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Multi-horizon modeling in hydro power planning

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Abstract

In this paper, a novel modeling framework is proposed, the *multi-horizon modeling* approach. This approach allows a very detailed and transparent modeling of many problems in hydro power planning by simultaneously being computationally very efficient. The model is applied to a complex pumped storage hydro power plant in a liberalized market environment in order to give decision support for the self-scheduling of it. The modeling framework is compared to three alternative state-of-the-art modeling approaches. The results suggest that multi-horizon models are especially valuable for the modeling of hydro power plants with different types of reservoirs.

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Keywords: medium-term; hydro scheduling; multi-horizon; stochastic dual dynamic programming;

1. Introduction

Hydro power plants (HPPs) in the Alps, and in other regions with similar topology, have typically two different types of reservoirs. The first type is storage reservoirs, which store water inflows in order to be able to produce during dry seasons. The second type of basins is balancing reservoirs. In comparison with the storage reservoirs, they are very small. Their role is to balance out water flows, i.e. to provide a necessary amount of water for generation or pumping. Whereas the filling of storage reservoirs define the medium-term operation strategy of a HPP, the balancing reservoirs are very important to consider in the daily operation of it. In order to consider both types of reservoirs in a medium-term hydro power planning (MTHP) optimization, a short time step is often modeled. Alternatively, the balancing reservoirs can also be aggregated in some way in order to reduce the computational complexity of solving the problem.

The proposed modeling approach can be said to lie in between these two ideas, having both a short time step and some aggregation in the same model. The model is as such, that the operating policies are calculated for daily time stages (and not hourly) only for the storage reservoirs (and not for the balancing reservoirs), which is the aggregation part of the model. This multi-stage stochastic program is decomposed into *interstage* and *intrastage* subproblems. The daily interstage problem handles the water management of the storage reservoirs. The intrastage subproblems, on the other hand, manages the hourly operation of the plant. In this subproblem, the balancing reservoirs and the hourly electricity market are considered and therefore a short time step for the operation of the plant can be considered.

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The nested decision structure, see Fig. 1 for two examples of it, can be depicted as a *multi-horizon decision tree* [1]. Thus, the modeling based on such a structure, is proposed to be called *multi-horizon modeling*.

In this paper the multi-horizon modeling approach is compared with three traditional modeling solutions. The different models are discussed and applied to a MTHP problem of a complex hydro power plant in Switzerland. All of the models are solved by dualized *stochastic dual dynamic programming (SDDP)* in order to cope with the complex structure and non-concave value functions of the HPP.

The contribution of this paper is the evaluation of the multi-horizon modeling concept against traditional modeling approaches. The application of such models allows very detailed and computationally efficient formulations of many different hydro power planning problems. It outperforms traditional modeling techniques considerably. The reasons are manyfold, but the most important ones are that both dynamic programming and mathematical solvers are combined in an efficient way, and that physical differences of reservoirs are exploited which is, to the best of the authors' knowledge, an original idea.

In the literature about MTHP, most work apply traditional models, i.e. to either consider a short time step for all of the reservoirs or to aggregate the balancing reservoirs. The most important works, which use models similar in spirit to multi-horizon ones, are as follows: In [3], *inter-* and *intrastage* problems are termed explicitly and used in a decoupled way for a MTHP optimization. Similarly, as in [4,5], price duration curves are used to model the short-term bidding. Thus, smaller basins would have to be aggregated in such approaches. In [2], the idea of inter- and intrastage problems are applied in order to incorporate day-ahead and intra-day bidding in a MTHP optimization. As in [6], the bidding is modeled as specifying a supply curve with fixed price points. Similar to the previous mentioned works, smaller basins would have to be aggregated in such an approach or be considered as state variables, which is one of the variants evaluated in this paper here.

In our previous works [7–11], multi-horizon models are used for different applications but are not compared to traditional modeling techniques. Additionally, apart from [11], an algorithm based on stochastic dynamic programming (SDP) is used instead of SDDP.

Nomenclature

t	$\in \{0, ..., T\}$, time stage [day]
τ	$\in \{1, \ldots, 24\}$, intrastage time stage [h]
N_t	Number of data samples per time stage (representing interstage uncertainty)
$v^{stor}(t), v^{bal}(\tau)$	Filling of storage/balancing reservoirs [1000m^3]
$Q(t, v^{stor})$	Value function [€]
$u(\tau), p(\tau)$	Electricity generation in turbines / pumped electricity [MW]
$f_u(u), f_p(p)$	Function: used/produced energy to water flow [1000m^3]
$a(\tau), o(\tau)$	Charges from upstream reservoirs / overflow (spillage) [1000m^3]
$c(\tau)$	Electricity market price [€/MW]
$m(\tau)$	Position in energy market [MW]
s	$\in \mathcal{S}$, intrastage scenarios
$\mathcal{A}_\tau$	$\subseteq \mathcal{S}$, bundle of intrastage scenarios with the same decisions up to some stage τ
Λ_τ	Set of all bundles in a stage τ, which can also be subject to interstage uncertainty
$U(\mathcal{A}_\tau)$	Set of bundles, i.e. children of the bundle $\mathcal{A}_\tau$

2. Multi-horizon modeling approach

The idea of the proposed modeling approach is that storage and balancing reservoirs are considered both with their inherent dynamics. Further, a decomposition of the problem into inter- and intrastage subproblems makes an efficient implementation of the model possible. The proposed model can be described as follows:

1. Interstage problem (master problem):
   - is formulated dynamically with daily time stages and storage reservoirs as state variables.
   - is possibly subject to interstage uncertainty

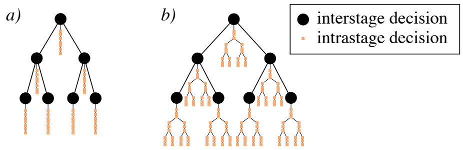

Fig. 1. Two examples of multi-horizon decision trees. *a*) The instrastage uncertain data is revealed at once at the beginning of the intrastage problem. Therefore, the intrastage problem is deterministic. *b*) The intrastage uncertain data in the intrastage problem is revealed every three hours for the next three hours, thus, it is a stochastic problem. Note that in both examples the interstage problem is subject to uncertainty.

- output are operation policies (water values) and profit-to-go function.

2. Intrastage problem:
   - has hourly time steps and state variables are the balancing reservoirs.
   - is formulated as a multistage stochastic program with possibly stochastic water inflows and market prices.
   - decision variables are the day-ahead market bidding and production operation.

It is clear that the overall problem is considered stochastically but note that the intrastage subproblems themselves are also stochastic multistage problems. The disclosure of uncertain realizations of inflows and prices can be modeled differently, which leads to the nested decision trees as depicted in Fig. 1.

The proposed model formulation mimics the way operators typically think about a MTHP problem, which is water release or filling of the storage reservoirs under unknown optimal short-term operation. Therefore, the policies suggested by such a model will support operators particularly.

Because of the decomposition of the problem, the water balance for the balancing reservoirs is not respected from one interstage to the other. That is, the fillings at the beginning and end of each intrastage subproblem are given and fixed beforehand. Thus, the model neglects that with balancing reservoirs water from one day to the next one could be stored. This simplification is meaningful if there is a strong daily autocorrelation in the operation pattern of the balancing reservoirs. In Switzerland this is the case because of the high influence of the demand on the electricity market prices and because of temperature driven water inflows.

The formulation is very flexible in how to model the disclosure of stochastic processes in the intrastage problem. For instance, the water inflow process can be modeled as being disclosed once every day and would, therefore, be known 24 hours ahead. A day-ahead market can be modeled similarly whereas a possible intra-day market is preferably subject to an intrastage uncertainty, e.g. a disclosure of the prices of and for every three hours as depicted in Fig. 1 *b*).

Algorithms based on SDP (as in [7–10]) or SDDP (as in [11]) can be applied for solving the overall problem. The multi-stage stochastic program in the intrastage problem, however, is formulated as its deterministic equivalent and a commercial solver is used to solve it. Therefore, the exploitation of the strengths of dynamic programming and efficient mathematical solvers is possible. That is, the curse of dimensionality in time is avoided by the decomposition in time. But, this is done only to daily time stages because the deterministic equivalent of a stochastic problem with hourly stages and a daily time horizon is manageable by mathematical solvers very efficiently. [12]

Note that the balancing reservoirs are not part of the state vector of the interstage problem. Therefore, the number of balancing reservoirs complicates only the intrastage problem, but, only moderately. More important is the modeling of the information disclosure in the intrastage scenario tree. The size of the variable vectors, and with that also the number of constraints in the intrastage problem, depend on the sum of the number of bundles (nodes in the tree) for each time step $\sum_\tau |\Lambda_\tau|$.

3. Mathematical model and solution methodologies

In order to formulate the mathematical model, the state variables, which are the fillings of the reservoirs, are separated into storage $v^{stor}(t)$ and balancing $v^{bal}(t)$ reservoir variables. Then, the recursive optimality equation is:

$$Q_t(v^{stor}_{t-1}) = \frac{1}{N_t} \sum_{j=1}^{N_t} \left(\max_{u,p,o,v,m} \sum_{\tau=1}^{24} \mathop{\mathbb{E}}_{\mathcal{A}_\tau \in \Lambda^j_\tau} [c(\tau)m(\tau)] + Q_{t+1}(v^{stor,j}_t) \right) \tag{1}$$

$$\text{s.t.:}\begin{cases} v^{bal}_{\mathcal{B}_\tau} = v^{bal}_{\mathcal{A}_{\tau-1}} - o_{\mathcal{B}_\tau} - f_u(u_{\mathcal{B}_\tau}) + f_p(p_{\mathcal{B}_\tau}) + a_{\mathcal{B}_\tau}\ ,\ \forall \mathcal{A}_{\tau-1} \in \Lambda^j_{\tau-1},\ \forall \mathcal{B}_\tau \in U(\mathcal{A}_{\tau-1}),\ \forall \tau & \text{(1a)} \\ v^{stor,j}_t = v^{stor,j}_{t-1} - \sum_{\tau=1}^{\mathcal{T}} \left[f_u(u^s_\tau) - f_p(p^s_\tau) - a^s_\tau \right] - o^j_t\ ,\ \forall s \in \mathcal{S}^j & \text{(1b)} \\ m_{\mathcal{A}_\tau} = u_{\mathcal{A}_\tau} - p_{\mathcal{A}_\tau}\ ,\ \forall \mathcal{A}_\tau \in \Lambda^j_\tau,\ \forall \tau & \text{(1c)} \\ 0 \le u, p, o, v^{stor}, v^{bal} \le ub_t\ ,\ lb_t \le m \le ub_t & \text{(1d)} \end{cases}$$

There are N_t number of possible intrastage subproblems. Note that even if $N_t = 1$, the intrastage subproblem can still involve multiple scenarios since it is based on a possibly stochastic scenario tree.
The optimal value of the intrastage subproblem, $\max \sum_{\tau=1}^{24} \mathbb{E}\left[c(\tau)m(\tau)\right]$, can be seen as an estimation of the short-term revenue. It is formulated with the help of scenario trees with the intrastage time stage $\tau \in \{1, \ldots, 24\}$, where the time duration is one hour. The short-term revenue, therefore, is the expected product of prices $c(\tau)$ and market positions $m(\tau)$ over all bundles $\mathcal{A}_\tau$. The market position m is defined in the constraints of the intrastage subproblem in (1c) as the difference of electricity generation and pumping. It is the amount of energy which is sold or bought back at the day-ahead spot market. This variable would not be needed for the considered application, but it makes the formulation more streamlined, especially for extensions of the model, like the consideration of multiple markets.
The intrastage subproblem is subject to the water balance constraints. For the balancing reservoirs, these constraints have to hold for each hour τ, which is specified in (1a). For the storage reservoirs, however, the water balance is maintained only for the interstage t in (1b). Therefore, the filling of the storage reservoirs are not known at an hourly resolution and only the sum over the hourly in- and outflows for each intrastage scenario s is used for calculating the reservoir filling $v^{stor,j}_t$ at the end of an interstage t. Note that this filling varies depending on the scenario j but has to stay the same for each intrastage scenario $s \in \mathcal{S}^j$, similar to a here-and-now decision. This also prevents having to calculate the profit-to-go Q_{t+1} multiple times.
Finally, all variables are positive except for the market position m. The market position can be bounded in order to introduce some crude form of risk control.

The problem (1) can be solved with SDP as done in [10]. In this paper, however, SDDP is used, which makes particularly sense if the model consists of more than a few storage reservoirs. Since some of the intrastage decisions are integer ones (e.g. due to forbidden zones for pumping and other operation rules), dualized SDDP is applied to solve the problem similarly as it was done in our previous work [11]. Further, although multi-horizon models would allow stochastic intrastage subproblems, in this paper, a relatively simple deterministic intrastage problem is considered, as depicted in Fig. 1 *a*). Note that stochasticity is still considered but only at the interstage level. This model is referred in the course of this paper as the *multi-horizon* model.

4. Evaluation against three traditional procedures

There are two obvious modeling alternatives when dealing with storage and balancing reservoirs. The first alternative is to model an hourly time step, which allows the consideration of a detailed HPP model. The second alternative is to aggregate balancing reservoirs in a way that a longer time step is sufficient and, thus, less computational effort is needed. Both alternatives are used in academia and practice. A third alternative builds up also on the decomposition of inter- and intrastage subproblems, however, it does not exploit the different types of reservoirs.
The alternative models are explained next and then compared with the multi-horizon model. The models are applied to a Swiss hydro power plant with 10 reservoirs depicted in Fig. 2. 4 reservoirs can be considered as storage reservoirs with capacities of 15 to 94 Mio m^3 and typically emptied once per year. The other 6 reservoirs have capacities of 0.02 to 0.08 Mio m^3. They can be emptied within a few hours and, therefore, are regarded as balancing reservoirs. Finally, there are aggregated units of 12 turbines and 8 pumps installed with a total capacity of 1.125 GW and 424 MW respectively.

4.1. 1. Alternative: MTHP models with hourly time steps

Although hourly time steps seem to be the natural choice for modeling a MTHP problem with balancing reservoirs, a closer look reveals many difficulties of such an approach. A model with hourly time steps, only storage reservoirs

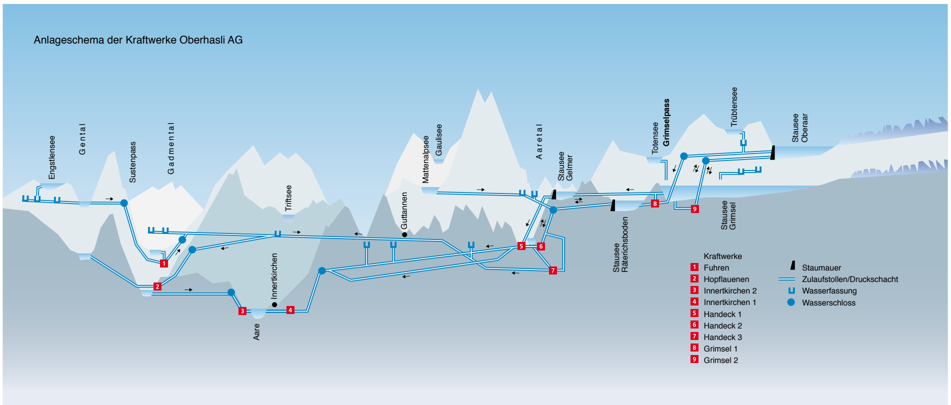

Fig. 2. Overview of the Kraftwerke Oberhasli AG (KWO) hydro power plant.[1]

as state variables, and without considering balancing reservoirs somehow is not reasonable. Because in such a case the HPP model would have to be aggregated, a detailed analysis of it does not make sense. Therefore, the only choice is to consider the balancing reservoirs as well. For a model with hourly consideration of both storage and balancing reservoirs the application of SDDP will be troublesome. The reason for this is that the fillings of balancing reservoirs are very volatile (in a MTHP perspective) and therefore difficult to predict. In such a setting, the convergence of SDDP will be poor and an application of SDP is more meaningful. As a conclusion, one can say that a model with hourly time steps only make sense if the balancing reservoirs are also considered as states and if the model is solved with SDP.

However, also SDP is troublesome to use in the setting considered in this paper. The more states, the more SDP suffers from the curse of dimensionality. Additionally, the discretization of the states has to be relatively fine in order to be able to estimate profit-to-go functions properly for hourly time steps. Such an approach would require vast computational resources for the here discussed power plant and, therefore, SDP-algorithms are not considered further.

A relative simple trick allows the usage of SDDP even for hourly time steps, which is a candidate model for the discussed HPP. If the fillings of very volatile reservoirs are neglected in the calculation of the value function, then, the convergence of SDDP is less of an issue. In principle, this means to consider only the storage reservoirs as state variables, whereas the balancing ones are considered but only with a constant filling. Since even hourly fillings of the storage reservoirs will not fluctuate much, they can be predicted well and SDDP can be applied.

This model will be referred in this paper as the *hourly model*. The recursive optimality equation of it looks as follows:

$$Q_t(v_{t-1}^{stor}) = \frac{1}{N_t} \sum_{j=1}^{N_t} \left(\max_{u,p,o,v,m} \; c_t^j m_t^j + Q_{t+1}(v_t^{stor,j}) \right) \tag{2}$$

$$\text{s.t.:} \begin{cases} v_t^{stor,bal,j} = v_{t-1}^{stor,bal,j} - o_t - f_u(u_t) + f_p(p_t) + a_t & \text{(2a)} \\ v_t^{bal,j} = v_{t-1}^{bal,j} = v_0^{bal} & \text{(2b)} \\ m_t = u_t - p_t & \text{(2c)} \\ 0 \le u, p, o, v^{stor,j}, v^{bal,j} \le ub_t \;,\; lb_t \le m \le ub_t & \text{(2d)} \end{cases}$$

In comparison to the problem (1), t has a time duration of one hour. Therefore, the subproblem for a scenario j is very small and consists of one decision per pump, turbine, etc. Further, the water balances of both balancing and storage reservoirs are assured in the same way in (2a). In the course of the algorithm, the fillings of the balancing reservoirs are not tracked, since they are not part of the state space. Therefore, their filling is kept constant in (2b) and is not important for the outcome of the optimization.

[1] Kraftwerke Oberhasli AG, "anlageschema-kwo.pdf", Retrieved from http://www.kwo.ch, November 2015.

The implementation of such an algorithm is quite similar to implementing a multi-horizon approach. The key difference is that there are more time steps but the subproblems are smaller.

Note that whereas the balancing reservoirs are considered in the optimization and no aggregation of turbines and pumps are necessary, the fillings of these reservoirs stay constant and no water can be stored. Therefore, short-term operation is modeled unrealistically. Further, this hourly stage-wise independent formulation allows only a poor representation of the MTHP problem. That is because in a recursive dynamic programming algorithm, there is only the knowledge of the expected future. When there are no price or water inflows states, then the autoregressive nature of such processes are not exploited. For instance, in such a model, relative high prices in the evening because of high demand can not be anticipated.

4.2. 2. Alternative: MTHP model with daily time steps

Another option to model the MTHP problem for a complex HPP is by performing a temporal aggregation of it to daily time steps. For a time horizon in MTHP problems of typically a few years, such an aggregation is appropriate. However, an aggregated time step leads also to the need of aggregating the market and the HPP model. Aggregating an hourly electricity market to a daily one or even further can be meaningful. Possibilities of how to do that in this context are also shown in [10]. In contrast to the market model, the implications on the HPP model can be more awkward. A daily view on balancing reservoirs is not reasonable. Such reservoirs are then often aggregated, which mostly also implies an aggregation of turbines and pumps which is troublesome, especially for the here considered complex power plant.

Out of these reasons, a similar trick as the one applied to the hourly model can be chosen: The balancing reservoirs are considered, but they are not modeled as state variables. The filling of these balancing reservoirs stays therefore the same, but no aggregation of turbines and pumps is necessary. Further, since a daily time step is modeled, only the daily average values of production and market decisions are considered. This model will be referred in the paper as the *daily model*.

With these modifications, the mathematical model is the same as (2) with the time step t modeling a duration of one day. Whereas previously, the constant filling of the balancing reservoirs led to model inaccuracies, this issue is here less prominent since the detailed operation is not considered explicitly. On the other hand, the performance of the HPP may be under- or overestimated considerably with a daily view.

4.3. 3. Alternative: MTHP model with daily time steps and intrastage subproblems

The decomposition of a problem into inter- and intrastage problems allows a third alternative modeling approach. The idea is to consider all reservoirs as states in a daily interstage problem as also done in [2]. This model is similar to the proposed multi-horizon model in equation (1), however, the physical differences of the reservoirs are not exploited. This model will be referred in this paper as the *daily intrastage* model. Its mathematical model can be formulated as follows:

$$Q_t(v_{t-1}) = \frac{1}{N_t} \sum_{j=1}^{N_t} \left(\max_{u,p,o,v,m} \sum_{\tau=1}^{24} \mathop{\mathbb{E}}_{\mathcal{A}_\tau \in \Lambda_\tau^j} \left[c(\tau) m(\tau) \right] + Q_{t+1}(v_t^j) \right) \tag{3}$$

$$\text{s.t.:} \begin{cases} v_\tau^j = v_{\tau-1}^j - o_\tau^j - f_u(u_\tau^j) + f_p(p_\tau^j) + a_\tau^j \ , \ \forall \tau & \text{(3a)} \\ v_{\tau=0}^j = v_{t-1} & \text{(3b)} \\ v_t^j = v_{\tau=24}^j & \text{(3c)} \\ m_\tau = u_\tau - p_\tau \ , \ \forall \tau & \text{(3d)} \\ 0 \le u, p, o, v \le ub_t \ , \ lb_t \le m \le ub_t & \text{(3e)} \end{cases}$$

In comparison to the formulation (1), the model does not differentiate between storage and balancing reservoirs. Therefore, the value of the fillings in the balancing reservoirs are considered in the calculation of the value function $Q(t, v)$. On the other hand, the water balance of the storage reservoirs are formulated in hourly resolution in (3a). As in (1), the value function is evaluated only in daily resolution. The linkage between the inter- and intrastage problem

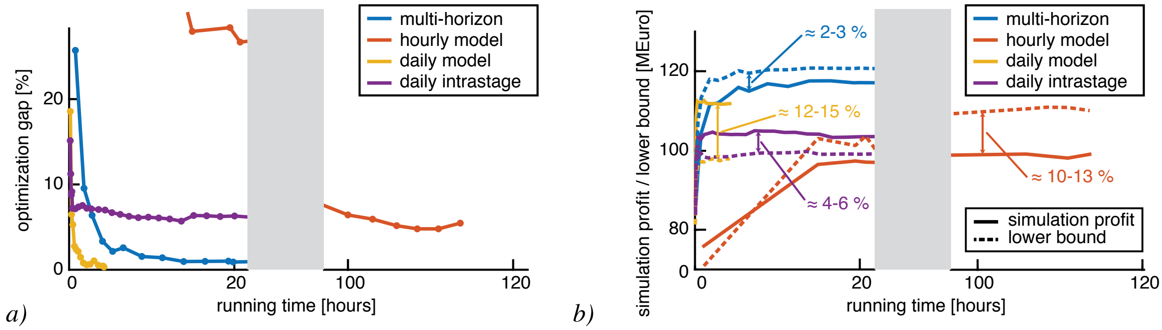

Fig. 3. Comparison of computational complexity and modeling accuracy: *a*) Optimization gap as the difference of the lower and upper bound of the SDDP algorithms in %. After less than 12 hours all methods saturated only the hourly formulation needed a few days. Note that the gap of the daily intrastage model stayed around a relative high 6 %. *b*) Simulation profit (solid lines) and lower bound of the SDDP algorithms (dashed lines). Stabilization of the lower bound and simulation profit indicates a converged algorithm and, respectively, that the operation policies are not improving anymore. Further, a small difference between simulation profit and lower bound points to high model accuracy.

is performed in the constraints (3b) and (3c).

This daily intrastage model appropriately allows only deterministic intrastage problems whereas in the multi-horizon formulation (1) also stochastic ones would be possible (as depicted in Fig. 1 *b*)). This is because the introduction of stochasticity would complicate the problem here to a great extent, since for each intrastage scenario a different profit-to-go value $Q_{t+1}(v_t)$ would have to be calculated.[2] However, in the setting here, a deterministic intrastage problem is sufficient.

Whereas from the modeling point of view the model is more precise than the formulation (1), the complexity to solve such a problem is relatively high. First, because of a higher number of state variables and second, because the daily filling of balancing reservoir are not important from a MTHP perspective and, therefore, SDDP will have difficulties in converging to stable filling values (as it will be shown later). Hence, such a model seems to be more appropriate for HPPs with only one type of reservoirs.

4.4. Comparison of computational complexity and modeling accuracy

Multi-horizon models as well as the three introduced traditional models are now compared regarding the computational complexity, if the models are optimized by a SDDP algorithm. The implementation was done in Matlab and solved on a computer with two Intel Xeon E5 processors at 2.2 GHz. 16 individual processor cores were available in total but only 8 of them were used. Linear, quadratic, and mixed-integer linear programs were solved by the mathematical solver IBM ILOG CPLEX 12.4.

To keep the computational burden low as well as to achieve a meaningful comparison, all of the optimizations are performed without considering different scenarios of water inflows and market prices, although they would allow it. The models are formulated for a time horizon of one year. Dualized SDDP algorithms (since no stochasticity is considered, strictly speaking DDP algorithms) are used with a cutting plane selection method based on level one dominance and maturity of the cuts (see also [13–16]).

Fig. 3 *a*) depicts the development of the optimization gap over running time of the SDDP algorithms. It is not clear a priori which optimization gaps relate to good operation policies. However, some structural insights about the convergence and the computational requirements of the different methods can be given.

After around 30 hours, the gap of the multi-horizon formulation did not improve anymore but stayed at a 0.5 %,

[2] This would require that the cutting planes would have to be considered multiple times within each intrastage subproblem. Therefore, if stochastic intrastage scenarios would be needed, a better modeling formulation would be to consider shorter time stages t and to consider the additional uncertainty as interstage scenarios.

indicating an almost optimal solution to the formulated problem. In comparison, the formulation of the hourly model, due to a higher number of subproblems and lower convergence rate, needed more than 4 days of running time until the optimization gap stabilized at around 5 %. Notable was also the high memory requirement because of hourly sets of hypercuts, which are very similar but, nevertheless, have to be stored individually.

In the daily formulation, the subproblems to solve are of similar size as the ones of the hourly formulation but there are 24 times less of them. Additionally, the convergence of the algorithm should be slightly better. These findings were qualitatively confirmed. After only 2 hours and approximately 50 iterations, the optimization gap converged to 0.5 %. Interestingly, the daily intrastage formulation converged almost as fast, with a gap of 7 % in half an hour of running time. However, the further convergence was very slow with a gap of still around 6 % after two days, which confirms the difficulties in finding optimal balancing reservoir fillings.

In order to analyze the modeling accuracy an operation simulation was performed. The simulation is constructed out of sequential hourly optimizations. In the hourly optimizations, the water values, depending on the method hourly or daily ones for storage or all reservoirs, are used as water opportunity costs and a mixed-integer linear program is formulated to find the optimal hourly operation. Note that such an operation simulation is much more realistic than the forward step of the SDDP algorithm of the different models.

For a proper operation simulation study, the simulation would have to be repeated for different sampled uncertain data. Nevertheless, here are presented only the results from one simulation run. Further, the chosen sample of uncertain data is the one which was already used in the optimization. The reason for performing such an in-sample simulation is to be able to compare how well the approaches approximate the original problem. Practical experience would have to answer the questions if a close representation of the model is necessary and how valuable their policies are.

Fig. 3 *b*) shows the lower bound of the algorithms (dashed lines), which are the operation values estimated by the SDDP forward steps. In addition, the evolutions of the operation simulation profits is depicted (solid lines). These profits increase in running time of the SDDP algorithms because better policies can be used. However, note that the simulation profits can not be compared directly among the different methods, because a higher simulation profit could also be due to overfitting.[3] Further, interesting to note is that in all simulations a comparable small amount of spillage was present. So the reason for higher simulation profits is primarily because of a different use of the water in time.

Out of Fig. 3 *b*), several questions regarding the modeling accuracy of the different models can be answered. First, a stabilization of the lower bound indicates convergence of the method. In line with the discussion of the computational complexity, the daily model convergence in less than one hour whereas the multi-horizon and daily intrastage model need a couple of hours until their lower bound stabilizes. Again, the hourly model requires a few days.

Second, the stabilization of the simulation profit indicates operation policies which are not improving anymore. Interestingly, the further improvement of the different optimization gaps indeed leads to better policies. Therefore, similar observations as given before are valid regarding the necessary running time.

Finally and most importantly, a smaller difference between the simulation profit and the lower bound indicates a better model accuracy. The daily model lead to a undervaluation of the performance of more than 10 %. This result may suggest that the HPP is operationally quite flexible. Therefore, the performance of the plant depends more on being able to take the opportunities on the hourly market, which is not well modeled with a daily view on it, and not so much on restrictive hourly water balances.

On the other hand, the difference of the hourly model was also more than 10 %, however, it overvalued the performance. This could point to a unnecessary detailed optimization of a poor model.

The difference for the daily intrastage model of 4-6 % is much better. The model undervalues the performance where the reason could be the relative high optimization gap. In comparison, the difference in the multi-horizon model is only 2-3 % which demonstrates its accuracy and superiority to the other models. The model slightly overvalues the performance which could be due to the deterministic model of the intrastage problem.

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[3] Consider for instance a deterministic optimization. A policy based on it would lead to much more profit for the given scenario, however, most probably not for different possible scenarios because the policy has been overfitted to the given scenario.

5. Concluding remarks

In this paper, a novel modeling approach, the multi-horizon modeling, was proposed and evaluated against traditional modeling approaches. For a medium-term hydro power planning optimization of complex hydro power plants with a few reservoirs of different sizes, multi-horizon models are a very efficient modeling approach both from the modeling as well as from the computational point of view.

It was shown that the traditional models either suffer from model inaccuracies or high computational complexity. A daily aggregated model performed reasonably well, but it neglects hourly flexibility leading to an underestimation of the operation opportunities. But an extension of it to hourly time steps was even more troublesome, which could be explained by the issue of optimizing a poor model in a more detailed way. The third alternative approach decomposes the problem into inter- and intraday problems similarly to multi-horizon models. However, because it does not exploit the different physical types of reservoirs, it is difficult to close the optimization gap which leads to suboptimal solutions.

Multi-horizon models, on the other hand, showed a number of advantages. They are more realistic, have less modeling issues since no aggregation is necessary and have a computational complexity which is almost as good as the one from aggregated models. Therefore, proper constructed multi-horizon models solved with stochastic dual dynamic programming are applicable for complex medium-term hydro power planning problems possibly solvable in less than a day and simultaneously allow many modeling details.

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