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Heterogeneous values of time in a multi-modal context: An activity- and agent-based simulation approach

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1 Abstract

Heterogeneity of user preferences can play a crucial role in the economic welfare evaluation and transportation policy and project appraisal. In this work an activity- and agent based simulation framework is applied to study effects of optimal congestion pricing on welfare and equity in a multi-modal context and with heterogeneous values of time. Bridging the gap between definition of value of time and schedule delay heterogeneity in a single trip scheduling bottleneck model and its inclusion in an activity-based context, this work focuses on two major aspects: impact of degree of heterogeneity on benefits from congestion pricing and impact of availability of alternatives on equity and consumer welfare.

The use of an activity- and agent based simulation framework MATSim enables integrated multi-modal approach, capturing effects of congestion pricing on an individual level. Accounting for service level of public transportation and associated capacity restrictions as well as crowding effects in the evaluation of congestion charging policy proves to be crucial for accurate identification of winners and losers of such policy. Based on a multi-modal corridor scenario, results of this study indicate that increasing heterogeneity in presence of multiple modes with different travel cost leads to beneficial self-organization effects and therewith diminishing gains of congestion pricing policies. Furthermore, changes in consumer welfare for individuals with different values of time are highly dependent on availability of alternative mode. This might have important implications on transport policy design in urban context and its impact on economic and social inequality.

2 Introduction

Addressing divergent mobility and transportation needs of growing, highly diverse urban population in modern cities and metropolitan areas is a major challenge for transportation planners and policy makers. Thereby, the heterogeneity of user preferences plays a crucial role in outcomes of design and evaluation of infrastructure projects and policy measures. Only by controlling for the different user as well as trip characteristics, equity implications and winners and loser groups can be adequately identified. Failure to address the diverse population needs and neglect of distributional effects is one of major reason for struggle of pricing and demand management policies to win public support.

One of the most important figures in transportation economics and major source of the heterogeneity among travellers is the monetary value placed on the time gains resulting from reduced travel times, It is commonly referred to as value of time (VoT) or value of travel time savings (VTTS) (Small and Verhoef, 2007). Discussed in more detail in Section 4, values of time are often linked to personal or household incomes, but can also vary with travel conditions, trip characteristics and personal preferences.

Major effects of value of time heterogeneity on outcomes of economic policy and project appraisal have been shown in various publications. Among others Bates (1996), Arnott et al. (1988) and van den Berg and Verhoef (2011a) highlight that type and extent of heterogeneity strongly influence welfare effects of congestion tolling and different tolling schemes and forcefully argue for the need to include it in models with trip-timing choice. The presence of an alternative mode adds an additional degree of complexity to the evaluation of pricing policies with heterogeneous traveller preferences, with only few publications addressing this question (e.g. van den Berg and Verhoef, 2013).

Working with highly skewed value of time distributions and multiple levels of heterogeneity within analytical frameworks and aggregated demand models is often challenging. In particular, real world scenarios involving multi-modal urban environment with populations of several million people analytical approach struggles to provide level of detail and accuracy desired by planners and policy makers. Thereby, an agent-based modelling approach, widely discussed in the literature during last two decades, represents a promising alternative to address the challenges associated with travellers heterogeneity. Operating on the level of individuals, with attached socio-economic and demographic characteristics, it inherently provides the necessary structure for accounting for personal user preferences. However, despite of being advocated as one of major advantages and strengths of an agent-based modelling approach, so far only limited effort has been made to truly include heterogeneous values of time into the models and simulation frameworks. This is partially due to the lack of disaggregated data and estimates on individual utilities attached to work and leisure activities and those variations within the population, time of day or activity type on the one hand, and the challenges in software design on the

other.

Most of the approaches to understand the interplay between pricing policies and heterogeneous user preference use dynamic economic models of traffic congestion, building upon and extending the classical bottleneck model presented by Vickrey (1969). van den Berg (2014), van den Berg and Verhoef (2013) and van den Berg and Verhoef (2011a) represent just a selection of more recent publications and are based on a single-trip model dynamic bottleneck model.

Activity-based approach to transport modelling, often used in an agent-based simulation context, takes a different approach and focuses on individual activity chains as drivers of travel demand. Thereby marginal utilities of activities play a determining role in trip timing decisions. Research by Jenelius et al. (2011) focuses on analytical derivation of traveller delay cost and value of time in presence of scheduling flexibility for an activity-based two trip model. Building on top of previous work by Ettema and Timmermans (2003), Jenelius et al. (2011) provide a generalization of a single-trip model using marginal activity utility functions for preference representation. However, the simple model restricted to two departure times as decision variables, discussing the inclusion of other dimensions as mode, route and destination choice as desirable but challenging. More recently, Li et al. (2014) use an activity-based approach to incorporate commuters day-long activity schedule and time decisions into the Vickrey's bottleneck model for the morning and evening trips. The modelling approach based on the utility maximization, is analytically explored for a special case of constant marginal utilities.

This paper goes a step further and uses a multi agent-based simulation framework to investigate impact of value of time and schedule delay heterogeneity on economic welfare and distributional effects of congestion pricing policy. In particular, this work focuses on presence of heterogeneity in a multi-modal context, with detailed modelling of interaction between private and public transport as well as capacity constrained public transport vehicles with crowding effects. Using a multi-modal corridor scenario in a multi-agent and activity-based transport simulation framework MATSim (MATSim, 2015), differences in values of time are modelled based on individual household income. Thereby, availability of a bus service as an additional alternative has significant impact on level and distribution of gains and losses resulting from introduction of congestion pricing policy.

3 Methodology

Transportation demand originates from travel decisions of individuals, who use provided transport supply to move from one location to another, mainly for sake of performing some activity, which could not have been performed otherwise. Arising from combination of travel choices made on an individual level, properties and behaviour of

transportation system as a whole represent an emergent phenomena within a complex system. Agent-based modelling and simulation approach provides a framework for modelling and studying of such complex systems composed of autonomous, interacting agents. In particular, given the vast range of personal socio-economic characteristics and individual preferences influencing travel behaviour, agent-based simulation possesses essential properties for study of user heterogeneity and associated emergent phenomena arising from collective behaviour. This work adopts and extends Multi-Agent Transport Simulation (MATSim) framework (MATSim, 2015), which is described in detail below.

3.1 Multi Agent Transport Simulation (MATSim)

Multi-Agent Transport Simulation (MATSim) framework integrates travel demand based on individual activity schedules with simulation-based dynamic traffic assignment. One of its major strengths is its capability for detailed modelling and simulation of multi-modal networks. Joint simulation of private and public transport based on the queuing model allows time-dependent calculation of travel times accounting for spill-over effects and direct interaction of private and public transport. By modelling interaction dynamics based on the actual physical properties of vehicles and links, it captures queueing at the end of links, enabling to accurately model congestion dynamics.

Based on a co-evolutionary algorithm, agents alter their behaviour from iteration to iteration, evaluating new routes, alternative transport modes and departure times in the process. Thereby, each agent tries to find an optimal daily schedule, which maximizes its utility function. Following each iteration of the queue-based network assignment, the activity scheduling and travel choices of each agent are evaluated and scored, enabling generation of an individual choice set. The selection of travel alternatives from the choice set of each agent is performed using a random utility model. As described in detail by Nagel and Flötteröd (2009), after a number of iterations individual utilities converge and the system reaches a stable agent-based Stochastic User Equilibrium (SUE). MATSim features a modular architecture, allowing for flexible management, adoption and extension of behavioural features and individual choice dimensions. For the single corridor scenario used in this work two choice dimension modules are relevant: departure time and mode choice.

Departure time choice enables agents to alter their departure times from the activities. Selected agents modify their departure times and activity durations of a daily plan randomly within a pre-defined time window. For the simulation set-up presented here, a time window of ± 60 min is used for single modification of the departure time.

If an agent is selected for mode choice, it can change the mode of its journeys in the next iteration. As the mode choice has to be consistent (taking bus in the morning to work and car back home is improbable, due to non-availability of the vehicle at the work

place), mode choice is altered at a sub-tour level (round-trip). For each mode route is generated based on classical Dijkstra Algorithm for private transport and multi-node Dijkstra routing algorithm of public transport (Rieser, 2010). However, given a simple network configuration of a corridor, route choice does not feature as an independent choice dimension at this stage. More details on the architecture and functionality of modules handling these choice dimensions within the MATSim framework can be found in Balmer et al. (2008) or MATSim (2015).

Incorporating heterogeneous value of time preferences in MATSim framework requires assigning each agent an individual utility function, or as it is referred to in MATSim language a scoring function, which evaluates daily schedule performance based on individual attributes. As this work focuses on income related variations in values of travel time, household income is attached to each agent, enabling its direct incorporation into value of time and schedule delay within the individuals scoring function. Section 4.1 describes the concept behind value of time variations in an agent-based context in detail. Same individual values of time have also to be taken into account in the routing module during the shortest path calculations in order to ensure consistency between routing and scoring.

3.2 Public Transport in MATSim

MATSim provides a fully integrated simulation of public transport operations, based on the detailed model of interactions between passengers, buses, trains and private transport vehicles. Each bus or train vehicle agent has its physical characteristics such as size, capacity, number of seats and numbers of doors, associated with it and moves on the road or rail network according to the queue-based traffic dynamics. Buses interacts with cars and therefore, in absence of dedicated bus lanes, are subject to congestion delays. Vice versa, a bus stopping at a bus stop without the bus bay on a one-lane street, will delay following cars during the passengers dwelling process. In case a vehicle is full, further boardings are denied, leaving travellers at the station to wait for the next bus or train. The duration of dwell process itself depends not only on number of passengers boarding and alighting, but also on number of bus doors and total bus occupancy. Thereby, a model presented in Sun et al. (2014) is applied, which uses data from the electronic smart card fare collection system in Singapore to study dwell process dynamics. The high level of detail in modelling of public transport is crucial to accurately capture interplay between public and private transport in case of shared road space as well as crowding effects and dynamic phenomena such as bus bunching.

3.3 Implementation of congestion pricing

Finding an optimal control strategy in a complex system and steer it towards an optimal state can be a highly challenging task. In economics, internalization of external cost is often considered to be a powerful policy for achieving more optimal outcome and increasing social welfare. As transport infrastructure is a public good with limited capacity, number of externalities on other users and non-users occur. Most prominent negative externalities are congestion, pollution, changes in land value and safety hazards.

The concept of first-best congestion pricing is based on the idea of internalization of congestion delay externality. Charging road user a toll equal to the cost she imposes on all other travellers on the same route by adding to the congestion delay, leads to an efficient allocation of network capacity among users. Adding this cost, commonly referred to as *marginal external congestion cost* or *mecc*, to each travellers trip cost, minimizes the total travel time in the network and lets the system converge to the state equivalent to the system optimum, as defined by Wardrop (1952).

As number of more recent publications demonstrate (e.g. Yang and Huang (1998); Safirova et al. (2007)), *mecc* can be computed on a link-by-link basis through the network without the need to account for effects of toll on the one link on the other links in the network. This property significantly simplifies the implementation of an optimal congestion pricing within the large-scale agent-based simulation framework. A *mecc* based pricing approximation for an agent-based queuing model was initially derived by Lämmel and Flötteröd (2009) and later refined by Lämmel (2011). Thereby, a simplified approximation of *mecc* based on the assumption of stationary flow through the time of existence of the queue leads to the following definition:

$$mecc_l(t_0) \approx t_l^{end}(t_0) - t_l^{lv}(t_0) \quad (1)$$

with $mecc_l(t_0)$ denoting the external cost that one additional agent causes by entering a link l at the time t_0 . $t_l^{end}(t_0)$ denotes the time at which the congestion, that the "causative" agent contributed to by entering a link at t_0 , dissolves. And $t_l^{lv}(t_0)$ is the time at which the "causative" agent enters the bottleneck at the end of the link l (for detailed derivation see Lämmel, 2011). In other words, under the assumption of constant, maximal outflow rate at the link l , *mecc* equals to the time the link l remains congested after the "causative" agent passed through it. Testing this computationally inexpensive method with continuous evaluation of social cost for each agent on every link in the network, Lämmel and Flötteröd (2009) and Lämmel (2011) present simulation results for an optimization of routing in an evacuation scenario supporting the efficiency of this approach.

The implementation of the presented marginal social cost pricing approach in MATSim framework in context of this work is performed by closely following the algorithm pre-

sented in Lämmel (2011, chap. 3.1) and under consideration of practical implementation issues discussed in Lämmel (2011, chap. 4.1.3) . Thereby, time bins of 5 min were chosen for aggregation of tolls, representing a reasonable time resolution for stable convergence of toll values.

It is important to note, that while this approach represents an approximation of first-best congestion pricing. The assumption of stationary flow is valid, as long as no spill back occurs. With spill back, the congestion charges are likely to be overestimated as double-charging on the bottleneck link and links further upstream might occur. Furthermore, using 5 min time bins results in extension of delay cost occurred only for a short period of time to the whole duration of a time bin, also leading to potential overcharging.

3.4 Economic Evaluation

As pointed out in the Introduction, central focus of this work lies in the assessment of social and economic impact of urban transport policies as in particular congestion pricing given a population of users with heterogeneous preferences. Benefits and losses of policy in question have to be quantified and assessed from societal as well as individual points of view. Thereby, defining an adequate economic evaluation methodology, capable to comprehensively capture policy impact on different levels is crucial for the quality of outcomes and robustness of conclusions derived.

Expected Maximum Utility (EMU) approach allows straightforward calculation of consumer surplus, which is consistent with the discrete choice theory, underlying agents daily schedule selection in MATSim. Challenges associated with EMU calculation, alternative welfare indicators as well as transition to social welfare calculation in presence of public transport operations are discussed in following.

3.4.1 Expected Maximum Utility

Mostly used in simulation, the discrete choice modelling approach enables the emergence of complex system behaviour based on behaviour and preferences of individual travellers. Rich body of work related to the applied welfare analysis based on random utility models in general and discrete choice modelling approach in particular (de Jong et al., 2005; Train, 2003; Ben-Akiva and Lerman, 1985; Small and Rosen, 1981; McFadden, 1981) provides provides a natural and consistent way for welfare and benefit evaluation by considering utilities of all alternatives available to the individual traveller. Referred to as *Expected Maximum Utility* (EMU) or as interpreted by Ben-Akiva and Lerman (1985), the systematic component of the maximum utility as a measure of accessibility, for a

logit model it is defined as

$$V_J = \frac{1}{\mu} \cdot \ln \sum_{j=1}^J e^{\mu V_j}, \quad (2)$$

with V_i being a deterministic utility of an alternative i and J the size of individuals choice set. μ is the scale parameter of the disturbance term and ϵ and can be understood as the degree of decision makers rationality or the ability of the user to distinguish between the utilities of different alternatives (see Section 2.3 in Kichhöfer (2014) for detailed explanation). Being dependent on the size of the choice set, the logarithmic formulation reflects the idea of decreasing marginal utility of additional alternatives.

Under the assumption of marginal utility of income staying constant over changes from the particular policy, the expected change in the consumer welfare as result of policy introduction for a traveller n is formulated in the equation 3, with α_n indicating marginal utility of income and superscripts 0 and 1 referring to the states before and after the change (de Jong et al. (2005)).

$$\Delta E(CS_n) = \frac{1}{\alpha_n} \frac{1}{\mu} \left[\ln \sum_{j=1}^{J^1} e^{\mu V_j^1} - \ln \sum_{j=1}^{J^0} e^{\mu V_j^0} \right]. \quad (3)$$

Extending the EMU formulation for the change in individual welfare of an agent n to a population of N individuals, the total change in the consumer welfare is computed as a sum of individual changes (equation 4).

$$\Delta W = \sum_{n=1}^N \Delta E(CS_n). \quad (4)$$

Challenges in Choice-Set Generation

One of the main premisses in the discrete choice modelling framework is the assumption of independence of irrelevant alternatives (IIA), where adding additional alternatives should not change the decision makers choice for any existing alternative. This is a strong condition to be satisfied, in particular in cases when potential set of alternatives is very large. Hall (2003, Chapter 2) discusses this intricate problem in detail for departure time and route choice. Main challenge thereby is containing number of possible choice alternatives while establishing independence between relevant potential choices.

MATSim can be considered as multidimensional choice set generator, which generates and evaluates number of alternative daily plans by varying existent plans in predefined dimensions (e.g. route, mode, departure time or location choice) with every iteration. With a commonly used "best score" criteria for selection of daily plans, only a limited number (typically 5) of best performing plans are kept as a part agents choice set in its memory, discarding other, worse performing alternatives. Though this guarantees a smooth and stable conversion of the overall system, with the increasing number of iterations plans in agents memory tend to become very similar, violating the IIA condition. Though identified and partially addressed in recent publication (Oliveros, 2013; Nagel et al., 2014; Grether, 2014), a cohesive multi-modal and time-dynamic approach to this problem requires further research and in-depth evaluation, as alternative choice sets may alter the stability and convergence process of the simulation

To circumvent this problem and ensure compliance with the IIA condition, a slightly alternative solution for generation of a choice set suitable for welfare evaluation is proposed and applied in this work. For each agent, the chosen plan of the final iteration of the simulation run is picked and used for a rule-based definition of alternatives. Thereby departure time alternatives are defined following the approach by Antoniou et al. (1997), where authors used set of five alternatives for each departure.

In an activity-based context, trip departure times are essentially activity end times. Therefore a choice alternative is determined not by a single departure time, but by a set of all departure times through the day, resulting in exponential growth of the choice set with increasing number of activities during the day. For the home - work - home activity chain considered in this study, departure times with +1h and -1h relative to the observed chosen alternative are considered. Applied for morning and evening commute, this results in a set of 9 possible departure time combinations. Thereby combinations, where work durations is altered by two hours: earlier departure in the morning (-1h) and later in evening (+1h), or later departure in the morning (+1h) and earlier in the evening (-1h) are discarded as being too substantial and therefore improbable schedule variations. Generating these alternatives for the car and bus modes, results in a total of 14 possible alternatives of a daily schedule. In case walking appears to be a realistic alternative (travel duration less than 1h), it is as well added to the choice set. As walking is unaffected by traffic and crowding conditions, only one utility maximizing set of departure time is chosen. This results in a choice set of 14 to 15 alternatives for each agent, with size of the choice set of an individual agents remaining constant in all scenarios. Though this appears to be a fairly large number, taking into account that it is a full day schedule and looking on it from single trip perspective, leaves us with maximum 7 choices for the morning commute (3 departure time for car and bus, and one for walking) and dependent on the first choice between 3 and 1 alternatives for the evening commute. These appear to be fairly realistic numbers, in line with number of choices considered by other studies.

Given the one chosen alternative and 13 to 14 non-chosen for each agent, the utility

of non-chosen alternatives is evaluated based on travel times of the last iteration. This is basically equivalent to the state, where only the agent of interest would change to a different plan without his decision having an effect on system state as whole. Simulation of these 13 to 14 non chosen alternatives for each agent with keeping behaviour of all the other agents constant, would result in maximum of $8000 \times 14 = 112'000$ simulation runs, which given a simulation time of about 1 min per run, is infeasible to execute. Therefore, to score all the alternatives, an approach based on a pseudo-simulation, presented by Fourie et al. (2013), is applied. Thereby, travel times from the network conditions and travel times from the last iterations are used to score each possible alternative.

3.4.2 Generalized cost and realized utility in an agent-based SUE

In an agent-based simulation framework each agent follows the goal of maximising its utility given a personal objective function, also referred to as utility function in discrete choice theory and or scoring function in a MATSim context. In an activity based model the utility function commonly incorporates utilities gained from activity performance and (dis)utilities associated with travelling. In context of MATSim's co-evolutionary optimization algorithm, each agent's experienced utility in the course of the simulated day is calculated at the end of each iteration. Sum of all utilities from the chosen alternatives across agents, can be interpreted as a form of generalised cost given the overall system state in this iteration and is referred to as $\sum Realized\ utility$ (Zöllig and Axhausen, 2012). In MATSim framework, average of $\sum realized\ utilities$, also called simulation score, is often used as a measure of convergence and stability of the stochastic user equilibrium. As it directly measures the actual utilities of chosen alternatives, it also can be adopted as qualitative indicator of overall economic performance. (Zöllig and Axhausen, 2012), for example, use it next to EMU calculation for assessment of infrastructure investments with an agent-based accessibility approach.

3.4.3 Public Transport Operation Cost

Providing a high service level of public transportation comes at a cost. Operating a bus line with higher headway requires more investments in buses, more manpower and results in more vehicle kilometres. At the same time, as demonstrated in following sections, level of public transportation service and availability of alternatives to the car mode has a decisive impact on economic benefits of congestion pricing in a multi-modal urban environment and in particular in presence of heterogeneous values of time. However, conducting welfare evaluations and analysing sensitivity of pricing policy effects in presence of varying bus service frequencies, requires to account for operational as well as capital cost of service provision.

Matching the travel behaviour parameters, which are based on the survey data from

Sydney Tirachini et al. (2014), formula and parameters for cost estimates of bus operations in the corridor scenario are borrowed from national guidelines for transport management in Australia Australian Transport Council (2006) and were previously used for simulation-based public transport fare and frequency optimization by Kaddoura et al. (2015). The total cost C of bus operations for one day are calculated according to equation 5.

$$C = (d_{vkm} \cdot c_{vkm} + t_{vh} \cdot c_{vh}) \cdot O + N_v \cdot c_{vday}, \quad (5)$$

with d_{vkm} as total vehicle kilometres per day, c_{vkm} monetary cost per km, t_{vh} total operational vehicle hours, c_{vh} monetary cost of vehicle operation per hour, O factor for overhead cost, N_v total number of vehicles and c_{vday} daily capital cost. Thereby the first part of the equation accounts for variable operational cost and the second part of the fixed cost. The daily unit cost c_{vday} and the cost per vehicle kilometre c_{vkm} are dependent on the vehicle capacity, with cost functions derived from linear regression and shown together with other parameters in Table 1.

Table 1: Bus operation cost according to Australian Transport Council (2006)

c_{vkm}	$0.006 \cdot \text{capacity} + 0.513$ [\$/vkm]
c_{vday}	$1.6064 \cdot \text{capacity} + 22.622$ [\$/ vday]
c_{vh}	33 [\$/ vh]
O	1.21

3.4.4 Social welfare and consumer surplus

In this work, social welfare is defined as sum of consumer benefits, monetary payments made by travellers for the usage of transportation infrastructure and cost of bus operations, as presented in equation 6. This definition is based on an idealized assumption of zero transaction cost and no losses as a result of monetary transfers from travellers to the bus and road operators. Money collected from congestion charging and public transport is assumed to be returned to society in one form or another, incurring no additional toll collection or administration cost in the process.

$$\begin{aligned} \text{Social Welfare} = & \text{Consumer Welfare} + \text{Fare Revenue} \\ & + \text{Toll Revenue} - \text{PT Operation Cost} \quad (6) \end{aligned}$$

3.5 Simulation methodology

The simulation methodology draws on wide body of experience of previous MATSim based studies and related research. The initial demand for all scenarios is based on a set of relaxed plans with car as a single mode and one plan per agent. After introduction of new bus service frequency or congestion pricing policy, 1000 iterations are performed to allow the system to reach a state of equilibrium. In the first 800 iterations, agents generate and evaluate new plans with the two major replanning strategies: mode choice and departure time choice. During this plan generation process the share of agents trying out new plans is linearly decreasing, with total of 40% in the first iteration and no new plans generated after iteration 800. This process is referred to as "linear annealing" and prevents extreme system state changes after disabling of the plan generation process. In the last 200 iterations, agents choose from the existing 5 plans in there memory with probabilities given by the logit model.

4 Value of time and money

As pointed out in the Introduction, valuation of travel time savings and its distribution across population can play a determining role in outcomes of economic project and policy appraisal and evaluation. One of the main challenge thereby is the high inter- as well as intra personal variations in values of time among individuals. Theoretical framework for valuation of time builds on models of time and income allocation, as initially presented by Becker (1965) and later refined and extended by i.a. DeSerpa (1971), Jara-Diaz (2003). An detailed overview of the theory and state of the art of estimation methodologies for value of travel time savings can be found in Small and Verhoef (2007), Small (2012) and Börjesson and Eliasson (2014).

Traditionally the values of time are estimated using travel diaries, micro-census data or other stated preference (SP) surveys (Jara-Diaz and Guevara, 2003, e.g.). Alternatively few studies were performed based on revealed preference (RP) observations or the combination and comparison of SP and RP sources (e.g. Brownstone and Small, 2005). The Swiss study by Axhausen et al. (2008) obtains not only high variation in valuation of travel time savings based on the trip purpose but also show significant influence of personal income and trip distance. Hess et al. (2008) produce more stable and reliable results by combing evidence from four different studies conducted in Switzerland.

Focusing only on income - dependent heterogeneity of values of time in this work, continuous interaction formulation used by Axhausen et al. (2008) is adopted in order to incorporate income parameter into the utility function. Therewith, for the (dis)utility of

monetary travel cost τ follows

$$TC(inc, \tau) = \beta_{mon} \left(\frac{inc}{\widehat{inc}} \right)^{\lambda_{inc, mon}} \cdot \tau, \quad (7)$$

with inc being the income variable influencing the sensitivity to monetary expenses β_{mon} and \widehat{inc} being a sample mean value used as a reference. In the subsequent part of the paper, the subscripts in $\lambda_{inc, mon}$ will be omitted, substituting $\lambda_{inc, mon}$ with λ for better readability. The income dependent factor in the formulation above will be referred to as "income-sensitivity factor".

4.1 Heterogeneity in an activity-based modelling context

An agent-based framework with personal utility function for every economic agent is in particularly suitable for the incorporation of heterogeneity on individual and trip levels into the simulation model.

As indicated in the previous section, the factors for (dis)utilities from travelling, activities or monetary expenses can vary from agent to agent. Kickhöfer (2014), Kickhöfer et al. (2011), Kickhöfer et al. (2010) previously addressed heterogeneity in user perception of monetary expenses on travel within MATSim context. Focusing on economic policy appraisal, the marginal utility of money was multiplied by an income-dependent term to reflect differences of users in perception of monetary travel expenses. More recently Nagel et al. (2014) used heterogeneous values of time and therewith varying sensitivity to road tolls to demonstrate the benefits of adding randomness to routing, when faced complex interactions between toll levels and values of time. However, in all this approaches (dis)utilities travel time and activity performance were left homogeneous among population. This work takes a different approach by incorporating heterogeneous values of time into individuals utilities of activity performance and (dis)utility of travelling, as presented in following.

Detailed review of utility functions for activity performance and travel (dis)utilities can be found in Nagel and Flötteröd (2012). Jumping straight to the definition of marginal values of travel time savings in MATSim (Nagel et al., 2014), mVTTS for an agent a are defined as opportunity cost of time divided by the marginal value of money β_a^{mon} :

$$mVTTS_a = \frac{mUTTS_a}{\beta_a^{mon}} = \frac{-\beta_a^{trv} + \beta_a^{act} \cdot \frac{t_{typ}}{t}}{\beta_a^{mon}}, \quad (8)$$

with t as actual activity durations and t_{typ} as typical or ideal activity duration. Following the logic of activity based modelling approach, it can be argued, that the differences in

variations of value of time across the population result from varying utilities of activities. Translating this argumentation into an activity-based modelling framework, the heterogeneous values of time should be reflected in variations of β^{act} among agents. Thereby the higher willingness to pay for the reduction of travel times emerges intrinsically from the higher activity utilities and travel (dis)utilities of agents with high personal or household income.

Following the definition of (marginal) value of travel time savings and adding heterogeneity from (7) into it, for the heterogeneous utility factor of money for an agent a becomes:

$$\beta_a^{mon} = \beta^{mon} \left(\frac{inc}{\overline{inc}} \right)^{\lambda_{inc,mon}} \quad (9)$$

Substituting (9) into (8) and rearranging the heterogeneity multiplier for the monetary cost into the nominator, follows:

$$\begin{aligned} mVTTs &= \frac{-\beta_a^{trv} + \beta_a^{act} \cdot \frac{t_{typ}}{t}}{\beta_a^{mon}} \\ &= \frac{-\beta_a^{trv} + \beta_a^{act} \cdot \frac{t_{typ}}{t}}{\beta^{mon} \left(\frac{inc}{\overline{inc}} \right)^{\lambda_{inc,mon}}} \\ &= \frac{-\beta_a^{trv} \left(\frac{inc}{\overline{inc}} \right)^{-\lambda_{inc,mon}} + \beta_a^{act} \left(\frac{inc}{\overline{inc}} \right)^{-\lambda_{inc,mon}} \cdot \frac{t_{typ}}{t}}{\beta^{mon}}. \end{aligned} \quad (10)$$

Under the assumption of an activity duration being in the neighbourhood of its typical duration, linearisation around $t = t_{typ}$ makes the mVTTs independent of the duration of the activity following the trip. As Nagel et al. (2014) point out, using this approximation substantially simplifies a robust software design.

Combining the homogeneous utility parameters for travel and activities with the heterogeneity factors, leads to:

$$mVTTs = \frac{-\hat{\beta}_a^{trv} + \hat{\beta}_a^{act}}{\beta^{mon}}, \quad (11)$$

with

$$\hat{\beta}_a^{trv} = \beta_a^{trv} \left(\frac{inc}{\overline{inc}} \right)^{-\lambda_{inc,mon}} \quad (12)$$

$$\hat{\beta}^{act} = \beta^{act} \left(\frac{inc}{\overline{inc}} \right)^{-\lambda_{inc,mon}} . \quad (13)$$

As equation 11 shows, the above formulation adjust the marginal utilities of activity performance and travelling for each individual, keeping the marginal utility of monetary travel expenses constant across the population. This transformation does not change individuals mVTTS, but has an important behavioural effect. Becoming intrinsic property of the model, heterogeneity in VTTS does not anymore directly depend on level of monetary travel expenses. It also paves the way towards activity specific marginal utilities as well as potential inclusion of budgeting into the simulation framework. Moreover, as it is demonstrated in the following section, this approach facilitates inclusion of schedule delay heterogeneity in an activity-based context as it intrinsically emerges from income dependent variation of the marginal activity utility.

5 Schedule delay

In the dynamic models of congestion schedule delay commonly refers to the time difference between preferred arrival and actual arrival time at an activity or preferred and actual departure time from an activity. The cost associated with these, for travellers undesired time deviations, is referred to as the schedule delay cost. Affected by a broad range of factors, schedule delay cost display a wide inter- and intra-personal variability, leading to challenges in study and quantification of its distributions. However, as recent studies show (van den Berg, 2014; van den Berg and Verhoef, 2013, 2011a,b, e.g.), heterogeneity in schedule delay cost can have significant influence on welfare and distributional effects in transport planning and policy design, and therefore requires more attention from modellers and practitioners.

5.1 Being Early or Late – Modelling and Behaviour

Schedule delay cost are commonly divided into schedule delay early (β) – arriving at an activity location before the desired activity can be started and the schedule delay late (γ) – arriving later than planned.

Work activity has traditionally been the most prominent example for illustration and study of schedule delay cost. The often rigid working hours in developed parts of the world lead to the high penalization of arriving at workplace too late or leaving them too early. Number of publications studied the schedule delay in the context of the

morning commute problem, using the same exact time preference for the arrival at the work place among all travellers. Thereby the starting point is commonly the bottleneck model presented by Vickrey (1969) and its extension with heterogeneous preferences or additional choice dimensions (see Small (2012) for overview). Comparatively little however was written about the evening peak, where traveller's deviation from preferred departure-time lead to additional cost. Vickrey (1973), Fargier (1983) and later de Palma and Lindsey (2002) compared the morning and evening commutes and demonstrated that the convenient symmetry of the two peaks in case of identical travellers, breaks down when heterogeneous trip timing preferences, values of time and schedule delay cost are considered. Yet, only little attention was given to the interdependency of morning and evening peaks and the whole day schedule dynamics. As activities often require a minimal duration for their performance, delays tend to propagate through the day and often have implications on all consecutive activities.

As schedule delay cost for commuting trips depend on individuals job and function, a direct relation with the personal value of time and therewith correlation with income is a common assumption. People with a high income tend to have more flexibility and power to adjust there personal schedule, while workers with lower income, often employed in service industry, healthcare, education or manufacturing industries, tend to be bound by strict opening and shift timings.

5.2 Schedule delay in an agent – based framework

Accurately capturing different levels and dimensions of user's preference heterogeneity among individuals and trips in a single model is a challenging task. From the transport economics perceptive, the ratios of individual's value of time, schedule delay early and schedule delay late play a determining role in trip scheduling and is therewith of major relevance for the welfare evaluation. Three forms of user's scheduling preference heterogeneity are commonly considered in the literature: proportional heterogeneity, α -heterogeneity and γ -heterogeneity (van den Berg (2014)). Being defined by the ratios of value of time and schedule delay cost $\mu = \frac{\alpha}{\beta}$, $\eta = \frac{\gamma}{\beta}$ and $\lambda = \frac{\alpha}{\gamma}$, different modelling forms try to capture varying trade-offs and user preferences between queuing and arriving before or after the preferred arrival time at given activity location.

In this paper, the activity-based framework is linked with the proportional form of heterogeneity. It is based on the assumption that values of time and schedule delay vary proportionally over travellers, following a certain distribution function. Consequently the ratios of value of time and schedule delay μ, η and λ remain constant for all travellers. Initially discussed by Vickrey (1973), it commonly follows the argumentation that people with higher income have higher value of income and therewith corresponding higher cost of schedule delay early and late. The major drawback of this heterogeneity form, is that it does not account for different flexibilities of schedule based on other factors as income.

For agent- and activity-based framework MATSim, the relation between marginal utilities as used in the simulation framework and values of time α and schedule delay early β and late γ from Vickrey's bottleneck model with homogeneous user preferences were discussed by Nagel and Flötteröd (2009) and are summarized in equation 14,

$$\begin{aligned}\alpha &= mVTTs \cdot \beta^{mon} = -\beta^{trv} + \beta^{act} \\ \beta &= \beta^{act} \\ \gamma &= \beta^{late}.\end{aligned}\tag{14}$$

Following the concept of heterogeneity with income depended utility of activity performance as presented above, transition to proportional heterogeneity in schedule delay is straightforward. By defining β^{late} in the equation (14) as $\beta^{late} = \beta_{const}^{late} \cdot \hat{\beta}_a^{act}$, for heterogeneous values of time and schedule delay follows:

$$\begin{aligned}\alpha &= -\beta^{trv} + \beta^{act} = -\beta_{const}^{trv} \cdot \left(\frac{inc}{\overline{inc}}\right)^{-\lambda_{inc,mon}} + \beta_{const}^{act} \cdot \left(\frac{inc}{\overline{inc}}\right)^{-\lambda_{inc,mon}} \\ \beta &= \beta^{act} = \beta_{const}^{act} \cdot \left(\frac{inc}{\overline{inc}}\right)^{-\lambda_{inc,mon}} \\ \gamma &= \beta_{const}^{late} \cdot \beta^{act} = \beta_{const}^{late} \cdot \beta_{const}^{act} \cdot \left(\frac{inc}{\overline{inc}}\right)^{-\lambda_{inc,mon}}\end{aligned}\tag{15}$$

From the equations (15), it is easy to see, that ratios $\mu = \frac{\alpha}{\beta}$ and $\eta = \frac{\gamma}{\beta}$ stay constant for all agents, with α , β and γ varying proportionally with the income factor. This corresponds to the definition of proportional heterogeneity. The value of β_{const}^{late} is determined by the ratio $\eta = \frac{\gamma}{\beta} = 3.9$ as in by Arnott et al. (1990).

6 Experimental Scenario Set-up

6.1 Supply

For the initial evaluation of presented methodology and effects of heterogeneous travellers, a rather simple, multi-modal corridor scenario is chosen. Essentially limiting agents choice dimensions to departure time and mode choice, allows for better isolation of effects from users heterogeneous values of time as well the first-best road pricing approximation. At the same time, limiting agents to 2 degrees of freedom enables drawing of parallels between analytical economic models and agent-based simulation methodology.

The 20km long corridor, with distribution of home on one and work locations on the other end, consist of three lanes in each direction with flow capacity of 800 vehicles per hour and lane. A bus line in each direction is operating along the full corridor length, with bus stops located each 600 meters. The headway in each direction is 5 min during the course of day.

A sketch of the scenario set-up can be found in figure 1

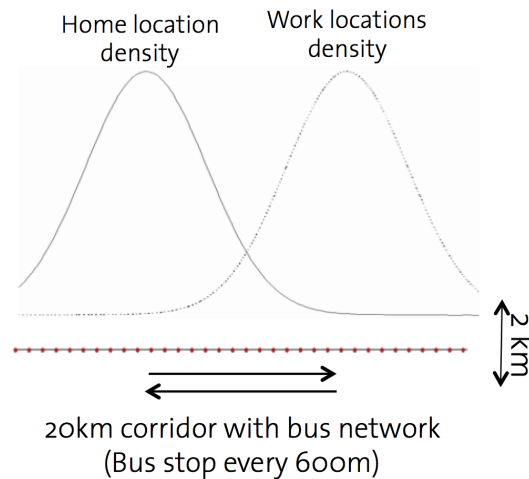


Figure 1: Corridor scenario set-up, with bus stops locations and distribution of home and work locations

6.2 Travel demand and behavioural parameters

The agent population consist of 8'000 agents, all with a same daily home - work - home activity chain. The home locations of agents follow a normal distribution along the west side of the corridor, with $\mu = 6.67km$ and standard deviation $\sigma = 3.33km$. On the other side, the work locations are distributed around $\mu = 13.33km$ with the same standard deviation $\sigma = 3.33km$. Furthermore, both home and work locations are also uniformly distributed on the north and south side of the corridor, with maximal distance of 1km to each side. This results in maximal bee line distance to the closet but stop of 1.04km.

In the agent based context, the preferred arrival and departure times to and from activities are determined by typical activity duration parameters as well as additional constrain parameters for the time intervals during which each activity can actually be performed (also called facility opening times). An additional set of parameters: "latest start time" and "earliest end time" helps to translate the definition of schedule delay cost to the activity-based framework. Arrival at an activity location after "latest arrival time" or departure from an activity before the "earliest departure time" induces additional schedule delay penalty. Table 2 presents activity timing constrains as defined in the

corridor scenario. The behavioural parameters for the corridor scenario are borrowed

Table 2: Activity constrains

Activity	Typical duration	Opening time	Latest start time	Earliest end time	Closing time
Home	14h	-	-	-	-
Work	9.5h	8.00am	9.00am	6.00pm	7.00pm

from the enriched, agent- and activity-based Sioux Falls model, as presented in Chakirov and Fourie (2014) and initially estimated by Tirachini et al. (2014) from results of stated choice survey conducted in Sydney in 2009 Hensher et al. (2011). Table 3 summarizes the parameters used for this study.

Inherently, the marginal travel time related disutility coefficients estimated in traditional discrete-choice models combines the opportunity cost time and the additional disutility caused by the travel time with the corresponding mode. Applying this behavioural parameters in an activity-based model requires to split the estimated utility parameters into two components and assign a separate utility for activity performance and a disutility for travel time (Kickhöfer, 2014; Kickhöfer et al., 2011). As the "doing nothing" situation, which occurs in case of early arrival at an activity location or departure after the closing time, corresponds to the disutility of schedule delay early β , the split of marginal travel time related disutility for car mode is defined by the ratio $\frac{\alpha}{\beta} = 2$ (Arnott et al., 1990).

Table 3: Behavioural and monetary simulation parameters for the corridor scenario

Parameter	Value	Parameter	Value
β_{act}	+ 0.48 [utlis/h]	PT Fare	2 \$ / trip
$\beta_{tr,car}$	- 0.48 [utlis/h]	Car cost per km	0.2 \$ / km
$\beta_{tr,pt}$	-0.66 [utlis/h]	Parking cost	6\$ / trip (= 12\$ / day)
$\beta_{tr,walk}$	-1.401 [utlis/h]		
$\beta_{wait,pt}$	-1.458 [utlis/h]		
β_{cost}	-0.062 [utils/\$]		
$\beta_{0,car}$	-0.562 [utils]		
$\beta_{0,pt}$	-0.124 [utils]		
$\beta_{0,walk}$	0.0 [utils]		

6.3 Socio-economic characteristics

In this work household income represents the single socio-economic characteristics attached to the agent population. Thereby incomes of agents in the corridor scenario are generated using the income distribution from the synthetic population of the Sioux Falls scenario presented by Chakirov and Fourie (2014). In case of Sioux Falls scenario

the distribution emerges intrinsically during the iterative proportional fitting inflation procedure. In order to recreate the same incomes distribution, a log-normal probability density function was fitted to the Sioux Falls income distribution and used to randomly draw incomes for the 8000 agents of the corridor scenario. Figure Fig. 2 shows the resulting income histogram incomes. No correlation between personal income and home or work locations exists and therewith the daily commute distance is independent of the income. Furthermore, car ownership is not specifically modelled in the corridor scenario and is equal to 100%.

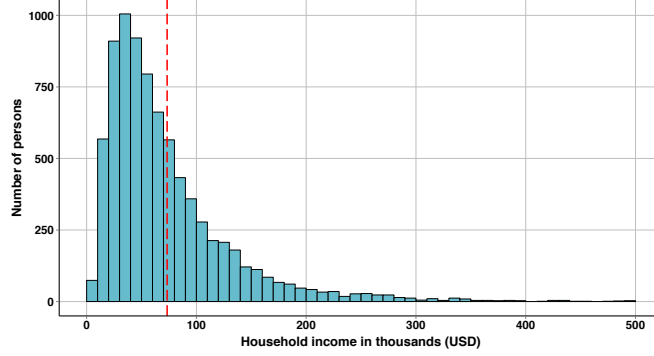


Figure 2: Histogram of incomes with dashed line indicating the mean of 73'320 USD.

Following the formulation presented in equation 7, the degree of heterogeneity and the sensitivity of the utility function to income can be controlled by adjusting the parameter λ . Using a scale factor n for variation of λ , the contribution of travel cost to the utility term becomes:

$$TC(inc, \tau) = \frac{\beta_{mon}}{m} \left(\frac{inc}{\overline{inc}} \right)^{n \cdot \lambda} \cdot \tau \quad \text{with} \quad m = \frac{1}{N} \sum_{p=1}^N \left(\frac{inc_p}{\overline{inc}} \right)^{n \cdot \lambda} \quad (16)$$

with τ being the amount of monetary expenses and m the normalization factor equal to the average of the income dependent correction term. As the mean value of the income sensitivity factor is $\neq 1$, introducing it with a constant β_{mon} , which was estimated separately, leads to an increase in mean of value of time. Therefore, normalization term m is used to ensure comparability between the homogeneous reference case and various heterogeneity scenarios, keeping the average value of time for all scenarios constant.

For heterogeneous (dis)utilities of activity performance and travelling accordingly follows:

$$\hat{\beta}^{trv} = \beta_a^{trv} \cdot m \cdot \left(\frac{inc}{\overline{inc}} \right)^{-n \cdot \lambda} \quad \hat{\beta}^{act} = \beta^{act} \cdot m \cdot \left(\frac{inc}{\overline{inc}} \right)^{-n \cdot \lambda} \quad (17)$$

The meaning of n for scaling of the parameter λ can be interpreted in two ways. On the one side, scaling λ can be seen as change in sensitivity for perception of the monetary travel cost expenses to income. This sensitivity can vary based on number of economic and cultural characteristics of a particular geographic region or even for the same person dependent on a trip purpose. As Axhausen et al. (2008) observe, the λ for business trips is significantly higher as for commuting trip and almost corresponds to $n=5$ in this study.

On the other side, slightly rewriting equation 16, as follows

$$TC(inc, \tau) = \frac{\beta_{mon}}{m} \left(\frac{inc^n}{inc} \right)^\lambda \cdot \tau = \frac{\beta_{mon}}{m} \left(\frac{inc_{new}}{inc_{new}} \right)^\lambda \cdot \tau \quad (18)$$

allows to interpret different n - factors as variations in spread of the underlying income distribution and therewith varying inequality. As a common measure of inequality, a Gini-coefficient indicates the deviation of distribution of incomes from perfectly equal distribution, with 0 in case to perfect equality of incomes and 1 the population where one person is receiving all the income (Gini, 1921; Cowell, 2011). Geometrically, it can be visualized as area difference between integral of the real cumulative income percentage curve and the integral of the perfectly equal cumulative income percentage curve. Such plots also referred to as Lorenz curves and are presented in Figure 3 for different n -factors and the underlying income distribution of the corridor scenario. It is interesting to note that Gini-coefficient of 0.20 ($n=0.5$) is rather close to a more equal country as Sweden, 0.39 ($n=1$) is slightly under the US average and close to the Gini estimates for UK and 0.69 ($n=2$) is only slightly above the Gini-coefficient estimates for South Africa (World Bank, 2015; Central Intelligence Agency, 2013). The scenarios of $n=0$ is the homogeneous user case and $n=3$, $n=5$ can be considered as extreme reference cases, or as scenarios with not only high income inequality, but also higher sensitivity to travel cost. Such situation could be expected in places, where the share of transport cost of the total living expenses is especially high.

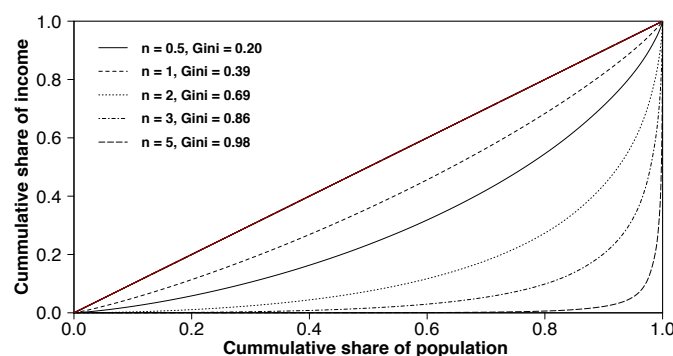


Figure 3: Lorenz curves and Gini coefficients for different heterogeneity parameters.

The bean plot in Figure 4 shows the effective value of time distributions dependent on the heterogeneity factor n and based on behavioural parameters and the income distribution as presented in Section 6.2. It highlights the growing gap between minimum to maximum values of α as the mean remains constant. In the remainder of this work, expressions such as degree of heterogeneity, spread in values of time or increasing n -factor are used interchangeably.

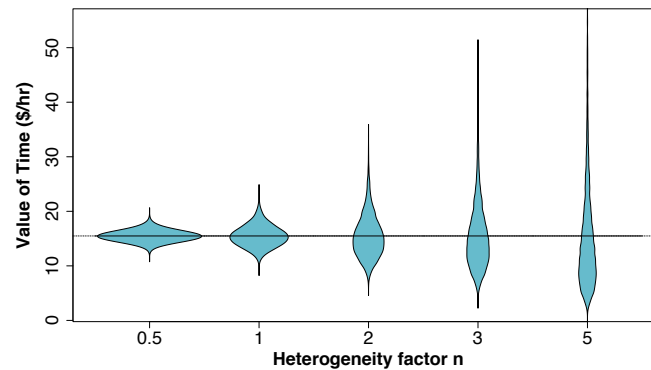


Figure 4: Distribution of Value of Time for different heterogeneity factors (car mode)

7 Simulation Results and Discussion

As this work focuses on better understanding gains or loses of congestion tolling based on availability of alternative transport modes and heterogeneity in values of time and schedule delay, simulation set with all possible combinations of parameters presented in Table 4 are conducted. With heterogeneity degree $n = 0$ being equal to homogeneous users, total of 48 simulation runs were performed.

In order to better understand the effect of alternatives available to travellers on outcomes of pricing policies, experiments with bus headways of 2 min, 5 min, 10 min and no bus operations at all are conducted. Given a capacity of 90 passengers per bus, the used service headways translate into the overall throughput of the bus line of 2700, 1080, 540 and 0 passengers per hours, respectively.

Based on agent- and activity-based simulation approach, the results presented here are derived from fundamentally different methodology as in case of trip-based bottleneck model. Therefore direct comparisons of the results should be enjoyed with caution. However, such comparison can also be highly beneficial for bridging the gap between analytically robust transport economics approach, but based on highly simplified assumptions and agent-based simulation approach allowing for detailed modelling of travel behaviour and physical interactions, but more challenging for generalization of simulation results.

Table 4: Simulation sets

Heterogeneity degree n	Bus headway	Heterogeneity type	Pricing
(0)	2 min	homogeneous	no road pricing
0.5	5 min	proportional	congestion charge
1	10 min		
2	no service		
3			
5			

In following, sensitivity of economic welfare and distributional effects of congestion pricing policies to varying parameters of transportation supply and demand, as listed in Table 4, are evaluated. Starting with the unimodal case and extending it to a multi-modal scenario, the importance of integrated transport policy modelling and planning is highlighted and discussed. Table 5 provides overview of abbreviations used in figures and tables presented in the course of this discussion.

Table 5: Overview of abbreviations and terminology

Scenarios		
n	Factor determining the level of heterogeneity	
NCP	No Congestion Pricing	
CP	Congestion Pricing	
Evaluation measures		
CW	Consumer welfare	Consumer welfare (also known as consumer surplus or user benefits) calculate as average EMU per person.
SW	Social Welfare	Sum of consumer welfare (CW), cost of bus operations and monetary revenues from congestion charge and bus fares.
RCW	Realized Consumer Welfare	Realized consumer welfare, averaged over last 100 iterations.
RSW	Realized Social Welfare	Sum of Realized Consumer Welfare (RCW), cost of bus operations and monetary revenues from congestion charge and bus fares, averaged over last 100 iterations.

7.1 Social and Consumer Welfare

Total social and consumer welfare represent fundamental aggregated indicators for policy and project evaluation. As discussed in detail in Section 3.4, calculation of consumer welfare is performed using the *Expected Maximum Utility* approach, based on the chosen daily schedule in the last iterations as well as a choice set of 13-14 non-chosen alternatives. Total social welfare is calculated as sum of consumer welfare, cost of public transport operations and monetary revenue from toll and fare collection. It is important to highlight, that the utilities of chosen and non-chosen alternatives used in the calculation of the EMU results from the system state in the last iteration of the evaluated simulation run. Same applies to the monetary revenues from pricing and public transport fares. Though, the simulation methodology presented in Section 3 ensures stable convergence of the stochastic user equilibrium, minor variations from iterations to iteration remain. In particular in process of comparison of two simulation run, unfavourable superposition of such stochastic variations can hide more subtle changes in social and consumer welfare indicators.

In order to better understand the degree and impact of stochastic variations, average of realized social and user benefits from a set of last 100. iteration runs, is evaluated for the unimodal scenario. Averaging over multiple iterations, during which travellers only choose from the existing daily plans in their memory, enables to smooth out effects of stochasticity, providing additional qualitative indicator in support of EMU - based social and consumer welfare evaluation.

7.1.1 Unimodal scenario - car only

In a unimodal scenario car is the only mode available to the commuters. In absence of a bus service as a viable alternative, departure time choice represents the single degree of freedom along which an agent can optimize its behaviour. It is also important to note, that without congestion charging and no bus operations in place, social and consumer welfare are identical.

No congestion pricing (NCP)

Figure 5 depicts social and consumer welfare before and after introduction of congestion pricing dependent on degree of heterogeneity n , comparing the three types of value of time heterogeneity discussed above: proportional, α and γ . Solid lines represent the scenarios before congestion pricing and dashed lines after the pricing policy is introduced.

Table 6 summarizes changes in social and consumer welfare resulting from congestion pricing in presence of proportional heterogeneity, as visualized in 5. From the Table 6

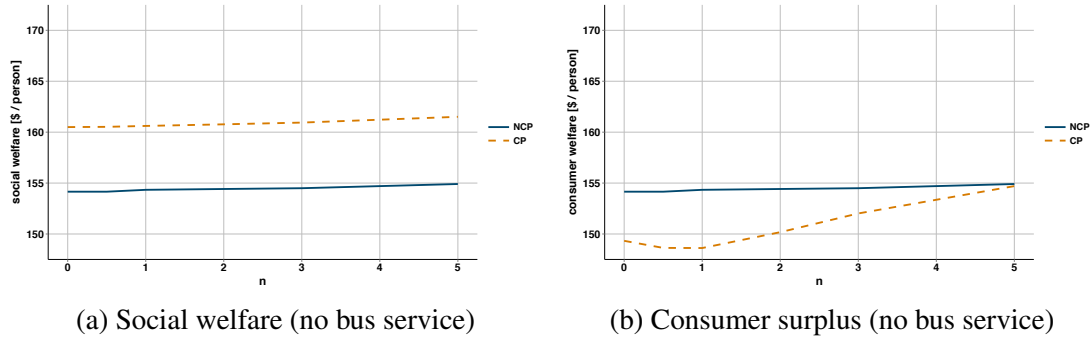


Figure 5: Effect of congestion pricing on social welfare and consumer surplus for different degrees of heterogeneity n and different types of schedule-delay heterogeneity, with only car mode available. Base case ($n = 0$) is the scenario with homogeneous travellers.

however, it is easier to note the subtle increase in social welfare with higher n values, resulting in welfare for $n=5$ being 0.5% larger than in homogeneous case. This minor increase is an artefact of generation and evaluation of alternative daily planes with the non-linear activity utility function, using the Logsum term for welfare calculation. Realized welfare averaged over multiple iterations and summarized in Table 7, does not exhibit the same increase. Therefore, it can be concluded that without congestion pricing, degree of proportional heterogeneity does not affect social and consumer welfare. This independence of welfare on proportional heterogeneity is expected. Without time depended monetary travel expenses, introduction of proportional heterogeneity does not alter individuals ratios of value of time and schedule delay early and late, and therefore provides no incentives for agents to change there travel behaviour compared to the homogeneous values of time scenario. It is also in line with the publications based on the bottleneck-model mentioned above (van den Berg, 2014; van den Berg and Verhoef, 2013, 2011a,b). All of these publications agree, that introducing proportional heterogeneity does not change the departure and travel time patterns.

Table 6: Effects of congestion charge on social and consumer welfare (logsum) for proportional heterogeneity in the unimodal "car only" scenario

n	SW Prop. (p.p)	CW Prop. (p.p)	Δ SW prop.	Δ CW prop.
0	154.2 \$	154.2 \$	4.12 %	- 3.13 %
0.5	154.2 \$	154.2 \$	4.13 %	- 3.58 %
1	154.3 \$	154.3 \$	4.06 %	- 3.70 %
2	154.4 \$	154.4 \$	4.11 %	- 2.75 %
3	154.5 \$	154.5 \$	4.17 %	- 1.60 %
5	154.9 \$	154.9 \$	4.26 %	- 0.13 %

Congestion pricing (CP)

Introduction of congestion pricing has significant effect on social welfare, increasing it by around 4 % for all degrees of heterogeneity (Figure 5). Though differences between varying degrees of heterogeneity scenarios appear to be of minor scale compared to total gains of congestion pricing, it is worth to discuss these effects at this point, as they become more pronounce after the introduction of mode choice as an additional choice dimension.

For proportional heterogeneity, a slight increase in welfare gain with increasing degree of heterogeneity can be presumed mainly based on scenarios with wider spread in values of time $n=3$ and $n=5$ (Table 6, Δ SW prop.). For lower degrees of proportional heterogeneity, stochastic variations in SUE outweigh this effect. Averaged over last 100 iterations, realized values of social welfare in Table 7 (avg. Δ RSW) confirm this assumption. Therefore, it can be concluded, that imposing congestion pricing on population of travellers with proportionally heterogeneous values of time, allows for more efficient self-organization of travel and departure times as in case of identical value of time for all users.

One of the most interesting results however, emerges from analysing effects of congestion pricing on consumer welfare under heterogeneous preferences. For homogeneous travellers, congestion pricing has a negative effect leading to an average loss of 3.13 %. Even though congestion pricing almost eliminates travel delays, monetary toll payments of second-best congestion charging scheme exceed gains from eradication of excessive congestion, causing average loss for the consumer. Most surprisingly however is the rapidly diminishing average consumer loss with increasing degree of heterogeneity. For extreme case of proportional heterogeneity ($n=5$), second-best congestion charge has almost no effect on consumer welfare (Δ CW = -0.13%), substantially increasing the social welfare at the same time. This outcome is primary due to the strong distributional effects, with high value of time travellers disproportionately gaining from the congestion eliminating pricing policy while schedule delay losses of the commuters with low value of time contributes only little to the overall welfare.

According to dynamic bottle model of traffic congestion (Small and Verhoef, 2007), first-best pricing of a single bottleneck does not affect consumer welfare. The toll paid by travellers is equal to the monetary value of time savings resulting from elimination of congestion delays, leaving the generalised price of travel constant. For the second-best pricing however, which is commonly studied in presence of an untolled alternative, such as untolled parallel road or lane, results diverge. Conventional static models of congestion tend to predict losses in average consumer welfare and for the majority of users from introduction of first- and second-best pricing policies with and without presence of heterogeneity (Small and Yan, 2001; de Palma and Lindsey, 2002; Verhoef and Small, 2004; van den Berg and Verhoef, 2011b). For the dynamic congestion model however, the impact of pricing policy on consumer welfare before returning toll revenues to the user can be either negative (van den Berg and Verhoef, 2011a) or positive (van den

Table 7: Effects of congestion charge on *realized social welfare and consumer surplus* for proportional heterogeneity and "car only" scenario, averaged over last 100 iterations

n	avg. RSW (p.p)	avg. RCS (p.p)	avg. Δ RSW	avg. Δ RCW	avg. Toll revenue
0	126.9 \$	126.9 \$	+ 4.7 %	- 4.5 %	92,979 \$
0.5	126.8 \$	126.8 \$	+ 4.8 %	- 4.1 %	90,419 \$
1	126.8 \$	126.8 \$	+ 4.9 %	- 4.0 %	90,054 \$
2	126.8 \$	126.8 \$	+ 5.3 %	- 3.9 %	82,919 \$
3	126.9 \$	126.9 \$	+ 5.7 %	- 1.6 %	73,653 \$
5	126.9 \$	126.9 \$	+ 6.5 %	+ 0.8 %	58,056 \$

Berg and Verhoef, 2011b, 2013). This appears to be highly dependent on dimensions of heterogeneity in value of time and schedule delay taken into account. These findings are also confirmed by van den Berg (2014), who investigated performance of different models of course tolling in presence of heterogeneous user preferences.

As discussed above, the overall impact of heterogeneity on social welfare changes from congestion pricing in a unimodal scenario appears to be rather minor, while it is more significant for consumer welfare. Yet, the rather small scale of pricing policy effects amounting to few percentage points is eye-catching. Being highly dependent on the scenario set-up, double digit percentage point gains are commonly seen in bottleneck model based scenarios (van den Berg and Verhoef, 2013, 2011b). However, the social and consumer welfare in traditional models only capture the time and monetary cost of travel. Welfare gains are expressed as percentage reduction of total travel cost. In contrast, in an activity-based context social and consumer welfare are assessed based on sum of utility gains in activity performance and utility losses from travelling in the course of one day. Therefore, changes in welfare from pricing policies need to be considered not as percentage of travel cost but as of total utility earned through the day.

7.1.2 Multi-modal scenario with varying levels of bus service

Studies of pricing and optimization policies based on simplified, uni-modal models are helpful to gain valuable fundamental insights and understanding of system dynamics. However, considering transport modes in isolation neglects number of important factors acting on transport demand and behaviour in a context of dense, modern urban environments. Shifting the focus of transport planning practice towards individual well-being, opportunities and inclusion, requires comprehensive integrated system modelling approaches. Presence of alternative transportation modes can have a significant impact on welfare and distributional effects of pricing policies. Therefore, moving to a multi-modal approach with high degree of realism is crucial for provision of valuable and applicable insights to planners and practitioners.

In order to correctly capture welfare effects of varying levels of bus service, it is important to account for operation and capital cost of providing the service. Therefore, model and cost estimates presented by Australian Transport Council (2006) and discussed in Section 3.4 are applied. As only morning and evening peak hours and commuting trips are considered, cost of bus operations from 6am - 10am and 5pm - 21pm are taken into account. The capital cost are accounted for in full, neglecting the use of the same vehicles through the day and serving a wider rider ship of non-commuting passengers. Accounting for potential delays along the route and proving time buffer for on-time service in the opposite direction, total time of 1h is assumed for a vehicle and driver to serve the 20km corridor in one direction. The resulting cost estimates are presented in Table 8

In this set-up, the bus fare is kept constant at 2 \$. This fare amount can not cover the bus operation cost for any of the three bus service frequencies. This implies the case of public transport subsidies from other revenues sources.

Table 8: Bus operation cost

Headway (min)	Number of vehicles	Fixed cost	Operational Cost	Total cost
10	12	2006 \$	6280 \$	8286 \$
5	24	4013 \$	12559\$	16572 \$
2	60	10032 \$	31398 \$	41430 \$

In presence of alternative mode, relative gains from congestion pricing decrease with proportional heterogeneity, even though the absolute social welfare under congestion pricing increases, following the same pricing effect on travellers with different values of time as described in Section 7.1.1. The welfare increasing effect of proportional heterogeneity under pricing is offset by self-sorting effects in presence of imperfect mode substitutes. Availability of viable alternatives to the car mode leads to strong efficiency gains through self-selection without pricing and therewith reduction of potential for benefits from congestion pricing policy. This effect becomes more pronounce as level and capacity of bus service increases.

These results match observation by van den Berg and Verhoef (2013), who find relative efficiency of welfare maximising road toll decreasing in presence of a rail alternative. They also do not contradict findings by van den Berg and Verhoef (2011b), who observe the relative efficiency gains from proportional heterogeneity on for parallel tolled and non-tolled alternatives. The welfare increase under heterogeneity is still observed, but is dominated by welfare increase without congestion pricing.

The even more critical effect of increased bus headways, is the direction of change in consumer welfare with introduction of the congestion pricing policy. Dependent on the level of bus service, changes on consumer welfare can be either positive or negative, which is crucial for potential support and implementation of the policy. Figure 7 visualizes changes in social and consumer welfare after introduction of congestion pricing

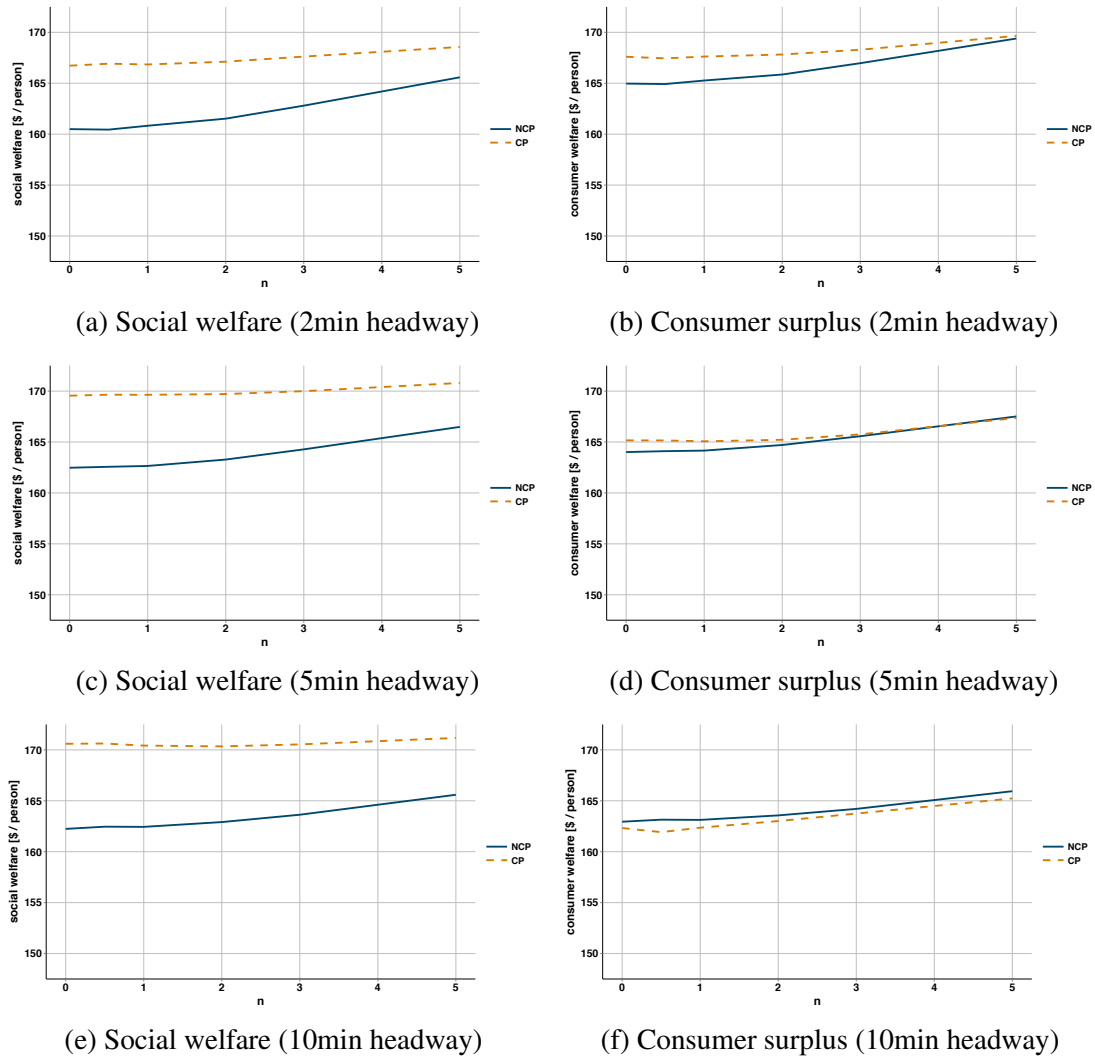


Figure 6: Effect of first-best pricing on social welfare and consumer surplus for different degrees of heterogeneity n . Base case ($n = 0$) is the scenario with homogeneous travellers

policy dependent on bus service frequency and degree of proportional heterogeneity. Highlighting the findings depicted above based on Figure 6, it offers a new perspective on the sensitivity of the pricing welfare gains to the two parameters. In case of social welfare, the maximal gains from congestion pricing policy are achieved not in presence biggest delays in absence of any public transport alternative, but in presence of moderate congestion and low frequency bus service operating with a 10 min headway. However, with growing spread in values of time and schedule delay, the largest welfare gains of congestion pricing are observed in the scenario with the biggest congestion due to the absence of any bus service. Furthermore, from the twisted shape at the transition from low bus frequencies to no bus service scenario, the interplay of two opposing effects becomes apparent: the increasing gains of pricing in presence of proportional

heterogeneity, combined with stronger effect of self-organization from spread in values of time in a multi-modal case without pricing. Lowest pricing gains can be observed, as to be expected for high degree of heterogeneity combined with high bus service frequency. The changes in consumer welfare, before redistribution of toll revenues, are shown in Figure 7 (b). In this graph non-linear dependency on bus frequency and heterogeneity is striking. In absence of any alternative to the car mode, the spread in values of time has profound effects on consumer welfare. Given everyone that has the same preferences and no alternative mode is available, consumers suffer a substantial loss from the pricing policy. However, with growing gap between higher and lower travel time valuations these gap vanishes and the change in consumer welfare turns positive. Same occurs for the increasing level of bus service, where even low frequency service substantially reduces aggregate consumer losses from congestion pricing. This creates a plateau, where given availability of bus alternative, the change in aggregate consumers welfare as a result of congestion pricing turns rather insensitive to the level of user heterogeneity.

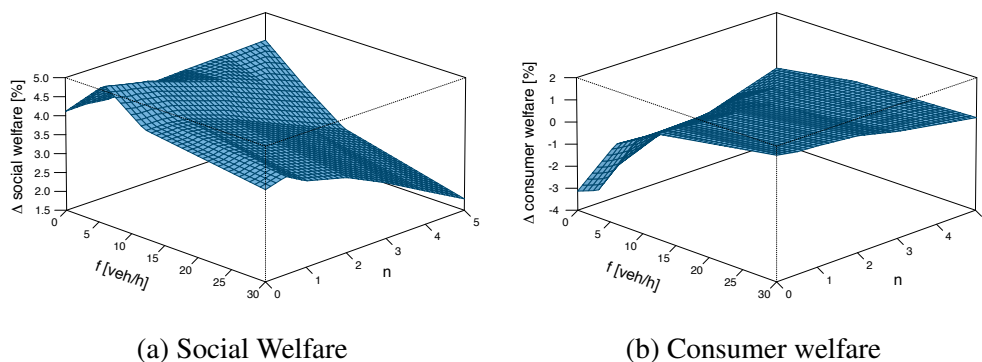
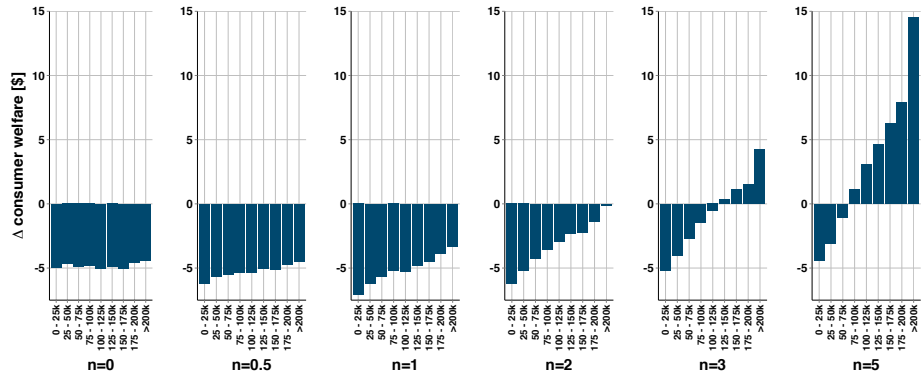


Figure 7: Welfare effects of congestion pricing, dependent on availability of public transport with proportional heterogeneity

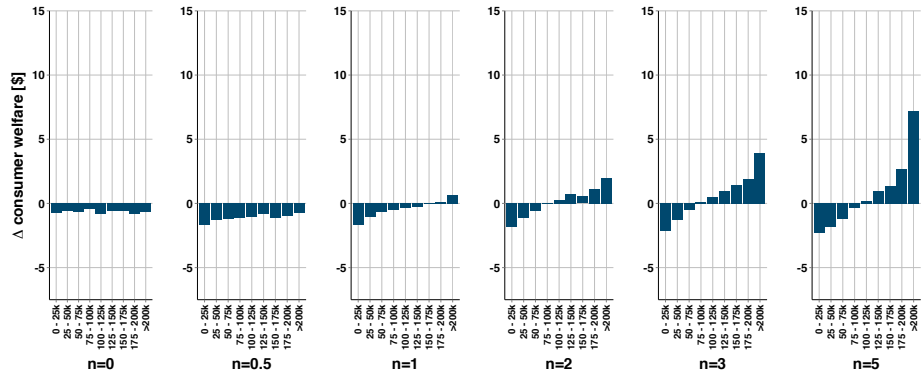
7.2 Distributional Effects

For the decision making process in favour or against certain policy implementation as well as public discussion around it, distributional effects often play more crucial role, than the aggregated social and consumer welfare gains of the policy in question. In context of pricing policies, the direct impact on welfare of individuals or groups characterized by certain socio-economic characteristics or locations of there activities can be especially evident, while redistribution benefits often take time to materialise. Given the dependency of values of time and schedule delay on household incomes in the presented approach, it is interesting to look at the relation between income and change in consumer welfare as a result of a congestion pricing policy, directly. Figure 8 visualizes gains and loses of different income groups for all scenarios with varying degrees of heterogeneity and levels of bus service. For all cases with any degree of heterogeneity high income groups with high values of time are better off compared to the lower income

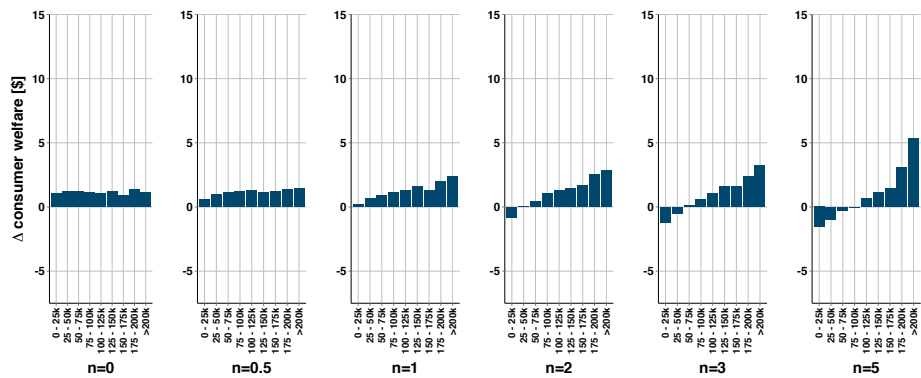
groups. As already seen above the sign and extend of consumer welfare changes is strongly dependent degree on availability of alternative transport mode and the degree of heterogeneity. Another characteristic of welfare change distribution is the narrowing gap between winners and losers with increasing service level of public transportation. The most surprising effect, however, appears to be the disappearing benefits and increasing loses of low-income groups with growing heterogeneity in all multi-modal scenarios. This is solely due to the increasing bus readership and associated crowding effects. With increase in degree heterogeneity, growing share of population has very low values of time leading to increase in bus ridership, with bus mode share among low-income groups reaching nearly 100% percent. As the introduction of congestion pricing pushes the bus mode shares even higher, crowding induced delays cause loses to the existing bus riders. The ability to adequately capture this effect originates from the detailed simulation of physical passenger - bus interactions, such as dwell times and bus capacity constraints, implemented within the MATSim framework.



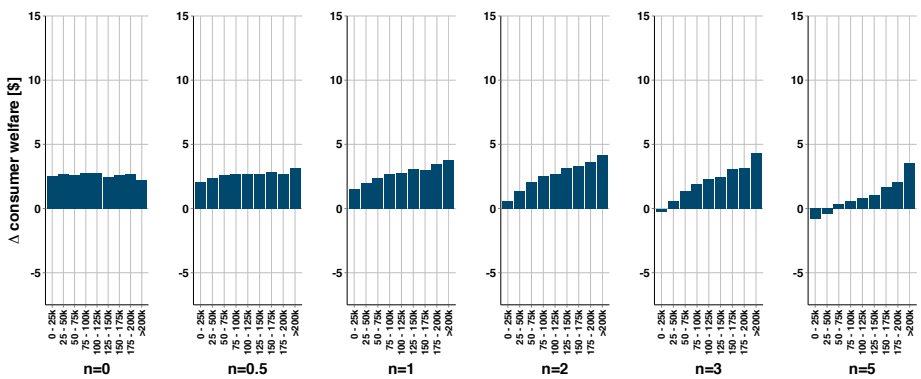
(a) no bus service



(b) 10 min



(c) 5 min



(d) 2 min

Figure 8: Changes in consumer welfare dependent on income after introduction of congestion pricing for different levels of proportional heterogeneity

8 Conclusion

The effects heterogeneous user preferences on social welfare and consumer benefits in presence of alternative modes can have major consequences for transportation policy design and project evaluation. Using only average values does not account for self-organizing effects in case of multiple choice dimensions and can significantly over- and underestimate the social and consumer welfare changes. Furthermore, the benefit of adding capacity to the public transport network is undervalued with homogeneous users as the effects of self-organization are not taking into account. On the contrary, the gains from introduction of congestion pricing policy could be overvalued. Furthermore, consumer benefits and therewith public acceptance of congestion pricing policy appears to be strongly dependent on availability of an alternative mode, to which drivers tolled away from the road, can divert.

This highlights the importance of joint evaluation of congestion pricing policy and public transport operations. Given fixed public transport fare, optimal operations frequency with heterogeneity can be higher than with homogeneous users. Furthermore, even if an increase in level of public transport service might have a negative effect on social welfare when evaluated as a stand alone policy, packaging it with congestion pricing policy can not only increase social but also consumer welfare and turn out to be pareto improving. This findings strengthen the case for use of multi-modal models and heterogeneous values of time for urban transport policy design and evaluation.

One of major challenges in transfer of such models into practice lies in cost associated with collection of data required for estimation of disaggregated values of time based on socio-economic characteristics, activity types, trip characteristics etc. More general, a set-up of a large scale agent based model is a laborious task with extensive data requirements on transportation infrastructure, building stock, population statistics, travel behaviour as well as residential-, business-, work- and education locations (Erath et al., 2012). But once such model is established, it provides a variety of benefits for scenario-based analysis and due to the large amount of disaggregated data incorporated in it, allows a wide range of applications. As shown in this paper, agent- and activity based simulation approach opens up new prospects for pricing policy design and evaluation. In particular, its ability to account for travel demand patterns of economic agents on individual scale including their socio-demographic attributes, makes it a highly suitable and attractive tool for evaluation of transportation policies on city scale.

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