Algorithmisches Differenzieren

ABHANDLUNG

zur Erlangung
des Titels eines Doktors der Mathematik
der
EIDGENÖSSISCHEN TECHNISCHEN HOCHSCHULE
ZÜRICH

vorgelegt von

JOHANN JOSS
dipl. Math. ETH
geboren am 3. Oktober 1944
von Konolfingen (Kt. Bern)

Angenommen auf Antrag von

Prof. Dr. P. J. Huber, Referent
Prof. Dr. P. Henrici, Korreferent

Abstract

A new method to compute derivatives, called algorithmic
differentiation, is presented. It is in a certain sence
intermediate between numerical and formal differentiation,
but it avoids their shortcomings. (Lach of arithmetic
precision, respectively exponential increase in the length
of the formulae.)
A program to compute the value of a funktion f is transformed
by algorithmic differentiation in a new program which computes
simultaneously f' and f.
The underlying ideas are that a machine with the help of
the four operations +, -, *, / and the relations =, $\neq$, >, $\geqslant$, $\leqslant$, <
can only represent piecewise fractional relational function.
The second idea is that the problem can be reduced by the chain
rule to differentiation of expressions with only two operands.
The simultaneous computation of f' and f requires at most 6
times as many arithmetic operations as what is required to
compute f alone and the whole computational effort is on
the average increased by a factor 3.