 0.002 |
| γ | 5% | C_F | 10, 75 |

Table 9.5: Normalized consequence models.

9.2.2.2.2 Load Process

For the reliability analysis, the load process was modelled as a Poisson spike process with Gumbel distributed spike intensities occurring once every year. The coefficient of variation of the spike intensities is taken as 50%, and the characteristic value used in the design of the beam is taken as the 98% quantile in accordance with the Swiss structural design code SIA 160:1989 and most other modern design codes. The reliability analysis accounts for uncertainties associated with the time to corrosion initiation and resistance degradation.

9.2.2.3 Consequence Models

Based on Sloth, Jensen, and Faber (2002), Faber and Sørensen (2002), Malioka (2002), and Stewart (2000), simple consequence models are established for the different types of costs. The costs are normalized with respect to the construction costs. As seen in Table 9.5, the costs are assumed to be constant over time, i.e. net of inflation. For the present example, the expected revenue and the decommissioning costs are independent of the design variable. Hence, these may be neglected and the maximization of the benefit is in this case equivalent to the minimization of the life cycle costs.

The construction costs are influenced by the concrete cover in several ways. Firstly, the concrete cover influences directly the amount of concrete utilized for the structure. Secondly, it influences the dead load of the structure and for higher dead loads, a higher resistance is required in order to guarantee the same reliability. Thirdly, an increasing concrete cover may increase the height of the structure and therefore the internal moment arm; hence, the structural resistance may be increased. In the following, a simple construction cost model is used, which assumes that the latter two effects do not change the amount of required reinforcing steel significantly, and that the construction costs are a function of the concrete material utilized. Then, the construction costs can be approximated sufficiently accurate by a linear function, where $C_{C,z}$ accounts for the proportional increase of the construction costs due to increasing concrete cover.

$$E[C_C](\mathbf{z}, \mathbf{i}, d) = 1 + E_X[C_{C,d}]z \quad (9.43)$$

9.2.2.4 Results

Figure 9.13 shows the expected life cycle costs and the expected values of its cost components as a function of the mean concrete cover $E[z]$. It is not surprising that the construction cost

increases linearly and the expected repair costs decrease exponentially with increasing concrete cover. The inspection costs are constant because the inspection plan is not changed. The expected failure costs attain a maximum at $E[z] = 40$ mm. For concrete covers smaller than 40

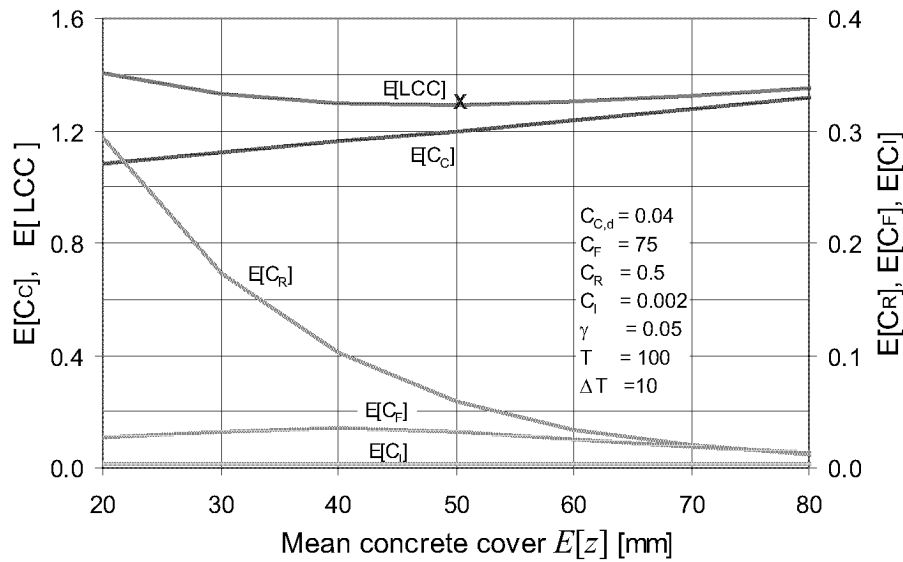


Figure 9.13: Expected costs as a function of the mean concrete cover $E[z]$.

mm, visual corrosion and repairs are more likely and lead to repairs and thus to a more reliable structure. For concrete covers larger than $E[z] = 40$ mm, the reinforcement is better protected and corrosion will occur later. For the considered cost model and inspection plan, the expected life cycle costs are minimal for $E[z] = 50$ mm. It is seen that the expected life cycle costs for $E[z] = 50$ mm and 60 mm are not much different to the costs obtained with the optimal value. For this reason, one might want to choose another concrete cover than the optimal one. However, if a less conservative design is chosen, lower construction costs are contrasted to much higher expected repair costs. For $E[z] = 40$ mm, the expected repair costs rise about 70% with respect to the optimal concrete cover. If $E[z] = 60$ mm is chosen, then up-front costs have to be covered, which are not compensated.

The curve showing the expected life cycle costs $E[LCC]$ in Figure 9.13 is taken and plotted in Figure 9.14 together with 7 others. Each curve represents the expected life cycle costs for a specific configuration of the associated consequences. They have in common that the repair costs are equal to half of the construction costs, i.e. $C_R = 0.5$.

It is seen that a significant increase in the failure costs from $C_F = 10$ to $C_F = 75$ influences the expected life cycle costs relatively little. However, increasing the failure costs results in a larger optimal concrete cover. Far more important are the construction and the repair costs. For increasing variable construction costs (increasing $C_{c,d}$), a smaller concrete cover is optimal. Table 9.6 summarizes the optimal concrete cover for different configurations of the associated consequences. The upper half shows the optimal values for failure costs equal to ten times the construction costs, whereas the lower part summarizes the optimal concrete cover for failure costs of 75 times the construction costs. The third column $C_R = 0.50$ corresponds to the optimum values obtained from Figure 9.14. In Table 9.6, concrete covers larger than 80 mm were not considered. It is seen that a more conservative design is optimal for increasing failure costs and

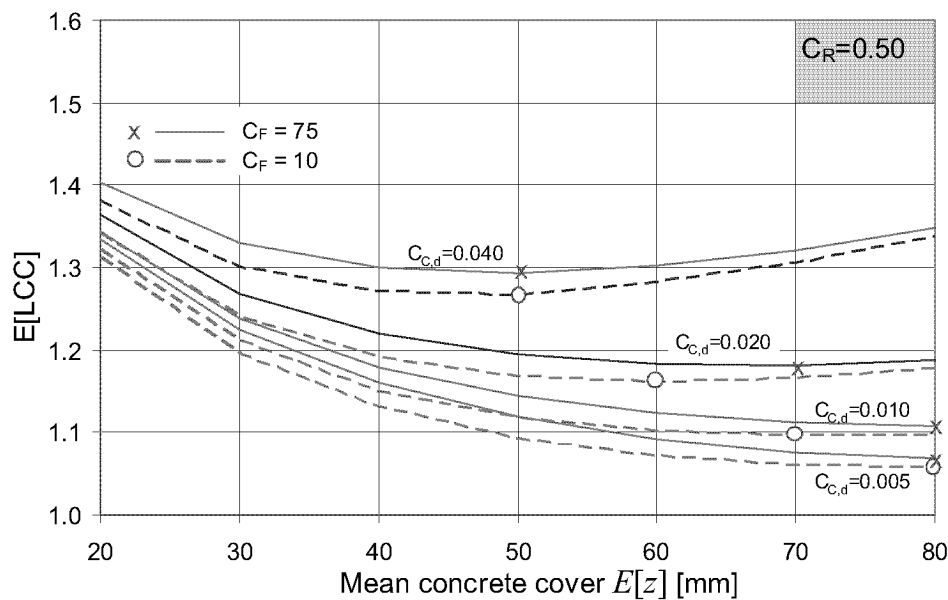


Figure 9.14: Parametric study for repair cost $C_R = 0.5$.

increasing repair costs as well. For increasing construction costs, a less conservative design is optimal.

It should be pointed out that the results obtained from this example are of a qualitative character as a consequence of the simplified modeling. For instance, the concrete cover's function for fire safety and bonding between concrete and reinforcement are neglected. However, the approach outlined for the optimal design of concrete structures can be extended to incorporate a more realistic modeling and also include the spatial variability of deterioration processes.

9.3 Summary

The preceding principal studies illustrate the application of the framework that is described in Chapter 8. It can be applied for the optimal design of civil engineering structures and decision-making. Similarly, this approach was utilized to study the optimal choice of efficient fire safety measures or the optimal point for a replacement of an obsolete structure, see Faber et al. (2004) and Kübler (2002).

It was shown that the consideration of revenues and different reconstruction strategies may lead to different designs. Moreover, it was shown that the underlying process determining structural failure may be considered as a stationary or non-homogenous Poisson process. By means of the non-homogenous Poisson process, also structures subject to deterioration processes may be described. But it is noted that not only the influence of the deterioration process on the structural reliability can be addressed. Also the influence of the deterioration on the probability of an indication for deterioration can be modelled so that the optimal inspection plan can be identified as well.

The illustrated studies show that the framework can be applied generally to any type of civil engineering structure, material or technology. However, before optimal design parameters can

| $C_F = 10$ | $E[d]$ [mm] | | | |
|------------|--------------|--------------|--------------|--------------|
| $C_{C,d}$ | $C_R = 0.25$ | $C_R = 0.50$ | $C_R = 0.75$ | $C_R = 1.00$ |
| 0.005 | 70 | 80 | 80 | 80 |
| 0.010 | 60 | 80 | 80 | 80 |
| 0.020 | 50 | 70 | 70 | 80 |
| 0.040 | 30 | 50 | 60 | 60 |
| $C_F = 75$ | $E[d]$ [mm] | | | |
| $C_{C,d}$ | $C_R = 0.25$ | $C_R = 0.50$ | $C_R = 0.75$ | $C_R = 1.00$ |
| 0.005 | 80 | 80 | 80 | 80 |
| 0.010 | 70 | 70 | 80 | 80 |
| 0.020 | 50 | 60 | 70 | 70 |
| 0.040 | 30 | 50 | 60 | 60 |

Table 9.6: Optimal mean concrete cover with regard to the associated consequences.

be determined, the relevance of the influencing aspects needs to be discussed and then taken into account within the assessment of the expected life cycle benefit. A relevant issue is the inclusion or non-inclusion of reconstructed structures.

9.3.1 Reconstructed Structures

For special types of civil engineering structures the additional consideration of revenues can be relevant for the structural design. Then the most general approach the maximization of the expected life cycle benefit has to be used. However, most commonly, the expected life cycle costs are evaluated for the optimal design of structures. It is shown that this approach also leads to the maximization of the expected life cycle benefit, if revenues are independent of the decision/ design variable. This is the case if failed structures are reconstructed which can be assumed, if the activity supported by the structure is essential for the owner or society.

As seen in Figure 9.9, where $E[LCC]$ is drawn for different reconstruction strategies, the inclusion of reconstructed structures in the life cycle modelling increases the optimal design value, in the specific case the reserve strength ratio RSR . This is also reflected by the values in Table 9.2.

To be sure, it is true that the real designs of future structures are not known when considering reconstructions in the life cycle costs. However, the design basis remains the same with increased knowledge of resistances, load processes and design concepts.

Another point that questions the inclusion of reconstructions is that with increasing wealth higher levels of reliability become affordable. The latter would lead to smaller failure costs of reconstructed structures and subsequently to smaller optimal design value. This means that the identification of the optimal design using the expected life cycle costs determines an upper reliability bound for the optimal design, if it is assumed that failed structures are reconstructed with the same level of reliability.

10 Code Calibration

Structural design codes are efficient tools for managing risks associated with civil engineering facilities. The aim of structural design codes is to maximize the utility, which is associated with the scope of the design codes, e.g. specific civil engineering facilities. As civil engineering structures need to be economic and at the same time reliable, the code committee has to balance the competing characteristics, namely economic efficiency and safety. The present chapter illustrates the code calibration procedure on the basis outlined in Chapter 8.

Practically all structural design codes are of a deterministic character and they utilize the partial safety factor concept to ensure safety. But the modern structural design codes also allow for a probabilistic verification of structural reliability, see e.g. the Eurocodes EN 1990:2002, the Swisscodes SIA 260:2003 and ISO 2394:1998. These codes consider both approaches, the deterministic and the probabilistic evaluation of safety, as being equivalent.

Even if both codes are considered to be equivalent, for safety verifications only a probabilistic code is able to provide a rational background to a deterministic code and not vice versa. Moreover, a probabilistic code is able to combine the different sources of uncertainty in a consistent way. It also provides a more realistic representation of the engineering problem; and finally, a quantitative discussion of structural safety is only possible, if the engineering problem is formulated probabilistically.

On the other hand, deterministic design codes using partial safety factors provide a time efficient and therefore, cost efficient way to verify structural reliability. In addition, deterministic codes do not require engineers to have a profound knowledge of probabilistic design.

Despite the advantages of probabilistic codes, the major part of safety verification will still be made using deterministic design codes. However, probabilistic codes are increasingly used for the design of extraordinary structures, see Vrouwenvelder (2002).

In the following the code calibration procedure for structural design codes using partial safety factors, e.g. Eurocodes or Swisscodes, is introduced. The same procedure can also be applied to codes using load and resistance factor design (LRFD), which was introduced by Ravindra and Galambos (1978).

10.1 Calibration Procedure

A systematic code calibration procedure is proposed by Lind (1978a) and Lind (1978b), see also Melchers (1999) and Faber and Sørensen (2002). A systematic approach can be subdivided in five steps, namely:

1. Definition of scope,
2. Code objective,
3. Frequency of demand,

4. Code space metric and
5. Selection of the best code format.

10.1.1 Definition of Scope

It is impossible to account for all possible structural design situations by one single structural design code such that the objective of the code is met exactly. An optimal set of partial safety factors for a specific limit state is not generally optimal for another. An illustrative example according to Ditlevsen (1997) is given in Section 10.4. Therefore, the scope has to be limited and determined.

For instance, a deterministic code may be calibrated for a specific type of structure such as dolos armours for rubble mound breakwater Sørensen, Kroon, and Faber (1994). On the other hand, a code may comprise a variety of structural types (buildings, bridges, etc.) different materials (steel, concrete, timber, etc.) assembled with different technologies (reinforced or prestressed concrete) so that the structure withstands loads (wind, live load, etc.) at different geographic locations safely and is reliable against different failure modes (shear, bending, buckling, etc.). The parametered set of structures, limit states etc. may be summarized in a set, called the data set, Lind (1978a).

10.1.2 Code Objective

Defining the objective of the code is the second step. For instance, the maximal probability of failure for a specific structural type may be set to a predefined value p_f^t . Or the objective may be to maximize the expected utility associated with the civil engineering facilities or the activity they support.

If the optimal safety level for a specific class has already been derived, the objective can be expressed to have constant reliability under constant consequence conditions. But applying the same level of safety over the whole data set would contradict the *principle of marginal return*. For instance, if a screw is the critical component of a series system and failure consequences are high, then it is optimal to design the screw to a higher safety level, e.g. when compared to a steel beam, because the initial costs of the screw are relatively low.

10.1.3 Limit States and their Relative Frequency

Based on the predefined scope of the code, the relevant limit states are identified, which are used in the calibration process. It is noted that generally, the differentiation of the continuous spectrum of failures into ultimate and serviceability limit states is a gross simplification. Nonetheless, it is a meaningful method.

From experience it is known that some of the identified limit states occur more often than others. The relative frequency w_i of the i^{th} limit state expresses the importance of the particular limit state (with $\sum_i w_i = 1$). By means of w_i this importance can be considered in the code metric, whereby \mathbf{w} is the vector that contains all w_i .

| Relative Cost of Safety Measure | Ultimate Limit State | | |
|---------------------------------------|-------------------------------|----------|-------|
| | Expected Failure Consequences | | |
| | Minor | Moderate | Large |
| Large | 3.1 | 3.3 | 3.7 |
| Normal | 3.7 | 4.2 | 4.4 |
| Small | 4.2 | 4.4 | 4.7 |

Table 10.1: Target reliability levels for Ultimate Limit State (ULS) according to the JCSS Probabilistic Model Code for a reference period of one year.

| Relative Cost of Safety Measure | Serviceability Limit State |
|---------------------------------------|----------------------------------|
| High | 1.3 |
| Moderate | 1.7 |
| Low | 2.3 |

Table 10.2: Target reliability levels for Serviceability Limit State (SLS) according to the JCSS Probabilistic Model Code for a reference period of one year.

10.1.4 Code Metric

The code space metric formulates the objective of the code mathematically and measures the closeness of a particular code format to this objective. As discussed in Section 10.1.1, the objective of the code can be founded on a reliability or on a risk-based formulation.

10.1.4.1 Reliability-Based Metric

A reliability-based metric is meaningful, if the target reliability level is known for a specific class of structures under constant exposures and consequences. For instance, target reliability levels are identified in the Eurocodes EN 1990:2002 (see also JCSS (1996)) or by the JCSS¹ Probabilistic Model Code JCSS (2001b). Tables 10.1 and 10.2 summarize target levels of reliability for the ultimate limit state (ULS) and serviceability limit state (SLS). The tables have been taken from the JCSS Probabilistic Model Code JCSS (2001b). The JCSS classifies the expected failure consequences into three categories using ρ , the ratio between the total cost (construction costs plus failure costs) and construction costs with:

$$\begin{aligned} \rho < 2: & \quad \text{Minor consequences,} \\ 2 < \rho < 5: & \quad \text{Moderate consequences and} \\ 5 < \rho < 10: & \quad \text{Large consequences.} \end{aligned}$$

If ρ is larger than 10, the JCSS recommends to carry out a full cost benefit analysis. In this case, consequences are extreme and the project might have a negative utility. In such a case, the project should not be carried out.

¹Joint Committee on Structural Safety, www.jcss.ethz.ch.

The vector β^t contains the target reliability levels β_i^t for each design situation i . If the β_i^t have been identified, the code metric can be chosen. A crude but simple approach is to choose the code format which minimizes the maximal difference of the reliability level to the target reliability index, see Equation 10.1. However, this equation does not consider the relative importance of the design situation i which is expressed by w_i . Generally, Equation 10.2 is used for reliability based code calibration. It considers all design situations i defined within the scope of the code (including materials, technology, loads, consequences, etc.). The relative frequency w_i of the design situation weights the difference between the achieved and target level. Here, the factor n is larger than one. $n > 2$ penalizes large deviations from the target level disproportionately and with $n = 2$ the least square method is obtained.

$$M(\gamma, \beta^t, \mathbf{w}) = \max_i |\beta_i(\gamma) - \beta_i^t| \quad (10.1)$$

$$M(\gamma, \beta^t, \mathbf{w}) = \sum_i w_i |\beta_i(\gamma) - \beta_i^t|^n \quad (10.2)$$

Finally, the best code format is obtained by minimization of the code metric over the space of meaningful $\gamma \in \Gamma$, whereas Γ is a set defined by the lower and upper limits γ_j^l, γ_j^u of the partial safety factors γ_j . Then the optimization formulation is given as

$$\begin{aligned} \min_{\gamma} M(\gamma, \beta^t, \mathbf{w}) \\ \text{s.t. } \gamma \in \Gamma. \end{aligned} \quad (10.3)$$

Inclusion of other boundary conditions is conceivable. For instance, it may be added that a minimum reliability level is always ensured. Instead of using the reliability index β , the minimization problem can also be formulated using the failure probability P_f by means of Equation 3.16, see Sørensen et al. (1994). In addition, design situations may involve more than one design variable. In this case, the mapping of a specific γ to the set of design variables can be ambiguous. However, this problem may be solved using the outcome of another optimization problem, see Sørensen et al. (1994).

Although codes may be calibrated on the basis of a target reliability, the main objective of structural codes in general is the maximization of the societal utility. Safety is balanced with economic efficiency.

10.1.4.2 Expected Utility Maximization

In accordance with Equation 8.9 the code calibration decision problem can be formulated as

$$\begin{aligned} \max_{\gamma} M(\gamma, \beta^t, \mathbf{w}, C) \\ \text{s.t. } \gamma \in \Gamma, \end{aligned} \quad (10.4)$$

where $M(\gamma, \beta^t, \mathbf{w}, C)$ is the expected utility $E[u(\gamma, \beta^t, \mathbf{w}, C)]$ associated with $\gamma, \beta^t, \mathbf{w}$, and the consequences C . An illustrative example is given in Lind (1978b), see also Ditlevsen and Madsen (2005). Here, the expected utility associated with structural types represented by the scope of the code can be formulated in terms of expected total cost C_T .

$$C_T = C_C + C_F P_f \quad (10.5)$$

C_T is calculated from the construction cost C_C , the failure probability P_f and the expected failure costs C_F . The latter may include immaterial losses. In general, it is possible to extend this equation to consider the complete life cycle, as given by Equation 8.9. In the neighborhood of the optimal reliability, any construction cost relation can be approximated by

$$C_I = a(1 + b\beta). \quad (10.6)$$

Moreover, Lind approximates $P_f = \Phi(-\beta)$ by $P_f = c e^{-\frac{\beta}{d}}$ so that the expected utility is formulated as

$$C_T(\beta) = a(1 + b\beta) + C_F c e^{-\frac{\beta}{d}}. \quad (10.7)$$

Differentiation of $C_T(\beta)$ with respect to β yields the minimal total cost and the relation

$$C_F = \frac{abd}{c} e^{\frac{\beta^t}{d}}, \quad (10.8)$$

from which the optimal reliability β^t can be derived. Implementing Equation 10.8 into 10.7 and taking the difference of $C_T(\beta)$ and $C_T(\beta^t)$, the following metric is obtained.

$$M = \frac{C_T(\beta) - C_T(\beta^t)}{abd} = \frac{\beta - \beta^t}{d} - 1 + e^{-\frac{\beta - \beta^t}{d}} \quad (10.9)$$

Obviously, the metric is independent of the factors a, b, c and C_F , but these values are needed to determine β^t . For $10^{-6} \leq P_f \leq 10^{-3}$, $d \approx 0.23$ is a good approximation, Ditlevsen and Madsen (2005). In Figure 10.1, it is seen that the metric represented by Equation 10.9 is not symmetrical. Reliability indices smaller than the target value are more penalized than larger values. Hereby, the engineers intuitive preference to overdesign at low additional cost is implicitly accounted for, Lind (1978b).

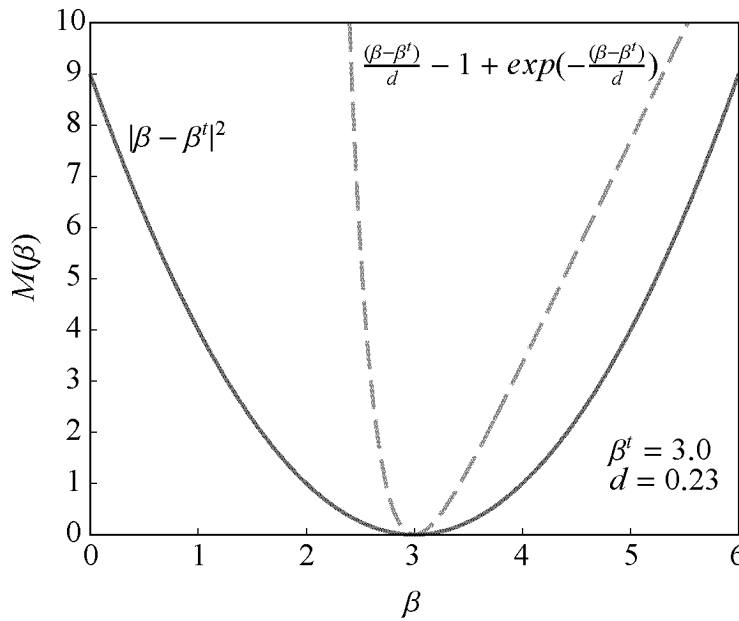


Figure 10.1: Code metric for reliability and risk-based code calibration.

10.1.5 Selection of Best Code Format

The most suitable set of safety factors for a code format may be derived by carrying out steps 1 to 4 of the code calibration process. However, first, a code format has to be defined, as well as the underlying probabilistic model.

10.1.5.1 Code Format

The code format describes the limit state by means of design equations, characteristic values for the variables \mathbf{x}_k and safety factors γ . Empirical relations may be used as design equations $d(\mathbf{x}_k, \gamma, \mathbf{z})$ with the vector \mathbf{z} containing the design variables. But if available, they should be replaced by physical models. The code format also defines characteristic values in terms of return periods or the quantile of the cumulative distribution function considering a specific reference period. If design codes consider more than two variable loads, which can act simultaneously on the structure, it is required to consider the likelihood of the possible load combinations. This is accounted for by using load combination factors Ψ_0 , Ψ_1 and Ψ_2 , see also Figure 10.2. These factors are also summarized in γ .

When faced with the question which code format to choose, the most important question to answer is the determination of the number of safety factors. Utilizing many factors, a flexible code is obtained, which better approaches the objective of the code. However, the flexibility has to be paid for by a higher level of complexity. For ordinary structural design codes, the objective of the code can only be fulfilled exactly at the expense of an unacceptable level of complexity. Structural design codes must retain a level of simplicity, not for the engineers' comfort but also to limit gross errors.

10.1.5.2 Probabilistic Model

The equivalent to the design equation $d(\mathbf{x}_k, \gamma, \mathbf{z})$ is the limit state function $g(\mathbf{x}, \gamma, \mathbf{z})$. The latter is evaluated probabilistically. If possible, the limit state function should be given a physical meaning according to the current state of knowledge. It is not necessary that the design equations and limit state functions are based on the same physical model. For instance, the technological development implied in the evolution of structural design codes can only be quantified if they are verified against a common basis, i.e. the same limit state function.

After having defined \mathbf{z} for a given γ with the design equation $d(\mathbf{x}_k, \gamma, \mathbf{z})$, the corresponding probability of failure can be evaluated using the limit state function $g(\mathbf{x}, \gamma, \mathbf{z})$ and methods of structural reliability as discussed in Chapter 3. Therefore, it is needed to assign a probability structure to the relevant random variables \mathbf{X} in terms of $F_{\mathbf{X}}(\mathbf{x})$.

If there is more than one variable load, the likelihood of combined occurrences has to be modelled. Initially, Turkstra's rule, which is not conservative, was utilized. Ferry Borges-Castanheta's load model is more suitable, see Thøft-Christensen and Baker (1982). Another helpful reference not least for probabilistic modeling of load combinations but also for load and resistance variables is the JCSS Probabilistic Model Code, JCSS (2001b).

Finally, from a set of conceivable simple code formats, the format is chosen which is not too complex and, which approaches the objective of the code best. A comprehensive code calibration also identifies the sensitivity of the structural code with regard to the determined safety factors and decisive parameters. In the following, the partial safety factor format utilized by the Eurocodes is introduced.

10.2 The Code Format of the Eurocodes

The Eurocodes distinguishes between ultimate and serviceability limit states (ULS) and (SLS). SLS describes limit states relevant for the serviceability of the structure such as deflections but also the comfort of its users. Moreover, it ensures the durability of the structure. The ULS considers limit states, which are relevant for the structural integrity and the safety of persons. Moreover, the verification of ULS can further be classified into four categories, namely:

1. Loss of equilibrium of the structure or parts thereof, which can be considered as a rigid body,
2. Structural failure due to insufficient structural resistance,
3. Failure due to insufficient soil resistance and
4. Failure due to fatigue.

Both limit states (ULS and SLS) are formulated as design equations which are based on the partial safety factor format. In the Eurocodes, design equations are found for resistances R_d , load effects E_d and serviceability criteria C_d . Finally, the design is verified if the design value of the resistance R_d is larger than the design value of the load effect E_d (ULS) or if the design value E_d is smaller than C_d (SLS). The aim of fulfilling the ultimate as well as the serviceability limit states is that thereby Equation 8.9 is maximized.

$$E_d \leq R_d \quad (\text{ULS}) \quad (10.10)$$

$$E_d \leq C_d \quad (\text{SLS}) \quad (10.11)$$

In the following, the code format is introduced for the ultimate limit state considering the design of ordinary structures subject to ordinary loads. The design verification for SLS and extraordinary design situations is straightforward, see EN 1990:2002.

The design value of the resistance is given by

$$R_d = \frac{1}{\gamma_{Rd}} d_R \left(\eta_i \frac{X_{k,i}}{\gamma_{m,i}} \right) \quad (10.12)$$

$$= d_R \left(\eta_i \frac{X_{k,i}}{\gamma_{M,i}} \right), \quad (10.13)$$

where $X_{k,i}$ is the characteristic value of the i^{th} material or soil property contributing to the resistance. γ_{Rd} is the partial safety factor considering the uncertainty of the model, η_i accounts for effects of time, environment, temperature, size, etc. and $\gamma_{m,i}$ is the partial safety factor for the uncertainty regarding the material property. Generally, R_d is calculated using $\gamma_{M,i}$ the resistance partial safety factor. $\gamma_{M,i}$ is obtained by multiplication of γ_{Rd} and $\gamma_{m,i}$. Also η_i may be included in $\gamma_{M,i}$. In the most simple case one obtains

$$R_d = \frac{R_k}{\gamma_M}. \quad (10.14)$$

From Equation 10.14 it is seen that the characteristic value of the resistance R_k is divided by the partial safety factor γ_M to obtain the design value R_d .

To obtain design variables of loads, e.g. variable loads Q_d , the characteristic value Q_k is multiplied by the partial safety factors γ_Q . The different definitions of load and resistance safety factors permit one to obtain – with the current definition of characteristic values – safety factors which are larger than one.

$$Q_d = \gamma_Q Q_k \quad (10.15)$$

Combining the relevant design loads, the design value of the load effect E_d is obtained. In the following equation, \oplus means *combined with* and \bigoplus *combined load effect of*, see EN 1990:2002 or Rackwitz (2004).

$$E_d = d_E \left(\bigoplus_{j \geq 1} \gamma_{G,j} G_{k,j} \oplus \gamma_P P \oplus \left[\gamma_{Q,j} Q_{k,j} \oplus \bigoplus_{m \neq j} \gamma_{Q,m} \psi_{0,m} Q_{k,m} \right] \right) \quad (10.16)$$

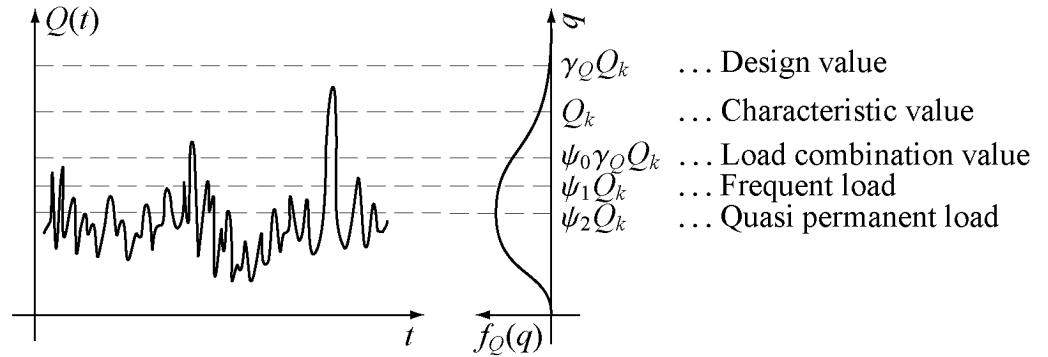


Figure 10.2: Representative values of variable loads.

Equation 10.16 makes use of different values of variable loads, namely characteristic value, design value and load combination value. The representation of these is illustrated in Figure 10.2 by means of the realization of a variable load and the corresponding arbitrary point in time distribution $f_Q(q)$. In the figure, it is seen that the characteristic value corresponds to a specific fractile of $f_Q(q)$. If $f_Q(q)$ is the annual extreme value distribution, then Q_k is given by $F_Q^{-1}(p_k)$ with p_k as the probability corresponding to the fractile. Generally, $p_k = 0.98$ for variable loads, which corresponds to a return period of $T_R = 50$ years.

$$T_R = \frac{1}{1 - p_k} = \frac{1}{1 - F_Q(Q_k)} \quad (10.17)$$

For accidental loads, the characteristic value may be given by a higher return period, e.g. characteristic earthquake loads are characterized with a return period of 475 years. For permanent loads G and prestressing P , the median can be used as the characteristic value. The design value is obtained by multiplying the characteristic value by the partial safety factor, whereas the load combination value is obtained, if the design load is multiplied by the load combination factor ψ_0 with $0 \leq \psi_0 \leq 1$. This factor takes into account the likelihood and intensity of a combined occurrence. For the verification of serviceability, the Eurocode introduces the frequent load $\psi_1 Q_k$ and

| Code | Material | γ_M |
|------|------------------------------------|------------|
| EC2 | Concrete | 1.50 |
| | Reinforcing and prestressing steel | 1.15 |
| | Unreinforced concrete | 1.80 |
| EC3 | Steel | 1.10 |
| | Joining elements | 1.25 |
| EC4 | Steel | 1.10 |
| | Concrete | 1.50 |
| | Reinforcing steel | 1.15 |
| EC5 | Timber | 1.30 |
| | Steel | 1.15 |
| EC6 | masonry | 1.70 |
| | Fastener | 2.50 |
| | Steel joining elements | 1.10 |

Table 10.3: Partial safety factors for resistances for ULS design situation.

| Load | Material | γ_M |
|--------------|-------------|------------|
| Permanent | unfavorable | 1.35 |
| | favorable | 1.00 |
| Prestressing | unfavorable | 1.20 |
| | favorable | 0.90 |
| Variable | unfavorable | 1.50 |
| | favorable | 0.00 |

Table 10.4: Partial safety factors for loads for ULS design situation.

quasi permanent load $\psi_2 Q_k$. Tables 10.3, 10.4 and 10.5 summarize the relevant safety factors for the most frequent design situations.

The safety factors considering accidental loading, as well as the other ULS limit states (geotechnical, fatigue and loss of equilibrium) and serviceability (SLS) are found in EN 1990:2002.

10.3 Classical Interpretation of Partial Safety Factors

From Equation 10.15, it is seen that the partial safety factors for loads are given by

$$\gamma_i = \frac{x_{d,i}}{x_{k,i}}. \quad (10.18)$$

In the classical interpretation, see JCSS (1996), the design variable is equivalent to the variable's component x_i^* of the design point \mathbf{x}^* . \mathbf{x}^* is the most likely point of failure. Considering α_{x_i} as the sensitivity of the limit state function with respect to x_i and β the reliability index, the design point $x_{d,i}$ in the standard normal space is given by $u_{x_i} = \alpha_{x_i} \beta$. If $p_{k,i}$ is the quantile defining the characteristic value $x_{k,i} = F_{X_i}^{-1}(p_{k,i})$, the partial safety factor can be written as

$$\gamma_i = \frac{F_{X_i}^{-1}(\Phi(-\alpha_{x_i} \beta))}{F_{X_i}^{-1}(p_{k,i})}. \quad (10.19)$$

| Action | ψ_0 | ψ_1 | ψ_2 |
|---|----------|----------|----------|
| Imposed loads in buildings, category (see EN 1991-1-1) | | | |
| Category A : domestic, residential areas | 0.7 | 0.5 | 0.3 |
| Category B : office areas | 0.7 | 0.5 | 0.3 |
| Category C : congregation areas | 0.7 | 0.7 | 0.6 |
| Category D : shopping areas | 0.7 | 0.7 | 0.6 |
| Category E : storage areas | 1.0 | 0.9 | 0.8 |
| Category F : traffic area, vehicle weight ≤ 30 kN | 0.7 | 0.7 | 0.6 |
| Category G : traffic area, 30 kN $<$ vehicle weight ≤ 160 kN | 0.7 | 0.5 | 0.3 |
| Category H : roofs | 0.0 | 0.0 | 0.0 |
| Snow loads on buildings (see EN 1991-1-3) | | | |
| Finland, Iceland, Norway, Sweden | 0.7 | 0.5 | 0.2 |
| Remainder of CEN Member States, altitude $H > 1000$ m a.s.l. | 0.7 | 0.5 | 0.2 |
| Remainder of CEN Member States, altitude $H \leq 1000$ m a.s.l. | 0.5 | 0.2 | 0.0 |
| Wind loads on buildings (see EN 1991-1-4) | 0.6 | 0.2 | 0 |
| Temperature (non-fire) in buildings (see EN 1991-1-5) | 0.6 | 0.5 | 0.0 |

NOTE: The load combination factors ψ_0 , ψ_1 and ψ_2 may be set by the National annex.

Table 10.5: Load combination factors.

This means that the partial safety factor is clearly identified by the distribution function $F_{X_i}(x)$, the sensitivity α_{x_i} , the reliability index β and $p_{k,i}$.

For instance, partial safety factors for normally distributed load variables are as follows.

$$\gamma_i = \frac{\mu_{X_i} - \alpha_{x_i}\beta\sigma_{X_i}}{\mu_{X_i} - k_{k,i}\sigma_{X_i}} \quad (10.20)$$

$$= \frac{1 - \alpha_{x_i}\beta V_{X_i}}{1 - k_{k,i}V_{X_i}} \quad (10.21)$$

With μ_{X_i} , σ_{X_i} , V_{X_i} as the mean, standard deviation and coefficient of variation of X_i . $k_{x,i}$ is the coefficient corresponding to the characteristic value of the normal distribution. For permanent loads X_i , $p_{k,i} = 50\%$ so that $k_{x,i} = 0$ and for normally distributed variable loads X_i , $p_{k,i} = 98\%$, i.e. $k_{x,i} = 2.05$.

Partial safety factors should be larger than one. Therefore, partial safety factors for resistances are defined as

$$\gamma_i = \frac{x_{k,i}}{x_{d,i}}. \quad (10.22)$$

If the resistance is lognormally distributed, the partial safety factor is given by following equations.

$$\gamma_i = \frac{x_{k,i}}{\mu_{X_i} \exp[-\frac{1}{2} \ln(1 + V_{X_i}^2) - \alpha_{x_i}\beta [\ln(1 + V_{X_i}^2)]^{1/2}]} \quad (10.23)$$

$$\approx \frac{\mu_{X_i} \exp(-k_{x,i}V_{X_i})}{\mu_{X_i} \exp(-\alpha_{x_i}\beta V_{X_i})} \quad (10.24)$$

$$\approx \exp[(\alpha_{x_i}\beta - k_{x,i})V_{X_i}] \quad (10.25)$$

| Variable | α_{x_i} |
|--------------------------------|----------------|
| Dominating resistance variable | 0.8 |
| All other resistance variables | ≈ 0.4 |
| Dominating load variable | 0.7 |
| All other load variables | ≈ 0.4 |

 Table 10.6: Approximative values for the sensitivity factor α_{x_i} .

In the Eurocodes, characteristic values for resistances are defined by the 5% fractile so that $k_{x,i} = 1.64$. For Weibull distributed resistances the partial safety factor is given by

$$\gamma_i = \frac{[-\ln(\Phi(\Phi^{-1}(p_{k,i})))]^{1/k}}{[-\ln(\Phi(\alpha_{x_i}\beta))]^{1/k}}, \quad (10.26)$$

where k is the form factor of the Weibull distribution.

As an approximation, the values given in Table 10.6 may be used for α_{x_i} , see Rackwitz (2004) and EN 1990:2002.

In addition to the partial safety factors, the *central safety factor* γ_c and the *characteristic safety factor* γ_k can be defined, see Cornell (1969a), Cornell (1969b) and Melchers (1999). Therefore, the load variable S and the resistance variable R are needed together with the approximation $(\sigma_R^2 + \sigma_S^2)^{1/2} \approx \alpha(\sigma_R + \sigma_S)$ to obtain the following approximations for γ_c and γ_k .

$$\gamma_c = \frac{\mu_R}{\mu_S} = \frac{1 + \alpha_S \beta V_S}{1 + \alpha_R \beta V_R} \quad (10.27)$$

$$\gamma_k = \frac{R_k}{S_k} = \frac{\mu_R(1 - k_R V_R)}{\mu_S(1 - k_S V_S)} = \frac{(1 - k_R V_R)}{(1 + \alpha_R \beta V_R)} \frac{(1 + \alpha_S \beta V_S)}{(1 - k_S V_S)} \quad (10.28)$$

It is immediately seen that the characteristic safety factor is simply the product $\gamma_k = \gamma_R \gamma_S$ so that again the partial safety factor format is obtained, see also Figure 10.3.

$$\frac{R_k}{\gamma_R} \geq \gamma_S S_k \quad (10.29)$$

The classical interpretation of partial safety factors is easy to use and gives partial safety factors an illustrative representation. For a given limit state function, partial safety factors can be calculated so that the target reliability level is achieved exactly. The biggest advantage is that approximate partial safety factors can be readily obtained without a thorough structural reliability analysis. Only the distribution functions, the target reliability level β_t , the characteristic values $x_{k,i}$ and the sensitivities α_{x_i} have to be known. For the latter, valid approximations are given in Table 10.6. On the other hand, it is seen that these safety factors depend on the sensitivities referring to limit state functions. That is, if different limit states are covered within the scope of the code, this method becomes impractical.

10.4 The Disadvantage of an Extensive Scope

In Ditlevsen (1997) and Ditlevsen and Madsen (2005) an illustrative example is given that shows why an extensive scope of a structural design code is not able to achieve the objective of a code exactly. In Figure 10.4a two limit state functions $g_1(\mathbf{u})$ and $g_2(\mathbf{u})$ are illustrated together with

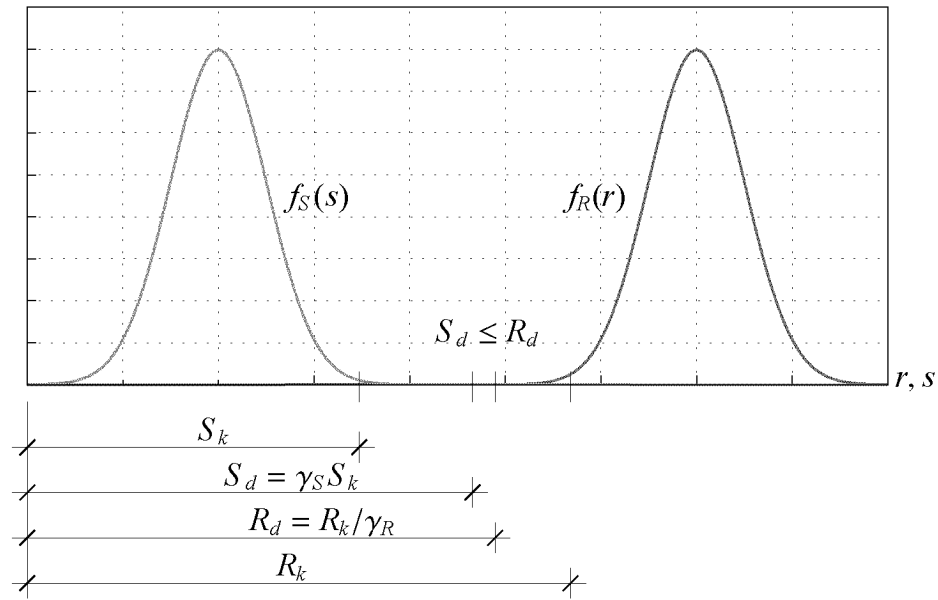


Figure 10.3: Partial safety factor concept.

their design points D_1 and D_2 which lie on the circle representing the same reliability β . The corresponding vectors from the origin to the design points are $\alpha_1\beta$ and $\alpha_2\beta$. It is seen that the intersection point D of the $g_1(\mathbf{u})$ and $g_2(\mathbf{u})$ can be used as a common design point and partial safety factors may be calculated. Hereby, partial safety factors are derived for both limit states. $\tilde{\alpha}\beta$ is the vector from the origin to the new design point D .

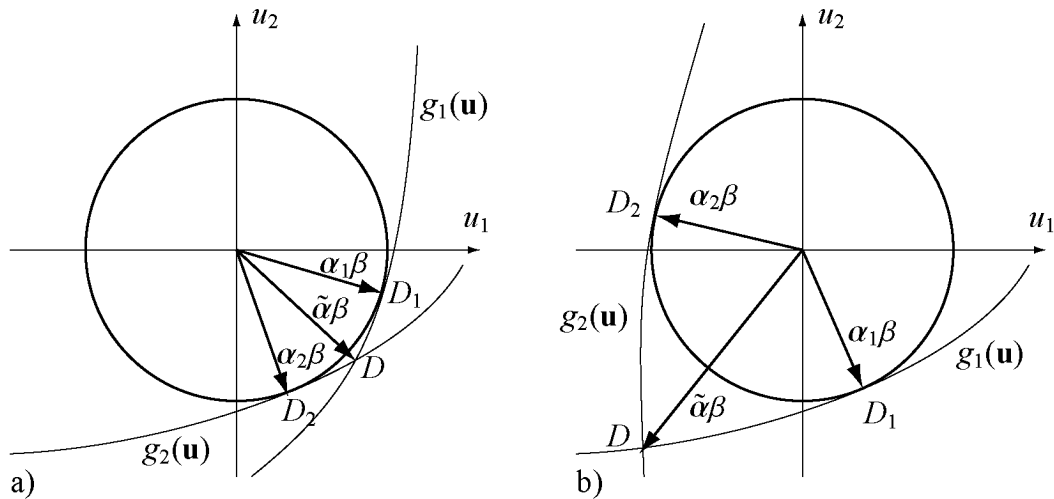


Figure 10.4: Replacement vector $\tilde{\alpha}$.

However, if the scope covers more than two different limit states, then the objective cannot be fulfilled exactly. Figure 10.4b shows that the approximation error can be considerable when the scope of the code is extended. As an example, Ditlevsen (1997) considers the design of

reinforced concrete. In particular the design against bending failure of a beam and compression failure of a short column is considered. In the case of bending failure, the yield strength of the reinforcement u_1 is the dominant resistance variable and u_2 the compression strength of the concrete does not contribute much. Hence, D_1 is in the vicinity of $(-\beta, 0)$. In the case of a short column failure the main resistance variable is the compressive strength of the concrete and yield strength is of minor importance; D_2 is close to $(0, -\beta)$. This situation is illustrated in Figure 10.4b. Adding a third limit state function, the objective of the code cannot be fulfilled exactly. It can only be approximated. Two ways to reduce the error are possible. Firstly, the scope of the code can be limited so that two codes are set up, one for $g_1(\mathbf{u})$ and one for $g_2(\mathbf{u})$, or different sets of partial safety factors may be used. If both options are considered to be impractical, the approximation error has to be accepted.

10.5 Should Safety Factors be Calibrated for Plastic or Elastic Limit States?

For which limit state should the safety factors of structural design codes be calibrated, for the elastic or the plastic limit? The answer to this question depends on the material itself. If in reality the considered material behaves linearly elastic, the answer is simple. Only design involving the elastic limit is appropriate. However, the consequences of an abrupt failure may be different to ductile failure and should be accounted for in the code calibration process, e.g. by means of a higher target reliability.

If after linear elastic deformation the material exhibits ductile behaviors, should the structure that is built with a ductile material be calculated according to the plastic or elastic limit state? The ductile behavior describes reality better and therefore the material law accounting for this should be applied. On the other hand, designers might question, if it is appropriate to decrease the structural reliability by designing according to plasticity theory. Ditlevsen (1997) states that many structures have already been built using plasticity theory as the design basis; the implied reliability reduction is accepted by the public, which has not detected higher failure rates of structures or other weaknesses. Therefore, limit state formulations must use plasticity theory, if the material is appropriately described thereby.

If the limit state is defined in the code calibration process as the event of structural failure, i.e. structural collapse, then for ductile materials plasticity theory is to be applied in the limit state formulation. However, is it possible to calibrate partial safety factors so that safety verifications might for convenience be made on the basis of the the elastic limit, in the way that the plastic limit is considered? That this is impossible is illustrated by the following simple example.

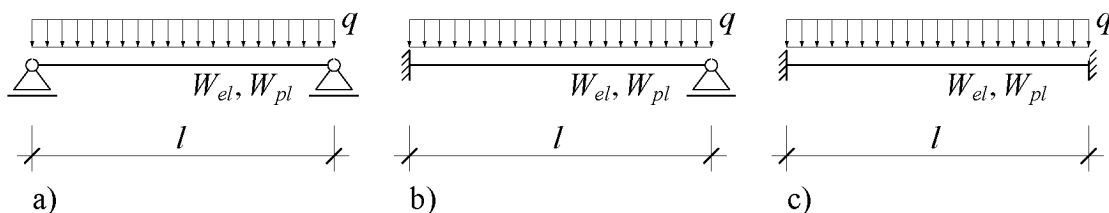


Figure 10.5: System A, B and C.

| Method | A | B | C |
|--------|-----------------------------|------------------------------|------------------------------|
| E – E | $\frac{gl^2}{8}$ | $\frac{gl^2}{8}$ | $\frac{gl^2}{12}$ |
| E – P | $\frac{gl^2}{8\kappa_{pl}}$ | $\frac{gl^2}{8\kappa_{pl}}$ | $\frac{gl^2}{12\kappa_{pl}}$ |
| P – P | $\frac{gl^2}{8\kappa_{pl}}$ | $\frac{gl^2}{12\kappa_{pl}}$ | $\frac{gl^2}{16\kappa_{pl}}$ |

Table 10.7: Required elastic moment M_{el}

| Method | A | B | C |
|--------|-------------------------|--------------------------|--------------------------|
| E – E | $\frac{1}{\kappa_{pl}}$ | $\frac{2}{3\kappa_{pl}}$ | $\frac{3}{4\kappa_{pl}}$ |
| E – P | 1 | $\frac{2}{3}$ | $\frac{3}{4}$ |
| P – P | 1 | 1 | 1 |

Table 10.8: Reduction factor γ_{rd} for equivalent design and reliability.

Consider the three simple mechanical systems as shown in Figure 10.5. Figure 10.5a shows system A, a simple supported beam loaded with the uniformly distributed load q . The other systems (system B and C) shown in Figures 10.5b and 10.5c are also loaded with q . While the beam of B is simply supported on one side and clamped on the other, the other beam C is clamped on both ends. All three beams are of length l and reach the elastic limit at M_{el} and the plastic limit at M_{pl} . The ratio between M_{pl} and M_{el} is given by $\kappa_{pl} = M_{pl}/M_{el}$, with $\kappa_{pl} \geq 1$.

For all three systems the maximal load can be calculated using a linear elastic and a plastic strain stress relation. These loads may then be compared with the elastic and plastic resistance. Comparing the elastic resistance with the elastic load is identified by "E – E", whereas "E – P" indicates that the elastic load is compared with the plastic resistance. Finally "P – P" means that both load and resistance are evaluated on the basis of the plasticity theory.

Making use of $M_{el} = \frac{M_{pl}}{\kappa_{pl}}$, the required elastic resistance is calculated for each evaluation method ("E – E", "E – P" and "P – P") and system (A, B and C). The results are summarized in Table 10.7. Hence, if q , l and κ_{pl} are given, the cross-section of the beam may be selected to fulfill the safety requirement.

If it is agreed that the design method "P – P" represents the failure limit state best, then a reduction factor γ_{rd} could be introduced that reduces the design resulting from "E – E" or "E – P" so that the same design and reliability is achieved. The reduction factor γ_{rd} is simply obtained by dividing the required M_{el} obtained from "P – P" with the corresponding values of Table 10.7. The factors are summarized in Table 10.8.

From Table 10.8 it is seen that the variation of γ_{rd} is large. The factor varies across systems, design methods and types of cross-section represented by κ_{pl} . If it is agreed that the plasticity theory is the more realistic description of the material's behavior, there is no meaningful way to adjust safety factors for elastic design equations without introducing an unacceptable level of complexity. In this case, the design of ductile behaving structures using a linear elastic material behavior is tantamount to wasting money because structures are too conservatively designed.

Moreover, from Table 10.8 it is seen that the elastic design of systems A and B provides design variables which are κ_{pl} and $\frac{3}{2}\kappa_{pl}$ larger than required. This shows that B is more reliable than A. This means that for the same failure consequences, an elastic design leads to different levels of

reliability across systems and cross-sections.

10.6 Updating of Characteristic Values

Characteristic values of load and resistance variables are generally defined by means of quantiles of the underlying probability distribution function $f_X(x)$, see Section 10.2. The distribution of the variable X is fully described by the distribution type and the parameters θ . The parameters $\theta = (\theta_1, \theta_2, \dots, \theta_m)^T$ may be, e.g. the mean $\theta_1 = \mu_X$ and standard deviation $\theta_2 = \sigma_X$ of the considered variable. For a given θ the distribution may also be denoted with $f_X(x|\theta)$. The parameters may also be uncertain and can be described by a distribution function $f'_\theta(\theta)$. The prime identifies the distribution as a prior distribution which considers knowledge available a priori.

If new information becomes available in terms of observations $\mathbf{o} = (o_1, o_2, \dots, o_n)^T$, the parameters may be updated by the calculation of the posterior distribution $f''_\theta(\theta)$.

$$f''_\theta(\theta) = \frac{L(\mathbf{o}|\theta)f'_\theta(\theta)}{\int_{-\infty}^{\infty} L(\mathbf{o}|\theta)f'_\theta(\theta)d\theta} \quad (10.30)$$

In this equation, $L(\mathbf{o}|\theta)$ is the likelihood of the observations, which is given by

$$L(\mathbf{o}|\theta) = \prod_{i=1}^n f_X(o_i|\theta). \quad (10.31)$$

After having calculated the posterior distribution, the predictive distribution $F_X(x|\mathbf{o})$ may be obtained by the following integration.

$$F_X(x|\mathbf{o}) = \int_{-\infty}^{\infty} F_X(x|\theta)f''_\theta(\theta)d\theta \quad (10.32)$$

Finally, the updated characteristic value is obtained as

$$x_k^u = F_X^{-1}(p_k|\mathbf{o}), \quad (10.33)$$

with $F_X^{-1}(p_k|\mathbf{o})$ as the inverse of $F_X(x|\mathbf{o})$ and p_k the probability defining the characteristic value.

For the above procedure, there are distributions such that the posterior distribution is of the same type as the prior distribution. Such distributions are called *conjugate prior distributions* and analytical solutions are available, see e.g. JCSS (2001a).

10.6.1 Example: Updating of Yield Strength

The yield strength distribution is a priori modelled to be normally distributed with a known standard deviation of $\sigma_{f_y} = 15$ (all values are given in MPa). Based on experience, the mean value is modelled as a normally distributed variable with the mean value $\mu_{\mu_{f_y}} = 345$ and standard deviation $\sigma_{\mu_X} = 10$. In addition, experiments are carried out leading to the following observations: $\mathbf{o} = (364, 338, 356, 366, 351)^T$.

Applying Equations 10.30, 10.32 and 10.33, the characteristic value is updated from $f_{yk} = 315$ MPa to $f_{yk} = 325$ MPa, see Figure 10.6. In addition, the coefficient of variation is reduced from

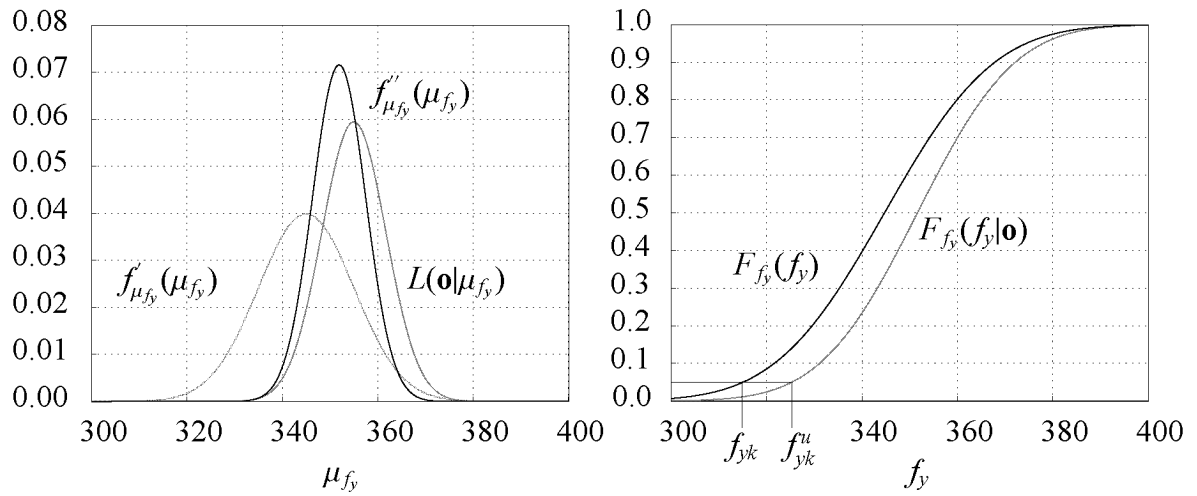


Figure 10.6: Prior, posterior and predictive distribution.

| | original | optimized | updated & optimized |
|------------|----------|-----------|---------------------|
| γ_R | 1.10 | 1.03 | 1.02 |
| γ_G | 1.35 | 1.25 | 1.27 |
| γ_Q | 1.50 | 1.61 | 1.65 |

Table 10.9: Optimized partial safety factors using CodeCal.

$V_{f_y} = 0.052$ to $V_{f_y}^u = 0.046$. This means that by updating, not only the characteristic value is better predicted, but also variation may often be reduced, which leads to a higher reliability, if the same partial safety factors are applied. On the other hand, safety factors may be updated as well to maintain the same reliability.

With $\alpha_{f_y} = 0.8$, $\beta = 4$, $k_{f_y} = 1.64$ and the inverse of Equation 10.21, the partial safety factors are obtained as:

$$\gamma_M = 1.10 \quad \text{and} \quad (10.34)$$

$$\gamma_M^u = 1.08. \quad (10.35)$$

Finally, the increase in f_{yk} and the decrease of the partial safety factor γ_M lead to a 4.8% reduction of the required design variable, e.g. cross-section.

Figure 10.7 illustrates the results using the more refined reliability-based code calibration procedure which is implemented in the JCSS code calibration program CodeCal, see Faber, Kübler, and Köhler (2003). Basically, Equation 10.2 is used with $n = 2$. The figure shows the original distribution of the reliability index β_{or} for partial safety factors as applied in the current Eurocodes. Here, the reliability index is shown for different values of α , the ratios of permanent load G to total load $G + Q$. Calibrating the partial safety factors using CodeCal, the distribution of β_{opt} is obtained. Especially for high fractions of permanent load (α close to one), it is seen that the obtained distribution is much closer to the target reliability $\beta_t = 4.0$. Table 10.9 shows that, depending on the choice of approach, the partial safety factors may differ considerably. The table also shows that updating may reduce the safety factor of the resistance which is achieved by an increase of the load factors. In Figure 10.7 it is seen that the reliability obtained after updating and optimization is practically identical to the optimized level without optimization. For the

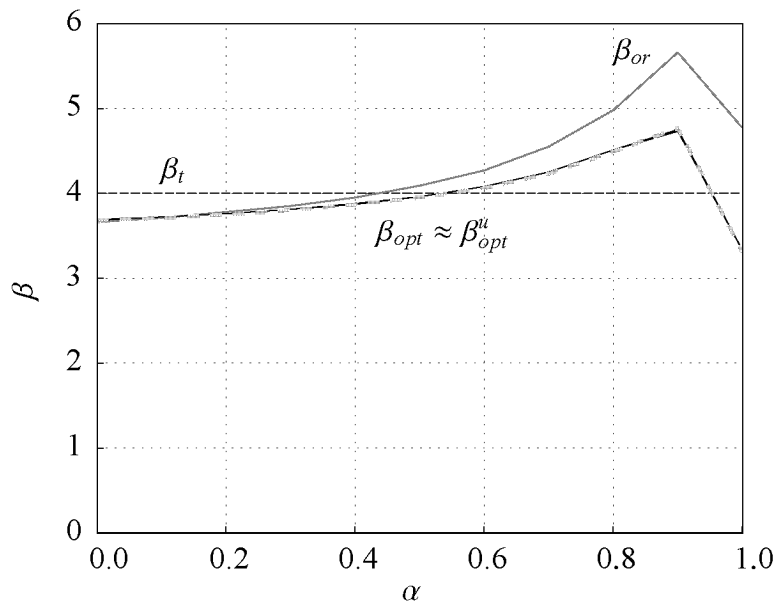


Figure 10.7: Code calibration using CodeCal03.

calibration procedure, the relative frequencies w_i are assumed to be uniformly distributed, the permanent load is taken to be normally distributed with a coefficient of variation of $V_G = 0.10$ and the variable load is Gumbel distributed with $V_Q = 0.4$.

10.7 Summary

Structural design codes are time and cost-efficient tools to verify the safety of civil engineering structures. This makes structural design codes a crucial component in managing risks associated with civil engineering facilities.

First a code calibration procedure is introduced consisting of five steps, namely: the definition of the scope of the code, the formulation of the objective of the code, the determination of the relative frequency of the limit states, selection of a code metric expressing the objective of the code mathematically and finally, the selection of the best code format.

As an example, the code format of the Eurocodes is introduced and thereafter the classical interpretation of partial safety factors is given. By means of the classical interpretation, it is shown that an extensive scope is in conflict with a good approximation of the objective of the code.

Moreover, it is shown that if it is agreed that the plastic limit states represent a more realistic description of the failure states, then partial safety factors should be calibrated on the basis of plasticity theory. Otherwise, a non-uniform level of reliability is obtained for the same associated consequences.

Finally, it is shown how new information may be incorporated. Additional observations may be used to update the prior distribution of the parameters of a basic random variable, the predictive distribution, characteristic values and safety factors.

11 Summary and Conclusions

11.1 Summary

The optimal management of civil engineering facilities requires decision-making. For instance, the optimal structural design deals with the choice of the most appropriate design concept, construction method, and not least the determination of design variables, such as the cross-sectional area or the section modulus of a steel beam. But decisions have to be made not only in the design stage. The whole life cycle of the structure needs to be considered and decisions have to be made throughout the whole life cycle of the considered facility/ structure.

The present work, which is subdivided into three parts, addresses the basic principles of decision-making in civil engineering. Part I (Chapter 2 and 3) reviews the fundamentals, namely decision theory and uncertainty modeling, whereas Part II (Chapter 4–7) is concerned with the assessment and modeling of consequences and preferences. Finally, Part III (Chapter 8–10) puts together the different components of decision-making and illustrates its applicability by means of principal studies.

A rational basis for decision-making is decision theory, which is introduced in Chapter 2. The chapter first classifies decision theory into different categories and discusses which of them are particularly relevant for engineering decision-making. This includes the differentiation between descriptive and normative decision theory or decision-making under uncertainty, risk or certainty. If uncertainty is expressed in terms of probabilities, the two strong concepts, namely decision theory and Bayes' theorem can be joined. The decision basis thus obtained is also known as Bayesian decision theory and it permits one to integrate new information into the decision process and to evaluate the value of information. This makes it a crucial tool for the assessment of existing structures and risk-based inspection and maintenance planning. Moreover, it is shown how decision-making in civil engineering is carried out within a risk management framework.

For decision-making under risk, the involved uncertainty needs to be expressed in terms of probabilities. Chapter 3 first categorizes uncertainties into aleatoric and epistemic. Then, three possible interpretations of probabilities are given, namely the classical, the frequentistic and the subjective interpretation. Engineering problems involve all three of them and they can be consistently combined, which is also referred to as Bayesian modeling. In order to quantify uncertainty by using statistical methods requires that the event of interest is observable. However, this is seldom the case in civil engineering. Such events can be modelled using methods for time-variant and -invariant structural reliability analysis.

Chapter 4 starts with a categorization of the consequences. Mostly, they are subdivided into consequences to humans, environment, economy and cultural assets. Moreover, consequences can also be differentiated, whether they are substitutable i.e. material or immaterial consequences. They can also be differentiated, whether they constitute direct or indirect consequences. However, the latter differentiation involves a subjective consideration of what is interpreted as being directly or indirectly related. Independent of the used categorization scheme, it is helpful

to categorize consequences into different types that are mutually exclusive. This allows one to obtain the total consequence by simply adding up the different types of consequences within the introduced framework. The framework is illustrated by giving an overview of the consequences associated with the structural failure of the WTC Twin Towers. This assessment highlights the importance of the inclusion of follow-up consequences.

The original derivation of the life quality index (LQI) is given in Chapter 5 together with other published formulations of the LQI. Then, the correlation between GDP per capita and the life expectancy is studied, which is observed for many countries. It is shown that it is possible to construct a framework, which argues that the observed correlation results from economic behavior of rational decision-making. That is, the decision maker chooses the best combination of life expectancy and consumption that is affordable. In addition, the different LQI formulations are checked against plausibility considerations with regard to possible combinations of leisure and the gross domestic product. Several LQI formulations are rejected for not appropriately reflecting the preferences of the public. The remaining LQI formulation is then investigated, whether it is possible to derive an optimal consumption of leisure. The obtained optimal consumption is described by a path which is compared with empirical data. Generally, a tendency supporting the study can be observed.

In Chapter 6, acceptance criteria are derived based on the life quality index. Acceptable life saving costs are derived which can be considered within a risk-based decision framework. The life saving costs can be interpreted as the costs that an activity or safety measure can be more expensive, if it saves an anonymous person's life. Even if this basis for decision-making is rejected, the implied life saving costs can be calculated for any decisions involving risk to life. This also includes decisions made in the past. The chapter closes with the introduction of other commonly applied acceptance criteria, namely the Farmer diagram, the fatal accident rate and structural design codes.

Even if it is agreed on the basis of utilitarian decision-making, the identification of the optimal decision is sensitive to the modeling of risk aversion, a controversially discussed concept. Chapter 7 summarizes the main reasons for risk aversion. It introduces the Arrow and Pratt measures to quantify risk aversion and shows that the LQI formulations comprise a risk averse attitude. This is illustrated by means of a simple example that shows the ability of the LQI to describe the self-preservation of individuals or societies. Finally, it is mentioned that inappropriately modelled risk aversion is penalized by not having considered opportunities.

Then Chapter 8 puts together the discussed components for engineering decision-making into one decision framework. The objective of this framework can be formulated using different approaches. It is argued that the most general approach is to assess the expected life cycle benefit. This approach is equivalent to decision-making according to the expected net present value. However, if not all uncertainties are expressed in terms of probabilities, decision-making under uncertainties can be studied after having accounted for the quantified uncertainty. However, if expert judgment is available, the remaining uncertainty can be expressed in terms of subjective probabilities. This approach allows e.g. code committees to consistently combine all available knowledge. The chapter also discusses criticisms that are directed towards risk and reliability assessments with regard to the inclusion of gross errors and the interpretation of calculated probabilities. Finally, discounting is briefly addressed.

By means of principal studies, Chapter 9 illustrates the application of the framework outlined in Chapter 8. The first principal study shows the optimal design of an offshore oil production facility, where the effect of different design approaches are investigated together with differ-

ent reconstruction considerations of failed structures. In addition, the optimal design together with the inspection and maintenance plan is identified for a concrete bridge that is subject to deterioration.

The work closes with Chapter 10, which illustrates how structural design codes can be calibrated on the basis outlined in Chapter 8.

11.2 Conclusions and Outlook

A focus of the work is to present decision theory as a profound and applicable basis for engineering decision-making and the management of risks associated with civil engineering facilities and to discuss its basic components. From the foregoing chapters, conclusions can be drawn with regard to three directions, namely considering the appropriate modeling of the life cycle of a structure, the assessment of consequences and the modeling of preferences.

11.2.1 Life Cycle Modeling

There are approaches, which aim to achieve the maximization of the utility of a civil engineering facility. The most general approach aims to maximize the *expected life cycle benefit* of the considered structure. It is shown that if another approach is used, which minimizes the expected life cycle costs, the expected life cycle benefit is not automatically maximized. Only, if the expected revenue is independent of the decision/ design variable, do both approaches identify the same outcome.

Generally, it is considered within the life cycle modeling of structures that if a structure fails, the activity supported by the structure is stopped. This simplification was already improved by Rosenblueth and Mendoza (1971), who implemented a reconstruction strategy within the life cycle modeling that always reconstructs failed structures. However, this reconstruction strategy represents another extreme, which might not be appropriate, e.g. for the optimal design of offshore oil production facilities. For such structures it is shown that reconstruction of a failed structure is a decision itself, which can a priori be considered for the optimal design and it is shown that this consideration may lead to a different optimal design. However, only a single possible reconstruction decision is considered. The introduced approach can be extended to also include additional reconstruction decisions.

Nonetheless, the reconstruction strategy to reconstruct failed structures introduced by Rosenblueth and Mendoza (1971) is meaningful for many civil engineering facilities. In this case, the expected revenue becomes independent of the structural design, and the minimization of the expected life cycle costs becomes an equivalent design approach to maximize the expected life cycle benefit. Moreover it can be argued that the consideration of this reconstruction strategy identifies an upper bound for the optimal design variable.

In addition, the effect of deterioration processes can be incorporated into the life cycle modeling which considers that failed structures will be reconstructed. Hereby, the effect of deterioration can be assessed on the basis of the inspection results and the residual structural reliability. The latter can be described by means of time-variant structural reliability analysis and the renewal theory. The renewal density is a function of the last repair or last reconstruction, which can be described using the backward recurrence time, which is a random quantity. In order to

keep the computation time at a reasonable level, a sensible assumption was made with regard to the backward recurrence time.

In addition, it is identified that a realistic life cycle modeling also requires an appropriate consideration of the whole system (in particular for large systems), the spatial variability of mechanical properties and exposures and an appropriate modeling of interest/ discount rates. All are relevant topics for further research.

11.2.2 Assessment of Consequences

Besides the modeling of events that may occur, the associated consequences need to be quantified, as well. A framework to assess these consequences is outlined with a special focus on indirect consequences that are often neglected. As an example, consequences due to business interruption losses are reviewed. If the adverse event affects a large number of business entities, such consequences can be assessed by calculating the lost value added which is adjusted for depreciation. These socioeconomic consequences and others have been assessed for the failure of the WTC Twin Towers, where it is clearly seen that follow-up consequences may constitute a considerable and decisive part.

In addition, the multiplier effect should be taken into account, which also represents follow-up consequences on the economy. However, the assessment of this factor requires a great deal of expertise in the field of economics, which is why this influence is generally neglected.

Another form of consequences are immaterial consequences. They can be assessed if the preferences of the decision maker are known.

11.2.3 Modeling of Preferences

The gross domestic product (GDP) is the best known social indicator, but it only covers a specific aspect of the performance of a society. A more representative description of the performance of a society is obtained, if a basket of indicators is considered. Assigning to such a basket a utility a new indicator a compound social indicator is obtained. Such an indicator is the life quality index. It is composed of the GDP per capita, the life expectancy and the work fraction. This indicator is reviewed in the light of microeconomic consumption theory.

On the basis of the microeconomic consumption theory, a framework is constructed that provides an explanation of the observed correlation between the life expectancy and the GDP per capita. It is shown that the observed correlation could result from a decision process aiming to achieve the highest life quality with the available resources. Therefore, simple assumptions are made with regard to the technologically achievable life expectancy and the fraction of the GDP per capita that is spent on safety.

It seems that this framework can explain why the observed correlation between the life expectancy and the GDP per capita does not need to be considered in the LQI derivation. Moreover, it might contribute to an empirical proof that public decision-making is already made on the basis that safety and wealth are exchangeable goods.

With regard to the work fraction, it is generally agreed, that individuals optimize it to obtain an the maximal life quality. In this regard, the LQI is investigated with regard to labor supply. The derived optimal path of the work fraction is a straight line and independent of the GDP per capita. The so obtained labor supply curve is compared with empirical data. Generally, developed countries show a tendency that supports the framework.

A further development of the LQI should aim to incorporate both optimization formulations into a single optimization problem. This means, the optimization problem should determine the work fraction w – and therewith also the recreation fraction $r = 1 - w$ – and the amount spent on safety c_l so that the LQI is maximized. Mathematically, this is formulated as:

$$\max_{w, c_l} L(w, c_l, c_c) \quad (11.1)$$

$$\text{s.t. } g - c_l - c_c = 0$$

$$g - pw = 0 \quad (11.2)$$

$$w + r - 1 = 0. \quad (11.3)$$

A meaningful LQI formulation then shows developments for w and c_l that can be observed using empirical data. In particular, the possibility of a nonlinear relation between g and w is of interest, because it is observed that developing countries show generally higher values for w . Also a better representation of the technology curve $l_l(c_l)$ may be found by calculating the efficiency of safety measures.

From the LQI the equivalent willingness to spend resources to avoid an adverse event can be calculated. For instance, the amount can be determined to avoid a reduction in leisure. But also the willingness for safety investments can be calculated in terms of acceptable life saving costs. Several formulations have been reviewed. They can be distinguished whether they account for changes in life expectancy that are marginal or not. Two measures that account for marginal changes differ by a factor of two, since the annual costs – which are roughly the same – are either multiplied by the life expectancy or half the value. Compared to the accuracy with which probabilities can be assessed in many practical risk assessments, this difference seems to be small. But it clearly shows that the LQI research is still in progress.

A Basic Macroeconomic Concepts

In the foregoing chapters, concepts known from economics were utilized to quantify material and immaterial consequences. For the reason of completeness, the concepts are introduced in the present and subsequent annex. Whereas Annex B describes the consumption theory of microeconomics, the present annex focuses on concepts of macroeconomics, such as the gross domestic product and the multiplier effect. The present chapter only provides an introduction to the economic concepts utilized in the present thesis. The interested reader can find more on economics, e.g. in Samuelson and Nordhaus (2001), Varian (2003) or Höfert (2001).

A.1 Gross Domestic Product

Social indicators are used to assess the state of societies. There are many societal indicators e.g. population growth, workforce, unemployment rate, inflation rate, etc. But the most known indicator is the *gross domestic product* (GDP). The GDP is used to indicate the prosperity or performance of societies¹, however, it is only able to focus on a specific aspect of the performance of society. It neglects measures such as life expectancy, political system, literacy, education, recreation time, safety etc. In order to overcome this, compound indicators may be constructed considering more than just one indicator. A compound indicator is e.g. the human development index or the life quality index, see Chapter 5.

The GDP is defined as the value of all goods and services produced within a specific geographic region within a specific period – typically one year. The nominal GDP considers the goods and services at the actual market prices, whereas the real GDP refers its prices to a base year. This eliminates the effect of inflation.

There are two possible ways to determine the GDP: the *flow-of-product* and the *earnings or costs* approach. Both approaches are equivalent and lead to the same GDP. For the purpose of illustration, Figure A.1 shows an economy circuit of a simple economy. The simplified economy consists of households and enterprises. The households offer productive factors at the factor markets, such as labor, land, capital, etc. These factors are sold to enterprises, which purchase them to produce goods and services. The circuit is closed by the sale of the goods and services to the households at the product markets. The upper loop of Figure A.1 measures the GDP with the flow-of-product approach, whereas the lower loop permits one to determine the GDP using the earnings or costs approach. Figure A.1 shows an oversimplified economy but it covers the main aspects.

The flow-of-product approach sums up all final products, which are purchased and used by consumers. The market value of these goods and services corresponds to the GDP. By using the flow-of-product approach, the GDP can further be divided into personal con-

¹A society is considered as a group of persons within a geographic region e.g. a country or a group of countries.

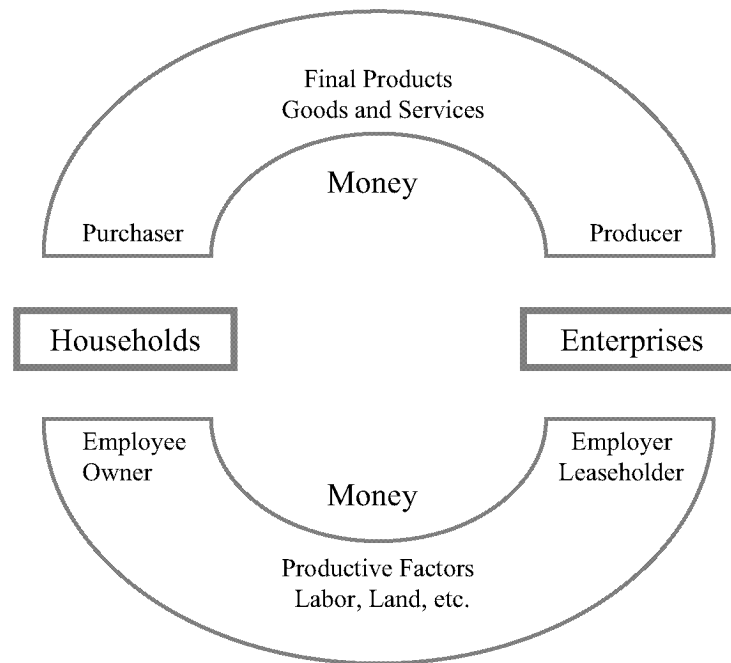


Figure A.1: A simple macroeconomic circuit.

sumption C , gross private domestic investment I , government consumption and investment purchase G and net export X .

$$GDP = C + I + G + X \quad (\text{A.1})$$

The earnings or costs approach determines the GDP by analyzing the costs for the production of final products:

1. wages, salaries and supplements,
2. net interests,
3. rental income,
4. indirect business taxes,
5. depreciation,
6. income of unincorporated enterprises and
7. corporate profits enterprises.

Besides the GDP, there are two other related measures, the net domestic product (NDP) and the gross national product (GNP).

The GDP is the total output produced with labor and capital which is located within the considered geographic region, whereas the gross national product (GNP) is the total output produced by labor and capital which is owned by a society of the considered region.

In contrast to the NDP, the GDP considers the gross investment, e.g. new buildings, new machines; however, it neglects depreciation, e.g. obsolete machines put out of service, old

computers, etc. The NDP is obtained by subtracting depreciation from the GDP. Generally, the NDP is a better measure of the output of an economy but the GDP is used more often. The reason for this is because depreciation is difficult to assess and needs to be estimated. Based on experience, the NDP amounts to around 90% of the GDP, Samuelson and Nordhaus (1989).

A.1.1 Value Added

Assessing the GDP, the problem of double counting may arise. This problem is best illustrated by means of a simple example. Assume a simple economy which only consists of households and two enterprises, namely a construction enterprise and a brickyard. The first enterprise constructs buildings worth 1 million USD each year. Therefore, it utilizes materials worth 0.5 million USD, which are produced by the second enterprise. The GDP is defined as the total production of final goods and services. In the present example, this is 1 million USD (the market value of the buildings). It excludes intermediate goods, in the present example, the bricks, which are produced by the second enterprise. When the cost approach is used one may tend to add the costs of both enterprises together. In this case, one obtains 1.5 million USD. However, national accounting considers the value added of the enterprises. This is the difference in the sales and the purchases. Utilizing this concept, we obtain a value added of 0.5 million USD for each enterprise and finally a GDP of $0.5 + 0.5 = 1$ million USD. The value added of an enterprise producing intermediate goods is not doubly counted.

A.1.2 Shortcomings

The gross domestic product is the soundest indicator for the total output of an economy; however, there are shortcomings. For instance, it does not consider activities at home which produce goods and services, such as housekeeping and neighborly help, neither of them is reported in payrolls. Neither does the GDP reflect illegal employment and illegal activities, e.g. drug dealing, smuggling, etc. However, these shortcomings do not seem to be important, in particular in the light of consequence assessment for engineering decision-making.

A.2 Multiplier Effect

The multiplier model is the simplest model and at the same time the most influential one to describe the effect of extrinsic influences upon an economy. Despite its simplicity, this model is able to describe the performance of the economy when it is subjected to economic impacts. Even if the model is extended, the essence remains valid. The simplified economy model is based on the following assumptions:

1. It is assumed that labor force is always available. This means that there are always unemployed persons or that people from other regions can be attracted for work.
2. Most crucial is the assumption that influences from financial markets and monetary policy on the economy are neglected. This does also imply that interest rates remain unchanged.
3. Trade with other countries is omitted.
4. The aggregate supply side is not considered.

5. Finally, investment is treated as an exogenous force, which is modelled to be independent of the GDP.

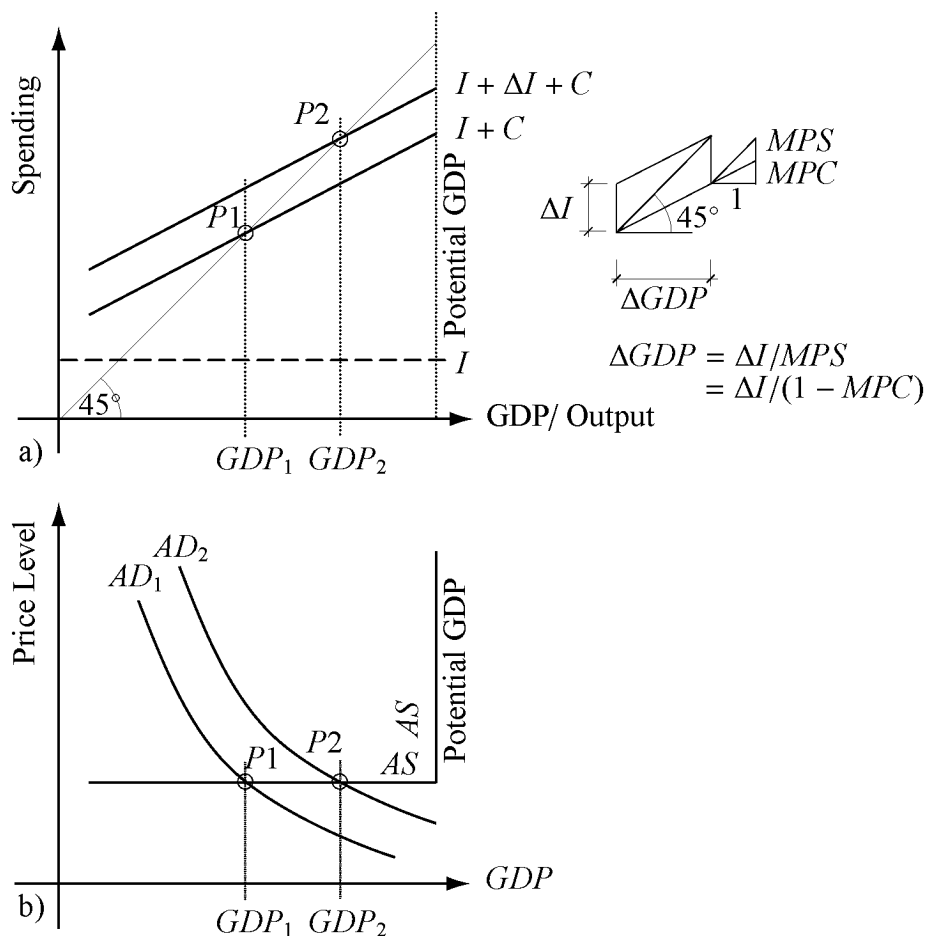


Figure A.2: Multiplier effect.

Considering these assumptions, the total spending of society can be formulated as a function of the output, i.e. GDP. The total spending S consists of the investment I , and the consumption C . Whereas the investment is assumed to be independent of the GDP, the consumption is modelled as a linear function of the GDP.

$$S = I + C(GDP) \tag{A.2}$$

$$= I + C_0 + MPC \cdot GDP \tag{A.3}$$

Here, C_0 is a constant and MPC is the marginal propensity to consume. It is the extra amount that people consume, when they receive an extra dollar of disposable income, or in this model an extra dollar of GDP. The economy is in equilibrium, when the total spending is equal to the total output, i.e. $S = GDP$. Substituting $S = GDP$ into equation A.3 one obtains

$$GDP_1 = \frac{1}{1 - MPC}(I + C_0). \tag{A.4}$$

In Figure A.2a this equilibrium is indicated by the point $P1$. If an increase of investment of ΔI is considered, a new equilibrium is obtained indicated by $P2$ with GDP equal to

$$GPD_2 = \frac{1}{1 - MPC}(I + \Delta I + C_0). \quad (A.5)$$

Taking the difference of Equation A.4 and A.5 one obtains

$$\Delta GPD = \frac{1}{1 - MPC}\Delta I. \quad (A.6)$$

This equation illustrates that if the investment is increased by ΔI , the GDP is increased by $1/(1 - MPC)$ times ΔI . $m = 1/(1 - MPC)$ is the multiplier which gives the economic model its name. As MPC is smaller than one, the multiplier is larger than one. For instance, if $MPC = 2/3$, the multiplier becomes 3, which means that an exogenous influence of ΔI leads to a GDP change of $3 \cdot \Delta I$.

Figure A.2a illustrates the multiplier model by means of the spending curves as a function of the GDP. The economy is in equilibrium, if spending is equal to the output. Graphically, this is represented by the intersection of the spending curve with the bisector of the first quadrant. Figure A.2b illustrates the multiplier model using aggregate demand and aggregate supply curves.

Another illustrative approach to the multiplier model considers an unemployed person, e.g. a mason who is hired to construct buildings. It is assumed that the mason obtains 10,000 USD as salary. When he has a marginal propensity to consume of $2/3$, he consumes 6,666 USD in goods and services. Persons offering these goods and services consume from the amount they obtain two thirds, as well. This process continues indefinitely. Finally, the total output may be written as

$$\Delta GPD = \Delta I + MPC \cdot \Delta I + MPC^2 \cdot \Delta I + \dots = \Delta I \sum_{n=0}^{\infty} MPC^n \quad (A.7)$$

$$= \Delta I \frac{1}{1 - MPC}. \quad (A.8)$$

To obtain Equation A.8, the geometric series $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$ has been utilized.

Generally, by means of improved models for aggregate supply and aggregate demand curves, more accurate multipliers may be derived at the expense of having more complex models, a loss of insight and more difficult calculations.

A.3 Summary

The GDP is the value of all goods and services produced within a specific geographic region and a specific period – typically one year. It is the most well known societal indicator, which is used to quantify the prosperity or wealth of society; however, it is only able to describe a specific aspect of the overall performance of society. Other measures, such as life expectancy, political system, literacy, education, recreation time, safety etc. are also meaningful. In order to overcome this, compound indicators may be constructed considering more than just one single indicator. A compound indicator e.g. is the human development index or the life quality index.

A Basic Macroeconomic Concepts

If the earnings or costs approach is selected to assess the GDP, the problem of double counting must be considered. To circumvent this, the value added of each producer is assessed so that intermediate goods are excluded.

Besides the GDP, there are other related measures such as the net domestic product (NDP) and the gross national product (GNP). In contrast to the GDP, the GNP considers the output by labor and capital which belongs to persons and bodies of the considered region, whereas the GDP and NDP is related to goods and services produced in the considered region. The NDP is simply the GDP minus depreciation. This makes it a more reasonable measure to assess e.g. business interruption losses; however, depreciation is difficult to assess and therefore, the GDP is used more frequently.

The multiplier model is the simplest model to describe extrinsic influences on an economy. Despite its simplicity, it is able to describe the economy's performance, when subjected to economic impacts. According to this model, an exogenous influence of ΔI leads to a GDP change of $m \cdot \Delta I$, where m is the multiplier giving the model its name.

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The present chapter gives a brief introduction to the consumption theory of microeconomics. It follows closely the outline given in Varian (2003), which provides more insight for the interested reader, see also Samuelson and Nordhaus (2001) or Höfert (2001).

Consumption theory covers the field of economics describing the economic behavior of consumers. The basis of consumption theory can be summarized in a single sentence:

Consumption theory describes the behavior of consumers who choose from all possible goods the *best* combination they can *afford*.

To make quantitative predictions of the behavior of consumers, it is needed to model quantitatively what consumers consider as *best* and what they can *afford*. What they can afford is described in the following section. What they consider as *best* is described subsequently. If these two items are quantified, the choice of consumers can be described and predicted.

B.1 Budget Limitation

What is affordable? A consumer is always restricted to a given budget $b \in \mathbb{R}_+$. Using this budget he or she may choose to invest a part or the total budget to invest in products, i.e. goods and services. In the following, services are considered to be included when we talk about goods. By choosing several goods, a basket may be composed represented by the vector $\mathbf{x}^T = (x_1, x_2, \dots, x_n)$. $x_i \in \mathbb{R}_+$ is the quantity of the i^{th} good in the basket, i.e. $\mathbf{x} \in \mathbb{R}_+^n$, with $n \in \mathbb{N}^*$. The prices associated with these goods can be summarized in another vector $\mathbf{p}^T = (p_1, p_2, \dots, p_n)$. For instance, x_1 may represent 10 m³ of concrete at a cost of $p_1 = 200$ USD per m³ concrete so that the total costs are 2,000 USD. In the same way, the total costs for the basket are obtained by the scalar product $\mathbf{x}^T \mathbf{p}$. As the consumer cannot spend more money than is available, he is restricted by the budget as follows.

$$p_1 x_1 + p_2 x_2 + \dots + p_n x_n \leq b \tag{B.1}$$

$$\mathbf{x}^T \mathbf{p} - b \leq 0. \tag{B.2}$$

Equation B.2 represents the budget limitation and defines the set $\mathfrak{A} = \{\mathbf{x} \in \mathbb{R}_+^n | \mathbf{x}^T \mathbf{p} - b \leq 0\}$ of affordable baskets or respectively the affordable combinations of goods.

Figure B.1 illustrates the budget limitation for the case of two goods. The linear function

$$\mathbf{x}^T \mathbf{p} - b = 0 \tag{B.3}$$

divides the first quadrant into two sets. The combination of goods above the line is not affordable with the given budget b , whereas the baskets of goods below are affordable. The intersections of the budget function with the axes are given by b/p_i , which represents the maximum possible amount of each good that can be purchased with the given budget. Finally, the impossible set

is indicated because negative values are excluded by definition. Or has someone ever bought or produced the amount of -5 m^3 concrete¹?

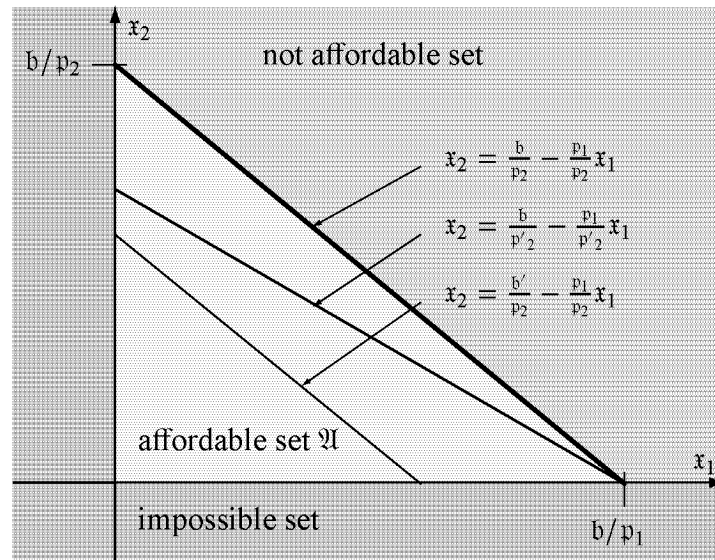


Figure B.1: Budget limitation.

In Figure B.1, it is also illustrated that a change in the budget – e.g. a reduction to b' – is represented by a parallel shift of the budget constraint. In contrast to this, a price change of a single good changes only the intersection point of the budget constraint with the axis of the considered good. The other intersection points remain unchanged. In the figure, a price increase of p_2 to p'_2 is illustrated. It is obvious that a proportional change of all prices may be represented by an equivalent budget change, e.g. due to an adjustment of the value added tax.

In economics, the changes of taxes, subsidies and rationing is studied to indicate their influence on the budget constraint and the thereby associated consumer demand, see e.g. Varian (2003).

In Figure B.1, it is seen that the budget constraints slope is given by

$$\frac{dx_2}{dx_1} = -\frac{p_1}{p_2}. \quad (\text{B.4})$$

This ratio is also known as *opportunity cost* of consuming good 1. In order to consume more of good 1, consumption of good 2 has to be reduced by $dx_2 = -\frac{p_1}{p_2} dx_1$. This means, by giving up the opportunity to consume good 2, the value of good 1 is evaluated. Therefore, the opportunity costs for normal goods are always negative.

B.2 Consumers' Preferences

What consumers can afford was studied in the preceding section. The present section describes what consumers consider as best or preferred goods.

¹Not to be confused with producing 5 m^3 concrete less.

From a set of goods summarizing all n relevant goods, a consumer may compose a basket \mathbf{x} . In the following figures, the basket is reduced to contain only two different goods for the reason of graphical illustration. But the application to more than two goods is straightforward. This is because the most relevant good can be considered as good 1 and all other goods are summarized in good 2, i.e. good 2 can be a basket itself. Furthermore, it is considered that all goods are readily available, although a generalization is straightforward.

Given two baskets $\mathbf{c}^T = (c_1, c_2)$ and $\mathbf{d}^T = (d_1, d_2)$ we consider that the consumer, who is a decision maker, may always state his preferences so that he or she indicates, whether he or she:

- prefers \mathbf{c} to \mathbf{d} : $\mathbf{c} > \mathbf{d}$,
- prefers \mathbf{d} to \mathbf{c} : $\mathbf{c} < \mathbf{d}$ or
- is indifferent between \mathbf{c} and \mathbf{d} : $\mathbf{c} \sim \mathbf{d}$.

Moreover, to act rationally the consumer is assumed to behave according to the stated axioms in Section 2.3.3.

B.2.1 Indifference Curve

Indifference curves are the graphical illustration of a set of baskets. These baskets have in common that the consumer is indifferent to them.

To construct indifference curves one starts at a specific basket, say (x_1, x_2) . Then the consumer is asked how good 2 has to be changed by Δx_2 , if good 1 is changed by Δx_1 , so that the two baskets (x_1, x_2) and $(x_1 + \Delta x_1, x_2 + \Delta x_2)$ are indifferent. Restarting from $(x_1 + \Delta x_1, x_2 + \Delta x_2)$ this can be continued, or an other arbitrary basket (x'_1, x'_2) may be selected as a new starting point. In this way, a figure representing the preferences of the consumer is obtained. An important property of indifference curves is that they cannot cross each other. For the mathematical proof, see Varian (2003).

B.2.1.1 Different Types of Goods

Figure B.2 shows indifference curves for different types of goods. Firstly, goods that perfectly substitute each other are illustrated. For instance, if the consumer would like to have constructed several buildings in order to lease them. In general, he or she may regard a timber structure (good 1) as a perfect substitute for a masonry structure (good 2). In this case, the consumer is only interested in the amount of gross floor area $x_1 + x_2$ the buildings provide, and he or she is willing to substitute one good with the other at a constant ratio, e.g. 1:1. Therefore, the indifference curve has a constant slope and is a linear function.

On the top right hand side, indifference curves of perfect complements are shown. It is L-shaped because the consumer prefers to consume the goods in a constant ratio, e.g. cement and gravel to produce concrete.

Figure B.2c illustrates the preferences with regard to an unwanted good, which is consumed if the consumer is compensated by a preferred good. On the right of this figure, the indifference curves of a neutral and a preferred good are drawn in Figure B.2d.

For the exterior walls of his building a consumer may find a 6 cm concrete wall to be too unreliable but a 2 m thick one as too expensive. In between, there is an optimum, otherwise called: a satiation point.

All these cases shown in Figure B.2a-e represent possible cases – for more details see Varian (2003) – but in consumer theory they are not considered as *normal*. Indifference curves of

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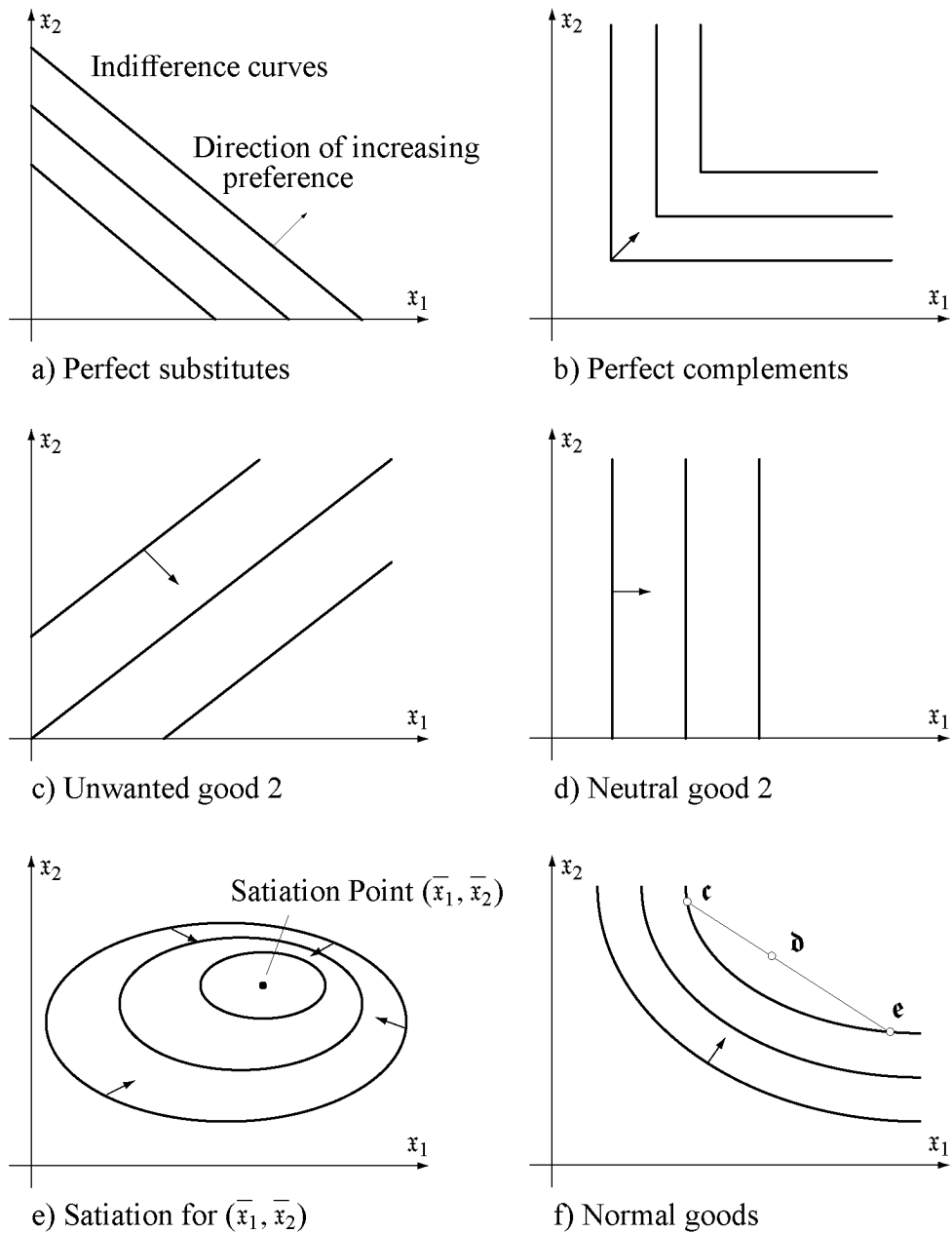


Figure B.2: Indifference curves for different types of goods.

normal goods are illustrated in Figure B.2f. No unwanted goods are considered and no satiation is achieved. The latter means that *more is more*. This results in monotonicity of the preferences and in turn implies a negative slope of the indifference curve.

In the majority of cases, indifference curves of normal goods are convex. They are convex, if the goods are liked to be consumed at the same time, otherwise the curves are concave. Varian (2003) gives an example of goods with concave indifference curves but mentions that the same goods may yield convex indifference curves, if the decision process considers a different, i.e. longer, time period.

Mathematically, \mathbf{e} defines a convex set, if $\mathbf{e} \geq \mathbf{c}$ and if \mathbf{e} is a linear combination of the two indifferent baskets \mathbf{c} and \mathbf{d} , i.e. $\mathbf{e} = t\mathbf{c} + (1 - t)\mathbf{d}$ with $t \in [0, 1]$, see also Figure B.2f.

B.2.2 Marginal Rate of Substitution

The marginal rate of substitution (MRS) is the slope of the indifference curve, i.e.:

$$MRS = dx_2/dx_1. \quad (\text{B.5})$$

It describes how much the consumer is willing to substitute a little more consumption of good 2 for a little less of good 1. Instead of using money, he or she is willing to pay with good 2 to obtain more of good 1.

Another interpretation is obtained if all other goods except the considered one are combined as good 2. In this case, the MRS indicates the *marginal willingness to pay* for good 1. That is: How much money the consumer is willing to pay for a little more Δx_1 of good 1. Here, the terms *marginal* and *willingness* are emphasized. Therefore, the additional amount Δx_1 needs to be marginal, i.e. little, in the sense that a linear approximation of the indifference curve is appropriate. Moreover, this means that the MRS represents a price that a consumer is willing to pay. But this price can be very different from the actual market price.

From Figure B.2f, it is seen that for normal goods, i.e. convex indifference curves, the marginal rate of substitution is negative. Moreover, it is seen that for strictly convex indifference curves the MRS approaches zero for increasing x_1 . This is called a *diminishing marginal rate of substitution* and shows an often observed phenomenon: The more you have of good 1, the more you are willing to exchange it for good 2. Consider the value of a glass of water when you are at a lake as compared to the situation when you are in the desert. Also recall the legend of king Midas².

B.3 Utility

Preferences are the basic concepts needed to understand decision-making. Indifference curves as well as utility functions are a measure to quantify them. That is, utility is only a possibility of describing preferences. It maps preferences ordering for baskets into real numbers. To be a valid representation of the preferences, the utility function must fulfill $u(\mathbf{c}) > u(\mathbf{d})$ if and only if $\mathbf{c} \succ \mathbf{d}$. For a given utility level z , the corresponding indifference curve is determined by following set $\{\mathbf{x} \in \mathbb{R}_+^n | u(\mathbf{x}) = z\}$.

B.3.1 Ordinal and Cardinal Utility

In decision theory as outlined in Chapter 2 cardinal utilities are used. In order to study the behavior of consumers, it is sufficient to consider utilities as being ordinal. When cardinal utilities are needed, the utility differences assigned to baskets are significant. However, in the case of ordinal utilities, only the preference ordering is important. Therefore, any monotonic transformation $t(u)$ of the utility u yields the same preference ordering, e.g. $t = u^3$. For valid transformations of cardinal utilities, see Section 2.3.4.

²King Midas wished from Dionysus that everything he touches should become gold. Dionysus fulfilled his wish, but also released him after Midas realized his bad choice.

B.3.2 Cobb-Douglas

Originally, the Cobb-Douglas function was used to study production performance, see also Section 5.1.3. Today, in microeconomics, it is a frequently used function to describe consumer preferences. The function is given by

$$u(x_1, x_2) = x_1^\beta x_2^\gamma. \quad (\text{B.6})$$

It is equivalent to

$$u(x_1, x_2) = x_1^\alpha x_2^{(1-\alpha)}, \quad (\text{B.7})$$

when the ordinal utility concept is utilized. To obtain Equation B.7, the $(\beta + \gamma)^{\text{th}}$ root is taken from Equation B.6, with $\alpha = \frac{\beta}{\beta+\gamma}$.

B.3.3 Marginal Utility

Marginal utility MU_i with respect to good i describes the change in utility, when the consumption of good i is infinitesimally changed.

$$MU_i = \frac{\partial u(\mathbf{x})}{\partial x_i} \quad (\text{B.8})$$

The total change in utility is

$$du = \sum_i^n \frac{\partial u(\mathbf{x})}{\partial x_i} dx_i : \quad (\text{B.9})$$

In order to obtain an the indifference curves du has to be zero. Allowing only goods 1 and 2 to be changed one obtains

$$MRS = \frac{dx_2}{dx_1} = -\frac{\partial u(\mathbf{x})/\partial x_1}{\partial u(\mathbf{x})/\partial x_2} \quad (\text{B.10})$$

$$= -\frac{MU_1}{MU_2} \quad (\text{B.11})$$

For the Cobb-Douglas utility function we obtain

$$MU_1 = \alpha x_1^{\alpha-1} x_2^{1-\alpha} \quad (\text{B.12})$$

$$MU_2 = (1 - \alpha) x_1^\alpha x_2^{-\alpha} \quad (\text{B.13})$$

$$MRS = -\frac{\alpha}{(1 - \alpha)} \frac{x_2}{x_1}. \quad (\text{B.14})$$

B.3.3.1 Gossen's First Law or the Law of Diminishing Marginal Utility

Considering normal goods, it is assumed that a satiation point exists only at infinity. Therefore, the marginal utility is non-negative.

$$\frac{\partial u(\mathbf{x})}{\partial x_i} \geq 0 \quad (\text{B.15})$$

However, it is generally observable that with increasing x_i , the marginal utility assigned by consumers decreases due to increasing satiation. This fundamental law postulated by Gossen is also known as the law of diminishing marginal utility. This implies that:

$$\frac{\partial^2 u(\mathbf{x})}{\partial x_i^2} \leq 0. \quad (\text{B.16})$$

Only if the quantity of the considered good converges to infinity, the marginal utility, as well as its derivative converge to zero. From the equations above, we obtain for the Cobb-Douglas utility function the condition that the parameter α is an element of the set $[0, 1]$.

Whereas the law of diminishing marginal utility is not necessarily needed to model consumption behavior, it is required, if cardinal utilities are utilized.

B.4 Optimal Choice

Varian (2003) describes the choice for the different types of goods as outlined above. In the following, just the important case of normal goods is considered. It is illustrated in Figure B.3.

Three baskets namely \mathbf{c} , \mathbf{d} and \mathbf{e} belong to the affordable set \mathfrak{A} indicated by the shaded area. Basket \mathbf{f} is not affordable. Moreover, the budget restriction is drawn by the black line together with the indifference curves representing different levels of utilities. The arrow shows the direction of increasing preferences or utility.

Considering basket \mathbf{c} , it is seen that this basket is affordable without spending the total budget. However, is the remaining money saved? If the saved money is considered as a good it can be summarized in a basket, which is then represented by good 2. Therefore, the baskets lying on the budget line are considered only. Starting from the budget line's right end we can consider basket \mathbf{d} and move it to the left. As it moves to the left, the utility of the basket increases and it is more preferred than the baskets to its right. Arriving at the point indicated as basket \mathbf{e} , a further move leftwards would yield a less preferred basket. Hence, basket \mathbf{e} is affordable and the most preferred among the baskets within the set \mathfrak{A} . Therefore, the optimal quantities of the goods are x_1^* and x_2^* .

In Figure B.3, it is seen that the highest affordable indifference curve has at the optimal point the same slope as the budget line. If this would not be the case, the curves would cross each other. This in turn implies that by starting from the optimum and by moving along the budget restriction in one direction, a more preferred basket could be obtained. This however, contradicts the definition of an optimum. Therefore, the slope condition is a necessary condition. But it is not a sufficient one, see Varian (2003) for examples. Mathematically, the tangent condition is formulated by

$$\frac{\partial u(\mathbf{x})/\partial x_1}{\partial u(\mathbf{x})/\partial x_2} = \frac{p_1}{p_2} \quad (\text{B.17})$$

or

$$MRS = -\frac{p_1}{p_2}. \quad (\text{B.18})$$

What is the interpretation of Equation B.18? It was mentioned earlier that the *MRS* is the rate at which the consumer is willing to exchange good 1 for good 2. However, the market offers the

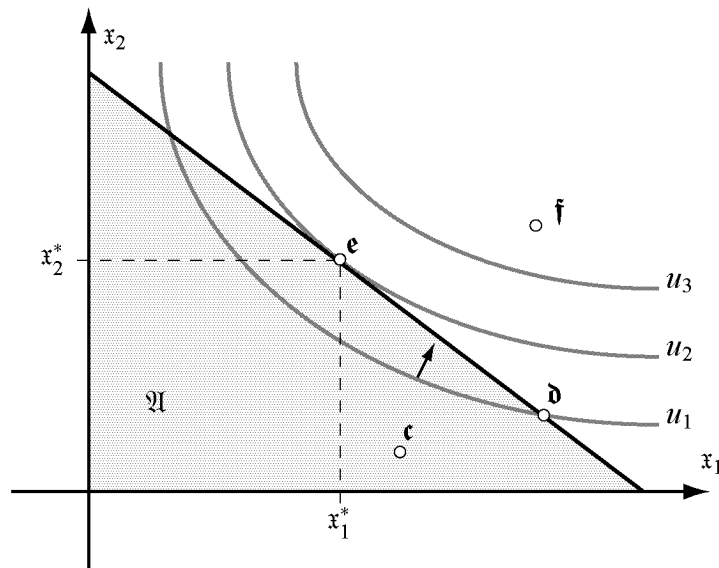


Figure B.3: Optimal choice.

opportunity to exchange it at the ratio $-\frac{p_1}{p_2}$. If they are not equal, e.g. $MRS > -\frac{p_1}{p_2}$ the consumer would like to exchange good 1 to obtain more of good 2.

In addition, the optimal decision process can also be formulated as an optimization problem as follows.

$$\begin{aligned} \max_{\mathbf{x}} u(\mathbf{x}) \\ \text{s.t. } \mathbf{x}^T \mathbf{p} - b = 0 \end{aligned} \quad (\text{B.19})$$

This problem can be solved e.g. using Lagrange multipliers. The Lagrangian function \mathcal{Q} is given by

$$\mathcal{Q} = u(\mathbf{x}) - \lambda(\mathbf{x}^T \mathbf{p} - b), \quad (\text{B.20})$$

where the utility function is augmented with the budget constraint and the Lagrange multiplier λ . The optimum is the point for which the $n + 1$ equations are fulfilled.

$$\frac{\partial \mathcal{Q}}{\partial x_i} = 0, \quad i = 1, \dots, n \quad (\text{B.21})$$

$$\frac{\partial \mathcal{Q}}{\partial \lambda} = 0 \quad (\text{B.22})$$

For the Cobb-Douglas utility function with $n = 2$ as given by Equation B.7, the optimal choice is obtained to:

$$x_1 = \alpha \frac{b}{p_1} \quad (\text{B.23})$$

$$x_2 = (1 - \alpha) \frac{b}{p_2}. \quad (\text{B.24})$$

It is seen that if α is an element of $[0, 1]$, it represents the fraction of the budget spent on good 1. The complete derivation is given in Varian (2003). With the Equations B.23 and B.24 the demand of the consumer for the individual goods is quantified, given a certain budget and market prices. These functions are also called *demand functions*.

B.5 Goods Categorized According to the Demand of Consumers

Generally, the demand for a specific good may be written as

$$x_i = x_i(\mathbf{p}, b). \quad (\text{B.25})$$

If the demand for a specific good is plotted as a function of the budget b , the so called *Engel curve* is obtained. For *normal* goods the demand is increasing with an increasing budget. If the demand for the good increases more than proportionally, the good is called a *superior* or *luxury* good. On the other hand, the demand for *inferior* goods decreases with increasing budgets. There are also *neutral* goods and goods that first behave like normal and then like inferior goods, see Figure B.4.

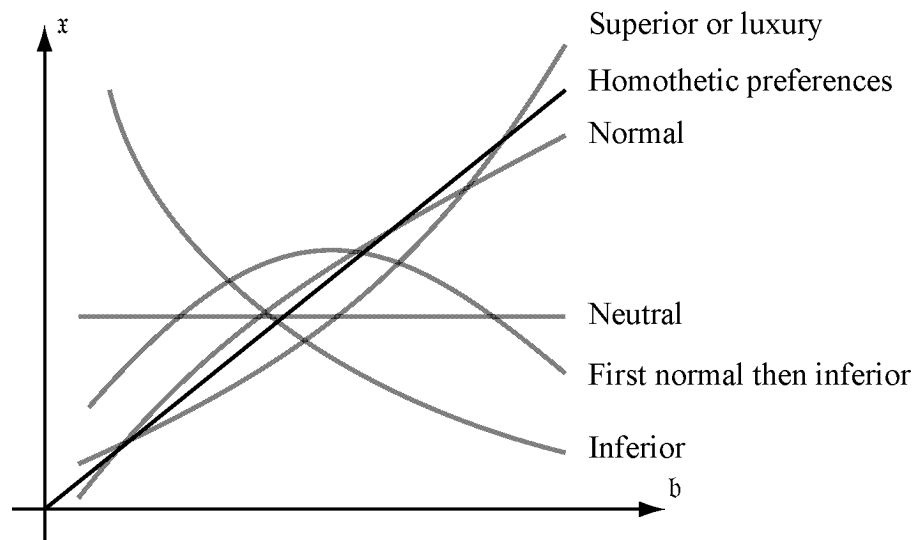


Figure B.4: Engel curves for different goods.

In addition to the budget, the price may change as well. Generally, the demand of a good decreases if the price increases. However, there are rare goods which behave differently: Their demand increases, if their price increases. Such goods are called *Giffen* goods. Due to the price change, the available budget might be decreased so that other goods are not affordable any more. Their reduction is then compensated by the good which has increased in price.

B.5.1 Homothetic Preferences

Preferences are called *homothetic*, if and only if $(x_1, x_2) > (y_1, y_2)$ and $(t \cdot x_1, t \cdot x_2) > (t \cdot y_1, t \cdot y_2)$, with $t \in \mathbb{R}$. This means that the preferences of the consumer depend only on the ratio of good 1 to good 2. Such preferences reveal a linear Engel curve. Examples of homothetic preferences are the cases of perfect substitute, perfect complement or preferences according to the Cobb-Douglas utility function.

B.6 Summary

Consumption theory assumes that from all possible goods the consumer chooses the *best* combination he or she can *afford*. The budget limitation helps one to describe what affordable is. In addition, the preferences of the consumer need to be quantified in order to characterize what he or she considers to be preferred. For this reason, indifference curves and utility functions can be used. For the latter, the distinction between ordinal and cardinal utility is possible; however, for the description of the consumer's choice, ordinal utilities are sufficient.

Having described the budget constraint and quantified the preferences, the optimal choice of goods may be identified. To identify the optimum, it is necessary but not sufficient to identify the point where the slope of the indifference curve is equal to the slope of the budget constraint.

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Curriculum Vitae

Oliver Kübler

Born 18 May 1974 in Schaffhausen, Switzerland

Citizen of Germany

- | | |
|-------------|--|
| 1993 | Abitur Technisches Gymnasium Waldshut, Germany |
| 1993 – 1994 | Internship Architekt Klaus Mannig, Lottstetten, Germany |
| 1994 – 2000 | Diploma in civil engineering University of Karlsruhe, Germany |
| 1997 – 1998 | Student exchange Institute National des Sciences Appliquées de Lyon, France |
| 2000 – 2005 | Research engineer and doctoral studies Institute of Structural Engineering at ETH Zurich, Switzerland |
| Since 2006 | Bridge engineer Major bridges, COWI A/S, Kongens Lyngby, Denmark |