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Essays in Public and Environmental Economics

Household Heterogeneity and the Distributional Effects of Climate Policy

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Summary

This thesis contains four essays on the distributional effects of environmental policies. Importantly, heterogeneous economic burdens can hinder policy implementation, and may call for redistributive measures to compensate households that are disproportionately impacted. This thesis therefore addresses questions such as the following: Are rich or poor households more impacted by environmental policy? How is the optimal environmental policy affected by social equity concerns? How will changing household consumption patterns interact with climate policy in developing economies?

The essays that compose this thesis study the effects of government intervention in a general equilibrium framework. The analysis thus not only captures the effects of policies on regulated activities, but also on the broader economy through the interaction of markets and on the origination and spending of income. A general equilibrium assessment is especially important in the case of climate policy, as the current economic system is still broadly reliant on energy from fossil sources. Methodologically, this thesis employs both analytical and computable general equilibrium models. The former, to derive insights in a general but stylized setting, and the latter, to estimate effects in a specific context, based on a more detailed representation of the economy.

The first three essays deal with positive aspects of environmental policy, that is, the consequences of policy are studied without ranking alternatives based on social desirability. Such studies are intended to provide economic intuition about both the aggregate and distributional effects of policy, including the effect of alternative allocations of revenues from pollution pricing, and to provide government decision-makers with a map of the policy space in which they operate. The fourth essay presents a normative analysis of environmental policy. The goal is hence to identify the preferred policy amongst alternative environmental tax and revenue redistribution schemes, given the social objectives of the government.

The first essay examines how the general equilibrium incidence of an environmental tax depends on the effect of different incomes and preferences of heterogeneous households on aggregate outcomes. It develops a model in the public economics tradition following Harberger, with general forms of preferences and substitution between capital, labor, and pollution in production that captures the impact of household heterogeneity and interactions with production characteristics on the general equilibrium. Failing to incorporate household heterogeneity is theoretically shown to affect incidence results qualitatively. This aggregation bias can be quantitatively important for assessing the incidence of a carbon tax, mainly by affecting the returns to factors of production such as capital and labor. The findings are robust to a number of extensions including alternative revenue recycling schemes, pre-existing taxes, non-separable utility in pollution, labor-leisure choice, and multiple commodities.

The second essay connects the first and third, by comparing incidence results derived in a linearized, Harberger-type analysis and results based on methodologies that capture the economy's exact response to policy. The essay studies the implications of the linearity assumption for the incidence of price- and quantity-based environmental policies. It proves analytically that changes in output prices and the pollution level following an environmental tax increase are overestimated in the

linearized case, whereby the bias on pollution changes is larger than the bias on price changes. Subsequent numerical analysis provides quantitative results that are in line with these theoretical findings. The linearized approach overstates the regressivity of a given environmental tax increase, and underestimates the regressivity of a policy targeting a given pollution reduction. The pattern of incidence remains nonetheless similar. The results therefore support the use of the Harberger approach as a method to assess the incidence of environmental policies, whilst pointing to sources of potential systematic bias.

The third essay quantifies how energy demand and associated CO_2 emissions are affected by income-driven shifts in consumption patterns in China, the world's largest emerging economy. Incorporating empirically-derived Engel curves within a general equilibrium model, the essay finds that, relative to projections based on standard assumptions of unitary income elasticity, direct household CO_2 emissions in 2030 are 61% lower, and national emissions are reduced by 8%. This has important implications for the welfare consequences of climate policy. The average welfare costs of climate policy decrease by more than half, with losses more evenly distributed across households. This is driven by the easing of policy stringency and convergence in the carbon intensity of household consumption baskets as incomes rise. The results point to non-homothetic household preferences as an important determinant of climate policy costs in developing economies, where incomes, energy demand and emissions are rising rapidly.

The fourth essay examines differentiated pollution pricing in the presence of social equity concerns using both theoretical and numerical general equilibrium analyses in an optimal taxation framework. The essay first theoretically studies the optimality conditions for sectoral pollution taxes and redistribution of pollution tax revenues. Heterogeneity in private preferences and endowments across households interacts with social preferences implying non-uniform pollution taxes at the social optimum. Taxes should be differentiated according to households' consumption characteristics and to raise revenues for targeted transfers to households. Quantitatively assessing the scope for differentiated carbon taxes in the context of the U.S. economy, this essay finds that optimal sectoral carbon taxes differ widely across sectors, even when social inequality aversion is relatively low. The optimal policy differentiates taxes to strongly increase revenues for redistribution to lower income households. Considering non-optimal redistribution schemes, the scope for tax differentiation is somewhat diminished but remains substantial. Relative to uniform carbon pricing, incidence patterns across household expenditure groups for a given redistribution scheme can vary qualitatively when allowing for differentiated taxes.

Riassunto

Questa tesi contiene quattro saggi sugli effetti distributivi della politica ambientale. Una distribuzione eterogenea dei costi può ostacolare l'attuazione della politica, inoltre che richiedere possibili misure redistributive per compensare i consumatori più affetti. Questa tesi si occupa quindi di domande quali le seguenti: Quali consumatori sono più negativamente affetti dalla politica ambientale: quelli ricchi o quelli poveri? Qual'è la politica ambientale ottimale, dati gli obiettivi sociali del governo? Come interagiranno l'evoluzione delle abitudini dei consumatori con la politica climatica nelle economie in via di sviluppo?

I saggi che compongono questa tesi studiano gli effetti di un intervento del governo sull'equilibrio economico generale. L'analisi coglie quindi non solo gli effetti sulle attività regolamentate, ma anche sull'intera economia, attraverso l'interazione dei mercati, e sui redditi. Tale approccio è particolarmente importante nel caso della politica climatica, dato che l'attuale sistema economico è ancora largamente dipendente da fonti di energia fossili. Metodologicamente, questa tesi si avvale sia di modelli di equilibrio generale analitici che computazionali. I primi servono ad ottenere un'intuizione economica in un contesto generale benché stilizzato; i secondi a stimare gli effetti in un contesto specifico, sulla base di una rappresentazione più dettagliata dell'economia.

I primi tre saggi analizzano aspetti positivi della politica ambientale. Le conseguenze di tale politica sono quindi studiate senza valutare alternative in base alla loro desiderabilità sociale. Questi studi sono intesi a fornire un'intuizione economica degli effetti aggregati e distributivi della politica, incluso l'effetto di utilizzi alternativi degli introiti dalla tassazione dell'inquinamento, così come fornire al governo una mappa dello spazio politico nel quale opera. Il quarto saggio presenta un'analisi normativa della politica ambientale. L'obiettivo è quindi di identificare la politica migliore fra schemi alternativi di tassazione ambientale e di redistribuzione degli introiti fiscali, dati gli obiettivi sociali del governo.

Il primo saggio esamina come l'incidenza di una tassa ambientale dipende dall'effetto delle caratteristiche di consumatori eterogenei (in termini di redditi e preferenze) sui risultati aggregati. Questo saggio sviluppa un modello nella tradizione di Harberger con forme generali di preferenze e produzione caratterizzata da sostituzione tra capitale, lavoro, ed inquinamento. Il modello cattura l'impatto dell'eterogeneità fra consumatori e le interazioni con le caratteristiche produttive sull'equilibrio economico generale. In una prima parte teorica, si dimostra che un'analisi che non rappresenta l'eterogeneità fra consumatori porta a risultati qualitativamente diversi. In una seconda parte applicata, si mostra che questa differenza può essere quantitativamente importante per la valutazione dell'incidenza di una tassa sulle emissioni di anidride carbonica, soprattutto tramite l'influsso sul rendimento dei fattori di produzione quali capitale e lavoro. I risultati sono robusti ad una serie di estensioni tra le quali modi alternativi di redistribuire gli introiti fiscali, altre tasse preesistenti, preferenze non separabili nell'inquinamento, scelta fra tempo libero e lavoro, ed un maggior numero di settori.

Il secondo saggio collega il primo ed il terzo. Il saggio confronta risultati di incidenza derivati nel contesto di un'analisi linearizzata nella tradizione di Harberger con risultati basati su metodolo-

gie che catturano la reazione esatta dell'economia. Il saggio studia le implicazioni di un'analisi linearizzata per l'incidenza di politiche ambientali che mirano sia ad implementare una data tassa sull'inquinamento che una data riduzione dell'inquinamento. Il saggio dimostra analiticamente che un'analisi linearizzata sopravvaluta il cambiamento dei prezzi e del livello di inquinamento a seguito di un dato aumento della tassa ambientale. L'imprecisione è maggiore per il cambiamento del livello di inquinamento rispetto al cambiamento dei prezzi. I risultati quantitativi confermano i risultati teorici. La regressività di un dato aumento della tassa sull'inquinamento è sopravvalutata dall'approccio linearizzato. Una data riduzione dell'inquinamento appare dall'altro canto meno regressiva rispetto a risultati basati su un'analisi che va oltre agli effetti lineari. L'incidenza rimane però simile in entrambe i casi. I risultati sostengono quindi l'approccio nella tradizione di Harberger per la valutazione dell'incidenza di politiche ambientali, indicando al contempo possibili fonti sistematiche di imprecisione.

Il terzo saggio quantifica come l'evoluzione delle abitudini di consumo e l'eterogeneità fra consumatori influenzano sia le emissioni di CO_2 che le conseguenze economiche della politica climatica in Cina, la più grande economia emergente al mondo. Il saggio sviluppa un metodo per incorporare curve di Engel derivate empiricamente in un modello di equilibrio economico generale e proietta sia il consumo energetico futuro che le emissioni di CO_2. Rispetto a proiezioni basate sull'ipotesi standard di preferenze omotetiche, le emissioni dirette di CO_2 causate dai consumatori sono inferiori del 61%, e le emissioni nazionali sono ridotte dell'8%. I costi della politica climatica, misurati in termini di benessere economico, si riducono di oltre la metà, e le perdite sono più ugualmente distribuite fra consumatori. Questi risultati sono causati dall'allentamento della severità della politica climatica e da una convergenza dell'intensità di CO_2 del consumo in seguito al rapido aumento dei redditi. Questo saggio mette in risalto preferenze non omotetiche quali fattore importante per i costi della politica climatica nelle economie in via di sviluppo, dove i redditi, il consumo di energia, e le emissioni sono in rapida crescita.

Il quarto saggio illustra come le tasse ottimali sull'inquinamento possono essere differenziate fra settori economici, a causa di considerazioni sociali riguardo all'uguaglianza distributiva, avvalendosi di modelli di equilibrio economico generale teoretici e computazionali. Questo saggio studia dapprima condizioni generali per l'ottimalità di tasse sull'inquinamento e redistribuzione degli introiti fiscali. Redditi e preferenze private eterogenei interagiscono con le preferenze pubbliche nel motivare tasse ambientali differenziate. Da un lato, le tasse sull'inquinamento sono differenziate in modo ottimale per aumentare le entrate fiscali a scopi redistributivi; dall'altro, in base alle differenti preferenze dei consumatori. Il saggio calibra in seguito un modello computazionale con dati reali per quantificare questi effetti nel contesto dell'economia degli Stati Uniti d'America. I risultati mostrano che le tasse ottimali sono fortemente differenziate, anche quando l'avversione del governo alla disuguaglianza è relativamente bassa. La politica ottimale differenzia le tasse sull'inquinamento per aumentare significativamente gli introiti fiscali, che vengono poi redistribuiti ad i consumatori a basso reddito. Considerando schemi redistributivi non ottimali la differenziazione diminuisce, rimanendo comunque significativa. L'incidenza per un dato schema redistributivo e tasse ambientali differenziate può differire qualitativamente dal caso con tasse ambientali uniformi.

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Giacomo Schwarz

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General introduction

Motivation

"Tackling climate change is a shared mission for mankind (...) Let us join hands to the establishment of an equitable and effective global mechanism on climate change" Xi Jinping (2015), President of the People's Republic of China.

Environmental issues such as global climate change are among the most pressing challenges for modern society. Anthropogenic greenhouse gas emissions are at a historic high, and keep on growing. Driven by the increase in the atmospheric concentration of CO_2—the major greenhouse gas—the Earth has gained substantial energy over the last decades (Stocker et al., 2013). Substantial warming has already taken place. The sea level has risen, snow and ice have diminished, and the frequency and severity of many extreme weather events have increased (Pachauri and Meyer, 2014).[1] Continued emissions will exacerbate these adverse effects, with important consequences for ecosystems and economies. The urgency of the climate problem is compounded by the fact that CO_2 remains in the atmosphere for centuries or more once emitted (Archer and Brovkin, 2008). Failing to keep greenhouse gas concentrations below dangerous levels will therefore have long-lasting consequences.

From the economic perspective, environmental problems in perfectly competitive economies reflect failures of the market to price pollution (Pigou, 1920). Such market failures call for government intervention. The government can either implement market-based policies such as pollution taxes or trading schemes, or non-market-based measures such quotas or standards. Due to the large informational requirements of non-market-based measures, especially in the case of economy-wide issues such as the emissions of greenhouse gases, market-based approaches are often preferable.

Wide-spread concerns with global climate change have lead to the recent implementation of a number of market-based policies, including the EU Emissions Trading System (ETS) and the Californian carbon market. Others are planned, such as the Chinese national ETS and other initiatives across the globe (World Bank Group, 2016). The 2015 Paris Climate Agreement, which commits all countries to act to mitigate CO_2 emissions and that entered into force on November 4, 2016 (UNFCCC, 2016), provides further momentum for reducing future greenhouse gas emissions. The commitments under the Paris Agreement are, however, likely insufficient to keep emissions below the level that would cause a median warming below 2 degrees Celsius above pre-industrial levels (Rogelj et al., 2016). More ambitious action will therefore be necessary in the future.

Although environmental regulation is motivated by the societal benefit of reduced pollution, such policies are not necessarily equitable. Given the large heterogeneity in the economic characteristics of households and in the distribution of adverse effects from deteriorating environmental quality, equity aspects are of major concern for both national and international environmental policy. Pol-

[1] Some degree of uncertainty regarding these changes remains. A large part of these changes has, however, been found to be at least *likely*, and some are *virtually certain* (Stocker et al., 2013; Pachauri and Meyer, 2014).

lution pricing tends, for example, to raise the relative price of goods consumed more intensively by low-income households. Environmental policy may therefore exacerbate equity concerns that already exist in the absence of pollution pricing, such as those associated with income inequality. It is therefore fundamentally important, firstly, to understand how the economic burdens of policy are distributed, an secondly, to assess the scope for corrective redistribution (given society's objectives).

Since Keynes, economics has distinguished between *positive* and *normative* analysis. As Friedman later specified, the task of positive economics is to "provide a system of generalizations that can be used to make correct predictions about the consequence of any change in circumstances" (p.4, Friedman (1953)). Normative economics, on the other hand, is concerned with "criteria of what ought to be" (p. 34, Keynes (1955)).

This work addresses both positive and normative questions regarding the distributional effects of market-based policies to address environmental externalities, such as the following:

- Are rich or poor households more impacted by environmental policy?
- Can theoretical issues such as the household aggregation problem affect real-world tax incidence?
- How is the optimal environmental policy affected by social equity concerns?
- How will changing household consumption patterns interact with climate policy in developing economies?

To answer such questions, this thesis employs quantitative economic models. Imposing structural assumptions based on economic first principles enables an *ex-ante* evaluation of policy. This is especially important for the assessment of policies that do not represent incremental additions to existing regulation—such as ambitious climate policy—for which econometric evaluations based on historic data are either not possible, or not particularly informative.

Quantitative economic models for the assessment of energy and environmental policies often adopt a general equilibrium (GE) framework. This includes a number of integrated assessment models, employed to calculate optimal pathways for carbon prices (Nordhaus, 1992; Nordhaus and Yang, 1996) and a wide range of energy-economic modeling studies to evaluate real-world climate policy proposals (Böhringer et al., 2009; Rausch and Karplus, 2014; Zhang et al., 2016). A GE framework enables analysis which goes beyond the direct effect of regulating a certain activity, by capturing feedbacks on the supply and demand equilibrium in multiple markets, mediated by the effect on prices, and on the the origination and spending of income.[2] Such an approach is especially important in the context of climate policy, as the current economy is still broadly dependent on energy from fossil sources. Climate policy therefore affects all sectors in the economy, either directly or indirectly, calling for an economy-wide assessment of economic costs.

[2]In general equilibrium, households are assumed to maximize utility, subject to their budget constraint, and firms are assumed to maximize profits. Equilibrium prices are such that markets for goods and productive factors clear. An introduction to GE theory can be found in Starr (2011).

This thesis employs both analytical and computable GE models.[3] The former are useful to build intuition on the relevant effects at play. Analytical GE models are intentionally simple, in order to be tractable, and to focus attention on a limited number of relevant effects. The latter can be highly complex. Computable GE models enable a representation of the economy that is closer to the level of detail often desired by policy-makers. Common features of such models are multiple productive sectors, often with a detailed representation of the most relevant industries depending on the question at hand (such as, in the case of climate policy, extractive and energy-intensive activities), intra- and international trade, investment and government expenditures, pre-existing government policies, as well as the dynamics of labor, capital, natural resources, and technological change. Such features are calibrated to real-world production and consumption data, as well as observed growth rates. Compared to analytical GE models, the insights provided by computable GE models are, however, necessarily derived in the context of specific data and assumptions about the underlying economic behavior of households and firms.

Environmental policy is often long-term, and this is especially the case for climate policy. Dynamic effects can, to a certain degree, be captured within static models by—for example—appropriately representing substitution in production and consumption. Some effects, however, such as the evolution of household consumption patterns with rising income levels, necessarily require a dynamic framework. The third essay, which focuses on this effect, employs a dynamic GE model. The other essays are based on static GE models.

This thesis studies four important issues regarding the distributional effects of environmental policy: the household aggregation bias, nonlinearities in the economy's response to policy, income-dependent consumption patterns and differentiated pollution pricing.

First, general equilibrium studies often represent the private demand side of the economy as a single consumer. This simplifying assumption is justified when the focus is on the supply side of the economy, but is less justifiable when considering questions that regard household demand and distributional effects of policy. Indeed, households tend to be heterogeneous along numerous dimensions, both between and within nations, for example in terms of preferences and the associated patterns of consumption, income levels, and the composition of incomes between different productive factors and government transfers. The first essay shows how incidence estimates may be biased when the economy is modeled as comprising a single, representative household. Consequently, the other essays in this thesis model household heterogeneity explicitly.

Second, tax incidence analysis relies extensively on linearization, whereby the local properties of an equilibrium are employed to estimate the economy's response to discrete perturbations. Such an approach is, for example, adopted in the first essay. Algorithmic approaches such as those adopted in the third essay, on the other hand, approximate the exact response of the economy. In view of the nonlinearities which are expected to characterize the economy's response to exogenous changes, the suitability of the linearized approach is not guaranteed *a priori*. The second essay builds upon the first to address this issue in the context of environmental policy. Its findings support the use

[3] Computable GE models solve for the economy's equilibrium algorithmically. This approach enables economic analysis that would not be possible based on purely analytical methods. Shoven and Whalley (1992) provides an overview of computable GE modeling.

of the linearized approach, with some important qualifications.

Third, economic models often assume household preferences to be homothetic, and thus household consumption patterns to be income-independent. This assumption implies constant, unitary income elasticities—an assumption that is often found to be inconsistent with empirical estimates. The first and the third essays incorporate non-homothetic preferences into analytical and computable GE models. In the case of the first essay, which employs a static model, the results are found to be relatively unaffected by such an extension. In the third essay, which models the dynamic evolution of a rapidly developing economy, non-homothetic preferences, calibrated to econometrically estimated income-consumption relationships, are found to strongly drive results. Comparing the two essays highlights an intuitive result: accounting for non-homothetic preferences is most important when representing large increases in household incomes, as those driven by the process of economic development in developing and fast-growing economies.

Fourth, the literature on the equity aspects of environmental policy has generally assumed that pollution prices are uniform across sectors in the economy. This assumption is based on the principle of equalizing marginal abatement costs in production, according to which uniform pricing is the most cost-efficient way to control pollution. Equity concerns are then usually addressed through redistribution alone. The fourth essay relaxes the assumption of uniformity, and investigates optimal environmental policy from the point of view of a government with social equity concerns. The theoretical and quantitative findings indicate that equity concerns can justify differentiated pollution prices across productive sectors to pursue social objectives.

Scientific contribution

This thesis contributes to the public economics literature on topics of environmental tax incidence, household heterogeneity, applied policy analysis, and optimal environmental taxation.

Essay 1

The first essay is based on a joint paper with Sebastian Rausch of ETH Zurich.[4] This essay examines how the general equilibrium incidence of an environmental tax depends on the effect of different incomes and preferences of heterogeneous households on aggregate outcomes. I develop a Harberger (1962)-type model with general forms of preferences and substitution between capital, labor, and pollution in production that captures the impact of household heterogeneity and interactions with production characteristics on the general equilibrium. I theoretically show that failing to incorporate household heterogeneity can qualitatively affect incidence. I quantitatively illustrate that this aggregation bias can be important for assessing the incidence of a carbon tax, mainly by affecting the returns to factors of production. The findings are robust to a number of

[4] The article on which this essay is based is published in *Journal of Public Economics, Volume 138, June 2016, 43-57* together with Sebastian Rausch (SR). A previous version appeared as CER-ETH Working Paper 16/230 in January 2016. Giacomo A. Schwarz (GS) and SR contributed equally to the research design. GS is the sole contributor to the analysis. GS and SR contributed equally to the exposition.

extensions including alternative revenue recycling schemes, pre-existing taxes, non-separable utility in pollution, labor-leisure choice, and multiple commodities.

It is well-known that preferences of heterogeneous households cannot in general be aggregated (Polemarchakis, 1983). The literature on environmental tax incidence nonetheless extensively represents the demand side of the economy as a single representative household. This includes a number of analytical (Fullerton and Heutel, 2007, 2010; Fullerton et al., 2012) and numerical (Metcalf et al., 2008; Araar et al., 2011; Dissou and Siddiqui, 2014) general equilibrium models. As noted by Kortum (2010), acknowledging heterogeneity in tastes undercuts the representative consumer framework that is used to calculate the general equilibrium effects on output and factor prices in incidence analysis. Although some references model heterogeneous households explicitly (Fullerton and Monti, 2013; Rausch et al., 2010a,b), to the best of my knowledge none study the implications of the household aggregation bias for environmental tax incidence. This essay contributes to the public economics literature by attempting to fill this gap.

First, I theoretically investigate the implication of the household aggregation problem for the incidence of environmental taxes, i.e., to what extent incidence results derived from a general equilibrium analysis which ignores household heterogeneity are biased. Second, I apply the heterogeneous household model to quantitatively assess how the aggregation bias affects equilibrium outcomes and the incidence of a tax on carbon dioxide (CO_2) emissions for the case of the United States. I assess the incidence on the sources and uses side of income, and explore how sensitive results are with respect to key characteristics governing households' and firms' behavior. I provide examples of conditions for households' and firms' characteristics under which the aggregation bias does or does not matter. For example, with limited substitutability between inputs of capital, labor, and pollution in production, factor and output price changes can be reversed, in turn yielding qualitatively different incidence results among poor and rich households.

I quantitatively illustrate that the aggregation bias for empirically motivated cases can be important for assessing the incidence of a carbon tax. As the aggregation bias on welfare is largely caused by the aggregation bias on the returns to factors of production, it mainly affects the sources of income. Additionally, I find that most of the variation in welfare impacts when altering production and household characteristics is driven by sources side impacts, and may even lead to a reversal of the incidence pattern across households. The analysis thus points to the importance of including sources of income impacts for tax incidence analysis. I also find that household heterogeneity in the elasticities of substitution in utility magnifies the aggregation bias due to heterogeneity in expenditure and income patterns. In this essay's static model, heterogeneity in income elasticities has a smaller effect compared to heterogeneity in substitution elasticities.

The findings are robust to a number of extensions including alternative revenue recycling schemes, pre-existing taxes, non-separable utility in pollution, labor-leisure choice, and multiple commodities. Any extension of the model obviously produces quantitatively different results but the point of the essay that household heterogeneity affects equilibrium and hence the incidence of environmental taxes remains. In fact, I argue that the case for the aggregation bias is strengthened rather than weakened.

Essay 2

Harberger (1962)-type tax incidence analysis relies on the linearized properties of an economy to assess the effects of discrete tax changes. The second essay studies the implications of the linearity assumption for the incidence of price- and quantity-based environmental policies. I prove analytically that changes in output prices and the pollution level following an environmental tax increase are overestimated in the linearized case, whereby the bias on pollution changes is larger than the bias on price changes. Subsequent numerical analysis provides quantitative results that are in line with these theoretical findings. The linearized approach overstates the regressivity of a given environmental tax increase and underestimates the regressivity of a policy targeting a given pollution reduction.

The Harberger (1962) model has been widely employed to study the effect of taxes on the economy (McLure, 1975). To overcome the limitations of the linearity assumption, Shoven (1976) employs an algorithmic approach to approximate the exact solution for a discrete change in the tax rate on capital income, and compares it to linearized results from Harberger (1966). The linearized approach is found to overstate the effects of a tax increase, both in terms of price changes and resource misallocation. The magnitude of the results in the two approaches are, however, found to be similar, thus lending credibility to results based on linearization. Harberger-type models have been recently employed to study environmental taxation (Fullerton and Heutel, 2007, 2010; Fullerton and Monti, 2013; Rausch and Schwarz, 2016). A number of features common to these models, however, differ from the original Harberger model—including the lack of a resource constraint for pollution, the larger number of substitution possibilities and the externality-correcting motive of pollution taxation—and may in principle lead to differing conclusions compared to Shoven (1976). In order for the results of Harberger-type studies of environmental tax incidence to be credibly employed to assess the effects of real-world policy interventions, a study of the suitability of linearized properties to approximate discrete changes in environmental taxes is therefore needed. To the best of my knowledge, no such study has been performed to date. This essay is an attempt to fill this gap.

I formulate a simple two sector two factor analytical general equilibrium model which is locally identical to Rausch and Schwarz (2016) and which, in a special case, can be solved analytically for discrete changes in the pollution tax. The analytical solution for the economy's exact response is a methodological contribution that goes beyond the approach of Shoven (1976). Beyond this special case, I approximate the exact solution algorithmically, analogously to Shoven (1976). For this purpose, I develop a simple Computable General Equilibrium (CGE) model with the same production and utility functions as the analytical model. I calibrate both the linearized results and the CGE model to the same data, employed previously in the context of Harberger-type environmental tax incidence analysis, thus delivering estimates of the effect of a pollution tax on output prices and returns to capital and labor in the two approaches, as well as the corresponding patterns of incidence.

I first consider the change in output and factor prices, and assess the bias introduced by the linearized approach. In the case of Cobb-Douglas production and identical Cobb-Douglas preferences across households I show analytically that, following an increase in the pollution tax rate, the linearized approach overstates price increases as well as the pollution reduction. On the other hand, the linearized approach understates the price change for a policy targeting a given pollution reduction. For a range of cases, the quantitative results are in line with these theoretical findings.

I then quantify the incidence of these price- and quantity-based environmental policies across households. I find that, for the price-based policy, the incidence results in linearized analysis are more regressive compared to the exact results, as the overestimated price changes magnify relative differences between households. For the quantity-based policy, incidence results in the linearized approach are instead less regressive, since, for a fiven tax change, the linearized approach overestimates pollution reductions more than price changes. Although the results differ quantitatively between the linearized and the exact approaches, they are qualitatively similar.

Essay 3

The third essay is based on joint research with Justin Caron of HEC Montréal and Valerie J. Karplus of the Massachusetts Institute of Technology.[5] I quantify how energy demand and associated CO_2 emissions are affected by income-driven shifts in consumption patterns in China, the world's largest emerging economy. Incorporating empirically-derived Engel curves within a general equilibrium model, I find that, relative to projections based on standard assumptions of unitary income elasticity, direct household CO_2 emissions in 2030 are 61% lower, and national emissions are reduced by 8%. This has important implications for the welfare consequences of climate policy. The average welfare costs of climate policy decrease by more than half, with losses more evenly distributed across households. This is driven by the easing of policy stringency and convergence in the carbon intensity of household consumption baskets as incomes rise. The results point to non-homothetic household preferences as an important determinant of climate policy costs in developing economies, where incomes, energy demand and emissions are rising rapidly.

The foundational work of Engel (as cited in Chai and Moneta (2010)), Working (1943) and Leser (1963) has established income as an important determinant of household consumption patterns. A large body of literature has focused on estimating income elasticities of household demand for goods and services (Houthakker, 1957; Carliner, 1973; Branch, 1993; Haque, 2005), often finding that they deviate from unity and vary by income level. A growing literature expounds on the relationship between income and patterns of household energy demand using 'micro' household-level data (Branch, 1993; Reinders et al., 2003; Filippini and Pachauri, 2004). By capturing the dynamics of adoption, these studies allow for differentiation between the intensive and the extensive margins of energy use. Only a small number of studies for China use micro data, due in part to the limited availability of household level surveys. Cao et al. (2014) exploit survey data collected by China's National Bureau of Statistics (NBS) to estimate the relationship between income and energy demand and provide strong evidence that income elasticity varies across income groups. They don't, however, estimate flexible Engel curves which span the full income spectrum.

[5] Giacomo A. Schwarz (GS), Justin Caron (JC) and Valerie J. Karplus (VK) contributed equally to the research design. GS is the sole contributor to the modeling analysis. JC is the sole contributor to the econometric analysis. GS is the main contributor to the exposition, with significant contributions from JC and VK.

Furthermore, while the survey contains a very large number of observations, it is limited to urban households in a limited number of provinces.

Numerous energy- or climate-policy specific economic models assume that consumption scales proportionally with income, due to a number of reasons including computational tractability and data limitations, thus contradicting the empirical findings discussed above. This includes static (Fullerton and Heutel, 2007; Fullerton and Monti, 2013; Rausch et al., 2011; Zhang et al., 2013) and dynamic (Goettle et al., 2009; Rausch et al., 2010b; Williams III et al., 2015) analyses. Models who do implement non-unitary income elasticities usually do not use flexible demand systems, and when they employ energy-specific income elasticities, these are either not directly estimated or assume values that are extrapolated from other sectors (Hertel, 1997; van der Mensbrugghe, 2010; Chen et al., 2015). I am not aware of economic modeling studies that rely on micro data to estimate energy-specific Engel curves and integrate them within a general equilibrium economy-wide model, nor of studies that focus specifically on the importance of income-dependent shifts in consumption patterns in determining climate policy costs and incidence. This essay aims at addressing these issues.

I contribute to the empirical literature by estimating Engel curves for China based on micro-data for energy use, by exploiting the large existing variation in household incomes and allowing for flexible functional forms and controlling for important co-variates. The estimation results indicate that income elasticities vary widely with income and differ significantly from unity at all levels, for most goods. I find relatively low income elasticities for energy consumption, except for gasoline and diesel and central heating (particularly for rural households). Coal consumption decreases with income—particularly for urban households—implying negative income elasticity. The same holds for bottled gas at high income levels.

I also contribute to the energy-economic modeling literature by developing a novel approach to calibrate recursive-dynamic general equilibrium (GE) models to any consumption path as a function of between-period changes in income. I then explore the implications of the empirically-derived estimates in a GE model of China that separately resolves urban and rural households in each province, enabling an analysis that captures two important dimensions of policy targeting in China (Ming and Zhao, 2006; Zhang et al., 2013). I isolate the effect of income-driven shifts in consumption patterns by comparing results for calibrated non-homothetic preferences to results for homothetic preferences. I then decompose the general equilibrium effects of income-driven shifts in consumption patterns on the distribution of the policy's welfare costs across households. I isolate the effect on the distribution of impacts due to changes in household incomes (which I refer to as "income side" impacts) and changes in the relative prices of consumption goods caused by policy (which I refer to as "consumption side" impacts).

The calibration alters projections of energy demand, CO_2 emissions, and the magnitude and distribution of climate policy impacts. Comparing results for calibrated non-homothetic preferences to results for homothetic preferences, I identify four main results. First, the estimated Engel curves, once incorporated into the GE model, translate into considerably lower total demand for energy in China and lower associated CO_2 emissions than if preferences are assumed to be unit-elastic. I find, second, that ignoring income-driven shifts in consumption patterns considerably over-estimates the average costs of reaching a given emissions target. Third, I find that the variation in welfare impacts is reduced when accounting for non-homothetic preferences. This finding is driven by the lower carbon price caused by the lower baseline emissions, which mutes the variation of both consumption and income side impacts, the lower correlation between these two channels, and a rapid convergence of the carbon intensity of consumption baskets across households as incomes grow (caused by low income elasticities of energy goods and rapid growth in household income levels). Fourth, income-driven consumption shifts cause the ranking between relative winners and losers under policy to differ compared to the case with homothetic preferences, driven by the differing temporal evolution of consumption patterns across households and interactions with the changes in relative prices of consumption goods under policy.

In summary, this essay combines, and contributes to, the literature on the relationship between household income and expenditures, and on empirical policy analysis. This essay's framework enables an understanding of both the role of the household consumption shifts on the economy's general equilibrium, and of their interactions with the impacts of climate policy, both at an aggregate and a household level.

Essay 4

The fourth essay is based on joint research with Sebastian Rausch and Jan Abrell of ETH Zurich.[6] This essay examines differentiated pollution pricing in the presence of social equity concerns using both theoretical and numerical general equilibrium analyses in an optimal taxation framework. I first theoretically study the optimality conditions for sectoral pollution taxes and redistribution of pollution tax revenues. Household heterogeneity in preferences and endowments interacts with social preferences implying non-uniform pollution taxes at the social optimum. Taxes should be differentiated according to households' consumption characteristics and to raise revenues for targeted transfers to households. Quantitatively assessing the scope for differentiated carbon taxes in the context of the U.S. economy, I find that optimal sectoral carbon taxes differ widely across sectors, even when social inequality aversion is relatively low. The optimal policy differentiates taxes to strongly increase revenues for redistribution to lower income households. Considering non-optimal redistribution schemes, the scope for tax differentiation is somewhat diminished but remains substantial. Relative to uniform carbon pricing, incidence patterns across household expenditure groups for a given redistribution scheme can vary qualitatively when allowing for differentiated taxes.

This essay contributes to the public economics literature in several ways. First, the essay is related to the literature on optimal environmental taxation following the seminal contribution by Pigou (1920) according to which an externality should be priced at its marginal social damage. Subsequent

[6] At the time of printing, the article based on this essay (entitled "Social Equity Concerns and Differentiated Environmental Taxes", together with Jan Abrell and Sebastian Rausch) was under review at the *American Economic Journal: Economic Policy*. The preprint version appeared as CER-ETH Working Paper 16/262 in November 2016. Giacomo A. Schwarz (GS), Jan Abrell (JA) and Sebastian Rausch (SR) contributed equally to the research design. GS is the sole contributor to the theoretical analysis, and a secondary contributor to the numerical analysis (to which JA is the primary contributor). GS and SR contributed equally to the exposition.

literature found that interactions between environmental taxes and the broader fiscal system as well as the use of tax revenues to reduce pre-existing distortionary taxes can modify the Pigouvian pricing rule (see, for example, Bovenberg and de Mooij, 1994; Bovenberg and van der Ploeg, 1994; Parry, 1995; Bovenberg and Goulder, 1996). By focusing on efficiency, this literature abstracts from heterogeneous households and social equity concerns. Moreover, it assumes a single polluting sector or uniform emissions pricing across multiple economic activities.

Second, studies concerned with pollution tax differentiation across sectors have also focused on efficiency aspects. Boeters (2014) and Landis et al. (2016) find that the cost of climate policy can be lowered by differentiating carbon taxes in light of pre-existing non-environmental tax distortions. In an open economy context and for unilateral environmental policy, international market power, terms-of-trade effects, and emissions leakage have been shown to imply pollution taxes to optimally differ across sectors (Hoel, 1996; Böhringer and Rutherford, 1997; Böhringer et al., 2014; Boeters, 2014). I contribute by examining how optimal pollution taxes should be differentiated across sectors due to social equity concerns.

Third, others have investigated the role of equity concerns within an optimal taxation framework. A large body of literature, not directly related to addressing an environmental externality, has established the result that optimal commodity and income taxation is in general affected by household heterogeneity and social preferences (Diamond and Mirrlees, 1971). In a framework that brings together revenue-raising and externality-correcting motives for optimal tax policy, Sandmo (1975) and Cremer et al. (2003) find that the optimal pollution tax rate in an economy with one polluting good depends on social preferences and household characteristics. Equity concerns also motivate non-linear consumption taxes (Cremer et al., 1998, 2003), thus leading to differentiated tax rates among consumers. Again, the possibility of pollution taxes that are differentiated across sectors is ruled out.

Fourth, equity aspects of environmental policy are often studied outside an optimal taxation framework. Here, it is common to assess the distributional outcomes of environmental policy without ranking alternative outcomes based on social desirability or deriving optimal pollution tax rates in light of social preferences (i.e., thus adopting a positive rather than a normative perspective). For example, Poterba (1991) and Fullerton and Heutel (2010) find that energy (gasoline and carbon) taxes are strongly regressive. Bento et al. (2009) find that the incidence of an increase in the gasoline tax depends crucially on how revenues are recycled. In other instances, the amount of revenue collected through the environmental policy has been found to be insufficient to alter incidence in desirable ways. For example, Fullerton and Monti (2013) find that low-wage earners are more impacted by pollution taxes, even when they receive all the tax revenues. Bovenberg et al. (2005) examine the efficiency cost of meeting distributional objectives across industries for emissions taxes. While this strand of the literature has highlighted important trade-offs between efficiency and equity, it has restricted its attention to uniform pollution taxes and has not analyzed the question of optimal pollution pricing in the presence of social equity concerns.

The theoretical analysis in this essay identifies the motives for optimally differentiating pollution taxes across sectors, absent efficiency-related motives such as international trade and pre-existing taxes. If households have different tastes and there exist social equity concerns, then taxes should be differentiated according to households' consumption characteristics, in order to shift the burden of taxation towards households with lower social weights. Moreover, pollution taxes should be differentiated to raise revenues for targeted transfers to households with high social weights.

The quantitative analysis supports the theoretical findings. The amount of sectoral carbon tax differentiation is substantial, even for relatively low degrees of social inequality aversion. Decomposing the different motives for tax differentiation, I find that the deviation from uniform pollution pricing is to a large extent driven by the motive to enhance the amount of tax revenues available for redistribution to lower income households. Pollution taxes are higher for sectors with relatively steep marginal abatement cost, as this enables raising higher tax revenues for a given amount of pollution. Hence, I find that efficiency of abatement in production is strongly sacrificed in favor of generating high tax revenues that can be spent on targeted transfers to address equity concerns.

Besides examining optimal policies consisting of differentiated pollution taxes and transfers, situations in which a specific redistribution scheme is already in place and cannot be altered may be the more relevant setting for real-world environmental policy. I analyze the optimal amount of tax differentiation as well as the incidence impacts for three non-optimal revenue redistribution schemes based on either equal per-capita transfers, income-, or consumption-based recycling. While I find that for these transfer schemes the case for tax differentiation is somewhat diminished (relative to the optimal policy), I show that optimal carbon taxes are still strongly differentiated across sectors for transfer schemes which allocate a large fraction of the tax revenue to lower income households (as, for example, under per-capita or consumption-based redistribution). In terms of incidence impacts by household expenditure deciles, optimally differentiated taxes yield more progressive (less regressive) outcomes relative to uniform pollution pricing. In general, I find that the difference in the incidence between uniform and optimally differentiated pollution pricing is larger for transfer schemes which a better suited to address inequality, i.e. distributing larger parts of the revenue to lower income households

Policy implications

Beyond its academic contributions, this thesis has a number of implications for policy:

- The results from all four essays indicate that, if the redistribution of carbon tax revenues is ignored, low-income households (both in the United States and China) tend to be disproportionately impacted by climate policy, driven by their higher expenditure shares on carbon-intensive goods. These findings imply that policy-makers should consider devising redistributive measures to compensate low-income households in order to, first, avoid a distribution of impacts that may be socially undesirable and, second, to stem potential opposition to the implementation of a policy that may be perceived as unfair. More stringent climate policy in the future will magnify such issues.

- As carbon intensive industries tend to be capital intensive, climate policy is expected to depress returns to capital relative to wages, thus affecting owners of capital (i.e., high-income households) disproportionately. The first essay indicates that part of the regressivity of U.S. climate policy due to the carbon intensity of household consumption patterns may indeed be offset by the relative impact on capital versus labor. At the same time, it indicates that such relative burdens are sensitive to uncertain parameters describing the economy's response to tax increases. In addition, the ownership of productive capital is notoriously hard to measure accurately at the household level, adding a further layer of uncertainty to uses side incidence estimates. These findings may nonetheless contribute to shift the public perception beyond uses side considerations alone, by highlighting a channel that, under climate policy, may disproportionately affect high-income households.

- The third essay finds that, in the Chinese context, heterogeneity in climate policy impacts on regional economies (caused by differences in the carbon intensity of production and in the relative importance of extractive industries) can strongly determine the distributional impacts across households. These findings indicate that the incidence question on the sources side should not only be concerned with the relative ownership of capital or labor across households, but also with the exposure to regional effects of climate policy on the returns to such productive factors. These include the disproportionate impact on wages in regions with particularly dirty production, and the loss in natural resource rents. Chinese policy-makers should thus consider devising regional compensation mechanisms, in addition to the income-based considerations motivated in the first point above. It should, however, be noted that the regional disparity of impacts relies strongly on assumptions about the mobility of productive factors in the economy. Labor, for example, is assumed in the model to be immobile across provinces in China. Relaxing this assumption would likely reduce the regional inequality of policy impacts.

- The first and the fourth essays illustrate how the use of pollution tax revenues can be just as important, or even more important, than relative price changes in determining relative winners and losers under climate policy. For the U.S. economy, returning revenues in proportion to dirty consumption, compared to income-proportional redistribution (or, equivalently, no redistribution), is found to cause incidence across expenditure deciles to be less regressive. Per-capita redistribution makes the environmental tax strongly progressive, i.e., it largely benefits low- relative to high-income households. These findings indicate that potential concerns regarding the regressivity of climate policy across expenditure deciles can likely be entirely addressed through redistribution. It should be noted that other studies have found tax revenues from climate policy to be insufficient to compensate low-skilled workers, thus highlighting the dependence of the efficacy of redistribution to modify incidence patterns on the dimension along which incidence is considered.

- The econometric estimation in the third essay finds that household energy consumption in China is expected to increase less than proportionally as income levels grow. Future CO_2 emissions in China may therefore be lower than expected from analyses based on the common simplifying assumption of income-independent composition of household consumption. This has direct implications for national climate policy, as less emissions reductions would then be necessary in order to achieve a given emissions target. It also has implications for local environmental policy, as less mitigating action may be necessary for local air pollution than expected. This last result may, however, be at least partially offset by the overproportional increase in expenditure on transportation as income levels rise. It should also be noted that projections in this essay ignore a number of relevant effects that may affect policy recommendations such as, first, migration and, second, potential changes in household preferences unrelated to income.

- The modeling results in the third essay highlight that accounting for the seldom considered effects of income-driven shifts in household consumption patterns strongly affects both the average and the variability of the estimated climate policy costs in China. Taken together, the results thus suggest that the case for climate policy in China may be stronger than previous projections suggest. It should, however, be noted that these conclusions rest importantly on the assumption that climate policy targets a given emissions trajectory. They would likely still hold, albeit in a less strong form, if climate policy were instead to target a given reduction in carbon intensity of GDP, or a given carbon price trajectory.

- Beyond China, the results in the third essay indicate that both the aggregate cost and the distribution of burdens from environmental policy depend crucially on how household consumption patterns change with rising income levels. Taking such effects into account is especially important for policymaking in rapidly developing economies, where such income-driven consumption shifts will likely play a more important role, already over relatively short time horizons. The applicability of the results for China to other contexts, however, strongly depends on the local characteristics of households. If, for example, low-income households in an other developing economy burn less coal domestically, low income elasticities for coal consumption would have a weaker effect on emissions compared to the case of China.

- The forth essay finds that, if the government is averse to inequality, carbon taxes should optimally be lower on sectors producing goods consumed more intensively by low-income households, and higher on sectors with rapidly increasing marginal abatement cost curves, in order to increase the amount of tax revenues for redistribution to low-income households. These insights highlight the fact that considering the redistributive and the pricing parts of environmental policy separately may preclude socially preferable outcomes. The recommendation to consider both aspects of environmental policy contemporaneously, however, is made without considering the feasibility of such a decision-making process.

- Although the results in the fourth essay indicate that differentiated carbon taxes may represent a possible means to address social equity concerns, in addition to environmental goals, a number of considerations should be made before recommending real-world carbon prices to be differentiated across sectors. First, achieving the social optimum in this framework relies on assumptions of perfect information, i.e., that the regulator can exactly foresee the economy's response to policy. Although further research may deliver some operational rules to enable real-world policy to approximate this ideal, it can unlikely be achieved in practice. Second, in order to focus on the effect of social equity concerns on pollution price differentiation, the fourth essay abstracts from pre-existing taxes and international trade—efficiency-motivated drivers of differentiation. If accounted for, these drivers would likely interact with social equity concerns, modifying the optimal pollution prices. Optimal pollution prices would, however, most likely still be differentiated. Third, the normative analysis carried out in this essay studies the optimal policy given the government's social objectives. It does not, however, prescribe what those objectives should be. The government may well apply equal social weights to all households, in which case social preferences would not motivate pollution price differentiation.

- Beyond the study of optimal environmental policy, the fourth essay takes a step in the direction of real-world policy-making by representing political constraints on redistribution. In such cases, the degree of tax differentiation is found to depend on how well the constrained redistribution scheme approximates the optimal one. Importantly, for an inequality-averse government, per-capita redistribution—which arguably best reflects real-world constraints on the redistribution of pollution taxation—exhibits a large degree of tax differentiation at the social optimum. This already holds for relatively low degrees of inequality aversion. These findings show that pollution price differentiation motivated by social equity concerns is robust to real-world constraints, thus indicating that it may be worthy of consideration for real-world policy-making. The caveats discussed in the previous point, however, still apply.

1 Household heterogeneity, aggregation, and the distributional impacts of environmental taxes[1]

[1]The article on which this essay is based is published in *Journal of Public Economics, Volume 138, June 2016, 43-57* (http://dx.doi.org/10.1016/j.jpubeco.2016.04.004) together with Sebastian Rausch.

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Abstract

This paper examines how the general equilibrium incidence of an environmental tax depends on the effect of different incomes and preferences of heterogeneous households on aggregate outcomes. We develop a Harberger-type model with general forms of preferences and substitution between capital, labor, and pollution in production that captures the impact of household heterogeneity and interactions with production characteristics on the general equilibrium. We theoretically show that failing to incorporate household heterogeneity can qualitatively affect incidence. We quantitatively illustrate that this aggregation bias can be important for assessing the incidence of a carbon tax, mainly by affecting the returns to factors of production. Our findings are robust to a number of extensions including alternative revenue recycling schemes, pre-existing taxes, non-separable utility in pollution, labor-leisure choice, and multiple commodities.

1.1 Introduction

The public acceptance for environmental taxes depends crucially on their distributional consequences. A plethora of applied research in public and environmental economics has investigated the incidence of environmental taxes in various policy settings. Not seldom, however, the empirical evidence whether a specific tax is regressive or not is mixed–even if the incidence of a given tax instrument is analyzed in a similar or identical policy context. Differences arise because the incidence analysis does not consider all relevant channels through which an environmental tax affects market outcomes (see, e.g., Atkinson and Stiglitz (1980) and Fullerton and Metcalf (2002) for a discussion of incidence impacts in the public finance literature).[2] One important channel which is typically omitted by general equilibrium analyses that employ a single, representative household model is the impact of household heterogeneity on the market equilibrium. Despite the high policy relevance and academic interest for understanding the distributional consequences of price-based pollution controls, an analysis of the effect of household aggregation on tax incidence is lacking.

This paper develops a theoretical Harberger (1962)-type general equilibrium model of the incidence of an environmental tax featuring heterogeneous households, general forms of preferences, differential spending and income patterns, differential factor intensities in production, and general forms of substitution among inputs of capital, labor, and pollution. Its purpose is two-fold. First, we theoretically investigate the implication of the household aggregation problem for the incidence of environmental taxes, i.e., to what extent incidence results derived from a general equilibrium analysis which ignores household heterogeneity are biased. In the absence of identical homothetic preferences for each individual or homothetic preferences and collinear initial endowment vectors (i.e., identical income shares), aggregated preferences depend on the distribution of income (Polemarchakis, 1983).[3] Thus acknowledging heterogeneity in tastes undercuts the representative consumer framework that is used to calculate the general equilibrium effects on output and factor prices (Kortum, 2010). Second, we apply the heterogeneous household model to quantitatively assess how the aggregation bias affects equilibrium outcomes and the incidence of a tax on carbon dioxide (CO_2) emissions for the case of the United States. We assess the incidence on the sources and uses side of income, and explore how sensitive results are with respect to key characteristics governing households' and firms' behavior.

Our main finding is that the household aggregation problem can have important implications for assessing the incidence of environmental taxes: basing the analysis on a single, representative

[2] Environmental taxes often appear to be regressive on the "uses side of income" as they affect more heavily the welfare of the poorest households than of the richest ones, since poorer households spend a larger fraction of their income on polluting goods (e.g., energy or electricity). "Sources side of income" impacts can dampen or even offset the regressive incidence on the uses side to the extent that environmental tax policies affect the returns to factors of production that are disproportionately owned by richer households and used intensively in the production of dirty relative to clean industries (e.g., capital). The regressivity of many environmental taxes on the uses side, including carbon pricing in the context of climate policy, constitutes a serious concern for policymakers and has been investigated extensively in the literature (Poterba, 1991; Metcalf, 1999; Fullerton et al., 2012). Gasoline taxes are generally found to be progressive on the uses side (Sterner, 2012). More recently, work by Fullerton and Heutel (2007), Araar et al. (2011), and Rausch et al. (2011) has also scrutinized the sources side impacts of carbon taxation.

[3] On a more fundamental conceptual level, and not related to the incidence of (environmental) taxation, the aggregation problem for heterogeneous consumers in general equilibrium models has been studied by Ackermann (2002) based on prior work by Rizvi (1994) and Martel (1996).

household model as opposed to an analysis that integrates household heterogeneity can yield both qualitatively and quantitatively different conclusions. Assuming homothetic preferences, we show that the impact of household heterogeneity on the equilibrium can be characterized by two statistical quantities which capture the degree of household heterogeneity in terms of household preferences and income shares. These metrics provide an intuitive way to express the discrepancy in results obtained under a case with heterogeneous households and a case with identical households. We provide examples of conditions for households' and firms' characteristics under which the aggregation bias does or does not matter. For example, with limited substitutability between inputs of capital, labor, and pollution in production, factor and output price changes can be reversed, in turn yielding qualitatively different incidence results among poor and rich households. Moreover, we find that there exist for any benchmark economy, described by data on production and distributions of consumption and income among households, values of production elasticities such that household aggregation leads to reversed factor price changes. We find that for non-homothetic preferences the burden of an environmental tax on factors of production can be qualitatively different as compared to a case with homothetic preferences.

We quantitatively illustrate that the aggregation bias for empirically motivated cases can be important for assessing the incidence of a carbon tax. As the aggregation bias on welfare is largely caused by the aggregation bias on the returns to factors of production, it mainly affects the sources of income. Additionally, we find that most of the variation in welfare impacts when altering production and household characteristics is driven by sources side impacts, and may even lead to a reversal of the incidence pattern across households. Our analysis thus points to the importance of including sources of income impacts for tax incidence analysis. We also find that household heterogeneity in the elasticities of substitution in utility magnifies the aggregation bias due to heterogeneity in expenditure and income patterns. In our static model, heterogeneity in income elasticities has a smaller effect compared to heterogeneity in substitution elasticities.

Our findings are robust to a number of extensions including alternative revenue recycling schemes, pre-existing taxes, non-separable utility in pollution, labor-leisure choice, and multiple commodities. Any extension of the model obviously produces quantitatively different results but the point of the paper that household heterogeneity affects equilibrium and hence the incidence of environmental taxes remains. In fact, we argue that the case for the aggregation bias is strengthened rather than weakened.

Our paper builds on a small but growing literature that uses analytical general equilibrium models to study the incidence of environmental taxes. Our model builds on a series of influential papers by Fullerton and others (Fullerton and Heutel, 2007, 2010; Fullerton et al., 2012; Fullerton and Monti, 2013) that extend the Harberger (1962) model and previous theoretical work by Rapanos (1992, 1995) to develop a model which represents pollution as an input along with capital and labor and that allows for general forms of substitution between inputs. We extend the single-consumer model presented in Fullerton and Heutel (2007) to include heterogeneous households. We additionally incorporate non-homothetic preferences. By fully integrating household heterogeneity, our paper also differs from the contributions in Fullerton and Heutel (2010) and Fullerton et al. (2012) that use price impacts derived from the single-consumer model in Fullerton and Heutel (2007) to determine the burdens of a carbon tax using household survey data. Fullerton and Monti (2013) integrate two types of households into an analytical general equilibrium model and investigate the distributional impacts of a pollution tax swap (recycling revenues through a wage tax of low-income workers). They do not, however, study the impact of household heterogeneity on equilibrium outcomes.

Our analysis is also related to the literature that uses computational methods to assess the distributional impacts of environmental taxes. A widespread approach is to employ Input-Output analysis to derive price changes for different consumer goods and then calculate tax burdens for households based on micro-household survey data.[4] Common to these studies is that they adopt a partial equilibrium perspective that does not consider behavioral changes and focuses on the uses sides of the incidence only. A few papers use numerical general equilibrium models with a single, representative consumer to derive price impacts on commodity and factor prices. Metcalf et al. (2008) carry out an analysis of carbon tax proposals and find that a carbon tax is highly regressive but that the regressivity is reduced due to sources side effects to the extent that resource and equity owners bear some fraction of the tax burden. Similarly, Araar et al. (2011) and Dissou and Siddiqui (2014) use price effects to assess the distributional impacts of a carbon tax. None of these studies, however, captures the impact of household heterogeneity on equilibrium outcomes.

Lastly, a few papers integrate heterogeneous households into a numerical general equilibrium framework. For example, Rausch et al. (2010a,b) investigate the incidence of a U.S. carbon tax in a model with nine households representing different income classes and find that the overall impact is neutral to modestly progressive due to sources side effects (assuming that government transfers to households are indexed to inflation). Williams III et al. (2015) and Chiroleu-Assouline and Fodha (2014) employ calibrated overlapping generations models to assess the distributional incidence across generations. A major weakness of analyses based on numerical simulation models is, however, their reliance on specific functional forms with limited forms of substitution. In contrast, our paper studies environmental tax incidence in a theoretical setup with general forms of substitution in production and consumption.

The remainder of this paper is organized as follows. Section 1.2 presents the model. Section 1.3 derives closed-form expressions to assess the incidence of an environmental tax change, and presents and interprets our theoretical results. Section 1.4 uses an empirically calibrated version of the model to quantitatively study the aggregation bias. Section 1.5 provides evidence that the aggregation bias remains relevant when extending the core model in a number of important directions. Section 1.6 concludes. Appendixes A.1 to A.3 contain additional derivations and proofs for our results.[5]

[4] Examples include Robinson (1985) who studies the distributional burden of industrial abatement in the U.S. economy and Poterba (1991) who focuses on the incidence of U.S. gasoline taxes. Bull et al. (1994) and Hassett et al. (2009) compare a tax based on energy content and a tax based on carbon, and Metcalf (1999, 2009a) analyze a revenue-neutral package of environmental taxes, including a carbon tax, an increase in motor fuel taxes, and taxes on various stationary source emissions. Dinan and Rogers (2002) assess the efficiency and distributional impacts of a U.S. cap-and-trade program for CO_2 emissions, and Mathur and Morris (2014) investigate the distributional effects of a carbon tax in broader U.S. fiscal reform. Other works study the incidence impacts of greenhouse gas emissions pricing policies across household income groups for different countries (e.g., Labandeira and Labeaga (1999) for Spain, Callan et al. (2009) for Ireland, and Jiang and Shao (2014) for China).

[5] Appendixes A.4 and A.5 provide supplementary analysis on incidence results for alternative revenue recycling

1.2 Model

We consider a static and closed economy with two sectors and two factors of production. A "clean" good is produced using capital and labor, and a "dirty" good is produced using capital, labor and pollution. Capital and labor are supplied inelastically and are mobile across sectors. The government taxes pollution, returning the revenue lump-sum to households. Our general equilibrium model follows closely Harberger (1962) and Fullerton and Heutel (2007) but differs in two important aspects. First, we introduce heterogeneous households that differ in terms of their preferences and income patterns derived from endowments of capital and labor. Second, we generalize the representation of household behavior by allowing for non-homothetic preferences. Using log-linearization, we analytically solve for first-order changes in equilibrium prices and quantities following an exogenous change in the pollution tax rate. Our model enables us to quantify the general equilibrium incidence of the environmental tax in the context of an economy with no a-priori restrictions placed on the number and characteristics of households.

The clean sector production function $X = X(K_X, L_X)$ and the dirty sector production function $Y = Y(K_Y, L_Y, Z)$ are assumed to exhibit constant returns to scale, where K_X, K_Y, L_X, and L_Y are the quantities of capital and labor used in each sector.[6] The total amounts of factors of production in the economy are exogenously given and fixed: $K_X + K_Y = \bar{K}$ and $L_X + L_Y = \bar{L}$. Totally differentiating the resource constraints yields:

$$\hat{K}_X \frac{K_X}{\bar{K}} + \hat{K}_Y \frac{K_Y}{\bar{K}} = 0 \tag{1.1}$$

$$\hat{L}_X \frac{L_X}{\bar{L}} + \hat{L}_Y \frac{L_Y}{\bar{L}} = 0 \,, \tag{1.2}$$

where a hat denotes a proportional change, e.g., $\hat{K}_X \equiv dK_X/K_X$. Pollution ($Z$) has no equivalent resource constraint and is a choice of the dirty sector. To ensure a finite use of pollution in equilibrium, we assume a pre-existing positive tax on pollution, $\tau_Z > 0$.

Firms in sector X can substitute between factors in response to changes in the wage rate (w) and capital rental rate (r) according to an elasticity of substitution in production, σ_X. Differentiating the definition for σ_X yields:

$$\hat{K}_X - \hat{L}_X = \sigma_X(\hat{w} - \hat{r}) \,. \tag{1.3}$$

The production decision of firms in sector Y depends additionally on the pollution price they face, which is given by the pollution tax rate τ_Z. We model the choice between the three inputs of capital, labor and pollution by means of the Allen elasticities e_{ij} between inputs i and j (Allen, 1938). The 3×3 matrix of Allen elasticities is symmetric (i.e., $e_{ij} = e_{ji}$), its diagonal entries are less or equal to zero (i.e., $e_{ii} \leq 0$), and at most one of the three independent off-diagonal elements can be negative. Furthermore, e_{ij} is positive whenever inputs i and j are substitutes, and negative

schemes as well as derivations and proofs for the model extensions.

[6] Note that the production side of our model is the same as for the single-consumer model of Fullerton and Heutel (2007). In describing production we thus follow closely the model description in Fullerton and Heutel (2007, pp. 574-75).

whenever they are complements. Totally differentiating input demand functions for sector Y, which describe the dirty sector's cost minimization problem, and dividing by the appropriate input level, yields:[7]

$$\hat{K}_Y - \hat{Z} = \theta_{YK}(e_{KK} - e_{ZK})\hat{r} + \theta_{YL}(e_{KL} - e_{ZL})\hat{w} + \theta_{YZ}(e_{KZ} - e_{ZZ})\hat{\tau}_Z \tag{1.4}$$

$$\hat{L}_Y - \hat{Z} = \theta_{YK}(e_{LK} - e_{ZK})\hat{r} + \theta_{YL}(e_{LL} - e_{ZL})\hat{w} + \theta_{YZ}(e_{LZ} - e_{ZZ})\hat{\tau}_Z\,, \tag{1.5}$$

where θ_{mn} is the share of sector m's revenue paid to factor n, e.g. $\theta_{XK} = \frac{rK_X}{p_X X}$. Let p_X and p_Y denote output prices for X and Y, respectively. Under the assumption of perfect competition, the following expressions hold:

$$\hat{p}_X + \hat{X} = \theta_{XK}(\hat{r} + \hat{K}_X) + \theta_{XL}(\hat{w} + \hat{L}_X) \tag{1.6}$$

$$\hat{p}_Y + \hat{Y} = \theta_{YK}(\hat{r} + \hat{K}_Y) + \theta_{YL}(\hat{w} + \hat{L}_Y) + \theta_{YZ}(\hat{\tau}_Z + \hat{Z}) \tag{1.7}$$

$$\hat{X} = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X \tag{1.8}$$

$$\hat{Y} = \theta_{YK}\hat{K}_Y + \theta_{YL}\hat{L}_Y + \theta_{YZ}\hat{Z}\,. \tag{1.9}$$

Households, indexed by $h = \{1, \ldots, H\}$, maximize utility by choosing optimal consumption of goods X and Y subject to an income constraint.[8] Each household inelastically supplies fixed factor endowments $\bar{K}^h$ and $\bar{L}^h$ which satisfy the following relations: $\sum_h \bar{K}^h = \bar{K}$ and $\sum_h \bar{L}^h = \bar{L}$. Income for household h is therefore given by $M^h = w\bar{L}^h + r\bar{K}^h + \xi^h \tau_Z Z$, where ξ^h is the share of the pollution tax revenue redistributed lump-sum to household h. Since the tax revenue is returned entirely to households, it follows that $\sum_h \xi^h = 1$.

Following Hicks and Allen (1934), we parameterize non-homothetic consumer preferences for the two goods using the elasticity of substitution between goods X and Y in utility σ^h, and the income elasticities of demand for goods X and Y, denoted by $E^h_{X,M}$ and $E^h_{Y,M}$ respectively.[9] Appendix A.1 derives the following expressions for changes in demand by household h in response to output and factor price changes:

$$\hat{X}^h - \hat{Y}^h = \sigma^h(\hat{p}_Y - \hat{p}_X) + (E^h_{Y,M} - E^h_{X,M})(\alpha^h \hat{p}_X + (1-\alpha^h)\hat{p}_Y - \hat{M}^h) \tag{1.10}$$

$$\hat{X}^h = -(\alpha^h E^h_{X,M} + (1-\alpha^h)\sigma^h)\hat{p}_X - ((1-\alpha^h)E^h_{X,M} - (1-\alpha^h)\sigma^h)\hat{p}_Y + E^h_{X,M}\hat{M}^h\,, \tag{1.11}$$

with $\hat{M}^h = \hat{w}\frac{w\bar{L}^h}{M^h} + \hat{r}\frac{r\bar{K}^h}{M^h} + \frac{\xi^h\tau_Z Z}{M^h}(\hat{\tau}_Z + \hat{Z})$.

Finally, totally differentiating the market clearing conditions for the two consumption goods, $X =$

[7] Appendix A in Fullerton and Heutel (2007) derives Equations (1.4)-(1.9).

[8] We assume that pollution, or environmental quality, is separable in utility, thus not influencing the optimal consumption choice. Note that the incidence analysis carried out in this paper focuses on utility derived from market consumption only.

[9] Homothetic preferences are represented by the special case $E^h_{X,M} = E^h_{Y,M} = 1$. In this case the first-order behavior of households can be sufficiently described by σ^h, as for example in Fullerton and Heutel (2007).

$\sum_h X^h$ and $Y = \sum_h Y^h$, yields:

$$\hat{X} = \sum_h \frac{X^h}{X} \hat{X}^h \tag{1.12}$$

$$\hat{Y} = \sum_h \frac{Y^h}{Y} \hat{Y}^h \,. \tag{1.13}$$

Equations (1.1)–(1.13) are $11 + 2H$ equations in $11 + 2H$ unknowns ($\hat{K}_X$, $\hat{K}_Y$, $\hat{L}_X$, $\hat{L}_Y$, $\hat{w}$, $\hat{r}$, $\hat{p}_X$, $\hat{X}$, $\hat{p}_Y$, $\hat{Y}$, $\hat{Z}$, $H \times \hat{X}^h$, $H \times \hat{Y}^h$). Following Walras' Law, one of the equilibrium conditions is redundant, thus the effective number of equations is $10+2H$. We choose X as the numéraire good, which implies $\hat{p}_X = 0$. The square system of model equations then endogenously determines all the above unknowns as functions of benchmark parameters (characterizing the equilibrium before the tax change), behavioral parameters (elasticities of production and consumption), and the exogenous positive change in the pollution tax ($\hat{\tau}_Z > 0$).

1.3 Analytical results and interpretations

When solving for the model unknowns as functions of the exogenous tax change, we are ultimately interested in the distributional incidence of the environmental tax. Let v^h denote the indirect utility function of household h, and dv^h the change in utility from consumption caused by an increase in the pollution tax rate by $d\tau_Z$.[10]

To compare the welfare impacts of an increase in the pollution tax across households, we express utility changes in monetary terms relative to income: $\frac{dv^h}{M^h \partial_{M^h} v^h}$ measures the amount of income which would cause a change in utility equal to dv^h at prices prior to the tax change, expressed relative to the income of household h. To isolate the distributional dimension from the economy-wide cost of the tax, we focus on the welfare impact of each household relative to the average welfare change. This ensures that results do not depend on the choice of numéraire. We can then write the welfare impact of household h relative to the average economy-wide monetary loss per unit of income as:[11]

$$\begin{aligned} \Phi^h &\equiv \frac{dv^h}{M^h \partial_{M^h} v^h} - \frac{1}{\sum_{h'} M^{h'}} \sum_{h'} \frac{dv^{h'}}{\partial_{M^{h'}} v^{h'}} \\ &= \underbrace{-(\gamma - \alpha^h)\hat{p}_Y}_{=\text{Uses of income impact}} + \underbrace{(\theta_L^h - \theta_L)\hat{w} + (\theta_K^h - \theta_K)\hat{r} + (\theta_Z^h - \theta_Z)(\hat{\tau}_Z + \hat{Z})}_{=\text{Sources of income impacts}} , \end{aligned} \tag{1.14}$$

[10] Fullerton (2011) provides a taxonomy of six channels of distributional effects of environmental policy. Our analysis is focused on the impacts of environmental taxes caused by higher prices of polluting goods, changes in relative returns to factors like capital and labor and the allocation of pollution tax revenues. It does not consider distributional impacts arising from the benefits from improvements in environmental quality, temporary effects during the transition, and capitalization of all those effects into prices of land, corporate stock, or house values. Also, the uses side in our analysis could be more general if consumption were disaggregated into more than two goods, and the sources side could be extended to represent in more detail the ownership of factors of production (e.g. natural resources, or skilled vs. unskilled labor).

[11] Recall that p_X is the numéraire. Then $dv^h = \partial_{p_Y} v^h dp_Y + \partial_{M^h} v^h dM^h = \partial_{p_Y} v^h p_Y \hat{p}_Y + \partial_M^h v^h (\hat{w} w \bar{L}^h + \hat{r} r \bar{K}^h + \xi^h \tau_Z Z(\hat{\tau}_Z + \hat{Z}))$. Roy's identity (i.e., $\partial_{p_Y} v^h = -Y^h \partial_{M^h} v^h$) then delivers the above equation.

where $\theta_K^h \equiv \frac{r\bar{K}^h}{M^h}$, $\theta_L^h \equiv \frac{w\bar{L}^h}{M^h}$ and $\theta_Z^h \equiv \frac{\xi^h \tau_Z Z}{M^h}$ are the capital and labor income shares of household h, and $\theta_K \equiv \frac{r\bar{K}}{p_X X + p_Y Y}$, $\theta_L \equiv \frac{w\bar{L}}{p_X X + p_Y Y}$, $\theta_Z \equiv \frac{\tau_Z Z}{p_X X + p_Y Y}$ and $\gamma \equiv \frac{p_X X}{p_X X + p_Y Y}$ are the value shares of capital, labor, tax revenues and the clean sector in the economy.

The welfare decomposition underlying Equation (1.14) enables an intuitive economic interpretation of the various channels through which household characteristics determine incidence in our analysis. On the one hand, for given changes in goods and factors prices, variation in impacts across households arises for two reasons. First, households differ in how they spend their income. For a given increase in the price of the dirty good ($\hat{p}_Y > 0$), consumers of the dirty good are more negatively impacted as compared to consumers of the clean good. This impact is referred to as the uses of income impact. Second, in a general equilibrium setting, a pollution tax also impacts factor prices. Households which rely heavily on income from the factor whose price falls relative to the other will be adversely impacted compared to the average household. These impacts, together with the impacts arising from the specific tax redistribution scheme, are referred to as sources of income impacts.

Since output and factor price changes are not independent of households' characteristics, two additional and less direct determinants of incidence emerge from the expression (1.14). First, in an economy with heterogeneous households, output and factor prices are not independent of the distribution of households' consumption profiles and factor endowments across the population; welfare changes for a given household type do not only depend on its own characteristics but also on those of other households in the economy. Second, even in an economy with identical households, the specifics of the household's behavioural response to price and income changes can affect equilibrium outcomes.

Appendix A.2 derives the following general solutions for $\hat{p}_Y$, $\hat{w}$ and $\hat{r}$ following a change in τ_Z:

$$\hat{p}_Y = \frac{(\theta_{YL}\theta_{XK} - \theta_{YK}\theta_{XL})\theta_{YZ}}{D}\left[A(e_{ZZ} - e_{KZ}) - B(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)(\delta - \sum_h \frac{\phi_Z^h}{\theta_{YZ}})\right]\hat{\tau}_Z + \theta_{YZ}\hat{\tau}_Z \tag{1.15a}$$

$$\hat{w} = \frac{\theta_{XK}\theta_{YZ}}{D}\left[A(e_{ZZ} - e_{KZ}) - B(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)(\delta - \sum_h \frac{\phi_Z^h}{\theta_{YZ}})\right]\hat{\tau}_Z \tag{1.15b}$$

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D}\left[A(e_{ZZ} - e_{KZ}) - B(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)(\delta - \sum_h \frac{\phi_Z^h}{\theta_{YZ}})\right]\hat{\tau}_Z\,, \tag{1.15c}$$

where $\gamma_K \equiv \frac{K_Y}{K_X}$, $\gamma_L \equiv \frac{L_Y}{L_X}$, $\beta_L \equiv \theta_{XL}\gamma_L + \theta_{YL}$, $\beta_K \equiv \theta_{XK}\gamma_K + \theta_{YK}$, $A \equiv \gamma_L\beta_K + \gamma_K(\beta_L + \theta_{YZ} - \sum_h \phi_Z^h)$, $B \equiv \gamma_K\beta_L + \gamma_L(\beta_K + \theta_{YZ} - \sum_h \phi_Z^h)$, $C \equiv \beta_K + \beta_L + \theta_{YZ} - \sum_h \phi_Z^h$, $D \equiv C\sigma_X + A[\theta_{XK}\theta_{YL}(e_{KL} - e_{ZL}) - \theta_{XL}\theta_{YK}(e_{KK} - e_{ZK})] - B[\theta_{XK}\theta_{YL}(e_{LL} - e_{ZL}) - \theta_{XL}\theta_{YK}(e_{LK} - e_{ZK})] - (\gamma_K - \gamma_L)(\theta_{XK}(\theta_{YL}\delta - \sum_h \phi_L^h) - \theta_{XL}(\theta_{YK}\delta - \sum_h \phi_K^h))$. The remaining expressions depend explicitly on household characteristics: $\phi_L^h \equiv (1 - \frac{\alpha^h}{\gamma})E_{X,M}^h \frac{w\bar{L}^h}{p_Y Y} + \frac{Y^h}{Y}(E_{Y,M}^h - E_{X,M}^h)\frac{w\bar{L}^h}{M^h}$, $\phi_K^h \equiv (1 - \frac{\alpha^h}{\gamma})E_{X,M}^h \frac{r\bar{K}^h}{p_Y Y} +$

$\frac{Y^h}{Y}(E^h_{Y,M} - E^h_{X,M})\frac{r\bar{K}^h}{M^h}$, $\phi^h_Z \equiv (1 - \frac{\alpha^h}{\gamma})E^h_{X,M}\frac{\xi^h \tau_Z Z}{p_Y Y} + \frac{Y^h}{Y}(E^h_{Y,M} - E^h_{X,M})\frac{\xi^h \tau_Z Z}{M^h}$ and $\delta \equiv \sum_h \frac{Y^h}{Y}\Big(\sigma^h + (\frac{\alpha^h}{\gamma} - 1)(\sigma^h - E^h_{X,M}) + (E^h_{Y,M} - E^h_{X,M})(1 - \alpha^h)\Big)$.[12]

While the interpretation of the general solution is limited by its complexity, it is apparent from the analytical expressions above that going beyond a single consumer and introducing multiple, heterogeneous households with non-homothetic preferences into the model in general has a first-order impact on the market equilibrium, and thus on the incidence results following Equation (1.14).

By considering expressions (1.15a)–(1.15c) one can identify the following two effects, which have also previously been identified in the context of the Harberger (1962) model. The $(\gamma_K - \gamma_L)(\delta - \sum_h \frac{\phi^h_Z}{\theta_{YZ}})$ term in Equations (1.15b) and (1.15c) represents the *output effect*: the tax on sector Y reduces output, and consequently depresses the returns to the factor used intensively in the dirty sector. The sign of the output effect follows this intuition only if the denominator D is positive, which in general is not the case, even for identical households and homothetic preferences (Fullerton and Heutel, 2007). Introducing multiple, heterogeneous households and non-homothetic preferences adds another layer of complexity to this indeterminacy, since $\delta - \sum_h \frac{\phi^h_Z}{\theta_{YZ}}$ cannot in general be signed, whereas this expression is positive for identical households with homothetic preferences.[13] The other terms in Equations (1.15b) and (1.15c) embody the *substitution effects*, which reflect the reaction of firms to factor price changes. Again, while for the case with identical households and homothetic preferences the constants A and B can be signed as positive, this is not the case in our more general model. The substitution effect thus also bears a greater degree of indeterminacy as compared to the Fullerton and Heutel (2007) model.

To better understand the various effects at work, it is necessary to depart from the generality of the above expressions. We therefore consider a series of special cases in which we impose restrictions on household and production characteristics in order to seek definitive results for the changes in prices and returns to factors of production, and therefore better understand the implications for incidence. First, we present a special case for production under which household characteristics have no impact on price changes. Second, we consider cases which allow for full household heterogeneity in terms of preferences and income patterns but where preferences are assumed to be homothetic. Third, the role of non-homothetic preferences is investigated for cases with identical households. These special cases highlight the interaction of production and household characteristics in determining the changes in output and factor prices, and consequently incidence.

[12] Note that in general $\hat{w} = -\frac{\theta_{XK}}{\theta_{XL}}\hat{r}$. Thus, in order to understand the burden of the change in the pollution tax on the returns to factors of production, it is sufficient to study the change in the returns to capital, keeping in mind that–given our choice of the numéraire good–$\hat{w}$ always has the opposite sign as $\hat{r}$.

[13] It should be noted that the term $\delta - \sum_h \frac{\phi^h_Z}{\theta_{YZ}}$ is a non-trivial generalization of the expression $(\sigma_U N + J)$ in Equation (16) in Fullerton and Monti (2013) from the case of two households, homothetic preferences, and identical σ^h among households. This generalization is critical for comparing models with a different degree of household heterogeneity.

1.3.1 Equal factor intensities in production

Consider first the case in which both industries have the same factor intensities, i.e., both are equally capital and labor intensive. Under this assumption, the price changes derived from a model with heterogeneous households are identical to those derived from a single household model.

Proposition 1.1 *Assume both sectors have the same factor intensities, i.e., $\gamma_K = \gamma_L$. Then, $\hat{p}_Y$, $\hat{w}$ and $\hat{r}$ are independent of household characteristics and depend only on production parameters.*

Proof. If $\gamma_K = \gamma_L$, then $A = B = \gamma_K C$. It then follows from Equations (1.15a)–(1.15c) that all terms containing household characteristics in the expressions for $\hat{p}_Y$, $\hat{w}$ and $\hat{r}$ cancel out. □

Proposition 1.1 implies that in the case of equal factor intensities across industries, price changes derived from a single household model with homothetic preferences are sufficient to determine incidence of an environmental tax, even in an economy with different household types. Intuitively, as long as factor intensities are equal, changes in demands for X and Y do not affect *relative* demands for capital and labor, thus implying that relative factor prices are unaffected. Factor price changes in our linearized model are thus determined by the "first-order" response of firms alone, as accounting for "first-order" household behavioral responses in combination with "first-order" firm responses would capture a second-order effect. The sign of factor price changes therefore depends only on production characteristics. Incidence remains in general undetermined, since it depends on how these price changes affect individual households, as determined by their income and expenditure shares.

1.3.2 Heterogeneous households with homothetic preferences

To provide a clear intuition of the effect of household heterogeneity on the general equilibrium (beyond the case with equal factor intensities in production), we restrict our attention in this section to the case with homothetic preferences. We also consider a specific allocation scheme for the pollution tax revenues, with revenues distributed in proportion to income ($\xi^h = M^h/(p_X X + p_Y Y)$). Since in this case the income shares from pollution are identical across all households (i.e., $\theta_Z^h \equiv \theta_Z$, $\forall h$), one can see from Equation (1.14) that incidence is not affected by the tax revenue. This case therefore allows for an analysis of the incidence impacts per se, as given by the changes in consumer prices and returns to factors of production alone.

For homothetic preferences, the heterogeneity of households can be described by the households' population distribution of the three following household characteristics: (i) expenditure shares α^h, (ii) income shares θ_L^h, and (iii) elasticities of substitution in utility σ^h.[14] Accordingly, we can summarize household heterogeneity by the following two quantities. First, we measure the degree in which expenditure and income patterns are correlated. To this end, we define the covariance

[14]Note that, for given ξ^h, a given θ_L^h uniquely determines θ_K^h.

between the expenditure share of the clean good and the labor income share as:

$$cov(\alpha^h, \theta_L^h) \equiv \sum_h (\alpha^h - \gamma) M^h (\theta_L^h - \theta_L) .$$

The covariance is, for example, positive if households who earn an above average share of their income from labor (i.e., $\theta_L^h > \theta_L$) spend an above average share of their income on the clean good (i.e., $\alpha^h > \gamma$).

Second, we quantify the interaction between expenditure shares α^h and substitution elasticities σ^h by defining the effective elasticity of substitution between clean and dirty goods in utility as:

$$\rho \equiv \frac{1}{p_Y Y} \sum_h (1 - \alpha^h) M^h \left(\frac{\alpha^h}{\gamma} (\sigma^h - 1) + 1 \right) .$$

ρ can be interpreted as a generalized weighted average of the σ^h's.[15]

Proposition 1.2 proves that the two quantities $cov(\alpha^h, \theta_L^h)$ and ρ are indeed sufficient to fully characterize the impact of household heterogeneity on equilibrium prices and the level of pollution. For homothetic preferences, the system of Equations (1.15a)–(1.15c) characterizing price changes in the general case simplifies to the following expressions, where the expression for $\hat{w}$ has been omitted due to its simple relationship to $\hat{r}$ (see Appendix A.3.1 for the derivation):

$$\hat{p}_Y = \frac{(\theta_{YL}\theta_{XK} - \theta_{YK}\theta_{XL})\theta_{YZ}}{D_H} \left[A_H(e_{ZZ} - e_{KZ}) - B_H(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)\rho \right] \hat{\tau}_Z + \theta_{YZ}\hat{\tau}_Z \tag{1.16a}$$

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_H} \left[A_H(e_{ZZ} - e_{KZ}) - B_H(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)\rho \right] \hat{\tau}_Z , \tag{1.16b}$$

where $A_H \equiv \gamma_L\beta_K + \gamma_K(\beta_L + \theta_{YZ})$, $B_H \equiv \gamma_K\beta_L + \gamma_L(\beta_K + \theta_{YZ})$, $C_H \equiv \beta_K + \beta_L + \theta_{YZ}$, $D_H \equiv C_H\sigma_X + A_H\left(\theta_{XK}\theta_{YL}(e_{KL} - e_{ZL}) - \theta_{XL}\theta_{YK}(e_{KK} - e_{ZK})\right) - B_H(\theta_{XK}\theta_{YL}(e_{LL} - e_{ZL}) - \theta_{XL}\theta_{YK}(e_{LK} - e_{ZK})) - (\gamma_K - \gamma_L)\rho(\theta_{XK}\theta_{YL} - \theta_{XL}\theta_{YK}) - (\gamma_K - \gamma_L)\frac{cov(\alpha^h, \theta_L^h)}{\gamma p_Y Y}$.

Proposition 1.2 then follows directly:

Proposition 1.2 *If preferences are homothetic, the impact of household heterogeneity on output and factor price changes in equilibrium only depends on two quantities describing individual households' characteristics: (i) the covariance between the expenditure share of the clean good and the labor income share, $cov(\alpha^h, \theta_L^h)$, and (ii) the effective elasticity of substitution between clean and dirty goods in utility, ρ.*

Proof. Equations (1.16a)–(1.16b). □

Using the quantities $cov(\alpha^h, \theta_L^h)$ and ρ, we can now investigate a key question of the paper: under what conditions are price and pollution changes from an economy populated by heterogeneous households with homothetic preferences identical to those derived from an economy with a single

[15] To see this, consider the case with equal expenditure shares across households, i.e. $\alpha^h = \gamma$, $\forall h$. Then, $\rho = \sum_h M^h\sigma^h / \sum_h M^h$.

representative household? The next proposition describes conditions in terms of household preferences and income patterns under which models with and without household heterogeneity yield identical equilibrium outcomes.

Proposition 1.3 *Assume homothetic preferences and (i) identical expenditure shares ($\alpha^h = \gamma, \forall h$) or (ii) identical income shares ($\theta_L^h = \theta_L, \forall h$). Then, output and factor price changes are identical to those for a single household characterized by homothetic preferences, clean good expenditure share γ, and elasticity of substitution between clean and dirty goods in utility equal to the effective elasticity ρ.*

Proof. Either of the above assumptions (i) and (ii) implies $cov(\alpha^h, \theta_L^h) = 0$. From Equations (1.16a)–(1.16b) it is then easy to see that price changes are identical to those derived for an economy with a single consumer with homothetic preferences, clean good expenditure share γ, and elasticity of substitution in utility ρ. □

It follows that in the case with homothetic preferences and either identical expenditure shares or identical income shares (or both), households behave in the aggregate as a single representative household characterized by an elasticity of substitution in utility given by ρ. In the case with identical expenditure shares, the effective elasticity is equal to the weighted average of the individual households' substitution elasticities: $\rho = \frac{1}{\sum_h M^h} \sum_h M^h \sigma^h$. The resulting aggregate behavior is thus completely independent of patterns of income from capital and labor, and does not depend on the number of households. This, however, no longer holds if households have identical income shares but exhibit heterogeneity on the expenditure side. In the latter case, the value of ρ depends on the interaction between expenditure shares α^h and the substitution elasticities of individual households σ^h: if households with an above average expenditure share on the dirty good have higher substitution elasticities, the corresponding single household responds in a more price-elastic manner as compared to a case with the same σ^h's but α^h's that are identical across households.

Proposition 1.3 motivates the definition of ρ as well as its interpretation as the "effective" elasticity of substitution between clean and dirty goods: when $cov(\alpha^h, \theta_L^h) = 0$–that is when either the households are identical on the expenditure or the income side (or both)–then in the aggregate, households *effectively* behave like a single household with substitution elasticity ρ. While Proposition 1.3 describes the conditions for household heterogeneity which allow for consumer aggregation, it is clear that in the context of empirical incidence analysis household characteristics most likely violate these conditions. A central question for incidence analysis therefore is to investigate to what extent household heterogeneity can affect output and factor price changes.

Proposition 1.4 *Assume different factor intensities (i.e., $\gamma_K \neq \gamma_L$) and correlated income and consumption patterns (i.e., $cov(\alpha^h, \theta_L^h) \neq 0$). Assume homothetic, unit-elastic preferences (i.e., $\sigma^h = 1, \forall h$). Then, for any observed consumption and production decisions before the tax change, there exist production elasticities (i.e., σ_X and e_{ij}) such that the relative burden on factors of production is of opposite sign compared to the single-consumer model based on the same production data.*

Proof. See Appendix A.3.2. □

Proposition 1.4 proves that in the presence of heterogeneous households the sources of income impacts from a pollution tax not only differ quantitatively but can yield qualitatively different results when relying on factor price changes derived from a single-household model. Importantly, the possibility of reversed factor price changes does not depend on a particular distribution of households' characteristics as long as the covariance between income and expenditure patterns is non-zero. $cov(\alpha^h, \theta_L^h) \neq 0$ seems to be the empirically relevant case since $cov(\alpha^h, \theta_L^h) = 0$ describes the case in which households are identical or their consumption and income patterns are completely uncorrelated. Proposition 1.4 thus highlights how the incidence of environmental taxes among heterogeneous households may be qualitatively affected by the impact of household heterogeneity on equilibrium outcomes.

To further illustrate the range of (differing) equilibrium outcomes which depend on the nature and degree of household heterogeneity, we provide an example for a special case of our simple economy.

Proposition 1.5 *Assume homothetic, unit-elastic preferences (i.e., $\sigma^h = 1$), Leontief technologies in clean and dirty good production (i.e., $\sigma_X = e_{ij} = 0$), and that the dirty sector is relatively capital-intensive (i.e., $\gamma_K > \gamma_L$). Then, the following holds:*[16]

(i) if consumers are identical on the sources or uses side of income, or both: $\hat{p}_Y = 0$, $\hat{w} > 0$, and $\hat{r} < 0$.

(ii) If labor ownership and clean good consumption have a negative covariance, then $\hat{p}_Y > 0$, $\hat{w} > 0$ and $\hat{r} < 0$.

(iii) If labor ownership and clean good consumption have a positive covariance, then $\hat{p}_Y < 0$, $\hat{w} > 0$, $\hat{r} < 0$ if the covariance is low *(i.e., $D_{H,1} > 0$), and $\hat{p}_Y > 0$, $\hat{w} < 0$, $\hat{r} > 0$ if the covariance is* high *(i.e., $D_{H,1} < 0$).*

Proof. Given the above assumptions, price changes assume the following form:

$$\hat{p}_Y = -\frac{cov(\alpha^h, \theta_L^h)}{D_{H,1}\gamma p_Y Y}\theta_{YZ}\hat{\tau}_Z \tag{1.17a}$$

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_{H,1}}\hat{\tau}_Z\,, \tag{1.17b}$$

where $D_{H,1} \equiv (\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL}) - \frac{cov(\alpha^h, \theta_L^h)}{\gamma p_Y Y}$. □

Proposition 1.5 illustrates that, depending on assumptions about heterogeneity of households' expenditure and income patterns, almost any combination of $\hat{p}_Y \gtrless 0$, $\hat{w} \gtrless 0$, $\hat{r} \gtrless 0$ may arise. This suggests that a pollution tax change can lead to qualitatively different incidence results on the uses and sources side of income. Lastly, note that one can easily show that for a model with a single

[16] Note that for the case where the dirty sector is relatively labor-intensive (i.e., $\gamma_K < \gamma_L$), the sign of all the results in Proposition 1.5 is the opposite.

household and Leontief production, $\hat{p}_Y = 0$. Hence, Proposition 1.5 provides cases in which price changes derived from an economy with heterogeneous households cannot arise in a single-consumer economy with the same production characteristics. This additionally supports our argument that consistently integrating household heterogeneity in general equilibrium analyses is important.

1.3.3 Identical households with non-homothetic preferences

Our results have so far proven that household heterogeneity can have a qualitative impact on the market equilibrium following an increase in a pollution tax, with implications for incidence. We now abstract from household heterogeneity in order to focus on the effect of non-homothetic preferences on the equilibrium.

As the following special case illustrates, accounting for non-homothetic preferences can also qualitatively affect price changes in equilibrium. Assume that all cross-price elasticities have the same positive value c: $\sigma^h = \sigma_X = e_{KL} = e_{KZ} = e_{LZ} \equiv c > 0$. Price changes are then of the following form:

$$\hat{p}_Y = -\frac{\theta_{XK}\theta_{XL}\gamma\theta_{YZ}}{D_{ID}}[(\gamma_K - \gamma_L)^2(E_{Y,M} - E_{X,M})]\hat{\tau}_Z + \theta_{YZ}\hat{\tau}_Z \tag{1.18a}$$

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_{ID}}[(\gamma_K - \gamma_L)(E_{Y,M} - E_{X,M})(1-\gamma)]\hat{\tau}_Z\,, \tag{1.18b}$$

where $E^h_{X,M} \equiv E_{X,M}$ and $E^h_{Y,M} \equiv E_{Y,M}\ \forall h$, $D_{ID} \equiv C_{ID}+A_{ID}\theta_{XL}+B_{ID}\theta_{XK}+(\gamma_K-\gamma_L)^2\theta_{XK}\theta_{XL}\frac{\gamma}{1-\gamma}$, $A_{ID} \equiv \gamma_L\beta_K + \gamma_K(\beta_L + \theta_{YZ} + (E_{X,M} - E_{Y,M})\frac{\tau_Z Z}{p_X X + p_Y Y})$, $B_{ID} \equiv \gamma_K\beta_L + \gamma_L(\beta_K + \theta_{YZ} + (E_{X,M} - E_{Y,M})\frac{\tau_Z Z}{p_X X + p_Y Y})$, $C_{ID} \equiv \beta_K + \beta_L + \theta_{YZ} + (E_{X,M} - E_{Y,M})\frac{\tau_Z Z}{p_X X + p_Y Y}$.

In order to determine the sign of the above price changes, we define the following *Condition 1*: $D_{ID} > 0$. Condition 1 holds if the expenditure share on the clean good increase with income ($E_{X,M} > E_{Y,M}$). It also holds when the clean good expenditure share decreases with income ($E_{Y,M} > E_{X,M}$), but the difference between the income elasticities is not too large. We can then prove that a wide range of possible combinations of output and factor price changes are possible in this special case, depending on the preference parameters.

Proposition 1.6 *Assume identical households and equal cross-price elasticities ($\sigma^h = \sigma_X = e_{KL} = e_{KZ} = e_{LZ} \equiv c > 0$). Then, the following holds:*

(i) If preferences are homothetic, then $\hat{p}_Y = \theta_{YZ}\hat{\tau}_Z$, and $\hat{w} = \hat{r} = 0$.

(ii) Assume that the dirty sector is relatively capital-intensive (i.e. $\gamma_K > \gamma_L$).[17]

(a) If Condition 1 holds, then for $E_{Y,M} > E_{X,M}$: $\hat{p}_Y < \theta_{YZ}\hat{\tau}_Z$, $\hat{w} > 0$ and $\hat{r} < 0$, and for $E_{Y,M} < E_{X,M}$: $\hat{p}_Y > \theta_{YZ}\hat{\tau}_Z$, $\hat{w} < 0$ and $\hat{r} > 0$.

[17] Note that for the case with $\gamma_K < \gamma_L$, the results for $\hat{w}$ and $\hat{r}$ are of opposite signs to the analogous expressions in Proposition 1.6 (ii). The results for $\hat{p}_Y$ remain unchanged, as long as factor intensities differ ($\gamma_K \neq \gamma_L$).

(b) If Condition 1 does not hold, then for $E_{Y,M} > E_{X,M}$: $\hat{p}_Y > \theta_{YZ}\hat{\tau}_Z$, $\hat{w} < 0$ and $\hat{r} > 0$, and for $E_{Y,M} < E_{X,M}$: $\hat{p}_Y < \theta_{YZ}\hat{\tau}_Z$, $\hat{w} > 0$ and $\hat{r} < 0$.

Proof. Equations (1.18a)–(1.18b). For (i): use $E_{Y,M} = E_{X,M}$. □

We have therefore illustrated that there exist cases where the relative burden on factors of production depends on the interaction between production characteristics and the income elasticities of demand for the clean and the dirty goods. It follows that, by extending the Fullerton and Heutel (2007) model to incorporate household heterogeneity and non-homothetic preferences, we have added two dimensions that can both qualitatively alter the economy's reaction to an exogenous increase in the pollution tax. Both features are therefore in general significant for incidence.

1.4 Numerical analysis

In this section, we apply the heterogeneous household model to quantitatively assess how the aggregation bias affects equilibrium outcomes and the incidence of a tax on carbon dioxide (CO_2) emissions for the case of the United States. We assess the incidence on the sources and uses side of income, and explore how sensitive results are with respect to key characteristics governing households' and firms' behavior.

1.4.1 Data and calibration

In order to situate our study in the context of the literature, we calibrate our model to data used previously for a two-sector general equilibrium environmental tax incidence analysis. For this purpose, we chose the production and consumption data of Fullerton and Heutel (2010). They aggregate a data set of the U.S. economy to a '"dirty" and a "clean" sector, where the dirty sector comprises the highly CO_2-intensive industries (electricity generation, transportation and petroleum refining). As in Fullerton and Heutel (2010) we assume an initial and pre-existing carbon tax of \$15 per metric ton of CO_2. Our comparative-static analysis considers a 100% increase in the carbon tax.

All prices in the benchmark are normalised to one, and quantities are normalised such that the total value of the economy is equal to one, i.e., $p_X X + p_Y Y = 1$. Calibrated values for outputs and inputs are then as follows: $X = 0.929$, $L_X = 0.579$, $L_Y = 0.029$, $K_X = 0.350$, $K_Y = 0.037$, and $Z = 0.005$. Households are grouped by annual expenditure deciles,[18] and data for expenditures by clean and dirty goods as well as capital and labor income are shown in Table 1.1. Note that our analysis abstracts from government transfers.

Incorporating heterogeneous households in a calibrated general equilibrium model of the U.S. economy requires that—at the aggregate level—data describing household consumption and income

[18] It is well-known in the literature on tax incidence that absent a fully dynamic framework, categorizing households by expenditure deciles is a better proxy for lifetime income as compared to a ranking based on annual income deciles (see, for example, Poterba, 1991; Fullerton and Heutel, 2010).

Table 1.1: Household expenditures on clean and dirty goods and household income by source for annual expenditure deciles (in % of total expenditure for a given household group)

Expenditure	Income sources		Expenditures by commodity	
decile h	Labor	Capital	Clean	Dirty
1	42.8	13.5	85.5	14.5
2	74.5	13.8	84.8	15.2
3	86.3	16.2	85.4	14.6
4	103.5	18.0	86.1	13.9
5	108.8	20.4	86.8	13.2
6	114.4	29.4	87.7	12.3
7	118.8	31.2	88.5	11.5
8	120.0	38.4	89.2	10.8
9	124.6	45.1	90.7	9.3
10	93.4	54.7	94.1	5.9

Notes: Household data is based on the "Consumer Expenditure Survey" (CEX) data as shown in Fullerton and Heutel (2010).

are consistent with the production data on output by sector and aggregate, economy-wide factor income. To reconcile data sources, we adjust the household data to be consistent with aggregate production data while preserving the relative characteristics of household expenditures across expenditure deciles. More specifically, data adjustments for each expenditure decile are as follows. First, we scale income to mach expenditure while keeping fixed the decile's capital-to-labor ratio. Second, we scale the capital ownership of all deciles by a common factor in order for aggregate household income by factor to match production side data, whilst preserving the relative capital ownership amongst deciles. Third, we perform an analogous scaling for consumption of the dirty good. This procedure yields consistent household and production data which is used to calibrate the general equilibrium model.

For our central case parametrization of production elasticities we follow Fullerton and Heutel (2010) assuming $\sigma_X = 1$, $e_{KL} = 0.1$, $e_{KZ} = 0.2$, and $e_{LZ} = -0.1$. This implies that capital is a better substitute for pollution than labor. For the single household model, Fullerton and Heutel (2010) assume that the elasticity of substitution between the clean and the dirty good in utility is unity, and that preferences are homothetic. Our central case is based on analogous assumptions for each household group, i.e., $\sigma^h = 1$ and $E^h_{X,M} = E^h_{Y,M} = 1$, $\forall h$. Note that while these parameter choices reflect central case assumptions, we perform extensive sensitivity analysis to check for the size of the aggregation bias and the incidence patterns from increases in the pollution tax.

1.4.2 Size of the aggregation bias and implications for incidence analysis

From the theoretical analysis we know that heterogeneous households and non-homothetic preferences can have a significant effect on price changes following an increase in the pollution tax. We now measure the aggregation bias introduced by modeling an economy comprising heterogeneous households as an economy with a single representative household. We first compute the price changes following a change in the pollution tax from the heterogeneous household model with expenditure and income patterns calibrated based on the data shown in Table 1.1. These price changes are then compared with price changes derived from a model calibrated to the same aggregate data but with a single representative household.[19]

Biased price changes translate into biased welfare results. To quantify this bias, we define the "Welfare Aggregation Bias", Γ, as:

$$\Gamma = \Omega^{-1} \sum_h \frac{M^h}{\sum_{h'} M^{h'}} \left| \Phi^h - \Phi^h_{Aggregate} \right| , \tag{1.19}$$

where h and h' are indexes for expenditure deciles and Φ^h is the household-level welfare impact as given by Equation (1.14). $\Phi^h_{Aggregate}$ is also derived from Equation (1.14) but uses instead price changes which are derived from the model with a single household representing aggregate demand.[20] Dividing by $\Omega \equiv \sum_h \frac{M^h}{\sum_{h'} M^{h'}} \left|\Phi^h\right|$ expresses the aggregation bias as a share relative to the average welfare impact across households.

Γ yields a measure of the average difference in welfare impacts derived under the consistent approach and the generally biased representative household approach. Γ is greater or equal to zero as it is defined as the weighted average of the absolute value of the difference between Φ^h and $\Phi^h_{Aggregate}$. If $\Gamma = 0$, the welfare results derived under the two approaches are identical. If $\Gamma > 0$, then there is a bias on the household-level welfare impacts when employing the representative household approach, and therefore the pattern of incidence will in general be biased.

Given the considerable uncertainty surrounding both the household survey data as well as household and production side parameters, we investigate a range of alternative cases around our central case assumptions which are based on observed data for the U.S. economy and parameter assumptions from the literature (see Section 1.4.1). First, "cov_{Low}" and "cov_{High}" represent cases where the covariance measure is respectively halved and doubled relative to the central case "cov_{Base}", representing cases where there is respectively less and more heterogeneity in expenditure and income shares among households. Second, we consider different assumptions with respect to higher-order properties of households' utility functions by introducing heterogeneity in the price and income elasticities of demand across households. A case labeled "ρ_{Low}" and "ρ_{High}" assumes that poorer households in lower expenditure deciles are described by a smaller and larger elasticity of substitution between clean and dirty goods relative to the richer households, respectively. We interact different cases regarding household characteristics with alternative assumptions about the production side, i.e., cases which differ with respect to the substitutability between capital and labor in the clean sector (σ_X) and between capital, labor, and pollution in the dirty sector ($e_{K/LZ}$). Table 1.2 reports the aggregation bias in terms of both price changes and welfare for these cases. The following key

[19] To focus on the incidence effects due to goods and factor price changes only, we here assume that the pollution tax revenue is redistributed in proportion to income. We consider alternative revenue recycling schemes in Section 1.5.

[20] This aggregate household is assumed to be characterized by an elasticity of substitution in utility between clean and dirty consumption and income elasticities that are given by the expenditure-weighted average of the elasticities of individual deciles, i.e., $\sigma_{Aggregate} = \frac{1}{\sum_{h'} M^{h'}} \sum_h M^h \sigma^h$ and $E^{Aggregate}_{X/Y,M} = \frac{1}{\sum_{h'} M^{h'}} \sum_h M^h E^h_{X/Y,M}$.

insights emerge.

First, comparing price changes from the aggregate household and heterogeneous household models, the aggregation bias on the returns to capital is larger than on the price of the dirty good; the aggregation bias for $\hat{r}$, i.e., the percentage difference between price changes, can be up to 38% (for "cov_{High}", "rho_{High}", and $\sigma_X = 1.5$) whereas for $\hat{p}_Y$ it is negligible for all cases. The reason is that $\hat{p}_Y$ is dominated by the "direct" cost pass-through effect which is represented by the term $\theta_{YZ}\hat{\tau}_Z$ in Equation (1.15a) (see also Fullerton and Heutel, 2010). The output and substitution effects arising in general equilibrium are only a fraction of the total change in $\hat{p}_Y$ but fully determine $\hat{r}$ and $\hat{w}$ (see first line of Equation (1.15a) and Equations (1.15b) and (1.15c)). As the cost pass-through is independent of household characteristics, the aggregation bias manifests itself only through the general equilibrium effects which explains why the relative impact of the aggregation bias for $\hat{p}_Y$ is smaller than for the factor price changes.

Second, the aggregation bias on prices for ρ_{Base} (which corresponds to $\sigma^h = 1$, $\forall h$) is small compared to the other cases. This translates into a smaller welfare aggregation bias Γ. When substitution elasticities are identical across households, for a given increase in the price of the dirty good, households all substitute the same percentage of dirty good consumption with clean consumption. Abstracting from changes in income, it then follows that the aggregate change in consumption is the same as for a representative household with the same substitution elasticity. The numerical results show that in this case other effects that may depend on household heterogeneity are not of particular significance.

Third, we find that, for a given covariance between income and expenditure patterns, the returns to capital are decreasing in the effective elasticity ρ. Intuitively, the reaction of aggregate demand to an increase of the price of the dirty good is disproportionately affected by the households that consume the dirty good more intensively. For ρ_{High}, these households' demand is more price elastic than the average demand, hence aggregate demand will react more elastically to an increase in the price of the dirty good as compared to the single consumer. This in turn depresses demand for the dirty good more, leading to a decrease in both the price of the dirty good and the returns to the factor which is used intensively in the dirty industry, i.e. capital. An analogous explanation holds true for the ρ_{Low} case.

Fourth, the changes in the return to capital are increasing in the absolute value of the covariance for ρ_{Low}, and decreasing in the absolute value of the covariance for ρ_{High}. A higher covariance means that households consuming an above-average share of the dirty good consume even more. This in turn magnifies the above-mentioned impact of the effective elasticity ρ on the determination of equilibrium price changes. Finally, we find that the aggregation bias is not much affected by introducing heterogeneity in the income elasticities of consumption (which we therefore do not show in Table 1.2). This points to the fact that heterogeneity in price effects dominates heterogeneity in income effects in determining aggregate consumption behavior.

In summary, we find that the effect of the aggregation bias for the empirically motivated cases shown in Table 1.2 is non-negligible, especially for changes in returns to factors of production. Household heterogeneity in the elasticities of substitution in utility magnifies the aggregation bias

Table 1.2: Price changes and welfare aggregation bias for alternative assumptions about household heterogeneity and production characteristics

	Aggregate household model	Heterogeneous household model									
		cov_{Base}		cov_{Low}				cov_{High}			
		ρ_{Base}		ρ_{Low}		ρ_{High}		ρ_{Low}		ρ_{High}	
	$\hat{r}$	$\hat{r}$	Γ	$\hat{r}$	Γ	$\hat{r}$	Γ	$\hat{r}$	Γ	$\hat{r}$	Γ
Substitutability between capital and labor in the clean sector											
$\sigma_X = 1.5$	-0.08	-0.08	0.0	-0.07	1.4	-0.09	1.4	-0.05	3.2	-0.11	3.4
$\sigma_X = 1$	-0.12	-0.12	0.0	-0.10	2.2	-0.13	2.3	-0.08	5.0	-0.16	5.3
$\sigma_X = 0.5$	-0.23	-0.23	0.2	-0.21	5.1	-0.26	5.5	-0.15	10.6	-0.31	12.4
Substitutability between capital, labor, and pollution in the dirty sector											
$e_{K/LZ} = \pm 0.5$	0.11	0.11	0.0	0.13	1.6	0.10	1.7	0.15	3.9	0.07	4.3
$e_{K/LZ} = \mp 0.5$	-0.58	-0.58	0.6	-0.57	5.4	-0.59	5.1	-0.54	9.7	-0.62	10.2

Notes: $\hat{r}$ is expressed as the percentage change relative to the price level before the pollution tax increase. Price changes for the dirty good are virtually identical across the cases shown here and are hence not shown. Γ is expressed as a percentage share.

due to heterogeneity in expenditure and income patterns. In our static model, heterogeneity in income elasticities has a smaller effect compared to heterogeneity in substitution elasticities.

Lastly, Table 1.3 presents selected cases for which the aggregation bias is sufficiently large to cause incidence patterns to be qualitatively different, changing the incidence shape from "U" to inverted "U" and reversing the sign of the welfare impact for some households. The wide variation in welfare impacts across deciles in these cases emphasizes the fact that within the range of possible values of household and production parameters there exist equilibria in which the economy is particularly sensitive to an increase in the pollution tax. Although these cases are relatively "distant" to our central case assumptions, they illustrate the pitfalls in assessing distributional impacts of an environmental tax in a model with a single, representative consumer.

1.4.3 Applying the heterogeneous household model: distributional impacts of a U.S. carbon tax

We now use our calibrated model to assess the incidence of a U.S. carbon tax. Importantly, we maintain our assumption that the carbon tax revenue is recycled in proportion to income thereby abstracting from differential impacts among households due to revenue recycling. This allows us to focus on the relative importance of channels for incidence which are affected by the household aggregation bias, i.e. consumer and factor price changes.[21]

[21] Our analysis should thus not be interpreted as a comprehensive assessment of a specific U.S. carbon tax policy proposal with specific provisions for revenue recycling. Of course, as documented by the large literature on the distributional impacts of carbon taxation, the way the revenues are recycled can importantly alter the incidence pattern across households (see, for example, Bento et al., 2009; Rausch et al., 2010b; Mathur and Morris, 2014;

Table 1.3: Selected cases for which welfare aggregation bias is "large", i.e. incidence results across household groups differ qualitatively due to the aggregation bias

Expenditure	Case1		Case 2		Case 3	
decile	Φ^h	$\Phi^h_{Aggregate}$	Φ^h	$\Phi^h_{Aggregate}$	Φ^h	$\Phi^h_{Aggregate}$
1	-0.15	-0.21	0.16	0.48	0.56	-0.67
2	0.21	-0.36	3.06	5.95	5.35	-6.03
3	0.23	-0.32	3.01	5.83	5.23	-5.89
4	0.31	-0.30	3.37	6.48	5.77	-6.49
5	0.29	-0.25	3.05	5.84	5.19	-5.83
6	0.12	-0.15	1.45	2.79	2.50	-2.81
7	0.14	-0.10	1.34	2.56	2.26	-2.54
8	0.01	-0.02	0.16	0.30	0.27	-0.31
9	-0.03	0.09	-0.63	-1.23	-1.11	1.26
10	-0.36	0.40	-4.16	-8.02	-7.15	8.04

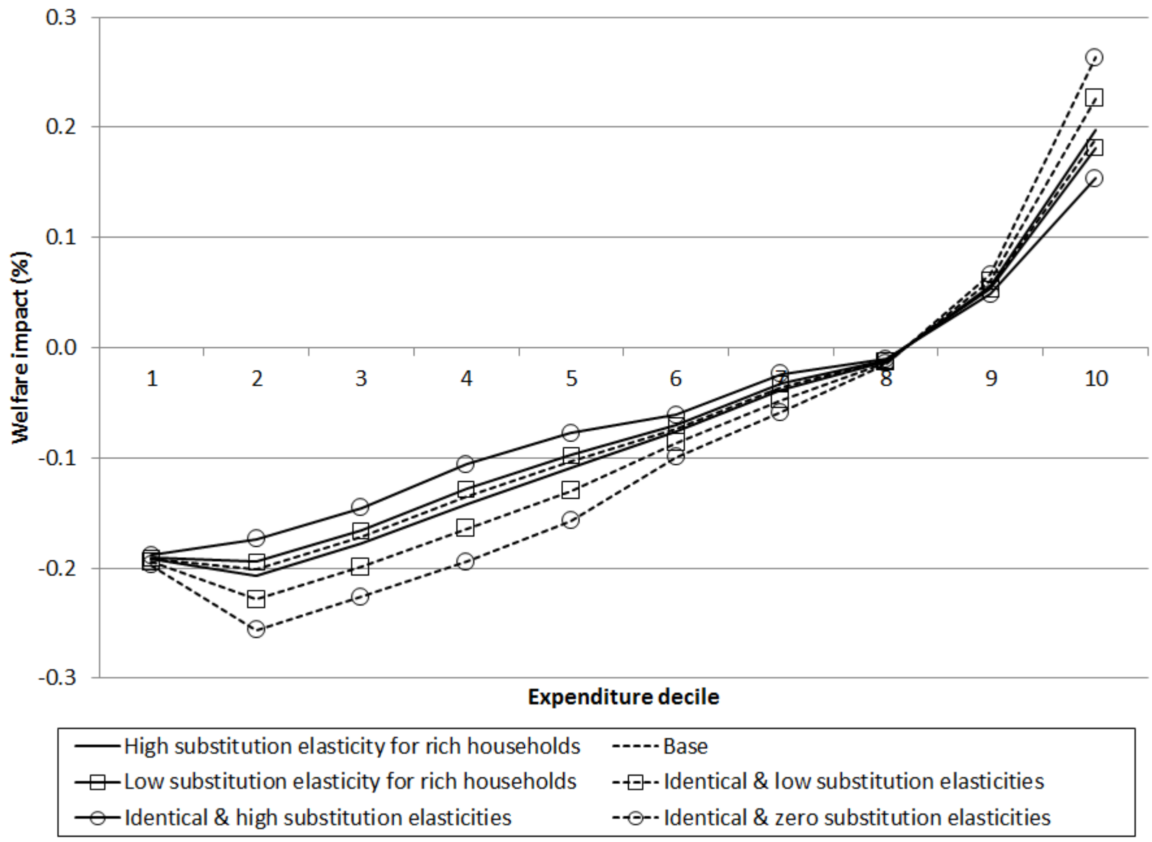
Notes: Cases are defined as follows. Case 1: $\sigma_X = 0$, $\sigma^h = 2$, for $h = 1, \ldots, 5$, $\sigma^h = 0$, for $h = 6, \ldots, 10$, $e_{KL} = 0.1$, $e_{KZ} = 0.5$, and $e_{LZ} = 0.4$. Case 2: Leontief production, σ^h as for ρ_{low}, $E^h_Y = 2$, for $h = 1, \ldots, 7$, and $E^h_Y = 0$, for $h = 8, \ldots, 10$. Case 3 corresponds to the case in Proposition 1.4: $\sigma_X = 0$, $\sigma^h = 1$, $e_{KL} = -0.145$, $e_{KZ} = e_{LZ} = 0$, and $E^h_Y = 1$.

We explore the robustness of the incidence result through "piecemeal" sensitivity analysis by varying household and production elasticities. For each case, we identify the relative importance of uses and sources effects of income. Figure 1.1a displays welfare impacts for a range of cases which vary household characteristics around the base case. We assume different values for σ^h, the elasticity of substitution in utility between clean and dirty goods. For "low" and "high" substitution cases for rich households, we set σ^h for different household groups as in ρ_{High} and ρ_{Low}, respectively. For cases with identical "zero", "low", and "high" substitution elasticities the following values are assumed, respectively: $\sigma^h = 0$, $\sigma^h = 0.5$, and $\sigma^h = 1.5$, $\forall h$. In all cases, household expenditure and income shares are left unchanged.

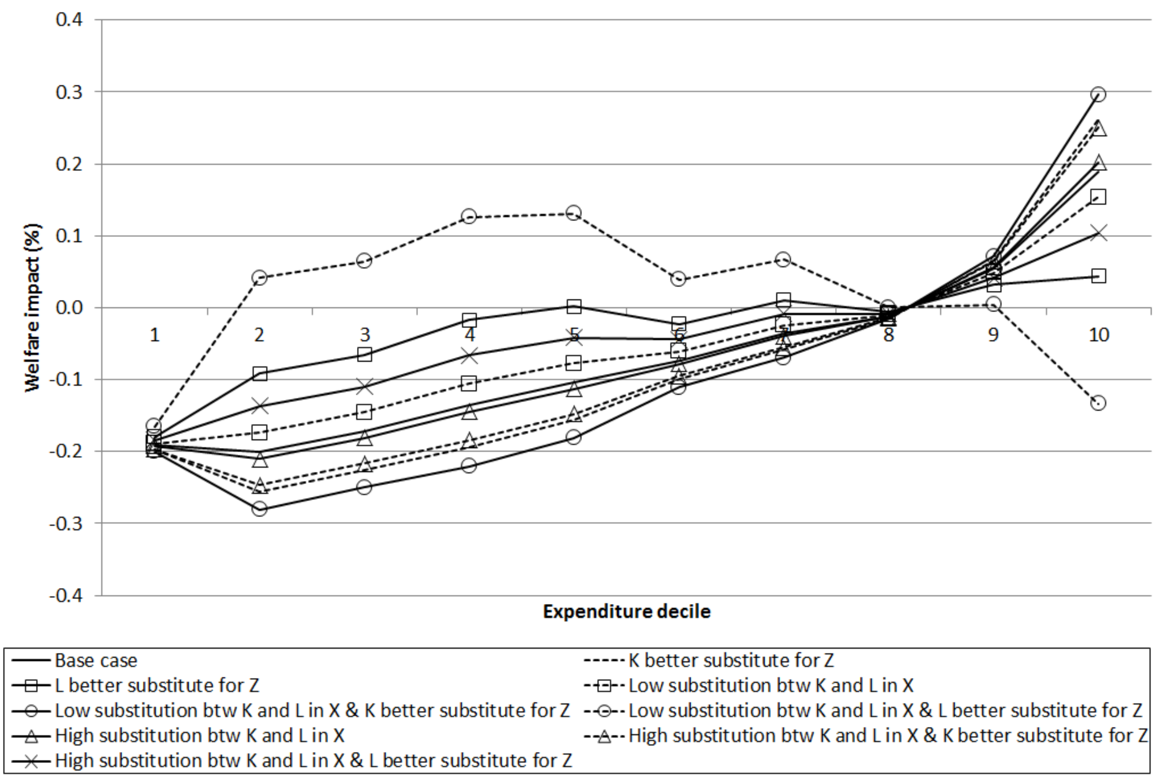
From Figure 1.1a it is evident that a carbon tax is regressive in the base case, and that this result is robust to varying household characteristics. Even if households are more able to substitute away from the taxed dirty good, as reflected by high σ^h's, the carbon tax puts disproportionately large burdens on households in lower expenditure deciles. The incidence is slightly more regressive for low values of σ^h as compared to cases with high values for σ^h. This is driven by the fact that for relatively low σ^h's, the burden from higher prices for the dirty good is borne to a larger extent by consumers, hence falling more heavily on those household groups that spend a relatively large fraction of their income on the dirty good. At the same time, as consumers are less able to substitute away from the dirty good, the reduction in the dirty sector output, Y, is relatively smaller, hence the return to capital, the factor used intensively in the production of Y, decreases by less. This explains why the welfare losses on the sources side of richer households with relatively

Williams III et al., 2015). To illustrate this point in the context of our model, Appendix A.4 contains supplementary analysis which considers two additional revenue recycling schemes. A first case assumes that the revenue is distributed in proportion to the consumption of the dirty good reflecting concerns about offsetting adverse impacts for poorer households. The resulting incidence pattern looks more neutral when compared to Figure 1.1. A second case considers distributing the carbon revenue equally among households on a per capita basis, resulting in a sharply progressive outcome.

Figure 1.1: Welfare impacts (Φ^h) of increased pollution tax across annual expenditure deciles



(a) Alternative assumptions about household characteristics



(b) Alternative assumptions about production characteristics

high capital income shares (i.e., deciles 9 and 10) get smaller as σ^h decreases. For $\sigma^h = 0$ rich households experience gains, relative to the average household, on both the uses and sources side.

Figure 1.1b displays welfare impacts for a range of cases which vary production characteristics around our base case assumptions. Cases shown vary either the elasticity of substitution between capital and labor in clean production, σ_X (halving and doubling the value from the base case), the substitutability between capital and labor vis-à-vis pollution, or a combination of the two. The case "K better substitute for Z" assumes $e_{KZ} = 0.5$ $e_{LZ} = -0.5$, and the case "L better substitute for Z" assumes $e_{KZ} = -0.5$ $e_{LZ} = 0.5$.

The following insights emerge from Figure 1.1b. First, while for the majority of cases the carbon tax is found to be regressive, there is considerable variation in welfare impacts depending on production parameters. Second, the pattern of distributional impacts depends largely on the substitutability of inputs in the production of the dirty good. If capital is a better substitute for pollution than labor, then the carbon tax is regressive, due to the regressivity of both the uses and the sources of income incidence. On the sources of income side, as the burden on factor prices falls on labor rather than capital, poorer households with high labor income shares experience large welfare losses, while richer households with high capital income shares experience larger relative gains. In contrast, the carbon tax is less regressive and can even in some cases be inversely U-shaped if labor is a relatively good substitute for pollution vis-à-vis capital, due to the progressivity of the sources of income incidence when the burden falls on capital rather than on labor. Third, higher values of σ_X imply flatter incidence curves, since this dampens the burden on the returns to the factors of production.

For the cases shown in Figure 1.1, Tables 1.4 and 1.5 decompose welfare impacts into uses and sources side impacts. For the range of household and production characteristics that we consider, we find that uses side effects are markedly regressive and that there is relatively little variation in the size of uses side impacts for a given household group. The sources side impacts on the other hand tend to be mostly neutral or progressive, driven by the fact that burdens mostly fall on capital, and are much more sensitive to behavioural parameters as compared to the uses side impacts.[22]

To summarize, while we find evidence that a carbon tax itself–i.e., ignoring differential impacts among households from revenue recycling–can be regressive, sensitivity analysis on production and household characteristics illustrates that other incidence outcomes (inverted U shape and progressive across the top five expenditure deciles) may be possible. As the aggregation bias on welfare is largely caused by the aggregation bias on the returns to factors of production, it mainly affects the sources of income. We also find that most of the variation in welfare impacts is driven by sources side impacts. Our analysis thus points to the importance of including sources of income impacts for tax incidence analysis.

[22] Note that the small variation in impacts for the first and eighth expenditure deciles reflects that these households have a capital-labor ratio which is similar to the sample's average. Hence, the sources side impacts relative to the average are small for these two deciles.

Table 1.4: Household welfare impacts (Φ^h) by expenditure decile (in %) by uses and sources side of income for alternative household characteristics

Expenditure	Uses side	Sources side					
Decile	*All cases*[a]	Central case ($\sigma^h = 1$)	ρ_{low}	ρ_{high}	$\sigma^h = 1.5$	$\sigma^h = .5$	$\sigma^h = 0$
1	-0.19	0.00	0.00	0.00	0.00	0.00	0.00
2	-0.23	0.03	0.02	0.03	0.05	0.00	-0.03
3	-0.20	0.03	0.02	0.03	0.05	0.00	-0.03
4	-0.16	0.03	0.02	0.04	0.06	0.00	-0.03
5	-0.13	0.03	0.02	0.03	0.05	0.00	-0.03
6	-0.09	0.01	0.01	0.02	0.02	0.00	-0.01
7	-0.05	0.01	0.01	0.01	0.02	0.00	-0.01
8	-0.01	0.00	0.00	0.00	0.00	0.00	0.00
9	0.06	-0.01	0.00	-0.01	-0.01	0.00	0.01
10	0.23	-0.04	-0.03	-0.04	-0.07	0.00	0.04

Notes: Cases shown in columns are identical to cases in Figure 1.1a. [a]Uses side impacts are virtually identical for all the cases, hence only one column is shown.

Table 1.5: Household welfare impacts (Φ^h) by expenditure decile (in %) by uses and sources side of income for alternative production characteristics

Expenditure	Uses side	Sources side								
Decile	*All cases*[a]	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	-0.19	0.00	0.00	0.01	0.00	0.00	0.02	0.00	0.00	0.01
2	-0.23	0.03	-0.03	0.13	0.05	-0.05	0.26	0.02	-0.02	0.09
3	-0.20	0.03	-0.02	0.13	0.05	-0.05	0.26	0.02	-0.02	0.09
4	-0.16	0.03	-0.03	0.14	0.06	-0.05	0.28	0.02	-0.02	0.10
5	-0.13	0.03	-0.03	0.13	0.05	-0.05	0.26	0.02	-0.02	0.09
6	-0.09	0.01	-0.01	0.06	0.02	-0.02	0.12	0.01	-0.01	0.04
7	-0.05	0.01	-0.01	0.06	0.02	-0.02	0.11	0.01	-0.01	0.04
8	-0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00
9	0.06	-0.01	0.01	-0.03	-0.01	0.01	-0.05	0.00	0.00	-0.02
10	0.23	-0.04	0.03	-0.18	-0.07	0.07	-0.35	-0.02	0.02	-0.12

Notes: Cases shown in columns are identical to cases in Figure 1.1b. [a]Uses side impacts are virtually identical for all the cases, hence only one column is shown. Columns are defined as follows: (1)=central case, (2)=K better substitute for Z ($e_{KZ} = 0.5$ and $e_{LZ} = -0.5$), (3)=L better substitute for Z ($e_{KZ} = -0.5$ and $e_{LZ} = 0.5$), (4)=Low substitution between K and L in sector X ($\sigma_X = 0.5$), (5)=Low substitution between K and L in sector X and K better substitute for Z, (6)=Low substitution between K and L in sector X and L better substitute for Z (7)=High substitution between K and L in sector X ($\sigma_X = 1.5$), (8)=X more price elastic and K better substitute for Z, (9)=High substitution between K and L in sector X and L better substitute for Z.

1.5 Extensions

In this section, we extend our analysis in a number of directions going beyond the stylized setup of our core model to check for the robustness of our results. As one would expect, any extension of the model produces different quantitative results. The point of the paper, however, that household heterogeneity affects equilibrium and hence the incidence of environmental taxes remains. In fact, we find that the case for the aggregation bias is strengthened rather than weakened since extending the analysis creates additional dimensions along which households may differ. Alongside the effects previously identified for our core model these extensions introduce new channels through which household heterogeneity affects the general equilibrium. In turn, we find that in general these channels affect the results. We briefly summarize the main findings for each extension here, while the detailed analysis is documented in Appendix A.5.

1.5.1 Alternative revenue recycling schemes

Our analysis so far has assumed that the environmental tax revenue is distributed in a way that abstracts from differential impacts among households, i.e. in proportion to income. Redistributing the tax revenue in a non-neutral manner introduces an additional channel of heterogeneity on the sources of income side. This could potentially affect how household heterogeneity impacts equilibrium outcomes. We consider two alternative ways of recycling the carbon tax revenue: a first case assumes distribution in proportion to dirty good consumption and a second case assumes that the revenue is distributed on an equal per capita basis. We find that price changes for both $\hat{r}$ and $\hat{p}_Y$ are very similar among alternative revenue recycling cases indicating that the impact of household heterogeneity on the equilibrium outcome is largely independent of the way the environmental tax revenue is redistributed.

1.5.2 Pre-existing, non-environmental taxes

Accounting for pre-existing taxes on capital and labor in the benchmark, analogous to Fullerton and Heutel (2007), modifies the production cost shares now including tax payments ($\theta_{YK} \equiv \frac{r(1+\tau_K)K_Y}{p_Y Y}$, and similarly for θ_{YL}, θ_{XK} and θ_{XL}) as well as the households' income constraints now including tax revenues as new sources of income. As long as the revenue from capital and labor taxes is also distributed in proportion to income, there is no additional effect of household heterogeneity on price changes as heterogeneity in terms of both uses and sources side is unchanged. In this case, all Propositions 1.1–1.6 remain valid. Distributing capital and labor tax revenue in a non-neutral way will introduce additional heterogeneity on the sources side. In this case, Propositions 1.1 and 1.6 still hold true and price changes for $\hat{r}$ and $\hat{p}_Y$ are quantitatively similar (analogously to our findings in Section 1.5.1).

1.5.3 Non-separable utility in pollution

With non-separable utility, consumption of clean and dirty goods in general depends on the level of pollution: $X^h = X^h(p_X, p_Y, M^h, Z)$ and $Y^h = Y^h(p_X, p_Y, M^h, Z)$. The change in the pollution level following a pollution tax increase can thus affect the equilibrium behavior of households. Aggregate economy outcomes therefore now depend on the household-level responses to changes in pollution as well as the interaction with other household characteristics. This introduces an additional dimension of heterogeneity to the extent that households have different preferences about pollution. This effect can be captured by introducing a new quantity that describes the interaction between expenditure patterns and pollution elasticities (similar to the effective elasticity of substitution between clean and dirty goods in utility ρ). All Propositions 1.1–1.6 can then be straightforwardly extended to account for the new pollution channel whilst maintaining the effects previously shown. In general, the overall effect of the impact of household heterogeneity on equilibrium outcomes may lead to a smaller or larger aggregation bias compared to the case with separable utility in pollution.

1.5.4 Labor-leisure choice

An important dimension along which households can differ is their valuation of leisure time resulting in differences with respect to the elasticity of labor supply. Incorporating endogenous labor supply significantly enhances the complexity of studying the impact of household heterogeneity of equilibrium outcomes as it affects both how income is earned and spent. To keep the theoretical analysis tractable, we restrict our attention to Cobb-Douglas utility and assume that in the benchmark households dedicate an equal fraction of their productive time to leisure. We find that results are mainly similar with new parameters summarizing the additional channels of household heterogeneity as well as the aggregate impact of labor-leisure choice on the general equilibrium. Proposition 1.1 is identical. Proposition 1.2 is analogous accounting in addition for interactions between leisure choice and expenditure and income patterns. Proposition 1.3 is analogous with the presence of a term that reflects the impact of average expenditure share of leisure on aggregate outcomes. Propositions 1.4 and 1.5 are analogous, too. For the special case of Cobb-Douglas utility, we thus find that the effect of household heterogeneity is similar to the case without labor-leisure choice; where it differs it can be understood in terms of additional terms reflecting interactions between the various types of heterogeneity (i.e., labor-leisure choice, expenditures and income patterns). Whether or not the aggregation bias is quantitatively smaller or larger would depend on the specific parametrization.

1.5.5 More than two sectors

Closely based on Fullerton and Heutel (2007), our analysis assumed a highly aggregated sectoral representation which is also in line with much of the literature following Harberger (1962). Including more sectors can obviously affect the aggregation bias as it enables representing household heterogeneity along more dimensions. With a finer sectoral resolution, it is, for example, conceivable that poorer households may have higher expenditure shares on some dirty goods and lower expenditure shares on some others when compared to richer households. The problem is further compounded by the possibility that different polluting goods may be produced with different capital and labor intensities, interacting with the sources of income incidence. As the aggregation bias is determined by the interaction between household and production side characteristics, the impact of going from two to multiple sectors on the aggregation bias is thus in general not clear-cut. For a special case, one can nevertheless show that the aggregation bias remains important for assessing the incidence of environmental taxes in a setting which includes an arbitrary number of sectors. Analogous to Proposition 1.5 with Leontief technologies, we find that the value of the covariance between the ownership of labor and consumption of each dirty good across households can reverse the sign of the factor price changes.

1.6 Conclusion

This paper has theoretically and quantitatively examined how the incidence of an environmental tax depends on how different incomes and preferences of heterogeneous households affect aggregate equilibrium outcomes. To this end, we have developed a simple theoretical Harberger-type model that allows for heterogeneous households, general forms of preferences, differential spending and income patterns, differential factor intensities in production, and general forms of substitution among inputs of capital, labor and pollution.

We have shown that ignoring the household aggregation problem can have important implications for analyzing the incidence of environmental taxes. Our theoretical analysis provides an intuitive way to characterize the degree of household heterogeneity and the impact of heterogeneity on equilibrium outcomes. We have provided conditions under which the household aggregation bias is large and incidence results vary substantially and can be reversed depending on the distribution of households' expenditure and income shares. We have also characterized conditions for which the household aggregation problem is muted. We have calibrated our model based on empirical parameter values to quantitatively assess the household aggregation problem for the example of a U.S. carbon tax. We find that the magnitude of the aggregation bias is non-negligible and that incidence patterns for household income groups may even be affected qualitatively, changing the incidence from "U" to an inverted "U" shape and reversing the sign of the welfare impact for some households. We find that most of the variation in welfare impacts is driven by sources side impacts. As the aggregation bias on welfare is largely caused by the aggregation bias on the returns to factors of production, it mainly affects the sources of income. Our analysis thus points to the importance of including sources of income impacts for tax incidence analysis. Finally, our findings are robust to extending our model in a number of directions, including alternative revenue recycling schemes, pre-existing taxes, non-separable utility in pollution, labor-leisure choice, and multiple commodities. In fact, we find that the case for the aggregation bias is strengthened rather than weakened.

Beyond the model extensions considered here, and based on the rich literature that followed the original Harberger (1962) article, the analysis could be extended in many additional ways allowing, for example, for imperfect factor mobility, increasing returns to scale, capital accumulation and economic growth, international trade in goods and factors, other factors of production, intermediate inputs, and government transfers. Any such addition to this model would indeed affect the quantitative results, but they are studied elsewhere, and they would not affect the point of this paper that household heterogeneity affects the general equilibrium incidence of environmental taxation.

2 Linearized vs. exact incidence analysis: the case of environmental policy

Abstract

Harberger-type tax incidence analysis relies on the linearized properties of an economy to assess the effects of a discrete tax change. I study the implications of the linearity assumption for the incidence of price- and quantity-based environmental policies. I prove analytically that changes in output prices and the pollution level following an environmental tax increase are overestimated in the linearized case, whereby the bias on pollution changes is larger than the bias on price changes. Subsequent numerical analysis provides quantitative results that are in line with these theoretical findings. The linearized approach overstates the regressivity of a given environmental tax increase, and underestimates the regressivity of a policy targeting a given pollution reduction.

2.1 Introduction

The Harberger (1962) model has been widely employed to study the effect of taxes on the economy (McLure, 1975). In this literature, the local properties of an initial equilibrium are obtained by linearizing equations describing the economy. Local responses to exogenous changes, such as for example price changes following a differential tax increase, are then often employed to study the effect of a discrete tax change. To overcome the limitations of the linearity assumption, Shoven (1976) employs an algorithmic approach to approximate the exact solution for a discrete change in the tax rate on capital income, and compares it to linearized results from Harberger (1966). The linearized approach is found to overstate the effects of a tax increase, both in terms of price changes and resource misallocation. The magnitude of the results in the two approaches are, however, found to be similar, thus lending credibility to results based on linearization.

Harberger-type models have been recently employed to study environmental taxation. Fullerton and Heutel (2007) extend Harberger (1962) by allowing for one sector to employ pollution as an input to production, in addition to capital and labor, and consider the effect of a pollution tax increase on relative prices and pollution. Subsequent studies have built upon this work by, for example, assessing incidence of a pollution tax across heterogeneous household types (Fullerton and Heutel, 2010), studying the relative burden of environmental taxation on high-skilled and low-skilled labor (Fullerton and Monti, 2013) and assessing the effect of the household aggregation problem for environmental tax incidence (Rausch and Schwarz, 2016).

A number of features common to models following Fullerton and Heutel (2007) differ from the original Harberger model, and may in principle lead to differing conclusions compared to Shoven (1976). First, whilst in the Harberger model the taxed input—capital—is subject to a resource constraint, this is generally not the case for pollution, which reacts endogenously to the tax change. Second, models following Fullerton and Heutel (2007) allow for more substitution possibilities in production, as pollution, in addition to capital and labor, is represented as an input. Third, beyond considering the incidence of a given pollution tax increase (which I refer to as the *price-based* policy), it is also of interest to study the distributional effects of achieving a given environmental target (the *quantity-based* policy). For the taxation of capital, quantity-based policies are, however, not meaningful, as they are motivated by the externality-correcting motive of pollution taxes. In order for the results of Harberger-type studies of environmental tax incidence to be credibly employed to assess the effects of real-world policy interventions, a study of the suitability of linearized properties to approximate discrete changes in environmental taxes is therefore needed. To the best of my knowledge, no such study has been performed to date. This essay is an attempt to fill this gap.

I formulate a simple two sector two factor analytical general equilibrium model which is locally identical to Rausch and Schwarz (2016) (with household preferences restricted to the homothetic case, and with equal cross-price substitution elasticities in production) and which, in a special case, can be solved analytically for discrete changes in the pollution tax. The analytical solution for the economy's exact response is a methodological contribution that goes beyond the approach of Shoven (1976). Beyond this special case, I approximate the exact solution algorithmically, analogously to Shoven (1976). For this purpose, I develop a simple Computable General Equilibrium

(CGE) model with the same production and utility functions as the analytical model. I calibrate both the linearized results and the CGE model to the same data, employed previously in the context of Harberger-type environmental tax incidence analysis, thus delivering estimates of the effect of a pollution tax on output prices and returns to capital and labor in the two approaches, as well as the corresponding patterns of incidence.

I first consider the change in output and factor prices, and assess the bias introduced by the linearized approach. In the case of Cobb-Douglas production and identical Cobb-Douglas preferences across households I show analytically that, following an increase in the pollution tax rate, the linearized approach overstates price increases as well as the pollution reduction. On the other hand, the linearized approach understates price changes for a policy targeting a given pollution reduction. For a range of cases, the quantitative results are in line with these theoretical findings.

I then quantify the incidence of these price- and quantity-based environmental policies across households. I find that, for the price-based policy, the incidence results in linearized analysis are more regressive compared to the exact results, as the overestimated price changes magnify relative differences between households. For the quantity-based policy, incidence results in the linearized approach are instead less regressive, since, for a given tax change, the linearized approach overestimates pollution reductions more than price changes. Although the results differ quantitatively between the linearized and the exact approaches, they are found to be qualitatively similar.

The rest of this essay is organized as follows. Section 2.2 presents the setup. Section 2.3 presents the analytical and numerical results. Section 2.4 concludes.

2.2 Setup

I consider a perfectly competitive economy with two sectors (X and Y), two factors of production (capital K and labor L), and an arbitrary number H of households $h \in \{1, 2, ..., H\}$.

Sector Y, which I refer to as the 'dirty' sector, employs pollution as an input to production, in addition to capital and labor. Sector X, which I refer to as the 'clean' sector, makes use of capital and labor only.[1] The government taxes pollution at a rate τ and returns the revenue entirely to households in proportion to their benchmark income.[2]

Production functions for the clean and the dirty sectors, as well as utility functions for each house-

[1] The model setup follows closely Fullerton and Heutel (2007) and Rausch and Schwarz (2016), in order to ensure the results are applicable to the Harberger-style environmental tax literature.

[2] This enables a focus on the incidence caused by changes in output prices and factors of production.

hold h are assumed to be of constant-elasticity-of-substitution (CES) form:

$$X(K,L) = A_X\left(\alpha_X K^{\frac{\sigma_X-1}{\sigma_X}} + (1-\alpha_X)L^{\frac{\sigma_X-1}{\sigma_X}}\right)^{\frac{\sigma_X}{\sigma_X-1}}$$

$$Y(K,L,Z) = A_Y\left(\alpha_Y K^{\frac{\sigma_Y-1}{\sigma_Y}} + \beta_Y Z^{\frac{\sigma_Y-1}{\sigma_Y}} + (1-\alpha_Y-\beta_Y)L^{\frac{\sigma_Y-1}{\sigma_Y}}\right)^{\frac{\sigma_Y}{\sigma_Y-1}}$$

$$U_h(X,Y) = A_h\left(\alpha_h X^{\frac{\sigma_C-1}{\sigma_C}} + (1-\alpha_h)Y^{\frac{\sigma_C-1}{\sigma_C}}\right)^{\frac{\sigma_C}{\sigma_C-1}},$$

where $A_X > 0$, $A_Y > 0$, and $A_h > 0$ define the units of the production and utility functions, and $\sigma_X > 0$, $\sigma_Y > 0$, and $\sigma_C > 0$ represent the elasticity of substitution in production amongst inputs and the elasticity of substitution in utility amongst clean and dirty consumption, respectively.

Capital and labor are supplied inelastically and are perfectly mobile across sectors. Markets for goods and factors are cleared:

$$X = \sum_h X^h \qquad Y = \sum_h Y^h \qquad K_X + K_Y = \sum_h K^h \equiv \bar{K} \qquad L_X + L_Y = \sum_h L^h \equiv \bar{L}\,,$$

where X^h and Y^h are the quantities of clean and dirty goods consumed by household h, K^h and L^h are the endowments of capital and labor of household h, and $K/L_{X/Y}$ are the quantities of capital and labor employed by sectors X and Y, respectively.

Firms pay the wage rate w, the capital rental rate r, and the pollution tax rate τ. Consumers pay the clean good price p_X and the dirty good price p_Y. The clean good price is chosen to be the numeraire (i.e., $p_X \equiv 1$).

From an initial competitive equilibrium with a non-zero pollution tax rate $\tau_0 > 0$, two approaches are employed to assess the effect of a tax increase. On the one hand, I consider the exact result for quantities of interest, such as the dirty good price $p_Y^E(\tau)$. On the other hand, I consider linearized quantities. In the linearized approach, the local properties of the equilibrium are used to calculate the effect of discrete changes in the tax rates as follows: $p_Y^L(\tau) = p_Y(\tau_0) + \frac{dp_Y}{d\tau}\Big|_{\tau_0}(\tau - \tau_0)$, and analogously for the other quantities of interest.

2.3 Findings

In Section 2.3.1, I consider a special case that can be solved analytically for exact prices and pollution level as functions of the pollution tax rate. The comparison to the linearized solutions provides insights that carry over to alternative cases considered in the numerical analysis in Section 2.3.2.

2.3.1 Analytical results

Assume Cobb-Douglas production and identical Cobb-Douglas preferences across households.[3] The linearized solutions then amount to the following (see Appendix C.1 for derivations):[4]

$$p_Y^L(\tau) = p_Y(\tau_0) + p_Y(\tau_0)\frac{\theta_{YZ}}{\tau_0}(\tau - \tau_0) \qquad Z^L(\tau) = Z(\tau_0) - \frac{Z(\tau_0)}{\tau_0}(\tau - \tau_0)\,.$$

The exact solutions, on the other hand, are as follows (see Appendix C.2 for derivations):

$$p_Y^E(\tau) = p_Y(\tau_0)(\frac{\tau}{\tau_0})^{\theta_{YZ}} \qquad Z^E(\tau) = Z(\tau_0)\frac{\tau_0}{\tau}\,.$$

In both cases, the wage rate and capital rental rate remain constant, $r^L(\tau) = r^E(\tau) = r(\tau_0)$ and $w^L(\tau) = w^E(\tau) = w(\tau_0)$.

From the analytical expressions above, it can easily be seen that, for a given pollution tax increase, the price of the dirty good increases more in the linearized compared to the exact case. At the same time, the pollution level decreases more in the linearized case, as stated in the following proposition:

Proposition 2.1 *Assume Cobb-Douglas production and identical Cobb-Douglas preferences. Then, for all pollution tax levels above the initial value (i.e., $\tau > \tau_0$), the price of the dirty good in the local approximation is higher than its exact value, and the pollution level is lower than its exact value (i.e., $p_Y^E(\tau) < p_Y^L(\tau)$ and $Z^E(\tau) > Z^L(\tau)$ for $\tau > \tau_0$).*

Proof. For Cobb-Douglas production and identical Cobb-Douglas preferences the following holds: $p_Y^L(\tau) - p_Y^E(\tau) = p_Y(\tau_0) + \theta_{YZ}\frac{p_Y(\tau_0)}{\tau_0}(\tau - \tau_0) - p_Y(\tau_0)\big(\frac{\tau}{\tau_0}\big)^{\theta_{YZ}} = p_Y(\tau_0)\big(1 - \theta_{YZ} + \frac{\theta_{YZ}}{\tau_0}\tau - \big(\frac{\tau}{\tau_0}\big)^{\theta_{YZ}}\big)$. For $\tau = \tau_0$ this is zero; for $\tau > \tau_0$ it is positive, since the derivative is positive: $\frac{d}{d\tau}(1 - \theta_{YZ} + \frac{\theta_{YZ}}{\tau_0}\tau - \big(\frac{\tau}{\tau_0}\big)^{\theta_{YZ}}) = \frac{\theta_{YZ}}{\tau_0}\big(1 - \big(\frac{\tau}{\tau_0}\big)^{\theta_{YZ}-1}\big) > 0$. Now consider pollution: $Z^E(\tau) - Z^L(\tau) = Z(\tau_0)\frac{(\tau-\tau_0)^2}{\tau\tau_0} > 0$. □

The linearized approach thus overstates both the price changes and the pollution reductions following an increase in the pollution tax. This result is in line with the findings of Shoven (1976) that the linearized approach overestimates the effects of a tax increase.

Intuitively, the reason for the overestimated effect of a tax increase is that the linearized approach ignores the following important effects. First, the effect of marginal tax increases on the dirty good price is not constant, but is rather decreasing with rising pollution tax rates, as the amount of pollution—and thus the pollution costs that are marginally passed through—decreases. For discrete tax changes, this leads to less than proportional increases in the price of the polluting good. Second, the effect of marginal tax increases on the pollution level is also decreasing with rising pollution taxes, since proportional changes in the level of pollution (caused by substitution

[3] Cobb-Douglas functions correspond to the limit of the CES functions as the elasticities of substitution go to 1.

[4] The local equilibrium behavior is derived following the approach of Fullerton and Heutel (2007) and Rausch and Schwarz (2016).

away from pollution) translate into smaller absolute changes, as the pollution level decreases. For discrete tax changes, this leads to less than proportional decreases in the pollution level.

In order to quantify the bias introduced by the linearized approach, it is useful to define two quantities. The first, which I refer to as the *bias on pollution changes*, amounts to the pollution change in the linearized approach relative to the exact pollution change: $\frac{Z^L(\tau)-Z(\tau_0)}{Z^E(\tau)-Z(\tau_0)}$. The second, which I refer to as the *bias on dirty good price changes*, is defined analogously, as follows: $\frac{p_Y^L(\tau)-p_Y(\tau_0)}{p_Y^E(\tau)-p_Y(\tau_0)}$. The following proposition shows that the bias on pollution is larger than the bias on prices:

Proposition 2.2 *Assume Cobb-Douglas production and identical Cobb-Douglas preferences. Then the bias on pollution changes is larger than the bias on dirty good price changes:* $\frac{Z^L(\tau)-Z(\tau_0)}{Z^E(\tau)-Z(\tau_0)} > \frac{p_Y^L(\tau)-p_Y(\tau_0)}{p_Y^E(\tau)-p_Y(\tau_0)}$.

Proof. First of all, note that $\frac{Z^L(\tau)-Z(\tau_0)}{Z^E(\tau)-Z(\tau_0)} = \frac{\tau}{\tau_0}$ and $\frac{p_Y^L(\tau)-p_Y(\tau_0)}{p_Y^E(\tau)-p_Y(\tau_0)} = \frac{\theta_{YZ}(\frac{\tau}{\tau_0}-1)}{(\frac{\tau}{\tau_0})^{\theta_{YZ}}-1}$. Hence, $\frac{Z^L(\tau)-Z(\tau_0)}{Z^E(\tau)-Z(\tau_0)} > \frac{p_Y^L(\tau)-p_Y(\tau_0)}{p_Y^E(\tau)-p_Y(\tau_0)}$ is equivalent to $\frac{\tau}{\tau_0} > \frac{\theta_{YZ}(\frac{\tau}{\tau_0}-1)}{(\frac{\tau}{\tau_0})^{\theta_{YZ}}-1}$. For $\tau > \tau_0$, this is in turn equivalent to $(\frac{\tau}{\tau_0})^{\theta_{YZ}+1} - \frac{\tau}{\tau_0} > \theta_{YZ}(\frac{\tau}{\tau_0} - 1)$. This last inequality holds, since the derivative of the left-hand side (i.e., $(\theta_{YZ}+1)(\frac{\tau}{\tau_0})^{\theta_{YZ}}(\frac{1}{\tau_0}) - \frac{1}{\tau_0}$) is greater than the derivative of the right-hand side (i.e., $\frac{\theta_{YZ}}{\tau_0}$), since $(\theta_{YZ}+1)(\frac{\tau}{\tau_0})^{\theta_{YZ}} > \theta_{YZ}+1$. For $\tau < \tau_0$ the proof follows analogously. □

The larger bias on pollution changes compared to price changes is due primarily to the fact that the price of pollution (i.e., the pollution tax) experiences a substantially larger proportional change compared to the dirty good price, since pollution only represents a fraction of the value of total inputs in production. Higher-order, non-linear effects, are therefore more important for substitution in production (which affects the pollution level directly) compared to substitution in consumption (which affects the output prices indirectly, through the interaction of demand and supply).

The results in Proposition 2.1 indicate that a given pollution tax increase leads to differing pollution levels in the linearized and the exact approaches. In order to compare the two approaches for the same environmental outcome, I now consider a quantity-based policy targeting a given level of pollution, in opposition to the above price-based policy. Since, from Proposition 2.2, the bias on pollution changes is larger than the bias on dirty good price changes, one would expect for the quantity-based policy to result in lower price changes in the linearized compared to the exact approach. This in indeed the case, as illustrated in the following proposition:

Proposition 2.3 *Assume Cobb-Douglas production and identical Cobb-Douglas preferences. For a given pollution target $\bar{Z}$ below the initial level (i.e., $\bar{Z} < Z(\tau_0)$), the associated dirty good price in the linearized approach is lower compared to the exact value (i.e., $p_Y^L(\tilde{\tau}) < p_Y^E(\tau)$, with τ and $\tilde{\tau}$ such that $Z^L(\tilde{\tau}) = Z^E(\tau) = \bar{Z}$).*

Proof. From $Z^E(\tau) = \bar{Z}$ it follows that $\tau = \frac{Z(\tau_0)\tau_0}{\bar{Z}}$. On the other hand, from $Z^L(\tilde{\tau}) = \bar{Z}$ it follows that $\tilde{\tau} = \tau_0 + \tau_0(1 - \frac{\bar{Z}}{Z(\tau_0)})$. Inserted into the respective expressions for p_Y^E and p_Y^L, the condition that $p_Y^E(\tau) > p_Y^L(\tilde{\tau})$ is equivalent to the following: $(\frac{Z(\tau_0)}{\bar{Z}})^{\theta_{YZ}} > 1+\theta_{YZ}(1-\frac{\bar{Z}}{Z(\tau_0)})$. This holds for any environmental target $\bar{Z}$ below the initial pollution level $Z(\tau_0)$. The Proposition thus holds. □

The above results indicate that, depending on whether price- or quantity-based environmental policies are compared in the linearized and the exact approaches, the bias on price changes introduced by the linearized approach will be qualitatively different. This has implications for the bias on incidence, as discussed in the following section.

2.3.2 Numerical results

I now quantify the bias introduced by the linearized approach in a framework that allows for the analysis to be extended beyond the special case studied analytically above. In this section, I approximate the exact solution algorithmically, by representing the same economy within a CGE model, and compare it with the linearized solution, derived in Appendix C.1. The CGE model's equilibrium conditions are expounded in Appendix C.3, and the code for the numerical implementation is reported in Appendix C.4.[5]

I calibrate both the CGE and the linearized model equations to the same benchmark data for production, consumption and income. I choose data that has already been used in the context of Harberger-type environmental incidence analysis, from Rausch and Schwarz (2016) (described in Section 1.4.1 in Essay 1), which represent heterogeneous expenditure deciles.

I consider two environmental policies: a price-based policy consisting of a 50% in the pollution tax rate, and a quantity-based policy requiring a 50% decrease in the level of pollution relative to the benchmark.

Table 2.1 reports the results. The findings from the analytical section apply to a number of cases beyond the special case of Cobb-Douglas production and identical Cobb-Douglas preferences considered above. The only qualitative difference from the previous special case is given by the factor prices, which are not constant beyond the Cobb-Douglas case. For wages and the capital rental rate, the bias introduced by the linearized approach, however, behaves analogously to the bias on the dirty good price, i.e. factor price changes in the linearized approach are overestimated for the price-based policy, and underestimated for the quantity-based policy.

I now consider the distributional impacts of the price- and quantity-based environmental policies in the exact and in the linearized approaches. I return pollution tax revenues in proportion to benchmark income, thus muting the channel of incidence driven by revenue redistribution, in order to focus on incidence determined by changes in relative consumption good and factor prices. In the exact approach, welfare impacts are directly computed in the CGE model as changes in real income, divided by the benchmark value. In the linearized approach, they are instead computed by multiplying price changes with benchmark expenditure shares, and factor price changes by benchmark factor income shares, as in Rausch and Schwarz (2016).

[5] Note that the numerical implementation is similar to that of stylized models such as those found in Markusen (2002).

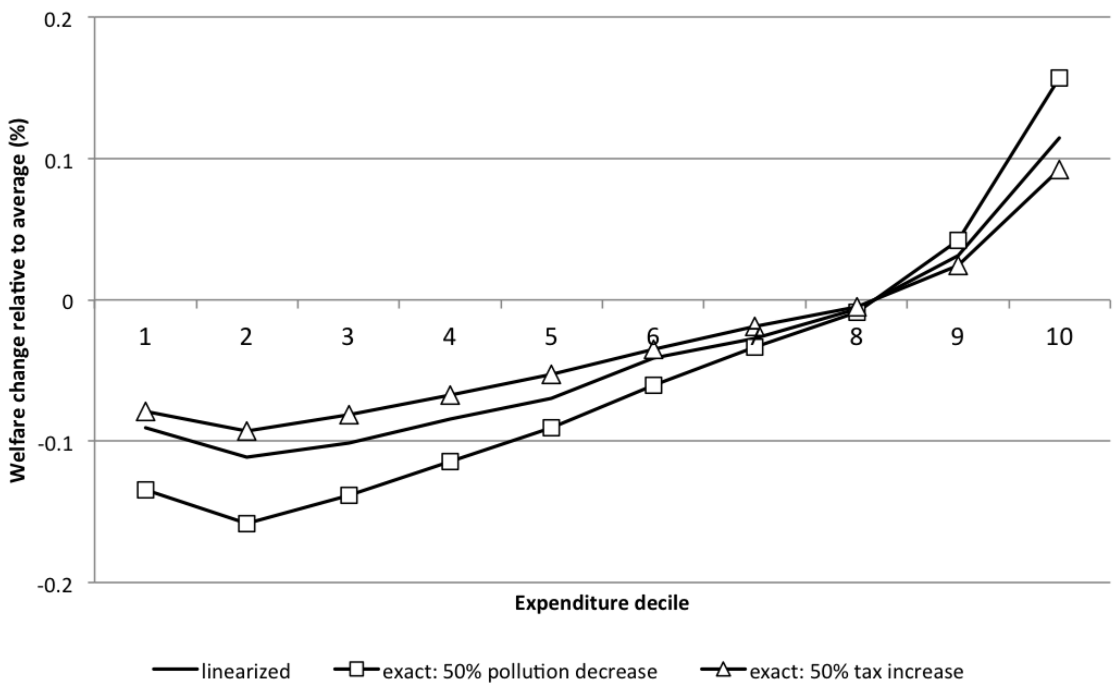
Figure 2.1 displays incidence patterns for both the price- and quantity-based environmental policy, for the case of Cobb-Douglas production and utilities. Unsurprisingly, biased price changes lead to biased incidence patterns: for the price-based policy, the linearized approach overstates the regressivity of the environmental tax, whilst it is understated for the quantity-based policy.

Table 2.1: Linearized (*Lin.*) versus algorithmic approximation of exact price and pollution changes for price- and quantity-based environmental policy

	Cobb-Douglas		$\sigma_{X/Y} = 1.5/0.5$		$\sigma_{X/Y} = 0.5/1.5$		$\sigma^h = 0.5$		$\sigma^h = 1.5$	
	Lin.	*Exact*	*Lin.*	*Exact*	*Lin.*	*Exact*	*Lin.*	*Exact*	*Lin.*	*Exact*
	Price-based: $\Delta\tau = 50\%$									
$\%\Delta p_Y$	3.61	2.97	3.60	3.27	3.64	2.73	3.63	2.99	3.60	2.96
$\%\Delta r$	0.00	0.00	-0.04	-0.03	0.10	0.07	0.05	0.04	-0.05	-0.04
$\%\Delta w$	0.00	0.00	0.02	0.02	-0.06	-0.04	-0.03	-0.03	0.03	0.03
$\%\Delta Z$	-50.0	-33.3	-26.7	-19.6	-73.3	-44.9	-48.3	-32.4	-51.7	-34.2
	Quantity-based: $\Delta Z = -50\%$									
$\%\Delta p_Y$	3.61	5.14	6.75	13.2	2.48	3.13	3.76	5.35	3.48	4.95
$\%\Delta r$	0.00	0.00	-0.07	-0.13	0.07	0.08	0.06	0.08	-0.05	-0.07
$\%\Delta w$	0.00	0.00	0.04	0.08	-0.04	-0.05	-0.03	-0.05	0.03	0.04

Notes: %Δ stands for the change from the initial (benchmark) value, expressed as a percentage share of the initial value.

Figure 2.1: Incidence across expenditure deciles for Cobb-Douglas technology and Cobb-Douglas utility



For the quantity-based policy, the lower regressivity in the linearized approach is due to the fact that the change in the dirty good price is underestimated compared to the exact case. The lower price change reduces relative differences in the welfare impacts across households, driven by relative differences in the dirty good expenditure shares.

For the price-based policy, on the other hand, price changes in the linearized approach are overestimated compared to the exact results. The higher prices magnify relative differences amongst households, thus leading to a more regressive outcome, compared to the exact approach.[6]

2.4 Conclusion

Harberger-type models tend to overstate the impacts of price-based environmental policies, and understate the impacts of quantity-based regulation, as the linearized approach overestimates price changes for price-based policies, and underestimates price changes for quantity-based policies. The reason lies in the higher bias on pollution reductions compared to price increases, for a given increase in the pollution tax, due to the stronger role of nonlinear substitution effects on the pollution level, compared to prices. The reversal of the bias on price changes causes incidence patterns to be less regressive for quantity-based policies and more regressive for price-based policies in the linearized compared to the exact approach.

Although the results between the linearized and the exact approaches are quantitatively different, they are found to be qualitatively similar, for a number of alternative behavioral parameters of households and firms. These findings support the use of the Harberger approach to assess environmental policies, whilst pointing to sources of possible systematic bias which differ for price- and quantity-based policies.

Possible directions for further work are, for example, the representation of labor-leisure choice and a higher level of sectoral disaggregation. From Shoven and Whalley (1972) and Shoven (1976), such extensions will likely increase the quantitative bias of the linearized approach, without, however, invalidating its qualitative results.

[6]It should be noted that the difference between incidence patterns in the two approaches captures the effect of both biased price changes *and* the use of a welfare measure that does not account for behavioral response in the linearized approach. West and Williams III (2004) find that, for given price and income changes, incidence measures that ignore households' demand response overstate the regressivity of environmental taxes. Hence, if only the bias on price changes were considered (i.e., if the behavioral response were also taken in account in the welfare measure of the linearized approach), then the difference between the incidence curves would be smaller for the price-based policy and larger for the quantity-based policy.

3 Income-dependent household energy consumption patterns: modeling non-homothetic preferences and implications for climate policy in China[1]

[1]Without implication, special thanks go to Lawrence H. Goulder, Sebastian Rausch, and the participants of the 22nd Annual Conference of the European Association of Environmental and Resource Economists, the ETH Simlab Workshop, the MIT Joint Program seminar and the 2015 China Energy and Climate Project Annual Meeting for helpful comments.

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Abstract

We quantify how energy demand and associated CO_2 emissions are affected by income-driven shifts in consumption patterns in China, the world's largest emerging economy. Incorporating empirically-derived Engel curves within a general equilibrium model, we find that, relative to projections based on standard assumptions of unitary income elasticity, direct household CO_2 emissions in 2030 are 61% lower, and national emissions are reduced by 8%. This has important implications for the welfare consequences of climate policy. The average welfare costs of climate policy decrease by more than half, with losses more evenly distributed across households. This is driven by the easing of policy stringency and convergence in the carbon intensity of household consumption baskets as incomes rise. Our results point to non-homothetic household preferences as an important determinant of climate policy costs in developing economies, where incomes, energy demand and emissions are rising rapidly.

3.1 Introduction

Household consumption in the developing world is seen as an increasingly important source of growth in global energy demand and emissions of CO_2, a major greenhouse gas linked to global climate change. Future patterns of growth in emerging economies will impact environmental quality and policy design. China is a case in point. Three decades of economic growth have lifted millions of households out of poverty but have also increased energy consumption, making China the world's largest energy user and emitter of CO_2.

Household income and consumption expenditures are well established as drivers of energy demand and CO_2 emissions (Wolfram et al., 2012). There is rich empirical evidence that elasticities vary widely across goods and income levels. Despite this, the relationship between consumption and energy use is typically represented in economic models used for projections and policy analysis using the simplifying assumption of unitary income elasticity (homothetic). For developing economies where wealth discrepancies are large, household energy sources are diverse, and income is growing rapidly, this assumption may be especially problematic and its implications for energy demand forecasting and policy design still poorly understood. Moreover, empirical estimates of Engel curves for emerging economies are rare, due to the paucity or poor quality of comprehensive micro data describing household consumption patterns.

In this essay, we estimate Engel curves for China by exploiting the large existing variation in household incomes and allowing for flexible functional forms and controlling for important co-variates. Engel curves for energy goods are estimated using a new energy-specific survey of Chinese household consumption. To capture the relationship between income and embodied energy use, we estimate Engel curves for non-energy goods using official statistics. These curves reveal that the income elasticity of most goods differs significantly from unity and varies across income levels. We then explore the implications of our empirically-derived estimates. We develop a novel approach to calibrate a recursive-dynamic general equilibrium (GE) models to estimated consumption pathways as a function of between-period changes in income.

We use a GE model of China that separately resolves urban and rural households in each province, enabling an analysis that captures two important dimensions of policy targeting in China (Ming and Zhao, 2006; Zhang et al., 2013). The GE model represents feedbacks between changing consumption patterns and supply, including production and trade flows, as well as interactions with energy and climate policy, which a partial equilibrium study of consumption shifts would not capture. It allows us to simulate a stylized climate policy consisting of national CO_2 emissions targets, implemented by a national CO_2 price, and examine a number of the "channels of climate policy incidence" documented by Fullerton (2011).[2] We compare outcomes based on the empirically-derived calibrated preferences, which describe non-homothetic behaviour and non constant income elasticities, with outcomes based on consumption patterns generated by a standard homothetic

[2]Our analysis captures five of the six "channels of incidence" from Fullerton (2011): (1) increased prices of carbon-intensive goods, (2) changes in relative returns to factors of production, (3) allocation of revenues from carbon pricing, (4) dynamic effects, (5) capitalization of all those effects into prices of land and other resources. We do not capture (6) benefits from improved environmental quality. A partial equilibrium analysis would only capture a subset of these channels.

constant-elasticity-of-substitution (CES) demand system with unitary income elasticity. We then decompose the general equilibrium effects of income-driven shifts in consumption patterns on the distribution of the policy's welfare costs across households. To do so, we disentangle the policy's welfare impacts due to changes in household incomes (which we refer to as "income side" impacts) and changes in the relative prices of consumption goods (which we refer to as "consumption side" impacts).

Our calibration alters projections of energy demand, CO_2 emissions, and the magnitude and distribution of climate policy impacts. Comparing results for calibrated non-homothetic preferences to results for homothetic preferences, we identify four main results. First, our estimated Engel curves, once incorporated into the GE model, translate into considerably lower total demand for energy in China and lower associated CO_2 emissions than if preferences are assumed to be unit-elastic. We find, second, that ignoring income-driven shifts in consumption patterns considerably overestimates the average costs of reaching a given emissions target. Third, the variation in welfare impacts is reduced considerably when accounting for non-homothetic preferences. This finding is driven by the lower carbon price caused by the lower baseline emissions, which mutes the variation of both consumption and income side impacts, as well as the lower correlation between these two channels. In addition, this result is driven by a rapid convergence of the carbon intensity of consumption baskets across households as incomes grow, caused by low income elasticities of energy goods and rapid growth in household income levels. Fourth, income-driven consumption shifts can significantly change the ranking between relative winners and losers under policy compared to the case with homothetic preferences, driven by the differing temporal evolution of consumption patterns across households and interactions with the changes in relative prices of consumption goods under policy.

Our results have implications for policy. Accounting for empirically calibrated non-homothetic preferences can substantially alter baseline projections for CO_2 emissions growth, as well as the magnitude and distribution of policy costs. The 2015 Paris Climate Agreement commits all countries to act to mitigate CO_2 emissions (UNFCCC, 2015). Economic cost is one measure used to evaluate national effort (Aldy et al., 2015), and distributional impacts are of major concern to policy makers (Metcalf et al., 2008; Rausch et al., 2011; Zhang et al., 2013). Our method introduces empirical realism into projections used to quantify effort measures for developing economies. For China, we find that existing policy evaluation exercises assuming unitary income elasticities may be biased in a way which is unfavorable for the perceived costs of climate policy. This finding may apply more broadly throughout the developing world, in the measure in which rising income levels cause consumption baskets to be both on average less carbon intensive, and to exhibit a lower variation of carbon intensity of consumption across households.

3.2 Estimating the relationship between consumption patterns and income

3.2.1 Empirical evidence for non-homothetic preferences

The foundational work of Engel (as cited in Chai and Moneta (2010)), Working (1943) and Leser (1963) has established income as an important determinant of household consumption patterns. A large body of literature has focused on estimating income elasticities of household demand for goods and services, including housing (Carliner, 1973), electricity (Branch, 1993), food/agricultural products (Haque, 2005; Chern et al., 2003), and studies that estimate elasticities for a wide range of consumption goods across multiple countries (Houthakker, 1957; Caron et al., 2014). Many estimates of income elasticities of demand for goods and services are found to deviate from unity and to vary by income level. For example, the income elasticity of food (Engel's Law, Houthakker (1957)) and clothing (Schwabe's Law, Haque (2005)) is generally found to be lower than one. In the case of China, Zhou et al. (2012) find evidence that, in the process of economic development, some luxury goods such as beef, fish and poultry turn into necessities, implying elasticities that move from above to below unity as income rises.

Many of the studies cited above rely on aggregate data to provide 'macro' estimates of the income-consumption relationship. For household energy demand, a growing literature expounds the relationship between income and consumption patterns using 'micro' household-level data. By capturing the dynamics of adoption, these studies allow for differentiation between the intensive and the extensive margins of energy use. Most studies use data for specific regions or countries, including the European Union (Reinders et al., 2003), the United States (Branch, 1993), Denmark (Munksgaard et al., 2000), but also a small but growing amount of developing world evidence such as India (Filippini and Pachauri, 2004), among other countries where the dynamics are likely very different. There is, for instance, growing evidence of an S-shaped relationship between household income and the adoption of energy-using consumer durables. Auffhammer and Wolfram (2014) document such a relationship using province-level data in China while Gertler et al. (2016) identify the relationship using micro-data from a Mexican conditional cash transfer program. These studies provide support for the conclusion that income elasticities of energy demand not only differ from unity but also vary by income level.

Only a small number of studies for China use micro data, due in part to the limited availability of household level surveys. One source is China's Urban Household Income and Expenditure Survey, a large data set focused on urban households collected by the National Bureau of Statistics (NBS). Cao et al. (2014) exploit these data to estimate the relationship between income and energy demand in an Almost-Ideal Demand System (AIDS) model. They provide strong evidence that income elasticity varies across income groups, but do not estimate flexible Engel curves which span the full income spectrum. While the survey contains a very large number of observations, it is limited to urban households in a limited number of provinces. As our objective is to simulate the national aggregate effect of income on energy consumption, we require observations from a more diverse set of households. Despite this, results in Cao et al. (2014) are generally in line with ours, although their income elasticity estimates for heat and gas are considerably higher (above one). Golley and Meng (2012) use the same survey and document a strongly declining relationship between income and the direct household emissions intensity. Interestingly, they also combine survey data with input-output and emission coefficients, finding evidence for flat or even slightly increasing indirect emissions intensities (especially at high income levels). Other studies rely on aggregated data made available by the NBS for consumption across household types. Wei et al. (2007) document the relationship between energy use and lifestyles choices, without, however, linking them to income.

Growing recognition that income elasticities differ from one have prompted a number of studies to move to flexible demand systems. Li et al. (2015) estimate an Exact Affine Stone Index (EASI) implicit Marshallian demand system (Lewbel and Pendakur, 2009) and document high-rank Engel curves for several non-energy goods in China.

3.2.2 Data and empirical strategy

This section provides an overview of the data sets and procedure underlying our estimation of the income-consumption relationship.

The central idea behind the estimation and our projections is to use household income (and its projected growth) as a predictor of household consumption patterns. The data we rely on are purely cross-sectional, so using these estimates as we do to extrapolate across time assumes that preferences are identical across Chinese households and stable over time.[3]

While we assume that households in all provinces will react in the same way to changes in income, expenditure shares are allowed to vary according to a number of covariates including cooling and heating degree days, household size, and prices. We also allow for different behavior on the part of urban and rural households. Estimates based on cross-sectional variation assume households to have had sufficient time to adapt energy-consuming capital to their conditions (income, climate, household size, etc.). Our methodology, thus, is best suited for projecting mid- to long-term changes in consumption patterns, which is the primary focus of this exercise. Based on contemporaneous income and consumption decisions, our approach does not, however, explicitly account for lifetime income.

We estimate the relationship between income and consumption patterns separately for energy goods and non-energy goods. Consumption of energy goods is estimated using a household-level survey collected by Renmin University (Zheng et al., 2014), the China Residential Energy Consumption Survey (CRECS). Household micro-data is scarce in China, and this energy-specific survey provides us with a detailed picture of the consumption of six types of energy goods $i \in I^{CRECS}$, which includes LPG, pipeline natural gas, gasoline and diesel, coal, central heating and electricity by 4600 households $h \in H^{CRECS}$ representing a large income spectrum in both rural and urban areas and

[3]It should be noted that we are not aware of any sufficiently long panel data that would enable us to test this assumption.

all provinces but Tibet. Figure 3.1 plots the geographical distribution of sampled households. Both urban and rural households span a large geographical area, across provinces with varying income levels. The survey was administered between 2012 and 2014.

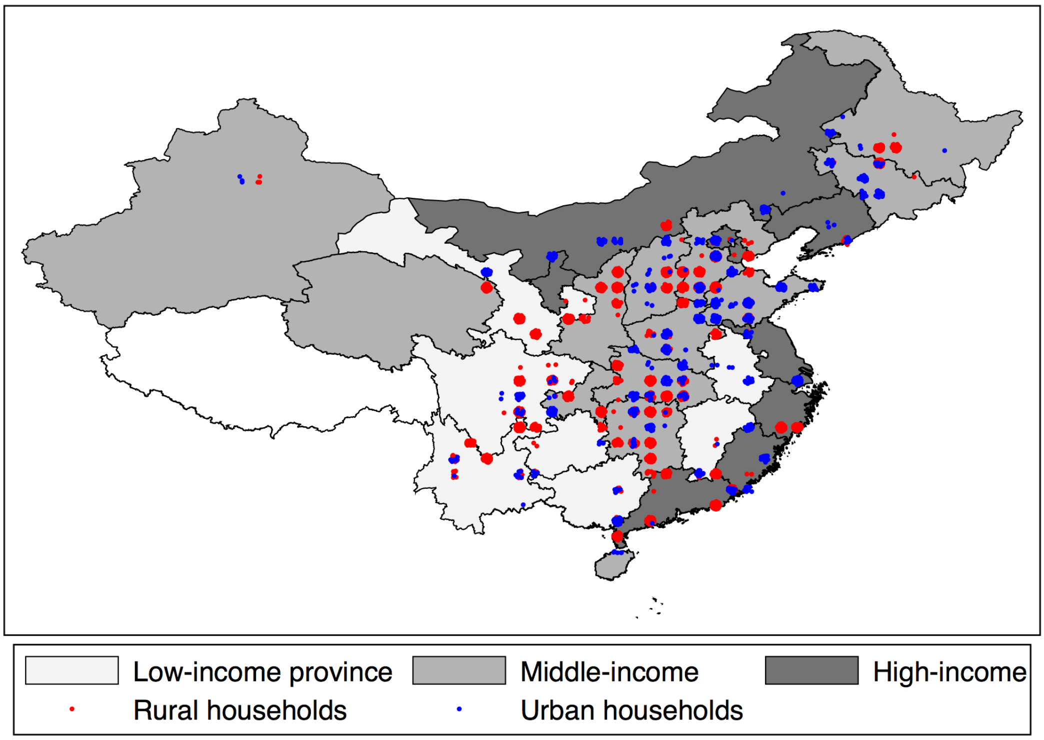


Figure 3.1: Geographical distribution of sampled households in household survey data (source: CRECS).

A significant share of household energy use is indirect and embodied in the consumption of non-energy goods. To capture the relationship between income and embodied energy, we estimate similar Engel curves for the eight aggregate consumption categories compiled by the Chinese National Bureau of Statistics (NBS) (the same as used in Wei et al. (2007) and Dai et al. (2011)[4]). While average consumption statistics are not a perfect substitute for household-level data, the data capture the large variation in consumption patterns and income found across 420 different types of representative households distinguished by province, seven income classes per province, and urban/rural types. The sectoral aggregation, while coarse, captures the observed shifts in consumption from agriculture and basic manufacturing towards services.

3.2.3 Estimation

We start by describing the estimation of energy consumption. First, we adjust each household h's nominal income I_{ihp} by the Stone price index to obtain a measure of real income which can be

[4] These categories are: food, housing, transportation, medical, education, services, clothing, and furniture. This is the most detailed disaggregation available for all provinces and income classes. We use 2012 statistics. The NBS does not make the raw survey data available, and we are not aware of any detailed consumption survey which covers households in all provinces and levels of income.

interpreted as 'implicit utility' in the Exact Affine Stone Index (EASI) demand system (Lewbel and Pendakur, 2009):

$$log\tilde{I}_{ihp} = \log I_{ihp} - \sum_i \theta_{ihp} \log P_{ip} - (1 - \sum_i \theta_{ihp}) \log CPI_p \,,$$

where P_{ip} represents the price of energy good i in Chinese province p and θ_{ihp} is good i's expenditure share. Price data for non-energy goods are unavailable and are approximated using consumer price indices by province. As noted by Lewbel and Pendakur (2009), real income and nominal income are strongly correlated. Our main explanatory variable of interest is adjusted household income $\tilde{I}_{ihp}$, but we include a number of controls: household size HHS_{ihp} and, at the provincial level, cooling- and heating degree days, CDD_p and HDD_p, and an indicator for being within the regions north of the Huai river where heating is required and available, HZ_p. All of the controls are at least partially correlated with income. Richer provinces have cheaper electricity prices (in real terms), for instance. Underlying our estimation of the relationship between income and household energy demand is a relationship between income and demand for energy services (cooking, heating, housing, transport, etc.). For our purposes, we do not need to separately estimate the determinants of energy services from those of actual energy use. For example, as households get richer, they might choose to live in a larger dwelling, which requires more energy to heat or cool. They might also choose to purchase a more energy-efficient heating or cooling system. We are only interested in the combined effect. Thus, we do not control for any household-specific characteristics such as dwelling size which may also be causally related to income. Given the small number of observations in some provinces, we do not include provincial fixed effects and simply pool our coefficients. Cross-provincial variation in energy demand is captured by temperature and prices.

The survey data include a large number of zero-consumption values, indicating that households do not necessarily consume all energy goods. The estimation of the relationship between income and consumption thus proceeds in two steps, where adoption rates for each of the energy types and the intensive margin of energy use are estimated separately. In both steps, we remain agnostic about the shape of the income-consumption function and estimate flexible functional forms which will allow for intrapolation within the GE model. The estimation equations are:

$$Pr(\theta_{ihp} > 0) = f^1(\tilde{I}_{ihp}) + g^1(CDD_p) + h^1(HDD_p) + i^1(HZ_p) + j^1(HHS_{ihp}) + \sum_i \beta_i^1 \log P_{ip} + \epsilon_{ihp}^1 \tag{3.1}$$

$$\theta_{ihp} = f^2(\tilde{I}_{ihp}) + g^2(CDD_p) + h^2(HDD_p) + i^2(HZ_p) + j^2(HHS_{ihp}) + \sum_i \beta_i^2 \log P_{ip} + \epsilon_{ihp}^2 \,, \tag{3.2}$$

where Equation (3.1) estimates the probability of adopting energy good i using probit. $\forall \theta_{ihp} > 0$, Equation (3.2) estimates the conditional expenditure shares using OLS. ϵ_{ihp}^1 ϵ_{ihp}^2 are residuals.[5] The

[5] The validity of our estimates depends on exogeneity of independent variables in both steps [$E(\epsilon_{ihp}^1 X_{ihp}) = 0$ and $E(\epsilon_{ihp}^2 X_{ihp}) = 0$]. Exogeneity is clear for the temperature-related variables. It is also plausible for the price variables, as these vary by province (and not by household) and are, in China, largely the product of government-regulated pricing and unlikely to depend directly on demand. Thus, the most critical assumption here is lack of causation

equations are estimated for each good $i \in I^{CRECS}$ and for urban and rural households separately.[6] $f(.)$, $g(.)$, $h(.)$ and $i(.)$ represent high-order polynomials, the best fitting combination of which is selected using the Akaike Information Criterion (AIC). Polynomials of up to 6 orders are allowed including log transformations and their polynomial transformations.[7]

The second step estimation of the intensive margin has been shown to closely approximate the flexible EASI demand system which allows for high-rank demand (see Lewbel and Pendakur (2009)) and is derived from utility-maximizing household behavior.[8] This flexible functional form embeds standard demand systems such as AIDS —if $\hat{f}(I_{ihp})$ is estimated as $\log(I_{ihp})$—or QUAIDS—if $\hat{f}(I_{ihp})$ is estimated as $\log(I_{ihp}) + (\log(I_{ihp}))^2$.

Income is found to be a significant determinant of demand for all goods at a significance level of at least p=0.01, except for coal consumption by urban households, for which there are very few observations. As usual in studies using household survey data to estimate demand systems, R2s are low (ranging from 0.08 for gasoline adoption from rural households to 0.53 for adoption of pipeline gas by urban households).

Using the estimates from Equations (3.1) and (3.2), we obtain the predicted probabilities of adoption for households of type $u \in \{\text{Urban, Rural}\}$, $\hat{Pr}(I)_{iu}$, and the predicted conditional expenditure shares for each good, $\hat{\theta}(I)_{iu}$, as functions of any income level I. Combining them provides estimates of the predicted relationship between average expenditure and income (Engel curves):[9]

$$\hat{E}(I)_{iup} := \hat{Pr}(I)_{iup}\,\hat{\theta}(I)_{iup}\, I \,. \tag{3.3}$$

These are displayed in Figure 3.2, evaluated at the mean of non-income covariates. We will refer to the average curves as $\hat{E}(I)_{iu}$. The figure shows that while the income-consumption paths are close to log-linear for some goods, non-linearity in most curves indicates that income elasticity usually varies with income. As noted by Gertler et al. (2016), estimated curves based on aggregated energy demand from multiple energy-using assets are not necessarily S-shaped. Our estimates imply relatively low income elasticities, except for gasoline and diesel and central heating (particularly for

between adoption and consumption of a particular energy good and income. We do not suggest a way to identify causality here, but refer to Gertler et al. (2016) who show that causality mainly goes from income to energy demand. In any case, if energy usage does lead to larger incomes, our income elasticity estimates would be biased upward. Our findings imply low income elasticity. Correcting for endogeneity would likely lead to yet lower estimates.

[6]Estimating the goods separately implies $[E(\epsilon_{ihp}\epsilon_{i'hp}) = 0]$ for all pairs of goods i and i'. We do not impose cross-restrictions on the coefficients. The large number of zero-consumption values prevent us doing so. However, Lewbel and Pendakur (2009) show that this is an acceptable approximation. Indeed, we find that the sum of the fitted expenditure shares sum to a value close to 1, even without the restriction, implying that the error terms are not significantly correlated between goods.

[7]This is implemented using a Multivariable fractional polynomial model which selects among all combination of the variable and its log at powers -3, -2, -1, -0.5, 0.5, 1, 2 and 3.

[8]Lewbel and Pendakur (2009) show that the EASI demand system can be closely approximated with a linearized ordinary least-squares estimation. Because of our two-step estimation procedure and the significant number of zero-consumption observations in our dataset, the exact estimation of EASI is not possible in this context.

[9]We implicitly assume no sample selection issues and no link between first and second step $[E(\epsilon^1_{ihp}\epsilon^2_{ihp}) = 0]$. We find evidence that unobserved variables may affect both steps, i.e. the residuals for both steps are found to be significantly correlated for most goods. This would be a problem if we were interested in the effect of income on consumption for those who consume a particular good. However, here we are focused on average expenditure by households at each given income level. Thus were a not concerned about sample selection problems in the classical sense.

rural households). Coal consumption decreases with income—particularly for urban households—implying negative income elasticity. The same holds for bottled gas at high income levels.

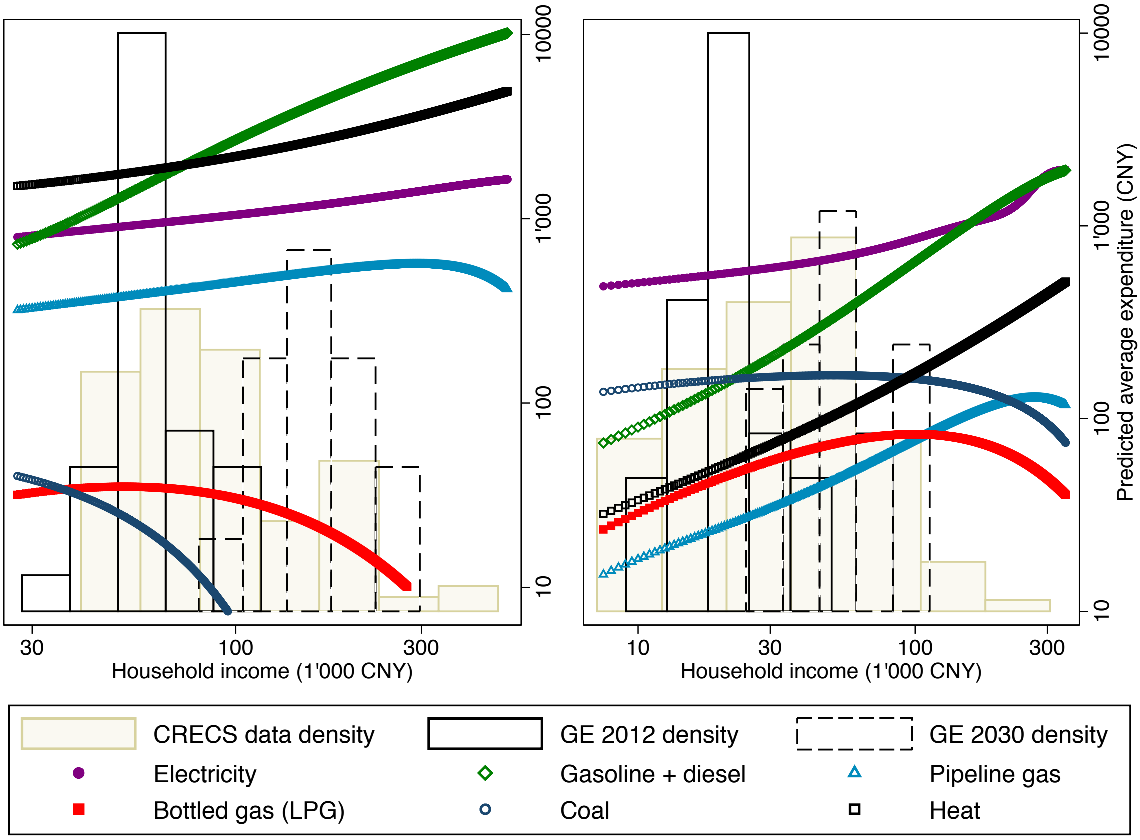


Figure 3.2: Estimated relationship between log household income and log energy expenditure (lines) and density of household incomes (histograms) for the sampled households from which estimates are derived (CRECS data density) and for households in the GE model for 2012 and 2030. Left panel: urban households; Right panel: rural households. Average expenditure is evaluated at the mean of the non-income covariates.

The heterogeneity in observed income levels in the CRECS data allows our projections to be in-sample to 2030, as they cover a larger range of incomes than the projected average incomes in the general equilibrium model employed in our analysis.[10] Of course we acknowledge that less is known about the expected behavior of the richer households and the prediction errors are likely larger at high incomes.

The Engel curve estimation for non-energy goods (based on NBS data) is similar to that for energy goods, albeit without the first step as we have full adoption of each category here. While we do not control for cross-province price differences, as price indices for these consumption categories are unavailable, evidence suggests that relative price differences between provinces are small. The

[10] Average disposable household income in the model's base year (2007) ranges from CNY 68128 for urban households in Shanghai to CNY 5292 for rural households in Sichuan. Between 2007 and the final year in consideration (2030), real income levels increase of a factor that varies between 6.9 (Inner Mongolia) and 3.7 (Xinjiang), due to different provincial growth rates (Projected provincial income growth rates between 2007 and 2030 range from annualized values of 8.8% to 5.8%, with a national population-weighted average of 7.1%), bringing the highest household incomes in the model in 2030 to CNY 292072 (urban households in Guangdong).

variation in incomes provided by the NBS data, which describe 7 income classes per province, also allows our projections to be within-sample to 2030.

3.3 General equilibrium modeling

While the above estimated Engel curves could be combined directly with income growth projections to provide estimates of growth in energy demand, such an approach would implicitly assume perfectly elastic supply, exogenous income levels, and would abstract from demand for energy as an intermediate in production. It would thus not capture the feedbacks from changing consumption patterns on prices through a realistic representation of supply, and on income, through changes in returns to factors of production (including the resource rents that, combined with demand, determine energy prices). It would also abstract from feedbacks on investment and patterns of trade, which are—in addition to household consumption—important sources of emissions. The consequences of climate policy should be assessed within a framework that captures these effects. This motivates our use of a general equilibrium model.

3.3.1 Existing energy-economic modeling approaches

The income-consumption relationship. Studies focused specifically on projecting future demand for energy services and associated emissions are widely based on aggregate macro data, which, contrary to micro household data, do not allow for actually identifying the relationship between income and consumption and do not take patterns of adoption into account. Many studies are based on time-series or panel reduced form projections. For China for example, Crompton and Wu (2005) projects future energy demand based on GDP, population and fuel prices. Auffhammer and Carson (2008) projects Chinese emissions based on provincial-level data.

Numerous energy- or climate-policy specific economic models assume that consumption scales proportionally with income, often for reasons of analytical and computational tractability and data limitations. Static analyses, including analytical general equilibrium models (Fullerton and Heutel, 2007; Fullerton and Monti, 2013) and computable general equilibrium models (Rausch et al., 2011; Zhang et al., 2013) hold production technologies and preferences constant when comparing outcomes in the presence and absence of policy. While instructive for developing intuition, these models are not designed to capture dynamics or to conduct long-term projections. Dynamic models (e.g., Goettle et al. (2009); Rausch et al. (2010b); Williams III et al. (2015)), by contrast, allow underlying features of the economy to evolve over time and interact with policy. These features, which are often empirically calibrated, include labor and capital productivity (Dai et al., 2011), capital stock accumulation and depreciation (Goulder and Summers, 1989), autonomous rates of energy efficiency improvement (Webster et al., 2008), and a rich representation of advanced technologies that can enter the market in response to policy (Jacoby et al., 2006). It is fair to suggest that most of the attention has been put on the supply side, with little attention given to household preferences.

Models implementing non-unitary income elasticities usually do not use flexible demand systems, and while some employ energy-specific income elasticities, these are often based on simple calibration with no empirical basis or on values that are extrapolated from other sectors. The GTAP model (Hertel, 1997; Aguiar et al., 2016), for instance, adopts the Constant Difference of Elasticities (CDE) demand system originally proposed by Hanoch (1975) and is calibrated to internally-consistent income elasticity estimates. These, however, are independent of the level of income and are estimated for broad consumption categories which do not separately identify energy. Other examples include MIT's EPPA model (Chen et al., 2015), which calibrates Stone-Geary preferences to estimated income elasticities for food and agricultural sectors only, the European Commission's GEM-E3 model (Capros et al., 2013) which also calibrates Stone-Geary preferences to income-independent estimates,[11] and the World Bank's ENVISAGE model, which incorporates a more flexible demand system, AIDADS, but relies on elasticity parameters that are only estimated for broad consumption categories and lump energy with other consumption goods (and imply implausibly high income elasticities for energy[12]).

Finally, integrated assessment models (IAMs), which combine economic models with climate models and are used to evaluate climate change policies—and thus require the modeling of future energy use—typically describe the economy as a single sector. To the best of our knowledge, none of these models explicitly allow for endogenous de-carbonization driven by shifts in consumption patterns (income-driven or not).[13] In single-sector models, this mechanism is not otherwise distinguishable from supply-side determinants such as autonomous energy intensity improvements.

To summarize, we are not aware of economic modeling studies that rely on micro data to estimate energy-specific income elasticities and integrate them within a general equilibrium economy-wide model, nor of studies which focus specifically on the importance of income-dependent shifts in consumption patterns in determining climate policy costs and incidence.

Household heterogeneity. Most models used for the evaluation of environmental or energy policy represent the demand side of the economy using a single representative household. Household heterogeneity may, however, affect aggregate economic outcomes and their dynamics. Variation in the rates of household income growth, for example, may interact with heterogeneous consumption patterns in determining the composition of total household consumption. This issue is exacerbated when preferences are non-homothetic and household incomes differ widely.

The representative household approach also does not enable a consistent evaluation of the distribution of economic burden caused by policy. We build on recent work to incorporate heterogeneous households into general equilibrium models for incidence analysis (Rausch et al., 2010a,b, 2011; Rausch and Schwarz, 2016), that has begun to address this issue.

[11] They do not document the source of the income elasticities they employ, but the virtual lack of variation of the estimates for most energy goods across a wide range of countries (see the table on p. 106 in Capros et al. (2013)) suggests that such estimates are not based on national-level household micro-data.

[12] See Table A8.3 of van der Mensbrugghe (2010).

[13] Examples of such models include the RICE/DICE (Nordhaus and Sztorc, 2013) model, Stanford's MERGE (Manne and Richels, 2004) model and FEEM's WITCH (Bosetti et al., 2007) model.

3.3.2 Model features

Our general equilibrium modeling framework extends the China Regional Energy Model (C-REM), a multi-region, multi-sector, recursive-dynamic global computable general equilibrium model with sub-national detail in China. Our model builds on Zhang et al. (2013) and Luo et al. (2016).[14] The model describes all 30 Chinese provinces, four international regions (the United States, Europe, other developing countries, and other developed countries), 13 sectors of the economy, including 5 energy goods and an energy-intensive industry sector, and 2 household types for the 2007 to 2030 period. The regional, sectoral and household aggregation scheme is shown in Table 3.1.

The model satisfies standard neoclassical assumptions, with households maximizing welfare for given incomes (derived from factors of production, taxes and government transfers) at given consumption prices, firms maximizing profits in perfectly competitive markets, subject to government taxation, and a government raising taxes to meet its budget. Starting from a benchmark year, the model solves for a series of equilibria at given time intervals, by imposing exogenous assumptions on the dynamics of the economy's endowments of productive factors, and on technological change.

Households are characterized by myopic expectations, and their behavior is thus optimal solely within, but not between, periods. Households are assumed to derive utility from savings, in addition to consumption and leisure. We do not consider household preference for pollution (emissions) abatement.

Within time periods, household utility is represented as a nested function. At the top level, a Cobb-Douglas utility function describes substitution between consumption, leisure and savings. This generates a positive labor supply elasticity through the labor-leisure trade-off. At the lower level, utility from consumption is represented by a Stone-Geary function of the individual consumption goods. For household type $\bar{h} \in H$ in region $r \in R$ at time t utility from consumption is thus assumed to be of the following form:[15]

$$U_{\bar{h}rt}(\mathbf{x}) = \left[\sum_j \tilde{\theta}_{j\bar{h}rt} \left(\frac{x_j - c^{\star}_{j\bar{h}rt}}{x_{j\bar{h}rt_0} - c^{\star}_{j\bar{h}rt}} \right)^{\rho} \right]^{1/\rho}, \quad \tilde{\theta}_{j\bar{h}rt} = \frac{p_{jrt_0}(x_{j\bar{h}rt_0} - c^{\star}_{j\bar{h}rt})}{I_{\bar{h}rt_0} - \sum_{j'} p_{j'rt_0} c^{\star}_{j'\bar{h}rt}}, \tag{3.4}$$

where $c^{\star}_{j\bar{h}rt}$ is the minimum level of required consumption of good $j \in J$ represented within the model, p_{jrt_0} the benchmark price, $x_{j\bar{h}rt_0}$ the benchmark consumption level, and $I_{\bar{h}rt_0}$ the benchmark income level, net of savings and leisure consumption. $\tilde{\theta}_{j\bar{h}rt}$ is the consumption share net of minimum consumption and ρ parametrizes the response of consumption to changes in relative prices.

Production of good j in region r (Y_{jr}) is assumed to exhibit constant returns to scale, with nested constant-elasticity-of-substitution (CES) functions representing differential substitution among inputs of intermediate goods (X_{jkr}), and productive factors capital (K_{jr}), labor (L_{jr}) and natural

[14] Zhang et al. (2013) first presented the static version of C-REM, and Luo et al. (2016) developed a recursive-dynamic extension. Our model description thus follows these references closely, as our core model (without disaggregated urban and rural households and with homothetic preferences) is similar.

[15] In the model, household utility from consumption is nested, as expounded in Appendix D.3. For simplicity, we abstract here from nesting.

Table 3.1: C-REM regions, commodities and households

Regions $r \in R$				**Commodities** $j \in J$	
AH	Anhui	JS	Jiangsu	AGR	Agriculture, forestry, livestock
BJ	Beijing	JX	Jiangxi	COL	Coal mining and processing
CQ	Chongqing	LN	Liaoning	CON	Construction
FJ	Fujian	NM	Inner Mongolia	CRU	Crude petroleum products
GD	Guangdong	NX	Ningxia	EIS	Energy intensive industries
GS	Gansu	QH	Qinghai	ELE	Electricity and heat
GX	Guanxi	SC	Sichuan	GAS	Natural gas products
GZ	Guizhou	SD	Shandong	MAN	Other manufacturing industries
HA	Henan	SH	Shanghai	OIL	Oil refining, cooking & nuclear fuels
HB	Hubei	SN	Shaanxi	OMN	Minerals and other mining
HE	Hebei	SX	Shanxi	SER	Services
HI	Hainan	TJ	Tianjin	TRN	Transportation and post
HL	Heilongjiang	XJ	Xinjiang	WTR	Water
HN	Hunan	YN	Yunnan		
JL	Jilin	ZJ	Zhejiang	**Household types** $\bar{h} \in H$	
EUR	Europe			hh1	Urban (China)
ODC	Other Developed Countries			hh2	Rural (China)
ROW	Rest of the World			hh	Undifferentiated (outside China)
USA	United States				

Notes: Other Developed Countries include Australia, Canada, Japan, New Zealand and South Korea.

resources (R_{jr}), comprising land, fossil fuel, wind and hydropower resources:

$$Y_{jr} = F_{jr}\left(L_{jr}, K_{jr}, R_{jr}; X_{1jr}, ..., X_{Jjr}\right). \tag{3.5}$$

The nesting structure for each sector's production function is expounded in Appendix D.3. Trade in goods and services is determined according to the Armington assumption of differentiated products by origin. Within China, labor is assumed to be mobile across sectors but not provinces, and capital is assumed to be mobile across both sectors and provinces. Outside China, both labor and capital are assumed to be immobile between regions. In both cases, natural resources are modeled as sector-specific factors.

Appendix D.3 provides a detailed description of the economy's equilibrium conditions. The equilibrium is determined numerically by formulating a mixed complementarity problem (MCP) (Mathiesen, 1985; Rutherford, 1995), which is solved using MPSGE (Rutherford, 1999)—a mathematical programming system—and the PATH solver (Dirkse and Ferris, 1995).

Between periods, growth in region r's labor endowment ($\bar{L}_r$) is driven by the combination of population growth and labor productivity growth. Capital stock dynamics ($\bar{K}_{rt}$) are determined by the accumulation of new capital through investment ($Y_{INV,rt}$), which amounts to aggregate household savings, and by the depreciation of existing capital:

$$\bar{L}_{rt_{l+1}} = \left(1 + g_{rt_l}\right)^{t_{l+1}-t_l} \cdot \bar{L}_{rt_l} \tag{3.6}$$

$$\bar{K}_{rt_{l+1}} = S_{r,t_l} + (1-\delta)^{t_{l+1}-t_l} \cdot \bar{K}_{rt_l}\,, \tag{3.7}$$

where δ is the depreciation rate, and g_{rt} is the sum of the population and the labor productivity growth rates.

Fossil fuel resources in each period are depleted to account for fossil fuel consumption in the previous period. Other sector-specific resources are assumed to grow at an exogenous rate. We furthermore assume energy efficiency to improve over time independently of changes in relative prices, through long-run autonomous energy efficiency improvement (AEEI) (Paltsev et al., 2005).

3.3.3 Calibration

Starting from the base year (the calibration of which is described in the paragraph entitled "Base year calibration" below), the economy's evolution is determined by recursively updating economic variables as described in the paragraphs entitled "Dynamic calibration" and "Income-dependent consumption patterns".

Base year calibration. Sub-national detail in China is parametrized using the full set of 2007 provincial input-output tables and provincial energy balance tables made available by the National Bureau of Statistics of the People's Republic of China (NBS) (2008, 2011). Each province comprises a representative urban and rural household, $\bar{h} \in \{\text{Urban, Rural}\}$, calibrated to provincial-level urban and rural consumption data. While household types likely also differ with respect to the sources from which they derive income, lack of data forces us to assume the composition of income for each household in a given province to be identical to the provincial average.

International regions are comprised of a single representative household. Economic and energy data as well as trade flows among all regions are parametrized using the GTAP (Global Trade Analysis Project) data base version 8 (Narayanan et al., 2012).

Calibration of the substitution elasticities in the model's production functions is based on the MIT Emissions Projection and Policy Analysis (EPPA) model (Paltsev et al., 2005): substitution between capital and labor is Cobb-Douglas; a value of 0.5 is assumed for both the elasticity of substitution between electricity and the fossil fuels composite, and between the value added and the energy composites in production. Following Caron et al. (2015), Armington trade elasticities are calibrated using estimates from GTAP, but the relative values of domestic and international elasticities are adjusted such that, in China, goods from other provinces are seen as closer substitutes to local (i.e. from within the province) goods than internationally sourced goods, which generates a border effect.

Dynamic calibration. Starting from the base year 2007, we run our model to generate projections for 2010, 2015, 2020, 2025 and 2030.

The initial population growth rate in China is assumed to equal 0.5% per year (United Nations, 2011). The initial labor productivity growth rate is calibrated such that it reproduces observed provincial economic growth rates between 2007 and 2010. Similar to Paltsev et al. (2005), we then assume an S-shaped evolution from the initial growth rate to the long-term steady state:

$$g_{rt} = (g_{rt_0} - g_{rT})\frac{1+\alpha}{1+\alpha e^{\beta(t-t_0)}} + g_{rT}\,, \tag{3.8}$$

where g_{rT} is the long-term (100 years) growth rate, assumed to be 2% per year. We set the values of α and β to 0.3 and 0.1. We assume the growth rate of land resources to be 2% per year. The savings rate is calibrated to base year data.

Physical energy quantities are obtained by province by first calibrating the model to observed economic growth, then adjusting energy efficiency to match 2010 observations for energy quantities based on National Bureau of Statistics of the People's Republic of China (NBS) (2012). The long-run AEEI for all sectors is assumed to be approximately 2% per year in Chinese provinces (Cao and Ho, 2010) and about 1% per year outside of China. In order to isolate the role of changing household consumption patterns, we assume no energy efficiency improvements on the part of households.

Income-dependent consumption patterns. Our main methodological contribution is an approach to capture income-dependent consumption patterns in a recursive-dynamic general equilibrium model, based on a hybrid demand system. The approach stems from the observation that changes in income in this class of models are primarily driven by productivity growth between periods whereas the within-period changes in income (due to energy or climate policies, for instance) are comparatively small. Allowing for flexibility in the shape of income-consumption relationships is thus most important for between-period changes. Our approach is furthermore based on our assumption that consumer behavior is myopic.[16] It allows us to calibrate the model to any empirically-estimated consumption path as a function of between-period changes in income. For a fixed set of relative prices, household expenditure shares follow the estimated Engel curves as income increases between periods. Within-period changes in income are captured by the Stone-Geary demand system which is appropriate for capturing the effect of comparatively small changes in income.[17]

For households within China, we proceed as follows.[18] First, we obtain the income of households of type u in province p at time t (I_{upt}) at benchmark prices, net of savings and demand for leisure. For

[16]It should be noted that our approach would not be suited for rational expectations models, as in our framework preference parameters are updated between model periods, and are thus not fixed in time.

[17]Chen et al. (2015) also employ an iterative calibration procedure for Stone-Geary preferences. Our approach, however, differs significantly, as we target flexible Engel curves, rather than point estimates of income elasticities, and we account for the effect of the preference calibration on real household income levels. The scope of our application also differs largely, as we target a wide range of goods, while Chen et al. (2015) calibrate income elasticities for food and agricultural products only.

[18]Outside China, preferences are assumed to be homothetic. The corresponding minimum consumption levels are thus assumed to be zero (i.e., $c^{\star}_{j\bar{h}rt} = 0$ for all regions r outside China).

this, we run the model with homothetic preferences (i.e., with minimum consumption parameters $c^{\star}_{jupt}$ set to zero), and then divide nominal income levels by each consumer's price index.

Second, we project consumption at benchmark prices for each good j, point in time t, household type u and province p, in order to follow the estimated income-expenditure relationships. The aggregation of goods in the GE model differs from that of the data used for estimation described in Section 3.2. As is usually the case, aggregation schemes from input-output tables do not match those of consumption data derived from household surveys. We thus map the relationships between income and expenditure estimated in Equation (3.3) ($\hat{E}(I)_{iu}$) to corresponding relationships for goods in the GE model ($\hat{E}(I)_{ju}$). Some goods in the model are composites of goods used for the estimation. The Engel curve for natural gas (GAS), for example, is constructed by aggregating the estimated curves for bottled gas and pipeline natural gas. Other goods in the model map directly, such as the agricultural sector (AGR), for which the Engel curve corresponds to the estimated curve for food.[19]

We use the Engel curves $\hat{E}(I)_{ju}$ to look up the expenditures corresponding to the level of household income: $\hat{E}(I_{upt})_{ju}$. We then compute the proportional change in consumption for each good, and scale the level of benchmark consumption, x_{jupt_0}, by this factor. This delivers the demand projections y_{jupt} at benchmark prices:

$$y_{jupt} = \frac{\hat{E}(I_{upt})_{ju}}{\hat{E}(I_{upt_0})_{ju}} x_{jupt_0} \, . \tag{3.9}$$

Third, we calibrate consumer preferences such that at the level of income I_{upt} and benchmark prices, consumption for each good j is equal to the projected (empirically-grounded) level y_{jupt}. The proposition below shows that Stone-Geary preferences can indeed be calibrated as desired.

Proposition 3.1 *Assume $t > t_0$. The following calibration ensures that at benchmark prices and at the income prevailing at time t, I_{upt}, the consumption of each good corresponds to the projections based on the estimated Engel curves, y_{jupt}:*

$$c^{\star}_{jupt} = \left(x_{jupt_0} \frac{I_{upt}}{I_{upt_0}} - y_{jupt} \right) \left(\frac{I_{upt}}{I_{upt_0}} - 1 \right)^{-1} . \tag{3.10}$$

Proof. At benchmark prices, the demand function for good j by household u in province p at time t is given by

$$x_{jupt}(\mathbf{p}_{rt_0}, I) = (x_{jupt_0} - c^{\star}_{jupt}) \frac{I - \sum_{j'} p_{j'pt_0} c^{\star}_{j'upt}}{I_{upt_0} - \sum_{j'} p_{j'pt_0} c^{\star}_{j'upt}} + c^{\star}_{jupt} \, . \tag{3.11}$$

Since $\sum_j p_{jpt_0} y_{jupt} = I_{upt}$, the following holds for the $c^{\star}_{jupt}$, from Equation (3.10): $\sum_j p_{jpt_0} c^{\star}_{jupt} = 0$. For this calibration, the subsistence consumption parameters at benchmark prices sum to zero in any

[19] There are some sectors for which we cannot construct a corresponding Engel curve based on our estimates. This concerns four of the twelve goods in the GE model which are primarily used as intermediate goods (CRU, OMN, EIS and WTR). For these goods, household consumption is very small and we assume it to scale in proportion to income.

given period, since household expenditure across all goods equals income. We use this to simplify Equation (3.11): $x_{jupt}(\mathbf{p}_{pt_0}, I \equiv I_{upt}) = (x_{jupt_0} - c^\star_{jupt})\frac{I_{upt}}{I_{upt_0}} + c^\star_{jupt} = x_{jupt_0}\frac{I_{upt}}{I_{upt_0}} + c^\star_{jupt}(1 - \frac{I_{upt}}{I_{upt_0}})$, then insert Equation (3.10), thus obtaining the following equality: $x_{jupt}(\mathbf{p}_{pt_0}, I \equiv I_{upt}) = y_{jupt}$. □

Fourth, we run the model with the newly calibrated preferences, obtaining new real income levels I'_{upt}. These are not generally equal to initial income (i.e., $I'_{upt} \neq I_{upt}$), both because the same prices and nominal income levels correspond to different levels of real income for different utility functions, and because different preferences will lead to a different market equilibrium, influencing prices and income. We therefore re-calibrate household preferences following steps two and three above, and re-run the model iteratively until real income has converged.[20] The resulting preferences are such that, at the levels of real income at a given time t and for benchmark prices, consumption levels correspond exactly to the projections based on the estimated Engel curves.

The use of Stone-Geary preferences in our hybrid approach is motivated by the fact that its implementation within a general equilibrium model is simple,[21] and delivers an approximation of the local properties of the Engel curve estimates. Additionally, since proportional changes in utility levels are equivalent—in welfare terms—to proportional changes in income at benchmark prices[22] (as our calibration ensures that $\sum_j p_{jpt_0} c^\star_{jupt} = 0$), our approach provides a metric to consistently evaluate the welfare impact of policy shocks across periods.

3.4 Baseline projection

We first summarize the effect of income-driven changes in consumption patterns on baseline projections of economic activity, energy use, and emissions. In order to isolate the role of these changes, we compare the results for calibrated non-homothetic preferences (which we will refer to as the *Calibrated* case) and the results for an alternative counterfactual case in which preferences are assumed to be homothetic.[23] We refer to this counterfactual as the *Proportional* case, since homothetic preferences imply proportional income-consumption relationships for all goods.

3.4.1 The composition of household consumption

Table 3.2 displays the composition of household consumption at the national level, both in the base year and in the final year of our simulation, 2030. In the proportional case, the average expenditure shares on energy goods as well as on agricultural goods increase over time. This indicates that the effect of rising energy prices, driven by the depletion of fossil fuel reserves, and rising agricultural goods prices, caused by the relative scarcity of land, outweighs the effect of substitution away from these increasingly expensive goods in determining household expenditure shares. Accounting for the empirically calibrated income-expenditure relationships reverses this trend, both for agricultural

[20] Convergence in our specific case is achieved after four iterations, as detailed in Appendix D.1.

[21] See Markusen and Rutherford (1995) for an example in a simple static CGE model.

[22] Equivalently, proportional changes in utility levels correspond to the equivalent variation at benchmark prices, divided by benchmark income.

[23] This corresponds to the case with minimum consumption parameters $c^\star_{j\bar{h}rt}$ set to zero, implying within-period CES preferences calibrated to shares that remain unchanged between periods.

goods as well as for all energy goods with the exception of oil. While the average consumption share of oil in the final year is higher than in the base year, it is nonetheless lower than in the proportional baseline. These results illustrate that income-driven shifts away from energy and agricultural goods have a stronger effect on consumption patterns than price-driven effects, due to the low income elasticities for these goods and the rapid growth in household income levels.

Manufacturing, transportation, and services represent a roughly constant share of average household consumption under homothetic preferences, as they see modest fluctuations in prices. However, the calibrated non-homothetic preference projection shows manufacturing shrinking in relative importance within household consumption baskets, while the average share of high income elasticity services strongly increases and that of transport more than doubles.

Table 3.2 also shows that in the proportional case, the standard deviation (across provinces and household types) of the expenditure shares of energy and agricultural goods increase in time, as differences between households are magnified by the increasing prices, while for other goods the values are roughly constant. The variation in household expenditure shares thus increases on average, suggesting divergence in relative consumption baskets across households.

The story is different in the calibrated case: the standard deviation in expenditure shares is lower for most goods, with the notable exception of transportation and services. As certain goods gain and others lose in importance on average, differences between households are in the first case amplified and, in the second, reduced. It is therefore not clear *a priori* if changes in the composition of consumption driven by rising incomes will lead to a divergence or a convergence in the energy and emissions-intensity of consumption baskets and therefore in the distribution of welfare impacts caused by climate policy-driven changes in relative prices of consumption goods. We investigate this question later in the essay.

3.4.2 Energy use and emissions

Table 3.3 displays China's demand for energy for each energy type, in 2007 and in 2030, for both the calibrated and proportional cases. In the calibrated case, household demand for energy is dramatically reduced compared to the proportional case. The relative reduction in total national demand for energy is smaller: household demand for energy only represents a fraction of national demand, which also includes intermediate demand (as substantial part of which goes to exports), government, and investment demand, all of which are particularly high in China.

Total household consumption of coal in 2030 is projected to be lower than its base year value, as expected given that its Engel curve estimates suggest it to be an inferior good for all but the poorest rural households, and is 81% lower compared to the prediction with homothetic preferences. The least affected energy good, refined oil, sees its household demand reduced by approximately 5% relative to the baseline projection with homothetic preferences. Overall, household demand for energy in 2030 is 58% lower than in the proportional case. National demand is 6% lower.

Changes in household energy demand by type between the proportional and calibrated cases do not translate exactly to equal changes in national demand. There are two reasons for the discrepancy.

Table 3.2: Baseline household expenditure share by good (% of total consumption)

	Average			Standard deviation		
	2007	2030		2007	2030	
Preferences:		Proportional	Calibrated		Proportional	Calibrated
COL	1.0	1.4	0.3	1.1	1.6	0.5
GAS	0.2	0.3	0.1	0.4	0.6	0.2
OIL	2.1	3.1	2.9	1.6	2.5	2.3
ELE	3.4	3.5	1.2	2.1	2.2	0.8
AGR	15.3	17.2	12.6	8.6	9.4	7.7
MAN	32.1	31.0	24.4	8.5	8.3	7.0
TRN	2.6	2.6	6.1	1.2	1.2	2.9
SER	35.6	33.7	43.6	8.5	8.5	9.8

Notes: Averages and standard deviations computed across provinces and household types, weighted by population.

Table 3.3: Baseline national and household-level demand for (secondary) energy by energy type

	Quantity								Price index		
	Household demand				National demand						
	2007	2030			2007	2030			2030		
Prefs.:		Prop.	Calib.	Diff.		Prop.	Calib.	Diff.	Prop.	Calib.	Diff.
COL	77	303	58	-81.0%	836	2332	2215	-5.0%	1.57	1.43	-8.5%
GAS	15	69	24	-65.2%	72	292	259	-11.4%	1.13	1.05	-6.7%
OIL	46	180	170	-5.5%	504	1183	1209	+2.2%	1.72	1.73	+0.6%
ELE	73	366	135	-63.1%	519	1816	1599	-12.0%	0.91	0.88	-3.0%

Notes: Units: Mtce (Quantity); prices relative to 2007 levels (Price index). Note that COL, GAS, and OIL data excludes demand from the electricity sector for these inputs, to avoid double counting. Price index constructed as an average of regional prices relative to benchmark, weighted by provincial energy consumption of each energy type.

Table 3.4: Baseline national and household-level emissions (Billion tons CO_2/year)

	Direct household emissions			National emissions		
Preferences:	Proportional	Calibrated	Difference	Proportional	Calibrated	Difference
2007	0.8	0.8	0.0%	6.4	6.4	0.0%
2010	1.0	0.8	-19.7%	7.7	7.5	-2.3%
2015	1.4	0.8	-38.4%	10.0	9.5	-4.9%
2020	1.9	0.9	-50.1%	12.7	11.9	-6.4%
2025	2.3	1.0	-56.5%	14.8	13.7	-7.2%
2030	**2.7**	**1.1**	**-60.8 %**	**16.3**	**15.0**	**-7.5%**

Notes: Direct household emissions comprise emissions from the domestic combustion of fossil fuels, as well as the emissions embodied in electricity consumed by households.

First, lower demand on the part of households depresses prices for most energy goods, causing an increase in the intermediate demand from the other sectors in the economy (which use energy goods as inputs). This mitigates the effect of income-driven consumption shifts. Exports are also, to a lesser extent, affected.

Second, input-output relationships imply that, abstracting from effects on relative prices, differences in demand for non-energy goods affect demand for energy goods as production inputs. Refined oil is, for example, an input to the transportation sector, the share of which in household consumption more than doubles between 2007 and 2030: while direct household demand for oil is slightly reduced by the consumption shift relative to the proportional case, the national demand for oil is higher. This is also reflected in the price of oil, which increases slightly in the calibrated relative to the proportional case.

Changes in the household consumption of energy and non-energy goods affect the levels of CO_2 emissions in the Chinese economy, both directly through emissions caused by the burning of fossil fuels and the consumption of electricity at the household level, and indirectly through changes in the composition of production to satisfy changing consumer preferences for non-energy goods. Table 3.4 summarizes these changes. In 2030, direct household CO_2 emissions fall by 1.67 billion tons as a result of reduced household consumption of fossil fuels and electricity, a 61% reduction compared to the homothetic case. National CO_2 emissions in the final year are roughly 8% lower compared to the proportional case, corresponding to a reduction of 1.22 billion tons of CO_2 emissions—just slightly larger than Japan's emissions in 2011.

3.5 Policy analysis

We now study the interaction between income-driven changes in the composition of consumption and climate policy. For that we design a simple stylized policy which is in line with current policy proposals. We consider a set of national CO_2 emissions targets, implemented by an economy-wide national CO_2 price. As above, we compare the outcomes of the proportional and calibrated cases.

By design, we choose emissions levels under policy to be identical in both cases. This allows a direct comparison of the costs of achieving a fixed environmental outcome. Since baseline emissions are lower in the calibrated case, the absolute emissions reductions will be lower compared to the case with homothetic preferences. Income-driven shifts in the composition of household consumption thus indirectly affect the stringency of the climate policy.

Figure 3.3 illustrates baseline emissions for the calibrated and proportional cases, as well as the targeted emissions path under policy. The CO_2 emissions targets correspond to an emissions trajectory that results from simulating a 4% yearly reduction in CO_2 intensity in the proportional case, consistently with current target emissions paths to achieve a CO_2 emissions plateau by 2030 (Zhang et al., 2016). This climate policy only targets fossil fuel-based CO_2 emissions, such that the results of the following sections can be considered similar to those of a policy that would target energy use directly (which is associated with other externalities such as local air pollution).

The allocation of the revenues from carbon pricing can have important consequences for incidence. In our analysis, unless specified otherwise, we consider a redistribution scheme in which the revenue is returned to households in each province lump sum proportional to their baseline income. Such revenue recycling does not affect the distribution of income, shutting off this channel of incidence and keeping the focus on differences in household characteristics.

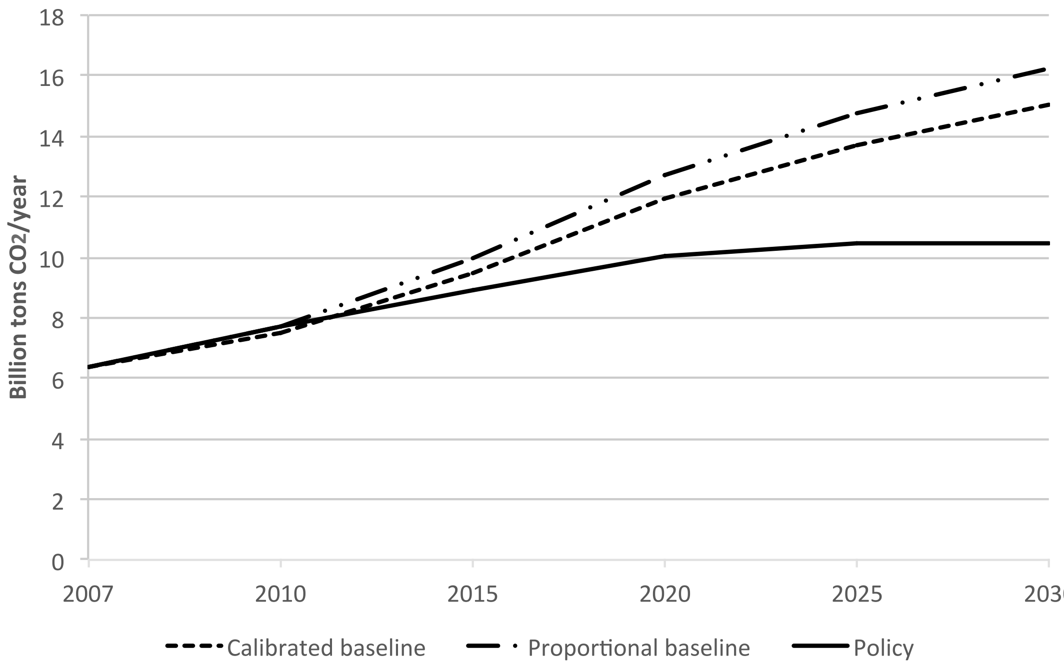


Figure 3.3: National CO_2 emissions for baseline and policy, for homothetic (i.e., Proportional) and for non-homothetic (i.e., Calibrated) preferences.

3.5.1 Energy and emissions reductions, carbon prices, and average policy impacts

Table 3.5 reports the effect of climate policy on national emissions, as well as the associated carbon price and policy impacts.

As expected, the emissions reduction (in tons) required to reach this emissions path is smaller than in the proportional case. In 2030 the difference amounts to 1.2 billion tons of CO_2 per year—21% less than the necessary emissions reductions which a model with homothetic preferences would suggest. The reduced stringency of policy translates to a 35% lower carbon price in 2030.[24]

The policy significantly affects energy use, as can be seen from Figure 3.4. We focus on the results for calibrated preferences. Primary energy demand in 2030 is reduced by 28% relative to the baseline. Under the policy, coal use peaks around 2025, while energy consumption from oil, gas and renewable sources continues to increase, albeit from a smaller base. By comparing the 28% reduction in primary energy to the 30% reduction in CO_2 emissions in the policy against

[24] The carbon price corresponds to the shadow price of the model's carbon constraint.

Table 3.5: The effect of climate policy on national emissions, the associated carbon price and the average consumption loss

	Emissions					Policy impacts	
	Targets	Reductions		Price		Consumption loss	
Prefs.:		Calibrated	Proportional	Calibrated	Proportional	Calibrated	Proportional
2015	8.9	0.6	1.1	8.1	16.0	0.3%	0.9%
2020	10.0	1.9	2.7	26.9	42.7	1.1%	2.6%
2025	10.4	3.3	4.3	49.5	74.3	2.0%	4.6%
2030	**10.5**	**4.6**	**5.8**	**72.7**	**111.7**	**2.9%**	**6.3%**

Notes: Units: Emissions: Billion tons CO_2/year. Price: USD per ton of CO_2. Averages are weighted by population. Here and elsewhere in the essay, an exchange rate of 7.6 CNY per USD has been applied (2007 average).

the baseline projection, we find that substitution away from energy use rather than substitution among energy sources is the main driving factor in the overall reduction of carbon emissions in our simulations.

We measure welfare impacts (expressed as equivalent variation relative to real income) in terms of changes in welfare from consumption and will thus also simply refer to it as *consumption loss*.[25]

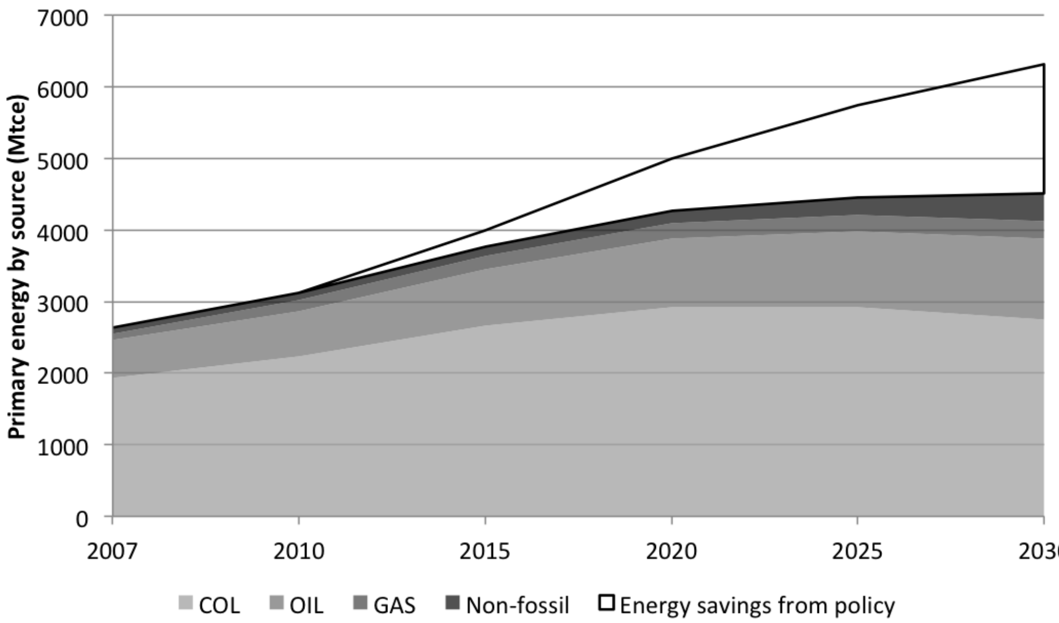


Figure 3.4: Primary energy by energy source for calibrated preferences.

[25]We report the loss in utility from consumption rather than loss in total utility, which in the model also includes utility from leisure and savings. These are introduced into utility to generate positive within-period labor and capital supply elasticities, and their contribution to welfare is thus arbitrary. In any case, we find that both the patterns of incidence and the effect of our preference calibration on the distribution of impacts are robust to the use of total household welfare.

Another plausible measure of household welfare would include government expenditures. Such a metric attenuates welfare losses in virtually all provinces, compared to private utility alone, whilst leaving the pattern of incidence largely unchanged. The effect of income-driven household consumption shifts on the distribution of welfare impacts is also not qualitatively different.

Assuming a 4% annual discount rate, the net present value of the total consumption loss to 2030, accounting for the recycled revenue, amounts to 395 billion 2007 USD for the calibrated case. Cumulative emissions reductions amount to 43 billion tons of CO_2. The average welfare cost thus amounts to 9.2 USD per ton abated. In the proportional case, on the other hand, the costs amount to 17.8 USD per ton abated, reflecting rapidly increasing marginal abatement costs.[26]

The average consumption loss under policy, for each model period following the implementation of the policy, is reported in Table 3.5.[27] The results show that ignoring income-driven shifts overestimates the average welfare costs of reaching a given emissions trajectory: higher baseline emissions in the proportional case lead to a more stringent policy, associated with higher carbon prices. In addition, the income-driven shift away from energy goods reduces the CO_2 intensity of household consumption, such that households are less exposed to the carbon price.

3.5.2 Distributional impacts of policy

We start by describing the distribution of welfare impacts from policy. The paragraphs entitled "The effect of changing consumption patterns on the variation of policy impacts" and "Relative winners and losers" will then examine the drivers of these results, based on a welfare decomposition expounded in the paragraph entitled "Decomposing the variation in welfare impacts".

We first describe the distribution of welfare impacts by pooling households into subsets, allowing differentiation along various policy-relevant dimensions. We focus first on average consumption loss within subgroups in the calibrated preferences case. Results are displayed in Table 3.6.

Under our assumption of income-proportional redistribution of CO_2 pricing revenues, we find that rural households are on average more negatively affected than urban households; households in low income provinces are more affected than households in medium and high income provinces (as can also be seen for average provincial-level impacts in the left panel of Figure 3.5); households in provinces with high coal production more affected than households in provinces with low coal production. From the right panel of Figure 3.5, we can also see that households in provinces with a high carbon intensity of GDP are impacted more strongly. The geographic distribution of impacts also varies widely (as can also be seen from Figure 3.6): Western provinces suffer the most, Eastern provinces suffer the least, with Central provinces (with the exception of Shanxi, a major coal producer and exporter, which is highly affected) falling in between.

We also investigate sensitivity to redistributing the revenues from CO_2 pricing in proportion to provincial emissions rather than household income. Results are reported in the middle columns of Table 3.6. The difference in welfare impacts between relative winners and losers is lower for pollution-proportional than for income-proportional redistribution, as provinces with high intensity

[26] For comparison, Rausch and Karplus (2014), also using a 4% discount rate, find for the US that cumulative CO_2 reductions of 50 billion tons by 2050 result in 2005 net present welfare costs of approximately 5 USD per ton. For 100 billion tons of cumulative reductions, they find welfare costs of roughly 15 USD per ton. The magnitude of welfare costs in our model is comparable to these results.

[27] The consumption loss is weighted by population. Note that since household incomes differ across provinces and household types, population weighted % changes differ from the % changes in the totals.

Table 3.6: % consumption loss with respect to the baseline for alternative redistribution schemes of revenues from CO_2 pricing, population-weighted average values (standard deviations in brackets), with households divided into subsets

Preferences:	Calibrated				Proportional	
Redistribution:	Income-proportional		Pollution-proportional		Income-proportional	
	2015	2030	2015	2030	2015	2030
All households	0.3	2.9	0.3	2.6	0.9	6.3
	(1.0)	(5.1)	(0.5)	(2.9)	(2.0)	(8.1)
Urban	0.2	2.3	0.1	2.2	0.7	5.0
Rural	0.4	3.4	0.4	3.0	1.2	7.5
East	0.0	0.7	0.0	0.7	0.3	2.9
Central	0.6	4.6	0.5	4.2	1.5	9.0
Central*	0.3	2.9	0.4	3.5	0.9	6.7
West	0.5	4.1	0.4	3.5	1.3	8.2
Low coal production	-0.3	0.0	0.0	1.9	-0.4	0.9
Medium coal production	0.3	2.8	0.2	2.2	1.0	6.6
High coal production	2.7	15.9	1.5	9.3	5.8	26.8
High income	0.1	1.9	0.1	2.0	0.5	4.4
Medium income	0.2	2.2	0.1	1.8	0.8	5.5
Low income	0.6	4.5	0.5	4.1	1.5	9.0

Notes: Coal production: High (above 5 % value of base year output from COL), Low (below 1 % value of base year output from COL). Eastern provinces: BJ, FJ, GD, HI, HE, JS, LN, SD, SH, TJ, ZJ; Central provinces: AH, HL, HA, HN, HB, JX, JL, NM, SX; Western provinces: CQ, GS, GX, GZ, NX, QH, SN, SC, XJ, YN. Central*: without SX.

of emissions, which tend to be more negatively affected by the policy, benefit from a higher share of revenues from the carbon price.

Compared to calibrated preferences, average consumption losses in the proportional case are more pronounced in almost all of the considered subsets. The difference in impacts between subsets is also larger. As an example, low income households in the calibrated case suffer consumption loss which is 2.6% higher than high income households in 2030, instead of 4.6% in the proportional case.

Turning to the distribution of these losses across all households, Figure 3.7 displays the impacts of policy across the whole set of urban and rural representative households in each province, in 2015 and 2030. In both years, under proportional preferences, the figure shows that the average consumption loss is not only higher, but that losses are also more spread out.

As can be seen on the second line of Table 3.6, standard deviations increase in time in all cases, as the increasing stringency of the climate policy magnifies relative differences among household types and provinces. For a given year, the variation of impacts is reduced under the emissions-based allocation of carbon price revenues, confirming that this alternative redistribution scheme is more neutral. Standard deviations in the calibrated case are lower than in the proportional case, implying that—compared to an incidence analysis that would abstract from the income-driven consumption shifts—the variation of welfare impacts is lower. The following sections further decompose these

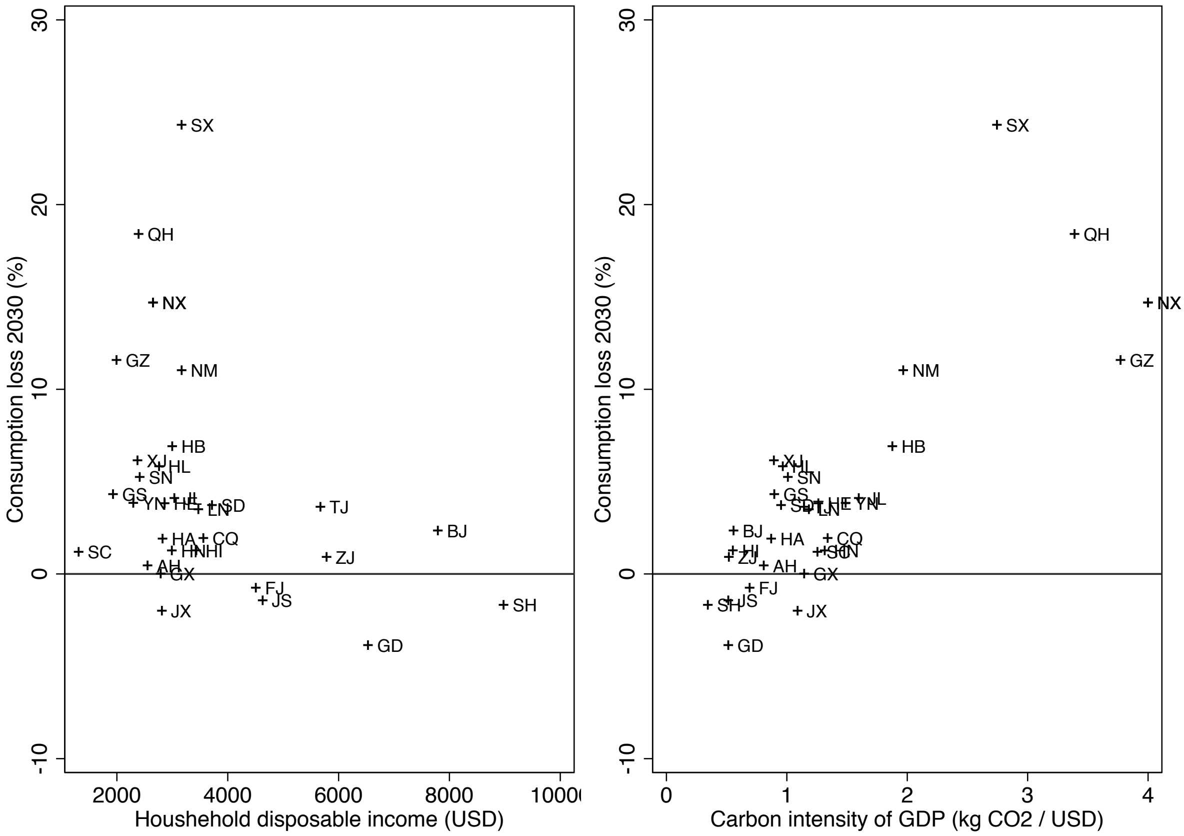


Figure 3.5: Relationship between consumption loss in 2030 and base-year household disposable income / carbon intensity of GDP for calibrated preferences and income-proportional revenue redistribution.

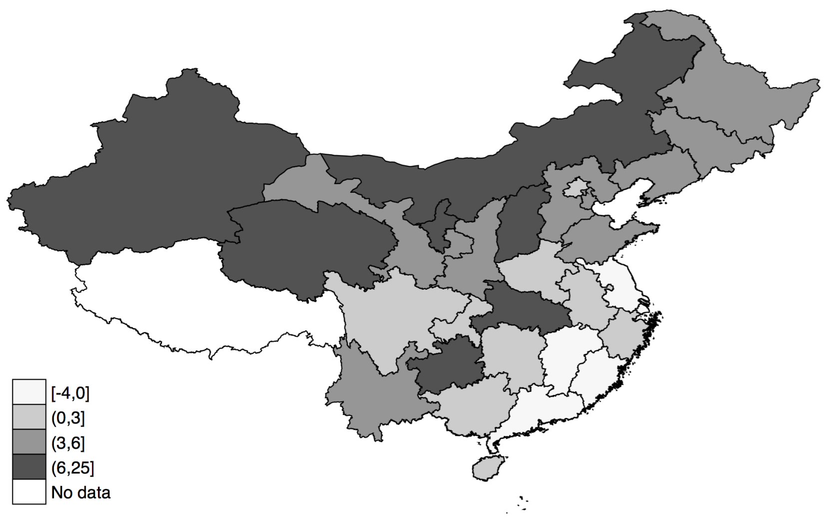
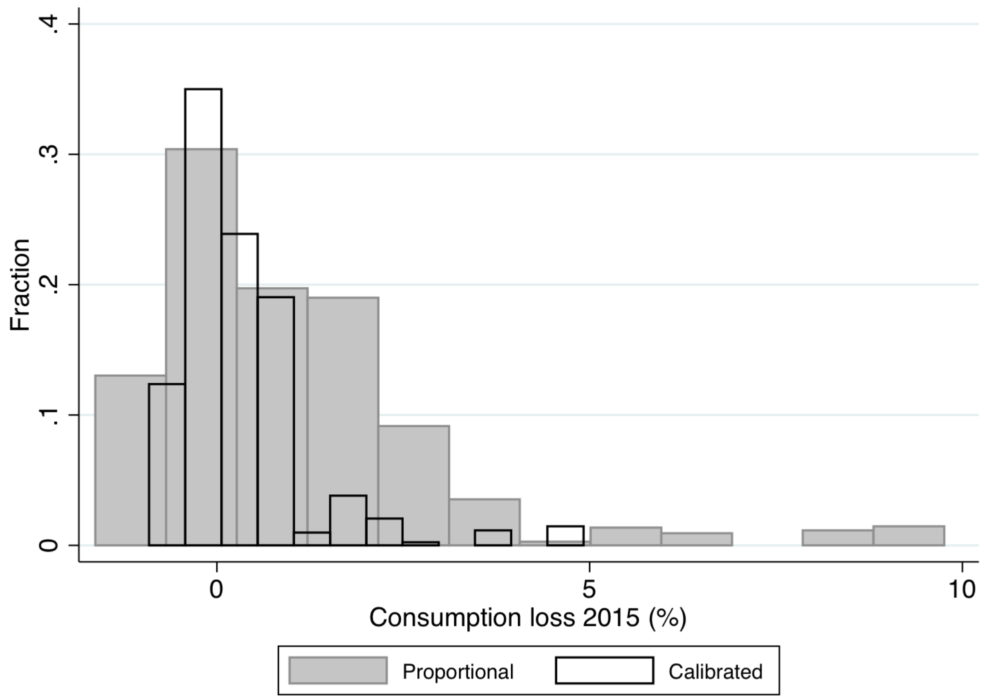
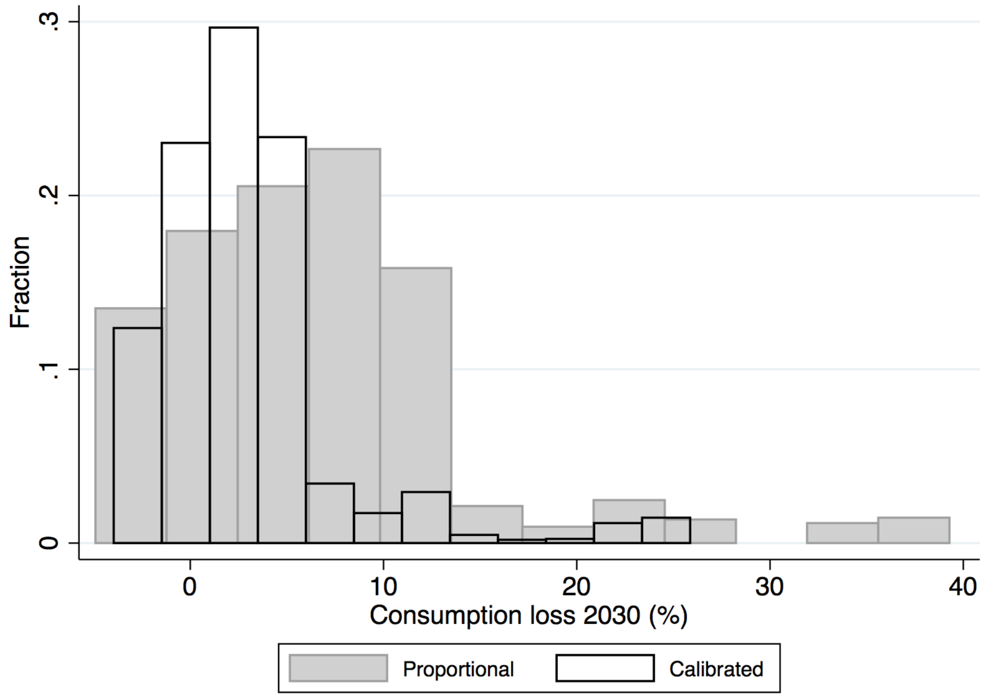


Figure 3.6: Geographic distribution of % consumption loss in 2030 for calibrated preferences and income-proportional revenue redistribution.



(a) 2015



(b) 2030

Figure 3.7: Population-weighted distribution of welfare impacts (across provinces and urban/rural types) of policy for calibrated and proportional preferences.

results.

Decomposing the variation in welfare impacts. The welfare results displayed so far are general equilibrium estimates and correspond to the equivalent variation associated with changes in utility from consumption, relative to real baseline income. In order to identify the drivers of variation in welfare impacts, we perform an *ex-post* decomposition which approximates the equivalent variation using the change in consumer surplus caused by changes in prices, quantities consumed and income, as in West and Williams III (2004). We then measure the contribution of each of these components to the variation in impacts across households under both preference calibrations.

Similarly to Williams III et al. (2015), we assume the demand curve to be linear in the relevant range, such that the change in consumer surplus relative to income is given by:

$$\frac{\Delta S}{I} = \underbrace{-\sum_j \frac{\Delta p_j}{p_j}\theta_j}_{=\text{First order consumption}} \underbrace{-\frac{1}{2}\sum_j \frac{\Delta p_j}{p_j}\frac{\Delta x_j}{x_j}\theta_j}_{=\text{Second order consumption}} + \underbrace{\frac{\Delta I}{I}}_{=\text{Income}}, \tag{3.12}$$

where province and household indices are suppressed for simplicity, $\Delta I = I^{Policy} - I$, with I the baseline income (net of savings and leisure consumption) and I^{Policy} the income under the policy, $\theta_j = \frac{p_j x_j}{I}$ the baseline expenditure share on good j, $\frac{\Delta p_j}{p_j}$ the proportional change in the price of good i and $\frac{\Delta x_j}{x_j}$ the proportional change in the consumption of good j.[28] We find that the change in consumer surplus from Equation (3.12) delivers a good approximation of the general equilibrium estimates of equivalent variation, for the proportional as well as the calibrated cases.[29]

The first two terms on the right-hand side of Equation (3.12) vary across households because of differences in impacts caused by policy-driven changes in relative consumption good prices. We will refer to these two terms as "consumption side" impacts. The third term varies because of differing proportional income changes. We will refer to this term as "income side" impacts. It should be noted that the goal of the decomposition is not to compare the magnitude of consumption and income side impacts at the household level, which are not meaningful (Fullerton and Metcalf, 2002), but to capture the variation in each term across households. Furthermore, distributional impacts are only caused by changes in relative prices, as the choice of numeraire is of no economic consequence.

The distribution of the first term ("first-order consumption") across households captures differences in welfare impacts caused by policy-induced changes in relative consumption good prices and differences in consumption patterns among households, holding consumption quantities and incomes constant. More precisely, this term reflects the within-household covariance between proportional price changes and consumption shares across sectors: households that more intensively consume goods experiencing larger price increases under the policy will be more adversely affected. If the price changes were the same in all provinces, and if all households were to consume goods with the

[28] Similarly to Williams III et al. (2015) we normalize prices such that the average consumer price index, $p_{avg,t}$, remains constant. It is defined as follows: $p_{avg,t} := \sum_{jup} \frac{x_{jupt}}{\sum_{u'p'} I_{u'p't}} p_{jpt}$, where I_{upt} is the (pre-policy) income of household type u in province p at time t and x_{jupt} is the (pre-policy) consumption of good j.

[29] This can be seen in Figure D.1 in the Appendix.

same intensity (i.e., assuming equal expenditure shares across all households), then the variability in impacts through this channel would be trivially zero.

The second term ("second-order consumption") captures second order effects that mitigate the first order term through changes in consumption caused by the policy: households can at least partially substitute away from consuming goods experiencing a relative price increase.

The policy affects household incomes in many ways (among others through returns to factors of production and recycled carbon price revenue). Further decomposing the third term ("income side") is not the focus of our essay, but we are interested in comparing the variation in income-side effects to the variation in consumption-side effects, for the cases with homothetic and non-homothetic preferences.

Equation (3.12) highlights a number of channels through which income-driven consumption shifts may affect the distribution of welfare impacts: expenditure shares (θ), price changes under policy ($\frac{\Delta p}{p}$), income changes ($\frac{\Delta I}{I}$) and behavioral responses ($\frac{\Delta x}{x}$). We distinguish two types of effects. On the one hand, household expenditure shares differ between the calibrated and proportional cases. This will cause given changes in relative consumption good prices under policy to translate to different distributions of welfare impacts. We will refer to this as the *direct effect* of changing consumption patterns.

On the other hand, income-driven consumption shifts affect baseline emissions, and hence the absolute amount of emissions reductions under the policy. The differing stringency will in turn affect consumption good price and income changes under policy, through the economy-wide CO_2 price. In addition, abstracting from differences in the absolute emissions reductions between the calibrated and the proportional cases, non-homothetic preferences also affect equilibrium prices through differing patterns of demand compared to the homothetic case, and hence consumption good and factor price changes under policy. We will refer to these effects linked to differing relative consumption good prices and incomes between the calibrated and proportional cases collectively as the *indirect effect* of changing consumption patterns.

Whereas the direct effect affects the consumption side only, the indirect effect affects both the consumption and the income side.

The effect of changing consumption patterns on the variation of policy impacts. The standard deviations of each term in Equation (3.12) across all households in the model (i.e., both urban and rural households in all provinces) are summarized in Table 3.7 and identify the channels which drive the heterogeneous burdens of climate policy. We observe that, for both 2015 and 2030, the income channel has the highest standard deviation, followed by the first order consumption channel. The variation introduced through the second-order consumption channel is significantly smaller, and will thus be ignored in the following. All standard deviations are increasing in time, driven by the increasing stringency of the policy.

We also find that both consumption and income channels exhibit lower variability under calibrated preferences, although the difference is greater for the consumption channel. On the income side,

Table 3.7: % consumption loss ($-\Delta S/I$): standard deviations by channel.

	1st order consumption		2nd order consumption		Income	
Preferences:	Calib.	Prop.	Calib.	Prop.	Calib.	Prop.
2015	0.2	0.5	0.0	0.0	1.0	1.9
2030	1.1	3.1	0.1	0.6	5.2	7.8

differences between the proportional and calibrated cases are caused by the indirect effect only. The main driver of this result is the lower carbon price in the calibrated case caused by the lower policy stringency, which moderates differences between households.

The finding that changing consumption patterns affect the variation of impacts on the consumption side more than on the income side suggests that the direct effect, in addition to the indirect effect, further moderates the variation of consumption side impacts.

To single out the direct effect of changing consumption patterns on the distribution of welfare impacts of policy, we compute the variability of impacts in a hypothetical economy in which relative goods prices, carbon prices, and incomes are unaffected by household preferences. In order to compare the magnitude of the direct effect across time we furthermore assume that relative price changes are fixed at the 2030 level, thus abstracting from the increasing policy stringency. We therefore compute the standard deviation, across both urban and rural households in each province, of the following measure:

$$\sum_j \left.\frac{\Delta p_j^P}{p_j^P}\right|_{2030} \left.\theta_j^{P/C}\right|_t, \tag{3.13}$$

where $\theta_j^{P/C}$ is the baseline expenditure shares for good j in the proportional (P) and calibrated (C) cases, respectively, and $\left.\frac{\Delta p_j^P}{p_j^P}\right|_{2030}$ is the proportional change in the price of good j in 2030, for the proportional case. Table 3.8 displays the results.

Table 3.8: % consumption loss through first-order consumption effects, holding price changes fixed at 2030 proportional case values: standard deviations

Preferences:	Proportional			Calibrated		
Year:	2007	2015	2030	2007	2015	2030
	2.4	2.6	**3.1**	2.4	1.7	**1.5**

For fixed policy-driven price changes, the variation in direct consumption impacts of climate policy in the proportional case increases in time. This result reflects the finding from Section 3.4.1 that, in the proportional case, the variation in household expenditure shares increases slightly, due to relative price changes in the baseline.

The picture in the calibrated case is very different: the direct consumption impacts of policy converge rapidly. This implies that income-driven changes in household consumption patterns are significantly stronger than the offsetting effect of changes in relative consumption good prices.

Overall, differences in the temporal evolution of consumption patterns causes the standard deviation to be reduced by more than half relative to the proportional case—from 3.1% to 1.5%. Comparing the 1,5% standard deviation in 2030 to the 1.1% from Table 3.7 suggests that a large part of the difference in the variation of consumption side impacts between the calibrated and proportional cases is due to the direct effect and is thus independent of differences in relative consumption good price changes under policy (caused, among other things, by the differing policy stringency).

Since relative price changes caused by policy are—to a first approximation—proportional to differences in embodied carbon intensity, the convergence in impacts due to the direct effect implies that differences in the embodied energy and carbon intensity of consumption baskets across households decline. In other words, household consumption patterns converge, from the point of view of embodied carbon emissions, due to changing consumption patterns driven by rising income levels in China.

Finally, in addition to the variation of impacts through each channel, the variation of total impacts is also determined by the correlation between the channels. In the proportional case, the correlation between the first order consumption and income channels is strong and positive (0.39): households that experience above-average income losses are also more impacted on the consumption side. In the calibrated case, on the other hand, this relationship is reversed: the correlation is negative (-0.26), thus further reducing the variation of total impacts compared to the proportional case.

At least two effects drive the sign reversal. First, provinces that are more negatively affected on the income side tend to consume a higher fraction of energy goods. The consumption shift disproportionately reduces the expenditure shares on energy of these households, thus reducing the correlation between income and consumption channels. Second, the policy depresses prices of non-energy goods more in provinces that are more negatively affected on the income side. This effect interacts positively with the consumption shift, as expenditure shares on most non-energy goods increase with rising income levels.

Relative winners and losers. The welfare decomposition of Equation (3.12) also allows a detailed description of the factors that determine the ranking between more and less affected household types under climate policy, and how this ranking is influenced by the income-driven shifts in the composition of household consumption.

On the income side, the ranking is mostly unaffected by household preferences, as illustrated in Figure 3.8. This is also reflected by Spearman's coefficient of rank correlation of income side impacts between the calibrated and proportional cases, that amounts to 0.990. The indirect effect of shifting consumption patterns affects household types similarly, despite the lower standard deviation found above.

Still, interesting patterns emerge. Households in heavy coal-producing provinces such as Shanxi (SX), Ningxia (NX), Guizhou (GZ), and Inner Mongolia (NM) are more negatively impacted on the income side than the national average. Climate policy, by depressing demand for coal, strongly impacts returns to factors of production in these provinces, reducing household incomes. Households in relatively developed provinces such as Guangdong (GD) and Shanghai (SH), on the other

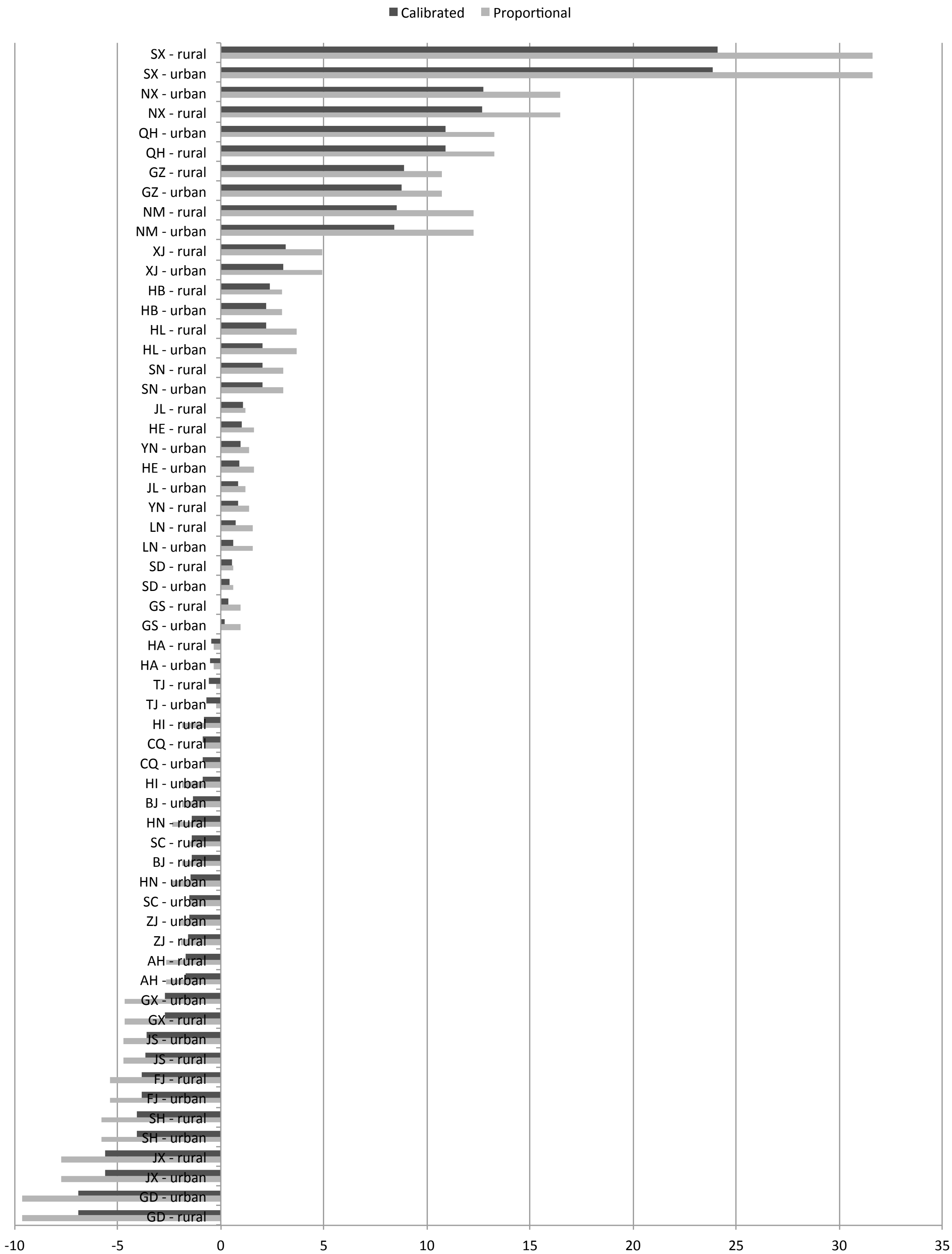


Figure 3.8: % welfare loss in 2030 through the income channel (relative to the population-weighted average impact through this channel).

hand, experience increases in income relative to the national average, due to the relatively clean output mix in these provinces. Within provinces, urban and rural households are affected equally through the income channel,[30] since the composition of their income is assumed to be identical in the model. We are unaware of data describing the differential sourcing of income within provinces.

On the consumption side, rank correlation between proportional and calibrated preferences is only 0.714 for the first order consumption channel (see Figure 3.9). The differences in rankings between the two cases are driven by differences in the temporal evolution of consumption patterns between households, as well as the interaction between income-driven consumption shifts and changes in relative consumption good prices under policy at the provincial level.

Differences in expenditure shares between the proportional and the calibrated cases, which are determined by the direct effect of changing consumption patterns only, are driven by the interaction between the shape of the Engel curves and differences in the dynamics of income growth across households. As an example, consider rural households in Inner Mongolia (NM) and Jilin (JL). In the proportional case, rural households in Inner Mongolia are projected to spend 5% of their disposable income on coal in 2030, whilst in Jilin this figure is 2%. In the calibrated case, both households spend about 1% of their disposable income on coal. Rural household incomes, which are similar for both provinces in the base year, are almost twice as high for Inner Mongolia in the final year, compared to Jilin. Combined with the roughly flat Engel curve for coal over the relevant range, this explains the stronger reduction of expenditure shares on coal for Inner Mongolia. As a consequence, whilst consumption side impacts in the proportional case are much larger in Inner Mongolia compared to Jilin, they are similar in the calibrated case. In fact, Inner Mongolia is slightly *less* impacted than Jilin in the calibrated case. The income driven consumption shifts thus alter the ranking between the two provinces.

Within most provinces, rural households are more affected by the first-order consumption channel than urban households. Since both household types face the same prices within a given province, these differences are driven by expenditure shares alone. The higher impacts on rural households through this channel are explained by higher relative consumption of coal and other energy goods and lower consumption of services, and other systematic differences in consumption patterns.

Between provinces, differences in impacts are additionally determined by differences in the relative price changes caused by policy. Consider the extreme example of Shanxi (SX). The large impact of policy on the energy-intensive industries in Shanxi depresses demand for labor, driving down the local wage rate. This in turn reduces costs in labor intensive sectors such as services, which experiences the largest price decrease relative to all other provinces. In the calibrated case, these price changes, combined with the relatively high share of household expenditure on services in Shanxi, cause households in Shanxi to be less adversely impacted than the average Chinese household. In the proportional case, on the other hand, rural households are more adversely impacted than the average Chinese household, due to the relatively high expenditure shares on coal, which counterbalance the beneficial effect of cheaper services under the policy. The example of Shanxi illustrates how

[30] Small and virtually insignificant differences are caused by the labor-leisure choice being differently affected for urban and rural households within a given province, due to the non-constant marginal utility of income dedicated to consumption.

changing consumption patterns cause households that are amongst the most affected on the income side to be amongst the least affected on the consumption side, thus exemplifying the reversal in the correlation between consumption and income side channels identified in the paragraph entitled "The effect of changing consumption patterns on the variation of policy impacts" above.

In summary, non-unitary income elasticities have a qualitatively different effect on the ranking of relative winners and losers on the consumption side impacts compared to the income side. Consumption side impacts are the main driver of changes in the relative ranking, compared to the case with homothetic preferences, due mostly to the direct effect of changing consumption patterns.

3.6 Conclusions

The relationship between household income and consumption is an important determinant of energy use. We show that improvements in the specification of this relationship can significantly alter projections of energy use and CO_2 emissions. It can also have large impacts on the magnitude, distribution, and dynamics of policy incidence. Based on our study of China, our analysis shows that these effects may be especially large in developing economies where income levels are changing dramatically and unevenly over relatively short time horizons.

In China, income-driven shifts in the composition of household consumption may have large impacts on energy and emissions projections *in the absence of policy*. With calibrated preferences, our baseline projection for China yields 61% lower direct household CO_2 emissions and 8% lower national emissions, driven by significantly reduced household energy demand. Absent general equilibrium feedbacks this reduction would be even more pronounced.

With the lower baseline under calibrated preferences, reaching a given target will require a lower CO_2 price as households "outgrow" some of the most CO_2 intensive energy consumption choices, implying that costs of climate change mitigation will be lower. The distributional impacts of policy are also considerably less pronounced. Three effects drive this result. First, lower carbon prices due to the easing of policy stringency mitigate differences between households. Second, rapid convergence in the carbon intensity of household consumption baskets as incomes rise leads to a convergence in the consumption side impacts of carbon pricing. Third, the lower correlation between consumption and income side impacts, caused by interactions between income-driven changes in consumption patterns and changes in relative consumption good prices under policy, further reduces the variation in welfare impacts.

Taken together, our results suggests that the case for climate policy in China may be stronger than previous projections suggest. Failing to capture this realism may erroneously weaken the case for ambitious climate policy. Viewed in this light, the extra investment in representing micro-level evolution of household consumption seems very worthwhile. Our findings furthermore highlight the value of accounting for consumption shifts in a general equilibrium framework, which enables the comparison of consumption *and* income side effects, and their interactions. While our application is focused on pricing CO_2 emissions, our insights are also relevant for assessing the impact of a wide range of public policies which target or interact with patterns of household consumption, directly or indirectly.

Beyond China, our exercise suggests more generally that models—which widely rely on unitary income elasticities—may be systematically misrepresenting the evolution of energy demand and emissions as household income rises in developing countries, yielding misleading policy prescriptions.

Our work could be extended in several ways. Due to a lack of data, differences in ownership of factors of production between urban and rural households in each province are currently omitted from our analysis. Distributional effects driven by changes in relative returns to factors of production therefore arise only between but not within provinces. A differentiated representation of factor ownership within provinces would improve upon the representation of this channel of incidence. Another direction for further work would be to incorporate estimates of the benefits from improved air quality and avoided climate change, a channel of incidence which our analysis does not capture. The reliance on purely cross-sectional data, and the fact that uncertainty in the empirical estimates is not incorporated into the simulation exercise, are further limitations that could be addressed in extensions of this work. Another extension could be to carry out similar analysis in a perfect-foresight model. This would, however, require an adjustment to the methodology developed in this essay.

Our analysis has focused on a policy which targets fixed emissions levels. Future research could study alternative policy objectives, such as targeting given absolute emissions reductions, or a given carbon price trajectory. Such policies would lead to average welfare impacts that would be closer between the homothetic and the non-homothetic cases. The variation of welfare impacts would, however, likely remain significantly lower in the non-homothetic case, as suggested by the rapid convergence in the carbon intensity of household consumption baskets with rising incomes.

Our approach for calibrating household preferences to estimated Engel curves for a wide range of goods within a recursive-dynamic GE model could be replicated or approximated in other countries, as long as reliable Engel curve estimates from micro-data are available. Numerous countries exhibit disparities in income which are comparable to those observed in China (Xie and Zhou, 2014). In such cases, the income spectrum for sufficiently large cross sections of the populations should be broad enough to perform in-sample projections of household consumption patterns such as the one in this essay. Our analysis represents a first step in this direction.

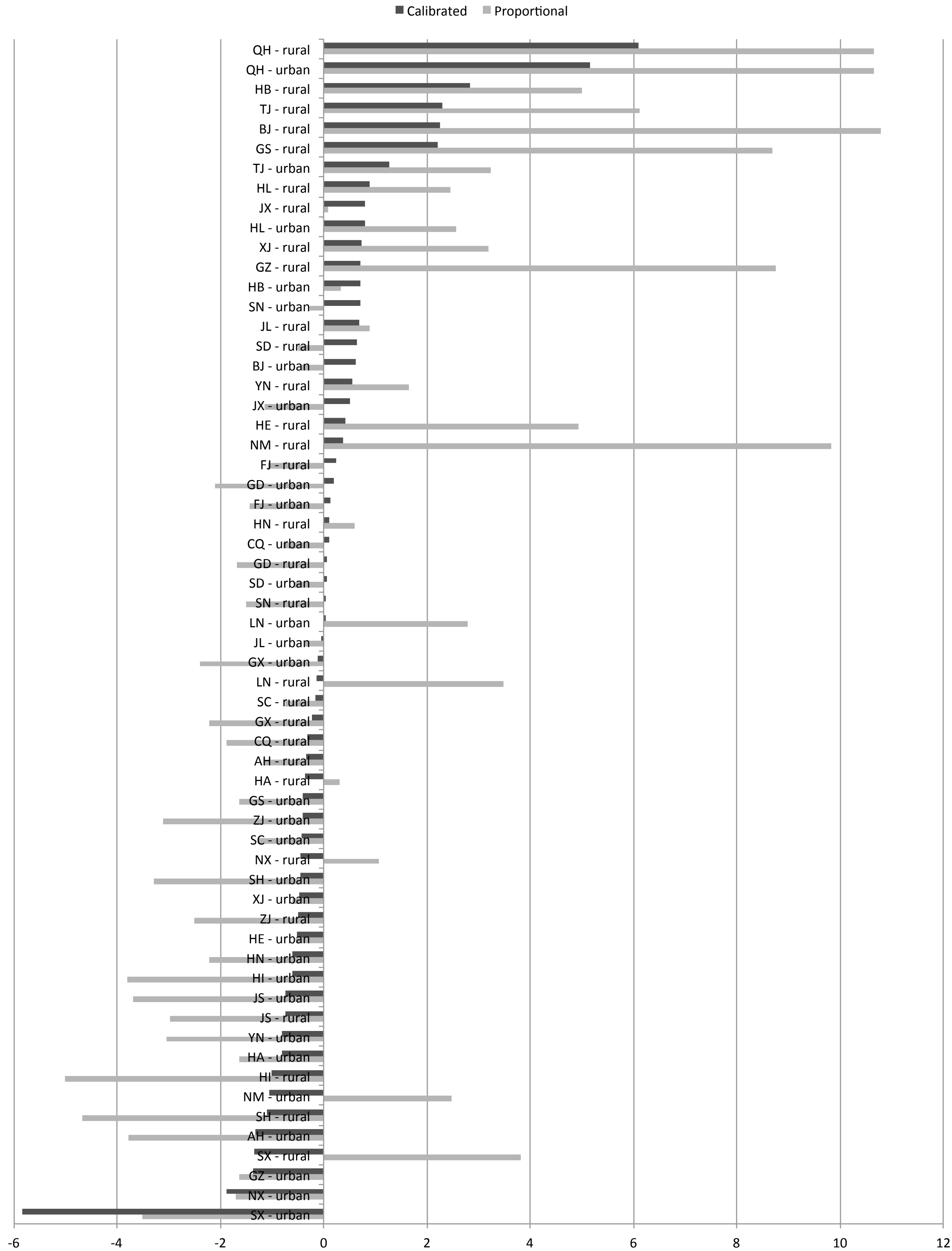


Figure 3.9: % welfare loss in 2030 through the 1st order consumption channel (relative to the population-weighted average impact through this channel).

4 Everybody pays the same?—Distributional equity and non-uniform environmental taxes[1]

[1] At the time of printing, the article based on this essay (entitled "Social Equity Concerns and Differentiated Environmental Taxes", together with Jan Abrell and Sebastian Rausch) was under review at the *American Economic Journal: Economic Policy*.

Abstract

This essay examines differentiated pollution pricing in the presence of social equity concerns using both theoretical and numerical general equilibrium analyses in an optimal taxation framework. We first theoretically study the optimality conditions for sectoral pollution taxes and redistribution of pollution tax revenues. Household heterogeneity in preferences and endowments interacts with social preferences implying non-uniform pollution taxes at the social optimum. Taxes should be differentiated according to households' consumption characteristics and to raise revenues for targeted transfers to households. Quantitatively assessing the scope for differentiated carbon taxes in the context of the U.S. economy, we find that optimal sectoral carbon taxes differ widely across sectors, even when social inequality aversion is relatively low. The optimal policy differentiates taxes to strongly increase revenues for redistribution to lower income households. Considering non-optimal redistribution schemes, the scope for tax differentiation is somewhat diminished but remains substantial. Relative to uniform carbon pricing, incidence patterns across household expenditure groups for a given redistribution scheme can vary qualitatively when allowing for differentiated taxes.

4.1 Introduction

Controlling pollution or carbon dioxide (CO_2) emissions with market-based regulatory instruments such as taxes or tradable permit systems has been shown to be cost-effective and efficient (Goulder and Parry, 2008; Metcalf, 2009b). At the same time, the public acceptance of such policies depends crucially on their distributional consequences (Atkinson and Stiglitz, 1980; Fullerton and Metcalf, 2002). While efficiency and equity are fundamentally linked, the issue of addressing unintended distributional outcomes of market-based environmental policy—among, for example, heterogeneous groups of industries, households, or countries—is often analyzed in isolation from efficiency. A large literature in environmental and public economics elucidates important trade-offs between efficiency and equity (for example, Poterba, 1989; Bovenberg et al., 2005; Bento et al., 2009; Rausch et al., 2010b; Sterner, 2012; Fullerton and Monti, 2013) but largely focuses on the issue of how the revenues from market-based regulation—that is typically based on *uniform* pollution pricing following the principle of equalizing marginal abatement cost in production—can be used to alter incidence outcomes. In an economy with heterogeneous households and social equity concerns, however, efficiency and equity can generally not be separated. This raises two fundamental questions. First, is uniform pollution pricing optimal in light of social equity concerns? Second, if not, how is the optimal differentiation of pollution taxes linked to social and private preferences?

We examine these questions using both theoretical and numerical general equilibrium analysis within the context of an optimal taxation framework. Our analysis rests on three basic premises that reflect, in our view, relevant aspects of the real-world setting faced by environmental tax policy in many countries. First, while it is in principle possible to fully address social equity concerns by means of appropriate lump-sum income redistribution, we abstract from such transfers since they are likely infeasible in practice (Feldstein, 1972). We instead focus on the design of environmental policy that is constrained to using the revenues raised from pollution taxes for redistributive purposes. Second, we focus on analyzing revenue-neutral tax policies which do not affect the government budget, i.e. pollution tax revenues are returned lump-sum to consumers. Third, we consider pollution tax differentiation among industry sectors. We abstract from direct pollution taxes on private consumption. This is motivated, on the one hand, by the observation that pollution is typically taxed at the source which generally lies at the industry level. Taxing pollution downstream at the level of consumption would raise the issue of accurately determining indirect (embodied) pollution in multiple consumption goods. On the other hand, while for some pollutants, such as CO_2 emissions, household emissions constitute a large fraction of economy-wide emissions, taxing these sources is politically contentious. Taken together, these premises lead us to consider the problem of optimal sectoral pollution pricing and optimal revenue redistribution in presence of social equity concerns.

We first theoretically study the optimality conditions for sectoral pollution taxes and government transfers. Household heterogeneity in preferences and endowments interacts with social equity concerns implying that marginal abatement costs absent social equity concerns are in general not equalized at the social optimum. We identify the motives that affect the sectoral marginal social cost of abatement. Heterogeneity in households' preferences implies a higher marginal social abatement cost for sectors which produce output that is consumed more intensively by households with higher social weights. Marginal social cost of abatement are instead lowered if factor income losses for households with high social weights are relatively small. In addition, the marginal social cost of abatement in a given sector depends on the social value of the pollution tax revenue raised per united of abated pollution. To the extent that these effects vary across sectors, the equalization of sectoral marginal social abatement cost implies that marginal abatement cost absent social equity concerns are not equalized, thus necessitating differentiated pollution pricing at the social optimum.

We use analytical examples to further examine the conditions under which uniform taxes are not optimal and how taxes should be differentiated to improve social welfare. If households have different tastes and there exist social equity concerns, then taxes should be differentiated according to households' consumption characteristics, in order to shift the burden of taxation towards households with lower social weights. Moreover, pollution taxes should be differentiated to raise revenues for targeted transfers to households with high social weights.

To quantitatively assess the relative importance of the different motives for tax differentiation in an empirical setting, we complement our theoretical analysis with numerical simulations. We develop a numerical framework that casts the problem of optimal sectoral pollution pricing with social equity concerns in the context of a numerical general equilibrium model that embodies firms' and households' behavioral equilibrium responses to pollution taxes, while at the same time satisfying market and aggregate economy constraints. More specifically, we focus on the issue of climate policy aimed at reducing CO_2 emissions through price-based market regulation looking at the case of the United States. To this end, we calibrate the numerical model to observed production, consumption, and national as well as household-level income data for the U.S. economy. We assume that the regulating entity—faced with the problem of designing the optimal carbon pricing policy—is averse to social (income) inequality.

We find that the amount of sectoral carbon tax differentiation is substantial, even for relatively low degrees of social inequality aversion (unsurprisingly, the degree of tax differentiation increases with higher social inequality aversion). Optimally differentiated carbon taxes significantly raise the amount of tax revenues collected which are then redistributed to lower income households. Importantly, this largely reduces inequality to levels which would not be attainable under uniform carbon pricing.

Decomposing the different motives for tax differentiation, we find that the deviation from uniform pollution pricing is to a large extent driven by the motive to enhance the amount of tax revenues available for redistribution. Pollution taxes are higher for sectors with relatively steep marginal abatement cost as this enables raising higher tax revenues for a given amount of pollution. Hence, we find that efficiency of abatement in production is strongly sacrificed in favor of generating high tax revenues that can be spent on targeted transfers to address equity concerns. While we find that the pattern of sectoral tax differentiation due to the heterogeneity in households' preferences is in line with the insights derived from the theoretical analysis, the quantitative importance of the preference effect in driving overall tax differentiation is dominated by the revenue-raising motive.

Besides examining optimal policies consisting of differentiated pollution taxes and transfers, situations in which a specific redistribution scheme is already in place and cannot be altered may be the more relevant setting for real-world environmental policy. We analyze the optimal amount of tax differentiation as well as the incidence impacts for three non-optimal revenue redistribution schemes based on either equal transfers to each household, income-, or consumption-based recycling. While we find that for these transfer schemes the case for tax differentiation is somewhat diminished (relative to the optimal policy), we show that optimal carbon taxes are still strongly differentiated across sectors for transfer schemes which allocate a large fraction of the tax revenue to lower income households (as, for example, under equal or consumption-based redistribution). In terms of incidence impacts by household expenditure deciles, optimally differentiated taxes yield more progressive (less regressive) outcomes relative to uniform pollution pricing. In general, we find that the difference in the incidence between uniform and optimally differentiated pollution pricing is larger for transfer schemes which a better suited to address inequality, i.e. distributing larger parts of the revenue to lower income households

This essay contributes to the existing literature in several ways. First, the essay is related to the literature on optimal environmental taxation following the seminal contribution by Pigou (1920) according to which an externality should be priced at its marginal social damage. Subsequent literature has explored a variety of reasons for deviating from this principle. Interactions between environmental taxes and the broader fiscal system as well as the use of tax revenues to reduce pre-existing distortionary taxes have been shown to modify (or generalize) the Pigouvian pricing rule (see, for example, Bovenberg and de Mooij, 1994; Bovenberg and van der Ploeg, 1994; Parry, 1995; Bovenberg and Goulder, 1996). By focusing on efficiency, this literature abstracts from heterogeneous households and social equity concerns. Moreover, it assumes a single polluting sector or uniform emissions pricing across multiple economic activities.

Second, studies concerned with pollution tax differentiation across sectors have also focused on efficiency aspects. Boeters (2014) and Landis et al. (2016) find that the cost of climate policy can be lowered by differentiating carbon taxes in light of pre-existing non-environmental tax distortions. In an open economy context and for unilateral environmental policy, international market power, terms-of-trade effects, and emissions leakage have been shown to imply pollution taxes to optimally differ across sectors. Hoel (1996) argues that leakage in an incomplete international climate agreement motivates carbon tax differentiation, if import and export tariffs on all goods are not allowed. Böhringer and Rutherford (1997) point to competitiveness arguments based on comparative advantage that can justify exemptions for energy- and export-intensive sectors under otherwise uniform carbon pricing in unilateral climate policy. Böhringer et al. (2014) study optimally differentiated sectoral emission prices, motivated by leakage and terms-of-trade effects, and find that the welfare gains compared to uniform emissions pricing are modest. Similarly, Boeters (2014) finds that market power in export markets gives rise to carbon taxes being optimally differentiated across sectors. We contribute by examining how optimal pollution taxes should be differentiated across sectors due to social equity concerns.[2]

[2] To isolate the role of social equity concerns, we have deliberately chosen to abstract from the motives for sectorally differentiated environmental taxes mentioned above. We leave for future research the investigation of how these motives interact with social equity concerns.

Third, we are not the first to investigate the role of equity concerns within an optimal taxation framework. A large body of literature, not directly related to addressing an environmental externality, has established the result that optimal commodity and income taxation is in general affected by household heterogeneity and social preferences (Diamond and Mirrlees, 1971). In a framework that brings together revenue-raising and externality-correcting motives for optimal tax policy, Sandmo (1975) and Cremer et al. (2003) find that the optimal pollution tax rate in an economy with one polluting good depends on social preferences and household characteristics. Equity concerns also motivate non-linear consumption taxes (Cremer et al., 1998, 2003), thus leading to differentiated tax rates among consumers. Again, the possibility of pollution taxes that are differentiated across sectors is ruled out.[3]

Fourth, equity aspects of environmental policy are often studied outside an optimal taxation framework. Here, it is common to assess the distributional outcomes of environmental policy without ranking alternative outcomes based on social desirability or deriving optimal pollution tax rates in light of social preferences (i.e., thus adopting a positive rather than a normative perspective). For example, Poterba (1991) and Fullerton and Heutel (2010) find that energy (gasoline and carbon) taxes are strongly regressive. Fullerton and others have used analytical general equilibrium models building on Harberger (1962) to investigate the incidence of environmental taxes (Fullerton and Heutel, 2007; Fullerton et al., 2012). Bento et al. (2009) find that the incidence of an increase in the gasoline tax depends crucially on how revenues are recycled. In other instances, the amount of revenue collected through the environmental policy has been found to be insufficient to alter incidence in desirable ways. For example, Fullerton and Monti (2013) find that low-wage earners are more impacted by pollution taxes, even when they receive all the tax revenues. Bovenberg et al. (2005) examine the efficiency cost of meeting distributional objectives across industries for emissions taxes. While this strand of the literature has highlighted important trade-offs between efficiency and equity, it has restricted its attention to uniform pollution taxes and has not analyzed the question of optimal pollution pricing in the presence of social equity concerns.

The remainder of the essay proceeds as follows. Section 4.2 derives and discusses our theoretical results on optimal sectoral pollution pricing. Section 4.3 presents our quantitative, empirical framework to examine optimal sectoral carbon taxation under social equity concerns in the context of a numerical general equilibrium analysis of the U.S. economy. Section 4.4 presents and discusses our main findings from the numerical simulations. Section 4.5 concludes.

[3] Outside of the optimal taxation framework, Mayeres and Proost (2001) study welfare-improving revenue-neutral marginal tax reforms for an economy with multiple households in the presence of an externality. While they highlight the importance of distributional considerations, their analysis focuses on marginal tax reforms in a setting with only one polluting good.

4.2 Theoretical framework and results

4.2.1 Basic setup

We consider a perfectly competitive and static economy comprising N sectors indexed $n \in \{1, 2, .., N\}$ and H households indexed $h \in \{1, 2, .., H\}$. We assume that the economy is closed (i.e., no international trade) and that there are no pre-existing taxes.[4] Production processes employ constant-returns-to-scale technologies. $M \leq N$ *dirty* sectors indexed by $i \in \{1, ..., M\}$ emit an uniformly dispersed pollutant (such as, e.g., CO_2) as part of the production process. Z_i denotes emissions of sector i. Firms maximize profits at given product and factor prices.

Households derive utility from consumption and dis-utility from pollution. We abstract from labor-leisure choice. The budget of household h, M_h, comprise factor income (F_h) and government transfers (T_h): $M_h = F_h + T_h$. Households are heterogeneous in two fundamental aspects. First, heterogeneous preferences are captured by their respective indirect utility function

$$V_h(\mathbf{p}, M_h, Z) \tag{4.1}$$

where $\mathbf{p}$ is the vector of consumer prices and $Z := \sum_i Z_i$ is the total amount of pollution. Second, households differ in terms of the level and composition of income which they receive from inelastically supplying factors of production to firms and from government transfers T_h. Households maximize utility by choosing consumption of sectoral goods at given prices and budgets.

The government seeks to maximize social welfare which is given by the following Bergson-Samuelson welfare functional:

$$W = W(V_1(\mathbf{p}, M_1, Z), \ldots, V_h(\mathbf{p}, M_h, Z), \ldots, V_H(\mathbf{p}, M_H, Z)), \tag{4.2}$$

by choosing a public policy $\{\tau_i, T_h\}$, $\forall i \in \{1, ..., M\}$, $\forall h \in \{1, 2, .., H\}$ consisting of sector-specific pollution taxes and household-specific transfers. τ_i can in general be either input or output taxes chosen such that total emissions do not to exceed a maximum level $\tilde{Z}$. If $\tilde{Z}$ is high enough, the government policy endogenously determines the optimal level of pollution.[5] Household transfers have to be fully financed out of the revenue from pollution taxes, i.e. $\sum_h T_h \leq \sum_i \tau_i Z_i$.

We consider two alternative government problems. In the fully optimal case ($\{\tau_i, T_h\}$), the government chooses optimal taxes and transfers. A constrained-optimal case ($\{\tau_i, T_h := \xi_h \sum_i \tau_i Z_i\}$) considers the situation in which the government can choose optimal pollution taxes but is constrained by an exogenously given transfer scheme for redistributing the pollution tax revenue, represented by fixed redistribution shares $\{\xi_h\}$. Considering the case of a fixed transfer scheme is useful as it

[4] As has been reviewed by the literature highlighted in Section 4.1, there exist various motives for tax differentiation related to pre-existing fiscal distortions and international trade. To focus on the implications of social equity concerns for tax differentiation, we abstract from such effects. We leave for future research the investigation of how these motives for tax differentiation may interact with social equity concerns.

[5] We introduce a pollution constraint $\tilde{Z}$ to be able to consider non-optimal levels of pollution. This is useful in models that do not explicitly account for dis-utility from pollution, such in our numerical analysis in Sections 4.3 and 4.4

enables examining cases beyond the fully optimal case which may reflect the situation of real-world policy that a given redistribution scheme is already in place (e.g., per-capita tax rebates or recycling in proportion to income).[6]

4.2.2 Optimal policies

In the fully optimal case where pollution taxes and transfers can be chosen, the government solves the following problem:

$$\begin{aligned} max_{\{\tau_i \geq 0, T_h \geq 0\}} W \quad s.t. \quad & \sum_h T_h \leq \sum_i \tau_i Z_i \quad (\mu) \\ & \sum_i Z_i \leq \tilde{Z} \qquad\quad (\epsilon)\ , \end{aligned} \tag{4.3}$$

where $\mu \geq 0$ denotes the marginal value of public funds and $\epsilon \geq 0$ is the shadow value of the pollution constraint.

Assuming an interior optimum, the optimality conditions for sectoral pollution taxes are (see Appendix E.1 for derivations):[7]

$$MSD = \underbrace{-\underbrace{\frac{\sum_h \sum_n \beta_h X_{nh} \partial_{\tau_i} p_n}{\partial_{\tau_i} Z}}_{=\text{Preference effect}} + \underbrace{\frac{\sum_h \beta_h \partial_{\tau_i} F_h}{\partial_{\tau_i} Z}}_{=\text{Factor income effect}} + \underbrace{\mu \frac{\partial_{\tau_i} \sum_j \tau_j Z_j}{\partial_{\tau_i} Z}}_{=\text{Revenue redistribution effect}}}_{=:MSC_i} \quad \forall\, i \in \{1, \ldots, M\}\,, \tag{4.4}$$

where β_h denotes the *direct social marginal utility of income* accruing to household h (Mayeres and Proost, 2001) and is given by $\beta_h := \lambda_h \partial_{V_h} W$ with $\lambda_h := \partial_{M_h} V_h$ denoting the private marginal utility of income.

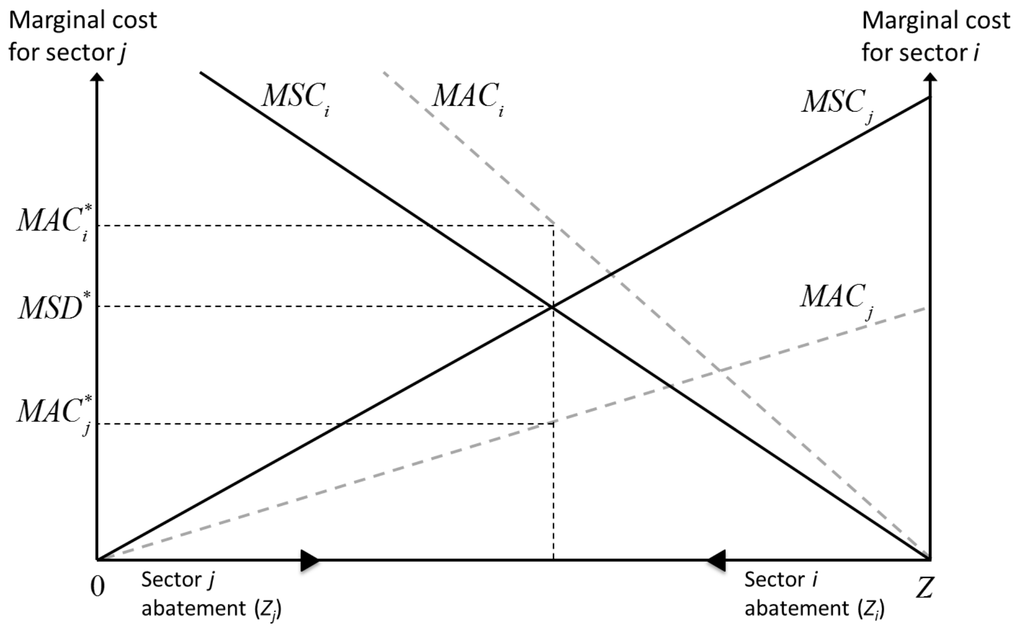
The *marginal social damage of pollution* is defined as: $MSD := \epsilon - \sum_h \partial_{V_h} W \partial_Z V_h$. Note that MSD increases in the shadow value of the pollution constraint (ϵ)[8] and that the terms $\partial_{V_h} W$ $(= \beta_h/\lambda_h)$ express the social weighting of the tax-induced change in household h's utility. Also note that for the case of a uniformly dispersed pollutant such as CO_2 that we consider here, MSD is independent from the polluting source (sector). The right-hand side (RHS) of Equation (4.4) is the *marginal social cost of abatement induced by a pollution tax on sector i* (MSC_i). It is thus straightforward to see that the following standard result holds:

[6] In addition, optimal tax rates in the fully optimal case can be conveniently analyzed by looking at tax rates in the constrained-optimal case with the appropriate fixed redistribution shares. Denote by $\{\bar{\tau}_i, \bar{T}_h\}$ the unconstrained optimum, and $\{\tau_i^\star\}$ the constrained optimum (with associated transfers $T_h^\star := \xi^h \sum_i \tau_i^\star Z_i^\star$, where $Z_i^\star$ is the pollution emitted by sector i at the constrained optimum). If the fixed transfer scheme implements the optimal redistribution, i.e. $\xi_h = \bar{\xi}_h := \frac{\bar{T}_h}{\sum_{h'} \bar{T}_{h'}}$, then the constrained-optimum coincides with the fully optimal case $\tau_i^\star = \bar{\tau}_i$ and $T_h^\star = \bar{T}_h$. This holds true because, in such a constrained-optimal case, the government optimizes over a subset of taxes and transfers which contains the policy choices that are optimal over the whole set.

[7] We employ the short-hand notation $\partial_X \equiv \frac{\partial}{\partial X}$ for partial derivatives.

[8] If the pollution level at the optimum is below the cap or—equivalently—for the case of unconstrained pollution, the shadow value of the pollution constraint is zero; it is positive if the constraint is binding.

Figure 4.1: Sectoral pollution pricing with and without social equity concerns



Notes: MSC (MAC) denote the marginal social cost of abatement taking into account (ignoring) social equity concerns.

Proposition 4.1 *Optimal pollution taxes equalize the marginal social cost of abatement across sectors (i.e., $MSC_i = MSC_j$, $\forall i, j$).*

Proof. From the conditions in Equation (4.4), since $MSD = MSC_i$ and $MSD = MSC_j$, $\forall i$ and $\forall j$, it follows that $MSC_i = MSC_j$. □

Proposition 4.1 can be viewed as a generalized version of the equi-marginal principle which takes into account the presence of social equity concerns for heterogeneous types of households. Figure 4.1 graphically depicts this situation where at the optimum, sectoral pollution taxes are set such that the marginal social cost of abatement across sectors i and j are equalized (and correspond to the marginal social damage MSD^*). While this general optimality principle is of course well-known, the central theme of this essay is to understand how social equity concerns modify the rules for optimal sectoral pollution pricing. For example, is uniform pollution pricing across sectors still optimal if the society is inequality averse? If non-uniform pricing is optimal, in what ways do sectoral pollution taxes have to be differentiated and what determines the magnitude of tax differentiation?

Intuitively, the answer depends on whether and how marginal social cost of abatement differ when social equity concerns are taken into account or left out. When social equity concerns are not considered, we simply refer to marginal social abatement cost as *marginal abatement cost* (MAC). As portrayed in Figure 4.1, if MSC and MAC differ, then it is likely no longer optimal to uniformly tax pollution in both sectors: equalizing marginal social costs across sectors ($MSC_i = MSC_j$ at MSD^*) then implies that $MAC_i^* \neq MAC_j^*$, thus requiring differentiated sectoral tax rates.

The MSC on the RHS of Equation (4.4) comprise three components that expound how household heterogeneity in preferences and endowments together with social preferences for equity determine various effects at play for optimal sectoral pollution taxes. Note that the marginal effect of tax increases on pollution is in general expected to be negative (i.e., $\partial_{\tau_i} Z < 0$). Divisions by $\partial_{\tau_i} Z$ normalizes abatement costs by the amount of pollution abated.

First, there is a *Preference effect* which captures the effect of consumption choices of households on marginal social cost which are driven by the preferences of heterogeneous households and the interaction with social preferences for equity.[9] The term $\sum_n X_{nh} \partial_{\tau_i} p_n$ captures the marginal impact of the tax in sector i on the consumption bundle of household h. If positive, it implies that a tax increase causes the consumption bundle to become more costly, in turn leading to a positive contribution to MSC_i. Weighting with β_h reflects the fact that the contribution of the *Preference effect* to marginal social cost in sector i is smaller for households who are associated with a lower direct social marginal utility of income.

Second, the *Factor income effect* captures how MSC are affected through pollution tax-induced changes in households' factor incomes. Intuitively, if factor income of household h decreases in response to a marginal increase in the pollution tax in sector i, then the marginal social costs are increased. Again, the abatement costs are lower if households which are more negatively impacted through this channel are associated with lower values of β_h.

Third, there is a *Revenue redistribution effect* which reflects the social value of revenue raised. If the marginal effect of τ_i on total tax revenue ($\sum_i \tau_i Z_i$) is positive, then overall abatement cost through the tax are reduced. Intuitively, the magnitude of this channel depends on the marginal value of public funds μ.

To see how household heterogeneity and social equity concerns cause the MSC to differ from the MAC, consider first the case without social concerns, i.e., with equal social weights across households: $\beta_h = \beta$, $\forall h$. The *Preference effect* is then equal to $\beta \sum_n (X_n \partial_{\tau_i} p_n)/(\partial_{\tau_i} Z)$ and the *Factor income effect* is $\beta \partial_{\tau_i} F/(\partial_{\tau_i} Z)$.

With social equity, i.e. with $\beta_h \neq \beta_{h'}$ for some h, h', the corresponding expressions in general differ, as in general

$$\sum_h \beta_h X_{nh} \neq (\sum_h \beta_h)(\sum_h X_{nh}) \qquad \text{and} \qquad \sum_h \beta_h F_h \neq (\sum_h \beta_h)(\sum_h F_h)\,.$$

The inequalities above are caused by the interaction between social weights and household heterogeneity in consumption and income. The RHS can be interpreted as a social weighting of the different components of the MAC. This weighting varies across sectors, as opposed to the case without social equity concerns. Due to the *Preference effect* a sector receives a relatively higher weight if households consuming its output are associated with higher direct social marginal utilities of income. Even if households have identical preferences but differ in terms of factor incomes, the marginal abatement cost absent social equity concerns is likely to differ across sectors due to the

[9] It should be noted that the numerator in the first term can also be expressed as follows: $\sum_n \partial_{\tau_i} p_n (\sum_h \beta_h X_{nh})$. The term in brackets is proportional to the *distributional characteristic* of good j as in Mayeres and Proost (2001). Analogous quantities have proven to be of relevance in previous literature, such as, e.g., Ahmad and Stern (1984) and Feldstein (1972). In cases where $\partial_{\tau_i} p_n = 1$ for $i = n$ and 0 for $i \neq n$, such as in a partial equilibrium analysis with linear taxes, the distributional characteristic of good i is the only term on the RHS of Equation (4.4) containing consumption characteristics of households.

Factor income effect.

In addition, there is another effect related to redistribution of the tax revenues which can cause the marginal social cost to deviate from the marginal abatement cost beyond the interaction between β and heterogeneous household characteristics. The importance of the *Revenue redistribution effect* relative to the other effects on the RHS of Equation (4.4) depends on the marginal value of public funds μ which in turn depends on the social weighting. This motivates a closer look at the redistributive part of the optimal policy.

The conditions for optimally distribution tax revenues across households are given by (see Appendix E.1):

$$\mu \geq \underbrace{(1-D_h)^{-1}(\underbrace{\beta_h}_{\substack{\text{Direct}\\ \text{effect}}} \underbrace{-\sum_{n,h'}\beta_{h'}X_{nh'}\partial_{T_h}p_n + \sum_{h'}\beta_{h'}\partial_{T_h}F_{h'}}_{\substack{\text{Indirect consumption and}\\ \text{factor income effects}}} - \underbrace{MSD\,\partial_{T_h}Z}_{\substack{\text{Indirect pollution}\\ \text{effect}}})}_{\text{Marginal social benefit of redistributing revenue to household } h} \quad \forall\, h \in \{1,\ldots,H\}\,, \tag{4.5}$$

where $D_h = \sum_i \tau_i \partial_{T_h} Z_i$ stands for the increase in revenue caused by the increased spending following an increase in transfers to household h.[10] Equation (4.5) holds with equality if transfers to household h are positive. Conditions in Equation (4.5) simply state that the socially optimal redistribution of tax revenues across households is achieved when (1) for each household the marginal social benefit of transfers (RHS) are less-equal to the marginal value of public funds (LHS) and (2) the marginal social benefit of transfers are equalized across all households receiving non-zero households.

The first term in parentheses represents a direct effect: the benefit of redistributing tax revenue to household h increases for higher values of the household's direct social marginal utility of income β_h. The remaining terms in Equation (4.5) are indirect effects which affect all households in the economy. The second and third terms indicate the marginal effect of redistributing tax revenue to household h on prices and returns to factors, and their impact on household consumption patterns and factor incomes, respectively. The fourth term captures the marginal effect on the pollution level, and the resulting damages. Division by the term $1 - D_h$ indicates that, as the household h spends the extra revenue it receives, this will affect tax revenue, which (in the case of increased revenues, i.e., positive D_h) increase the social value of redistribution.

From the conditions in Equation (4.5), we can determine the pattern of optimal transfers as follows:

Proposition 4.2 *Assume identical homothetic preferences across households. Then, if household h has a direct social marginal utility of income which is lower than the maximum value (i.e., $\beta_h < max_{\{h'\}}\beta_{h'}$), it receives zero transfers at the optimum (i.e., $T_h = 0$).*

Proof. Since with identical homothetic preferences increasing the income through transfers has the

[10] Note that $D_h < 1$, since the additional tax revenue given to household h results in a less than proportional increase in tax revenues.

same effect on prices, factor incomes and pollution levels for all households, Equation (4.5) assume the following form: $\mu \geq \frac{\beta_h + A}{B}$ with $B > 0$. If $\beta_h < max_{\{h'\}}\beta_{h'}$, then $\mu \geq \frac{max_{\{h'\}}\beta_{h'}+A}{B} > \frac{\beta_h+A}{B}$. Since $\mu > \frac{\beta_h+A}{B}$, it follows that $T_h = 0$. □

Proposition 4.2 implies that if households have identical consumption characteristics, the pollution tax revenues are optimally redistributed to the households with the highest social marginal utility of income. As an example, if the direct social marginal utility of income is a decreasing function of income—as in the case of an inequality-averse government—then all the tax income is redistributed to the poorer households. If the tax revenue is sufficient to raise the income of the poorest household to the same level as the second-poorest, then both households have the same direct social marginal utility of income, and both hence receive positive transfers. It also implies that the richest household only receives non-zero transfers if the tax revenue is sufficient to equalize the income of all households.

To the extent that differentiated pollution taxes increase tax revenues and there exist social equity concerns, Proposition 4.2 suggest that it may be optimal to use non-uniform sectoral polluting pricing to increase the revenues to increase transfers to households with the highest social weighting—thus trading off abatement efficiency in production and social welfare improvements through targeted transfers. Thus, if even if households have identical preferences and factor incomes, it may be optimal to differentiate sectoral pollution taxes based on considerations about the optimal redistribution of tax revenues when social weights are differentiated.

4.2.3 Optimal sectoral pollution taxes with non-optimal revenue redistribution

In real-world policy making, the government may not be able to implement optimal transfers, i.e. it may likely be constrained by a given revenue redistribution scheme that is already in place (such as, for example, per-capita tax rebates or recycling revenues through cutting personal income taxes). It is thus useful to consider the case in which the government problem involves choosing taxes for a fixed and non-optimal revenue redistribution scheme. Moreover, examining such cases enables us to isolate the effect of heterogeneity in private preferences and its interactions with social preferences on optimally differentiated pollution taxes, by considering a specific redistribution scheme that mutes the revenue redistribution effect.

Let $\{\xi_h\}_{h=1}^{H}$ describe a fixed and revenue-neutral transfer scheme which redistributes shares of the total pollution tax revenue to households according to $T_h = \xi_h \sum_i \tau_i Z_i$ with $\sum_h \xi_h = 1$. Given ξ_h, the government chooses sectoral pollution tax rates τ_i to solve the following problem:

$$max_{\{\tau_i \geq 0\}} W \quad s.t. \quad \sum_i Z_i \leq \tilde{Z} \quad (\epsilon) \ . \tag{4.6}$$

Assuming an interior solution, one can derive similar optimality conditions for sectoral pollution taxes as in Equation (4.3) with μ being replaced by $\sum_h \beta_h \xi_h$ (see Appendix E.2 for the derivations).[11]

[11] Appendix E.3 illustrates the equivalence of the fully optimal and the constrained optimal problems for the

Deviations of ξ_h from the optimal transfer scheme, as determined by Proposition 4.2, affect the relative importance of the revenue redistribution channel for differentiation pollution tax rates. As is borne out by the following proposition, the revenue distribution motive is muted if transfers are proportional to income:

Proposition 4.3 *Assume that (1) the relative composition of factor income does not vary across households (i.e., $F_{hf} = \phi_h F_f$, where F_{hf} is household h's ownership of factor f, ϕ_h is a fixed share, and F_f is the total factor income of factor f), (2) household preferences are identical and homothetic, and (3) pollution tax revenues are redistributed in proportion to household income (i.e., $\xi_h = M_h / \sum_{h'} M_{h'}$). Then, optimal pollution taxes are uniform across sectors.*

Proof. For equal and homothetic preferences, $MSD = \frac{\sum_h \beta_h \xi_h (\partial_{\tau_i} M - M \sum_n \alpha_n \partial_{\tau_i} p_n)}{\partial_{\tau_i} Z}$ with $\alpha_n := X_{nh}/M_h = X_{nh'}/M_{h'}$ and $M = \sum_h M_h$. The RHS of the above equation is identical to the case of a single household economy with direct social marginal utility of income given by the weighted average of the values for the identical households (i.e., $\beta \equiv \sum_h \beta_h \xi_h$). □

Note that the result of optimal uniform pollution taxes in Proposition (4.3) even holds when allowing for the presence of social equity concerns. Assumptions (1) and (2) shut off the *Preference effect* and *Factor income effect* in Equation (4.3). Proposition 4.3 then essentially states that the *Revenue redistribution effect* is also muted for income-proportional transfers. Intuitively, for income-proportional redistribution the share of each household's factor income in the economy's total is equal to the household's share of transfers in the total. A given household's income will therefore only be increased if the total amount of pollution tax revenues increases more than the loss in total factor income. The income-proportional redistribution scheme thus invests each household in the trade-off between increased tax revenues and decreased aggregate factor incomes. The result indicates that the increase in tax revenues achieved by differentiating pollution taxes is outweighed by the loss in total factor incomes.

Other redistribution schemes, including the optimal one, are likely to results in differentiated pollution taxes at the optimum due to the revenue redistribution motive. This is because other schemes expose different households unequally to the tax revenue/factor income tradeoff discussed above. For example, a per-capita transfer scheme allows low-income households to benefit more from the increased tax revenues compared to income-proportional redistribution. This creates a social incentive to differentiate taxes to raise revenues, in the measure in which low-income households are given higher social weights compared to high-income households. These insights again emphasizes the interaction between optimal polluting pricing and the redistributive side of the optimal policy, while illustrating that both aspects of the policy have to be considered at the same time.

4.2.4 When are differentiated pollution taxes better?

While the results above have allowed us to identify the channels that drive differentiated pollution taxes in the presence of social equity concerns, we now examine in more depth the conditions under

appropriate revenue redistribution scheme.

which uniform taxes are not optimal and how taxes should be differentiated to improve welfare. To derive results, we need to impose more specific assumptions on the structure of the economy within the general setting considered before.

We assume two sectors labeled X and Y–both polluting (i.e., $N = M = 2$)–and one factor of production (labor), which is supplied inelastically and mobile across sectors. Labor is subject to a resource constraint: $L_X + L_Y = \bar{L}$. There are two households (i.e., $H = 2$) labeled by A and B. Households have separable utility in pollution and Cobb-Douglas utility in consumption. We model pollution as an input to production, i.e. production functions are $X = X(L_X, Z_X)$ and $Y = Y(L_Y, Z_Y)$ where L_X, L_Y, Z_X and Z_Y are the quantities of labor and pollution used in each sector. The government returns the pollution tax revenues in a lump-sum manner to households according to a given and fixed redistribution scheme (ξ_h).

To analyze when differentiated pollution taxes are better, we consider pollution-neutral tax swaps starting from an initial situation of uniform pollution taxes ($\tau_X = \tau_Y \equiv \tau$). From Proposition 4.1 we know that if sectoral pollution tax rates are not optimal then the marginal social cost of abatement differ across sectors. Consider a pollution-neutral tax swap from i to j (i.e., $d\tau_j > 0$, $d\tau_i < 0$ and $dZ = 0$). The following then holds (see Appendix E.4.1 for derivations):

$$dW = -\partial_{\tau_j} Z(MSC_i^{\xi} - MSC_j^{\xi})d\tau_j \,, \tag{4.7}$$

where MSC_i^{ξ} denotes the marginal social cost of abatement in sector i given a redistribution scheme ξ. Assuming $\partial_{\tau_j} Z < 0$, it therefore follows that the tax swap is welfare-improving if $MSC_j^{\xi} < MSC_i^{\xi}$ and it is welfare-reducing when the opposite holds. If $MSC_j^{\xi} < MSC_i^{\xi}$, the tax on j should therefore be raised and the tax on i lowered.

To proceed, we need to derive expressions for MSC_i^{ξ} which take into account the equilibrium responses of the economy when marginally changing pollution tax rates. We perform comparative static analysis adopting the standard approach in the literature for analytically solving general equilibrium models by linearization (Harberger, 1962; Fullerton and Heutel, 2007). We then use the local properties of the equilibrium to evaluate Equation (4.7). Appendix E.4 shows that the condition $MSC_X^{\xi} < MSC_Y^{\xi}$ for a welfare-improving tax swap from Y to X is equivalent to:[12]

$$\Delta\Bigg\{\left(\frac{\sigma}{\tau L_Y(\delta_X - \delta_Y)} + \frac{\sigma - 1}{M} + \sum_h \frac{1-\alpha_h}{1-\gamma}\frac{\xi_h}{M}\right) \underbrace{\sum_h \beta_h M_h\left(\frac{\alpha_h}{\gamma} - 1\right)}_{\substack{\text{Social weighting of household} \\ \text{consumption patterns}}} + (1-\sigma)\underbrace{\sum_h \beta_h\left(\xi_h - \frac{M_h}{M}\right)}_{\substack{\text{Social weighting of} \\ \text{redistribution scheme}}}\Bigg\} < 0 \,, \tag{4.8}$$

where $\delta_i := \frac{Z_i}{L_i}$ is the pollution intensity of sector i, total income $M := \sum_h M_h$, σ is the elasticity of substitution in production,[13] $\alpha_h := \frac{p_X X_h}{M_h}$ is the expenditure share of household h on good X,

[12] Assuming $\partial_{\tau_X} Z < 0$ and $\partial_{\tau_Y} Z < 0$, i.e. pollution is reduced by raising pollution taxes.

[13] For reasons of tractability, we assume here that at the equilibrium point σ is uniform across sectors. This does

$\gamma := \frac{p_X X}{M}$ is the economy's aggregate expenditure share on good X. $\Delta := \tau L_Y(\delta_X - \delta_Y)[1 + \tau L_Y(\delta_X - \delta_Y)\sum_h \frac{1-\alpha_h}{1-\gamma}\frac{\xi_h}{M}]^{-1}$.

The LHS of Equation (4.8) represents the difference in sectoral *MSC*; it implies that the larger the difference, the larger is the social welfare improvement from the pollution-neutral tax swap. One can now see that the previously identified effects related to social weighting, differences in households' preferences, and redistribution of tax revenues determine whether and to what extent the tax swap moving toward differentiated taxes is welfare-improving.[14]

First, the larger the heterogeneity in households' expenditure shares (i.e., the larger $\left(\frac{\alpha_h}{\gamma} - 1\right)$), the larger is the difference in sectoral *MSC*. In contrast, if households spend their income in same proportions on different goods (i.e., $\frac{\alpha_h}{\gamma} - 1 = 0$), then the motive for tax differentiation to the *Preference effect* is absent.

Second, the more the revenue redistribution scheme deviates from the income-proportional scheme (i.e., the larger $\left(\xi_h - \frac{M_h}{M}\right)$), the larger is the difference in sectoral *MSC* and hence the larger is the *Revenue redistribution motive* for tax differentiation. In contrast, if $\xi_h - \frac{M_h}{M} = 0$, then the redistribution motive is absent (as implied by Proposition 4.3).

Third, the effects described in the previous two points are only present if social weighting differs across households. For a given redistribution and household expenditure pattern, the magnitude of the two effects is increasing in the difference of the social weights. If social weighting is equal across households ($\beta_h = \beta$, $\forall h$), then it is easy to see that the LHS of Equation (4.8) is zero, implying that differentiating taxes does not improve welfare.

Fourth, the relative importance of the *Preference effect* and the *Revenue redistribution effect* is determined by the interaction with production side characteristics (captured by the substitutability σ and differences in pollution intensity across sectors $\delta_X - \delta_Y$) as well as interactions between the two channels (captured by $\sum_h(1 - \alpha_h)\xi_h$).

How should sectoral pollution taxes be differentiated to enhance welfare? In answering this question, it is useful to decompose the two effects at play. By assuming income-proportional redistribution, we can first consider the case in which only the *Preference effect* is present. The following proposition clarifies how tax should be differentiated for given social weights and household consumption patterns:

Proposition 4.4 *Given initially uniform pollution taxes and income-proportional redistribution (i.e., $\xi_h = \frac{M_h}{M}$), a pollution-neutral tax swap from sector Y to sector X (i.e., $d\tau_Y < 0$, $d\tau_X > 0$, and $dZ = 0$) is welfare-improving if and only if the household with the higher expenditure share on X has a lower direct social marginal utility of income (i.e. $\alpha_h > \alpha_{h'}$ and $\beta_h < \beta_{h'}$).*

Proof. See Appendix E.4.3. □

Proposition 4.4 implies that if households have different tastes and there exist social equity concerns,

not rule out that sectors can differ in terms of the factor intensities.

[14] Note that assuming a single factor of production in the economy implies that factor income of different households is affected in the same proportion. The motive for tax differentiation due to affecting factor income differently across households is thus not present in the setup considered in Section 4.2.4.

then uniform sectoral pollution taxes are not optimal. Taxes should be differentiated according to households' consumption characteristics, in order to shift the burden of taxation towards households with lower direct social marginal utility of income.[15] More specifically, it suggests that the tax rate on a sector whose output is consumed more intensively by households with a higher social weight should be lower compared to other sectors.

To analyze how pollution taxes should be differentiated due to the *Revenue redistribution effect*, we assume that consumption patterns are identical across households and that revenue redistribution is done according to the optimal scheme (following Proposition 4.2). The following proposition then holds:

Proposition 4.5 *Assume initially uniform pollution taxes, identical household preferences, unequal social weighting (i.e.,* $\beta_A \neq \beta_B$*), and pollution tax revenues redistributed to the household with the higher* β*. Then, a pollution-neutral tax swap (i.e.,* $dZ = 0$*) is welfare-improving if and only if it increases the pollution tax revenue* $T = \tau_X Z_X + \tau_Y Z_Y$*.*

Proof. See Appendix E.4.4. □

Proposition 4.5 simply states that for the *Revenue redistribution effect* to drive a deviation from uniform pollution pricing, the tax differentiation has to be such that the pollution tax revenues are increased. If a tax swap reduces tax revenues, it cannot be welfare-improving because less revenues are available to distribute to households with high social weights.

4.3 Quantitative framework for empirical analysis

To quantitatively assess the scope for optimal pollution pricing and redistribution in an empirical setting, we complement our theoretical analysis with numerical simulations. We develop a numerical framework that casts the problem of optimal sectoral pollution pricing with social equity concerns in the context of a numerical general equilibrium (GE) model that embodies firms' and households' behavioral equilibrium responses to emissions taxes as well as satisfying cross-market and aggregate economy restrictions. Importantly, the numerical model extends the theoretical example presented previously in Section 4.2.4 by incorporating multiple households, multiple factors of production, multiple polluting sectors including energy-sector detail, intermediate inputs, and CO_2 emissions derived from burning multiple fossil fuels in production and consumption. We calibrate our model to the case of the U.S. economy.

This section (1) describes the general structure of the numerical framework detailing our computational strategy, (2) provides an overview of the numerical GE model, and (3) describes data and calibration.

[15] The result from Proposition 4.4 that goods which are consumed more intensively by households that contribute less to social welfare should be subject to higher tax rates is analogous to the findings of Diamond and Mirrlees (1971) in the context of optimal commodity taxation.

4.3.1 General structure and computational strategy

The numerical model maximizes a social welfare function by choosing non-negative carbon taxes for each sector i, τ_i, and non-negative transfers to each household h, T_h, subject to three sets of constraints:

$$\begin{aligned} \max\ & W(V_1(\mathbf{p}, M_1), ..., V_H(\mathbf{p}, M_H)) \\ & \mathbf{p}, \mathbf{x} \in \mathcal{A} \\ & \tilde{Z} \geq \sum_i Z_i \\ & \sum_h T_h = \sum_i \tau_i Z_i \\ & T_h, \tau_i \geq 0\,. \end{aligned} \tag{4.9}$$

First, $\mathcal{A}$ represents the set of the feasible equilibrium allocations consisting of prices $\mathbf{p}$ and quantities $\mathbf{x}$ derived from the numerical GE component of the model. Second, economy-wide CO_2 emissions as given by the sum of sectoral emissions cannot exceed an exogenously given and fixed target $\tilde{Z}$. Third, the total value of lump-sum transfers to households ($\sum_h T_h$) is equal to the revenues from sectoral carbon taxes ($\sum_i \tau_i Z_i$).

The problem in Equation (4.9) represents a *Mathematical Program under Equilibrium Constraints* (MPEC), i.e. a bi-level optimization problem which maximizes an objective function subject to a lower-level constraint set that contains an equilibrium problem (Luo et al., 1996). We cast the general equilibrium problem in the lower-level part as a mixed complementarity problem (MCP) (see e.g. Mathiesen, 1985; Rutherford, 1995) solving for primal and dual variables (i.e., quantities and prices). The advantage of this approach is that it naturally accommodates equilibria with corner solutions; here, optimal zero sectoral carbon taxes and household transfers. We solve the MPEC in Equation (4.9) using the NLPEC solver in the General Algebraic Modeling System (GAMS). The remainder of this section describes in more detail the structure and specification of the general problem laid out in Equation (4.9).

4.3.2 Social welfare function

Following Cremer et al. (2003), we consider the case of an inequality-averse government by evaluating policies in light of the following iso-elastic social welfare function:

$$W[V_1(\mathbf{p}, M_1), \ldots, V_H(\mathbf{p}, M_H)] \equiv \frac{1}{1-\eta} \sum_h \pi^h (V_h(\mathbf{p}, \frac{M_h}{\pi^h}))^{1-\eta} \quad \eta \neq 1 \quad \text{and} \quad 0 \leq \eta < \infty\,, \tag{4.10}$$

where V^h is the indirect utility function of households of type h. Note that we exclude environmental damages or benefits entering households' utility or social welfare. π^h is the population share of household h. η is the "inequality aversion index". The value of η reflects the desired degree of redistribution in the economy: higher values of η mean that the society cares more about equality.

With the social welfare function in Equation (4.10) and given homothetic household preferences (i.e., $V_h(\mathbf{p}, M_h) = M_h V_h(\mathbf{p}, 1)$), the direct marginal social utility of income for type h is given by:

$$\beta^h = \frac{\pi^h}{M_h^\eta}\left(\frac{V_h(\mathbf{p}, 1)}{\pi^h}\right)^{1-\eta}. \tag{4.11}$$

For $\eta > 0$, households with lower incomes are associated with a higher direct marginal social utility of income.[16] For $\eta = 0$, the social welfare function is utilitarian with $W = \sum_h M^h V_h(p, 1)$. For this case, if tastes are equal across households (i.e., $V_h(p, 1) = V_{h'}(p, 1)$, $\forall h, h'$), then social weights are uniform: $\beta_h = \beta$, $\forall h$.

4.3.3 Feasible allocations $\mathcal{A}$ based on general equilibrium model

The set of feasible allocations $\mathcal{A}$ is defined by the equilibrium conditions of the numerical GE model for the U.S. economy. We formulate the GE model as a system of nonlinear inequalities and characterize the economic equilibrium by two classes of conditions: zero-profit and market-clearing. Zero-profit and market-clearing conditions exhibit complementarity with respect to quantities $\mathbf{x}$ and prices $\mathbf{p}$, respectively. We now describe the structure and decisions problems of economic agents (firms and households) to derive the conditions that define $\mathcal{A}$.[17]

We consider a closed and static economy with perfectly competitive output and factor markets. Production of final output in each sector $i \in I$ is characterized by a three-stage process.

At the first stage, final output Y_i is produced using the following constant-returns-to-scale production technology that combines intermediate inputs from other sectors j, M_{ji}, together with a sector-specific composite, V_i:

$$Y_i = \left[\theta_i^Y (\min\{\theta_{1i}^M M_{1i}, \ldots, \theta_{ji}^M M_j i, \ldots, \theta_{Ii}^M M_{Ii}\})^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \theta_i^Y) V_i^{\frac{\sigma_i - 1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i - 1}}, \tag{4.12}$$

where θ_i^Y and θ_{ji}^M are share parameters and $\sigma_i > 0$ denotes the elasticity of input substitution.

At the second stage, the composite input V_i is produced using inputs of capital (K), labor (L), and an sector-specific aggregate of energy inputs (E_i) according to a nested constant-elasticity-of-substitution (CES) function:

$$V_i = \left[\theta_i^V (\theta_i^K K^{\lambda_i} + (1 - \theta_i^K) L^{\lambda_i})^{\frac{(\kappa_i - 1)\lambda_i}{\kappa_i (1 - \lambda_i)}} + (1 - \theta_i^V) E_i^{\frac{\kappa_i - 1}{\kappa_i}}\right]^{\frac{\kappa_i}{\kappa_i - 1}}, \tag{4.13}$$

where θ_i^K and θ_i^V are share parameters and κ_i and λ_i denote respective elasticity of input substitution parameters. Labor and capital inputs are assumed to be perfectly mobile across sectors.

[16] Note that $\eta \to \infty$ represents that case of a Rawlsian social welfare function.

[17] Appendix E.5 contains a more detailed exposition of the equilibrium conditions for the numerical general equilibrium model, including the definition of the model parameters employed below.

At the third stage of production in sector i, primary energy inputs R_{ei}, with fossil fuel input $e \in \{Coal, Natural\ gas, Crude\ oil, Refined\ oil\}$ and electricity, B_i, are combined according to the following nested CES function:

$$E_i = \left[\theta_i^E B_i^{\frac{\nu_i - 1}{\nu_i}} + (1-\theta_i^E)(\sum_e \theta_{ie}^R R_{ie}^{\frac{\mu_i - 1}{\mu_i}})^{\frac{(\nu_i-1)\mu_i}{\nu_i(\mu_i-1)}}\right]^{\frac{\nu_i}{\nu_i - 1}}, \tag{4.14}$$

where θ_i^E and θ_{ei}^R are share parameters and μ_i and ν_i denote respective elasticity of input substitution parameters. Electricity and primary energy inputs are treated in separate nests to distinguish differences in substitution possibilities.

Carbon emissions are modeled as an input into production and are directly associated with using the amount R_{ei} of fossil fuel e in the production of sector i.[18] Given fuel-specific carbon coefficients ϕ_e, the carbon emissions (pollution) caused by burning fuel e in sector i are thus given by

$$Z_i \equiv \sum_e \phi_e R_{ei}\,. \tag{4.15}$$

Taxing carbon at the rate τ_i would thus increase the cost of using R_{ei} units of fossil fuel by $\tau_i Z_i = \tau_i \sum_e \phi_e R_{ie}$. As energy inputs become more costly following a sectoral carbon price, firms can substitute away by adjusting the input mix at each of the three stages the production process.

Firms producing sectoral outputs maximize profits under perfect competition:

$$\max_{\{M_1\ldots,M_I,V_i\}} p_i^Y Y_i - p_i^V V_i - \sum_j p_j^M M_{ji} \tag{4.16}$$

subject to Equations (4.12)–(4.14) and taking prices of Y_i, V_i, and M_{ij}, denoted by p_i^Y, p_i^V, and p_i^M, respectively, as given. Optimal cost-minimizing behavior of firms can be summarized by the unit cost function for sector i, denoted by $c_i(\mathbf{p})$.

Households of type $h \in H$ maximize utility from consuming sectoral outputs C_{ih}:[19]

$$\max_{\{C_{1h}\ldots,C_{Ih}\}} U_h = \left[\sum_i \theta_{ih}^U C_{ih}^{\frac{\rho_h - 1}{\rho_h}}\right]^{\frac{\rho_h}{\rho_h - 1}}, \tag{4.17}$$

where θ_{ih}^U and ρ_h denote share and elasticity of substitution parameters, respectively, subject to a budget constraint:

$$M_h = T_h + r\,\omega_h^K + w\,\omega_h^L\,, \tag{4.18}$$

stating that income is derived from transfer income T_h and from inelastically supplying endowments of capital (ω_h^K) and labor (ω_h^L) to firms at respective market prices r and w. Optimal utility-maximizing behavior for households of type h can be summarized by the unit expenditure function

[18] Our model thus abstracts from process-based carbon emissions. While it would be straightforward to expand the model in this direction, it would not affect the insights we derive from our numerical analysis.

[19] For simplicity, we abstract here from the nesting structure of household utility from consumption, which is expounded in Appendix E.5.

$c_h(\mathbf{p})$ which is related to the indirect utility function according to $U_h c_h(\mathbf{p}) \equiv V_h^{-1}(\mathbf{p}, M_h)$. Let p_h^U denote the associated price index for utility.

In equilibrium, the zero-profit conditions for sectoral production and aggregation of consumption goods in household utility determine the equilibrium quantities $\mathbf{q}(\tau_j)$:[20]

$$c_i(\mathbf{p}(\tau_j)) \geq p_i^Y \quad \perp \quad Y_i \geq 0 \;\; \forall i \tag{4.19}$$

$$c_h(\mathbf{p}(\tau_j)) \geq p_h^U \quad \perp \quad U_h \geq 0 \;\; \forall h\,. \tag{4.20}$$

The equilibrium formulation is completed by adding market-clearing conditions which determine prices $\mathbf{p}(\tau_j)$.[21] The factor markets for capital and labor services, respectively, are in equilibrium if:

$$\sum_h \omega_h^L \geq \sum_i \frac{\partial c_i(\mathbf{p}(\tau_j))}{\partial w} Y_i \quad \perp \quad w \geq 0 \tag{4.21}$$

$$\sum_h \omega_h^K \geq \sum_i \frac{\partial c_i(\mathbf{p}(\tau_j))}{\partial r} Y_i \quad \perp \quad r \geq 0\,. \tag{4.22}$$

The sum of intermediate input demands for sectoral output in production and consumption demands by households cannot exceed supply of sectoral output:

$$Y_i \geq \sum_j \frac{\partial c_j(\mathbf{p}(\tau_j))}{\partial p_{ij}^{YE}} Y_i + \sum_h \frac{\partial c_h(\mathbf{p}(\tau_j))}{\partial p_i^Y} U_h \quad \perp \quad p_i^Y \geq 0 \;\; \forall i\,, \tag{4.23}$$

where p_{ij}^{YE} denotes the carbon tax inclusive price for commodity i employed in sector j. Finally, the market for utility is in equilibrium if:

$$U_h \geq \frac{M_h}{p_h^U} \quad \perp \quad p_h^U \geq 0 \;\; \forall h\,. \tag{4.24}$$

In summary, conditions (4.18) to (4.24) jointly define the set of feasible equilibrium allocations $\mathcal{A}$.[22]

4.3.4 Data and calibration

This section details the data sources and procedure used to calibrate the multi-sector multi-household model to data for the U.S. economy. First, we briefly explain how production and consumption technologies and the input-output structure of the economy are calibrated based on social accounting matrix data and external estimates about elasticity parameters. Second, we describe the specification of household behavior as well as the benchmark patterns of expenditures

[20] We use the perpendicular sign $\perp$ to denote the complementarity relation between a function $F\colon \mathbb{R}^n \longrightarrow \mathbb{R}^n$ and a variable $z \in \mathbb{R}^n$ such that $F(z) \geq 0$, $z \geq 0$, and $z^T F(z) = 0$: $F(z) \geq 0 \perp z \geq 0$.

[21] Applying the envelope theorem, we can derive the demand for a particular commodity used in production (consumption) based on the partial derivative of the unit cost (expenditure) function with respect to the input price.

[22] Note that for reasons of brevity, we have omitted here the equilibrium conditions for a number of price and quantity variables associated with explicitly including the lower levels of sectoral production and household consumption in the equilibrium formulation.

and incomes for the heterogeneous households. Third, we discuss our choice of the social inequality aversion parameter η.

Matching social accounting matrix data. The calibration of the numerical model follows the standard procedure in applied general equilibrium modeling (see, for example, Rutherford, 1995; Harrison et al., 1997; Böhringer et al., 2016) according to which production and consumption technologies are calibrated to replicate a single-period reference equilibrium consistent with the Social Accounting Matrix (SAM) data for a given year. We use SAM data from the most recent version (version 9) of the database from the Global Trade Analysis Project (GTAP, Aguiar et al., 2016) describing the U.S. economy in the year 2009. Importantly, these data contain detailed information on carbon emissions as well as physical energy flows differentiated by primary and secondary energy carrier.

The ten goods categories shown in Table 4.1 are an aggregation of the 57 commodities in the GTAP database.[23] The aggregation is guided by the idea to keep sufficient detail with respect to the supply of energy (electricity as well as four primary energy sectors including *Coal*, *Natural gas*, *Crude oil*, *Refined oil products*) and the use of energy in the production of energy-intensive goods and services (such as *Energy-intensive goods* and *Transportation*) as well as other major sectors (*Manufacturing products*, *Services*, and *Agricultural products*). To facilitate calibrating the model as a closed economy without pre-existing tax distortions, we have removed international trade, taxes and transfers, and government spending from the GTAP dataset. Factors of earnings in our dataset comprise capital and labor.

Parametrization of household heterogeneity. Data on household expenditures (by goods category) and income sources is based on the U.S. Consumer Expenditure Survey (CES) from the U.S. Bureau of Labor Statistics. We divide households into expenditure deciles (i.e., $\pi_h = 0.1$, $\forall h$) and use the data from Rausch et al. (2011) who have used the same categories of households and goods as we do in this essay.

In order to represent heterogeneous households within the general equilibrium model, household-level and aggregate data must be consistent. For this, we balance household data to match aggregate consumption and income values as follows. First, we calibrate household consumption by multiplying aggregate values by each deciles' expenditure shares in total expenditure on each good. Second, we calibrate household factor incomes by multiplying aggregate income of each factor by decile income shares (i.e., capital and labor) in total income of that factor. Third, differences between calibrated household expenditures and incomes are reconciled by adding income of each factor in proportion to the factors' share in aggregate income, thus delivering a very close approximation of the decile's capital-to-labor ratio from the household-level data.

Table 4.1 shows the household data used in the model. Several dimensions of household heterogeneity are particularly noteworthy here. First, households exhibit a wide variation in income, with

[23] The exact aggregation schemes for sectors and regions and the aggregated benchmark data is available on request from the authors.

the top decile earning more than five times more than the bottom decile. Importantly, the observed disparity of incomes results in differing marginal social utilities of income across households, i.e. β_h in Equation (4.11). Second, expenditure shares vary mostly monotonically across deciles. For *Agricultural products*, *Electricity*, *Natural gas*, and *Manufacturing products* expenditure shares are higher for lower expenditure deciles whereas for the goods categories *Energy-intensive products*, *Refined oil products*, *Services*, and *Transportation* expenditure shares decline increase with income. For all deciles, *Coal* and *Crude oil* represent negligible shares of household expenditure. Third, the composition of household income in terms of the capital-labor split also varies across deciles, with capital income representing the highest share of income for the two top deciles. Fourth, although expenditure shares for *Electricity* and *Natural gas* are slightly larger for lower deciles as compared to higher deciles, the share of embodied emissions in consumption largely increases with income being more than three times larger when comparing the top to the bottom decile.

Table 4.1: Household characteristics in benchmark data by expenditure decile

	Expenditure deciles									
	1	2	3	4	5	6	7	8	9	10
Share of expenditures (in % of economy-wide expenditures)										
	4.1	6.0	7.4	8.3	8.9	9.1	10.7	12.2	13.5	19.8
Expenditure shares by good category (in % of total expenditures for decile)										
Agricultural products	15.2	12.8	12.0	10.7	10.0	8.4	7.8	6.9	4.6	2.9
Coal	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Crude oil	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Energy-intensive goods	0.7	1.2	1.2	1.2	1.4	1.7	2.2	2.9	4.5	5.9
Electricity	1.8	1.7	1.5	1.5	1.4	1.4	1.3	1.2	1.0	0.8
Natural gas	0.2	0.2	0.2	0.2	0.1	0.2	0.1	0.1	0.1	0.1
Manufacturing products	40.5	34.3	32.1	28.4	26.6	22.3	20.8	18.3	12.3	7.8
Refined oil products	0.9	1.0	1.0	1.1	1.1	1.2	1.2	1.2	1.3	1.2
Services	39.5	47.6	50.7	55.5	58.0	63.4	65.1	67.9	74.5	79.7
Transportation	1.1	1.2	1.3	1.4	1.3	1.5	1.5	1.5	1.6	1.5
Income by primary factor (in % of decile income)										
Capital	28.3	27.0	26.6	25.3	25.5	23.4	24.1	26.4	33.2	47.6
Labor	71.7	73.0	73.4	74.7	74.5	76.6	75.9	73.6	66.8	52.4
CO_2 emissions embodied in consumption (as % share of total emissions)[a]										
	4.8	6.7	8.0	8.9	9.3	9.3	10.7	12.0	12.8	17.7

Notes: Data is rounded to one decimal place, i.e. expenditure shares for crude oil and coal are very small but non-zero. [a]To calculate embodied emissions, we adopt the approach described in Böhringer et al. (2016) to recursively solve an input-output version of our numerical general equilibrium model using a diagonalization algorithm.

Choice of social inequality aversion parameter η. Pinning down the inequality aversion parameter η in the iso-elastic social welfare function (4.10) is fraught with difficulties. Based on estimates for η for the context of France by Cremer et al. (2003) we carry out our numerical analysis for low and high values, i.e. $\eta = 0.1$ and $\eta = 1.9$. While higher values for η may be conceivable, we will see that $\eta = 1.9$ already yields strong results in terms of optimally differentiated sectoral carbon taxes as well as optimal redistribution.

4.4 Simulation Results

This section presents our results based on numerical simulations using the quantitative model to explore optimal sectoral pollution taxes and redistribution schemes. We start by looking at the optimal case and decompose the magnitude of different channels driving tax differentiation previously identified in the theoretical analysis. We then investigate optimal tax policies for given redistribution schemes and compare incidence impacts across household expenditure groups under uniform and differentiated pollution pricing. In our central case simulations, we assume that pollution is capped at 20% below the benchmark.

4.4.1 Optimal policies

Table 4.2 shows optimal sectoral carbon prices for different degrees of social inequality aversion. Several insights emerge. First, the degree of the social inequality aversion determines the amount of tax differentiation. For $\eta = 0$ sectoral carbon prices are almost identical.[24] The higher η, the more unequal social weights become and the larger is the differentiation of sectoral carbon taxes. It is evident that already a relatively low social inequality aversion ($\eta = 0.1$) brings about a significant deviation from uniform pollution pricing, i.e. the standard deviation of sectoral carbon taxes is 21.3 and the minimum and maximum tax rates are 20.9 and 119.1, respectively.

Second, as social equity concerns become more important (i.e., with increasing η), raising a higher amount of pollution tax revenues is optimal because it allows to a larger extent to implement targeted transfers to households with a higher social weight. Figure 4.2 shows the transfers (as a share of total revenue) to households by expenditure decile. With social equity concerns, optimal transfers imply higher shares of transfers received by low income households that have a relatively higher β. As the pollution tax revenues increase, sufficient revenue is available to address equity concerns of multiple household groups beyond the lowest expenditure decile. Given that the pollution level is capped, raising large revenues is achieved through generally higher carbon prices, as illustrated by the mean carbon tax.

Third, for high social inequality aversion ($\eta = 1.9$) the optimal pollution tax for some sectors is zero while other sectors face sizable tax rates. The reasons that drive the specific pattern of sectoral pollution taxes are discussed below when decomposing the different motives for tax differentiation. While the very high pollution taxes rates for some sectors reflect that social equity concerns are a powerful driver of tax differentiation, less extreme tax differentiation would be observed when optimal pollution taxes are set for a given, non-optimal redistribution schemes which may reflect constraints on implementing fully optimal transfers in real-world policy settings.

Fourth, the inequality of real household incomes, as measured by the Gini coefficient, decreases under optimally differentiated pollution taxes as compared to uniform pollution pricing. Not surprisingly, this decrease is larger, the larger is social inequality aversion. Importantly, differentiated

[24] This case is close to equal social weighting but β_h is not identical across households as there exist differences in households' tastes. See the definition of β_h in Equation (4.11).

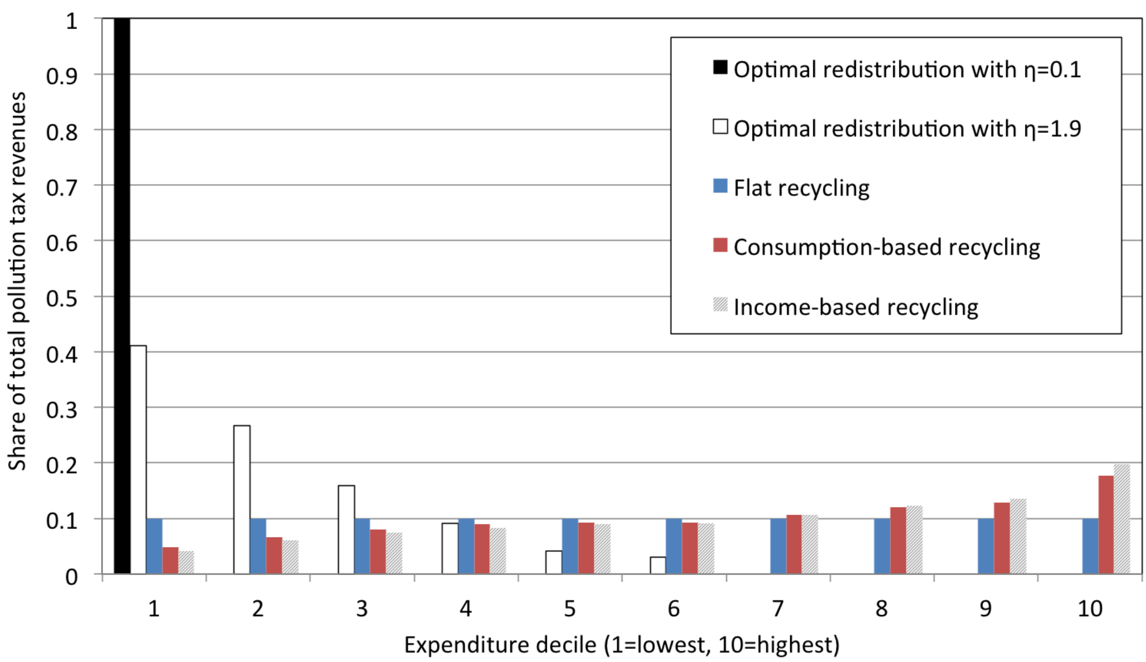
pollution pricing achieves a more equitable (i.e., socially optimal) outcome that cannot be attained under uniform pollution pricing and optimal transfers.

Table 4.2: Optimal sectoral carbon prices (US$/ton CO_2), tax revenues, and change in inequality for different social inequality aversion

	Social inequality aversion		
	$\eta = 0$	$\eta = 0.1$	$\eta = 1.9$
Summary statistics for carbon taxes			
Mean	32.4	41.5	365.7
Standard deviation	0.2	21.3	937.2
Sectoral carbon taxes			
Agricultural products	32.3	27.9	3.9
Coal	32.2	20.9	0.0
Crude oil	32.3	25.2	0.0
Energy-intensive goods	32.3	32.1	0.0
Electricity	32.3	30.0	4.8
Natural gas gas	32.2	23.0	0.0
Manufacturing goods products	32.3	27.4	0.0
Refined oil products	33.3	119.1	690.5
Services	32.3	27.9	0.0
Transportation	32.6	57.2	2904.1
Pollution tax revenues (in billion US$)	107.8	137.9	1214.2
Change in Gini coefficient (in % relative to uniform taxes)[a]	0.0	-1.5	-43.3

Notes: [a]Gini coefficients are based on real household income. The change in the Gini coefficient is computed relative to a policy which restricts pollution taxes to be uniform but allows for optimal transfers.

Figure 4.2: Transfers of pollution tax revenues for optimal and non-optimal redistribution policies



4.4.2 Decomposing the importance of different motives for tax differentiation

We now examine how much of the differentiation of sectoral pollution taxes as shown in Table 4.2 is driven by the three channels (*Preference effect*, *Revenue redistribution effect*, *Factor income effect*) we have identified previously. For decomposing these effects, we build on our theoretical results derived in Section 4.2. More specifically, we use Proposition 4.3 which states three conditions under which uniform pollution pricing is optimal. Assuming that one of these conditions is not met enables us to isolate the impact through this channel on non-uniform pollution pricing.

Preference effect. To isolate tax differentiation due to the *Preference effect*, we assume income-proportional redistribution (i.e., condition (3) in Proposition 4.3) and identical composition of factor income across households (i.e., condition (1) in Proposition 4.3).

Table 4.3 shows optimal sectoral pollution taxes due to the *Preference effect* for different degrees of social inequality aversion. Several insights emerge. First, the goods consumed intensively by poorer households—who receive a larger social weight β by an inequality-averse government—are taxed at a lower rate compared to goods for which the opposite holds. Looking at the expenditure shares by good category shows that "Services", "Transportation", and "Energy-intensive goods" are among the goods that are consumed more intensively by higher expenditure deciles whereas low-income households spend a large share of their income on "Agricultural products", "Manufacturing products", and "Electricity" (see Table 4.1). The sectoral differentiation of tax rates due to the *Preference effect* mostly reflects this pattern of expenditure shares. The intuition from Proposition 4.4, derived in the context of our analytical example, thus carries over to the more general setting of our numerical framework. In addition, our general equilibrium framework with multiple sectors also picks up the intermediate input-output structure of the economy. Thus, there exists some differentiation between "Coal" and "Crude oil" although there are virtually not consumed directly by households. Second, while there clearly is a differentiation of sectoral tax rates due to the *Preference effect*, the standard deviation is considerably smaller as compared to the case which includes all channels. This suggests that the other motives are important drivers as well. Third, the fact that the mean carbon price only increases slightly with a higher social inequality aversion reflects the absence of the revenue redistribution motive.

Revenue redistribution effect. To isolate the *Revenue redistribution effect*, we assume identical household preferences (i.e., condition (2) in Proposition 4.3) and identical composition of factor income across households (i.e., condition (1) in Proposition 4.3).

Table 4.4 shows optimal sectoral pollution taxes due to the *Revenue redistribution effect* for different degrees of social inequality aversion. First, we see that the *Revenue redistribution effect* gives rise to a substantial differentiation of sectoral pollution taxes. As under the *Revenue redistribution effect* the goal is to increase the pollution tax revenues, sectors with relatively steep marginal abatement cost are taxed heavily as for the given cap this drives up carbon prices. We

Table 4.3: Optimal sectoral carbon prices (US$/ton CO_2) for different social inequality aversion due to *Preference effect*

	Social inequality aversion		
	$\eta = 0$	$\eta = 0.1$	$\eta = 1.9$
Summary statistics for carbon taxes			
Mean	32.5	32.6	34.5
Standard deviation	0.0	0.4	5.2
Sectoral carbon taxes			
Agricultural products	32.5	31.1	29.4
Coal	32.5	32.9	32.6
Crude oil	32.5	33.0	33.5
Energy-intensive goods	32.5	33.1	39.2
Electricity	32.5	32.4	30.1
Natural gas gas	32.5	32.6	33.9
Manufacturing goods products	32.5	32.1	30.6
Refined oil products	32.5	33.0	34.1
Services	32.5	33.3	42.0
Transportation	32.5	32.9	41.3

find that CO_2 emissions in "Transportation" and "Refined oil products" are taxed at relatively high rates. This is driven by the fact that substitution between energy and non-energy inputs is relatively low in the transportation sector and that demand for "Transportation" and "Refined oil products" is relatively inelastic. Thus, to achieve a given emissions reductions through abatement in these sectors can only be achieved with relatively high pollution taxes. Second, the higher the social inequality aversion, the more pronounced is the differentiation of taxes. For $\eta = 1.9$, the pollution tax revenues are almost entirely raised by taxing "Transportation" and "Refined oil products" only. Here, efficiency in abatement is largely sacrificed in favor of raising high revenues for targeted transfers to enable addressing social inequality concerns. $\eta = 0.1$ represents an intermediate case in which the revenue-raising motive is already present, as is reflected by above-average pollution taxes on "Transportation" and "Refined oil products". This, however, has to be traded-off against productive efficiency in abatement where the latter is relatively more important given the lower degree of social inequality aversion. Hence, pollution tax rates for other sectors are also substantial for $\eta = 0.1$ and are close to the those in the case with uniform pollution pricing. Third, when comparing the case in which all three channels are present (Table 4.2) with the case in which only the *Revenue redistribution effect* is active (Table 4.4), it is evident that a similar pattern and standard deviation of sectoral pollution taxes emerges. This indicates that the differentiation of sectoral pollution taxes is to a large extent driven by the *Revenue redistribution effect*.

Factor income effect. To isolate the *Factor income effect*, we assume identical household preferences (i.e., condition (2) in Proposition 4.3) and redistribution in proportion to benchmark income (i.e., an approximation of condition (3) in Proposition 4.3).[25]

We find that tax differentiation due to the *Factor income effect* is relatively weak. The standard

[25] Unlike for the *Preference effect* and the *Revenue redistribution effect*, the decomposition of the *Factor income effect* is imperfect because in Proposition 4.3 relaxing the assumption (1) of identical composition of factor income whilst at the same time assuming a fixed and income-proportional redistribution scheme is not possible. Thus, the effect we derive is conflated with the *Revenue redistribution effect*.

Table 4.4: Optimal sectoral carbon prices (US$/ton CO_2) and pollution tax revenue for different social inequality aversion due to the *Revenue redistribution effect*

	Social inequality aversion		
	$\eta = 0$	$\eta = 0.1$	$\eta = 1.9$
Summary statistics for carbon taxes			
Mean	32.5	41.0	377.9
Standard deviation	0.0	16.5	1043.9
Sectoral carbon taxes			
Agricultural products	32.5	28.8	0.0
Coal	32.5	20.3	0.0
Crude oil	32.5	24.8	0.0
Energy-intensive goods	32.5	31.0	0.0
Electricity	32.5	29.8	0.6
Natural gas gas	32.5	22.7	0.0
Manufacturing goods products	32.5	27.2	0.0
Refined oil products	32.5	118.9	13.6
Services	32.5	26.7	0.0
Transportation	32.5	56.0	3267.7
Pollution tax revenues raised (billion US$)	107.9	136.2	1254.8

deviation of sectoral pollution taxes for $\eta = 0.1$ and $\eta = 1.9$ is 0.5 and 4.7, respectively. While standard deviations are comparable in magnitude to those under the *Preference effect*, it is important to note that these numbers also pick up parts of the *Revenue redistribution effect* which cannot be cleanly removed. That indeed the *Revenue redistribution effect* is driving tax differentiation here, can be seen by the fact that "Transportation" and "Refined oil products" are taxed at relatively high rates, in turn largely determining the standard deviation. The fact that the *Factor income effect* is relatively weak is unsurprising given that the composition of factor incomes across household groups in Table 4.1 is relatively similar.[26]

4.4.3 Pollution tax differentiation for non-optimal and fixed redistribution schemes

Besides analyzing government policies which comprise optimal pollution taxes *and* optimal transfers, the more relevant situation for real-world environmental policy may be one in which a specific redistribution scheme is already in place or favored over the extreme pattern of transfers under the optimal policy. We therefore now investigate the extent of optimal tax differentiation under the constraint of a given fixed and sub-optimal redistribution scheme. Following Bento et al. (2009), we consider three alternative revenue recycling schemes: "*Flat recycling*" returns revenues in equal amounts to every household; "*Income-based recycling*" returns revenues in proportion to benchmark income; "*Consumption-based recycling*"[27] returns revenues according to each household's share of aggregate dirty consumption in the benchmark.[28]

[26] Including government transfers that are unrelated to pollution tax rebates in the analysis may introduce additional heterogeneity on the sources of income side. This, and in general, more heterogeneity with respect to the composition of factor income may thus increase the scope for tax differentiation through this channel.

[27] The "*Consumption-based recycling*" is analogous to the "VMT-based" recycling in Bento et al. (2009).

[28] We define dirty consumption by using a comprehensive measure of embodied CO_2 emissions that captures direct and indirect emissions in household consumption (see Table 4.1).

Table 4.5 reports the optimal differentiation of sectoral pollution taxes for these three redistribution schemes and different degrees of social inequality aversion. First, the differentiation of sectoral pollution taxes is the largest for the *Flat recycling* and the lowest for *Income-based recycling*. The reason is that the transfers associated with *Flat recycling* are the closest to the optimal transfers among the three schemes considered here—as can be seen from Figure 4.2. Being closer to the optimal transfers means that the *Revenue redistribution effect* is stronger, hence implying a relatively larger tax differentiation to increase tax revenues; this is reflected by the fact that tax revenues as well as the mean carbon tax rate are largest under *Flat recycling*. The relatively low tax differentiation under *Income-based recycling* is due to the fact that this redistribution scheme is closest to a scheme that shuts off the *Revenue redistribution effect*. *Consumption-based recycling* represents an intermediate case reflecting the fact that the consumption of lower expenditure deciles is more carbon-intensive. Hence, as is evident from Figure 4.2, the distribution of transfers across expenditure deciles deviates from the one under the *Income-based recycling* in the direction of *Flat recycling*.

Table 4.5: Optimal differentiation of sectoral pollution taxes for alternative non-optimal and fixed redistribution schemes and different degree of social inequality aversion

	Social inequality aversion	
	$\eta = 0.1$	$\eta = 1.9$
Flat recycling		
Mean pollution tax (US$/ton CO_2)	34.7	132.9
Standard deviation of sectoral pollution taxes	5.0	232.3
Pollution tax revenues raised (billion US$)	115.1	441.4
Consumption-based recycling		
Mean pollution tax (US$/ton CO_2)	33.1	41.7
Standard deviation of sectoral pollution taxes	1.4	21.5
Pollution tax revenues raised (billion US$)	110.1	138.4
Income-based recycling		
Mean pollution tax (US$/ton CO_2)	32.8	36.9
Standard deviation of sectoral pollution taxes	0.7	10.6
Pollution tax revenues raised (billion US$)	109.0	122.6

Second, the difference in tax differentiation between the alternative redistribution schemes are magnified with increasing social inequality aversion (i.e., comparing $\eta = 1.9$ to $\eta = 0.1$). As social equity concerns become more important, the increasing difference in the standard deviation between the three schemes reflects their differences in terms of the relative importance of the *Revenue recycling effect*.

Third, despite our finding that these policy-relevant redistribution schemes strongly diminish the *Revenue recycling effect*—which we identified as a main driver for differentiation of sectoral pollution tax in the optimal policy—optimal pollution taxes may still be strongly differentiated (i.e., standard deviation as high as 232.3). This is due to the fact that the *Revenue redistribution effect* can still play an important role if a given redistribution scheme ensures a sufficiently high share of transfers to lower income households (who receive relatively high social weights under an inequality-averse government).

4.4.4 Incidence impacts of optimally differentiated taxes with non-optimal revenue redistribution

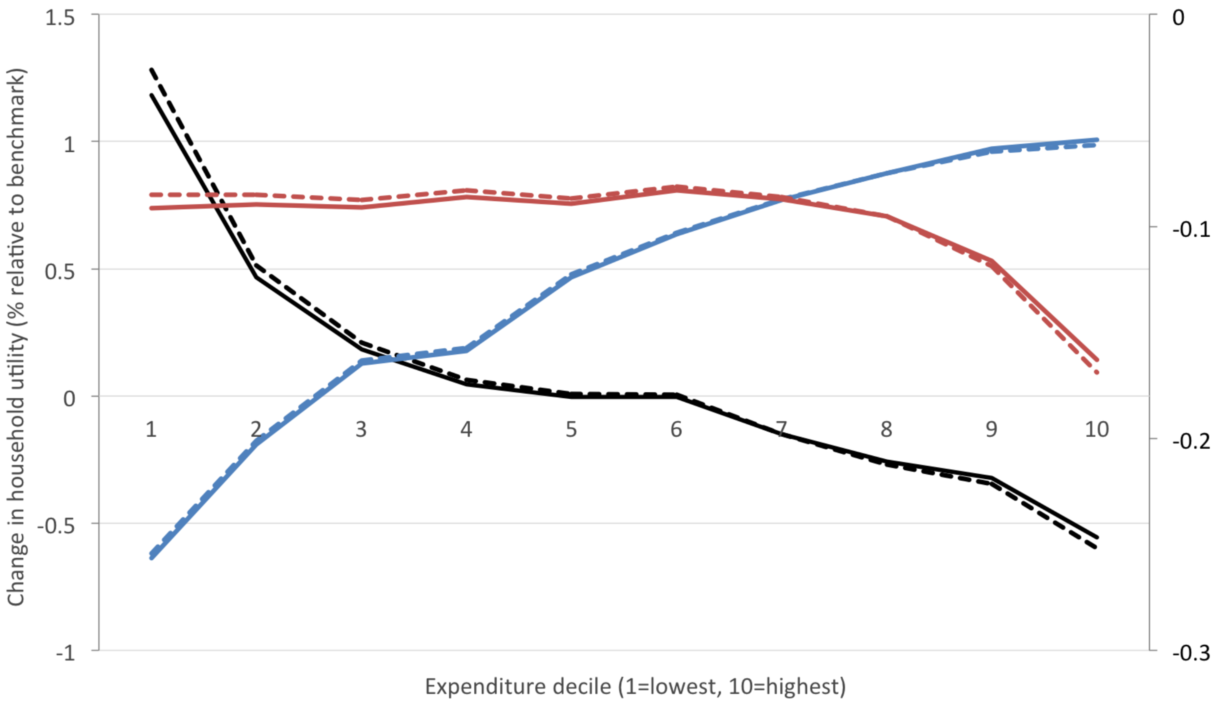
An important and policy-relevant question is to examine how households are affected by implementing pollution control policies that are based on optimally differentiated taxes. For this purpose, we compare the household-level welfare impacts under uniform or differentiated pollution pricing to a no-policy benchmark without pollution taxes. To stay in the space of policies that bear some realism, we investigate this question by continuing to focus on the three recycling schemes considered above.

Figure 4.3 shows the incidence across expenditure deciles for uniform and differentiated pollution taxes for the alternative revenue recycling schemes; Panel (a) and (b) considers low and high values for social inequality aversion, respectively. First, as documented by the large literature on the distributional impacts of carbon taxation, we find that the way the revenues are recycled can importantly alter the incidence pattern across households (see, for example, Bento et al., 2009; Rausch et al., 2010b; Mathur and Morris, 2014; Williams III et al., 2015). In line with the previous literature, we find that for uniform pollution pricing *Flat recycling* yields sharply progressive outcomes. *Income-based recycling* is regressive as poorer households spend a larger fraction of their income on pollution goods while the revenue rebates in proportion to (benchmark) income have a neutral effect. *Consumption-based recycling* represents an intermediate case which we find here to be neutral to somewhat progressive (for higher incomes). Note that these results, obtained under uniform pollution pricing, are not affected by social equity concerns (as the transfer scheme is exogenously fixed and tax differentiation is ruled out).

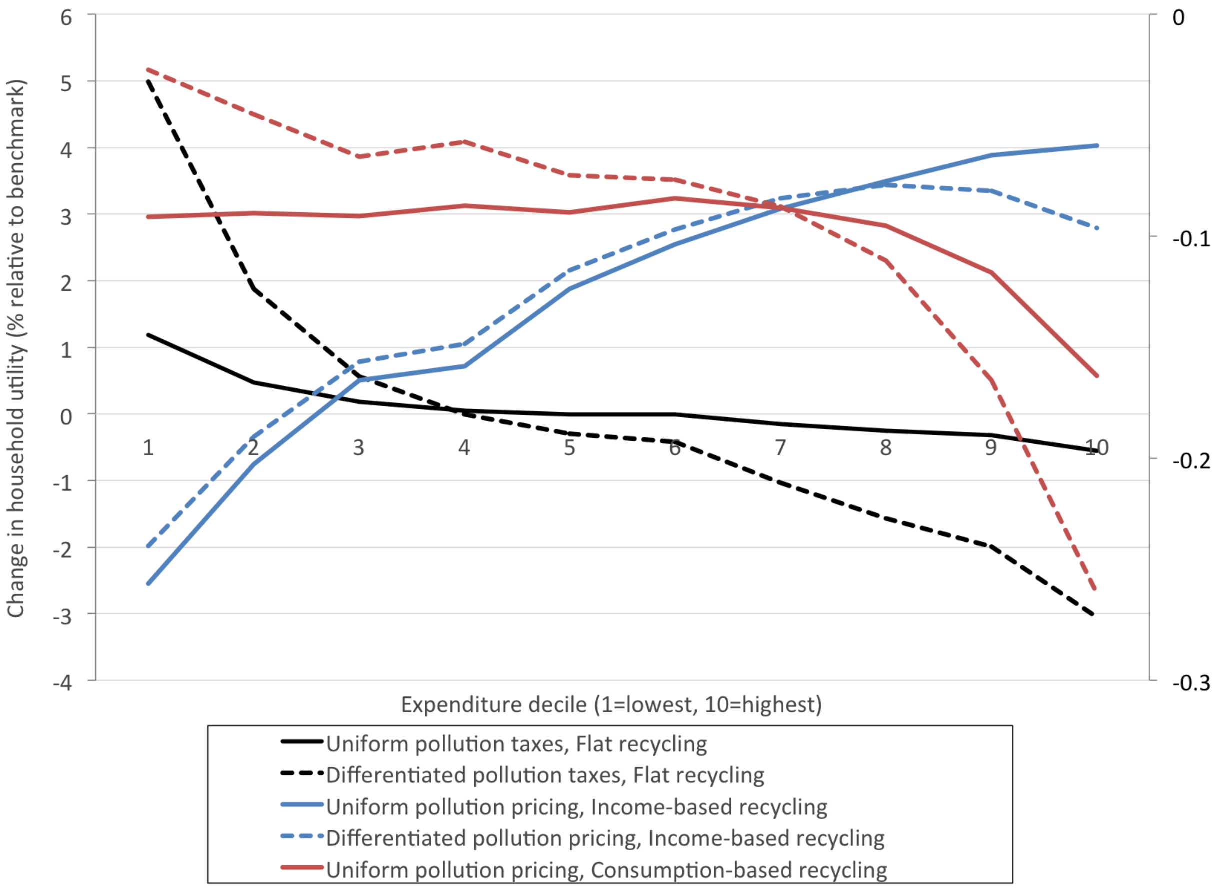
Based on Figure 4.3, the following insights emerge when comparing the incidence under differentiated versus uniform sectoral pollution taxes. First, for all three transfer schemes, optimally differentiated pollution taxes lead to an incidence pattern which is more progressive (less regressive) relative to uniform pollution taxes. Allowing for optimally differentiated pollution taxes brings the policy closer to the fully optimal policy with optimal pollution taxes *and* transfers. Deviating from uniform pollution pricing thus allows the government to reduce inequality through the three channels analyzed above (*Preference effect*, *Factor income effect*, and *Revenue redistribution effect*). Second, not surprisingly, the impact on the incidence pattern increases with higher social inequality aversion. Third, how much inequality can be reduced with differentiated pollution taxes depends on the given transfer scheme. In general, transfer schemes which are closer to the optimal one—thus better capturing the *Revenue recycling effect*—are better suited to address inequality thus leading to larger differences in the incidence pattern. Thus, allowing for differentiated pollution taxes in combination with *Flat recycling* has the largest effect on incidence (in line with our findings in Section 4.4.3). For the same reason, the incidence patterns are affected the least under *Income-based recycling*. Fourth, both under *Consumption-based* and *Income-based* recycling the qualitative pattern of the incidence is altered for parts of the income spectrum. With relatively high inequality aversion (i.e., $\eta = 1.9$), the incidence profile under *Consumption-based recycling* for expenditure deciles 1 to 7 changes from neutral to progressive while under *Income-based recycling* the incidence across the top three expenditure deciles changes from regressive to progressive.

Figure 4.3: Incidence by expenditure decile for non-optimal redistribution schemes: *Flat recycling* (primary axis), *Income-based recycling* and *Consumption-based recycling* (secondary axis)

(a) Social inequality aversion $\eta = 0.1$



(b) Social inequality aversion $\eta = 1.9$



4.5 Concluding remarks

This essay has investigated differentiated pollution pricing in the presence of social equity concerns. To this end, we employed both theoretical and numerical general equilibrium analyses in an optimal taxation framework. We illustrated how household heterogeneity in preference and endowments interacts with social equity concerns, thus causing marginal abatement cost absent social equity concerns not to be equalized at the social optimum, thus motivating pollution price differentiation. Relative to a case with uniform pollution pricing, tax rates should be differentiated to increase social welfare by shifting the burden of environmental policy towards households with low social weights, and by increasing the amount of pollution tax revenue for targeted transfers to households with high social weights. In the context of the U.S. economy, we found that optimal carbon taxes for an inequality-averse government differ significantly across sectors, even when social inequality aversion is relatively low. The degree of tax rate differentiation decreases for non-optimal redistribution schemes, whilst remaining significant. Relative to uniform carbon pricing, incidence patterns across household income groups for a given redistribution scheme can vary qualitatively when allowing for differentiated taxes.

Our study has a number of implications for policy-making. First, our findings illustrate how pollution taxation can serve distributional objectives, in addition to environmental goals. Second, social equity concerns should not be considered separately from efficiency aspects of environmental policy. It may indeed be optimal to sacrifice productive efficiency of pollution abatement to shield some households from increases in prices of consumption goods and decreases in returns to factors, as well as to raise pollution tax revenues for targeted transfers to households. Third, in the presence of political constraints on redistribution, the degree of tax differentiation depends on how well the constrained redistribution scheme approximates the optimal one. Importantly, for an inequality-averse government, equal redistribution across households—which arguably best reflects real-world constraints on the redistribution of pollution taxation—exhibits a large degree of tax differentiation at the social optimum.

Our study could be extended in a number of ways. On the one hand, a more detailed understanding of the relationships between optimal sectoral pollution taxes and the economy's characteristics may yield policy prescriptions to guide the implementation of optimal pollution pricing. On the other hand, future research could incorporate international market power, terms-of-trade effects and pre-existing non-environmental taxes. Such efficiency-based tax differentiation motives would likely interact with the equity-driven effects studied here, without, however, changing the basic result that household heterogeneity and social equity concerns cause optimal pollution prices to be differentiated across sectors.

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Appendices

A Appendix for Essay 1

A.1 Derivation of Equations (1.10) and (1.11)

Consider the household demand functions $X = X(p_X, p_Y, M)$ and $Y = Y(p_X, p_Y, M)$, where the household index h is omitted for simplicity. Define the income elasticities of demand of good X and Y as $E_{X,M} = \frac{M}{X}\frac{\partial X}{\partial M}$ and $E_{Y,M} = \frac{M}{Y}\frac{\partial Y}{\partial M}$, respectively. Let $E_{X,p_X} = -\frac{p_X}{X}\frac{\partial X}{\partial p_X}$ and $E_{Y,p_X} = -\frac{p_X}{Y}\frac{\partial Y}{\partial p_X}$ denote the respective own price elasticities of demand. As shown in Hicks and Allen (1934), at the equilibrium solution the following conditions hold: $E_{X,p_X} = \alpha E_{X,M} + (1-\alpha)\sigma$, $E_{Y,p_X} = \alpha E_{Y,M} - \alpha\sigma$, $E_{X,p_Y} = (1-\alpha)E_{X,M} - (1-\alpha)\sigma$, $E_{Y,p_Y} = (1-\alpha)E_{Y,M} + \alpha\sigma$, where σ is the elasticity of substitution between clean and dirty consumption in utility.

Using these four conditions, changes in household h's demand for good X and Y given changes in the prices of goods and factor prices can be expressed, respectively, as:

$$\begin{aligned}\hat{X}^h &= \frac{1}{X^h}(p_X\partial_{p_X}X^h\hat{p}_X + p_Y\partial_{p_Y}X^h\hat{p}_Y + M^h\partial_{M^h}X^h\hat{M}^h)\\ &= -E^h_{X,p_X}\hat{p}_X - E^h_{X,p_Y}\hat{p}_Y + E^h_{X,M}\hat{M}^h\\ &= -(\alpha E^h_{X,M} + (1-\alpha)\sigma^h)\hat{p}_X - ((1-\alpha)E^h_{X,M} - (1-\alpha)\sigma^h)\hat{p}_Y + E^h_{X,M}\hat{M}^h\,, \end{aligned} \tag{A.1}$$

and

$$\begin{aligned}\hat{Y}^h &= \frac{1}{Y^h}(p_X\partial_{p_X}\hat{p}_X + p_Y\partial_{p_Y}\hat{p}_Y + M^h\partial_{M^h}\hat{M}^h)\\ &= -E^h_{Y,p_X}\hat{p}_X - E^h_{Y,p_Y}\hat{p}_Y + E^h_{Y,M}\hat{M}^h\\ &= -(\alpha E^h_{Y,M} - \alpha\sigma^h)\hat{p}_X - ((1-\alpha)E^h_{Y,M} + \alpha\sigma^h)\hat{p}_Y + E^h_{Y,M}\hat{M}^h\,. \end{aligned} \tag{A.2}$$

A.2 Derivation of price and pollution changes in general solution (Equations (1.15a)–(1.15c))

Subtract Eq. (1.8) from Eq. (1.6) and Eq. (1.9) from Eq. (1.7), to obtain:

$$\hat{p}_X = \theta_{XK}\hat{r} + \theta_{XL}\hat{w} \tag{A.3}$$

$$\hat{p}_Y = \theta_{YK}\hat{r} + \theta_{YL}\hat{w} + \theta_{YZ}\hat{\tau}_Z\,. \tag{A.4}$$

Substitute Eqs. (1.12) and (1.13) into Eqs. (1.8) and (1.9):

$$\sum_h \frac{X^h}{X}\hat{X}^h = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X \tag{A.5}$$

$$\sum_h \frac{Y^h}{Y}\hat{Y}^h = \theta_{YK}\hat{K}_Y + \theta_{YL}\hat{L}_Y + \theta_{YZ}\hat{Z}\,. \tag{A.6}$$

Solve Eq. (1.10) for $\hat{Y}^h$ and insert the result into Eq. (A.6). Rearrange to obtain:

$$\frac{1}{Y}\sum_h Y^h\Big(\sigma^h(\hat{p}_Y-\hat{p}_X)+(E^h_{Y,M}-E^h_{X,M})(\alpha^h\hat{p}_X+(1-\alpha^h)\hat{p}_Y-\hat{M}^h)\Big)=$$
$$\sum_h\frac{Y^h}{Y}\hat{X}^h-\theta_{YK}\hat{K}_Y-\theta_{YL}\hat{L}_Y-\theta_{YZ}\hat{Z}\,.$$

From Eq. (A.5), insert the following on the right-hand side of the equality: $+0=\theta_{XK}\hat{K}_X+\theta_{XL}\hat{L}_X-\sum_h\frac{X^h}{X}\hat{X}^h$ and use the fact that X is chosen to be the numéraire, thus yielding:

$$\frac{1}{Y}\sum_h Y^h\Big(\sigma^h(\hat{p}_Y)+(E^h_{Y,M}-E^h_{X,M})((1-\alpha^h)\hat{p}_Y-\hat{M}^h)\Big)=$$
$$\sum_h\frac{M^h}{p_YY}(1-\frac{\alpha^h}{\gamma})\hat{X}^h+\theta_{XK}\hat{K}_X+\theta_{XL}\hat{L}_X-\theta_{YK}\hat{K}_Y-\theta_{YL}\hat{L}_Y-\theta_{YZ}\hat{Z}\,. \tag{A.7}$$

Eliminate $\hat{X}^h$ from Eq. (A.7) by using Eq. (1.11), then insert the explicit expression for the budget change $\hat{M}^h$:

$$\hat{p}_Y\delta=\sum_h\phi^h_L\hat{w}+\sum_h\phi^h_K\hat{r}+\sum_h\phi^h_Z\hat{\tau}_Z$$
$$+\theta_{XK}\hat{K}_X+\theta_{XL}\hat{L}_X-\theta_{YK}\hat{K}_Y-\theta_{YL}\hat{L}_Y+(\sum_h\phi^h_Z-\theta_{YZ})\hat{Z}\,. \tag{A.8}$$

Next, solve Eqs. (1.1) and (1.2) for $\hat{K}_X$ and $\hat{L}_X$, and insert them into (A.8). Furthermore, insert Eq. (A.4) to eliminate $\hat{p}_Y$, thus obtaining:

$$(\sum_h\phi^h_Z-\theta_{YZ})\hat{Z}=(\delta\theta_{YK}-\sum_h\phi^h_K)\hat{r}+(\delta\theta_{YL}-\sum_h\phi^h_L)\hat{w}+(\delta\theta_{YZ}-\sum_h\phi^h_Z)\hat{\tau}_Z$$
$$+\hat{K}_Y(\theta_{XK}\gamma_K+\theta_{YK})+\hat{L}_Y(\theta_{XL}\gamma_L+\theta_{YL})\,. \tag{A.9}$$

Solve Eqs. (1.4) and (1.5) for $\hat{K}_Y$ and $\hat{L}_Y$, and insert them into Eq. (A.9). This yields:

$$-C\hat{Z}=(-\sum_h\phi^h_K+\theta_{YK}(\delta+\beta_K(e_{KK}-e_{ZK})+\beta_L(e_{LK}-e_{ZK})))\hat{r}$$
$$+(-\sum_h\phi^h_L+\theta_{YL}(\delta+\beta_K(e_{KL}-e_{ZL})+\beta_L(e_{LL}-e_{ZL})))\hat{w}$$
$$+(-\sum_h\phi^h_Z+\theta_{YZ}(\delta+\beta_K(e_{KZ}-e_{ZZ})+\beta_L(e_{LZ}-e_{ZZ})))\hat{\tau}_Z\,. \tag{A.10}$$

Next eliminate $\hat{Z}$. To achieve this, substitute Eqs. (1.1) and (1.2) into Eq. (1.3), obtaining:

$$-\gamma_K\hat{K}_Y+\gamma_L\hat{L}_Y=\sigma_X(\hat{w}-\hat{r})\,. \tag{A.11}$$

Substituting Eqs (1.4) and (1.5) into Eq. (A.11) yields:

$$\sigma_X(\hat{w}-\hat{r})=(\gamma_L-\gamma_K)\hat{Z}+\theta_{YK}(\gamma_L(e_{LK}-e_{ZK})\hat{r}-\gamma_K(e_{KK}-e_{ZK}))\hat{r}$$
$$\theta_{YL}(\gamma_L(e_{LL}-e_{ZL})\hat{w}-\gamma_K(e_{KL}-e_{ZL}))\hat{w}+$$
$$\theta_{YZ}(\gamma_L(e_{LZ}-e_{ZZ})\hat{\tau}_Z-\gamma_K(e_{KZ}-e_{ZZ}))\hat{\tau}_Z\,. \tag{A.12}$$

Now solve Eq. (A.12) for $\hat{Z}$ and equate to Eq. (A.10):

$$\Big((\gamma_K-\gamma_L)(-\sum_h\phi^h_K+\theta_{YK}\delta)+C\sigma_X+\theta_{YK}[-A(e_{KK}-e_{ZK})+B(e_{LK}-e_{ZK})]\Big)\hat{r}$$
$$+\Big((\gamma_K-\gamma_L)(-\sum_h\phi^h_L+\theta_{YL}\delta)-C\sigma_X+\theta_{YL}[-A(e_{KL}-e_{ZL})+B(e_{LL}-e_{ZL})]\Big)\hat{w}$$
$$=\Big((\gamma_L-\gamma_K)(-\sum_h\phi^h_Z+\theta_{YZ}\delta)+\theta_{YZ}[-A(e_{ZZ}-e_{KZ})+B(e_{ZZ}-e_{LZ})]\Big)\hat{\tau}_Z \tag{A.13}$$

Eqs. (A.3) and (A.13) are two equations in two unknowns, $\hat{r}$ and $\hat{w}$. Solve Eq. (A.3) for $\hat{w}$ and substitute into Eq. (A.13), solving for $\hat{r}$. Inserting $\hat{r}$ into Eq. (A.3) and Eq. (A.4) then delivers $\hat{w}$ and $\hat{p}_Y$, respectively. These expressions correspond to Eqs. (1.15a)–(1.15c).

A.3 Special cases and proofs

A.3.1 Derivation of Equations (1.16a)–(1.16b)

In the case of homothetic preference, $E^h_{X,M} = E^h_{Y,M} = 1$. We can therefore simplify some of the terms that reflect the heterogeneity of preferences in Eqs. (1.15a)–(1.15c) as follows:

$$\sum_h \phi^h_Z = \sum_h (1 - \frac{\alpha^h}{\gamma})\xi^h \frac{\tau_Z Z}{p_Y Y} = \frac{\tau_Z Z}{p_X X p_Y Y} \sum_h (\gamma - \alpha^h) M^h = 0\,,$$

$$\sum_h \phi^h_L = \sum_h (1 - \frac{\alpha^h}{\gamma}) \frac{w\bar{L}^h}{p_Y Y} = \frac{1}{\gamma p_Y Y} \sum_h (\gamma - \alpha^h) M^h \theta^h_L = -\frac{cov(\alpha^h, \theta^h_L)}{\gamma p_Y Y}\,,$$

$$\sum_h \phi^h_K = \sum_h (1 - \frac{\alpha^h}{\gamma}) \frac{r\bar{K}^h}{p_Y Y} = \sum_h (1 - \frac{\alpha^h}{\gamma}) \frac{M^h - w\bar{L}^h - \xi^h \tau_Z Z}{p_Y Y} = \frac{cov(\alpha^h, \theta^h_L)}{\gamma p_Y Y}\,,$$

$$\delta \equiv \rho := \frac{1}{p_Y Y} \sum_h (1 - \alpha^h) M^h \left(\frac{\alpha^h}{\gamma}(\sigma^h - 1) + 1 \right) \geq \frac{1}{\gamma p_Y Y} \sum_h (1 - \alpha^h) M^h (\gamma - \alpha^h) = \frac{1}{\gamma p_Y Y} \sum_h M^h (\gamma - \alpha^h)^2 \geq 0\,.$$

Inserting these simplified expressions into the system of Eqs. (1.15a)–(1.15c) delivers Eqs. (1.16a)–(1.16b).

A.3.2 Proof of Proposition 1.4

If preferences are homothetic and unit-elastic, the change in returns to capital is given by:

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_{H,2}} [A_H(e_{ZZ} - e_{KZ}) - B_H(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)]\hat{\tau}_Z \tag{A.14}$$

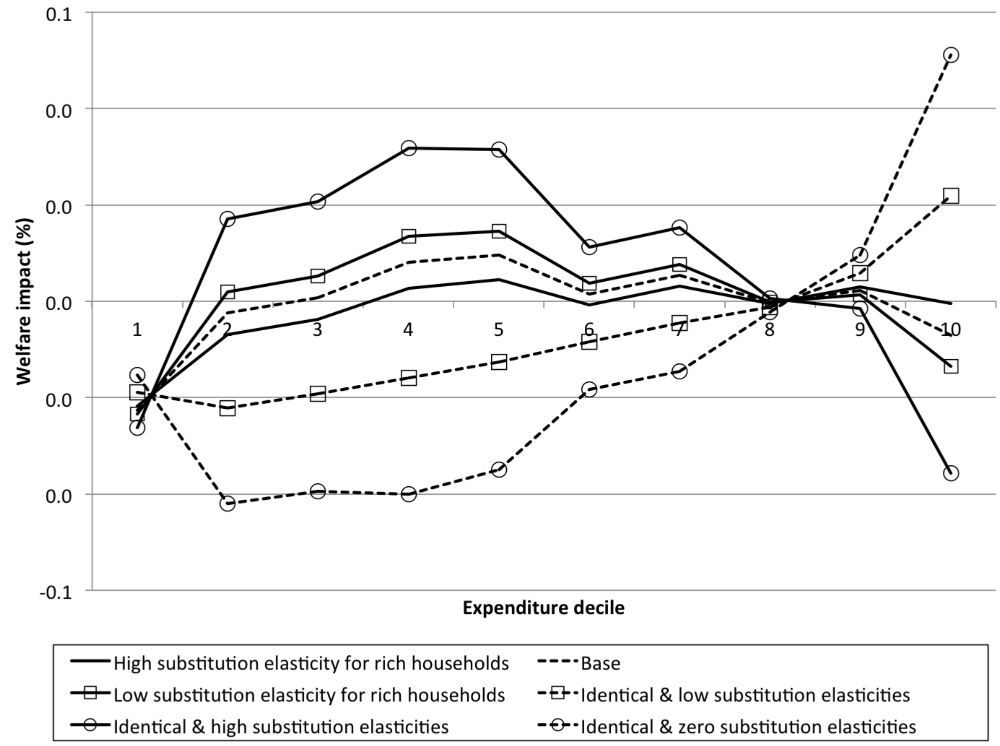
where $D_{H,2} = C_H\sigma_X + e_{KL}[A_H\theta_{YL} + B_H\theta_{YK}] + e_{LZ}[B_H\theta_{XK}(\theta_{YZ} + \theta_{YL}) - A_H\theta_{XK}\theta_{YL}] + e_{KZ}[A_H\theta_{XL}(\theta_{YZ} + \theta_{YK}) - B_H\theta_{XL}\theta_{YK}] + (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL}) - (\gamma_K - \gamma_L)\frac{1}{p_Y Y \gamma} cov(\alpha^h, \theta^h_L)$.

Since income and expenditure patterns are assumed to be correlated, the last term in $D_{H,2}$—which is the only term depending on household characteristics other than the aggregate ones—is non-zero. Note that on the other hand, for a single consumer, this term equals zero. It thus follows that one can choose Allen elasticities such that the sign is reversed when setting the last term to zero, i.e., when considering the model with a single consumer. An example of such a choice would be $\sigma_X = e_{KZ} = e_{LZ} = 0$ and $-[A_H\theta_{YL} + B_H\theta_{YK}]e_{KL} \in \Big(min\{(\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL}) - (\gamma_K - \gamma_L)\frac{1}{p_Y Y\gamma} cov(\alpha^h, \theta^h_L), (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL})\}, max\{(\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL}) - (\gamma_K - \gamma_L)\frac{1}{p_Y Y\gamma} cov(\alpha^h, \theta^h_L), (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL})\}\Big)$. As the numerator in Eq. (A.14) depends only on aggregate household characteristics, its value will be identical in both the heterogeneous and the single consumer case. It thus follows—for the given choice of Allen elasticities—that the signs of $\hat{w}$ and of $\hat{r}$ are reversed as compared to the model with a single household with homothetic preferences.

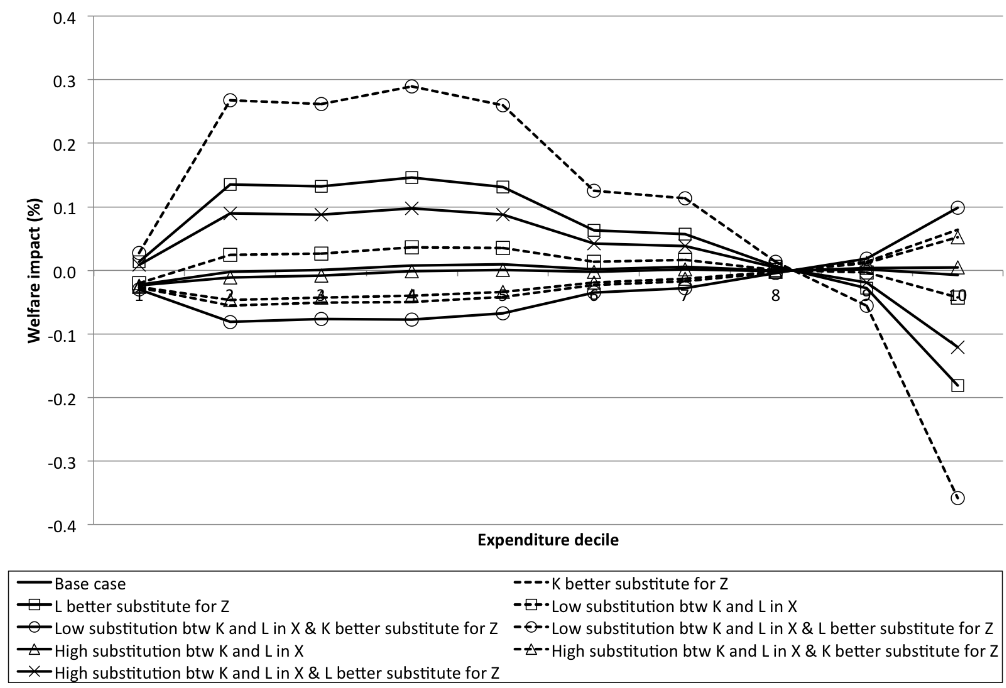
This online appendix contains supplementary analysis (1) for Section 1.4.3 exploring the distributional impacts among households under alternative revenue recycling schemes and (2) for Section 1.5 providing additional propositions that are analogous to Propositions 1.1–1.6 for multiple extensions of the core model (pre-existing taxes on capital and labor, non-separable utility in pollution, labor-leisure choice, an arbitrary number of commodities, and alternative revenue recycling schemes).

A.4 Incidence for alternative carbon tax revenue recycling schemes

Figure A.1: Welfare impacts (Φ^h) of increased pollution tax across annual expenditure deciles; revenues allocated in proportion to dirty good consumption

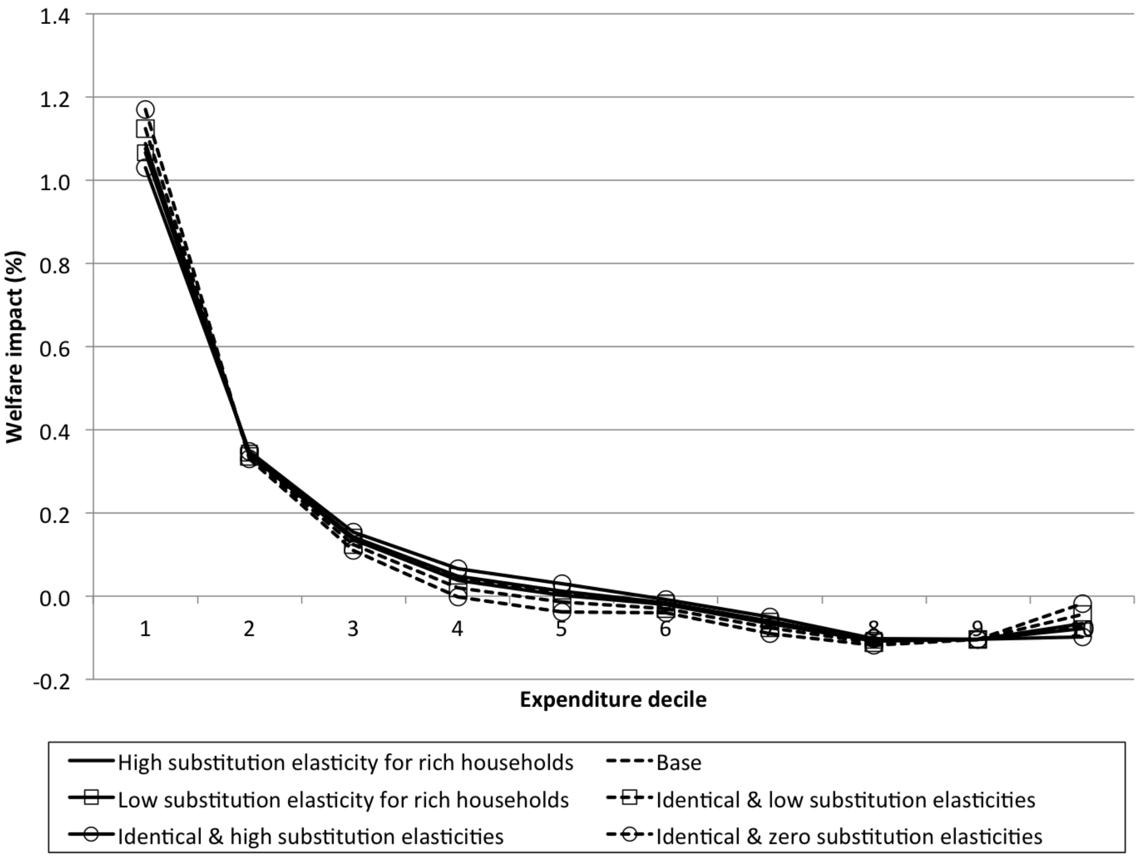


(a) Alternative assumptions about household characteristics

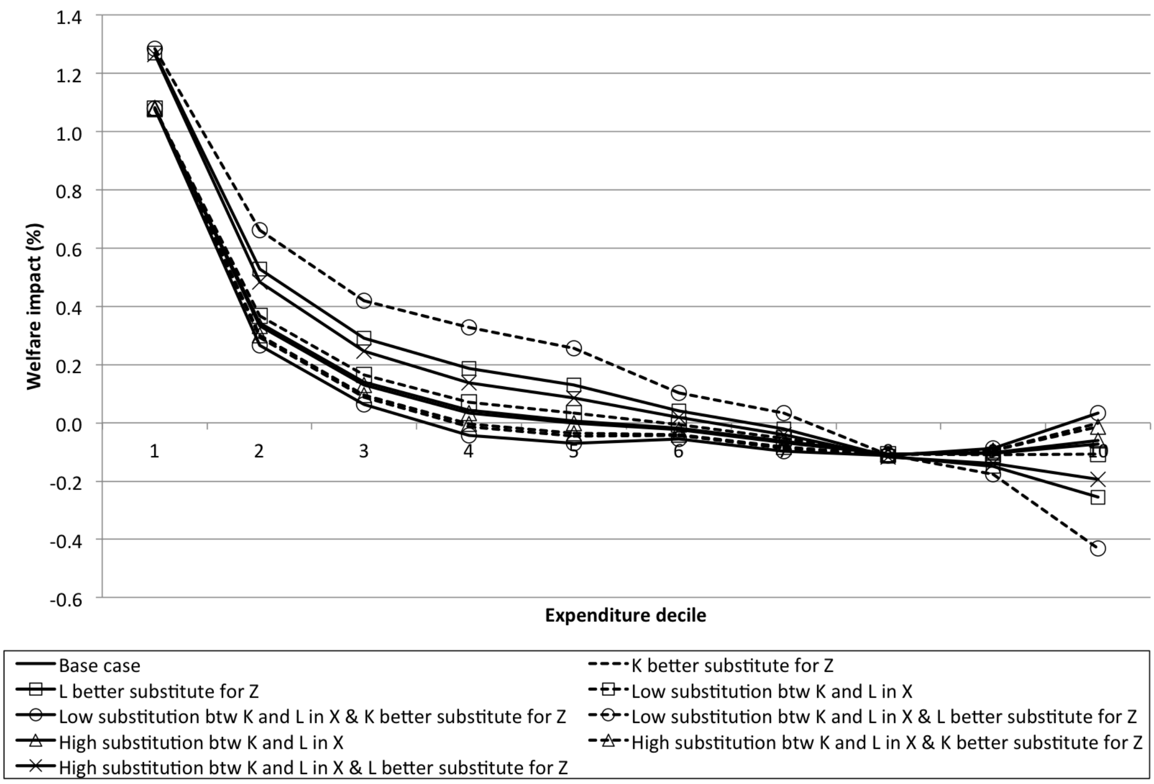


(b) Alternative assumptions about production characteristics

Figure A.2: Welfare impacts (Φ^h) of increased pollution tax across annual expenditure deciles; revenues allocated on per-capita basis



(a) Alternative assumptions about household characteristics



(b) Alternative assumptions about production characteristics

A.5 Extensions

A.5.1 Alternative revenue recycling schemes

Table A.1 reports price changes for alternative revenue recycling schemes. As is evident the price changes for both $\hat{r}$ and $\hat{p}_Y$ are very similar among alternative revenue recycling cases indicating that the impact of household heterogeneity on the equilibrium outcome is largely independent of the way the environmental tax revenue is redistributed.

Table A.1: Price changes for alternative revenue recycling schemes

COV_{Base}		COV_{Low}				COV_{High}			
ρ_{Base}		ρ_{Low}		ρ_{High}		ρ_{Low}		ρ_{High}	
$\hat{p}_Y$	$\hat{r}$	$\hat{p}_Y$	$\hat{r}$	$\hat{p}_Y$	$\hat{r}$	$\hat{p}_Y$	$\hat{r}$	$\hat{p}_Y$	$\hat{r}$
Redistribution proportional to income									
7.20	-0.12	7.20	-0.10	7.19	-0.13	7.21	-0.08	7.18	-0.16
Redistribution proportional to dirty good consumption									
7.20	-0.12	7.20	-0.10	7.19	-0.13	7.21	-0.07	7.19	-0.16
Redistribution on per capita basis									
7.20	-0.12	7.20	-0.10	7.19	-0.13	7.21	-0.07	7.19	-0.16

Notes: $\hat{r}$ and $\hat{p}_Y$ are expressed as the percentage change relative to the price level before the pollution tax increase. The results in the table are based on the central case assumptions for production side characteristics.

A.5.2 Pre-existing, non-environmental taxes

Accounting for pre-existing taxes on capital and labor in the benchmark, analogous to Fullerton and Heutel (2007), modifies the cost shares now including tax payments ($\theta_{YK} \equiv \frac{r(1+\tau_K)K_Y}{p_Y Y}$, and similarly for θ_{YL}, θ_{XK} and θ_{XL}) as well as the households' income constraint now including tax revenues as new sources of income:

$$M^h = w\bar{L}^h + r\bar{K}^h + \xi^h \tau_Z Z + \xi^h_K \tau_K K + \xi^h_L \tau_L L\,,$$

where τ_K and τ_L denote the *ad valorem* tax rate on capital and labor, respectively, and ξ^h, ξ^h_K, and ξ^h_L are the shares of total revenue from pollution, capital and labor taxes redistributed to household h, respectively.

We find that as long as the revenue from capital and labor taxes is also distributed in proportion to income, there is no additional effect of household heterogeneity on price changes as heterogeneity in terms of both uses and sources side is unchanged. In this case, all Propositions 1.1–1.6 remain valid. Distributing capital and labor tax revenue in a non-neutral way will introduce additional heterogeneity on the sources side. In this case, Propositions 1.1 and 1.6 still hold true and price changes for $\hat{r}$ and $\hat{p}_Y$ are quantitatively similar (analogously to Section 1.5.1). This can be seen as follows. The budget change following a change in the pollution tax is now given by:

$$\hat{M}^h = \hat{w}\frac{w\bar{L}^h}{M^h} + \hat{r}\frac{r\bar{K}^h}{M^h} + \frac{\xi^h \tau_Z Z}{M^h}(\hat{\tau}_Z + \hat{Z}) + \frac{\xi^h_K r \tau_K K}{M^h}(\hat{r} + \hat{K}) + \frac{\xi^h_L w \tau_L L}{M^h}(\hat{w} + \hat{L})\,.$$

Since the total amounts of capital and labor in the economy are assumed to be exogenously given and fixed, it follows that $\hat{K} = 0$ and $\hat{L} = 0$. Hence:

$$\hat{M}^h = \hat{w}\left(\frac{w(\bar{L}^h + \xi^h_L \tau_L L)}{M^h}\right) + \hat{r}\left(\frac{r(\bar{K}^h + \xi^h_K \tau_K K)}{M^h}\right) + \frac{\xi^h \tau_Z Z}{M^h}(\hat{\tau}_Z + \hat{Z})\,.$$

This expression is formally identical to the budget change without capital and labor taxes, with $\bar{L}^h$ replaced by $\bar{L}^h + \xi_L^h \tau_L L$ and $\bar{K}^h$ replaced by $r(\bar{K}^h + \xi_K^h \tau_K K)$. It therefore follows that the model results are identical to (15a)-(15c), with the following changes:

$$\phi_L^h \to \phi_{OT,L}^h \equiv (1 - \frac{\alpha^h}{\gamma}) E_{X,M}^h \frac{w(\bar{L}^h + \xi_L^h \tau_L L)}{p_Y Y} + \frac{Y^h}{Y}(E_{Y,M}^h - E_{X,M}^h) \frac{w(\bar{L}^h + \xi_L^h \tau_L L)}{M^h} \tag{A.OT.1}$$

$$\phi_K^h \to \phi_{OT,K}^h \equiv (1 - \frac{\alpha^h}{\gamma}) E_{X,M}^h \frac{r(\bar{K}^h + \xi_K^h \tau_K K)}{p_Y Y} + \frac{Y^h}{Y}(E_{Y,M}^h - E_{X,M}^h) \frac{r(\bar{K}^h + \xi_K^h \tau_K K)}{M^h} . \tag{A.OT.2}$$

From the above considerations, it is straightforward to derive the following propositions which are analogous to Propositions 1.1–1.6 in the paper. We use the label "OT" to enable comparison between original propositions and the propositions based on the model with pre-existing, other taxes.

Proposition 1.1.OT *Assume non-zero ad valorem taxes on capital and labor inputs in production. Then Proposition 1.1 holds.*

Proof. Analogous to the proof of Proposition 1.1. □

Proposition 1.2-5.OT *Assume non-zero ad-valorem taxes on capital and labor inputs in production. Assume that all tax revenue is redistributed in proportion to benchmark income:* $\xi^h = \xi_K^h = \xi_L^h \equiv \frac{M^h}{p_X X + p_Y Y}$. *Then Propositions 1.2–1.5 hold.*

Proof. With $\phi_{OT,L}^h$ as in Eq. (A.OT.1) and $\phi_{OT,K}^h$ as in Eq. (A.OT.2) it follows that $\sum_h \phi_{OT,L}^h = \sum_h (1 - \frac{\alpha^h}{\gamma}) \frac{w\bar{L}^h}{p_Y Y}$ and $\sum_h \phi_{OT,K}^h = \sum_h (1 - \frac{\alpha^h}{\gamma}) \frac{r\bar{K}^h}{p_Y Y}$. These expressions are identical to the case with homothetic preferences and zero taxes on capital and labor. Hence the price changes are also identical. □

Proposition 1.6.OT *Assume non-zero ad valorem taxes on capital and labor inputs in production. Then Proposition 1.6 holds.*

Proof. Since for identical households the ϕ_{OT}^h expressions are zero, it follows that the taxes on capital and labor have no impact on the results. □

A.5.3 Non-separable utility in pollution

With non-separable utility, consumption of clean and dirty goods in general depends on the level of pollution: $X^h = X^h(p_X, p_Y, M^h, Z)$ and $Y^h = Y^h(p_X, p_Y, M^h, Z)$. Eqs. (A.1) and (A.2) then become

$$\hat{X}^h = -(\alpha E_{X,M}^h + (1-\alpha)\sigma^h)\hat{p}_X - ((1-\alpha)E_{X,M}^h - (1-\alpha)\sigma^h)\hat{p}_Y + E_{X,M}^h \hat{M}^h + E_{X,Z}^h \hat{Z}$$

$$\hat{Y}^h = -(\alpha E_{Y,M}^h - \alpha\sigma^h)\hat{p}_X - ((1-\alpha)E_{Y,M}^h + \alpha\sigma^h)\hat{p}_Y + E_{Y,M}^h \hat{M}^h + E_{Y,Z}^h \hat{Z} ,$$

where $E_{X,Z}^h \equiv \frac{Z}{X^h}\partial_Z X^h$ and $E_{Y,Z}^h \equiv \frac{Z}{Y^h}\partial_Z Y^h$ can, respectively, be interpreted as the pollution elasticity of clean and dirty consumption. Eqs. (1.10) and (1.11) can then be written as:

$$\hat{X}^h - \hat{Y}^h = \sigma^h(\hat{p}_Y - \hat{p}_X) + (E_{Y,M}^h - E_{X,M}^h)(\alpha^h \hat{p}_X + (1-\alpha^h)\hat{p}_Y - \hat{M}^h) + (E_{X,Z}^h - E_{Y,Z}^h)\hat{Z} \tag{A.NS.1}$$

$$\hat{X}^h = -(\alpha E_{X,M}^h + (1-\alpha)\sigma^h)\hat{p}_X - ((1-\alpha)E_{X,M}^h - (1-\alpha)\sigma^h)\hat{p}_Y + E_{X,M}^h \hat{M}^h + E_{X,Z}^h \hat{Z} . \tag{A.NS.2}$$

In order analyze our propositions, we first need to derive the price changes for the case with non-separable utility in pollution. They turn out to be identical to those for separable preferences, up to the coefficients A, B and C, which become

$$A \to A_{NS} \equiv A - \gamma_K \sum_h \phi^h_{NS} \qquad B \to B_{NS} \equiv B - \gamma_L \sum_h \phi^h_{NS} \qquad C \to C_{NS} \equiv C - \sum_h \phi^h_{NS}\,, \tag{A.NS.3}$$

with $\phi^h_{NS} = \frac{M^h}{p_Y Y}(1 - \frac{\alpha^h}{\gamma})E^h_{X,Z} + \frac{Y^h}{Y}(E^h_{Y,Z} - E^h_{X,Z})$. The next subsection derives this result.

Derivation of price changes for non-separable utility in pollution. Solve Eq. (A.NS.1) for $\hat{Y}^h$ and insert the result into Eq. (A.6). Rearrange to obtain:

$$\frac{1}{Y}\sum_h Y^h\Big(\sigma^h(\hat{p}_Y - \hat{p}_X) + (E^h_{Y,M} - E^h_{X,M})(\alpha^h \hat{p}_X + (1-\alpha^h)\hat{p}_Y - \hat{M}^h) + (E^h_{X,Z} - E^h_{Y,Z})\hat{Z}\Big) =$$

$$\sum_h \frac{Y^h}{Y}\hat{X}^h - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y - \theta_{YZ}\hat{Z}\,.$$

From Eq. (A.5) insert the following on the right-hand side of the equality: $+0 = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \sum_h \frac{X^h}{X}\hat{X}^h$ and use the fact that X is the numéraire, thus yielding:

$$\frac{1}{Y}\sum_h Y^h(\sigma^h(\hat{p}_Y) + (E^h_{Y,M} - E^h_{X,M})((1-\alpha^h)\hat{p}_Y - \hat{M}^h) + (E^h_{X,Z} - E^h_{Y,Z})\hat{Z}) =$$

$$\sum_h \frac{M^h}{p_Y Y}(1 - \frac{\alpha^h}{\gamma})\hat{X}^h + \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y - \theta_{YZ}\hat{Z}\,. \tag{A.NS.4}$$

Eliminate $\hat{X}^h$ from Eq. (A.NS.4) by using Eq. (A.NS.2), then insert the explicit expression for the budget change $\hat{M}^h$:

$$\hat{p}_Y\delta = \sum_h \phi^h_L \hat{w} + \sum_h \phi^h_K \hat{r} + \sum_h \phi^h_Z \hat{\tau}_Z\,,$$

$$+\,\theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y + \Big(\sum_h(\phi^h_{NS} + \phi^h_Z) - \theta_{YZ}\Big)\hat{Z}\,. \tag{A.NS.5}$$

with $\phi^h_{NS} = \frac{M^h}{p_Y Y}(1 - \frac{\alpha^h}{\gamma})E^h_{X,Z} + \frac{Y^h}{Y}(E^h_{Y,Z} - E^h_{X,Z})$. Next, solve Eqs. (1.1) and (1.2) for $\hat{K}_X$ and $\hat{L}_X$, and insert them into Eq. (A.NS.5). Furthermore, insert Eq. (A.4) to eliminate $\hat{p}_Y$, thus obtaining:

$$(\sum_h(\phi^h_{NS} + \phi^h_Z) - \theta_{YZ})\hat{Z} = (\delta\theta_{YK} - \sum_h \phi^h_K)\hat{r} + (\delta\theta_{YL} - \sum_h \phi^h_L)\hat{w} + (\delta\theta_{YZ} - \sum_h \phi^h_Z)\hat{\tau}_Z$$

$$+\,\hat{K}_Y(\theta_{XK}\gamma_K + \theta_{YK}) + \hat{L}_Y(\theta_{XL}\gamma_L + \theta_{YL})\,. \tag{A.NS.6}$$

Solve Eqs. (1.4) and (1.5) for $\hat{K}_Y$ and $\hat{L}_Y$, and insert them into Eq. (A.NS.6). This yields:

$$-\,C_{NS}\hat{Z} = (-\sum_h \phi^h_K + \theta_{YK}(\delta + \beta_K(e_{KK} - e_{ZK}) + \beta_L(e_{LK} - e_{ZK})))\hat{r}$$

$$+\,(-\sum_h \phi^h_L + \theta_{YL}(\delta + \beta_K(e_{KL} - e_{ZL}) + \beta_L(e_{LL} - e_{ZL})))\hat{w}$$

$$+\,(-\sum_h \phi^h_Z + \theta_{YZ}(\delta + \beta_K(e_{KZ} - e_{ZZ}) + \beta_L(e_{LZ} - e_{ZZ})))\hat{\tau}_Z\,, \tag{A.NS.7}$$

with $C_{NS} = \beta_K + \beta_L + \theta_{YZ} - \sum_h(\phi^h_Z + \phi^h_{NS})$. Next eliminate $\hat{Z}$. To achieve this, substitute Eqs (1.1) and (1.2) into Eq. (1.3), obtaining:

$$-\,\gamma_K\hat{K}_Y + \gamma_L\hat{L}_Y = \sigma_X(\hat{w} - \hat{r})\,. \tag{A.NS.8}$$

Substituting Eqs. (1.4) and (1.5) into Eq. (A.NS.8) yields:

$$\sigma_X(\hat{w} - \hat{r}) = (\gamma_L - \gamma_K)\hat{Z} + \theta_{YK}(\gamma_L(e_{LK} - e_{ZK})\hat{r} - \gamma_K(e_{KK} - e_{ZK}))\hat{r}$$

$$\theta_{YL}(\gamma_L(e_{LL} - e_{ZL})\hat{w} - \gamma_K(e_{KL} - e_{ZL}))\hat{w} +$$

$$\theta_{YZ}(\gamma_L(e_{LZ} - e_{ZZ})\hat{\tau}_Z - \gamma_K(e_{KZ} - e_{ZZ}))\hat{\tau}_Z\,. \tag{A.NS.9}$$

Now solve Eq. (A.NS.9) for $\hat{Z}$ and equate to Eq. (A.NS.7):

$$\Big((\gamma_K - \gamma_L)(-\sum_h \phi_K^h + \theta_{YK}\delta) + C_{NS}\sigma_X + \theta_{YK}[-A_{NS}(e_{KK} - e_{ZK}) + B_{NS}(e_{LK} - e_{ZK})]\Big)\hat{r}$$

$$+\Big((\gamma_K - \gamma_L)(-\sum_h \phi_L^h + \theta_{YL}\delta) - C_{NS}\sigma_X + \theta_{YL}[-A_{NS}(e_{KL} - e_{ZL}) + B_{NS}(e_{LL} - e_{ZL})]\Big)\hat{w}$$

$$= \Big((\gamma_L - \gamma_K)(-\sum_h \phi_Z^h + \theta_{YZ}\delta) + \theta_{YZ}[-A_{NS}(e_{ZZ} - e_{KZ}) + B_{NS}(e_{ZZ} - e_{LZ})]\Big)\hat{\tau}_Z\,, \qquad \text{(A.NS.10)}$$

with $A_{NS} \equiv -(\gamma_K - \gamma_L)\beta_K + C_{NS}\gamma_K = A_P - \gamma_K \sum_h \phi_{NS}^h$ and $B_{NS} \equiv (\gamma_K - \gamma_L)\beta_L + C_{NS}\gamma_L = B_P - \gamma_L \sum_h \phi_{NS}^h$. Eq. (A.NS.10) is formally identical to Eq. (A.13), with the coefficients A, B and C replaced by A_{NS}, B_{NS} and C_{NS}. It therefore follows that price changes will also be identical up to the value of these coefficients.

Results. As can be seen from above, the change in the pollution level following a pollution tax increase can affect price changes and hence the equilibrium behavior of households. This introduces an additional dimension of heterogeneity to the extent that households have different preferences about pollution.

From the above considerations, it is straightforward to derive the following propositions which are analogous to Propositions 1.1–1.6 in the paper. We use the label "NS" to enable comparison between the original propositions and the propositions based on the model with non-separable pollution.

Equal factor intensities in production

Proposition 1.1.NS *Assume non-separable utility from pollution. Then Proposition 1.1 holds.*

Proof. If $\gamma_K = \gamma_L$, then from the proof of Proposition 1.1, we know that it then follows that $A = B = \gamma_K C$. This implies that $A - \gamma_K \sum_h \phi_{NS}^h = B - \gamma_L \sum_h \phi_{NS}^h = \gamma_K(C - \sum_h \phi_{NS}^h)$ which in turn is equivalent to $A_{NS} = B_{NS} = \gamma_K C_{NS}$. It then follows that all the terms containing household characteristics in the expressions for the price changes drop out. ◻

Heterogeneous households with homothetic preferences

In this paragraph, assume that the pollution tax revenue is returned to households in proportion to income. Now define the *effective pollution elasticity of clean consumption* $\Xi \equiv \sum_h \frac{X^h}{X} E_{X,Z}^h$. It then follows that:

$$\sum_h \phi_{NS}^h = -\frac{1}{1-\gamma}\Xi\,,$$

using the fact that, from the budget constraint, the following holds: $E_{Y,Z}^h = -\frac{\alpha^h}{1-\alpha^h} E_{X,Z}^h$. Using this, the analogues to Propositions 1.2–1.5 hold.

Proposition 1.2.NS *Assume non-separable utility from pollution. It then follows that, in addition to $cov(\alpha^h, \theta_L^h)$ and ρ, Proposition 1.2 is extended to include the effective pollution elasticity of clean consumption, Ξ.*

Proof. The proof is analogous to the one for Proposition 1.2 accounting for the new term Ξ. ◻

Proposition 1.3.NS *Assume non-separable utility from pollution. Then, in addition to γ and ρ, the single household with homothetic preferences in Proposition 1.3 is also characterized by a pollution elasticity of clean consumption equal to the effective elasticity Ξ.*

Proof. For the assumptions in Proposition 1.3 it is straightforward to see that price changes are identical to those derived for an economy with a single consumer with homothetic preferences, clean good expenditure share γ, elasticity of substitution in utility ρ, and pollution elasticity of clean consumption Ξ. ◻

Proposition 1.4.NS *Assume non-separable utility from pollution. Then the analogue of Proposition 1.4 holds, with the single consumer being characterized by a pollution elasticity of clean consumption given by the effective pollution elasticity* Ξ.

Proof. The proof of Proposition 1.4.NS carries through analogously to the one for Proposition 1.4. However, since now $A_{NS,H}\theta_{YL} + B_{NS,H}\theta_{YK}$ could in principle be equal to zero, it is necessary to additionally show that the other two coefficients multiplying the e's in $D_{NS,H,2}$ cannot also be zero. It then follows that, if $A_{NS,H}\theta_{YL} + B_{NS,H}\theta_{YK} = 0$, then one can construct the example analogously, based on e_{LZ} or e_{KZ}. To show this, assume that $A_{NS,H}\theta_{YL}+B_{NS,H}\theta_{YK} = 0$. It follows that $B_{NS,H}\theta_{XK}(\theta_{YZ}+\theta_{YL})-A_{NS,H}\theta_{XK}\theta_{YL} = \theta_{XK}B_{NS,H}$ and $A_{NS,H}\theta_{XL}(\theta_{YZ}+\theta_{YK})-B_{NS,H}\theta_{XL}\theta_{YK} = \theta_{XL}A_{NS,H}$, therefore in order to be both zero, the following must hold: $A_{NS,H} = 0$ and $B_{NS,H} = 0$. This in turn implies $A_H = \gamma_K \sum_h \phi^h_{NS}$ and $B_H = \gamma_L \sum_h \phi^h_{NS}$, which in turn implies $\frac{A_H}{\gamma_K} = \frac{B_H}{\gamma_L}$. Inserting the explicit expressions for A_H and B_H delivers: $\gamma_L\gamma_L\beta_K + \gamma_L\gamma_K\beta_L = \gamma_K\gamma_K\beta_L + \gamma_K\gamma_L\beta_K \Leftrightarrow (\gamma_L - \gamma_K)\frac{\beta_K}{\gamma_K} = (\gamma_K - \gamma_L)\frac{\beta_L}{\gamma_L} \Leftrightarrow (\gamma_L - \gamma_K)(\theta_{XK} + \frac{\theta_{YK}}{\gamma_K}) = (\gamma_K - \gamma_L)(\theta_{XL} + \frac{\theta_{YL}}{\gamma_L})$. Since we are assuming that $\gamma_K \neq \gamma_L$, it follows that the last equality is equivalent to $(\theta_{XK} + \frac{\theta_{YK}}{\gamma_K}) = -(\theta_{XL} + \frac{\theta_{YL}}{\gamma_L})$, which is a contraction. □

Proposition 1.5.NS *Assume non-separable utility from pollution. Then Proposition 1.5 holds.*

Proof. Since only A, B and C are affected by this model extension, and since they all multiply with elasticities that are zero, it follows that price changes in this special case are identical to those in the original model. □

Identical households with non-homothetic preferences

Proposition 1.6.NS *Assume non-separable utility from pollution. Then Proposition 1.6 holds, with the coefficients in Condition 1 are generalised as follows:* $A_{ID} \to A_{ID} + \frac{\gamma_K}{1-\gamma}E_{X,Z}$, $B_{ID} \to B_{ID} + \frac{\gamma_L}{1-\gamma}E_{X,Z}$ *and* $C_{ID} \to C_{ID} + \frac{1}{1-\gamma}E_{X,Z}$.

Proof. This follows from Eq. (A.NS.3), using the fact that $E_{Y,Z} = -\frac{\gamma}{1-\gamma}E_{X,Z}$. □

Proposition 1.6.NS illustrates that while extending the model to allow for non-separability of utility in pollution can affect the quantitative parameter values at which the model behavior switches, it does not change the qualitative behavior of the model.

A.5.4 Labor-leisure choice

An important dimension along which households can differ is their valuation of leisure time resulting in differences with respect to the elasticity of labor supply. Incorporating endogenous labor supply significantly enhances the complexity of studying the impact of household heterogeneity of equilibrium outcomes as it affects both how income is earned and spent. To keep the theoretical analysis tractable, we restrict our attention to Cobb-Douglas utility:

$$U^h(X^h, Y^h, l^h) = X^{\eta^h_X} Y^{\eta^h_Y} l^{1-\eta^h_X-\eta^h_Y},$$

where income is given by $M^h = w(T^h - l^h) + r\bar{K}^h + \tau_Z Z\xi^h$. T^h represents household h's endowment of productive time.[1] We further assume that in the benchmark, households dedicate an equal fraction of their productive time to leisure: $\frac{l^h}{T^h} \equiv \mathcal{L}$, $\forall h$.

Using the first-order conditions, the demand functions are:

$$X^h = \frac{\eta^h_X}{p_X}(wT^h + r\bar{K}^h + \xi^h\tau_Z Z) \qquad Y^h = \frac{\eta^h_Y}{p_Y}(wT^h + r\bar{K}^h + \xi^h\tau_Z Z) \qquad l^h = \frac{1-\eta^h_X-\eta^h_Y}{w}(wT^h + r\bar{K}^h + \xi^h\tau_Z Z)\,.$$

[1] Differences in T^h could be viewed as reflecting differences in labor productivity across households.

It then follows that

$$\hat{X}^h = -\hat{p}_X + \hat{w}\frac{wT^h}{M^h + wl^h} + \hat{r}\frac{r\bar{K}^h}{M^h + wl^h} + \frac{\xi^h \tau_Z Z}{M^h + wl^h}(\hat{\tau}_Z + \hat{Z}) \tag{A.LL.1}$$

$$\hat{Y}^h = -\hat{p}_Y + \hat{w}\frac{wT^h}{M^h + wl^h} + \hat{r}\frac{r\bar{K}^h}{M^h + wl^h} + \frac{\xi^h \tau_Z Z}{M^h + wl^h}(\hat{\tau}_Z + \hat{Z})$$

$$\hat{l}^h = -\hat{w} + \hat{w}\frac{wT^h}{M^h + wl^h} + \hat{r}\frac{r\bar{K}^h}{M^h + wl^h} + \frac{\xi^h \tau_Z Z}{M^h + wl^h}(\hat{\tau}_Z + \hat{Z})\,. \tag{A.LL.2}$$

We now need to modify our model equations as follows: Eq. (1.11) is replaced by Eq. (A.LL.1), and Eq.(1.2) is replaced by:

$$\hat{L}_X\frac{L_X}{L} + \hat{L}_Y\frac{L_Y}{L} = -\frac{1}{L}\sum_h l^h\hat{l}^h\,. \tag{A.LL.3}$$

In order analyze our propositions, we first need to derive the price changes for the case with labor-leisure choice. They turn out to be identical to those for a model with exogenous labor supply, up to the value of the ϕ parameters, which are extended as follows:

$$\phi_K^h \to \phi_{LL,K}^h \equiv \left(1 - \frac{wl^h}{wT^h + r\bar{K}^h + \tau_Z Z\xi^h}\right)\phi_K^h - \frac{wl^h}{wT^h + r\bar{K}^h + \tau_Z Z\xi^h}\frac{r\bar{K}^h}{p_X X}$$

$$\phi_L^h \to \phi_{LL,L}^h \equiv \left(1 - \frac{wl^h}{wT^h + r\bar{K}^h + \tau_Z Z\xi^h}\right)\frac{wT^h}{w(T^h - l^h)}\phi_L^h + \frac{wl^h}{p_X X}\frac{r\bar{K}^h + \tau_Z Z\xi^h}{wT^h + r\bar{K}^h + \tau_Z Z\xi^h}$$

$$\phi_Z^h \to \phi_{LL,Z}^h \equiv \left(1 - \frac{wl^h}{wT^h + r\bar{K}^h + \tau_Z Z\xi^h}\right)\phi_Z^h - \frac{wl^h}{p_X X}\frac{\xi^h \tau_Z Z}{wT^h + r\bar{K}^h + \tau_Z Z\xi^h}\,.$$

The next subsection derives these results.

Derivations for labor-leisure choice

Price changes

Up until Eq. (A.7), the derivation is analogous, yielding the following expression:

$$\hat{p}_Y = \sum_h \frac{M^h}{p_Y Y}(1 - \frac{\alpha^h}{\gamma})\hat{X}^h + \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y - \theta_{YZ}\hat{Z}\,. \tag{A.LL.4}$$

Eliminate $\hat{X}^h$ from Eq. (A.LL.4) by using Eq. (A.LL.1) :

$$\hat{p}_Y = \hat{w}\sum_h \frac{M^h}{M^h + wl^h}\frac{wT}{wL^h}\phi_L^h + \hat{r}\sum_h \frac{M^h}{M^h + wl^h}\phi_K^h + \hat{\tau}_Z\sum_h \frac{M^h}{M^h + wl^h}\phi_Z^h$$

$$+ \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y + \hat{Z}\left(\sum_h \frac{M^h}{M^h + wl^h}\phi_Z^h - \theta_{YZ}\right), \tag{A.LL.5}$$

where $L^h = T^h - l^h$ and the ϕs are evaluated for homothetic preferences. Next, solve Eqs. (1.1) and (A.LL.3) for $\hat{K}_X$ and $\hat{L}_X$, and insert them into Eq. (A.LL.5). Furthermore, insert Eq. (A.4) to eliminate $\hat{p}_Y$, thus obtaining:

$$\left(\sum_h \frac{M^h}{M^h + wl^h}\phi_Z^h - \theta_{YZ}\right)\hat{Z} = \left(\theta_{YK} - \sum_h \frac{M^h}{M^h + wl^h}\phi_K^h\right)\hat{r} + \left(\theta_{YL} - \sum_h \frac{M^h}{M^h + wl^h}\frac{wT^h}{wL^h}\phi_L^h\right)\hat{w} + \left(\theta_{YZ} - \sum_h \frac{M^h}{M^h + wl^h}\phi_Z^h\right)\hat{\tau}_Z$$

$$+ \hat{K}_Y(\theta_{XK}\gamma_K + \theta_{YK}) + \hat{L}_Y(\theta_{XL}\gamma_L + \theta_{YL}) + \frac{\theta_{XL}}{L_X}\sum_h l^h\hat{l}^h\,. \tag{A.LL.6}$$

Now eliminate $\hat{l}^h$ by substituting Eq. (A.LL.2) into Eq. (A.LL.6) :

$$\left(\sum_h \phi_{LL,Z}^h - \theta_{YZ}\right)\hat{Z} = \left(\theta_{YK} - \sum_h \phi_{LL,K}^h\right)\hat{r} + \left(\theta_{YL} - \sum_h \phi_{LL,L}^h\right)\hat{w} + \left(\theta_{YZ} - \sum_h \phi_{LL,Z}^h\right)\hat{\tau}_Z +$$

$$+ \hat{K}_Y(\theta_{XK}\gamma_K + \theta_{YK}) + \hat{L}_Y(\theta_{XL}\gamma_L + \theta_{YL})\,, \tag{A.LL.7}$$

where $\phi_{LL,K}^h \equiv \frac{M^h}{M^h+wl^h}\phi_K^h - \frac{wl^h}{p_X X}\frac{r\bar{K}^h}{M^h+wl^h}$, $\phi_{LL,L}^h \equiv \frac{M^h}{M^h+wl^h}\frac{wT^h}{wL^h}\phi_L^h + \frac{wl^h}{p_X X}\frac{r\bar{K}^h+\tau_Z Z\xi^h}{M^h+wl^h}$ and $\phi_{LL,Z}^h \equiv \frac{M^h}{M^h+wl^h}\phi_Z^h - \frac{wl^h}{p_X X}\frac{\xi^h\tau_Z Z}{M^h+wl^h}$. Eq. (A.LL.7) is formally identical to Eq. (A.9), with the exception of the ϕs. It therefore follows that the resulting price changes are also formally identical to Eqs. 1.15a–(1.15c) up to the value of the parameters ϕ.

Household parameters

Using the fact that $cov(\alpha^h,\beta^h) = \sum_h M^h\alpha^h\beta^h - M\alpha\beta$ and $\sum_h M^h\alpha^h\beta^h\gamma^h = cov(\alpha^h,\beta^h,\gamma^h) + \alpha cov(\beta^h,\gamma^h) + \beta cov(\alpha^h,\gamma^h) + \gamma cov(\alpha^h,\beta^h) + M\alpha\beta\gamma$.

$$\sum_h \phi^h_{LL,L} = \sum_h (1-\lambda^h)\frac{wT}{w(T-l^h)}(1-\frac{\alpha^h}{\gamma})\frac{w\bar{L}^h}{p_Y Y} + \frac{cov(\lambda^h,\theta^h_K)}{p_X X} + \frac{\lambda\theta_K}{\gamma} + \frac{\tau_Z Z}{p_X X}\lambda =$$
$$\frac{1}{1-\mathcal{L}}\frac{1}{\gamma p_Y Y}\sum_h M^h\theta^h_L\lambda^h\alpha^h - \frac{1}{1-\mathcal{L}}\frac{cov(\theta^h_L,\alpha^h)}{\gamma p_Y Y} - \frac{1}{1-\mathcal{L}}\frac{cov(\theta^h_L,\lambda^h)}{p_Y Y} + \frac{cov(\lambda^h,\theta^h_K)}{p_X X}$$
$$-\frac{1}{1-\mathcal{L}}\frac{(p_X X + p_Y Y)}{p_Y Y}\theta_L\lambda + \frac{\lambda}{\gamma}(1-\theta_L)\,,$$

hence

$$\sum_h \phi^h_{LL,L} = \frac{cov(\theta^h_L,\lambda^h,\alpha^h) + \theta_L cov(\lambda^h,\alpha^h) + (\lambda-1)cov(\theta^h_L,\alpha^h)}{(1-\mathcal{L})\gamma p_Y Y} - \frac{cov(\lambda^h,\theta^h_L)}{p_X X} - \frac{\lambda(\theta_L-1)}{\gamma}\,.$$

Analogously:

$$\sum_h \phi^h_{LL,K} = \frac{-cov(\theta^h_L,\lambda^h,\alpha^h) + \theta_K cov(\lambda^h,\alpha^h) - (\lambda-1)cov(\theta^h_L,\alpha^h)}{\gamma p_Y Y} + \frac{cov(\lambda^h,\theta^h_L)}{p_X X} - \frac{\lambda\theta_K}{\gamma}\,.$$

Using the above expressions consider the ϕ parameters as they appear in the expressions for the price changes:

$$\sum_h \phi^h_{LL,Z} = \frac{\tau_Z Z}{p_X X p_Y Y}cov(\lambda^h,\alpha^h) - \frac{\tau_Z Z}{p_X X}\lambda \tag{A.LL.8}$$

and

$$\theta_{XL}\sum_h \phi^h_{LL,K} - \theta_{XK}\sum_h \phi^h_{LL,L} =$$
$$-\frac{1}{\gamma p_Y Y}\frac{1}{(1-\mathcal{L})}\Big((1-\mathcal{L}\theta_{XL})cov(\theta^h_L,\lambda^h,\alpha^h) + (\theta_K(\mathcal{L}\theta_{XL}-1)+\theta_{XK}(1-\theta_Z))cov(\lambda^h,\alpha^h) + (\lambda-1)(1-\mathcal{L}\theta_{XL})cov(\theta^h_L,\alpha^h)\Big)$$
$$+\frac{cov(\lambda^h,\theta^h_L)}{p_X X} - \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK})\,, \tag{A.LL.9}$$

where $\lambda^h \equiv \frac{wl^h}{wT^h + r\bar{K}^h + \tau_Z Z\xi^h}$, $\lambda \equiv \frac{1}{\sum_{h'} M^{h'}}\sum_h \lambda^h M^h = \frac{1}{p_X X + p_Y Y}\sum_h \lambda^h M^h$, and the covariance of three variables is defined analogously to the definition for two variables in our paper. Note that, in the following, we will refer to λ^h as household h's *expenditure share on leisure*.

Results. We find that results are mainly similar with new parameters summarizing the additional channels of household heterogeneity as well as the aggregate impact of labor-leisure choice on the general equilibrium. Proposition 1.1 is identical, Proposition 1.3 is analogous with presence of a term that reflects the impact of average expenditure share of leisure on aggregate outcomes, and Proposition 1.2 is analogous accounting in addition for interactions between leisure choice and expenditure and income patterns. Propositions 1.4 and 1.5 are analogous, too. For the special case of Cobb-Douglas utility, we thus find that effect of household heterogeneity is similar to the case without labor-leisure choice; where it differs it can be understood in terms of additional terms reflecting interactions between the various types of heterogeneity (labor-leisure choice, expenditures and income patterns). Whether or not the aggregation bias is quantitatively smaller or larger depends on specific parametrization. The following subsection provide detailed analysis supporting the above statements. We use the label "LL" to enable comparison between the original propositions and the propositions based on the model with leisure.

Equal factor intensities in production

Proposition 1.1.LL *Assume the model with labor-leisure choice and Cobb-Douglas utility. Then Proposition 1.1 holds.*

Proof. If $\gamma_K = \gamma_L$, then from the proof of Proposition 1.1, we know that it then follows that $A = B = \gamma_K C$. This implies that $A_{LL} = B_{LL} = \gamma_K C_{LL}$. It then follows that all the terms containing household characteristics in the expressions for the price changes drop out. □

Heterogeneous households with homothetic preferences

Proposition 1.2.LL *Assume the model with labor-leisure choice, equal benchmark share of leisure time across households ($\frac{l^h}{L^h} = \mathcal{L}$, $\forall h$), and Cobb-Douglas utility. Then, in addition to $cov(\alpha^h, \theta_L^h)$ and ρ, Proposition 1.2 is extended to include $cov(\alpha^h, \lambda^h)$, $cov(\theta_L^h, \lambda^h)$ and $cov(\theta_L^h, \lambda^h, \alpha^h)$.*

Proof. Eqs. (A.LL.8) and (A.LL.9). □

Proposition 1.3.LL *Assume the model with labor-leisure choice, equal benchmark share of leisure time across households ($\frac{l^h}{L^h} = \mathcal{L}$, $\forall h$), and Cobb-Douglas utility. If income shares are identical across households ($\theta_L^h = \theta_L$, $\forall h$), then output and factor price changes are identical to those for a single household characterized by Cobb-Douglas preferences, clean good expenditure share γ, an elasticity of substitution between clean and dirty goods in utility equal to the effective elasticity ρ, and expenditure share on leisure given by the income-weighted average of the shares across households, λ.*

Proof. Eqs. (A.LL.8) and (A.LL.9). Consider furthermore the following: $\lambda^h = \frac{wl^h}{M^h + wl^h} = \frac{wl^h}{M^h}\frac{M^h}{M^h + wl^h} = \frac{wl^h}{M^h}(1-\lambda^h) = \frac{\mathcal{L}}{1-\mathcal{L}}\frac{wL^h}{M^h}(1-\lambda^h)$ using $l^h = \frac{\mathcal{L}}{1-\mathcal{L}}L^h$. Rewrite the above equality, therefore obtaining: $\lambda^h = \frac{\theta_L^h}{(\frac{1-\mathcal{L}}{\mathcal{L}} + \theta_L^h)}$. It therefore follows that, in the case where labor income shares are identical across households ($\theta_L^h = \theta_L$, $\forall h$), the same holds for the λ^hs, thus implying $cov(\lambda^h, \alpha^h) = 0$. □

Proposition 1.4.LL *Assume the model with labor-leisure choice, equal benchmark leisure time across households ($\frac{l^h}{L^h} = \mathcal{L}$, $\forall h$), and Cobb-Douglas utility. Assume different factor intensities (i.e., $\gamma_K \neq \gamma_L$), constant expenditure shares across households (i.e., $\alpha^h = \gamma$, $\forall h$) and non-zero covariance between labor income shares and expenditure shares on leisure (i.e., $cov(\lambda^h, \theta_L^h) \neq 0$). Then, for any observed consumption and production decisions before the tax change, there exist production elasticities (i.e., σ_X and e_{ij}) such that the relative burden on factors of production is opposite compared to the model with a single consumer, coupled to the same production side data and characterized by an expenditure share on leisure given by the income-weighted average of shares across households, λ.*

Proof. For the above assumptions, the change in the return on capital is given by:

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_{LL,1}}\left[A_{LL,1}(e_{ZZ} - e_{KZ}) - B_{LL,1}(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)(1 + \frac{1-\gamma}{\gamma}\lambda)\right]\hat{\tau}_Z\,,$$

where $A_{LL,1} = \gamma_L\beta_K + \gamma_K(\beta_L + \theta_{YZ} + \frac{\tau_Z Z}{p_X X}\lambda)$, $B_{LL,1} = \gamma_K\beta_L + \gamma_L(\beta_K + \theta_{YZ} + \frac{\tau_Z Z}{p_X X}\lambda)$, $C_{LL,1} = \beta_K + \beta_L + \theta_{YZ} + \frac{\tau_Z Z}{p_X X}\lambda$, $D_{LL,1} = C_{LL,1}\sigma_X + e_{KL}[A_{LL,1}\theta_{YL} + B_{LL,1}\theta_{YK}] + e_{LZ}[B_{LL,1}\theta_{XK}(\theta_{YL} + \theta_{YZ}) - A_{LL,1}\theta_{XK}\theta_{YL}] + e_{KZ}[A_{LL,1}\theta_{XL}(\theta_{YK} + \theta_{YZ}) - B_{LL,1}\theta_{XL}\theta_{YK}] + (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL} + \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK})) - (\gamma_K - \gamma_L)\frac{cov(\lambda^h, \theta_L^h)}{p_X X}$. Analogously to the proof of Proposition 4, one parameter choice that leads to the reversal of factor price changes between the heterogeneous household model and the single household model is the following: $\sigma_X = e_{KZ} = e_{LZ} = 0$ and $-[A_{LL,1}\theta_{YL} + B_{LL,1}\theta_{YK}]e_{KL} \in \Big(min[(\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL} + \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK})) - (\gamma_K - \gamma_L)\frac{cov(\lambda^h, \theta_L^h)}{p_X X}, (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL} + \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK}))], max[(\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL} + \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK})) - (\gamma_K - \gamma_L)\frac{cov(\lambda^h, \theta_L^h)}{p_X X}, (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL} + \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK}))]\Big)$. □

Proposition 1.5.LL *Assume the model with labor-leisure choice, equal benchmark leisure time across households ($\frac{l^h}{L^h} = \mathcal{L}, \forall h$), and Cobb-Douglas utility. Assume Leontief technologies in clean and dirty good production (i.e., $\sigma_X = e_{ij} = 0$), and that the dirty sector is relatively more capital intensive (i.e., $\gamma_K > \gamma_L$), such that the following holds: $(\theta_{YL}\theta_{XK} - \theta_{YK}\theta_{XL})\frac{p_Y Y}{p_X X} + \frac{1}{\gamma}(\theta_K + \theta_Z\theta_{XK}) = 0$. Then:*

- *(i) if consumers are identical on the sources and uses side of income: $\hat{p}_Y = 0$, $\hat{w} > 0$ and $\hat{r} < 0$.*
- *(ii) if consumers are identical on the uses side of income, and the θ_L^h and λ^hs have* low *covariance (i.e., $D_{LL} > 0$), then $\hat{p}_Y < 0$, $\hat{w} > 0$ and $\hat{r} < 0$.*
- *(iii) if consumers are identical on the uses side of income, and the θ_L^h and λ^hs have* high *covariance (i.e., $D_{LL} < 0$), then $\hat{p}_Y > 0$, $\hat{w} < 0$ and $\hat{r} > 0$.*

Proof. Price changes assume the following form:

$$\hat{p}_Y = \frac{\theta_{YZ}}{D_{LL}}\Big((\theta_{YL}\theta_{XK} - \theta_{YK}\theta_{XL})\frac{1-\gamma}{\gamma}\lambda + \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK}) - \frac{cov(\lambda^h, \theta_L^h)}{p_X X}\Big)\hat{\tau}_Z$$

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_{LL}}(1 + \frac{p_Y Y}{p_X X}\lambda)\hat{\tau}_Z\,,$$

where $D_{LL} = (\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL}) + \frac{\lambda}{\gamma}(\theta_K + \theta_Z\theta_{XK}) - \frac{cov(\lambda^h, \theta_L^h)}{p_X X}$. □

A.5.5 More than two sectors.

Our analysis so far assumed a highly aggregated sectoral representation. Including more sectors can obviously affect the aggregation bias as it enables representing household heterogeneity along more dimensions. With a finer sectoral resolution, it is, for example, conceivable that poorer households have higher expenditure shares on some dirty goods and lower expenditure shares on some others when compared to richer households. The problem is further compounded by the possibility that different polluting goods are likely to be produced with different capital and labor intensities. As the aggregation bias is determined by the interaction between household and production side characteristics, the impact of going from two to multiple sectors on the aggregation bias is thus in general not clear-cut.

We show for a special case with Leontief technologies in production that the aggregation bias can still be important for assessing the incidence of environmental taxes in a setting which includes an arbitrary number of dirty sectors J, denoted by the index j. Analogous to Proposition 1.5 with Leontief technologies, we find that the covariance between the ownership of labor and consumption of each dirty good across households can reverse the sign of the factor price changes. We use the label "MC" to enable comparison between the original propositions and the propositions based on the model with multiple polluting commodities.

Proposition 1.5.MC *Assume Cobb-Douglas preferences and Leontief technologies in clean and dirty production sectors. Assume furthermore that each dirty sector j is more capital-intensive than the economy-wide average (i.e., $\frac{K_{Y_j}}{L_{Y_j}} > \frac{\bar{K}}{\bar{L}}, \forall j$), and that every dirty sector j is more capital intensive than the clean sector (i.e., $\frac{K_{Y_j}}{L_{Y_j}} > \frac{K_X}{L_X}, \forall j$). Then, the following holds:*

- *(i) If consumers are identical on the sources or uses side of income, or both: $\hat{w} > 0$ and $\hat{r} < 0$.*
- *(ii) If labor ownership and dirty good consumption (for each dirty good j) have a positive covariance, then $\hat{w} > 0$ and $\hat{r} < 0$.*
- *(iii) If labor ownership and dirty good consumption (for each dirty good j) has a negative covariance, then $\hat{w} > 0$ and $\hat{r} < 0$ if covariance is* low *(i.e. $D_J > 0$), and $\hat{w} < 0$, $\hat{r} > 0$ if covariance is* high *(i.e. $D_J < 0$).*

Proof. For $J \geq 1$, we derive in the subsection "Derivations" below the following expression for the rental rate of capital:

$$\hat{r} = -\frac{\theta_{XL} \sum_{n=1}^{J} \theta_{Y_n Z_n} \left(\frac{K_{Y_n}}{\bar{K}} - \frac{L_{Y_n}}{\bar{L}} \right)}{D_J} \hat{\tau}_Z \,,$$

where $D_J \equiv \sum_{n=1}^{J} \left(\frac{K_{Y_n}}{\bar{K}} - \frac{L_{Y_n}}{\bar{L}} \right) \left(\theta_{XL}\theta_{Y_n K_n} - \theta_{XK}\theta_{Y_n L_n} - cov(\frac{\alpha^h}{p_X X} - \frac{\alpha_n^h}{p_{Y_n} Y_n}, \theta_L^h) \right)$. For $J = 1$, this is identical to the case considered in Proposition 1.5. Consider the above equation for $\hat{r}$, bearing in mind that if labor ownership and dirty good consumption have a positive covariance for each good j, then the labor ownership and clean good consumption have negative covariance, since $\alpha^h = 1 - \sum_j \alpha_j^h$. Furthermore $\theta_{XL}\theta_{Y_j K_j} - \theta_{XK}\theta_{Y_j L_j} = \theta_{XL}\theta_{XK}(\frac{\theta_{Y_j K_j}}{\theta_{XK}} - \frac{\theta_{Y_j L_j}}{\theta_{XL}}) = \theta_{XL}\theta_{XK}\frac{p_X X}{p_Y Y}(\frac{K_{Y_j}}{K_X} - \frac{L_{Y_j}}{L_X})$. This expression is positive if every dirty sector is more capital intensive than the clean sector. □

Derivations

The equilibrium conditions (1.1)–(1.13) for the model with J dirty sectors and one clear sector are given by:

$$\hat{K}_X \frac{K_X}{\bar{K}} + \sum_l \hat{K}_{Y_l} \frac{K_{Y_l}}{\bar{K}} = 0 \tag{A.MC.1}$$

$$\hat{L}_X \frac{L_X}{\bar{L}} + \sum_j \hat{L}_{Y_l} \frac{L_{Y_l}}{\bar{L}} = 0 \tag{A.MC.2}$$

$$\hat{K}_X - \hat{L}_X = 0 \tag{A.MC.3}$$

$$\hat{K}_{Y_j} - \hat{Z}_j = 0 \quad \forall j \tag{A.MC.4}$$

$$\hat{L}_{Y_j} - \hat{Z}_j = 0 \quad \forall j \tag{A.MC.5}$$

$$\hat{p}_X + \hat{X} = \theta_{XK}(\hat{r} + \hat{K}_X) + \theta_{XL}(\hat{w} + \hat{L}_X) \tag{A.MC.6}$$

$$\hat{p}_{Y_j} + \hat{Y}_j = \theta_{Y_j K_j}(\hat{r} + \hat{K}_{Y_j}) + \theta_{Y_j L_j}(\hat{w} + \hat{L}_{Y_j}) + \theta_{Y_j Z_j}(\hat{\tau}_Z + \hat{Z}_j) \quad \forall j \tag{A.MC.7}$$

$$\hat{X} = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X \tag{A.MC.8}$$

$$\hat{Y}_j = \theta_{Y_j K_j}\hat{K}_{Y_j} + \theta_{Y_j L_j}\hat{L}_{Y_j} + \theta_{Y_j Z_j}\hat{Z}_j \quad \forall j \tag{A.MC.9}$$

$$\hat{X}^h - \hat{Y}_j^h = \hat{p}_{Y_j} \quad \forall h, j \tag{A.MC.10}$$

$$\hat{X}^h = \hat{M}^h \quad \forall h \tag{A.MC.11}$$

$$\hat{X} = \sum_h \frac{X^h}{X}\hat{X}^h \tag{A.MC.12}$$

$$\hat{Y}_j = \sum_h \frac{Y_j^h}{Y_j}\hat{Y}_j^h \quad \forall j \,, \tag{A.MC.13}$$

with $\hat{M}^h = \hat{w}\frac{w\bar{L}^h}{M^h} + \hat{r}\frac{r\bar{K}^h}{M^h} + \frac{\tau_Z Z}{p_X X + p_Y Y}(\hat{\tau}_Z + \frac{\sum_j Z_j \hat{Z}_j}{Z})$. Eqs. (A.MC.1)–(A.MC.13) are $6 + 5J + H + JH$ equations in $6 + 5J + H + JH$ unknowns ($\hat{K}_X$, $J \times \hat{K}_{Y_j}$, $\hat{L}_X$, $J \times \hat{L}_{Y_j}$, $\hat{w}$, $\hat{r}$, $\hat{X}$, p_x, $J \times \hat{p}_{Y_j}$, $J \times \hat{Y}_j$, $J \times \hat{Z}_j$, $H \times \hat{X}^h$, $J \times H \times \hat{Y}_j^h$). Following Walras' Law, one of the equilibrium conditions is redundant, thus the effective number of equations is $5 + 5J + H + JH$. We choose X as the numéraire good, thus delivering a square system of equations. The equilibrium solutions are therefore fully determined as functions of the exogenous tax increase $\hat{\tau}_Z > 0$.

In order to derive the factor price changes, start by subtracting Eq. (A.MC.8) from Eq. (A.MC.6) and Eq. (A.MC.9) from Eq. (A.MC.7):

$$0 = \theta_{XK}\hat{r} + \theta_{XL}\hat{w} \tag{A.MC.14}$$

$$\hat{p}_{Y_j} = \theta_{Y_j K_j}\hat{r} + \theta_{Y_j L_j}\hat{w} + \theta_{Y_j Z_j}\hat{\tau}_Z \quad \forall j \,. \tag{A.MC.15}$$

Substitute Eq. (A.MC.12) into Eq. (A.MC.8) and Eq. (A.MC.13) into Eq. (A.MC.9):

$$\sum_h \frac{X^h}{X}\hat{X}^h = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X \tag{A.MC.16}$$

$$\sum_h \frac{Y_j^h}{Y_j} \hat{Y_j}^h = \theta_{Y_jK_j}\hat{K}_{Y_j} + \theta_{Y_jL_j}\hat{L}_{Y_j} + \theta_{Y_jZ_j}\hat{Z}_j \quad \forall j\,. \tag{A.MC.17}$$

Solve Eq. (A.MC.10) for Y_j^h and insert into Eq. (A.MC.17):

$$\frac{1}{Y_j}\sum_h Y_j^h \hat{p}_{Y_j} = \frac{1}{Y_j}\sum_h Y_j^h \hat{X}^h - \theta_{Y_jK_j}\hat{K}_{Y_j} - \theta_{Y_jL_j}\hat{L}_{Y_j} - \theta_{Y_jZ_j}\hat{Z}_j \quad \forall j\,. \tag{A.MC.18}$$

From Eq. (A.MC.16) insert the following on the right-hand side of the equality: $0 = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \sum_h \frac{X^h}{X}\hat{X}^h$ and use the fact that $\frac{1}{Y_j}\sum_h Y_j^h \hat{p}_{Y_j} = \hat{p}_{Y_j}$, thus yielding:

$$\hat{p}_{Y_j} = \sum_h \left(\frac{Y_j^h}{Y_j} - \frac{X^h}{X}\right)\hat{X}^h + \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{Y_jK_j}\hat{K}_{Y_j} - \theta_{Y_jL_j}\hat{L}_{Y_j} - \theta_{Y_jZ_j}\hat{Z}_j \quad \forall j\,. \tag{A.MC.19}$$

Eliminate $\hat{X}^h$ from Eq. (A.MC.19) by using Eq. (A.MC.11), then insert the explicit form of the budget change $\hat{M}^h$:

$$\hat{p}_{Y_j} = \hat{w}\sum_h \phi_{Lj}^h + \hat{r}\sum_h \phi_{Kj}^h + \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{Y_jK_j}\hat{K}_{Y_j} - \theta_{Y_jL_j}\hat{L}_{Y_j} - \theta_{Y_jZ_j}\hat{Z}_j \quad \forall j\,, \tag{A.MC.20}$$

where $\phi_{Lj}^h = (\frac{Y_j^h}{Y_j} - \frac{X^h}{X})\frac{w\bar{L}^h}{M^h}$, $\phi_{Kj}^h = (\frac{Y_j^h}{Y_j} - \frac{X^h}{X})\frac{r\bar{K}^h}{M^h}$ and using the fact that $\sum_h(\frac{Y_j^h}{Y_j} - \frac{X^h}{X})\frac{\tau_Z Z}{p_X X + p_Y Y} = 0$. Now solve Eqs. (A.MC.1) and (A.MC.2) for $\hat{K}_X$ and $\hat{L}_X$ and insert them into Eq. (A.MC.20). Furthermore, insert Eq. (A.MC.15) to eliminate $\hat{p}_{Y_j}$, thus obtaining:

$$-\theta_{Y_jZ_j}\hat{Z}_j = (\theta_{Y_jK_j} - \sum_h \phi_{Kj}^h)\hat{r} + (\theta_{Y_jL_j} - \sum_h \phi_{Lj}^h)\hat{w} + \theta_{Y_jZ_j}\hat{\tau}_Z + \hat{K}_{Y_j}\theta_{Y_jK_j} + \sum_l \hat{K}_{Y_l}\theta_{XK}\frac{K_{Y_l}}{K_X} + \hat{L}_{Y_j}\theta_{Y_jL_j} + \sum_l \hat{L}_{Y_l}\theta_{XL}\frac{L_{Y_l}}{L_X} \quad \forall j\,. \tag{A.MC.21}$$

Solve Eqs. (A.MC.4) and (A.MC.5) for $\hat{K}_{Y_j}$ and $\hat{L}_{Y_j}$, and insert them into Eq. (A.MC.21). This yields:

$$-\hat{Z}_j - \sum_l \left(\theta_{XK}\frac{K_{Y_l}}{K_X} + \theta_{XL}\frac{L_{Y_l}}{L_X}\right)\hat{Z}_l = (-\sum_h \phi_{Kj}^h + \theta_{Y_jK_j})\hat{r} + (-\sum_h \phi_{Lj}^h + \theta_{Y_jL_j})\hat{w} + \theta_{Y_jZ_j}\hat{\tau}_Z \quad \forall j\,. \tag{A.MC.22}$$

Next eliminate the $\hat{Z}$s. To achieve this, substitute Eqs. (A.MC.1) and (A.MC.2) into Eq. (A.MC.3), obtaining:

$$-\sum_l \frac{K_{Y_l}}{K_X}\hat{K}_{Y_l} + \sum_l \frac{L_{Y_l}}{L_X}\hat{L}_{Y_l} = 0\,. \tag{A.MC.23}$$

Substituting Eqs. (A.MC.4) and (A.MC.5) into Eq. (A.MC.23) yields:

$$\sum_l \left(-\frac{K_{Y_l}}{K_X} + \frac{L_{Y_l}}{L_X}\right)\hat{Z}_l = 0\,. \tag{A.MC.24}$$

Now insert Eq. (A.MC.24) into Eq. (A.MC.22):

$$\hat{r}(\theta_{Y_jK_j} - \sum_h \phi_{Kj}^h) + \hat{w}(\theta_{Y_jL_j} - \sum_h \phi_{Lj}^h) + \hat{\tau}_Z\theta_{Y_jZ_j} = -\sum_l \frac{K_{Y_l}}{K_X}\hat{Z}_l - \hat{Z}_j \quad \forall j\,. \tag{A.MC.25}$$

Now combine the above J equations in Eq. (A.MC.25) in order to be able to solve for the factor price changes. To do so, multiply each equation by an unknown parameter A_j and sum over j:

$$\hat{r}\sum_j (A_j\theta_{Y_jK_j} - A_j\sum_h \phi_{Kj}^h) + \hat{w}\sum_j (A_j\theta_{Y_jL_j} - A_j\sum_h \phi_{Lj}^h) + \hat{\tau}_Z\sum_j A_j\theta_{Y_jZ_j} = -\sum_l \left(\left(\sum_j A_j\right)\frac{K_{Y_l}}{K_X} + A_l\right)\hat{Z}_l\,. \tag{A.MC.26}$$

It therefore follows that if the right-hand side is zero, then the $\hat{Z}$s drop out of Eq. (A.MC.26). As an ansatz, require the following, which will then make the right-hand side of Eq. (A.MC.26) zero due to Eq. (A.MC.24):

$$\left(\sum_l A_l\right)\frac{K_{Y_j}}{K_X} + A_j = \frac{K_{Y_j}}{K_X} - \frac{L_{Y_j}}{L_X} \quad \forall j\,. \tag{A.MC.27}$$

In order to solve for the set of As that satisfies Eq. (A.MC.27), sum Eq. (A.MC.27) over j, and relabel indices to obtain the following (using the notation $K_Y \equiv \sum_l K_{Y_l}$ and analogous notation for the other aggregate variables):

$$(\sum_l A_l) = \frac{K_X}{\bar{K}}(\frac{K_Y}{K_X} - \frac{L_Y}{L_X}) \,. \tag{A.MC.28}$$

Insert Eq. (A.MC.28) back into Eq. (A.MC.27), and solve for A_j:

$$A_j = \frac{K_{Y_j}}{K_X} - \frac{L_{Y_j}}{L_X} - \frac{K_{Y_j}}{\bar{K}}(\frac{K_Y}{K_X} - \frac{L_Y}{L_X}) = \frac{\bar{L}}{L_X}\left(\frac{K_{Y_j}}{\bar{K}} - \frac{L_{Y_j}}{\bar{L}}\right) \quad \forall j \,. \tag{A.MC.29}$$

For the A coefficients as in Eq. (A.MC.29), Eq. (A.MC.26) then becomes:

$$\left(\sum_j A_j(\theta_{Y_jK_j} - \sum_h \phi^h_{Kj})\right)\hat{r} + \left(\sum_j A_j(\theta_{Y_jL_j} - \sum_h \phi^h_{Lj})\right)\hat{w} = -\hat{\tau}_Z \sum_j A_j\theta_{Y_jZ_j} \,. \tag{A.MC.30}$$

Solve Eq. (A.MC.14) for $\hat{w}$ and substitute into Eq. (A.MC.30), thus obtaining:

$$\hat{r} = -\frac{\theta_{XL}\sum_j A_j\theta_{Y_jZ_j}}{\sum_j A_j\left(\theta_{XL}\theta_{Y_jK_j} - \theta_{XK}\theta_{Y_jL_j} - \theta_{XL}\sum_h \phi^h_{Kj} + \theta_{XK}\sum_h \phi^h_{Lj}\right)}\hat{\tau}_Z \,. \tag{A.MC.31}$$

This then delivers the expression for $\hat{r}$, using the fact that

$$-\theta_{XL}\sum_h \phi^h_{Kj} + \theta_{XK}\sum_h \phi^h_{Lj} = \sum_h\left((\frac{Y^h_j}{Y_j} - \frac{X^h}{X})(\theta^h_L - \theta_L)\right) = \frac{1}{p_{Y_j}Y_j}cov(\alpha^h_j, \theta^h_L) - \frac{1}{p_XX}cov(\alpha^h, \theta^h_L) \,. \tag{A.MC.32}$$

B Supplement for Essay 1[2]

It should be noted that the condition $e_{ii} \leq 0$ for the diagonal elements of the matrix of Allen elasticities is not automatically satisfied. This has not been accounted for explicitly in Essay 1. I clarify here the implications of this restriction, which—although worthy of note—is of no consequence for the paper's results.[3]

First, the cases considered in the sensitivity analysis in the bottom half of Table 1.2 are analogous to those considered by Fullerton and Heutel (2010) (cases (2) and (3) in Table 4). Case (3) (i.e., $e_{K/LZ} = \mp 0.5$), however, leads to dirty-sector elasticities that do not satisfy the requirement $e_{ii} \leq 0$. As our analysis is based on the same production data as Fullerton and Heutel (2010), this issue also applies to our case. In order to satisfy this requirement, whilst obtaining virtually identical quantitative results (in Table 1.2, as well as in Figure 1.1b and Table 1.5), the case $e_{K/LZ} = \mp 0.5$ can be replaced with $e_{KZ} = -0.3$ and $e_{LZ} = 0.9$.

Second, replacing the case $e_{K/LZ} = \mp 0.5$ with $e_{KZ} = -0.3$ and $e_{LZ} = 0.9$ in Figures A.1b and A.2b delivers quantitative results that are not identical, but nonetheless deliver the same qualitative incidence patterns.

Third, it should be noted that Proposition 1.4 does not guarantee that the production elasticities which correspond to the reversal in the sign of factor changes satisfy the condition $e_{ii} \leq 0$. This requirement is, however, satisfied for a wide range of cases, as is, for example, illustrated in Proposition 1.5 (for any benchmark with *high* covariance).

Fourth, Case 3 in Table 1.3 does not satisfy the requirement that $e_{ii} \leq 0$. Other cases, however, exist, in addition to Cases 1 and 2, which do satisfy this requirement and that at the same time deliver a "large" welfare aggregation bias. One such case is, for exampe, the following: Leontief production, σ^h as for ρ_{high}, $E_Y^h = 0$, for $h = 1, \ldots, 7$, and $E_Y^h = 2$, for $h = 8, \ldots, 10$.

[2]Note that this material is separate from the article on which Essay 1 is based, published in *Journal of Public Economics, Volume 138, June 2016, 43-57* (http://dx.doi.org/10.1016/j.jpubeco.2016.04.004) together with Sebastian Rausch.

[3]As an aside, a typo in footnote 20 should be corrected: E_Y should replace $E_{X/Y}$.

C Appendix for Essay 2

C.1 Analytical results for linearized approach

Starting from a competitive equilibrium with a given tax rate, consider a small tax increase. Results then follow from Rausch and Schwarz (2016) (Eqs. (1.16a), (1.16b) and (A.10)), with household preferences assumed to be homothetic, and cross-price elasticities in production assumed to be equal, corresponding to the local properties of the economy's CES utility and production functions:[4]

$$\hat{p}_Y = \frac{(\theta_{YL}\theta_{XK} - \theta_{YK}\theta_{XL})\theta_{YZ}}{D}(\gamma_L - \gamma_K)(\sigma_Y - \rho)\hat{\tau} + \theta_{YZ}\hat{\tau}$$

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D}(\gamma_L - \gamma_K)(\sigma_Y - \rho)\hat{\tau}$$

$$\hat{w} = \frac{\theta_{XK}\theta_{YZ}}{D}(\gamma_L - \gamma_K)(\sigma_Y - \rho)\hat{\tau}$$

$$\hat{Z} = -\frac{1}{C}[(\theta_{YK}\rho - \frac{cov(\alpha^h, \theta_L^h)}{\gamma p_Y Y})\hat{r} + (\theta_{YL}\rho + \frac{cov(\alpha^h, \theta_L^h)}{\gamma p_Y Y})\hat{w} + (\theta_{YZ}\rho + (\beta_K + \beta_L)\sigma_Y)\hat{\tau}] =$$

$$-\frac{1}{C}\Big(\theta_{YZ}\rho + (\beta_K + \beta_L)\sigma_Y + \big(\theta_{XL}(\frac{cov(\alpha^h, \theta_L^h)}{\gamma p_Y Y} - \theta_{YK}\rho) + \theta_{XK}(\frac{cov(\alpha^h, \theta_L^h)}{\gamma p_Y Y} + \theta_{YL}\rho)\big)\frac{\theta_{YZ}}{D}(\gamma_L - \gamma_K)(\sigma_Y - \rho)\Big)\hat{\tau}\,,$$

where hats represent proportional changes, e.g., $\hat{p}_Y = \frac{dp_Y}{p_Y}$, and with the following benchmark quantities: clean and dirty sector output value $p_X X$ and $p_Y Y$, where p_X and p_Y are the price of the clean and dirty goods, respectively; household income levels M^h; household clean good expenditure shares $\alpha^h \equiv \frac{p_X X^h}{M^h}$; value share of the clean sector in the economy $\gamma \equiv \frac{p_X X}{p_X X + p_Y Y}$; household income shares from labor $\theta_L^h \equiv \frac{w L^h}{M^h}$; value share of labor in the economy $\theta_L \equiv \frac{w\bar{L}}{p_X X + p_Y Y}$; $\gamma_K = \frac{K_Y}{K_X}$ and $\gamma_L = \frac{L_Y}{L_X}$; and composite parameters $cov(\alpha^h, \theta_L^h) \equiv \sum_h (\alpha^h - \gamma) M^h (\theta_L^h - \theta_L)$, $\rho \equiv \frac{1}{p_Y Y}\sum_h (1 - \alpha^h) M^h \left(\frac{\alpha^h}{\gamma}(\sigma_C - 1) + 1\right)$, and $D \equiv C\sigma_X + \sigma_Y(A\theta_{XL} + B\theta_{XK}) - (\gamma_K - \gamma_L)\rho(\theta_{XK}\theta_{YL} - \theta_{XL}\theta_{YK}) - (\gamma_K - \gamma_L)\frac{cov(\alpha^h, \theta_L^h)}{\gamma p_Y Y}$ with $A \equiv \gamma_L\beta_K + \gamma_K(\beta_L + \theta_{YZ})$, $B \equiv \gamma_K\beta_L + \gamma_L(\beta_K + \theta_{YZ})$, $C \equiv \beta_K + \beta_L + \theta_{YZ}$, $\beta_K = \theta_{XK}\gamma_K + \theta_{YK}$ and $\beta_L = \theta_{XL}\gamma_L + \theta_{YL}$.

The following thus holds:

$$p_Y^L(\tau) = p_Y(\tau_0) + \Big(\frac{(\theta_{YL}\theta_{XK} - \theta_{YK}\theta_{XL})\theta_{YZ}}{D}(\gamma_L - \gamma_K)(\sigma_Y - \rho) + \theta_{YZ}\Big)\frac{p_Y(\tau_0)}{\tau_0}(\tau - \tau_0) \tag{B.1a}$$

$$r^L(\tau) = r(\tau_0) - \frac{\theta_{XL}\theta_{YZ}}{D}(\gamma_L - \gamma_K)(\sigma_Y - \rho)\frac{r(\tau_0)}{\tau_0}(\tau - \tau_0)\,, \tag{B.1b}$$

$$w^L(\tau) = w(\tau_0) + \frac{\theta_{XK}\theta_{YZ}}{D}(\gamma_L - \gamma_K)(\sigma_Y - \rho)\frac{w(\tau_0)}{\tau_0}(\tau - \tau_0)\,, \tag{B.1c}$$

$$Z^L(\tau) = Z(\tau_0) - \frac{1}{C}\Big(\theta_{YZ}\rho + (\beta_K + \beta_L)\sigma_Y$$

$$+\big(\theta_{XL}(\frac{cov(\alpha^h, \theta_L^h)}{\gamma p_Y Y} - \theta_{YK}\rho) + \theta_{XK}(\frac{cov(\alpha^h, \theta_L^h)}{\gamma p_Y Y} + \theta_{YL}\rho)\big)\frac{\theta_{YZ}}{D}(\gamma_L - \gamma_K)(\sigma_Y - \rho)\Big)\frac{Z(\tau_0)}{\tau_0}(\tau - \tau_0)\,. \tag{B.1d}$$

C.2 Analytical results for exact approach

For Cobb-Douglas utility and production technologies, the model equations are as follows.

Household demand functions:

$$X = \frac{\alpha M}{p_X} \tag{B.2}$$

[4] Note that the assumption of equal cross-price substitution elasticities is automatically satisfied in the clean sector, since it only employs two inputs. In dirty production, this assumption corresponds to $e_{KL} = e_{KZ} = e_{LZ} \equiv \sigma_Y \geq 0$, where σ_Y is the elasticity of substitution of the CES production function describing the dirty sector.

$$Y = \frac{(1-\alpha)M}{p_Y} \tag{B.3}$$

where the income M is given by $M \equiv r\bar{K} + w\bar{L} + \tau Z$. Clean sector demand functions:

$$K_X = \frac{\alpha_X}{r} p_X X \tag{B.4}$$

$$L_X = \frac{(1-\alpha_X)}{w} p_X X \tag{B.5}$$

Dirty sector demand functions:

$$K_Y = \frac{\alpha_Y}{r} p_Y Y \tag{B.6}$$

$$Z = \frac{\beta_Y}{\tau} p_Y Y \tag{B.7}$$

$$L_Y = \frac{1-\alpha_Y-\beta_Y}{w} p_Y Y \tag{B.8}$$

Market clearing conditions:

$$L_X + L_Y = \bar{L} \tag{B.9}$$

$$K_X + K_Y = \bar{K} \tag{B.10}$$

Production technologies:

$$X = A_X K_X^{\alpha_X} L_X^{1-\alpha_X} \tag{B.11}$$

$$Y = A_Y K_Y^{\alpha_Y} Z^{\beta_Y} L_Y^{(1-\alpha_Y-\beta_Y} \tag{B.12}$$

Eqs. (B.4) and (B.5) into Eq. (B.11):

$$X = A_X \left(\frac{\alpha_X}{r} p_X X\right)^{\alpha_X} \left(\frac{1-\alpha_X}{w} p_X X\right)^{1-\alpha_X} = A_X \left(\frac{\alpha_X}{r}\right)^{\alpha_X} \left(\frac{1-\alpha_X}{w}\right)^{1-\alpha_X} p_X X \tag{B.13}$$

Set p_X as the numeraire ($p_X \equiv 1$), thus obtaining:

$$r^{\alpha_X} w^{1-\alpha_X} = A_X {\alpha_X}^{\alpha_X} (1-\alpha_X)^{1-\alpha_X} \tag{B.14}$$

Now substitute Eqs. (B.6), (B.7) and (B.8) into Eq. (B.12):

$$Y = A_Y \left(\frac{\alpha_Y}{r} p_Y Y\right)^{\alpha_Y} \left(\frac{\beta_Y}{\tau} p_Y Y\right)^{\beta_Y} \left(\frac{(1-\alpha_Y-\beta_Y)}{w} p_Y Y\right)^{1-\alpha_Y-\beta_Y} \tag{B.15}$$

thus

$$r^{\alpha_Y} \tau^{\beta_Y} w^{1-\alpha_Y-\beta_Y} = p_Y \alpha_Y^{\alpha_Y} \beta_Y^{\beta_Y} (1-\alpha_Y-\beta_Y)^{1-\alpha_Y-\beta_Y} \tag{B.16}$$

Now substitute Eqs. (B.5) and (B.8) into Eq. (B.9), using $\frac{p_Y Y}{p_X X} = \frac{1-\alpha}{\alpha}$ from Eq. (B.2) and Eq. (B.3):

$$\bar{L} = \frac{1-\alpha_X}{w} p_X X + \frac{1-\alpha_Y-\beta_Y}{w} p_Y Y = \left(\frac{1-\alpha_X}{w} + \frac{1-\alpha_Y-\beta_Y}{w} \frac{1-\alpha}{\alpha}\right) p_X X \tag{B.17}$$

Analogously substitute Eqs. (B.4) and (B.6) into Eq. (B.10):

$$\bar{K} = \frac{\alpha_X}{r} p_X X + \frac{\alpha_Y}{r} p_Y Y = \left(\frac{\alpha_X}{r} + \frac{\alpha_Y}{r} \frac{1-\alpha}{\alpha}\right) p_X X \tag{B.18}$$

Divide Eq. (B.17) by Eq. (B.18), thus obtaining:

$$\frac{w\bar{L}}{r\bar{K}} = \frac{\alpha(1-\alpha_X) + (1-\alpha)(1-\alpha_Y-\beta_Y)}{\alpha\alpha_X + (1-\alpha)\alpha_Y} \tag{B.19}$$

Insert Eq. (B.19) into Eq. (B.14), thus obtaining $w = const$, which then implies $r = const$. From Eq. (B.16) it then follows that $p_Y = const \cdot \tau^{\beta_Y}$.

For the level of pollution, insert Eq. (B.3) into Eq. (B.7) and rearrange, to obtain:

$$Z = \frac{const}{\tau} \tag{B.20}$$

The following thus follows for price and pollution changes:

$$p_Y^E(\tau) = p_Y(\tau_0)\left(\frac{\tau}{\tau_0}\right)^{\theta_{YZ}} \qquad r^E(\tau) = r(\tau_0) \qquad w^E(\tau) = w(\tau_0) \qquad Z^E(\tau) = Z(\tau_0)\frac{\tau_0}{\tau} \tag{B.21}$$

C.3 Equilibrium conditions for numerical general equilibrium model

The CGE model I employ in this essay is intentionally very simple.[5]. Tables C.1 and C.2 define the parameters and variables in the model.

Zero-profit conditions are given by the following equations:[6]

$$\begin{aligned} c_h &\geq PC_h & \perp \quad & C_h \geq 0 \quad \forall\, h \\ c_i &\geq PY_i & \perp \quad & Y_i \geq 0 \quad \forall\, i\,. \end{aligned}$$

Market clearing conditions are given by the following equations:

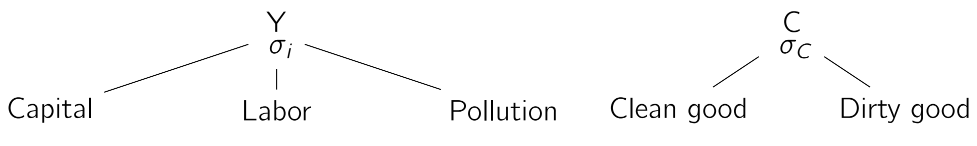
$$\begin{aligned} \sum_h \omega_{Lh} &\geq \sum_i \frac{\partial c_i}{\partial PL} Y_i & \perp \quad & PL \geq 0 \\ \sum_h \omega_{Kh} &\geq \sum_i \frac{\partial c_i}{\partial PK} Y_i & \perp \quad & PK \geq 0 \\ Y_i &\geq \sum_h \frac{\partial c_h}{\partial PY_i} C_h & \perp \quad & PY_i \geq 0 \qquad \forall i \\ C_h &\geq \frac{M_h}{PC_h} & \perp \quad & PC_h \geq 0 \qquad \forall h\,, \end{aligned}$$

where the income of household h given as the sum of income from factor endowments (ω) and transfers: $M_h = PL\omega_{Lh} + PK\omega_{Kh} + T_h$. Transfers to household h are given by a constant fraction θ_h^Z of pollution tax revenues: $T_h = \theta_h^Z(PZ \sum_i \frac{\partial c_i}{\partial PZ})$.

The price of pollution PZ equals the carbon tax rate: $PZ = \tau$.

Figure C.1: Structure of production and private consumption

(a) Production (b) Final consumption



The expenditure function for household h is a CES function without nesting, represented in Figure C.1b. It is defined as:[7]

$$c_h := \left[\sum_i \theta_{ih} PY_i^{1-\sigma_C}\right]^{\frac{1}{1-\sigma_C}} .$$

[5]The description below follows closely the description of other (more complex) models of this type such as, for example, in Caron and Rausch (2013), Abrell and Rausch (2016), and Abrell et al. (2016).

[6]The perpendicular sign $\perp$, following the conventional notation (Rutherford, 1995), denotes complementarity between equilibrium condition and variable: given a function $F\colon \mathbb{R}^n \longrightarrow \mathbb{R}^n$, find $z \in \mathbb{R}^n$ such that $F(z) \geq 0$, $z \geq 0$, and $z^T F(z) = 0$, or, in compact notation, $F(z) \geq 0 \perp z \geq 0$.

[7]Note that benchmark prices are normalized to one.

According to the nesting structure given in Figure C.1a, unit cost functions for production activity i is given as:

$$c_i := \left[\theta_i^L PL^{1-\sigma_i} + (1-\theta_i^L-\theta_i^Z)PL^{1-\sigma_i} + \theta_i^Z PZ^{1-\sigma_i}\right]^{\frac{1}{1-\sigma_i}} .$$

Table C.1: Sets, prices, and quantity variables

Symbol	Description
Sets	
$i \in I$	Commodities (*clean* and *dirty*)
$h \in H$	Household
Prices and quantities	
C_h	Private consumption index household h
M_h	Private income of household h
Y_i	Production index sector i
PL	Wage rate in region r
PC_h	Consumer price index for household h
PK	Capital rental rate
PY_i	Commodity i output price
T_h	Transfer to household h
PZ	Carbon price

Table C.2: Model parameters

Symbol	Description
Elasticity of substitution parameters	
σ_C	Consumption
σ_i	Production in sector i
Other parameters	
θ_{ih}^C	Expenditure share of commodity i in total expenditure of household h
θ_i^L	Share of labor cost in production i
θ_i^Z	Share of pollution cost in production i
ω_{Kh}	Capital endowment household h
ω_{Lh}	Labor endowment household h
τ	Carbon tax

C.4 GAMS code for CGE model

```
$title  CGE model to replicate analytical model

*===========================================================================================
*   DATA
*===========================================================================================

* Households (expenditure deciles)
set h /h1*h10/;

parameter
      cprice          Carbon price
      cqty            Carbon cap

* Production parameters

      valX            Benchmark value of sector X
      valY            Benchmark value of sector Y
      valXK           Benchmark value paid to capital by sector X
      valXL           Benchmark value paid to labor by sector X
      valYK           Benchmark value paid to capital by sector Y
      valYL           Benchmark value paid to labor by sector Y
      valYZ           Benchmark value paid to pollution by sector Y

* Heterogeneous household parameters

      exp(h)          Total benchmark consumption by household h
      expX(h)         Benchmark consumption of good X by household h
      expY(h)         Benchmark consumption of good Y by household h
      incK(h)         Benchmark income from capital for household h
      incL(h)         Benchmark income from labor for household h
      incZ(h)         Benchmark income from pollution for household h

* Elasticity parameters of firms and households

      sigmay          Top level elasticity of substitution for firm y
      sigmax          Top level elasticity of substitution for firm x
      sigmah          Elasticity of substitution of households

* Reporting parameters

      PK_cprice       Percentage change in price of capital in price-based policy
      PL_cprice       Percentage change in price of labor in price-based policy
      PY_cprice       Percentage change in price of sector Y good in price-based policy
      Z_cprice        Percentage change in pollution level in price-based policy
      U_cprice(h)     Percentage change in utility of household h in price-based policy
      PK_cqty         Percentage change in price of capital in quantity-based policy
      PL_cqty         Percentage change in price of labor in quantity-based policy
      PY_cqty         Percentage change in price of sector Y good in quantity-based policy
      Z_cqty          Percentage change in pollution level in quantity-based policy
      U_cqty(h)       Percentage change in utility of household h in quantity-based policy;

cprice = 1;
cqty = 1;

* Initialize production parameters

valX =   0.92861619;
valY =   0.07138381;
valXK =  0.34962402;
valXL =  0.57899217;
valYK =  0.03726235;
valYL =  0.02896088;
valYZ =  0.00516059;

* Initialize household parameters

exp("h1") =  0.02611600;
exp("h2") =  0.04519300;
exp("h3") =  0.05902200;
exp("h4") =  0.07150700;
exp("h5") =  0.08008100;
exp("h6") =  0.08901200;
exp("h7") =  0.10659500;
exp("h8") =  0.12725700;
exp("h9") =  0.15475500;
exp("h10") = 0.24046200;

expX("h1") =  0.02354921;
```

```
expX("h2") =  0.04053682;
expX("h3") =  0.05318107;
expX("h4") =  0.06476982;
expX("h5") =  0.07291596;
expX("h6") =  0.08159089;
expX("h7") =  0.09828598;
expX("h8") =  0.11794119;
expX("h9") =  0.14499965;
expX("h10") = 0.23084558;

expY("h1") =  0.00256679;
expY("h2") =  0.00465618;
expY("h3") =  0.00584093;
expY("h4") =  0.00673718;
expY("h5") =  0.00716504;
expY("h6") =  0.00742111;
expY("h7") =  0.00830902;
expY("h8") =  0.00931581;
expY("h9") =  0.00975535;
expY("h10") = 0.00961642;

incK("h1") =  0.00980673;
incK("h2") =  0.01106067;
incK("h3") =  0.01460821;
incK("h4") =  0.01658963;
incK("h5") =  0.01980109;
incK("h6") =  0.02849896;
incK("h7") =  0.03472099;
incK("h8") =  0.04831141;
incK("h9") =  0.06440675;
incK("h10") = 0.13908193;

incL("h1") =  0.01617450;
incL("h2") =  0.03389911;
incL("h3") =  0.04410920;
incL("h4") =  0.05454835;
incL("h5") =  0.05986664;
incL("h6") =  0.06005369;
incL("h7") =  0.07132391;
incL("h8") =  0.07828887;
incL("h9") =  0.08954963;
incL("h10") = 0.10013915;

incZ("h1") =  0.000134774;
incZ("h2") =  0.000233223;
incZ("h3") =  0.000304588;
incZ("h4") =  0.000369018;
incZ("h5") =  0.000413265;
incZ("h6") =  0.000459354;
incZ("h7") =  0.000550093;
incZ("h8") =  0.000656721;
incZ("h9") =  0.000798627;
incZ("h10") = 0.001240926;

* Initialize elasticity parameters

sigmax = 1;
sigmay = 1;
sigmah = 1;

* Flags for unit elasticities

parameter
        sigmax_isone
        sigmay_isone
        sigmah_isone;

sigmax_isone = 0;
sigmay_isone = 0;
sigmah_isone = 0;

sigmax_isone$(sigmax = 1) = 1;
sigmay_isone$(sigmay = 1) = 1;
sigmah_isone$(sigmah = 1) = 1;

*==========================================================================================
* MCP MODEL
*==========================================================================================

NONNEGATIVE VARIABLES
        X               activity level for clean production
        Y               activity level for dirty production
```

```
        U(h)           activity level for welfare "production" for household h
        PX             price of good X
        PY             price of good Y
        PU(h)          price of a unit of welfare
        PL             price of labor
        PK             price of capital
        PZ             price of pollution
        CONS(h)        income of household h

EQUATIONS
        PRF_X          zero profit for sector X
        PRF_Y          zero profit for sector Y
        PRF_U(h)       zero profit for sector U(h)

        MKT_X          supply-demand balance for commodity X
        MKT_Y          supply-demand balance for commodity Y
        MKT_L          supply-demand balance for primary factor L
        MKT_K          supply-demand balance for primary factor K
        MKT_U(h)       supply-demand balance for demand of household h

        I_CONS(h)      income definition for household h

        AUX_CARB_P     auxiliary equation for carbon emissions - price
        AUX_CARB_Q     auxiliary equation for carbon emissions - quantity;

*  Zero profit inequalities

        PRF_X..        (1 - sigmax_isone)*( (valXL/valX)*PL**(1-sigmax)
                                  + (valXK/valX)*PK**(1-sigmax)
                                  )**(1/(1 - sigmax + sigmax_isone))
                       + sigmax_isone*(PL**(valXL/valX)*PK**(valXK/valX))
                                                =G=
                                                   PX;

        PRF_Y..        (1 - sigmay_isone)*( (valYL/valY)*PL**(1-sigmay)
                                  + (valYK/valY)*PK**(1-sigmay)
                                  + (valYZ/valY)*PZ**(1-sigmay)
                                  )**(1/(1 - sigmay + sigmay_isone))
                       + sigmay_isone*( PL**(valYL/valY)
                                      *PK**(valYK/valY)
                                      *PZ**(valYZ/valY))
                                                =G=
                                                   PY;

        PRF_U(h)..     (1 - sigmah_isone)*( (expX(h)/exp(h))*PX**(1-sigmah)
                                  + (expY(h)/exp(h))*PY**(1-sigmah)
                                  )**(1/(1 - sigmah + sigmah_isone))
                       + sigmah_isone*(PX**(expX(h)/exp(h))*PY**(expY(h)/exp(h)))
                                                =G=
                                                   PU(h);

*  Market clearance inequalities

        MKT_X..        sum(h, exp(h))*X
                                                =G=
                                                   sum(h, exp(h)*U(h)*(PU(h)/PX)**(sigmah));

        MKT_Y..        sum(h, exp(h))*Y
                                                =G=
                                                   sum(h, exp(h)*U(h)*(PU(h)/PY)**(sigmah));

        MKT_U(h)..     exp(h)*U(h)
                                                =E=
                                                   CONS(h)/PU(h);

        MKT_L..        valXL + valYL
                                                =G=
                                                   X*valXL*(PX/PL)**(sigmax)
                                                 + Y*valYL*(PY/PL)**(sigmay);

        MKT_K..        valXK + valYK
                                                =G=
                                                   X*valXK*(PX/PK)**(sigmax)
                                                 + Y*valYK*(PY/PK)**(sigmay);

*  Income balance equations

        I_CONS(h)..    CONS(h)
                                                =E=
                                                   incL(h)*PL
                                                 + incK(h)*PK
                                                 + incZ(h)*Y*(PY/PZ)**(sigmay)*PZ;
```

```
*  Auxiliary equations

        AUX_CARB_P..  cprice
                                                            =E=
                                                                 PZ;
        AUX_CARB_Q..  cqty
                                                            =G=
                                                                 Y*(PY/PZ)**(sigmay);

MODEL CGE /PRF_X.X, PRF_Y.Y, PRF_U.U,
           MKT_X.PX, MKT_Y.PY, MKT_L.PL, MKT_K.PK,
           MKT_U.PU, I_CONS.CONS, AUX_CARB_P.PZ /;

MODEL CGE_QTY /PRF_X.X, PRF_Y.Y, PRF_U.U,
           MKT_X.PX, MKT_Y.PY, MKT_L.PL, MKT_K.PK,
           MKT_U.PU, I_CONS.CONS, AUX_CARB_Q.PZ /;

* Chose a numeraire

        PX.FX = 1;

*Set initial value of variable:

        X.L=1; Y.L=1; U.L(h)=1;
        PY.L=1; PK.L=1; PL.L=1; PU.L(h)=1; PZ.L=1;
        CONS.L(h)=exp(h);

*==========================================================================================
* BENCHMARK VERIFICATION
*==========================================================================================

SOLVE CGE USING MCP;

*==========================================================================================
* COUNTERFACTUAL SCENARIOS
*==========================================================================================

* Price-based policy: increase the price of carbon by 50%

cprice = 1.5;
SOLVE CGE USING MCP;

* Report results

PK_cprice = 100*(PK.L - 1);
PL_cprice = 100*(PL.L - 1);
PY_cprice = 100*(PY.L - 1);
U_cprice(h) = 100*(U.L(h) - 1);
Z_cprice =  100*(Y.L*(PY.L/PZ.L)**(sigmay) - 1);

display  PY_cprice, PK_cprice, PL_cprice, Z_cprice, U_cprice;

*Re-set initial value of variable:

        X.L=1; Y.L=1; U.L(h)=1;
        PY.L=1; PK.L=1; PL.L=1; PU.L(h)=1; PZ.L=1;
        CONS.L(h)=exp(h);

* Quantity-based policy: 50% reduction in pollution level

cqty = 0.5;
SOLVE CGE_QTY USING MCP;

* Report results

PK_cqty = 100*(PK.L - 1);
PL_cqty = 100*(PL.L - 1);
PY_cqty = 100*(PY.L - 1);
U_cqty(h) = 100*(U.L(h) - 1);
Z_cqty =  100*(Y.L*(PY.L/PZ.L)**(sigmay) - 1);

display  PY_cqty, PK_cqty, PL_cqty, Z_cqty, U_cqty;
```

D Appendix for Essay 3

D.1 Issues specific to our model and data

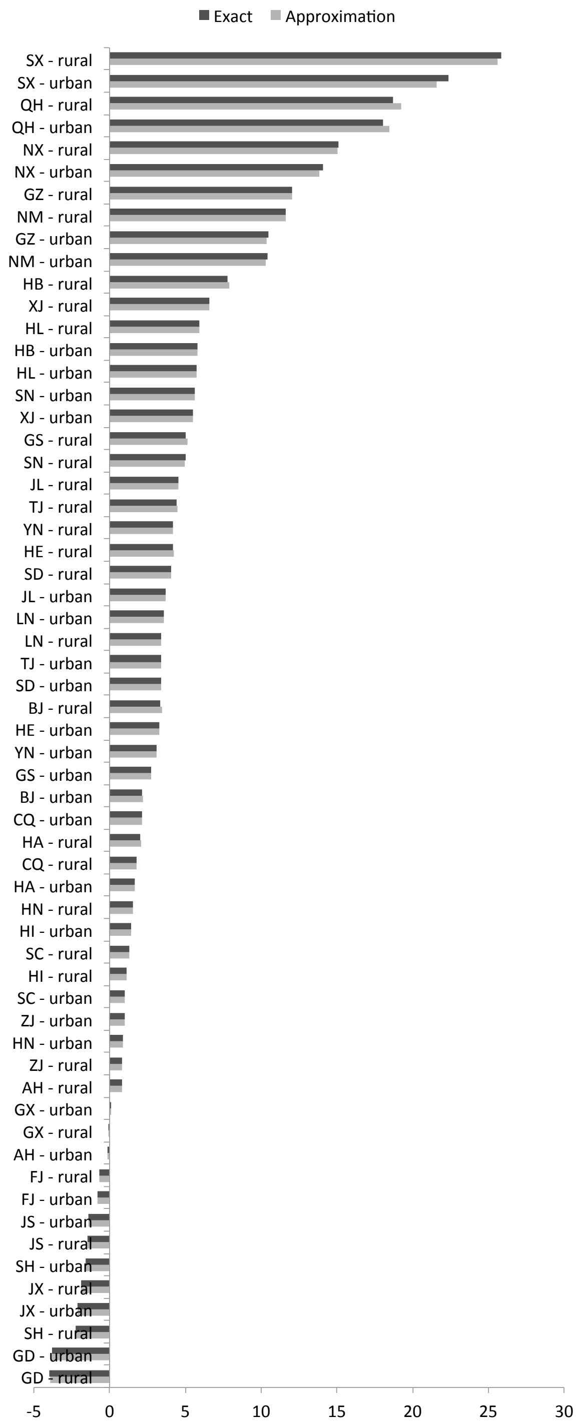
Our model's benchmark year is 2007, but the CRECS survey as well as the NBS data we use to estimate the income curves are from 2012. In order to use the estimated Engel curves consistently in the model, we therefore convert 2007 to 2012 prices and then look up the consumption at various income levels denominated in 2012 prices. The implicit GDP deflator is used to determine the rate of inflation (The World Bank, 2015).

Our general method to calibrate preferences to Engel curves is limited to cases where the projected consumption of a good remains above the benchmark consumption level (i.e., $y_{jupt} \geq x_{jupt_0}$).[8] This holds for a given good j if its income elasticity of consumption is always positive. This is also the case if the income elasticity is initially positive, but later becomes negative as income grows—as long as the implied level of consumption never goes below the level in the benchmark year. For our model and data, however, consumption of COL and GAS for some provinces and households is decreasing to the point where the consumption levels drop below 2007 levels by 2030. For these goods, we thus resort to an alternative implementation of changing consumption patterns. We do this by scaling benchmark consumption shares to target projected consumption at benchmark prices and projected real income. For all other goods, we employ the calibration described in Section 3.3.3.

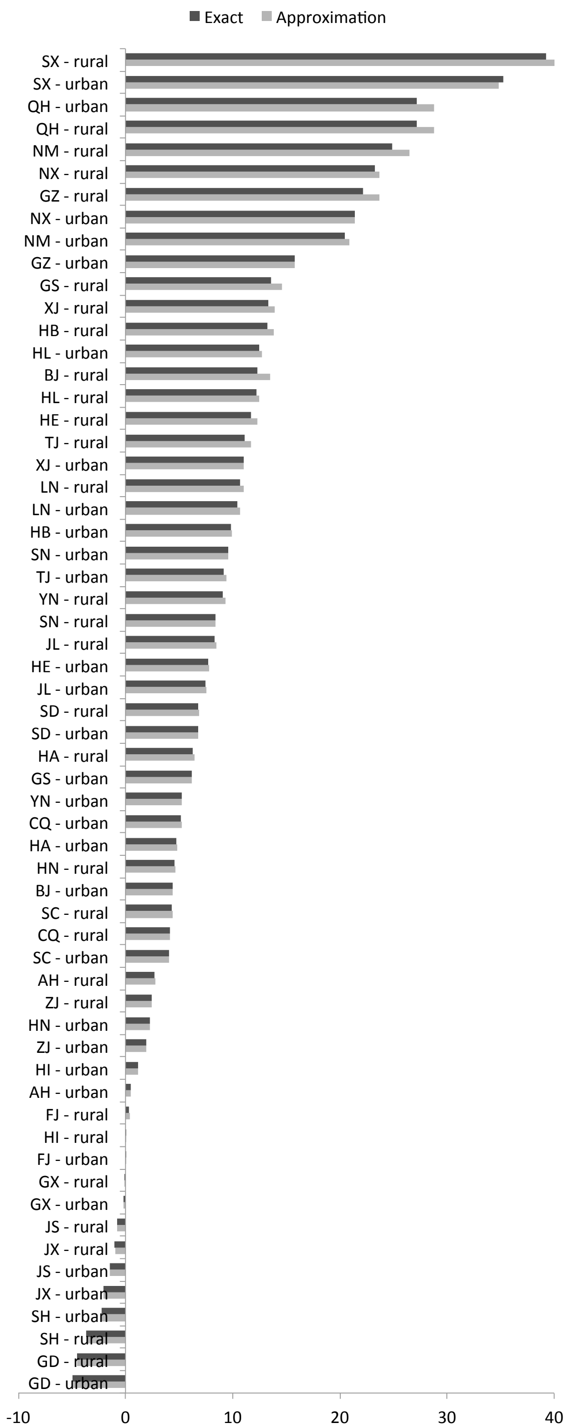
Regarding the convergence of the iterative routine involved in the calibration of preferences, we find that the difference in real income between the second and the first iteration of the calibration process varies widely by province and household type, with a magnitude relative to the 2030 real income as large as 10.5%. This difference drops sharply, and the fourth iteration delivers differences that are significantly lower than 0.1%, at which point we terminate the iteration.

[8] If this condition does not hold, the calibration would then imply $c^{\star}_{jupt} > x_{jupt_0}$, and as a consequence $\tilde{\theta}_{jupt} < 0$, which cannot hold.

D.2 Additional figures



(a) Calibrated



(b) Proportional

Figure D.1: % consumption loss in 2030: comparison between exact measure (equivalent variation) and approximation (consumer surplus), for the calibrated and the proportional cases.

D.3 Equilibrium conditions for numerical general equilibrium model

We formulate our model as a non-linear complementarity problem (Rutherford, 1995).[9] The economic equilibrium is characterized by a system of non-linear inequalities, comprizing zero profit and market clearing conditions.[10] Zero-profit conditions are complementary to quantity variables, and market clearing conditions are complementary to price variables. Tables 3.1 and D.1 to D.3 define the parameters and variables in the model.

We start by defining the unit cost functions c, which enter the zero profit conditions. Household welfare is assumed to consist of a Cobb-Douglas function of household consumption, leisure, and savings. The associated cost function therefore assumes the following form:

$$c^{W}_{\bar{h}r} := P_{\bar{h}s}^{\tilde{\theta}^{W,CON}_{\bar{h}r}} \, P_{INV,r}^{\theta^{W,SAV}_{\bar{h}r}} \, PL_r^{1-\tilde{\theta}^{W,CON}_{\bar{h}r}-\theta^{W,SAV}_{\bar{h}r}} .$$

The unit cost function for utility from consumption is defined as (Figure D.2):[11]

$$c^{C}_{\bar{h}r} := \left[\tilde{\theta}^{TRN}_{\bar{h}r} \left(\frac{PAT_{TRNr}}{\overline{\overline{pat}}_{TRNr}} \right)^{1-\sigma_{ctop}} + \left(1-\tilde{\theta}^{TRN}_{\bar{h}r}\right) (c^{NTRN}_{\bar{h}r})^{1-\sigma_{ctop}} \right]^{\frac{1}{1-\sigma_{ctop}}}$$

where

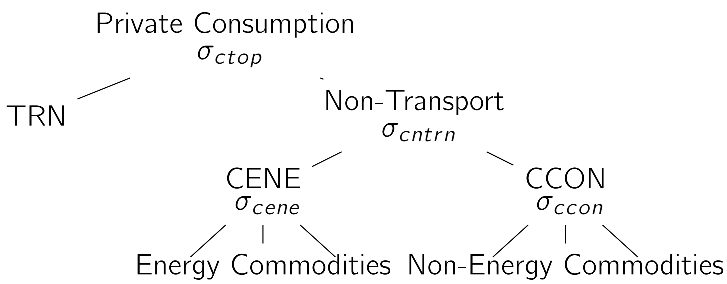
$$c^{NTRN}_{\bar{h}r} := \left[\tilde{\theta}^{CON}_{\bar{h}r} (c^{CENE}_{\bar{h}r})^{1-\sigma_{cntrn}} + \left(1-\tilde{\theta}^{CON}_{\bar{h}r}\right) (c^{CCON}_{\bar{h}r})^{1-\sigma_{cntrn}} \right]^{\frac{1}{1-\sigma_{cntrn}}}$$

$$c^{CENE}_{\bar{h}r} := \left[\sum_{j \in ENE} \tilde{\theta}^{CENE}_{j\bar{h}r} \left(\frac{PAT_{jr}}{\overline{\overline{pat}}_{jr}} \right)^{1-\sigma_{cene}} \right]^{\frac{1}{1-\sigma_{cene}}}$$

$$c^{CCON}_{\bar{h}r} := \left[\sum_{j \in NENE \setminus \{TRN\}} \tilde{\theta}^{CCON}_{j\bar{h}r} \left(\frac{PAT_{jr}}{\overline{\overline{pat}}_{jr}} \right)^{1-\sigma_{ccon}} \right]^{\frac{1}{1-\sigma_{ccon}}} ,$$

where PAT_{jr} denote carbon cost inclusive Armington prices.[12]

Figure D.2: Structure of private consumption



For government and investment consumption, unit cost functions c^G_r and c^I_r are defined similarly as for pricate

[9]As this part of the model is based on Zhang et al. (2013), the model description will follow the previous exposition closely. It is also similar to other descriptions of models of this type such as, for example, in Caron and Rausch (2013), Abrell and Rausch (2016), and Abrell et al. (2016).

[10]In general, a non-linear complementarity problem assumes the following form (Rutherford, 1995): given a function $F\colon \mathbb{R}^n \longrightarrow \mathbb{R}^n$, find $z \in \mathbb{R}^n$ such that $F(z) \geq 0$, $z \geq 0$, and $z^T F(z) = 0$, or, in compact notation, $F(z) \geq 0 \perp z \geq 0$, indicating complementarity between equilibrium condition and variable.

[11]Prices denoted with an upper bar stand for tax-inclusive benchmark values. Note that, for simplicity, we abstract in the notation from the fact that such prices are differentiated across agents, due to differentiated tax rates. θ generally refers to share parameters.

[12]The carbon cost is added to PA in proportion to the good's carbon intensity.

consumption (Figure D.3):

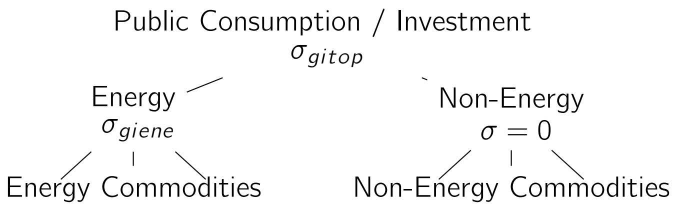
$$c_r^{G/I} := \left[\theta_r^{G/ICON}(c_r^{G/ICENE})^{1-\sigma_{gitop}} + \left(1-\theta_r^{G/ICON}\right)(c_r^{G/ICCON})^{1-\sigma_{gitop}}\right]^{\frac{1}{1-\sigma_{gitop}}}$$

where

$$c_r^{G/ICENE} := \left[\sum_{j\in ENE} \theta_{jr}^{G/ICENE}\left(\frac{PAT_{jr}}{\overline{\overline{pat_{jr}}}}\right)^{1-\sigma_{giene}}\right]^{\frac{1}{1-\sigma_{giene}}}$$

$$c_r^{G/ICCON} := \sum_{j\in NENE} \theta_{jr}^{G/ICCON}\left(\frac{PAT_{jr}}{\overline{\overline{pat_{jr}}}}\right).$$

Figure D.3: Structure of government consumption and investment



Unit cost functions for production activities $j \in$ {EIS, MAN, WTR, CON, TRN, SER, AGR, OMN} are given as (Figures D.4 and D.5):

$$c_{jr} := \left[\sum_{j'\in NENE} \theta_{j'jr}^{TOP}\left(\frac{PAT_{j'r}}{\overline{\overline{pat_{j'r}}}}\right)^{1-\sigma_{ytop}} + \theta_{jr}^{RES}\left(\frac{PS_{jr}(1+ts_{jr})}{\overline{\overline{ps_{jr}}}}\right)^{1-\sigma_{ytop}} + \left(1-\theta_{jr}^{RES} - \sum_{j'\in NENE}\theta_{j'jr}^{TOP}\right)(c_{jr}^{VAE})^{1-\sigma_{ytop}}\right]^{\frac{1}{1-\sigma_{ytop}}}$$

where

$$c_{jr}^{VAE} := \left[\theta_{jr}^{VAE}(c_{jr}^{VA})^{1-\sigma_{vae}} + \left(1-\theta_{jr}^{VAE}\right)(c_{jr}^{ENE})^{1-\sigma_{vae}}\right]^{\frac{1}{1-\sigma_{vae}}}$$

$$c_{jr}^{VA} := \left[\theta_{jr}^{VA}\left(\frac{(1+tl_{jr})PL_r}{\overline{\overline{pl_{jr}}}}\right)^{1-\sigma_{va}} + \left(1-\theta_{jr}^{VA}\right)\left(\frac{(1+tk_{jr})PK_r}{\overline{\overline{pk_{jr}}}}\right)^{1-\sigma_{va}}\right]^{\frac{1}{1-\sigma_{va}}}$$

$$c_{jr}^{ENE} := \left[\theta_{jr}^{ENE}\left(\frac{PAT_{ELE,r}}{\overline{\overline{pat_{ELE,r}}}}\right)^{1-\sigma_{ene}} + \left(1-\theta_{jr}^{ENE}\right)(c_{jr}^{FOF})^{1-\sigma_{ene}}\right]^{\frac{1}{1-\sigma_{ene}}}$$

$$c_{jr}^{FOF} := \left[\sum_{j'\in FOF} \theta_{j'jr}^{FOF}\left(\frac{PAT_{j'r}}{\overline{\overline{pat_{j'r}}}}\right)^{1-\sigma_{fof}}\right]^{\frac{1}{1-\sigma_{fof}}}.$$

Note that for Chinese provinces p, there is a single capital rental rate, since capital markets within China are assumed to be integrated. Thus $PK_p \equiv PK^{CHN}$, $\forall p \in$ CHN.

Figure D.4: Structure of production for $j \in$ {EIS, MAN, WTR, CON, TRN, SER}

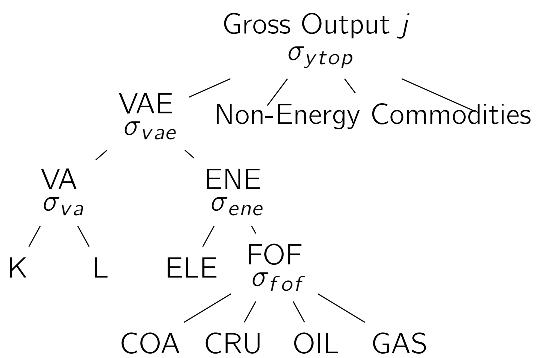
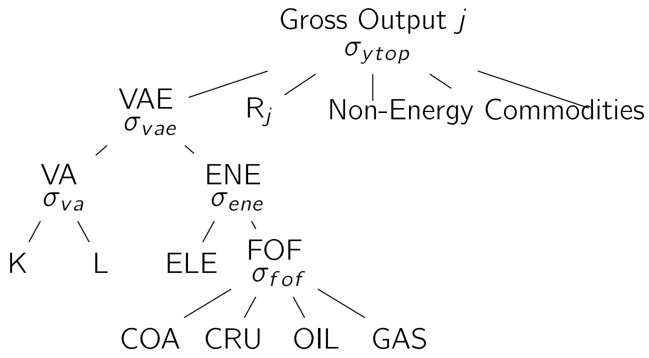
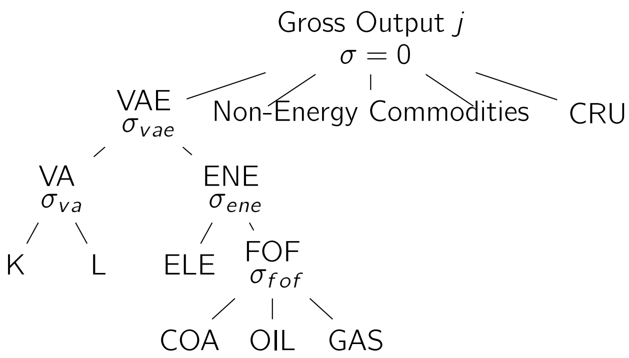


Figure D.5: Structure of production for $j \in$ {AGR, OMN}



Unit cost functions for production activity $j \in$ {OIL} are analogous to those for $i \in$ {EIS, MAN, ... } from above, with the top level elasticity set to zero (Figure D.6):

Figure D.6: Structure of production for $j \in$ {OIL}



The unit cost function for production activity $j \in$ {ELE} is given as (Figure D.7):

$$c_{jr} := min\Big\{c_{NUC,r},\ c_{HYD,r},\ c_{WYD,r},\ c^{CGO}_{r}\Big\}$$

where

$$c_{nhw,r} := \left[\theta^{TOP}_{nhw,r}\left(\frac{(1+tsn_{nhw,r})PS_{nhw,r}}{\overline{\overline{psnhw_{nhw,r}}}}\right)^{1-\sigma_{nhwtpo}} + \left(1-\theta^{TOP}_{nhw,r}\right)\left(c^{VAM}_{nhw,r}\right)^{1-\sigma_{nhwtop}}\right]^{\frac{1}{1-\sigma_{nhwtop}}}$$

$$c^{VAM}_{nhw,r} := \theta^{SER}_{nhw,r}\left(\frac{PAT_{SER,r}}{\overline{\overline{pat_{SER,r}}}}\right) + \left(1-\theta^{SER}_{nhw,r}\right)c^{VA}_{nhw,r}$$

$$c^{VA}_{nhw,r} := \left[\theta^{VA}_{nhw,r}\left(\frac{(1+tl_{nhw,r})PL_r}{\overline{\overline{pl_{nhwr}}}}\right)^{1-\sigma_{va}} + \left(1-\theta^{VA}_{nhw,r}\right)\left(\frac{(1+tk_{nhw,r})PK_r}{\overline{\overline{pk_{nhw,r}}}}\right)^{1-\sigma_{va}}\right]^{\frac{1}{1-\sigma_{va}}}$$

and

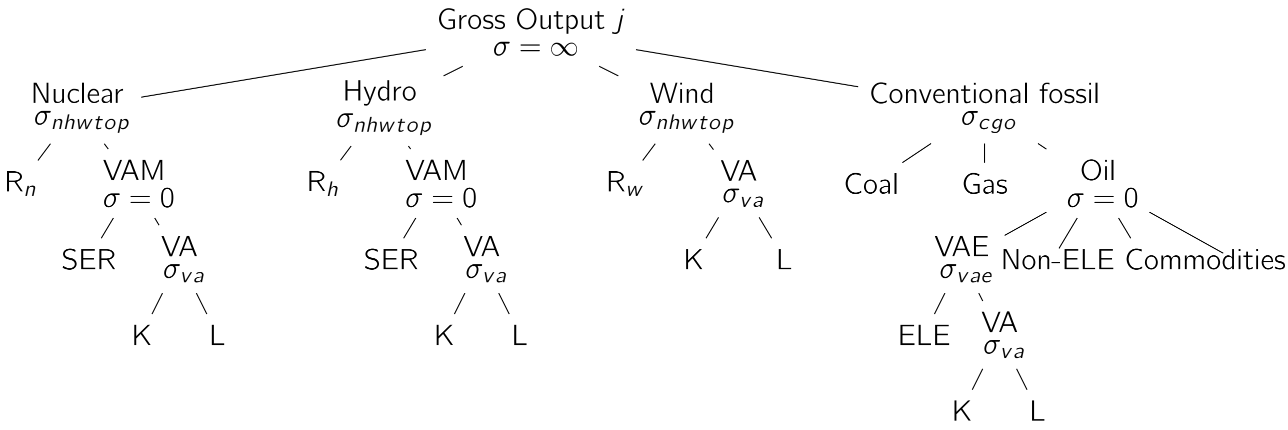
$$c^{CGO}_{r} := \left[\sum_{cgo\in CGO}\theta^{CGO}_{cgo,r}\left(c^{CGO}_{cgo,r}\right)^{1-\sigma_{cgo}}\right]^{\frac{1}{1-\sigma_{cgo}}}$$

$$c^{CGO}_{cgo,r} := \sum_{j\in J\setminus\{ELE\}}\theta_{j,cgo,r}\frac{PAT_{jr}}{\overline{\overline{pat_{jr}}}} + \left(1-\sum_{j\in J\setminus\{ELE\}}\theta_{j,cgo,r}\right)c^{VAE}_{cgo,r}$$

$$c^{VAE}_{cgo,r} := \left[(1-\theta^{VA}_{cgo,r})\left(\frac{PAT_{ELE,r}}{\overline{\overline{pat_{ELE,r}}}}\right)^{1-\sigma_{vae}} + \theta^{VA}_{cgo,r}\left(c^{VA}_{cgo,r}\right)^{1-\sigma_{vae}}\right]^{\frac{1}{1-\sigma_{vae}}}$$

$$c^{VA}_{cgo,r} := \left[\theta^{L}_{cgo,r}\left(\frac{(1+tl_{cgo,r})PL_r}{\overline{\overline{pl_{cgo,r}}}}\right)^{1-\sigma_{va}} + \left(1-\theta^{L}_{cgo,r}\right)\left(\frac{(1+tk_{cgo,r})PK_r}{\overline{\overline{pk_{cgo,r}}}}\right)^{1-\sigma_{va}}\right]^{\frac{1}{1-\sigma_{vae}}}$$

Figure D.7: Structure of production for $j \in \{\text{ELE}\}$



Unit cost functions for production activities $j \in \{\text{COL, CRU, GAS}\}$ are given as (Figure D.8):

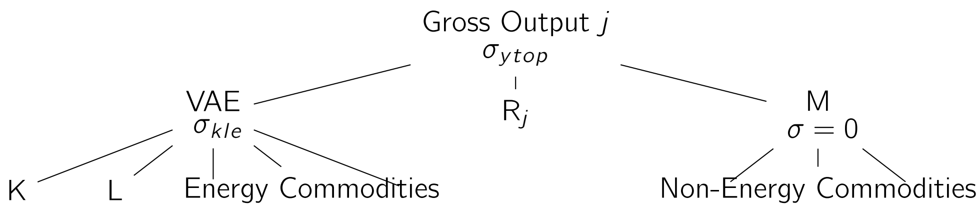
$$c_{jr} := \left[\theta_{jr}^{MAT}\left(c_{jr}^{MAT}\right)^{1-\sigma_{ytop}} + \theta_{jr}^{RES}\left(\frac{PS_{jr}(1+ts_{jr})}{\overline{ps_{jr}}}\right) + \left(1-\theta_{jr}^{MAT}-\theta_{jr}^{RES}\right)\left(c_{jr}^{VAE}\right)^{1-\sigma_{ytop}}\right]^{\frac{1}{1-\sigma_{ytop}}}$$

where

$$c_{jr}^{MAT} := \sum_{j' \in NENE} \theta_{j'jr}^{CMAT}\frac{PAT_{j'r}}{\overline{pat_{j'r}}}$$

$$c_{jr}^{VAE} := \left[\theta_{jr}^{LVAE}\left(\frac{(1+tl_{jr})PL_r}{\overline{pl_{jr}}}\right)^{1-\sigma_{kle}} + \theta_{jr}^{KVAE}\left(\frac{(1+tk_{jr})PK_r}{\overline{pk_{jr}}}\right)^{1-\sigma_{kle}} + \sum_{j' \in ene}\theta_{j'jr}^{EVAE}\left(\frac{(1+ti_{j'r})PAT_{j'r}}{\overline{pat_{j'r}}}\right)^{1-\sigma_{kle}}\right]^{\frac{1}{1-\sigma_{kle}}}$$

Figure D.8: Structure of production for $j \in \{\text{COAL, CRU, GAS}\}$



Transporting commodity j from region r to region r' requires services from the transportation sector. The import price for commodity j transported from region r to region r' therefore amounts to: $(1+te_{jr})\,P_{jr} + \phi_{jrr'}^{T}PT$, for routes involving international transport and $(1+te_{jr})\,P_{jr} + \phi_{jrr'}^{T}P_{TRN,r}$, for domestic routes, where te_{jr} is the export tax collected in region r and $\phi_{jrr'}^{T}$ stands for the amount of (international or domestic) transport commodity needed to transport commodity j. For simplicity, we abstract from import subsidies.

The unit cost function for the Armington commodity in region r is (Figures D.9 and D.10):

$$c_{jr}^{A} := \left[\theta_{jr}^{A}\left(c_{jr}^{D}\right)^{1-\sigma_{a,j}} + \left(1-\theta_{jp}^{A}\right)\left(c_{jp}^{M}\right)^{1-\sigma_{a,j}}\right]^{\frac{1}{1-\sigma_{a,j}}}$$

The unit cost function for the imported composite in China province p is:

$$c_{jp}^{M} := \left[\sum_{s}\theta_{jsp}^{M}\left(\left(1+tm_{jr}^{INT}\right)\frac{(1+te_{js})\,P_{js}+\phi_{jsp}^{T}PT}{\overline{pm_{jsp}}}\right)^{1-\sigma_{f,j}}\right]^{\frac{1}{1-\sigma_{f,j}}}$$

The unit cost function for the domestic composite in China province p is:

$$c_{jp}^{D} := \left[\theta_{jp}^{D}\left((1+td_{jr})\,\frac{P_{jp}}{\overline{pd}_{jp}}\right)^{1-\sigma_{c,j}} + \left(1-\theta_{jp}^{D}\right)\left(c_{jp}^{DC}\right)^{1-\sigma_{c,j}}\right]^{\frac{1}{1-\sigma_{c,j}}}$$

with

$$c_{jp}^{DC} := \left[\sum_{p'}\theta_{jp'p}^{DC}\left(\left(1+tm_{jp}^{CHN}\right)\frac{(1+te_{jp'})\,P_{jp'}+\phi_{jp'p}^{T}P_{TRN,p'}}{\overline{pm}_{jp'p}}\right)^{1-\sigma_{c,j}}\right]^{\frac{1}{1-\sigma_{c,j}}} .$$

The unit cost function for the imported composite for international region s is

$$c_{js}^{M} := \left[\theta_{js}^{M}\left(c_{js}^{MC}\right)^{1-\sigma_{c,j}} + \left(1-\theta_{js}^{M}\right)\left(c_{js}^{MNC}\right)^{1-\sigma_{c,j}}\right]^{\frac{1}{1-\sigma_{c,j}}}$$

with

$$c_{js}^{MC} := \left[\sum_{p}\theta_{jps}^{MC}\left(\left(1+tm_{js}^{CHN}\right)\frac{(1+te_{jp})\,P_{jp}+\phi_{jps}^{T}PT}{\overline{pm}_{jps}}\right)^{1-\sigma_{c,j}}\right]^{\frac{1}{1-\sigma_{c,j}}}$$

$$c_{js}^{MNC} := \left[\sum_{s'}\theta_{js's}^{MNC}\left(\left(1+tm_{js}^{INT}\right)\frac{(1+te_{js'})\,P_{js'}+\phi_{js's}^{T}PT}{\overline{pm}_{js's}}\right)^{1-\sigma_{o,j}}\right]^{\frac{1}{1-\sigma_{o,j}}} .$$

The unit cost function for the domestic composite for international region s is simply

$$c_{js}^{D} := (1+td_{jr})\,\frac{P_{js}}{\overline{pd}_{js}}$$

Figure D.9: Aggregation of local, domestic, and foreign varieties of good j for China province p

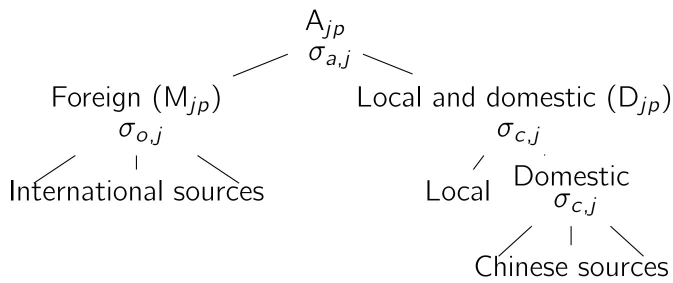
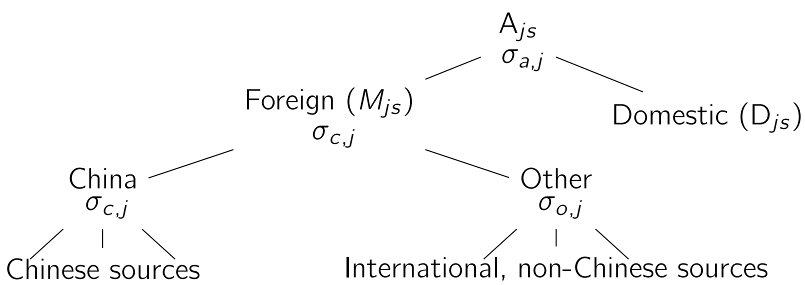


Figure D.10: Aggregation of domestic and foreign varieties of good j for international region s



International transport services are produced according to a Cobb-Douglas function of transport services from each region:

$$c^{T} := \prod_{r} P_{TRN,r}^{\theta_r^T},$$

where θ_r^T is the cost share of region r transport commodity in the international transport services composite.

The model's **zero-profit conditions** are then given by:

$$
\begin{array}{lll}
c^W_{\bar{h}r} \geq PW_{\bar{h}r} & \perp \quad W_{\bar{h}r} \geq 0 & \forall \bar{h}, r \\
c^C_{\bar{h}r} \geq P_{\bar{h}r} & \perp \quad Y_{\bar{h}r} \geq 0 & \forall \bar{h}, r \\
c^G_r \geq P_{GOV,r} & \perp \quad Y_{GOV,r} \geq 0 & \forall r \\
c^I_r \geq P_{INV,r} & \perp \quad Y_{INV,r} \geq 0 & \forall r \\
c_{jr} \geq (1 - to_{jr})P_{jr} & \perp \quad Y_{jr} \geq 0 & \forall j, r \\
c^A_{jr} \geq PA_{jr} & \perp \quad A_{jr} \geq 0 & \forall j, r \\
c^D_{jr} \geq PD_{jr} & \perp \quad D_{jr} \geq 0 & \forall j, r \\
c^M_{jr} \geq PM_{jr} & \perp \quad M_{jr} \geq 0 & \forall j, r \\
c^T \geq PT & \perp \quad T \geq 0 & \forall r
\end{array}
$$

By use of Shephard's lemma, **market clearing equations** are given by:

$$
\begin{array}{lll}
Y_{jp} \geq \sum_s \frac{\partial c^M_{js}}{\partial P_{jp}} M_{js} + \sum_{p'} \frac{\partial c^D_{jp'}}{\partial P_{jp}} D_{jp} + \frac{\partial c^T}{\partial P_{jp}} T & \perp \quad P_{jp} \geq 0 & \forall j, p \\
Y_{js} \geq \sum_p \frac{\partial c^M_{jp}}{\partial P_{js}} M_{jp} + \sum_{s' \neq s} \frac{\partial c^M_{js'}}{\partial P_{js}} M_{js'} + \frac{\partial c^D_{js}}{\partial P_{js}} D_{js} + \frac{\partial c^T}{\partial P_{js}} T & \perp \quad P_{js} \geq 0 & \forall j, s \\
Y_{INV,r} \geq \sum_{\bar{h}} \frac{\partial c^W_{\bar{h}r}}{\partial P_{INV,r}} W_{\bar{h}r} & \perp \quad P_{INV,r} \geq 0 & \forall r \\
Y_{\bar{h},r} \geq \sum_{\bar{h}} \frac{\partial c^W_{\bar{h}r}}{\partial P_{\bar{h},r}} W_{\bar{h}r} & \perp \quad P_{\bar{h},r} \geq 0 & \forall \bar{h}, r \\
D_{jr} \geq \frac{\partial c^A_{jr}}{\partial PD_{jr}} D_{jr} & \perp \quad PD_{jr} \geq 0 & \forall j, r \\
M_{jr} \geq \frac{\partial c^A_{jr}}{\partial PM_{jr}} M_{jr} & \perp \quad PM_{jr} \geq 0 & \forall j, r \\
A_{jr} \geq \sum_j^{\prime} \frac{\partial c_{j'r}}{\partial PA_{jr}} Y_{j'r} + \sum_{\bar{h}} \frac{\partial c^C_{\bar{h}r}}{\partial PA_{jr}} C_r + \frac{\partial c^G_r}{\partial PA_{jr}} G_r + \frac{\partial c^I_r}{\partial PA_{jr}} I_r & \perp \quad PA_{jr} \geq 0 & \forall j, r
\end{array}
$$

$$
\begin{array}{lll}
\overline{L}_r \geq \sum_j \frac{\partial c_{jr}}{\partial PL_r} Y_{ir} + \sum_{\bar{h}} \frac{\partial c^W_{\bar{h}r}}{\partial PL_r} W_{\bar{h}r} & \perp \quad PL_r \geq 0 & \forall r \\
\overline{K}_s \geq \sum_j \frac{\partial c_{js}}{\partial PK_s} Y_{js} & \perp \quad PK_s \geq 0 & \forall s \in INT \\
\overline{K}_p \geq \sum_{jp} \frac{\partial c_{jp}}{\partial PK^{CHN}} Y_{jp} & \perp \quad PK^{CHN} \geq 0 & \\
\sum_{\bar{h}} \overline{R}_{j\bar{h}r} \geq \frac{\partial c_{jr}}{\partial PS_{jr}} Y_{jr} & \perp \quad PS_{jr} \geq 0 & \forall j, r \\
\sum_{\bar{h}} \overline{R}_{nhw,\bar{h}r} \geq \frac{\partial c_{nhw,r}}{\partial PS_{nhw,r}} Y_{nhw,r} & \perp \quad PS_{nhw,r} \geq 0 & \forall nhw, r \\
T \geq \sum_{j'r} \frac{\partial c^A_{j'r}}{\partial PT} A_{j'r} & \perp \quad PT \geq 0 & \forall r
\end{array}
$$

$$
\begin{array}{lll}
W_{\bar{h}r} \geq \frac{INC^C_{\bar{h}r}}{P_{\bar{h}r}} & \perp \quad PW_{\bar{h}r} \geq 0 & \forall \bar{h}, r \\
Y_{GOV,r} \geq \frac{INC^G_r}{P_{GOV,r}} & \perp \quad P_{GOV,r} \geq 0 & \forall r \, ,
\end{array}
$$

where $\overline{L}_r \equiv \sum_{\bar{h}} \overline{L}_{\bar{h}r}$, $\overline{K}_r \equiv \sum_{\bar{h}} \overline{K}_{\bar{h}r}$, and $\overline{L}_{\bar{h}r}$ $\overline{K}_{\bar{h}r}$, $\overline{R}_{j\bar{h}r}$ and $\overline{R}_{nhw,\bar{h}r}$, are consumers' endowments of labor, capital, and natural resources, respectively.

Household income in the model amounts to factor income net of a lump sum payment to the local government and of payment for minimum consumption. Government income is the sum of all tax revenues:

$$
\begin{aligned}
INC^{C}_{\bar{h}r} :=& PL_r \overline{L}_{\bar{h}r} + PK_r \overline{K}_{\bar{h}r} + \sum_j PS_{jr} \overline{R}_{j\bar{h}r} + \sum_{nhw} PS_{nhw,r} \overline{R}_{nhw,\bar{h}r} - htax_{\bar{h}r} - \sum_j PAT_{jr} c^{\star}_{j\bar{h}r} \\
INC^{G}_{p} :=& \sum_j Y_{jp} \left[tl_p PL_p \frac{\partial c_{jp}}{\partial PL_p} + tk_p PK_p \frac{\partial c_{jp}}{\partial PK_p} + ts_p PS_{jp} \frac{\partial c_{jp}}{\partial PS_{jp}} + \sum_{nhw} tsn_{nhw,p} PS_{nhw,p} \frac{\partial c_{jp}}{\partial PS_{nhw,p}} \right] \\
&+ \sum_j to_{jp} P_{jp} Y_{jp} + \sum_j td_{jp} P_{jp} \frac{\partial c^{D}_{jp}}{\partial P_{jp}} D_{jp} \\
&+ \sum_{j,p' \neq p} tm^{CHN}_{jp} \left[(1 + te_{jp'}) P_{jp'} + \phi^{T}_{jp'p} P_{TRN,p} \right] \frac{\partial c^{D}_{jp}}{\partial P_{jp'}} D_{jp} + \sum_{js} tm^{INT}_{jp} \left[(1 + te_{js}) P_{js} \phi^{T}_{jsp} PT \right] \frac{\partial c^{M}_{jp}}{\partial P_{js}} M_{jp} \\
&+ \sum_{j,p' \neq p} te_{jp} P_{jp} \frac{\partial c^{D}_{jp'}}{\partial P_{jp}} D_{jp'} + \sum_{j,s} te_{jp} P_{jp} \frac{\partial c^{M}_{js}}{\partial P_{jp}} M_{js} \\
&+ \sum_{\bar{h}} htax_{\bar{h}p} \\
INC^{G}_{s} :=& \sum_j Y_{js} \left[tl_s PL_r \frac{\partial c_{js}}{\partial PL_s} + tk_s PK_s \frac{\partial c_{js}}{\partial PK_s} + ts_s PS_{js} \frac{\partial c_{js}}{\partial PS_{js}} + \sum_{nhw} tsn_{nhw,s} PS_{nhw,s} \frac{\partial c_{js}}{\partial PS_{nhw,s}} \right] \\
&+ \sum_j to_{js} P_{js} Y_{js} + \sum_j td_{js} P_{js} \frac{\partial c^{D}_{js}}{\partial P_{js}} D_{js} \\
&+ \sum_{jp} tm^{CHN}_{js} \left[(1 + te_{jp}) P_{jp} + \phi^{T}_{jps} PT \right] \frac{\partial c^{M}_{js}}{\partial P_{jp}} D_{js} + \sum_{j,s' \neq s} tm^{INT}_{js} \left[(1 + te_{js'}) P_{js'} \phi^{T}_{js's} PT \right] \frac{\partial c^{M}_{js}}{\partial P_{j,s'}} M_{js} \\
&+ \sum_{jp} te_{js} P_{js} \frac{\partial c^{M}_{jp}}{\partial P_{js}} M_{jp} + \sum_{j,s' \neq s} te_{js} P_{js} \frac{\partial c^{M}_{js'}}{\partial P_{js}} M_{js'} \\
&+ \sum_{\bar{h}} htax_{\bar{h}s} \, .
\end{aligned}
$$

Table D.1: Sets, price and quantity variables

Symbol	Description
Sets	
$g \in G$	Sectors (commodity production, private and public consumption, investment)
$j \in J \subset G$	Commodities
$s \in INT \subset R$	International regions
$p \in CHN \subset R$	Chinese provinces
$nhw \in NHW$	Nuclear, hydro and wind generated electricity
$cgo \in CGO$	Coal, Gas, Oil generated electricity
$ENE \subset J$	Energy commodities
$FOF \subset ENE$	Fossil fuel commodities
$NENE \subset J$	Non-energy commodities
Quantities and Prices	
Y_{gr}	Production index of sector g in region r
A_{jr}	Armington index of commodity j in region r
M_{jr}	Imports of commodity j in region r
YT	International transportation services
$W_{\bar{h}r}$	Welfare of consumer $\bar{h}$ in region r
P_{gr}	Domestic output price for sector g in region r
PA_{jr}	Armington price of commodity j in region r
PD_{jr}	Price of domestic composite of commodity j in region r
PM_{jr}	Price of imported composite of commodity j in region r
$PAT_{j,g,r}$	Tax and carbon price inclusive Armington price of j, input to sector g, in region r
PT	Price index of international transport services
$PW_{\bar{h}r}$	Welfare index for consumer $\bar{h}$ in region r
PK_r	Capital rental rate in region r
PL_r	Wage rate in region r
PS_{jr}	Price for sector-specific resource for sector j in region r
$PS_{nhw,r}$	Resource price for nhw generated electricity in region r
P_r^{CGO}	Price of conventionally generated electricity in region r
$P_{nhw,r}$	Price of electricity of type nhw in region r
$INC_{\bar{h}r}^C$	Private income of consumer $\bar{h}$ in region r
INC_r^G	Government income in region r

Table D.2: Elasticities of substitution

Symbol	Value	Description
σ_{ctop}	1.0	Top level private consumption (transport vs. non-transport)
σ_{cntrn}	0.25	Energy vs. non-transport consumption composite
σ_{cene}	0.4	Energy composite in consumption
σ_{ccon}	0.25	Non-transport composite in consumption
σ_{gitop}	1.0	Top level public consumption/investment (energy vs. non-energy)
σ_{giene}	1.0	Energy composite in public consumption/investment
σ_{ytop}	0.5	Top level (material vs. value added/energy inputs)
σ_{vae}	0.5	Value added vs. energy composite
σ_{kle}	1.0	Capital vs. labor and energy commodities
σ_{va}	1.0	Value added composite
σ_{ene}	0.5	Electricity vs. fuels composite
σ_{fof}	1.0	Fossil fuels in production
σ_{cgo}	5.0	Conventional fossil electricity production composite
σ_{nhwtop}	0.25	Top-level in NHW production (resource vs. value added and services composite)
$\sigma_{a,j}$	0.9 - 11.9	Foreign (M) vs. domestic (D) commodities
$\sigma_{o,j}$	0.9 - 11.9	Foreign, non-Chinese commodities
$\sigma_{c,j}$	1.8 - 30.9	Chinese commodities

Table D.3: Other model parameters

Symbol	Description
$htax_{\bar{h}r}$	Direct tax from household $\bar{h}$ to local government in region r
$\overline{pat_{jr}}$	Tax-inclusive reference Armington price of commodity j in region r
$\overline{pl_{jr}}$	Tax-inclusive reference price for labor in production of commodity j in region r
$\overline{pk_{jr}}$	Tax-inclusive reference price for capital in production of commodity j in region r
$\overline{ps_{jr}}$	Tax-inclusive reference price resources in production of commodity j in region r
$\overline{psnhw_{nhw,r}}$	Tax-inclusive reference price resources in production of *nhw* generated electricity in region r
$\overline{pm_{jrr'}}$	Tax-inclusive import price for commodity j shipped from region r to region r' in region r
tl_{jr}	Labor use tax in production of commodity j in region r
tk_{jr}	Capital use tax in production of commodity j in region r
to_{jr}	Output tax for commodity j in region r
ts_{jr}	Use tax for sector j specific resource in region r
$tsn_{nhw,r}$	Use tax for *nhw* generated electricity specific resource in region r
te_{jr}	Export tax for commodity j in region r
td_{jr}	Domestic tax rate for commodity j in region r
tm_{jr}^{CHN}	Tax rate for imports of commodity j from provinces in China to region r in region r
tm_{jr}^{INT}	Tax rate for imports of commodity j from international regions to region r in region r
$c_{j\bar{h}r}^{\star}$	Min. consumption level (Stone-Geary parameter) of commodity j for household $\bar{h}$ in region r
$\theta_{\bar{h}r}^{W,SAV}$	Share of savings in top-level utility of household $\bar{h}$ in region r
$\tilde{\theta}_{\bar{h}r}^{W,CON}$	Share of consumption (net of min. consumption) in top-level utility of household $\bar{h}$ in r
$\tilde{\theta}_{\bar{h}r}^{TRN}$	Exp. share (net of min. consumption) of transport in total exp. of household $\bar{h}$ in region r
$\tilde{\theta}_{\bar{h}r}^{CON}$	Exp. share (net of min. consumption) of energy commodities in non-TRN exp. in region r
$\tilde{\theta}_{j\bar{h}r}^{CENE}$	Exp. share (net of min. consumption) of commodities j in total energy exp. in region r
$\tilde{\theta}_{j\bar{h}r}^{CCON}$	Exp. share (net of min. consumption) of comm. j in total non-energy, non-TRN exp. in r
$\theta_{r}^{G/ICON}$	Exp. share of energy commodities in government/investment exp. in region r
$\theta_{jr}^{G/ICENE}$	Exp. share of commodities j in total energy exp. of government/investment sector in r
$\theta_{jr}^{G/ICCON}$	Exp. share of commodities j in total non-energy exp. of government/investment sector in r
$\theta_{j'jr}^{TOP}$	Share of non-energy commodity j' in top-level production of commodity j in region r
θ_{jr}^{RES}	Share of resources in top-level production of commodity j in region r
θ_{jr}^{VAE}	Share of value-added cost in value-added/energy cost bundle in production of comm. j in r
θ_{jr}^{VA}	Share of labor cost in value added cost bundle in production of commodity j in region r
θ_{jr}^{ENE}	Share of electricity in energy bundle in production of commodity j in region r
$\theta_{j'jr}^{FOF}$	Share of commodity j' cost in fossil fuel bundle in production of commodity j in region r
θ_{jr}^{A}	Share domestic (including local for China) commodity j in top-level Armington composite
θ_{jsp}^{M}	Share of j imports from s in total international imports of commodity j to province p
$\theta_{jp'p}^{D}$	Share of j imports from province p' in total domestic imports of commodity j to province p
θ_{jp}^{D}	Domestic share of j in total domestic and local composite of commodity j in province p

Notes: The production share parameters above are defined for commodities $j \in$ {EIS, MAN, WTR, CON, TRN, SER, AGR, OMN}. For the remaining commodities, share parameters are defined analogously, but are omitted here for lack of space. Following the same logic, we omit the shares in the Armington aggregation for international regions.

E Appendix for Essay 4

E.1 Derivations: optimal taxes and optimal transfers

Consider the maximization problem

$$max_{\{\tau_i, T_h\}} W \quad s.t. \quad \sum_h T_h \leq \sum_i \tau_i Z_i \quad \& \quad \sum_i Z_i \leq \tilde{Z}\,.$$

The Lagrangian is

$$\mathcal{L} = W + \mu\Big(\sum_i \tau_i Z_i - \sum_h T_h\Big) + \epsilon\Big(\tilde{Z} - \sum_i Z_i\Big).$$

The first order conditions are as follows:

$$\partial_{\tau_i}\mathcal{L} \leq 0 \quad \tau_i \geq 0 \quad and \quad \tau_i \partial_{\tau_i}\mathcal{L} = 0 \quad \forall i \tag{E.1}$$

$$\partial_{T_h}\mathcal{L} \leq 0 \quad T_h \geq 0 \quad and \quad T_h \partial_{T_h}\mathcal{L} = 0 \quad \forall h \tag{E.2}$$

$$\partial_{\mu}\mathcal{L} \geq 0 \quad \mu \geq 0 \quad and \quad \mu\partial_{\mu}\mathcal{L} = 0$$
$$\partial_{\epsilon}\mathcal{L} \geq 0 \quad \epsilon \geq 0 \quad and \quad \epsilon\partial_{\epsilon}\mathcal{L} = 0\,.$$

Start from the optimality conditions for taxes, i.e., Eq. (E.1). Assuming an interior optimum in the tax rates, by use of Roy's identity (i.e., $\partial_{p_i} V_h = -X_{ih}\lambda_h$), and since $\partial_{\tau_i} M_h = \partial_{\tau_i} F_h$, they are equivalent to the following:

$$MSD = -\frac{\sum_h \sum_n \beta_h X_{nh} \partial_{\tau_i} p_n}{\partial_{\tau_i} Z} + \frac{\sum_h \beta_h \partial_{\tau_i} F_h}{\partial_{\tau_i} Z} + \frac{\mu \partial_{\tau_i} \sum_j \tau_j Z_j}{\partial_{\tau_i} Z}\,.$$

Now consider the optimality condition for transfers, i.e., Eq. (E.2). Using the fact that $\partial_{T_{h'}} T_h = 1$ if $h = h'$ and $= 0$ if $h \neq h'$, as well as Roy's identity, in addition to $\sum_i \tau_i \partial_{T_h} Z_i \leq 1$ (which holds since the extra tax revenue given to household h results in a less than proportional increase in tax revenues), this is equivalent to

$$\mu \geq (1 - \sum_i \tau_i \partial_{T_h} Z_i)^{-1}(\beta_h - \sum_{n,h'} \beta_{h'} X_{nh'} \partial_{T_h} p_n + \sum_{h'} \beta_{h'} \partial_{T_h} F_{h'} - MSD\, \partial_{T_h} Z)\,.$$

E.2 Derivations: optimal taxes and fixed revenue redistribution scheme

For a given redistribution scheme with fixed shares ξ_h, consider the maximisation problem:

$$max_{\{\tau_i\}} W \quad s.t. \quad \sum_i Z_i \leq \tilde{Z}\,.$$

The Lagrangian is:

$$\mathcal{L} = W + \epsilon\Big(\tilde{Z} - \sum_i Z_i\Big).$$

The optimality condition for taxes (assuming an interior solution) can be rewritten as follows:

$$MSD = -\frac{\sum_h \sum_n \beta_h X_{nh} \partial_{\tau_i} p_n}{\partial_{\tau_i} Z} + \frac{\sum_h \beta_h \partial_{\tau_i} F_h}{\partial_{\tau_i} Z} + \frac{(\sum_h \beta_h \xi_h) \partial_{\tau_i} \sum_j \tau_j Z_j}{\partial_{\tau_i} Z} \tag{E.3}$$

E.3 Equivalence of optimality conditions for appropriate redistribution scheme

This section illustrates the equivalence of the optimality conditions for the case with optimal transfers, and the case with a given fixed redistribution scheme, when the fixed scheme is such that it allows for the solution with optimal transfers. Start by establishing the relationship between partial derivatives in the two approaches:

$$\begin{aligned}
\partial_{\tau_i} p_n(\tau) &= \partial_{\tau_i} p_n(\tau, T) + \sum_h \partial_{T_h} p_n(\tau, T) \partial_{\tau_i} T_h(\tau) \\
\partial_{\tau_i} F_h(\tau) &= \partial_{\tau_i} F_h(\tau, T) + \sum_{h'} \partial_{T'_h} F_h(\tau, T) \partial_{\tau_i} T_{h'}(\tau) \\
\partial_{\tau_i} Z(\tau) &= \partial_{\tau_i} Z(\tau, T) + \sum_h \partial_{T_h} Z(\tau, T) \partial_{\tau_i} T_h(\tau)
\end{aligned}$$

where $\partial_{\tau_i} T_h(\tau) = \xi_h \partial_{\tau_i} \sum_j \tau_j Z_j(\tau)$. Insert into Eq. (E.3):

$$\begin{aligned}
MSD = &\frac{1}{\partial_{\tau_i} Z + (\sum_h \xi_h \partial_{T_h} Z) \partial_{\tau_i} \sum_j \tau_j Z_j(\tau)} \Bigg\{ -\sum_h \beta_h \sum_n X_{nh} \partial_{\tau_i} p_n + \sum_h \beta_h \partial_{\tau_i} F_h \\
&+ \Bigg(-\sum_h \beta_h \sum_n X_{nh} \sum_{h'} \partial_{T_{h'}} p_n \xi_{h'} + \sum_h \beta_h \sum_{h'} \partial_{T_{h'}} F_h \xi_{h'} + \sum_h \beta_h \xi_h \Bigg) \partial_{\tau_i} \sum_j \tau_j Z_j(\tau) \Bigg\},
\end{aligned}$$

where quantities are functions of τ and T, unless otherwise specified,

Since $Z(\tau) = Z(\tau, T(\tau))$, $\partial_{\tau_i} \sum_j \tau_j Z_j(\tau) = \partial_{\tau_i} \sum_j \tau_j Z_j(\tau, T) + \sum_j \sum_h \tau_j \partial_{T_h} Z_j(\tau, T) \xi_h (\partial_{\tau_i} \sum_k \tau_k Z_k(\tau))$, it follows that $\partial_{\tau_i} \sum_j \tau_j Z_j(\tau) = \frac{\partial_{\tau_i} \sum_j \tau_j Z_j(\tau,T)}{1 - \sum_j \sum_h \tau_j \xi_h \partial_{T_h} Z_j(\tau,T)} \equiv \frac{\partial_{\tau_i} \sum_j \tau_j Z_j(\tau,T)}{1 - \sum_h \xi_h D_h}$. Therefore:

$$\begin{aligned}
MSD = &\frac{1 - \sum_h \xi_h D_h}{\partial_{\tau_i} Z(1 - \sum_h \xi_h D_h) + (\sum_h \xi_h \partial_{T_h} Z) \partial_{\tau_i} \sum_j \tau_j Z_j} \Bigg(-\sum_h \beta_h \sum_n X_{nh} \partial_{\tau_i} p_n + \sum_h \beta_h \partial_{\tau_i} F_h \Bigg) \\
&+ \frac{\partial_{\tau_i} \sum_j \tau_j Z_j}{\partial_{\tau_i} Z(1 - \sum_h \xi_h D_h) + (\sum_h \xi_h \partial_{T_h} Z) \partial_{\tau_i} \sum_j \tau_j Z_j} \Bigg(\sum_h \beta_h \xi_h - \sum_h \beta_h \sum_n X_{nh} \sum_{h'} \xi_{h'} \partial_{T_{h'}} p_n + \sum_h \beta_h \sum_{h'} \xi_{h'} \partial_{T_{h'}} F_h \Bigg)
\end{aligned} \quad \text{(E.4)}$$

We now show that, for the appropriate redistribution scheme, the above equation is equivalent to the optimality conditions for the fully optimal case from Eqs. (4.4) and (4.5). At the optimum, a subset of households indexed by k receives all the transfers, and share the same direct social marginal utility of income β. Thus chose the redistribution scheme such that $\xi_k \neq 0$ and $\beta_k = \beta \ \forall k$, and $\xi_h = 0$ for all other households. For this redistribution scheme, multiply Eq. (4.5) with equality by ξ_h and sum over h (this can be done since the inequalities are multiplied by zero on both sides when they don't hold with equality). This delivers:

$$\mu = \frac{1}{1 - \sum_h \xi_h D_h} \Bigg(\sum_h \xi_h \beta_h - \sum_{h'} \beta_{h'} \sum_n X_{nh'} \sum_h \xi_h \partial_{T_h} p_n + \sum_{h'} \beta_{h'} \sum_h \xi_h \partial_{T_h} F_{h'} - MSD \sum_h \xi_h \partial_{T_h} Z \Bigg)$$

Insert this into Eq. (4.4), and rearrange:

$$\begin{aligned}
MSD = &\frac{1 - \sum_h \xi_h D_h}{\partial_{\tau_i} Z(1 - \sum_h \xi_h D_h) + (\sum_h \xi_h \partial_{T_h} Z) \partial_{\tau_i} \sum_j \tau_j Z_j} \Bigg(-\sum_h \beta_h \sum_n X_{nh} \partial_{\tau_i} p_n + \sum_h \beta_h \partial_{\tau_i} F_h \Bigg) + \\
&\frac{\partial_{\tau_i} \sum_j \tau_j Z_j}{\partial_{\tau_i} Z(1 - \sum_h \xi_h D_h) + (\sum_h \xi_h \partial_{T_h} Z) \partial_{\tau_i} \sum_j \tau_j Z_j} \Bigg(\sum_h \beta_h \xi_h - \sum_{h'} \beta_{h'} \sum_n X_{nh'} \sum_h \xi_h \partial_{T_h} p_n + \sum_{h'} \beta_{h'} \sum_h \xi_h \partial_{T_h} F_{h'} \Bigg)
\end{aligned} \quad \text{(E.5)}$$

Eq. (E.5) is identical to Eq. (E.4). It thus follows that, for a redistribution scheme that allows for the optimum, the optimality conditions in the two approaches are identical.

E.4 Derivations: pollution-neutral tax swaps

E.4.1 Derivation of Equation (4.7)

A differential, pollution neutral (i.e., $dZ = 0$) change in social welfare can be written as follows, by means of Roy's identity:

$$dW = \sum_h \beta_h [-\sum_n X_{nh} dp_n + dM_h] \,.$$

Express differential quantities in terms of pollution tax rate changes and rearrange, using the fact that (due to pollution neutrality of the tax swap) $d\tau_i = -(\partial_{\tau_j} Z / \partial_{\tau_i} Z) d\tau_j$, thus obtaining:

$$dW = -\partial_{\tau_j} Z \Bigg(\frac{1}{\partial_{\tau_i} Z} \sum_h \beta_h [-\sum_n X_{nh} \partial_{\tau_i} p_n + \partial_{\tau_i} M_h] - \frac{1}{\partial_{\tau_j} Z} \sum_h \beta_h [-\sum_n X_{nh} \partial_{\tau_j} p_n + \partial_{\tau_j} M_h] \Bigg) d\tau_j \,.$$

This implies

$$dW = -\partial_{\tau_j} Z (MSC_i^{\xi} - MSC_j^{\xi}) d\tau_j \,.$$

E.4.2 Derivation of Equation (4.8)

Total differentiation of the labor delivers the following relationship:[13]

$$\hat{L}_X \frac{L_X}{\bar{L}} + \hat{L}_Y \frac{L_Y}{\bar{L}} = 0 \,, \tag{E.6}$$

where hats represent proportional changes, e.g., $\hat{L}_X \equiv \frac{dL_X}{L_X}$. Production levels adjust following changes in the wage rate and in the pollution tax rates, according to the following relationships:

$$\hat{Z}_i - \hat{L}_i = \sigma(\hat{w} - \hat{\tau}_i) \,, \tag{E.7}$$

where σ is the elasticity of substitution in production (which is assumed to be to be uniform across sectors at the equilibrium point, for reasons of tractability). Assuming perfect competition, the following furthermore holds (analogously to Fullerton and Heutel (2007)):

$$\hat{p}_X + \hat{X} = \theta_{XL}(\hat{w} + \hat{L}_X) + \theta_{XZ}(\hat{\tau}_X + \hat{Z}_X) \tag{E.8}$$

$$\hat{p}_Y + \hat{Y} = \theta_{YL}(\hat{w} + \hat{L}_Y) + \theta_{YZ}(\hat{\tau}_Y + \hat{Z}_Y) \tag{E.9}$$

$$\hat{X} = \theta_{XL} \hat{L}_X + \theta_{XZ} \hat{Z}_X \tag{E.10}$$

$$\hat{Y} = \theta_{YL} \hat{L}_Y + \theta_{YZ} \hat{Z}_Y \,, \tag{E.11}$$

where θ_{XL} represents the share of revenue from sector X paid to labor, and the other θ parameters are defined analogously. Household budget: $M_h = w\bar{L}_h + T_h$, where $T_h = \xi_h(\tau_X Z_X + \tau_Y Z_Y)$ is the amount of tax revenue redistributed to household h (where $\xi_A + \xi_B = 1$). Changes in household demand are therefore as follows:

$$\hat{X}_h - \hat{Y}_h = \hat{p}_Y - \hat{p}_X \tag{E.12}$$

$$\hat{X}_h = -\hat{p}_X + \hat{M}_h \,, \tag{E.13}$$

[13] It should be noted that the following linearized model equations follow closely Fullerton and Heutel (2007) and Rausch and Schwarz (2016).

where $\hat{M}_h = \hat{w}\frac{w\bar{L}_h}{M_h} + \frac{\xi_h}{M_h}\tau\Big(Z_X(\hat{\tau}_X+\hat{Z}_X)+Z_Y(\hat{\tau}_Y+\hat{Z}_Y)\Big)$. Finally, total differentiation of the market clearing conditions delivers the following:

$$\hat{X} = \frac{X_A}{X}\hat{X}_A + \frac{X_B}{X}\hat{X}_B \tag{E.14}$$

$$\hat{Y} = \frac{Y_A}{Y}\hat{Y}_A + \frac{Y_B}{Y}\hat{Y}_B\,. \tag{E.15}$$

The above Eqs. (E.6) to (E.15) represent 13 equations in 13 unknowns ($\hat{L}_X$, $\hat{L}_Y$, $\hat{Z}_X$, $\hat{Z}_Y$, $\hat{w}$, $\hat{p}_X$, $\hat{p}_Y$, $\hat{X}$, $\hat{Y}$, $\hat{X}_A$, $\hat{X}_B$, $\hat{Y}_A$, $\hat{Y}_B$) and two exogenous variables ($\hat{\tau}_X$ and $\hat{\tau}_Y$). One equation is redundant, and we choose the wage rate to be the numeraire (hence $\hat{w} = 0$), therefore delivering a square system of 12 equations that we can solve for 12 unknowns, in terms of two exogenous tax rate changes.

To solve the linearized model equations, start by subtracting Eqs. (E.10) and (E.11) from Eqs. (E.8) and (E.9) , thus obtaining:

$$\hat{p}_X = \theta_{XZ}\hat{\tau}_X \tag{E.16}$$

$$\hat{p}_Y = \theta_{YZ}\hat{\tau}_Y\,. \tag{E.17}$$

Insert Eq. (E.13) into Eq. (E.12), then insert the result into Eq. (E.15):

$$\hat{Y} = -\hat{p}_Y + \frac{Y_A}{Y}\hat{M}_A + \frac{Y_B}{Y}\hat{M}_B\,.$$

Insert the explicit expression for the change in the household budget, then insert Eq. (E.7) :

$$\hat{Y} = -\hat{p}_Y + (\frac{Y_A}{Y}\frac{\xi_A}{M_A} + \frac{Y_B}{Y}\frac{\xi_B}{M_B})\Big(\tau Z_X(\hat{L}_X + \hat{\tau}_X(1-\sigma)) + \tau Z_Y(\hat{L}_Y + \hat{\tau}_Y(1-\sigma))\Big). \tag{E.18}$$

Insert Eqs. (E.18) and (E.7) into Eq. (E.9):

$$(\frac{Y_A}{Y}\frac{\xi_A}{M_A} + \frac{Y_B}{Y}\frac{\xi_B}{M_B})\Big(\tau Z_X(\hat{L}_X + \hat{\tau}_X(1-\sigma)) + \tau Z_Y(\hat{L}_Y + \hat{\tau}_Y(1-\sigma))\Big) = \hat{L}_Y + \theta_{YZ}\hat{\tau}_Y(1-\sigma)\,.$$

Now insert Eq. (E.6) and solve for $\hat{L}_Y$:

$$\hat{L}_Y = \frac{\Phi\tau Z_X\hat{\tau}_X(1-\sigma) + (\Phi\tau Z_Y - \theta_{YZ})\hat{\tau}_Y(1-\sigma)}{1-\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})}\,,$$

with $\Phi := \frac{Y_A}{Y}\frac{\xi_A}{M_A} + \frac{Y_B}{Y}\frac{\xi_B}{M_B}$. Using Eq. (E.6), we then also find that

$$\hat{L}_X = -\frac{L_Y}{L_X}\frac{\Phi\tau Z_X\hat{\tau}_X(1-\sigma) + (\Phi\tau Z_Y - \theta_{YZ})\hat{\tau}_Y(1-\sigma)}{1-\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})}\,.$$

Eq. (E.7) then implies:

$$\hat{Z}_Y = \hat{\tau}_X\frac{\Phi\tau Z_X(1-\sigma)}{1-\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})} + \hat{\tau}_Y\Big(\frac{(\Phi\tau Z_Y - \theta_{YZ})(1-\sigma)}{1-\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})} - \sigma\Big). \tag{E.19}$$

and

$$\hat{Z}_X = \hat{\tau}_X\Big(\frac{L_Y}{L_X}\frac{\Phi\tau Z_X(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})-1} - \sigma\Big) + \hat{\tau}_Y\frac{L_Y}{L_X}\frac{(\Phi\tau Z_Y - \theta_{YZ})(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X\frac{L_Y}{L_X})-1}\,. \tag{E.20}$$

Now note that the expression for MSC^{ξ} for our model can be written as follows:

$$MSC_X^{\xi} = \frac{\sum_h \beta_h(X_h\partial_{\tau_X}p_X + Y_h\partial_{\tau_X}p_Y - \xi_h(Z_X + \tau_X\partial_{\tau_X}Z_X + \tau_Y\partial_{\tau_X}Z_Y))}{-\partial_{\tau_X}Z}\,, \tag{E.21}$$

and

$$MSC_Y^{\xi} = \frac{\sum_h \beta_h (X_h \partial_{\tau_Y} p_X + Y_h \partial_{\tau_Y} p_Y - \xi_h (Z_Y + \tau_X \partial_{\tau_Y} Z_X + \tau_Y \partial_{\tau_Y} Z_Y))}{-\partial_{\tau_Y} Z} . \tag{E.22}$$

Based on the solutions to our linearized model equations, we can now calculate the partial derivatives in Eqs. (E.21) and (E.22), as follows. Start by expressing Eqs. (E.16) and (E.17) in term of total differentials:

$$dp_X = \frac{Z_X}{X} d\tau_X , \qquad dp_Y = \frac{Z_Y}{Y} d\tau_Y .$$

Analogously for Eqs. (E.19) and (E.20):

$$dZ_X = d\tau_X \frac{Z_X}{\tau}\left(\frac{L_Y}{L_X}\frac{\Phi\tau Z_X(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X \frac{L_Y}{L_X}) - 1} - \sigma\right) + d\tau_Y \frac{Z_X}{\tau}\frac{L_Y}{L_X}\frac{(\Phi\tau Z_Y - \theta_{YZ})(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X \frac{L_Y}{L_X}) - 1} ,$$

and

$$dZ_Y = d\tau_X \frac{Z_Y}{\tau}\frac{\Phi\tau Z_X(1-\sigma)}{1 - \Phi(\tau Z_Y - \tau Z_X \frac{L_Y}{L_X})} + d\tau_Y \frac{Z_Y}{\tau}\left(\frac{(\Phi\tau Z_Y - \theta_{YZ})(1-\sigma)}{1 - \Phi(\tau Z_Y - \tau Z_X \frac{L_Y}{L_X})} - \sigma\right).$$

By comparing coefficients with the generic form of the total derivative of the quantities above, we therefore find that, at a market equilibrium with uniform pollution tax rates ($\tau_X = \tau_Y \equiv \tau$) the following holds:

$$\partial_{\tau_X} p_X = \frac{Z_X}{X} , \qquad \partial_{\tau_Y} p_X = 0 , \qquad \partial_{\tau_X} p_Y = 0 , \qquad \partial_{\tau_Y} p_Y = \frac{Z_Y}{Y} ,$$

and

$$\partial_{\tau_X} Z_X = \frac{Z_X}{\tau}\left(\frac{L_Y}{L_X}\frac{\Phi\tau Z_X(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X \frac{L_Y}{L_X}) - 1} - \sigma\right), \qquad \partial_{\tau_Y} Z_X = \frac{Z_X}{\tau}\frac{L_Y}{L_X}\frac{(\Phi\tau Z_Y - \theta_{YZ})(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X \frac{L_Y}{L_X}) - 1} ,$$

$$\partial_{\tau_X} Z_Y = \frac{Z_Y}{\tau}\frac{\Phi\tau Z_X(1-\sigma)}{1 - \Phi(\tau Z_Y - \tau Z_X \frac{L_Y}{L_X})} , \qquad \partial_{\tau_Y} Z_Y = \frac{Z_Y}{\tau}\left(\frac{(\Phi\tau Z_Y - \theta_{YZ})(1-\sigma)}{1 - \Phi(\tau Z_Y - \tau Z_X \frac{L_Y}{L_X})} - \sigma\right).$$

From the above expressions, we can calculate the partial derivatives of total pollution:

$$\partial_{\tau_X} Z = \partial_{\tau_X} Z_X + \partial_{\tau_X} Z_Y = (\frac{Z_X}{\tau}\frac{L_Y}{L_X} - \frac{Z_Y}{\tau})\frac{\Phi\tau Z_X(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X \frac{L_Y}{L_X}) - 1} - \frac{Z_X}{\tau}\sigma \tag{E.23}$$

and

$$\partial_{\tau_Y} Z = \partial_{\tau_Y} Z_X + \partial_{\tau_Y} Z_Y = (\frac{Z_X}{\tau}\frac{L_Y}{L_X} - \frac{Z_Y}{\tau})\frac{(\Phi\tau Z_Y - \theta_{YZ})(1-\sigma)}{\Phi(\tau Z_Y - \tau Z_X \frac{L_Y}{L_X}) - 1} - \frac{Z_Y}{\tau}\sigma \tag{E.24}$$

The expressions for MSC^{ξ} can therefore be expressed as follows:

$$MSC_X = \frac{\sum_h \beta_h \left(X_h \frac{Z_X}{X} - \xi_h (Z_X + \tau \partial_{\tau_X} Z)\right)}{-\partial_{\tau_X} Z} , \qquad MSC_Y = \frac{\sum_h \beta_h \left(Y_h \frac{Z_Y}{Y} - \xi_h (Z_Y + \tau \partial_{\tau_Y} Z)\right)}{-\partial_{\tau_Y} Z} ,$$

with $\partial_{\tau_X} Z$ and $\partial_{\tau_Y} Z$ as in Eqs. (E.23) and (E.24), respectively.

Now consider the following case: $MSC_X^{\xi} < MSC_Y^{\xi}$. Assuming $\partial_{\tau_X} Z < 0$ and $\partial_{\tau_Y} Z < 0$, this corresponds to the following:

$$\sum_h \beta_h \left(Z_X \partial_{\tau_Y} Z (\frac{X_h}{X} - \xi_h) - Z_Y \partial_{\tau_X} Z (\frac{Y_h}{Y} - \xi_h)\right) > 0 .$$

Insert Eqs. (E.23) and (E.24) and rearrange, using $\frac{X_h}{X} \equiv \frac{\alpha_h M_h}{\gamma M}$, $\frac{Y_h}{Y} \equiv \frac{(1-\alpha_h)M_h}{(1-\gamma)(M)}$ and $p_X X + p_Y Y = M$:

$$\Delta\Bigg\{\bigg(\frac{\sigma}{\tau L_Y(\delta_X - \delta_Y)} + \frac{\sigma - 1}{M} + \sum_h \frac{1-\alpha_h}{1-\gamma}\frac{\xi_h}{M}\bigg)\sum_h \beta_h M_h\bigg(\frac{\alpha_h}{\gamma} - 1\bigg) \\ +(1-\sigma)\sum_h \beta_h\bigg(\xi_h - \frac{M_h}{M}\bigg)\Bigg\} < 0\,.$$

E.4.3 Proof of Proposition 4.4

Pollution neutrality implies $d\tau_Y = -d\tau_X \partial_{\tau_X} Z/\partial_{\tau_Y} Z$. The requirement that $d\tau_Y < 0$ and $d\tau_X > 0$ thus implies that $\partial_{\tau_X} Z/\partial_{\tau_Y} Z > 0$. We start by showing that $\partial_{\tau_X} Z$ and $\partial_{\tau_Y} Z$ cannot simultaneously be positive. It then follows that both must be negative.

Start by noting that, for the redistribution scheme in Proposition 4.4, $1+\Phi\tau(Z_X\frac{L_Y}{L_X} - Z_Y) > 0$ and $\Phi\tau Z_Y - \theta_{YZ} < 0$. Assume on the one hand $\partial_{\tau_X} Z > 0$. This implies $(\frac{Z_X}{\tau}\frac{L_Y}{L_X} - \frac{Z_Y}{\tau})(\sigma - 1) > 0$. It then follows that $\partial_{\tau_Y} Z = (\frac{Z_X}{\tau}\frac{L_Y}{L_X} - \frac{Z_Y}{\tau})\frac{(\Phi\tau Z_Y - \theta_{YZ})(\sigma-1)}{1+\Phi(\tau Z_X \frac{L_Y}{L_X} - \tau Z_Y)} - \frac{Z_Y}{\tau}\sigma < 0$. Assume on the other hand $\partial_{\tau_Y} Z > 0$. This implies $(\frac{Z_X}{\tau}\frac{L_Y}{L_X} - \frac{Z_Y}{\tau})(\sigma - 1) < 0$, which then implies $\partial_{\tau_X} Z = (\frac{Z_X}{\tau}\frac{L_Y}{L_X} - \frac{Z_Y}{\tau})\frac{\Phi\tau Z_X(\sigma-1)}{1+\Phi(\tau Z_X \frac{L_Y}{L_X} - \tau Z_Y)} - \frac{Z_X}{\tau}\sigma < 0$.

Since $\partial_{\tau_X} Z$ and $\partial_{\tau_Y} Z$ are both negative, it follows that the tax swap is welfare-improving if and only if $MSC_X^{\xi} < MSC_Y^{\xi}$. For the assumptions in Proposition 4.4, this is equivalent to $\beta_A(\alpha_A - \gamma) < \beta_B(\alpha_A - \gamma)$ (since the case with Leontief production is excluded by requiring that $\partial_{\tau_X} Z$ and $\partial_{\tau_Y} Z$ are both negative, as can be seen from Eqs. (E.23) and (E.24); furthermore, $M_A(\alpha_A - \gamma) = M_B(\gamma - \alpha_B)$ and $1+\big(\frac{Y_A}{Y}\frac{\xi_A}{M_A} + \frac{Y_B}{Y}\frac{\xi_B}{M_B}\big)(\tau Z_X \frac{L_Y}{L_X} - \tau Z_Y) \equiv 1 + \frac{\tau Z_X}{M_A+M_B}\frac{L_Y}{L_X} - \frac{\tau Z_Y}{M_A+M_B} > 1 - \frac{\tau Z_Y}{M_A+M_B} > 0$). The case with $\alpha_A > \alpha_B$ (equivalent to $\alpha_A > \gamma$) and $\beta_A < \beta_B$ as well as the case $\alpha_B > \alpha_A$ (equivalent to $\alpha_A < \gamma$) and $\beta_B < \beta_A$ satisfy this relation, but no other combination of the relations between αs and βs does. Hence, Proposition 4.4 follows.

E.4.4 Proof of Proposition 4.5

Assume without loss of generality that the household with the higher β is A. Pollution neutrality implies $d\tau_Y = -d\tau_X \frac{\partial_{\tau_X} Z}{\partial_{\tau_Y} Z}$. Therefore the requirement that $d\tau_Y < 0$ and $d\tau_X > 0$ implies that $\frac{\partial_{\tau_X} Z}{\partial_{\tau_Y} Z} > 0$. Since $\partial_{\tau_X} Z$ and $\partial_{\tau_Y} Z$ cannot both be positive (analogously to the proof of Proposition 4.4), it follows that both are negative. The tax swap will therefore be welfare improving if and only if $MSC_X^{\xi} < MSC_Y^{\xi}$. For the above assumptions, this is equivalent to $(Z_X\frac{L_Y}{L_X} - Z_Y)(1-\sigma) < 0$ (since $1+\big(\frac{Y_A}{Y}\frac{\xi_A}{M_A} + \frac{Y_B}{Y}\frac{\xi_B}{M_B}\big)(\tau Z_X\frac{L_Y}{L_X} - \tau Z_Y) \equiv 1 + \frac{Y_A}{Y}\frac{1}{M_A}(\tau Z_X \frac{L_Y}{L_X} - \tau Z_Y) > 0$). Now note that the change in the total pollution tax revenue T can be expressed as follows: $dT = \frac{d\tau_X}{\partial_{\tau_Y} Z}(Z_X \partial_{\tau_Y} Z - Z_Y \partial_{\tau_X} Z)$. Insert Eqs. (E.23) and (E.24), and substitute $\Phi = (1-\gamma)/p_Y Y$, thus obtaining the following: $dT = (\frac{Z_X}{\tau}\frac{L_Y}{L_X} - \frac{Z_Y}{\tau})(1-\sigma)\frac{d\tau_X}{\partial_{\tau_Y} Z}\bigg(\frac{Z_X \theta_{YZ}}{1+(1-\gamma)\frac{\tau Z_X}{p_Y Y}\frac{L_Y}{L_X} - (1-\gamma)\theta_{YZ}}\bigg)$. It therefore follows that $(Z_X\frac{L_Y}{L_X} - Z_Y)(1-\sigma) < 0$ is equivalent to $dT > 0$. Hence, Proposition 4.5 follows.

E.5 Equilibrium conditions for numerical general equilibrium model

Tables E.1, E.2, and E.3 define the parameters and variables in the model.[14] Consider first the **unit cost functions** $c_i(\mathbf{p})$ and $c_h(\mathbf{p})$.

The unit cost function for utility of household h is defined as (see Figure E.1):[15]

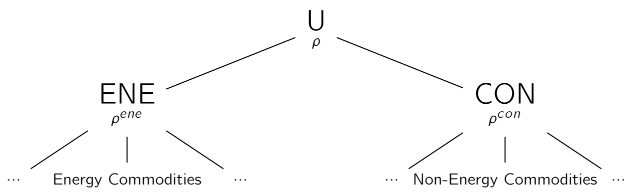
$$c_h := \left[\theta_h^U (c_h^{ENE})^{1-\rho_h} + \left(1-\theta_h^U\right)(c_h^{CON})^{1-\rho_h}\right]^{\frac{1}{1-\rho_h}}$$

where

$$c_h^{UENE} := \left[\sum_{i \in ene} \theta_{ih}^{ENE} PY_i^{1-\rho_h^{ene}}\right]^{\frac{1}{1-\rho_h^{ene}}}$$

$$c_h^{UCON} := \left[\sum_{i \in con} \theta_{ih}^{CON} PY_i^{1-\rho_h^{con}}\right]^{\frac{1}{1-\rho_h^{con}}} \quad ,$$

Figure E.1: Structure of private consumption



The unit cost for production activity i is defined as (see Figure E.2):

$$c_i := \left[\theta_i^Y \sum_{j \in mat} \theta_{ji}^M PYE_{ji}^{1-\sigma_i} + \left(1-\theta_i^Y\right)\left(c_{ir}^V\right)^{1-\sigma_i}\right]^{\frac{1}{1-\sigma_i}}$$

where

$$c_i^V := \left[\theta_i^V \left(c_i^{VA}\right)^{1-\kappa_i} + \left(1-\theta_i^V\right)\left(c_i^E\right)^{1-\kappa_i}\right]^{\frac{1}{1-\kappa_i}}$$

$$c_{ir}^{VA} := \left[\theta_i^K \; PK^{1-\lambda_i} + \left(1-\theta_i^K\right) PL^{1-\lambda_i}\right]^{\frac{1}{1-\lambda_i}}$$

$$c_i^E := \left[\theta_i^E PYE_{ELE,i}^{1-\nu_i} + \left(1-\theta_i^E\right)\left(c_i^R\right)^{1-\nu_i}\right]^{\frac{1}{1-\nu_i}}$$

$$c_i^R := \left[\sum_{j \in e} \theta_{ei}^R PYE_{ei}^{1-\mu_i}\right]^{\frac{1}{1-\mu_i}} \quad ,$$

The investment commodity is produced according to a Cobb-Douglas technology:[16]

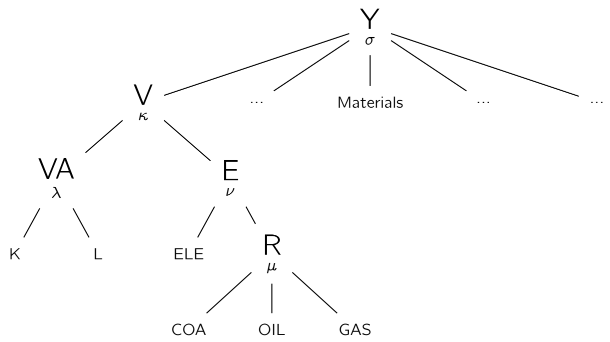
$$c^I := \prod_i PY_i^{\theta_i^I} \,.$$

[14]The exposition in this section is similar to other descriptions of models of this type such as, for example, in Caron and Rausch (2013), Abrell and Rausch (2016), and Abrell et al. (2016).

[15]As we abstract from pre-existing taxation already prevalent in the benchmark equilibrium, benchmark prices are normalized to one. θ generally refers to share parameters.

[16]Note that in the main text investment is not mentioned explicitly, as it is not central to our analysis. Households are assumed to consume a fixed amount of investment, which is imputed in proportion to benchmark expenditures.

Figure E.2: Structure of production



The model's **zero-profit conditions** are then given by:

$$
\begin{aligned}
c_h &\geq PU_h && \perp \quad U_h \geq 0 && \forall h \\
c_i &\geq PY_i && \perp \quad Y_i \geq 0 && \forall i \\
c^I &\geq PI && \perp \quad I \geq 0 &&
\end{aligned}
$$

Using Shepard's lemma, **market clearing equations** become:

$$
\begin{aligned}
Y_i &\geq \sum_j \frac{\partial c_j}{\partial PYE_{ij}} Y_j + \sum_h \frac{\partial c_h}{\partial PY_i} U_h + \frac{\partial c^I}{\partial PY_i} I && \perp \quad PY_i \geq 0 && \forall i \\
I &\geq \sum_h \bar{i}_h && \perp \quad PI \geq 0 && \\
\sum_h \omega_h^L &\geq \sum_i \frac{\partial c_i}{\partial PL} Y_i && \perp \quad PL \geq 0 && \\
\sum_h \omega_h^K &\geq \sum_i \frac{\partial c_i}{\partial PK} Y_i && \perp \quad PK \geq 0 && \\
U_h &\geq \frac{M_h}{PU_h} && \perp \quad PU_h \geq 0 && \forall h\,,
\end{aligned}
$$

where PYE_{ij} denotes the carbon tax inclusive price for commodity i employed in sector j. The carbon cost is added to PY_i in proportion to the good's carbon intensity and depending on the carbon tax rate in sector j.

Household income in defined factor income net of investment (savings), plus government transfers:

$$M_h := PL\omega_h^L + PK\omega_h^K - PI\bar{i}_h + T_h\,.$$

Constraints in the optimization problem (4.9) can be expressed as

$$
\begin{aligned}
\tilde{Z} &\geq \sum_{e,i} \phi_e \frac{\partial c_i}{\partial PYE_{ei}} Y_i \\
\sum_h T_h &= \sum_{e,i} \tau_i \phi_e \frac{\partial c_i}{\partial PYE_{ei}} Y_i
\end{aligned}
$$

Table E.1: Sets, prices, and quantity variables

Symbol	Description
Sets	
$h \in H$	Households
$i \in I$	Commodities
$con \subset I$	Non-energy consumption commodities
$ene \subset I$	Energy consumption commodities
$e \subset I$	Fossil fuel input commodities
$mat \subset I$	Material input commodities
Prices and quantities	
U_h	Private consumption index of household h
M_h	Private income of household h, net of investment
I	Investment consumption index
Y_i	Production index of sector i
PL	Wage rate
PU_h	Consumer price index for household h
PI	Investment consumption price index
PK	Capital rental rate
PY_i	Commodity i output price
T_h	Transfer to household h
τ_i	Carbon tax on sector i emissions

Table E.2: Elasticities of substitution

Parameter	Description	Value
Production		
σ_i	Top level (materials vs. energy/value-added composite)	0.0
κ_i	Value-added vs. energy composite	0.5
λ_i	Capital vs. labor	0.8
ν_i	Primary energy vs. electricity	1.2
μ_i	Fossil fuels	0.7
Consumption		
ρ_h	Top level (energy vs. non-energy)	0.0
ρ_h^{ene}	Energy commodities	0.7
ρ_h^{con}	Non-energy commodities	0.7

Table E.3: Other model parameters

Symbol	Description
$\bar{i}_h$	Reference investment (savings) level household of *h*
ω_h^L	Labor endowment of household *h*
ω_h^K	Capital endowment of household *h*
θ_h^U	Expenditure share on energy commodities in total expenditure of household *h*
θ_{ih}^{ENE}	Expenditure share on commodity *i* in total energy expenditure of household *h*
θ_{ih}^{CON}	Expenditure share of commodity *i* in total non-energy expenditure of household *h*
θ_i^Y	Share of material inputs in top-level production of commodity *i*
θ_{ji}^M	Share of material input *j* in total materials cost in production of commodity *i*
θ_i^V	Share of value-added cost in commodity *i* value-added/energy composite
θ_i^K	Share of capital cost in commodity *i* value added composite
θ_i^E	Share of electricity cost in commodity *i* energy composite
θ_{ei}^R	Share of fossil fuel *e* cost in commodity *i* fossil fuel composite
θ_i^I	Expenditure share on commodity *i* in investment consumption
ϕ_e	Carbon coefficient of fuel *e*
π_h	Share of household *h* in total population

E.6 GAMS code for CGE model

E.6.1 Main file

```
*#################################################################
*@                  DATA UPLOAD AND CALIBRATION
*#################################################################
*@@ ------------------------ DATA UPLOAD --------------------------
* Data upload
$batinclude dataload_hh "%datadir%%dataset%"

*@@ ------------------------ CALIBRATION -------------------------
* Calibration
$include calibration

*@@ --------------- FLAGS AND POLICY PARAMETERS ----------------
$include policies

*#################################################################
*@                        MODEL DEFINITION
*################################################################
$include model_MPS
$include model_MPEC

*#################################################################
*@                           SCENARIOS
*#################################################################
$include %scenariodir%%scenario%\%scenario%
```

E.6.2 Data upload

```
$ontext
File name: dataload_hh
$offtext

$if "a%1" == "a" $abort "##### dataload.gms: Argument missing: $batinclude dataload DATASET #####"
$set dataset %1
$if not exist %dataset%.gdx $abort "##### dataload.gms: Dataset %dataset%.gdx is missing #####"

*#########################################################################
*@                          SET PARAMETER DEFINITION
*#########################################################################
set
    g         global set of agent accounts
    i(g)      commodities
    f         factors
    h(g)      households
;
alias(i,j), (g,gg), (i, jj), (f,ff), (h,hh);

parameter
   c0(h)              benchmark total consumption [billion $]
   cd0(i,h)           benchmark household demand [billion $]
   pc0(i,h)           benchmark consumption price
   tc(i,h)            final consumption tax [%]
   fs0(f,h)           benchmark factor endowment [billion $]
   htax0(h)           benchmark lumpsum tax  [billion $]
   sav0(h)            benchmark savings by household [billion $]
   factsh(h)          factor income share
   taxsh0(h)          benchmark Share of (pollution) tax revenue returned to household h
   popsh(h)           population share of households of type h in total population

   g0                 benchmark total government consumption [billion $]
   gd0(i)             benchmark government demand [billion $]
   pg0(i)             benchmark government price
   tg(i)              public consumption tax [%]

   inv0               benchmark total investment [billion $]
   invd0(i)           benchmark invesment demand [billion $]
   pinv0(i)           benchmark investment imput price
   tinv(i)            investment input tax [%]

   y0(i)              benchmark total output [billion $]
   d0(i)              benchmark output to domestic market [billion $]
```

```
   e0(i)              benchmark output to exports [billion $]
   id0(i,j)           benchmark intermediate input [billion $]
   fd0(f,i)           benchmark factor demand [billion $]
   pi0(i,j)           benchmark intermediate input price
   pf0(f,i)           benchmark factor input prices
   ti(i,j)            intermediate input tax [%]
   tf(f,i)            factor use taxes [%]
   to(i)              output tax [%]

   a0(i)              benchmark Armington supply [billion $]
   m0(i)              benchmark imports [billion $ 2004]
   tm(i)              import tax [%]
   pm0(i)             benchmark import price
   bop0               benchmark balance of payment [billion $]

   co20(i,g)          benchmark carbon emissions [Gt]
   ef0(i,g)           benchmark energy flows [EJ]
;

$gdxin %dataset%
$load g i f h
$loaddc c0 cd0 pc0 tc fs0 htax0 sav0 factsh taxsh0 popsh
$loaddc g0 gd0 pg0 tg
$loaddc inv0 invd0 pinv0 tinv
$loaddc y0 d0 e0 id0 fd0 pi0 pf0 ti tf to
$loaddc a0 m0 tm pm0 bop0
$loaddc co20 ef0


*###########################################################################
*@                              BALANCE CHECKS
*###########################################################################
parameter
    rod             number of digits for rounding of check parameters /8/
    check_zpf(*)    check zero profit (should be zero)
    check_inc(*)    check income balances (should be zero)
    check_mkt(*)    check market clearing conditions (should be zero)
    check_inv       check I-S balance (should be zero)
;

check_zpf(i) = round( (1-to(i))*(d0(i) + e0(i))
                 - ( sum(j, pi0(j,i)*id0(j,i))
                     + sum(f, pf0(f,i)*fd0(f,i))
                   ), rod);

check_zpf("INV") = round( inv0 - sum(i, invd0(i)*pinv0(i)), rod);

check_inc(h) = round(sum(f, fs0(f,h))
                    - sum(i, pc0(i,h)*cd0(i,h))
                    - sav0(h) - htax0(h)
                    , rod);

check_inc("Gov") = round(
                    sum(i, y0(i)*to(i))
                    + sum((i,j), ti(i,j)*id0(i,j))
                    + sum((f,i), tf(f,i)*fd0(f,i))
                    + sum((i,h), tc(i,h)*cd0(i,h))
                    + sum(i, tg(i)*gd0(i))
                    + sum(i, tinv(i)*invd0(i))
                    + sum(i, tm(i)*m0(i))
                    + sum(h, htax0(h)) + bop0
                    - sum(i, pg0(i)*gd0(i))
                    , rod);

check_mkt(i) = round(a0(i) - sum(j, id0(i,j)) - sum(h, cd0(i,h)) - gd0(i) - invd0(i) ,rod);
check_inv = round( inv0 - sum(h, sav0(h)));

abort$( abs(sum(h, check_inc(h))) gt 1e-5
        or abs(check_inv)  gt 1e-5
        or abs(check_Inc("gov")) gt 1e-5
        or sum(i, abs(check_Zpf(i))) gt 1e-5
        or sum(i, abs(check_mkt(i))) gt 1e-5 ) "#### INITIAL DATASET UNBALANCED ####",
        check_Zpf, check_Mkt, check_inc, check_mkt, check_inv;
```

E.6.3 Calibration

```
$ontext
File name: calibration
$offtext
```

```
*###############################################################################
*@                               NESTING SETS
*###############################################################################
set
    ele(i)     electricity
    /ele/
    fof(i)     non electricty non crude oil energy commodites
    /gas, coa, p_c/
    mat(i)     all remaining commodities
    cene(i)    household energy commodits (incl. electricity)
    /ele, gas, coa, p_c/
    ccon(i)    household nen-energy consumption
    ee(i)      emission related energy commodities
    /gas, coa, p_c/
;

mat(i)$(not ele(i) and not fof(i)) = yes;
ccon(i)$(not cene(i)) = yes;

*###############################################################################
*@                               ELASTICITIES
*###############################################################################
parameter
   gammaY_out(i)     transformation elasticity production domest exports
   sigmaY_top(i)     substitution elasticity production top level
   sigmaY_vae(i)     substitution elasticity production value added-energy
   sigmaY_va(i)      substitution elasticity production value added
   sigmaY_ene(i)     substitution elasticity production electricity
   sigmaY_fof(i)     substitution elasticity production fossil fuels

   sigmaC_top(h)     substitution elasticity consumption top level
   sigmaC_con(h)     substitution elasticity consumption non-energy consumption
   sigmaC_ene(h)     substitution elasticity consumption energy consumption
   sigmaC_sav(h)     substitution elasticity savings and consumption

   sigmaT_dm(i)      substitution elasticity trade domesic import

   sigmaI            substitution elasticity investment
   sigmaG            substitution elasticity government

;

gammaY_out(i) = 2;
sigmaY_top(i) = 0;
sigmaY_vae(i) = 0.5;
sigmaY_va(i)  = 0.8;
sigmaY_ene(i) = 1.2;
sigmaY_fof(i) = 0.7;
sigmaC_top(h) = 0;
sigmaC_con(h) = 0.7;
sigmaC_ene(h) = 0.7;
sigmaC_sav(h) = 1;
sigmaT_dm(i)  = 3;
sigmaI = 1;
sigmaG = 0;

*###############################################################################
*@                               SHARE PARAMETERS
*###############################################################################
alias(i, jj);
parameter
*    production
    c0_Y(i)             benchmark unit cost sector i
    thetaOUT_D(i)       value share production domestic in output
    thetaOUT_E(i)       value share production export in output
    thetaTOP_VAE(i)     value share production value added energy in top level
    thetaTOP(j,i)       value share production commodities top level
    thetaVA(f,i)        value share production factors in values added
    thetaVAE_VA(i)      value share production value added in VAE nest
    thetaVAE_ENE(i)     value share production energy aggregate in VAE nest
    thetaENE_ELE(i)     value share production electricity in energy nest
    thetaENE_FOF(i)     value share production fossil fules in energy nest
    thetaFOF(j,i)       value share production primary fuel in fossil fuel nest

*    consumption
    thetaC_TOP_CON(h)   value share consumption non-energy consumption in top level
    thetaC_TOP_ENE(h)   value share consumption energy consumption in top level
    thetaC_CON(i,h)     value share consumption non-energy commodities
    thetaC_ENE(i,h)     value share consumption energy commodities
    thetaC_SAV(h)       value share consumption savings in income
```

```
*    government, investment, and trade
     thetaG(i)           value share public consumption
     thetaI(i)           value share investment
     thetaDM_D(i)        value share domestic in Armington aggregation
     thetaDM_M(i)        value share import in Armington aggregation
;

* production
c0_Y(i)$y0(i) = [ sum(j, pi0(j,i)*id0(j,i)) + sum(f, pf0(f,i)*fd0(f,i)) ]/y0(i);
thetaOUT_D(i)$y0(i) = d0(i)/y0(i);
thetaOUT_E(i)$y0(i) = e0(i)/y0(i);
thetaTOP_VAE(i)$[ sum(j, pi0(j,i)*id0(j,i)) + sum(f, pf0(f,i)*fd0(f,i)) ] =
  [ sum(j$(not mat(j)), pi0(j,i)*id0(j,i)) + sum(f, pf0(f,i)*fd0(f,i)) ]/[ sum(j, pi0(j,i)*id0(j,i)) + sum(f, pf0(f,i)*fd0(f,i)) ];
thetaTOP(j,i)$(mat(j) and [ sum(jj, pi0(jj,i)*id0(jj,i)) + sum(f, pf0(f,i)*fd0(f,i)) ]) = pi0(j,i)*id0(j,i)/
  [ sum(jj, pi0(jj,i)*id0(jj,i)) + sum(f, pf0(f,i)*fd0(f,i)) ];
thetaVAE_VA(i)$[ sum(j$(not mat(j)), pi0(j,i)*id0(j,i)) + sum(f, pf0(f,i)*fd0(f,i)) ] = [sum(f, pf0(f,i)*fd0(f,i)) ]/
  [ sum(j$(not mat(j)), pi0(j,i)*id0(j,i)) + sum(f, pf0(f,i)*fd0(f,i)) ];
thetaVAE_ENE(i)$[ sum(j$(not mat(j)), pi0(j,i)*id0(j,i)) + sum(f, pf0(f,i)*fd0(f,i)) ] =
  [ sum(j$(not mat(j)), pi0(j,i)*id0(j,i)) ]/ [ sum(j$(not mat(j)), pi0(j,i)*id0(j,i)) + sum(f, pf0(f,i)*fd0(f,i)) ];
thetaVA(f,i)$sum(ff, pf0(ff,i)*fd0(ff,i)) = pf0(f,i)*fd0(f,i) /sum(ff, pf0(ff,i)*fd0(ff,i));
thetaENE_ELE(i)$sum(j$(not mat(j)), pi0(j,i)*id0(j,i)) = sum(j$ele(j), pi0(j,i)*id0(j,i) )/ sum(j$(not mat(j)), pi0(j,i)*id0(j,i));
thetaENE_FOF(i)$sum(j$(not mat(j)), pi0(j,i)*id0(j,i)) = sum(fof, pi0(fof,i)*id0(fof,i) )/ sum(j$(not mat(j)), pi0(j,i)*id0(j,i));
thetaFOF(j,i)$(fof(j) and sum(fof, pi0(fof,i)*id0(fof,i))) = pi0(j,i)*id0(j,i)/sum(fof, pi0(fof,i)*id0(fof,i));

* consumption
thetaC_ENE(i,h)$cene(i) = pc0(i,h)*cd0(i,h)/sum(cene, pc0(cene,h)*cd0(cene,h));
thetaC_CON(i,h)$ccon(i) = pc0(i,h)*cd0(i,h)/sum(ccon, pc0(ccon,h)*cd0(ccon,h));
thetaC_TOP_CON(h)$sum(i, pc0(i,h)*cd0(i,h)) = sum(ccon, pc0(ccon,h)*cd0(ccon,h))/sum(i, pc0(i,h)*cd0(i,h));
thetaC_TOP_ENE(h)$sum(i, pc0(i,h)*cd0(i,h)) = sum(cene, pc0(cene,h)*cd0(cene,h))/sum(i, pc0(i,h)*cd0(i,h));
thetaC_SAV(h) = sav0(h)/(c0(h) + sav0(h));

* government, investment, and trade
thetaG(i)$sum(j, pg0(j)*gd0(j)) = pg0(i)*gd0(i)/sum(j, pg0(j)*gd0(j));
thetaI(i)$sum(j, pinv0(j)*invd0(j)) = pinv0(i)*invd0(i)/sum(j, pinv0(j)*invd0(j));
thetaDM_D(i)$(d0(i) + pm0(i)*m0(i)) = d0(i)/(d0(i) + pm0(i)*m0(i));
thetaDM_M(i)$(d0(i) + pm0(i)*m0(i)) = pm0(i)*m0(i)/(d0(i) + pm0(i)*m0(i));

*##############################################################################
*                                  CARBON
*##############################################################################
parameter
    co2coef(i,g)  carbon coefficient
    carblim0      benchmark carbon emissions [Gt]
    carblimS0(g)  benchmark emissions by sector [Gt]
;

co2coef(i,g) = 0;

co2coef(ee,j)$id0(ee,j)= co20(ee,j)/id0(ee,j);
co2coef(ee,h)$cd0(ee,h) = co20(ee,h)/cd0(ee,h);
co2coef(ee,"G")$gd0(ee) = co20(ee,"G")/gd0(ee);

carblim0 = sum((ee,g), co20(ee,g));
carblimS0(g) = sum(ee, co20(ee,g));
```

E.6.4 Flags and policy parameters

```
$ontext
File name: policies
$offtext

parameter
    ineq_aversion   coefficient of inequality aversion
    carblim         carbon limit (zero offsets restriction)
    carblimS(g)     carbon limit by agent (zero offsets restriction)
    taxsh(h)        Share of (pollution) tax revenue returned to household h
    revRec          flag for revenue recycling
*                   1     lumpsum recycling
*                   else: no recycling i.e. increase in government consumption
    pcarbDiff       flag for carbon tax differentiation
*                   0     uniform carbon tax
*                   1     differentiated by agent
*                   else: carbon price differentiated by agent and input commodity
    sav_clos        flag for savings closure
*                   0     constant savings
*                   1     Keynesian closure (constant fraction of income)
;

* Coefficient of inequality aversion
```

```
ineq_aversion = 0.1;

* initialize policies
carblim = 0;
carblimS(g) = 0;

* set revenue recycling scheme
revRec = 1;

* by default no tax differentiation
pcarbDiff = 0;

* by default constant savings
sav_clos = 0;

* by default set share of pollution tax to benchmark scheme
taxsh(h) = factsh(h);
```

E.6.5 MPSGE model

```
$ontext
File name: model_MPS
$offtext

$ontext
*###################################################################
*@                         ECONOMY DEFINITION
*###################################################################
$model:basic_MPS
$sectors:
    Y(i)$y0(i)                ! commodity production
    A(i)$a0(i)                ! armington commodities
    C(h)                      ! consumption
    GC$g0                     ! public consumption
    INV                       ! investment

$commodities:
    PD(i)$y0(i)               ! Price domestic products
    PFX$bop0                  ! Exchange rate
    PA(i)$a0(i)               ! Price Armington commodities
    PF(f)                     ! Price factors
    PC(h)                     ! Price consumer bundle
    PG                        ! Price government consumption bundle
    PINV                      ! Price investment bundle
    PCARB$carblim             ! Price carbon uniform
    PCARBS(g)$carblimS(g)     ! Price carbon by agent

$consumers:
    INC(h)                    ! Income representative agent
    GINC                      ! Income government

$auxiliary:
    LSMULT                    ! Lumpsum recycling multiplier

*###################################################################
*@                         FUNCTION DEFINITION
*###################################################################
*@@ ----------------------- PRODUCTION ---------------------------
$prod:Y(i)$y0(i)    s:sigmaY_top(i)         t:gammaY_out(i)
+                   vae:sigmaY_vae(i)
+                   va(vae):sigmaY_va(i)       ene(vae):sigmaY_ene(i)
+                   fof(ene):sigmaY_fof(i)
+                   ee.tl(fof):0
    O:PD(i)                                      Q:d0(i)
+                                                A:GINC          T:to(i)
    O:PFX$bop0                                   Q:e0(i)
+                                                A:GINC$to(i)    T:to(i)
    I:PA(j)$(not ee(j))                          Q:id0(j,i)      P:pi0(j,i)      ene:$ele(j)
+                                                A:GINC          T:ti(j,i)
    I:PA(ee)                                     Q:id0(ee,i)     P:pi0(ee,i)     ee.tl:
+                                                A:GINC$ti(ee,i) T:ti(ee,i)
    I:PCARB#(ee)$carblim                         Q:co20(ee,i)    P:1e-6          ee.tl:
    I:PCARBS(i)#(ee)$carblimS(i)                 Q:co20(ee,i)    P:1e-6          ee.tl:
    I:PF(f)                                      Q:fd0(f,i)      P:pf0(f,i)      va:
+                                                A:GINC$tf(f,i)  T:tf(f,i)

*@@ ------------------ ARMINGTON AGGREGATION ----------------------
$prod:A(i)$a0(i)     s:sigmaT_dm(i)
    O:PA(i)                           Q:a0(i)
```

```
    I:PD(i)                            Q:d0(i)
    I:PFX$bop0                         Q:m0(i)       P:pm0(i)
+                                      A:GINC$tm(i) T:tm(i)

*@@ ------------------------- INVESTMENT ------------------------------
$prod:INV      s:sigmaI
    O:PINV                             Q:inv0
    I:PA(i)                            Q:invd0(i)      P:pinv0(i)
+                                      A:GINC$tinv(i) T:tinv(i)

*@@ ------------------------- GOVERNMENT ------------------------------
$prod:GC$g0        s:sigmaG      ee.tl:0
    O:PG                               Q:g0
    I:PA(i)$(not ee(i))                Q:gd0(i)          P:pg0(i)
+                                      A:GINC$tg(i)      T:tg(i)
    I:PA(ee)                           Q:gd0(ee)         P:pg0(ee)       ee.tl:
+                                      A:GINC$tg(ee)     T:tg(ee)
    I:PCARB#(ee)$carblim               Q:co20(ee,"G")    P:1e-6          ee.tl:
    I:PCARBS("G")#(ee)$carblimS("G")   Q:co20(ee,"G")    P:1e-6          ee.tl:

$demand:GINC
    D:PG$g0                            Q:g0
    E:PF("LAB")                        Q:(sum(h, taxsh(h)))  R:LSMULT
*   trade balance is constant
    E:PFX$bop0                         Q:(bop0)
    E:PCARB$carblim                    Q:carblim
    E:PCARBS(g)$carblimS(g)            Q:carblimS(g)
*   create fake demand if not government consumption exist (taxes removed)
    D:PG$(not g0)                      Q:1
    E:PG$(not g0)                      Q:1

*@@ ------------------------- HOUSEHOLD ------------------------------
$prod:C(h)    s:sigmaC_top(h)
+             con:sigmaC_con(h)      ene:sigmaC_ene(h)
+             ee.tl(ene):0
    O:PC(h)                                          Q:c0(h)
    I:PA(i)$(not ee(i))                              Q:cd0(i,h)        P:pc0(i,h)       con:$ccon(i)
+                                                    A:GINC$tc(i,h)    T:tc(i,h)        ene:$cene(i)
    I:PA(ee)                                         Q:cd0(ee,h)       P:pc0(ee,h)      ee.tl:
+                                                    A:GINC$tc(ee,h)   T:tc(ee,h)
    I:PCARB#(ee)$carblim                             Q:co20(ee,h)      P:1e-6           ee.tl:
    I:PCARBS(h)#(ee)$carblimS(h)                     Q:co20(ee,h)      P:1e-6           ee.tl:

$demand:INC(h)        s:sigmaC_sav(h)
    D:PC(h)                            Q:c0(h)
    E:PF(f)                            Q:fs0(f,h)
    E:PF("LAB")                        Q:(-taxsh(h))    R:LSMULT
*   invesment demand is constant
    E:PINV$(sav_clos eq 0)             Q:(-sav0(h))
    D:PINV$(sav_clos eq 1)             Q:(sav0(h))

*@@ ------------------------- CONSTRAINTS ----------------------------
$constraint:LSMULT
*   revenue recycling with government consumption
    (GC - 1)$(revRec eq 1 and g0)
*   revenue recycling without government consumption
    + (     (PCARB*carblim)$carblim
         + sum(g$carblimS(g),PCARBS(g)*carblimS(g))
         + (PFX*bop0)$bop0
*         + sum(h, PC(h)*taxsh(h)*LSMULT)
         + sum(h, PF("LAB")*taxsh(h)*LSMULT)
    )$(revRec eq 1 and not g0)
*   No revenue recycling, i.e., hold transfer constant in absolute terms
    + (LSMULT - sum(h,htax0(h)))$(revRec ne 1)    =E=      0;

$offtext
$sysinclude mpsgeset basic_MPS

* set a flag that variables are already defined
$set VariablesDefined yes
```

E.6.6 MPEC model

```
$ontext
File name: model_MPEC
$offtext

*###########################################################################
*@                          DEFINITION VARIABLES AND EQUATIONS
```

```
*#############################################################################
* if MPSGE model is run before dont define variables again
$ifi %VariablesDefined%=="yes" $goto alreadyDefined
Positive Variable
    Y(i)        index production
    A(i)        index armington
    C(h)        index consumption
    GC          index government consumption
    INV         index investment

    PD(i)       price domestic products
    PFX         price exchange rate
    PA(i)       price armington composite
    PF(f)       price factors
    PC(h)       price consumption bundle
    PG          price public consumption bundle
    PINV        price invsetment bundle
    PCARB       price carbon emissions
    PCARBS(g)   price carbon emissions by agents

    INC(h)      income household
    GINC        income government
;

Variable
    LSMULT  lumsum recycling multiplier
;
$label alreadyDefined

Free Variable
    WELFARE     objective value
;

positive variable
    TCARB(i,g)  carbon tax by agent and input
    TRANS(h)    transfer share to household h
;

equation
*   MPEC upper level
    obj                 objective definition
    mkt_PCARB           market-clearing carbon
    res_uniform         restriction for uniform carbon price
    res_diffAgent       restriction for agent specific carbon price (but uniform across inputs)
    res_TRANS           restriction that transfer multipliers add to one

*   CGE lower level
    zpf_Y(i)            zero-profits production
    zpf_A(i)            zero-profits armington
    zpf_C(h)            zero-profits consumption
    zpf_GC              zero-profits public consumption
    zpf_INV             zero-profits investment

    mkt_PD(i)           market-clearing domestic products
    mkt_PFX             market-clearing foreign exchange
    mkt_PA(i)           market-clearing armington composite
    mkt_PF(f)           market-clearing factors
    mkt_PC(h)           market-clearing consumption
    mkt_PG              market-clearing government consumption
    mkt_PINV            market-clearing investment

    def_INC(h)          definition consumer income
    def_GINC            definition government income

    res_REVR            restricition revenue reycling
;

* Define index over households (to hold carbon price constant across households)
parameter index_hh(g);
index_hh(g)=0;
index_hh(h)=1;

*#############################################################################
*@                                  FUNCTIONAL FORMS
*#############################################################################
* aliases only used in macro definitions
alias(i,i__), (f,f__);

*@@ ------------------------------ PRODUCTION --------------------------------
* carbon inclusive price
$macro PACO2(j,i) (\
      (1+ti(j,i))*PA(j) + co2coef(j,i)*TCARB(j,i)$ee(j)\
```

```
)

* unit revenue function
$macro revenueY(i) (\
    ( thetaOUT_D(i)*PD(i)**(1+gammaY_out(i)) + (thetaOUT_E(i))*PFX**(1+gammaY_out(i)) )**(1/(1+gammaY_out(i))) \
)

* domestic supply function
$macro supD(i) (\
   Y(i)*d0(i)*(   revenueY(i) / PD(i) )**(-gammaY_out(i))\
)

* export function
$macro supE(i) (\
    Y(i)*e0(i)*( revenueY(i) / PFX )**(-gammaY_out(i))\
)

* top-level cost function
$macro costY(i) (\
      c0_Y(i) * \
        ( sum(i__$mat(i__), thetaTOP(i__,i)*( PACO2(i__,i)/ pi0(i__,i) )**(1-sigmaY_top(i))) \
          + thetaTOP_VAE(i) * CY_VAE(i)**(1-sigmaY_top(i))  \
        )**(1/(1-sigmaY_top(i))) \
)

* cost function value added energy aggregate
$macro CY_VAE(i) (\
     [ thetaVAE_VA(i)*CY_VA(i)**(1-sigmaY_vae(i)) + thetaVAE_ENE(i)*CY_ENE(i)**(1-sigmaY_vae(i))    \
     ]**(1/(1-sigmaY_vae(i)))\
)

* cost function value addded
$macro CY_VA(i) (\
   [ sum(f__, thetaVA(f__,i)*( (1+tf(f__,i))*PF(f__)/pf0(f__,i) )**(1-sigmaY_VA(i))) \
    ]**(1/(1-sigmaY_VA(i))) \
)

* cost function energy bundle
$macro CY_ENE(i) (\
   [ sum(i__$ele(i__), thetaENE_ELE(i)* ( PACO2(i__,i)/pi0(i__,i) )**(1-sigmaY_ene(i))) \
     + thetaENE_FOF(i)*CY_FOF(i)**(1-sigmaY_ene(i)) \
   ]**(1/(1-sigmaY_ene(i)))\
)

* cost function fossil fuels
$macro CY_FOF(i) (\
    sum(i__$fof(i__), thetaFOF(i__,i)*( PACO2(i__,i)/pi0(i__,i) )**(1-sigmaY_fof(i)) \
    )**(1/(1-sigmaY_fof(i)))\
)

* factor demand
$macro FD(f,i) (\
    Y(i)*fd0(f,i)\
          *( costY(i)/(c0_Y(i)*CY_VAE(i)) )**sigmaY_TOP(i) \
          *( CY_VAE(i)/CY_VA(i) )**sigmaY_VAE(i) \
          *( CY_VA(i)*pf0(f,i)/((1+tf(f,i))*PF(f)) )**sigmaY_VA(i) \
)

* intermediate demand
$macro ID(j,i) (\
    Y(i)*id0(j,i) \
    * { [ ( costY(i)*pi0(j,i)/(c0_Y(i)*PACO2(j,i)) )**sigmaY_TOP(i) ]$mat(j) \
       +[ ( costY(i)/(c0_Y(i)*CY_VAE(i)) )**sigmaY_TOP(i) \
        *( CY_VAE(i)/CY_ENE(i) )**sigmaY_VAE(i) \
        *( CY_ENE(i)*pi0(j,i)/(PACO2(j,i)) )**sigmaY_ENE(i) ]$ele(j) \
      +[ ( costY(i)/(c0_Y(i)*CY_VAE(i)) )**sigmaY_TOP(i) \
        *( CY_VAE(i)/CY_ENE(i) )**sigmaY_VAE(i) \
        *( CY_ENE(i)/CY_FOF(i) )**sigmaY_ENE(i)  \
        *( CY_FOF(i)*pi0(j,i)/(PACO2(j,i)) )**sigmaY_FOF(i) ]$fof(j) \
    }\
)

*@@ ----------------------------- ARMINGTON --------------------------------
* cost function armington composite
$macro costA(i) (\
    [ thetaDM_D(i)*PD(i)**(1-sigmaT_dm(i)) + thetaDM_M(i)*((1+tm(i))*PFX/pm0(i))**(1-sigmaT_dm(i))\
    ]**(1/(1-sigmaT_dm(i))) \
)

* Armington demand domestic commodities
$macro AD_D(i) (\
    A(i)*d0(i)*(costA(i)/PD(i))**sigmaT_dm(i) \
```

```
)

* Armington demand imports
$macro AD_M(i) (\
    A(i)*m0(i)*(costA(i)*pm0(i)/((1+tm(i))*PFX))**sigmaT_dm(i) \
)

*@@ ---------------------------------------- CONSUMPTION ----------------------------------------
* armington price incl carbon
$macro PACO2H(i,h) (\
    (1+tc(i,h))*PA(i) + TCARB(i,h)*co2coef(i,h)$ee(i)\
)

* expenditure function
$macro costC(h) (\
    ( thetaC_TOP_CON(h)*CC_CON(h)**(1-sigmaC_top(h)) \
      + thetaC_TOP_ENE(h)*CC_ENE(h)**(1-sigmaC_top(h)) \
    )**(1/(1-sigmaC_top(h)))\
)

* cost function non-energy consumption
$macro CC_CON(h) (\
     sum(i__$ccon(i__), thetaC_CON(i__,h)* ( PACO2H(i__,h)/pc0(i__,h) )**(1-sigmaC_con(h)) \
     )**(1/(1-sigmaC_con(h)))\
)

* cost function energy consumption
$macro CC_ENE(h)(\
     sum(i__$cene(i__), thetaC_ENE(i__,h)* ( PACO2H(i__,h)/pc0(i__,h) )**(1-sigmaC_ene(h)) \
     )**(1/(1-sigmaC_ene(h)))\
)

* final demand functions
$macro CD(i,h) ( \
     C(h)*cd0(i,h)\
    *{ [ ( costC(h)/CC_ENE(h))**sigmaC_top(h) \
         *( CC_ENE(h)*pc0(i,h)/(PACO2H(i,h)))**sigmaC_ene(h) ]$cene(i) \
      + [ ( costC(h)/CC_CON(h) )**sigmaC_top(h) \
         *( CC_CON(h)*pc0(i,h)/(PACO2H(i,h)))**sigmaC_con(h) ]$ccon(i) \
    }  \
)

*@@ ------------------------------INVESTMENT / GOVERNMENT ----------------------------------------
* armington price incl carbon
$macro PACO2G(i) (\
    (1+tg(i))*PA(i) + TCARB(i,"G")*co2coef(i,"G")$ee(i)\
)
* expenditure function government
$macro costG (\
    [sum( i__, thetaG(i__)*(PACO2G(i__)/pg0(i__))**(1-sigmaG)   )**(1/(1-sigmaG))]$(sigmaG <> 1) \
    + [prod(i__, (PACO2G(i__)/pg0(i__))**thetaG(i__) )]$(sigmaG eq 1) \
)

* government demand
$macro GD(i) (\
   GC*gd0(i)*( costG*pg0(i)/(PACO2G(i)))**sigmaG \
)

* cost function investment
$macro costI (\
    [sum( i__, thetaI(i__)*((1+tinv(i__))*PA(i__)/pinv0(i__))**(1-sigmaI)   )**(1/(1-sigmaI))]$(sigmaI <> 1) \
    + [prod(i__, ((1+tinv(i__))*PA(i__)/pinv0(i__))**thetaI(i__))]$(sigmaI eq 1) \
)

* demand function investment
$macro INVD(i) (\
   INV*invd0(i)*( costI*pinv0(i)/((1+tinv(i))*PA(i)) )**sigmaI \
)


*###############################################################################
*@                              EQUATION ASSIGNMENT
*###############################################################################

*---------------------------------------- MPEC UPPER LEVEL ----------------------------------------
* MPEC objective: Welfare maximization

obj..
WELFARE                          =E=      10*(1/(1 - ineq_aversion))*
                                          sum(h, popsh(h)*(C(h)*c0(h)*(1/popsh(h)))**(1 - ineq_aversion))
;
```

```
* market clearing carbon
mkt_PCARB..
    carblim                    =E=     sum((ee,i), co2coef(ee,i)*ID(ee,i))
                                       + sum((ee,h), co2coef(ee,h)*CD(ee,h))
                                       + sum(ee, co2coef(ee,"G")*GD(ee))
;

* restriction for uniform carbon price
* coal carbon price in electricity sector taken as normalization
res_uniform(i,g)$(pcarbDiff eq 0 and co2coef(i,g) gt 0 and y0(i))..
    sum((j,gg)$co2coef(j,gg), TCARB(j,gg))
                                =E=    TCARB(i,g)*sum((j,gg)$co2coef(j,gg), 1)
;

* restriction for agent specific carbon price (but uniform across inputs)
res_diffAgent(i,g)$(pcarbDiff eq 1 and co2coef(i,g) gt 0 and y0(i))..
    (sum(j$co2coef(j,g), TCARB(j,g)))$(index_hh(g)=0)
+       (sum((j,h)$co2coef(j,h), TCARB(j,h)))$(index_hh(g)=1)
                               =E=    (TCARB(i,g)*sum(j$co2coef(j,g), 1))$(index_hh(g)=0)
+                                     (TCARB(i,g)*sum((j,h)$co2coef(j,h), 1))$(index_hh(g)=1)
;

* restriction transfer multipliers add to one
res_TRANS..
    sum(h, TRANS(h))           =E=     1
;

*------------------------------ CGE LOWER LEVEL ------------------------------
* zero-profits production
zpf_Y(i)$y0(i)..
   costY(i)                    =G=      (1-to(i))*revenueY(i)
;

* zero-profits Armington composite
zpf_A(i)$a0(i)..
    costA(i)                   =G=      PA(i)
;

*zero-profits private consumption
zpf_C(h)..
    costC(h)                    =G=      PC(h)
;

* zero profits government consumption
zpf_GC$g0..
    costG                      =G=      PG
;

* zero profits investment
zpf_INV..
    costI                      =G=      PINV
;

* market clearing Armington composite
mkt_PA(i)$a0(i)..
    A(i)*a0(i)                 =G=      sum(h,CD(i,h)) + GD(i)
                                        + sum(j, ID(i,j)) + INVD(i)
;

* market clearing factors
mkt_PF(f)..
    sum(h, fs0(f,h))           =G=       sum(i, FD(f,i))
;

* market clearing domestic comoditites
mkt_PD(i)$y0(i)..
    supD(i)                    =G=       AD_D(i)
;

* market clearing consumer bundle
mkt_PC(h)..
    c0(h)*C(h)*PC(h)           =G=       INC(h)
                                         - thetaC_SAV(h)*INC(h)$(sav_clos eq 1)
;

* market clearing public consumption bundle
mkt_PG$g0..
    g0*GC*PG                   =G=       GINC
;

* market clearing investment
```

```
mkt_PINV..
    inv0*INV                   =G=      sum(h, sav0(h)$(sav_clos = 0)
                                              + thetaC_SAV(h)*INC(h)$(sav_clos eq 1))
;

* market clearing foreign exchange
mkt_PFX..
   sum(i$y0(i), supE(i))       =G=     sum(i$m0(i), AD_M(i))
                                       - bop0
;

* defintion consumer income
def_INC(h)..
    INC(h)                     =E=     sum(f, PF(f)*fs0(f,h))
*                                       - PC(h)*taxsh(h)*LSMULT
                                       - PF("LAB")*TRANS(h)*LSMULT
*                                      investment is constant
                                       - PINV*sav0(h)$(sav_clos eq 0)
;

* defintion government income
def_GINC..
    GINC                       =E=     sum(i,
                                          to(i)*(PD(i)*supD(i) + PFX*supE(i))
                                          + sum(h, tc(i,h)*PA(i)*CD(i,h))
                                          + tg(i)*PA(i)*GD(i)
                                          + tinv(i)*PA(i)*INVD(i)
                                          + tm(i)*PFX*AD_M(i)
                                          + sum(f, tf(f,i)*PF(f)*FD(f,i))
                                       )
                                       + sum((i,j), ti(j,i)*PA(j)*ID(j,i))
                                       + sum(h, PF("LAB")*TRANS(h)*LSMULT)
*                                      trade balance is constant
                                       + PFX*bop0
*                                      carbon income
                                       + sum((j,i)$co2coef(j,i), TCARB(j,i)*co2coef(j,i)*ID(j,i))
                                       + sum((i,h)$co2coef(i,h), TCARB(i,h)*co2coef(i,h)*CD(i,h))
                                       + sum(i$co2coef(i,"G"), TCARB(i,"G")*co2coef(i,"G")*GD(i))
;

* government expenditure are constant
res_REVR..
*   revenue recycling with government consumption
    (GC - 1)$(revRec eq 1 and g0)
*   revenue recycling without government consumption
    + (     sum((j,i)$co2coef(j,i), TCARB(j,i)*co2coef(j,i)*ID(j,i))
          + sum((i,h)$co2coef(i,h), TCARB(i,h)*co2coef(i,h)*CD(i,h))
          + sum(i$co2coef(i,"G"), TCARB(i,"G")*co2coef(i,"G")*GD(i))
          + (PFX*bop0)$bop0
*           + sum(h, PC(h)*taxsh(h)*LSMULT)
          + sum(h, PF("LAB")*TRANS(h)*LSMULT)
    )$(revRec eq 1 and not g0)
    + (LSMULT - 1)$(revRec ne 1)    =E=       0;
;

*##############################################################################
*                                MODEL ASSIGNMENT/ INTITAL VALUES
*##############################################################################
model basic_MPEC basic model MPEC formulation
      /
*       upper level: objective and carbon market clearing
        obj, mkt_PCARB, res_uniform, res_diffAgent, res_TRANS
*       lower level: CGE
        zpf_Y.Y, zpf_A.A, zpf_C.C, zpf_GC.GC, zpf_INV.INV
        mkt_PA.PA, mkt_PF.PF,  mkt_PD.PD,
        mkt_PINV.PINV, mkt_PFX.PFX, mkt_PC.PC, mkt_PG.PG,
        def_INC.INC, def_GINC.GINC
        res_REVR.LSMULT/
;
```

E.6.7 Initial values for MPEC model parameters

```
$ontext
File name: init_MPEC
$offtext

*##########################################################################
*                               RELEASE PREVIOUS BOUNDS
*##########################################################################
```

```
Y.LO(i) = 0;
A.LO(i) = 0;
C.LO(h) = 0;
GC.LO   = 0;
INV.LO  = 0;
PD.LO(i)= 0;
PFX.LO  = 0;
PA.LO(i)= 0;
PF.LO(f)= 0;
PC.LO(h)= 0;
PG.LO   = 0;
PINV.LO = 0;
LSMULT.LO = -inf;
TCARB.LO(i,g) = 0;
TRANS.LO(h) = 0;

Y.UP(i) = +inf;
A.UP(i) = +inf;
C.UP(h) = +inf;
GC.UP   = +inf;
INV.UP  = +inf;
PD.UP(i)= +inf;
PFX.UP  = +inf;
PA.UP(i)= +inf;
PF.UP(f)= +inf;
PC.UP(h)= +inf;
PG.UP   = +inf;
PINV.UP = +inf;
TCARB.UP(i,g) = +inf;
TRANS.UP(h) = 1;

*###############################################################################
*                              INITITAL VALUES
*###############################################################################
Y.L(i) = 1;
A.L(i) = 1;
C.L(h) = 1;
GC.L   = 1;
INV.L  = 1;
PD.L(i)= 1;
PFX.L  = 1;
PA.L(i)= 1;
PF.L(f)= 1;
PC.L(h)= 1;
PG.L   = 1;
PINV.L = 1;
TCARB.L(i,g) = 0;

GINC.L = g0;
INC.L(h) = c0(h);
LSMULT.L = sum(h,htax0(h));
TRANS.L(h) = taxsh0(h);

*###############################################################################
*                              FIX VARIABLES
*###############################################################################
* fix government sector if no expenditure exist
PG.FX$(not g0) = 0;
GC.FX$(not g0) = 0;
TRANS.FX(h) = taxsh0(h);

*###############################################################################
*                              NUMERAIRE
*###############################################################################
PINV.FX = 1;
```

E.6.8 Optimal redistribution scenario

```
$ontext
File name: Optimal
$offtext

$set ownpath %system.FP%

*###################################################
*@         SET POLICY VARIABLES AND SWITCHES
*###################################################
* 20% emission reduction
carblim = 0.8*carblim0;
```

```
display carblim;

* transfer scheme same as in benchmark
taxsh(h) = taxsh0(h);

* lumpsum revenue recycling
revRec = 1;

*#####################################################
*@                        RUN MODEL
*#####################################################
parameter
    report      reporting parameter carbon prices and overview
    emissions   reporting parameter emissions
;

*@@ ----------------- uniform carbon price ----------------
$include init_MPEC.gms
pcarbDiff = 0;

* activate transfer optimization
TRANS.LO(h) = 0;
TRANS.UP(h) = 1;

solve basic_MPEC using MPEC maximizing welfare;
$batinclude %ownpath%report uniform

* verification with MPSGE model
Pcarb.L = sum((i,g), TCARB.L(i,g))/sum((i,g)$co2coef(i,g), 1);
taxsh(h) = TRANS.L(h);

basic_MPS.iterlim = 0;
$include basic_MPS.gen
solve basic_MPS using MCP;
abort$(abs(basic_MPS.objval) gt 1e-4) "#### MPSGE MODEL DOES NOT VERIFY MPEC SOLUTION FOR UNIFORM TAX ####",
     basic_MPS.objval, report, emissions, carblim0, carblim, carblimS, carblimS0 ;

*@@ ------------- agent specific carbon price -----------
pcarbDiff = 1;

$set scen AgentSpecific_Benchmark
carblim = 0.8*carblim0;
pcarbDiff = 1;

$include init_MPEC.gms

* activate transfer optimization
TRANS.LO(h) = 0;
TRANS.UP(h) = 10;

solve basic_MPEC using MPEC maximizing welfare;
$batinclude %ownpath%report AgentSpecific_Benchmark

* Verification with MPSGE model
carblim = 0;
PCARB.L = 0;
$ondotl
carblimS(i) =  sum((j)$co2coef(j,i), co2coef(j,i)*ID(j,i));
carblimS(h) = sum(i$co2coef(i,h), co2coef(i,h)*CD(i,h));
carblimS("G") = sum(i$co2coef(i,"G"), co2coef(i,"G")*GD(i));
$offdotl
PcarbS.L(g) = 0;
PcarbS.L(g)$sum(i$co2coef(i,g), 1) = sum(i$co2coef(i,g), TCARB.L(i,g))/sum(i$co2coef(i,g), 1);
taxsh(h) = TRANS.L(h);

basic_MPS.iterlim = 0;
$include basic_MPS.gen
solve basic_MPS using MCP;
abort$(abs(basic_MPS.objval) gt 1e-4) "#### MPSGE MODEL DOES NOT VERIFY MPEC SOLUTION FOR DIFFERENTIATED TAX
(benchmark start case) ####",
     basic_MPS.objval, report, emissions, carblimS, carblimS0 ;
```

E.6.9 Flat recycling scenario

```
$ontext
File name: Per_Capita
$offtext

$set ownpath %system.FP%
```

```
*#####################################################
*@         SET POLICY VARIABLES AND SWITCHES
*#####################################################
* 20% emission reduction
carblim = 0.8*carblim0;

* lumpsum revenue recycling
revRec = 1;

* Per-capita revenue redistribution
taxsh(h)$(card(h)=10) = 0.1;

*#####################################################
*@                    RUN MODEL
*#####################################################
parameter
    report      reporting parameter carbon prices and overview
    emissions   reporting parameter emissions
;

*@@ --------------- uniform carbon price -------------
$include init_MPEC.gms

TRANS.FX(h) = taxsh(h);
TRANS.L(h) = taxsh(h);

pcarbDiff = 0;

solve basic_MPEC using MPEC maximizing welfare;
$batinclude %ownpath%report uniform

* verification with MPSGE model
Pcarb.L = sum((i,g), TCARB.L(i,g))/sum((i,g)$co2coef(i,g), 1);
basic_MPS.iterlim = 0;
$include basic_MPS.gen
solve basic_MPS using MCP;
abort$(abs(basic_MPS.objval) gt 1e-4) "#### MPSGE MODEL DOES NOT VERIFY MPEC SOLUTION FOR UNIFORM TAX ####",
     basic_MPS.objval, report, emissions, carblim0, carblim, carblimS, carblimS0 ;

*@@ ------------ agent specific carbon price ----------
pcarbDiff = 1;
solve basic_MPEC using MPEC maximizing welfare;
$batinclude %ownpath%report AgentSpecific

* verification with MPSGE model
* offset uniform carbon cosntraint in MPSGE mode
carblim = 0;
PCARB.L = 0;
$ondotl
carblimS(i) =  sum((j)$co2coef(j,i), co2coef(j,i)*ID(j,i));
carblimS(h) = sum(i$co2coef(i,h), co2coef(i,h)*CD(i,h));
carblimS("G") = sum(i$co2coef(i,"G"), co2coef(i,"G")*GD(i));
$offdotl
PcarbS.L(g)$sum(i$co2coef(i,g), 1) = sum(i$co2coef(i,g), TCARB.L(i,g))/sum(i$co2coef(i,g), 1);
basic_MPS.iterlim = 0;
$include basic_MPS.gen
solve basic_MPS using MCP;
abort$(abs(basic_MPS.objval) gt 1e-4) "#### MPSGE MODEL DOES NOT VERIFY MPEC SOLUTION FOR DIFFERENTIATED TAX
(non-benchmark start case) ####",
     basic_MPS.objval, report, emissions, carblimS, carblimS0 ;

*@@ ----------- agent specific carbon price ----------
pcarbDiff = 1;

$set scen AgentSpecific_Benchmark
carblim = 0.8*carblim0;
pcarbDiff = 1;

$include init_MPEC.gms

TRANS.FX(h) = taxsh(h);
TRANS.L(h) = taxsh(h);

solve basic_MPEC using MPEC maximizing welfare;
$batinclude %ownpath%report AgentSpecific_Benchmark

* Verification with MPSGE model
carblim = 0;
PCARB.L = 0;
$ondotl
carblimS(i) =  sum((j)$co2coef(j,i), co2coef(j,i)*ID(j,i));
```

```
carblimS(h) = sum(i$co2coef(i,h), co2coef(i,h)*CD(i,h));
carblimS("G") = sum(i$co2coef(i,"G"), co2coef(i,"G")*GD(i));
$offdotl
PcarbS.L(g) = 0;
PcarbS.L(g)$sum(i$co2coef(i,g), 1) = sum(i$co2coef(i,g), TCARB.L(i,g))/sum(i$co2coef(i,g), 1);
basic_MPS.iterlim = 0;
$include basic_MPS.gen
solve basic_MPS using MCP;
abort$(abs(basic_MPS.objval) gt 1e-4) "#### MPSGE MODEL DOES NOT VERIFY MPEC SOLUTION FOR DIFFERENTIATED TAX
(benchmark start case) ####",
      basic_MPS.objval, report, emissions, carblimS, carblimS0 ;
```

E.6.10 Consumption-based recycling scenario

```
$ontext
File name: Dirty_Consumption_Prop
$offtext

$set ownpath %system.FP%

*######################################################
*@          SET POLICY VARIABLES AND SWITCHES
*######################################################
* 20% emission reduction
carblim = 0.8*carblim0;

* lumpsum revenue recycling
revRec = 1;

* Share of embodied carbon emissions in consumption
taxsh('h1') = 0.0478;
taxsh('h2') = 0.0667;
taxsh('h3') = 0.0797;
taxsh('h4') = 0.0890;
taxsh('h5') = 0.0925;
taxsh('h6') = 0.0929;
taxsh('h7') = 0.1066;
taxsh('h8') = 0.1198;
taxsh('h9') = 0.1278;
taxsh('h10')= 0.1772;

*######################################################
*@                   RUN MODEL
*######################################################
parameter
    report      reporting parameter carbon prices and overview
    emissions   reporting parameter emissions
;


*@@ ---------------- uniform carbon price ---------------
$include init_MPEC.gms

TRANS.FX(h) = taxsh(h);
TRANS.L(h) = taxsh(h);


pcarbDiff = 0;

solve basic_MPEC using MPEC maximizing welfare;
$batinclude %ownpath%report uniform

* verification with MPSGE model
Pcarb.L = sum((i,g), TCARB.L(i,g))/sum((i,g)$co2coef(i,g), 1);
basic_MPS.iterlim = 0;
$include basic_MPS.gen
solve basic_MPS using MCP;
abort$(abs(basic_MPS.objval) gt 1e-4) "#### MPSGE MODEL DOES NOT VERIFY MPEC SOLUTION FOR UNIFORM TAX ####",
      basic_MPS.objval, report, emissions, carblim0, carblim, carblimS, carblimS0 ;


*@@ ------------- agent specific carbon price -----------
pcarbDiff = 1;
solve basic_MPEC using MPEC maximizing welfare;
$batinclude %ownpath%report AgentSpecific

* verification with MPSGE model
* offset uniform carbon cosntraint in MPSGE mode
carblim = 0;
```

```
PCARB.L = 0;
$ondotl
carblimS(i) =  sum((j)$co2coef(j,i), co2coef(j,i)*ID(j,i));
carblimS(h) = sum(i$co2coef(i,h), co2coef(i,h)*CD(i,h));
carblimS("G") = sum(i$co2coef(i,"G"), co2coef(i,"G")*GD(i));
$offdotl
PcarbS.L(g)$sum(i$co2coef(i,g), 1) = sum(i$co2coef(i,g), TCARB.L(i,g))/sum(i$co2coef(i,g), 1);
basic_MPS.iterlim = 0;
$include basic_MPS.gen
solve basic_MPS using MCP;
abort$(abs(basic_MPS.objval) gt 1e-4) "#### MPSGE MODEL DOES NOT VERIFY MPEC SOLUTION FOR DIFFERENTIATED TAX
(non-benchmark start case) ####",
     basic_MPS.objval, report, emissions, carblimS, carblimS0 ;


*@@ ------------ agent specific carbon price ----------
* starts from benchmark

pcarbDiff = 1;

$set scen AgentSpecific_Benchmark
carblim = 0.8*carblim0;
pcarbDiff = 1;

$include init_MPEC.gms

TRANS.FX(h) = taxsh(h);
TRANS.L(h) = taxsh(h);

solve basic_MPEC using MPEC maximizing welfare;
$batinclude %ownpath%report AgentSpecific_Benchmark

* Verification with MPSGE model
carblim = 0;
PCARB.L = 0;
$ondotl
carblimS(i) =  sum((j)$co2coef(j,i), co2coef(j,i)*ID(j,i));
carblimS(h) = sum(i$co2coef(i,h), co2coef(i,h)*CD(i,h));
carblimS("G") = sum(i$co2coef(i,"G"), co2coef(i,"G")*GD(i));
$offdotl
PcarbS.L(g) = 0;
PcarbS.L(g)$sum(i$co2coef(i,g), 1) = sum(i$co2coef(i,g), TCARB.L(i,g))/sum(i$co2coef(i,g), 1);
basic_MPS.iterlim = 0;
$include basic_MPS.gen
solve basic_MPS using MCP;
abort$(abs(basic_MPS.objval) gt 1e-4) "#### MPSGE MODEL DOES NOT VERIFY MPEC SOLUTION FOR DIFFERENTIATED TAX
(benchmark start case) ####",
     basic_MPS.objval, report, emissions, carblimS, carblimS0 ;
```

E.6.11 Income-based recycling scenario

```
$ontext
File name: Income_Prop
$offtext

$set ownpath  %system.FP%

*#####################################################
*@         SET POLICY VARIABLES AND SWITCHES
*#####################################################
* 20% emission reduction
carblim = 0.8*carblim0;

* lumpsum revenue recycling
revRec = 1;

*#####################################################
*@                    RUN MODEL
*#####################################################
parameter
    report     reporting parameter carbon prices and overview
    emissions  reporting parameter emissions
;

*@@ ---------------- uniform carbon price ---------------
$include init_MPEC.gms

TRANS.FX(h) = taxsh(h);
TRANS.L(h) = taxsh(h);
```

```
pcarbDiff = 0;

solve basic_MPEC using MPEC maximizing welfare;
$batinclude %ownpath%report uniform

* verification with MPSGE model
Pcarb.L = sum((i,g), TCARB.L(i,g))/sum((i,g)$co2coef(i,g), 1);
basic_MPS.iterlim = 0;
$include basic_MPS.gen
solve basic_MPS using MCP;
abort$(abs(basic_MPS.objval) gt 1e-4) "#### MPSGE MODEL DOES NOT VERIFY MPEC SOLUTION FOR UNIFORM TAX ####",
     basic_MPS.objval, report, emissions, carblim0, carblim, carblimS, carblimS0 ;

*@@ ------------ agent specific carbon price ----------
pcarbDiff = 1;
solve basic_MPEC using MPEC maximizing welfare;
$batinclude %ownpath%report AgentSpecific

* verification with MPSGE model
* offset uniform carbon cosntraint in MPSGE mode
carblim = 0;
PCARB.L = 0;
$ondotl
carblimS(i) =  sum((j)$co2coef(j,i), co2coef(j,i)*ID(j,i));
carblimS(h) = sum(i$co2coef(i,h), co2coef(i,h)*CD(i,h));
carblimS("G") = sum(i$co2coef(i,"G"), co2coef(i,"G")*GD(i));
$offdotl
PcarbS.L(g)$sum(i$co2coef(i,g), 1) = sum(i$co2coef(i,g), TCARB.L(i,g))/sum(i$co2coef(i,g), 1);
basic_MPS.iterlim = 0;
$include basic_MPS.gen
solve basic_MPS using MCP;
abort$(abs(basic_MPS.objval) gt 1e-4) "#### MPSGE MODEL DOES NOT VERIFY MPEC SOLUTION FOR DIFFERENTIATED TAX
(non-benchmark start case) ####",
     basic_MPS.objval, report, emissions, carblimS, carblimS0 ;

*@@ ------------ agent specific carbon price ----------
pcarbDiff = 1;

$set scen AgentSpecific_Benchmark
carblim = 0.8*carblim0;
pcarbDiff = 1;

$include init_MPEC.gms

TRANS.FX(h) = taxsh(h);
TRANS.L(h) = taxsh(h);


solve basic_MPEC using MPEC maximizing welfare;
$batinclude %ownpath%report AgentSpecific_Benchmark

* Verification with MPSGE model
carblim = 0;
PCARB.L = 0;
$ondotl
carblimS(i) =  sum((j)$co2coef(j,i), co2coef(j,i)*ID(j,i));
carblimS(h) = sum(i$co2coef(i,h), co2coef(i,h)*CD(i,h));
carblimS("G") = sum(i$co2coef(i,"G"), co2coef(i,"G")*GD(i));
$offdotl
PcarbS.L(g) = 0;
PcarbS.L(g)$sum(i$co2coef(i,g), 1) = sum(i$co2coef(i,g), TCARB.L(i,g))/sum(i$co2coef(i,g), 1);
basic_MPS.iterlim = 0;
$include basic_MPS.gen
solve basic_MPS using MCP;
abort$(abs(basic_MPS.objval) gt 1e-4) "#### MPSGE MODEL DOES NOT VERIFY MPEC SOLUTION FOR DIFFERENTIATED TAX
(benchmark start case) ####",
     basic_MPS.objval, report, emissions, carblimS, carblimS0 ;
```

Giacomo A. Schwarz

Education

ETH ZURICH — **ZURICH, CH**
PhD in Economics, Dept. of Management, Technology and Economics — 2013 - current

- **Massachusetts Institute of Technology (MIT)**, visiting student: China Energy and Climate Project — 2014 - 2015
- **London School of Economics (LSE)**, summer school in Beijing: 'China's Environmental Challenges' — 2013

UNIVERSITY OF CAMBRIDGE — **CAMBRIDGE, UK**
Master of Advanced Study in Mathematics, Dept. of Applied Mathematics and Theoretical Physics — 2011 - 2012

ETH ZURICH — **ZURICH, CH**
BSc in Physics, Dept. of Physics — 2008 - 2011

LICEO LUGANO 1 — **LUGANO, CH**
High School Diploma — 2003 - 2007

Experience

ETH ZURICH — **ZURICH, CH**
Research and Teaching Assistant, Centre for Energy Policy and Economics — 2012 - 2014 / 2014 - current

MCKINSEY & COMPANY — **ZURICH, CH**
Intern, Management Consulting — 2016

BSI BANK, GENERALI GROUP — **LUGANO, CH**
Intern, Asset Management — 2012

ETH ZURICH — **ZURICH, CH**
Teaching Assistant, Department of Mathematics — 2010

Publications

- 'Household heterogeneity, aggregation, and the distributional impacts of environmental taxes', *Journal of Public Economics*, Volume 138, June 2016, 43-57, together with Sebastian Rausch

Conference Presentations

- 22nd Annual Conference of the European Association of Environmental and Resource Economists, Zurich, 2016
- 21st Annual Conference of the European Association of Environmental and Resource Economists, Helsinki, 2015
- 18th Annual Conference on Global Economic Analysis, Global Trade Analysis Project, Melbourne, 2015
- 34th International Energy Workshop, Abu Dhabi, 2015
- 38th International Association for Energy Economics International Conference, Antalya, 2015
- 5th World Congress of Environmental and Resource Economists, Istanbul, 2014
- 33rd International Energy Workshop, Beijing, 2014
- Annual Congress of the Swiss Society of Economics and Statistics, Bern, 2014

Awards

- 'Ken Pearson Scholarship', Victoria University, Australia, 2015
- Student Paper Award, International Association for Energy Economics Conference, Turkey, 2015
- 'Doc.Mobility' Scholarship, Swiss National Science Foundation, 2014
- Prize scholarship, Churchill College, 2012
- Polya Prize, ETH Mathematics department, 2012

Languages

- Languages: Italian (native), English (native), German (fluent), French (conversational), Russian (beginner)